Final Exam for ATW Fall-2022

Petros Elia, elia@eurecom.fr

February 7th, 2023

Instructions

- Exercises fall in categories of 1-point, 3-point, 4-point, and 5-point exercises.
- Total of $(14 \times 1 + 3 \times 3 + 1 \times 4) +$ one extra credit exercise for 5 points = total $= 27 + 5 = 32$ points (where naturally the 100% grade corresponds to 27 points).
- The last exercise is for extra credit.
- Open book and open class notes are allowed (including notes taken by students during exam). No other notes are allowed.
- Each answer should be clearly written, and the solution should be developed in detail.
- Mathematical derivations need to show all steps that lead to the answer.
- Partial credit will be given for incomplete solutions.
- There is NO penalty for incorrect solutions.

Hints - equations - conventions:

- Notation
  - R represents the rate of communication in bits per channel use (bpcu),
  - ρ represents the SNR (signal to noise ratio),
  - w will denote additive noise which will be distributed as a circularly symmetric Gaussian random variable $\mathbb{C}\mathcal{N}(0, N_0)$. If N_0 is not specified, then set $N_0 = 1$,
  - Remember: for a given signal-to-noise ratio (SNR), then SNR in dB is simply $10 \log_{10} SNR$
  - SISO stands for single-input single-output, MISO stands for multiple-input single output, SIMO stands for single-input multiple output, MIMO stands for multiple input multiple output.
  - CSIT stands for channel state information at the transmitter, while CSIR stands for channel state information at the receiver.
  - AWGN stands for additive white Gaussian noise.
- GOOD LUCK!!

EXAM PROBLEMS

1) *(1 point).* Consider a SISO setting, with no fading. Consider that the maximum possible rate (i.e., the capacity) is equal to 3 bpcu. What is the minimum SNR required to achieve this rate?

2) *(1 point).* Consider a SISO fading channel with no CSIT. Imagine that we are not happy with a probability of error $P_{err} \approx (SNR)^{-1}$. Suggest various ways we can decrease this down to $P_{err} \approx (SNR)^{-4}$.

3) *(1 point).* What are some of the advantages of MISO vs. SIMO?

4) *(1 point).* In multi-user MISO settings, describe some of the advantages and some of the disadvantages of matched filtering vs. zero forcing.

5) *(1 point).* Provide a real-life scenario that entails an AWGN channel and one that entails fast fading.

6) *(1 point).* What is the main reason for using OFDM? (1 line)

7) *(1 point).* What is the complication that prevents interference alignment from being widely used in practice? (1 line)

8) *(1 point).* What does channel hardening mean in Massive MIMO? (1-2 lines)

9) *(1 point).* What does favorable propagation mean in Massive MIMO and what does it imply for receiver and transmitter design? (2-3 lines)

10) *(1 point).* How would Cell-Free Massive MIMO allow us to reduce energy consumption? (1 line)

11) *(1 point).* In a single-user MIMO channel, how much diversity gain would we be able to get if we employed a transmitter with 4 transmit antennas and a receiver with 2 receive antennas?

12) *(1 point).* In a single-user MISO channel, how much multiplexing gain would we be able to get if we employed a transmitter with 2 transmit antennas?

13) *(1 points).* Consider a SISO fading channel with coherence time equal to $T_c = 10$ milli-seconds. Imagine that our boss imposes a delay constraint of 50ms.
    - What type of diversity do we expect to get here: time diversity or spatial diversity?

- What is the maximal such diversity?

14) *(1 points).* Consider communication over a quasi-static 2×1 MISO fading channel. Assume that you must draw symbols from 16-QAM.
    - What is the diversity and the rate (in bpcu) given by the Alamouti code? Justify your answer.

