Advanced Topics in Wireless: Exam Questions

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1 Read this

Duration of the exam is 2 hours, all documents are permitted. Don't forget to justify where necessary (concisely - a few words is enough!) your answer (and prove mathematically where possible). Use your own words and do not copy sentences from a book/notes.
Also: read carefully the questions!

2 General Context

We consider a cellular network, where, in each cell, a base station (BTS) communicates with a number of mobile terminals (MT) either in uplink or in downlink. The BTS is equipped with M antennas, and each MT is equipped with N antennas. The exam has two parts. The first is on qualitative nature and the second is algorithmic and quantitative.

3 Qualitative questions

Exercise 1 *We consider the use of various MIMO strategies for improving the system performance, as shown in class. Please answer WRONG or CORRECT to each of these statements. When possible, justify the answer with a couple of words (be very concise).*

- *To get diversity gain from multiple transmitters, the transmitters need to know the channel coefficients.*
- *To get beamforming gain from multiple transmitters, the transmitters dont need to know the channel coefficients.*
- *The diversity gain with 3 transmitters, 1 receiver, at 10e-4 bit error rate, with QPSK modulation, is less than 25dB.*
- *The diversity gain difference from 1 to two antennas is less than from 2 to 3 antennas.*

- *The multiplexing gain in a single user MIMO channel decreases with the line of sight component.*
- *The beamforming gain in MIMO in not very sensitive to the presence of a line of sight component.*
- *The beamforming gain in a system with M transmit antennas and 1 receive antenna is the same as the beamforming gain with 1 transmit and M receive antennas (assuming channel knowledge at the transmitter and the receiver).*
- *In a multiuser MIMO channel with a base station equipped with M antennas, the base communicates with mobiles, each equipped with $N = 1$ antennas. Then the base can serve $M+1$ mobiles at the same time without interference.*
- *In a multiuser MIMO channel with a base station equipped with M antennas, the base communicates with mobiles, each equipped with 2 antennas. Then the base can serve $2M$ mobiles at the same time without interference.*

4 Quantitative questions

4.1 Notations for signals and channels

We consider the baseband representation of a signal being transmitted from the BTS to a single MT. The transmitted symbol sequence, denoted $s(l)$ is drawn from a complex constellation with variance: $E|s(l)|^2 = \sigma_s^2$, representing the transmit power.
The symbols are sent into a frequency-non-selective (no ISI) MIMO channel represented by the $N \times M$ matrix: $\mathbf{H}$.
The link between the j-th BTS antenna and the i-th MT antenna is denoted by the complex gain $\mathbf{H}_{ij} = h_{ij}$.
The matrix is normalized so that all links have unit variance $E|h_{ij}|^2 = 1$.
We assume the channel are all uncorrelated (between users, between antennas).

4.2 Beamforming on the downlink

We assume a single user setup whe the BTS communicates with one MT only.
The modulation symbols are sent or converted into coded symbols before launching on the transmit antennas. The possibly coded symbol (intended to the MT) on the j-th transmit antenna at the BTS, at time n, is denoted $c_j(n)$.
The received signal at the MT at time n is obtained by:

$$\mathbf{y}(n) = \mathbf{H} \begin{pmatrix} c_1(n) \\ c_2(n) \\ \vdots \\ c_M(n) \end{pmatrix} \frac{1}{\sqrt{M}} + \mathbf{b}(n) \tag{1}$$

where $\mathbf{b}(n)$ is an additive noise vector caused by receiver electronics and interference. We assume the noise to be spatially and temporally white, with each component having power $E|b_i(n)|^2 = \sigma_b^2$, $i = 1..N$. The factor $\sqrt{M}$ is used at the transmitter to limit and normalize the power independently of the number of antennas.

