Advanced Topics in Wireless

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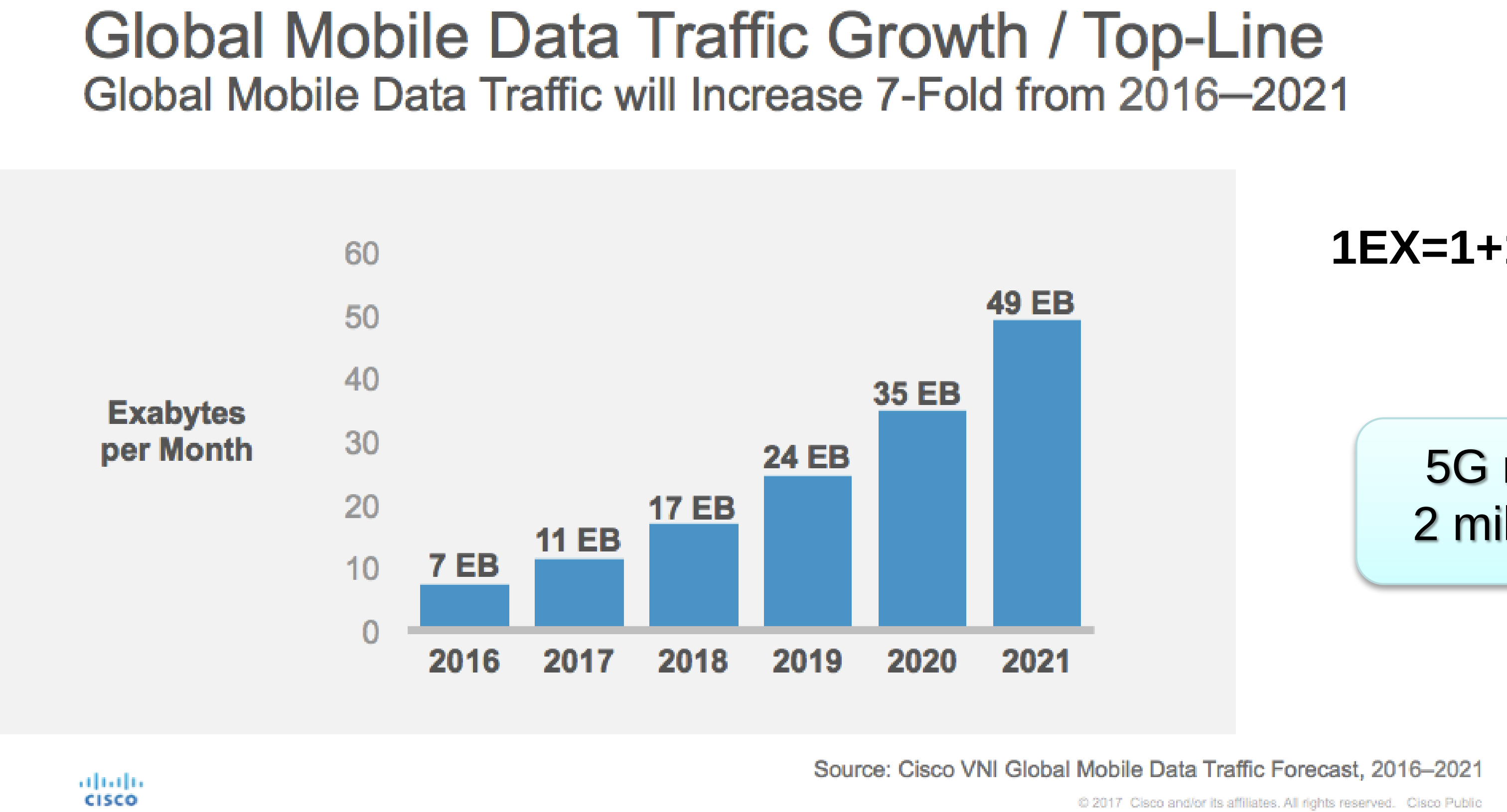
Heinrich Hertz (1857-1894)

"I do not think that the wireless waves I have discovered will have any practical application." **(H.R.Hertz)**

Mobile data storm



1EX=1+18 zeroes

5G node density:
2 millions per km2

-

A technology perspective from 1G to 6G

1980	1992	2003	2011	2020	?
10kb/s	100kb/s	10Mb/s	100 Mb/s	1Gb/s	?
1G	2G	3G	4G	5G	6G

Analog voice

Low speed data

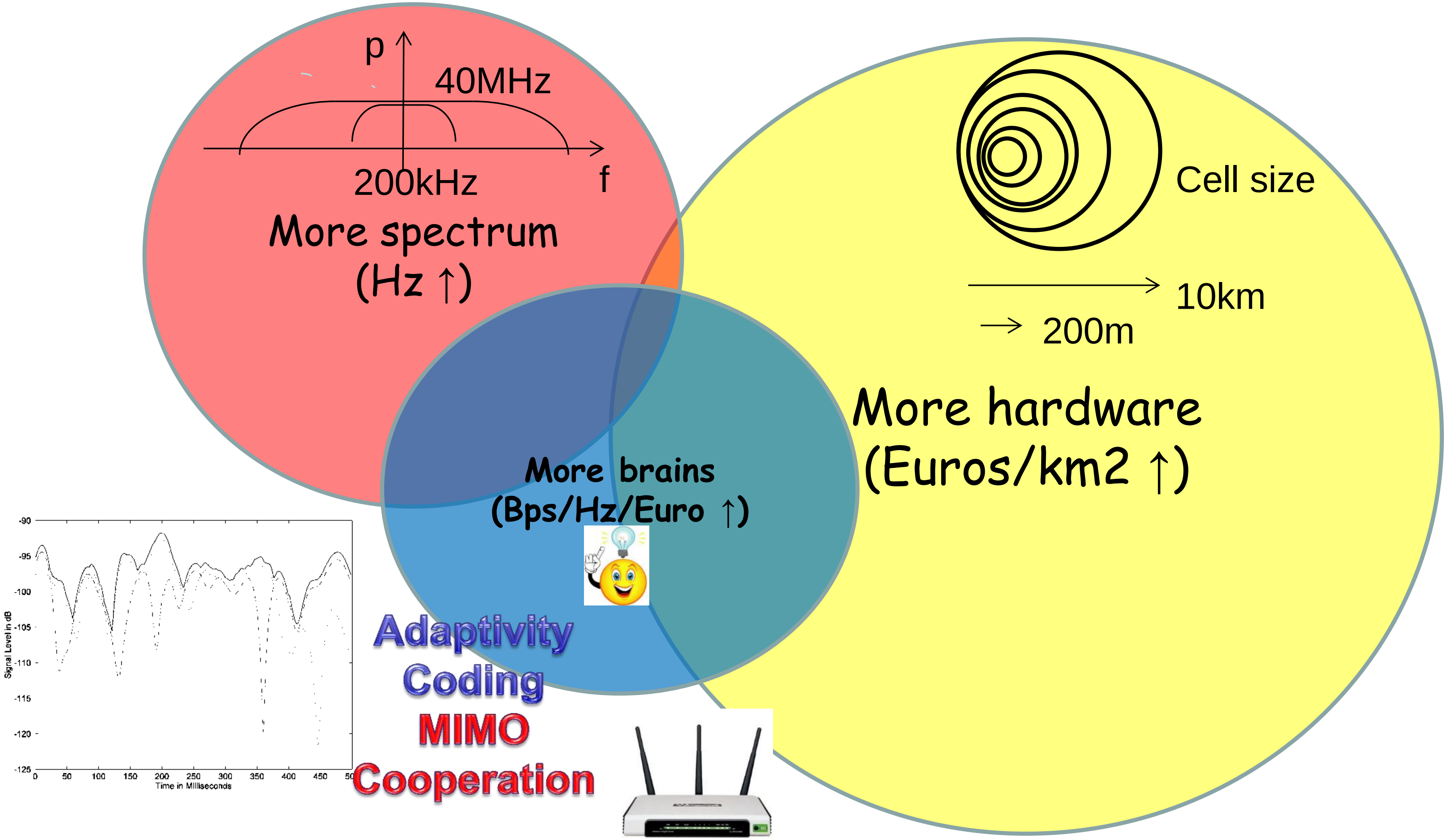
Digital voice
Digital signal processing

High speed data
OFDM
Smart Antennas

Mission critical services
New 5G enabling technologies

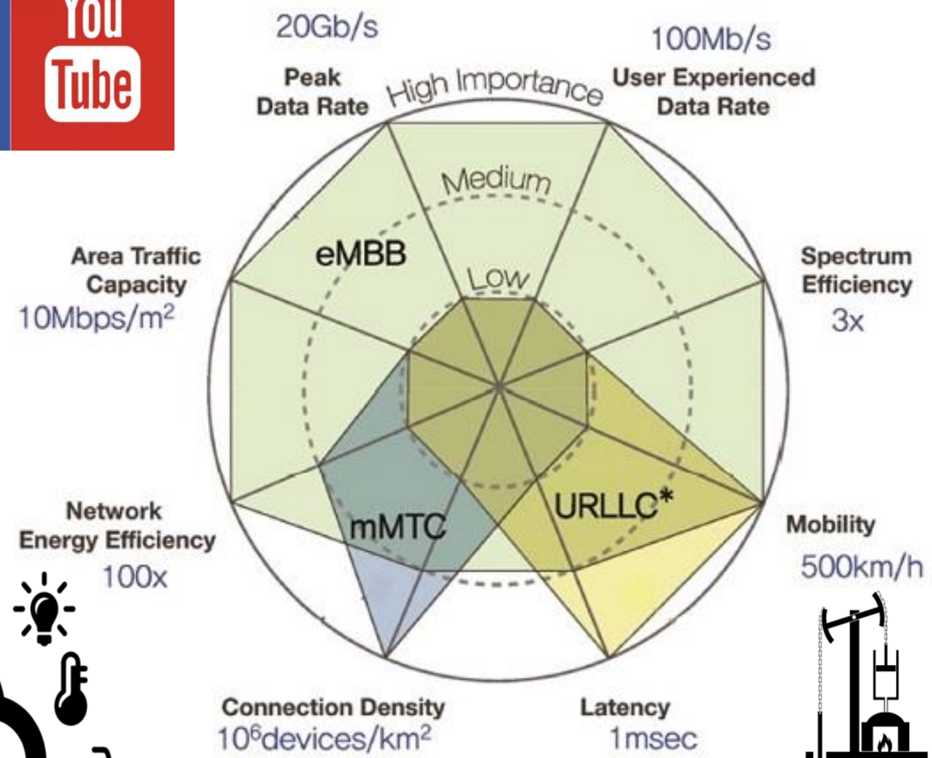
Holography, VR
Artificial intelligence
self-organized network
Open-source for radio
Tera-Herz

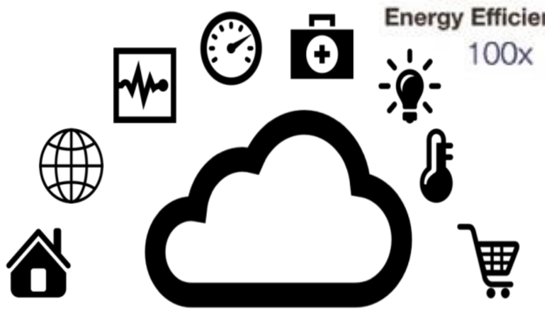
How did we gain a performance factor X100 000 ?

