CHAPTER 11
Trading Strategies Involving Options

Practice Questions

Problem 11.8.
Use put–call parity to relate the initial investment for a bull spread created using calls to the initial investment for a bull spread created using puts.

A bull spread using calls provides a profit pattern with the same general shape as a bull spread using puts (see Figures 11.2 and 11.3 in the text). Define p_1 and c_1 as the prices of put and call with strike price K_1 and p_2 and c_2 as the prices of a put and call with strike price K_2. From put-call parity

$$p_1 + S = c_1 + K_1 e^{-rT}$$

$$p_2 + S = c_2 + K_2 e^{-rT}$$

Hence:

$$p_1 - p_2 = c_1 - c_2 - (K_2 - K_1)e^{-rT}$$

This shows that the initial investment when the spread is created from puts is less than the initial investment when it is created from calls by an amount $(K_2 - K_1)e^{-rT}$. In fact as mentioned in the text the initial investment when the bull spread is created from puts is negative, while the initial investment when it is created from calls is positive.
The profit when calls are used to create the bull spread is higher than when puts are used by $(K_2 - K_1)(1 - e^{-rT})$. This reflects the fact that the call strategy involves an additional risk-free investment of $(K_2 - K_1)e^{-rT}$ over the put strategy. This earns interest of

$$(K_2 - K_1)e^{-rT}(e^{rT} - 1) = (K_2 - K_1)(1 - e^{-rT}).$$

Problem 11.9.
Explain how an aggressive bear spread can be created using put options.

An aggressive bull spread using call options is discussed in the text. Both of the options used have relatively high strike prices. Similarly, an aggressive bear spread can be created using put options. Both of the options should be out of the money (that is, they should have relatively low strike prices). The spread then costs very little to set up because both of the puts are worth close to zero. In most circumstances the spread will provide zero payoff. However, there is a small chance that the stock price will fall fast so that on expiration both options will be in the money. The spread then provides a payoff equal to the difference between the two strike prices, $K_2 - K_1$.

Problem 11.10.
Suppose that put options on a stock with strike prices \$30 and \$35 cost \$4 and \$7, respectively. How can the options be used to create (a) a bull spread and (b) a bear spread? Construct a table that shows the profit and payoff for both spreads.

A bull spread is created by buying the $30 put and selling the $35 put. This strategy gives rise to an initial cash inflow of $3. The outcome is as follows:

Stock Price	Payoff	Profit
$S_T \geq 35$	0	3
$30 \leq S_T < 35$	$S_T - 35$	$S_T - 32$
$S_T < 30$	–5	–2

A bear spread is created by selling the $30 put and buying the $35 put. This strategy costs $3 initially. The outcome is as follows

Stock Price	Payoff	Profit
$S_T \geq 35$	0	–3
$30 \leq S_T < 35$	$35 - S_T$	$32 - S_T$
$S_T < 30$	5	2

Problem 11.11.
Use put–call parity to show that the cost of a butterfly spread created from European puts is identical to the cost of a butterfly spread created from European calls.

Define c_1, c_2, and c_3 as the prices of calls with strike prices K_1, K_2 and K_3. Define p_1, p_2 and p_3 as the prices of puts with strike prices K_1, K_2 and K_3. With the usual notation

$$c_1 + K_1 e^{-rT} = p_1 + S$$

$$c_2 + K_2 e^{-rT} = p_2 + S$$

$$c_3 + K_3 e^{-rT} = p_3 + S$$

Hence

$$c_1 + c_3 - 2c_2 + (K_1 + K_3 - 2K_2)e^{-rT} = p_1 + p_3 - 2p_2$$

Because $K_2 - K_1 = K_3 - K_2$, it follows that $K_1 + K_3 - 2K_2 = 0$ and

$$c_1 + c_3 - 2c_2 = p_1 + p_3 - 2p_2$$

The cost of a butterfly spread created using European calls is therefore exactly the same as the cost of a butterfly spread created using European puts.

