CHAPTER 17

THE GREEKS

FIN2325 with Dr. Velthuis

Today

- Option Delta
  - Delta hedging
- Option Gamma
- Option Vega
- Theta and Rho
- *Game*

Measures of Risk

- Equities
  - Beta

- Bonds
  - Duration

- Derivatives
  - Greeks

Option Greeks

- Each Greek Letter measures a different dimension to the risk in an option position

- The aim of a trader is to manage the Greeks so that all risks are acceptable

- Most applicable to:
    - market makers in options on an exchange
    - over the counter traders working for financial institutions

Example

- A bank has sold for \$300,000 a European call option on 100,000 shares of a non-dividend-paying stock
- S = 49, K = 50, r = 5%, σ = 20%,

  T = 20 weeks, μ = 13%
- The Black-Scholes value of the option is \$240,000
- How does the bank hedge its risk?

NAKED & COVERED POSITIONS

- Naked position
  - Take no action
- Covered position
  - Buy 100,000 shares today

- Both strategies leave the bank exposed to significant risk

Stop-Loss Strategy

This involves:

- Buying 100,000 shares as soon as price reaches $50
- Selling 100,000 shares as soon as price falls below $50

This deceptively simple hedging strategy does not work well

Black-Scholes Model

- Call and put option prices depend on five key factors:
  - S (and Div or q), K, σ, r, T
  - K is known in advance
- We can use the BS model to measure the sensitivity of the price to each risk factor by computing first (and second) derivatives

"adjusted probability"

$$c = S\underbrace{N(d_1)} - Ke^{-rT}\underbrace{N(d_2)}$$

risk-neutral prob. of exercising call option

$$p = Ke^{-rT}N(-d_2) - SN(-d_1)$$

where $$d_1 = \frac{\ln(S/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

$\frac{\partial c}{\partial S}$

TAKING DERIVATIVES…

$\frac{\partial p}{\partial S}$

Greek letter	Risk source	Call option	Put option
Delta, Δ	S'	$N(d_1)$ between $(0,1)$	$N(d_1)-1$ between $(-1,0)$
Gamma, Γ	S'' or Δ'	$\frac{N'(d_1)}{S\sigma\sqrt{T}}$	$\frac{N'(d_1)}{S\sigma\sqrt{T}}$
Theta, Θ	T'	$\frac{SN'(d_1)\sigma}{2\sqrt{T}}-rKe^{-rT}N(d_2)$	$-\frac{SN'(d_1)\sigma}{2\sqrt{T}}+rKe^{-rT}N(-d_2)$
Vega, υ	σ'	$S\sqrt{T}N'(d_1)$	$S\sqrt{T}N'(d_1)$
Rho, ρ	R'	$KTe^{-rT}N(d_2)$	$-KTe^{-rT}N(-d_2)$

where N'() is the normal pdf

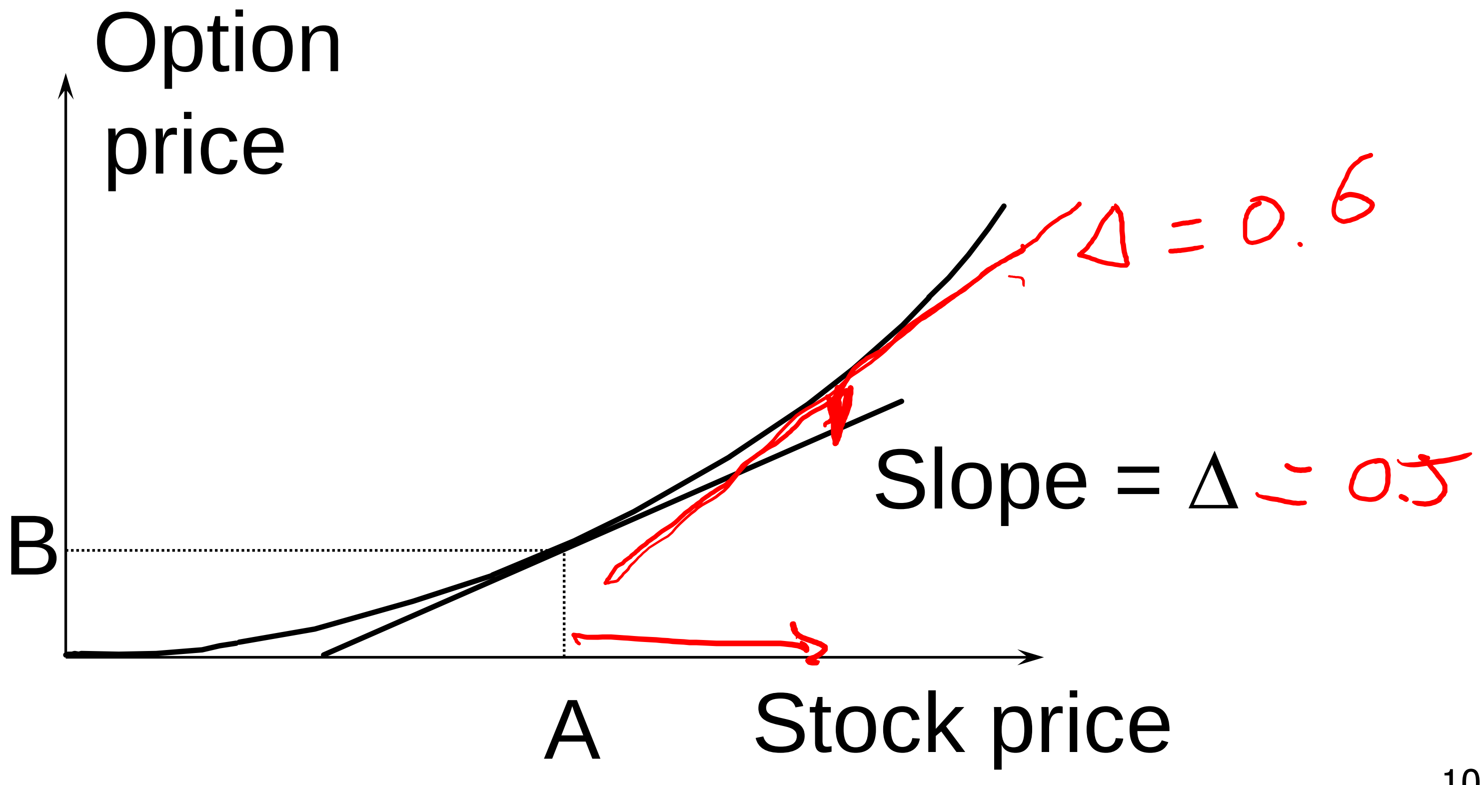
and $d_1 = \frac{\ln(S/K)+(r+\sigma^2/2)T}{\sigma\sqrt{T}}$

$$d_2 = d_1 - \sigma\sqrt{T}$$

Use DerivaGem to plot each measure against one of the key input factors

DELTA

- Delta (Δ) is the rate of change of the option price with respect to price of the underlying asset



DELTA

- The delta of a European call on a non-dividend-paying stock is $N(d_1)$

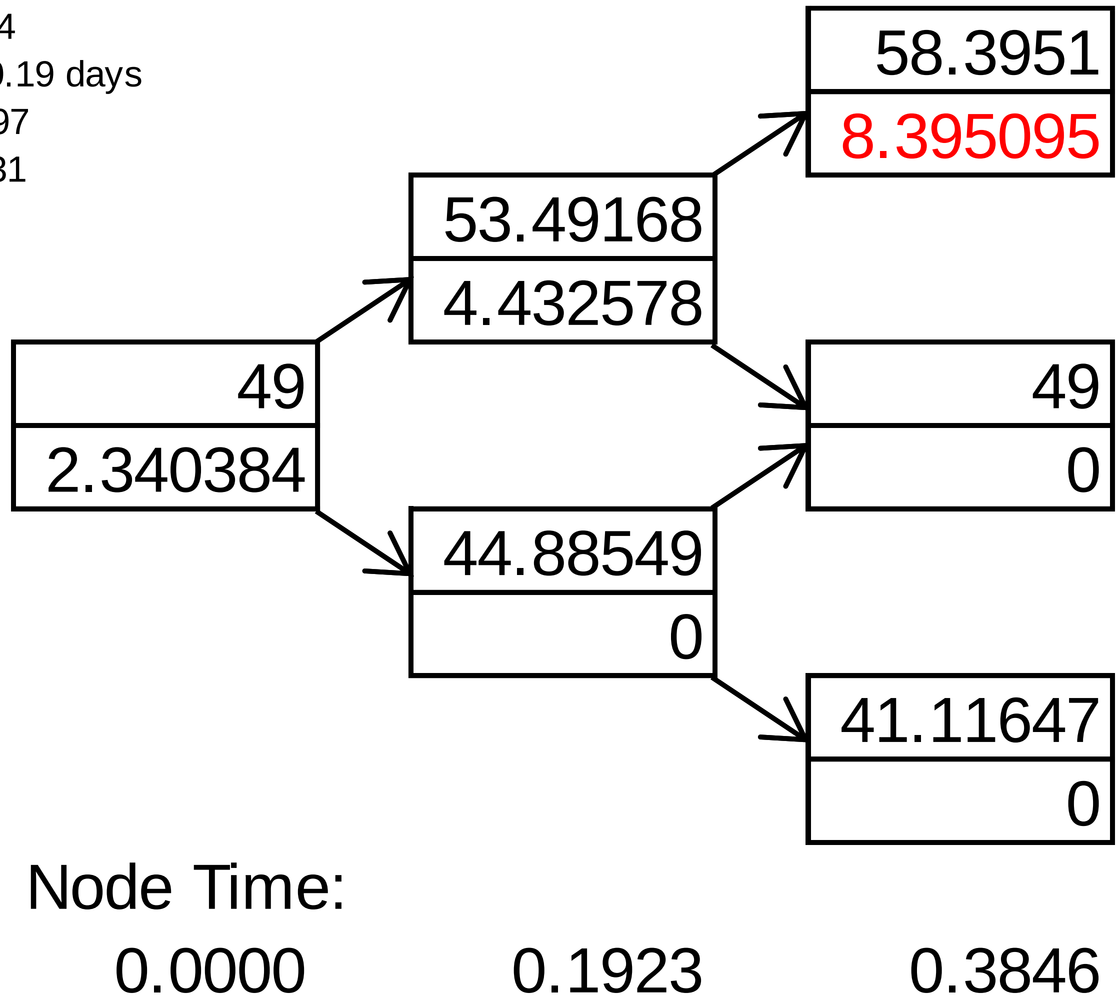
- The delta of a European put on the stock is

$$[N(d_1) - 1]$$

- In the example, what is the delta of the option?