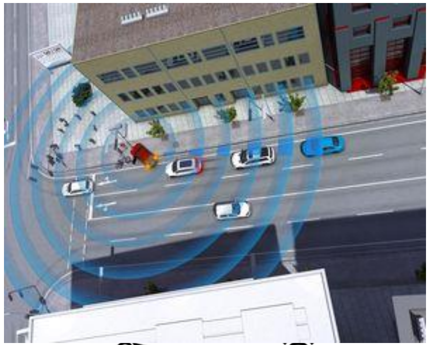


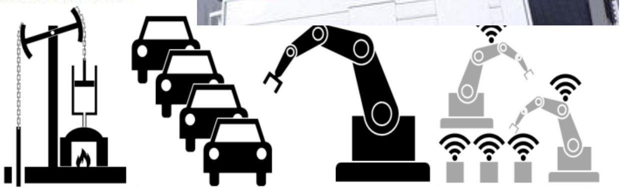
New services and performance requirements

Network Slicing!



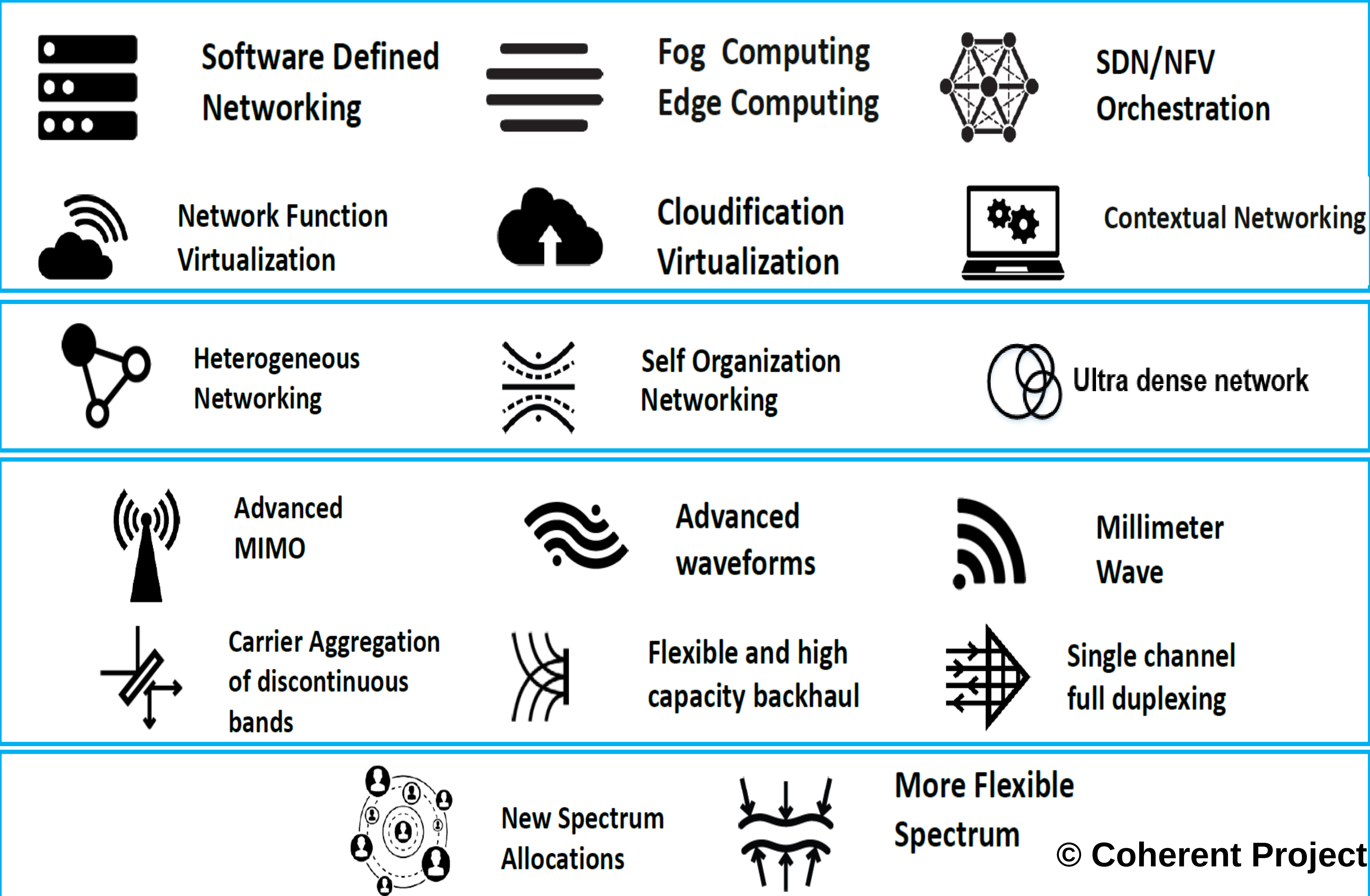






Lesson1: Make network ultra-flexible!
Lesson 2: Bring network closer to user!

Source: 5GPPP Metis II white paper 5G RAN Architecture and Functional Design

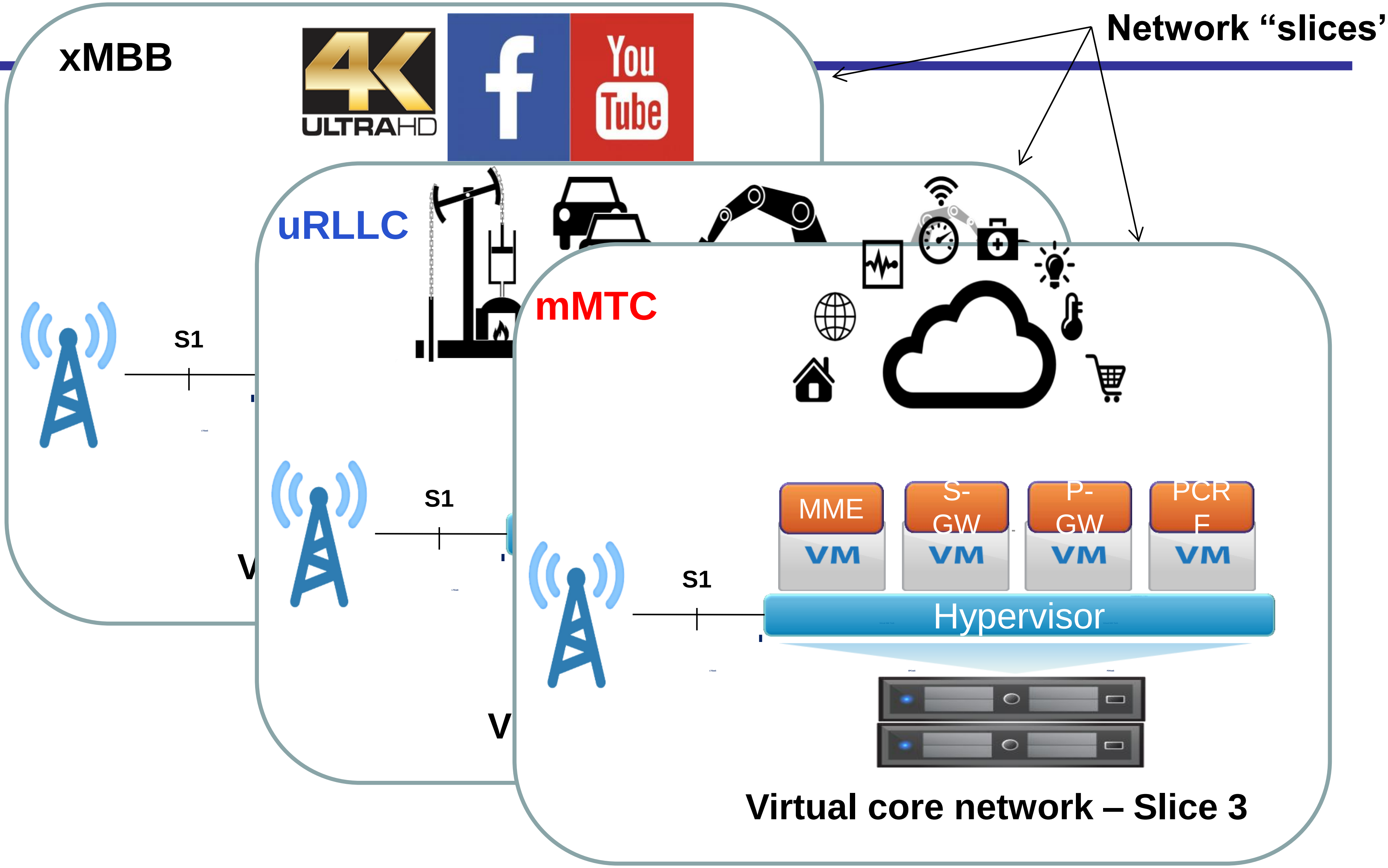


5G Technology Enablers

Some hot wireless design topics

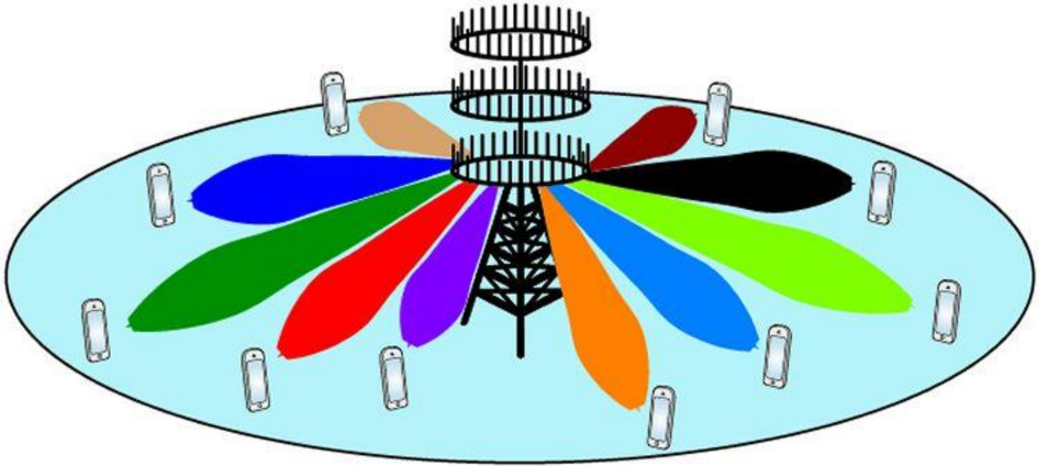
- Network softwarization and slicing
- Millimeter-wave communications
- X-MIMO (X=multiuser, network, massive, metasurface, etc.)
- New Radio interface for URLLC (ultra-reliable low latency)
- Open source (not a technology but an unstoppable trend)

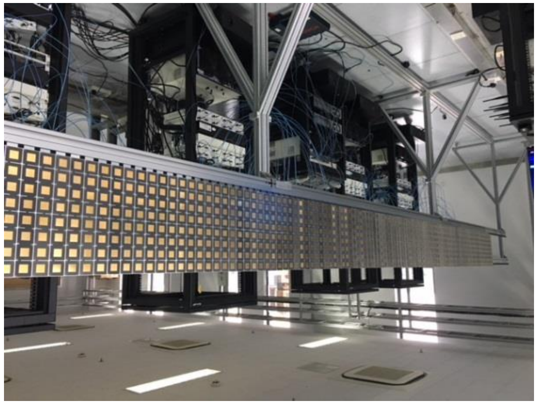
Network slicing:
Each service gets a virtual dedicated mobile network