Problem 11.12.
A call with a strike price of $60 costs $6. A put with the same strike price and expiration date costs $4. Construct a table that shows the profit from a straddle. For what range of stock prices would the straddle lead to a loss?

A straddle is created by buying both the call and the put. This strategy costs $10. The profit/loss is shown in the following table:

Stock Price	Payoff	Profit
$S_T > 60$	$S_T - 60$	$S_T - 70$
$S_T \leq 60$	$60 - S_T$	$50 - S_T$

This shows that the straddle will lead to a loss if the final stock price is between $50 and $70.

Problem 11.13.
Construct a table showing the payoff from a bull spread when puts with strike prices K_1 *and* K_2 *are used* $(K_2 > K_1)$.

The bull spread is created by buying a put with strike price K_1 and selling a put with strike price K_2. The payoff is calculated as follows:

Stock Price	Payoff from Long Put	Payoff from Short Put	Total Payoff
$S_T \geq K_2$	0	0	0
$K_1 < S_T < K_2$	0	$S_T - K_2$	$-(K_2 - S_T)$
$S_T \leq K_1$	$K_1 - S_T$	$S_T - K_2$	$-(K_2 - K_1)$

Problem 11.14.
An investor believes that there will be a big jump in a stock price, but is uncertain as to the direction. Identify six different strategies the investor can follow and explain the differences among them.

Possible strategies are:

Strangle
Straddle
Strip
Strap
Reverse calendar spread
Reverse butterfly spread

The strategies all provide positive profits when there are large stock price moves. A strangle is less expensive than a straddle, but requires a bigger move in the stock price in order to provide a positive profit. Strips and straps are more expensive than straddles but provide bigger profits in certain circumstances. A strip will provide a bigger profit when there is a large downward stock price move. A strap will provide a bigger profit when there is a large upward stock price move. In the case of strangles, straddles, strips and straps, the profit increases as the size of the stock price movement increases. By contrast in a reverse calendar spread and a reverse butterfly spread there is a maximum potential profit regardless of the size of the stock price movement.

Problem 11.15.
How can a forward contract on a stock with a particular delivery price and delivery date be created from options?

Suppose that the delivery price is K and the delivery date is T. The forward contract is created by buying a European call and selling a European put when both options have strike price K and exercise date T. This portfolio provides a payoff of $S_T - K$ under all circumstances where S_T is the stock price at time T. Suppose that F_0 is the forward price. If $K = F_0$, the forward contract that is created has zero value. This shows that the price of a call equals the price of a put when the strike price is F_0.

Problem 11.16.

"A box spread comprises four options. Two can be combined to create a long forward position and two can be combined to create a short forward position." Explain this statement.

A box spread is a bull spread created using calls and a bear spread created using puts. With the notation in the text it consists of a) a long call with strike K_1, b) a short call with strike K_2, c) a long put with strike K_2, and d) a short put with strike K_1. a) and d) give a long forward contract with delivery price K_1; b) and c) give a short forward contract with delivery price K_2. The two forward contracts taken together give the payoff of $K_2 - K_1$.

Problem 11.17.

What is the result if the strike price of the put is higher than the strike price of the call in a strangle?

The result is shown in Figure S11.1. The profit pattern from a long position in a call and a put is much the same when a) the put has a higher strike price than a call and b) when the call has a higher strike price than the put. But both the initial investment and the final payoff are much higher in the first case.

