From Torch to Projector: Fundamental Tradeoff of Integrated Sensing and Communications

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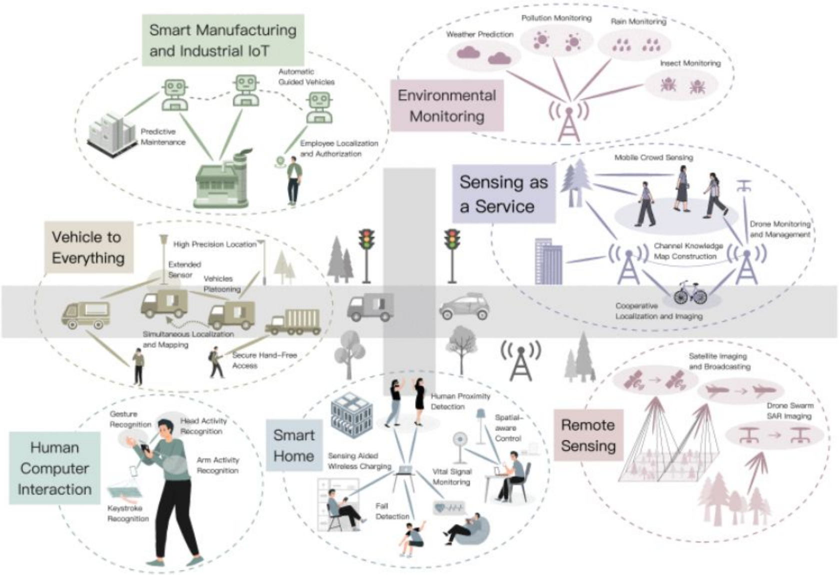
Structure of the presentation

- What is ISAC
- The Torch Metaphor and motivation for the Projector
- DRT and ST in PRACTICAL ISAC SYSTEMS
- Video demo of ISAC

What is ISAC

- Integrated Communication and sensing



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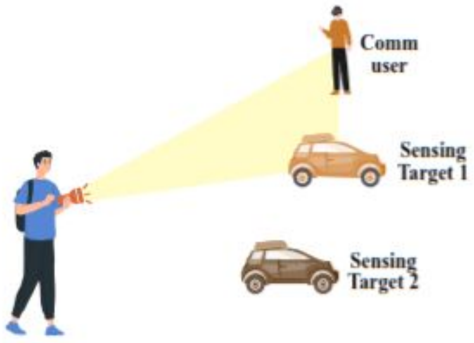
The Torch Metaphor

$$\mathbf{Y}_{\mathrm{c},n} = \boldsymbol{H}_{\mathrm{c}}\mathbf{X}_n + \mathbf{Z}_{\mathrm{c},n},$$
$$\mathbf{Y}_{\mathrm{s},n} = \eta\boldsymbol{H}_{\mathrm{s}}\mathbf{X}_n + \mathbf{Z}_{\mathrm{s},n},$$

System model

$$R = \lim_{N\to\infty} \frac{1}{N} \log M_N$$
$$E = \lim_{N\to\infty} \frac{1}{N} \log \frac{1}{\delta_N}$$

Performance metrics



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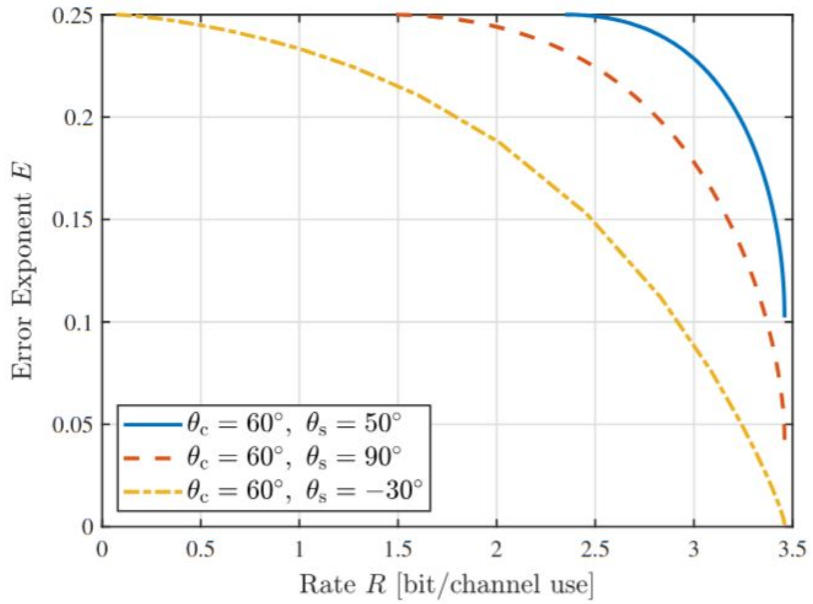
The Torch Metaphor