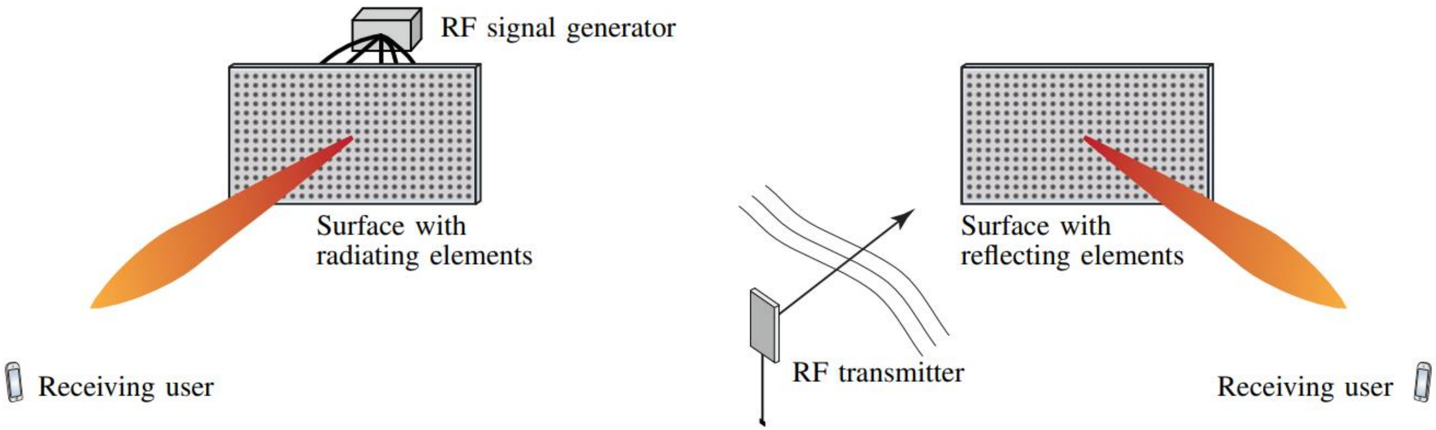
Massive MIMO for 5G (multiple-input multiple-output)

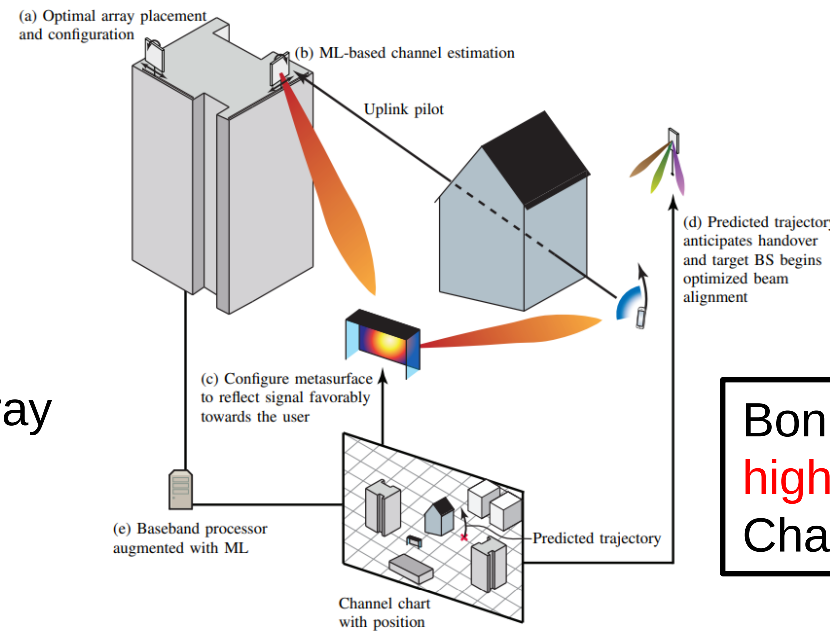
- Multi-element antenna panels (64,128 elements)
- Highly directive radio transmission
- Reduced interference
- Reduced transmission power
- Spatial multiplexing





Next MIMO revolution (for 6G): Ultra directivity via smart surfaces



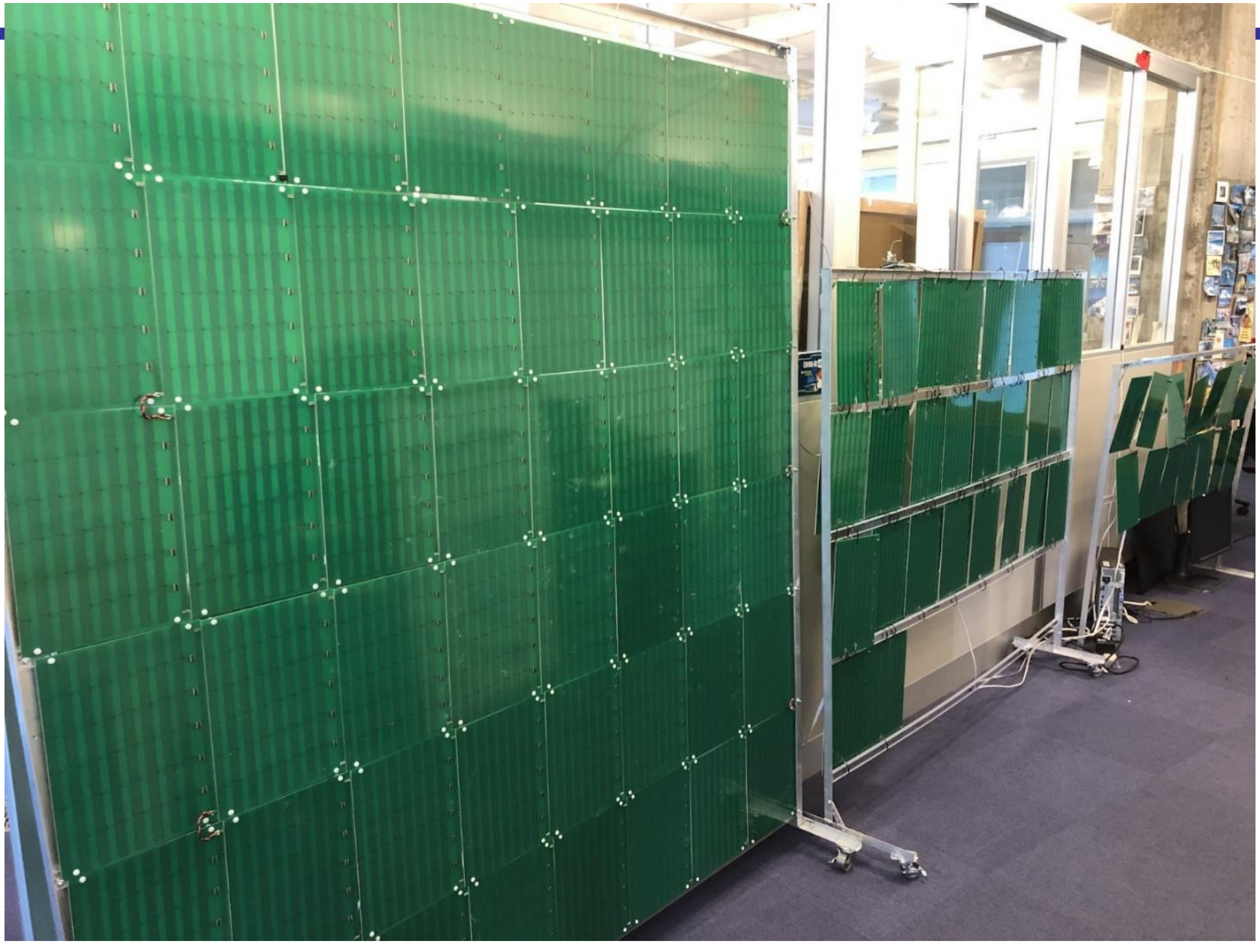


1 Full digital array
2 Phase shifter array
3 Switched reflector array
4 Meta material array

Bonus +++:
high-resolution positioning!
Challenge: (AI-driven) control

[Bjornson et al, “massive MIMO is a reality, what is next?”, 2019]

Ultra directivity via smart surfaces



6M2 surface
3.7K elements
Switched reflectors
2.5GHz
10-20x SNR gain (indoor)

[Arun et al (MIT), May 2019]

ATW Course Structure

- 50% academic lectures
- 50% experts from Industry
- In-class exercises
- 1 lab session (MATLAB based)
- 1 Interactive session (paper presentations)
- 2hr Written exam (on academic part only)

Expert Talks from Industry (attendance mandatory)

- 22 oct: Mr. Alain Sultan (ETSI)
- 5 nov: Dr. Faycal Ait Aoudia (NOKIA Bell Labs)
- 19 nov: Dr. Stefania Sesia (RENAULT)
- 3 dec: Dr. Maxime Guillaud (HUAWEI)
- 17 dec: Dr. Christophe Le Martret (THALES)
- 7 janvier 2020: Dr. Stefania Sesia (RENAULT Software Labs)

Pre-requisites/advisable

- Digital Communications
- Statistical Signal Processing
- Probabilities and Stochastic Processes
- Networking

If you are unsure please meet me after class!

Advanced Topics in Wireless

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October 4, 2019

Course Outline

- General introduction to the course
- Challenges of wireless network design
- Introduction to MIMO networks and network-MIMO
- The degrees of freedom of wireless networks

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General introduction to the course

- General information
- Supporting material for the course
- Some useful definitions
- Some useful acronyms

General information

This course aims at giving understanding in

- Basic MIMO theory
- single user and multiuser communication techniques using MIMO
- Cooperation for interference management
- Basic understanding in some emerging topics for 5G

Pre-requisites/advisable

- Digital Communications and basic information theory
- Statistical Signal Processing
- Basic wireless communication design and modeling

Quick math backgrounder

- Notations
- Orthogonality, special matrices
- Matrix rank
- Eigenvalue decomposition
- Singular value decomposition

Mathematical notations

- u: scalar
- $\mathbf{u}$: vector
- $\mathbf{U}$: Matrix
- $\mathbf{U}^T$: Transpose operator
- $\mathbf{U}^*$: Complex-conjugate operator
- $\mathbf{U}^H$: Transpose conjugate operator (i.e. $\mathbf{U}^H = \mathbf{U}^{*T}$
- $\mathbf{U}^{\#}$: Pseudo-inverse operator
- $E()$: Expectation operator
- $\mathbf{I}_N$: identity matrix of size $N \times N$
- $(x)^+$: x if x is positive, zero otherwise.

Orthogonality

Vector Orthogonality: Let $\mathbf{u} = [u_1, .., u_N]$ and $\mathbf{v} = [v_1, .., v_N]$ be complex vectors of size N. $\mathbf{u}$ and $\mathbf{v}$ are *orthogonal* iff:

$$\mathbf{u}^H \mathbf{v} = \sum_{i=1}^{N} u_i^* v_i = 0$$

Special matrices

Hermitian: Matrix $\mathbf{U}$ is *hermitian* iff

$$\mathbf{U} = \mathbf{U}^H$$

Unitary: Matrix $\mathbf{U} = [\mathbf{u}_1, \mathbf{u}_2, .., \mathbf{u}_N]$ is *unitary* (or "orthogonal") iff

$$\mathbf{U}^H \mathbf{U} = \mathbf{I}_N$$

which means that vectors $\mathbf{u}_1, \mathbf{u}_2, .., \mathbf{u}_N$ are *unit norm* and orthogonal to each other.

Matrix rank

The rank r of matrix $\mathbf{U} = [\mathbf{u}_1, \mathbf{u}_2, .., \mathbf{u}_K]$ of size $N \times K$ is defined by the dimension spanned by its columns $\mathbf{u}_1, \mathbf{u}_2, .., \mathbf{u}_K$ (or by its rows).

$$r = \mathrm{dimension}(\mathrm{span}(\mathbf{u}_1, \mathbf{u}_2, .., \mathbf{u}_K))$$

Therefore $r \leq \min(N, K)$.
If $r = \min(N, K)$ the matrix is said to be "full rank".
The matrix is left (resp. right) invertible iff it is full column (resp. row) rank .

