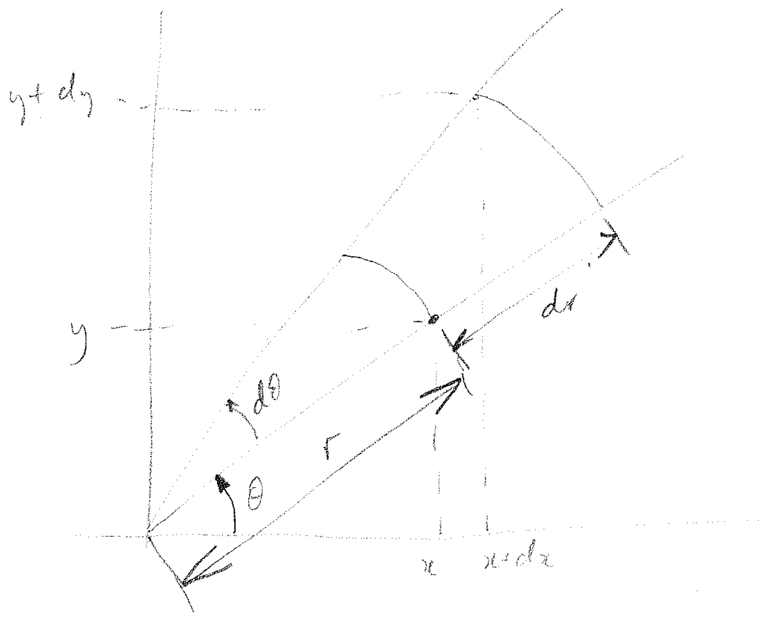
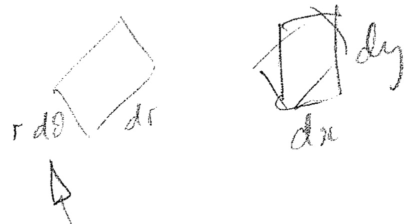
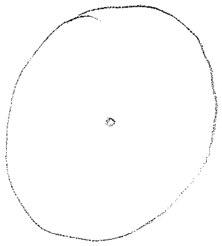






2π radians correspond to
a circumference of 2πr

dθ radians correspond to a
circumference of $\frac{\partial\theta}{2\pi}$ of $2\pi r$

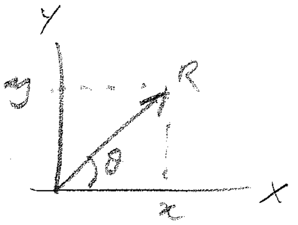
$= d\theta . r$

& $$\frac{\partial(x,y)}{\partial(r,\theta)} = \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{dx}{\partial\theta} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial\theta} \end{vmatrix} = \frac{dx}{\partial r}\cdot\frac{dy}{\partial\theta} - \frac{\partial x}{\partial\theta}\cdot\frac{\partial y}{\partial r}$$

$$= r$$

$$f_{XY}(x,y) = \frac{\exp(-(x^2+y^2)/2\sigma^2)}{2\pi\sigma^2}$$

$$R = \sqrt{X^2+Y^2} \qquad \theta = \tan^{-1}\left(\frac{Y}{X}\right)$$



$$X = R\cos\theta = g_1^{-1}(R,\theta)$$
$$Y = R\sin\theta = g_2^{-1}(R,\theta)$$

$$\cos\theta = \frac{x}{R}$$
$$\sin\theta = \frac{y}{R}$$

Under the transform, the infinitesimal area $dx\,dy$ in the x,y plane transforms to the area $r\,dr\,d\theta$ in the $r\theta$ plane:

$$\frac{\partial(x,y)}{\partial(r,\theta)} = \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} \end{vmatrix}$$

$$= \begin{vmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & +r\cos\theta \end{vmatrix}$$

$$= r\cos^2\theta + r\sin^2\theta$$

$$= r$$

$$f_{UV}(r,\theta) = f_{XY}(x,y)\left|\frac{\partial(x,y)}{\partial(r,\theta)}\right|_{\substack{x=g_1^{-1}(r,\theta)\\ y=g_2^{-1}(r,\theta)}}$$

$$= \frac{\exp(-r^2/2\sigma^2)\,r}{2\pi\sigma^2}$$