FORMAL METHODS WITH DYNAMIC AGENT SAFETY LOGIC

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by

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ABSTRACT

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Chapter 1

Introduction

In this doctoral thesis, I present a logic for reasoning about safety-critical information flow among machines and humans. The thesis advances the domain of modal logic by developing a rich and expressive logic suitable for reasoning about real humans in real situations, which in turn provides a new tool for formal methods researchers interested in developing safe human-machine hybrid systems. Thus, the thesis is interdisciplinary, pulling from fields as diverse as philosophy, game theory, computer science, and safety engineering.

The logic, which I call Dynamic Agent Safety Logic (DASL), is based on the logical foundations of game theory, in which models of agency formally capture how knowledge, rationality, and action relate to each other. Game theory presents a model that, given a description of a scenario, allows one to deduce what actions are dictated by a given theory of rationality. The standard game-theoretic inference works as follows:

$$Knowledge\_of\_Situation \wedge Rationality \Rightarrow Good\_Action.$$

One can read this as, "if an agent has knowledge of a situation (*e.g.* a game), and the agent is rational, then the agent executes a good action. In game theory, the important terms are suitably formalized for mathematical treatment. Knowledge is assumed to be perfect, rationality is defined as the maximization of some measure of utility, and good actions are those that bring about the outcomes with the most possible utility payoffs. The definitions make the inference an analytic truth.

Empirically, however, humans frequently deviate from the prescribed behavior. Looking at the above formula, we can ask a question: what can we infer when an agent fails to execute the prescribed action, as when pilots provide unsafe control inputs to their aircrafts? We can answer this question by examining the contrapositive of the above game-theoretic inference:

$$\neg Good\_Action \Rightarrow \neg(Knowledge\_of\_Situation \wedge Rationality),$$

or equivalently,

$$\neg Good\_Action \Rightarrow \neg Knowledge\_of\_Situation \vee \neg Rationality.$$

With a bit more Boolean manipulation, we have the following:

$$\neg Good\_Action \wedge Rationality \Rightarrow \neg Knowledge\_of\_Situation.$$

This can be read, "If an agent is rational but executes a bad action, then the agent lacked knowledge of the situation." Thus, embedded in the classical game-theoretic model of agency is a logical inference from bad action to missing knowledge. This makes intuitive sense upon reflection. If someone is rational, yet they commit an irrational (read: "bad") action, then it must be the case that they didn't know some crucial information. With this insight in hand, I identify a logic in which the above inference is sound, with details about which particular pieces of information are missing from an agent's knowledge base when she executes a bad action. Again, it should not be surprising that such a logic exists, because classical game theory already posits a *logical* relationship between knowledge of particular propositions and particular actions.

I have formally captured such inferences with DASL, where a rational agent executes a bad action, and from this we can infer which safety-critical information they are missing. This can be done at run-time, as demonstrated herein by a prototype that uses the Z3 theorem prover to compute relevant part of the inference, formalized as a set of clauses in first order logic. The prototype satisfies an information assurance property not yet treated formally by the literature, but done so here with DASL. The property formalizes the idea that safety-critical information should not fail to reach a human and inform her actions. This is more than the information assurance property of *availability*, because it is available to her only in a passive sense. It must be actively and specifically *delivered* by ensuring that non-critical information does not compete for the human's awareness at critical moments. Formally specifying this property is another contribution of this thesis.

I apply these formal ideas to the domain of aviation safety, specifically incidents involving pilot errors that contribute to fatal mishaps. This is both an important area of research, where advancements can help save lives, but it also satisfies some desirable properties as a domain. When a new modal logic is developed, it is usually first applied to simple, closed domains, like games, involving simple agents with relatively few choices compared to the rich variability one can find in real life. Similarly, the environmental factors modeled in these situations are usually quite closed. To extend these logics into a realistic domain involving complex human agents and environments is quite a leap. The cockpit is a nice step between the two, because the relevant environment is not simple, but not as rich as a more general environment. The environment that must be modeled consists of discrete instruments with a somewhat limited range of possible values. The agents are real humans, but concerned with a limited number of actions involving inputs that manipulate the airplane's flight. Another advantage of the human agents in aviation is that they are highly trained, and so meet a level of rationality in the problem space that is not quite the full blown rationality of game theoretic agents, but is certainly better than an average human navigating a random problem encountered in the real world. Thus, human pilots in the cockpit are a Goldilocks zone of human agency that is more realistic than the agents in most game examples, but not quite as complex as humans in the more general problem space of reality writ large.

By developing a logic that can model the information flow in these situations, we advance the project of formally reasoning about human agency. Credit for initiating this project belongs to many researchers over the years, especially philosophers in the analytic tradition concerned with analyzing the epistemology and metaphysics of agency in a rigorous fashion. I have been most influenced by the works of the Amsterdam school of modal logic, led by Professor Johan van Benthem, where the efforts center around rich combinations of modal logics in order to model human agency. In [20], they develop a modal logic for reasoning about the knowledge and decision-making of agents in games, and in [30], van Benthem explores similar themes around information flow and interaction.

The history of research in this area dates back to the 1950s and '60s in the development of temporal logic (or tense logic) by Arthur Prior [21], a graph theoretic semantics by Saul Kripke [22], and logics for belief and knowledge due to work by Rescher [50], von Wright [52], and Hintikka [34]. These logics formalize reasoning about what was true, what will be true at some point, what is always going to be true, what is believed to be true, and what is known to be true. Each of these modifiers is a truth modality, and hence they each constitute a modal logic. The Amsterdam school and others built on these methods, especially the work by Patrick Blackburn [10], which illustrates the general features of any modal logic as a tool for reasoning about systems that can be modeled as graphs from an *internal* perspective, whereas first order logic reasons about such systems from an external perspective. A first order formula might say what is true of the object x, from a global perspective, but modal logic allows us to formalize truth from x's perspective. If the first order formula is $\forall x, \exists y: \ R(x,y) \wedge P(y) \Rightarrow P(x)$, the corresponding modal formula would simply be $\Diamond_R p \Rightarrow p$. They both say the same thing: For any node x in the graph, if it can reach a node y by relation R, and y is a node where the proposition "y is P" holds, then "x is P" holds. The former explicitly quantifies over the nodes, and the latter does so implicitly through the semantics, to be described later.

While these developments occurred in what might be called the philosophical branch of modal logic research, researchers in economics explored the mathematical foundations of game theory. Aumann, in [48], showed the axioms of epistemic logic that must be assumed in order for classical game theoretic results to hold. The agents in classical games, otherwise known as *homo economicus*, are ideally rational, with perfect knowledge of their situations. We will meet these axioms and modify them for our purposes later.

The final foundational school of modal logic comes from theoretical computer science, where computer programs are modeled as state transition diagrams. As a program executes, the computer transitions from state to state, where each state is a collection of values assigned to variables, and each transition is a simple action executed. As this formalization lends itself to graph theoretic representation, it lends itself to formalization in modal logic, per Blackburn's insight. There are two main approaches to applying modal logic to the analysis of programs. The first is a static approach, where the entirety of the program's execution tree is modeled at once. Transitions from each state are captured by temporal logic. A program might be formalized in temporal logic, and the following theorem might be proven of it: *At the source of the execution graph, it will always be true that bad event B does not occur.*

The second approach is dynamic, where each simple transition action A or B gets a modal operator, which allows us to reason about what happens after every execution of subprocess A, or after some executions of subprocess B, etc. A state of the system is modeled, and it is updated based on the effects of the action modalities, making its representation in memory more efficient. The two approaches are extensionally equivalent, but they differ in flavor and ease of expression.

One thing that is easy to do with dynamic modal logic but somewhat complicated in the static approach is to model actions and knowledge. An epistemic logic is modeled by a static Kripke structure, and in the dynamic case this structure changes as the agents act and learn different things. The static approach requires a grand two dimensional Kripke structure with one dimension capturing the epistemic relations at a moment, and one dimension capturing the temporal relation as actions move forward through time. For an example of this approach, see John Horty's [35]. For the dynamic approach, see van Ditmarsch *et al.* [37].

Van Benthem and the Amsterdam school identified these various threads dispersed around campuses and saw how they related to each other, and how they might be fruitfully combined for various ends. As philosophers, they were mostly concerned with using the rich tools from economics and computer science to analyze human agency robustly and accurately. Modal logics offer tremendous expressive power at often a lower cost than first- or second order logic, because modal logics take an internal view of the graphs they reason about and are defined by. This often means that a powerful, useful modal logic can be defined that is also sound, complete, and *decidable*. So, by carefully defining the modal operators for, say, knowledge, preference, and action, a modal logic of game theory can be developed; not just the epistemic aspect of the agents, but the games themselves, which van Benthem calls a Theory of Play.

Applications of the Theory of Play have thus far been limited to relatively simple, artificial examples, in the same way methods in genetics research are often developed on fruit flies. In this thesis, I continue this work by extending application of the methods to richer real-world cases of humans in cockpits, which for reasons mentioned earlier make good cases for early forays into the formal modeling of human behavior. Just as genetics methods mature and eventually apply to humans, so must modal logic methods mature and apply to real humans in the world. It turns out, as this thesis demonstrates, that using modal logic to analyze systems with real human components yields new information assurance insights. The information assurance property that falls out of the formal analysis of humans in cockpits is interesting, but should not be surprising.

Information assurance properties have thus far dealt with systems whose components are entirely machine. Thus, properties like *availability* assume that by guaranteeing the broadcast or even unicast of information suffices to guarantee that the information is useful to the receiver. If the component receiving the information is another machine, this assumption typically holds. However, we can see how this assumption might be violated in cases where the resources of the receiving device are overwhelmed, as is the case in a denial of service (DOS) attack. Merely making critical information available to the receiving device does not guarantee that it can receive it and make use of it. We would typically say that in this case, the availability property failed in the receiving device, as it had insufficient resources for processing the critical information. What would be called a denial of service attack in a machine to machine system is called *information overload* when the receiver is a human. If we are modeling the sender and receiver as part of a larger system, and the receiver happens to be a human, it does not make sense to try to increase the availability of the brain's information processing resources by adding to them. Instead, it makes sense to throttle down the competing but less critical information in order to ensure that the critical information reaches the receiver (human brain). I call this property *delivery*.

Dynamic Agent Safety Logic allows us to reason about which critical information is not being delivered to the human's brain. Because we can deduce which safety-critical information is missing from her knowledge base, we can automatically act to correct this failure of delivery, and therefore build systems that have a high assurance that the delivery property is satisfied. I apply this technique to aviation safety as a formal method, but in principle it could be applied to other domains of human agency that meet certain conditions. Some examples that strike me as plausible include doctors and nurses in emergency rooms, cybersecurity analysts monitoring network traffic alerts, or power plant operators. During crises, these environments can quickly become saturated with alarms, and humans quickly suffer from information overload. If actions can be properly related to instruments such that unsafe actions can be detected, then the agent's missing knowledge can be deduced and rectified.

In what follows, I will describe the relevant background material in Section 2, including the foundations of game theory and the logical models of agency informing my developments. In Section 3, I present the logic DASL, and prove that it is sound and complete. In Section 4, I illustrate its application to three aviation mishaps, formalized in the Coq Proof Assistant. In Section 5, I formally specify the property of *delivery*, and present a run-time monitor applied to the previously formalized case studies.

Chapter 2

Background

This chapter describes the context in which my research makes advances. Section 2.1 lays out the basics of game theory. Section 2.6 introduces Dynamic Epistemic Logic, upon which the logic presented in this thesis is based. Section 2.7 describes the state of formal methods.

2.1 Game Theory

Game theory is a mathematical model for strategic reasoning. Strategic reasoning refers to the way an agent reasons in situations where her payoffs depend on the actions of other agents in addition to her own, and in which she knows about these dependencies. For turn-based games, the mathematical structure employed is a *game tree*, where each node represents a player's turn, and each edge the transition via a player's action. The leaves of the tree represent the payoffs each player receives at the end of the game. This paper is not concerned with the games themselves, but rather with the underlying assumptions about agency that entail their solutions. We briefly illustrate these underlying assumptions with the following example.

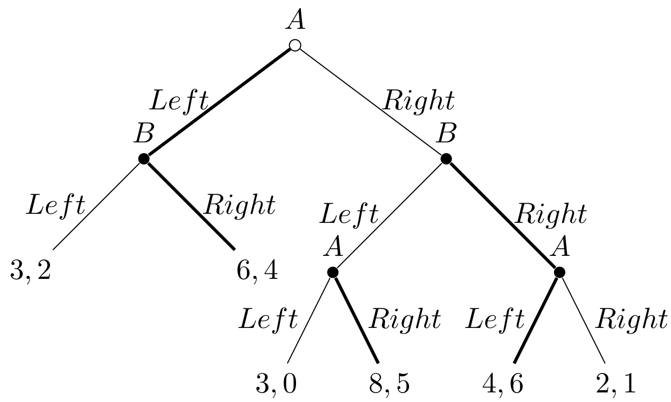


Figure 2.1: A game between players A and B

We can see in the figure below that the first player to act is Player A, at the root node. Her choices are to move *Left* or *Right.* Player B faces similar choices at the resulting nodes, and the players alternate turns until the game ends and they receive their payoffs, listed *(A,B)*. Briefly glancing through the outcomes, it looks like A should aim for the node with payoff *(8,5)*, because 8 is the highest payoff available to A. However, the official solution to the game is for A to first go *Left*, and then for B to go *Right*, resulting in a payoff of *(6,4)*, where both get less than the intuitively appealing outcome! Why is this so?

Game theory makes strong assumptions about agent knowledge and rationality. The solution to this game is reached through an algorithm called *backward induction* [20]. The players reason by starting at each end node and looking to the immediate parent node, and asking what the deciding player will do at that node, assuming she will choose the path with the highest payoff. So, at the bottom right of the figure, player A is to act, and she can go *Left* for a payoff of 4, or *Right* for a payoff of 2. So, she will go *Left*, illustrated by the bold line. The end nodes not selected are subsequently eliminated. This process is repeated at each end node. Then it is recursively applied up the tree. So along the right branch, player B decides between *Left* for a payoff of 5, or *Right* for a payoff of 6, because B knows that A is rational, and he knows how she will act at each end node. A, at the root, then must choose between *Left* for a payoff of 6, or *Right* for a payoff of 4, because she knows that B is rational, and knows that B knows that she is rational. The explanation begins to illustrate the assumptions game theory makes about each player's knowledge. In fact, this only scratches the surface.

Game theory, and classical economics in general, makes the following assumptions about agent knowledge, formalized in epistemic logic [48].

Agency Model in Classical Game Theory.

