Hierarchical Frequency Hopping Technique for Heterogeneous Multi-Tier Networks

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Abstract

Future wireless communication networks are characterized by the heterogeneous multi-tier infrastructure, which require the various levels of quality-of-services (QoSs) for different tiers. In this paper, we propose a novel type of frequency hopping (FH) with hierarchical level of Hamming correlations values (i.e., hierarchical FH for short). A construction algorithm of hierarchical FH sequence set (FHS) is proposed and its hierarchical Hamming property is demonstrated by an example. As a study case, the developed FHS set is imposed in asynchronous and heterogeneous multi-tier uplinks networks. The simulated results reveal that the proposed hierarchical FHS can provide multi-level bit-error-rate (BER) for various tiers networks; meanwhile, guarantee the superior transmission quality by significantly suppressing the inter- and intra- tier interferences.

1 Introduction

In the fifth cellular network and beyond (5G/B5G), a heterogeneous multi-tier architecture which consists of macro-cells (MC) and small cells (SC, including micro-cells, pico-cells

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and femto-cells) is the fundamental network infrastructure [1]. In the multi-tier networks, hierarchical quality-of-services (QoSs) which refer to the error-rate, spectral efficiency, latency and so forth, are required for various tiers, as illustrated in Fig. 1. For example, the network tier serving for the connected autonomous vehicles should require much higher QoS than the network tier serving for the wireless personal applications of pedestrians. Actually, in the PHY the multi-QoSs always refers to multi-level BERs. Besides, due to the natures of heterogeneous architecture, multi-tier interferences including the inter-tier and intra-tier interferences, are the critical challenges that degrade the performance.

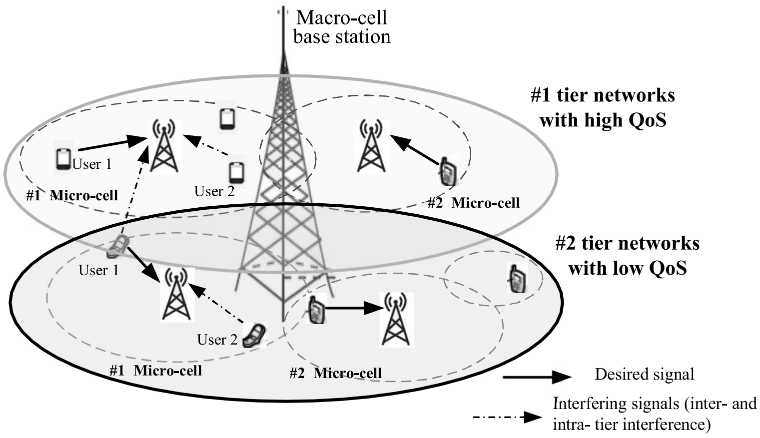

Figure 1: The infrastructure of heterogeneous multi-tier networks with hierarchical QoSs requirements. The desired signal may be impaired by the inter- and intra- tiers interferences.

The traditional pseudo-random FH technique and its FH sequence set (FHS) [2, 3, 4, 5, 6, 7] can efficiently combat various interference, channel fading and jamming attacks; however, it is unable to provide hierarchical QoSs, due to the fact that each available frequency-slot in these pseudo-random FHS sets just follows a near unform-distribution. The uniform-distribution property determines that FHS has only single-level values of Hamming cross-/auto-correlations (i.e., frequency-hits). Thus the traditional FH is not suitable to the heterogeneous multi-tier networks. It is well known that the BER performance of FH multi-access (FHMA) system is closely related to the Hamming cross-correlation value. By our previous study [8], the multi-QoSs goal for heterogeneous multi-tier can be achieved by delicately controlling the values of Hamming cross-correlations (e.g., frequency-hit rate). Taking the case of two-level hierarchical FHS set as the instance, the tier with high-QoS FHS (i.e., HQoS-FHS or HQoS-tier) should have the lower value of Hamming correlation, while the tier with low-QoS FHS (i.e., LQoS-FHS or LQoS-tier) is opposite. In addition, both high-QoS and low-QoS tiers should efficiently reduce the inter- and intra- tier interferences.

The prototype of two-level hierarchical FHS is firstly developed for smart grid communication networks[8]. The smart grid communications require the various QoSs of the data transmission for diversified power services. Based on this inspiration, recently a new FH technique with two-level hierarchy was proposed and adopted by users located in cell edge to attain the prioritized radio-access [9]. However, these hierarchical FHSs have the drawback: they may experience all frequency-hits in the entire FHS period at some large access delays (i.e., asynchronous access), thus leading to the worst error-rate. Since the heterogeneous multi-tier networks always follows the asynchronous access, the aforementioned FHSs are not suitable to the heterogeneous multi-tier scenarios.

