Cyclical Labor Market Sorting*

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Abstract

We consider sorting in the labor market, that is, whether high or low productivity workers and firms tend to match with each other, and how this varies over time using U.S. linked employer-employee data. Composition changes of workers and firms move in opposite directions over the business cycle. During and after recessions, low-rank workers are less likely to work, while the employment share of low-rank firms increases. The agreement between worker and firm ranks increases in the early stages of labor market downturns. We consider these findings in the context of a model of cyclical labor market sorting.

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1 Introduction

It is commonly said that during and after recessions, overqualified workers get stuck in low-paying jobs. Studies by Kahn (2010) and Oreopoulos, von Wachter, and Heisz (2012) have shown that college graduates obtain relatively low-skill jobs during labor market downturns. This disconnect between workers and their best job matches is what Barlevy (2002) called the "sullying" effect of recessions. Barlevy (2002) also emphasized that, during labor market downturns, a lower rate of voluntary quits for better employment can cause workers to spend more time in worse matches.

Such a sullying effect of recessions contrasts with the more conventional "cleansing" effect.[1] This mechanism suggests that, during economic downturns, the least productive jobs are destroyed. This cleansing mechanism implies that the remaining jobs will be (at least relatively) more productive. There are thus two plausible competing channels through which economic downturns might affect job match quality. However, little is known empirically about how economic downturns affect the quality distributions of workers and firms, and the sorting of workers between firms.

In this paper, we provide evidence on the cleansing and sullying effects of recessions on workers, firms, and their sorting in the labor market. We use matched employer-employee data to implement several methods of ranking workers and firms to establish how labor market sorting (i.e., the degree to which low- vs. high-rank workers work at low vs. high-rank firms) varies over the business cycle. We find that, regardless of the ranking method, recessions are times when the employment distribution shifts towards high-rank workers. This cleansing effect on the worker distribution is fairly intuitive. Somewhat more surprising are the firm dynamics, where we find evidence of a sullying effect. The firm quality distribution shifts down in recessions, as low-rank firms take a larger share of employment. Although several mechanisms are at work, positive sorting strengthens during recessions. We consider these cleansing and sullying effects of recessions in the context of the model of cyclical labor market sorting proposed by Lise and Robin (2017).

We present evidence on how labor market sorting varies over the business cycle. To do so, we rely on the insights of many contributions on sorting in the labor market that exploit the unique properties of universe-level linked employer-employee data. We implement four methods of ranking workers and firms using quarterly linked employer-employee data for 11 U.S. states from 1994 to

[1] See Caballero and Hammour (1994). Note that Barlevy (2002) considered both the cleansing and sullying effects of recessions.

2014. Each of these methods involves ordering workers and firms along a univariate, time-invariant ranking.[2] In other words, we assume that workers and firms are of high or low intrinsic rank along a single dimension. In total, we implement four methods of ranking workers and firms using our linked employer-employee data. We rank workers based on the time spent in employment vs. nonemployment, as well as by their average earnings when working. Following Bagger and Lentz (2019), we rank firms based on the share of their hires poached from other firms. Motivated by the recent work of Bartolucci, Devicienti, and Monzón (2018) and Haltiwanger, Hyatt, and McEntarfer (2018), we also rank firms by labor productivity (revenue per worker, with industry adjustments to capture differences in value added). We also concurrently rank workers and firms by assuming that earnings are an additive function of a worker effect and a firm effect as in Abowd, Kramarz, and Margolis (1999). Finally, we concurrently rank workers and firms by implementing a ranking algorithm that follows Hagedorn, Law, and Manovskii (2017) and Lopes de Melo (2018), whose methods are motivated by labor market search models. We focus on cyclical changes in composition and sorting using employment-weighted terciles (i.e., low, middle, and high) of the respective worker and firm rank distributions.

All four methods of ranking workers and firms yield qualitatively similar results on worker composition, firm composition, and sorting. Low-rank (lowest tercile) workers are most affected by labor market downturns. Although both low-rank and high-rank (highest tercile) workers have fewer net flows from nonemployment in worse labor markets, the declines in the nonemployment transition rate are more severe for low-rank workers. Thus, recessions are times when the composition of the workforce shifts away from low-rank workers. Every percentage point increase in the unemployment rate is associated with a quarterly 0.114 to 0.449 percentage point decline in the employment share of low-rank workers. This result can be characterized as a cleansing effect. During labor market downturns, more productive workers are better able to compete for scarce jobs. Many potential job opportunities for low-rank workers are no longer profitable in worse states of the economy. This result echoes work by Oi (1962) and van Ours and Ridder (1995) on the cyclicality of worker employment by skill and education.

During labor market downturns, sullying effects characterize changes in firm composition. Especially in times of high unemployment that follow recessions, the employment share of low-rank firms increases. Every additional percentage point in the HP-detrended unemployment rate is associated with a 0.032 to 0.063 percentage point increase in the employment share of firms ranked in the lowest

[2] See Baley, Figueiredo, and Ulbricht (2020) for a multi-dimensional model of cyclical sorting.

tercile. We show the central importance of the cyclical job ladder, which drives the observed countercyclical increase in employment at low-rank firms through the differential poaching margin response of low- versus high-rank firms. Among low-rank firms, net hires from poaching *increase* by 0.051 to 0.068 percentage points with each additional percentage point in the HP-detrended unemployment rate, while among high-rank firms, net poaching hires *decrease* by 0.044 to 0.065 percentage points. This differential poaching response of 0.095 to 0.130 percentage points strongly favors low-rank firms. In contrast, we find that the net nonemployment hiring of low-rank and high-rank firms adjust similarly in times of high unemployment. Therefore, the increase in the employment share of low-rank firms in times of high unemployment can be attributed to changes in net poaching flows. Our paper links the cyclical job ladder, considered by Haltiwanger et al. (2018), Haltiwanger, Hyatt, and McEntarfer (2018), and Moscarini and Postel-Vinay (2018), to employment composition by firm rank. During labor market downturns, workers spend more time in worse jobs.

Cyclical changes in labor market sorting naturally follow from these composition changes. Labor market downturns are times when low-rank workers are less likely to work at high-rank firms. Specifically, a one percentage point increase in the unemployment rate is associated with a 0.077 to 0.181 percentage point decrease in the share of low-rank workers at high-rank firms. This change for low-rank workers is driven by a slowdown in the job ladder: differential changes in net poaching flows into low- vs. high-rank firms are larger than differential changes in net nonemployment flows. This decline in the share of low-rank workers at high-rank firms strengthens the agreement between worker and firm ranks. We also find that high-rank workers are more likely to work at low-rank firms during labor market downturns, which weakens the agreement between worker and firm ranks. A one percentage point increase in the unemployment rate is associated with a 0.023 to 0.098 percentage point increase in the share of high-rank workers at low-rank firms. The slowdown in the job ladder drives this change as well. Overall, the agreement between worker and firm ranks increases slightly during recessions.

We consider our findings in the context of the model of cyclical labor market sorting proposed by Lise and Robin (2017). Their model includes heterogeneous worker and firms, on-the-job search, and business cycles. We draw two main lessons. First, in order to generate countercyclical cleansing of the worker distribution and sullying of the firm distribution, workers must drive the match value of output. This is consistent with the empirical literature that uses linked employer-employee data to estimate additive models of worker and firm effects starting with Abowd, Kramarz, and Margolis

(1999). Second, the set of firms in operation needs to be stable over time. During downturns, workers spend longer in worse matches, but the worst matches are eliminated from the economy. If the set of firms that find it profitable to produce shifts dramatically during recessions, the cleansing effect will dominate. In order for recessions to increase the employment share of low-rank firms, the sullying effect must dominate the cleansing effect.

2 Data

2.1 Source data

The Longitudinal Employer-Household Dynamics (LEHD) linked employer-employee data allows us to explore the cyclical behavior of labor market composition and sorting. These labor income records are collected for the purpose of administering the unemployment insurance system, and thus the income records cover nearly all private sector employment as well as state and local (but not federal) government workers, see Abowd et al. (2009). Because different states enter the LEHD microdata at different times, we use a consistent set of eleven states with data available from 1994-2014.[3]

We follow the approach to measuring employment transitions in Hyatt et al. (2014).[4] This involves considering the set of jobs (i.e., distinct employer-employee combinations) that span two consecutive quarters. A worker's "dominant job" is the employer at which that worker earns the most among all such consecutive quarter jobs. Following those definitions, when a worker's dominant employer changes without a gap in earnings, the worker undergoes an employer-to-employer transition. If the worker has one or more quarters without earnings, then any flows into or from employment are considered flows from and into nonemployment, respectively.

2.2 Ranking workers and firms

We rank workers and firms in four different ways, roughly following different strands of the literature on labor market sorting.[5] All ranks are calculated on an employment-weighted basis, and use real 2014 dollars.

[3] These states are CA, CO, ID, IL, KS, MD, MT, NC, OR, WA, and WI.
[4] For exact definitions, see Appendix A.
[5] We provide a summary of each method here. For additional details on our ranking methods, see Appendix B.

We start by ranking workers and firms based on simple summary statistics. Our first method ranks workers and firms in ways that do not rely directly on observed earnings. We rank firms based on the share of new hires that come from other firms vs. from nonemployment, following Bagger and Lentz (2019).[6] A firm's poaching (i.e., employer-to-employer transition) hires as a share of all hires is a rough metric for how desirable a firm is as an employer. This measure, in principle, reflects wage and salary compensation, as well as nonwage amenities. To rank workers, we use the fraction of their careers that they spend in employment vs. nonemployment. We count workers who are more frequently employed as being more productive.[7] Specifically, we regress employment on a set of year of birth by quarter dummies, separately by gender, and then rank workers based on the average of the residuals from that regression. This method also yields ranks that have a straightforward interpretation in the model of Lise and Robin (2017). These methods identify both the workers who are more likely to encounter a productive match with the firms operating in the economy, and the firms who are likely to offer workers a more productive job. These methods of ranking workers and firms also run quickly in our model-simulated environment. For these reasons, we target these moments in our quantitative exercise in Section 4.

Our second method ranks workers by earnings and firms by labor productivity. To rank workers, we use the average residual from regressing earnings on year of birth by quarter dummies. Note that this measure of average regression-adjusted worker earnings also provides the initial guess of a worker's rank in our third and fourth ranking methods. For firms, we use revenue from the U.S. Census Bureau's Business Register, in the spirit of the recent work by Haltiwanger et al. (2017).[8] We use this firm-level revenue data to calculate a firm's average deviation over time from the employment-weighted industry average revenue per worker. We then obtain a measure of labor productivity by adding this firm-level measure to industry-level value added per worker as published by the Bureau of Economic Analysis.

Our third method jointly ranks workers and firms using a model that assumes earnings are an additive function of a firm effect and a worker effect, as in Abowd, Kramarz, and Margolis (1999). Card,

[6] Appropriate caution in interpreting our results is warranted because Bagger and Lentz (2019) do not consider aggregate uncertainty.

[7] This method serves as a measure for worker quality because workers with less to gain from working might spend less time doing so. Bagger and Lentz (2019) use unemployment duration as a method of ranking workers in Section 4.2.1 of their paper, although they do not place as much emphasis on this method of ranking workers as they do their poaching hire method of ranking firms.

[8] Differences between our revenue measure and that of Haltiwanger et al. (2017) include our use revenue data starting in 1994 for all industries, as well as our imputation of missing data. See Appendix Section B.2.2.

Cardoso, and Kline (2016) recently used this framework to measure the degree of sorting in the labor market. Because we observe workers at different parts of their life-cycles, we first regress earnings on a set of year of birth by quarter in time dummies (e.g., born in 1965 and working in 1997Q1). Then, following Guimarães and Portugal (2010), we apply an iterative method to the residuals from this regression to update worker effects, firm effects, and birth cohort by quarter effects.

Fourth, we apply a technique inspired by the recent work of Hagedorn, Law, and Manovskii (2017) and Lopes de Melo (2018). These papers provide solutions to the inconsistency between the identification assumptions of Abowd, Kramarz, and Margolis (1999) and standard models of labor market search.[9] We initially rank workers by their average lifetime earnings, but then iteratively re-rank workers who are employed at the same firm so as to maximize the likelihood that a higher ranked worker earns more than a lower ranked worker when employed by the same firm. Firms are afterwards ranked based on the surplus paid to workers, measured as the difference between employees' received earnings relative to their reservation wages (the minimum earnings received by workers of a given rank). Firms with a greater difference between earnings paid and the reservation wage have a greater surplus from a match and are considered more productive.

3 Empirical evidence on composition and sorting

3.1 Overview and notation

In this section, we document how the sorting of workers into firms of different ranks varies over the business cycle. We consider the share of employment of workers and firms of different ranks, and the relative frequency of particular combinations of worker and firm ranks (i.e., the degree of sorting), and how this changes over time. We also measure the worker flows into and from nonemployment and poaching flows across firms that account for these changes. We characterize the health of the labor market using the difference of the unemployment rate from its HP trend, as well as the first difference in the unemployment rate, following Haltiwanger et al. (2018). These transformations of the unemployment rate serve as our cyclical indicators. The first-difference of the unemployment rate surges during NBER recessions. The difference in unemployment from its HP trend is a measure of

[9] Readers should note that the random search models proposed by Hagedorn, Law, and Manovskii (2017) and Lopes de Melo (2018) do not consider aggregate uncertainty and, therefore, appropriate caution is required in interpreting our cyclical results.

times of low vs. high unemployment. We rank firms and workers into three terciles: low, middle, and high based on an employment-weighted ranking of workers and firms across all quarters.

We introduce some notation to document how employment evolves over time, which builds on the framework of Haltiwanger et al. (2018). Let E_{ijt} denote the number of workers of rank tercile i working at firms of rank tercile j at time t. Employment for each worker i, firm j bin changes from time $t-1$ to t due to separations to nonemployment N^s_{ijt}, hires (accessions) from nonemployment N^a_{ijt}, separations from poaching (i.e., employer-to-employer transitions) P^s_{ijt}, and poaching hires P^h_{ijt}. Specifically, the change in employment can be expressed as

$$\Delta E_{ijt} = E_{ijt} - E_{ijt-1} = N^a_{ijt} - N^s_{ijt} + P^a_{ijt} - P^s_{ijt}. \quad (1)$$

The change in employment for any worker-firm group can be expressed as the sum of net hires from nonemployment $N^a_{ijt} - N^s_{ijt}$ and net hires from poaching $P^a_{ijt} - P^s_{ijt}$. We further express the sum of workers of rank i across firms of any rank at time t as $E_{i\bullet t}$, and analogously express totals for firm rank j as $E_{\bullet jt}$. Total employment at time t, $E_{\bullet\bullet t}$ is written E_t. Note that poaching flows do not change the total employment of any worker rank and so $\Delta E_{i\bullet t} = N^a_{i\bullet t} - N^s_{i\bullet t}$. This is because an employer-to-employer transition implies a separation of a worker of rank i from one employer and a hire of a worker of that same rank at a different employer. Net poaching flows, however, can affect the composition of firms.

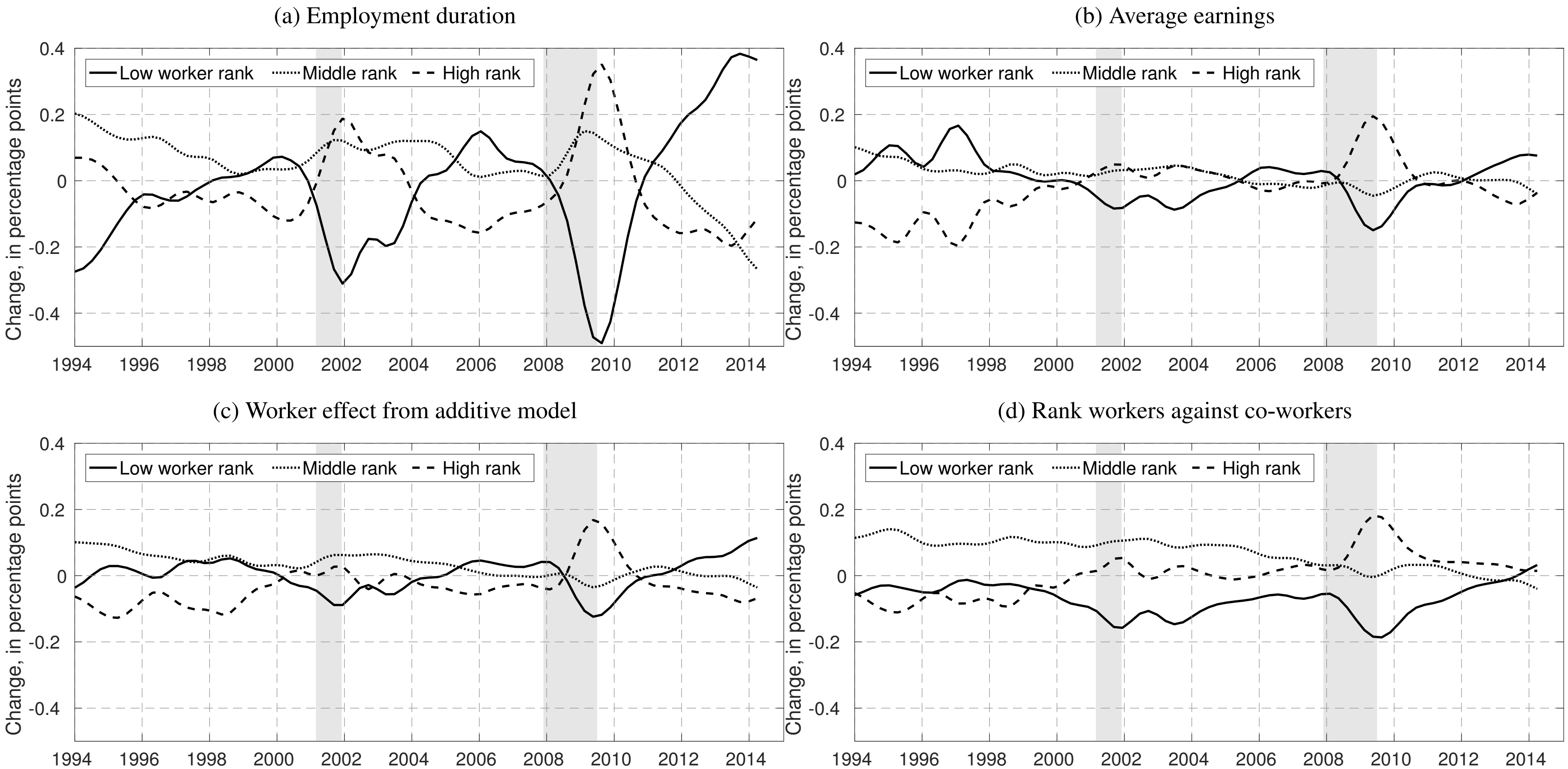
3.2 Worker and firm composition

3.2.1 Worker composition

We now document how worker composition evolves over time. By construction, low-, middle-, and high-rank workers will on average each have a share of one-third. However, in any given quarter the shares of employment in these terciles can differ from one-third. From one quarter to another, workers with a time-invariant rank enter and leave employment, and these transitions determine how the employment shares of these different groups evolve over time. For example, if more workers of high-rank enter employment than other groups, their employment share will increase in that period.

The evolution of the employment shares of low, middle, and high-rank workers is shown in Figure 1. We plot the quarterly change in the share of employment of workers of different ranks, using each

Figure 1: Changes in worker rank shares



Notes: Shaded regions indicate recessions. Data seasonally adjusted and Henderson-filtered using X-11.

Table 1: Changes in worker rank shares and the unemployment rate

Worker tercile	Employment duration	Average earnings	Additive model worker effects	Rank workers vs. co-workers
	Difference in unemployment from its HP trend			
Low	-11.2***	-3.0**	-3.2**	-2.7***
	(2.7)	(1.4)	(1.2)	(1.0)
High	7.4***	2.1	2.4**	2.2**
	(2.0)	(1.4)	(1.1)	(0.9)
	First-difference of the unemployment rate			
Low	-44.9***	-13.9***	-12.6***	-11.4***
	(5.0)	(3.2)	(2.7)	(2.3)
High	31.6***	16.9***	14.8***	12.9***
	(3.9)	(2.7)	(2.2)	(1.7)

Notes: We regress the change in share of employment on the seasonally-adjusted unemployment rate, seasonal dummies, and a time trend. *, **, and *** indicate statistical significance at 10%, 5%, and 1%, respectively. Standard errors are in parentheses. To avoid excessive decimal places, the dependent variables range from $[-100, 100]$, while the cyclical indicators range from $[-1, 1]$.

of our four ranking methods. In terms of the notation introduced in Section 3.1, we plot $E_{i\bullet t}/E_t - E_{i\bullet t-1}/E_{t-1}$. A positive value indicates that a rank tercile increases its share of employment, while a negative value indicates that a rank tercile loses some of its share.

