Advanced Topics in Wireless: Exam Questions

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1 Read this

Duration of the exam is 2 hours, all documents are permitted. Don't forget to justify where necessary (concisely - a few words is enough!) your answer (and prove mathematically where possible). Use your own words and do not copy sentences from a book/notes.
Also: read carefully the questions!

2 General Context

We consider a cellular network, where, in each cell, a base station (BTS) communicates with a number of mobile terminals (MT) either in uplink or in downlink. The BTS is equipped with M antennas, and each MT is equipped with N antennas. The exam has two parts. The first is on qualitative nature and the second is algorithmic and quantitative.

3 Qualitative questions

Exercise 1 *We consider the use of various MIMO strategies for improving the system performance, as shown in class. Please answer WRONG or CORRECT to each of these statements. No explanation needed.*

- *To use space-time codes, the transmitters need to know the channel coefficients.*
- *To use spatial multiplexing techniques, the transmitters always need to know the channel coefficients.*
- *The optimal form of spatial multiplexing (optimal is in the sense of maximum sum rate) uses equal power assigned to all the streams.*
- *The diversity gain from 1 to 2 antennas is about 10dB is almost all situations.*

- *Consider a MIMO channel with a strong line of sight component: in this case there is receive beamforming gain but no transmit beamforming gain.*
- *The multiplexing gain in a multi-user MIMO channel is usually independent on the existence of the line of sight component.*
- *In MIMO channel with spatial multiplexing, the MMSE receiver is the optimal form of receiver.*
- *In a multiuser MIMO channel with a base station equipped with M antennas, the base communicates with mobiles, each equipped with 2 antennas. Then the base can serve $2M$ mobiles at the same time without interference.*

Exercise 2 *Consider the table given in the separate (last) page. It presents different configurations of number of antennas for a single user MIMO system and for a multi-user system. For example, "3x2" means 3 transmit antennas and 2 receive antennas, "downlink 2 users 3x2" means the base has 3 antennas and communicates with 2 users with 2 antennas each ON THE DOWNLINK. "uplink 3 users 3x4" means the base has 4 antennas and communicates with 3 users with 3 antennas each ON THE UPLINK. In the multiuser case, the multiplexing order refers to the total number of streams that the base can send, across all users.*
It specifies for each case a scenario where the channel coefficient information is known at the receiver (CSIR) and/or at the receiver (CSIT). For example "NO CSIT" means the transmitter does not have channel knowledge.
Please complete the missing boxes by indicating for each case (without any explanation) respectively, the order of multiplexing possible (e.g. factor of 2, factor of 3, etc..), the total order of diversity, and the beamforming gain **expressed in dB**.

4 Quantitative questions

4.1 Notations for signals and channels

We consider the baseband representation of a signal being transmitter from the BTS to a single MT. The transmitted symbol sequence, denoted $s(l)$ is drawn from a complex constellation with variance: $E|s(l)|^2 = \sigma_s^2$, representing the transmit power.
The symbols are sent into a frequency-non-selective (no ISI) MIMO channel represented by the $N \times M$ matrix: $\mathbf{H}$.
The link between the j-th BTS antenna and the i-th MT antenna is denoted by the complex gain $\mathbf{H}_{ij} = h_{ij}$.
The matrix is normalized so that all links have unit power $E|h_{ij}|^2 = 1$.
We assume the channel are all uncorrelated (between users, between antennas).

4.2 Received signal for TDMA downlink

We assume a TDMA setup, and examine the system during a single time slot. Which means that during that time the MIMO system is a single user MIMO system between the base and the selected mobile.
The modulation symbols are sent or converted into coded symbols before launching on the transmit antennas. The coded symbol (intended to the MT) on the j-th transmit antenna at the BTS, at time n, is denoted $c_j(n)$.
The received signal at the MT at time n is obtained by:

$$\mathbf{y}(n) = \mathbf{H} \begin{pmatrix} c_1(n) \\ c_2(n) \\ \vdots \\ c_M(n) \end{pmatrix} \frac{1}{\sqrt{M}} + \mathbf{b}(n) \tag{1}$$

where $\mathbf{b}(n)$ is an additive noise vector caused by receiver electronics and interference. We assume the noise to be spatially and temporally white, with each component having power $E|b_i(n)|^2 = \sigma_b^2$, $i = 1..N$. The factor $\sqrt{M}$ is used at the transmitter to limit and normalize the power independently of the number of antennas.