In this paper, we will propose a *strong* two-level hierarchical FH technique, which can provide optimal two-level Hamming correlations in the asynchronous access, also offer the low intra- and inter- tier interferences to heterogeneous multi-tier networks. The construction algorithm of such a hierarchical FH pattern is proposed via the series of mathematical transformations based on a given optimal pseudo-random FHS set. As a study case, the developed FHS set is imposed into the FH-based OFDM in asynchronous heterogeneous multi-tier networks, and the multi-level BERs are investigated and verified by extensive simulations.

2 Requirement of Hierarchical FHS Set

First, we introduce the definition of Hamming correlation with regard to FHS set.

Definition 1. Let $\mathbb{S} = \{\mathbf{s}^{(k)} | k = 1, 2, \cdots, K\}$ denote an FHS set with K sequences over a given frequency set $\mathbb{F} = \{f_1, f_2, \cdots, f_q\}$ with size q, where $\mathbf{s}^{(k)} = \left(s_0^{(k)}, s_1^{(k)}, \cdots, s_{L-1}^{(k)}\right)$, is the k-th sequence with length L. The Hamming correlation function of $\mathbf{s}^{(u)}$ and $\mathbf{s}^{(v)}$ at the integer delay τ is defined as

$$H_{uv}(\tau | \mathbf{s}^{(u)}, \mathbf{s}^{(v)}) = \sum_{i=0}^{L-1} h\left[s_i^{(u)}, s_{i+\tau}^{(v)}\right], \mathbf{s}^{(u,v)} \in \mathbb{S} \tag{1}$$

where $h[x, y] = 1$ for $x = y$ denotes the frequency-slot x colliding with another one y, whilst $h[x, y] = 0$ for $x \neq y$ denotes hit-free. The subscript addition $(\cdot)_{i+\tau}$ is performed modulo L. Further, for $u = v$, $H_{uu}(\cdot)$ denotes Hamming auto-correlation, and $H_{uv}(\cdot)$ denotes Hamming cross-correlation for $u \neq v$.

From this definition, the Hamming correlation function denotes the total number of the frequency hits over a whole length of sequence L at the relative delay τ. The Hamming correlation function determines the capability of anti-interference, anti-jamming and so forth, which is the most critical properties for FH multi-access (FHMA) systems.

Next, we define the maximum Hamming (out-of-phase) auto-correlation and cross-correlation of $\mathbb{S}$ as follows, respectively.

$$\begin{aligned} H_a(\mathbb{S}) &= \max\left\{H_{uu}(\tau) | \mathbf{s}^{(u)} \in \mathbb{S}, 0 < |\tau| \leq L-1\right\}, \\ H_c(\mathbb{S}) &= \max\left\{H_{uv}(\tau) | \mathbf{s}^{(u)}, \mathbf{s}^{(v)} \in \mathbb{S}, u \neq v, |\tau| \leq L-1\right\}. \end{aligned} \tag{2}$$

For ease understanding, an two-level hierarchical FHS set $\mathbb{S}$ is briefly introduced in this section, which can be easily extend to the generalized case of multi-level case. According to the various levels of Hamming correlation values, the FHS set $\mathbb{S}$ can be separated into two disjoint subsets

$$\mathbb{S} = \{\mathbb{S}_1; \mathbb{S}_2\}, \mathbb{S}_1 \cap \mathbb{S}_2 = \varnothing, \tag{3}$$

where $\mathbb{S}_1$ with low Hamming cross-correlation value applied to the high QoS (HQoS) tier and $\mathbb{S}_2$ with high Hamming cross-correlation value applied to the low QoS (LQoS) tier, that is, $H_c(\mathbb{S}_1) < H_c(\mathbb{S}_2)$.

To realize the optimal performance in FHMA systems, it is generally desired that $\mathbb{S}_1$ and $\mathbb{S}_2$ have the following requirements under the given number of frequency slot q.

- The size of single FHS set $\mathbb{S}_1$ (and $\mathbb{S}_2$) should be as large as possible.
- The length of sequence in $\mathbb{S}_1$ (and $\mathbb{S}_2$) should be as long as possible.
- The Hamming cross- and (out-of-phase) auto- correlation values within $\mathbb{S}_1$ (and $\mathbb{S}_2$) should be as small as possible.
- The Hamming cross-correlation value of $\mathbb{S}_1$ is lower than that of $\mathbb{S}_2$.
- The Hamming cross-correlation between $\mathbb{S}_1$ and $\mathbb{S}_2$ should be as small as possible.

