CHAPTER 10

PROPERTIES OF STOCK OPTIONS

SUMMARY

- Factors Affecting Option Prices
- Notation
- Put-call Parity
- Upper and Lower Bounds for Option Prices
- Early Exercise

Factors Affecting Option Prices

1. The current stock price
2. The strike price
3. The time to maturity
4. The volatility of stock price
5. The risk-free rate
6. Dividends

Notation

- t : now
- T : expiration day
- S : current stock price (S_t)
- K : strike price
- S_T : stock price at T
- r : risk-free rate (nominal)
- C : American call price
- P : American put price
- c : European call price
- p : European put price
- σ : volatility

Effect of Variables on Option Pricing: *Which are positively related?*

Variable	C	P
S		
K		
T-t		
σ		
r		
Dividend		

(all other things being equal)

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Stock Price

- A call option's payoff is
  - $\max(S_T - K, 0)$
  - $S_T - K$ if a call option is in the money

- Call options are more valuable if:
  - The stock price increases (fix the strike price)
  - Let $K = \$100$
  - $S_{T1} = \$120$, $S_{T2} = \$125$
  - $Call_1 = \$120 - 100 = \$20$, $Call_2 = \$125 - 100 = \$25$

Strike Price

- A call option's payoff is
  - $\max(S_T - K, 0)$
  - $S_T - K$ if a call option is in the money

- Call options are more valuable if:
  - The strike price decreases (fix the stock price)
  - Let $S_T = \$100$
  - $K_1 = \$90$, $K_2 = \$80$
  - $Call_1 = \$100 - 90 = \$10$, $Call_2 = \$100 - 80 = \$20$

Stock Price

- A put option's payoff is
  - $\max(K - S_T, 0)$
  - $K - S_T$ if a put option is in the money

- Put options are more valuable if:
  - The stock price decreases (fix the strike price)
  - Let K = $100
  - S_{T1} = $90, S_{T2} = $80
  - Put_1=$100-90=$10, Put_2=$100-80=$20

Strike Price

- A put option's payoff is
  - $\max(K-S_T, 0)$
  - $K-S_T$ if a put option is in the money

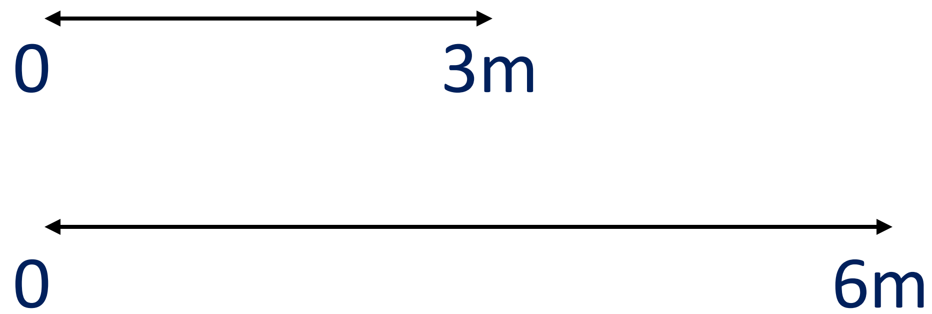
- Put options are more valuable if:
  - The strike price increases (fix the stock price)
  - Let S_T = \$100
  - K_1 = \$110, K_2 = \$120
  - Put_1=\$110-100=\$10, Put_2=\$120-100=\$20