Cyclical changes in worker composition are similar in direction across the different ranking methods, but the largest changes are found when we rank workers by their employment duration in Panel 1(a). Panel 1(b) shows how the shares of the employment evolve when workers are ranked by average earnings. Panel 1(c) shows how the shares evolve when workers are ranked based on the worker effect from our additive model of earnings with worker and firm effects. Panel 1(d) shows how composition evolves for workers initially ranked by average earnings, but then re-ranked to ensure that more productive workers at the same firm earn more than their less productive co-workers. The changes in the shares of these terciles are very small, with the share of workers of a given type never moving up or down by more than 0.5 percentage points over the span of a quarter. It is apparent from Figure 1 that the middle and late periods of economic expansions are times when low-rank workers gain as a share of employment, and the share of high-rank workers declines. During and after recessions, the employment share of high-rank workers increases, as that of low-rank workers decreases.

Table 1 shows how the shares of employment by worker rank change with our cyclical indicators.[10] Specifically, we regress $E_{i\bullet t}/E_t - E_{i\bullet t-1}/E_{t-1}$ on our seasonally-adjusted cyclical indicators, seasonal dummies, and a time trend.[11] This table summarizes the cyclical features of Figure 1. Changes in worker composition are greatest during recessions, rather than the times of high unemployment that follow recessions. Consistent with these features of Figure 1, point estimates in Table 1 are greater in magnitude for the first-difference of the unemployment rate than the deviation of the unemployment rate from its HP trend. Specifically, a one percentage point increase in the unemployment rate is associated with a decline of 0.114 to 0.449 percentage points in the share of low-rank workers, and a 0.129 to 0.316 percentage point increase in the share of high-rank workers. Every additional percentage point in the HP-detrended unemployment rate is associated with a decline in the low-rank worker share of 0.027 to 0.112 percentage points, and an increase in the high-rank worker share of 0.021 to 0.074 percentage points. Consistent with Figure 1, workers ranked by employment duration show larger cyclical changes than when ranked by other methods.

Table 2 explores the transition dynamics that underlie these cyclical shifts in employment composition by worker rank. Specifically, it shows how net hiring from nonemployment changes with labor market conditions. The net nonemployment variable we define is $(N^a_{i\bullet t} - N^s_{i\bullet t})/((E_t + E_{t-1})/2)$.

Cyclical changes in net nonemployment transitions are concentrated among low-rank workers. When the unemployment rate increases by one percentage point, net nonemployment flows of low-rank workers decline by 0.487 to 0.830 percentage points, while flows of high-rank workers decline by 0.028 to 0.278 percentage points. As with worker composition, changes in net nonemployment flows are more closely aligned to recessions (i.e., the first-difference of the unemployment rate) than times of high unemployment (i.e., HP-detrended unemployment). An additional percentage point in the HP-detrended unemployment rate is associated with a decline of 0.131 to 0.214 percentage points in the share of low-rank workers with a low employment duration, but a decline of only 0.017 to 0.083 percentage points for high-rank workers. This exercise highlights the mechanisms that generate cyclical changes in employment shares in Table 1. Because the high-rank worker tercile has a smaller

[10]We present the results of robustness exercises for our worker composition results in Appendix Tables C1 and C3. Appendix Table C1 presents results when we exclude workers with relatively few employment observations. Appendix Table C3 presents results when we only rank workers using pre-recession employment data, and only rely on the years during and after recessions to measure employment composition of each so-defined subset of workers. In both robustness exercises, results are similar in sign but smaller in magnitude than in Table 1.

[11]Similar specifications have been used to measure the cyclicality of job ladders in the labor market by, among others, Haltiwanger et al. (2018).

Table 2: Net nonemployment hiring by worker rank and unemployment

Worker tercile	Employment duration	Average earnings	Additive model worker effects	Rank workers vs. co-workers
		Difference in unemployment from its HP trend		
Low	-21.4***	-13.5***	-13.9***	-13.1***
	(5.1)	(3.3)	(2.9)	(2.8)
High	-1.7	-7.7***	-8.0***	-8.3***
	(1.6)	(2.7)	(2.4)	(2.3)
		First-difference of the unemployment rate		
Low	-83.0***	-56.4***	-51.3***	-48.7***
	(10.0)	(6.0)	(5.5)	(5.3)
High	-2.8	-27.8***	-23.0***	-25.6***
	(4.0)	(6.1)	(5.8)	(5.4)

Notes: We regress net hires on the seasonally-adjusted unemployment rate, seasonal dummies, and a time trend. *, **, and *** indicate statistical significance at 10%, 5%, and 1%, respectively. Standard errors are in parentheses. To avoid excessive decimal places, the dependent variables range from $[-100, 100]$, while the cyclical indicators range from $[-1, 1]$.

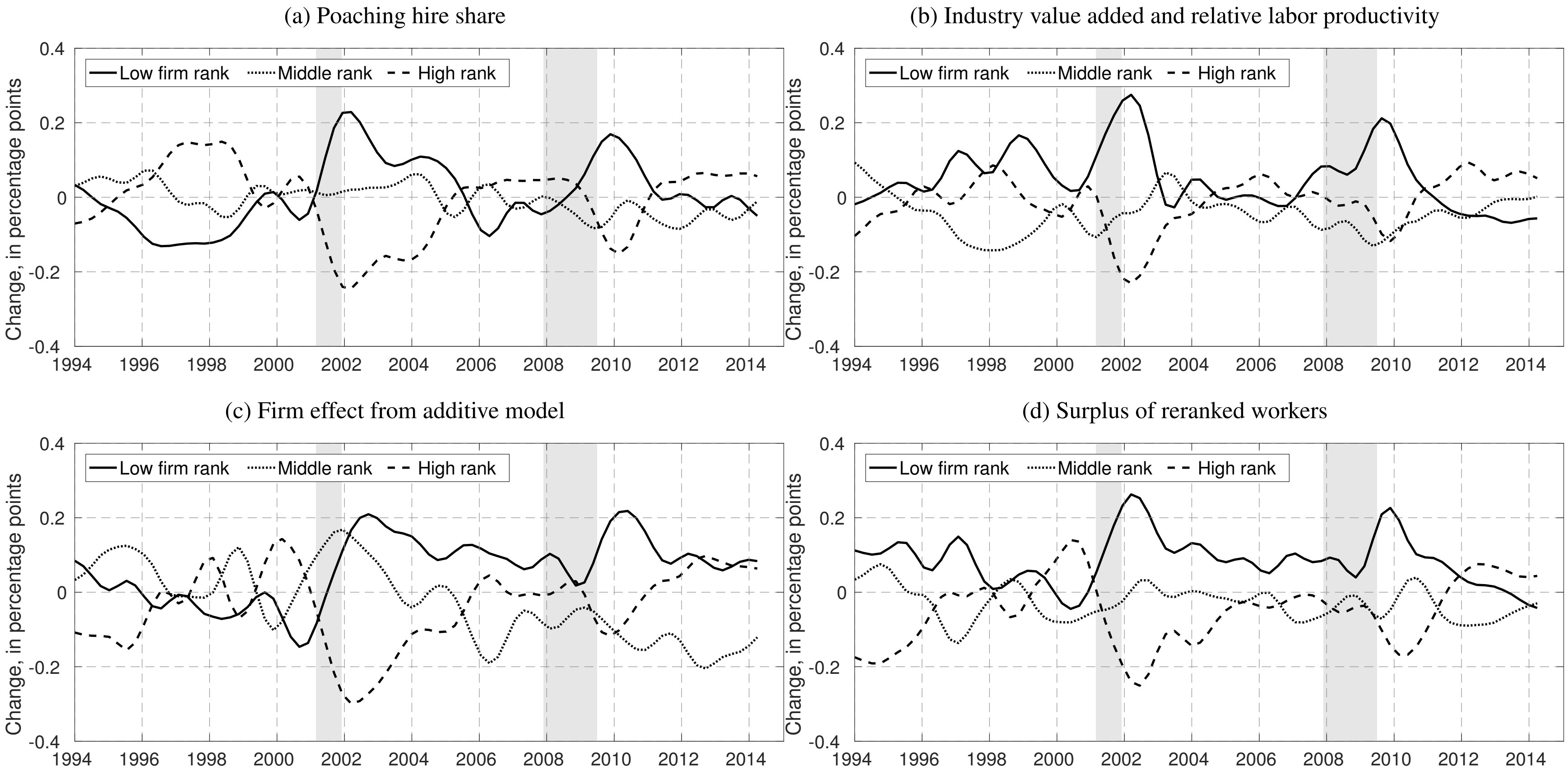
countercyclical decline in net nonemployment flows, its share of employment increases.

Our results on cyclical worker composition have an intuitive interpretation. In all four of our ranking methods, recessions are times when the employment distribution shifts away from low-rank workers and towards high-rank workers. This result can be understood as a cleansing effect on the worker distribution. During economic expansions, increasing employment requires hiring relatively unproductive workers. When the economy contracts, there are fewer jobs available. The more productive workers are better able to compete for scarce jobs. Therefore, the employment share of more productive workers increases while that of less productive workers declines.

3.2.2 Firm composition

We now explore how the employment shares of differently ranked firms change over time. Figure 2 shows quarterly changes in firm composition over time, using each of our four ranking methods. In the average quarter, each tercile accounts for one-third of employment, but this share changes over time. We plot the change in employment share $E_{\bullet jt}/E_t - E_{\bullet jt-1}/E_{t-1}$ for each firm tercile j. Panel 2(a) shows how firm composition evolves when firms are ranked by poaching hire share. Panel 2(b) shows how firm composition evolves when firms are ranked by labor productivity. Panel 2(c) shows

Figure 2: Change in firm rank shares



Notes: Shaded regions indicate recessions. Data seasonally adjusted and Henderson-filtered using X-11.

how firm composition evolves when firms are ranked by their estimated effect from an additive model. Panel 2(d) shows how firm composition evolves when firms are ranked by match surplus implied by the earnings of workers re-ranked against their co-workers.

Most of the changes in firm composition in Figure 2 are small, with each tercile's share rarely changing by more than 0.2 percentage points. The exceptions occur during and after each of the two recessions. In expansions, the high-rank firm tercile slowly increases its share of employment, and the share of low-rank firms decreases. During and after the 2001 and 2007-2009 recessions, employment quickly shifted away from high-rank firms and toward low-rank firms.

Table 3 measures how firm composition varies with the unemployment rate.[12] Each specification regresses the outcome of interest $E_{\bullet jt}/E_t - E_{\bullet jt-1}/E_{t-1}$ on the unemployment rate (either the deviation from its HP trend or its first-difference), as well as a linear time trend and seasonal dummies. Labor market downturns are associated with an increase in the employment share of low-rank firms, and a corresponding decline for high-rank firms. The change in employment composition by firm rank is qualitatively consistent across ranking methods when we use the HP-detrended unemployment rate as our cyclical indicator, i.e., when we measure times of low vs. high unemployment. For every additional percentage point of the unemployment rate relative to its HP trend, the employment share of low-rank firms increases by 0.032 to 0.063 percentage points, and that of high-rank firms decreases by 0.039 to 0.072 percentage points. A percentage point increase in the unemployment rate is associated with an increase in the employment share of low-rank firms of 0.002 to 0.149 percentage points, and a decrease in the share of high-rank firms of 0.066 to 0.090 percentage points.

The countercyclical increase in the employment share of low-rank firms is a robust empirical finding, and is consistent with the evidence presented by Haltiwanger et al. (2018). Note that these four ranking methods rely on three distinct sets of variables. The poaching hire share ranking method uses transitions of workers between firms and from nonemployment to employment. The labor productivity measure uses firm-level revenue and industry-level value added. The other two firm ranking methods rely on earnings. Despite substantial differences in how each ranking method is constructed, each method approximates a firm's rank in the job ladder.

[12] We present the results of robustness exercises for our firm composition results in Appendix Tables C2 and C4. Appendix Table C2 presents results when we exclude firms who employ relatively few workers. Appendix Table C4 presents results when we only rank firms using pre-recession employment data, and only rely on the years during and after recessions to measure employment composition of each so-defined subset of firms. In both robustness exercises, results are similar in sign but smaller in magnitude than in Table 3. Note that the regression results use HP-detrended unemployment in Appendix Table C4 are not statistically different from zero.

Table 3: Changes in firm rank shares and the unemployment rate

Firm tercile	Poaching share of hires	Labor productivity	Additive worker & firm effects	Surplus of reranked workers
	Difference in unemployment from its HP trend			
Low	6.3***	3.2*	5.5***	5.4***
	(2.2)	(1.7)	(1.5)	(1.4)
High	-6.9***	-3.9***	-6.6***	-7.2***
	(1.8)	(1.4)	(1.8)	(1.7)
	First-difference of the unemployment rate			
Low	12.0**	14.9***	2.0	9.0**
	(5.5)	(3.9)	(3.9)	(3.6)
High	-8.9*	-9.0***	-8.5*	-6.6
	(4.7)	(3.4)	(4.7)	(4.5)

Notes: We regress the change in share of employment on the seasonally-adjusted unemployment rate, season dummies, and a time trend. *, **, and *** indicate statistical significance at the 10%, 5%, and 1%, respectively. Standard errors are in parentheses. To avoid excessive decimal places, the dependent variables range from $[-100, 100]$, while the cyclical indicators range from $[-1, 1]$.

We now explore the role of the cyclical job ladder in explaining the countercyclical increase in the employment share of low-rank firms. Table 4 measures how poaching and nonemployment transitions for firms of different ranks vary with the unemployment rate. Our dependent variables are $(N^a_{\bullet jt} - N^s_{\bullet jt})/((E_t + E_{t-1})/2)$ for nonemployment and $(P^a_{\bullet jt} - P^s_{\bullet jt})/((E_t + E_{t-1})/2)$ for poaching.[13] In weaker labor markets, net hiring from nonemployment declines for both low-rank and high-rank firms. Low-rank firms mostly exhibit a larger decline than high-rank firms, although it is usually not possible to reject equality of the coefficients. When the unemployment rate increases by one percentage point, high-rank firms decrease net hiring from nonemployment by 0.301 to 0.380 percentage points, while low-rank firms increase decrease net hiring by 0.375 to 0.512 percentage points. Nonemployment hiring changes are smaller using HP-detrended unemployment as the cyclical indicator. For every additional percentage point of the unemployment rate relative to its HP trend, net hiring from nonemployment declines by 0.091 to 0.109 percentage points for high-rank firms, and declines by 0.102 to 0.120 for low-rank firms.

Table 4 also measures how net poaching for firms of different ranks vary with the unemployment rate. Net poaching flows of high-rank and low-rank firms move in opposite directions in response to

[13] Appendix Figure C2 shows the time series of the net nonemployment measure for each firm tercile, and Appendix Figure C3 shows analogous time series for the net poaching measure.

Table 4: Change in net hiring by firm rank and unemployment

Firm tercile	Poaching share of hires	Labor productivity	Additive model firm effects	Surplus of reranked workers
	Difference in unemployment from its HP trend			
Nonemployment				
Low	-10.2**	-12.0***	-11.9***	-11.9***
	(3.9)	(2.5)	(3.2)	(2.9)
High	-9.1***	-10.1***	-10.9***	-10.7***
	(2.1)	(2.7)	(2.8)	(2.7)
Poaching				
Low	6.0***	5.1***	6.8***	6.5***
	(0.9)	(0.9)	(1.0)	(1.0)
High	-6.5***	-4.4***	-6.2***	-5.9***
	(1.0)	(0.8)	(0.9)	(0.9)
	First-difference of the unemployment rate			
Nonemployment				
Low	-40.4***	-37.5***	-51.2***	-46.6***
	(8.9)	(5.4)	(6.0)	(5.7)
High	-30.1***	-38.0***	-35.1***	-33.3***
	(4.5)	(5.8)	(6.3)	(6.1)
Poaching				
Low	12.7***	12.0***	13.7***	14.8***
	(2.3)	(2.1)	(2.6)	(2.5)
High	-12.7***	-9.7***	-12.3***	-11.2***
	(2.6)	(2.1)	(2.5)	(2.6)

Notes: We regress net hires on the seasonally-adjusted unemployment rate, seasonal dummies, and a time trend. *, **, and *** indicate statistical significance at 10%, 5%, and 1%, respectively. Standard errors are in parentheses. To avoid excessive decimal places, the dependent variables range from $[-100, 100]$, while the cyclical indicators range from $[-1, 1]$.

unemployment. To understand this result, note that high-rank firms tend to gain employment through poaching, while low-rank firms tend to lose employment through poaching. In weaker labor markets, workers move from low-rank to high-rank firms at a slower pace. Therefore, net poaching flows of high-rank firms decline, while net poaching flows of low-rank firms increase. Specifically, when the unemployment rate increases by one percentage point, net hiring through poaching of high-rank firms declines by 0.097 to 0.127 percentage points. For low-rank firms, it increases by 0.120 to 0.148 percentage points. For every additional percentage point in the HP-detrended unemployment rate, net poaching of high-rank firms declines by 0.044 to 0.065 percentage points, and that of low-rank firms

Table 5: Net poaching and nonemployment: high minus low firm tercile

	Employment duration	Average earnings	Additive model worker effects	Rank workers vs. co-workers
	Difference in unemployment from its HP trend			
Poaching	-12.5***	-9.5***	-13.0***	-12.5***
	(1.7)	(1.6)	(1.9)	(1.9)
Nonemp.	1.1	1.9	0.9	1.1
	(2.9)	(1.7)	(1.9)	(1.9)
	First-difference of the unemployment rate			
Poaching	-25.2***	-21.7***	-26.1***	-26.0***
	(4.7)	(4.0)	(5.1)	(4.9)
Nonemp.	10.4	-0.5	16.4***	13.6***
	(7.0)	(4.3)	(4.3)	(4.4)

Notes: We regress the net hire differential on the seasonally-adjusted unemployment rate, seasonal dummies, and a time trend. *, **, and *** indicate statistical significance at 10%, 5%, and 1%, respectively. Standard errors are in parentheses. To avoid excessive decimal places, the dependent variables range from $[-100, 100]$, while the cyclical indicators range from $[-1, 1]$.

increases by 0.051 to 0.068 percentage points.

The transition dynamics in Table 4 highlight the role of the cyclical job ladder in changes in employment composition by firm rank. In order for there to be countercyclical increases in the employment share of low-rank firms, the cyclical response of poaching must be greater than that of nonemployment.[14] The net poaching response dominates in the times of high unemployment that follow recessions. We consider the differential poaching and nonemployment hiring responses of low- and high-rank firms in Table 5. Using the HP-detrended unemployment rate as the cyclical indicator yields a differential net poaching response that favors low-rank firms by 0.095 to 0.130 percentage points. Meanwhile, the differential net nonemployment response, which favors high-rank firms, is small in comparison: only 0.009 to 0.019 percentage points. Therefore, in times of high unemployment, cyclical changes in relative employment are driven by differences in net poaching.

[14]This relationship between the cyclical job ladder and employment composition helps interpret Figure 2, which shows that the employment share of low-rank firms has a larger increase during and after the 2001 recession than the 2007-2009 recession. This is despite the fact that the latter recession was more severe, both in terms of output and in the associated decline in the health of the labor market. Appendix Figures C2 and C3 show net nonemployment and net poaching hires, respectively, for each firm rank tercile. Changes in net poaching are similar in the 2001 and 2007-2009 recessions. However, the nonemployment response to the 2007-2009 recession is much larger than that of the 2001 recession. The countercyclical shift in employment toward low-ranked firms is determined by the differential poaching response (which favors low-rank firms) relative to the differential nonemployment response (which favors high-rank firms). Therefore, the employment share of low-rank firms is greater in the 2001 recession relative to the 2007-2009 recession.

In recessions, the slowdown of the job ladder competes with an effect from nonemployment transitions. Net nonemployment hiring of both low-rank and high-rank firms declines, but the net nonemployment hiring of high-rank firms usually declines by less. Table 5 shows that, using the first difference of the unemployment rate, the differential response of the net nonemployment margin of high-rank firms to low-rank firms is -0.005 to 0.164 percentage points. This result is consistent with a cleansing effect of recessions: relatively productive businesses are less affected. However, there is a sullying effect that more than offsets this cleaning effect. The corresponding differential net poaching response of low-rank firms relative to high-rank firms is between 0.217 to 0.261 percentage points.

In summary, we find that the employment share of low-ranked firms increases in times of high unemployment that follow recessions. This countercyclical sullying of the firm distribution contrasts with the countercyclical cleansing of the worker distribution that we documented in Section 3.2.1. The cyclical job ladder drives the changes in the employment share of low-rank firms, consistent with the earlier evidence of Haltiwanger et al. (2018), Haltiwanger, Hyatt, and McEntarfer (2018), and Moscarini and Postel-Vinay (2018).