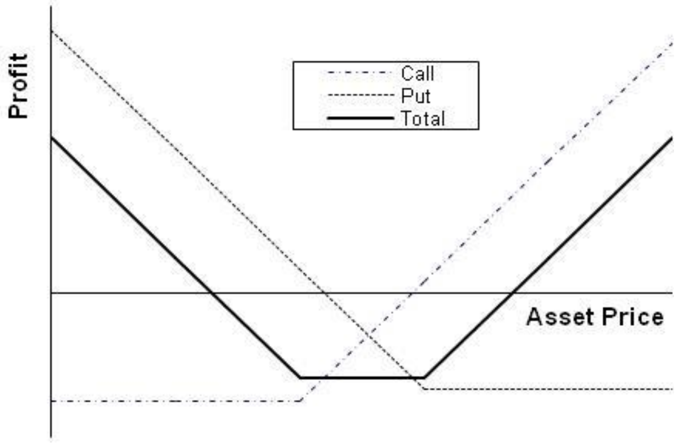

Figure S11.1 Profit Pattern in Problem 11.17

Problem 11.18.

A foreign currency is currently worth \$0.64. A one-year butterfly spread is set up using European call options with strike prices of \$0.60, \$0.65, and \$0.70. The risk-free interest rates in the United States and the foreign country are 5% and 4% respectively, and the volatility of the exchange rate is 15%. Use the DerivaGem software to calculate the cost of setting up the butterfly spread position. Show that the cost is the same if European put options are used instead of European call options.

To use DerivaGem select the first worksheet in DG400f.xls and choose Currency as the

Underlying Type. Select Black-Scholes European as the Option Type. Input exchange rate as 0.64, volatility as 15%, risk-free rate as 5%, foreign risk-free interest rate as 4%, time to exercise as 1 year, and exercise price as 0.60. Select the button corresponding to call. Do not select the implied volatility button. Hit the *Enter* key and click on calculate. DerivaGem will show the price of the option as 0.0618. Change the exercise price to 0.65, hit *Enter*, and click on calculate again. DerivaGem will show the value of the option as 0.0352. Change the exercise price to 0.70, hit *Enter,* and click on *Calculate*. DerivaGem will show the value of the option as 0.0181.
Now select the button corresponding to put and repeat the procedure. DerivaGem shows the values of puts with strike prices 0.60, 0.65, and 0.70 to be 0.0176, 0.0386, and 0.0690, respectively.
The cost of setting up the butterfly spread when calls are used is therefore

$$0.0618 + 0.0181 - 2 \times 0.0352 = 0.0095$$

The cost of setting up the butterfly spread when puts are used is

$$0.0176 + 0.0690 - 2 \times 0.0386 = 0.0094$$

Allowing for rounding errors, these two are the same.

Problem 11.19
An index provides a dividend yield of 1% and has a volatility of 20%. The risk-free interest rate is 4%. How long does a principal-protected note, created as in Example 11.1, have to last for it to be profitable for the bank issuing it? Use DerivaGem.

Assume that the investment in the index is initially $100. (This is a scaling factor that makes no difference to the result.) DerivaGem can be used to value an option on the index with the index level equal to 100, the volatility equal to 20%, the risk-free rate equal to 4%, the dividend yield equal to 1%, and the exercise price equal to 100. For different times to maturity, *T*, we value a call option (using Black-Scholes European) and calculate the funds available to buy the call option (=100-100$e^{-0.04\times T}$). Results are as follows:

Time to maturity, T	*Funds Available*	*Value of Option*
1	3.92	9.32
2	7.69	13.79
5	18.13	23.14
10	32.97	33.34
11	35.60	34.91

This table shows that the answer is between 10 and 11 years. Continuing the calculations we find that if the life of the principal-protected note is 10.35 year or more, it is profitable for the bank. (Excel's Solver can be used in conjunction with the DerivaGem functions to facilitate calculations.)

Further Questions

Problem 11.20
A trader creates a bear spread by selling a six-month put option with a $25 strike price for $2.15 and buying a six-month put option with a $29 strike price for $4.75. What is

the initial investment? What is the total payoff when the stock price in six months is (a) $23, (b) $28, and (c) $33.

The initial investment is $2.60. (a) $4, (b) $1, and (c) 0.