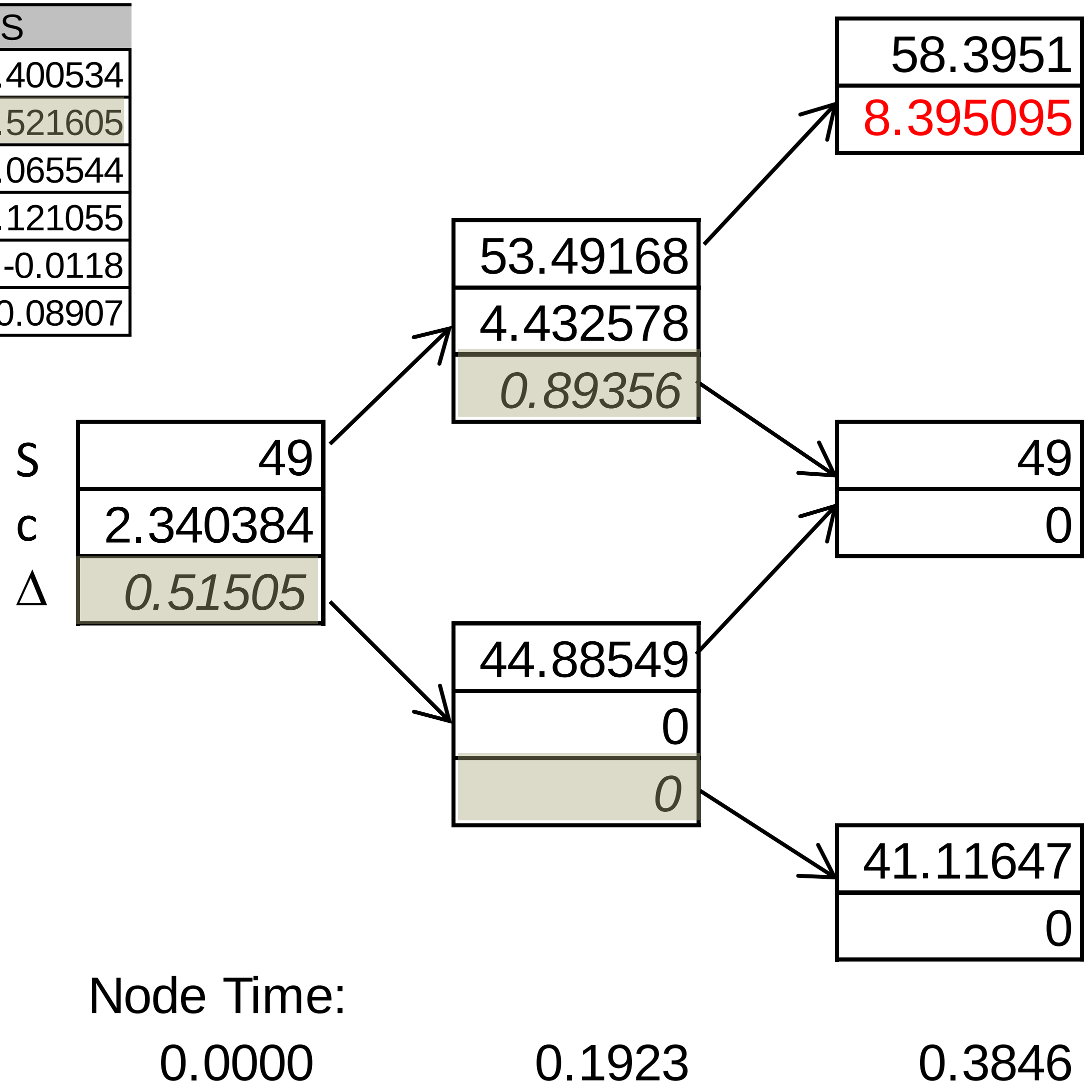
Delta (Binomial Tree)

Strike price = 50
Discount factor per step = 0.9904
Time step, dt = 0.1923 years, 70.19 days
Growth factor per step, a = 1.0097
Probability of up move, p = 0.5331
Up step size, u = 1.0917
Down step size, d = 0.9160

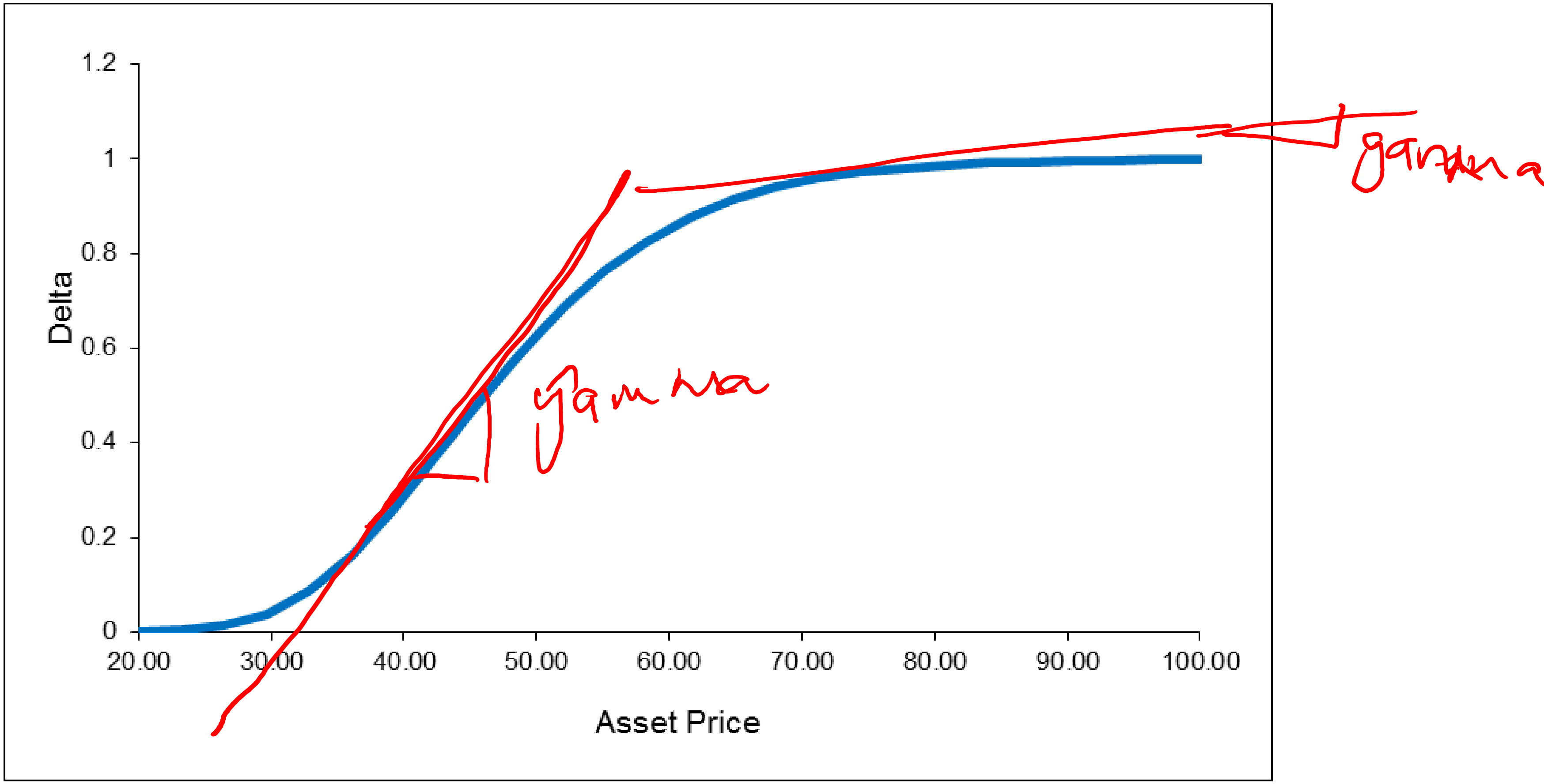


Delta (Binomial Tree)

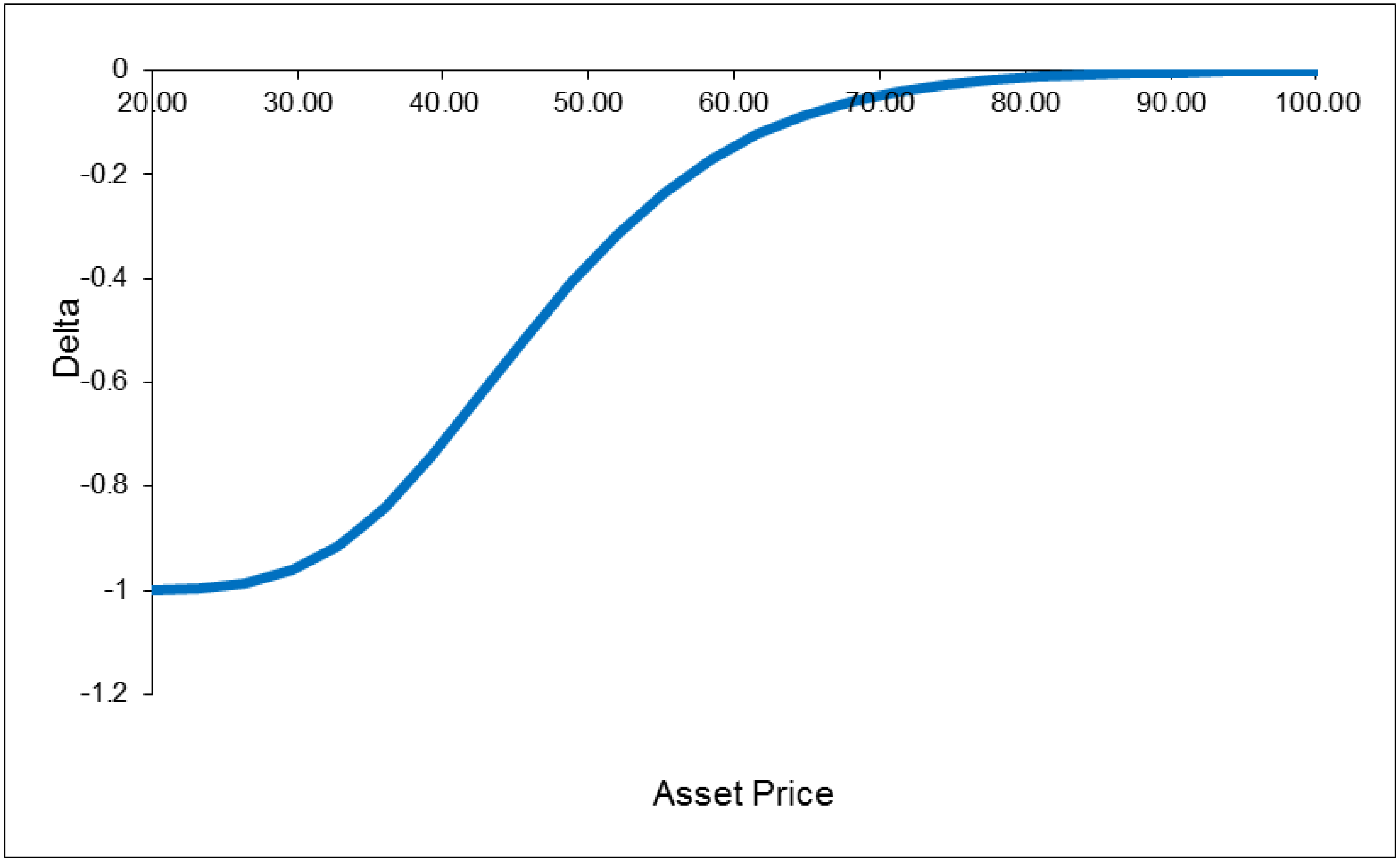
	Binomial	B-S
Price:	2.340384	2.400534
Delta (per $):	0.515045	0.521605
Gamma (per $ per $):	0.10343	0.065544
Vega (per %):	0.10886	0.121055
Theta (per day):	-0.01667	-0.0118
Rho (per %):	0.088807	0.08907



Delta for Call Option: S_0=K=50, σ = 25%, r = 5%, T = 1



DELTA FOR PUT OPTION: S_0=K=50, σ = 25%, r = 5%, T = 1



Delta Hedging

- Example:
  - The delta of the bank's short position of option is

    - 0.522 * 100,000 = -52,200
  - Thus, 52,200 shares need to be purchased $(\Delta_S = 1)$
  - After hedging, the net delta of the total portfolio is zero
    → delta neutral

$$\Delta_c \cdot W_c + \Delta_S \cdot W_S = -0.522 \times 100{,}000 + 1 \times 52{,}200$$

$$= 0$$

Delta of a Portfolio

- The delta of a portfolio Π can be computed from the deltas of the individual securities in the portfolio

$$\Delta_\Pi = \sum_{i=1}^{n} w_i \Delta_i = w_1\Delta_1 + w_2\Delta_2 \cdots$$

where

- Δ_Π is the delta of the portfolio
- Δ_i is the delta of security i
- w_i is the quantity of security *i* in the portfolio

Delta of a Portfolio: Example

- Suppose a financial institution has the following three positions in options on a stock:
  - Long position in 100,000 call options, K = 55, T = 3 months, delta = 0.533
  - Short position in 200,000 call options, K = 56, T = 5 months, delta = 0.468
  - Short position in 50,000 put options, K = 56, T = 2 months, delta = -0.508
- The delta of the whole portfolio is:

$$100{,}000 \times 0.533 - 200{,}000 \times 0.468 - 50{,}000 \times (-0.508) = -14{,}900$$

