Q) Given that 'f' and 'g' are convex functions, prove that $f-g$ is not necessarily convex.

Sol: We know that for any convex function 'h',

$$h(\lambda x + (1-\lambda)y) \leq \lambda h(x) + (1-\lambda) h(y)$$

for all $x, y \in \mathbb{R}$ and $\lambda \in [0,1]$.

Now consider a f^n : $h(x) = f(x) - g(x)$

Let us say that this func. is a convex f^n

$\Rightarrow \exists\ t \in [0,1]$ s.t.

$$h(tx + (1-t)y) \leq t\,h(x) + (1-t)\,h(y)$$

$$\Rightarrow f(tx + (1-t)y) - g(tx + (1-t)y)$$
$$\leq t[f(x) - g(x)] + (1-t)[f(y) - g(y)]$$

but since 'f' is convex, we know that

$$f(tx + (1-t)y) \leq t f(x) + (1-t) f(y)$$

$$\Rightarrow -g(tx + (1-t)y) \leq -t g(x) - (1-t) g(y)$$

$$\Rightarrow g(tx + (1-t)y) \geq t g(x) + (1-t) g(y)$$

this contradicts the fact that 'g' is convex

ie. $g(tx + (1-t)y) \leq t g(x) + (1-t) g(y)$

$\Rightarrow$ '$f-g$' need not be convex if 'f' & 'g' are convex.