Problem 11.21
A trader sells a strangle by selling a call option with a strike price of $50 for $3 and selling a put option with a strike price of $40 for $4. For what range of prices of the underlying asset does the trader make a profit?
The trader makes a profit if the total payoff is less than $7. This happens when the price of the asset is between $33 and $57.

Problem 11.22.
Three put options on a stock have the same expiration date and strike prices of $55, $60, and $65. The market prices are $3, $5, and $8, respectively. Explain how a butterfly spread can be created. Construct a table showing the profit from the strategy. For what range of stock prices would the butterfly spread lead to a loss?

A butterfly spread is created by buying the $55 put, buying the $65 put and selling two of the $60 puts. This costs $3+8-2\times5=\$1$ initially. The following table shows the profit/loss from the strategy.

Stock Price	Payoff	Profit
$S_T \geq 65$	0	-1
$60 \leq S_T < 65$	$65-S_T$	$64-S_T$
$55 \leq S_T < 60$	S_T-55	S_T-56
$S_T < 55$	0	-1

The butterfly spread leads to a loss when the final stock price is greater than $64 or less than $56.

Problem 11.23.
A diagonal spread is created by buying a call with strike price K_2 and exercise date T_2 and selling a call with strike price K_1 and exercise date $T_1(T_2 > T_1)$. Draw a diagram showing the profit from the spread at time T_1 when (a) $K_2 > K_1$ and (b) $K_2 < K_1$.

There are two alternative profit patterns for part (a). These are shown in Figures S11.2 and S11.3. In Figure S11.2 the long maturity (high strike price) option is worth more than the short maturity (low strike price) option. In Figure S11.3 the reverse is true. There is no ambiguity about the profit pattern for part (b). This is shown in Figure S11.4.

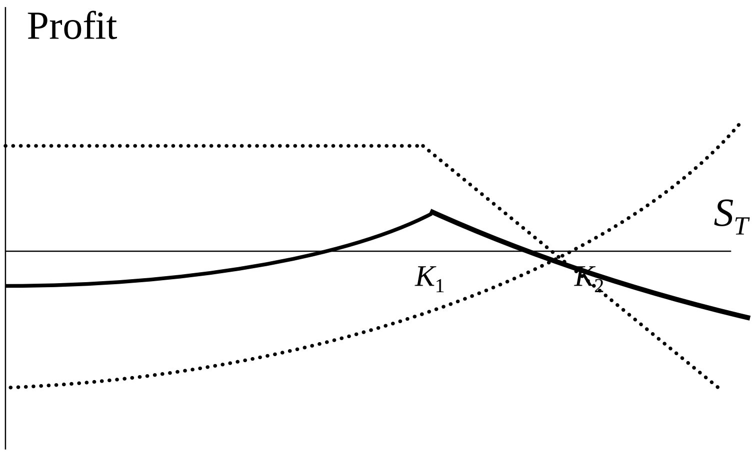

Figure S11.2: Investor’s Profit/Loss in Problem 11.23a when long maturity call is worth more than short maturity call

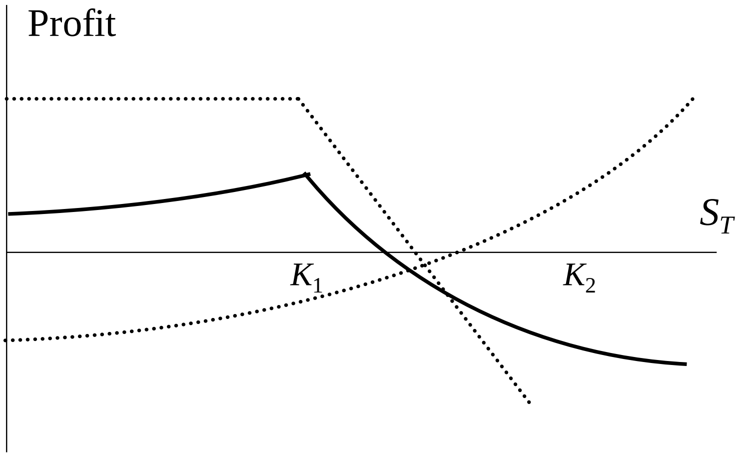

Figure S11.3 Investor’s Profit/Loss in Problem 11.23b when short maturity call is worth more than long maturity call

