PROJECT REPORT

MULTI-AGENT REINFORCEMENT LEARNING VIA ADAPTIVE KALMAN TEMPORAL DIFFERENCE AND SUCCESSOR REPRESENTATION

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Reinforcement Learning Challenges

- Fixed Reward Function
- Overfitting, High sensitivity and …

Kalman an Idea to Solve The Challenges

- Online Learning and …

INTRODUCTION MARI

Questions!

INTRODUCTION MARI

Challenges!

Problem Formulation

SINGLE AGENT REINFORCEMENT LEARNING

$$Q_\pi(\boldsymbol{s}, a) = \mathbb{E}\left\{\sum_{k=0}^{T} \gamma^k r_k | \boldsymbol{s}_0 = \boldsymbol{s}, a_0 = a, a_k = \pi(\boldsymbol{s}_k)\right\}$$

SINGLE AGENT REINFORCEMENT LEARNING

$$Q_\pi(\mathbf{s},a) = \mathbb{E}\left\{\sum_{k=0}^{T} \gamma^k r_k | \mathbf{s}_0 = \mathbf{s}, a_0 = a, a_k = \pi(\mathbf{s}_k)\right\}$$

$$a_k = \arg\max_{a \in \mathcal{A}} Q_{\pi^*}(\mathbf{s}_k, a).$$

TD LEARNING

$$Q_{\pi^*}(\boldsymbol{s}_k, a_k) = Q_{\pi^*}(\boldsymbol{s}_k, a_k) + \alpha\left(r_k + \gamma \max_{a \in \mathcal{A}} Q_{\pi^*}(\boldsymbol{s}_{k+1}, a) - Q_{\pi^*}(\boldsymbol{s}_k, a_k)\right)$$

TD LEARNING

$$Q_{\pi^*}(\boldsymbol{s}_k, a_k) = Q_{\pi^*}(\boldsymbol{s}_k, a_k) + \alpha\left(r_k + \gamma \max_{a \in \mathcal{A}} Q_{\pi^*}(\boldsymbol{s}_{k+1}, a) - Q_{\pi^*}(\boldsymbol{s}_k, a_k)\right)$$

$$a_k = \arg\max_{a \in \mathcal{A}} Q_{\pi^*}(\boldsymbol{s}_k, a)$$

MULTI AGENT SETTING

Agent i, for $(1 \leq i \leq N)$

$$\mathbb{S} = \{\mathcal{S}^{(1)}, \ldots, \mathcal{S}^{(N)}\}$$

$$\mathbb{A} = \{\mathcal{A}^{(1)}, \ldots, \mathcal{A}^{(N)}\}$$

$$\mathbb{Z} = \{\mathcal{Z}^{(1)}, \ldots, \mathcal{Z}^{(N)}\}$$

$$r^{(i)} : \mathbb{S} \times \mathcal{A}^{(i)} \rightarrow \mathbb{R}$$

$$R^{(i)} = \sum_{t=0}^{T} \gamma^t (r^{(i)})^t$$

MULTI AGENT SR

$$\boldsymbol{M}_{\pi^{(i)}}(\boldsymbol{s}^{(i)}, \boldsymbol{s}'^{(i)}, a^{(i)}) = \mathbb{E}\left[\sum_{k=0}^{T} \gamma^k \mathbb{1}[\boldsymbol{s}_k^{(i)} = \boldsymbol{s}'^{(i)}] | \boldsymbol{s}_0^{(i)} = \boldsymbol{s}^{(i)}, a_0^{(i)} = a^{(i)}\right]$$

$$\begin{aligned}\boldsymbol{M}^{\text{new}}_{\pi^{(i)}}(\boldsymbol{s}_k^{(i)}, \boldsymbol{s}'^{(i)}, a_k^{(i)}) &= \boldsymbol{M}^{\text{old}}_{\pi^{(i)}}(\boldsymbol{s}_k^{(i)}, \boldsymbol{s}'^{(i)}, a_k^{(i)}) + \\ &\alpha\left(\mathbb{1}[\boldsymbol{s}_k^{(i)} = \boldsymbol{s}'^{(i)}] + \gamma \boldsymbol{M}_{\pi^{(i)}}(\boldsymbol{s}_{k+1}^{(i)}, \boldsymbol{s}'^{(i)}, a_{k+1}^{(i)}) - \boldsymbol{M}^{\text{old}}_{\pi^{(i)}}(\boldsymbol{s}_k^{(i)}, \boldsymbol{s}'^{(i)}, a_k^{(i)})\right)\end{aligned}$$

$$Q_{\pi^{(i)}}(\boldsymbol{s}_k^{(i)}, a_k^{(i)}) = \sum_{\boldsymbol{s}'^{(i)} \in \mathcal{S}^{(i)}} \boldsymbol{M}(\boldsymbol{s}_k^{(i)}, \boldsymbol{s}'^{(i)}, a_k^{(i)}) R^{(i)}(\boldsymbol{s}'^{(i)}, a_k^{(i)})$$

MULTI AGENT SR

$$\boldsymbol{M}_{\pi^{(i)}}(\boldsymbol{s}^{(i)}, \boldsymbol{s}'^{(i)}, a^{(i)}) = \mathbb{E}\left[\sum_{k=0}^{T} \gamma^k \mathbb{1}[\boldsymbol{s}_k^{(i)} = \boldsymbol{s}'^{(i)}] | \boldsymbol{s}_0^{(i)} = \boldsymbol{s}^{(i)}, a_0^{(i)} = a^{(i)}\right]$$

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$$Q_{\pi^{(i)}}(\boldsymbol{s}_k^{(i)}, a_k^{(i)}) = \sum_{\boldsymbol{s}'^{(i)} \in \mathcal{S}^{(i)}} \boldsymbol{M}(\boldsymbol{s}_k^{(i)}, \boldsymbol{s}'^{(i)}, a_k^{(i)}) R^{(i)}(\boldsymbol{s}'^{(i)}, a_k^{(i)})$$