3.3 Cyclical worker-firm rank agreement

We now characterize cyclical sorting in the labor market by measuring the correlation between worker and firm ranks. We move from characterizing cyclical changes in workers and firms considered independently to considering the dynamics of worker-firm rank combinations. Joint worker-firm dynamics largely follow from the composition changes that we document above.

3.3.1 Employment shares of worker-firm rank combinations

We begin our analysis by considering how the shares of employment for combinations of the worker and firm tercile ranks evolve with the labor market conditions. Table 6 shows how sorting varies with the unemployment rate. Specifically, the dependent variables in our regressions are $E_{ijt}/E_t - E_{ijt-1}/E_{t-1}$ for each worker tercile i and firm tercile j. In weaker labor markets, the employment share of low-rank workers at high-rank firms declines. A one percentage point increase in the unemployment rate is associated with a 0.065 to 0.181 decline in the share of employment of such matches, and an additional percentage point in the HP-detrended unemployment rate is associated with a decline of 0.019 to 0.060 percentage points. This effect increases the agreement between worker and

Table 6: Changes in worker-firm rank shares and unemployment

	Employment & poaching share	Earnings & productivity	Additive worker & firm effects	Ranked workers & surplus
	Difference in unemployment from its HP trend			
Low-rank firms &				
Low-rank workers	-0.7	-0.7	0.9	1.1
	(1.3)	(1.0)	(1.0)	(0.7)
High-rank workers	2.6***	1.7***	2.0***	1.8***
	(0.9)	(0.6)	(0.5)	(0.5)
High-rank firms &				
Low-rank workers	-6.6***	-1.9***	-3.0***	-3.0***
	(1.3)	(0.6)	(0.7)	(0.7)
High-rank workers	1.9**	-0.4	-1.2	-1.3*
	(0.8)	(0.7)	(1.0)	(0.7)
	First-difference of the unemployment rate			
Low-rank firms &				
Low-rank workers	-8.3***	3.4	-0.8	0.9
	(3.0)	(2.5)	(2.3)	(1.7)
High-rank workers	9.8***	5.0**	2.3*	4.6***
	(1.9)	(1.4)	(1.3)	(1.3)
High-rank firms &				
Low-rank workers	-18.1***	-7.7***	-9.1***	-6.5***
	(3.1)	(1.4)	(1.6)	(1.7)
High-rank workers	11.0***	4.1***	5.7**	4.1**
	(1.6)	(1.7)	(2.3)	(1.7)

Notes: We regress the change in employment on the seasonally-adjusted unemployment rate, seasonal dummies, and a time trend. *, **, and *** indicate statistical significance at 10%, 5%, and 1%, respectively. Standard errors are in parentheses. To avoid excessive decimal places, the dependent variables range from $[-100, 100]$, while the cyclical indicators range from $[-1, 1]$.

firm ranks and therefore strengthens sorting. An analogous countercyclical change weakens sorting: high-rank workers are more likely to work at low-rank firms. A one percentage point increase in the unemployment rate is associated with an increase of the share of employment of high-rank workers at low-type firms of 0.023 to 0.098 percentage points, and an additional percentage point in the HP-detrended unemployment rate is associated with an increase of 0.017 to 0.026 percentage points. Therefore, the association between firm rank and worker rank strengthens, on net, among these margins. This suggests a modest increase in worker-firm rank agreement in weaker labor markets.

These cyclical changes in labor market sorting follow from the composition changes that we documented in Sections 3.2.1 and 3.2.2, and also provide insights into how these composition changes occur. Labor market downturns are times when low-rank workers are less likely to work. The decline in the employment share of low-rank workers is concentrated at high-rank firms, and thus increases the agreement between worker rank and firm rank. By contrast, the increase in the employment share of high-rank workers during economic downturns is concentrated at low-rank firms. This countercyclical change weakens positive sorting. The decline of low-rank workers at high-rank firms is larger than the increase of high-rank workers at low-rank firms, and so agreement increases on net.

3.3.2 The poaching and nonemployment margins of sorting

We now explore the role of poaching and nonemployment hiring in generating the changes in employment of particular worker-firm combinations. This analysis provides insight into what drives countercyclical increases in worker-firm rank agreement. The cleansing effect drives countercyclical changes along the nonemployment margin. In contrast, changes related to poaching imply a sullying effect. These cleansing and sullying mechanisms determine changes in sorting.

Table 7 shows how the net nonemployment hiring propensity for each worker-firm combination varies with our cyclical indicators. We use $(N^a_{ijt} - N^s_{ijt})/((E_{ijt} + E_{ijt-1})/2)$ as our dependent variable. This allows us to assess whether, for workers of a given rank, there is a differential response by firm rank, or whether these changes are spread rather evenly across firms of different ranks.[15]

There is some evidence of a cleansing effect that removes high-rank workers from low-rank firms. When we rank workers by employment duration and firms by poaching hire share, we find essentially no response of high-rank workers at either low- or high-rank firms. The other three ranking methods show a modest differential. Specifically, when the unemployment rate increases by one percentage point, the share of high-rank workers at high-rank firms declines by 0.621 to 0.737 percentage points, compared with 0.790 to 1.145 at low-rank firms. These results provide evidence that the cleansing effect of recessions may disproportionately affect worker-firm matches in which the ranks disagree. There is less evidence of a cleansing effect on worker-firm matches when we use HP-detrended unemployment rate as our cyclical indicator.

[15]This change in the denominator is needed in order to assess cleansing versus sullying effects. Although our worker and firm terciles each have one-third of employment on average, this does not imply that the intersections of these tercile groups each has one-ninth of employment. In particular, there are relatively few low-rank workers at high-rank firms. In order to measure the differential response, e.g., of low-rank workers at firms of low-rank vs. high-rank, we need to make this adjustment to our denominator.

Table 7: Net nonemployment hires for worker-firm rank shares and unemployment

	Employment & poaching share	Earnings & productivity	Additive worker & firm effects	Ranked workers & surplus
	Difference in unemployment from its HP trend			
Low-rank firms &				
Low-rank workers	-57.4***	-43.8***	-41.7***	-40.6***
	(16.4)	(8.7)	(9.0)	(8.6)
High-rank workers	-6.8	-25.2**	-25.0**	-27.6***
	(11.0)	(10.8)	(11.1)	(9.1)
High-rank firms &				
Low-rank workers	-69.5***	-41.8***	-45.8***	-45.7***
	(15.2)	(10.8)	(12.2)	(10.1)
High-rank workers	-4.5*	-25.4***	-28.7***	-27.6***
	(2.7)	(7.9)	(8.4)	(7.9)
	First-difference of the unemployment rate			
Low-rank firms &				
Low-rank workers	-218.6***	-126.1***	-154.6***	-149.0***
	(35.2)	(19.9)	(17.4)	(16.6)
High-rank workers	-3.5	-79.0***	-114.5***	-98.8***
	(26.9)	(25.8)	(24.9)	(20.7)
High-rank firms &				
Low-rank workers	-251.1***	-182.7***	-200.6***	-158.4***
	(30.6)	(19.7)	(22.8)	(21.0)
High-rank workers	-5.4	-73.7***	-62.1***	-68.1***
	(6.6)	(18.7)	(20.9)	(19.3)

Notes: We regress net nonemployment hires on the seasonally-adjusted unemployment rate, seasonal dummies, and a time trend. *, **, and *** indicate statistical significance at 10%, 5%, and 1%, respectively. Standard errors are in parentheses. To avoid excessive decimal places, the dependent variables range from $[-100, 100]$, while the cyclical indicators range from $[-1, 1]$.

Table 8 shows the net poaching response, using $(P^a_{ijt} - P^s_{ijt})/((E_{ijt} + E_{ijt-1})/2)$ as our dependent variable. The net poaching flows of low-rank workers at high-rank firms decline substantially in labor market downturns. Specifically, a one percentage point increase in the unemployment rate is associated with a decline in net poaching flows for low-rank workers at high-rank firms of 0.454 to 0.771 percentage points. In contrast, net poaching flows of low-rank workers at low-rank firms increase by 0.305 to 0.458 percentage points. This slowdown of the job ladder is less severe for high-rank workers. Net poaching of high-rank workers at high-rank firms declines by 0.131 to 0.206

Table 8: Net poaching hires for worker-firm rank shares and unemployment

	Employment & poaching share	Earnings & productivity	Additive worker & firm effects	Ranked workers & surplus
	Difference in unemployment from its HP trend			
Low-rank firms &				
Low-rank workers	22.1***	10.3***	16.5***	17.0***
	(3.1)	(2.9)	(2.5)	(2.6)
High-rank workers	9.1***	20.5***	21.4***	17.1***
	(2.1)	(3.1)	(3.3)	(2.7)
High-rank firms &				
Low-rank workers	-37.6***	-18.8***	-28.5***	-24.1***
	(5.2)	(3.1)	(5.0)	(4.2)
High-rank workers	-7.4***	-8.2***	-12.4***	-10.3***
	(1.4)	(1.8)	(2.0)	(1.7)
	First-difference of the unemployment rate			
Low-rank firms &				
Low-rank workers	45.8***	30.5***	34.5***	38.2***
	(8.3)	(5.3)	(6.7)	(6.7)
High-rank workers	17.7***	34.7***	31.8***	38.4***
	(5.3)	(8.7)	(9.4)	(7.0)
High-rank firms &				
Low-rank workers	-76.2***	-45.4***	-77.1***	-49.6***
	(14.2)	(7.6)	(11.6)	(10.9)
High-rank workers	-13.1***	-16.1***	-19.2***	-20.6***
	(3.6)	(4.7)	(5.7)	(4.5)

Notes: We regress net poaching on the seasonally-adjusted unemployment rate, season dummies, and a time trend. *, **, and *** indicate statistical significance at the 10%, 5%, and 1%, respectively. Standard errors are in parentheses. To avoid excessive decimal places, the dependent variables range from $[-100, 100]$, while the cyclical indicators range from $[-1, 1]$.

percentage points, while that of low-rank firms declines by 0.177 to 0.384 percentage points.

The slowdown in movement from low-rank to high-rank firms is greater in magnitude for low-rank workers than for high-rank workers. Therefore, low-rank workers are especially unlikely to move to high-rank firms during recessions. Overall, this evidence suggests that the countercyclical shift of low-rank workers at high-rank firms is driven by a slowdown in the job ladder. As in Haltiwanger, Hyatt, and McEntarfer (2018), the fact that this slowdown affects employment of both low-rank workers and high-rank workers at high-rank firms suggests that workers of all ranks agree on which firms are relatively desirable workplaces.

These results on the cyclical changes in net poaching and net nonemployment flows for worker-firm rank groups help illustrate the role of the cyclical job ladder in increasing and decreasing the agreement between worker and firm ranks. In the times of high unemployment that follow recessions, net poaching basically drives all changes in worker-firm rank agreement. During recessions, net nonemployment transitions have some have explanatory effect, particularly for high-rank workers at low-rank firms. According to our additive model, the nonemployment differential explains at most 29% of the shift in low-rank workers' employment share away from high-ranked firms and towards low-ranked firms.[16] Therefore, the countercyclical decline in the share of low-rank workers at high-rank firms is mostly due to sullying effects. And similarly, most of the countercyclical increase in the agreement between worker rank and firm rank can be attributed to sullying rather than cleansing effects. Thus, the cyclical job ladder drives changes in sorting.

3.3.3 Worker-firm rank correlation and unemployment

In order to characterize the degree to which sorting varies with the unemployment rate, we also consider how the correlation between worker and firm ranks, varies with the unemployment rate.[17] Regression evidence is shown in Table 9. Specifically, we allow workers and firms to be in one of 50 employment-weighted rank bins, and we measure the correlation between worker rank and firm rank for those bins. This correlation, which is in the interval $[-1,1]$, is calculated separately for each quarter in the data, and serves as the dependent variable for regressions on our two cyclical indicators.[18] Overall, the measured correlation between worker rank and firm rank increases in labor market downturns. For example, a one percentage point increase in the unemployment rate is associated with an increase of 0.012 in the correlation between a worker's rank measured by employment duration and a firm's rank measured by the poaching hire share. This relationship is stronger when we use the first-difference of the unemployment rate as our cyclical indicator, which suggests that the correlation between worker ranks and firm ranks increases during recessions more than in the times of high

[16] See the results for the additive model in Tables 7 and 8 where we use the first-difference of the unemployment rate as the cyclical indicator, $(2.006-1.546)/(2.006-1.546+0.345+0.771) \approx 0.29$. Ranking workers by employment duration and firms by their poaching hire share, we obtain an estimate of $(2.511-2.186)/(2.511-2.186+0.458+0.762) \approx 0.21$.

[17] For the full set of correlations between worker and firm ranks, see Appendix Table C5.

[18] Note that to avoid excess decimal places in Table 9, we multiply the correlations by 100.

Table 9: Relationship between worker-firm correlations and the unemployment rate

	Employment & poaching share	Earnings & productivity	Additive worker & firm effects	Ranked workers & surplus
	Difference in unemployment from its HP trend			
Rank correlation	-0.0	0.2*	0.3	0.3***
	(0.2)	(0.1)	(0.2)	(0.1)
	First-difference of the unemployment rate			
Rank correlation	1.2**	0.8***	1.7***	-0.4
	(0.5)	(0.1)	(0.4)	(0.3)

Notes: We regress the correlation between worker and firm ranks for each quarter on the seasonally-adjusted unemployment rate, season dummies, and a linear time trend. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively. Standard errors are in parentheses. To avoid excessive decimal places, the dependent variables range from $[-100, 100]$, while the cyclical indicators range from $[-1, 1]$.

unemployment that follow.

To interpret the results of Table 9, it is important to consider the results on changes in the employment of different worker-firm rank combinations that we discussed in Section 3.3.1. Countercyclical changes include some that strengthen and others that weaken the degree of agreement between worker and firm ranks. Our results show that, overall, positive sorting tends to strengthen during labor market downturns.

3.4 Summary of empirical findings

Our empirical evidence shows how labor market composition and sorting change with aggregate conditions. All four of our ranking methods deliver similar results. During recessions, the employment share of low-rank workers declines while that of high-rank workers increases. This change can be attributed to the differential net nonemployment transitions of low- vs. high-rank workers. Although workers of all ranks are less likely to work during economic downturns, low-rank workers are especially unlikely to work. Thus, worker composition can be characterized by a countercyclical cleansing effect as relatively unproductive workers leave employment during downturns.

Cyclical changes in firm composition are quite different. During economic downturns, the employment share of low-rank firms increases. This is true whether we rank firms by poaching hire share, labor productivity, or transformations of worker earnings. This increase in the employment share of low-rank firms is driven by the countercyclical decline in net poaching from low-rank to high-rank firms. During expansions, high-rank firms poach workers away from low-rank firms as workers move up the job ladder. But during downturns, the job ladder shuts down, and relative employment increases for low-ranked firms.

This countercyclical cleansing of the worker distribution and sullying of the firm distribution drive changes in labor market sorting. As low-rank firms and high-rank workers have an increasing share of employment during labor market downturns, the share of such job matches naturally increases. This weakens the degree of sorting and is driven by a sullying effect. We also find that low-rank workers are less likely to work at high-rank firms, which strengthens sorting. This change can mostly be attributed to the slowdown of the job ladder and hence also appears to be a sullying effect. The decline of low-rank workers at high-rank firms dominates, and the measured agreement between worker rank and firm rank increases during recessions.

4 A Model of heterogeneous workers and firms

In this section, we use a model of labor market sorting to interpret the facts documented in Section 3, focusing on the cyclical changes in worker and firm composition. Specifically, we seek to understand the mechanisms that may drive countercyclical shifts in employment toward high-rank workers and low-rank firms. Such effects matter for sorting because these composition changes drive the observed changes in the worker-firm rank distribution. Before describing the details of the model it is useful to discuss the intuition.

Consider the worker distribution. The worker distribution will be cleansed in downturns if the marginal matches tend to be those with low-type workers. To take an extreme case, if match output is almost entirely a function of worker type then in a recession the dissolved matches will be almost entirely those with low-type workers, as opposed to low-type firms. Then the question becomes whether a such a worker-centric production function is consistent with sullying of the firm distribution in recessions. We argue that it is, and that it is driven by on-the-job search. Moscarini and Postel-Vinay (2013) have shown that the sullying of the firm distribution is consistent with a model where there are heterogeneous firms and on-the-job search.[19] In their model, lower recruiting during a

[19]Cairó, Hyatt, and Zhao (2018) also show that this mechanism can operate in a simplified version of Lise and Robin (2017) that abstracts from worker heterogeneity and only considers firm heterogeneity.

recession leads to fewer poaching losses for low-type firms, allowing them to grow relative to high-type firms. This poaching mechanism can operate under any amount of firm heterogeneity. Thus it is consistent with a match output function that is mostly (though not completely) a function of worker type. We show that model estimation implies a production function that is driven by worker type.

But will the sullying effect dominate the firm distribution? In order to do so, this sullying effect of recessions must overcome the cleansing effect. In the models of Barlevy (2002) and Moscarini and Postel-Vinay (2013), on-the-job search is the mechanism for the sullying effect of recessions. During labor market downturns, the quit rate is lower, and so workers spend more time in lower-value matches. However, if a match has a value that is too low, then it may not be able to offer compensation to the worker that provides more value than unemployment. A decrease in such relatively unproductive job matches is the cleansing effect of recessions. In the framework that follows, either the cleansing or sullying effect can dominate the firm distribution.

4.1 Model environment

We work with the model proposed by Lise and Robin (2017). Their model includes aggregate shocks, worker heterogeneity, firm heterogeneity, and on-the-job search. Despite its richness, the model can be solved relatively easily via the simulated method of moments. We briefly describe the main features here, see Lise and Robin (2017) for details.

Time is discrete and goes on forever. There is a fixed mass of workers. Workers are indexed by $x \in [0,1]$. Firms (jobs) are indexed by $y \in [0,1]$.[20] Jobs may be vacant or filled. Maintaining a vacant job costs $c(v(y))$, which is exogenous to the firm. When matched with a worker, a job produces flow output $f(x,y,z)$ per period, where z is the productivity shock. Workers search while unemployed, and search with a lower intensity s while matched. Search is random, and the number of meetings is determined by a Cobb-Douglas meeting function that takes searching workers and vacancies as its inputs. Matches are dissolved at an exogenous rate δ. Matches may also dissolve endogenously, as aggregate shocks make existing matches unprofitable or outside offers result in poaching losses.

The aggregate productivity shock z_t evolves exogenously according to, e.g., an AR(1). In period t

[20] Note that there is a distinction between a worker or firm's type, and the rank a worker or firm will receive from a ranking method, as argued by Eeckhout and Kircher (2011). Papers such as Hagedorn, Law, and Manovskii (2017) propose methods of recovering worker and firm types via a ranking method, but except in specific model environments there is not a one-to-one correspondence between observable (estimated) rankings and unobservable worker and firm types.

the aggregate state is summarized by z_t and the distribution of workers across job types y. The timing is as follows. At the beginning of each period z changes from z_{t-1} to z_t. Next, exogenous separations occur at rate δ. Endogenous separations also occur, dissolving matches with negative expected surplus. Then, given the aggregate state, firms decide how many vacancies to post. Unemployed and employed workers meet vacancies according to an aggregate meeting function. When a worker and firm meet they decide whether to match and at what wage. Finally, production takes place and wages are paid.

A key feature of the Lise and Robin (2017) model is wage setting. Wages are renegotiated only when one party can credibly threaten to dissolve the match if the wage goes unchanged. This may occur if the aggregate state changes, changing match production and/or the outside options. It may also occur if the worker receives a job offer from another firm. When a firm meets an unemployed worker, the firm makes a take it or leave it offer of an initial wage. The worker must accept the offer or refuse and remain unemployed. In equilibrium the firm will offer a wage that delivers nothing more than the worker's reservation value, and the firm will extract all the expected surplus of the match.[21] When an employed worker meets a second firm, the two firms are put into Bertrand competition. Each firm will try to offer a wage that barely exceeds the value delivered by their competitor. The outcome is that the worker will end up working for the firm that has the highest match surplus with the worker, and will receive the full value of the surplus with the losing firm.[22]

Under the wage bargaining outlined above, match surplus is independent of the other equilibrium variables. In particular, let $b(x,z_t)$ be the flow value of unemployment. Lise and Robin show that match surplus $S(x,y,z_t)$ obeys

$$S(x,y,z_t) = f(x,y,z_t) - b(x,z_t) + \frac{1-\delta}{1+r}\mathbb{E}_t\left[\max\left\{S(x,y,z_{t+1}),0\right\}\right] \tag{2}$$

[21]This is a major difference between the model of Lise and Robin (2017) and that of Hagedorn, Law, and Manovskii (2017). In the latter model, workers exiting nonemployment obtain the full surplus of the job match. While this feature allows Hagedorn, Law, and Manovskii (2017) to rank workers by the wages that they are paid, introducing this into the Lise and Robin (2017) model would make aggregate uncertainty considerably less tractable. More generally, the second-price auction wage setting mechanism of Lise and Robin (2017) often yields a negative relationship between wages and worker productivity for nonemployment exiters, see Bagger and Lentz (2019). Ranking workers by nonemployment duration and firms by their poaching hire rank allows us to avoid this issue.