- The portfolio can be made delta-neutral by buying 14,900 shares

Delta Hedging

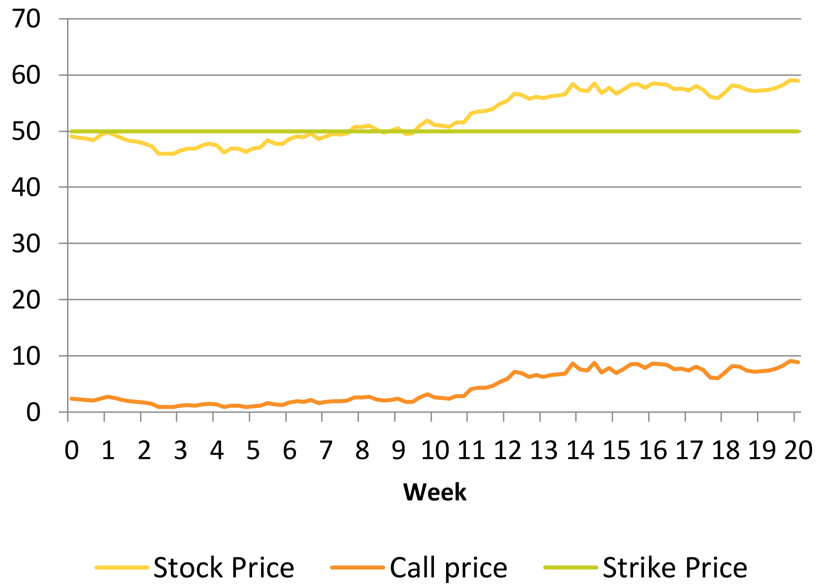
- Delta hedging involves maintaining a delta neutral portfolio
  - A portfolio with a delta of *zero* is referred to as being delta neutral

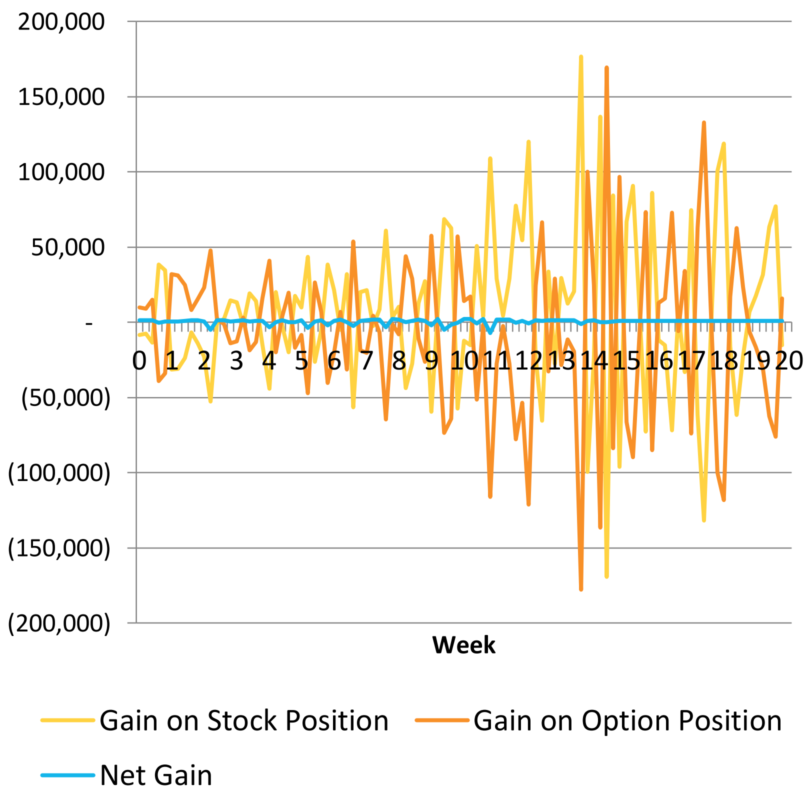
Delta Hedging

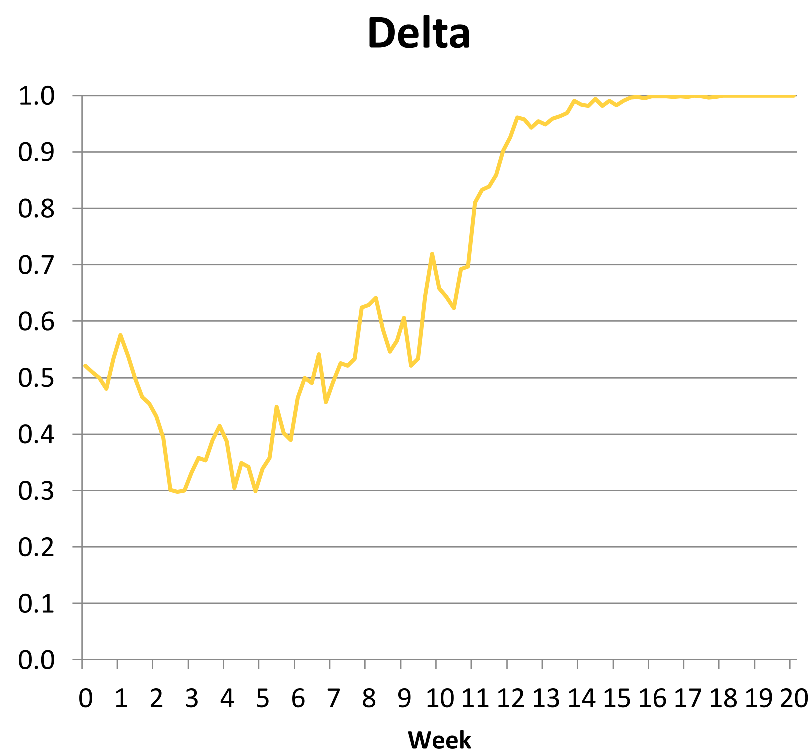
- Delta of option changes
- Delta-hedged portfolio remains delta neutral for only a short period of time
- The hedged position must be frequently rebalanced
- There are economies of scale when hedging a large portfolio consisting of many options

Delta Hedging

Daily Rebalancing







GAMMA $\frac{\partial^2 c}{(\partial S)^2}$

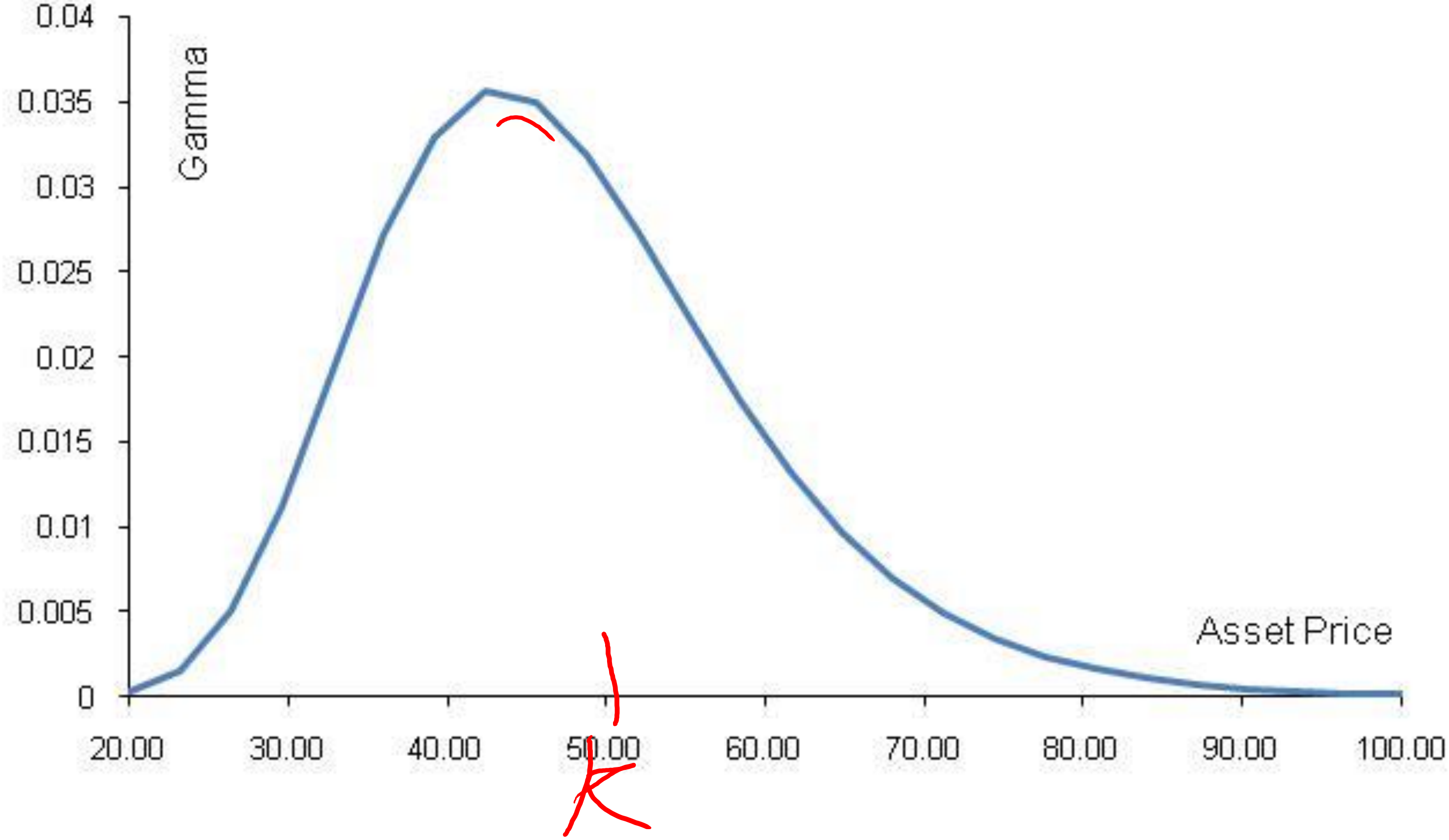
- Gamma (Γ) is the rate of change of delta (Δ) with respect to the price of the underlying asset

Gamma

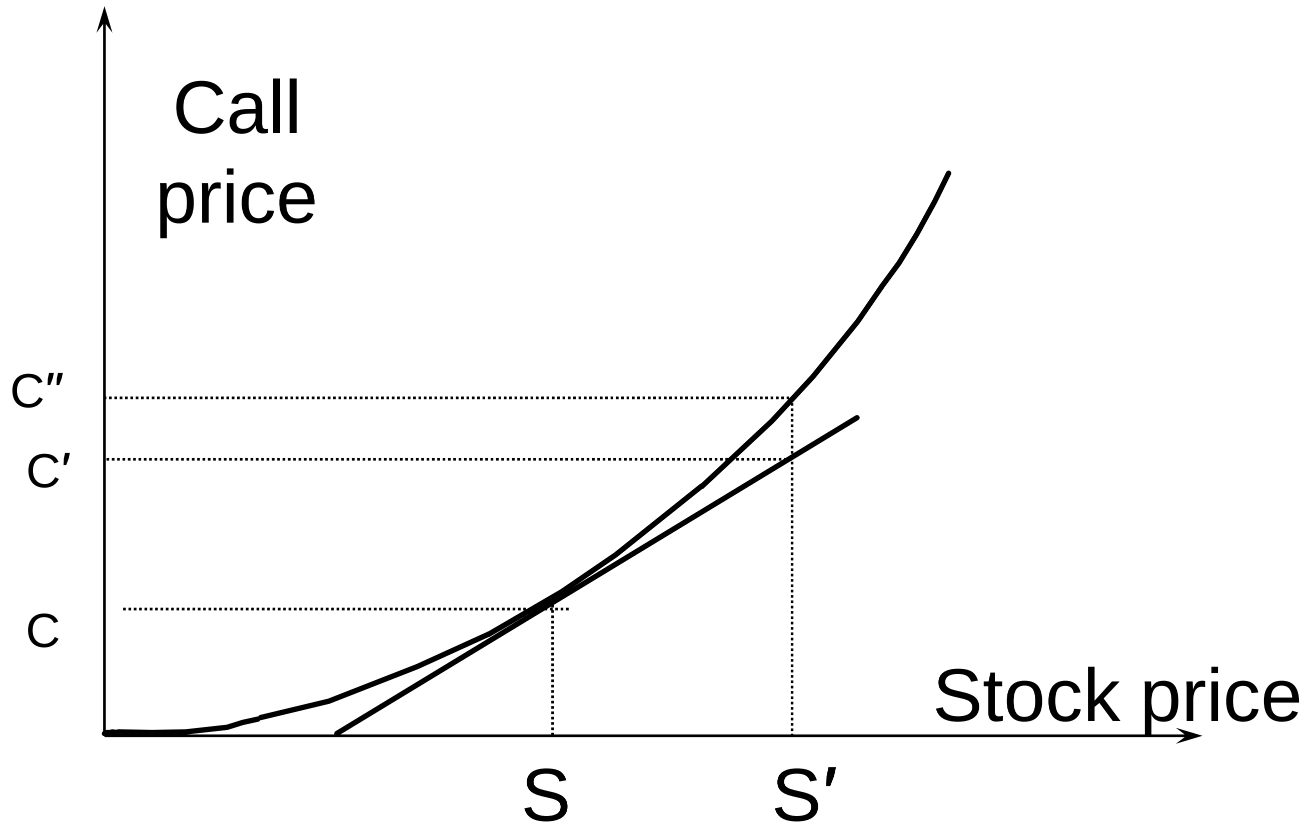
- If gamma is small, delta changes slowly.