The first three properties are the required ones for traditional pseudo-random FHS, while the remaining ones are the additional properties for the proposed FHS set. Based on the theory of the code design, meeting all above requirements will significantly increase the design difficulty.

3 Design and Analysis of Two-Level Hierarchy FH Pattern

In this section, we firstly propose a construction algorithm of FHS set with two-level hierarchy, i.e., $\mathbb{S} = \{\mathbb{S}_1; \mathbb{S}_2\}$, where $\mathbb{S}_1$ denotes the HQoS FHS set and $\mathbb{S}_2$ denotes the LQoS one. Then, an example is presented to verify its hierarchical Hamming correlation properties.

Before demonstrating the construction algorithm, we define the two-level hierarchical FHS sets $\mathbb{S}_1$ and $\mathbb{S}_2$ as follows respectively,

$$\mathbb{S}_1 = \left\{\mathbf{s}_1^{(k)} | k = 1, 2, \cdots, K_{S1}\right\}, \mathbb{S}_2 = \left\{\mathbf{s}_2^{(k)} | k = 1, 2, \cdots, K_{S2}\right\}, \tag{4}$$

where K_{S1} and K_{S2} denote the sizes of $\mathbb{S}_1$ and $\mathbb{S}_2$, respectively. The sequences in $\mathbb{S}_1$ and $\mathbb{S}_2$ are denoted as

$$\mathbf{s}_1^{(k)} = \left(u_0^{(k)}, u_1^{(k)}, \cdots, u_{L-1}^{(k)}\right), \quad \mathbf{s}_2^{(k)} = \left(w_0^{(k)}, w_1^{(k)}, \cdots, w_{L-1}^{(k)}\right), \tag{5}$$

where L is the length of these sequences.

Next, we define a cyclic v-digit(s) shift operation, where v is a non-negative integer. Given a row (or column) vector $\mathbf{g} = (g_0, g_1, \cdots, g_{L-1})$ with length L, the cyclic v-digit(s) shift operation on $\mathbf{g}$ is defined as

$$\mathbf{g}^{\langle v \rangle} := \ (g_v, g_{v+1}, \cdots, g_{L-1}, g_0, \cdots, g_{v-1}), \tag{6}$$

where the subscript addition $(\cdot)_{a+b}$ is performed modulo L.

3.1 Construction Algorithm

The two-level hierarchical FHS set $\mathbb{S} = \{\mathbb{S}_1; \mathbb{S}_2\}$ can be constructed via the following steps.
Step 1: Given a *prime FHS set* $\mathbb{C}$ with size K_c as following

$$\mathbb{C} = \left\{\mathbf{c}^{(k)} | k = 1, 2, \cdots, K_c\right\},$$
$$\mathbf{c}^{(k)} = \left(c_0^{(k)}, c_1^{(k)}, \cdots, c_{L_c-1}^{(k)}\right), c_l^{(k)} \in \mathrm{GF}(p), \tag{7}$$

where p is a prime and L_c is the length of prime FHS. Based on the properties of prime sequence, we have $L_c = p$ and $K_c = p - 1$.

In addition, we define a new subset $\overline{\mathbb{C}}$ which is obtained from $\mathbb{C}$ excluding $\mathbf{c}^{(k_1)}$, that is,

$$\overline{\mathbb{C}} = \mathbb{C} \backslash \mathbf{c}^{(k_1)}, \forall\ k_1 = 1, 2, \cdots, K_c, \tag{8}$$

where $\overline{\mathbb{C}}$ has length p and size $p - 2$. The sequence set $\overline{\mathbb{C}}$ will be utilized to construct the LQoS FHS set $\mathbb{S}_2$.
Step 2: Based on the selected sequence $\mathbf{c}^{(k_1)}$ from the *prime FHS set* $\mathbb{C}$, a new sequence set $\mathbb{G}$ is obtained as follows via performing the cyclic v-digit(s) shift operation on $\mathbf{c}^{(k_1)}$.

$$\mathbb{G} = \left\{\left(\mathbf{c}^{(k_1)}\right)^{<v>} | v = 0, 1, 2, \cdots, p-1\right\}. \tag{9}$$