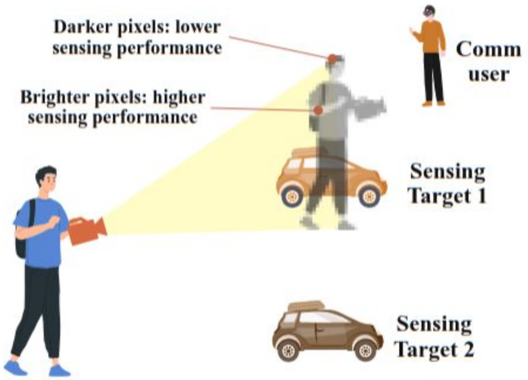


$$R \leqslant \log \left| \boldsymbol{I} + \sigma_c^{-2} \boldsymbol{H}_c \widetilde{\boldsymbol{R}}_{\mathbf{X}} \boldsymbol{H}_c^{\mathrm{H}} \right|,$$

$$E \leqslant \frac{1}{4} \mathrm{Tr} \left\{ \sigma_s^{-2} \boldsymbol{H}_s \widetilde{\boldsymbol{R}}_{\mathbf{X}} \boldsymbol{H}_s^{\mathrm{H}} \right\},$$

Towards The Projector

- Problems with The Torch metaphor:
  - Does the tradeoff equation hold in general?
  - If not, under what conditions?
  - => new approach is needed



- What is the <u>comms</u> optimal vs <u>sensing</u> optimal <u>waveform</u>?
  - Deterministic-random Tradeoff (DRT)

Torch to Projector Metaphor

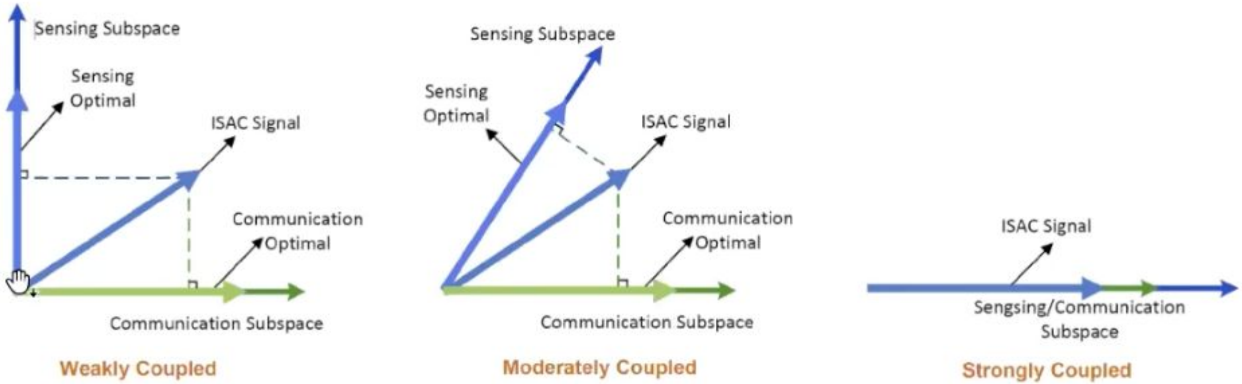
Aspect	Torch Metaphor	Projector Metaphor
Focus	Spatial energy distribution (beamforming).	Both spatial and statistical properties of signals.
Scope	Mainly applies to beamforming in ISAC.	Generalized framework for estimation and detection tasks.
Mathematical Tools	Intuitive analogy without rigorous structure.	Incorporates Fisher Information, CRB, and subspace projections.
Signal Properties	Energy spread (narrow vs. wide beam).	Decomposition into deterministic (sensing) and random (communication) components.
Empirical Distribution	Not addressed.	Explicitly considers the type (distribution) of the signal.
Performance Metrics	Implicitly connected to beamwidth.	Directly links to CRB, P_D, P_{FA}, and spectral efficiency.