MULTI AGENT SR

$$\boldsymbol{M}_{\pi^{(i)}}(\boldsymbol{s}^{(i)}, \boldsymbol{s}'^{(i)}, a^{(i)}) = \mathbb{E}\left[\sum_{k=0}^{T} \gamma^k \mathbb{1}[\boldsymbol{s}_k^{(i)} = \boldsymbol{s}'^{(i)}] | \boldsymbol{s}_0^{(i)} = \boldsymbol{s}^{(i)}, a_0^{(i)} = a^{(i)}\right]$$

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$$Q_{\pi^{(i)}}(\boldsymbol{s}_k^{(i)}, a_k^{(i)}) = \sum_{\boldsymbol{s}'^{(i)} \in \mathcal{S}^{(i)}} \boldsymbol{M}(\boldsymbol{s}_k^{(i)}, \boldsymbol{s}'^{(i)}, a_k^{(i)}) R^{(i)}(\boldsymbol{s}'^{(i)}, a_k^{(i)})$$

MAK-TD FRAMEWORK

$$Q_{\pi^{(i)*}}(\boldsymbol{s}_k^{(i)}, a_k^{(i)}) \approx r_k^{(i)} + \gamma \max_{a^{(i)} \in \mathcal{A}} Q_{\pi^{(i)*}}(\boldsymbol{s}_{k+1}^{(i)}, a^{(i)})$$

$$r_k^{(i)} = Q_{\pi^{(i)*}}(\boldsymbol{s}_k^{(i)}, a_k^{(i)}) - \gamma \max_{a^{(i)} \in \mathcal{A}} Q_{\pi^*}(\boldsymbol{s}_{k+1}^{(i)}, a^{(i)}) + v_k^{(i)}$$

v_k is modeled as a zero-mean normal distribution with variance of $R^{(i)}$

MAK-TD FRAMEWORK

$$Q_{\pi^{(i)*}}(\mathbf{s}_k^{(i)}, a_k^{(i)}) \approx r_k^{(i)} + \gamma \max_{a^{(i)} \in \mathcal{A}} Q_{\pi^{(i)*}}(\mathbf{s}_{k+1}^{(i)}, a^{(i)})$$

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v_k is modeled as a zero-mean normal distribution with variance of $R^{(i)}$

MAK-TD FRAMEWORK

$$Q_{\pi^{(i)}}(\boldsymbol{s}_k^{(i)}, a_k^{(i)}) = \boldsymbol{\phi}(\boldsymbol{s}_k^{(i)}, a_k^{(i)})^T \boldsymbol{\theta}_k^{(i)}$$

$$r_k^{(i)} = \left[\boldsymbol{\phi}(\boldsymbol{s}_k^{(i)}, a_k^{(i)})^T - \gamma \max_{a^{(i)} \in \mathcal{A}} \boldsymbol{\phi}(\boldsymbol{s}_{k+1}^{(i)}, a^{(i)})^T\right] \boldsymbol{\theta}_k^{(i)} + v_k^{(i)}$$

$$\boldsymbol{h}_k^{(i)} = \boldsymbol{\phi}(\boldsymbol{s}_k^{(i)}, a_k^{(i)}) - \gamma \max_{a^{(i)} \in \mathcal{A}} \boldsymbol{\phi}(\boldsymbol{s}_{k+1}^{(i)}, a^{(i)})$$

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MAK-TD FRAMEWORK

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$$r_k^{(i)} = [\boldsymbol{h}_k^{(i)}]^T \boldsymbol{\theta}_k^{(i)} + v_k^{(i)}$$

MAK-TD FRAMEWORK

$$r_k^{(i)} = [\boldsymbol{h}_k^{(i)}]^T \boldsymbol{\theta}_k^{(i)} + v_k^{(i)}$$

$$x_k = F_{k-1} x_{k-1} + G_{k-1} u_{k-1} + w_{k-1}$$

$$y_k = H_k x_k + v_k$$

$$E(w_k) = E(v_k) = 0 \quad E(w_k w_j^T) = Q_k \delta_{k-j} \quad E(v_k v_j^T) = R_k \delta_{k-j} \quad E(v_k w_j^T) = 0$$

MAK-TD FRAMEWORK

$$x_k = F_{k-1} x_{k-1} + G_{k-1} u_{k-1} + w_{k-1}$$

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$$r_k^{(i)} = [\boldsymbol{h}_k^{(i)}]^T \boldsymbol{\theta}_k^{(i)} + v_k^{(i)}$$

$$E(w_k) = E(v_k) = 0 \quad E(w_k w_j^T) = Q_k \delta_{k-j} \quad E(v_k v_j^T) = R_k \delta_{k-j} \quad E(v_k w_j^T) = 0$$

MAK-TD FRAMEWORK

$$x_k = F_{k-1}x_{k-1} + G_{k-1}u_{k-1} + w_{k-1}$$

$$y_k = H_k x_k + v_k \qquad r_k^{(i)} = [\boldsymbol{h}_k^{(i)}]^T \boldsymbol{\theta}_k^{(i)} + v_k^{(i)}$$

$$E(w_k) = E(v_k) = 0 \quad E(w_k w_j^T) = Q_k \delta_{k-j} \quad E(v_k v_j^T) = R_k \delta_{k-j} \quad E(v_k w_j^T) = 0$$

$$K_k = P_k^- H_k^T (H_k P_k^- H_k^T + R_k)^{-1} = P_k^+ H_k^T R_k^{-1}$$

$$\hat{x}_k^+ = \hat{x}_k^- + K_k (y_k - H_k \hat{x}_k^-)$$

$$P_k^+ = (I - K_k H_k) P_k^- = (I - K_k H_k) P_k^- (I - K_k H_k)^T + K_k R_k K_k^T$$

