Finale

The supervillain is attempting to solve the following problems. Get inside the villain's head and select the appropriate numerical method below for each problem. The options you have to choose from, listed in chronologically learned order, are

AK. Order Error	FS. Error	JS. Potential Energy
RS. Root Finding	AT. Optimization	RM. Interpolation
DE. Integration	FE. Project 3	AS. Runga-Kutta ODE Solvers
TE. BVP Solvers	OR. Finite Divided Difference Method	

i. The rate of change of panic amongst the students in the classroom.
ii. The volume of the classroom.
iii. The location of an unknown student given the location of all the other students.
iv. The maximum and minimum stress amongst the students.
v. The temperature throughout a student given the temperature on the student's head and feet.
vi. The number of exam questions the supervillain needs to have to make the students' grades all be 0

Solution

i. Finite Divided Difference Method - OR
ii. Integration – DE
iii. Interpolation – RM
iv. Optimization – AT
v. BVP Solvers – TE
vi. Root Finding – RS

Base

You are attempting to accurately find the cross-sectional area of the supervillain's base. Using satellite imagery, you take the following measurements.

Width	0	2	4	6	7	9	11
Height	0	5	4	5	2	1.7	0

Solution

3/8 + trapz + 1/3

$$(6-0)\frac{0+3*5+3*4+5}{8}+(7-6)\frac{5+2}{2}+(11-7)\frac{2+4*1.7+0}{6}=33.36667$$

Nefarious Calculations

The supervillain is getting a large truncation error in their most recent, evil calculation. They decided to reduce their step size to reduce this error. Given the truncation error of the specified numerical method is order h^3, if the supervillain divides the step size by the ones digit of his base's cross-sectional area, what is the truncation error divided by?

Solution

Cross-sectional area is 3 for ones digit

$$\left(\frac{h}{3}\right)^3 = \frac{h^3}{3^3} = \frac{h^3}{27}$$

27

Hideout

The following coordinates map out the layout of the supervillain's secret layer. Connect the coordinates using linear splines to determine the passageways within the layer.

X	3	1	1	3	3	1
Y	2	2	4	4	0	0

Solution

9

Hahahahaha!

The supervillain is laughing maniacally! The real rate of laughs can be found in the supervillain's **hideout**. Given your hand calculation's of laughs per minute is 8, and your Python calculation's of laughs per minute is 4.5, what is the round-off error?

Solution

Answer from hideout is 9

True Value = By-Hand Solution + Truncation Error

9 = 8 + Truncation Error -> Truncation Error = 1

True Value = Python Solution + Truncation Error + Round-Off Error

9 = 4.5 + 1 + Round-Off Error -> Round-Off Error = 10 – 5.5 – 1 = 3.5

Deflection Beam

A horizontal beam of length $12\ m$ is attached to the side of a building. The supervillain deflects the tip of the horizontal beam straight down $1\ m$ with a force of 1,000 N; there is no deflection of the tip in the horizontal direction. What is the potential energy of the beam if the beam has a cross sectional area of $0.56\ m^2$, length of $12\ m$, and modulus of elasticity of $25 \times 10^6\ Pa$?

Solution

Only movement in the y direction.

Work = Force * Distance = $1000 \times 1 = 1000$

$$PE_{Beam} = \frac{1}{2} k \Delta^2$$

$$k = \frac{EA}{L} = \frac{25000000\ (0.56)}{12}$$

$$\Delta = L - L'$$

$$L' = \sqrt{12^2 + 1^2}$$

PE = $PE_{Beam} - W$ = 9.230241196626025

Escape Route

The supervillain's escape route can be determined by an interpolating polynomial connecting all the following normalized coordinates. What will be the supervillain's latitude location given a longitude location of 2?

Longitude	0	3	5	9
Latitude	0	6	2	1

Solution

$f(0) = 0$	$f(3) = 6$	$f(5) = 2$	$f(9) = 1$	
$f[3,0] = \frac{6-0}{3-0} = \frac{6}{3} = 2$	$f[5,3] = \frac{2-6}{5-3} = -\frac{4}{2} = -2$	$f[9,5] = \frac{1-2}{9-5} = -\frac{1}{4} = -0.25$		
$f[5,3,0] = \frac{-2-2}{5-0} = -\frac{4}{5} = -0.8$	$f[9,5,3] = \frac{-0.25-2}{9-3} = -0.375$			
$f[9,5,3,0] = \frac{-0.375-(-0.8)}{9-0} = 0.04722222$				

$$f(x) = 0 + 2(x-0) - 0.8(x-0)(x-3) + 0.047\overline{2}(x-0)(x-3)(x-5)$$

$$f(2) = 5.88333\overline{3}$$

River Madness

The supervillain is attempting to disrupt the flow of a major river. The following data is from said river ($y =$ distance from bank, $H =$ depth, and $U =$ velocity of the water):

$y\ [m]$	0	1	3	5	7	8	9	10
$H\ [m]$	0	1	1.5	3	3.5	3.2	2	0
$U\ [m/s]$	0	0.1	0.12	0.2	0.25	0.3	0.15	0

What is the flow rate of the water the supervillain will need to divert from the river? Flow rate can be calculated by $\int H\ U\ dy$. Utilize all points but use the least accurate method from class.

