Midterm Take-Home Exam

December 20, 2024 (DUE January 10, 2025)

Instructions

- Open book and open class notes are allowed (including notes taken by students during exam). No other notes are allowed.
- Each answer should be clearly written, and the solution should be developed in detail.
- Mathematical derivations need to show all steps that lead to the answer.
- Partial credit will be given for incomplete solutions.
- There is NO penalty for incorrect solutions.

Hints - equations - conventions:

- Notation
  - R represents the rate of communication in bits per channel use (bpcu),
  - ρ represents the SNR (signal to noise ratio),
  - w will denote additive noise which will be distributed as a circularly symmetric Gaussian random variable $\mathbb{C}\mathcal{N}(0, N_0)$. If N_0 is not specified, then set $N_0 = 1$,
  - Remember: for a given signal-to-noise ratio (SNR), then SNR in dB is simply $10 \log_{10} SNR$
  - SISO stands for single-input single-output, MISO stands for multiple-input single output, SIMO stands for single-input multiple output, MIMO stands for multiple input multiple output.
  - MU stands for multi-user.
  - CSIT stands for channel state information at the transmitter, while CSIR stands for channel state information at the receiver.
  - AWGN stands for additive white Gaussian noise.
- GOOD LUCK!!

EXAM PROBLEMS

1) *(1 point)*. Consider a SISO setting, with no fading. Consider that the maximum possible rate (i.e., the capacity) is equal to 7 bpcu. What is the minimum SNR required to achieve this rate? Do you need CSIR?

2) *(1 point)*. Consider a SISO quasi-static fading channel with no CSIT. We wish to decrease the probability of error, from $P_{err} \approx (SNR)^{-1}$ to $P_{err} \approx (SNR)^{-4}$. Suggest various ways we can achieve this, based on what we have learned in class.

3) *(1 point)*. What are some of the advantages of MISO vs. SIMO, mentioned in class?

4) *(1 point)*. In a single-user MIMO channel, how much diversity gain would we be able to get if we employed a transmitter with 4 transmit antennas and a receiver with 2 receive antennas, when in fact the channel between the first transmit and receive antenna, is identical always to the channel between the first transmit and second receive antenna?

5) *(1 point)*. In a single-user MISO channel, how much multiplexing gain would we be able to get if we employed a transmitter with 2 transmit antennas?

6) *(1 points)*. Consider communication over a quasi-static 2×1 MISO fading channel. Assume that you must draw symbols from 16-QAM.
   - Can you name a space time code, that gives full diversity in this setting, and then describe the rate (in bpcu) of such a code.

7) *(3 points)*. In the context of various strategies, answer if each of the following statements are true or false, justifying briefly your answers.
   - In a MISO channel, we can get transmitter beamforming gain even without CSIT.
   - A base station equipped with 5 antennas in the downlink, can <u>simultaneously</u> serve up to 5 users (single receive antenna each).
   - A base station equipped with 5 antennas in the downlink, can <u>simultaneously</u> serve up to 10 users (two receive antennas each).
   - A base station equipped with 4 antennas in the downlink, can <u>simultaneously</u> serve up to 2 users (two receive antennas each).
   - Line of sight channels are detrimental for spatial multiplexing in both single-user and multiuser MIMO.
   - For a MIMO receiver using spatial multiplexing, the complexity of ZF receiver is more than the complexity of the maximum-likelihood receiver.
   - CSIT is easier to obtain than CSIR.
   - CSIT is of cardinal importance in multi-user MIMO.

8) *(2 points).* In a MU-MIMO channel, if I double the number of users I simultaneously serve, must I always halve the individual rate to each user? Justify your answer.

9) *(4 points).* Consider communication over the 2×1 quasi-static fading MISO channel, using a diagonal code (see below for details) such that the channel model is given by

$$\overbrace{\begin{pmatrix} y_1 & y_2 \end{pmatrix}}^{y}=\theta\overbrace{\begin{pmatrix} h_1 & h_2 \end{pmatrix}}^{h}\overbrace{\begin{pmatrix} x_1 & 0 \\ 0 & x_2 \end{pmatrix}}^{\mathcal{X}_{tr}}+\overbrace{\begin{pmatrix} w_1 & w_2 \end{pmatrix}}^{w}$$

where $h_i\sim\mathbb{C}N(0,1)$ and $w_i\sim\mathbb{C}N(0,1)$, and where θ is the power normalization factor that lets you regulate SNR.