[22]The exception is when the potential poaching firm cannot even offer enough to make on improvement on the worker's current wage. In this case the outside offer is not a credible alternative for the worker, and there is no renegotiation.

where $\frac{1}{1+r}$ is the discount factor. In this expression $f(x,y,z_t) - b(x,z_t)$ is the single period flow surplus of the match. It consists of match output, less the value the worker would derive from unemployment $b(x,z_t)$. The threat point of the firm is zero, since vacant jobs yield zero expected profit in equilibrium. The expectation on the right hand side of (2) is taken over future values of z_{t+1}. If the surplus is still positive in $t+1$ the match is still profitable, and yields the continuation value $S(x,y,z_{t+1})$. If the surplus of the match would become negative ($S(x,y,z_{t+1}) < 0$) then the match is dissolved and the continuation value is zero.

It is remarkable that the surplus depends only on z_t and not on the distribution of workers across firms and unemployment. As Lise and Robin (2017) explain, the split of the surplus will of course depend on distributions, but the total surplus need not. Their wage setting mechanism ensures that surplus is preserved under employer-to-employer transitions, because the original match serves as the (initial) reservation value of the new match. In addition, the value of unemployment is simple to calculate because the hiring firm takes all the expected surplus. The surplus equation can be solved simply by iterating until a fixed point is found. With the surplus equation in hand, the model equilibrium is easy to calculate. Most of the equilibrium equations are identities making sure that flows and stocks add up correctly. The reader is referred to Lise and Robin (2017) for further derivations.

4.2 Estimation

4.2.1 Data moments

Following Lise and Robin (2017), we estimate the model via the simulated method of moments (SMM). In the SMM objective function, we replace some of the Lise and Robin (2017) heterogeneity moments with our own LEHD-derived moments.[23] For the purposes of estimation we rank workers by nonemployment duration and firms by poaching hire shares, since these map most cleanly into the model framework and are also cheap to calculate on the model-simulated data. Construction of the model-implied moments is hampered by the computational cost of simulating a large panel of workers and firms on every iteration of the SMM solver. In the Lise and Robin (2017) model all workers of a given type have the same expected unemployment duration, and all firms of a given type have

[23] One technical issue involves the number of gridpoints used in estimation. Lise and Robin (2017) use 21 gridpoints for each of the worker and firm type distributions. Matching the cyclical worker and firm share moments using this limited number of grid points consistently created a weak Beveridge curve. In the estimation results presented here, we use 41 gridpoints for the worker type distribution and 31 for the firm type distribution. Of course, increasing the number of girdpoints in this way increases the time required for estimation.

the same expected poaching share. We can group the "true" worker and firm types into (population weighted) terciles based on these expected values.

Given a parameter guess, we order worker types by their expected unemployment duration. We divide worker types into terciles, based on average shares of employment, just as in the data. Similarly, we break the firm type distribution into terciles based on poaching share. Then we calculate the relationship of each worker tercile-firm tercile share with the first difference of the unemployment rate, as in column 1 of Tables 1 and 3. We substitute these new moments into the original ones used by Lise and Robin (2017), and do not target their moments on unemployment duration and productivity dispersion. We put a high subjective weight on the LEHD moments to be sure that they are influential in the estimation.

4.2.2 Parameterization

We parameterize the model following Lise and Robin (2017). The production function has 6 free parameters p_1 to p_6:

$$p(x,y,z) = z(p_1 + p_2x + p_3y + p_4x^2 + p_5y^2 + p_6xy). \tag{3}$$

The aggregate meeting function is Cobb-Douglas and transforms searching workers L_t and vacancies V_t into meetings according to $m(L_t,V_t) = \alpha L_t^{0.5} V_t^{0.5}$, with elasticity 0.5 and efficiency α is to be estimated. The convex vacancy posting cost function is $(1+c_1)^{-1} c_0 v^{1+c_1}$, where c_0 and c_1 are estimated. Exogenous job destruction δ and the relative intensity of on-the-job search s are also estimated. The worker type distribution is assumed to be Beta, with parameters β_1 and β_2. The flow utility of unemployment b for each is set to provide 70% of output from a worker's most productive match at aggregate state $z = 1$. Finally, the persistence of aggregate productivity (ρ) and its variability (σ) are also estimated. Thus there are a total of 15 parameters to be estimated.

Table 10: Parameter estimates

Parameter	Description	Estimate
α	Matching efficiency	0.668
s	Employed search effort	0.026
c_0	Vacancy cost constant	0.030
c_1	Vacancy cost curvature	0.080
δ	Exogenous separation rate	0.011
σ	Aggregate shock	0.073
ρ	Aggregate persistence	0.9997
β_1	Worker shape 1	2.577
β_2	Worker shape 2	11.270
p_1	Constant	0.003
p_2	Worker linear	1.998
p_3	Firm linear	-0.186
p_4	Worker quadratic	8.017
p_5	Firm quadratic	-1.744
p_6	Worker-firm interaction	6.517

4.3 Results

4.3.1 Parameter estimates

Table 10 shows our parameter estimates.[24] Employed workers exert search effort (s) that is only 2.6% that of unemployed workers. The parameter estimates $c_0 = 0.03$ and $c_1 = 0.08$ indicate that the vacancy cost function is increasing and convex. The shape parameters of the Beta distribution for workers, $\beta_1 < \beta_2$ implies that most of the mass of the worker distribution will be at lower values of the interval.

Table 11 shows how the estimated economy behaves with respect to the targeted moments. The simulated moments are generally close to the data. Table 11 also shows the LEHD moments. Each β_R^G is the coefficient from a regression of rank $R \in \{L, H\}$ for group $G \in \{worker, firm\}$ share of employment on the first difference of the unemployment rate. The cyclical composition moments are taken from column 1 of Tables 1 and 3. The implied moments from our estimation are in the second column of Table 11. On the worker side, the model matches the empirical pattern that the worker distribution shifts away from low-rank workers and toward high-rank workers. The LEHD data show a decline of 0.449 percentage points in the employment share of low-rank workers and an increase of 0.268 percentage points for high-rank workers. Our model estimates show a decline of 0.343

[24] For comparison of our results with Lise and Robin (2017), see Appendix D.

Table 11: Moments: data & model-implied

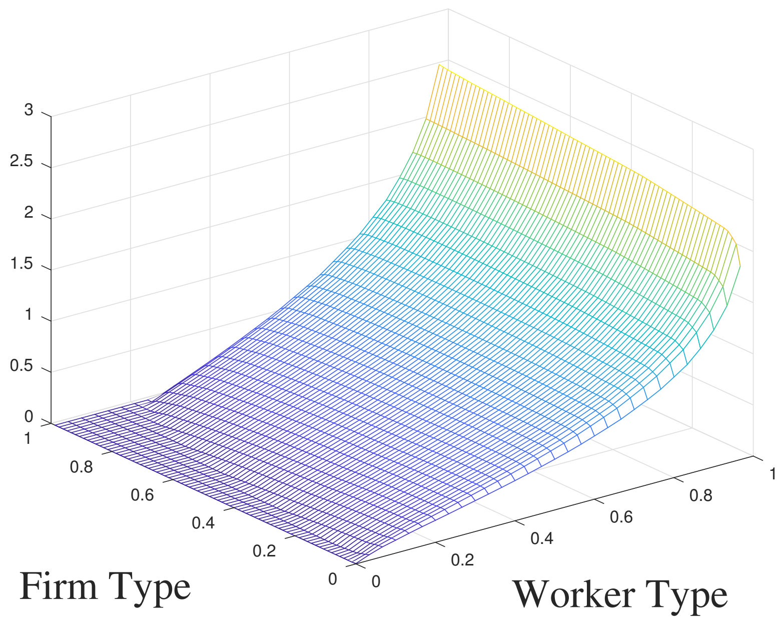
Moment	Data	Estimation
Moments from Lise and Robin (2017)		
$\mathbb{E}[U]$	0.058	0.060
$\mathbb{E}[UE]$	0.421	0.231
$\mathbb{E}[EU]$	0.025	0.015
$\mathbb{E}[EE]$	0.025	0.036
$\mathbb{E}[V/U]$	0.634	1.003
$\text{sd}[V]$	0.206	0.089
$\text{sd}[VA]$	0.033	0.033
$\text{autocorr}[VA]$	0.932	0.992
$\text{corr}[V,U]$	-0.846	-0.680
$\text{corr}[U,VA]$	-0.860	-0.979
$\text{sd}[U]$	0.191	0.150
$\text{sd}[UE]$	0.127	0.047
$\text{sd}[EU]$	0.100	0.159
$\text{sd}[EE]$	0.095	0.186
$\text{sd}[V/U]$	0.381	0.220
$\text{corr}[V,VA]$	0.721	0.749
$\text{corr}[UE,VA]$	0.878	0.166
$\text{corr}[EU,VA]$	-0.716	-0.946
$\text{corr}[UE,EE]$	0.695	-0.169
LEHD Moments		
β_L^{worker}	-44.9	-34.3
β_H^{worker}	31.6	17.6
β_L^{firm}	12.0	31.0
β_H^{firm}	-8.9	-59.1

Notes: β_R^G is the impact of a 1 percent change in the unemployment rate on the employment share of $G \in \{worker, firm\}$, $R \in \{L,H\}$.

percentage points in the employment share of low-rank firms and an increase of 0.176 percentage points in the employment share of high-rank workers.

On the firm side, a one percentage point increase in the unemployment rate is associated with a 10.796 percentage point decrease in the employment share of low-rank firms, and a 3.486 percentage point increase in the employment share of high-rank firms. When we target the cyclical firm and worker moments from LEHD data, we obtain something close to what the data describe: and increase of 0.310 percentage points in the employment share of low-rank firms and a 0.591 percentage point decline in the employment share of high-rank firms. The Lise and Robin (2017) framework therefore has the potential to demonstrate cleansing of the worker distribution and sullying of the firm

Figure 3: Model implied production function



Notes: Parameter estimates targeting LEHD moments. Worker and firm type distributions normalized to uniform.

distribution.[25]

4.3.2 What drives labor market composition and sorting?

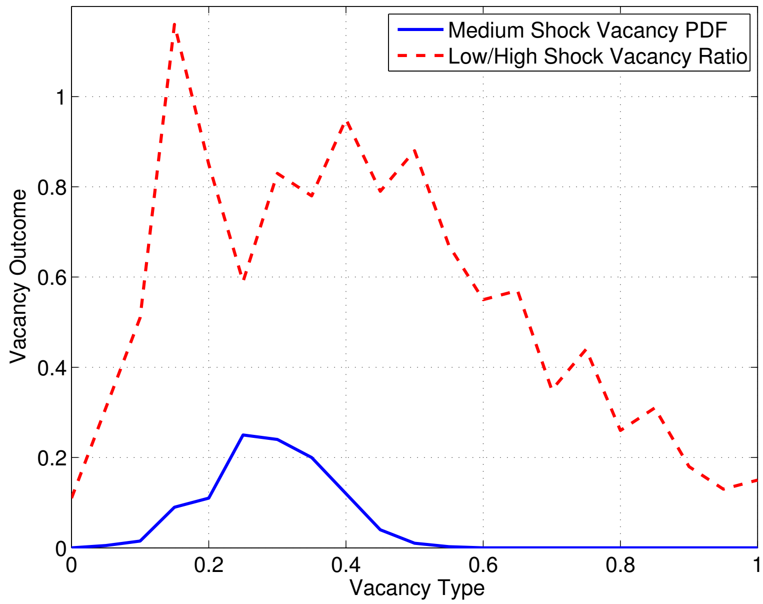
We have shown that the Lise and Robin (2017) model can produce a countercyclical increase in the share of firms with a high poaching rank. This exercise provides guidance on the mechanisms that generate cyclical changes in labor market composition and sorting that we observe in Section 3.

First, workers rather than firms must drive the match value of output. Figure 3 shows the estimated production functions. In Figure 3 the production function is normalized so that each increment along the worker (firm) type axis covers an equal fraction of the worker (firm) distribution. In other words, the worker and firm populations have been reindexed to be uniform distributions. It is apparent that both production function puts more weight on the worker type, and is nearly flat in firm type.[26] A

[25] Note that the Lise and Robin (2017) framework does not automatically generate our finding regarding cyclical changes in firm composition, see Appendix Table D.2. It is important that we target cyclical composition changes by worker and firm rank.

[26] Our worker and firm ranking method that follows Hagedorn, Law, and Manovskii (2017) provides a method of confirming this result. Inverting the surplus function, we can recover the implied match value of worker-firm output, see Appendix E. The results of this exercise are shown in Appendix Figure E1. Workers drive the match value of output, and there is an inflection among relatively high-type workers. This exercise therefore also suggests there are relatively large changes in output by worker type at the upper end than for middle-type workers.

Figure 4: Model implications for the cyclical distribution of vacancy posting



Notes: Vacancy posting distribution using Table 10 parameter estimates.

large empirical literature indicates that workers, rather than firms, drive the match value of output. Our results here are consistent with this finding. Abowd, Kramarz, and Margolis (1999) reported that workers, rather than firms, explained most of the variation in wages in their early analysis of linked employer-employee data. Most studies that followed have confirmed this, see Card et al. (2018) for a recent summary.

It is also key that firm entry and exit decisions not shift substantially by job ladder rank over the business cycle. Our cyclical share moments suggest that there is more, not less, employment at low-rank firms when unemployment is high. The behavior of low-rank firms is critical to obtain this result. Cyclical changes by firm type are characterized in Figure 4, which contains the distribution of vacancies that prevail in the long run under a zero shock (50th percentile), as well as the relative changes for low (25th percentile) vs. high (75th percentile) aggregate states of the economy. Note that the firm type is not to be confused with the value of the match. The highest value vacancies are found in the lower half of the firm type distribution, and reach a maximum at 0.25.

Figure 4 provides insights into the nature of countercyclical cleansing and sullying of low-rank firms. This can be seen in the measures of the low vs. high shock ratio of vacancies. Naturally, there are fewer vacancies with the aggregate state is low relative to when it is high, hence the ratio of vacancies is generally less than one. At the extremes of the distribution, firms post less than 20% of the vacancies in the low state than they do in the high aggregate state, however, Figure 4 also illustrates that there are approximately zero firms operating in this range of the distribution. The ratio of vacancies in the low vs. high state is also not monotonic, and what is especially interesting is a spike in the ratio of vacancies that occurs at the gridpoint in which the distribution of vacancies loses most of its mass. In other words, there is extra activity at the bottom of the job ladder: the set of firms that, when there is a high aggregate state, has an almost trivial mass since an offer to an employed worker has an approximately 100% chance of them being poached away. That the vacancy posting ratio exceeds one in a marginal case after targeting LEHD moments implies that some of the lowest-ranked firms are posting *more* vacancies in worse states of the economy than in better ones.

These results haves implications for how the low end of the wage offer distribution is determined. There are competing forces at work in the Lise and Robin (2017) model. First, there is the "cleansing" effect of recessions as in Caballero and Hammour (1994): when firms have relatively low value from producing, they are more likely to sensitive to macroeconomic shocks. This means that low aggregate states of the economy will drive low-value firms out of the firm distribution. The competing effect is described by Moscarini and Postel-Vinay (2013): the lowest ranked firms will obtain more value from posting a vacancy when they can hold on to their workers for longer. This mechanism can induce very low value firms to post relatively more vacancies. The latter mechanism can generate a spike in the vacancy posting distribution at its lowest end. When we match the model to the countercyclical increase in the employment share of low-rank firms that we observe in our linked employer-employee data, this sullying effect dominates.

5 Conclusion

In this paper, we use a number of recent methods that have been developed for ranking firms, workers, and the degree of sorting in the labor market via direct calculations on linked employer-employee data. Despite the fact that these different methods are often contrasted with each other due to their different findings regarding the nature and extent of sorting, we find that they share common cyclical properties. Low-rank workers are disproportionately affected by labor market downturns, in which their share of the workforce declines. In contrast, the share of employment in low-rank firms increases during and after labor market downturns. The reason for this is that the job ladder slows down substantially:

during economic expansions, high-rank firms rapidly poach workers away from low-rank firms, and so low-rank firms have a low size due to poaching. During economic contractions, both the job ladder and hiring from nonemployment slow, but the job ladder margin dominates in terms of cyclical employment composition at low vs. high-rank firms.

We find small countercyclical increases in the agreement between worker and firm ranks. During labor market downturns, there is a strong movement away from low-rank workers at high-rank firms. This is due to a dramatic slowdown in the job ladder for low-rank workers.

We evaluate our findings in the context of the model of cyclical labor market sorting proposed by Lise and Robin (2017). This framework can deliver countercyclical changes in worker and firm composition that mimic our empirical estimates. In order to produce countercyclical shifts away from less productive workers but toward less productive firms, workers must drive the match value of output. Furthermore, the sullying effect of recessions must be substantial if it is to overcome the cleansing effect.

References

[1] Abowd, John, Bryce Stephens, Lars Vilhuber, Fredrik Andersson, Kevin McKinney, Marc Roemer, and Simon Woodcock. 2009. "The LEHD Infrastructure Files and the Creation of the Quarterly Workforce Indicators." In *Producer Dynamics: New Evidence from Micro Data*, 68, Studies in Income and Wealth, ed. Timothy Dunne, J.Bradford Jensen and Mark J. Roberts, 149-230. Chicago: University of Chicago Press.

[2] Abowd, John, Francis Kramarz, and David Margolis. 1999. "High Wage Workers and High Wage Firms." *Econometrica* 67(2): 251-333.

[3] Bagger, Jesper, and Rasmus Lentz. 2019. "An Empirical Model of Wage Dispersion with Sorting." *Review of Economic Studies* 86(1): 153-190.

[4] Baley, Isaac, Ana, Figueiredo, and Robert Ulbricht. 2020. "Mismatch Cycles." Unpublished draft, Universitat Pompeu Fabra.

[5] Barlevy, Gadi. 2002. "The Sullying Effect of Recessions." *Review of Economic Studies* 69(1): 65-96.

[6] Bartolucci, Cristian, Francesco Devicienti, and Ignacio Monzón. 2018. "Identifying Sorting in Practice." *American Economic Journal: Applied Economics*, 10(4): 408-38.

[7] Caballero, Ricardo, and Mohamad Hammour. 1994. "The Cleansing Effect of Recessions." *American Economic Review* 84(5): 1350-1368.

[8] Cairó, Isabel, Henry Hyatt, and Nellie Zhao. 2018. "The U.S. Job Ladder in the New Millennium." Unpublished draft, U.S. Census Bureau.

[9] Card, David, Ana Cardoso, Joerg Heining, and Patrick Kline. 2018. "Firms and Labor Market Inequality: Evidence and Some Theory." *Journal of Labor Economics* 36(S1): S13-S69.

[10] Card, David, Ana Cardoso, and Patrick Kline. 2016. "Bargaining, Sorting, and the Gender Wage Gap: Quantifying the Effect of Firms on the Relative Pay of Women." *Quarterly Journal of Economics* 131(2): 633-686.

[11] Eeckhout, Jan, and Philipp Kircher. 2011. "Identifying Sorting - In Theory." *Review of Economic Studies* 78(3): 872-906.

[12] Guimarães, Paulo, and Pedro Portugal. 2010. "A Simple Feasible Procedure to Fit Models with High-Dimensional Fixed Effects." *The Stata Journal* 10(4): 628-649.

[13] Hagedorn, Marcus, Tzuo Law, and Iourii Manovskii. 2017. "Identifying Equilibrium Models of Labor Market Sorting." *Econometrica* 85(1): 29-65.

[14] Haltiwanger, John, Henry Hyatt, Lisa Kahn, and Erika McEntarfer. 2018. "Cyclical Job Ladders by Firm Size and Firm Wage." *American Economic Journal: Macroeconomics* 10(2): 52-85.

[15] Haltiwanger, John, Henry Hyatt, Erika McEntarfer. 2018. "Who Moves Up the Job Ladder?" *Journal of Labor Economics* 36(S1): S301-S336.

[16] Haltiwanger, John, Ron Jarmin, Robert Kulick, and Javier Miranda. 2017. "High Growth Young Firms: Contribution to Job, Output, and Productivity Growth." In: *Measuring Entrepreneurial Businesses: Current Knowledge and Challenges*, 75, Studies in Income and Wealth, ed. John Haltiwanger, Erik Hurst, Javier Miranda, and Antoinette Schoar, Chicago: University of Chicago Press: 11-62.

