CS-E4850 Computer Vision
Exercise Round 5

The exercises should be solved and the solutions submitted via Aalto JupyterHub by the deadline. Provide your solutions to the pen-and-paper exercises in a single PDF file. You may use e.g. LaTeX, but handwritten solutions are also fine if they are scanned to PDF format and clear to read. You do not need to solve all the tasks, as points will also be awarded for partial solutions.
This PDF contains the pen-and-paper exercises. This round also includes programming tasks in a separate Jupyter notebook. Complete the programming tasks and submit the notebook along with a single PDF of your pen-and-paper solutions.

Exercise 1. Total least squares line fitting. (Pen-and-paper problem, 10 points)
An overview of least squares line fitting is presented on the slide 13 of Lecture 4. Study it in detail and present the derivation with the following stages:

1) Given a line $ax+by-d=0$, where the coefficients are normalized so that $a^2+b^2=1$, show that the distance between a point (x_i, y_i) and the line is $|ax_i + by_i - d|$.

2) Thus, given n points (x_i, y_i), $i = 1, \ldots, n$, the sum of squared distances between the points and the line is $E = \Sigma_{i=1}^{n}(ax_i + by_i - d)^2$. In order to find the minimum of E, compute the partial derivative $\partial E/\partial d$, set it to zero, and solve d in terms of a and b.

3) Substitute the expression obtained for d to the formula of E, and show that then $E = (a\ b)U^\top U(a\ b)^\top$, where matrix U depends on the point coordinates (x_i, y_i).

4) Thus, the task is to minimize $||U(a\ b)^\top||$ under the constraint $a^2 + b^2 = 1$. The solution for $(a\ b)^\top$ is the eigenvector of $U^\top U$ corresponding to the smallest eigenvalue, and d can be solved thereafter using the expression obtained above in the stage two.