MAK-TD FRAMEWORK

$$r_k^{(i)} = [\boldsymbol{h}_k^{(i)}]^T \boldsymbol{\theta}_k^{(i)} + v_k^{(i)}$$

$$K_k = P_k^- H_k^T (H_k P_k^- H_k^T + R_k)^{-1} \quad = P_k^+ H_k^T R_k^{-1}$$

$$\hat{x}_k^+ = \hat{x}_k^- + K_k (y_k - H_k \hat{x}_k^-)$$

$$P_k^+ = (I - K_k H_k) P_k^- = (I - K_k H_k) P_k^- (I - K_k H_k)^T + K_k R_k K_k^T$$

$$\begin{aligned}
\boldsymbol{K}_k^{j\,(i)} &= \boldsymbol{P}_{(\boldsymbol{\theta},k|k-1)}^{(i)} \boldsymbol{h}_k^{(i)} \left(\boldsymbol{h}_k^{T\,(i)} \boldsymbol{P}_{(\boldsymbol{\theta},k|k-1)}^{(i)} \boldsymbol{h}_k^{(i)} + R^{j\,(i)}\right)^{-1} \\
\hat{\boldsymbol{\theta}}_k^{j\,(i)} &= \hat{\boldsymbol{\theta}}_{(k|k-1)}^{(i)} + \boldsymbol{K}_k^{j\,(i)} \left(r_k^{(i)} - \boldsymbol{h}_k^{T\,(i)} \hat{\boldsymbol{\theta}}_{(k|k-1)}^{(i)}\right) \\
\boldsymbol{P}_{\boldsymbol{\theta},k}^{j\,(i)} &= \left(\boldsymbol{I} - \boldsymbol{K}_k^{j\,(i)} \boldsymbol{h}_k^{T\,(i)}\right) \boldsymbol{P}_{(\boldsymbol{\theta},k|k-1)}^{T\,(i)} \left(\boldsymbol{I} - \boldsymbol{K}_k^{j\,(i)} \boldsymbol{h}_k^{T\,(i)}\right) + \boldsymbol{K}_k^{j\,(i)} R^{j\,(i)} \boldsymbol{K}_k^{j\,T\,(i)}
\end{aligned}$$

MAK-TD FRAMEWORK

$$x_k = F_{k-1}x_{k-1} + G_{k-1}u_{k-1} + w_{k-1}$$

$$\bar{x}_k = F_{k-1}\bar{x}_{k-1} + G_{k-1}u_{k-1}$$

$$P_k = F_{k-1}P_{k-1}F_{k-1}^T + Q_{k-1}$$

MAK-TD FRAMEWORK

$$\hat{\boldsymbol{\theta}}_k^{(i)} = \sum_{j=1}^{M} \omega^{j\,(i)} \hat{\boldsymbol{\theta}}_k^{j\,(i)}$$

$$\boldsymbol{P}_{\boldsymbol{\theta},k}^{(i)} = \sum_{j=1}^{M} \omega^{j\,(i)} \left(\boldsymbol{P}_{\boldsymbol{\theta},k}^{j\;\,(i)} + (\hat{\boldsymbol{\theta}}^{j\,(i)} - \hat{\boldsymbol{\theta}}^{(i)})(\hat{\boldsymbol{\theta}}^{j\,(i)} - \hat{\boldsymbol{\theta}}^{(i)})^T \right)$$

MAK-TD FRAMEWORK

$$\boldsymbol{\phi}(\mathbf{s}_k^{(i)}) = \left[\phi_1(\mathbf{s}_k^{(i)}), \phi_2(\mathbf{s}_k^{(i)}), \ldots, \phi_{N_b-1}(\mathbf{s}_k^{(i)}), \phi_{N_b}(\mathbf{s}_k^{(i)})\right]^T$$

$$\phi_n(\mathbf{s}_k^{(i)}) = \exp\left\{\frac{-1}{2}(\mathbf{s}_k^{(i)} - \boldsymbol{\mu}_n^{(i)})^T \boldsymbol{\Sigma}_n^{(i)^{-1}} (\mathbf{s}_k^{(i)} - \boldsymbol{\mu}_n^{(i)})\right\}$$

where $\boldsymbol{\mu}_n^{(i)}$ and $\boldsymbol{\Sigma}_n^{(i)}$ are the mean and covariance of $\phi_n(\mathbf{s}_k^{(i)})$, for $(1 \leq n \leq N_b)$

MAK-TD FRAMEWORK

$$\boldsymbol{\phi}(\mathbf{s}_k^{(i)}) = [\phi_1(\mathbf{s}_k^{(i)}), \phi_2(\mathbf{s}_k^{(i)}), \ldots, \phi_{N_b-1}(\mathbf{s}_k^{(i)}), \phi_{N_b}(\mathbf{s}_k^{(i)})]^T$$

$$\phi_n(\mathbf{s}_k^{(i)}) = \exp\{\frac{-1}{2}(\mathbf{s}_k^{(i)} - \boldsymbol{\mu}_n^{(i)})^T \boldsymbol{\Sigma}_n^{(i)^{-1}}(\mathbf{s}_k^{(i)} - \boldsymbol{\mu}_n^{(i)})\}$$

where $\boldsymbol{\mu}_n^{(i)}$ and $\boldsymbol{\Sigma}_n^{(i)}$ are the mean and covariance of $\phi_n(\mathbf{s}_k^{(i)})$, for $(1 \leq n \leq N_b)$

MAK-TD FRAMEWORK

$$\boldsymbol{\phi}(\boldsymbol{s}_k^{(i)}, a_k^{(i)}) = [\phi_{1,a_1}(\boldsymbol{s}_k^{(i)}), \ldots \phi_{N_b,a_1}(\boldsymbol{s}_k^{(i)}), \phi_{1,a_2}(\boldsymbol{s}_k^{(i)}), \ldots \phi_{N_b,a_{D(i)}}(\boldsymbol{s}_k^{(i)})]^T$$

$$\boldsymbol{\phi}(\boldsymbol{s}_k^{(i)}, a_k^{(i)}) = [0, \ldots 0, \phi_1(\boldsymbol{s}_k^{(i)}), \ldots, \phi_N(\boldsymbol{s}_k^{(i)}), 0, \ldots 0,]^T$$