(1) $\mathbf{K_i}\,(\varphi \Rightarrow \psi) \Rightarrow (\mathbf{K_i}\,\varphi \Rightarrow \mathbf{K_i}\,\psi)$

(2) $\mathbf{K_i}\,\varphi \Rightarrow \varphi$

(3) $\mathbf{K_i}\,\varphi \Rightarrow \mathbf{K_i}\,\mathbf{K_i}\,\varphi$

(4) $\neg\mathbf{K_i}\,\varphi \Rightarrow \mathbf{K_i}\,\neg\mathbf{K_i}\,\varphi$

(5) $\mathbf{C}_G((1) \wedge (2) \wedge (3) \wedge (4) \wedge (5))$.

This forms an idealized model of the knowledge component of classical game theory's agents. $\mathbf{K_i}$ is a modal operator for knowledge, and $\mathbf{K_i}\,\varphi$ reads, "agent i knows that φ." $\mathbf{C}_G$ is a modal operator for common knowledge, the fixpoint for "everyone in group G knows that everyone knows that..." The agents are logically omniscient due to (1), knowledge implies truth with (2), agents have *positive introspection* with (3), and *negative introspection* with (4). Assumptions (1), (3), (4), and (5) are somewhat dubious. The model also fails to formally represent other aspects of agency, like action and evaluation of outcomes. The model we propose makes weaker, more realistic assumptions about knowledge, includes a modal operator for belief, and formally represents action and the evaluation of actions as either safe or unsafe.

Recent work at the intersection of game theory and logic focuses on the information flow that occurs during games. Van Ditmarsch identifies a class of games called *knowledge games*, in which players have diverging information [31]. This slightly relaxes the assumption of classical game theory that players have common knowledge about each other's perfect information. Similarly, it invites logicians to study the information conveyed by the fact that an action is executed. For example, if the action is that agent 1 asks agent 2 the question, "p?", the information conveyed is that 1 does not know whether p, believes that 2 knows whether p, and after the action occurs, this information becomes publicly known. The logic modeling games of this kind is of particular interest to us, as we are concerned with identifying the knowledge and belief state of human pilots based on their actions.

The proceeding sections introduce the various logical systems that form a foundation for the work of this thesis, starting with modal logic in its traditional philosophical interpretation, and expanding to epistemic and doxastic logic. Then, we introduce dynamic logic, and its expansion into Dynamic Epistemic Logic and Public Announcement Logic.

2.2 Modal Logic

Philosophers going back to Aristotle have noted a distinction between contingent truth, possible truth, and necessary truth. Necessary truths could not have been otherwise. The definitions of natural numbers and the addition function guarantee that in all possible worlds, *2 + 2 = 4*. On a plane, the truths of Euclidean geometry are necessarily true. "There is life on Saturn's moon Enceladus" is a possible truth. "Enceladus has water on it" is a contingent truth. What about "Water is H20"? Modal logic allows us to reason about necessary and possible truths, through the use of modal operators. What follows is a brief illustration of the concepts of modal logic.

The modal logic for reasoning about metaphysical necessity is called *S5*. *S5* is the modal logic that grounds our intuitive reasoning in the following examples and proofs. Suppose p is the arithmetic expression "2 + 2 = 4". Obviously, p is true in the actual world. We can make the stronger claim that p is necessarily true, but what does this mean? As informal shorthands, initial attempts to define necessary truths might appeal, as I did above, to the claim that they could not possibly have been false. But then what do we mean by "possible"? The formal semantics for dealing with these questions comes from Saul Kripke, and we introduce that machinery in the next section. For now, we say a statement is necessarily true if and only if it is true in all possible worlds. The modal operator for necessity makes the formal statement: $\Box p$. Consider the following inference, where $\Rightarrow$ means "implies": $\Box p \Rightarrow p$. This reads, "Necessarily p implies p," or equivalently, "If p is necessarily true, then p is true." If p is necessarily true, is p true? Intuitively, the answer is 'yes', and indeed a modal logic of metaphysical necessity includes this axiom for all formulas φ: $\Box\varphi \Rightarrow \varphi$.

What other inferences can we make, based on our intuitive notion of necessity and possibility? What about "if p is true, then p is possibly true"? To formalize this, we need the modal operator for possibility: $\Diamond$. So, the modal formula would be $p \Rightarrow \Diamond p$. This seems true as well, and indeed it is an axiom: $\varphi \Rightarrow \Diamond\varphi$.

It is obvious then that $\Box\varphi \Rightarrow \Diamond\varphi$ is a theorem. This states that if something is necessarily true then it is possibly true. What about the other direction: $\Diamond\varphi \Rightarrow \Box\varphi$? It turns out this is not a theorem under the typical notions of necessity and possibility, nor under the axioms of *S5*. But this raises a question about how we would present a counterexample that disproves it. To do this, we need a semantics for the logic. The semantics we use are called *possible world semantics*, and they are due to Saul Kripke [22]. One would be hard-pressed to find a species of modal logic, whether in economics, computer science, or philosophy, that does not use possible world semantics in some form or another. Sometimes they are referred to as *Kripke semantics*.

In possible world semantics, a graph structure is created with worlds as nodes and accessibility relations among worlds as the edges in the graph. Propositional formulas are true or false at each world. These graph structures are typically called Kripke structures. We can define the following Kripke structure, $\mathcal{M} = \{W, R, V\}$, where W is a finite set of worlds, $\{w, v\}$, R is a binary accessibility relation defined on those worlds $\{(w, v), (v, w)\}$, meaning w has access to v and v has access to w, and V is a *valuation* function, which maps propositions to sets of worlds at which they are true. For example, if p is true at w, then $w \in V(p)$. In our model we only care about the proposition p, which now stands for some contingent proposition, like "all swans are white". Formally, we say $w \notin V(p)$ if not all swans are white in world w, denoted by $\neg p$, while $w \in V(p)$ if they are, denoted by p. The following figure illustrates $\mathcal{M}$:

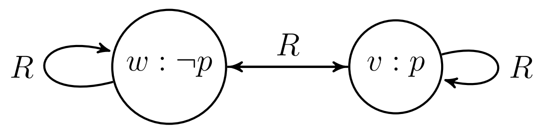


Figure 2.2: $\mathcal{M}$: A simple counterexample using possible world semantics.

According to possible world semantics, $\Box\varphi$ is true at a world w, written $w \models \Box\varphi$, if and only if for all worlds v such that $R(w, v)$ (v is related to w by the R relation), $v \models \varphi$. This says a formula is necessarily true at a world if and only if it is true at all worlds accessible by that world according to the underlying R relation. Similarly, $w \models \Diamond\varphi$ if and only if there is some world v such that $R(w, v)$ and $v \models \varphi$. In $\mathcal{M}$, w is R-accessible to itself, so there is a world accessible to w where p is false, and thus $w \models \neg\Box p$. However, since v is R-accessible to w, and $v \models p$, it is true that $w \models \Diamond p$. Thus, we have $w \models \Diamond p$ and $w \models \neg\Box p$, a negation of $\Diamond p \Rightarrow \Box p$, so it cannot be the case that $\Diamond\varphi \Rightarrow \Box\varphi$ is a theorem, for arbitrary formula φ.

S5, while a very important modal logic, is not the only one. The way it is distinguished from other modal logics, and indeed that way any are distinguishable from each other, is based entirely on the definition of R. For *S5*, R is a symmetric, transitive, reflexive binary relation on worlds. Thus, that is how "possibility" is formally defined, and by extension, "necessarily". We formally define these notions in the next section, along with the syntax and semantics of basic modal logic, sometimes call propositional modal logic.

2.3 Modal Logic Syntax and Semantics

This section formally defines the syntax, what the logic looks like, and the semantics, what the truth conditions are for the logic.

Recall that Boolean logic is a simple logic for reasoning about basic propositions using the logical connectives ‘and’, ‘or’, ‘not’, and ‘if...then’. It forms the foundation of most logics and has applications ranging from philosophy to circuit design. Propositions are represented as constants p, q and well-formed formulas of the language are constants and any proper combination of constants using the above logical connectives, represented symbolically as,

$$\varphi \overset{def}{=} p \mid \varphi \wedge \varphi \mid \varphi \vee \varphi \mid \neg\varphi \mid \varphi \rightarrow \varphi.$$

As illustrated in the previous section, modal logic adds to propositional logic with modal operators for necessary and possible truths. The syntax for propositional logic is extended in the following way to make modal logic:

$$\varphi \overset{def}{=} p \mid \varphi \wedge \varphi \mid \varphi \vee \varphi \mid \neg\varphi \mid \varphi \rightarrow \varphi \mid \Box\varphi \mid \Diamond\varphi.$$

The semantics for Boolean logic are simply truth tables for each connectives, which I will not reproduce here. Because the connectives are truth-functional, truth tables suffice. However, the operators in modal logic are not truth functional, and require more complex semantics, which we earlier mentioned are called possible world semantics. They are as follows.

For a Kripke model $\mathcal{M} = \{W, R, V\}$ such that W is a set of possible worlds, R is a binary relation on those worlds, and V is a valuation function mapping atomic propositions to the sets worlds satisfying them,

$$w \models p \textbf{ iff } w \in V(p)$$

$$w \models \neg\varphi \textbf{ iff } w \not\models \varphi$$

$$w \models \varphi \wedge \psi \textbf{ iff } w \models \varphi \textit{ and } w \models \psi$$

$$w \models \Box\varphi \textbf{ iff } \forall v,\ R(w,v) \textit{ implies } v \models \varphi.$$

$$w \models \Diamond\varphi \textbf{ iff } \exists v,\ R(w,v) \textit{ and } v \models \varphi.$$

The character of a modal logic is determined by the binary relation on worlds underlying the modal operators. The logic we used for our intuitive notion of necessity in the previous secion, *S5*, has a R relation in which every world is accessible to itself by the binary relation. It is also one in which the R relation is transitive and symmetric. These conditions are called a *frame* conditions, and the models that satisfy these conditions belong to said frame. All reflexive frames have the following frame conditions:

$$\forall x,\ R(x,x) \tag{2.1}$$

Likewise, all reflexive frames have the following axiom:

$$\Box\varphi \Rightarrow \varphi \tag{2.2}$$

It is not just a stipulation that all reflexive frames must have that axiom: They have that axiom *because* they have that frame condition. This is due to correspondence theory, which we explain later on.

Relaxing a frame condition changes the R relation, and in doing so changes the logic. If we remove the reflexivity condition, the above axioms are no longer axioms, and the above theorem no longer holds. This means that we can specify the axioms we want by specifying the frame condition on the accessibility relation. Each frame condition corresponds to a modal logic axiom. In addition to reflexivity, the other common frame conditions are as follows, with their corresponding modal logic axiom:

- Transitivity

$$\forall x, y, z \; R(x, y) \wedge R(y, z) \Rightarrow R(x, z) \tag{2.3}$$

$$\Box\varphi \Rightarrow \Box\Box\varphi \tag{2.4}$$

- Symmetry

$$\forall x, y \; R(x, y) \Rightarrow R(y, x) \tag{2.5}$$

$$\varphi \Rightarrow \Box\Diamond\varphi \tag{2.6}$$

- Euclidean

$$\forall x, y, z \; R(x, y) \wedge R(y, z) \Rightarrow R(x, z) \tag{2.7}$$

$$\Diamond\varphi \Rightarrow \Box\Diamond\varphi \tag{2.8}$$

- Seriality

$$\forall x \exists y, R(x, y) \tag{2.9}$$

$$\Box\varphi \Rightarrow \Diamond\varphi \tag{2.10}$$

We saw earlier that reflexive frames are also serial frames, because we proved that we could derive the axiom for serial frames as a theorem from just the axiom for reflexive frames.

These conditions are not exhaustive, but they represent some of the commonly combined conditions used to define axioms of different modal logics. By combining frame conditions, a modal operator is defined with the properties desired. For example, if we wish to define a modal operator for reasoning about the knowledge of ideally rational agents, as is done in Fagin *et. al* [32] and Hintikka [34], we impose the frame conditions of transitivity, reflexivity, and Euclidean. However, if we wish to develop a logic for belief, as the previously cited works also do, we must impose transitivity, Euclidean, and seriality. The reasons for this are explored in the following sections.

2.4 Epistemic Logic

This section presents epistemic logic, the logic for reasoning about the knowledge of ideally rational agents. They are ideally rational in the sense that there is no bound on how much they can be said to know, nor on what propositions they are aware of at any one time. Thus, they have no problem conceiving of every possible sequence of moves a game may consist of, evaluating all possible outcomes, and deducing the optimal sequence of moves in order to maximize their own utilities. They are even stronger than contemporary computers in this way, which are bounded by time and space, and so are unable to actually compute the ideal strategy for an otherwise solvable game like Go. An ideally rational agent can solve Go and compute the game tree all the way to its end.

The axioms that specify this level of knowledge are those introduced in Section 2.1, and recounted here without reference to common knowledge, which complicates things too much for our purposes:

Agency Model in Classical Game Theory.

(1) $\mathbf{K_i}\,(\varphi \Rightarrow \psi) \Rightarrow (\mathbf{K_i}\,\varphi \Rightarrow \mathbf{K_i}\,\psi)$

(2) $\mathbf{K_i}\,\varphi \Rightarrow \varphi$

(3) $\mathbf{K_i}\,\varphi \Rightarrow \mathbf{K_i}\,\mathbf{K_i}\,\varphi$

(4) $\neg\mathbf{K_i}\,\varphi \Rightarrow \mathbf{K_i}\,\neg\mathbf{K_i}\,\varphi$.

These are the axioms of *S5* modal logic, and from a certain perspective they make sense. The first axiom holds for all *normal* modal logics, and under the epistemic interpretation, it states that if an agent knows that φ implies ψ, and she knows φ, then she knows ψ. This is intuitive enough on the first pass. The second axiom states that a known proposition must be true, and finds ample support from philosophers devoted to studying the nature of knowledge. The third axiom, *positive introspection*, states that an agent knows something only if she knows *that* she knows it. Finally, *negative introspection* states that an agent does not know something only if she knows that she does not know it. Axioms (3) and (4) are clearly very strong notions of knowledge, and axiom (1) implies that agents are logically omniscient. Because these axioms are firmly rooted in the literature of formal epistemology, relaxing them in order to model more realistic agents requires a word or two. I spend this section introducing and defending my relaxation of the classical axiomatization.

2.5 Doxastic Logic

This section presents doxastic logic, the logic for reasoning about belief.