Solution

Use trapezoidal method for each segment

Solution = 4.0125

Vision Loss

The supervillain is trying to speed up the vision loss of the populace! As such, he has released a chemical in the water supply that makes the intraocular pressure of the populace to be $P(t) = 0.3\ t + 13$ where t is the age in years. Using the model and constants from your project 3, use 4 segments of trapezoidal method to calculate the vision loss percent at age 35.

Solution

Integral with 4 segments has 5 points.

The integral value is 90.

12.88797 %

Notes Fiasco

The supervillain throws your ME 355 notes out of a helicopter right before your exam. Given a second order drag coefficient of $0.25\ kg/m$ (Force of drag = $c_d\ v^2$) and a mass of $5\ kg$ (That's a lot of notes), what is the note's velocity after 1 second such that you can intercept them midair? Use Heun's methods with a $0.5\ s$ timestep. Assume gravity is 10 m/s^2. When developing your mathematical model, assuming the positive direction is straight down.

Solution

$$m\frac{dv}{dt} = \sum F = mg - c_d v^2 \rightarrow \frac{dv}{dt} = g - \frac{c_d}{m}\ v^2 = 10 - \frac{1}{20}v^2 = f(t,v)$$

$$t_0 = 0\ , v_0 = 0, h = 0.5$$

$$v_{i+1}^{temp} = v_i + f(t_i, v_i)h$$

$$v_{i+1} = v_i + \frac{\left(f(t_i, v_i) + f\left(t_{i+1}, v_{i+1}^{temp}\right)\right)}{2}h$$

$$v(0) = 0, v(0.5) = 4.6875, v(1) = 8.36901180, v(1.5) = 10.80628244, v(2) = 12.27068686159$$

Beam Madness

You are hanging on to a beam at x = 3 meters. The supervillain heats up the end close to you (3 meters away) to a temperature of 200 C! The heat transfer in the beam can be calculated by the mathematical model

$$\frac{d^2T}{dx^2} = h(T - T_a)$$

with the heat transfer coefficient (h) being 0.04 and the ambient temperature of the room (T_a) being 200 C. Given the beam is 4 meters long, what temperature must your heat resistant gloves be able to withstand? Use the finite-difference methods with a step size of 1. Assume the far end of the beam is touching a cold-storage freezer that is held at a constant -60 C.

Solution

$$\frac{T_{i-1} - 2T_i + T_{i+1}}{\Delta x^2} = h(T_i - T_a)$$

$$T_{i-1} - (2 + \Delta x^2\ h)T_i + T_{i+1} = -T_a \Delta x^2 h$$

$$T_{i-1} - 2.04T_i + T_{i+1} = -8$$

$$T_0 = 200$$

$$T_4 = -60$$

$$T = [200, 141.04, 79.72, 13.59, -60]$$

Boulder I

You see the supervillain at the top of a hill! As you race towards him, you run up a steep slope. Given your current position is x = 0 meters, what is the current slope of the hill you are standing at? Use the most accurate method you know!

Horizontal Distance, [m]	-4	0	4
Vertical Distance, [m]	90	50	30

Solution

$$\frac{30 - 90}{8} = 7.5$$

Boulder II

Indiana Jones style, the supervillain rolls a giant boulder down a slope towards you. Given you are standing at x = 0 meters and the following position data, what is the current slope of the terrain you are standing on? Use all data available.

Horizontal Distance, [m]	0	2	4
Vertical Distance, [m]	50	42	30

Solution

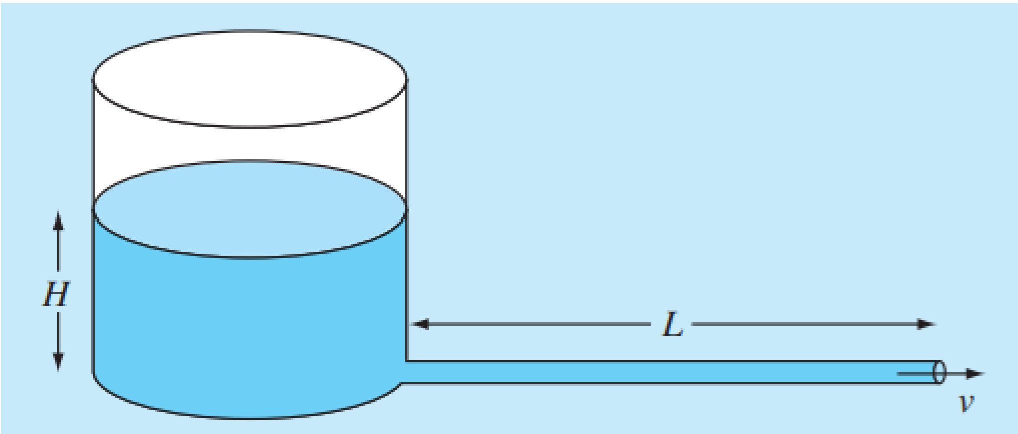
$$f'(0) = \frac{-30 + 4(42) - 3(50)}{2\ (2)} = -3$$