Matrix eigenvalue-decomposition

Let the $N \times N$ matrix $\mathbf{A}$. The EVD of this matrix is the set of unit-norm eigenvectors $\mathbf{u}_1$,..,$\mathbf{u}_N$ and eigenvalues λ_1,.., λ_N such that:

$$\mathbf{A}\mathbf{u}_i = \lambda_i \mathbf{u}_i$$

in other terms:

$$\mathbf{A} = [\mathbf{u}_1, .., \mathbf{u}_N] \begin{bmatrix} \lambda_1 & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \lambda_N \end{bmatrix} [\mathbf{u}_1, .., \mathbf{u}_N]^{-1}$$

i.e.:

$$\mathbf{A} = \mathbf{U}\mathbf{\Lambda}\mathbf{U}^{-1}$$

EVD of hermitian matrix

- Hermitian matrices are important. All covariance matrices, of the type $E(\mathbf{x}\mathbf{x}^H)$, are hermitian.
- The eigenvalues of complex hermitian matrices are positive real. The eigenvectors are orthogonal.

$$\mathbf{A} = \mathbf{A}^H$$

Then:

$$\mathbf{A} = \mathbf{U}\boldsymbol{\Lambda}\mathbf{U}^H$$

with $\lambda_i \geq 0$ and $\mathbf{U}^H\mathbf{U} = \mathbf{I}_N$

Let the $N \times K$ matrix $\mathbf{A}$. The SVD of this matrix is given by:

$$\mathbf{A} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^H$$

where

- $\mathbf{U} = [\mathbf{u}_1,..,\mathbf{u}_N]$ is the $N \times N$, unitary matrix, containing the *left singular vectors*
- $\mathbf{V} = [\mathbf{v}_1,..,\mathbf{v}_K]$ is the $K \times K$, unitary matrix, containing the *right singular vectors*
- $\boldsymbol{\Sigma}$ is the $N \times K$ matrix containing the *singular values*. Example for $N \geq K$:

$$\boldsymbol{\Sigma} = \begin{bmatrix} \sigma_1 & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \sigma_K \\ \mathbf{0} & & \mathbf{0} \end{bmatrix}$$

SVD versus EVD

Let the $N \times K$ matrix $\mathbf{A}$. The SVD relates to the EVD by the relation:

$$\mathbf{A}\mathbf{A}^H = \mathbf{U}\mathbf{\Sigma}^2\mathbf{U}^H$$

and

$$\mathbf{A}^H\mathbf{A} = \mathbf{V}\mathbf{\Sigma}^2\mathbf{V}^H$$

The left singular vectors of $\mathbf{A}$ are the eigen-vectors of hermitian matrix $\mathbf{A}\mathbf{A}^H$. The right singular vectors of $\mathbf{A}$ are the eigen-vectors of hermitian matrix $\mathbf{A}^H\mathbf{A}$.
Finally $|\sigma_i| = \sqrt{(\lambda_i(\mathbf{A}^H\mathbf{A}))} = \sqrt{(\lambda_i(\mathbf{A}\mathbf{A}^H))}$.

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- General introduction to the course
- Challenges of wireless network design
- Introduction to MIMO networks
- Introduction to MIMO networks and network-MIMO
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Challenges of wireless network design

- Metrics for wireless performance
- Challenges of the wireless channel
- State-of-the-art solutions

Metrics for wireless performance

Key metrics

- Rates (Bits/Sec)
- Range (kms), under a target rate constraint
- Mobility support (km/h), under a target rate constraint
- Latency (ms), under a target rate constraint

Perspective on Wireless Performance

- Coverage in km (matters most in early deployment stages)
- spectral efficiency in Bit/Sec/Hz/Cell (mature deployments):

$$SE = \frac{rM}{K} \tag{1}$$

M =average modulation order (bits/symbol), r = code rate (1/2, 3/4, ..), K = Frequency reuse or

$$SE = \frac{R}{KB} \tag{2}$$

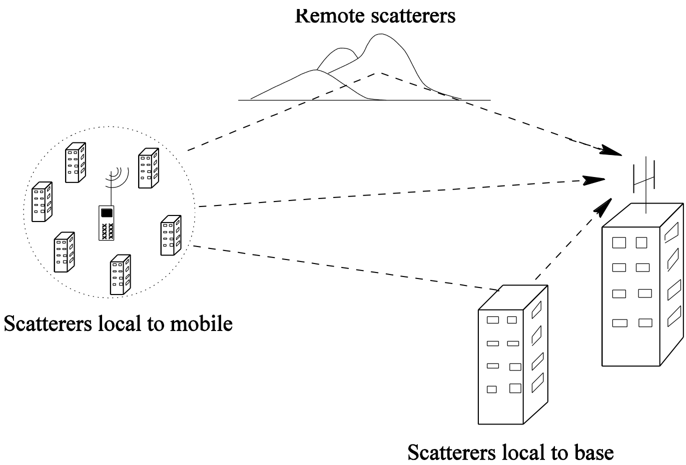
R =average user rate (Bits/sec) B = bandwidth

Challenges of the wireless channel

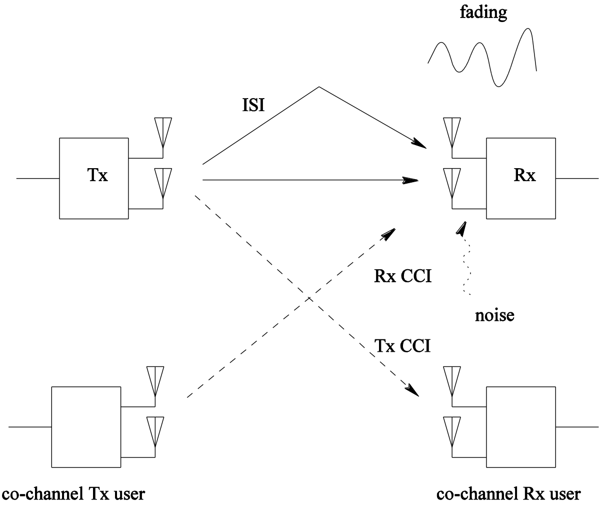
Wireless transmission introduces:

- Fading: multiple paths with different phases add up at the receiver, giving a random (Rayleigh/Ricean) amplitude signal.
- ISI: multiple paths come with various delays, causing intersymbol interference.
- CCI: Co-channel users create interference to the target user
- Noise: electronics suffer from thermal noise, limiting the SNR.
- Doppler: The channel varies over time, needs to be tracked.

Multipath Propagation



MIMO link diagram



Some solutions

Tricking the wireless channel to improve performance

- Advanced coding and filtering (turbo)
- Hybrid retransmission protocols
- Fast link adaptation
- multi-antenna (MIMO) techniques
- multi-user
  - filtering
  - scheduling
  - inter-cell coordination (for interference control)
  - cooperation

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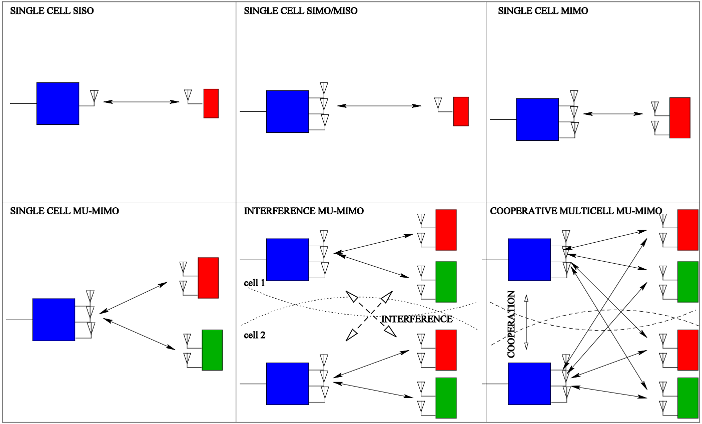
Using the space dimension (i.e. space-time processing)

- MIMO is multiple inputs multiple outputs
- Using antenna arrays permits to process radio signals in space, not only time.

...to improve performance in presence of fading/interference

- coverage
- quality (BER, MOS, outage)
- capacity: Bit/sec/Hz/BTS or # users/Hz/BTS
- peak data rates: Bit/sec

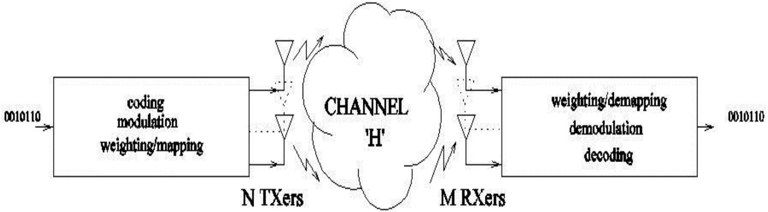
MIMO Configurations



We first focus on the single-user scenario:

- Multiple antennas at receive side
- Multiple antennas at transmit side (without feedback)
- Joint transmit-receive side without channel feedback
- Joint transmit-receive side with channel feedback

MIMO link diagram (single user)



Mapping the data to the multiple antennas

Key approaches

- 1 - Beamforming algorithms: To increase received SNR for desired directions/signatures.
- 2 - Diversity algorithms: To combat fading in order to work at less SNR.
- 3 - Interference mitigation: To maximally reuse the channel frequencies
- 4 - Spatial Multiplexing algorithms:
  - single-user multiplexing: to increase data speeds
  - multi-user multiplexing: to cell user capacity

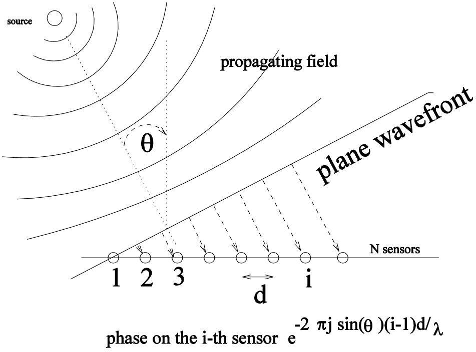
Want to combine all of these? Not possible!

Antennas are dimensions in a finite dimensional vector space, there is only a small finite number of ways of combining them.

Smart Antennas techniques at receive

- Steering vectors and signal formulation
- Information theoretic bounds
- Beamforming
- Diversity

Steering vector



$$\mathbf{h}(\theta) = [1, e^{-2\pi j \sin(\theta)\frac{d}{\lambda}}, e^{-2\pi j \sin(\theta)\frac{2d}{\lambda}}, .., e^{-2\pi j \sin(\theta)\frac{(N-1)d}{\lambda}}]^T$$

Math formulation

- $\mathbf{h} = [h_1, h_2, .., h_N]^T$ is the spatial signature of the sent signal $s(t)$
- $\mathbf{w} = [w_1, w_2, .., w_N]^T$ is the vector of antenna weights
- $y(t)$ is the (noisy) measured signal at the array output

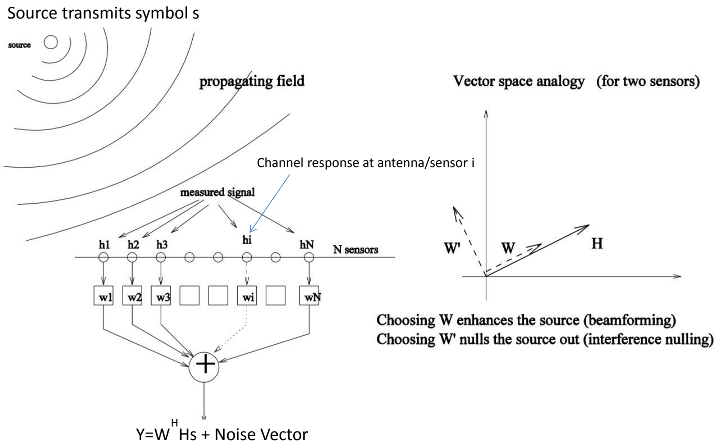
$$\mathbf{y}(t) = \mathbf{h}s(t) + \mathbf{n}(t)$$

$$z(t) = \mathbf{w}^H\mathbf{y}(t) = \mathbf{w}^H\mathbf{h}s(t) + \mathbf{w}^H\mathbf{n}(t)$$

Note 1: The use of transpose-conjugate ^{H} in applying filter $\mathbf{w}$ is a writing convention, used to mark the fact that the filtering operation introduces a new effective channel $\mathbf{w}^H\mathbf{h}$ given by the complex vector product between the filter and the actual channel $\mathbf{h}$.