2.6 Logical Framework

Recall that Boolean logic is a simple logic for reasoning about basic propositions using the logical connectives ‘and’, ‘or’, ‘not’, and ‘if...then’. It forms the foundation of most logics and has applications ranging from philosophy to circuit design. Propositions are represented as constants p, q and well-formed formulas of the language are constants and any proper combination of constants using the above logical connectives, represented symbolically as,

$$\varphi \stackrel{def}{=} p \mid \varphi \wedge \varphi \mid \varphi \vee \varphi \mid \neg\varphi \mid \varphi \rightarrow \varphi.$$

Boolean logic is limited in what it can express, however. Some situations require higher fidelity, for example when reasoning about knowledge. Consider the proposition, "If Alice knows p, then p is true." Philosophers of knowledge agree with this proposition, considering it a valid claim about knowledge, based on the principle that all known propositions are true. However, this validity cannot be faithfully represented in propositional logic as a general validity. Each sentence in the claim is assigned its own constant, resulting in a formalization like, "$a \rightarrow p$," with a representing "Alice knows p," and p representing "p is true." But $a \rightarrow p$ is not a validity of Boolean logic.

To represent reasoning about knowledge, and other similar domains requiring higher fidelity, epistemic operators are added to the basic Boolean logic, resulting in the following logic:

$$\varphi \stackrel{def}{=} p \mid \mathbf{K_i}\varphi \mid \neg\varphi \mid \varphi \wedge \varphi,$$

where $\mathbf{K_i}\varphi$ states that agent i *knows* that φ, allowing us to formally represent the above validity:

$$\mathbf{K_{Alice}}p \rightarrow p.$$

Semantics for epistemic logic are given by Kripke structures, which serve as models by which epistemic formulas are evaluated. At its core, a Kripke structure is a graph with nodes and edges, accompanied by a function determining which atomic propositions are true at which worlds. The nodes are normally thought of as possible worlds, or as possible states of the system being modeled. The edges are normally thought of as possibility relations among worlds or states. If, at a node representing a world w, agent A considers it possible that she is in world v, the Kripke semantics modeling this situation would have world w's node connected to world v's node by an edge representing A's epistemic possibility relation.

Formally, we say a Kripke structure is a *tuple* $\langle W,\ V,\ Agents,\ \{R_i | i \in Agents\}\rangle$, where W is a set of worlds, V is a function from propositional constants to sets of worlds satisfying the proposition, *Agents* is a set of agents, and each R_i is agent i's epistemic possibility relation.

The semantics are as follows, for worlds $w,\ v\ \in W$:

$$w \models p \textbf{ iff } w \in V(p)$$

$$w \models \neg\varphi \textbf{ iff } w \not\models \varphi$$

$$w \models \varphi \wedge \psi \textbf{ iff } w \models \varphi \textit{ and } w \models \psi$$

$$w \models \mathbf{K_i}\varphi \textbf{ iff } \forall v,\ wR_iv \textit{ implies } v \models \varphi.$$

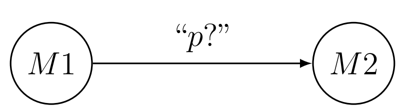
Early applications of epistemic logic in information security modeled system components as agents whose knowledge represented the information flowing to them. Recently, a modified version of epistemic logic known as dynamic epistemic logic has been used in information security to formally reason about security properties involving human components of systems. The research described in this proposal advances along similar lines, treating pilots as human components of safety critical aviation systems and using dynamic epistemic logic to reason about them. The next section describes the basic dynamic epistemic logic.

2.6.1 Dynamic Epistemic Logic

Dynamic Epistemic Logic (DEL) formalizes situations in which agents' epistemic states change over time, due to announcements or other informational events. [37] For example, if Alice truthfully and trustworthily communicates to Bob that φ, then after this informational even it is true that Bob knows φ. This situation cannot be modeled by the epistemic logic introduced in the previous section. To model it, we introduce the following formal machinery.

To capture informational events, we introduce the idea of relativizing a Kripke structure. In the previous example, if we model the Alice and Bob situation prior to Alice's communication, we can have a world w from which Bob considers φ- as well as $\neg\varphi$-worlds possible. However, after the informational event, Bob knows φ, so the model is *relativized* to a submodel in which only φ-worlds are accessible by Bob's epistemic possibility relation. Thus, after the informational event, the model transitions to a submodel with fewer edge relations.

The logic for reasoning about information flow in knowledge games is called Dynamic Epistemic Logic (DEL). As its name suggests, it combines elements of epistemic logic and dynamic logic. Epistemic logic is the static logic for reasoning about knowledge, and dynamic logic is used to reason about actions. In dynamic logic semantics, nodes are states of the system or the world, and relations on nodes are transitions via programs or actions from node to node. If we think of each node in dynamic logic as being a model of epistemic logic, then actions become relations on models, representing transitions from one multi-agent epistemic model to another. For example, if we have a static epistemic model $M1$ representing the knowledge states of agents 1 and 2 at a moment, then the action "p?" is a relation between $M1$ and $M2$, a new static epistemic model of 1's and 2's knowledge after the question is asked. All of this is captured by DEL.



The above figure illustrates the relationship between static epistemic models and dynamic logic models. As a purely dynamic model, the figure shows the action "p?" transitioning between nodes $M1$ and $M2$. If we were to zoom in on the nodes, we would see their structure as epistemic models, with their own nodes and edges, representing possible worlds and epistemic relations.

We are concerned with an additional element: the *safety* status of an action, and an agent's knowledge and belief about that. To capture this, we extend DEL and call the new logic Dynamic Agent Safety Logic (DASL), which we introduce in the next chapter. The next section lays out the state of formal methods involving human-machine systems.

2.7 Formal Methods

Formal methods increase confidence that hardware and software components function correctly [17]. They involve the application of mathematical techniques to the design and analysis of systems, usually the safety-critical components [15]. A standard approach is to develop an abstract specification of the software or hardware system, design the system to meet those specifications, and then use formal logic to prove that the designed system meets the desired safety specifications.

According to the Federal Aviation Administration (FAA), one of the top 12 causes of mistakes in the aviation workplace is a lack of awareness [1]. According to Boeing and the FAA, approximately 80 per cent of aviation accidents (including maintenance accidents) are due to human error [2, 3]. To increase safety in human factors, the industry relies on education, psychology, anthropometrics, safety engineering, and to a limited extent, computer science. The primary focus of computer science research regarding human factors is the design and testing of software systems that are easy and intuitive for humans to interact with.

Some researchers have sought to develop formal methods for mitigating human-induced sources of failure [14, 23]. Thus far this work has focused on the development of formal methods tools for analyzing human machine interaction during the design and specification phase, rather than at runtime. The goals have been to develop software, and techniques for verifying the correctness of that software, to avoid mode confusion, a type of pilot error wherein the pilot believes the autopilot is in one mode, when in reality it is in another. In these situations, the autopilot is not offering protections concerning flight control inputs like thrust and pitch, so the pilot risks providing dangerous inputs.

Butler, Miller, Potts, and Carreno [14] trace mode confusion to three sources:

1. poor display of automation state
2. unnecessarily complex automation
3. flight crew has an incorrect mental model of the state of the aircraft

They say human factors research focuses on mitigating item (1) above, and they develop formal models in the PVS automated theorem prover to address items (2) and (3). However, the formal models they develop are of the automation system, not of the human components.

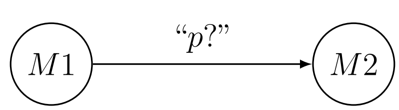
Similarly, Rushby [23] describes a class of errors he calls automation surprises, which are distinct but related to mode confusion. These occur when a pilot becomes surprised by the automated behavior of the system. He proposes a formal method addressing automation surprises that constructs a model of the system behavior, constructs a formal specification of a possible pilot's mental model of the system, and compares them for disagreement. His solution focuses on identifying potential automation surprises in the system, and both models are of the system behavior itself: one directly of the model, and one indirectly of the model, mediated through a hypothetical pilot's mind.

In each of the above cases, the formal efforts focus on modeling the system itself, rather than on the human component, and they address the problem at design time. This work complements theirs by constructing a formal model of the pilot herself, and addressing the problem at runtime. Similarly, this work aims to address a variety of problems due to a pilot's loss of situational awareness, including mode confusion. The next chapter introduces Dynamic Agent Safety Logic.

Chapter 3

Dynamic Agent Safety Logic

The logic for reasoning about information flow in knowledge games is called Dynamic Epistemic Logic (DEL). As its name suggests, it combines elements of epistemic logic and dynamic logic. Epistemic logic is the static logic for reasoning about knowledge, and dynamic logic is used to reason about actions. In dynamic logic semantics, nodes are states of the system (or of the world), and relations on nodes are transitions via programs or actions from node to node. If we think of each node in dynamic logic as being a model of epistemic logic, then actions become relations on models, representing transitions from one multi-agent epistemic model to another. For example, if we have a static epistemic model $M1$ representing the knowledge states of agents 1 and 2 at a moment, then the action "p?" is a relation between $M1$ and $M2$, a new static epistemic model of 1's and 2's knowledge after the question is asked. All of this is captured by DEL.



The above figure illustrates the relationship between static epistemic models and dynamic logic models. As a purely dynamic model, the figure shows the action "p?"

transitioning between nodes $M1$ and $M2$. If we were to zoom in on the nodes, we would see their structure as epistemic models, with their own nodes and edges, representing possible worlds and epistemic relations.

We are concerned with an additional element: the *safety* status of an action, and an agent's knowledge and belief about that. To capture this, we extend DEL and call the new logic Dynamic Agent Safety Logic (DASL). The remainder of this section presents DASL's syntax, semantics, and proves its soundness.

3.1 Syntax and Semantics

3.1.1 Syntax

The Dynamic Agent Safety Logic (DASL) used in this paper has the following syntax.

$$\varphi \ ::= \ p \mid \neg\varphi \mid \varphi \wedge \varphi \mid \mathbf{K_i}\,\varphi \mid \mathbf{B_i}\,\varphi \mid [\mathbf{i}, (\mathbf{A}, a)]\varphi \mid [\mathbf{i}, (\mathbf{A}, a), \mathbf{S}]\varphi,$$

where $p \in AtProp$ is an atomic proposition, $\mathbf{i}$ refers to $i \in Agents$, a is the name of an action, called an action token, belong to a set of such tokens, *Actions*, and $\mathbf{A}$ refers to an action structure. The knowledge operator $\mathbf{K_i}$ indicates that "agent i knows that ..." Similarly, the operator for belief, $\mathbf{B_i}$ can be read, "agent i believes that..." The notion of action tokens and structures will be defined in the semantics. The operators $[\mathbf{i}, (\mathbf{A}, a)]$ and $[\mathbf{i}, (\mathbf{A}, a), \mathbf{S}]$ are the dynamic operators for agent i executing action token a from action structure A in the former case, and doing so safely in the latter case. Note that the $\mathbf{S}$ in $[\mathbf{i}, (\mathbf{A}, a), \mathbf{S}]$ stands for 'safety', and is not a variable, whereas the $\mathbf{i}, (\mathbf{A}, a)$ are variables for agents, action structures, and action tokens, respectively. One can read the action operators as "after i executes a from A, φ holds.' We define the dual modal operators $\langle \mathbf{K}_i \rangle$, $\langle \mathbf{B}_i \rangle$, $\langle \mathbf{i}, (\mathbf{A}, a) \rangle$, and $\langle \mathbf{i}, (\mathbf{A}, a), \mathbf{S} \rangle$ in the usual way.

The semantics of DASL involve two structures that are defined simultaneously, one for epistemic models, and one for action structures capturing the transition relation among epistemic models. Additionally, we define numerous helper functions that straddle the division between metalanguage and object language.

3.1.2 Metalanguage

Kripke Model

A Kripke model $M \in Model$ is a tuple $\langle W, \{R_k^i\}, \{R_b^i\}, w, V\rangle$. It is a set of worlds, sets of epistemic and doxastic relations on worlds for agents, a world denoting the actual world, and a valuation function V mapping atomic propositions to the set of worlds satisfying them. Most readers will be somewhat familiar with epistemic logic, the logic for reasoning about knowledge. Doxastic logic is a similar logic for reasoning about belief[34].

Action Structure

An action structure $A \in ActionStruct$ is a tuple $\langle Actions,$ $\{\chi_k^i\}, \{\chi_b^i\}, a\rangle$. It is a set of action tokens, sets of epistemic and doxastic relations on action tokens for agents, and an action token, a, denoting an actual action token executed.

An action structure captures the associated subjective events of an action occurring, including how it is observed by various agents, incorporating their uncertainty. The action tokens are the actual objective events that might occur. For example, if I am handed a piece of paper telling me who won the Oscar for Best Actress, and I read it, and you see me read it, then the action structure will include possible tokens in which I read that each nominee has won, and you will consider each of these tokens to be possible. When I read the paper, I consider only one action token to be the one executed. This action structure represents that transition from one epistemic model, in which both of us considers all nominees the potential winner, to an epistemic model in which I know the winner and you still do not know the winner. We can think of the action structure A as the general action "Agent 1 reads the piece of paper" and the tokens as the specific actions "Agent 1 reads that nominee n has won the award."

Model Relation

Just as R_k^i denotes a relation on worlds, $[\![i,(A,a)]\!]$ denotes a relation on Kripke model-world pairs. It represents the relation that holds between M, w and M', w' when agent i executes action (A, a) at M, w and causes the world to transition to M', w'.

Precondition Function

The Precondition function, $\mathit{pre} :: \mathit{Actions} \mapsto \varphi$, maps an action to the formula capturing the conditions under which the action can occur. For example, if we assume agents tell the truth, then an announcement action has as a precondition that the announced proposition is true, as with regular Public Announcement Logic.

Postcondition Function

The Postcondition function, $\mathit{post} :: A \times \mathit{AtProp} \mapsto \mathit{AtProp}$, takes an action structure and an atomic proposition, and maps to the corresponding atomic proposition after the action occurs.

$$\mathit{post}(A,p) = p \text{ if } \mathit{update}(M,A,w,a,i) \models p, \text{ else } \neg p.$$

Update Function

The Update function, $update :: (Model \times ActionStruct \times W \times Actions \times Agents) \mapsto (Model \times W)$, takes a Kripke model M, an action structure A, a world from the Kripke model, an action token from the Action structure, and an agent executing the action, and returns a new Kripke model-world pair. It represents the effect actions have on models, and is more complicated than other DEL semantics in that actions can change the facts on the ground in addition to the knowledge and belief relations. It is a partial function that is defined iff a model-world pair satisfies the action's preconditions.