- However, if gamma is large, delta is highly sensitive to the change in the price of the underlying asset.

Gamma for Call or Put Option: $S_0=K=50$, $\sigma = 25\%$, $r = 5\%$, $T = 1$



Gamma Addresses Delta Hedging Errors Caused By Curvature



Making a Portfolio Gamma Neutral

- A position in the underlying asset itself has zero gamma, and cannot be used to change the gamma of a portfolio
- Thus, making a portfolio gamma neutral requires taking a position in another traded option
  - Side effect: including another traded option will change the delta of the portfolio.
  - So, to obtain delta neutrality, the position in the underlying asset has to be adjusted accordingly.

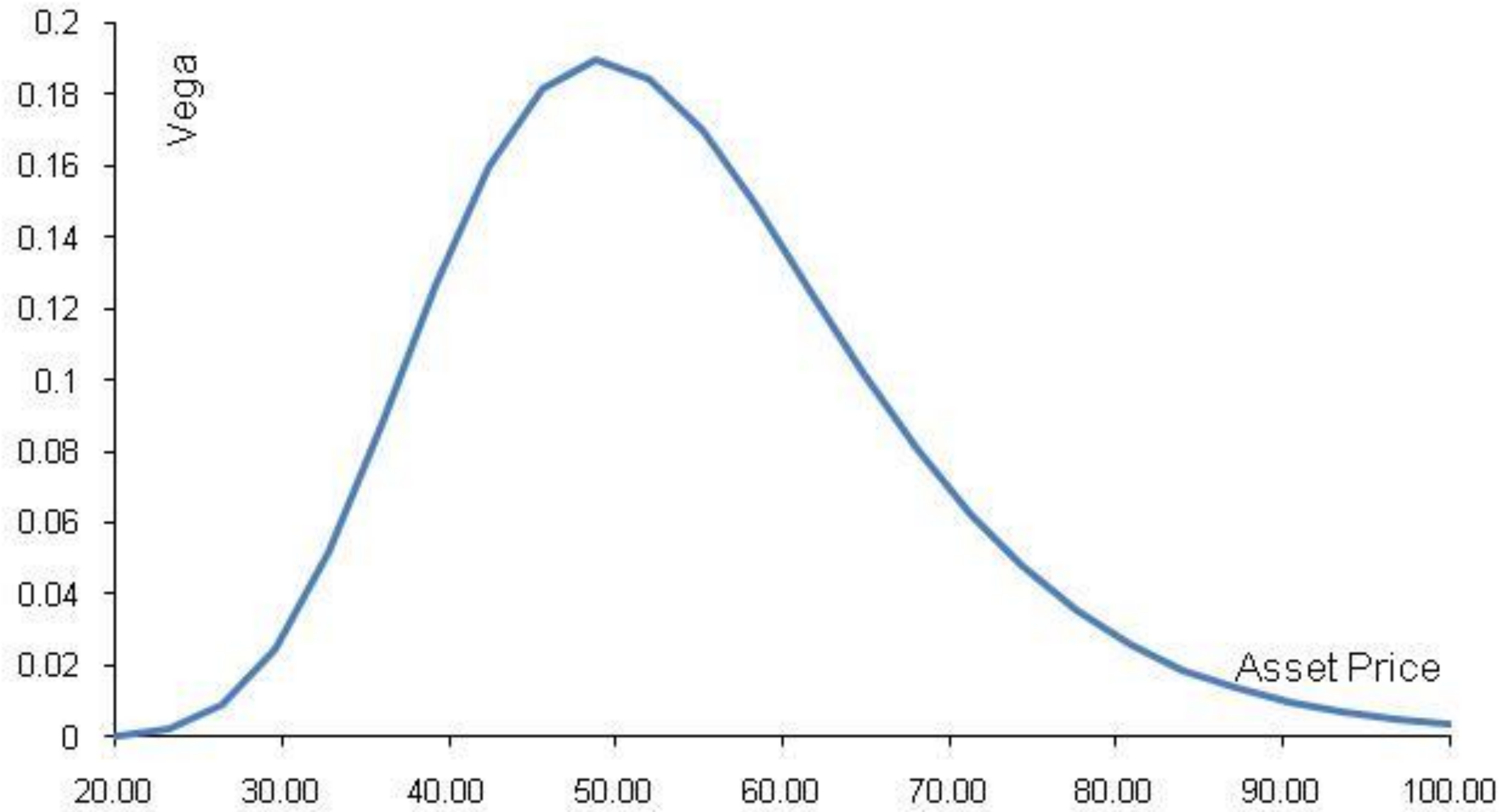
Making a Portfolio Gamma Neutral - Example

- A trader's portfolio is delta neutral and has a gamma of -3,000.
- The delta and gamma of a particular traded call option are 0.62 and 1.5, respectively.
- The trader wants to make the portfolio gamma neutral as well as delta neutral. He or She can:
  1. Make portfolio gamma neutral by buying 3,000/1.5 =2,000 options (20 contracts)
  2. Sell 2,000*0.62=1,240 units of the underlying asset to maintain delta neutrality

Vega

- Vega ($\mathcal{V}$) is the rate of change of the value of a derivatives portfolio with respect to volatility

Vega for Call or Put Option: $S_0 = K = 50$, $\sigma = 25\%$, $r = 5\%$, $T = 1$



Managing Vega

- A position in the underlying asset has zero vega
- To adjust vega, it is necessary to take a position in an traded option or other derivative

Managing Gamma & Vega Neutral

- A portfolio that is gamma neutral will not in general be vega neutral.
- To obtain both gamma neutral and vega neutral, at least two traded derivatives must be used

Hedging in Practice

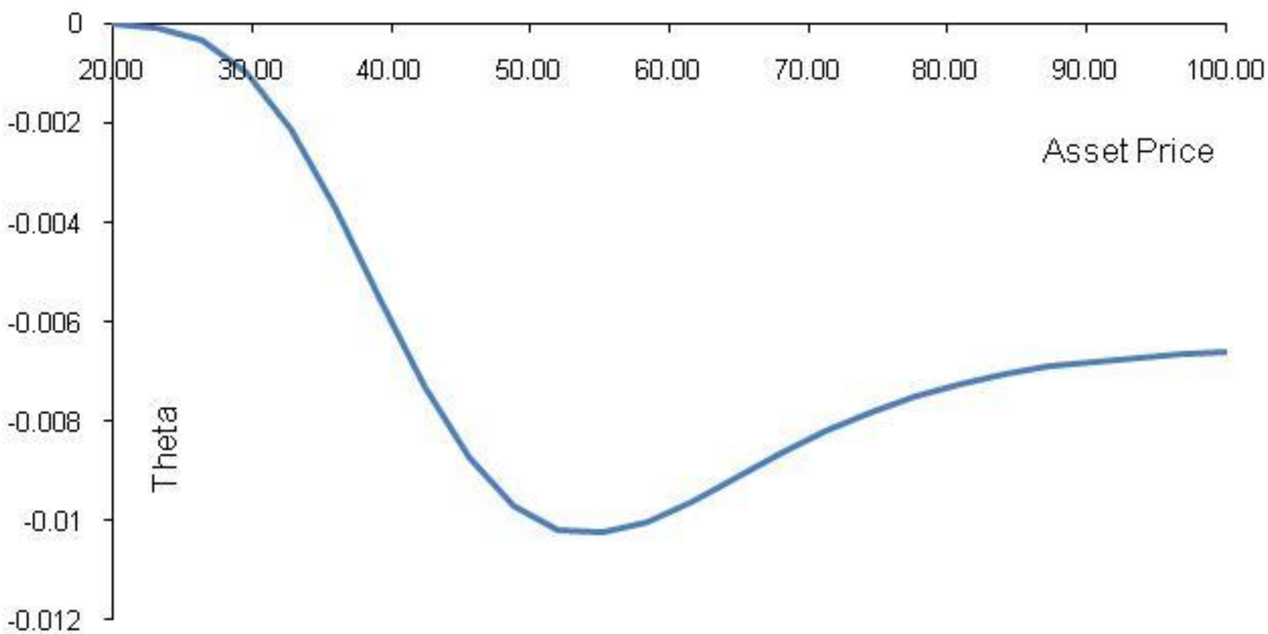
- Traders usually ensure that their portfolios are delta-neutral at least once a day
- Whenever the opportunity arises, they improve gamma and vega

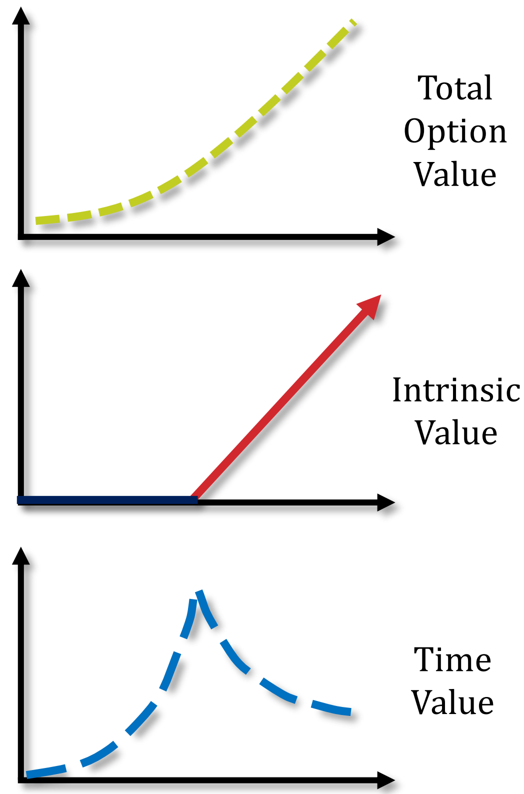
Theta

- Theta (Θ) of a derivative is the rate of change of the value of a derivative with respect to the passage of time