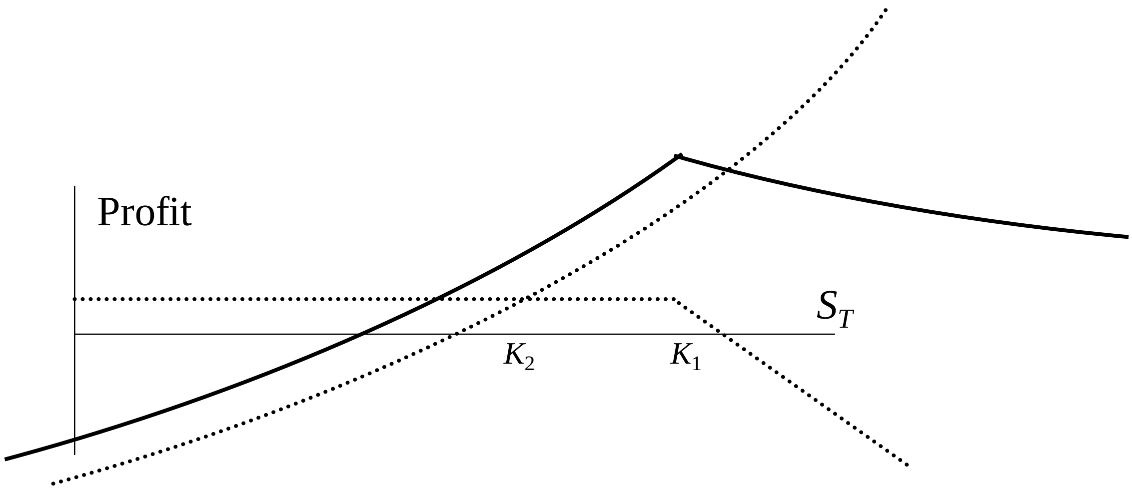

Figure S11.4 Investor's Profit/Loss in Problem 11.23b

Problem 11.24.

Draw a diagram showing the variation of an investor's profit and loss with the terminal stock price for a portfolio consisting of

a. *One share and a short position in one call option*
b. *Two shares and a short position in one call option*
c. *One share and a short position in two call options*
d. *One share and a short position in four call options*

In each case, assume that the call option has an exercise price equal to the current stock price.

The variation of an investor's profit/loss with the terminal stock price for each of the four strategies is shown in Figure S11.5. In each case the dotted line shows the profits from the components of the investor's position and the solid line shows the total net profit.