The set $\mathbb{G}$ can be written as the matrix with the column-wise manner, that is,

$$\mathbb{G} = [\mathbf{g}_0, \mathbf{g}_1, \cdots, \mathbf{g}_{p-1}]_{p \times p}, \tag{10}$$

where the i-th column vector in $\mathbb{G}$ $\mathbf{g}_i = [g_0^{(i)}, g_1^{(i)}, g_2^{(i)}, \cdots, g_{p-1}^{(i)}]^{\mathrm{T}}, g_l^{(i)} \in GF(p)$.
Step 3: Given a small positive integer $Z, 0 < Z < p$, we define Z non-negative integers $\{a_1, a_2, \cdots, a_Z | a_i > 1\}$, where $a_i \neq a_j$ if $i \neq j$. In addition, we select Z column-vectors as follows.

$$\begin{aligned}
\mathbf{u}_1 &= [p, p+1, \cdots, 2p-1]^{\mathrm{T}}, \\
\mathbf{u}_2 &= [2p, 2p+1, \cdots, 3p-1]^{\mathrm{T}}, \\
&\vdots \\
\mathbf{u}_Z &= [Zp, Zp+1, \cdots, (Z+1)p-1]^{\mathrm{T}}.
\end{aligned} \tag{11}$$

Based on (10) and (11), multiple matrices can be obtained as follows.

$$\begin{aligned}
\mathbf{G}_0 &= [\mathbf{g}_0, \mathbf{u}_1, \mathbf{u}_2, \cdots, \mathbf{u}_Z], \\
\mathbf{G}_1 &= [\mathbf{g}_1, \mathbf{u}_1^{\langle a_1 \rangle}, \mathbf{u}_2^{\langle a_2 \rangle}, \cdots, \mathbf{u}_Z^{\langle a_Z \rangle}], \\
&\vdots \\
\mathbf{G}_{p-1} &= \left[\mathbf{g}_{p-1}, \mathbf{u}_1^{\langle (p-1)a_1 \rangle}, \mathbf{u}_2^{\langle (p-1)a_2 \rangle}, \cdots, \mathbf{u}_Z^{\langle (p-1)a_Z \rangle}\right],
\end{aligned} \tag{12}$$

where $\mathbf{u}_i^{\langle v\rangle}$ denotes the cyclic v-digit(s) shifting operator on the column vector $\mathbf{u}_i$. Then a new matrix $\mathbb{S}_1$ can be obtained as follows via the cascading all matrices shown in (12).

$$\mathbb{S}_1 = [\mathbf{G}_0, \mathbf{G}_1, \mathbf{G}_2, \cdots, \mathbf{G}_{p-1}]_{p\times p(Z+1)}\,. \tag{13}$$

The matrix $\mathbb{S}_1$ by reforming it as a row-wise manner is namely the HQoS FHS set. The k-th row vector in $\mathbb{S}_1$ is the k-th HQoS FHS $\mathbf{s}_1^{(k)} = \left(u_0^{(k)}, u_1^{(k)}, \cdots, u_{L-1}^{(k)}\right)$.

Step 4: We select the first row vector of $\mathbb{S}_1$ (i.e., $\mathbf{s}_1^{(1)}$) as the base FHS to further generate the LQoS FHS set[1]. Based on the construction steps shown in (12) and (13), $\mathbf{s}_1^{(1)}$ can be also denoted as

$$\mathbf{s}_1^{(1)} = \left(g_0^{(0)}, u_0^{(1:Z)},\ g_0^{(1)}, u_0^{\langle a(1:Z)\rangle}, \cdots, g_0^{(p-1)}, u_0^{\langle (p-1)a(1:Z)\rangle}\right) \tag{14}$$

where $u_0^{(1:Z)}$ and $u_0^{\langle ia(1:Z)\rangle}$ denote these Z elements which are the first elements of column vectors in $[\mathbf{u}_1, \mathbf{u}_2, \cdots, \mathbf{u}_Z]$, and in $[\mathbf{u}_1^{\langle ia_1\rangle}, \mathbf{u}_2^{\langle ia_2\rangle}, \cdots, \mathbf{u}_Z^{\langle ia_Z\rangle}]$ as shown in (11), respectively.

Based on (14), we design a mapping operator $\mathcal{M}(g_0^{(i)})$ which maps the element $g_0^{(i)} \in GP(p)$ to a specific vector, that is,

$$\begin{aligned} \mathcal{M}(g_0^{(0)}):&\quad g_0^{(0)} \mapsto \left[g_0^{(0)}, u_0^{(1:Z)}\right], \\ \mathcal{M}(g_0^{(i)}):&\quad g_0^{(i)} \mapsto \left[g_0^{(i)}, u_0^{\langle ia(Z:1)\rangle}\right],\ i \neq 0, \end{aligned} \tag{15}$$

where the operator '$A \mapsto B$' denotes the entry A is replaced by B. $u^{\langle ia(Z:1)\rangle}$ denotes Z elements with the inverse order of $u_0^{\langle ia(1:Z)\rangle}$, that is, the first elements of column vectors in $[\mathbf{u}_Z^{\langle ia_Z\rangle}, \mathbf{u}_{Z-1}^{\langle ia_{Z-1}\rangle}, \cdots, \mathbf{u}_2^{\langle ia_2\rangle}, \mathbf{u}_1^{\langle ia_1\rangle}]$.