MAK-TD FRAMEWORK

$$L_k^{(i)} = \left(\boldsymbol{\phi}^T(\boldsymbol{s}_k^{(i)}, a_k)\,\boldsymbol{\theta}_k^{(i)} - r_k^{(i)}\right)^2$$

$$\begin{aligned}
\Delta\boldsymbol{\mu}^{(i)} &= -\frac{\partial L_k^{(i)}}{\partial \boldsymbol{\mu}^{(i)}} = -\frac{\partial L_k^{(i)}}{\partial Q_{\pi^*(i)}} \frac{\partial Q_{\pi^*(i)}}{\partial \boldsymbol{\phi}^{(i)}} \frac{\partial \boldsymbol{\phi}^{(i)}}{\partial \boldsymbol{\mu}^{(i)}} \\
\text{and } \Delta\boldsymbol{\Sigma}^{(i)} &= -\frac{\partial \boldsymbol{\Sigma}_k^{(i)}}{\partial \boldsymbol{\mu}^{(i)}} = -\frac{\partial L_k^{(i)}}{\partial Q_{\pi^*(i)}} \frac{\partial Q_{\pi^*(i)}}{\partial \boldsymbol{\phi}^{(i)}} \frac{\partial \boldsymbol{\phi}^{(i)}}{\partial \boldsymbol{\Sigma}^{(i)}}
\end{aligned}$$

MAK-TD FRAMEWORK

$$L_k^{(i)} = (\boldsymbol{\phi}^T(\boldsymbol{s}_k^{(i)}, a_k)\,\boldsymbol{\theta}_k^{(i)} - r_k^{(i)})^2$$

$$\begin{aligned}
\Delta\boldsymbol{\mu}^{(i)} &= -\frac{\partial L_k^{(i)}}{\partial \boldsymbol{\mu}^{(i)}} = -\frac{\partial L_k^{(i)}}{\partial Q_{\pi^*(i)}}\,\frac{\partial Q_{\pi^*(i)}}{\partial \boldsymbol{\phi}^{(i)}}\,\frac{\partial \boldsymbol{\phi}^{(i)}}{\partial \boldsymbol{\mu}^{(i)}} \\
\text{and } \Delta\boldsymbol{\Sigma}^{(i)} &= -\frac{\partial \boldsymbol{\Sigma}_k^{(i)}}{\partial \boldsymbol{\mu}^{(i)}} = -\frac{\partial L_k^{(i)}}{\partial Q_{\pi^*(i)}}\,\frac{\partial Q_{\pi^*(i)}}{\partial \boldsymbol{\phi}^{(i)}}\,\frac{\partial \boldsymbol{\phi}^{(i)}}{\partial \boldsymbol{\Sigma}^{(i)}}
\end{aligned}$$

MAK-TD FRAMEWORK

$$
\begin{aligned}
a_k^{(i)} &= \arg\max_a \left(\boldsymbol{h}_k^{(i)}(\boldsymbol{s}_k^{(i)}, a^{(i)}) R^{-1\,(i)} \boldsymbol{h}_k^{T\,(i)}(\boldsymbol{s}_k^{(i)}, a^{(i)}) \right) \\
&= \arg\max_a \left(\boldsymbol{h}_k^{(i)}(\boldsymbol{s}_k^{(i)}, a^{(i)}) \boldsymbol{h}_k^{T\,(i)}(\boldsymbol{s}_k^{(i)}, a^{(i)}) \right).
\end{aligned}
$$

Algorithm 1 THE PROPOSED MAK-TD FRAMEWORK

Learning Phase:

Set $\boldsymbol{\theta}_0, \boldsymbol{P}_{\theta,0}, \boldsymbol{F}, \boldsymbol{\mu}_{n,i_d}, \boldsymbol{\Sigma}_{n,i_d}$ for $n = 1, 2, \ldots, N$ and $i_d = 1, 2, \ldots, D$

Repeat (for each episode):

  Initialize $\boldsymbol{s}_k$

  Repeat (for each agent i):

    While $\boldsymbol{s}_k^{(i)} \neq \boldsymbol{s}_T$ **do**:

      $a_k^{(i)} = \arg\max_a \left(\boldsymbol{h}_k^{(i)}(\boldsymbol{s}_k^{(i)}, a^{(i)}) \boldsymbol{h}_k^{T(i)}(\boldsymbol{s}_k^{(i)}, a^{(i)}) \right)$

      Take action $a_k^{(i)}$, observe $\boldsymbol{s}_{k+1}^{(i)}, r_k^{(i)}$

      Calculate $\boldsymbol{\phi}^{(i)}(\boldsymbol{s}^{(i)}, a^{(i)})$ via Equations (22) and (23)

      $\boldsymbol{h}_k^{(i)}(\boldsymbol{s}_k^{(i)}, a_k^{(i)}) = \boldsymbol{\phi}^{(i)}(\boldsymbol{s}_k^{(i)}, a_k^{(i)}) - \gamma \arg\max_a \boldsymbol{\phi}^{(i)}(\boldsymbol{s}_{k+1}^{(i)}, a^{(i)})$

      $\hat{\boldsymbol{\theta}}_{(k|k-1)}^{(i)} = \boldsymbol{F}^{(i)} \hat{\boldsymbol{\theta}}_k^{(i)}$

      $\boldsymbol{P}_{(\theta,k|k-1)}^{(i)} = \boldsymbol{F}^{(i)} \boldsymbol{P}_{\theta,k-1}^{(i)} \boldsymbol{F}^{T(i)} + \boldsymbol{Q}^{(i)}$

      for $j = 1 : M$ **do**:

        $\boldsymbol{k}_k^{j\,(i)} = \boldsymbol{P}_{(\theta,k|k-1)}^{(i)} \boldsymbol{h}_k^{(i)} (\boldsymbol{h}_k^{T(i)} \boldsymbol{P}_{(\theta,k|k-1)}^{(i)} \boldsymbol{h}_k^{(i)} + R^{j\,(i)})^{-1}$