S&C Tradeoff as a Two-fold Tradeoff

when moving from P_{CS} to P_{SC} Two things occur

- Deterministic-Random Tradeoff (DRT): The randomness of the ISAC signal reduces

- Subspace Tradeoff (ST): The signal power moves from the comms subspace to the sensing subspace



ISAC Gain closely related to coupling strength

ISAC Signals

Signal Model

$$Y_c = H_c X + Z_c, \quad Y_s = H_s(\eta) X + Z_s$$

Parameters

- H_c, H_s: communication and sensing channels
- η: sensing parameters, e.g., angle, range, velocity, $\eta \sim p_\eta(\eta)$
- X: ISAC signal, $X \sim p_X(X)$
- $R_X = T^{-1} X X^H$: sample covariance matrix
- $\tilde{R}_X = \mathbb{E}(R_X)$: statistical covariance matrix

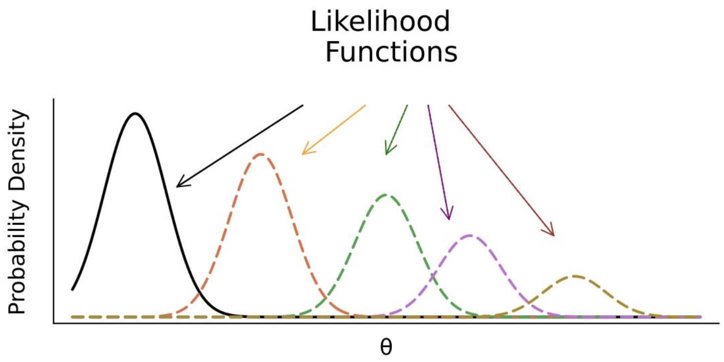
DRT - Deterministic-Random Trade Off

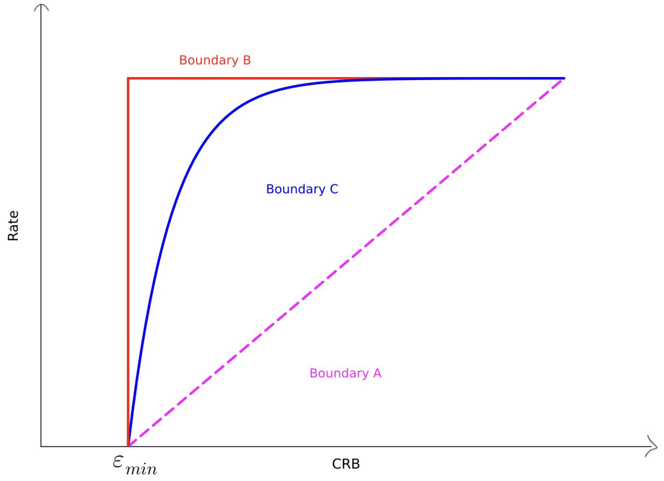
Randomness of the ISAC signal reduces when moving from P_{CS} to P_{SC}