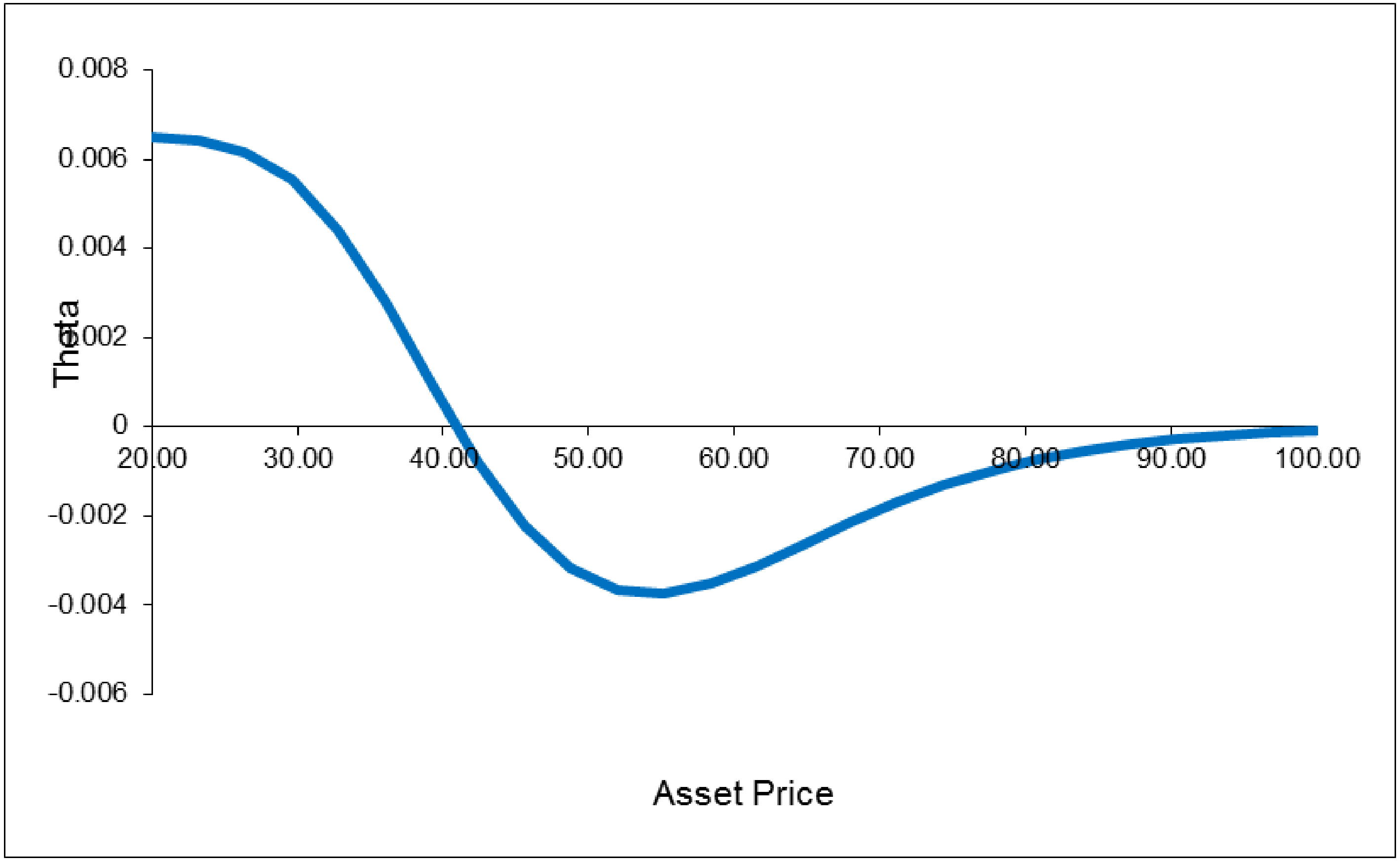
- Not really a risk factor
  - Deterministic
  - Time decay
    - If all the other factors remained the same, one month from now, the option would be worth less

Theta for Call Option: $S_0=K=50$, $\sigma = 25\%$, $R = 5\%$, $T = 1$





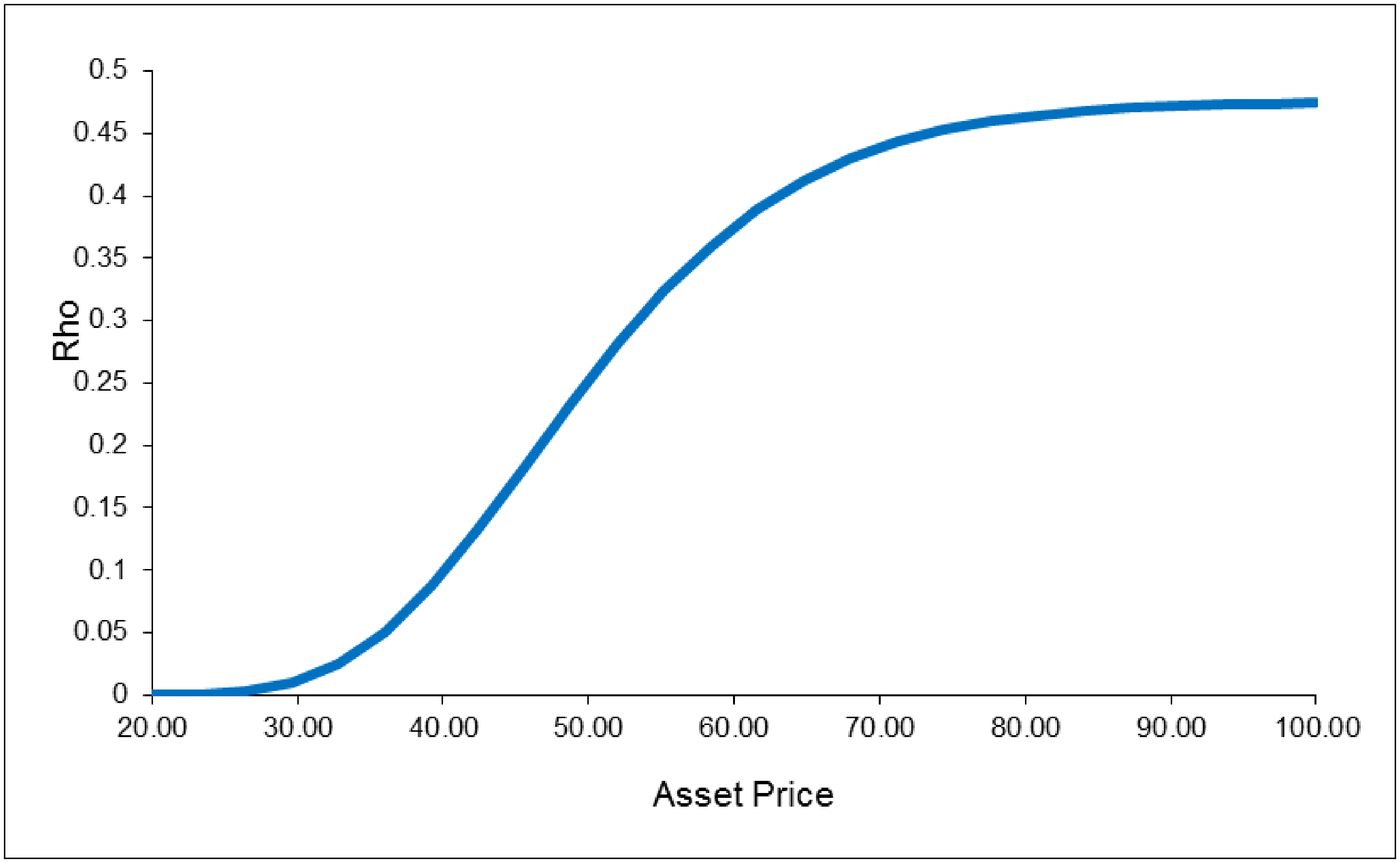
THETA FOR PUT OPTION: S_0=K=50, σ = 25%, r = 5%, T = 1



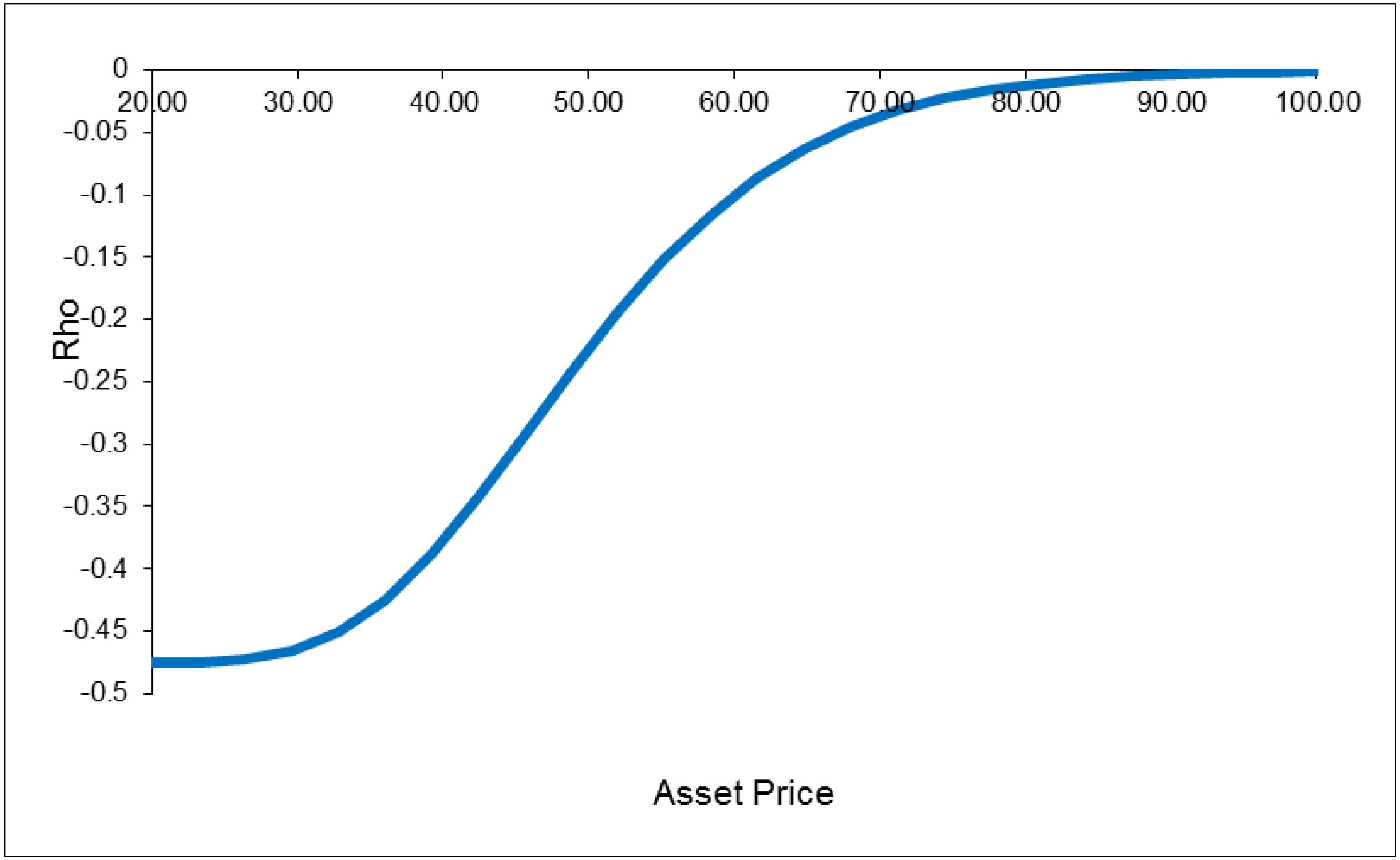
Rho

- Rho (ρ) is the rate of change of the value of a derivative with respect to the interest rate