Time to Expiration

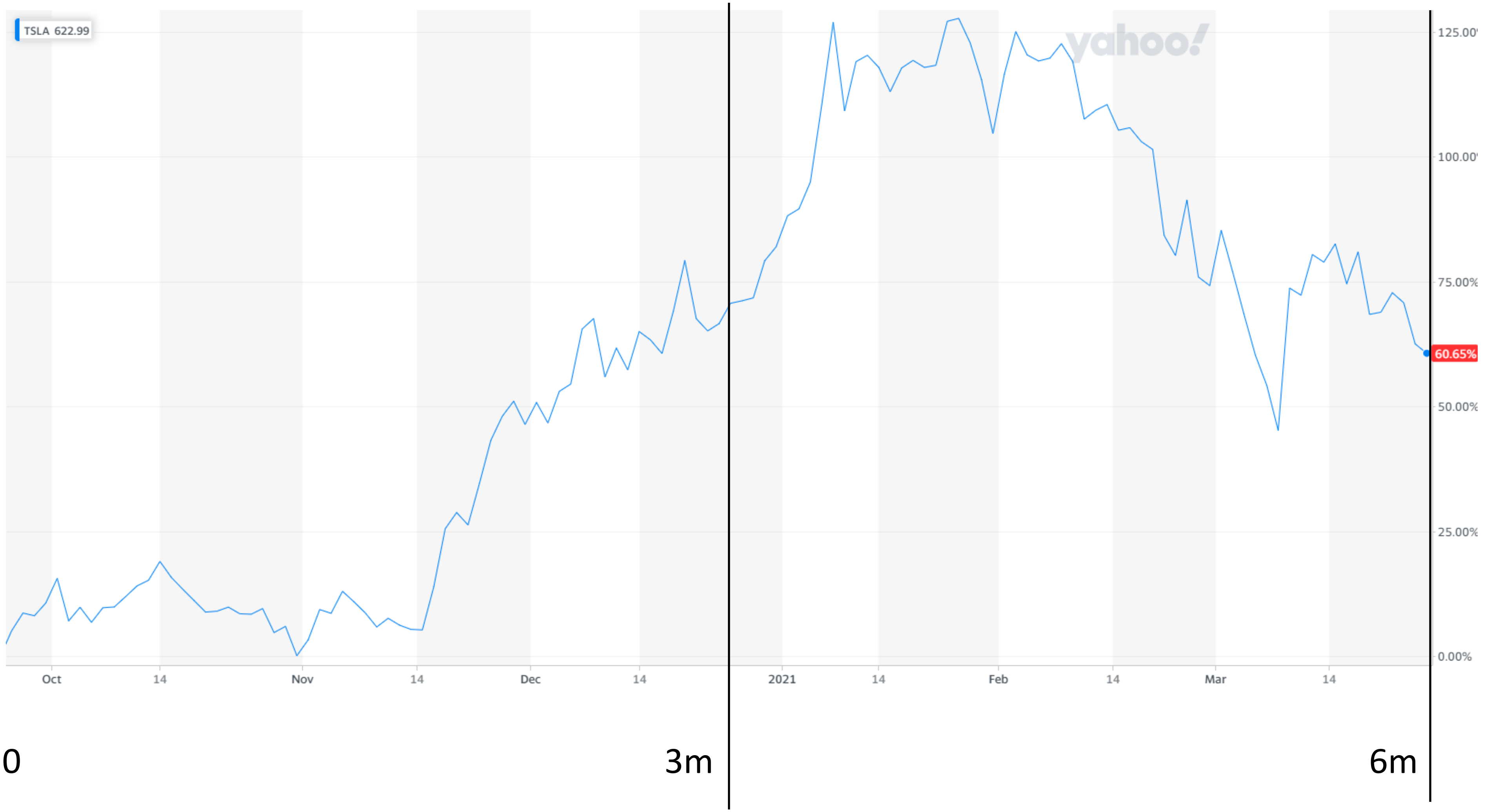
- For American calls and puts, options become more valuable if the maturity increases
  - Consider two options that differ only in the maturity
  - The owner of the long-life option can take part in all exercise opportunities as the owner of the short-life option, and more
- For European options, no conclusion

TIME TO EXPIRATION

- Two American options (A 3-month and a 6-month option)
- They can be exercised at any time up to the expiration day

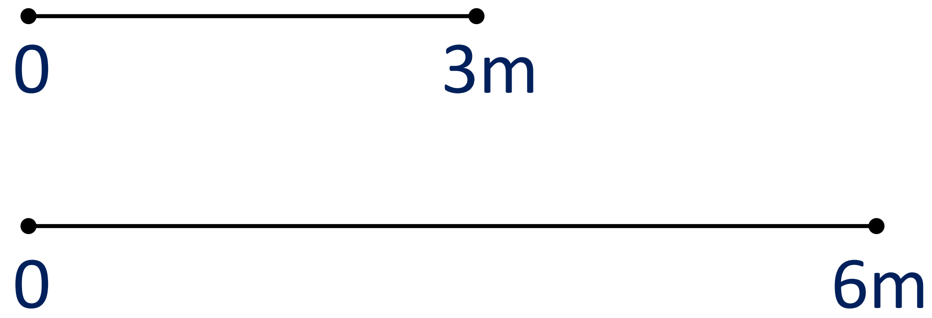


Time to Expiration



TIME TO EXPIRATION

- Two European options (a 3-month and a 6-month option)
- They can only be exercised on the expiration day



Volatility

- What is volatility
  - A measure of uncertainty
  - Unexpected fluctuations of returns
- If volatility increases, it is more likely that the stock price will go up or go down
- The owner of the call option will benefit from price increases but has limited downside risk in the event of price decreases
- Thus, the value of the call option increases as volatility increases

Volatility

- The owner of the put option will benefit from price decreases but has limited downside risk in the event of price increases
- Thus, the value of the put option increases as volatility increases

