

# Model Free Control

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Outline: 1. Introduction 2. On-policy Monte Carlo Control 3. On-policy Temporal Difference Learning 4. Off-policy Learning 5. Summary

## Model Free Reinforcement Learning.

- Last Lecture:

- **Model-Free Prediction**

- Estimate the value function of an unknown MDP

- This Lecture:

- **Model-Free Control**

- Optimise the value function of an unknown MDP

## Uses of Model-Free Control

Some example problems that can be modelled as MDPs.

- ✓ Elevator

- ✓ Quake

- ✓ Parallel parking

- ✓ Robocup Soccer

- ✓ Ship Steering

- ✓ Portfolio management

- ✓ Bioreactor

- ✓ Protein Folding

- ✓ Helicopter

- ✓ Robot walking

- ✓ Airplane trajectory

- ✓ Game of Go

- For most of these problems, either:

- MDP model is unknown, but experience can be sampled

- MDP model is known, but is too big to use, except by sampling

**Model Free Control** can solve these problems.

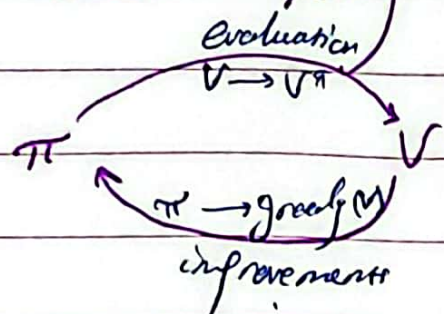
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# On and off-policy learning

- **On-policy learning**: 1) "Learn on the job"  
2) Learn about policy  $\pi$  from experience sampled from  $\pi$
- **Off-policy learning**: 1) "look over someone's shoulder"  
2) Learn about policy  $\pi$  from experience sampled from  $\mu$

## Generalized Policy Iteration (Refractor)



$$\pi^* \rightleftharpoons V^*$$

Policy Evaluation Estimate  $V_\pi$   
e.g. Iterative policy evaluation

Policy improvement Generate  $\pi' \geq \pi$   
e.g. Greedy policy improvement

## Generalized Policy Iteration with Monte-Carlo Evaluation

Policy Evaluation Monte-Carlo policy evaluation,  $V_\pi \approx \bar{V}_\pi$ ?

Policy improvement Greedy policy improvement? do not have exploration

needs dynamics MDP  $Q, V_\pi$

## Model Free Policy Iteration using Action-Value Function

- Greedy policy improvement over  $V_{\pi'}$  requires model of MDP

$$\pi'(s) = \arg \max_{a \in A} P_s^a + \rho_{ss'}^a V_{\pi'}(s')$$

- Greedy policy improvement over  $Q_{\pi', a}$  is model-free

$$\pi'(s) = \arg \max_{a \in A} Q_{\pi', a}(s, a)$$



## $\epsilon$ -Greedy Exploration

- Simplest idea for ensuring continual exploration
- All  $m$  actions are tried with non-zero probability
- with probability  $1-\epsilon$  choose the greedy action
- with probability  $\epsilon$  choose an action at random

$$\pi(a|s) = \begin{cases} \epsilon/m + 1-\epsilon & \text{if } a^*, \text{ argmax}_{a \in A} Q(s,a) \\ \epsilon/m & \text{otherwise} \end{cases}$$

## $\epsilon$ -Greedy Policy Improvement

Theorem:

For any  $\epsilon$ -greedy policy  $\pi$ , the  $\epsilon$ -greedy policy  $\pi'$  with respect to  $q_\pi$  is an improvement,  $V_{\pi'}(s) \geq V_\pi(s)$

$$q_{\pi'}(s, \pi'(s)) = \sum_{a \in A} \pi'(a|s) q_\pi(s, a)$$

$$= \frac{\epsilon}{m} \sum_{a \in A} q_\pi(s, a) + (1-\epsilon) \min_{a \in A} q_\pi(s, a)$$

$$\geq \frac{\epsilon}{m} \sum_{a \in A} q_\pi(s, a) + (1-\epsilon) \sum_{a \in A} \frac{\pi(a|s) - \epsilon/m}{1-\epsilon} q_\pi(s, a)$$

$\pi(a|s) \geq \epsilon/m$  this

$$= \sum_{a \in A} \pi(a|s) q_\pi(s, a) = V_\pi(s)$$

therefore from policy improvement theorem  $V_{\pi'}(s) \geq V_\pi(s)$



# GLIE

## Definition

Greedy in the Limit with Infinite Exploration (GLIE)

All state-action pairs are explored infinitely many times

$$\lim_{k \rightarrow \infty} N_k(s, a) = \infty$$

The policy converges on a greedy policy

$$\lim_{k \rightarrow \infty} \pi_k(a|s) = 1 \quad (a = \arg\max_{a' \in A} Q_k(s, a'))$$

For example,  $\epsilon$ -greedy is GLIE if  $\epsilon$  reduces to zero as  $\epsilon_k = \frac{1}{k}$

## GLIE Monte-Carlo Control

Sample  $k$ th episode using  $\pi$ :  $\{s_1, a_1, r_1, \dots, s_T\} \sim \pi$

For each state  $s_t$  and action  $a_t$  in the episode,

$$N(s_t, a_t) \leftarrow N(s_t, a_t) + 1$$

$$Q(s_t, a_t) \leftarrow Q(s_t, a_t) + \frac{1}{N(s_t, a_t)} (G_t - Q(s_t, a_t))$$

Improve policy based on new action-value function

$$\epsilon \leftarrow \frac{1}{k}$$

$$\pi \leftarrow \epsilon\text{-greedy}(Q)$$

## Theorem:

GLIE Monte-Carlo Control converges to the optimal action-value function

$$Q(s, a) \rightarrow q_*(s, a)$$



# Off policy Learning

- Evaluate target policy  $\pi(a|s)$  to compute  $U_{\pi}(s)$  or  $Q_{\pi}(s, a)$
- while following behaviour policy  $\mu(a|s)$ :  
 $\{S_1, A_1, R_1, \dots, S_T\} \sim \mu$
- why is this important?
- learn from observing humans or other agents
- Reuse experience generated from old policies  $\pi_1, \pi_2, \dots, \pi_{E-1}$
- learn about optimal policy while following, exploratory policy
- learn about multiple policies while following, one policy

## Importance Sampling

Estimate the expectation of a different distribution

$$\begin{aligned}
 E_{x \sim p}[f(x)] &= \sum p(x) f(x) \\
 &= \sum q(x) \frac{p(x)}{q(x)} f(x) \\
 &= E_{x \sim q} \left[ \frac{p(x)}{q(x)} f(x) \right]
 \end{aligned}$$

→ for off policy Monte-carlo

use returns generated from  $\mu$  to evaluate  $\pi$

weight return  $G_t$  according to similarity between policies  
 Multiply importance sampling corrections along whole episode

$$G_t^{\pi/\mu} = \frac{\pi(A_t|S_t) \pi(A_{t+1}|S_{t+1}) \dots \pi(A_T|S_T)}{\mu(A_t|S_t) \mu(A_{t+1}|S_{t+1}) \dots \mu(A_T|S_T)} G_t$$

update value towards ground return

$$V(S_t) \leftarrow V(S_t) + \alpha (G_t^{\pi/\mu} - V(S_t))$$

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Cannot use if  $\mu$  is zero when  $\sigma$  is non-zero

Defiance sampling and can dramatically increase variance  
variance

→ extremely high variance not for off policy you have to use TD

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