Rho for Call Option: $S_0=K=50$, $\sigma = 25\%$, $r = 5\%$, $T = 1$



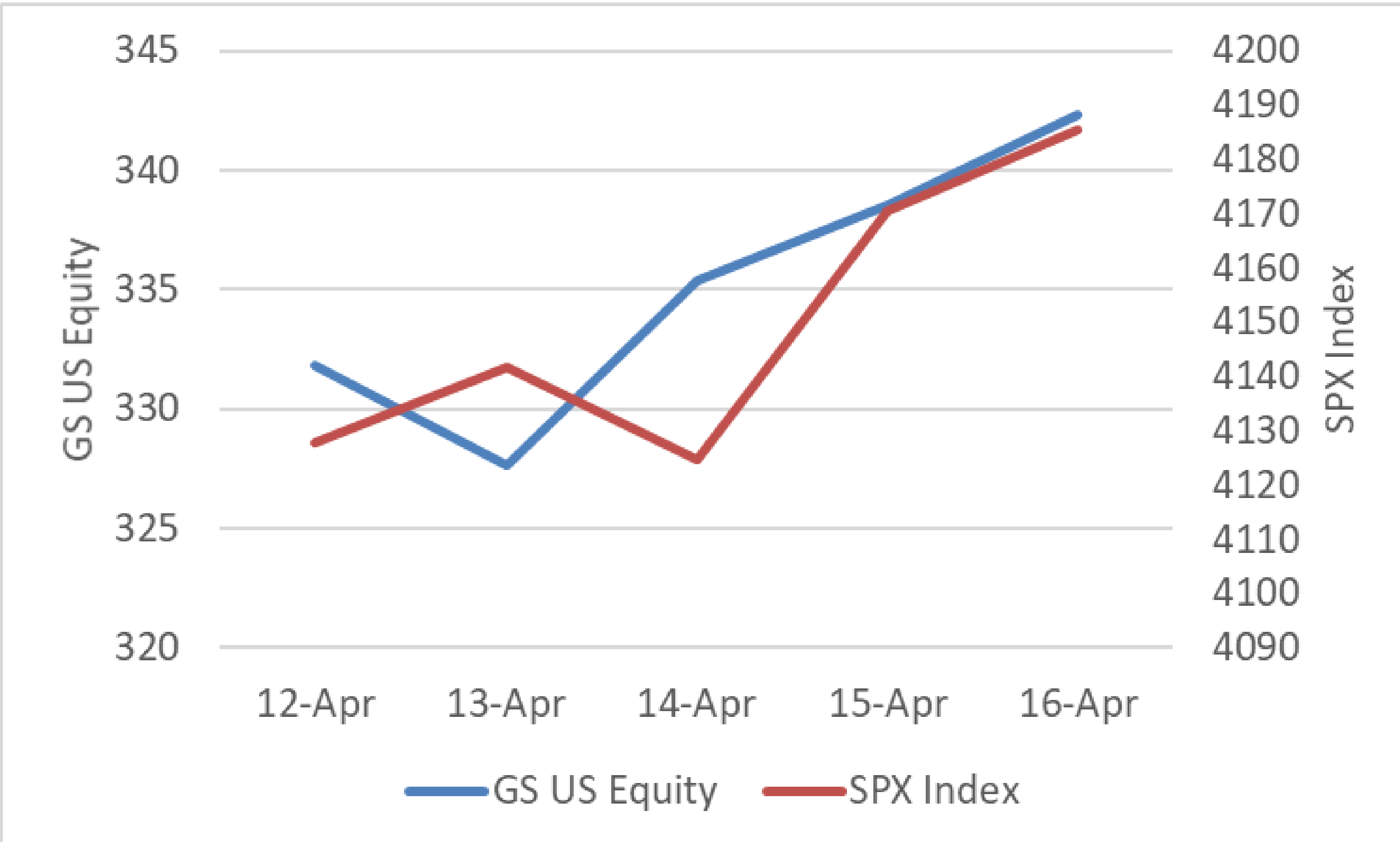
Rho for Put Option: $S_0=K=50$, $\sigma = 25\%$, $r = 5\%$, $T = 1$



The Greeks Options Game

- You have $10,000 and your goal is to grow your assets as much as possible during a one week period by speculating with call options. To do this, you have your eyes set on a company called Greek Salads Inc., which is about to release its Q1 earnings report.
- The first decision (*which option to buy*) is the most important one. After that, you can choose each day whether to exercise the options, sell the options, or wait until the next day.

RESULTS

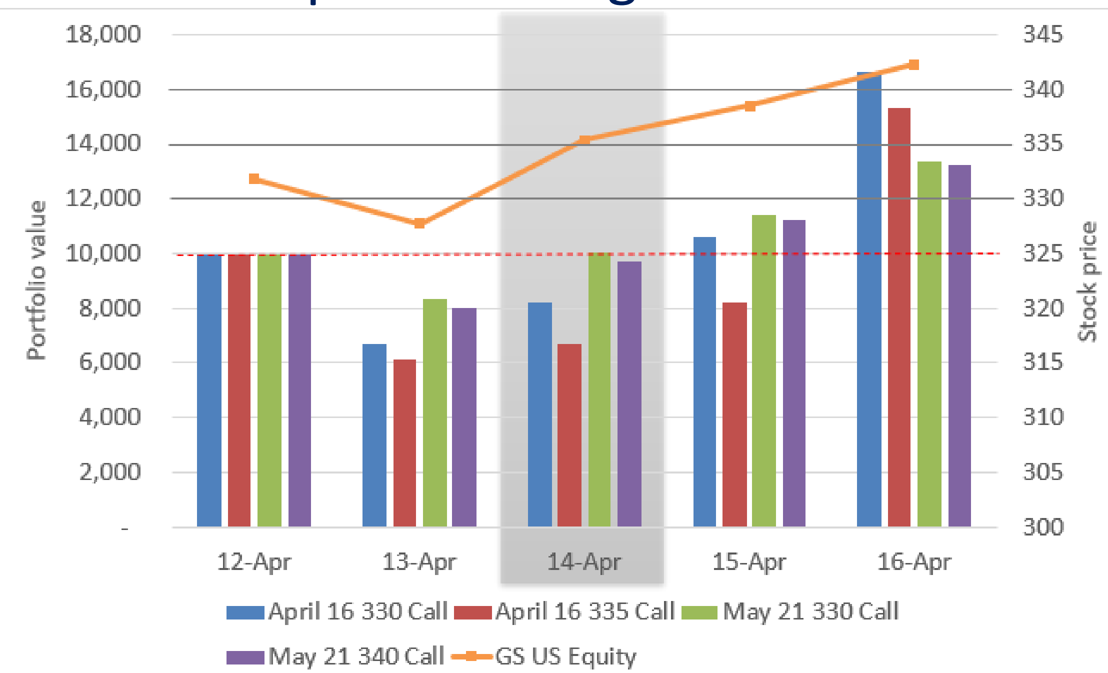

GS US Equity
SPX Index
345
340
335
330
325
320
4200
4190
4180
4170
4160
4150
4140
4130
4120
4110
4100
4090
12-Apr
13-Apr
14-Apr
15-Apr
16-Apr
GS US Equity
SPX Index

QUESTIONS

- *Monday*: What was your strategy?
  - Did you incorporate DerivaGem?
  - How did you decide on option prices / quantity vs. strike / T?
    - IV & Vega, T & Theta, Delta & Gamma… Strike vs cost, breakeven P
- *Tuesday*: did the drop scare you?
  - Loss already 20-40% after 1% drop in stock price
    - Timing is always hard to get right. This drop hurt our play quite a bit and our options didn’t completely recover on earnings day, even though the stock was net positive.
- *Wednesday*: Was the gain in the portfolio what you expected given the rise in the stock price to 335?
  - How do we reconcile the analyst forecasts with the option premiums that we faced?
- Did you continue to “play” after Wednesday 4/14?
- Did you try exercising any options?

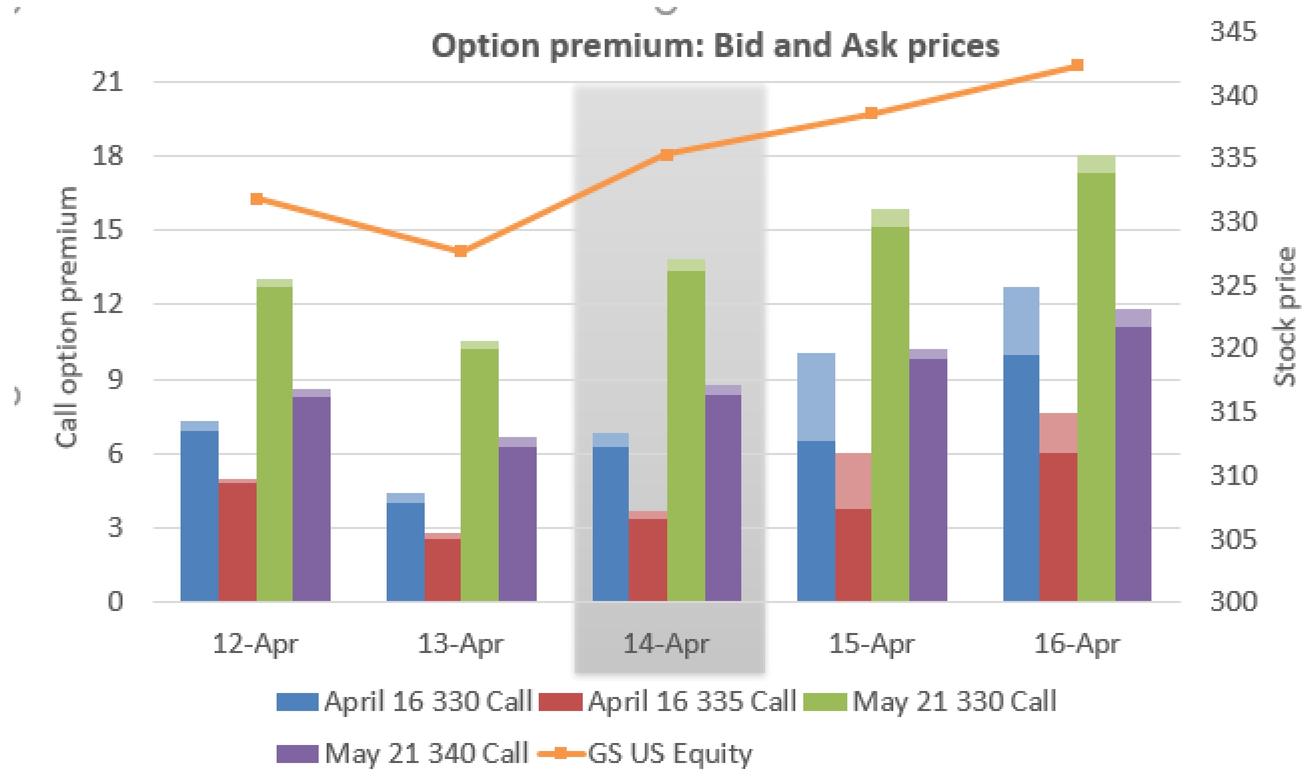
VOLATILITY REDUCTION VS. STOCK PRICE INCREASE

- The short-term options lost some value due to reduced volatility as well as time decay
  - This offset the increase in the stock price on 4/14
- Gains after April 14 are due to luck, not the earnings report (unless there was post-earnings announcement drift (PEAD))