	Sensing Metrics		Communication Metrics
Detection	- **Detection probability:** $P_D = \Pr(\mathcal{H}_1 \mid \mathcal{H}_1)$	**Efficiency**	- Spectral Efficiency (SE)
	- **False alarm probability:** $P_{FA} = \Pr(\mathcal{H}_1 \mid \mathcal{H}_0)$		- Energy Efficiency (EE)
Estimation	- **Mean Squared Error (MSE):** $\epsilon_\theta = \left(\mathbb{E}\left(\theta - \hat{\theta})^2\right)\right)$	**Robustness**	- Bit Error Rate (BER)
	- **Cramer-Rao Bound (CRB):** $\operatorname{var}(\hat{\theta}) \geq \frac{1}{-\mathbb{E}\left(\frac{\partial^2 \ln p(y_R;\theta)}{\partial \theta^2}\right)} \triangleq \mathrm{CRB}(\hat{\theta})$		- Symbol Error Rate (SER)
			- Frame Error Rate (FER)
Recognition	- **Recognition Accuracy**		

)

Cramer-Rao Bound (also the inverse FIM)



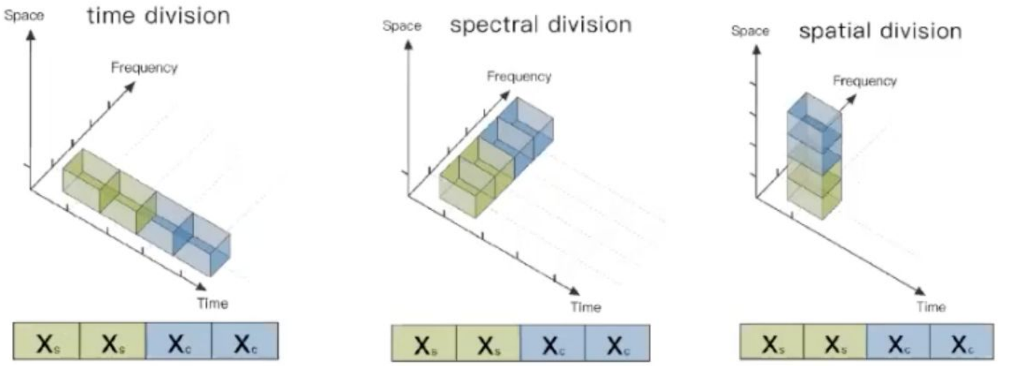


$$\underbrace{\mathbb{E}\left\{(\eta-\hat{\eta})(\eta-\hat{\eta})^H\right\}}_{\text{MSE}} \succeq \underbrace{J^{-1} = \left\{\mathbb{E}\left[\left(\frac{\partial^2 \ln p(Y,\eta)}{\partial\eta\partial\eta^H}\right)^2\right]\right\}^{-1}}_{\text{Inverse FIM}}$$

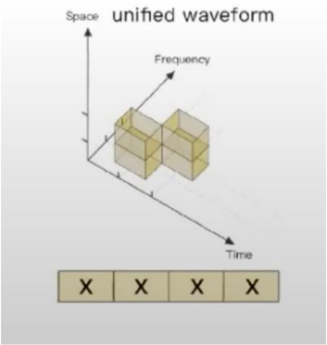
Trade-Off between sensing accuracy (minimizing CRB) and communication performance (rate)

Unified Waveform

Orthogonal Resource Allocation



Unified Waveform



Sensing-Optimal Waveform

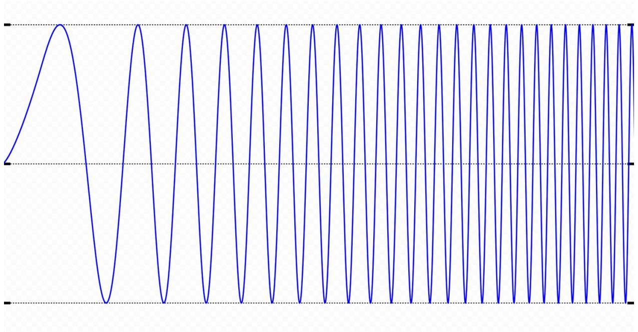
$$\mathbf{X} = \sqrt{T}(\tilde{\mathbf{R}}_X^{SC})^{\frac{1}{2}}\mathbf{Q} = \sqrt{T}\mathbf{U}_S\mathbf{\Lambda}_S^{\frac{1}{2}}\mathbf{Q}$$