[17] Hyatt, Henry, Erika McEntarfer, Kevin McKinney, Stephen Tibbets, and Douglas Walton. 2014. "Job-to-Job (J2J) Flows: New Labor Market Statistics from Linked Employer-Employee Data." *JSM Proceedings 2014*, Business and Economics Statistics Section: 231-245.

[18] Kahn, Lisa. 2010. "The Long-Term Labor Market Consequences of Graduating from College in a Bad Economy." *Labour Economics* 17(2), pp. 303-316.

[19] Lise, Jeremy, and Jean-Marc Robin. 2017. "The Macrodynamics of Sorting between Workers and Firms." *American Economic Review* 107(4): 1104-1135.

[20] Lopes de Melo, Rafael. 2018. "Firm Wage Differentials and Labor Market Sorting: Reconciling Theory and Evidence." *Journal of Political Economy* 126(1): 313-346.

[21] Moscarini, Giuseppe, and Fabien Postel-Vinay. 2013. "Stochastic Search Equilibrium." *Review of Economic Studies* 80(4): 1545-1581.

[22] Moscarini, Giuseppe, and Fabien Postel-Vinay. 2018. "The Cyclical Job Ladder." *Annual Review of Economics* 10(1): 165-188.

[23] Oi, Walter. 1962. "Labor as a Quasi-Fixed Factor." *Journal of Political Economy* 70(6): 538-555.

[24] Oreopoulos, Philip, Till von Wachter, and Andrew Heisz. 2012. "The Short- and Long-Term Career Effects of Graduating in a Recession." *American Economic Journal: Applied Economics* 4(1): 1-29.

[25] van Ours, Jan, and Geert Ridder. 1995. "Job Matching and Job Competition: Are Lower Educated Workers at the Back of Job Queues?" *European Economic Review* 39(9): 1717-1731.

Appendices

A Employment and transition definitions

We use 11 states of LEHD microdata that have data available for 1994-2014.[1] Our definitions follow the notation established by Abowd et al. (2009), augmented to include employer-to-employer transitions by Hyatt et al. (2014). The starting point is earnings for individual i from employer j in quarter t, denoted w_{ijt}. If an individual has no earnings from an employer in a given quarter, then the worker did not receive unemployment insurance taxable income from that employer during that quarter. Otherwise, if the worker did receive positive earnings from that employer ($w_{ijt} > 0$), then the worker worked for the employer. Earnings are in real 2014 dollars. The following definitions allow us to carefully measure employment and transitions in administrative records that lack start and end dates.

A.1 Employment concepts

We consider the jobs that span two consecutive quarters (often called "beginning of quarter" jobs). By definition, in such jobs the employee was employed by the employer at the time of the break between the quarters. This employment measure therefore may reasonably be interpreted as indicative of point-in-time employment. Formally, a worker is employed at the beginning of quarter t when

$$b_{ijt} = \begin{cases} 1, & \text{if } w_{ijt-1} > 0 \text{ and } w_{ijt} > 0 \\ 0, & \text{otherwise.} \end{cases}$$

For any two-quarter pair, we disambiguate the data by considering jobs that are maximal earning among all jobs a worker holds at the beginning of quarter t. To do so, the job with the greatest earnings summed across quarter $t-1$ and t is identified, as follows:

[1]Note that hours data are not available for any state but Washington for our 11 state set in the analysis time period, and we are not able to release any results for particular U.S. states in this paper.

$$
domb_{ijt} = \begin{cases} 1, & \text{if } b_{ijt} = 1 \text{ and} \\ & w_{ijt} + w_{ijt-1} > w_{ikt} + w_{ikt-1} \forall k \\ & \text{s.t. } b_{ikt} = 1 \text{ and} j \neq k \\ 0, & \text{otherwise.} \end{cases}
$$

The set of jobs defined in $domb_{ijt}$ are those we use in all of our empirical analysis. Such jobs are unique at the person-quarter level.

A.2 Transition concepts

We consider transitions between dominant job status across quarters. These are worker movements between employers, as well as into and from nonemployment.

We consider within-quarter transitions

$$
wq_{ijkt} = \begin{cases} 1, & \text{if } domb_{ijt} = 1 \text{ and } domb_{ikt+1} = 1 \\ & \text{and } j \neq k \\ 0, & \text{otherwise,} \end{cases}
$$

as well as adjacent quarter transitions

$$
aq_{ijkt} = \begin{cases} 1, & \text{if } domb_{ijt-1} = 1 \text{ and } domb_{ikt+1} = 1 \\ & \text{and } domb_{ilt} \neq 1 \forall l \text{ and } j \neq k \\ 0, & \text{otherwise.} \end{cases}
$$

Flows into persistent nonemployment in quarter t have full-quarter earnings when

$$en2\_doms2_{ijt} = \begin{cases} 1, & \text{if } domb_{ijt} = 1 \\ & \text{and } domb_{ilt+1} \neq 1 \forall l \\ & \text{and } domb_{imt+2} \neq 1 \forall m \\ 0, & \text{otherwise,} \end{cases}$$

Flows from persistent nonemployment into employment in quarter t have full quarter earnings when

$$ne2\_doma2_{ikt} = \begin{cases} 1, & \text{if } domb_{ikt+1} = 1 \\ & \text{and } domb_{ilt} \neq 1 \forall l \\ & \text{and } domb_{imt-1} \neq 1 \forall m \\ 0, & \text{otherwise,} \end{cases}$$

We also consider workers who did not change jobs, who are called "job stayers."

$$dombe_{ijt} = \begin{cases} 1, & \text{if } domb_{ijt} = 1 \text{ and } domb_{ijt+1} = 1 \\ 0, & \text{otherwise.} \end{cases}$$

There are, therefore, seven transition concepts: four for employer-to-employer transitions, two for transitions into and from nonemployment, and an exhaustive residual for those with dominant employers, job stayers.

In addition to these, we create an additional nonemployment hire measure that is useful when calculating a firm's rank when hiring from poaching. This measure excludes recalls

$$ne2\_norecall_{ikt} = \begin{cases} 1, & \text{if } domb_{ikt+1} = 1 \\ & \text{and } domb_{ilt} \neq 1 \forall l \\ & \text{and } domb_{imt-1} \neq 1 \forall m \\ & \text{and } domb_{ikt-2} \neq 1 \\ 0, & \text{otherwise.} \end{cases}$$

A.3 Aggregation

We consider the evolution of total consecutive quarter employment. For workers in group i and firms in group j, this is expressed as:

$$E_{ijt} = \sum_{ij} b_{ijt+1}.$$

Total employment evolves via poaching hires and hires from nonemployment. Total poaching hires for workers in group i and firms in group k are:

$$P^a_{ikt} = \sum_{ik} (wq_{ijkt} + aq_{ijkt}).$$

Total poaching separations for workers of group i from firms of group j are

$$P^s_{ijt} = \sum_{ij} (wq_{ijkt} + aq_{ijkt-1}).$$

Total nonemployment hires for workers of group i into firms of group k are

$$N^a_{ikt} = \sum_{ik} en2\_doma2_{ikt}.$$

Total nonemployment separations for workers of group i from firms of group j are

$$N^s_{ijt} = \sum_{ij} en2\_doms2_{ijt}.$$

References

Abowd, John, Bryce Stephens, Lars Vilhuber, Fredrik Andersson, Kevin McKinney, Marc Roemer, and Simon Woodcock. 2009. "The LEHD Infrastructure Files and the Creation of the Quarterly Workforce Indicators." In *Producer Dynamics: New Evidence from Micro Data*, 68, Studies in Income and Wealth, ed. Timothy Dunne, J.Bradford Jensen and Mark J. Roberts, 149-230. Chicago: University of Chicago Press.

Hyatt, Henry, Erika McEntarfer, Kevin McKinney, Stephen Tibbets, and Douglas Walton. 2014. "Job-to-Job (J2J) Flows: New Labor Market Statistics from Linked Employer-Employee Data." *JSM Proceedings 2014*, Business and Economics Statistics Section: 231-245.

B Worker ranking implementation details

We here describe in detail each of our four worker and firm ranking algorithms. Earnings are in logs throughout. Whenever earnings are employed in a ranking method, the earnings concept used in ranking is the same as that used to determine a worker's dominant employer in Appendix A, that is $w_{ijt} + w_{ijt-1}$.

B.1 Method 1: worker nonemployment duration and firm poaching hire share

Our first method of ranking workers and firms involves ranking methods that can be implemented quickly on administrative records data. Specifically, we rank firms on the basis of the share of the firm's hires that are poached away from other firms (versus hired from nonemployment), as higher productivity firms ought to obtain workers from other firms more frequently than lower productivity firms. Workers are ranked on the basis of the amount of time they spend employed, the assumption being that more productive workers are more likely to be employed rather than nonemployed.

B.1.1 Ranking firms by poaching share of hires

In a manner similar to Bagger and Lentz (2019), we rank firms according to each firm's share of hires that are poached from other firms (as opposed to being hired from nonemployment). We begin by identifying the total hires from either employment or from nonemployment for each firm in the 11 states of the LEHD microdata. We include as employer-to-employer transitions hires both same-quarter wq_{ijkt} and adjacent-quarter aq_{ijkt} transitions. A same-quarter transition occurs if the worker has positive earnings from both the previous and the new employer in the transition quarter. An adjacent-quarter transition occurs in period t if the worker both has positive earnings from the old employer, but not the new employer, in period t; and has positive earnings from the new employer, but not the old employer, in period $t+1$. For the calculation of a firm's nonemployment hires, we exclude all one-quarter recall hires, and so we use $ne2\_norecall_{ikt}$. We define a one-quarter recall hire as a three-quarter employment pattern of employment-to-nonemployment-to-employment, where the worker's dominant employer was the same in the first and last quarter and the worker was nonemployed for exactly one full calendar quarter in between.

We estimate each the poaching share of hires for firm k as the ratio of hires from other employers to total hires, as follows:

$$\frac{\sum_k wq_{ijkt} + aq_{ijkt}}{\sum_k ne2\_norecall_{ikt}}.$$

Firms are then rank ordered into 50 bins according to their poaching share.

B.1.2 Ranking workers by prime-age employment rates

We rank workers by their prime-age quarterly employment rate relative to the average employment rate for individuals born in the same year. For each worker, we construct a 0-1 employment indicator variable for every quarter that the worker is between the ages of 25 to 55 (inclusive). This employment indicator variable is set to one if the worker had positive earnings in that quarter and to zero if they were nonemployed for the entire calendar quarter.

We then divide workers into cohorts according to their year of birth. For every quarter, we compute the average employment rate of each birth cohort as the average of the employment indicator for all individuals in that birth cohort in the given quarter. For every quarter in which a worker is between the ages of 25-55, we calculate the deviation of the worker's employment indicator from the birth-cohort average employment rate for the given quarter. The worker's prime-age employment rate is simply the sum of the worker's deviations from the birth-cohort average divided by the number of observed quarters in the LEHD micro data for which the worker was between the ages of 25-55. The worker ranking is determined by a rank ordering of workers into 50 bins according to their prime-age quarterly employment rate.

B.2 Method 2: average earnings and labor productivity

B.2.1 Ranking workers based on average earnings

In our second method, we rank workers in a way that is motivated by the fact that high-type workers may exhibit higher average earnings. We simply rank workers by the average of their residual earnings after controlling for age and time-period fixed effects. Note that this is the initial guess of a worker's rank in our additive model (Method 3) and our reranking workers and surplus approach (Method 4).

B.2.2 Ranking firms based on revenue productivity

We use revenue data from the U.S. Census Bureau's Business Register to measure labor productivity, i.e., revenue-per-worker. We use all available revenue data from 1994-2014.[2] Each year, the Business Register reports total annual revenue for both the current year and the previous year. Thus, for any given firm and calendar year, we may have two reports of the firm's revenue data. In such cases, we given priority to the most recent report of the firm's revenue since this better reflects revisions to firms' filings. These data are Windsorized at both the top and bottom 1% of the revenue distribution.

Not all businesses have revenue data in all years. In some cases, a crosswalk was not available between the LEHD employer data and the Business Register (i.e., missing firm identifier), and in others revenue data was missing from the Business Register. We therefore impute these data elements when they are missing, assuming that they are missing-at-random within quarter, firm industry, size, and age categories.

Specifically, we assume that revenue is the following linear function of log firm size and age, estimated separately by quarter and four-digit NAICS code:

$$lp = \beta_0^a + \beta_1^a * firmsize + \beta_2^a * firmage + \beta_3^a * firmsize * firmage + \beta_4^a * firmsize^2 + \beta_5^a * firmage^2$$

where lp is log labor productivity, $firmage$ is log firm age, and $firmsize$ is log firm size.

The distribution of the Business Register revenue data shifts discontinuously upward around the year 2002, when the Business Register was redesigned. This is because additional data elements concerning revenue became available and more accurate totals are available. Since we do not want

[2] Recent work by Haltiwanger et al. (2017) uses the same source data to create firm-level measures of labor productivity for a shorter set of years, and a subset of industries.

the firms in more recent years to appear more productive simply because of a change in reporting, we also implement a simple imputation to adjust for this. The revenue data for 2000 is all provided under the old regime, that for 2002, all under the new, and the year 2001 is a mix of old and new. We therefore take all businesses that existed in the year 2000 and 2002 and use this as training data for imputation of

$$lp_n = \begin{array}{l} \beta_0^b + \beta_1^b * lp_o + \beta_2^b * lp_o^2 + \beta_3^b * firmsize + \beta_4^1 * firmage + \\ \beta_5^b * firmsize * firmage + \beta_6^b * firmsize^2 + \beta_7^b * firmage^2 \end{array}$$

where lp_n is 2002 revenue data and lp_o is revenue data from the year 2000 or earlier.

Having attached revenue to all firms in the LEHD data, we proceed in a simple manner to produce ranks firms based on revenue. We rank firms based on the residual firm productivity from year of entry by quarter by industry dummy variable regression. We then add this residual to the value-added per worker data as published by the Bureau of Economic Analysis to obtain a proxy for firm-level value added per worker. We then rank firms based on the average of this sum, over time.

B.3 Method 3: additive worker and firm effects

We estimate worker and firm fixed effects via an iterative algorithm that follows Guimarães and Portugal (2010). We fit the following model for earnings outcomes

$$W = B\xi + D\theta + F\psi$$

where W is the $N \times 1$ dimensional vector total earnings observations w_{ijt}, B is an $N \times G_B$ dimensional matrix of birth cohort by time fixed effects, D is an $N \times G_D$ dimensional matrix of person-specific fixed effects, and F is an $N \times G_F$ dimensional matrix of firm effects. Our goal is to recover the $1 \times G_B$ dimensional vector ξ of fixed effects for birth cohort c at time t ξ_{ct}, the $1 \times G_D$ dimensional vector θ of person-specific fixed effects, and the $1 \times G_F$ dimensional vector ψ of firm-specific fixed effects.

We can express the least-squares formula for this problem in terms of a cross-product matrix similar to Abowd, Kramarz, and Margolis (1999):

$$\begin{bmatrix} B'B & B'D & B'F \\ D'B & D'D & D'F \\ F'B & F'D & F'F \end{bmatrix} \begin{bmatrix} \xi \\ \theta \\ \psi \end{bmatrix} = \begin{bmatrix} B'W \\ D'W \\ F'W \end{bmatrix}$$

which, after rearranging terms, can be expressed as

$$\begin{bmatrix} B'B\xi + B'D\theta + B'F\psi = B'W \\ D'B\xi + D'D\theta + D'F\psi = D'W \\ F'B\xi + F'D\theta + F'F\psi = F'W \end{bmatrix}.$$

which is a system of three equations. Solving each of these independently yields

$$\begin{bmatrix} \xi = (B'B)^{-1}B'(W - D\theta - F\psi) \\ \theta = (D'D)^{-1}D'(W - B\xi - F\psi) \\ \psi = (F'F)^{-1}F'(W - B\xi + D\theta) \end{bmatrix}.$$

We iterate between these sets of equations to obtain the least squares solution. In fact, solving each of these equations can be done using group means since all of our independent variables are dummy variables. As the datasets we use in this analysis contains billions of person-quarter observations (e.g. 50 million workers times twenty quarters implies one billion person-quarter observations), omitting computational matrix inversion allows us to greatly speed up our computation time. To see how we can skip having a computer run a tediously slow regression program on our massive dataset, note that the first equation of our system specifies a separate indicator for each birth cohort c at each quarter in time,

$$\xi_{ct} = \frac{1}{\sum_{ij} \mathbb{1}(w_{ijct} > 0)} \sum_{ij} (w_{ijct} - \theta_i' d_i - \psi_j' f_j)$$

and for each worker i,

$$\theta_i = \frac{1}{\sum_{jct} \mathbb{1}(w_{ijct} > 0)} \sum_{jct} (w_{ijct} - \xi_{cb} b_{cb} - \psi_j' f_j)$$

and for each firm j,

$$\psi_j = \frac{1}{\sum_{ict} \mathbb{1}(w_{ijct} > 0)} \sum_{ict} (w_{ijct} - \xi_i' b_i - \theta_i' d_i).$$

In other words, these are the least squares solutions to a high-dimensional set of mutually exclusive indicator variables. The least squares solutions are sample means of residuals, which can be calculated directly without having a computer multiply or invert matrices.

We can now solve for θ_i, ψ_j, and ξ_{ct} for the universe of our 11 states of linked employer-employee data. We first compute the average log earnings of each birth cohort by time cell $\widehat{\xi}_{ct} = \Sigma_{ij} w_{ijct}$ of each worker, this is our initial guess of the birth cohort by time effect. We then proceed as follows.

1. Estimate the initial worker effects $\widehat{\theta}_i = w_{ijt} - \widehat{\xi}_{ct}$.
2. Estimate the intial firm effects $\widehat{\psi}_j = w_{ijt} - \widehat{\xi}_{ct} \widehat{\theta}_i$.
3. Update the birth cohort by time effects $\widehat{\xi}_{ct} = w_{ijt} - \widehat{\theta}_i - \widehat{\psi}_j$,
4. Update the worker effects $\widehat{\theta}_i = w_{ijt} - \widehat{\psi}_j - \widehat{\xi}_{ct}$.
5. Update the firm effects $\widehat{\psi}_j = w_{ijt} - \widehat{\theta}_i - \widehat{\xi}_{ct}$.
6. Proceed back to step 3 until a goodness-of-fit criterion is reached.

We then group each of the employment-weighted firm effects $\widehat{\psi}_j$, and the participation-weighted worker effects $\widehat{\theta}_i$ into terciles.

B.4 Method 4: worker reranking and surplus

We implement an algorithm for ranking workers and firms that borrows heavily from Hagedorn, Law, and Manovskii (2017). It is substantially simplified and was not intended to be a direct replication of this method.

B.4.1 Worker residuals for ranking

The first part of our algorithm calculates residual earnings that will then serve as the starting point for the ranking algorithm. We first calculate average log earnings by birth cohort c (specifically, year of birth) by quarter in time t. We then estimate an initial guess of worker productivity as the deviation of that worker's earnings from the birth cohort by time mean.

B.4.2 Reranking workers to minimize disagreement

We use the rank order of these residuals as the initial guess of a worker's rank, where workers with a higher residual earnings are more productive.

We then look at workers who are employed by the same firm. We evaluate the goodness of fit of our worker ranks as the fraction of the time that a higher ranked worker earns more at a particular firm than a lower ranked worker.

