Advanced Topics in Wireless: Exam Questions

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1 Read this

Duration of the exam is 2 hours, all documents are permitted. Don't forget to justify where necessary (concisely - a few words is enough!) your answer (and prove mathematically where possible). Use your own words and do not copy sentences from a book/notes.
Also: read carefully the questions!

2 General Context

We consider a cellular network, where, in each cell, a base station (BTS) communicates with a number of mobile terminals (MT) either in uplink or in downlink. The BTS is equipped with M antennas, and each MT is equipped with N antennas. The exam has two parts. The first is on qualitative nature and the second is algorithmic and quantitative.

3 Qualitative questions

Exercise 1 *We consider the use of various MIMO strategies for improving the system performance, as shown in class. Please answer WRONG or CORRECT to each of these statements. When possible, justify the answer with a couple of words (be very concise).*

- *The Alamouti space time code can be designed for a channel with an arbitrary number of receive antennas*
- *The complexity of decoding the Alamouti space time code is exponential with the number of receive antennas*
- *Beamforming at the receive side requires the knowledge of the channel at the transmitter*
- *It is possible to have full beamforming gain and full diversity gain with the same algorithm*

- *The diversity gain between 1 and 2 antennas is less than between 2 and 3 antennas.*
- *The presence of a line of sight component in the channel does not decrease the beamforming gain*
- *The presence of a line of sight component in the channel does not decrease the multiplexing gain in single user communication.*
- *The presence of a line of sight component in the channel does not decrease the multiplexing gain in multiple user communication.*
- *The MMSE is the optimal receiver algorithm (to minimize Bit Error Rate) in a MIMO channel with spatial multiplexing.*
- *The capacity of a MIMO channel using spatial multiplexing increases as log(min(M,N)) where M and N are the number of antennas at the transmitter and receiver respectively.*

4 Quantitative questions

4.1 Notations for signals and channels

We consider the baseband representation of a signal being transmitter from the BTS to a single MT. The transmitted symbol sequence, denoted $s(l)$ is drawn from a complex constellation with variance: $E|s(l)|^2 = \sigma_s^2$, representing the transmit power.
The symbols are sent into a frequency-non-selective (no ISI) MIMO channel represented by the $N \times M$ matrix: $\mathbf{H}$.
The link between the j-th BTS antenna and the i-th MT antenna is denoted by the complex gain $\mathbf{H}_{ij} = h_{ij}$.
The matrix is normalized so that all links have unit variance $E|h_{ij}|^2 = 1$.
We assume the channel are all uncorrelated (between users, between antennas).

4.2 Beamforming on the downlink with TDMA

We assume a TDMA setup, and examine the system during a single time slot. Which means that during that time the MIMO system is a single user MIMO system between the base and the selected mobile.
The modulation symbols are sent or converted into coded symbols before launching on the transmit antennas. The possibly coded symbol (intended to the MT) on the j-th transmit antenna at the BTS, at time n, is denoted $c_j(n)$.
The received signal at the MT at time n is obtained by:

$$\mathbf{y}(n) = \mathbf{H} \begin{pmatrix} c_1(n) \\ c_2(n) \\ \vdots \\ c_M(n) \end{pmatrix} \frac{1}{\sqrt{M}} + \mathbf{b}(n) \tag{1}$$

where $\mathbf{b}(n)$ is an additive noise vector caused by receiver electronics and interference. We assume the noise to be spatially and temporally white, with each component having power $E|b_i(n)|^2 = \sigma_b^2$, $i = 1..N$. The factor $\sqrt{M}$ is used at the transmitter to limit and normalize the power independently of the number of antennas.

Exercise 2 *We assuming M is arbitrary and $N = 1$. We assume the channel is not known at the transmitter and known at the receiver. The transmitter simply sends $c_j(n) = s(n)$ (i.e. the same symbol is repeated over all transmit antennnas). Compute the SNR at the receiver side. Analyze the diversity and beamforming gain of this technique.*

Exercise 3 *We want to use beamforming at the transmit side, we again assume M is arbitrary and $N = 1$. This time we assume the channel is known at the transmitter and at the receiver. Give the form taken by the $c_j(n)$ is optimal beamforming is used, and assuming the total transmit power is kept to σ_s^2. Give the SNR at the receiver, the diversity gain and the beamforming gain.*

4.3 MIMO spatial multiplexing

We want to use the multiple antennas for increasing the rate via spatial multiplexing. We assume $M = N = 2$ and a fixed realization of the channel $\mathbf{H}$.
The channel is given by

$$\mathbf{H} = \begin{pmatrix} 2 & 3 \\ -2 & 4 \end{pmatrix} \tag{2}$$

Exercise 4 *We assume the the channel is known at the transmitter (and at the receiver). Express the optimal transmit and receive strategy over this MIMO channel to maximize the rate, under the constraint that the total transmit power is $\sigma_s^2 = 1$. The noise power is assumed to be 0.1. Compute the optimal power allocation strategy and derive the maximum capacity.*

Exercise 5 *Explain the power allocation strategy in the two following limiting regimes of power: a) $\sigma_s^2 \to \infty$, and b) $\sigma_s^2 \to 0$*

Exercise 6 *Now we assume that the channel matrix is NOT known at the transmitter (only known at the receiver). In this case, give the equations for spatial multiplexing using V-BLAST receiver and signal ordering. Give the SNR obtained when decoding each symbol.*

4.4 Spatial Division Multiple Access (SDMA) in the uplink

We now consider a multiple access protocol where multiple terminals are allowed to access the channel and transmit to the base station simultaneously, **on the uplink channel**. We select K mobile terminals and let them transmit to the base. Each mobile has $N = 2$ antennas. The base still has M antennas.
Each terminal k transmits two independant symbols $s_k^1(n)$ and $s_k^2(n)$ at time n. The signal model for the (noisy) received signal vector at time n at the base station, denoted $y(n)$, is given bys

Exercise 7 *Write the signal model for the (noisy) received signal vector at time n at the base station, denoted $y(n)$.*

Exercise 8 *We assume linear zero-forcing beamforming at the base station receiver. write the equation for the estimated symbols after receiver processing. What is the SNR for each symbol? How many mobiles can this system support if one want to completely cancel the interference between symbols?*

Exercise 9 *We now assume that each user has two transmit antennas and knows the transmit channel coefficients. Assume that the users only to send one stream each. Propose a ways of making use of the two transmit antennas, exploiting the knowledge of the channel. Give the equation to obtain the optimal transmit beamformer at each user.*

4.5 Scheduling

We now consider the uplink situation with $M = 1$ at the base station and N is arbitrary, and K users in the network. We assume that each user knows the transmit channel to the base station. The base station also knows the channels.

Exercise 10 *How many users can be scheduled simultaneously without interference? What is the optimal way of combining the antennas at the user side?*
Give the optimal scheduling rule to maximize the system throughput. Provide the metric that should be maximized by the selected user.

Exercise 11 *What is the diversity order, after user selection, for the system above? Justify.*

THAT'S ALL FOLKS!