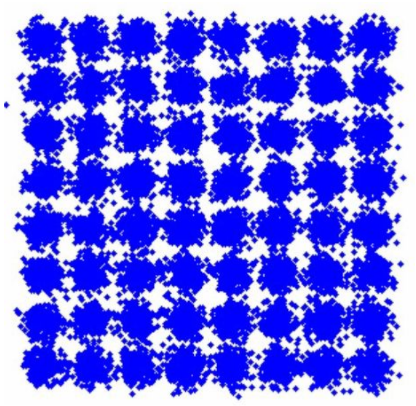
Communication-Optimal Waveform

$$\mathbf{X} = \sqrt{T}(\tilde{\mathbf{R}}_X^{CS})^{\frac{1}{2}}\mathbf{D} = \sqrt{T}\mathbf{U}_c\mathbf{\Lambda}_c^{\frac{1}{2}}\mathbf{D}$$

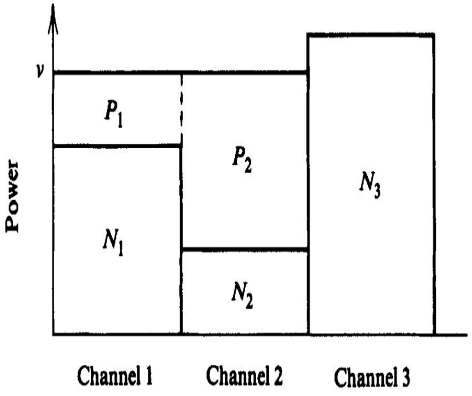
DRT And ST in Practical ISAC Systems

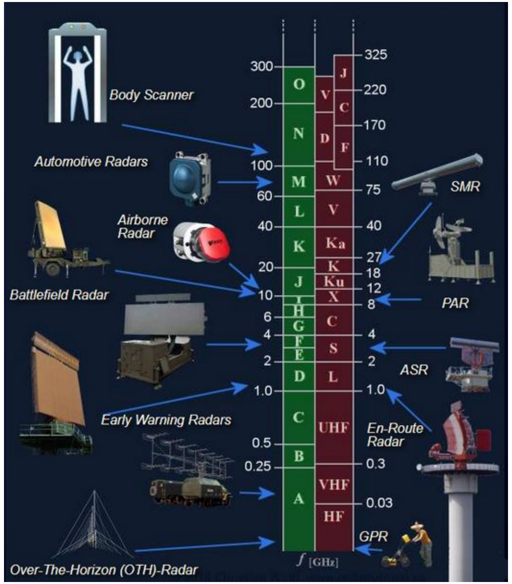
DRT: Determination and randomness tradeoff





ST: Subspace tradeoff —>communication subspace and sensing subspace





DRT: Sensing with Random Signals

Tools: Linear minimum MSE (LMMSE) estimator

· In precoding design the water-filling solution may not be optimal due to the randomness

Proposed 2 methods:

· Data-dependent design

Solve

$$\min_{\|\boldsymbol{W}_n\|_F^2 = P_{\mathrm{T}}} \mathrm{Tr}\left[\left(\boldsymbol{R}_{\mathbf{H}}^{-1} + \frac{1}{\sigma_{\mathrm{s}}^2 N_{\mathrm{s}}} \boldsymbol{W}_n \boldsymbol{S}_n \boldsymbol{S}_n^{\mathrm{H}} \boldsymbol{W}_n^{\mathrm{H}}\right)^{-1}\right]$$