We assume that wage observations are the true wages plus iid measurement error. So the observed wage of worker i at firm k in period t is

$$\hat{w}_{i,k,t} = w_{i,k} + \varepsilon_t$$

where $w_{i,k}$ is the true wage and ε_t is iid noise. Then $n_{i,k}$ is the completed tenure of the worker, the difference in observed wages is

$$\bar{w}_{i,k} - \bar{w}_{j,k} = w_{i,k} - w_{j,k} + \frac{1}{n_{i,k}} \sum_{t=1}^{n_{i,k}} \varepsilon_{i,k,t} - \frac{1}{n_{j,k}} \sum_{t=1}^{n_{j,k}} \varepsilon_{j,k,t}.$$

Suppose that the prior is

$$w_{i,k} \sim \mathcal{N}(\mu_0, \tau_0^2).$$

Then the posterior of $w_{i,k}$, given $Var(\varepsilon_t) = \sigma^2$ is

$$p(w_{i,k} | \bar{w}_{i,k}, n_{i,k}) = \mathcal{N}(\mu_n, \tau_n^2)$$

where μ_n is the precision-weighted average of the means

$$\mu_n = \frac{\frac{1}{\tau_0^2}\mu_0 + \frac{n_{i,k}}{\sigma^2}\bar{w}_{i,k}}{\frac{1}{\tau_0^2} + \frac{n_{i,k}}{\sigma^2}}$$

and

$$\frac{1}{\tau_n^2} = \frac{1}{\tau_0^2} + \frac{n_{i,k}}{\sigma^2}.$$

We assume an uninformative prior: $\tau_0^2 \to \infty$. The expressions simplify to

$$\mu_n = \bar{w}_{i,k}$$

and

$$\frac{1}{\tau_n^2} = \frac{n_{i,k}}{\sigma^2}.$$

The "posterior" densities are then

$$p(w_{i,k}|\bar{w}_{i,k}, n_{i,k}) = \mathcal{N}\left(\bar{w}_{i,k}, \frac{\sigma^2}{n_{i,k}}\right)$$

$$p(w_{j,k}|\bar{w}_{j,k}, n_{j,k}) = \mathcal{N}\left(\bar{w}_{j,k}, \frac{\sigma^2}{n_{j,k}}\right)$$

Since everything is independent, the difference in average wages is also normal:

$$p(w_{i,k} - w_{j,k}|\bar{w}_{i,k}, n_{i,k}, \bar{w}_{j,k}, n_{j,k}) = \mathcal{N}\left(\bar{w}_{i,k} - \bar{w}_{j,k}, \frac{\sigma^2}{n_{i,k}} + \frac{\sigma^2}{n_{j,k}}\right)$$

Then we can compute the probability that $w_{j,k} < w_{i,k}$ using the normal CDF:

$$\mathbb{P}(w_{j,k} < w_{i,k}) = \Phi\left(\frac{\bar{w}_{i,k} - \bar{w}_{j,k}}{\sqrt{\frac{\sigma^2}{n_{i,k}} + \frac{\sigma^2}{n_{j,k}}}}\right)$$

The true ranking of workers is given by $\Pi(i, j)$, where $\Pi(i, j) = 1$ if i is (strictly) preferred to j and $\Pi(i, j) = 0$ otherwise. Let $c(i, j)$ be the probability that $\Pi(i, j) = 1$.

If k is the only firm where i and j both worked, then

$$c(i, j) = \Phi\left(\frac{\bar{w}_{i,k} - \bar{w}_{j,k}}{\sqrt{\frac{\sigma^2}{n_{i,k}} + \frac{\sigma^2}{n_{j,k}}}}\right)$$

Otherwise, we set

$$c(i, j) = \prod_{k \in E(i,j)} \Phi\left(\frac{\bar{w}_{i,k} - \bar{w}_{j,k}}{\sqrt{\frac{\sigma^2}{n_{i,k}} + \frac{\sigma^2}{n_{j,k}}}}\right)$$

where $E(i, j)$ is the set of firms that have employed both i and j, and the product symbol should not be confused with the ranking $\Pi(i,j)$.

We estimate Π by choosing $\hat{\Pi}$ to maximize the number of so-defined correctly ranked workers. Specifically, we seek a transitive, complete ordering $\hat{\Pi}$ that solves

$$\arg\max_{\hat{\Pi}} \sum_{j=1}^{j=N} \sum_{i=j+1}^{N} \left\{ c(i,j)\hat{\Pi}(i,j) + c(j,i)\hat{\Pi}(j,i) \right\}$$

where

$$\begin{aligned} c(i,j) &= \prod_{k \in E(i,j)} \Phi\left(\frac{\bar{w}_{i,k} - \bar{w}_{j,k}}{\sqrt{\frac{\sigma^2}{n_{i,k}} + \frac{\sigma^2}{n_{j,k}}}} \right) \\ \bar{w}_{i,k} &= \frac{1}{n_{i,k}} \sum_{t=1}^{t=n_{i,k}} w_{i,k,t}. \end{aligned}$$

We start with an initial guess and make a single arbitrary move, and check the goodness-of-fit measure to see whether it improves. Our method is as follows:

1. Start with an initial ranking $\hat{\Pi}_0$. Note that i and j are worker names. Any ranking $\hat{\Pi}_n$ implies at function $r_n(i)$, which returns the rank (on $\{1,2,..N\}$) of the worker i.

2. Starting from a ranking $\hat{\Pi}_n$ choose a random worker name i from $\{1,2,..N\}$ and a random worker rank r from $\{1,2,..N\}$.

3. If changing the rank of worker i from $r_n(i)$ to r improves the fit, make this change. Otherwise do nothing.

4. Return to Step 2. Repeat until no more single move rerankings can be made, or some weaker condition is met.

Worker ranks are grouped into three employment-weighted groups: low, middle, and high.

B.4.3 Surplus-based firm ranking

Pool of nonemployed by worker type For each worker, we identify the worker as nonemployed in a given quarter if the quarter falls between the workers' first and last quarters of observed earnings and the worker had zero earnings for the quarter. We then sum the total number of nonemployed workers in each quarter for each estimated worker type $\hat{x}$. This corresponds to the pool of unemployed, $u(\hat{x})$, used in the Hagedorn, Law, and Manovskii (2017) IDNoise Algorithm.

The IDNoise algorithm To address noise in the classification of workers' types, Hagedorn, Law, and Manovskii (2017) propose an algorithm called IDNoise that aims to identify workers whose worker types are particularly unusual given the set of worker types employed by the workers' employers. Hagedorn, Law, and Manovskii (2017) assign these workers with noisy worker types to a set $\hat{\mathbb{N}}$. For each firm j, the IDNoise algorithm identifies $\hat{\mathbb{B}}(\hat{x}, j)$, a set of "cleaned" worker types that the firm hires from nonemployment. The algorithm works as follows for each firm j.

1. Compute the following four firm-specific variables:

   - $N(j)$: The number of workers hired from nonemployment by firm j
   - $p(\hat{x}, j)$: The number of workers of estimated type $\hat{x}$ hired from nonemployment by firm j
   - $\pi(\hat{x}, j)$: The theoretical fraction of workers of type $\hat{x}$ hired from nonemployment by firm j, which is a function of the types of workers that the firm hires and the relative number of this worker-type in the pool of nonemployed workers:

$$\pi(\hat{x}, j) = \frac{u(\hat{x})\mathbb{1}\left[p(\hat{x}, j) > 0\right]}{\sum_{\hat{x}} u(\hat{x})\mathbb{1}\left[p(\hat{x}, j) > 0\right]} \tag{4}$$

   - $F\left(p(\hat{x}, j), \pi(\hat{x}, j), N(j)\right)$: The probability of observing at most $p(\hat{x}, j)$ hires from nonemployment given the probability $\pi(\hat{x}, j)$ from $N(j)$ trials. Assuming that these hires from nonemployment are random draws from the pool of nonemployed workers matching the firm's worker types, $F\left(p(\hat{x}, j), \pi(\hat{x}, j), N(j)\right)$ is:

$$F\left(p(\hat{x}, j), \pi(\hat{x}, j), N(j)\right) = \sum_{i=0}^{p(\hat{x},j)} \begin{pmatrix} N(j) \\ i \end{pmatrix} \pi(\hat{x}, j)^{i} \left(1 - \pi(\hat{x}, j)\right)^{N(j)-i} \tag{5}$$

2. For each worker type $\hat{x}$, initialize $\hat{\mathbb{B}}(\hat{x}, j) = 1$ if the firm hires any workers of that estimated type $(p(\hat{x}, j) > 0)$

3. * for all worker types, $\hat{x}$, with $\hat{\mathbb{B}}(\hat{x}, j) = 1$

- If the worker type, $\hat{x}$, is the lowest (=1) or highest (=50) worker types and $F\left(p(\hat{x},j),\pi(\hat{x},j),N(j)\right) \leq 0.1$, then set $\hat{\mathbb{B}}(\hat{x},j)=0$ and return to *.

- For all other worker types, if either $\hat{\mathbb{B}}(\hat{x}-1,j)=0$ or $\hat{\mathbb{B}}(\hat{x}+1,j)=0$ and $F\left(p(\hat{x},j),\pi(\hat{x},j),N(j)\right) \leq 0.1$, then set $\hat{\mathbb{B}}(\hat{x},j)=0$ and return to *.

After computing the set of types hired by each firm, $\hat{\mathbb{B}}(\hat{x},j)$, a worker i, with estimated type $\hat{x}(i)$ is assigned to the set $\hat{\mathbb{N}}$ if they are ever employed by a firm j where $\hat{\mathbb{B}}(\hat{x}(i),j)=0$.

Identifying the reservation wage of each worker type When determining the reservation wages of each worker type, we follow Hagedorn, Law, and Manovskii (2017) in excluding the earnings histories of any worker i with a noisy worker type ($i \in \hat{\mathbb{N}}$). The reservation wage for each worker type $\hat{x}$ is calculated using the remaining workers as follows:

1. Construct the set $J(\hat{x})$ which consists of all firms j that hire any worker of type $\hat{x}$ from nonemployment.

2. For each firm $j \in J(\hat{x})$, compute $\bar{w}(\hat{x},j)$, the average wage paid by firm j to workers of type $\hat{x}$ hired from nonemployment.

3. We define the reservation wage for type $\hat{x}$, $w^r(\hat{x})$, is the 10th percentile of the set of $w(\hat{x},j)$ where $j \in J(\hat{x})$. Note that Hagedorn, Law, and Manovskii (2017) propose using the minimum average wage as the reservation wage, but we find that this is a very noisy signal, whereas the 10th percentile is smoothly increasing in worker type.

Ranking firms by their average wage premium Following Hagedorn, Law, and Manovskii (2017), we rank firms by the product of their average wage premium and their job filling rate. The average wage premium of firm j, $\Omega^u(j)$ is:

$$\Omega^u(j) = \sum_{\hat{x} \text{ s.t. } \hat{\mathbb{B}}(\hat{x},j)=1} \frac{\frac{u(\hat{x})}{U}\left(\bar{w}(\hat{x},j)-w^r(\hat{x})\right)}{\sum_{\hat{x} \text{ s.t. } \hat{\mathbb{B}}(\hat{x},j)=1}\frac{u(\hat{x})}{U}} \tag{6}$$

The job filling rate for firm j is a function of the probability that the firm encounters an unemployed worker, $\mathbb{M}_v$, times the probability that the worker's type, $x(i)$, matches the firm's set of acceptable worker types ($\hat{\mathbb{B}}(\hat{x}(i),j)=1$). Since the probability that a firm encounters an unemployed worker is constant across all firms, this is simply a scalar factor in the firm ranking and we thus ignore it. Calculate the probability that the encountered workers' type $x(i)$ matches the firm's set of acceptable worker types, $\tilde{q}^u(j)$, as:

$$\tilde{q}^u(j) = \sum_{\hat{x} \text{ s.t. } \hat{\mathbb{B}}(\hat{x},j)=1} \frac{u(\hat{x})}{U} \tag{7}$$

References

Abowd, John, Francis Kramarz, and David Margolis. 1999. "High Wage Workers and High Wage Firms." *Econometrica* 67(2): 251-333.

Guimarães, Paulo, and Pedro Portugal. 2010. "A Simple Feasible Procedure to Fit Models with High-Dimensional Fixed Effects." *The Stata Journal* 10(4): 628-649.

Hagedorn, Marcus, Tzuo Law, and Iourii Manovskii. 2017. "Identifying Equilibrium Models of Labor Market Sorting." *Econometrica* 85(1): 29-65.

Haltiwanger, John, Ron Jarmin, Robert Kulick, and Javier Miranda. 2017. "High Growth Young Firms: Contribution to Job, Output, and Productivity Growth." In: *Measuring Entrepreneurial Businesses: Current Knowledge and Challenges*, 75, Studies in Income and Wealth, ed. John Haltiwanger, Erik Hurst, Javier Miranda, and Antoinette Schoar, Chicago: University of Chicago Press: 11-62.

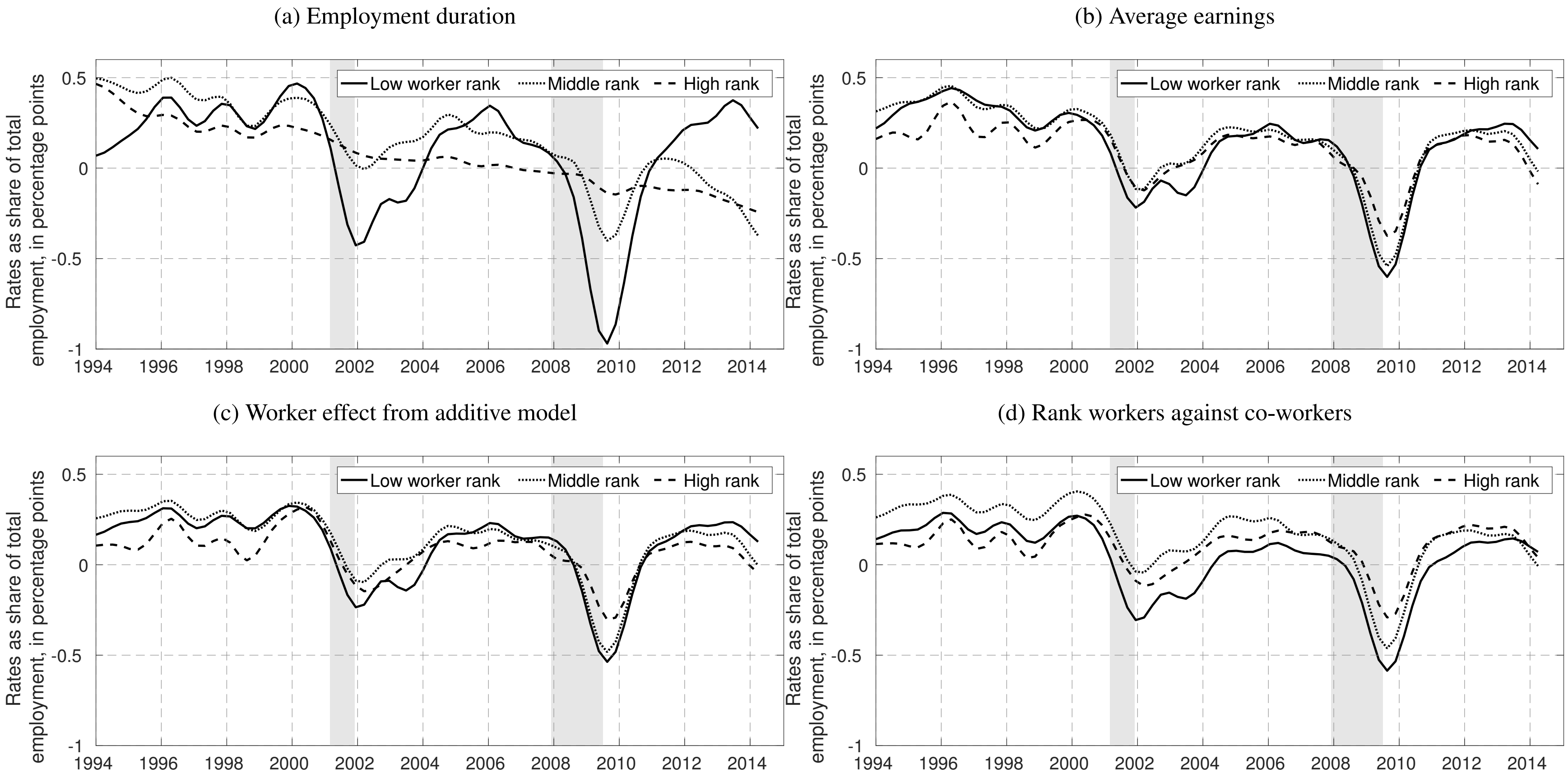
C Supplemental tables and figures

C.1 Poaching vs. nonemployment margins

These changes in employment shares by type are determined by labor market transitions into and out of nonemployment, as well as across employers.We show these transition rates in Figure C1. Figure C1 shows net hires from nonemployment by worker type. Net employment growth declines sharply during recessions for all three types of workers. The 2007-2009 recession has more of a decline in employment than the 2001 recession. However, for high productivity workers, especially in the 2007-2009 recession, their employment did not decline nearly as much as it did for the lower productivity groups. When considering the employment transitions across firms of different types, it is helpful to keep in mind the findings of Haltiwanger et al. (2018) that firms that are higher-ranked in the job ladder are net poachers, and that low-rank firms rely disproportionately on nonemployment to obtain their workers. Figure C2 shows net hires from nonemployment by firm type. There are level differences between the types of firms, with low-rank firms having more net hiring from nonemployment than the other two groups. Despite these level differences, the cyclicality is similar, with net nonemployment hiring falling sharply during the two recessions. Figure C3 shows net poaching by firm rank. Note that net poaching for each worker type is equal to zero by construction (each employer-to-employer transition contributes exactly one poaching gain and one poaching loss). Low-rank firms lose workers via poaching flows, and high-rank firms gain workers throughout the time period, but this movement away from low-rank firms and toward high-rank firms slows substantially during recessions.

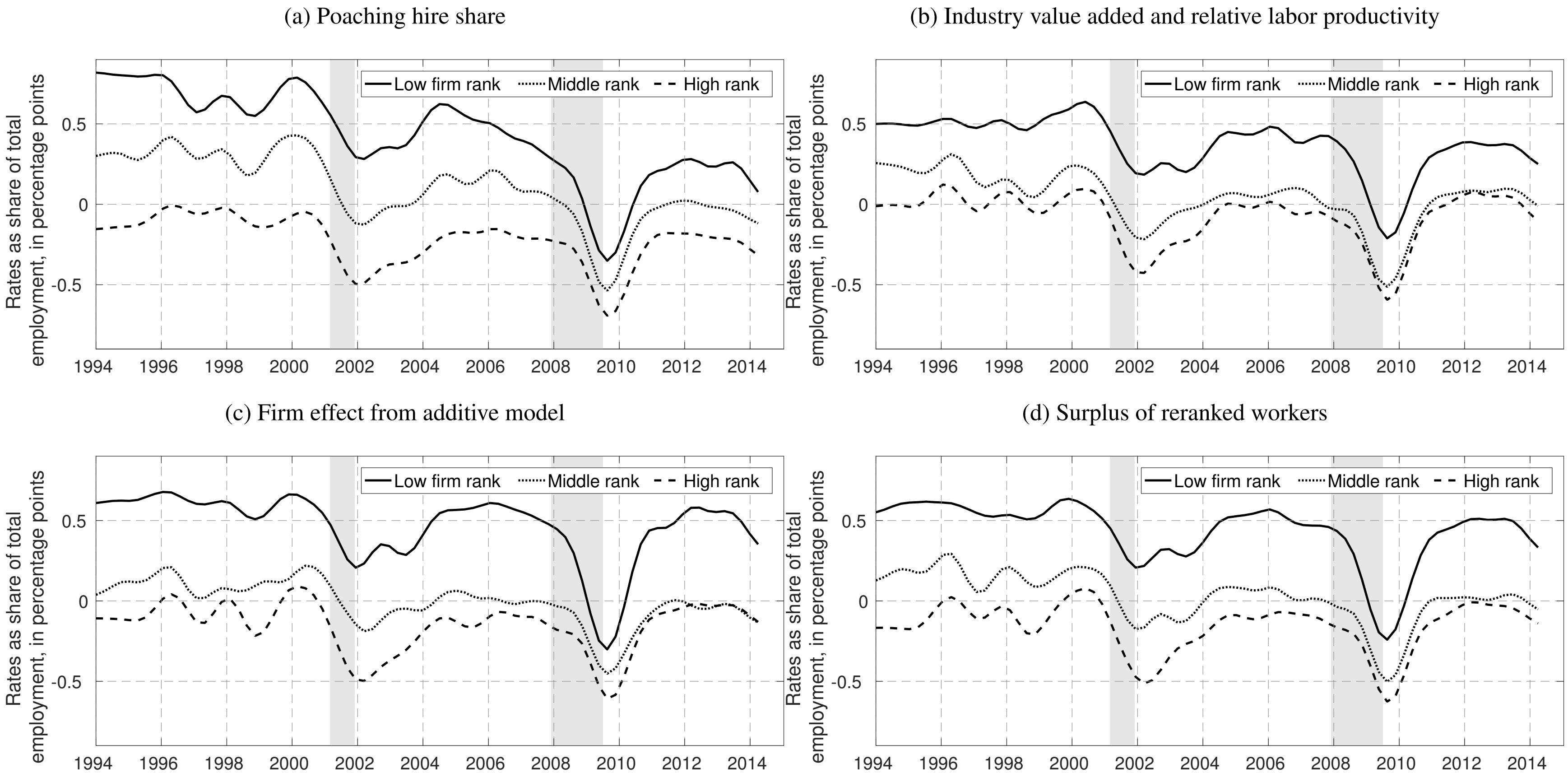
Figures C1, C2, and C3 help illustrate how the employment composition effect led to a larger build-up at low-rank firms in the wake of the 2001 recession than the 2007-2009 recession. Following Haltiwanger et al. (2018), in order to see a counter-cyclical build-up at the low-end of the job ladder, the “poaching margin” must overwhelm the “nonemployment margin.” In other words, the counter-cyclical decline in the movement of workers from low-rank firms to high-rank firms must be larger than the decline in nonemployment for low-rank firms. In the wake of the 2001 recession, there was relatively little change in the difference in nonemployment hiring for high- vs. low-rank firms and so the change in poaching dominates. However, in the 2007-2009 recession the excess nonemployment hiring by low-rank firms shut down, mitigating the build-up in the share of employment at low-rank firms.