15) *(3 points).* In the context of various strategies, answer if each of the following statements are true or false, justifying briefly your answers.
    - In a MISO channel, we can get transmitter beamforming gain even without CSIT.
    - A base station equipped with 5 antennas in the downlink, can simultaneously serve up to 5 users (single receive antenna each).
    - A base station equipped with 5 antennas in the downlink, can simultaneously serve up to 10 users (two receive antennas each).
    - A base station equipped with 4 antennas in the downlink, can simultaneously serve up to 2 users (two receive antennas each).
    - Line of sight channels are detrimental for spatial multiplexing in both single-user and multiuser MIMO.
    - For a MIMO receiver using spatial multiplexing, the complexity of ZF receiver is more than the complexity of the maximum-likelihood receiver.
    - CSIT is easier to obtain than CSIR.
    - CSIT is of cardinal importance in multi-user MIMO.

16) *(3 points).*
    - What are some reasons for transporting OFDM symbols in the frequency-domain in the O-RAN 7.2 fronthaul-protocol? (1-2 lines)
    - What is the purpose of the "control-plane" in a fronthaul protocol? (1 line)
    - Why should radio units be time and frequency synchronized? (1 line)
    - What are the main differences between CPRI and ECPRI?

17) *(3 points).* In the context of various strategies, answer the following with brief justification of your answers.
    - How the capacity of a SU-MIMO channel is achieved when the channel is known at both the transmitter and the receiver?
    - In a MU-MIMO channel, if I double the number of users I simultaneously serve, must I always half the individual rate to each user? Justify your answer.

18) *(4 points).* As in the matlab session, consider communication over the 3×1 quasi-static fading MISO channel, using a diagonal code (see below for details) such that the channel model is given by

$$\overbrace{\begin{pmatrix} y_1 & y_2 \end{pmatrix}}^{y} = \theta \overbrace{\begin{pmatrix} h_1 & h_2 \end{pmatrix}}^{h} \overbrace{\begin{pmatrix} x_1 & 0 \\ 0 & x_2 \end{pmatrix}}^{\mathcal{X}_{tr}} + \overbrace{\begin{pmatrix} w_1 & w_2 \end{pmatrix}}^{w}$$

where $h_i \sim \mathbb{C}N(0,1)$ and $w_i \sim \mathbb{C}N(0,1)$, and where θ is the power normalization factor that lets you regulate SNR.
    - Describe the ML decoding rule.

- Describe the cardinality[1] of code $\mathcal{X}_{tr}$ if you wish a rate of $R = 4$ bpcu.
- For a desired rate of $R = 8$ bpcu, and a desired $SNR = 10$ dB (where by SNR we mean the AVERAGE signal power divided by the noise unit power, under QAM) then what is the normalizing factor θ?
- Imagine that what you transmit (x_1, x_2) are independently chosen from 16-PAM, then
  - What is the rate of your code (in bpcu)?
  - What is the slope of your probability of error, in high SNR, if you plot on the y-axis the probability of error, in log scale ($\log_{10}$(Prob)), and the x-axis is the SNR, in dB?
- Imagine now that $\begin{pmatrix} x_1 & x_2 \end{pmatrix} = \begin{pmatrix} s_1 & s_2 \end{pmatrix} \cdot \mathbf{Q}$, where s_1, s_2 are independently chosen from a 64-QAM constellation, where the matrix $\mathbf{Q}$ is a randomly chosen orthogonal matrix. Then
  - What is the rate of your code?
  - What is the aforementioned slope of your probability of error?

19) *(Extra Credit: 5 points)*. Consider communication over a quasi-static 2×2 MIMO channel, utilizing the space-time code $\mathcal{X} = \{\mathbf{X}_1, \mathbf{X}_2, \mathbf{X}_3, \mathbf{X}_4\}$, where

$$\mathbf{X}_1 = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}, \ \mathbf{X}_2 = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}, \mathbf{X}_3 = \begin{bmatrix} -1 & -1 \\ 1 & -1 \end{bmatrix} \text{ and } \mathbf{X}_4 = \begin{bmatrix} -1 & 1 \\ -1 & -1 \end{bmatrix}$$

- What is the average SNR?
- What is the rate of the code in bpcu?
- What is the diversity gain of this code?
- What is the approximate (in the high SNR regime) probability of error of this code, if SNR is 30dB?

[1] By cardinality we mean the number of matrices that the code has.