        $\hat{\boldsymbol{\theta}}_k^{j\,(i)} = \hat{\boldsymbol{\theta}}_{(\theta,k|k-1)}^{(i)} + \boldsymbol{k}_k^{j\,(i)} (r_k^j - \boldsymbol{h}_k^{T(i)} \hat{\boldsymbol{\theta}}_{(k|k-1)}^{(i)})$

        $\boldsymbol{P}_{\theta,k}^{(i)} = (\boldsymbol{I} - \boldsymbol{K}_k^{j\,(i)} \boldsymbol{h}_k^{T(i)}) \boldsymbol{P}_{(\theta,k|k-1)}^{(i)} (\boldsymbol{I} - \boldsymbol{K}_k^{j\,(i)} \boldsymbol{h}_k^{T(i)})^T + \boldsymbol{K}_k^{j\,(i)} R^j \boldsymbol{K}_k^{j\,T(i)}$

      end for

```
Compute the value of c and w^{j(i)} by using Σ_{j=1}^{M} w^{j(i)} = 1 and Equation (19)
```

Compute the value of c and $w^{j^{(i)}}$ by using $\sum_{j=1}^{M} w^{j^{(i)}} = 1$ and Equation (19)

$\hat{\boldsymbol{\theta}}_k^{(i)} = \sum_{j=1}^{M} w^{j^{(i)}} \hat{\boldsymbol{\theta}}_k^{j^{(i)}}$

$\boldsymbol{P}_{\boldsymbol{\theta}_k}^{(i)} = \sum_{j=1}^{M} \omega^{j^{(i)}} \left(\boldsymbol{P}_{\boldsymbol{\theta},k}^{j^{(i)}} + (\hat{\boldsymbol{\theta}}^{j^{(i)}} - \hat{\boldsymbol{\theta}}^{(i)})(\hat{\boldsymbol{\theta}}^{j^{(i)}} - \hat{\boldsymbol{\theta}}^{(i)})^T \right)$

RBFs Parameters Update:

$L_k^{(i)} = (\boldsymbol{\phi}^T(\boldsymbol{s}_k^{(i)}, a_k)\, \boldsymbol{\theta}_k^{(i)} - r_k^{(i)})^2$

if $L_k^{(i)\frac{1}{2}} (\boldsymbol{\theta}_k^{(i)^T} \boldsymbol{\phi}(\cdot)) > 0$ **then**:

Update $\boldsymbol{\Sigma}_{n,a_d}$ via Equation (29)

else:

Update $\boldsymbol{\mu}_{n,a_d}$ via Equation (30)

end if

end while

Testing Phase:

Repeat (for each trial episode):

While $s_k \neq s_T$ **do**:

Repeat (for each agent):

$a_k = \arg\max_a \boldsymbol{\phi}(s_k, a)^T \boldsymbol{\theta}_k$

Take action a_k, and observe $\boldsymbol{s}_{k+1}, r_k$

Calculate Loss S_k for all agents

End While

MAK-SR FRAMEWORK

$$\boldsymbol{M}_{\pi^{(i)}}(\boldsymbol{s}^{(i)}, :, a^{(i)}) = \mathbb{E}\left[\sum_{k=0}^{T} \gamma^k \boldsymbol{\phi}(\boldsymbol{s}_k^{(i)}, a_k^{(i)}) | \boldsymbol{s}_0^{(i)} = \boldsymbol{s}^{(i)}, a_0^{(i)} = a^{(i)}\right]$$

$$r^{(i)}(\boldsymbol{s}_k^{(i)}, a_k^{(i)}) \approx \boldsymbol{\phi}(\boldsymbol{s}_k^{(i)}, a_k^{(i)})^T \boldsymbol{\theta}_k^{(i)}$$

$$Q(\boldsymbol{s}_k^{(i)}, a_k^{(i)}) = \boldsymbol{\theta}_k^{(i)T} \boldsymbol{M}(\boldsymbol{s}_k^{(i)}, :, a_k^{(i)})$$

$$\boldsymbol{M}_{\pi^{(i)}}(\boldsymbol{s}_k^{(i)}, :, a_k^{(i)}) \approx \boldsymbol{M}_k \boldsymbol{\phi}(\boldsymbol{s}_k^{(i)}, a_k^{(i)})$$

$$\boldsymbol{M}_{\pi^{(i)}}^{\text{new}}(\boldsymbol{s}_k^{(i)}, :, a_k^{(i)}) = \boldsymbol{M}_{\pi^{(i)}}^{\text{old}}(\boldsymbol{s}_k^{(i)}, :, a_k^{(i)}) + \alpha\left(\boldsymbol{\phi}^{(i)}(\boldsymbol{s}_k^{(i)}, a_k^{(i)}) + \gamma \boldsymbol{M}_{\pi^{(i)}}(\boldsymbol{s}_{k+1}^{(i)}, :, a_{k+1}^{(i)}) - \boldsymbol{M}_{\pi^{(i)}}^{\text{old}}(\boldsymbol{s}_k^{(i)}, :, a_k^{(i)})\right)$$

MAK-SR FRAMEWORK

$$\boldsymbol{M}_{\pi^{(i)}}(\boldsymbol{s}^{(i)}, :, a^{(i)}) = \mathbb{E}\left[\sum_{k=0}^{T} \gamma^k \boldsymbol{\phi}(\boldsymbol{s}_k^{(i)}, a_k^{(i)}) | \boldsymbol{s}_0^{(i)} = \boldsymbol{s}^{(i)}, a_0^{(i)} = a^{(i)}\right]$$

