REPORT

December 21, 2024

Midterm Take-Home Exam

December 20, 2024 (DUE January 10, 2025)

Instructions

- Open book and open class notes are allowed (including notes taken by students during exam). No other notes are allowed.
- Each answer should be clearly written, and the solution should be developed in detail.
- Mathematical derivations need to show all steps that lead to the answer.
- Partial credit will be given for incomplete solutions.
- There is NO penalty for incorrect solutions.

Hints - equations - conventions:

- Notation
  - R represents the rate of communication in bits per channel use (bpcu),
  - ρ represents the SNR (signal to noise ratio),
  - w will denote additive noise which will be distributed as a circularly symmetric Gaussian random variable $\mathbb{C}\mathcal{N}(0, N_0)$. If N_0 is not specified, then set $N_0 = 1$,
  - Remember: for a given signal-to-noise ratio (SNR), then SNR in dB is simply $10log_{10}SNR$
  - SISO stands for single-input single-output, MISO stands for multiple-input single output, SIMO stands for single-input multiple output, MIMO stands for multiple input multiple output.
  - MU stands for multi-user.
  - CSIT stands for channel state information at the transmitter, while CSIR stands for channel state information at the receiver.
  - AWGN stands for additive white Gaussian noise.
- GOOD LUCK!!

EXAM PROBLEMS

1) ***(1 point)***. Consider a SISO setting, with no fading. Consider that the maximum possible rate (i.e., the capacity) is equal to 7 bpcu. What is the minimum SNR required to achieve this rate? Do you need CSIR?

[ ]:

2) ***(1 point)***. Consider a SISO quasi-static fading channel with no CSIT. We wish to decrease the probability of error, from $P_{err} \approx (SNR)^{-1}$ to $P_{err} \approx (SNR)^{-4}$. Suggest various ways we can achieve this, based on what we have learned in class.

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3) ***(1 point)***. What are some of the advantages of MISO vs. SIMO, mentioned in class?

[ ]:

4) ***(1 point)***. In a single-user MIMO channel, how much diversity gain would we be able to get if we employed a transmitter with 4 transmit antennas and a receiver with 2 receive antennas, when in fact the channel between the first transmit and receive antenna, is identical always to the channel between the first transmit and second receive antenna?

[ ]:

5) ***(1 point)***. In a single-user MISO channel, how much multiplexing gain would we be able to get if we employed a transmitter with 2 transmit antennas?

[ ]:

6) ***(1 points)***. Consider communication over a quasi-static 2×1 MISO fading channel. Assume that you must draw symbols from 16-QAM.

- Can you name a space time code, that gives full diversity in this setting, and then describe the rate (in bpcu) of such a code.

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7) ***(3 points)***. In the context of various strategies, answer if each of the following statements are true or false, justifying briefly your answers.

- In a MISO channel, we can get transmitter beamforming gain even without CSIT.
- A base station equipped with 5 antennas in the downlink, can simultaneously serve up to 5 users (single receive antenna each).
- A base station equipped with 5 antennas in the downlink, can simultaneously serve up to 10 users (two receive antennas each).
- A base station equipped with 4 antennas in the downlink, can simultaneously serve up to 2 users (two receive antennas each).

- Line of sight channels are detrimental for spatial multiplexing in both single-user and multiuser MIMO.
- For a MIMO receiver using spatial multiplexing, the complexity of ZF receiver is more than the complexity of the maximum-likelihood receiver.
- CSIT is easier to obtain than CSIR.
- CSIT is of cardinal importance in multi-user MIMO.

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8) ***(2 points)***. In a MU-MIMO channel, if I double the number of users I simultaneously serve, must I always halve the individual rate to each user? Justify your answer.

[ ]:

9) ***(4 points)***. Consider communication over the 2×1 quasi-static fading MISO channel, using a diagonal code (see below for details) such that the channel model is given by

$$\overbrace{\begin{pmatrix} y_1 & y_2 \end{pmatrix}}^{y} = \theta \overbrace{\begin{pmatrix} h_1 & h_2 \end{pmatrix}}^{h} \overbrace{\begin{pmatrix} x_1 & 0 \\ 0 & x_2 \end{pmatrix}}^{\mathcal{X}_{tr}} + \overbrace{\begin{pmatrix} w_1 & w_2 \end{pmatrix}}^{w}$$

where $h_i \sim \mathbb{C}N(0,1)$ and $w_i \sim \mathbb{C}N(0,1)$, and where θ is the power normalization factor that lets you regulate SNR.

- Describe the ML decoding rule for this case.
- Describe the cardinality[1] of code $\mathcal{X}_{tr}$ if you wish a rate of $R = 4$ bpcu.
- For a desired rate of $R = 8$ bpcu, and a desired SNR $= 10$ dB (where by SNR we mean the AVERAGE signal power divided by the noise unit power, under QAM) then what is the normalizing factor θ ?
- Imagine that what you transmit (x_1, x_2) are independently chosen from 16-PAM, then
  - What is the rate of your code (in bpcu)?
  - What is the slope of your probability of error, in high SNR, if you plot on the y-axis the probability of error, in log scale ($\log_{10}$(Prob)), and the x-axis is the SNR, in dB?
- Imagine now that $(x_1 x_2) = s_1 s_2 \cdot \mathbf{Q}$, where s1,s2 are independently chosen from a 64-QAM constellation, where the matrix $\mathbf{Q}$ is a randomly chosen orthogonal matrix. Then
  - What is the rate of your code?
  - What is the aforementioned slope of your probability of error?

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10) ***(Extra Credit: 5 points)***. Consider communication over a quasi-static 2×2 MIMO channel, utilizing the space-time code $\mathcal{X} = \{\mathbf{X}_1, \mathbf{X}_2, \mathbf{X}_3, \mathbf{X}_4\}$, where

$$\mathbf{X}_1 = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}, \mathbf{X}_2 = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}, \mathbf{X}_3 = \begin{bmatrix} -1 & -1 \\ 1 & -1 \end{bmatrix} \text{and } \mathbf{X}_4 = \begin{bmatrix} -1 & 1 \\ -1 & -1 \end{bmatrix},$$

- What is the average SNR?

- What is the rate of the code in bpcu?
- What is the diversity gain of this code?
- What is the approximate (in the high SNR regime) probability of error of this code, if SNR is 30dB?

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