RISK-FREE RATE

- It has two effects $E(R_i) = r_f + \beta(E(R_m) - r_f)$
  - +/− As interest rates increase, the stock return tends to increase
  - − As interest rates increase, the PV of the any future cash flow decreases
- For call options, the first effect dominates
- \+ Thus, the value of the call increases as the risk-free rate increases (keeping everything else fixed, *incl. S*)
- For put options, both effects decrease the value of the option

DIVIDEND — S

- Dividend distribution will lead to a decline in the stock price
- Bad news for call owners and good news for put owners
  - The value of the call options decreases
  - The value of the put options increases

Effect of Variables on Option Pricing

	Eur.		Am.	
Variable	c	p	C	P
S	+	-	+	-
K	-	+	-	+
T-t	?	?	+	+
σ	+	+	+	+
r	+	-	+	-
Dividend	-	+	-	+

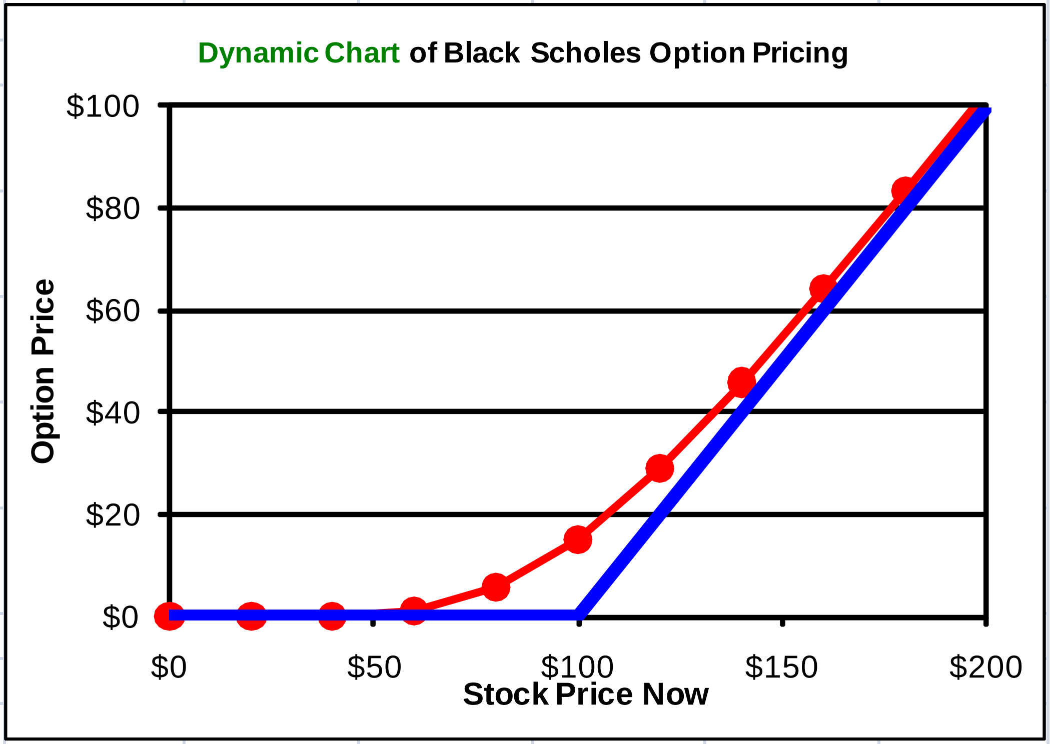
(all other things being equal)

Demonstration (Excel)

Black Scholes Option Pricing — Dynamic Chart — Call

Inputs

Option Type: 1=Call, 0=Put	1	1
Standard Dev - Annual (σ)	50%	5
Riskfree Rate- Annual (r)	5.0%	5
Exercise Price (X)	$100.00	100
Time To Maturity - Yrs (T)	0.50	5



Properties of option prices

- Two types of no-arbitrage conditions:
  1. Parity conditions – equivalence relations
  2. Boundary conditions – maximums and minimums of option premiums
- These don't depend on any option pricing model
- We assume contract size = 1 share

- Assumptions:
  - No transaction costs
  - All trading profits are subject to same tax rate
  - Borrowing and lending occurs at the risk-free rate

VALUE OF CALL + BOND

- Let's form a portfolio consisting of:
  - A European call option
    - Strike = K
    - Expiration = T
  - a risk-free bond
    - Principal = K
    - Maturity = T
    - Risk-free rate = r

- Value at t: $c + Ke^{-r(T-t)}$

- Value at T?