According to the above mapping operation, the element $c_i^{(k)}, k \neq k_1$ in the set $\overline{\mathbb{C}}$ as shown in (8) can be extended, that is, different values of $c_i^{(k)}$ are mapped to different sub-vectors as shown in (15). Then the LQoS FHS set $\mathbb{S}_2$ is obtained as following,

$$\begin{aligned} \mathbb{S}_2 =& \left\{\left(w_0^{(k)}, w_1^{(k)}, \cdots, w_L^{(k)}\right)\right\} \\ :=& \left\{\left(\mathcal{M}(c_0^{(k)}), \mathcal{M}(c_1^{(k)}), \cdots, \mathcal{M}(c_{p-1}^{(k)})\right)\right\}, k \neq k_1, \end{aligned} \tag{16}$$

where $\left(c_0^{(k)}, c_1^{(k)}, \cdots, c_{p-1}^{(k)}\right) \in \overline{\mathbb{C}}$. According to above construction, it is easy to obtain that the length of $\mathbb{S}_2$ is $L = p(Z+1)$ and the size of $\mathbb{S}_2$ is $K_{S2} = K_c - 2 = p - 2$.

3.2 An Example of Two-Level hierarchical FH Pattern

In this subsection, an example of two-level hierarchical FH pattern and its Hamming correlation properties are presented.

[1] Other row vector of $\mathbb{S}_1$ is applicable as well.

Table 1: The comparisons of Hamming correlations among the proposed FHS set and other FHS sets with Z=2.

	τ	**-7**	**-6**	**-5**	**-4**	**-3**	**-2**	**-1**	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**
Proposed	$H_{uv}(\tau\|\mathbb{S}_1)$	0	**5**	0	0	**5**	0	0	0	0	0	**5**	0	0	**5**	0
FHS	$H_{uv}(\tau\|\mathbb{S}_2)$	0	3	0	0	3	0	0	3	0	0	3	0	0	3	0
	$H_{uv}(\tau\|\mathbb{S}_1,\mathbb{S}_2)$	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	$H_{uu}(\tau)$	0	0	0	0	0	0	0	15	0	0	0	0	0	0	0
Multi-QoS	$H_{uv}(\tau\|\mathbb{S}_1)$	0	**15**	0	0	**15**	0	0	0	0	0	**15**	0	0	**15**	0
FHS in [8]	$H_{uv}(\tau\|\mathbb{S}_2)$	0	3	0	0	3	0	0	3	0	0	3	0	0	3	0
	$H_{uv}(\tau\|\mathbb{S}_1,\mathbb{S}_2)$	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	$H_{uu}(\tau)$	0	0	0	0	0	0	0	15	0	0	0	0	0	0	0
Trad.	$H_{uv}(\tau)$	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FHS in [5]	$H_{uu}(\tau)$	0	0	0	0	0	0	0	15	0	0	0	0	0	0	0

$\cdot\ u \neq v$.

Example 1: Let $Z=2, p=5$ and $[a_1,a_2]=[2,3]$, the HQoS FHS set $\mathbb{S}_1$ and the LQoS FHS set $\mathbb{S}_2$ can be generated as shown below.

$$\begin{aligned}
\mathbf{s}_1^{(1)} &= \{0,5,10,1,7,13,2,9,11,3,6,14,4,8,12\};\\
\mathbf{s}_1^{(2)} &= \{1,6,11,2,8,14,3,5,12,4,7,10,0,9,13\};\\
\mathbf{s}_1^{(3)} &= \{2,7,12,3,9,10,4,6,13,0,8,11,1,5,14\};\\
\mathbf{s}_1^{(4)} &= \{3,8,13,4,5,11,0,7,14,1,9,12,2,6,10\};\\
\mathbf{s}_1^{(5)} &= \{4,9,14,0,6,12,1,8,10,2,5,13,3,7,11\}.
\end{aligned}$$

$$\begin{aligned}
\mathbf{s}_2^{(1)} &= \{0,10,5,2,11,9,4,12,8,1,13,7,3,14,6\};\\
\mathbf{s}_2^{(2)} &= \{0,10,5,3,14,6,1,13,7,4,12,8,2,11,9\};\\
\mathbf{s}_2^{(3)} &= \{0,10,5,4,12,8,3,14,6,2,11,9,1,13,7\}.
\end{aligned}$$

From the example, we obtain the following parameters: $K_{S1}=5, K_{S2}=3, L=15$ and the size of available frequency-slots set $q=15$. In addition, the obtained FHSs have a good randomness since frequency-slot elements ($q=15$ frequency-slots) in each FHS evenly spread over the entire FHS set.

