In-Class Exercise: Queuing Models

A Supersonic Burger fast-food franchise is considering adding drive-through service. Data collected at other Supersonic drive drive-throughs in the area indicate that customer arrivals follow a Poisson probability distribution, with a mean arrival rate of 24 cars per hour, and that service times follow an exponential probability distribution. In the proposed drive-through system, arriving customers will place orders at an intercom station at the back of the parking lot and then drive to the service window to pay for and receive their orders. The following three service alternatives, which are in use at other Supersonic Burgers with drive-through service, are being considered:

1. A single-channel operation in which one employee fills the order and takes the money from the customer. The average service time for this alternative is 2 minutes.

M/M/1; λ = 24 customers/hour; μ = 2 minutes/customer = 30 customers/hour

1. A single-channel operation in which one employee fills the order while a second employee takes the money from the customer. The average service time for this alternative is 1.25 minutes.

M/M/1; λ = 24 customers/hour; μ = 1.25 minutes/customer = 48 customers/hour

1. A two-channel operation with two service windows and two employees. The employee at each service window fills the order and takes the money from the customer. The average service time for this alternative is 2 minutes.

M/M/2; λ = 24 customers/hour; μ = 2 minutes/customer = 30 customers/hour

1. For each alternative determine the probability that an arriving car will have to wait for service.
2. P(t>0) = Pn>0 = ρ = λ/μ = 24/30 = 0.80
3. P(t>0) = Pn>0 = ρ = λ/μ = 24/48 = 0.50
4. P(t>0) = ρ = λ/Kμ = 24/(2\*30) = 0.40
5. For each alternative determine the average number of cars waiting for service.
6. Ls = λ2/(μ(μ-λ)) = 24^2/(30(30-24)) = 3.2 cars
7. Ls = λ2/(μ(μ-λ)) = 24^2/(48(48-24)) = 0.5 cars
8. Ls = L-(λ/μ) = 0.9524 - (24/30) = 0.1524 cars
9. For each alternative determine the average time that a car spends in the drive-through system.
10. W = 1/(μ-λ) = 1/(30-24)) = 0.16 hours = 10 minutes
11. W = 1/(μ-λ) = 1/(48-24)) = 0.0416 hours = 2.5 minutes
12. W = L/λ = 0.9524/24 = 0.0397 hours = 2.382 minutes
13. For each alternative determine the average time that a car spends waiting to place an order.
14. Wq = λ/(μ(μ-λ)) = 24/(30(30-24)) = 0.13 hours = 8 minutes
15. Wq = λ/(μ(μ-λ)) = 24/(48(48-24)) = 0.2083 hours = 1.25 minutes
16. Wq = Lq/λ = 0.1524/24 = 0.00635 hours = 0.381 minutes
17. Because waiting time is costly in the fast-food business, Supersonic's management assigns a cost of $25 per hour to customer waiting time. From an expected-cost perspective, which of the three alternatives described above would you recommend if Supersonic drivethrough employees are paid $6.50 per hour and the operating cost per channel (equipment, space, etc.) is $20 per hour? Explain your answer.

Total System Cost = Total Service Cost + Total Waiting Cost

= KCS + (Avg # Customers/hours)(Avg Time in System)(CW)

= KCS + λWCW

= KCS + LCW

1. L = λ/(μ-λ) = 24/(30-24) = 4 customers

(1 server)($6.5/hour) + (4)($25) = $106.5/hour

1. L = λ/(μ-λ) = 24/(48-24) = 1 customer

(2 server)($6.5/hour) + (1)($25) = $38/hour

1. L = 0.9524 customers

(2 servers)($6.5/hour) + (0.9524)($25) = $36.81/hour

The two channel system is superior because the waiting cost penalty is far greater than the labor cost for the system.