Neural Network – Decision Trees

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EXERCISE 1. NEURAL NETWORKS

Consider the Boolean function $\varphi(x_1,x_2,x_3)$ = (x_1 AND NOT x_2) OR (x_2 AND NOT x_3).

Neurons use the transfer function $f(t) = \text{sign}(t) = \begin{cases} 0 & \text{if } t \le 0 \\ 1 & \text{if } t > 0 \end{cases}$

1. Is it possible that a single neuron with three binary inputs x_1,x_2,x_3 has an output which is identical to $\varphi(x_1,x_2,x_3)$? Motivate your answer.
2. Design a Multi-Layer Perceptron with three binary inputs x_1,x_2,x_3 so that its output is identical to $\varphi(x_1,x_2,x_3)$. Clearly indicate the number of layers, number of neurons, and connection weights and threshold for each neuron.

EXERCISE 2. NEURAL NETWORKS

Consider a single artificial neuron with three inputs, x_1, x_2, and x_3, and a sigmoid transfer function:

$$f(z) = \frac{1}{1+e^{-z}}$$

The output of the neuron is $y = f(w_0 + w_1 x_1 + w_2 x_2 + w_3 x_3)$.

1. Given that the values of x_1, x_2, and x_3 can either be 0 or 1, find values of the weights w_0, w_1, w_2 and w_3 so that the output of the neuron is greater than 0.5 if and only if the condition (x_1 AND x_2) OR x_3 is true.
2. What is the derivative of the sigmoid transfer function $f(z)$?
3. Assuming that we have a training example (x_1, x_2, x_3) for which the value of the expected output of the neuron is t, while the value of the actual output is o, what are the updates Δw_i of each weights w_0 w_1 w_2 w_3 when we apply the Back-Propagation algorithm to this training sample ?

EXERCISE 3. DECISION TREES

In this question we consider Decision-Tree learning based on the Boolean random variables X_1, X_2, X_3 and Y, for which we have the following samples:

	X_1	X_2	X_3	Y
e_1	true	true	false	false
e_2	false	true	false	true
e_3	false	true	true	true
e_4	false	false	true	false
e_5	true	false	false	false

1. What is the best attribute to split on first? Explain why.
2. Draw a decision tree that the Decision-Tree learning algorithm could build. For each node (including the leaves) show which samples are used to determine the classification at that node. (The root note of the tree will be labelled with the list of all of the samples).

3. consider the new sample:

e_6	false	false	false	false

Show how the above tree classifies that instance.

EXERCISE 4. DECISION TREES

You are stranded on a desert island. The only available food is composed of mushrooms of various types. Some types have been determined as poisonous and others as not (determined by your former companions' trials and errors), according to the following table:

Mushroom Type	Heavy	Spotted	Smooth	Poisonous
A	No	No	No	No
B	No	Yes	No	No
C	Yes	No	Yes	No
D	Yes	No	Yes	Yes
E	No	Yes	No	Yes
F	No	Yes	Yes	Yes
G	No	No	Yes	Yes
H	Yes	No	No	Yes
U	Yes	Yes	Yes	?
V	No	No	Yes	?
W	Yes	No	No	?

Note that $log_2(3) = 1.6$ and $log_2(5) = 2.3$.

1. What is the entropy of the random variable **Poisonous** ?
2. Which attribute should you use at the root of the decision tree ? What is the corresponding information gain ?
3. Based on the best attribute selected in the previous question, classify mushrooms U, V and W as poisonous or not.

END OF EXERCISES