$update(M, A, w, a, i) = (M', w')$ $where:$

1. $M = \langle W, \{R_k^i\}, \{R_b^i\}, w, V \rangle$
2. $A = \langle Actions, \{\chi_k^i\}, \{\chi_b^i\}, a, pre, post \rangle$
3. $M' = \langle W', \{R_k'^i\}, \{R_b'^i\}, w', V' \rangle$
4. $W' = \{(w, a) | w \in W, a \in Actions, \text{ and } w \models pre(a)\}$
5. $R_k'^i = \{((w, a), (v, b)) | wR_k^i v \text{ and } a\chi_k^i b\}$
6. $R_b'^i = \{((w, a), (v, b)) | wR_b^i v \text{ and } a\chi_b^i b\}$
7. $w' = (w, a)$
8. $V'(p) = post(A, p)$

Safety Precondition Function

The Safety Precondition Function, $pre_s :: Actions \mapsto \varphi$, is a more restrictive function than pre. Where pre returns the conditions that dictate whether the action is possible, pre_s returns the conditions that dictate whether the action is safely permissible. This function is the key reason the dynamic approach allows for easy inference from action to safety-critical information.

3.1.3 Semantics

The logic DASL has the following Kripke semantics.

$$
\begin{aligned}
M, w \models p &\text{ iff } \; w \in V(p) \\
M, w \models \neg\varphi &\text{ iff } \; M, w \not\models \varphi \\
M, w \models \varphi \wedge \psi &\text{ iff } \; M, w \models \varphi \text{ and } \; M, w \models \psi \\
M, w \models \mathbf{K_i}\,\varphi &\text{ iff } \; \forall v, \; wR^i_k v \; \text{ implies } \; M, v \models \varphi \\
M, w \models \mathbf{B_i}\,\varphi &\text{ iff } \; \forall v, \; wR^i_b v \; \text{ implies } \; M, v \models \varphi \\
M, w \models [\mathbf{i}, (\mathbf{A}, a)]\varphi &\text{ iff } \; \forall M', w', \; (M, w)[\![i, (A, a)]\!](M', w') \\
&\text{ implies } \; M', w' \models \varphi \\
M, w \models [\mathbf{i}, (\mathbf{A}, a), \mathbf{S}]\varphi &\text{ iff } \; \forall M', w', \; (M, w)[\![i, (A, a), S]\!](M', w') \\
&\text{ implies } \; M', w' \models \varphi
\end{aligned}
$$

The definitions of the dynamic modalities make use of a relation between two model-world pairs, which we now define.

$$
\begin{aligned}
(M, w)[\![i, (A, a)]\!](M', w') &\text{ iff } \; M, w \models pre(a) \\
&\text{ and } \; update(M, A, w, a, i) = (M', w') \\
(M, w)[\![i, (A, a), S]\!](M', w') &\text{ iff } \; M, w \models pre_s(a) \\
&\text{ and } \; update(M, A, w, a, i) = (M', w')
\end{aligned}
$$

3.1.4 Hilbert System

DASL is axiomatized by the following Hilbert system.

All propositional tautologies are axioms.

$\mathbf{K_i}$ is T (knowledge relation is reflexive)

$\mathbf{B_i}$ is KD45 (belief relation is serial, transitive, and Euclidean)

EP1: $\mathbf{K_i}\,\varphi \Rightarrow \mathbf{B_i}\,\varphi$

EP2: $\mathbf{B_i}\,\varphi \Rightarrow \mathbf{B_i}\,\mathbf{K_i}\,\varphi$

EP3: $\mathbf{B_i}\,\varphi \Rightarrow \mathbf{K_i}\,\mathbf{B_i}\,\varphi$

SP: $[\mathbf{i},(\mathbf{A},a)]\varphi \Rightarrow [\mathbf{i},(\mathbf{A},a),\mathbf{S}]\varphi$

PR: $\langle \mathbf{i},(\mathbf{A},a)\rangle\,\varphi \Rightarrow \mathbf{B_i}\,\langle \mathbf{i},(\mathbf{A},a),\mathbf{S}\rangle\,\varphi,$

plus the inference rules Modus Ponens and Necessitation for $\mathbf{K_i}$ and $\mathbf{B_i}$.

Above are the axioms characterizing the logic. Knowledge is weaker here than in most epistemic logics, and belief is standard [32]. They are related logically by EP(1-3), which hold that knowledge entails belief, belief entails that one believes that one knows, and belief entails than one knows that one believes. Finally, actions and safe actions are logically related by SP and PR, which hold that necessary consequences of *mere* action are also necessary consequences of *safe* actions, and that a pilot can execute an action only if he believes that he is executing a safe action.

Below are the axioms characterizing the reduction laws from the dynamic logic to a purely static logic through recursive application.

Aprop: $[\mathbf{i}, (\mathbf{A}, a)]p \Leftrightarrow (pre(a) \Rightarrow (post(A, p) \Rightarrow p))$

AN: $[\mathbf{i}, (\mathbf{A}, a)]\neg\varphi \Leftrightarrow (pre(a) \Rightarrow \neg[\mathbf{i}(\mathbf{A}, a)]\varphi)$

AC: $[\mathbf{i}, (\mathbf{A}, a)](\varphi \wedge \psi) \Leftrightarrow ([\mathbf{i}, (\mathbf{A}, a)]\varphi \wedge [\mathbf{i}, (\mathbf{A}, a)]\psi)$

AK: $[\mathbf{i}, (\mathbf{A}, a)]\mathbf{K_i}\,\varphi \Leftrightarrow (pre(a) \Rightarrow \bigwedge_{a\chi_k^i b} \mathbf{K_i}\,[\mathbf{i}, (\mathbf{A}, b)]\varphi)$

AB: $[\mathbf{i}, (\mathbf{A}, a)]\mathbf{B_i}\,\varphi \Leftrightarrow (pre(a) \Rightarrow \bigwedge_{a\chi_b^i b} \mathbf{B_i}\,[\mathbf{i}, (\mathbf{A}, b)]\varphi)$

Sprop: $[\mathbf{i}, (\mathbf{A}, a), \mathbf{S}]p \Leftrightarrow (pre_s(a) \Rightarrow (post(A, p) \Rightarrow p))$

SN: $[\mathbf{i}, (\mathbf{A}, a), \mathbf{s}]\neg\varphi \Leftrightarrow (pre_s(a) \Rightarrow \neg[\mathbf{i}(\mathbf{A}, a), \mathbf{S}]\varphi)$

SC: $[\mathbf{i}, (\mathbf{A}, a), \mathbf{S}](\varphi \wedge \psi) \Leftrightarrow ([\mathbf{i}, (\mathbf{A}, a), \mathbf{S}]\varphi \wedge [\mathbf{i}, (\mathbf{A}, a), \mathbf{S}]\psi)$

SK: $[\mathbf{i}, (\mathbf{A}, a), \mathbf{S}]\mathbf{K_i}\,\varphi \Leftrightarrow (pre_s(a) \Rightarrow \bigwedge_{aR_k'^i b} \mathbf{K_i}\,[\mathbf{i}, (\mathbf{A}, b), \mathbf{S}]\varphi)$

SB: $[\mathbf{i}, (\mathbf{A}, a), \mathbf{S}]\mathbf{B_i}\,\varphi \Leftrightarrow (pre_s(a) \Rightarrow \bigwedge_{aR_b'^i b} \mathbf{B_i}\,[\mathbf{i}, (\mathbf{A}, b), \mathbf{S}]\varphi)$

3.2 Soundness

Theorem 3.2.1 (Soundness). *Dynamic Agent Safety Logic is sound for Kripke structures with*

(1) reflexive R_k^i relations,

(2) serial, transitive, Euclidean R_b^i relations,

(3) which are partially ordered $(R_k^i \circ R_b^i) \subseteq R_b^i$, $(R_b^i \circ R_k^i) \subseteq R_b^i$, and $R_b^i \subseteq R_k^i$,

(4) $[\![i, (A, a), S]\!] \subseteq [\![i, (A, a)]\!]$ and

(5) $([\![i, (A, a), S]\!] \circ R_b^i) \subseteq [\![i, (A, a)]\!]$.

Proof. (1) *and* (2) correspond to the axioms that $\mathbf{K_i}$ is a T modality and $\mathbf{B_i}$ is a KD45 modality in the usual way. (3) corresponds to EP1, EP2, and EP3. Axioms AP through SB are reduction axioms. This leaves (4), corresponding to SP, and (5) which corresponds to PR. Here we will prove (5). Let M be a Kripke structure satisfying the five conditions above. Let A be an Action structure with a and i as its actual action token and agent.

We prove (5) via the contrapositive of PR: $\langle \mathbf{B}_i \rangle [\mathbf{i}, (\mathbf{A}, a), \mathbf{S}]\varphi \Rightarrow [\mathbf{i}, (\mathbf{A}, a)]\varphi$. Assume $M, w \models \langle \mathbf{B}_i \rangle [\mathbf{i}.(\mathbf{A}, a), \mathbf{S}]\varphi$. By the semantics of $\langle \mathbf{B}_i \rangle$, there exists a v, such that $wR^i_b v$ and $v \models [\mathbf{i}, (\mathbf{A}, a), \mathbf{S}]\varphi$. From the semantics, it follows that forall M', v', if $(M, v)[\![i, (A, a), S]\!](M', v')$ then $M', v' \models \varphi$. By slightly abusing the notation, and letting $(W, w)R^i_b(W, v)$ be equivalent to $wR^i_b v$, we can create the composed relation $([\![i, (A, a), S]\!] \circ R^i_b)$. It then holds, by condition (5), that $(M, w)([\![i, (A, a), S]\!] \circ R^i_b)(M', v')$ implies $(M, w)[\![i, (A, a)]\!](M', v')$. So, for all M', v', if $(M, w)[\![i, (A, a)]\!](M', v')$, then $M', v' \models \varphi$. So, $M, w \models [\mathbf{i}, (\mathbf{A}, a)]\varphi$. $\square$

Aprop : $\Rightarrow$. Assume $M, w \models [\mathbf{i}, (\mathbf{A}, a)]p$. We must show that $M, w \models pre(a) \Rightarrow (post(A, p) \Rightarrow$ By the semantics of $[\mathbf{i}, (\mathbf{A}, a)]$, for all (M', w'), if $M, w \models pre(a)$ and $update(M, A, w, a, i) = (M', w')$, then $M', w' \models p$. By definition of $post(A, p)$, if $update(M, A, w, a, i) = (M', w')$ and $M', w' \models p$, then $post(A, p) = p$. So, if $M, w \models pre(a)$, then $post(A, p) = p$, and thus $post(A, p) \Rightarrow p$.

$\Leftarrow$. Assume $M, w \models pre(a) \Rightarrow (post(A, p) \Rightarrow p)$. By the definition of $post(A, p)$, if $post(A, p) = p$ then $update(M, A, w, a, i) \models p$. So, if $M, w \models pre(a)$, then $update(M, A, w, a, i) \models p$. Therefore, $M, w \models [\mathbf{i}, (\mathbf{A}, a)]p$.

AN : $\Rightarrow$. Assume $M, w \models [\mathbf{i}, (\mathbf{A}, a)]\neg\varphi$. It suffices to show that $M, w \models pre(a) \Rightarrow \langle \mathbf{i}, (\mathbf{A}, a) \rangle \neg\varphi$. From the assumption and the semantics, for all (W', w'), if $(M, w)[\![i, (A, a)]\!](M', w')$ then $M', w' \models \neg\varphi$. So, if $M, w \models pre(a)$ and $update($ $M, A, w, a, i) = (M', w')$, then $M', w' \models \neg\varphi$. Assume $M, w \models pre(a)$, and it follows that $update(M, A, w, a, i)$ is defined, so there exists a M', w' such that $(M, w)[\![i, (A, a)]\!](M', w')$ and $update(M, A, i) = (M', w')$ and $M', w' \models \neg\varphi$. Therefore, $M, w \models pre(a) \Rightarrow \langle \mathbf{i}, (\mathbf{A}, a) \rangle \neg\varphi$.

$\Leftarrow$. Assume $M, w \models pre(a) \Rightarrow \neg[\mathbf{i}, (\mathbf{A}, a)]\varphi$. This is equivalent to $M, w \models pre(a) \Rightarrow \langle \mathbf{i}, (\mathbf{A}, a) \rangle \neg\varphi$. By the semantics, if $M, w \models pre(a)$, then there exists a (M', w') such that $(M, w)[\![i, (A, a)]\!](M', w')$ and $M', w' \models \neg\varphi$. The relation $[\![i, (A, a)]\!]$ is functional, so $\exists$ implies $\forall$. So, for all (M', w'), if $(M, w)[\![i, (A, a)]\!](M',$

w'), then $M', w' \models \neg\varphi$, and therefore $M, w \models [\mathbf{i}, (\mathbf{A}, a)]\neg\varphi$.

AC is obvious.

AK. For this proof, assume for simplicity, without loss of generality, that $Actions = \{a\}$.

$\Rightarrow$. Assume $M, w \models [\mathbf{i}, (\mathbf{A}, a)]\mathbf{K_i}\,\varphi$. Unfolding the semantics, for all (M', w'), if $(M, w) \models pre(a)$ and $update(M, A, w, a,$
$i) = (M', v')$, then $M', w' \models \mathbf{K_i}\,\varphi$. $M', w' \models \mathbf{K_i}\,\varphi$ iff for all $v \in W$, if $wR^i_k v$ and $M, v \models pre(a)$ and $update(M, A, v, a, i) = (M', v')$ and $a\chi^i_k a$, then $M', v' \models \varphi$. That is, $M, w \models \mathbf{K_i}\,[\mathbf{i}, (\mathbf{A}, a)]\varphi$.

$\Leftarrow$. Assume $M, w \models pre(a) \Rightarrow \mathbf{K_i}\,[\mathbf{i}, (\mathbf{A}, a)]\varphi$. We must show $M, w \models [\mathbf{i}, (\mathbf{A}, a)]$ $\mathbf{K_i}\,\varphi$. Thus, we must show $M, w \models pre(a)$ and $update(M, A, w, a, i) = (M', w')$ implies $M', w' \models \mathbf{K_i}\,\varphi$. So it suffices to show that if $update(M, A, w, a, i) = (M', w')$ and $M, w \models \mathbf{K_i}\,[\mathbf{i}, (\mathbf{A}, a)]\varphi$, then $M', w' \models \mathbf{K_i}\,\varphi$. Assume $update(M, A, w,$
$a, i) = (M', w')$ and $M, w \models \mathbf{K_i}\,[\mathbf{i}, (\mathbf{A}, a)]\varphi$. Then for all v, if $wR^i_k v$, then $M, v \models$ $[\mathbf{i}, (\mathbf{A}, a)]\varphi$. It follows that $M, v \models pre(a)$ and $update(M, A, v, a, i) = (M', v')$ implies $M', v' \models \varphi$. Since $wR^i_k v$ and $a\chi^i_k a$, it holds that $w' R^{i'}_k v'$. Thus, $M', w' \models \mathbf{K_i}\,\varphi$.

Proofs for **AB** through **SB** follow the above proofs exactly analogously. $\Box$

Assume $M, w \models \langle \mathbf{i}, a\rangle\, true$. By the semantics of $\langle \mathbf{i}, a\rangle$, $M, w \models pre(a)$ and $update(M, w, \chi) \models true$. Let $(M', w') = update(M, w, \chi)$. Then $M', w' \models true$. From (5) above, it holds that $R^i_b(w) \subseteq V^s(a)$. R_b is serial, so there is at least one such $v \in R^i_b$. Then $M, v \models pre_s(a)$. From (4), $M, v \models pre(a)$, so $update(M, v, \chi)$ is defined, call it (M'', v'). Because (M'', v') is defined, $M'', v' \models true$. So, $M, v \models$ $pre_s(a)$ and $update(M, v, \chi) \models true$. This holds for all v, such that $wR_b v$. Thus, $M, w \models \mathbf{B_i}\,\langle \mathbf{i}, a, \mathbf{S}\rangle\, true$. Therefore, $M, w \models \langle \mathbf{i}, a\rangle\, true \Rightarrow \mathbf{B_i}\,\langle \mathbf{i}, a, \mathbf{S}\rangle\, true$. $\Box$

Next we turn to completeness.

3.2.1 Completeness

Completeness proofs in contemporary modal logic research follow the following format. First, a *canonical model* is defined such that it belongs to the Kripke frames for which the logic under investigation is sound. The worlds of the canonical model are maximal consistent sets of formulas from the language. The valuation function is defined by membership to the worlds as maximal consistent sets. We define this formally below. The objective is to show that all formulas valid for the relevant Kripke frames are likewise deducible in the logic. Thus, the model combines the deducibility relation of the proof theory and the satisfaction relation of the model theory in order to show that valid formulas are deducible, which proves completeness of the logic.

The previous section proved soundness of DASL for frames with a reflexive R^i_k relation, a serial, transitive, Euclidean R^i_b relation, which are partially ordered ($(R^i_k \circ R^i_b) \subseteq R^i_b$, $(R^i_b \circ R^i_k) \subseteq R^i_b$, and $R^i_b \subseteq R^i_k$, and with action relations $[\![i,(A,a),S]\!] \subseteq [\![i,(A,a)]\!]$, and $([\![i,(A,a),S]\!] \circ R^i_b) \subseteq [\![i,(A,a)]\!]$. The canonical model we proceed to define will belong to this frame. Because it belongs to this frame, the logical closure of a set of formulas will consist of valid DASL inferences. This allows us to infer deducibility from validity. First we must define the notion of a maximal consistent set.

Informally, a maximal consistent set of formulas is one that is a subset of the well-formed formulas of the language $\mathcal{L}_{DASL}$ according to the syntax presented earlier, whose members are consistent with each other, such that adding even a single formula to the set would render the set consistency. Another way to phrase this last part is to say that our maximal consistent set is not a proper subset of any other consistent subset of $\mathcal{L}_{DASL}$.

Definition. Maximal Consistent Set.

For a set of formulas $\Gamma \subseteq \mathcal{L}_{DASL}$, Γ is maximal consistent iff

1. Γ is consistent: $\Gamma \nvdash \bot$.

2. Γ is maximal: there is no $\Gamma' \subseteq \mathcal{L}_{DASL}$ such that $\Gamma \subset \Gamma'$ and $\Gamma' \vdash \bot$.

A canonical model is a Kripke model of $\mathcal{L}_{DASL}$ with a set of worlds W^C that collectively satisfy all formulas from a maximal consistent set. For our formalism here, we treat worlds as identical to sets of formulas.

Definition. Canonical Model. A canonical model $M^C = \langle W^C,\ R^C_{k,i},\ R^C_{b,i},\ w,\ V^C\rangle$ is defined:

1. $W^C = \{\Gamma | \Gamma \text{ is maximal consistent relative to } \mathcal{L}_{DASL}\}$
2. $\Gamma R^C_{k,i} \Delta$ iff $\mathbf{K_i}\,\varphi \in \Gamma$ implies $\varphi \in \Delta$
3. $\Gamma R^C_{b,i} \Delta$ iff $\mathbf{B_i}\,\varphi \in \Gamma$ implies $\varphi \in \Delta$
4. $V^C(p) = \{\Gamma \in W^C \mid p \in \Gamma\}$

Proof. Completeness states that if a formula φ is valid, then it is deducible. The sketch for this proceeds proceeds by contraposition. So we must show that for every formula φ in the language $\mathcal{L}_{DASL}$, $\nvdash \varphi$ implies $\not\models \varphi$.

Theorem 3.2.2 (Completeness)**.** *The language of Dynamic Agent Safety Logic, $\mathcal{L}_{DASL}$, is complete for Kripke structures with*

(1) reflexive R^i_k relations,

(2) serial, transitive, Euclidean R^i_b relations,

(3) which are partially ordered $(R^i_k \circ R^i_b) \subseteq R^i_b$, $(R^i_b \circ R^i_k) \subseteq R^i_b$, and $R^i_b \subseteq R^i_k$,

(4) $[\![i, (A, a), S]\!] \subseteq [\![i, (A, a)]\!]$ and

(5) $([\![i, (A, a), S]\!] \circ R^i_b) \subseteq [\![i, (A, a)]\!]$.

The proof appeals to the following lemmas, proven in [37]:

Lemma 3.2.3 (Lindenbaum)**.** *Every consistent set of formulas is a subset of a maximal consistent set of formulas.*

If we begin with a consistent set of DASL formulas β, we construct a maximal consistent set Γ as follows:

1. enumerate the formulas of DASL $\varphi_0, \ldots \varphi_n, \ldots$
2. $\Gamma_0 \equiv \beta$
3. $\Gamma_{n+1} \equiv \{\varphi_n\}$ if $\Gamma_n \vdash \varphi_n$
4. $\Gamma_{n+1} \equiv \{\neg\varphi_n\}$ otherwise

Lemma 3.2.4 (Properties)**.** *If Γ and Δ are maximal consistent sets and β a consistent set, then:*

1. *Γ and Δ are deductively closed.*
2. *$\varphi \in \Gamma$ iff $\neg\varphi \notin \Gamma$.*
3. *If $\beta \nvdash \varphi$, then there exists a maximal consistent set Γ such that $\beta \subset \Gamma$ but $\varphi \notin \Gamma$*

Lemma 3.2.5 (Truth)**.** *For every $\varphi \in \mathcal{L}_{DASL}$, and every maximal consistent set Γ:*

$$\varphi \in \Gamma \textit{ iff } (M^C, \Gamma) \models \varphi$$

Lemma 3.2.6 (Canonicity)**.** *The canonical model satisfies the above frame conditions.*

With these lemmas, we assume $\nvdash \varphi$, and show that $\not\models \varphi$. Since $\nvdash \varphi$, the set $\{\neg\varphi\}$ is consistent. From Lindenbaum, $\{\neg\varphi\}$ is part of a maximal consistent set, call it Γ. From the Truth Lemma, $(M^C, \Gamma) \models \neg\varphi$. Therefore, $\not\models \varphi$.$\Box$

Because the static logic is complete and we have translation axioms that convert the dynamic formulas to equivalent static ones, we can conclude that the entirety of DASL is complete. In the next chapter we examine case studies and a mechanization of the logic.

Chapter 4

Case Studies

4.1 Case Study and Mechanization

In this section we apply the logic just developed to the formal analysis of the Air France 447 aviation incident, then mechanize the formalization in the Coq Proof Assistant. Our mechanization follows similar work by Maliković and Čubrilo [43, 44], in which they mechanize an analysis of the game of Cluedo using Dynamic Epistemic Logic, based on van Ditmarsch's formalization of the game [31]. It is commonly assumed that games must be adversarial, but this is not the case. Games need only involve situations in which players' payoffs depend on the actions of other players. Similarly, knowledge games need not be adversarial, and must only involve diverging information. Thus, it is appropriate to model aviation incidents as knowledge games of sorts, where players' payoffs depend on what others do, specifically the way the players communicate information with each other. The goal is to achieve an accurate situational awareness and provide flight control inputs appropriate for the situation. Failures to achieve this goal result in disaster, and often result from imperfect information flow. A formal model of information flow in these situations provides insight and allows for the application of formal methods to improve information flow during emergency situations.

4.1.1 Air France 447

This case study is based on the authoritative investigative report into Air France 447 performed and released by France's Bureau d'Enquêtes et d'Analyses pour la Sécurité de l'Aviation Civile (BEA), responsible for investigating civil aviation incidents and issuing factual findings[19]. The case is mechanized by instantiating, in Coq, the above logic to reflect the facts of the case. One challenge associated with this is that the readings about inputs present in aviation are often real values on a continuum, whereas for our purposes we require discrete values. We accomplish this by dividing the continuum associated with inputs and readings into discrete chunks, similar to how fuzzy logic maps defines predicates with real values[40].

Air France flight 447 from Rio de Janeiro, Brazil to Paris, France, June 1, 2009. The Airbus A330 encountered adverse weather over the Atlantic ocean, resulting in a clogged Pitot-static system. Consequently, the airspeed indicators delivered unreliable data concerning airspeed to the pilot flying, resulting in confusion. A chain of events transpired in which the pilot overcorrected the plane's horizontal attitude again and again, and continued to input nose up pitch commands, all while losing airspeed. Perhaps most confusing to the pilot was the following situation: the aircraft's angle of attack (AOA) was so high it was considered invalid by the computer, so no stall warning sounded until the nose pitched down into the valid AOA range, at which point the stall warning would sound. When the pilot pulled up, the AOA would be considered invalid again, and the stall warning would cease. The aircraft entered a spin and crashed into the ocean. Palmer [46] argues that had the pilot merely taken no action, the Pitot tubes would have cleared in a matter of seconds, and the autopilot could have returned to Normal Mode.

This paper will formalize an excerpted instance from the beginning of the case, involving an initial inconsistency among airspeed indicators, and the subsequent dangerous input provided by the pilot. Formalized in the logic, the facts of the case allow us to infer that the pilot lacked negative introspection about the safety-critical data required for his action. This demonstrates that the logic allows information about the pilot's situational awareness to flow to the computer, via the pilot's actions. It likewise establishes a safety property to be enforced by the computer, namely that a pilot should maintain negative introspection about safety-critical data, and if he fails to do so, it should be re-established as quickly as possible.

According to the official report, at 2 hours and 10 minutes into the flight, a Pitot probe likely became clogged by ice, resulting in an inconsistency between airspeed indicators, and the autopilot disconnecting. This resulted in a change of mode from Normal Law to Alternate Law 2, in which certain stall and control protections ceased to exist. The pilot then made inappropriate control inputs, namely aggressive nose up commands, the only explanation for which is that he mistakenly believed that the aircraft was in Normal Law mode with protections in place to prevent a stall. This situation, and the inference regarding the pilot's mistaken belief, is modeled in the following application and mechanization of the logic.

We first introduce a lemma relating belief to knowledge, and then formalize the critical moment.

Lemma 4.1.1 (Belief is epistemically consistent)**.** $\mathbf{B_i}\,\varphi \Rightarrow \neg\mathbf{K_i}\,\neg\varphi$.

Proof. From the fact that the belief modality is serial, it holds that

$$\mathbf{B_i}\,\varphi \Rightarrow \langle \mathbf{B}_i \rangle\,\varphi,$$

which is equivalent to

$$\mathbf{B_i}\,\varphi \Rightarrow \neg\mathbf{B_i}\,\neg\varphi.$$

Due to axiom EP1, it follows that

$$\mathbf{B_i}\,\varphi \Rightarrow \neg\mathbf{K_i}\,\neg\varphi. \qquad \square$$

We now formalize the critical moment.

1. $\neg(Mode = Normal)\ldots$———————————— configuration.
2. $\langle pilot, hardnoseup\rangle\ true$————————————- pilot input.
3. $\mathbf{B}_{pilot}\,(Mode = Normal)$——————————— from axiom PR, pre_s.
4. $\neg\mathbf{K}_{pilot}\,(Mode = Normal)$—————————— from axiom K-Reflexive.
5. $\mathbf{B}_{pilot}\,\mathbf{K}_{pilot}\,(Mode = Normal)$——————- from (3), axiom EP2.
6. $\neg\mathbf{K}_{pilot}\,\neg\mathbf{K}_{pilot}\,(Mode = Normal)$————- from (5), Lemma 4.1.1.
7. $\neg\mathbf{K}_{pilot}\,(Mode = Normal) \wedge \neg\mathbf{K}_{pilot}\,\neg\mathbf{K}_{pilot}\,(Mode = Normal)$—— from (4), (6).
8. $\neg(\neg\mathbf{K}_{pilot}\,(Mode = Normal) \Rightarrow \mathbf{K}_{pilot}\,\neg\mathbf{K}_{pilot}\,(Mode = Normal))$ —— from (7).

The crux of the case is that inconsistent information was being presented to the pilot, along with a cacophony of inconsistent alarms, and the pilot's control inputs indicated a lack of awareness of safety-critical information. A detailed analysis, using the Coq Proof Assistant and the logic developed by my research, will make explicit these failures of information flow, both from the computer to the pilots, between the pilot and co-pilot, and from the pilot to the computer. This will motivate the description of a prototype safety monitor that identifies and corrects information flow failures like those found in Air France 447.

4.1.2 Mechanization in Coq

Mechanical theorem proving divides into two categories, automated and interactive. Automated theorem proving combines a search algorithm with a proof checking algorithm to fully automate the process. The problem itself is undecidable in general, and human control is limited to the injection of hints prior to the algorithm's execution. Interactive theorem proving, however, combines human-directed search with a proof checking algorithm, allowing the human to have more control over the procedure. Coq is a tool that facilitates interactive theorem proving.

The underlying logic of Coq is called the Calculus of Inductive Constructions, a dependently-typed constructive logic. One uses Coq by formalizing the target logic and its semantics in Coq and using what are called *tactics* to manipulate proof objects. My project will implement the previously described Safe Dynamic Agency Logic in Coq and formally model the Air France 447 case, demonstrating the logic's ability to dynamically model safety-critical information flow in a real-world scenario. This will require translating the logic as it appears here and its metatheory into Coq, complete with tactics appropriate for the desired proofs and fully instantiated semantics, a process called *mechanization*.

The following mechanization demonstrates progress from the artificially simply toy examples normally analyzed in the literature to richer real-world examples. However, it does not represent the full richness of the approach. The actions and instrument readings mechanized in this paper are constrained to those most relevant to the case study. The approach is capable of capturing the full richness of all instrument reading configurations and actions available to a pilot. To do so, one needs to consult a flight safety manual and formally represent each action available to a pilot, and each potential instrument reading, according to the following scheme.

Before beginning, we note that our use of sets in the following Coq code requires the following argument passed to coqtop before executing: -impredicative-set. In CoqIDE, this can be done by selecting the 'Tools' dropdown, then 'Coqtop arguments'. Type in *-impredicative-set*.