Exercise 3 *We assume $M = 2$, N is arbitrary. We want to use the antennas for diversity purpose, using Alamouti code. We denote $\mathbf{h}_i$ the i-th column of channel matrix $\mathbf{H}$. We consider the transmission of a group of two successive QAM symbols $s(1), S(2)$.*
Express the received signal vector (over time and space dimension) in terms of the two input symbols $s(1)$ $s(2)$ and the channel vectors $\mathbf{h}_1$ and $\mathbf{h}_2$ and noise.
Give the equation for estimating the symbols. What is the diversity order? What is the beamforming gain in dB?

4.3 MIMO spatial multiplexing

We want to use the multiple antennas for increasing the rate via spatial multiplexing. We assume $M = N = 2$ and a fixed realization of the channel $\mathbf{H}$.

Exercise 4 *We now assume the the channel is known at the transmitter (and at the receiver). Express the general form of the optimal transmit and receive strategy over this MIMO channel to maximize the rate, under the constraint that the total transmit power is σ_s^2. 2×1 received signal vector $\mathbf{y}$ in terms of two input uncoded symbols $s(1)$, $s(2)$ and the channel matrix and noise.*
Please explain your response by giving the transmit signal structure and expression for the received signal, and processing at the receiver. Please explicit the optimal power allocation strategy.

Exercise 5 *The channel is given by*

$$\mathbf{H} = \begin{pmatrix} 3 & 1.5 \\ -2 & 1 \end{pmatrix} \tag{2}$$

What is for this channel the optimal transmit strategy and optimal power allocation? What will be the SNR after processing at the receiver?

Exercise 6 *The channel is now given by*

$$\mathbf{H} = \begin{pmatrix} 3 & 3 \\ -2 & 1 \end{pmatrix} \tag{3}$$

Now we assume that the channel matrix is NOT known at the transmitter. In this case, give the equations for spatial multiplexing using V-BLAST receiver and signal ordering. Give expressions for decoding both symbols.

4.4 Spatial Division Multiple Access (SDMA) in the uplink

We now consider a multiple access protocol where multiple terminals are allowed to access the channel and transmit to the base station simultaneously, **on the uplink channel**. We select K mobile terminals and let them transmit to the base. Each mobile has $N = 1$ antennas. the base still has M antennas.
Each terminal k transmits a symbols $s_k(n)$ at time n. The signal model for the (noisy) received signal vector at time n at the base station, denoted $y(n)$, is given by

$$y(n) = \mathbf{H}\mathbf{s}(n) + \mathbf{n}(n) \tag{4}$$

Where the channel over the K terminals is denoted by $\mathbf{H}$ and is therefore of size $M \times K$, and $\mathbf{s}(n)$ is the vector of symbols for all the users, of size $K \times 1$.

Exercise 7 *We assume linear zero-forcing beamforming at the base station receiver. write the equation for the estimated symbols after receiver processing. What is the SNR for each symbol? How many mobiles can this system support?*
Propose a more optimal non-linear receiver design to estimate all the user symbols instead of the ZF. Give the main equation(s).

Exercise 8 *We now assume that each user has two transmit antennas and knows the transmit channel coefficients. Assume that the users only to send one stream each. Propose a ways of making use of the two transmit antennas, exploiting the knowledge of the channel. Give the equation to obtain the optimal transmit beamformer at each user.*

4.5 Scheduling

We now consider the uplink situation with $M = 1$ at the base station and N is arbitrary, and K users in the network. We assume that each user knows the transmit channel to the base station. The base station also knows the channels.

Exercise 9 *How many users can be scheduled simultaneously without interference? What is the optimal way of combining the antennas at the user side?*
Give the optimal scheduling rule to maximize the system throughput. Provide the key equation(s).

Exercise 10 *What is the diversity order, after user selection, for the system above? Justify.*

THAT'S ALL FOLKS!

Antenna Configuration	CSIR	CSIT	Multiplexing order?	Diversity order?	Beamforming gain (dB)?
4x1	Yes	No			
2x1	No	Yes			
2x4	Yes	Yes			
3x3	Yes	No			
2x3	Yes	Yes			
4x2	Yes	No			
Uplink 2 users 2x6	Yes	No		IGNORE	IGNORE
Uplink 3 users 1x4	Yes	Yes		IGNORE	IGNORE
Downlink 5 users 1x2	Yes	No		IGNORE	IGNORE
Downlink 3 users 2x1	Yes	Yes		IGNORE	IGNORE
Downlink 2 users 2x2	Yes	No		IGNORE	IGNORE
Downlink 3 users 5x1	Yes	Yes		IGNORE	IGNORE
Downlink 2 users 3x2	Yes	No		IGNORE	IGNORE

Figure 1: Table for exercise 2