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$$Q(\boldsymbol{s}_k^{(i)}, a_k^{(i)}) = \boldsymbol{\theta}_k^{(i)T} \boldsymbol{M}(\boldsymbol{s}_k^{(i)}, :, a_k^{(i)})$$

$$\boldsymbol{M}_{\pi^{(i)}}(\boldsymbol{s}_k^{(i)}, :, a_k^{(i)}) \approx \boldsymbol{M}_k \boldsymbol{\phi}(\boldsymbol{s}_k^{(i)}, a_k^{(i)})$$

$$\boldsymbol{M}_{\pi^{(i)}}^{\text{new}}(\boldsymbol{s}_k^{(i)}, :, a_k^{(i)}) = \boldsymbol{M}_{\pi^{(i)}}^{\text{old}}(\boldsymbol{s}_k^{(i)}, :, a_k^{(i)}) + \alpha\left(\boldsymbol{\phi}^{(i)}(\boldsymbol{s}_k^{(i)}, a_k^{(i)}) + \gamma \boldsymbol{M}_{\pi^{(i)}}(\boldsymbol{s}_{k+1}^{(i)}, :, a_{k+1}^{(i)}) - \boldsymbol{M}_{\pi^{(i)}}^{\text{old}}(\boldsymbol{s}_k^{(i)}, :, a_k^{(i)})\right)$$

MAK-SR FRAMEWORK

$$\boldsymbol{M}_{\pi^{(i)}}(\boldsymbol{s}^{(i)}, :, a^{(i)}) = \mathbb{E}\left[\sum_{k=0}^{T} \gamma^k \boldsymbol{\phi}(\boldsymbol{s}_k^{(i)}, a_k^{(i)}) | \boldsymbol{s}_0^{(i)} = \boldsymbol{s}^{(i)}, a_0^{(i)} = a^{(i)}\right]$$

$$r^{(i)}(\boldsymbol{s}_k^{(i)}, a_k^{(i)}) \approx \boldsymbol{\phi}(\boldsymbol{s}_k^{(i)}, a_k^{(i)})^T \boldsymbol{\theta}_k^{(i)}$$

$$Q(\boldsymbol{s}_k^{(i)}, a_k^{(i)}) = \boldsymbol{\theta}_k^{(i)\,T} \boldsymbol{M}(\boldsymbol{s}_k^{(i)}, :, a_k^{(i)})$$

$$\boldsymbol{M}_{\pi^{(i)}}(\boldsymbol{s}_k^{(i)}, :, a_k^{(i)}) \approx \boldsymbol{M}_k\, \boldsymbol{\phi}(\boldsymbol{s}_k^{(i)}, a_k^{(i)})$$

$$\boldsymbol{M}_{\pi^{(i)}}^{\text{new}}(\boldsymbol{s}_k^{(i)}, :, a_k^{(i)}) = \boldsymbol{M}_{\pi^{(i)}}^{\text{old}}(\boldsymbol{s}_k^{(i)}, :, a_k^{(i)}) + \alpha\left(\boldsymbol{\phi}^{(\boldsymbol{i})}(\boldsymbol{s}_k^{(i)}, a_k^{(i)}) + \gamma \boldsymbol{M}_{\pi^{(i)}}(\boldsymbol{s}_{k+1}^{(i)}, :, a_{k+1}^{(i)}) - \boldsymbol{M}_{\pi^{(i)}}^{\text{old}}(\boldsymbol{s}_k^{(i)}, :, a_k^{(i)})\right)$$

MAK-SR FRAMEWORK

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MAK-SR FRAMEWORK

$$\hat{\boldsymbol{\phi}}(\boldsymbol{s}_k^{(i)}, a_k^{(i)}) = \boldsymbol{M}^{\text{new}}(\boldsymbol{s}_k^{(i)}, :, a_k^{(i)}) - \gamma \boldsymbol{M}(\boldsymbol{s}_{k+1}^{(i)}, :, a_{k+1}^{(i)}) + \boldsymbol{n}_k^{(i)}$$

$$\hat{\boldsymbol{\phi}}(\boldsymbol{s}_k^{(i)}, a_k^{(i)}) = \boldsymbol{M}_k \underbrace{\left[\boldsymbol{\phi}(\boldsymbol{s}_k^{(i)}, a_k^{(i)}) - \gamma \boldsymbol{\phi}(\boldsymbol{s}_{k+1}^{(i)}, a_{k+1}^{(i)})\right]}_{g_k^{(i)}} + \boldsymbol{n}_k^{(i)}$$

$$\hat{\boldsymbol{\phi}}(\boldsymbol{s}_k^{(i)}, a_k^{(i)}) = (\boldsymbol{g}^{(i)}{}_k^T \otimes \boldsymbol{I}) \boldsymbol{m}_k^{(i)} + \boldsymbol{n}_k^{(i)}$$

$$\boldsymbol{m}_{k+1}^{(i)} = \boldsymbol{m}_k^{(i)} + \boldsymbol{\mu}_k^{(i)}$$

MAK-SR FRAMEWORK

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MAK-SR FRAMEWORK

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$$\hat{\boldsymbol{\phi}}(\boldsymbol{s}_k^{(i)}, a_k^{(i)}) = (\boldsymbol{g}^{(i)}{}_k^T \otimes \boldsymbol{I})\boldsymbol{m}_k^{(i)} + \boldsymbol{n}_k^{(i)}$$

$$\boldsymbol{m}_{k+1}^{(i)} = \boldsymbol{m}_k^{(i)} + \boldsymbol{\mu}_k^{(i)}$$

Algorithm 2 THE PROPOSED MAK-SR FRAMEWORK

Learning Phase:
Initialize: $\theta_0, \boldsymbol{P}_{\theta,0}, \boldsymbol{m}_0, \boldsymbol{P}_{M,0}, \boldsymbol{\mu}_n$, and $\boldsymbol{\Sigma}_n$ for $n = 1, 2, \ldots, N$
Parameters: $Q_\theta, Q_M, \lambda_\mu, \lambda_\Sigma$, and $\{R_\theta^j, R_M^j\}$ for $j = 1, 2, \ldots, M$
Repeat (for each episode):
  Initialize $\boldsymbol{s}_k$
  Repeat (for each agent i):
    While $\boldsymbol{s}_k^{(i)} \neq \boldsymbol{s}_T$ **do**:
      Reshape $\boldsymbol{m}_k$ into $L \times L$ to construct 2-D matrix $\boldsymbol{M}_k$.
      $a_k^{(i)} = \arg\max_a \left(\boldsymbol{g}_k^{(i)}(\boldsymbol{s}_k^{(i)}, a) {\boldsymbol{g}_k^{(i)}}^T (\boldsymbol{s}_k^{(i)}, a^{(i)}) \right)$
      Take action $a_k^{(i)}$, observe $\boldsymbol{s}_{k+1}^{(i)}$ and $r_k^{(i)}$.
      Calculate $\boldsymbol{\phi}(\boldsymbol{s}_k^{(i)}, a_k^{(i)})$ via Equations (23) and (25).
      Update reward weights vector: Perform MMAE to update $\boldsymbol{\theta}_k^{(i)}$.
      Update SR weights vector: Perform KF on Equations (40) and (41) to update $\boldsymbol{m}_k^{(i)}$.
      Update RBFs parameters: Perform RGD on the loss function L_k to update $\boldsymbol{\Sigma}_n$ and $\boldsymbol{\mu}_n$.
    end while