We first formalize the set of agents.

```
Inductive Agents: Set := Pilot | CoPilot | AutoPilot.
```

Next we formalize the set of available inputs. These themselves are not actions, but represent atomic propositions true or false of a configuration.

```
Inductive Inputs : Set :=
HardThrustPlus | ThrustPlus
| HardNoseUp    | NoseUp
| HardWingLeft  | WingLeft
| HardThrustMinus | ThrustMinus
| HardNoseDown  | NoseDown
| HardWingRight | WingRight.
```

We represent readings by indicating which *side* of the panel they are on. Typically, an instrument has a left-side version, a right-side version, and sometimes a middle version serving as backup. When one of these instruments conflicts with its siblings, the autopilot will disconnect and give control to the pilot.

```
Inductive Side : Set := Left | Middle | Right.
```

We divide the main instruments into chunks of values they can take, in order to provide them with a discrete representation in the logic. For example, the reading *VertUp1* may represent a nose up reading between 0° and 10°, while *VertUp2* represents a reading between 11° and 20°.

```
Inductive Readings (s : Side) : Set :=
VertUp1 | VertUp2 | VertUp3 | VertUp4
| VertDown1 | VertDown2 | VertDown3 | VertDown4
| VertLevel | HorLeft1 | HorLeft2 | HorLeft3
| HorRight1 | HorRight2 | HorRight3 | HorLevel
| AirspeedFast1 | AirspeedFast2 | AirspeedFast3
| AirspeedSlow1 | AirspeedSlow2 | AirspeedSlow3
| AirspeedCruise| AltCruise | AltClimb | AltDesc | AltLand.
```

We define a set of potential modes the aircraft can be in.

```
Inductive Mode : Set := Normal | Alternate1 | Alternate2.
```

We define a set of global instrument readings representing the mode and all of the instrument readings, left, right, and middle, combined together. This represents the configuration of the instumentation.

```
Inductive GlobalReadings : Set := Global (m: Mode)
(rl : Readings Left)
(rm : Readings Middle)
(rr : Readings Right).
```

The set of atomic propositions we are concerned with are those representing facts about the instrumentation.

```
Inductive Atoms : Set :=
| M (m : Mode)
| Input (a : Inputs)
| InstrumentL (r : Readings Left)
| InstrumentM (r : Readings Middle)
| InstrumentR (r : Readings Right)
| InstrumentsG (g : GlobalReadings).
```

Next we follow Maliković and Čubrilo [43, 44] in defining a set *prop* of propositions in predicate calculus, distinct from Coq's built in type *Prop*. The definition provides constructors for atomic propositions consisting of particular instrument reading predicate statements, implications, propositions beginning with a knowledge modality, and those beginning with a belief modality. Interestingly, modal logic cannot be directly represented in Coq's framework [41]. We first define propositions in first-order logic, which we then use to define DASL. This appears to be the standard technique for mechanizing modal logics in Coq.

```
Inductive prop : Set :=
| atm : Atoms → prop
| imp: prop → prop → prop
| Forall : forall (A : Set), (A → prop) → prop
| K : Agents → prop → prop
| B : Agents → prop → prop
```

We use the following notation for implication and universal quantification.

```
Infix "==>" := imp (right associativity, at level 85).
Notation "\-/ p" := (Forall _ p) (at level 70, right associativity).
```

We likewise follow Maliković and Čubrilo [43, 44] by defining an inductive type *theorem* representing a theorem of DASL. The constructors correspond to the Hilbert system, either as characteristic axioms, or inference rules. The first three represent axioms for propositional logic, then the rule Modus Ponens, then the axioms for the epistemic operator plus its Necessitation rule, then the doxastic operator and its Necessitation rule. Do not confuse the Necessitation rules with material implication in the object language. The final constructors capture the axioms relating belief and knowledge. The axioms for dynamic modal operators are defined separately, and are not included here.

```
Inductive theorem : prop → Prop :=
| Hilbert_K: forall p q : prop, theorem (p ⟹ q ⟹ p)
| Hilbert_S: forall p q r : prop,
theorem ((p⟹ q⟹ r)⟹ (p⟹ q)⟹ (p⟹ r))
| Classic_NOTNOT : forall p : prop, theorem ((NOT (NOT p)) ⟹ p)
| MP : forall p q : prop, theorem (p ⟹ q) → theorem p → theorem q
| K_Nec : forall (a : Agents) (p : prop), theorem p → theorem (K a p)
| K_K : forall (a : Agents) (p q : prop),
theorem (K a p ⟹ K a (p ⟹ q) ⟹ K a q)
| K_T : forall (a : Agents) (p : prop), theorem (K a p ⟹ p)
| B_Nec : forall (a : Agents) (p : prop), theorem p → theorem (B a p)
| B_K : forall (a : Agents) (p q : prop),
theorem (B a p ⟹ B a (p ⟹ q) ⟹ B a q)
| B_Serial : forall (a : Agents) (p : prop),
theorem (B a p ⟹ NOT (B a (NOT p)))
| B_4 : forall (a : Agents) (p : prop), theorem (B a p ⟹ B a (B a p))
| B_5 : forall (a : Agents) (p : prop),
theorem (NOT (B a p) ⟹ B a (NOT (B a p)))
| K_B : forall (a : Agents) (p : prop), theorem (K a p ⟹ B a p)
| B_BK : forall (a : Agents) (p : prop), theorem (B a p ⟹ B a (K a p)).
```

We use the following notation for *theorem*:

```
Notation "|-- p" := (theorem p) (at level 80).
```

We encode actions as records in Coq, recording the acting pilot, the observability of the action (whether it is observed by other agents or not), the input provided by the pilot, and the preconditions for the action and the safety preconditions for the action, both represented as global atoms.

```
Record Action : Set := act {Ai : Agents; Aj : Agents; pi : PI;
input : Inputs; c : GlobalReadings;
c_s : GlobalReadings}.
```

The variable c holds the configuration representing the precondition for the action, while the variable c_s holds the configuration for the safety precondition. We encode the precondition and safety precondition functions as follows.

```
Function pre (a:Action) : prop := atm (InstrumentsG (c a)).
Function pre_s (a : Action) : prop := atm (InstrumentsG (c_s a)).
```

In the object language, the dynamic modalities of action and safe action are encoded as follows.

```
Parameter aft_ex_act : Action → prop → prop.
Parameter aft_ex_act_s : Action → prop → prop.
```

Many standard properties of logic, like the simplification of conjunctions, hypothetical syllogism, and contraposition, are encoded as Coq axioms. As an example, here is how we encode simplifying a conjunction into just its left conjunct.

```
Axiom simplifyL : forall p1 p2,
|−− p1 & p2 → |−− p1.
```

We formalize the configuration of the instruments at 2 hour 10 minutes into the flight as follows.

```
Definition Config_1 := (atm (M Alternate2)) &
(atm (InstrumentL (AirspeedSlow3 Left))) &
(atm (InstrumentM (AirspeedSlow3 Middle))) &
(atm (InstrumentR (AirspeedCruise Right))).
```

The mode is Alternate Law 2, and the left and central backup instruments falsely indicate that the airspeed is very slow, while the right side was not recorded, but because there was a conflict, we assume it remained correctly indicating a cruising airspeed.

The pilot's dangerous input, a hard nose up command, is encoded as follows.

```
Definition Input1 := act Pilot Pilot Pri HardNoseUp
(Global Alternate2 (AirspeedSlow3 Left)
(AirspeedSlow3 Middle)
(AirspeedCruise Right))
(Global Normal (AirspeedCruise Left)
(AirspeedCruise Middle)
(AirspeedCruise Right)).
```

The action is represented in the object language by taking the dual of the dynamic modality, $\neg[\mathbf{i}, (\mathbf{A}, a)]\neg True$, equivalently $\langle \mathbf{i}, (\mathbf{A}, a)\rangle\, True$, indicating that the precondition is satisfied and the action token is executed.

```
Definition Act_1 := NOT (aft_ex_act Input1 (NOT TRUE)).
```

The actual configuration satisfies the precondition for the action, but it is inconsistent with the safety precondition. The safety precondition for the action indicates that the mode should be Normal and the readings should consistently indicate cruising airspeed. However, in Config_1, the conditions do not hold. Thus, the action is unsafe. From the configuration and the action, DASL allows us to deduce that the pilot lacks negative introspection of the action's safety preconditions.

Negative introspection is an agent's awareness of the current unknowns. To lack it is to be unaware of one's unknown variables, so lacking negative introspection about one's safety preconditions is to be unaware that they are unknown.

```
Theorem NegIntroFailMode :
|-- (Config_1 ==>
Act_1 ==>
((NOT (K Pilot (pre_s(Action1)))) &
(NOT (K Pilot (NOT (K Pilot (pre_s(Action1)))))))).
```

In fact, in general it holds that if the safety preconditions for an action are false, and the pilot executes that action, then the pilot lacks negative introspection of those conditions. We have proven both the above theorem, and the more general theorem, in Coq.

```
Theorem neg_intro_failure :
forall (A Ao : Agents) (pi : PI) (inp : Inputs)
(m : Mode)
(rl : Readings Left) (rm : Readings Middle) (rr : Readings Right)
(ms : Mode)
(rls : Readings Left) (rms : Readings Middle) (rrs : Readings Right)
phi,
|-- (NOT
(aft_ex_act
(act A Ao pi inp (Global m rl rm rr) (Global ms rls rms rrs))
(NOT phi)) ⟹
NOT (atm (InstrumentsG (Global ms rls rms rrs))) ⟹
(NOT (K A (atm (InstrumentsG (Global ms rls rms rrs)))) &
(NOT (K A (NOT (K A (atm (InstrumentsG (Global ms rls rms rrs)))))))).
```

This indicates that negative introspection about safety preconditions is a desirable safety property to maintain, consistent with the official report's criticism that the Airbus cockpit system did not clearly display the safety critical information. The logic described in this research accurately models the report's findings that the pilot's lack of awareness about safety-critical information played a key role in his decision to provide unsafe inputs. Furthermore, the logic supports efforts to automatically infer which safety-critical information the pilot is unaware of and effectively display it to him.

The next section formalizes additional case studies in DASL.

4.2 Additional Case Studies

To illustrate the flexibility of this approach, we now formalize additional case studies in the logic. We analyze Copa Airlines flight 201 and Asiana Airlines flight 214.

4.2.1 Copa 201 and Asiana 214

Copa flight 201 departed Panama City, Panama for Cali, Colombia in June, 1992. Due to faulty wiring in the captain's Attitude Indicator, he incorrectly believed he was in a left bank position. In response to this, he directed the plane into an 80 degree roll to the right, which caused the plane to enter a steep dive. A correctly functioning backup indicator was available to the captain, and investigators believe that the captain intended to direct the backup indicator's readings to his own, but due to an outdated training module, the flip he switched actually sent his own faulty readings to the co-pilot's indicator. Approximately 29 minutes after takeoff, the plane crashed into the jungle and all passengers and crew perished. We formalize the moment at which the pilot provides the hard right roll input.

1. $(LeftAIHorLeft2) \wedge \neg(MiddleAIHorLeft2) \ldots$——— configuration.

2. $\langle pilot, hardwingright \rangle\ true$——————————- pilot input.

3. $\mathbf{B}_{pilot}\,(MiddleAIHorLeft2)$——————— from axiom PR, pre_s.

4. $\neg\mathbf{K}_{pilot}\,(MiddleAIHorLeft2)$————— from axiom K-Reflexive.

5. $\mathbf{B}_{pilot}\,\mathbf{K}_{pilot}\,(MiddleAIHorLeft2)$———— from (3), axiom EP2.

6. $\neg\mathbf{K}_{pilot}\,\neg\mathbf{K}_{pilot}\,(MiddleAIHorLeft2)$———— from (5), Lemma 4.1.1.

7. $\neg\mathbf{K}_{pilot}\,(MiddleAIHorLeft2) \wedge \neg\mathbf{K}_{pilot}\,\neg\mathbf{K}_{pilot}\,(MiddleAIHorLeft2)$—— from (4), (6).

8. $\neg(\neg\mathbf{K}_{pilot}\,(MiddleAIHorLeft2) \Rightarrow \mathbf{K}_{pilot}\,\neg\mathbf{K}_{pilot}\,(MiddleAIHorLeft2))$ —- from (7).

Asiana flight 214 from South Korea to San Francisco departed in the evening of July 6, 2013 and was schedule to land just before noon that morning [45]. The weather was good and air traffic control cleared the pilots to perform a visual approach to the runway. The plane came in short and crashed against an embankment in front of the runway, resulting in the deaths of three passengers and 187 injured. The National Transportation Safety Board (NTSB) investigation found that the captain had mismanaged the approach and monitoring of the airspeed, resulting in the plane being too high for a landing. Upon noticing this, the captain selected a flight mode (flight level change speed) which unexpectedly caused the plane to climb higher. In response to this, the captain disconnected the autopilot and pulled back on the thrust. This caused an autothrottle (A/T) protection to turn off, so when the captain pitched the nose down, the plane descended faster than was safe, causing it to come down too quickly and collide with the embankment in front of the runway. We will formalize the moment at which the pilot pitches the nose down.

1. $(A/T = \mathit{Off}) \wedge (\mathit{AirspeedSlow3})\ldots$ ———————— configuration.
2. $\langle \mathit{pilot}, \mathit{hardthrustminus}\rangle\ \mathit{true}$ ———————— pilot input.
3. $\mathbf{B}_{pilot}\,(A/T = On)$ ———————— from axiom PR, pre_s.
4. $\neg\mathbf{K}_{pilot}\,(A/T = On)$ ———————— from axiom K-Reflexive.
5. $\mathbf{B}_{pilot}\,\mathbf{K}_{pilot}\,(A/T = On)$ ———————— from (3), axiom EP2.
6. $\neg\mathbf{K}_{pilot}\,\neg\mathbf{K}_{pilot}\,(A/T = On)$ ———————— from (5), Lemma 4.1.1.
7. $\neg\mathbf{K}_{pilot}\,(A/T = On) \wedge \neg\mathbf{K}_{pilot}\,\neg\mathbf{K}_{pilot}\,(A/T = On)$ — from (4), (6).
8. $\neg(\neg\mathbf{K}_{pilot}\,(A/T = On) \Rightarrow \mathbf{K}_{pilot}\,\neg\mathbf{K}_{pilot}\,(A/T = On))$ — from (7).

The above formalizations follow the same format as that of Air France 447. A pilot provides an input whose safety precondition conflicts with one of the instruments in the configuration, and we infer that the pilot lacks negative introspection of the safety precondition. This is distinct from but related to the property that the pilots, in engaging in an unsafe action, are unaware of the unsafe instrument readings. We can capture this in the form of safety properties, which I turn to in the next section.