Computing payoff of long call + bond portfolio

1. Long call and long bond			
Current bond price	Bond Principal	Strike	Call premium
453.11	455	455	10.14
Stock price at maturity	Bond value	Payoff	Combined (c+B):
	at maturity	Long call	call+Bond
415			
420			
425			
430			
435			
440			
445			
450			
455			
460			
465			
470			
475			
480			

VALUE OF CALL + BOND

→ Value at T = $\underline{\max(S_T, K)}$

Value of portfolios at T

		$S_T > K$	$S_T < K$
Portfolio 1	Call option	$S_T - K$	0
	Zero-coupon bond	K	K
	Total	S_T	K

VALUE OF PUT + STOCK

- Let's form a portfolio consisting of:
  - a European put option
    - Strike = K
    - Expiration = T
  - a stock
    - Current price = S_t
    - Price at T = S_T

- Value at t: $p + S_t$

- Value at T: ?

Computing payoff of long put + stock portfolio

2. A protective put: Long stock and long put		
Current stock price	**Strike**	**Put premium**
450	**455**	**13.25**
Stock price at maturity	**Payoff**	**Combined (p+S):**
	Long put	**Protective put**
415		
420		
425		
430		
435		
440		
445		
450		
455		
460		

Value of Put + Stock

→ Value at T: $\max(S_T, K)$

Value of portfolios at T

		$S_T > K$	$S_T < K$
Portfolio 2	Put Option	0	$K - S_T$
	Share	S_T	S_T
	Total	S_T	K

$c = p + S_t - Ke^{-rT}$

Put-call Parity for European Options (No Dividends)

	Value of portfolios at T	$S_T > K$	$S_T < K$
Portfolio 1	Call option	$S_T - K$	0
	Zero-coupon bond	K	K
	Total	S_T	K
Portfolio 2	Put Option	0	$K - S_T$
	Share	S_T	S_T
	Total	S_T	K

- At T: value Portfolio 1 = value Portfolio 2 = $\max(S_T, K)$

They must therefore be worth the same today

- At t: $$c + Ke^{-r(T-t)} = p + S_t$$

Example: Arbitrage Opportunity

- A 1-month European call and put with a strike of \$455 are priced at \$10.14 and \$23.25, respectively. The risk-free rate is 5% and the current stock price is \$450. What is the arbitrage opportunity?
  - $\boxed{c + Ke^{-rT} \overset{<}{=} p + S}$

≠

long c + B — short p - S

An Example

Action	Cash Flow at t	Cash Flow at T	
		if $S_T = 500$	if $S_T = 400$
Buy Call	$-c_t = -10.14$	$S_T - K = 45$	0
Buy Bonds	$-Ke^{-r(T-t)} = -453.11$	$K = 455$	$K = 455$
Short Put	$p_t = 23.25$	0	$-(K - S_T) = -55$
Short Stock	$S_t = 450$	$-S_T = -500$	$-S_T = -400$
Total	$10.00 > 0$	0	0

Upper and Lower Bounds for Option Prices

- Next, we derive upper and lower bounds for option prices
- If an option price is above the upper bound or below the lower bound
  - There will be arbitrage opportunities

UPPER BOUNDS

- Call gives owner the right to buy 1 share at K:
  - No one would pay more than S for this right:

$$c \leq St \; and \; C \leq S_t$$

- American put gives right to sell 1 share at K at any time:
  - No one would pay more than K for option:

$$P \leq K$$

- European put gives right to sell 1 share at K at expiration:
  - No one would pay more than the PV of K today for this right:

$$p \leq Ke^{-rT}$$

Lower Bounds

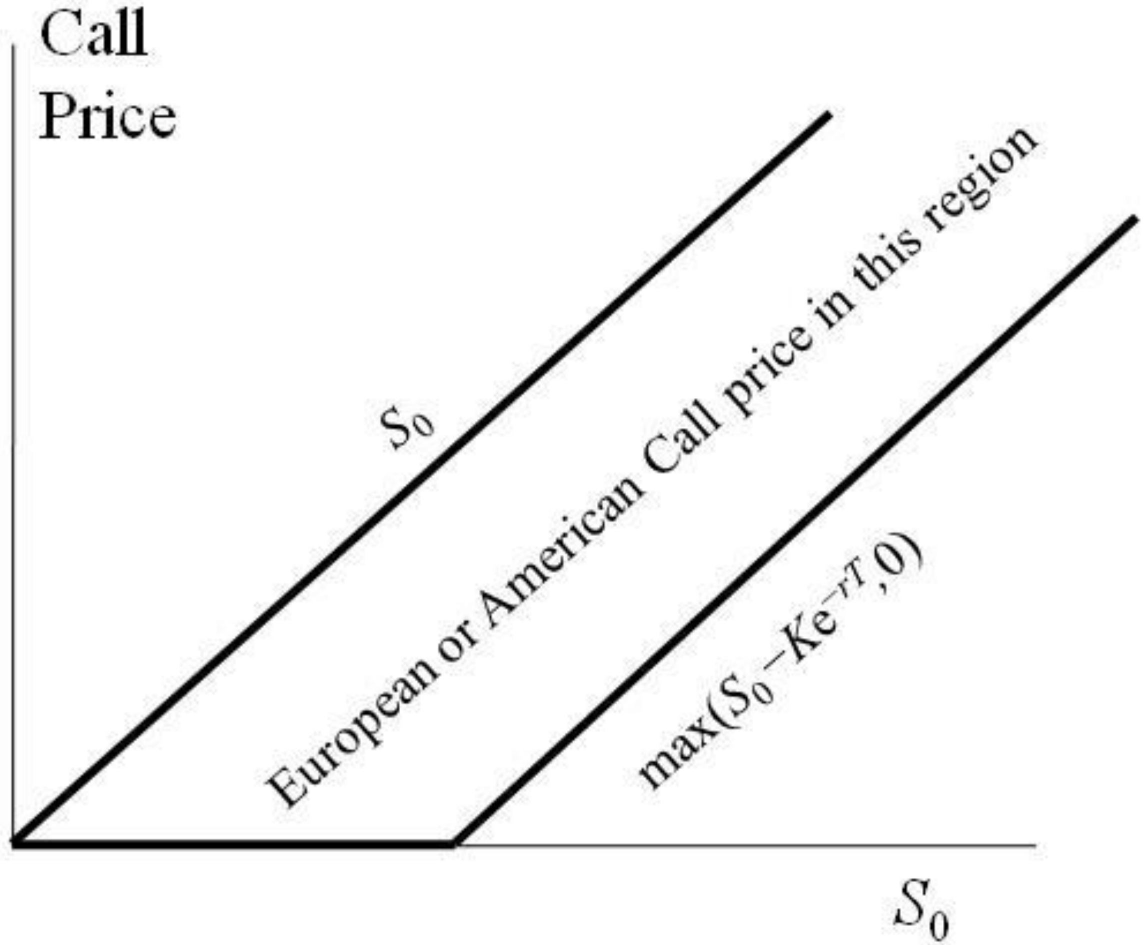
- Rewriting the put-call parity:

$$c = \cancel{p} + S_t - Ke^{-r(T-t)}$$
$$p = \cancel{c} + Ke^{-r(T-t)} - S_t$$

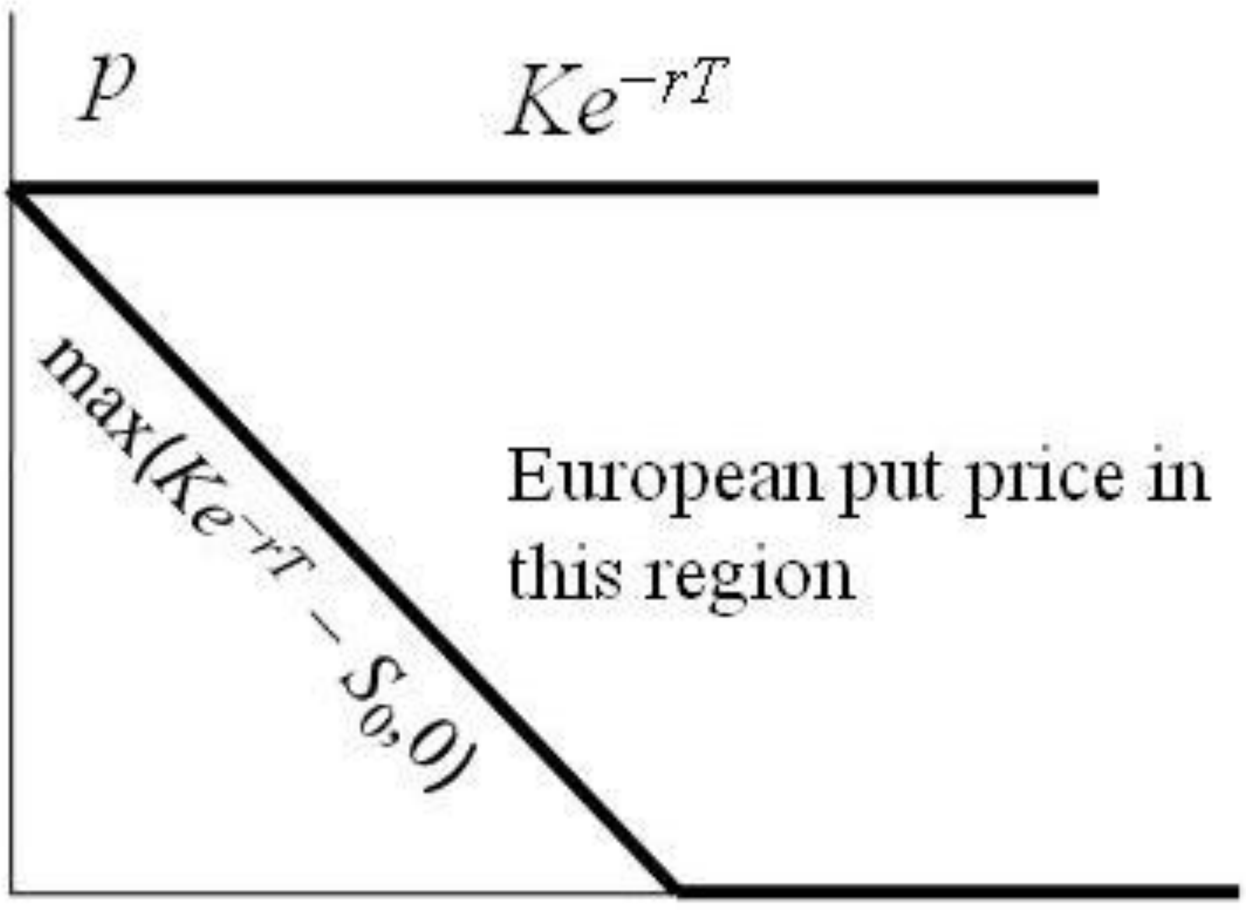
- Since options cannot have negative value:

$$c \geq S_t - Ke^{-r(T-t)}$$
$$p \geq Ke^{-r(T-t)} - S_t$$

Bounds for European Call Options (No Dividends)



Bounds for European Put Options (No Dividends)



AMERICAN VS. EUROPEAN OPTIONS

- American options are worth at least as much as the corresponding European options

$$c \le C$$
$$p \le P$$

Early Exercise of American Calls

- An American call option can be exercised prior to expiration.
  - However, it is never optimal to exercise an American call early if the underlying stock pays no dividends.

An Example

- Case 1: “I like the stock”
  - Assume you plan to exercise the call option, pay $455 and hold the stock for one more month
  - The alternative is to hold the call option instead

- Case 2: overpriced stock
  - Assume you believe the stock is overpriced now, and you plan to exercise the call option and sell the stock
  - The alternative is to sell the option instead

An Example

- $S_t = 500$
- $K = 455$
- $T\text{-}t = 1$ month
- $r = 5\%$
- The call option is in the money
- Should you exercise the American (call) option?

An Example

- Case 1: The trade-off when you like the stock
  - Recall $C_T = max\ (S_T - K, 0)$
  - If the stock price is above \$455 in one month
    - You exercise the call and get 1 share on the expiration day
    - By not exercising early, you pay \$455 one month later
  - If the stock price is below \$455 in one month
    - you will be glad that you did not exercise early
    - You can pay $S_T < 455$ and get 1 share in the stock market
  - Thus, there is no advantage to exercise the call early if you plan to hold the stock for the rest of the life of the option
    - Since the stock doesn't pay dividend, you don't miss out on any income

An Example

- <u>Case 2: The trade-off when you don't like the stock</u>
  - You get $45 if you exercise the call option now
  - The price obtained for the option will be greater than its intrinsic value, $45, since

$$C_t \geq c_t \geq S_t - Ke^{-r(T-t)}$$

$$= 500 - 455e^{-0.05*\frac{1}{12}} = \$46.89$$

  - You get $46.89 if you sell the option to another investor
  - You should not exercise the American call early!

Reasons for Not Exercising a Call Option Early

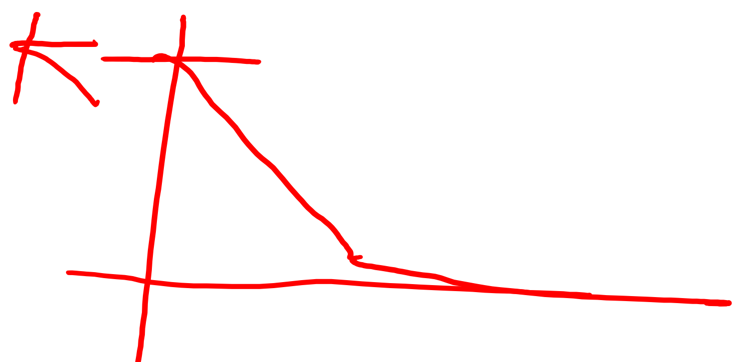
- Strike price is paid later
- Insurance element retained
  - Holding the call provides insurance against stock price falling below the strike price
- No income (dividend) is sacrificed