Water Tower I

The supervillain is attempting to release the water from a water tower, flooding the entire city! The velocity of the water (v) in meters per second discharged through cylindrical tank through a long pipe is computed using

$$v = \sqrt{2gH}\tanh\left(\frac{\sqrt{2gH}}{2L}t\right)$$

where $g = 9.81\ m/s^2$, $H =$ initial head (m), $L =$ pipe length (m), and $t =$ elapsed time (s). Determine the head needed to achieve a velocity of 4 m/s in 3 seconds for a 5-meter-long pipe using the bisection method with 2 iterations with boundary conditions [0, 8]. Note, 1 iteration would yield 4 as your root.



Solution

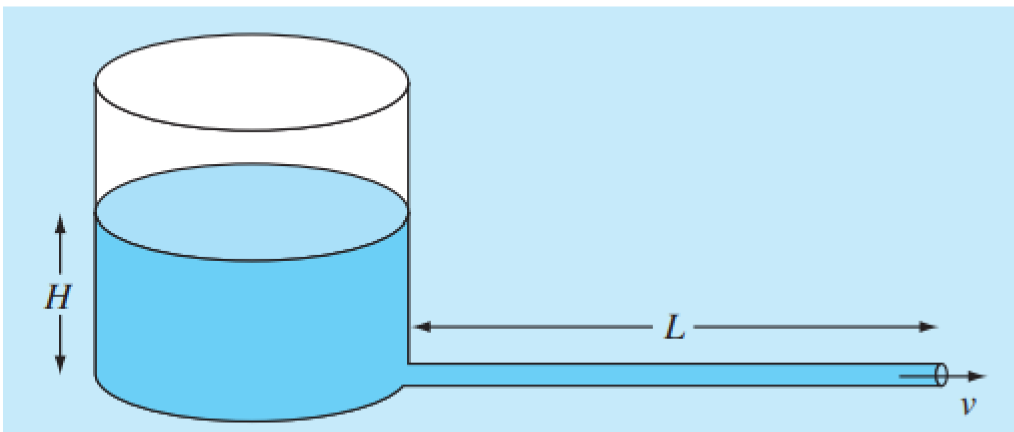
2

Water Tower II

The supervillain is attempting to release the water from a water tower, flooding the entire city! The velocity of the water (v) in meters per second discharged through cylindrical tank through a long pipe is computed using

$$v = \sqrt{2gH}\tanh\left(\frac{\sqrt{2gH}}{2L}t\right)$$

where $g = 9.81\ m/s^2$, $H =$ initial head (m), $L =$ pipe length (m), and $t =$ elapsed time (s). Determine the head needed to achieve a velocity 4 m/s in 3 seconds for a 5-meter-long pipe using one iteration of the Secant method. Use starting guesses of $H_0 = 0$ and $H_1 = 8$.



Solution

2.557

Laptop Rescue I

The supervillain has stolen your laptop with all your ME 355 code on it! He is looking to hang your laptop with a 10-foot rope on a beam of length L, deflected with a linear increasing distributed load. The deflection of the beam can be calculated by the mathematical model

$$y = \frac{w_0}{120EIL}(-x^5 + 2L^2x^3 - L^4x)$$

Given that $L = 500\ cm$, $E = 50{,}000\ kN/cm^2$, $I = 30{,}000\ cm^4$, and $w_0 = 2.5\ kN/cm$, at what position along the beam (x) will the supervillain hang the laptop such that it is closest to the lava pit below. Use the golden-section search for one iteration to solve this with bounds of [0, 500].

Solution

$$f(0) = 0, f(190.983) = -0.24, f(309.017) = -0.2049, f(500) = 0$$

$$190.983$$

Laptop Rescue II

The supervillain has stolen your laptop with all your ME 355 code on it! He is looking to hang your laptop with a 10-foot rope on a beam of length L, deflected with a linear increasing distributed load. The deflection of the beam can be calculated by the mathematical model

$$y = \frac{w_0}{120EIL}(-x^5 + 2L^2x^3 - L^4x)$$

Given that $L = 500\ cm$, $E = 50{,}000\ kN/cm^2$, $I = 30{,}000\ cm^4$, and $w_0 = 2.5\ kN/cm$, at what position along the beam (x) will the supervillain hang the laptop such that it is closest to the lava pit below. Use one iteration of Netwon's method with an initial guess of 300.

Solution

$$211.\bar{1}$$