Note 2: if receiver or transmitter moving, then $\mathbf{h} \rightarrow \mathbf{h}(t)$.

The directional beamforming concept



Key limits of beamforming/interference canceling

- A N-antenna beamformer can amplify one source (no interference) by a factor N in the average SNR.
- A N-antenna beamformer can extract one source and cancel out $N-1$ interferers simultaneously
- All N sources can be simultaneously extracted (assuming the other $N-1$ are viewed as interferers) by beamformer superposition.
- Transmit beamforming realizes the same benefits/gains at receive beamforming if CSI is given.
- These gains are robust to line-of-sight/rich multipath conditions.

Applying Interference Canceling Algorithms?

A difficult task..

- Concept similar to beamforming: except one tries to use weights *orthogonal* to interferer's channel vector.
- Tricky to implement from a system point of view because knowledge of interfering signature not usually available
- Performance highly dependent on specular nature of interference (a few strong better than many weak).
- Future systems, carrying bursty IP data, make interference less predictable.

The diversity principle

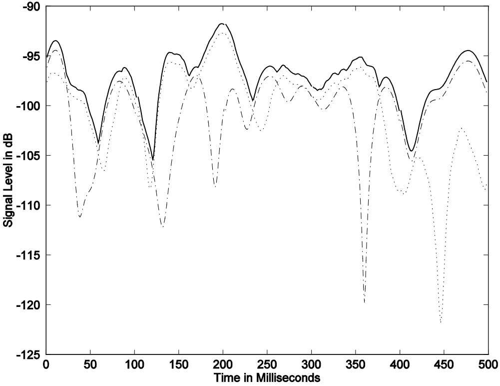
- In the presence of multipath, $|h(t)|$ can follow a Rayleigh distribution (i.e. $h(t) = h_r(t) + jh_i(t)$ is complex random Gaussian)
- $|h(t)|$ fades with non-negligible probability. But

$$SNR(t) = \frac{(|h_1(t)|^2 + .. + |h_N(t)|^2)\sigma_s^2}{\sigma_n^2} \quad \text{is zero iff } h_i(t) \text{ is zero for all } i\text{-s!} \tag{3}$$

$\Rightarrow$ Probability that SNR(t) $< \epsilon$, where ϵ is a chosen small threshold decreases exponentially with N!

The Power of Diversity

Diversity gain: reduction in SNR acceptable for the same BER target.



Diversity Gain

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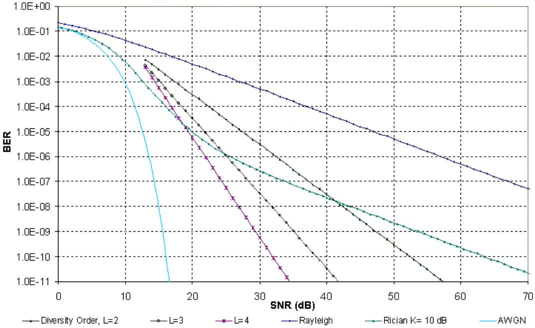


Diversity vs. beamforming gain

- Diversity gain can be 10dB-20dB with just 2 antennas (much larger than beamforming (array) gain: 3dB)
- Diversity gain can be used to extend the range or increase data rate if adaptive modulation is enabled.

At the transmitter (to be seen later)

- Like beamforming, diversity can be used on receiver *and* transmitter.
- On transmitter, if channel is unknown, *transformed domain* techniques or *space time coding* are used.

Multiple Antennas techniques at transmitter (MISO)

If channel coefficients known by transmitter (TDD or FDD with channel feedback)

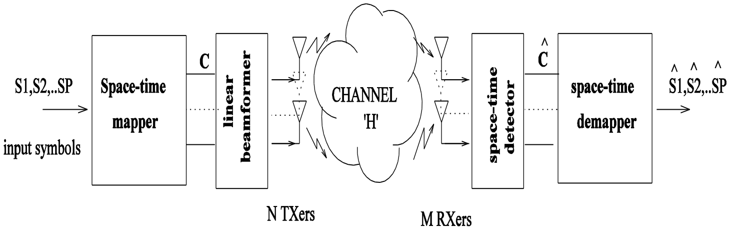
- Transmit beamforming can be used (same gains as receive beamforming!)

If channel coefficients not known by transmitter

- Transmit beamforming is not possible!
- Diversity gain IS possible! (use space time codes)

Origins

- Invented 1996 at ATT Labs, NJ. (treillis space time codes) (Tarokh,Seshadri,Calderbank)
- Low complexity space-time block codes invented at Bell Labs, NJ, USA ("Alamouti code").



General Model

1) P input (QAM) modulation symbols are mapped to a *codeword matrix* of size $N \times K$.

$$\{s_1, s_2, .., s_P\} \Rightarrow \mathbf{C} = \begin{bmatrix} c_1(1) & c_1(2) & \dots & c_1(K) \\ \vdots & \vdots & & \vdots \\ c_N(1) & c_N(2) & \dots & c_N(K) \end{bmatrix}$$

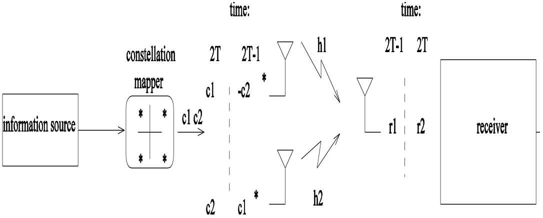
2) The codeword (possibly linearly transformed by some matrix $\mathbf{V}$) is launched into the channel. We receive:

$$\mathbf{Y} = \begin{bmatrix} h_{11} & \dots & h_{1N} \\ \vdots & & \vdots \\ h_{M1} & h_{M2} & h_{MN} \end{bmatrix} \mathbf{V} \begin{bmatrix} c_1(1) & c_1(2) & \dots & c_1(K) \\ \vdots & \vdots & & \vdots \\ c_N(1) & c_N(2) & \dots & c_N(K) \end{bmatrix} + \mathbf{Noise}.$$

3) For conventional space-time code, pick $P = K$. Said to be "rate one" (one symbol per channel use).

Space-Time Block Coding: Alamouti scheme ($\mathbf{V} = \mathbf{I}$)

2×1 case:



Space-Time block coding equations

$$\begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} -s_2^* & s_1^* \\ s_1 & s_2 \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} + \mathbf{n} \tag{4}$$

$$\begin{bmatrix} r_1^* \\ r_2 \end{bmatrix} = \begin{bmatrix} h_2^* & -h_1^* \\ h_1 & h_2 \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \tilde{\mathbf{n}} \tag{5}$$

$$[h_2 h_1^*] \begin{bmatrix} r_1^* \\ r_2 \end{bmatrix} = (|h_2|^2 + |h_1|^2)s_1 + n_1 \ \rightarrow \hat{s}_1 \tag{6}$$

$$[-h_1 h_2^*] \begin{bmatrix} r_1^* \\ r_2 \end{bmatrix} = (|h_2|^2 + |h_1|^2)s_2 + n_2 \ \rightarrow \hat{s}_2 \tag{7}$$

$$\tag{8}$$

Space-Time Block Coding: Arbitrary N

Theorem (Tarokh et al.):

- There exist full-rate orthogonal block codes for real-valued modulations for $N = 2$, 4, and 8 only. Beyond that, loss of rate is unavoidable.
- For complex (e.g. QAM), there exist full-rate orthogonal block codes for $N = 2$ only.

Orthogonal codes can be developed for $N > 2$ but with rate $P/K < 1$ only! Example for $N = 4$, rate is $4/8 = 1/2$:

$$\mathbf{C} = \begin{bmatrix} s_1 & -s_2 & -s_3 & -s_4 & s_1^* & -s_2^* & -s_3^* & -s_4^* \\ s_2 & s_1 & s_4 & -s_3 & s_2^* & s_1^* & s_4^* & -s_3^* \\ s_3 & -s_4 & s_1 & s_2 & s_3^* & -s_4^* & s_1^* & s_2^* \\ s_4 & s_3 & -s_2 & s_1 & s_4^* & s_3^* & -s_2^* & s_1^* \end{bmatrix} \tag{9}$$

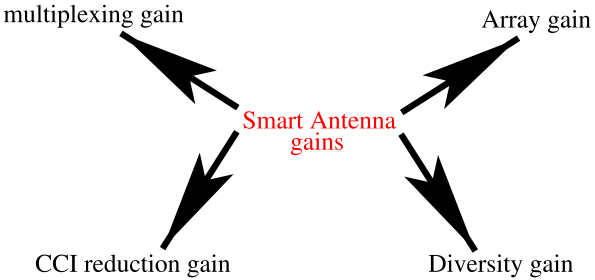
MIMO Systems

- "real" MIMO means nb TX antenna > 1 *and* nb of RX antennas > 1
- First introduced at Lucent (1996) and Stanford Univ. (1994).
- Exploit space dimension at both transmitter and receiver: Subscriber unit is also equipped with multiple antennas.
- Leads to hyper-diversity gains (joint TX-RX)
- Channel becomes matrix. Possibility to transmit on several so-called 'eigen-modes' simultaneously (spatial multiplexing gain).
- Exploits multipath instead of mitigating it.
- Can be combined with conventional strategies for beamforming and interference reduction.

The first MIMO (Lucent) prototype)

16-element array

Qualitative MIMO gains



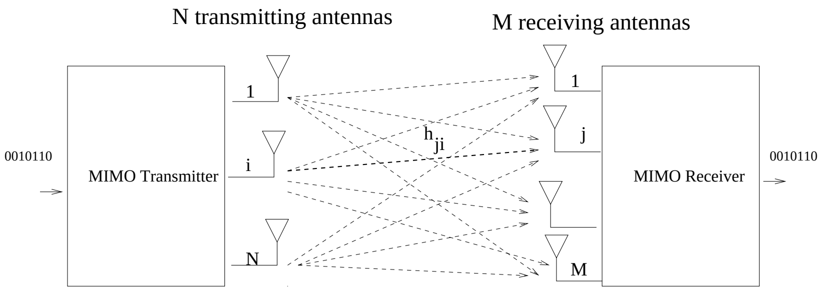
Quantitative MIMO Gains

- Array gain: $\log(M)$ dB on receive side, $log(N)$ dB on transmit side if transmit channel known.
- Diversity gain: Up to $M \times N$ orders of diversity.
- Spatial Multiplexing gain: Up to $\min(M, N)$ factor of data rate increase!

