Euler

May 27, 2025

[1]:
```
using Plots
```

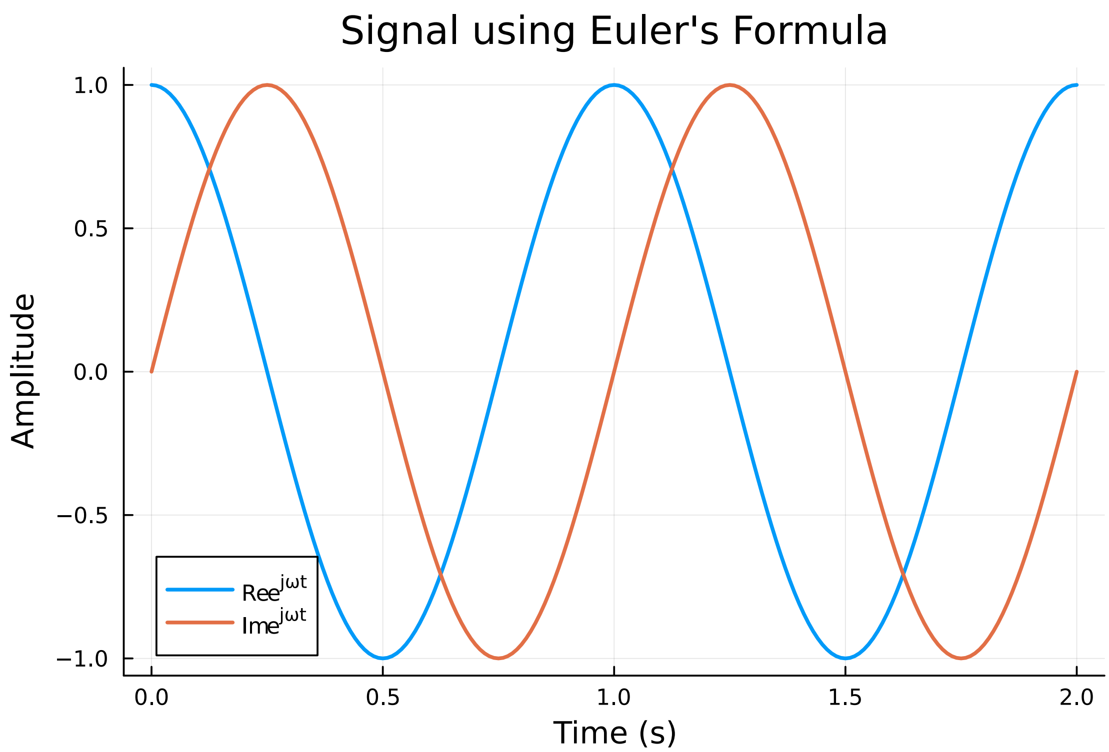
[2]:
```
# Parameters
  = 2           # Angular frequency (1 Hz)
t = 0:0.01:2     # Time vector from 0 to 2 seconds
j = √(Complex(-1.0))
```

[2]:
```
0.0 + 1.0im
```

[3]:
```
# Euler's formula: e^(j t) = cos( t) + j*sin( t)
# signal = exp.(j *   .* t)
signal = cos.(  .* t) + j*sin.(  .* t);
```

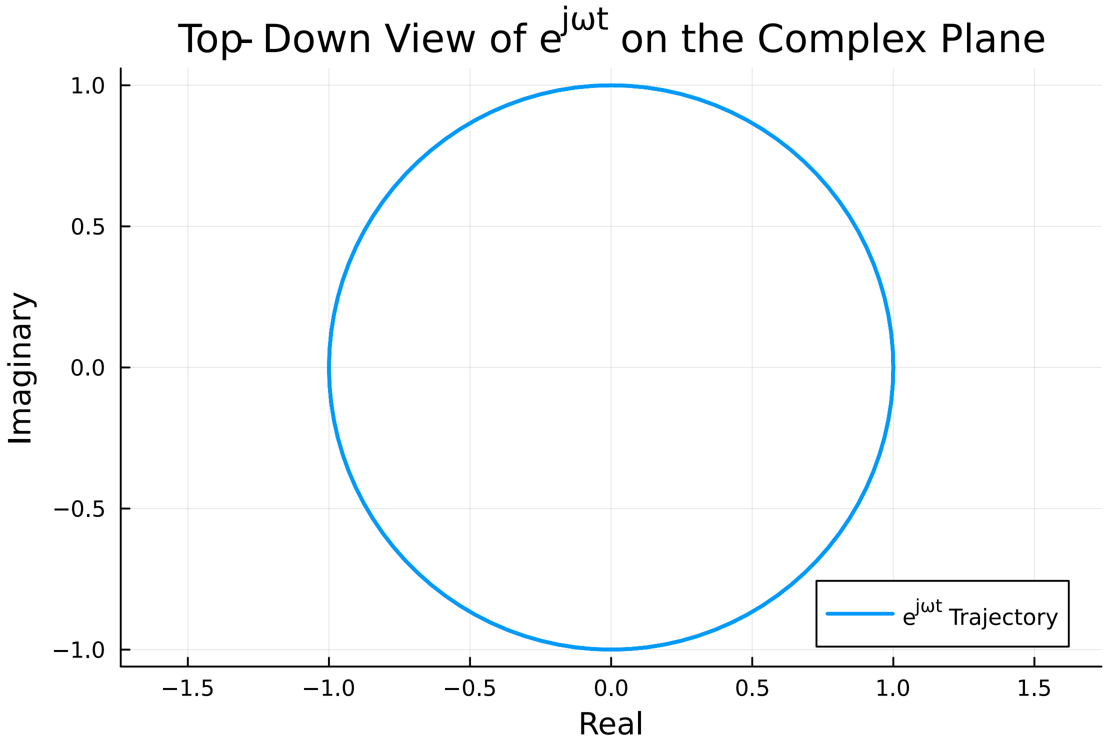
[4]:
```
# Plot real and imaginary parts
plot(t, real.(signal)
    , xlabel = "Time (s)", ylabel = "Amplitude"
    , title = "Signal using Euler's Formula"
    , label="Re{e^{j t}}", lw=2
)
plot!(t, imag.(signal), label="Im{e^{j t}}", lw=2)
```

[4]:



[5]:
```
# Plot in complex plane (top-down view)
plot(real.(signal), imag.(signal)
      , label="e^{j t} Trajectory"
      , xlabel="Real", ylabel="Imaginary"
      , title="Top-Down View of e^{j t} on the Complex Plane"
      , aspect_ratio=:equal, legend=:bottomright
      , lw=2)
```

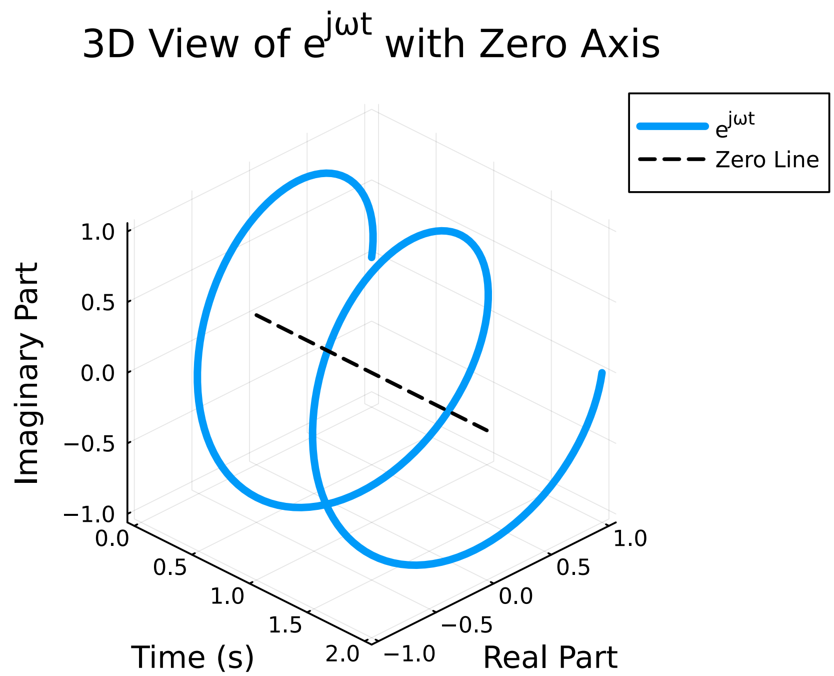
[5]:



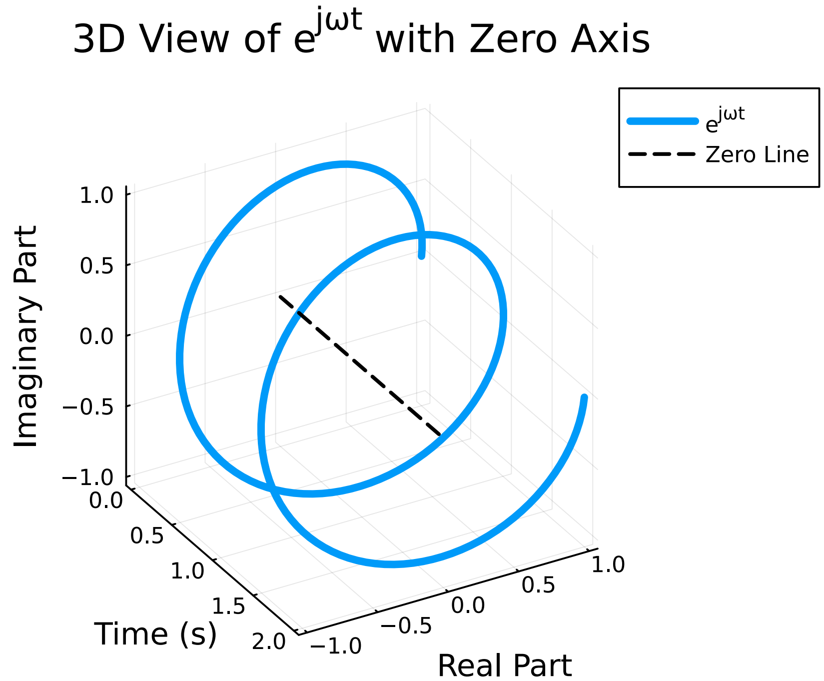
[6]:
```
# Extract components
x = t; y = real.(signal); z = imag.(signal);
```

[7]:
```
# 3D Plot of the signal
plt = plot(x, y, z
    , label="e^{j t}"
    , xlabel="Time (s)", ylabel="Real Part", zlabel="Imaginary Part"
    , title="3D View of e^{j t} with Zero Axis"
    , legend=:topright, lw=4
    , camera=(45, 30))

# Add line traversing through origin along time
plot!(plt, [t[1], t[end]], [0, 0], [0, 0],
    label="Zero Line",
    lw=2,
    linestyle=:dash,
    color=:black)
```

[7]:



[8]:
```
# 3D Plot of the signal
plt = plot(x, y, z
    , label="e^{j t}"
    , xlabel="Time (s)", ylabel="Real Part", zlabel="Imaginary Part"
    , title="3D View of e^{j t} with Zero Axis"
    , legend=:topright, lw=4
    , camera=(60, 30))

# Add line traversing through origin along time
plot!(plt, [t[1], t[end]], [0, 0], [0, 0],
    label="Zero Line",
    lw=2,
    linestyle=:dash,
    color=:black)
```
[8]:



Sinusoidal signals look like a coil in 3D — only specific combinations do, especially **complex exponentials** or **paired sinusoids with quadrature phase**.

0.0.1 Let's break it down:

When you do get a coil in 3D: You get a **3D helix (coil)** when you plot a signal like:

```
x(t) = t
y(t) = cos(t)
z(t) = sin(t)
```

This forms a **helix** because the signal traces a **circular path in the y-z plane** (via `cos` and `sin`), and moves **linearly in x (time)**.

This pattern arises in:

- **Euler's formula**: $e^{j\omega t} = \cos(\omega t) + j\sin(\omega t)$
- **Quadrature carriers** in communications (I/Q modulation)
- **Spinning phasors** in signal processing
- Circular/elliptical motion physics

0.0.2 When you don't get a coil:

Case 1: Pure real sinusoid

```
x(t) = t
y(t) = sin( t)
z(t) = 0
```

You just get a **wave along x-y**, flat in z → no coil.

Case 2: Arbitrary sinusoids not in quadrature

```
y(t) = sin( t)
z(t) = sin( t + )    # not exactly ± /2
```

You get **wobbly curves**, possibly **Lissajous figures** — not a helix.

Case 3: Mixed frequencies

```
y(t) = sin( t)
z(t) = sin( t)
```

The signal may trace **complex 3D shapes**, but not a simple coil.

0.0.3 Summary:

Signal Type	3D Shape
`cos( t) + j*sin( t)`	Helix / Coil
Pure `sin( t)` or `cos( t)`	Flat wave
`sin( t)`, `sin(2 t)`	Complex waveform
`sin( t)`, `cos( t + )` (/2)	Lissajous-like path