Experimental Results

Experimental Results

Experimental Results

Table 1. Total loss averaged across all the episodes and for all the four implemented scenarios.

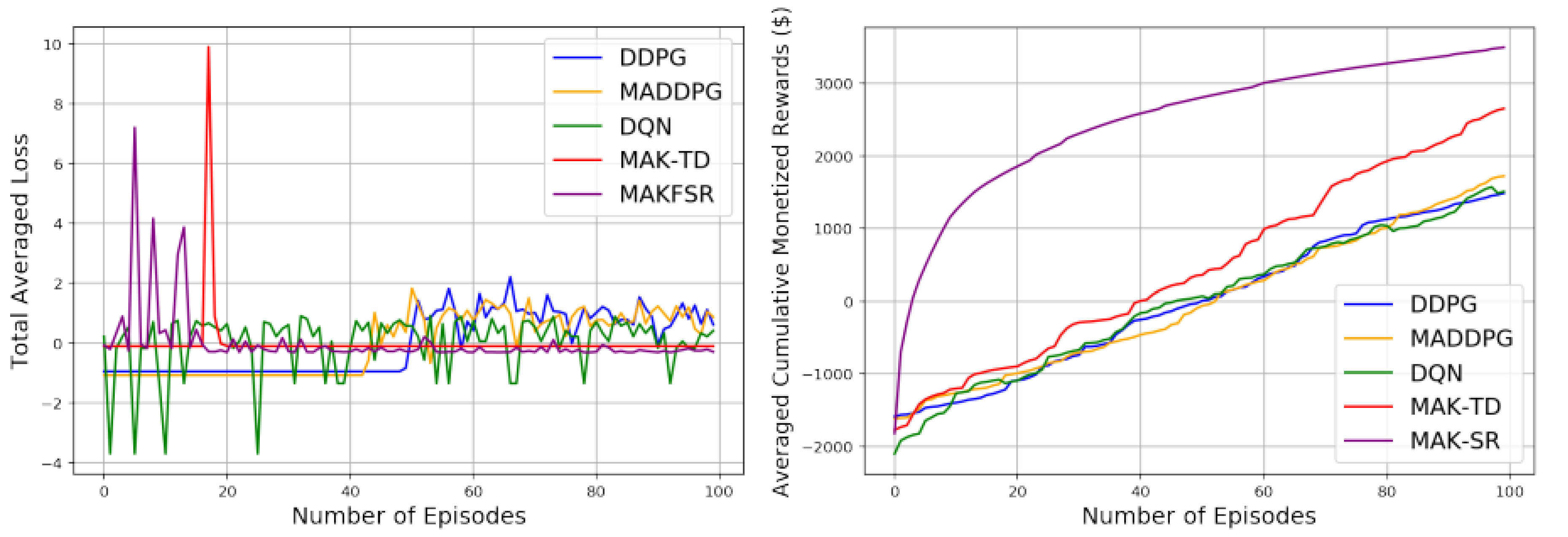
Environment	MAK-SR	MAK-TD	MADDPG	DDPG	DQN
Cooperation	8.93	2.4088	9649.84	10,561.16	10.93
Competition	0.43	4.9301	10,158.18	10,710.37	107.39
Predator–Prey 1v2	0.005	1.9374	6816.34	6884.33	8.21
Predator–Prey 2v1	8.87	1.2421	7390.18	6882.2	10.24

Experimental Results

Table 2. Total received reward by the agents averaged for all the four implemented scenarios.

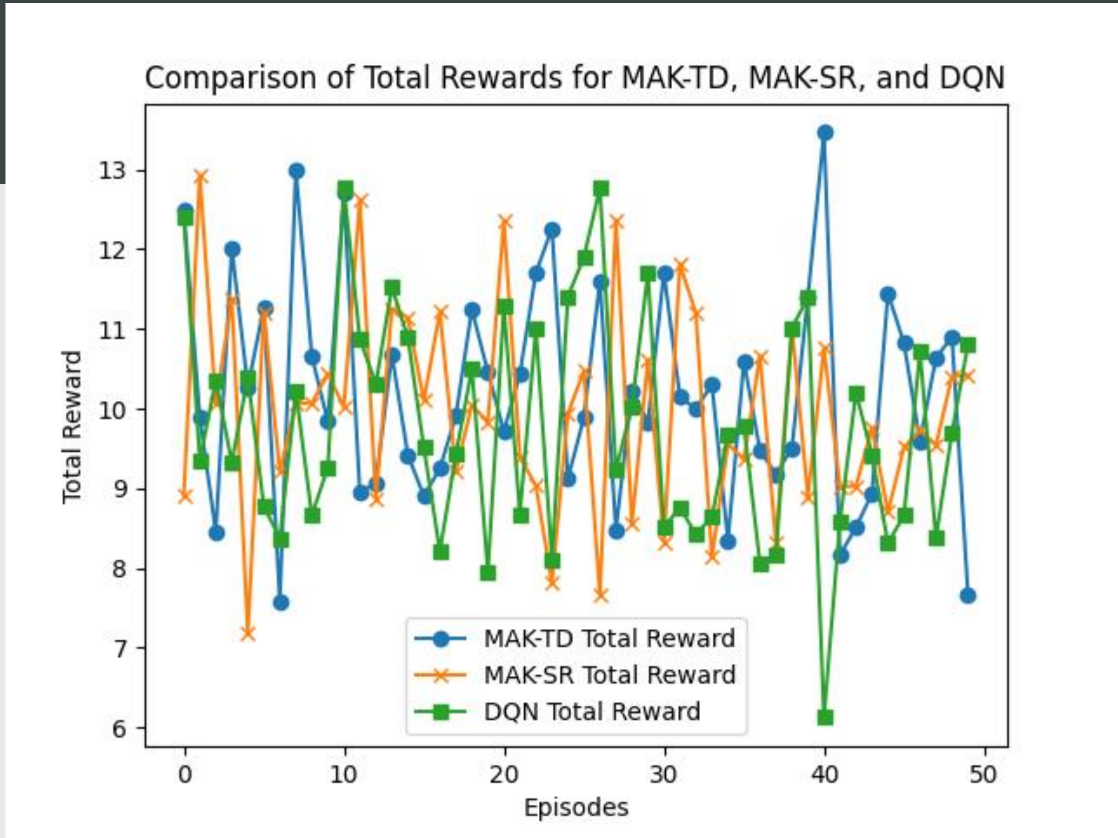
Environment	MAK-SR	MAK-TD	MADDPG	DDPG	DQN
Cooperation	−16.0113	−23.0113	−69.28	−66.29	−39.96
Competition	−0.778	−13.358	−63.30	−61.34	−14.49
Predator–Prey 1v2	−0.0916	−13.432	−46.17	−20.53	−23.451
Predator–Prey 2v1	−0.081	−17.0058	−55.69	−49.41	−44.32