Figure C1: Percent change in worker employment



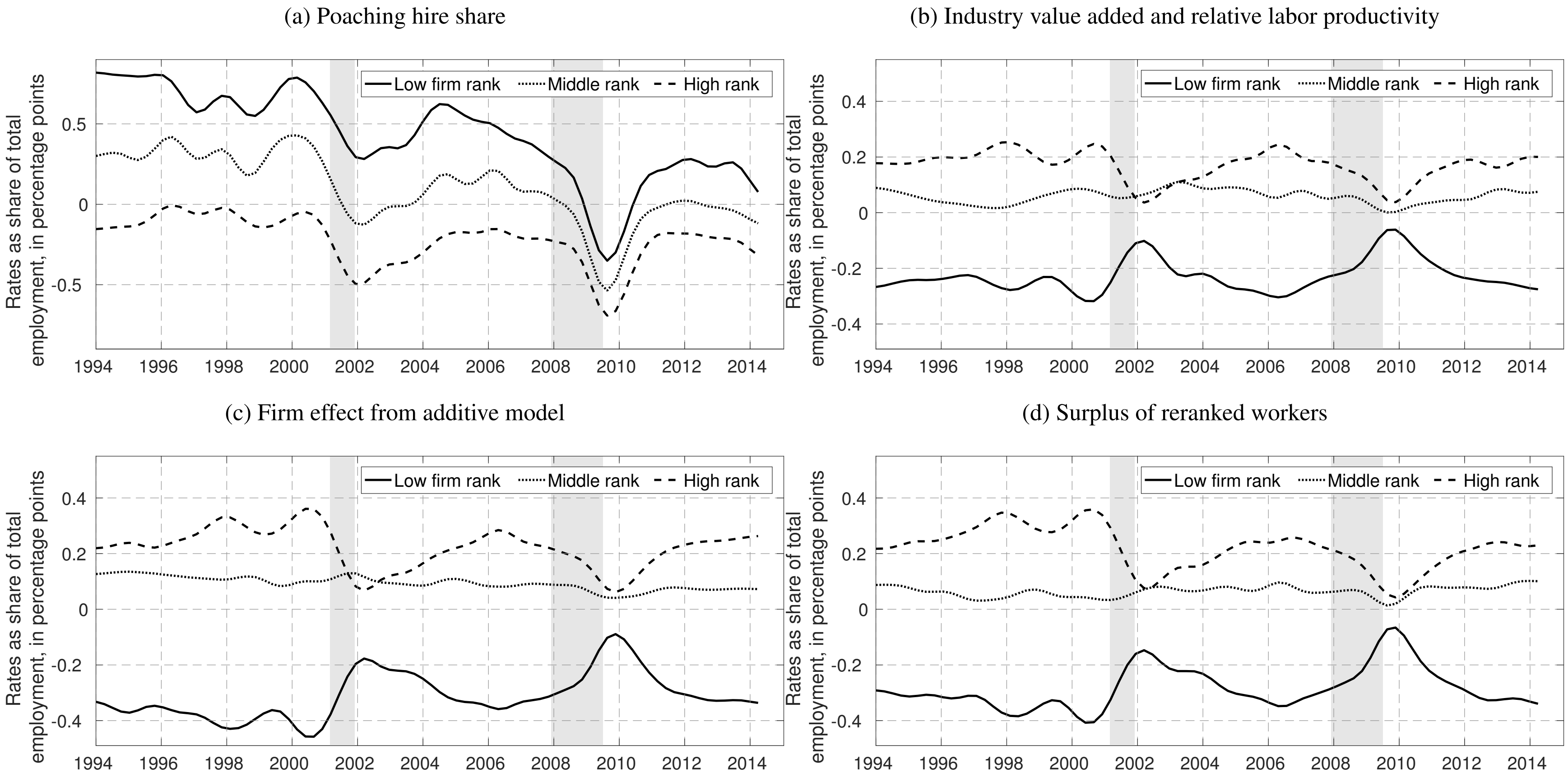
Notes: Shaded regions indicate recessions. Data seasonally adjusted and Henderson-filtered using X-11.

Figure C2: Percent change in firm employment: nonemployment



Notes: Shaded regions indicate recessions. Data seasonally-adjusted and Henderson-filtered using X-11.

Figure C3: Percent change in firm employment: poaching



Notes: Shaded regions indicate recessions. Data seasonally adjusted and Henderson-filtered using X-11.

Table C1: Changes in worker rank shares and the unemployment rate (twenty or more employees)

	Employment duration	Average earnings	Additive model worker effects	Rank workers vs. co-workers
	Difference in unemployment from its HP trend			
Low	-6.6***	-2.1**	-2.8**	-2.2***
	(1.5)	(0.8)	(0.8)	(0.8)
High	6.0***	1.3	2.3***	1.6**
	(1.2)	(0.9)	(0.8)	(0.8)
	First-difference of the unemployment rate			
Low	-20.9***	-4.7**	-8.3***	-7.9***
	(3.2)	(1.9)	(1.8)	(1.9)
High	16.4***	8.1***	10.7***	9.7***
	(2.9)	(1.9)	(1.7)	(1.6)

Notes: Estimates of change in share of employment on the seasonally-adjusted unemployment rate, season dummies, and a time trend. *, **, and *** indicate statistical significance at the 10%, 5%, and 1%, respectively. Standard errors are in parentheses. To avoid excessive decimal places, the dependent variables range from $[-100, 100]$, while the cyclical indicators range from $[-1, 1]$.

C.2 Robustness

In this subsection, we explore the robustness of our main findings regarding cyclical employment composition by worker rank and firm rank. While both exercises offer potential improvements to worker and firm ranks, they also apply only to a subset of workers and firms, and so composition results necessarily apply only to this subset. In the body of our paper, we focus on results that apply conventional methods to our matched employer-employee data, and therefore present results on the composition of all workers and all firms.

C.2.1 Minimum employment

Our first robustness exercise applies a minimum employment threshold to workers and firms. For workers, we require at least twenty quarters (five years) of employment. For firms, we require that firms employ at least twenty workers. These selection criteria may improve estimated worker and firm ranks for two reasons. First, the worker and firm ranks of our methods are estimate the permanent (time-invariant) component in determining earnings or productivity. Any time-varying transitory variation in earnings or productivity will bias our rank estimates of any particular worker or firm. Having

Table C2: Changes in firm rank shares and the unemployment rate (twenty or more employment)

	Poaching share of hires	Labor productivity	Additive worker & firm effects	Surplus of reranked workers
	Difference in unemployment from its HP trend			
Low	5.3***	4.7***	7.7***	5.4***
	(2.0)	(1.7)	(1.6)	(1.5)
High	-5.9***	-4.3**	-6.8***	-6.5***
	(1.8)	(1.6)	(2.0)	(1.8)
	First-difference of the unemployment rate			
Low	17.3***	10.7**	10.6**	7.3*
	(4.6)	(4.2)	(4.3)	(4.1)
High	-12.9***	-8.4***	-9.7*	-5.5
	(4.6)	(4.1)	(5.2)	(4.6)

Notes: Estimates of change in share of employment on the seasonally-adjusted unemployment rate, season dummies, and a time trend. *, **, and *** indicate statistical significance at the 10%, 5%, and 1%, respectively. Standard errors are in parentheses. To avoid excessive decimal places, the dependent variables range from $[-100, 100]$, while the cyclical indicators range from $[-1, 1]$.

a larger number of earnings observations for a particular worker or firm should reduce this bias. Our minimum employment thresholds may also mitigate a second sort of bias applies particularly to an additive model of worker and firm effects. Identification of an additive model of worker and firm effects comes from workers employed by multiple employers. Workers who are employed by relatively few employers may therefore induce "limited mobility bias," see Andrews et al. (2012).

Cyclical composition by worker rank is shown in Table C1. In worse labor markets, the employment composition shifts away from low-rank workers and toward high-rank workers. An additional percentage point of the unemployment rate above its HP trend is associated with a 0.021 to 0.066 percentage point decline in the employment share of low-rank workers, and a 0.013 to 0.060 percentage point increase in the employment share of high-rank workers. A one percentage point increase in the unemployment rate is associated with a 0.047 to 0.209 percentage point decline in the employment share of low-rank workers, and a 0.081 to 0.164 percentage point increase in the employment share of high-rank workers.

Cyclical composition by firm rank is shown in Table C2. In worse labor markets, the employment composition shifts away from high-rank firms and toward low-rank firm. An additional percentage point of the unemployment rate above its HP trend is associated with a 0.047 to 0.077 percentage point

Table C3: Changes in worker rank shares and the unemployment rate (pre-recession ranks)

	Employment duration	Average earnings	Additive model worker effects	Rank workers vs. co-workers
	Difference in unemployment from its HP trend			
Low	-0.2	-5.9**	-9.0***	-6.7**
	(11.6)	(2.9)	(3.1)	(2.6)
High	9.2	6.4***	10.2***	6.5***
	(8.4)	(2.1)	(2.5)	(1.8)
	First-difference of the unemployment rate (pre-recession ranks)			
Low	-23.4***	-6.0***	-7.3***	-6.3***
	(3.9)	(1.0)	(1.0)	(0.9)
High	15.5***	3.7***	5.4***	3.5***
	(3.0)	(0.8)	(0.9)	(0.7)

Notes: Estimates of change in share of employment on the seasonally-adjusted unemployment rate, season dummies, and a time trend. *, **, and *** indicate statistical significance at the 10%, 5%, and 1%, respectively. Standard errors are in parentheses. To avoid excessive decimal places, the dependent variables range from $[-100, 100]$, while the cyclical indicators range from $[-1, 1]$.

increase in the employment share of low-rank firms, and a 0.043 to 0.068 percentage point decrease in the employment share of high-rank firms. A one percentage point increase in the unemployment rate is associated with a 0.073 to 0.173 percentage point increase in the employment share of low-rank firms, and a 0.055 to 0.29 percentage point decrease in the employment share of high-rank firms.

These results are qualitatively similar, although smaller in magnitude, than those in Tables 1 and 3. One potential explanation for the difference between these results is that marginal workers and firms may be the most responsive to economic conditions. Marginal firms may employ relatively few workers, and marginal workers may tend to work less than other workers.

C.2.2 Pre-recession ranks

Our second robustness exercise further limits the ability of recessions to determine worker and firm ranks. Although our ranking strategies described in Appendix B control for time effects, there is the potential for cyclical changes in employment, earnings, hiring, and productivity to influence worker and firm rankings. We therefore separate the time periods used to calculate ranks from those used to assess cyclical composition changes. Specifically, we rank workers and firms based on observations

Table C4: Changes in firm rank shares and the unemployment rate (pre-recession ranks)

	Poaching share of hires	Labor productivity	Additive worker & firm effects	Surplus of reranked workers
	Difference in unemployment from its HP trend			
Low	3.1	3.2	1.3	-1.9
	(3.5)	(4.0)	(4.3)	(4.0)
High	-2.3	-5.2	-4.0	-0.7
	(3.5)	(4.2)	(5.5)	(4.7)
	First-difference of the unemployment rate			
Low	2.7*	4.8***	7.2***	5.9**
	(1.4)	(1.5)	(1.6)	(1.5)
High	-4.5*	-5.4***	-8.4***	-7.6***
	(1.3)	(1.6)	(2.0)	(1.7)

Notes: Estimates of change in share of employment on the seasonally-adjusted unemployment rate, season dummies, and a time trend. *, **, and *** indicate statistical significance at the 10%, 5%, and 1%, respectively. Standard errors are in parentheses. To avoid excessive decimal places, the dependent variables range from $[-100, 100]$, while the cyclical indicators range from $[-1, 1]$.

that preceded the 2001 recession, and 2007-2009 recession, and measure employment composition outcomes from 2001-2002, and 2008-2014, respectively.[3]

Cyclical composition by worker rank is shown in Table C3. In worse labor markets, the employment composition shifts away from low-rank workers and toward high-rank workers. An additional percentage point of the unemployment rate above its HP trend is associated with a 0.002 to 0.090 percentage point decline in the employment share of low-rank workers, and a 0.064 to 0.102 percentage point increase in the employment share of high-rank workers. A one percentage point increase in the unemployment rate is associated with a 0.060 to 0.234 percentage point decline in the employment share of low-rank workers, and a 0.037 to 0.155 percentage point increase in the employment share of high-rank workers.

Cyclical composition by firm rank is shown in Table C4. In worse labor markets, the employment composition shifts away from high-rank firms and toward low-rank firm. An additional percentage point of the unemployment rate above its HP trend is associated with a -0.019 to 0.032 percentage point increase in the employment share of low-rank firms, and a 0.007 to 0.052 percentage point decrease in the employment share of high-rank firms. Note that, for none of the HP detrended results,

[3]We thank Rasmus Lentz for suggesting this robustness exercise during his discussion of our paper at the October 2019 "Models of Linked Employer-Employee Data" conference.

are we able to reject zero at conventional significance levels A one percentage point increase in the unemployment rate is associated with a 0.027 to 0.072 percentage point increase in the employment share of low-rank firms, and a 0.045 to 0.084 percentage point decrease in the employment share of high-rank firms.

These results are qualitatively similar, although smaller in magnitude and statistical significance, than those in Tables 1 and 3. One potential explanation for the difference between these results is that only a subset of the quarters is employed, which lowers the power of our tests. Note that this exercise also changes the composition by necessarily excluding entrants. If new labor market entrants are especially like to start at the bottom of the job ladder in low-rank firms, this regression may miss one of the main channels that determines cyclical labor market sorting.

C.3 Correlations between worker and firm ranks

We measure the correlation between worker ranks and firm ranks from each of the four methods, and present these correlations in Table C5. A sizable literature exists on how these different methods yield different measures of labor market sorting, and so we do not expect perfect agreement.

The different methods of ranking worker and firms are positively correlated with each other, although correlations are generally less than 0.5. The revenue productivity measure has the lowest correlation with other ranking methods.

The different methods yield different correlations in the extent to which low- vs. high-rank workers are employed at low- vs. high-rank firms. The revenue productivity method produces the strongest correlation, at 0.35, while the poaching share and employment duration model produces the lowest correlation, at 0.22. The reranking and reservation wage method yields a correlation of 0.24, and our additive worker and firm effects method yields a correlation of 0.33.

The correlation between worker effects and firm effects in the additive model is larger than some early implementations of Abowd, Kramarz, and Margolis (1999) estimators on linked employer-employee data, which suggested that the correlation between worker type and firm type was close to zero. Our estimates are of the same order of magnitude but smaller than the recently proposed estimator of Bonhomme, Lamadon, and Manresa (2018), and much smaller than that of Borovičková and Shimer (2018). Using annual data for the U.S. in a similar time period, Lamadon, Mogstad, and Setzler report a correlation of 0.10 from estimation that follows Abowd, Kramarz, and Margolis (1999).

Table C5: Correlation of worker and firm ranks across methods

	Firm rankings				Worker rankings			
	Poaching Share	Labor Productivity	Additive Firm	Surplus	Employment	Earnings	Additive Worker	Reranking
Firm Rankings								
Poaching Share	1.00							
Labor Productivity	0.32	1.00						
Additive	0.43	0.43	1.00					
Surplus	0.44	0.54	0.77	1.00				
Worker Rankings								
Employment	0.22	0.08	0.16	0.18	1.00			
Earnings	0.23	0.31	0.36	0.51	0.30	1.00		
Additive	0.23	0.29	0.33	0.49	0.31	0.98	1.00	
Reranking	0.13	0.14	0.16	0.24	0.24	0.79	0.31	1.00

Notes: All correlations are statistically distinct from zero at the 0.0001 significance level.

Table C6: Correlation of additive model worker and firm ranks across implementation methods

Correlation	0.332	0.326	0.165	0.158	0.176	0.172	0.176	0.161	0.173	0.180	0.188
Control Variables											
Age quadratic and time FE	-	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
Birth year * time FE	Y	-	-	-	-	-	-	-	-	-	
Random sample											
10%	-	-	-	Y	Y	-	-	-	-	-	-
100%	Y	Y	Y	-	-	Y	Y	Y	Y	Y	Y
Full-quarters employed											
20+ Quarters	-	Y	-	-	-	-	-	-	-	-	-
Time range											
1994-2003	-	-	Y	Y	-	-	Y	-	-	-	-
2004-2014	-	-	-	-	Y	-	-	-	-	-	-
1994-2014	Y	Y	-	-	-	-	-	-	-	-	-
2004-2013	-	-	-	-	-	-	-	Y	-	-	-
1994-2013	-	-	-	-	-	Y	-	-	Y	Y	Y
Frequency											
Quarterly	Y	Y	Y	Y	Y	-	-	-	-	-	-
Annual	-	-	-	-	-	Y	Y	Y	Y	Y	Y
Earnings threshold											
$15,000 2014 dollars	-	-	-	-	-	-	-	-	Y	Y	-
Age restriction											
25 to 55	-	-	-	-	-	-	-	-	-	Y	-
Firm size restriction											
20+ workers	-	-	-	-	-	-	-	-	-	-	Y

Notes: Annual estimates end in 2013 because 2014 is only partially available.

We view our relatively large correlation as the effect of having a very large number of workers and firms, a relatively lengthy panel, and using quarterly rather than annual data. The use of quarterly data especially reduces the amount of "limited mobility bias" that can result in lower correlation estimates, see Andrews et al. (2012). We show in Table C6 that implementing our additive estimator on subsets of the data yields much smaller correlations between worker type and firm type. Comparing the correlations across columns also yields information about the relative effects of different commonly used sample selection techniques on worker-firm rank agreement in an additive framework.

References

Abowd, John, Francis Kramarz, and David Margolis. 1999. "High Wage Workers and High Wage Firms." *Econometrica* 67(2): 251-333.

Andrews, Martyn, Leonard Gill, Thorsten Schank, and Richard Upward. 2012. "High Wage Workers Match with High Wage Firms: Clear Evidence of the Effects of Limited Mobility Bias." *Economics Letters* 117(3): 824-287.

Bonhomme, Stéphane, Thibaut Lamadon, and Elena Manresa. 2018. "A Distributional Framework for Matched Employer-Employee Data." Unpublished draft, University of Chicago.

Borovičková, Katarína, and Robert Shimer. 2018. "High Wage Workers Work for High Wage Firms." Unpublished draft, University of Chicago.

Lamadon, Thibaut, Magne Mogstad, and Bradley Setzler. 2019. "Imperfect Competition, Compensating Differentials, and Rent Sharing in the U.S. Labor Market." Unpublished draft, University of Chicago.

D Comparison of LEHD moments with Lise and Robin (2017)

Table D.1 shows our parameter estimates, alongside those of Lise and Robin (2017). For the most part the estimates are qualitatively similar to each other: the parameters for employed worker search effort s, vacancy posting costs c_0 and c_1, the exogenous job destruction rate δ, and the aggregate state ρ and σ are nearly identical. Estimated matching efficiency is somewhat different, increasing from 0.497 in Lise and Robin (2017) to 0.680. The shape parameters of the Beta distribution for workers,

β_1 and β_2 are closer to each other when we target LEHD moments, which indicates a more symmetric distribution. The parameters for the production function itself exhibit relatively little change.

Table D.2 shows how the two estimated economies behave with respect to the Lise and Robin (2017) targeted moments. The targeted, simulated moments are generally close to each other (and the data), as would be expected when the estimated parameters are similar. There are some noteworthy differences among untargeted moments. When we target LEHD moments, we do not target the moments for particular unemployment durations. First, in our LEHD moments estimation, unemployment is a more persistent state than implied by the unemployment duration moments targeted by Lise and Robin (2017). Furthermore, we do not target the standard deviation of labor productivity or its evolution over time. We obtain a much higher standard deviation of labor productivity of 0.758 than the 0.494 implied by the data.[4]

Table D.2 also shows the LEHD moments. Each β is the coefficient from a regression of a worker tercile or firm tercile share of employment on the first difference of unemployment. For example, β_L^{worker} is the coefficient for the employment share of low-rank workers. The cyclical composition moments are taken from column 1 of Tables 1 and 3. The second column is the same quantities,

[4]We note that Lise and Robin (2017) obtain this labor productivity dispersion moment using Compustat data from Bloom et al. (2018). Compustat firms are larger and more stable than the typical business, see Davis et al. (2007). There is therefore reason to interpret the data moment on labor productivity as a lower bound.