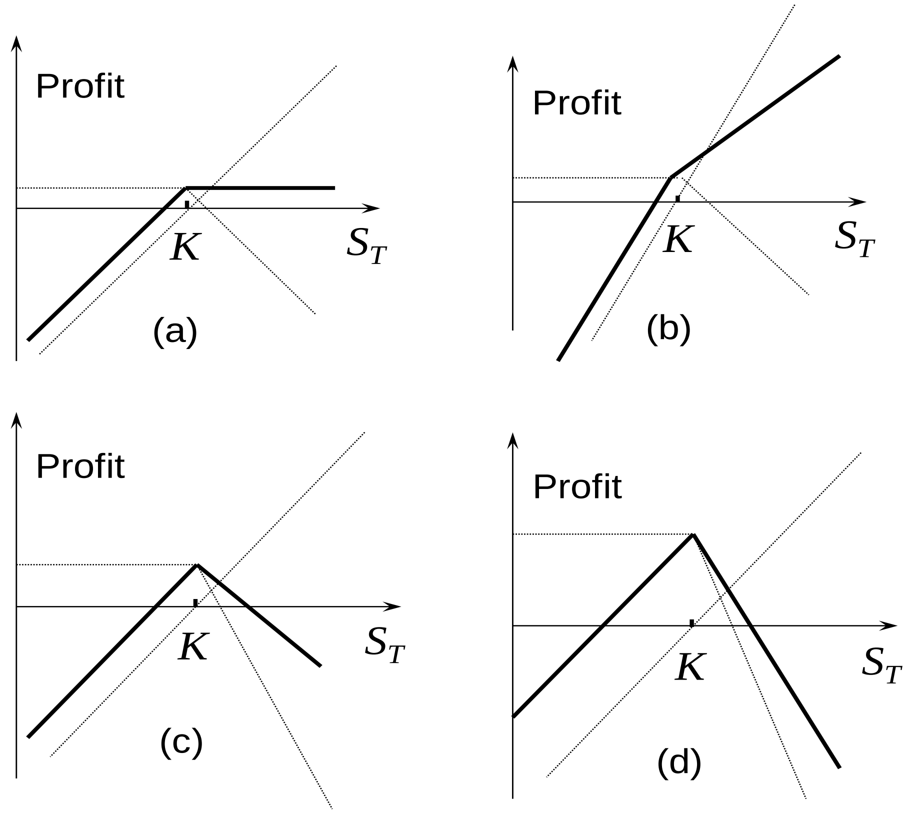

Figure S11.5 Answer to Problem 11.24

Problem 11.25.
Suppose that the price of a non-dividend-paying stock is \$32, its volatility is 30%, and the risk-free rate for all maturities is 5% per annum. Use DerivaGem to calculate the cost of setting up the following positions. In each case provide a table showing the relationship between profit and final stock price. Ignore the impact of discounting.

a. *A bull spread using European call options with strike prices of \$25 and \$30 and a maturity of six months.*
b. *A bear spread using European put options with strike prices of \$25 and \$30 and a maturity of six months*
c. *A butterfly spread using European call options with strike prices of \$25, \$30, and \$35 and a maturity of one year.*
d. *A butterfly spread using European put options with strike prices of \$25, \$30, and \$35 and a maturity of one year.*
e. *A straddle using options with a strike price of \$30 and a six-month maturity.*
f. *A strangle using options with strike prices of \$25 and \$35 and a six-month maturity.*

(a) A call option with a strike price of 25 costs 7.90 and a call option with a strike price of 30 costs 4.18. The cost of the bull spread is therefore $7.90-4.18=3.72$. The profits ignoring the impact of discounting are

Stock Price Range	*Profit*
$S_T \le 25$	–3.72
$25 < S_T < 30$	$S_T - 28.72$
$S_T \ge 30$	1.28

(b) A put option with a strike price of 25 costs 0.28 and a put option with a strike price of 30 costs 1.44. The cost of the bear spread is therefore $1.44 - 0.28 = 1.16$. The profits ignoring the impact of discounting are

Stock Price Range	*Profit*
$S_T \le 25$	+3.84
$25 < S_T < 30$	$28.84 - S_T$
$S_T \ge 30$	-1.16

(c) Call options with maturities of one year and strike prices of 25, 30, and 35 cost 8.92, 5.60, and 3.28, respectively. The cost of the butterfly spread is therefore $8.92 + 3.28 - 2 \times 5.60 = 1.00$. The profits ignoring the impact of discounting are

Stock Price Range	*Profit*
$S_T \le 25$	-1.00
$25 < S_T < 30$	$S_T - 26.00$
$30 \le S_T < 35$	$34.00 - S_T$

(d) Put options with maturities of one year and strike prices of 25, 30, and 35 cost 0.70, 2.14, 4.57, respectively. The cost of the butterfly spread is therefore $0.70 + 4.57 - 2 \times 2.14 = 0.99$. Allowing for rounding errors, this is the same as in (c). The profits are the same as in (c).