4.3 Safety Properties

Definition. Safety Negative Introspection (SNI). If a safety precondition does not hold, then agent knows that he does not know it to hold.

$$\neg pre_s(a) \Rightarrow \mathbf{K_i}\,\neg\mathbf{K_i}\,pre_s(a)$$

Definition. Safety-Critical Delivery (SCD). If a safety precondition is false, then agent knows that it is false.

$$\neg pre_s(a) \Rightarrow \mathbf{K_i}\, \neg pre_s(a)$$

Our above formalizations show that SNI is false when a pilot provides an unsafe input. Notice that SCD implies SNI.

Lemma 4.3.1 (SCD implies SNI)**.** *Safety-Critical Delivery (SCD) implies Safety Negative Introspection (SNI).*

Proof. It suffices to show that $\mathbf{K_i}\, \neg\varphi \Rightarrow \mathbf{K_i}\, \neg \mathbf{K_i}\, \varphi$. Assume $\mathbf{K_i}\, \neg\varphi$ holds. From EP1, it follows that $\mathbf{B_i}\, \neg\varphi$, and because knowledge is a normal modality, it follows that $\mathbf{K_i}\, \mathbf{B_i}\, \neg\varphi$ holds. From lemma 1, and again the fact that knowledge is normal modality, it follows that $\mathbf{K_i}\, \neg \mathbf{K_i}\, \neg\neg\varphi$, or equivalently, that $\mathbf{K_i}\, \neg \mathbf{K_i}\, \varphi$. □

However, the converse does not hold. We can satisfy SNI when the safety precondition is false, the agent knows that he doesn't know it, but doesn't know that it is false. A counterexample consists of a model with three worlds: {u,v}. Let φ be the safety precondition, with the following truth assignment: {False, True}. Let the epistemic relation include (u,v), (v,u), and the reflexive relations. Then at world u φ is false, and $\mathbf{K_i}\, \neg \mathbf{K_i}\, \varphi$ is true, but $\mathbf{K_i}\, \neg\varphi$ is false.

The formalizations show that from the pilot's unsafe action, it follows that he lacks negative introspection of the safety precondition.

$$\langle \mathbf{i}, a \rangle\, true \wedge \neg pre_s(a) \Rightarrow \neg \mathbf{K_i}\, \neg \mathbf{K_i}\, pre_s(a) \tag{4.1}$$

This situation violates SNI, because the pilot doesn't know that he doesn't know the safety precondition. Since SCD implies SNI, SCD is also violated.

$$\langle \mathbf{i}, a \rangle\, true \wedge \neg pre_s(a) \Rightarrow \neg \mathbf{K_i}\, \neg pre_s(a) \tag{4.2}$$

So, from an unsafe action, we can also infer that the pilot does not know that the safety precondition is false, a stronger conclusion.

Thus, by restoring knowledge that the safety precondition is false, it follows that either the safety precondition is true, or the unsafe action is not executed.

$$\mathbf{K_i}\, \neg pre_s(a) \Rightarrow \neg\langle \mathbf{i}, a\rangle\, true \vee pre_s(a) \tag{4.3}$$

The pilot's knowledge in the antecedent implies that the safety precondition is false, so this simplifies to:

$$\mathbf{K_i}\, \neg pre_s(a) \Rightarrow \neg\langle \mathbf{i}, a\rangle\, true. \tag{4.4}$$

This squares with the standard game theoretic inference, wherein a rational agent with knowledge of the situation executes a good action. Because our model of knowledge and rationality is weaker, we make the weaker claim that a minimally rational pilot with knowledge of the safety-critical information does not execute a bad action.

4.4 Decision Problem

An important extension of the foundational work provided by this paper is the construction of a system that takes advantage of the logic as a runtime safety monitor. It will monitor the pilot's control inputs and current flight configurations, and in the event that an action's safety preconditions do not hold, infer which instrument readings the pilot is unaware of and act to correct this. In order to avoid further information overload, the corrective action taken by the computer should be to temporarily remove or dim the non-safety-critical information from competition for the pilot's attention, until the pilot's unsafe control inputs are corrected, indicating awareness of the safety-critical information. Construction of a prototype of this system is underway. Here we define the decision problem and prove that it is NP-Complete. The decision problem we face is formalized as follows.

SD. Input: $a : Action$, $C : Configuration$, $pre_s : Action \mapsto Configuration$, $k : Int$.

Output: Is there a set of instruments I of size k from configuration C that falsifies $pre_s(a)$?

Theorem 4.4.1 (NP)**.** *SD is NP-Complete.*

Proof. First, we prove that SD is in NP by defining the decision problem SDV that takes input to SD and a *certificate* and verifies that the certificate falsifies $pre_s(a)$ in polynomial time. Let the certificate be a set of instruments I of size k from the configuration C. Note that $pre_s(a)$ has the form $(c_1 \wedge c_2 \wedge \ \ldots \ \wedge c_n) \vee (c_1' \wedge c_2' \wedge \ \ldots \ \wedge c_n') \vee \ \ldots$ The negation of this has the form $(\neg c_1 \vee \neg c_2 \vee \ \ldots \ \vee \neg c_n) \wedge (\neg c_1' \vee \neg c_2' \vee \ \ldots \neg c_n') \wedge \ \ldots,$ which is in Conjunctive Normal Form. Computing the negation of pre_s can be done in linear time. Next we take I and treat it as an assignment of values to instruments of the form $(i_1 \wedge i_2 \wedge \ \ldots \ \wedge i_n)$ for each $i \in I$. Then we check whether I satisfies $\neg pre_s(a)$ in polynomial time, treating I as the certificate for the SAT problem.

Second, we prove that SD is in NP-Hard. We do this by providing a polynomial time reduction from $nSAT$ to SD. Taking the input from $nSAT$, we let the size of the configuration $|C| = n$, that is, we let n be the number of instruments on the flight deck. The maximum size of any solution I to SD is therefore n. We let the $nSAT$ formula be the negation of $pre_s(a)$. We iterate over $k \in \{1..n\}$. Thus, for $nSAT$, the SD problem is run at most n times. Any input to $nSAT$ will be at least of size n, so the number of times SD is run is polynomial in the length of $nSAT$'s input. $\square$

It turns out that the Z3 Theorem Prover can solve this problem, which we turn to in the next chaper.

Chapter 5

Ensuring Delivery of Safety-Critical Information

Theorem 5.0.2 (Awareness of Safety-Critical Information)**.** *If a rational pilot* $\mathbf{i}$ *has awareness about the safety precondition of action a and the safety precondition is false, then* $\mathbf{i}$ *does not execute a.*

Proof.

1. $\neg\mathbf{K_i}\, pre_s(a) \Rightarrow \mathbf{K_i}\, \neg\mathbf{K_i}\, pre_s(a)$ .... Awareness Assumption
2. $\neg pre_s(a)$............................ Safety Precondition Assumption
3. $\neg pre_s(a) \Rightarrow \langle \mathbf{i}, \mathbf{a} \rangle\, \mathit{True} \Rightarrow \neg\mathbf{K_i}\, \neg\mathbf{K_i}\, pre_s(a)$........ Theorem 1
4. $\langle \mathbf{i}, \mathbf{a} \rangle\, \mathit{True} \Rightarrow \neg\mathbf{K_i}\, \neg\mathbf{K_i}\, pre_s(a)$............... 2, 3 Modus Ponens
5. $\neg\mathbf{K_i}\, pre_s(a)$.. 2, $\mathbf{K}$ reflexive
6. $\mathbf{K_i}\, \neg\mathbf{K_i}\, pre_s(a)$.. 1, 5 Modus Ponens
7. $\neg\langle \mathbf{i}, \mathbf{a} \rangle\, \mathit{True}$... 4, 6 Modus Tollens
8. $[\mathbf{i}, \mathbf{a}]\mathit{False}$.. Def. $\langle \mathbf{i}, \mathbf{a} \rangle$ $\square$

□

Thus, if the countermeasure succeeds in establishing the pilot's awareness of the safety-critical information's being false, it follows by the logic of DASL that the agent discontinues the unsafe action. The goal is to convince her that she does not know what she thinks she knows, like the airspeed, or the horizontal attitude.

The insight gained from the formal analysis allows us to increase aviation safety, but we require a means for computing the safety-critical information that the pilot is unaware of. We do this by encoding the actual configuration of the flight deck and the safety precondition of an input action as a set of constraints and a formula to be examined by a SMT solver, and then check whether it is satisfiable. If it is not, we are interested in retrieving the specific instrument readings that conflict with the safety precondition. In SMT solving, this collection of clauses is called the unsatisfiable core. In the next section, we describe the formal properties of the unsatisfiable core and other ways it is used in formal methods research.

Before providing background on the unsatisfiable core's role in other applications of formal methods, we will provide a bit more discussion on the safety precondition. For this paper, we take it as a given that the safety precondition of an action can be determined by formally representing the contents of flight safety manuals. The specificity of instructions one finds in aviation manuals is one of the principle reasons we have this confidence. Aviation safety frequently discusses the notion of a "flight envelop" as a range of conditions in which safe actions can be executed. In a sense, we are formalizing the flight envelop as constraints for a SMT solver. However, we do not represent the safety preconditions for actions as they would appear in a higher fidelity system, but rather approximate them with a little bit of common sense, in order to achieve a balance between illustration of the approach and brevity. Were this approach to be engineered for real-world applications, the safety preconditions, and indeed the instrument configurations, would no doubt be more complex.

5.1 SMT Solving and the Unsatisfiable Core

Satisfiability Modulo Theories (SMT) is a decision problem from theoretical computer science where a formula from first order logic, constrained by theories in the form of other first order formulas, is assessed for satisfiability. Tools exist to solve engineering problems amenable to formalization in first order logic with constraining theories.

Consider the following example. The first order logic formula is

$$(x < y) \wedge (y = 10) \wedge (x > 20) \wedge (z = 40).$$

The constraining theory is the standard set of axioms of arithmetic. The formula is unsatisfiable.

When a formula constrained by theories is unsatisfiable, the unsatisfiable core refers to the set of subformulas responsible for a clausal formula's being unsatisfiable. That is, the unsatisfiable core of the formula is the portion of it that conflicts with respect either to internal consistency or with the constraining theories. In the example above, the unsatisfiable core is the first three propositions. The unsatisfiable core is successfully applied as a formal method for engineering in the domains of hardware and software verification. When engineers design a chip, before fabricating it they need to verify that the design meets the specifications. One approach to this is to formalize the chip components as variables in a first order formula, and to formalize the specification as a set of constraints on the formula. Then, running the design and specification through a SMT solver, one can identify which aspects of the design are in conflict with the specification. Software is similarly formalized and analyzed during its specification stage.

Some recent work has applied SMT solving to what are known as cyber physical systems, or hybrid systems [38]. In particular, some hybrid systems are those that involve human machine interaction. Efforts to apply SMT solving to human machine interaction have succeeded in identifying execution paths the system can go down that could cause the human user to become confused. Furthermore, this approach has been applied to the domain of aviation safety. However, as yet no efforts have been undergone to apply SMT solving for the run-time diagnosis and error correction of human error in hybrid systems.

This work does so by leveraging the logical connection between a pilot's action, her knowledge and beliefs about the system, and the features of the system that make the action safe or unsafe. We model the conditions of the action's safe execution as a constraining theory of instrument readings, and the actual instrument readings as a first order formula to be tested for satisfiability modulo that theory. If the formula modeling the actual instrument readings is not satisfiable, we extract the unsatisfiable core. Based on the logical connection established by DASL, we identify the unsatisfiable core with the safety-critical information that the pilot is unaware of when she executes an unsafe action.

5.2 Encoding and Case Studies

We develop an encoding of the instruments and the safety preconditions of actions so that they can be fed into the Z3 SMT solver and, if the actual instrument readings conflict with the constraining safety precondition, their unsatisfiable core may be extracted [42]. The case studies we present as examples in this section are meant to illustrate the encoding and the usefulness of the approach. With length limitations, we include three case studies in order to show the versatility of the approach, but sacrifice the complexity of the representations. The actual values of the case studies have been simplified for presentation, and the encodings themselves have been abridged to include only enough of the instrumentation and safety preconditions so as to illustrate the approach.

The value of representing the safety precondition as a first order formula, rather than as a propositional formula, is that it is both more succinct and a more accurate representation of the way aviation safety experts reason about the relationship between instrument readings and actions. Because instruments have values drawn from the real number space, it is natural to represent them as variables over numbers, as is possible in first order logic. The previous approach using propositional logic that "chunks" the instruments into distinct predicates of values, like *airspeedlow*, *airspeedmedium*, and *airspeedhigh*, is a clunky way of achieving a less accurate goal.

The propositional approach unnecessarily imposes a trade-off between the space complexity of the encoding and its precision. For example, if each instrument is "chunked" into three predicates, that means there are three propositions for each instrument, a subformula specifying that their truth values are mutually exclusive (because a single instrument cannot have two different readings at the same time), and the safety precondition must explicitly encode each distinct combination of instrument readings that allow the action to be safely executed. Imagine a very precise "chunking" of the instrument space, say each natural number value gets its own proposition. Then there would have to be an explosion of clauses enumerating every permitting configuration of instrument readings. The files would be huge, and the computation would be less efficient.

In this section we describe the first order logic encoding of actions and their safety preconditions, and present case studies as examples. We begin with Air France 447[1].

[1] The source code for the case studies can be found here: https://github.com/sethahrenbach/UnsatCore

5.2.1 Air France 447 Encoded

Air France flight 447 from Rio de Janeiro, Brazil, to Paris, France, occurred June 1, 2009. The Airbus A330 encountered adverse weather over the Atlantic Ocean, resulting in a clogged Pitot-static system. Consequently, the airspeed indicators delivered unreliable data to the pilot flying, resulting in confusion. A chain of events transpired in which the pilot overcorrected the plane's horizontal attitude again and again, and continued to input nose up pitch commands, all while losing airspeed. Perhaps most confusing to the pilot was the following situation: the aircraft's angle of attack (AOA) was so high it was considered invalid by the computer, so no stall warning sounded until the nose pitched down into the valid AOA range, at which point the stall warning would sound. When the pilot pulled up, the AOA would be considered invalid again, and the stall warning would cease. The aircraft entered a spin and crashed into the ocean. Palmer [46] argues that had the pilot merely taken no action, the Pitot tubes would have cleared in a matter of seconds, and the autopilot could have returned to Normal Mode. The official Bureau d'Enquêtes et d'Analyses pour la Sécurité de l'Aviation Civile (BEA) investigation found that a procedure for safely dealing with unknown airspeeds was known to the pilots but not engaged in, and this may have saved the flight as well.

