CS-E4850 Computer Vision
Exercise Round 6

The exercises should be solved, and the solutions submitted via Aalto JupyterHub by the deadline. Provide your solutions to the pen-and-paper exercises in a single PDF file. You may use e.g. LaTeX, but handwritten solutions are also fine if they are scanned to PDF format and clear to read.
This PDF contains the pen-and-paper exercises. This round also includes a programming demo in a separate Jupyter notebook. You don't need to return the Jupyter notebook, but submit a single PDF of your pen-and-paper solutions.

Exercise 1. Least-squares fitting for affine transformations. (pen& paper, 10 points)
A brief overview of affine transformation estimation is presented on slides 17-19 of Lecture 5. Present a derivation and compute an example by performing the following stages:

a) Compute the gradient of the least squares error $E = \sum_{i=1}^{n} ||\mathbf{x}_i' - \mathbf{M}\mathbf{x}_i - \mathbf{t}||^2$ with respect to the parameters of the transformation (i.e. elements of matrix $\mathbf{M}$ and vector $\mathbf{t}$).

b) Show that by setting the aforementioned gradient to zero you will get an equation of the form $\mathbf{Sh} = \mathbf{u}$, where vector $\mathbf{h}$ contains the unknown parameters of the transformation, and 6×6 matrix $\mathbf{S}$ and 6×1 vector $\mathbf{u}$ depend on the coordinates of the point correspondences $\{\mathbf{x}_i', \mathbf{x}_i\}$, $i = 1, \ldots, n$.

c) Thus, one may solve the transformation by computing $\mathbf{h} = \mathbf{S}^{-1}\mathbf{u}$. Compute the affine transformation from the following point correspondences $\{(0,0) \rightarrow (1,2)\}$, $\{(1,0) \rightarrow (3,2)\}$, and $\{(0,1) \rightarrow (1,4)\}$.
(Hint: This calculation can be done with pen and paper but you may check the correct answer by running the function `affinefit` given for the third exercise round in `utils.py`. Another way to check the answer is to draw the point correspondences on paper and visually determine the correct solution.)

Tasks continue on the next page...

Exercise 2. Similarity transformation from two point correspondences. (pen&paper, 10 points)
A similarity transformation consists of rotation, scaling and translation and is defined in two dimensions as follows:

$$\mathbf{x}' = s\mathbf{R}\mathbf{x} + \mathbf{t} \quad \Leftrightarrow \quad \begin{pmatrix} x' \\ y' \end{pmatrix} = s \begin{pmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} t_x \\ t_y \end{pmatrix} \tag{1}$$

Describe a method for solving the parameters s, θ, t_x, t_y of a similarity transformation from two point correspondences $\{\mathbf{x}_1 \to \mathbf{x}'_1\}$, $\{\mathbf{x}_2 \to \mathbf{x}'_2\}$ using the following stages:

a) Compute the vectors $\mathbf{v}' = \mathbf{x}'_2 - \mathbf{x}'_1$ and $\mathbf{v} = \mathbf{x}_2 - \mathbf{x}_1$ and present a formula to recover the rotation angle θ from the corresponding unit vectors.

b) Compute the scale factor s as the ratio of the norms of vectors $\mathbf{v}'$ and $\mathbf{v}$.

c) After solving s and θ compute $\mathbf{t}$ using equation (1) and either one of the two point correspondences.

d) Use the procedure to compute the transformation from the following point correspondences: $\{(\frac{1}{2}, 0) \to (0, 0)\}$, $\{(0, \frac{1}{2}) \to (-1, -1)\}$.
(Hint: Drawing the point correspondences on a grid paper may help you to check your answer.)

Demo 1. Matlab demo of panoramic image stitching. (Just a demo, no points given)
Jupyter notebook contains an example that demonstrates robust feature-based panorama stitching, which was covered during the Lecture 5 (i.e. SURF feature extraction and matching and RANSAC based homography estimation). Check the example by running the notebook. Homography estimation is covered in Chapter 4 of the book by Hartley & Zisserman.

Demo 2. Properties of rotations and reflections. (Teacher will present, no points given)
Isometries are transformations that preserve distances. An isometry is presented as

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} \epsilon\cos(\theta) & -\sin(\theta) & t_x \\ \epsilon\sin(\theta) & \cos(\theta) & t_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} \mathbf{U} & \mathbf{t} \\ \mathbf{0} & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}, \tag{2}$$

where $\epsilon = \pm 1$. If $\epsilon = 1$, the isometry is orientation-preserving. If $\epsilon = -1$, the isometry is orientation-reversing.
a) Show that $\mathbf{U}$ is a pure rotation when $\epsilon = 1$ and a composition of rotation and reflection when $\epsilon = -1$.
b) Show that $\det \mathbf{U} = \epsilon$.
c) Show that $\mathbf{U}$ is orthogonal, i.e. $\mathbf{U}^\top \mathbf{U} = \mathbf{I}$.
d) Show that the orientation-preserving isometries form a group under the operation of matrix multiplication.