Exercise 2 *We assume M is arbitrary and $N = 1$. We assume the channel is not known at the transmitter and known at the receiver. The transmitter simply sends $c_j(n) = s(n)$ (i.e. the same symbol is repeated over all transmit antennnas). Compute the SNR at the receiver side. Analyze the diversity and beamforming gain of this technique.*

Exercise 3 *Assuming (just for this question) that the channel is pure line of sight. The channel vector is now given by*

$$\mathbf{H} = \left[1, e^{-2\pi j(d/\lambda)\sin\theta}, e^{-2\pi j(2d/\lambda)\sin\theta}, .., e^{-2\pi j((M-1)d/\lambda)\sin\theta}\right] \tag{2}$$

(steering vector), where λ is the wavelength and d is the inter-antenna spacing, and θ is the direction of the mobile. The channel vector is not known a priori. We want to use beamforming as a way to localize the mobile (i.e. estimate θ). Propose a method to localize the mobile by exploiting a combination of beamforming and feedback of SNR levels by the mobile to the base.

Exercise 4 *We want again to use beamforming at the transmit side, assume M is arbitrary and $N = 1$. But this time we assume the channel is known at the transmitter and at the receiver. Give the form taken by the $c_j(n)$ is optimal beamforming is used, and assuming the total transmit power is kept to σ_s^2. Give the SNR at the receiver, the diversity gain and the beamforming gain.*

4.3 MIMO spatial multiplexing

We want to use the multiple antennas for increasing the rate via spatial multiplexing. We assume $M = N = 2$ and a fixed realization of the channel $\mathbf{H}$.
The channel is given by

$$\mathbf{H} = \begin{pmatrix} 2 & 3 \\ 5 & 2 \end{pmatrix} \tag{3}$$

Exercise 5 *We assume that the channel matrix is NOT known at the transmitter (only known at the receiver). In this case, give the equations for spatial multiplexing using V-BLAST receiver and signal ordering. Give the SNR obtained when decoding each symbol.*

4.4 Spatial Division Multiple Access (SDMA) in the uplink

We now consider a multiple access protocol where multiple terminals are allowed to access the channel and transmit to the base station simultaneously, **on the uplink channel**. *We select K mobile terminals and let them transmit to the base. Each mobile has $N = 1$ antennas. The base still has M antennas.*
Each terminal k transmits one symbol $s_k(n)$ at time n.

Exercise ***6*** *Write the signal model for the (noisy) received signal vector at time n at the base station, denoted $y(n)$.*

Exercise ***7*** *We assume linear MMSE beamforming at the base station receiver. write the equation for the estimated symbols after receiver processing. How many mobiles can this system support if one wanted to completely cancel the interference between symbols (i.e using zero-forcing)?*

Exercise ***8*** *Assuming a maximum-likelihood receiver at the base station, give an estimate for the complexity of the receiver assuming 16QAM modulation is used at all users.*

Exercise ***9*** *Compare the above system with another one with only one user but equipped with K antennas instead of K users with one antenna each. Compare in terms of spectrum efficiency and robustness. Are the two system equivalent?*

Exercise ***10*** *We now assume that each user has two transmit antennas ($N = 2$) and knows the transmit channel coefficients. How many users can be supported while still canceling out the interference at the base station?*

Exercise ***11*** *We still assume that each user has two transmit antennas and knows their transmit channel coefficients. Assume that the users only to send one stream each. Propose a simple way of making use of the two transmit antennas, exploiting the knowledge of the channel.*

4.5 Scheduling on the downlink

We still consider the downlink situation but this time with $N = 1$ and M is arbitrary, and K users in the network. We assume the base station knows the channels.

Exercise ***12*** *Assuming TDMA scheduling (i.e. one user is selected at a time), what is the optimal scheduling rule for maximizing the data rate (give equation as function of channels). What is the diversity obtained after taking account of the user selection gain?*

Exercise ***13*** *Now assume we do scheduling of groups of users (SDMA). We limit the group size to two users. At any time slot, a group of two users is selected and transmit beamforming is applied at the base station towards the two users such that there is no interference at the user side. Give the optimal scheduling rule for maximizing the data rate (give equation as function of channels)*

THAT’S ALL FOLKS!