Constructing 0.012345679… from 0.111111… Without Fractions

Overview

This document records the full algorithmic construction discussed: how the repeating decimal 0.012345679… arises as the square of 0.111111…, using only decimal operations and without any reference to fractions such as 1/81.

Step 1: Decimal Input as a Process

We begin with the infinite decimal x = 0.111111…, understood operationally as an infinite stream of the digit 1. This is not interpreted as a fraction or limit, but as a rule: at every decimal place, write the digit 1.

Step 2: Decimal Squaring Algorithm

To compute x^2, we perform ordinary schoolbook multiplication of infinite decimals. At the n-th decimal place, the raw (pre-carry) contribution is the sum of all diagonal products whose indices add to n. Since every digit involved is 1, this diagonal sum equals n.

Step 3: Carry Propagation Rule

Decimal arithmetic requires normalization in base 10. At position n, the digit written is n mod 10, while the carry sent to the next position is floor(n/10). Incoming carries are added before reduction. This rule alone governs the entire process.

Explicit Digit and Carry Table (n = 1–12)

n	Raw diagonal sum	Digit written	Carry to next place
1	1	1	0
2	2	2	0
3	3	3	0
4	4	4	0
5	5	5	0
6	6	6	0
7	7	7	0
8	8	8	0
9	9	9	0
10	10	0	1
11	11 + 1 = 12	2	1
12	12 + 1 = 13	3	1

Emergence of the Repeating Cycle

Once carries begin at n = 10, they persist indefinitely. The interaction between the increasing diagonal sums and the constant carry of 1 forces the digits into a stable repeating cycle.

Observed Digit Pattern

The stabilized repeating block is: 012345679. The digit 8 does not appear in the repeating cycle because the carry suppresses it.

Final Result

The resulting decimal expansion of x^2 is therefore: 0.012345679012345679…

Conceptual Summary

This construction is entirely decimal and algorithmic. The repeating pattern is not imposed by division or rational representation, but emerges from diagonal summation and carry propagation. The identification with 1/81 is external to the derivation.