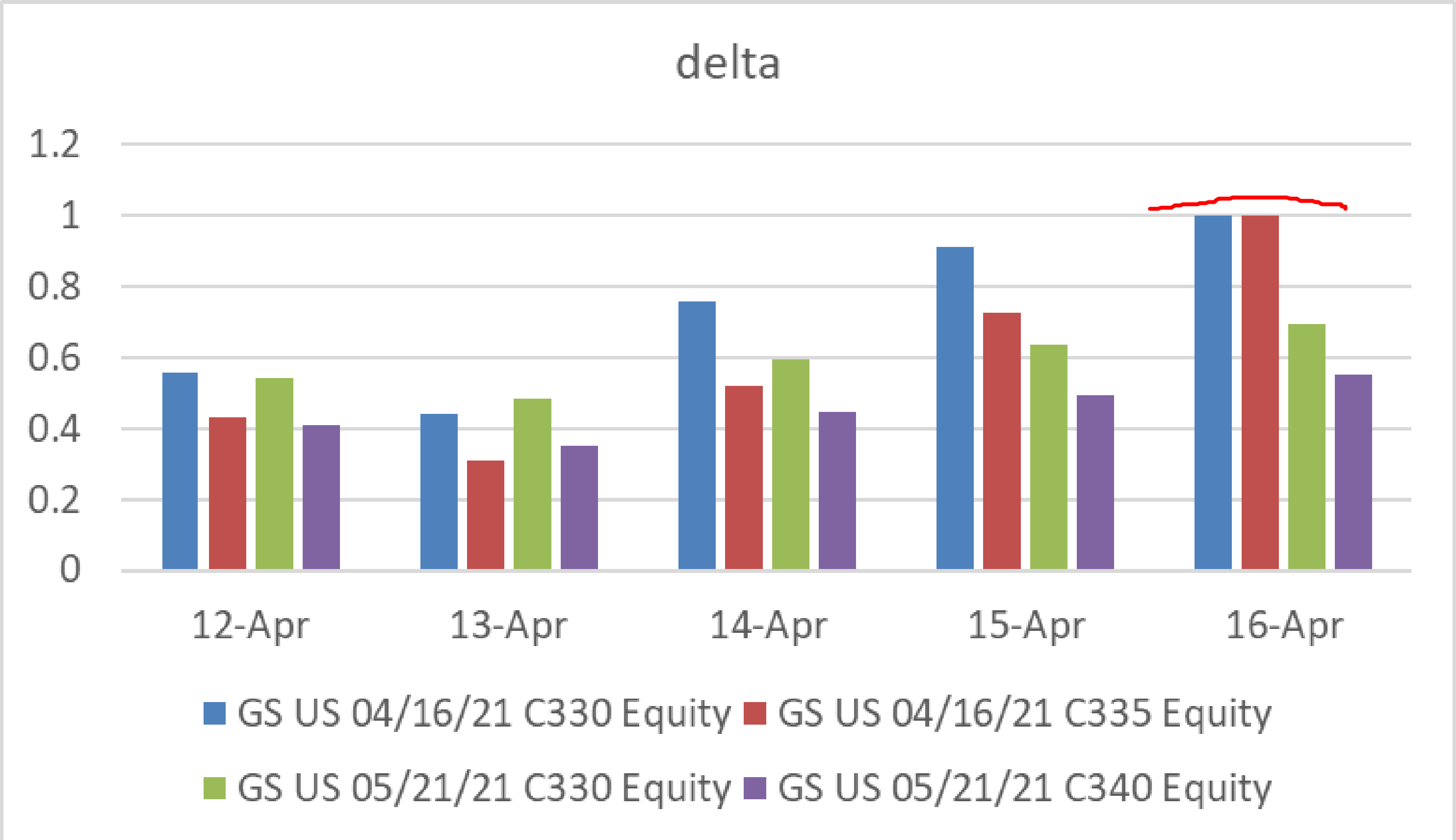
Option premiums

- Bid-ask spreads widen close to the expiration date, possibly because many investors want to avoid exercising the option (which requires funds)



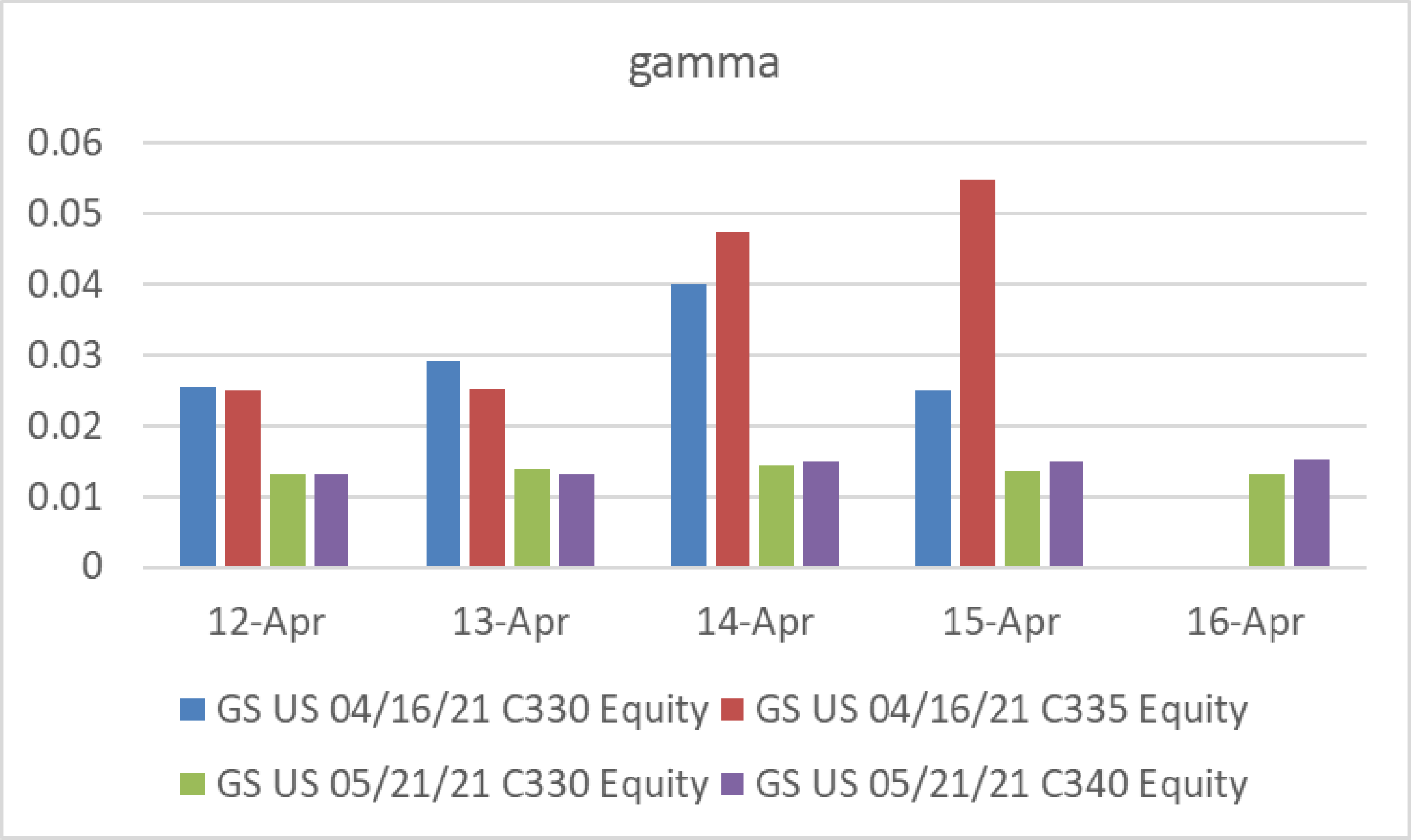
Delta increases with S

- Delta increases as it becomes more likely that the option will expire in-the-money.



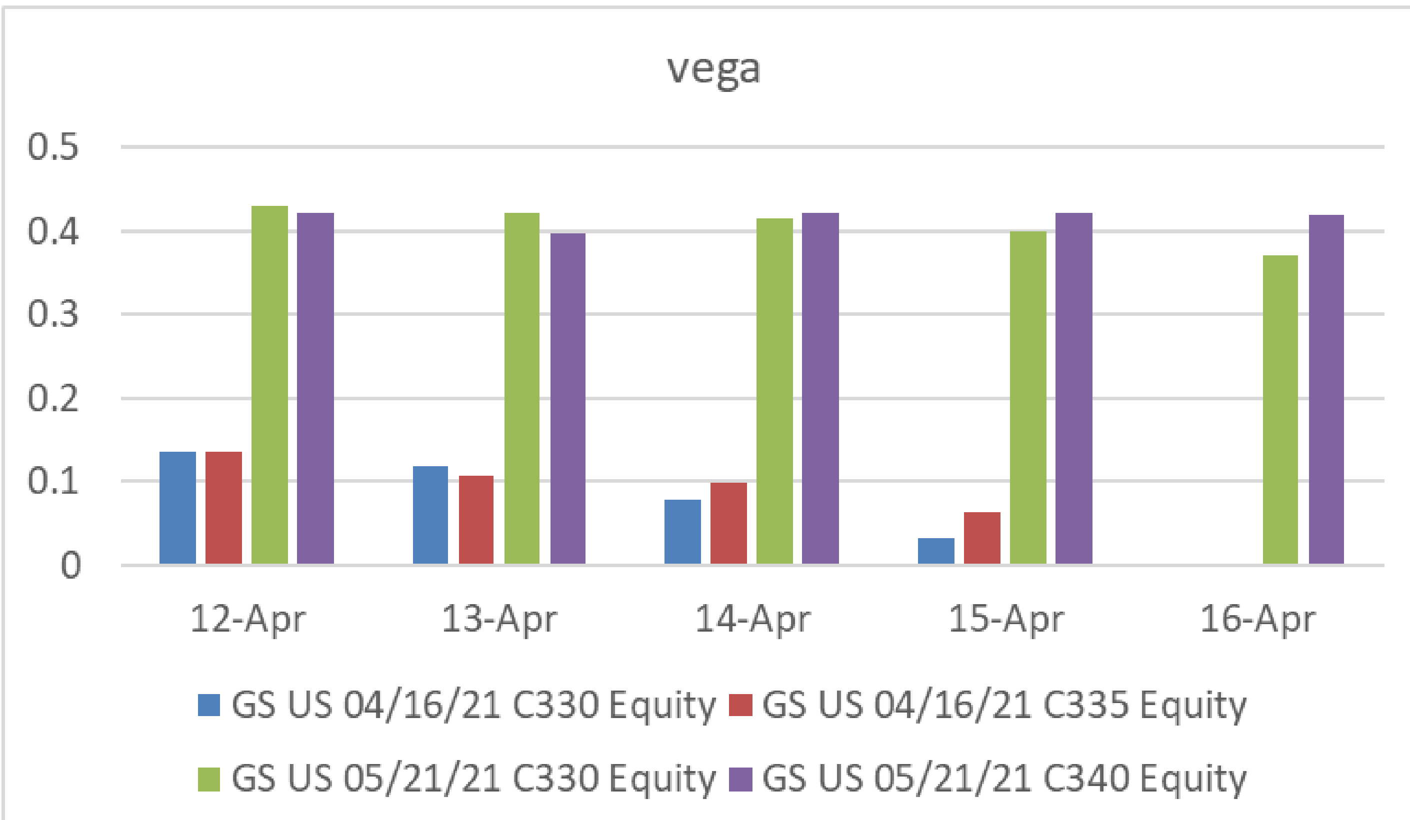
Gamma

- highest when the stock price is close to the strike price
- higher for options with less time to expiration



