

WORKSHEET

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Great question! Let's break it down.

0.0.1 What is a qubit?

A **qubit** (quantum bit) is the basic unit of quantum information. It's like a regular bit in classical computing, but it behaves very differently because it follows the rules of **quantum mechanics**.

0.0.2 Classical Bit vs. Qubit

1. Classical Bit:

- A classical bit can be **either** 0 or 1.

Example:

0 or 1

2. Qubit:

- A qubit can be **0**, **1**, or a combination (or “superposition”) of both.
- Mathematically, a qubit is represented as:

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

Here, α and β are complex numbers, and they tell you the probabilities of measuring the qubit as 0 or 1.

The rule is:

$$|\alpha|^2 + |\beta|^2 = 1$$

Example:

A qubit might look like this:

$$|\psi\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

If you measure this qubit, there's a ****50% chance**** of getting `0` and a ****50% chance**** of getting `1`.

0.0.3 Superposition

A qubit can exist in a state where it's both 0 and 1 at the same time, called **superposition**. However, when you measure the qubit, it will “collapse” to either 0 or 1.

0.0.4 Entanglement

When two or more qubits are **entangled**, they become linked in a way that measuring one affects the other. For example, if two qubits are in the entangled state:

$$|\phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

If you measure the first qubit and get 0, the second qubit will also be 0. If you measure the first qubit and get 1, the second will be 1. This happens even if the qubits are far apart!

0.0.5 Summary

- A **qubit** is like a bit but can be in a **superposition** of both 0 and 1.
- When qubits interact, they can become **entangled**, meaning their states are linked.
- This is what makes quantum computing so powerful and different from classical computing.

Yes! The **+** **operation** is a key factor that determines whether a quantum state is **entangled** or **independent** (separable). Let me explain why!

0.0.6 What happens when there's no + (as in $|00\rangle$)?

The state $|00\rangle$ can be written as:

$$|00\rangle = |0\rangle \otimes |0\rangle$$

Here: - The two qubits are in separate, independent states. The first qubit is in $|0\rangle$, and the second qubit is also in $|0\rangle$. - There is **no entanglement** between them because the state is simply a product of two individual qubits.

0.0.7 What happens when there's a + (superposition or entanglement)?

Consider a state like:

$$|\phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

Here's what changes: - The two qubits are now in a **superposition** of both $|00\rangle$ and $|11\rangle$. - This means the state cannot be factored into a product of two independent single-qubit states. - Measuring one qubit affects the other. For example: - If you measure the first qubit and get 0, the entire state collapses to $|00\rangle$, meaning the second qubit is also 0. - If you measure the first qubit and get 1, the state collapses to $|11\rangle$, meaning the second qubit is also 1.

This kind of state is **entangled**, and the key feature is the **+** in the superposition.

0.0.8 How does the + operation create entanglement?

The **+** in a superposition means that multiple possibilities are present in the state at once. If these possibilities involve correlations between two qubits (like both being 0 or both being 1), the qubits become **entangled**. This correlation can't be separated into independent parts.

0.0.9 Summary

- The state $|00\rangle$ has no $+$ operation, so the qubits are **independent**.
- The state $|\phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ uses a $+$ to create a superposition, making the qubits **entangled**.
- The $+$ **operation** is crucial in forming **entanglement** by linking the outcomes of measurements on multiple qubits.

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