Experimental Results



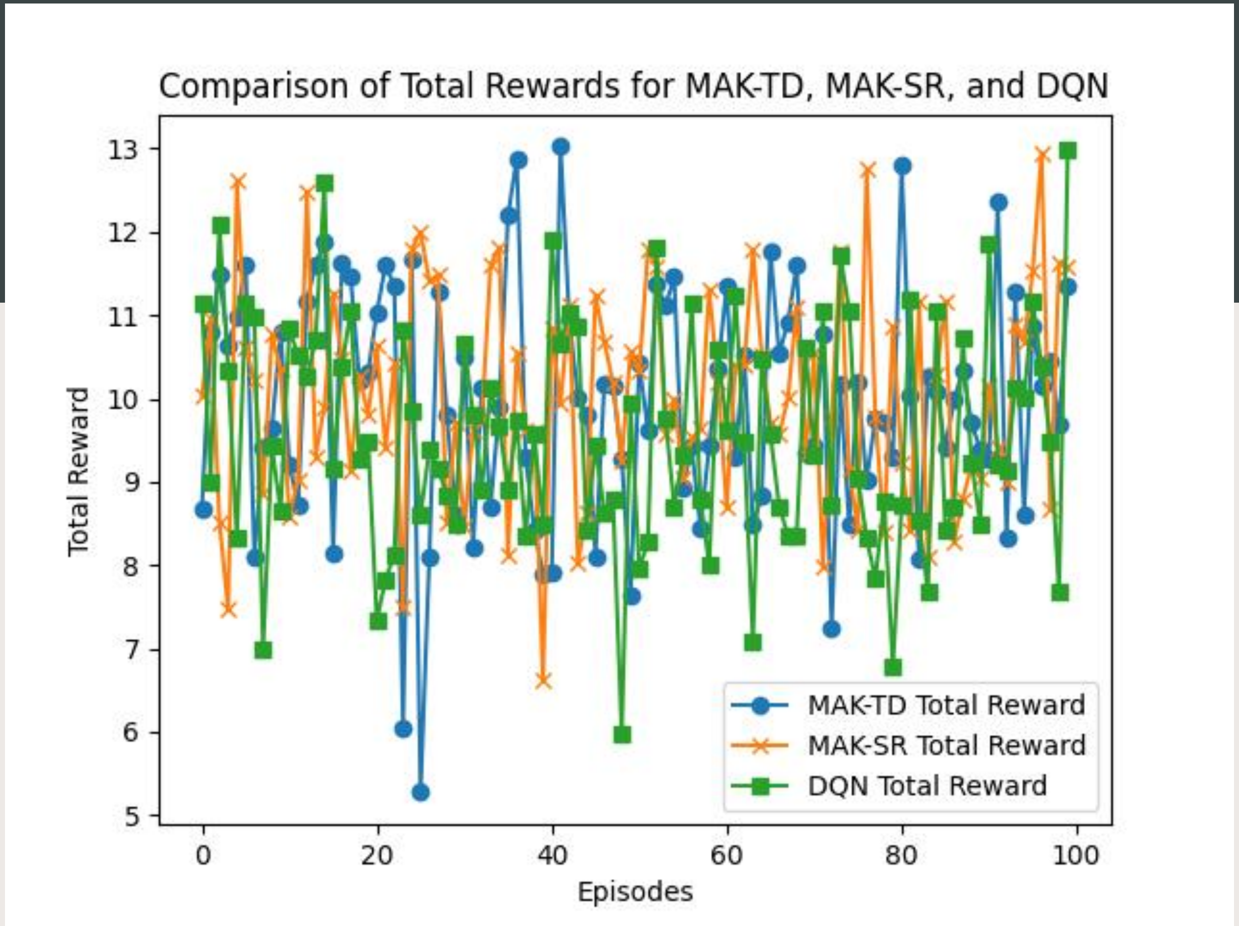
(a) (b)

Simulation Results




```
Final Total Rewards after 50 Episodes:
MAK-TD Total Reward: 510.10918848452957
MAK-SR Total Reward: 498.127397822994
DQN Total Reward: 488.5321211723451
```

Simulation Results



Final Total Rewards after 100 Episodes:
MAK-TD Total Reward: 996.1441199350996
MAK-SR Total Reward: 1003.7453232643612
DQN Total Reward: 955.8240584992651

Key Achievements:

- MAK-TD
- MAK-SR

CONCLUSIONS

THANK YOU!

Winter 2025