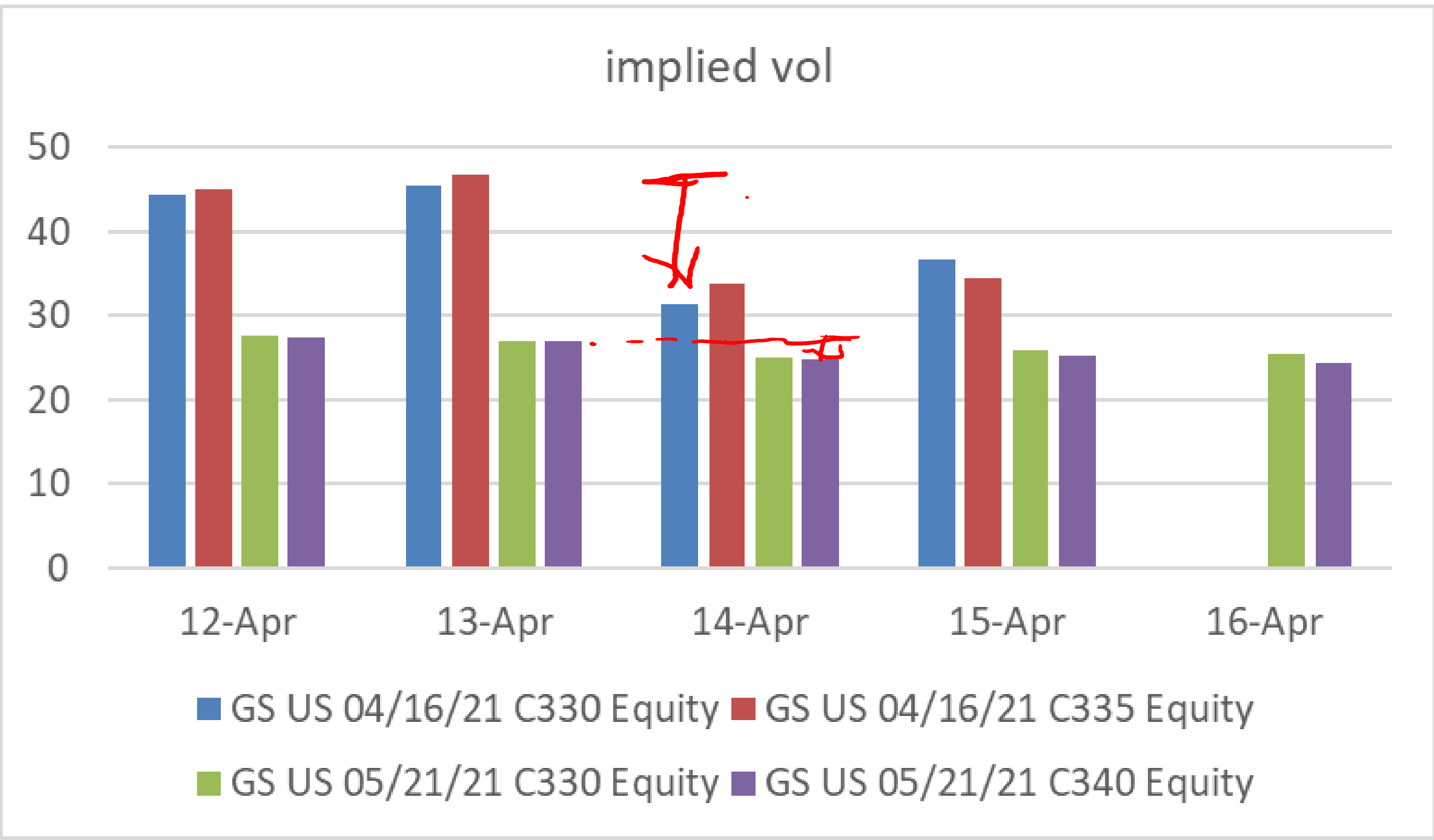
Vega

- Vega decreases over time for short-term options.
- Vega is higher for longer term options.



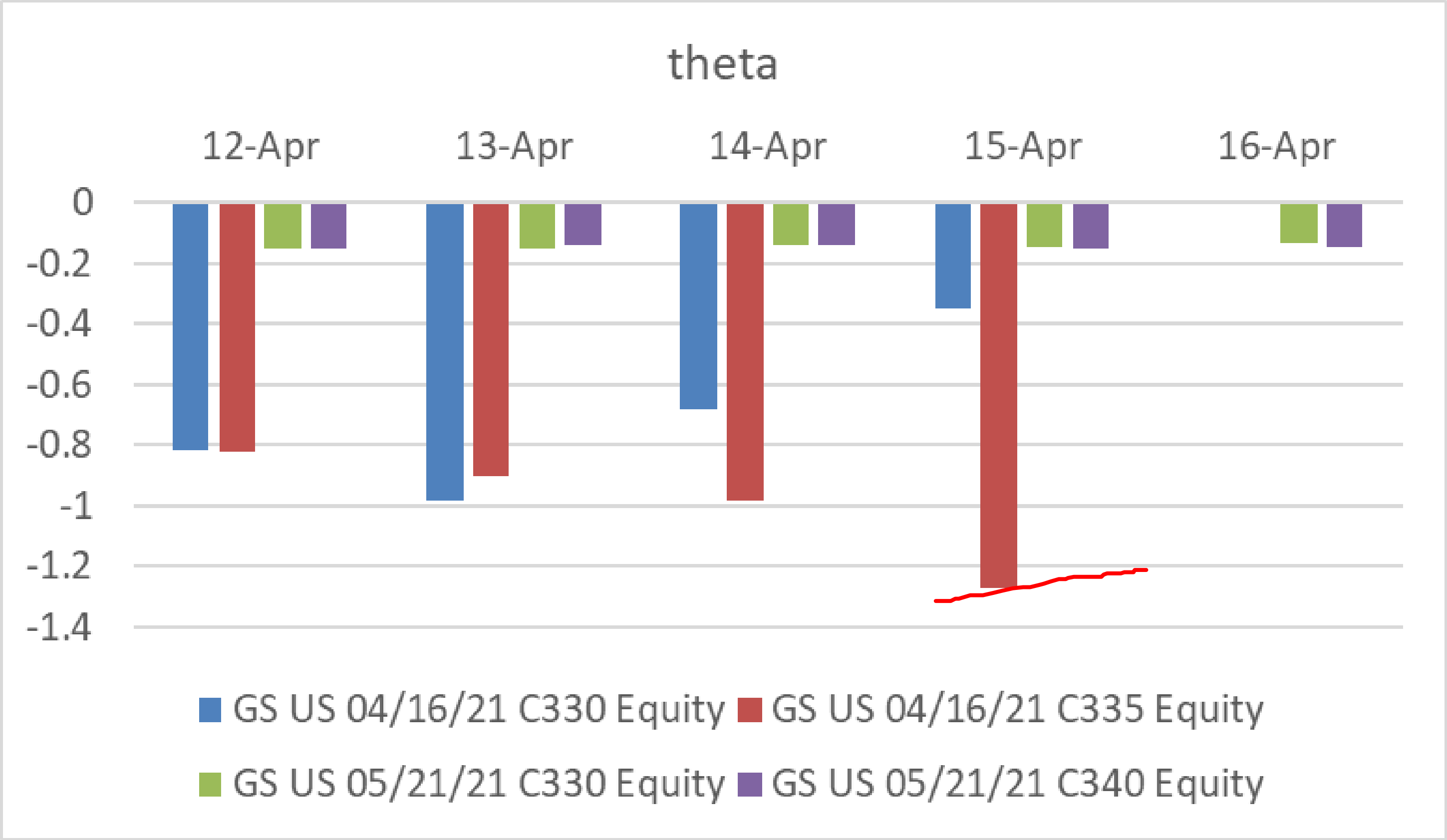
IMPLIED VOLATILITY

- IV of short-term options decreases after earnings report



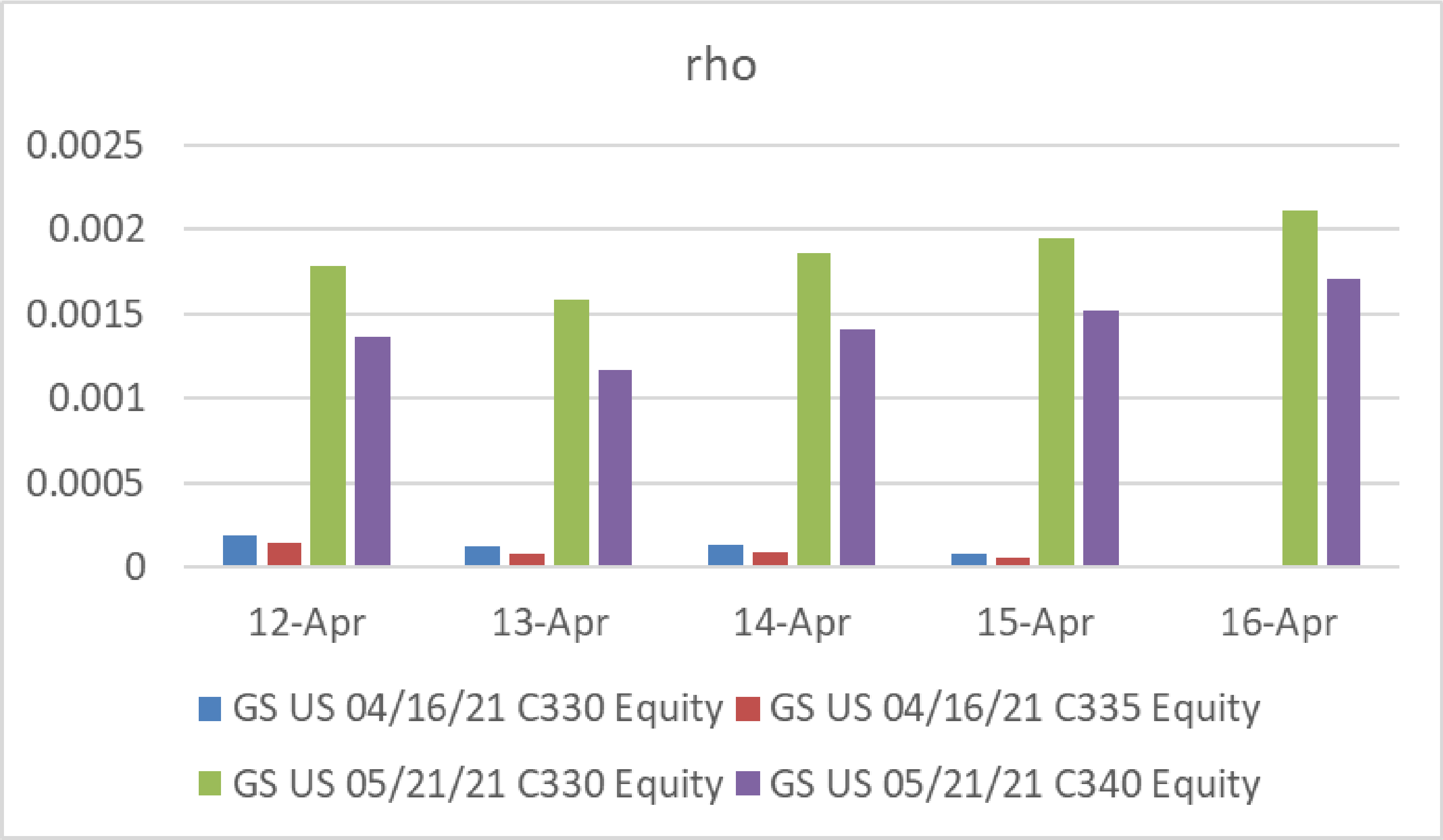
Theta (time decay)

- Highest for short term options
- Faster decay when the option is at-the-money



Rho

- higher for longer term options (1% incr in r → +$0.15)
- higher for options that are more in the money



Hedging vs Creation of an Option Synthetically

- When we are hedging we take positions that offset Δ, Γ, ν, etc.
- When we create an option synthetically we take positions that match Δ, Γ, & ν

Portfolio Insurance

- In October of 1987 many portfolio managers attempted to create a put option on a portfolio synthetically
- This involves initially selling enough of the portfolio (or of index futures) to match the Δ of the put option

Portfolio Insurance

- As the value of the portfolio increases, the Δ of the put becomes less negative and some of the original portfolio is repurchased
- As the value of the portfolio decreases, the Δ of the put becomes more negative and more of the portfolio must be sold

PORTFOLIO INSURANCE

The strategy did not work well on October 19, 1987...

- The market went down, and the trading rules of portfolio insurers dictated the sale of more securities, likely exacerbating the decline in equity prices

- It is dangerous to follow a particular trading strategy – even a hedging strategy – when many other market participants are doing the same thing
  - Think also about *gamma squeeze* and Gamestop call options

Finance: Art + Science

- Know when to trust models, and when not to
  - There are flaws to both human behavior and our modeling capabilities when put to financial use
  - We will have to see what the next overlooked source of risk will be…
    - tail risk that is hard to measure
    - black box machine learning models
    - overuse of leverage?
- You have to think about the entire system, how others will react, what you do differently, how your trades affect the market
  - E.g., portfolio insurance didn't work when everyone pursued it at the same time
  - In business and in finance, you are never making decisions in a vacuum
- Trillion Dollar Bet (LTCM) conclusion
  - 41:36-48:07 (6:30 min)

Summary

- **The Greeks can be used for risk management**
- Delta (Δ): change in option price when stock price changes
- Gamma (Γ): change in delta when stock price changes
- Vega (ν): change in option price when volatility changes
- Theta (Θ): change in option price when time to maturity changes
- Rho (ρ): change in option price when interest rate changes
- **An option's payoff can be replicated or offset through delta hedging**
    - It can also be used to generate a synthetic option
    - At times, as in 1987, this hedging strategy does not work

Finance 2325

Practice Questions:

Chapter 17: 2, 3, 4, 5, 6, 7, 8, 10, 24

Next class: Review

Come with questions ☺

Project: **due 12/9**