Caution:

- Not all these gains can be achieved at the same time!
- Specific algorithm will extract specific gains
- Channel models have a direct impact on the gains

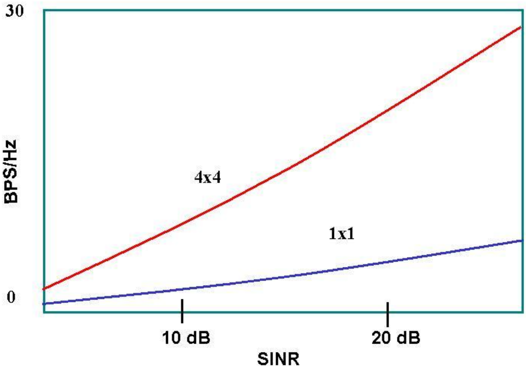
MIMO channel matrix



Example for 3×4 system:

$$\mathbf{H} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \\ h_{41} & h_{42} & h_{43} \end{bmatrix}$$

ideal=i.i.d. Rayleigh distributed

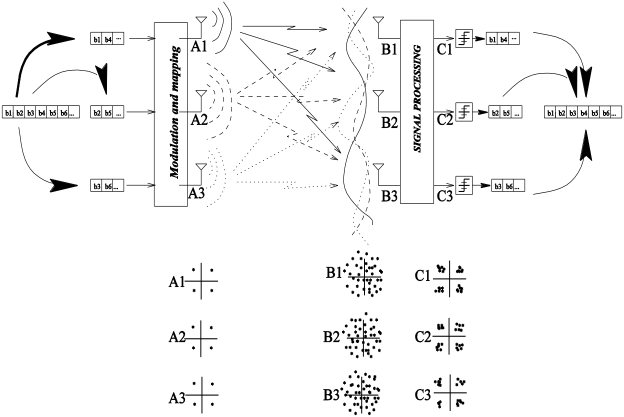


Principles

- Transmitter design works independent of number of RX antennas!
- Space-time code picks up $M * N$ orders of diversity
- Example: Alamouti code for 2×2 system (developped in class)

MIMO Spatial multiplexing

Spatial Multiplexing: We send multiple signals, the receiver learns the channel matrix and inverts it to separate the data.



Spatial multiplexing seen as a space-time code

Spatial multiplexing is equivalent to a space-time code with $P = KN$: $P = KN$ input (QAM) modulation symbols are mapped to a *codeword matrix* of size $N \times K$.

$$\{s_1, s_2, .., s_P\} \Rightarrow \mathbf{C} = \begin{bmatrix} c_1(1) & c_1(2) & \dots & c_1(K) \\ \vdots & \vdots & & \vdots \\ c_N(1) & c_N(2) & \dots & c_N(K) \end{bmatrix}$$

We receive:

$$\mathbf{Y} = \begin{bmatrix} h_{11} & \dots & h_{1N} \\ \vdots & & \vdots \\ h_{M1} & h_{M2} & h_{MN} \end{bmatrix} \begin{bmatrix} c_1(1) & c_1(2) & \dots & c_1(K) \\ \vdots & \vdots & & \vdots \\ c_N(1) & c_N(2) & \dots & c_N(K) \end{bmatrix} + \mathbf{Noise}.$$

A simple mapping example

The simplest mapping for spatial multiplexing is given by: $c_n(k) = s_{nk}$

$$\mathbf{Y} = \begin{bmatrix} h_{11} & h_{12} & .. \\ h_{21} & h_{22} & .. \\ \vdots & \vdots & \vdots \end{bmatrix} \begin{bmatrix} s_1 & s_{N+1} & .. \\ s_2 & s_{N+2} & .. \\ \vdots & \vdots & .. \\ s_N & s_{N+2} & .. \end{bmatrix} + \mathbf{n} \tag{10}$$

How it works

Example for 3x3:

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \underbrace{\begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix}}_{\mathbf{H}} \begin{bmatrix} s_1 \\ s_2 \\ s_3 \end{bmatrix} + \textbf{Noise} \tag{11}$$

$$\begin{bmatrix} \hat{s}_1 \\ \hat{s}_2 \\ \hat{s}_3 \end{bmatrix} = \mathbf{H}^{-1} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \tag{12}$$

Impact of channel model

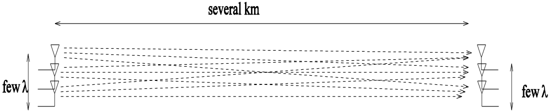
MIMO Performance very sensitive to channel matrix *invertibility*

The following degrades the conditioning of the channel matrix:

- Antenna correlation caused by:
  - small antenna spacing, or
  - small angle spread

  Line of sight component compared with multipath fading component:
  - multipath fading component, close to i.i.d. random, is well conditioned
  - Line of sight component is very poorly conditioned.

All of this fixed in the multi-user scenario!



The system

$$\mathbf{H} \approx \alpha \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

is near rank one (non invertible)!

$\Rightarrow$ Spatial multiplexing in single-user requires multipath to work!!!

Not so in multi-user MIMO :)

Spatial multiplexing receiver design

1. Linear receivers for BLAST (Zero-Forcing, MMSE)
2. Non linear receiver (SIC, ML)
3. Performance and complexity trade-offs

Zero-Forcing receiver

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & .. \\ h_{21} & h_{22} & .. \\ \vdots & \vdots & \vdots \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \\ \vdots \end{bmatrix} + \mathbf{n} \tag{13}$$

Zero Forcing implements matrix (pseudo)-inverse (ignores noise enhancement problems):

$$\hat{\mathbf{s}} = \mathbf{H}^{\#}\mathbf{y} \tag{14}$$

where:

$$\mathbf{H}^{\#} = (\mathbf{H}^H\mathbf{H})^{-1}\mathbf{H}^H$$

MMSE receiver

The MMSE receiver optimizes the following criterion:

$$\mathbf{W} := \text{argmin}\{E|\mathbf{W}^H\mathbf{y} - \mathbf{s}|^2\}$$

We find:

$$\hat{\mathbf{s}} = \mathbf{H}^H(\mathbf{H}\mathbf{H}^H + \mathbf{R}_n)^{-1}\mathbf{y} \tag{15}$$

where $\mathbf{R}_n$ is the noise/intf covariance.
This offers a compromise between residual interference between input signals and noise enhancement.

Non linear MIMO receivers

Maximum likelihood receiver:

- Optimum detection
- Exhaustive search. No iterative procedure for MIMO.
- Complexity exponential in QAM order and N.

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & .. \\ h_{21} & h_{22} & .. \\ \vdots & \vdots & \vdots \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \\ \vdots \end{bmatrix} + \mathbf{n} \tag{16}$$

Maximum Likelihood Solution:

$$\hat{\mathbf{s}} = \text{argmin} |\mathbf{y} - \mathbf{H}\mathbf{s}|^2$$

where $\mathbf{s}$ is searched over the modulation alphabet (e.g. 4QAM, 16QAM..)

Iterative procedure: V-BLAST

'Onion peeling' Solution (or 'Successive Interference Canceling'):

$$\begin{aligned}
\hat{s}_1 &= \mathbf{w}_1^H \mathbf{y} && (17)\\
\hat{\hat{s}}_1 &= \text{Slicer}(\hat{s}_1) && (18)\\
\mathbf{y}_1 &= \mathbf{y} - \mathbf{h}_1 \hat{\hat{s}}_1 && (19)\\
\hat{s}_2 &= \mathbf{w}_2^H \mathbf{y}_1 \quad \text{etc}
\end{aligned}$$

- Offers complexity linear in N
- Disadvantage: performance unequal on various stages (best on s_1..worst on s_N).
- input signals are ranked in power for better performance

Signal ordering for V-BLAST

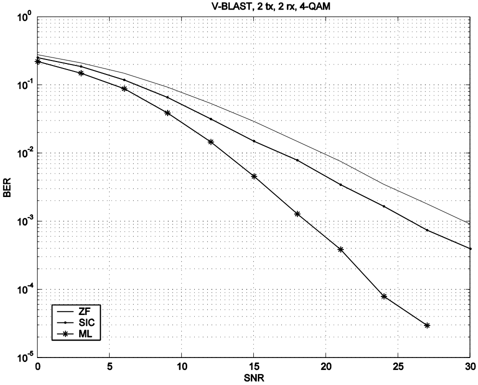
Idea: Decode first the streams which exhibit highest SNR so as to minimize the propagation of errors in later stages.
Example, at first stage: Let $\mathbf{G} = \mathbf{H}^{\#}$ be the $N \times M$ pseudo-inverse of the channel matrix.
Let $\mathbf{g}_i$ be the i-th row of $\mathbf{G}$. The SNR for $\hat{s}_i$ is inversely proportional to $\|\mathbf{g}_i\|^2$. Therefore we start with s_{i_0} where

$$i_0 = \mathrm{argmin}_{i=1:N}\{\|\mathbf{g}_i\|^2\}$$

and continue with s_{i_1} (second smallest row norm), etc.

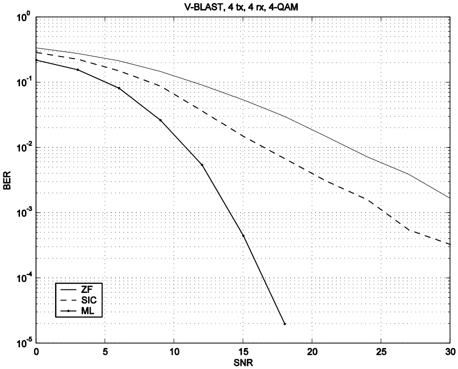
Performance comparison

BLAST zero-forcing vs. V-BLAST (SIC) vs BLAST-ML (2x2)



Performance comparison

BLAST zero-forcing vs. V-BLAST (SIC) vs BLAST-ML (4x4)

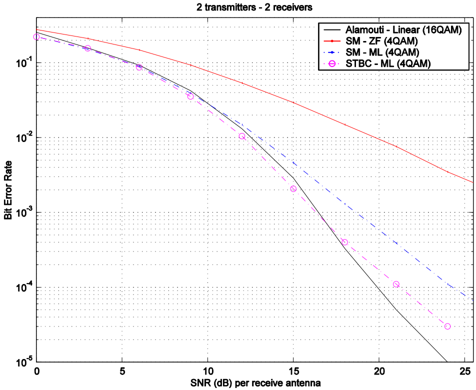


SM receiver trade-offs

- Linear MMSE receiver is simplest to compute and performs better than ZF receiver.
- MMSE receiver provides a diversity order only equal to $M - N + 1$. Very limiting for square systems.
- ML provides optimal performance and diversity for a SM receiver but complexity exponential with QAM order and N.
- V-BLAST provides good compromise between performance of ML and low complexity of a linear receiver.
- Issue with V-BLAST: Performance very unequal between different stages (last stage much better than first stage because of added diversity).

Performance comparison

Same rate comparison: Alamouti (1x16 QAM), BLAST zero-forcing, BLAST ML, STBC-BLAST-ML (all 2x4QAM).



- Considerations on channel feedback
- Information theoretic bounds
- Techniques

Channel knowledge via feedback

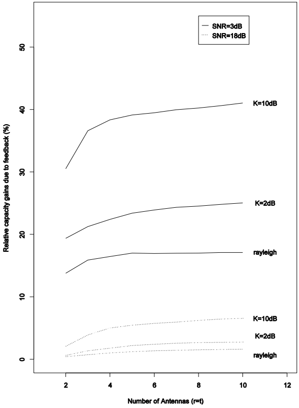
Ideal case:

- Channel is fully known: The same gain can be achieved on either transmit or receive beamforming.

Practical case (channel estimated):

- Receive side: Channel estimated from training sequence
- Transmit side:
  - FDD system with dedicated feedback channel: quality depends on feedback rate.
  - FDD system without feedback: partial knowldge obtained reciprocal channel component (angles, delays). However non faded paths on downlink can be faded on uplink. Robustness issues.
  - TDD system: Beamformer uses uplink channel IR estimates. Quality depends on "ping-pong time".

Gain of feedback vs. Rice K factor



MISO and MIMO techniques with feedback

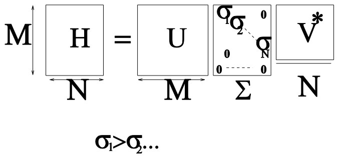
- MISO case: gains become identical to SIMO!
  - TX diversity
  - TX beamforming
- MIMO case
  - eigen-beamforming
  - Spatial waterfilling

Conclusion: feedback (CSIT) is useful to permit SNR (array) gain and smart power allocation at TX. Feedback gain thus vanishes at high SNR.