Table D.1: Parameter estimates

Parameter	Lise & Robin (2017) estimation	LEHD moments estimation
α	0.497	0.668
s	0.027	0.026
c_0	0.028	0.030
c_1	0.084	0.080
δ	0.013	0.011
σ	0.071	0.073
ρ	0.9997	0.9997
β_1	2.148	2.577
β_2	12.001	11.270
p_1	0.003	0.003
p_2	2.053	1.998
p_3	-0.140	-0.186
p_4	8.035	8.017
p_5	-1.907	-1.744
p_6	6.596	6.517

Table D.2: Moments: data & model-implied

Moment	Data	Lise & Robin (2017) estimation	LEHD moments estimation
	Moments from Lise and Robin (2017)		
$\mathbb{E}[U]$	0.058	0.059	0.060
$\mathbb{E}\left[U^{5p}\right]$	0.035	0.032	*0.047*
$\mathbb{E}\left[U^{15p}\right]$	0.018	0.018	*0.041*
$\mathbb{E}\left[U^{27p}\right]$	0.010	0.011	*0.039*
$\mathbb{E}[UE]$	0.421	0.468	0.231
$\mathbb{E}[EU]$	0.025	0.028	0.015
$\mathbb{E}[EE]$	0.025	0.025	0.036
$\mathbb{E}[V/U]$	0.634	0.744	1.003
$\mathbb{E}$[sd labor prod]	0.494	0.505	*0.758*
sd $[V]$	0.206	0.105	0.089
sd $[VA]$	0.033	0.034	0.033
autocorr $[VA]$	0.932	0.991	0.992
corr $[V,U]$	-0.846	-0.975	-0.680
corr $[U,VA]$	-0.860	-0.983	-0.979
sd $[U]$	0.191	0.203	0.150
sd $\left[U^{5p}\right]$	0.281	0.315	*0.152*
sd $\left[U^{15p}\right]$	0.395	0.413	*0.145*
sd $\left[U^{27p}\right]$	0.478	0.439	*0.141*
sd $[UE]$	0.127	0.127	0.047
sd $[EU]$	0.100	0.095	0.159
sd $[EE]$	0.095	0.112	0.186
sd $[V/U]$	0.381	0.306	0.220
sd [sd labor prod]	0.039	0.038	*0.036*
corr $[V,VA]$	0.721	0.996	0.749
corr $[UE,VA]$	0.878	0.978	0.166
corr $[EU,VA]$	-0.716	-0.910	-0.946
corr $[UE,EE]$	0.695	0.977	-0.169
corr [sd labor prod, VA]	-0.366	-0.361	*-0.458*
	LEHD Moments		
β_L^{worker}	-44.9	*-53.6*	-34.3
β_H^{worker}	31.6	*26.8*	17.6
β_L^{firm}	12.0	*-1079.6*	31.0
β_H^{firm}	-8.9	*348.6*	-59.1

Notes: Entries in italics are untargeted moments. β_R^G is the impact of a 1 percent change in the unemployment rate on the employment share of $G \in \{worker, firm\}$, $R \in \{L,H\}$.

calculated from simulated data using the moments from Lise and Robin (2017). The implied moments from our estimation are in the third column of Table D.2. On the worker side, the model consistently matches the empirical pattern that the worker distribution shifts away from low-rank workers and toward high-rank workers. Specifically, for every one percentage point increase in the unemployment rate, the Lise and Robin (2017) parameter estimates predict a decline in the employment share of low-rank workers of 0.536 percentage points and an increase in that of high-rank workers by 0.268 percentage points. These are very close to what we obtain from the LEHD data, which show a decline of 0.449 percentage points in the employment share of low-rank workers and an increase of 0.268 percentage points for high-rank workers. Our own estimates that target the worker and firm composition moments from LEHD data do no better: they show a decline of 0.343 percentage points in the employment share of low-rank firms and an increase of 0.176 percentage points in the employment share of high-rank workers.

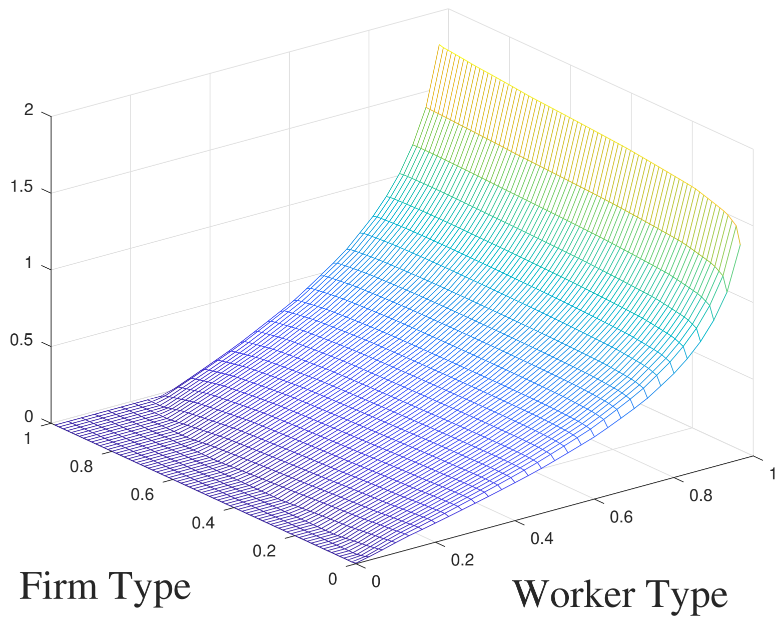
On the firm side, in the estimation in Lise and Robin (2017), the employment share of low-rank firms decreases, while that of high-rank firms increases. Specifically, a one percentage point increase in the unemployment rate is associated with a 10.796 percentage point decrease in the employment share of low-rank firms, and a 3.486 percentage point increase in the employment share of high-rank firms. This contrasts with the increase in the employment share of low-rank firms of 0.120 percentage points, and decline in that of high-rank firms of 0.089 percentage points. Thus, the Lise and Robin (2017) framework does not automatically generate our finding regarding cyclical changes in firm composition: cleansing effects dominate, and strongly so. However, when we target the cyclical firm and worker moments from LEHD data, we obtain something closer to what the data describe: and increase of 0.310 percentage points in the employment share of low-rank firms and a 0.591 percentage point decline in the employment share of high-rank firms. The Lise and Robin (2017) framework therefore has the potential to demonstrate cleansing of the worker distribution and sullying of the firm distribution.

Figure D.1 shows the estimated production functions. The Lise and Robin (2017) model estimates distribution of worker types, and given the parameters the masses of each type of firm are endogenous. Thus, to compare the production functions from two equilibria we need to normalize the worker and firm distributions. In Figure D.1 the production functions are normalized so that each increment along the worker (firm) type axis covers an equal fraction of the worker (firm) distribution. It is apparent that both production functions put more weight on the worker type, and are nearly flat in firm type.

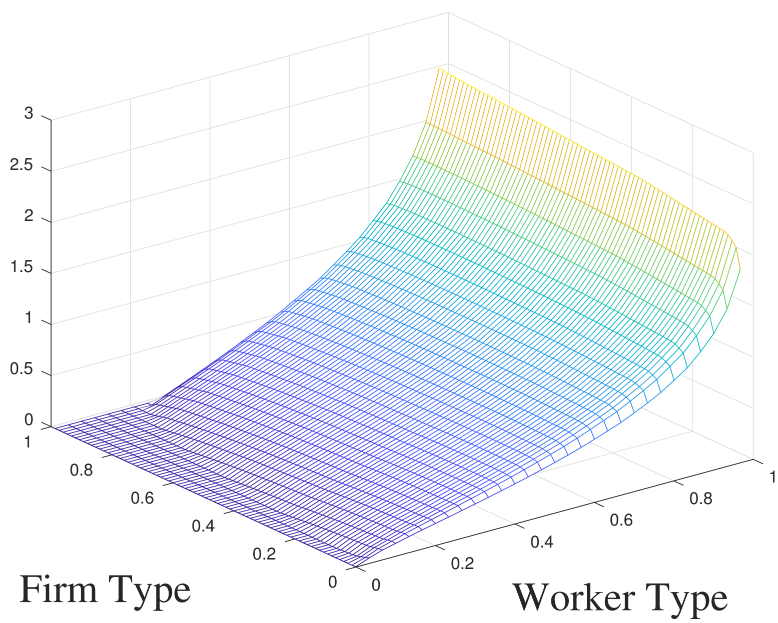
We show the shift in the worker type distribution in Figure D.2. This shows that, after targeting the LEHD moments, the worker distribution targets higher ranked workers. In order to understand the mechanics of sorting in the Lise and Robin (2017) model, it is helpful to keep in mind that a worker's

Figure D.1: Model implied production functions

(a) Lise & Robin (2017) parameter estimates

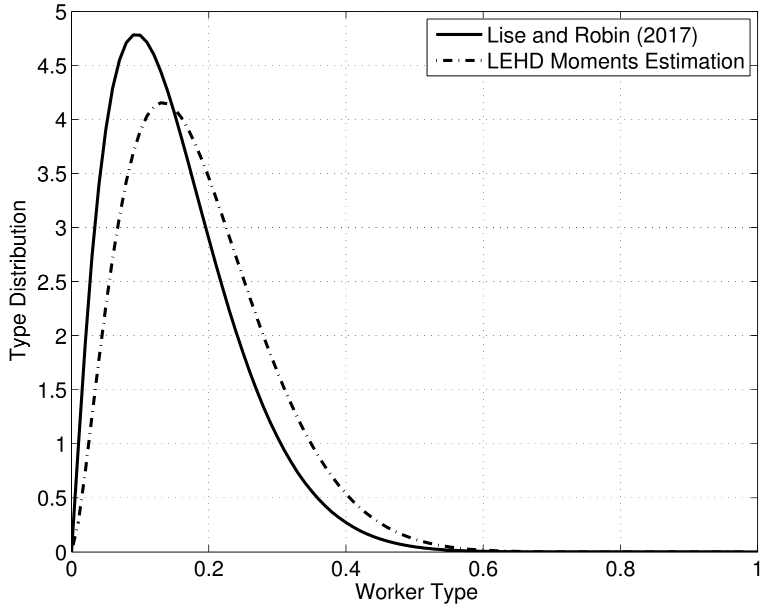


(b) Parameter estimates targeting LEHD moments



Notes: Worker and firm type distributions normalized to uniform.

Figure D.2: Worker distributions



optimal match is determined by the production function. Taking the first derivative of the production function with respect to firm type y yields

$$\frac{\partial}{\partial y}(p_1 + p_2 x + p_3 y + p_4 x^2 + p_5 y^2 + p_6 xy) = p_3 + 2p_5 y + p_6 x.$$

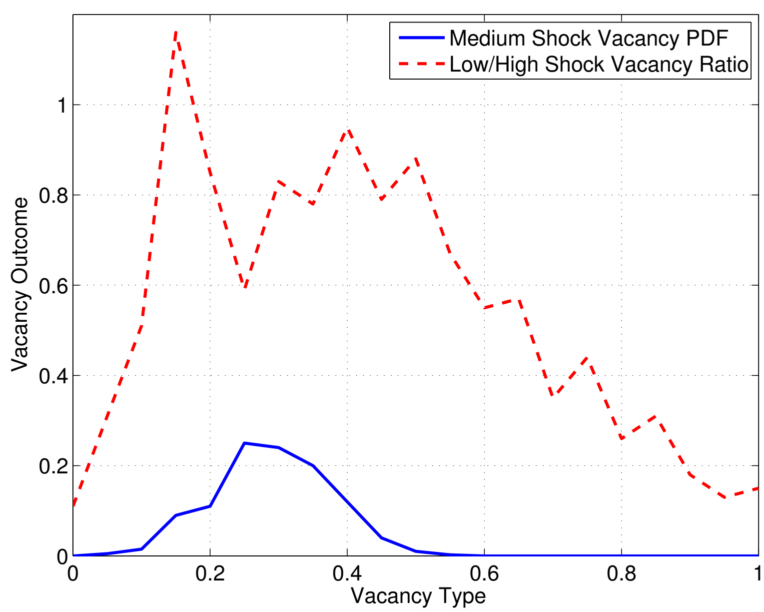
There are two regions in which monotonicity prevails. At the lower extreme workers for whom $x \leq p_3/p_6$ strictly prefer firms with a lower value of y and so monotonically move toward firms with $y = 0$. This occurs at value 0.021 in Lise and Robin (2017) and value 0.029 when we target the LEHD moments. Likewise, there is a region where workers monotonically move toward firms with rank $y = 1$, for workers with value $x \geq (p_3 + 2p_5)/p_6$. This occurs at value 0.60 in Lise and Robin (2017) and 0.56 when we target LEHD moments. For the region in between these values of x, the optimal firm is of type $y = (p_6 x - p_3)/(2p_5)$. Workers will gradually move toward these optimal matches as they move up the ladder. Figure D.2 shows that the share of workers in either region characterized by monotonicity is small in both Lise and Robin (2017) and when we target our LEHD moments.

The lower tail of the firm type distribution can be characterized in in Figure D.3, which contains the distribution of vacancies that prevail in the long run under a zero shock (50th percentile), as well as the relative changes for low (25th percentile) vs. high (75th percentile) aggregate states of the

Figure D.3: Model implications for the cyclical distribution of vacancy posting

(a) Lise and Robin (2017) estimation

(b) LEHD moment estimation



Notes: Vacancy posting distribution using Table 10 parameter estimates.

economy. This exercise is done both for the baseline Lise and Robin (2017) parameter estimates, as well as parameter estimates that target the new cyclical moments estimated on LEHD data. Note that in both cases, the firm rank is not to be confused with the value of the match. The highest value vacancies are found in the middle of the firm rank distribution, somewhere between 0.2 and 0.3 in both versions of the estimation. Targeting the LEHD moments changes the baseline distribution of vacancies: in Panel D.3(a), there is a much more pronounced left tail of low-rank firms, whereas in Panel D.3(b) this panel is less pronounced.

Figure D.3 also provides insights into the nature of countercyclical "cleansing" of low-rank firms. This can be seen in the measures of the low vs. high shock ratio of vacancies. Naturally, there are fewer vacancies with the aggregate state is low relative to when it is high, hence the ratio of vacancies is generally less than one. At the extremes of the distribution, firms post less than 20% of the vacancies in the low state than they do in the aggregate state, however, Figure D.3 also illustrates that there are approximately zero firms operating in this range of the distribution. The ratio of vacancies in the low vs. high state is also not monotonic, and what is especially interesting is a spike in the ratio of vacancies that occurs at the gridpoint in which the distribution of vacancies loses most of its mass. In other words, there is extra activity at the bottom of the job ladder: the set of firms that, when there is a high aggregate state, has an almost trivial mass since an offer to an employed worker has an approximately 100% chance of them being poached away. Note that this spike is apparent in the baseline estimates of Lise and Robin (2017).[5] However, when we we target the LEHD moments this spike in vacancy posting at the bottom of the firm type distribution is much more dramatic.

References

Bloom, Nicholas, Max Floetotto, Nir Jaimovich, Itay Saporta-Eksten, and Stephen Terry. 2018. "Really Uncertain Business Cycles." *Econometrica* 86(3): 1031-1065.

Davis, Steven, John Haltiwanger, Ron Jarmin, Javier Miranda, Christopher Foote, and Éva Nagypál. 2007. "Volatility and Dispersion in Business Growth Rates: Publicly Traded versus Privately Held Firms." *NBER Macroeconomics Annual 2006* Daron Acemoglu, Kenneth Rogoff and Michael Woodford, Eds., 107-179. Chicago: University of Chicago Press.

[5] This nonmonotonicity is less apparent in Lise and Robin (2017), Figure 3, Panel A, when they present similar results for a larger difference in aggregate states.

Lise, Jeremy, and Jean-Marc Robin. 2017. "The Macrodynamics of Sorting between Workers and Firms." *American Economic Review* 107(4): 1104-1135.

E Production function inversion estimation

To estimate the production implied from the reranking methodology of identifying worker and firm types, we employ the job surplus inversion method described in Hagedorn, Law, and Manovskii (2017). The production function is a function of the surplus, $S(\hat{x},\hat{y})$, generated by the matching of a worker of type $\hat{x}$ with a firm of type $\hat{y}$ plus the value of a vacancy to a firm of type $\hat{y}$, $V_v(\hat{y})$ and the value of unemployment to a worker of type $\hat{x}$, $V_u(\hat{x})$. These factors are weighted by the time-discount factor (β) and the job destruction rate (δ). Specifically, the productivity of a specific worker-firm match, $f(\hat{x},\hat{y})$ is:

$$f(\hat{x},\hat{y}) = (1-\beta(1-\delta))\, S(\hat{x},\hat{y}) + (1-\beta)V_v(\hat{y}) + (1-\beta)V_u(\hat{x}) \tag{8}$$

E.1 Value of unemployment by worker type

We estimate the value of unemployment, $V_u(\hat{x})$, by estimated worker type, $\hat{x}$, as the present discounted value of the minimum quarterly earnings from nonemployment accepted by workers of type $\hat{x}$ from any firm type. For every worker-firm type combination, we calculate $e^{10p}(\hat{x},\hat{y})$, the 10th percentile of residual earnings (after controlling for age). The value of unemployment to a specific worker type $\hat{x}$ is the minimum of the e^{10p} across all potential firm-types given the worker type.

$$V_u(\hat{x}) = \frac{1}{1-\beta} \min_{\hat{y}} e^{10p}(\hat{x},\hat{y}) \tag{9}$$

E.2 Value of employment by worker-firm combination

We estimate the value to a worker of being employed by worker-firm type combination $V_e(\hat{x},\hat{y})$ as the average across all observed jobs spells of the present discounted value of earnings of workers of type $\hat{x}$ over the job spells at firms of type $\hat{y}$. If $i(\hat{x},\hat{y})$ is an index of job spells of type $\hat{x}$ workers at type $\hat{y}$ firms and d_i is the duration of job spell i then $V_e(\hat{x},\hat{y})$ is:

$$V_e(\hat{x},\hat{y}) = \sum_{i(\hat{x},\hat{y})} \frac{1}{N_i} \sum_{t=0}^{d_i-1} \beta^t e_{it} + \beta^d V_u(\hat{x}) \tag{10}$$

E.3 Match surplus by worker-firm combination

We estimate the worker-firm type combination match surplus, $S(\hat{x},\hat{y})$, as the scaled difference between the value of a worker's value of being employed at a firm of a given type and the worker's value of employment, where the scaling factor is the measure of worker's bargaining power α. More specifically,

$$S(\hat{x},\hat{y}) = \frac{V_e(\hat{x},\hat{y}) - V_u(\hat{x})}{\alpha} \tag{11}$$

We use $\alpha = 0.5$, as in the model of Shimer and Smith (2000).

E.4 Vacancy value by firm type

We estimate the vacancy value to a firm of type $\hat{y}$, $V_v(\hat{y})$. The vacancy value is a function of the discount factor β, the worker's bargaining power α, the job destruction rate δ, and the average firm surplus for firms of type $\hat{y}$, $\Omega(\hat{y})$. The firm surplus is estimated using the surplus-based ranking method described in Appendix Section B.4.3.

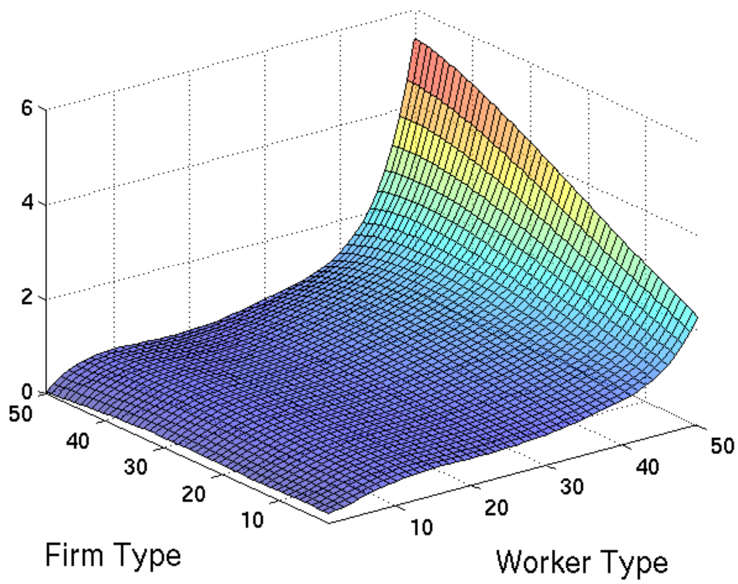
$$V_v(\hat{y}) = \frac{\beta}{1-\beta} \frac{1-\alpha}{\alpha} (1-\delta)\Omega(\hat{y}) \tag{12}$$

References

Hagedorn, Marcus, Tzuo Law, and Iourii Manovskii. 2017. "Identifying Equilibrium Models of Labor Market Sorting." *Econometrica* 85(1): 29-65.

Shimer, Robert, and Lones Smith. 2000. "Assortative Matching and Search." *Econometrica* 68(2): 343-369.

Figure E1: Production Function from Worker Reranking & Surplus Method



Notes: Worker and firm type distributions normalized to uniform.