(e) A call option with a strike price of 30 costs 4.18. A put option with a strike price of 30 costs 1.44. The cost of the straddle is therefore $4.18 + 1.44 = 5.62$. The profits ignoring the impact of discounting are

Stock Price Range	*Profit*
$S_T \le 30$	$24.38 - S_T$
$S_T > 30$	$S_T - 35.62$

(f) A six-month call option with a strike price of 35 costs 1.85. A six-month put option with a strike price of 25 costs 0.28. The cost of the strangle is therefore $1.85 + 0.28 = 2.13$. The profits ignoring the impact of discounting are

Stock Price Range	*Profit*
$S_T \le 25$	$22.87 - S_T$
$25 < S_T < 35$	-2.13
$S_T \ge 35$	$S_T - 37.13$

Problem 11.26

What trading position is created from a long strangle and a short straddle when both have the same time to maturity? Assume that the strike price in the straddle is halfway between the two strike prices of the strangle.

A butterfly spread (together with a cash position) is created.

Problem 11.27 (Excel file)

Describe the trading position created in which a call option is bought with strike price K_2 and a put option is sold with strike price K_1 when both have the same time to maturity and $K_2 > K_1$. What does the position become when $K_1 = K_2$?

The position is as shown in Figure S11.6 (for $K_1 = 25$ and $K_2 = 35$). It is known as a range forward and is discussed further in Chapter 15. When $K_1 = K_2$, the position becomes a regular long forward.

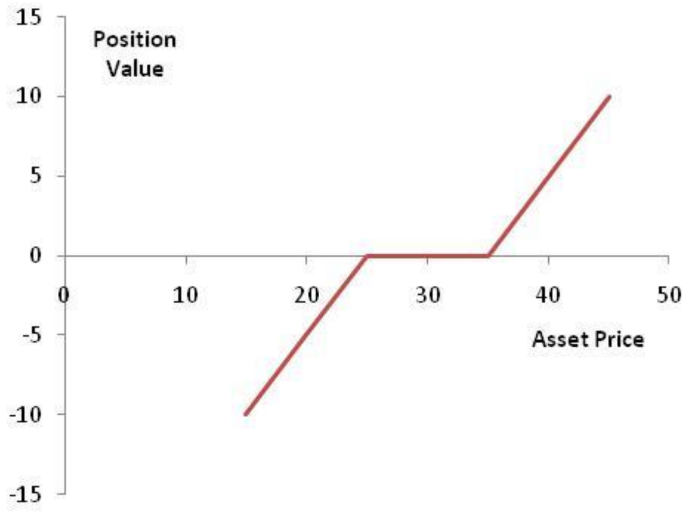

Figure S11.6 Trading position in Problem 11.27

Problem 11.28

A bank decides to create a five-year principal-protected note on a non-dividend-paying stock by offering investors a zero-coupon bond plus a bull spread created from calls. The risk-free rate is 4% and the stock price volatility is 25%. The low-strike-price option in the bull spread is at the money. What is the maximum ratio of the high strike price to the low strike price in the bull spread. Use DerivaGem.

Assume that the amount invested is 100. (This is a scaling factor.) The amount available to create the option is $100-100e^{-0.04\times5}=18.127$. The cost of the at-the money option can be calculated from DerivaGem by setting the stock price equal to 100, the volatility equal to 25%, the risk-free interest rate equal to 4%, the time to exercise equal to 5 and the exercise price equal to 100. It is 30.313. We therefore require the option given up by the investor to be worth at least $30.313-18.127 = 12.186$. Results obtained are as follows:

Strike	*Option Value*
125	21.12
150	14.71
175	10.29
165	11.86

Continuing in this way we find that the strike must be set below 163.1. The ratio of the high strike to the low strike must therefore be less than 1.631 for the bank to make a profit. (Excel's Solver can be used in conjunction with the DerivaGem functions to facilitate calculations.)