closed-form solution

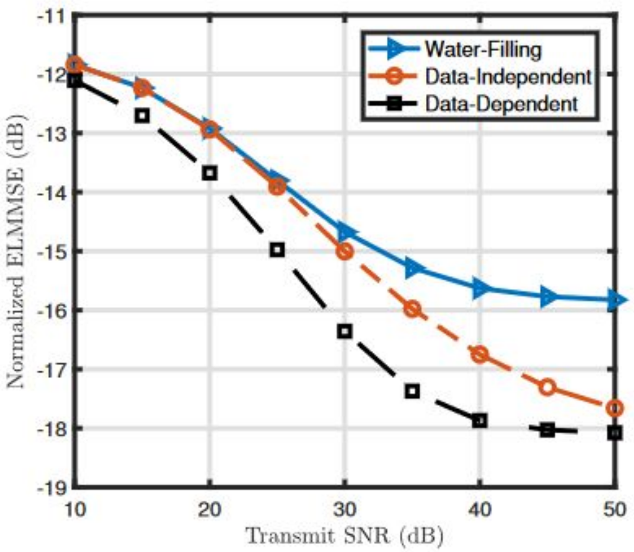
· Data independent design

Solve

$$\min_{\|\boldsymbol{W}_n\|_F^2 = P_{\mathrm{T}}} \mathbb{E}\left\{\mathrm{Tr}\left[\left(\boldsymbol{R}_{\mathbf{H}}^{-1} + \frac{1}{\sigma_{\mathrm{s}}^2 N_{\mathrm{s}}} \boldsymbol{W} \mathbf{S} \mathbf{S}^{\mathrm{H}} \boldsymbol{W}^{\mathrm{H}}\right)^{-1}\right]\right\}$$

SGD

DRT: Sensing with Random Signals



(b) $M = 64, N_s = 32, T = 32$

Data-dependent design

· Pros: Low LMMSE error

· Cons: Huge cost in computation

Data-independent design

· Pros: Easy to compute

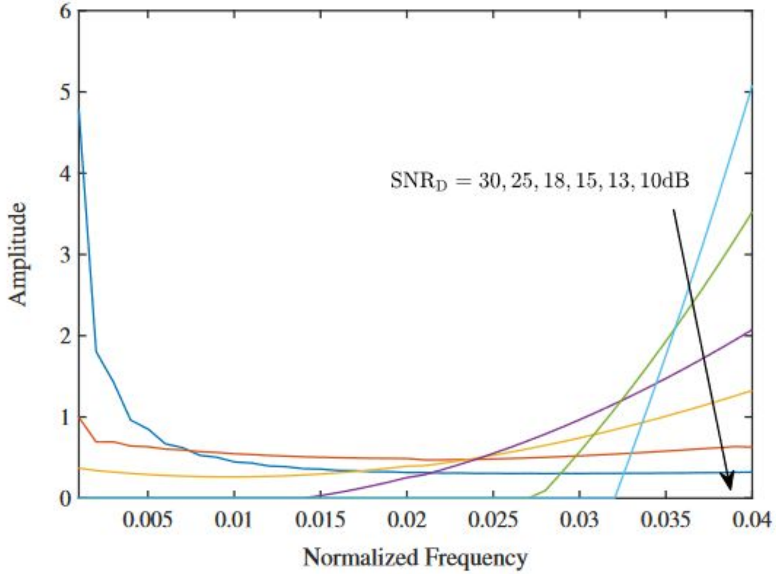
· Cons: Inferior performance (than data-dependent design)

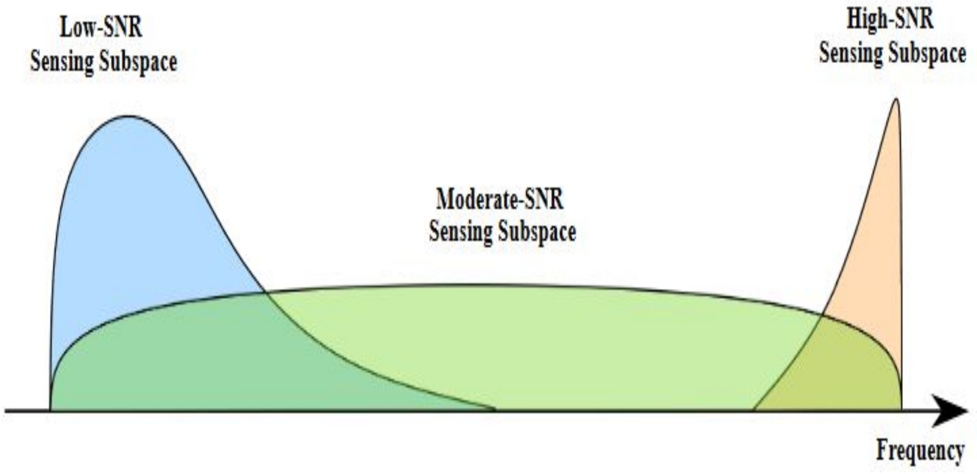
Frequency-domain ST: Valuating Sensing Resources