The typical Hamming correlations of the proposed $\{\mathbb{S}_1;\mathbb{S}_2\}$ and the comparisons among other FHS sets are shown in Table 1 in details. For fair comparisons, the FHS sets in Tab. 1 have the same parameters, i.e., q, and L. The construction algorithms of the previous two-level hierarchical FHS set and the traditional pseudo-random FHS set are referred to [8] and [5], respectively.

Observed from the Tab. 1, the Hamming correlation of $\mathbb{S}_1$ is equal to zero, and outperforms $\mathbb{S}_2$ for the time-shift $|\tau|\leq Z$ (i.e.,the quasi-synchronous multi-tier networks).For the asynchronous case ($Z<|\tau|<L$), the proposed FHS set gets the best performance due to lowest Hamming correlations. However, the previous two-level hierarchical FHS

set in [8] attains the whole hits (i.e., $H_{uv} = 15$) thus leading to the disastrous BER degradation, which is wholly unacceptable for the asynchronous two-tier networks. Besides, the Hamming cross-correlation of traditional pseudo-random FHS has the single value for any two FHSs (i.e., $H_{uv} \equiv 1$), which cannot provide various QoSs for two-tier networks.

Overall, via this example, the properties of the proposed FHS set meet the technique requirements of hierarchical FHS set mentioned in Section 2. The similar conclusions can be drawn for the general cases with other Zs, ps and $\{a_i\}_i$.

4 BERs of the proposed Hierarchical FHS set applied in heterogenous networks

4.1 Transceiver of hierarchical FH based OFDM System

In heterogeneous multi-tier networks, the FH based OFDM transmitter (i.e., FH/OFDM) with N_b branches is introduced, as shown in [10], except that the proposed hierarchical FHSs are integrated into OFDM sub-branches. In this transmitter, the entire bandwidth is evenly divided into N_b non-overlapped sub-bands $\{\mathbb{F}_l, l = 0, 1, \cdots, N_b - 1\}$, and each sub-band contains q frequency slots, i.e., $||\mathbb{F}_l|| = q$. The active sub-carriers of the l-th sub-branch are hopped within $\mathbb{F}_l$ according to the proposed hierarchical FHS set $\mathbb{S} = \{\mathbb{S}_1, \mathbb{S}_2\}$, which is as shown in Section 3. For simplicity, it is assumed that one OFDM symbol is transmitted during one hop interval T. Then, the transmitted signal of the k-th user during the n-th hopping interval can be written as

$$S^{(k)}(t) = \sum_{l=0}^{N_b-1} \sqrt{2P^{(k)}} d_l^{(k)}(n) \cos\left[j2\pi \left(f_l + \frac{s_n^{(k)}}{T} \right) t \right], \qquad nT \le t < (n+1)T, \tag{17}$$

where $d_l^{(k)}(n)$ denotes the baseband symbol on the l-th branch. In the following analysis, the binary PSK (BPSK) mapping scheme is employed, thus $d_l^{(k)}(n)$ is randomly generated the symbol from the alphabet set $\{-1, 1\}$. f_l is the first frequency slot in the l-th branch, which can be set as ql/T guaranteeing that the N_b sub-bands are not overlapped. $s_n^{(k)}$ denotes the instantaneous hopped-frequency slot of the k-th user, where $\{s_n^{(k)} | n = 0, 1, 2, \cdots\} \subset \mathbb{S}_1$ is adapted by users in the HQoS tier, and $\{s_n^{(k)} | n = 0, 1, 2, \cdots\} \subset \mathbb{S}_2$ is utilized by users in the LQoS tier. $P^{(k)}$ denotes the transmitting power of the k-th user.