MIMO Eigen-beamforming

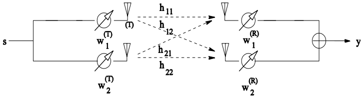
- Requires transmit channels knowledge
- Relies on singular value decomposition of $\mathbf{H}$ (SVD)
- Trade-off between diversity and rate easy to control

SVD of $\mathbf{H}$: $\mathbf{H} = \mathbf{U\Sigma V}^H$



Diversity maximization (I)

- We transmit one symbol s on the top right singular vector $\mathbf{v}_1$, and receive on $\mathbf{u}_1$:
- Rate is 1. Diversity order is maximum MN.



Diversity maximization (II)

Output signal:

$$y = \mathbf{u}_1^H \mathbf{H} \mathbf{v}_1 s + \mathbf{u}_1^H \mathbf{n}.$$

Output signal-to-noise ratio:

$$\mathrm{SNR}_{output} = \frac{\sigma_1^2 \sigma_s^2}{\sigma_n^2} = \sigma_1^2 \mathrm{SNR}_{input}$$

SNR gain is given maximum singular value!

MIMO Eigen-beamforming with multiplexing

- We transmit P symbols on the top P right singular vectors, and receive on the top P left.
- Rate is P. Diversity is maximum on top singular pair, then decreasing with singular value index.

$$\mathbf{Y} = [\mathbf{u}_1, .., \mathbf{u}_P]^H \mathbf{H} [\mathbf{v}_1, .., \mathbf{v}_P][s_1, .., s_P] + [\mathbf{u}_1, .., \mathbf{u}_P]^H \mathbf{n}.$$

Capacity-achieving eigen-beamforming

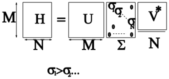
- We transmit $\min(M, N)$ symbols on each of the $\min(M, N)$ right singular vectors
- The power of each symbol is adjusted separately (with $\sqrt{P_i}$) according to the singular values
- Rate is $\min(M, N)$

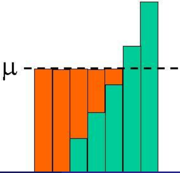
Assume $M \geq N$:

$$\mathbf{Y} = [\mathbf{u}_1, .., \mathbf{u}_N]^H \mathbf{H} [\mathbf{v}_1, .., \mathbf{v}_N][\sqrt{P_1} s_1, .., \sqrt{P_N} s_N] + [\mathbf{u}_1, .., \mathbf{u}_N]^H \mathbf{n}.$$

MIMO Waterfilling

$$\mathbf{Y} = [\mathbf{u}_1, .., \mathbf{u}_N]^H \mathbf{H} [\mathbf{v}_1, .., \mathbf{v}_N][\sqrt{P_1} s_1, .., \sqrt{P_N} s_N] + [\mathbf{u}_1, .., \mathbf{u}_N]^H \mathbf{n}.$$





$$P_k = (\mu - \frac{1}{\lambda_k})^+$$

Multi-user MIMO networks

- Motivations
- Introduction to multi-user techniques

Motivations

Multi-user makes certain things difficult:

- Dealing with users of unequal channel conditions (fairness issues).
- Mixing antenna filtering and scheduling problems into a harder problem.
- Multiple users can't cooperate as well as multiple antennas on a single device.
- Leads to multiple (rather than single) power constraints.

But others much easier !

- Provides multi-user diversity (less reliance on antenna diversity).
- Provides decorrelation of spatial signatures.
- Allows for user- (in addition to stream-) multiplexing.
- low rank channels no longer a problem but an advantage.

Introduction to multi-user MIMO techniques

In practice multi-user techniques involves interplay of:

- Admission control
- Multi-antenna combining (for MIMO case)
- Power control
- User scheduling

Multi-user spatial multiplexing

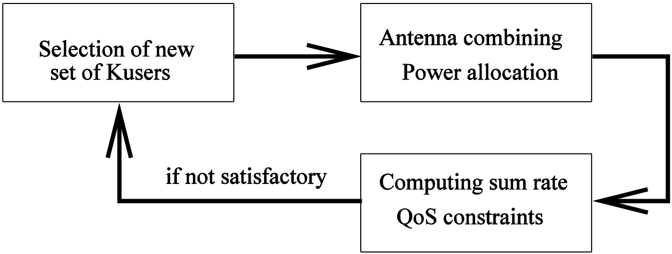
Definition: Spatial multiplexing across users

- $K \geq 1$ user signals are superposed (within the same unit of spectral resource)
- The K users are differentiated from their spatial signature

Multi-user MIMO can operate with:

- $K = 1$: at each time/slot, we deal with a single-user MIMO link.
- $K > 1$: The scheduler selects a group of K users
- With $K > 1$, the scheduler and antenna technique become *coupled* so as to maximize capacity

User group scheduling



Multi-user MIMO techniques

- Introduction
- The uplink problem
- The downlink problem

Introduction to multi-user MIMO techniques

Key design parameters

- Choice of objective function:
  - max sum rate
  - achieving per use rate targets
- Complexity:
  - pushed toward the base (rather than at the users!)
  - iterative/block based
  - linear vs. non linear (optimal)
- Channel state information (CSI):
  - CSI at RX, CSI or partial CSI at TX
  - CSI at RX, no CSI at TX

Received signal model at the base:

$$\mathbf{y} = \mathbf{H}^H \mathbf{X} + \mathbf{n} \tag{20}$$

with global uplink channel matrix:

$$\mathbf{H}^H = \left[\mathbf{H}_1^H, \mathbf{H}_2^H, .., \mathbf{H}_K^H\right] \tag{21}$$

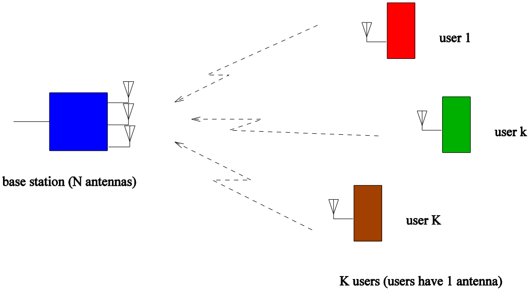
And global user transmit vector:

$$\mathbf{X} = \left[\mathbf{x}_1^T, \mathbf{x}_2^T, .., \mathbf{x}_K^T\right]^T \tag{22}$$

where vector $\mathbf{x}_k$ carries $m_k \leq M_k$ symbols per channel uses.

Multi-user MIMO: The uplink with $M_k = 1$

The traditional SDMA (space division multiple access) setup:



Each user transmits $x_k = \sqrt{q_k} s_k$ where q_k is the power level.

Non-linear methods for the MU-MIMO uplink with $M_k = 1$

We first consider single-antenna users ($M_k = 1$). i.e. each user transmits one stream of data ($m_k = 1$).
The optimal detection method is given by the maximum likelihood multi-user detector:
We form $\mathbf{s} = (s_1, s_2, .., s_K)^T$

$$\hat{\mathbf{s}} = \arg\min_{\mathbf{s}\in\mathcal{A}^K} \|\mathbf{y} - \mathbf{H}^H\sqrt{\mathbf{Q}}\mathbf{s}\|^2 \tag{23}$$

where $\mathcal{A}$ is the known symbol alphabet for each transmitted symbol
$\mathbf{Q} = \text{diag}(q_1, ..q_k)$ is the diagonal power allocation matrix.

Linear methods for the MU-MIMO uplink with $M_k = 1$

At the base, we perform linear filtering:

$$\mathbf{z} = \mathbf{W}^H \mathbf{y} = \mathbf{W}^H \mathbf{H}^H \sqrt{\mathbf{Q}} \mathbf{s} + \mathbf{W}^H \mathbf{n} \tag{24}$$

where $\mathbf{W}^H$ is complex of size $K \times N$

$$\mathbf{H}^H = \left[\mathbf{h}_1^H, \mathbf{h}_2^H, .., \mathbf{h}_K^H \right] \tag{25}$$

and $\mathbf{h}_k^H$ is the user k's channel vector of size $N \times 1$.

Multi-user MIMO uplink: The zero-forcing solution

Goal is to extract signal of each user, free of interference (SINR=SNR), with highest SNR:
We take:

$$\mathbf{W}^H = \sqrt{\mathbf{Q}}^{-1}(\mathbf{H}\mathbf{H}^H)^{-1}H \tag{26}$$

where $\mathbf{Q}$ is the power allocation matrix:

$$\mathbf{Q} = \begin{pmatrix} q_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & q_K \end{pmatrix} \tag{27}$$

We solve:

$$\mathbf{W} = \text{arg min} \quad E\|\mathbf{W}^H\mathbf{y} - [s_1, s_2, .., s_K]\,\|^2 \tag{28}$$

We obtain:

$$\mathbf{W} = (\mathbf{I} + \mathbf{H}^H\mathbf{Q}\mathbf{H})^{-1}\mathbf{H}^H\mathbf{A} \tag{29}$$

where $\mathbf{A}$ is a diagonal power normalization matrix matrix such that

$$\left[\mathbf{W}^H\mathbf{W}\right]_{k,k} = 1 \tag{30}$$

Note: the normalization leaves the SINR unchanged.

Uplink with multiple antennas per user

- User k has $M_k > 1$ antennas.
- Each user has three main strategies: (i) Beamforming, (ii) ST coding, and/or (iii) spatial multiplexing

(i) Beamforming:

- User needs CSIT
- Combined channel+beamformer becomes equivalent to $M_k = 1$ antenna channel

(ii) Space-time Coding:

- To combine user multiplexing with diversity at the level of each user

(iii) Spatial multiplexing

- User k can transmit m_k streams, with $m_k \leq M_k$, $\sum_k m_k \leq N$.
- Each stream seen as a "virtual single antenna user"
- Single antenna case applies, with modified power constraint

Multi-user MIMO: The downlink

Received signal model at user k:

$$\mathbf{y}_k = \mathbf{H}_k \mathbf{X} + \mathbf{n}_k \text{ where } \mathbf{X} = \sum_k \mathbf{X}_k \tag{31}$$

usng the global downlink channel matrix:

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_1 \\ \mathbf{H}_2 \\ \vdots \\ \mathbf{H}_K \end{bmatrix} \tag{32}$$

We have the global receive vector for all users:

$$\mathbf{y} = \left[\mathbf{y}_1^T, .., \mathbf{y}_K^T\right]^T = \mathbf{H} \sum_k \mathbf{X}_k + \mathbf{n} \tag{33}$$

where $\mathbf{X}_k$ is the signal vector designed to reach user k

Fundamental CSIT/performance trade-off

There exists an interesting trade-off between

(i) the capacity performance

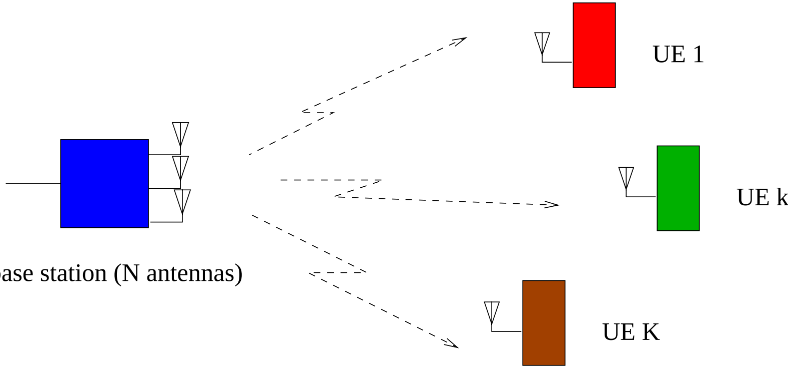
(ii) the number of antennas at the users

(iii) the need for CSIT.