Tools: Ziv-Zakai bound (ZZB) to mesure MSE

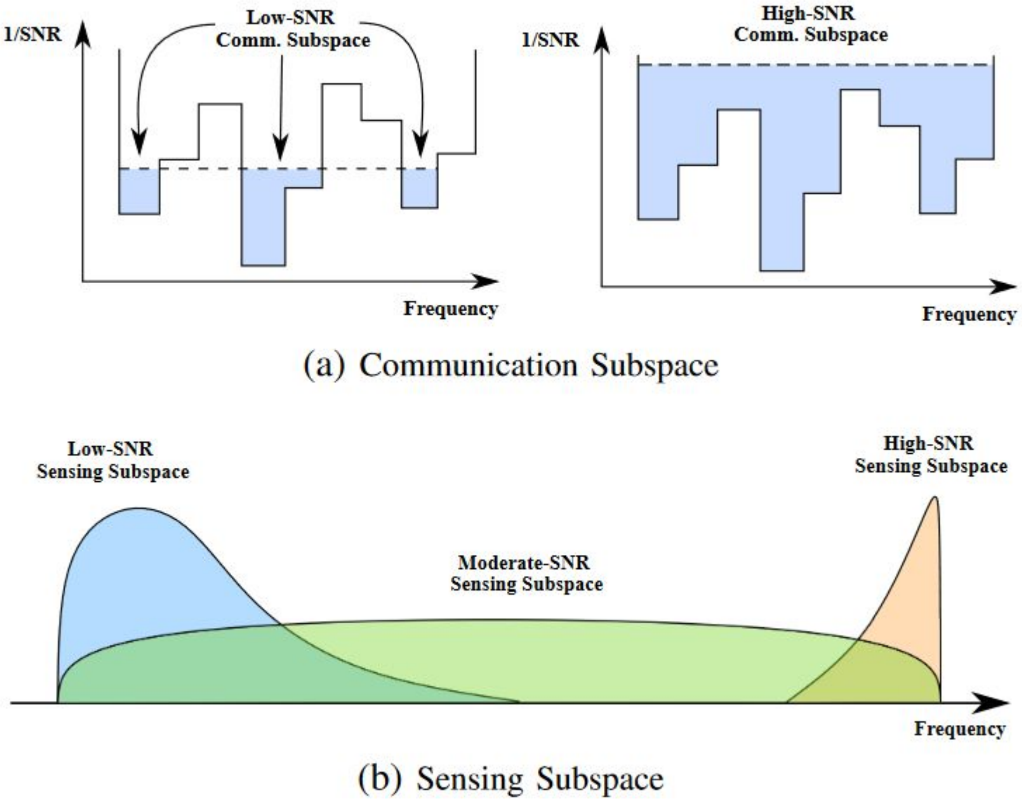
$$\mathbb{E}\left\{(d-\hat{d})^2\right\} \geqslant \int_0^{\epsilon_{\max}} xQ\left(\sqrt{2^{-1}\mathrm{SNR}(1-\widetilde{R}(x))}\right)\mathrm{d}x$$

Numerically computed PSDs of ZZB optimal waveforms in different SNRs





Frequency-domain ST: Different strategies in SNRs



Sum up to this paper

- Theoretical analysis
- Tradeoffs faced in ISAC being further clarified
- Designs to balance these tradeoffs might be the keys in future ISAC employment
- Lots of problems remaining to be solved