Advanced Topics in Wireless: Exam Questions

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1 Read this

Duration of the exam is 2 hours, all documents are permitted. Don't forget to justify where necessary (concisely - a few words is enough!) your answer (and prove mathematically where possible). Use your own words and do not copy sentences from a book/notes.
Also: read carefully the questions!

2 General Context

We consider a cellular network, where, in each cell, a base station (BTS) communicates with a number of mobile terminals (MT) either in uplink or in downlink. Each BTS is equipped with M antennas, and each MT is equipped with N antennas.
The exam has two parts. The first is on qualitative nature and the second is algorithmic and quantitative.

3 Qualitative questions

Exercise 1 *We consider the use of various MIMO strategies for improving the system performance, as shown in class. Please answer WRONG or CORRECT to each of these statements. When possible, justify the answer with a couple of words (be very concise).*

- *To get beamforming gain from multiple transmitters, the transmitters dont need to know the channel coefficients.*
- *The diversity gain with 3 transmitters, 1 receiver, at 10e-4 bit error rate, with QPSK modulation, is less than 25dB.*
- *The diversity gain going from 2 to 3 receive antennas is less than going from 3 to 4 receive antennas.*
- *The multiplexing gain in a single user MIMO channel decreases with the line of sight component.*

- *The beamforming gain in MIMO in very sensitive to the presence of a line of sight component.*

- *The beamforming gain in a system with M transmit antennas and 1 receive antenna is the same as the beamforming gain with 1 transmit and M receive antennas (assuming channel knowledge at the transmitter and the receiver).*

- *In a multiuser MIMO channel with a base station equipped with M antennas, the base communicates with mobiles, each equipped with $N = 1$ antennas. Then the base can only serve $M - 1$ mobiles at the same time without interference.*

- *In a multiuser MIMO channel with a base station equipped with M antennas, the base communicates with mobiles, each equipped with 2 antennas. Then the base can serve $2M$ mobiles at the same time without interference.*

- *In a network with two interfering cells, each cell having a base station with M antennas. Assume uplink and that the two base stations cannot coordinate themselves (no signaling between the cells), then the number of mobiles with 1 antenna each that the network can serve in total without interference is up to 2M.*

- *In a network with two interfering cells, each cell having a base station with M antennas. Assume uplink and that the base stations can coordinate themselves (unlimited signaling between the cells), then the number of mobiles with 1 antenna each that the network can serve in total without interference is up to 4M.*

4 Quantitative questions

4.1 Notations for signals and channels

We consider the baseband representation of a signal being transmitted from the BTS to a single MT. The transmitted symbol sequence, denoted $s(l)$ is drawn from a complex constellation with variance: $E|s(l)|^2 = \sigma_s^2$, also representing the **total transmit power**.
The symbols are sent into a frequency-non-selective (no ISI) MIMO channel represented by the $N \times M$ matrix: $\mathbf{H}$.
The link between the j-th BTS antenna and the i-th MT antenna is denoted by the complex gain $\mathbf{H}_{ij} = h_{ij}$.
The matrix is normalized so that all links have unit variance $E|h_{ij}|^2 = 1$.
We assume the channels are all uncorrelated (between users, between antennas).
The noise on each receive antenna is assumed to be white (uncorrelated across antennas and time) and of variance σ_n^2

4.2 Single user beamforming on the downlink with and without channel knowledge

We consider $M > 1$ and $N = 1$.

Exercise 2 *First we try to beamform from the transmitter without channel knowledge. On each transmit antenna we transmit the same symbol, properly weighted such that the total transmit power from the BTS is σ_s^2. Write the receive signal equation. Compute the SNR at the receiver side. Analyze the diversity and beamforming gain of this technique.*

Exercise 3 *We want again to use beamforming at the transmit side. But this time we assume the channel is known at the transmitter and at the receiver. Write the optimal beamforming solution (still under the same power constraint as above). Compute the SNR at the receiver, the diversity order and the beamforming gain.*

4.3 MIMO spatial multiplexing

We want to use the multiple antennas for increasing the rate via spatial multiplexing. We assume $M = N = 2$ and a fixed realization of the channel $\mathbf{H}$.
The channel is given by

$$\mathbf{H} = \begin{pmatrix} 2 & 3 \\ 5 & 6 \end{pmatrix} \tag{1}$$

Exercise 4 *We assume that the channel matrix is NOT known at the transmitter (only known at the receiver). In this case, give the equations for spatial multiplexing, still under the same total transmit power constraint as previously. Write decoding equations using successive interference canceler (i.e. SIC or "V-BLAST") receiver and signal ordering. Give the SNR obtained when decoding each symbol as a function of σ_n^2 and σ_s^2.*

4.4 Downlink Spatial Division Multiple Access (SDMA) for a single cell

We now consider a spatial multiple access protocol where multiple terminals are allowed to receive symbols from the base station simultaneously, **on the downlink channel**. We consider first a single cell (i.e. only one base station). We select K mobile terminals with single antenna ($N = 1$) and let them receive from the base. Each mobile k has $N = 1$ antennas and wants to receive symbol s_k from the base. The base still has $M > 1$ antennas. The $1 \times M$ channel vector to terminal k from the base is denoted by $\mathbf{h}_k^T$.

Exercise 5 *Assume the BS knows all the channels. Assume a linear precoder matrix W of size $M \times K$ is used. Write the signal model for the (noisy) received signal vector at the k-th mobile, denoted $\mathbf{y}_k$.*

Exercise 6 *We assume linear zero-forcing beamforming at the base station. Write the expression for the precoder matrix given that we want to maintain a total transmit power of σ_s^2 at the BS. Write the SNR are the mobile side. How many mobiles K can this system support if one wanted to completely cancel the interference between terminals?*

4.5 Downlink Spatial Division Multiple Access (SDMA) for two cells

We now consider a two cell system. In cell 1, one base station with $M = 2$ antennas wants to communicate with 2 mobiles located in that cell. Next to cell 1, another cell (cell 2) is placed with another base station with $M = 2$ antennas, trying to communicate with his own set of 2 mobiles. All mobile terminals want to receive one symbol each, on the downlink. Assume the same frequency is assigned in both cells so that they interfere with each other.

Exercise 7 *Assume the two cells cannot coordinate. Assume the BS in cell 1 uses downlink precoder $\mathbf{W}_1$, and BS in cell 2 uses downlink precoder $\mathbf{W}_2$. Write the received signal equation at both terminals of cell 1. Is it possible to design $\mathbf{W}_1$ and $\mathbf{W}_2$ to cancel out all interference?*

Exercise 8 *Assume the two BS are linked by a optical fiber link of near infinite capavity over which they can exchange any data. Write the received signal equation at both terminals of cell 1. Is it possible to design $\mathbf{W}_1$ and $\mathbf{W}_2$ to cancel out all interference? Write a possible solution for $\mathbf{W}_1$ and $\mathbf{W}_2$, as function of the channel vectors between all pairs of users and base stations.*

THAT'S ALL FOLKS!