In this paper, we assume that the received envelop of signal in base-station (BS) $\Gamma^{(k)} = \sqrt{2P^{(k)}}$ follows an i.i.d. Rayleigh distribution. In addition, a practical case of heterogeneous multi-tier networks with arbitrary access delays (i.e., asynchronous access mechanism) is considered in this paper, then the received signal at the BS is shown as

$$r(t) = \sum_{k=1}^{K} S^{(k)}(t - \tau_k) + \eta(t), \tag{18}$$

where K denotes the number of users in the entire multi-tier networks, including the high-QoS users and the low-QoS users). τ_k is the arbitrary access delay of the k-th user, which

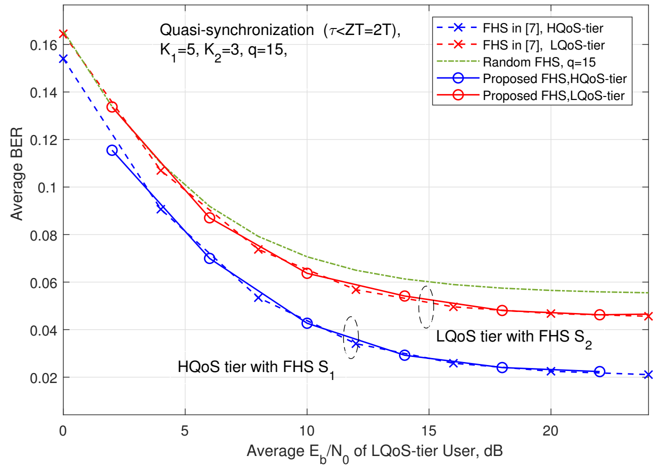

Figure 2: The BER comparisons between HQoS- and LQoS- tiers users in quasi-synchronous multi-tier networks by employing various FHSs.

uniformly distributed over duration of the entire sequence length. $\eta(t)$ represents the complex additional white Gaussian noise (AWGN) with two-sided power spectral density of $N_0/2$.

At the BS, the receiver structure of FH/OFDM system is shown in Fig. 2, which consists of N_b receiver branches. In each branch, the received signals $r(t)$ are first put into dehoppers, of which local frequencies are controlled by the given FHS. Then the low-pass-filter (LPF) with bandwith $2/T$ following the dehopper. Subsequently, the signal in each branch is processed by the correlator. The output of correlator is put into the demodulator.

4.2 BER analysis via simulations

In this section, the performance of the FH/OFDM system employing the developed hierarchical FHS set will be evaluated. To demonstrate the merits of our developed FHS set, the quasi-synchronous access mechanism (i.e., $D \leq ZT$) and the a-synchronous access mechanism (i.e., $D > ZT$) are adopted in multi-tier networks respectively in the following simulations. The employed two-level hierarchical FHS sets $\{\mathbb{S}_1; \mathbb{S}_2\}$ in the following simulations are as shown in *Example 1* in Section 3.2, where the parameters are $(q, K_1, K_2, L, Z) = (15, 5, 3, 15, 2)$.

The BER comparisons of the proposed FH/OFDM multi-tier networks employing the various FHS sets are plotted in Fig. 2 under quasi-synchronous mechanism and in Fig. 3 under asynchronous mechanism, respectively. For comparison, two types of classic FHS sets (i.e., the completely random FHS and the previous hierarchical FHS in [8]) are investigated as well under the same conditions, e.g., the number of frequency slots q, the length of FHS L. The system parameters are set as: $K_1 = 5, K_2 = 3$.

In Figs. 2-3, we find that our proposed hierarchical FHS and the previous FHS in [8] can attain two-level BERs for two-tiers networks but the random FHS set fails. In the

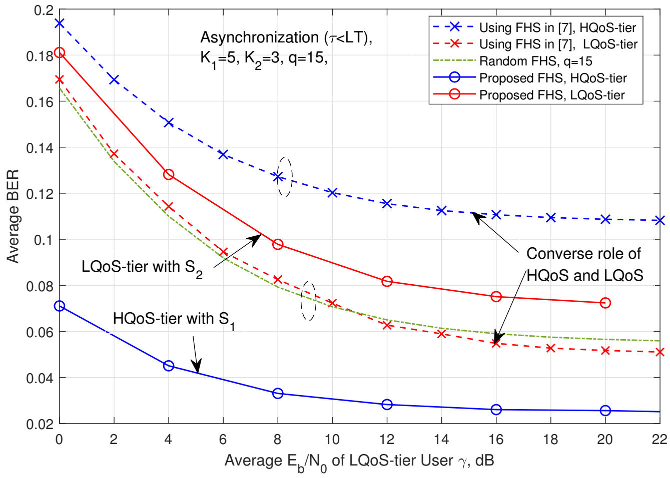

Figure 3: The BER comparisons between HQoS- and LQoS- tiers users in asynchronous multi-tier networks by employing various FHSs.