- Capacity scales with $\min(K, N)$ provided the base has CSIT.
- In the absence of CSIT, user multiplexing is generally not possible: The base does not know in which "direction" to form beams!
- This is contrast with single user MIMO where CSIT is not necessary to get multiplexing gain.
- One case where multiplexing gain is restored is when at least $M_k = \min(N, K)$ antennas are installed at each user (exercise!)

Multi-user MIMO: The downlink with $M_k = 1$

We first consider single antenna users ($M_k = 1$)



K users (UEs have 1 antenna each)

Signal model for MU-MIMO downlink beamforming

The base transmits signal vector $\mathbf{X} = \mathbf{W}\sqrt{\mathbf{Q}}\mathbf{s}$ where

- $\mathbf{W}$ is the $N \times K$ downlink beamformer and
- $\mathbf{s} = (s_1, .., s_K)^T$ contains the symbols.
- $\mathbf{Q} = \text{diag}(q_1, .., q_K)$ is the power allocation matrix.

The received signal at all users becomes:

$$\mathbf{y} = \mathbf{H}\mathbf{W}\sqrt{\mathbf{Q}}\mathbf{s} + \mathbf{n} \tag{34}$$

MU-MIMO downlink zero-forcing beamformer

Goal is to send an interference free signal to each user.
We compute, withouth power constraint:

$$\mathbf{W} = \mathbf{H}^H(\mathbf{H}\mathbf{H}^H)^{-1}\sqrt{\mathbf{Q}}^{-1} \quad \text{risk of explosion!} \tag{35}$$

With total power constraint P:

$$\mathbf{W} = \frac{1}{\sqrt{\lambda}}\mathbf{H}^H(\mathbf{H}\mathbf{H}^H)^{-1}\sqrt{\mathbf{Q}}^{-1} \tag{36}$$

where $\lambda = \frac{1}{P}\,\mathrm{Tr}\left[(\mathbf{H}\mathbf{H}^H)^{-1}\right]$

MU-MIMO downlink MMSE beamformer

Goal is to trade off interference at each user for *power usage* at the base station.
We solve:

$$\mathbf{W} = \arg\min \ E\|\mathbf{s} - \mathbf{y}\|^2 = \arg\min E\|\mathbf{s} - \mathbf{H}\mathbf{W}\sqrt{\mathbf{Q}}\mathbf{s}\|^2 \tag{37}$$

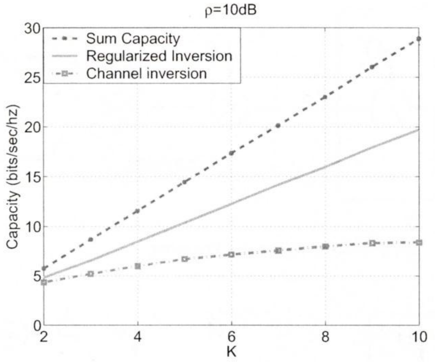
under total power constraint.
We obtain:

$$\mathbf{W} = \mathbf{H}^H(\alpha\mathbf{I} + \mathbf{H}\mathbf{H}^H)^{-1}\sqrt{\mathbf{Q}}^{(-1)} \tag{38}$$

where α (real, positive) is tuned such that $\mathrm{Tr}(\mathbf{W}\mathbf{Q}\mathbf{W}^H) = P$
Note: the uplink MMSE and downlink MMSE filters are different in general!

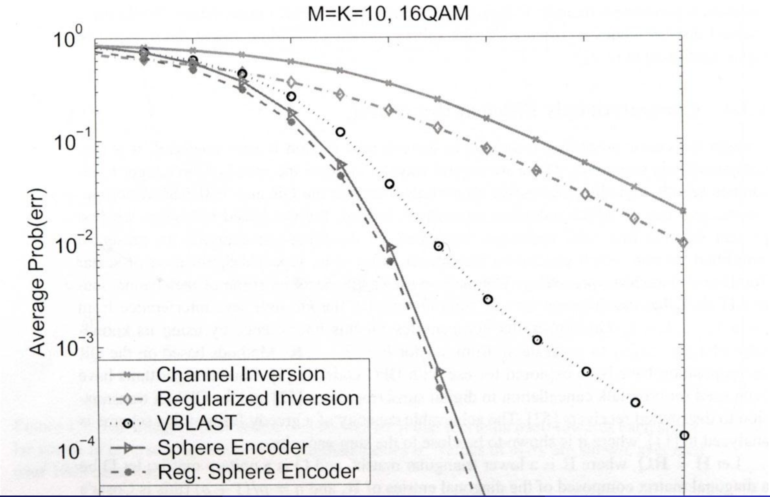
MU-MIMO downlink performance: linear methods

From Peel et al. 2005.



MU-MIMO downlink performance comparison

From Peel et al. 2005.



MU-MIMO downlink with multiple antennas per user

- User k has $M_k > 1$ antennas.
- The base has several possible strategies, including :

1. (1) $m_k = M_k$, i.e. base sends as many symbols as receive antennas. This assumes $N \geq \sum_k M_k$.
   - Sending one different symbol to each user antenna (e.g. in ZF or MMSE sense).
   - Block diagonalization: the base cancels inter-user but not intra-user interference. Users perform stream separation.
2. (2) $m_k = 1$, i.e. base sends as many symbols as users. This assumes $N \geq K$.
   - Each user performs MMSE RX beamforming *on the combined DL channel+filter*

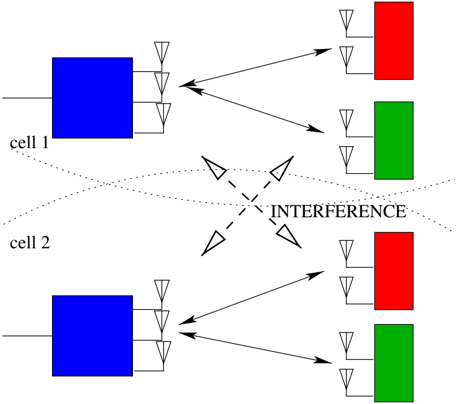
Multicell MU-MIMO

- The network contains several multiuser MIMO links, sharing the same resource.
- Neighboring coverage regions overlap each other.

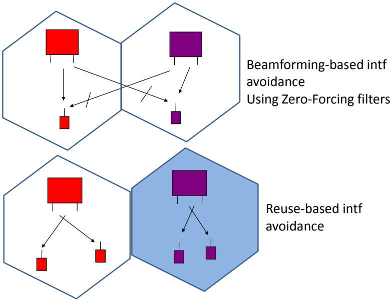
Two key approaches are possible

- MIMO links can be competing or cooperating.
- Cooperation is infrastructure based
- Unlike mobile relaying, infrastructure cooperation works with standard user devices
- No spectral efficiency consumed in BTS relaying (BTS are connected via high speed optical fibers)
- Cooperation can still however be limited by partial channel knowledge

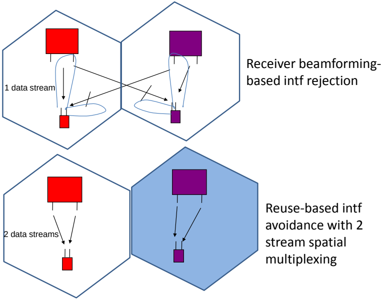
Multicell Interference



Exploiting spatial DoF wisely: reuse vs. beamforming

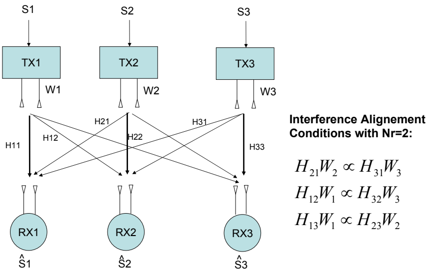


Exploiting spatial DoF wisely: The inefficiency of rejection



- Avoidance and rejection are not free: consume receiver's degrees of freedom
- Are there better ways to handle interference?

Coordination using multiple antenna: Alignement



Alignement can be carried out in space, frequency, time domains. A optimal DoF of 1/2 can be achieved (everyone gets half the cake) *[Maddah-Ali, Motahari, Khandani, Trans IT 2008] [Cadambe, Jafar, Trans IT 2008]*

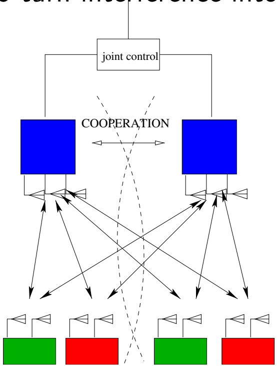
Interference Alignement: Algorithm design

Exploiting uplink downlink duality of alignement [Gomadam et al., 08]

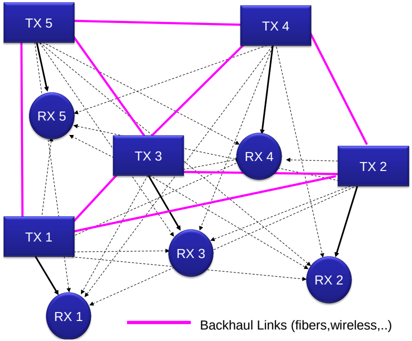
1. Let U_i be the receiver beamforming vector at user i.
2. Let I_i be the total noise summed at RX i, with covariance Q_i.
3. Take U_i as minimum eigenvector of Q_i, $\forall i$.
4. Use U_i as transmit beamforming vector from user i.
5. Take W_i as RX vector at base i, on reciprocal channel.
6. Find W_i as minimum eigenvector of noise covariance matrix at base i.
7. Back to step 2 and iterate.

Multicell MU-MIMO

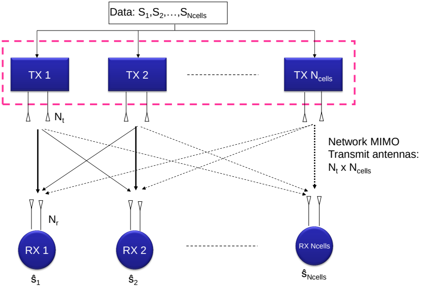
Multi-cell cooperation exploits overlap of coverage regions. We use signal processing at PHY layer to turn interference into a benefit for edge of cell users.



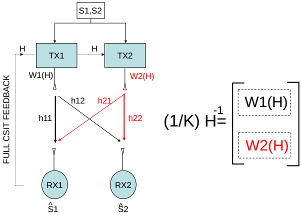
Multi-cell (Network) MIMO



Multi-cell (Network) MIMO

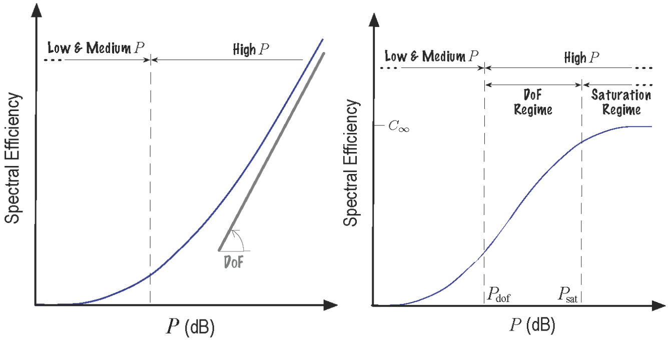


How does it work?



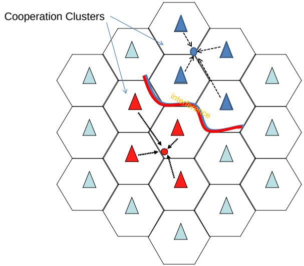
Modify standard MU-MIMO schemes to reflect per base power constraint (ZF, MMSE, non-linear precoding: Dirty Paper Coding, vector perturbation, ..)

Myth and Reality of Transmitter Cooperation



* *A. Lozano et al, "Fundamental limits of cooperation", IEEE Trans. On Information Theory, Sept. 2013.*

Multi-cell MIMO with clustering



Massive MIMO for 5G