$$\rightarrow C_t = c_t$$

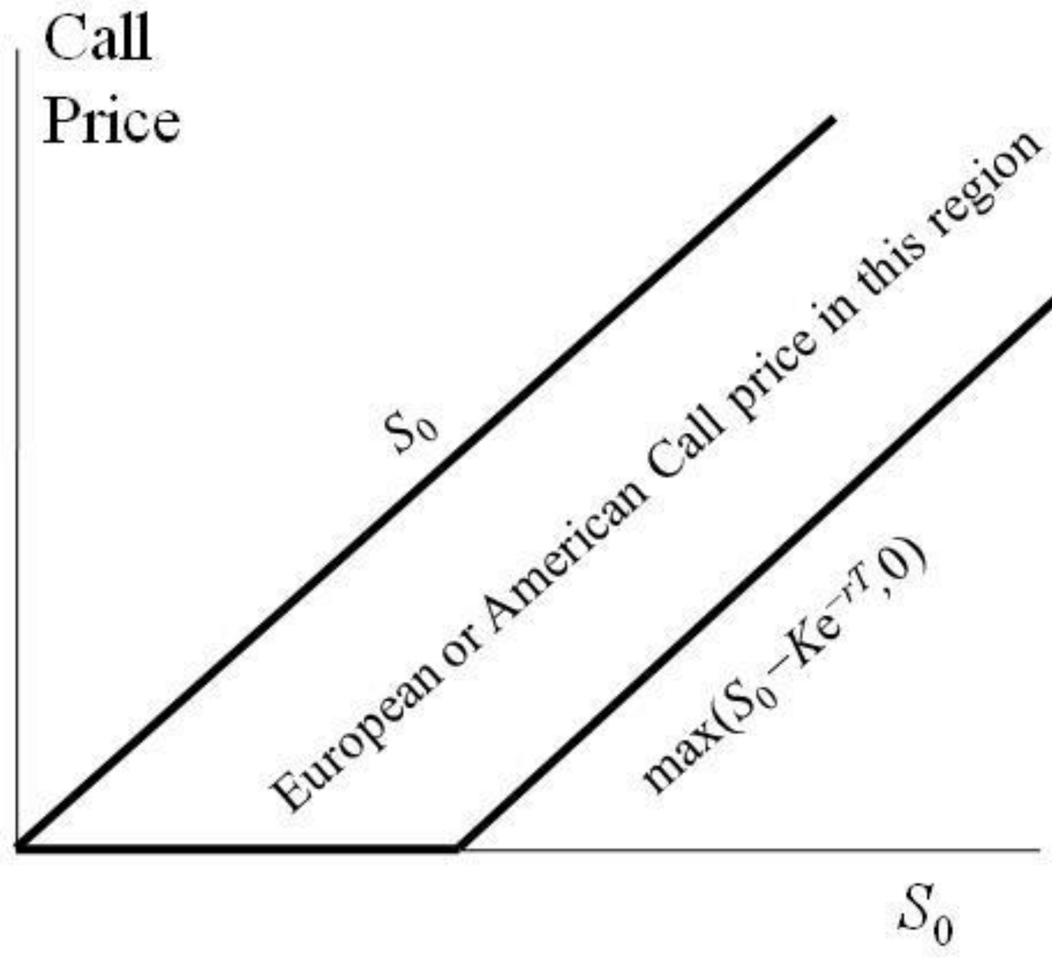
True for calls written on stocks that pay no dividends during the life of the option.