For this case study, we encode the segment of the mishap chain concerning the inconsistent airspeed indicator readings and the hard nose up pitch command that the pilot continuously executed. We begin by declaring the variables of the scenario.

```
;; Air France 447 Case
(set-option :produce-unsat-cores true)

;; right side instruments
(declare-const right-airspeed Int)
(declare-const right-pitch Int)
(declare-const right-bank Int)

;; middle instruments
(declare-const middle-airspeed Int)
(declare-const middle-pitch Int)
(declare-const middle-bank Int)

;; left side instruments
(declare-const left-airspeed Int)
(declare-const left-pitch Int)
(declare-const left-bank Int)
```

Next we encode the actual instrument readings during the flight. We give these clauses names, so that the unsatisfiable core extraction can refer to them if they are responsible for a logical conflict.

```
;; actual instrument readings
(assert (!(= left-airspeed 135) :named lair))
(assert (!(= right-airspeed 100) :named rair))
(assert (!(= middle-airspeed 100) :named mair))
(assert (!(= right-pitch 15) :named rpitch))
(assert (!(= middle-pitch 15) :named mpitch))
(assert (!(= left-pitch 15) :named lpitch))
(assert (!(= right-bank 15) :named rbank))
(assert (!(= middle-bank 15) :named mbank))
(assert (!(= left-bank 15) :named lbank))
```

Next, we encode the safety precondition of the hard nose up input action. We capture the requirement that a hard nose up should not be engaged in if the indicators for the same aspect of flight, like airspeed, are in disagreement. We also capture some upper and lower bounds between which the indicators must read in order for the input to be considered safe. Normally, these bounds are thought of as the flight envelope.

```
;; safety precondition of action: Hard Nose Up
(assert (= left-airspeed right-airspeed))
(assert (= left-airspeed middle-airspeed))
(assert (= middle-airspeed right-airspeed))
(assert (= right-pitch left-pitch))
(assert (= right-pitch middle-pitch))
(assert (= middle-pitch left-pitch))
(assert (< right-airspeed 700))
(assert (> right-airspeed 99))
(assert (< middle-airspeed 700))
(assert (> middle-airspeed 99))
(assert (< left-airspeed 700))
(assert (> left-airspeed 99))

(check-sat)
(get-unsat-core)
```

The final commands of the .smt2 file tell Z3 to check whether the formula is satisfiable, and if not, to retrieve and print the unsatisfiable core. The command line input and output for running Z3 on the file appear below.

```
input : z3 af447.smt2
output: unsat
output: (lair rair)
```

After downloading the Z3 executable[2] and creating the af447.smt2 file, we run Z3 on

[2]https://github.com/Z3Prover/z3/releases

the file in a single command line input, and receive two output lines. One line tells us that the formula is unsatisfiable, which means that there is some conflict between the actual instrument readings and the safety precondition for the action. The second line tells us which instrument readings are responsible for the conflict. In this case, the instrument readings *lair* and *rair* are determined to be the conflict. If we look up at the file contents, we see that the clauses with those labels capture the left airspeed indicator, reading 135, and the right airspeed indicator, reading 100. According to the safety precondition, these values should be equal. Otherwise, a hard nose up input is unsafe, because the airspeed is not known.

The unsatisfiable core of the formula, as extracted by the Z3 SMT solver, infers the safety-critical information of the case, because it is that pair of instrument readings that render the safety precondition false, by contradicting the condition that they be in agreement.

At this point, one might fairly wonder why *mair* is not likewise included in the set. This is because it is not necessary for identifying the source of the contradiction. Having found that *lair* and *rair* conflict, these suffice to serve as the unsatisfiable core. And indeed, it is irrelevant whether the core is identified as *lair* and *rair* or *lair* and *mair*, because either will adequately serve as the safety critical information. Both sufficiently demonstrate a conflict in the relevant indicators.

5.2.2 Copa Flight 201 Encoded

Copa flight 201 departed Panama City, Panama for Cali, Colombia in June, 1992 [**?**]. Due to faulty wiring in the captain's Attitude Indicator, he incorrectly believed he was in a left bank position. In response to this, he directed the plane into an 80 degree roll to the right, which caused the plane to enter a steep rolling dive. At some point one of the pilots switched the input to the backup Attitude Indicator to be that of the pilot's faulty one, furthering confusion. Approximately 29 minutes after takeoff, the plane crashed into the jungle and all passengers and crew perished. We formalize the moment at which the pilot provides the hard right roll input.

We encode the variables the same as before, and leave out the code to save space. A simplification of the actual instrument readings is encoded as follows.

```
;; actual instrument readings
(assert (!(= left-airspeed 135) :named lair))
(assert (!(= right-airspeed 135) :named rair))
(assert (!(= middle-airspeed 135) :named mair))
(assert (!(= right-pitch 15) :named rpitch))
(assert (!(= middle-pitch 15) :named mpitch))
(assert (!(= left-pitch 15) :named lpitch))
(assert (!(= right-bank -15) :named rbank))
(assert (!(= middle-bank 0) :named mbank))
(assert (!(= left-bank 0) :named lbank))
```

Then the safety precondition for a hard right bank would include the condition that the instrument readings agree on the plane's actual bank position.

```
;;safety precondition of action: Hard Right Bank
(assert (= left-airspeed right-airspeed))
(assert (= left-airspeed middle-airspeed))
(assert (= middle-airspeed right-airspeed))
(assert (= right-bank left-bank))
(assert (= right-bank middle-bank))
(assert (= middle-bank left-bank))
(assert (< right-airspeed 700))
(assert (> right-airspeed 99))
(assert (< middle-airspeed 700))
(assert (> middle-airspeed 99))
(assert (< left-airspeed 700))
(assert (> left-airspeed 99))

(check-sat)
(get-unsat-core)
```

When we save the file as copa201.smt2 and run Z3 on it, we see the following in the terminal.

```
input : z3 copa201.smt2
output: unsat
output: (rbank lbank)
```

Again, as before, the Z3 SMT solver allows us to infer which safety-critical information in particular the pilot is unaware of, because it is the information that conflicts with the safety precondition. What the pilot's action reveals is that he mistakenly believed, at least at first, that he was in a steep left bank, because he was relying on the faulty instrument in front of him. Had he grasped the safety-critical information, that the attitude indicators were in stark disagreement, he would have realized that he did not know the plane's bank position. In this case, safety procedure is to maintain a level horizontal input, absent other convincing evidence that a steep left bank is occurring, which there was not in this case. Before executing such an extreme action, proper cross-checking of other instruments must occur.

5.2.3 Asiana Flight 214 Encoded

Asiana flight 214 from South Korea to San Francisco departed in the evening of July 6, 2013 and was scheduled to land just before noon that morning [45]. Air traffic control cleared the pilots to perform a visual approach to the runway. The plane came in short and crashed against an embankment in front of the runway, resulting in the deaths of three passengers and 187 injured. The National Transportation Safety Board (NTSB) investigation found that the captain had mismanaged the approach and monitoring of the airspeed, resulting in the plane being too high for a landing. When the pilot realized the plane's glide path was too high, he accidentally disconnected the autopilot's monitoring of the airspeed. This caused an autothrottle (A/T) protection to turn off, so when he pitched the nose down, the plane descended faster than was safe, causing it to collide with the embankment in front of the runway. We will formalize the moment at which the pilot pitches the nose down, unaware that the autothrottle is turned off.

In encoding this state of affairs, we add a new global state variable to capture the autothrottle status.

```
;; Asiana 214 Case
(set-option :produce-unsat-cores true)

;; global state
(declare-const autothrottle Bool)
```

We likewise include right, middle, and left instruments for altitude in the variable declarations, omitted here for space. The encoding of the actual instrument readings is as follows.

```
;; actual instrument readings
(assert (!(= autothrottle false) :named at-status))
(assert (!(= right-alt 5000) :named ralt))
(assert (!(= middle-alt 5000) :named malt))
(assert (!(= left-alt 5000) :named lalt))

(assert (!(= left-airspeed 200) :named lair))
(assert (!(= right-airspeed 200) :named rair))
(assert (!(= middle-airspeed 200) :named mair))
```

Finally, for the action of a Hard Nose Down, we show how to encode a conditional constraint, that if the altitude is less than 10000 feet, then the autothrottle must be on in order for a Hard Nose Down to safely be executed. Logically, this conditional is equivalent to the proposition that either the altitude is greater than (or equal to, but we omit this in the formalization) 10000 feet, or the autothrottle is on.

```
;; safety precondition of action: Hard Nose Down
(assert (= left-alt right-alt))
(assert (= left-alt middle-alt))
(assert (= middle-alt right-alt))
(assert (= left-airspeed right-airspeed))
(assert (= left-airspeed middle-airspeed))
(assert (= middle-airspeed right-airspeed))
(assert (or (> right-alt 10000) (= autothrottle true)))



(check-sat)
(get-unsat-core)
```

When we save the file as asiana.smt2 and run Z3 on it, we see the following in the terminal.

```
input : z3 asiana.smt2
output: unsat
output: (at-status ralt)
```

This indicates that the safety-critical information is the combination of the altitude, here Z3 returns only the right side altimeter reading because it is sufficient, and the fact that the autothrottle is off, just as the NTSB report concluded.

5.3 The Unsatisfiable Core is the Safety-Critical Information

This section establishes the identification of the unsatisfiable core extracted from Z3 with the safety-critical information that a pilot is unaware of when she executes an unsafe action. To do this, we examine a case study formalized in DASL, and compare the results with the same case study encoded as an SMT problem that was run through Z3.

Consider the following formalization of Copa flight 201 in DASL. Line 1 represents a conjunction over the actual instrument readings, encoded as atomic propositions. The relevant propositions included in the text state that the right attitude indicator (*RightBank*) reads that the aircraft is pitching left considerably (*SteepLeft*), but that the left attitude indicator (*LeftBank*) reads level flight. We ignore the middle attitude indicator to try and keep it readable, but if it were included, it would be present in the conclusion as well. Line 2 expresses in DASL that the pilot executes a hard right bank action. Line 3 concludes with the theorem's application, which states that the pilot lacks awareness either the left indicator or the right indicator: she is unaware of the falsity of one of them. This analysis establishes that the attitude indicators' values together are a piece of safety-critical information, because it shows that the pilot engages in the unsafe action *only if* she remains unaware of them jointly.

1. $(RightBankSteepLeft) \wedge (LeftBankLevel)$ .... configuration.

2. $\langle pilot, hardrightbank\rangle\, true$ .... pilot input.

3. $\neg\mathbf{K}_{pilot}\,(LeftBankSteepLeft \vee RightBankLevel) \wedge$
   $\neg\mathbf{K}_{pilot}\,\neg\mathbf{K}_{pilot}\,(LeftBankSteepLeft \vee RightBankLevel)$ ...... 1,2,Th 1.

Compare this with the results of running the SMT encoding through Z3. The instrument readings construed as a first order forumla are unsatsifiable modulo the constraints imposed by the action's safety precondition. The unsatisfiable core identified by Z3, that is, the reason it is unsatisfiable, is because the right attitude indicator (*rbank*) and the left attitude indicator (*lbank*) are in disagreement. This is consistent with the DASL analysis, which identifies the left attitude indicator's value as the one the pilot is critically unaware of.

The way these two formalisms work together is that the DASL theorem establishes the connection between the action, pilot knowledge, and safety conditions, and the SMT solver performs the safety-critical inference in an efficient and expressive way. They are different in an interesting way, though. DASL works by identifying a proposition from the safety precondition of the action that is *false*, not currently present on the instrument panel. For example, a hard right bank action's safe execution requires the attitude indicators to be in agreement. If the attitude indicators disagree, and the pilot executes the action anyway, then DASL allows us to infer unawareness of the false propositions from the safety precondition. The SMT formalization operates by identifying which of the *true* propositions from the instrument panel conflict with the safety precondition of the action. Thus, the result of DASL: the pilot is unaware regarding the left indicator's being in a steep left bank or the right indicator's being level; Z3 infers that the actual instrument readings of the right airspeed indicator and the left airspeed indicator are the ones that the pilot needs to see. We built it this way so that the instruments themselves, not the safety precondition, are the output of the Z3 engine, for reasons to be discussed in the next section.

5.4 Application Design

This section briefly describes how this encoding of aviation activity and the Z3 theorem prover might be used in a runtime monitor application.

The application would execute the following algorithm in a loop.

Algorithm 1 Monitor Algorithm

```
while flying do
    smt2_actual_readings ← instrument readings
    smt2_safety_preconditions ← lookup_safety_precondition(flight control input)
    smt2_file ← concat(smt2_actual_readings, smt2_safety_precondtions)
    unsat_core ← Z3 smt2_file
    if unsat_core is empty then undim all
    else if unsat_core is not empty then
        map(dim, [x|x ∉ unsat_core])
```

This algorithm, though simple, represents a dramatic departure from contemporary safety practices, in that it aims to draw the pilot's attention to the safety-critical information by *dimming* the information that is not, at the moment, safety-critical. Dimming, of course, is not a technical term, and has not been defined. Its definition and implementation depends on the context and the technical details of the cockpit.

In most emergency situations, a number of alarms are going off at once, and the pilots suffer from information overload, which decreases their situational awareness. The safety-critical information is right in front of them, and perhaps even has an alarm and blinking light accompanying it. And yet they still crash. This algorithm addresses the problem by temporarily *removing* the alarms and blinking lights that are not related to the safety-critical information identified by Z3's returned unsatisfiable core, based on the encoding of the configuration and action. If the only alarm or blinking lights are those highlighting the fact that, say, the airspeed indicators are in disagreement, immediately following the pilot's action to dramatically reduce thrust, then the chances that he or she notices this problem must increase.

This runtime monitor will increase the likelihood that the pilot notices the safety-critical information. Noticing it, he or she will cease the unsafe action.

5.5 Related Work

Over the past decade related work in applying formal methods to the human component of aviation safety have advanced the state of the art and contributed to aviation safety [38, 23, 17, 14]. However, previous work applies formal methods to the specification stage of systems, typically using model checking or SAT/SMT solving to discover possible states of the system in which a human might become disoriented or confused. This work is important. The present work advances a different but related line of effort, applying formal methods to the run-time stage of systems, with the goal of diagnosing and correcting unsafe system behaviors as they occur, especially those caused by human confusion. To do this, we have taken a formal model of the pilot's reasoning in the form of DASL and used the resulting analysis to develop a tool for delivering safety-critical information to the pilot when it is needed.

Chapter 6

Summary and concluding remarks

Congratulations on completing your dissertation.

Appendix A

Title of first appendix

A.1 Section title

Here is some additional information which would have detracted from the point being made in the main article.

A.1.1 Subsection title

This section even has subtitles

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In another paragraph, you could say if/when you were married, what the name of your kids are, and what your plans are for after graduation if you choose. Take a look at other vita's from other dissertations for examples.