

Part A:

- 1.
2. Even when varying the step-size, (in vacuum), I did not observe any violation of energy conservation. I check this by varying the step-size, and computing the change in potential and kinetic energy, at the first and last integration points (output from 'vterm.cpp'):

```
Step size: 1.000e-02 s
Final coordinate (x,y,z): ( 0.00 m, 0.00 m, -0.00 m)
    Change in kinetic: +7.117357 J
    Change in potential: -7.117357 J
    Energy discrepancy: +0.000e+00 J
Step size: 1.668e-02 s
Final coordinate (x,y,z): ( 0.00 m, 0.00 m, -0.08 m)
    Change in kinetic: +7.224040 J
    Change in potential: -7.224040 J
    Energy discrepancy: -4.441e-15 J
Step size: 2.783e-02 s
Final coordinate (x,y,z): ( 0.00 m, 0.00 m, -0.20 m)
    Change in kinetic: +7.395514 J
    Change in potential: -7.395514 J
    Energy discrepancy: +5.329e-15 J
Step size: 4.642e-02 s
Final coordinate (x,y,z): ( 0.00 m, 0.00 m, -0.11 m)
    Change in kinetic: +7.275364 J
    Change in potential: -7.275364 J
    Energy discrepancy: +4.441e-15 J
Step size: 7.743e-02 s
Final coordinate (x,y,z): ( 0.00 m, 0.00 m, -0.76 m)
    Change in kinetic: +8.198037 J
    Change in potential: -8.198037 J
    Energy discrepancy: +0.000e+00 J
Step size: 1.292e-01 s
Final coordinate (x,y,z): ( 0.00 m, 0.00 m, -0.24 m)
    Change in kinetic: +7.448661 J
    Change in potential: -7.448661 J
    Energy discrepancy: +1.776e-15 J
Step size: 2.154e-01 s
Final coordinate (x,y,z): ( 0.00 m, 0.00 m, -0.69 m)
    Change in kinetic: +8.096227 J
    Change in potential: -8.096227 J
    Energy discrepancy: -1.776e-15 J
Step size: 3.594e-01 s
Final coordinate (x,y,z): ( 0.00 m, 0.00 m, -0.70 m)
    Change in kinetic: +8.110164 J
    Change in potential: -8.110164 J
    Energy discrepancy: +1.776e-15 J
Step size: 5.995e-01 s
Final coordinate (x,y,z): ( 0.00 m, 0.00 m, -2.05 m)
    Change in kinetic: +10.029784 J
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Change in potential: -10.029784 J
Energy discrepancy: -3.553e-15 J
Step size: 1.000e+00 s
Final coordinate (x,y,z): ( 0.00 m, 0.00 m, -14.62 m)
Change in kinetic: +27.908469 J
Change in potential: -27.908469 J
Energy discrepancy: +0.000e+00 J

```

3. Terminal velocity for the default drag / baseball size parameters can be determined by quitting iterations when the magnitude of acceleration drops below some arbitrarily small value (also from vterm.cpp):

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Number of points to reach terminal velocity: 210 (stopping condition: |a| < 1.0e-04
m/s^2)
terminal velocity (default parameters): 31.8000 m/s

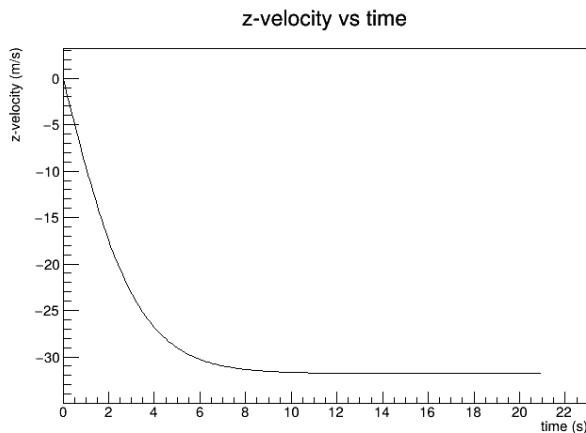
```

4. When the velocity of the baseball is very close to the terminal velocity, the following formula for $v(t)$ and $a(t)$ are reasonable, judging by the look of the plot that follows:

$$v(t) \simeq v_{\text{term.}} + e^{-\delta t}$$

$$a(t) \simeq \delta e^{-\delta t}$$

$$a(t) \simeq \delta \cdot |v_{\text{term.}} - v(t)|$$



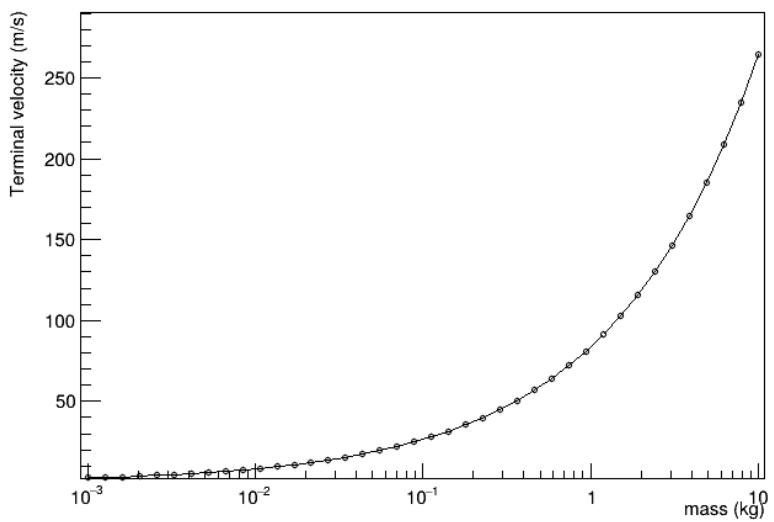
So, if you stop iterations after the magnitude of acceleration falls below some small number, we can use the following formula:

$$|a(t)| < \epsilon$$

$$|v_{\text{term.}} - v(t)| < \epsilon/\delta$$

And, judging by the plot, $\delta \sim O[1]$, so using $\epsilon = 10^{-4}$, we can say that our final value for v_{terminal} is accurate on the order 10^{-4} .

Baseball mass vs. Terminal Velocity



5.