Reasons for Exercising a Put Option Early

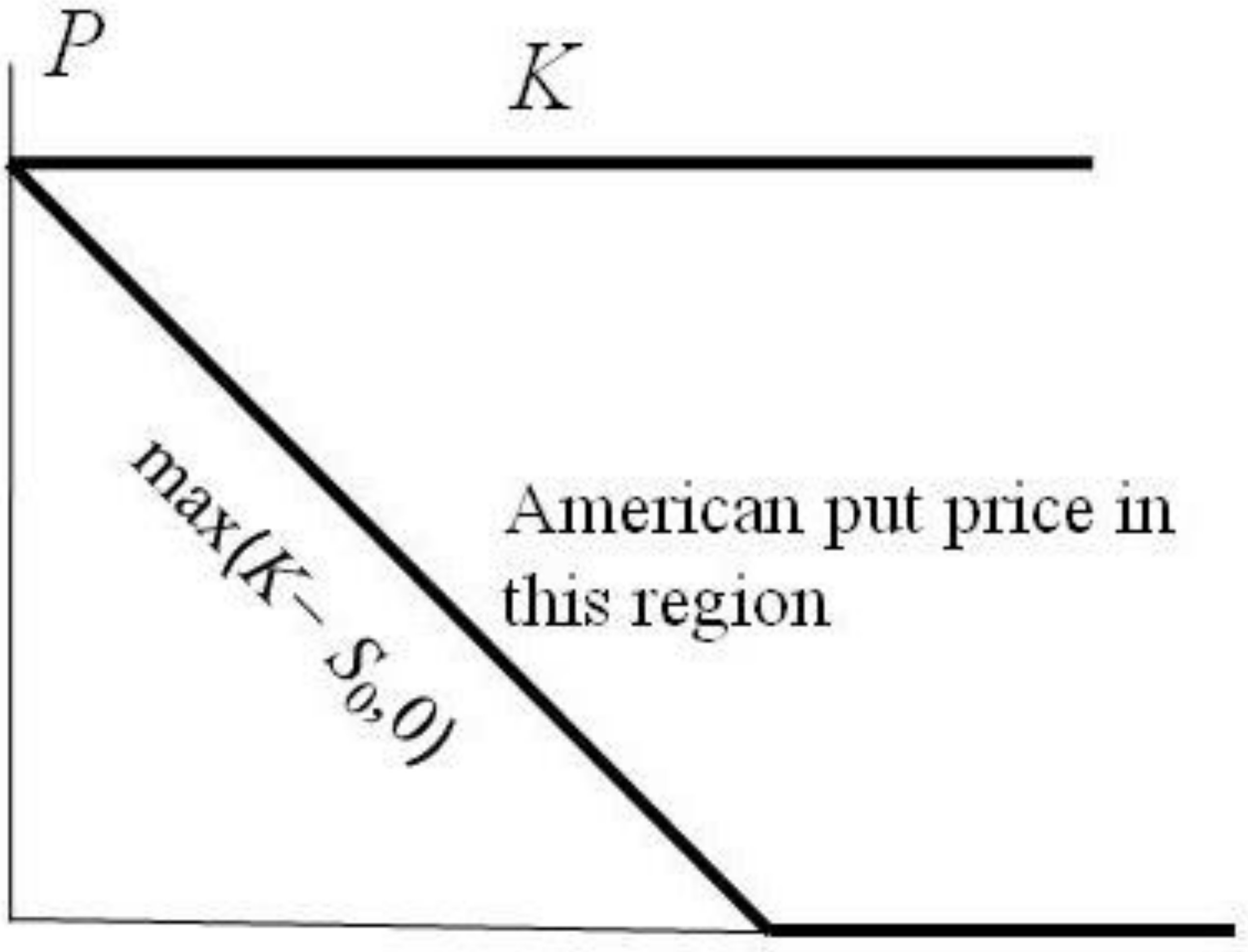
- An example:
  - K = 10
  - $S_t = 0$ bankrupt!
  - If exercise now, you get \$10 today
  - If exercise 3-month later, you get \$10 after 3 months
- Sometimes, it makes sense to forego the insurance that the put option provides and realize the strike price K immediately



Bounds for American Call Options (No Dividends)



Bounds for American Put Options (No Dividends)



Put-call Parity for American Options (No Dividends)

- Put call-parity for European options:

$$c + Ke^{-r(T-t)} = p + S$$

- Since $P > p$ and $C = c$, the put-call parity for American options must be:

$$C + Ke^{-r(T-t)} < P + S$$

Dividends and Boundaries

- Let D = PV of dividends paid over life of option.
  - Dividend $\Longrightarrow$ goes ex dividend before option expires.
  - Since dividends are bad for calls and good for puts, we subtract D from call boundaries and add D to put boundaries.
- New lower bounds:
  - Lower bound for call: $c \geq \max(S - Ke^{-rT} - D, 0)$
  - Lower bound for put: $p \geq \max(Ke^{-rT} - S + D, 0)$
  - Put-call parity: $c + Ke^{-rT} + D = p + S$

Summary

- Call option prices increase with the stock price, volatility, time to expiration, and risk-free rate. They are negatively related to the strike price, dividends.
- Put option prices increase with the strike price, volatility, time to expiration, and dividends. They are negatively related to the stock price and risk-free rate.
- Put-call parity is given by:

$$c_t + Ke^{-r(T-t)} = p_t + S_t \text{ (European options)}$$

$$C_t + Ke^{-r(T-t)} < P_t + S_t \text{ (American options)}$$

- Early exercise of a call on a non-dividend paying stock is never optimal

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Homework 12

Chapter 10. Properties of stock options

Questions 10.1, 10.6, 10.7, 10.10, 10.23