- Describe the ML decoding rule for this case.
- Describe the cardinality[1] of code $\mathcal{X}_{tr}$ if you wish a rate of $R=4$ bpcu.
- For a desired rate of $R=8$ bpcu, and a desired $SNR=10$ dB (where by SNR we mean the AVERAGE signal power divided by the noise unit power, under QAM) then what is the normalizing factor θ?
- Imagine that what you transmit (x_1,x_2) are independently chosen from 16-PAM, then
  - What is the rate of your code (in bpcu)?
  - What is the slope of your probability of error, in high SNR, if you plot on the y-axis the probability of error, in log scale ($\log_{10}$(Prob)), and the x-axis is the SNR, in dB?
- Imagine now that $\begin{pmatrix} x_1 & x_2 \end{pmatrix}=\begin{pmatrix} s_1 & s_2 \end{pmatrix}\cdot\mathbf{Q}$, where s_1,s_2 are independently chosen from a 64-QAM constellation, where the matrix $\mathbf{Q}$ is a randomly chosen orthogonal matrix. Then
  - What is the rate of your code?
  - What is the aforementioned slope of your probability of error?

10) *(Extra Credit: 5 points).* Consider communication over a quasi-static 2×2 MIMO channel, utilizing the space-time code $\mathcal{X}=\{\mathbf{X}_1,\mathbf{X}_2,\mathbf{X}_3,\mathbf{X}_4\}$, where

$$\mathbf{X}_1=\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix},\ \mathbf{X}_2=\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix},\mathbf{X}_3=\begin{bmatrix} -1 & -1 \\ 1 & -1 \end{bmatrix}\text{ and }\mathbf{X}_4=\begin{bmatrix} -1 & 1 \\ -1 & -1 \end{bmatrix}$$

- What is the average SNR?
- What is the rate of the code in bpcu?
- What is the diversity gain of this code?
- What is the approximate (in the high SNR regime) probability of error of this code, if SNR is 30dB?

11) *(1 point).* Consider the following distributed setup with $N=3$ workers, as shown in Figure 1. There are $K=3$ datasets, W_1, W_2, and W_3, each of size 100 MBytes and each cache has a storage capacity of 200 MBytes.
   a) How does one need to distribute the datasets across the workers to ensure that the master node can recover all the datasets from any 2 workers?
   b) Assume that there is a delay constraint of 10 milli-seconds (ms) allowed for the master node to receive all the information. What is the total rate R (in <u>received</u> bits per milli-second) with which the master node will be receiving data, in order for it to successfully recover W_1, W_2, W_3?

[1]By cardinality we mean the number of matrices that the code has.

c) Let us now assume that the master node also has memory and can cache one dataset. What is the total rate of transmission the master needs from the workers to successfully recover W_1, W_2, W_3?

d) Comment on the role of memory on the transmission rate from the workers.

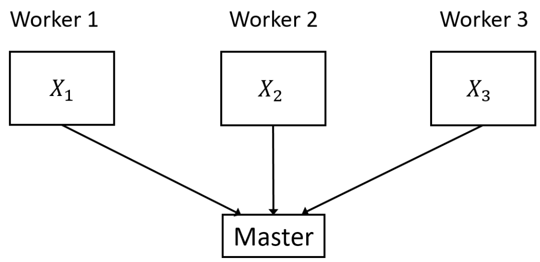

Fig. 1: A distributed computing scenario.

12) *(1 point).* Name two applications that in your opinion popularized Reinforcement Learning in scientific communities working on Artificial Intelligence?

13) *(1 point).* AlphaZero is one of the most exciting real-world applications that works using Reinforcement Learning. Name the two algorithms that are used by AlphaZero based on which this software achieves superior performance compared to humans.

14) *(1 point).* Name two classes based on which one can distinguish exact dynamic programming algorithms?

15) *(1 point).* What is the difference between deterministic and stochastic dynamic programming algorithms?

16) *(5 points).* Quantum. The two components of the state $|\theta\rangle = \frac{\sqrt{3}}{2\sqrt{2}}|11\rangle + \frac{\sqrt{5}}{2\sqrt{2}}|00\rangle$ are shared amidst Alice and Bob. In other words, Alice has the first component and Bob has the second. Alice performs a measurement with two outcomes $\{+1, -1\}$. Outcome $+1$ is associated with operator $|+\rangle\langle+|$ and Outcome -1 is associated with $|-\rangle\langle-|$. Bob performs a measurement with two outcomes $\{+1, -1\}$. Outcome $+1$ is associated with operator $|0\rangle\langle 0|$ and Outcome -1 is associated with $|1\rangle\langle 1|$.

a) Before the measurement is performed, is the joint state entangled or separable?
b) Compute the probability of Alice observing -1 and Bob observing $+1$.
c) Compute the probability of Alice and Bob BOTH observing -1.
d) Compute the probability of Alice and Bob BOTH observing $+1$.
e) Identify the post measurement state when Alice observes $+1$ and Bob observes -1.