Advanced Topics in Wireless: Exam Questions

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1 Read this

Duration of the exam is 2 hours, all documents are permitted. Don't forget to justify where necessary (concisely - a few words is enough!) your answer (and prove mathematically where possible). Use your own words and do not copy sentences from a book/notes.
Also: read carefully the questions!

2 General Context

We consider a cellular network, where, in each cell, a base station (BTS) communicates with a number of mobile terminals (MT). The BTS is equipped with M antennas, and each MT is equipped with N antennas.
The exam has two parts. The first is on qualitative nature and the second is algorithmic and quantitative.

3 Qualitative questions

Exercise 1 *We consider the use of various MIMO strategies for improving the system performance, as shown in class. Please answer WRONG or CORRECT to each of these statements. No explanation needed.*

1. *The gain of using multiple transmit antennas is always the same as the gain from using multiple receive antennas.*

2. *The beamforming gain in dB is proportional to the number of antennas*

3. *The diversity gain with 4 transmitters, 1 receiver, at 10e-6 bit error rate, with QPSK modulation, is more than 30dB.*

4. *The multiplexing gain in a single user MIMO channel is equal to the product between number of transmit and receive antennas.*

5. *The beamforming gain in MIMO in not very sensitive to the presence of a line of sight component.*
6. *Feedback of channel state information to the transmitter is more important to have when the Ricean factor is high.*
7. *In a multiuser MIMO channel with a base station equipped with M antennas, the base communicates with mobiles, each equipped with $N = 1$ antennas. Then the base can serve at most $M - 1$ mobiles at the same time without interference.*
8. *In a multiuser MIMO channel with a base station equipped with M antennas, the base communicates with mobiles, each equipped with 2 antennas. Then there is no gain compared with the system in the previous bullet point (i.e. users with 1 antenna each).*

Exercise 2 *Consider the table given in the separate page (Fig.1) It presents different configurations of number of antennas for a single user MIMO system and for a multi-user system. For example, "3x2" means 3 transmit antennas and 2 receive antennas, "2 users 3x2" means the base has 3 antennas and communicates with 2 users with 2 antennas each ON THE DOWNLINK. In the multiuser case, the multiplexing order refers to the total number of streams that the base can send, across all users.*
It specifies for each case a scenario where the channel coefficient information is known at the receiver (CSIR) and/or at the receiver (CSIT). For example "NO CSIT" means the transmitter does not have channel knowledge.
Please complete the missing boxes by indicating for each case (without any explanation) respectively, the order of multiplexing possible (e.g. factor of 2, factor of 3, etc..), the total order of diversity, and the beamforming gain expressed in dB.

4 Quantitative questions

4.1 Notations for signals and channels

We consider the baseband representation of a signal being transmitted from the BTS to a single MT initially, and then from multiple MT in later questions. Each symbol at symbol-period l is denoted $s(l)$ and is drawn from a complex constellation with variance: $E|s(l)|^2 = \sigma_s^2$, also representing the **total transmit power**.
The symbols are sent into a frequency-non-selective (no ISI) MIMO channel represented by the $N \times M$ matrix: $\mathbf{H}$.
The link between the j-th BTS antenna and the i-th MT antenna is denoted by the complex gain $\mathbf{H}_{ij} = h_{ij}$.
The matrix is normalized so that all links have unit variance $E|h_{ij}|^2 = 1$.
We assume the channels are all uncorrelated (between users, between antennas).
The noise on each receive antenna is assumed to be white (uncorrelated across antennas and time) and of variance σ_n^2

4.2 Beamforming on the downlink with and without channel knowledge

We consider transmission towards a single MT, $M > 1$ and $N = 1$. First we try to beamform from the transmitter without channel knowledge. On each transmit antenna we transmit the same symbol, properly normalized such that the total transmit power from the BTS is σ_s^2.