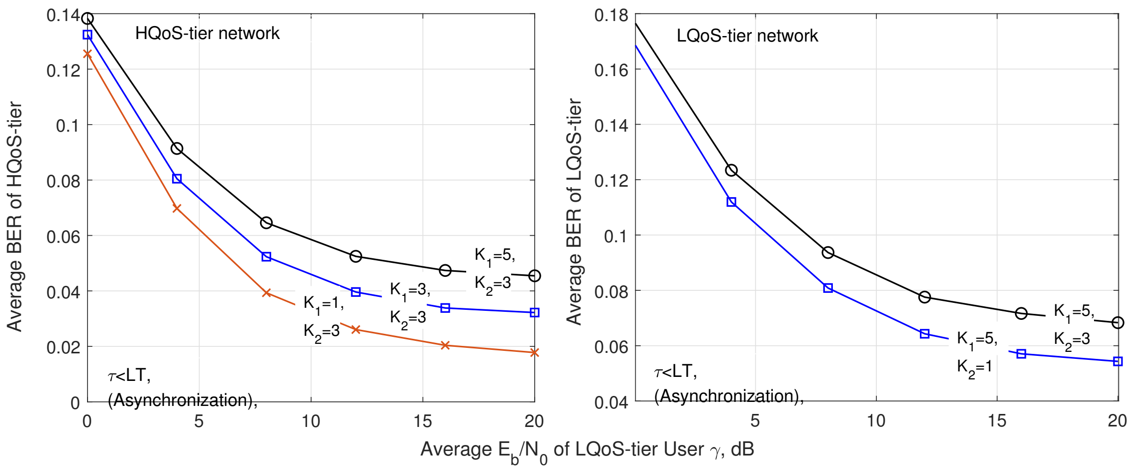

Figure 4: The average BERs of the proposed hierarchical FHSs with various number of users K_1 and K_2.

quasi-synchronization, the performance of our proposed FHS set coincides with that of FHS in [8], since these two types of FHS sets have the same Hamming correlation value when $D < 2T$ which is as shown in Tab. 1. However as for the a-synchronization scenario shown in Fig. 3, the previous FHS in [8] can not provide two-level QoSs, the FHS set $\mathbb{S}_1$ even is inferior to $\mathbb{S}_2$ due to the whole-hits occurrence of $\mathbb{S}_1$ at some large access delays, e.g., $H_{uv}(\mathbb{S}_1||\tau| = 3, 6) = L = 15$. By employing the FHS set proposed in this paper, the two-level QoSs target can readily restore, as illustrated in Fig. 3.

The average BERs of the HQoS- and LQoS- tiers with various number of users K_1 and K_2 are plotted in Fig. 4. The left sub-figure is for the HQoS-tier BERs, and the right one is for the LQoS-tier BERs. The results in these figure can also quantify the impacts of intra- and inter- tiers interferences on the BERs of HQoS- and LQoS- tiers due to the frequency-hits of FHS set. Here, we will take the figure of HQoS-tier case as example to explain the BER behavior. Obtained from this sub-figure, the BER follows descent as the K_1 decreases due to the reductions of the multi-user interference contributed from intra-tier. The BER behavior of LQoS-tier follows the same trend as that of HQoS-tier, of which explanation is omitted due to the limited space.

5 Conclusions

This paper is dedicated to the design and analysis of a hierarchical FHS set and its application to heterogeneous multi-tier FH/OFDM networks. As an improvement of the traditional FHS set with single-level Hamming correlation, the hierarchical FHS set attains two-level Hamming correlations to match the two-level QoSs requirement. The newly developed FHS set $\mathbb{S} = \{\mathbb{S}_1; \mathbb{S}_2\}$ enjoys the following properties: the FHS subset $\mathbb{S}_1$ with lower frequency-hit rate is applicable to HQoS-tier networks, while the FHS set with higher frequency-hit rate $\mathbb{S}_2$ is applicable to LQoS-tier networks. The more attractive merits are that the FH subsets $\mathbb{S}_1$ and $\mathbb{S}_2$ possess the minimum Hamming correlation for the large access delay, which is very helpful to the asynchronous multi-tier networks.

To evaluate the enabling of two-level QoSs via the proposed FHS set, we investigated the FH/OFDM system employed with such an FHS set in multi-tier uplinks. The simulation results have shown that, in aid of the hierarchical FHS set, the FH/OFDM system conveniently implements the multi-level BERs target, meanwhile, eliminating the intra- and inter-tier interference efficiently. In our future work, some interesting topics following this paper will be studied, such as, the bounds on the Hammming correlation of the multi-level hierarchy FHS, the construction algorithm of generalized multi-level hierarchy FHS and so forth.

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