Exercise 3 *Write the receive signal equation at the MT. Compute the SNR at the receiver side. Analyze the diversity and beamforming gain of this technique.*

Exercise 4 *We want again to use beamforming at the transmit side. But this time we assume the channel is known at the transmitter and at the receiver. Write the optimal beamforming solution (still under the same power constraint as above). Compute the SNR at the receiver, the diversity order and the beamforming gain.*

4.3 MIMO spatial multiplexing

We want to use the multiple antennas for increasing the rate via spatial multiplexing. We assume $M = N = 2$ and a fixed realization of the channel $\mathbf{H}$.
The channel is given by

$$\mathbf{H} = \begin{pmatrix} 3 & 1 \\ 4 & 2 \end{pmatrix} \tag{1}$$

Exercise 5 *We assume that the channel matrix is NOT known at the transmitter (only known at the receiver). In this case, give the equations for spatial multiplexing, still under the same total transmit power constraint as previously. Write decoding equations using successive interference canceler (i.e. SIC or "V-BLAST") receiver and signal ordering. Give the SNR obtained when decoding each symbol as a function of σ_n^2 and σ_s^2.*

Exercise 6 *Now assume a linear zero-forcing receiver is used instead of the above receiver. Compute the SNR when decoding each symbol and compare with the result above with the SIC receiver. Interpret the difference.*

4.4 Uplink Spatial Division Multiple Access (SDMA) for a single cell

We now consider a multiple access protocol where multiple terminals are allowed to access the channel and transmit to the base station simultaneously, **on the uplink channel**. We consider first a single cell (i.e. only one base station). We select K mobile terminals and let them transmit to the base. Each mobile k has $N = 1$ antennas and wants to send symbol s_k to the base. The base still has $M > 1$ antennas. The $M \times 1$ channel vector from terminal k to the base is denoted by $\mathbf{h}_k$.

Exercise 7 *Write the signal model for the (noisy) received signal vector at the base station, denoted* $\mathbf{y}$.
We assume linear zero-forcing beamforming at the base station receiver. write the equation for the estimated symbols after receiver processing. How many mobiles K *can this system support if one wanted to completely cancel the interference between terminals?*

4.5 Uplink Spatial Division Multiple Access (SDMA) for two cells

We now consider a two cell system. In cell 1, one base station with $M > 1$ antennas wants to communicate with K mobiles located in that cell. Next to cell 1, another cell (cell 2) is placed with another base station with $M > 1$ antennas, trying to communicate with his own set of K mobiles. All mobile terminals want to send one symbol each, on the uplink. The two cells can be assigned orthogonal frequencies, in which case there is no intercell interference, or they can be assign the same frequency, causing intercell interference. Our goal is to understand which strategy gives the highest spectrum efficiency. The two base stations are not linked to each other, initially.
The channel vector from mobile k in cell i towards the base station in cell j is denoted by $\mathbf{h}_k^{ij}$. The symbol sent by mobile k in cell i is denoted by s_k^i. The noise vector at base station i is denoted $\mathbf{n}^k$.

Exercise 8 *Assume the same frequency is assigned in both cells. Write the received signal equations are both base stations 1 and 2. Assuming the use of zero-forcing beamforming at the base stations, how many mobiles in total can be supported in the two-cell system as a function of* M*, while canceling out all interference?*

Exercise 9 *Assume now that* **different frequencies** *are assigned in both cells. Write the received signal equations are both base stations 1 and 2. Assuming the use of zero-forcing beamforming at the base stations, how many mobiles in total can be supported in the two-cell system as a function of* M*, while canceling out all interference? Compare the spectrum efficiency of this system with the one above with same frequency. What is the advantage of this system over the one above?*

Exercise 10 *We now assume that both base stations are linked together with an optical fiber allowing joint processing of both base station signals. Assuming the use of joint zero-forcing beamforming combining all* $2M$ *base station antennas, how many mobiles in total can be supported in the two-cell system as a function of* M*, while canceling out all interference? What could be a limitation of such a system?*

THAT'S ALL FOLKS!

Antenna Configuration	**CSIR**	**CSIT**	**Multiplexing order?**	**Diversity order?**	**Beamforming gain (dB)?**
1x1	Yes	No			
1x2	No	Yes			
2x2	Yes	Yes			
2x2	Yes	No			
2x4	Yes	Yes			
4x2	Yes	Yes			
4x2	Yes	No			
2x4 with Line of sight	Yes	Yes			
2 users 1x2	Yes	No		IGNORE	IGNORE
2 users 2x1	Yes	No		IGNORE	IGNORE
2 users 2x2	Yes	No		IGNORE	IGNORE
2 users 2x1	Yes	Yes		IGNORE	IGNORE
2 users 3x2	Yes	No		IGNORE	IGNORE

Figure 1: