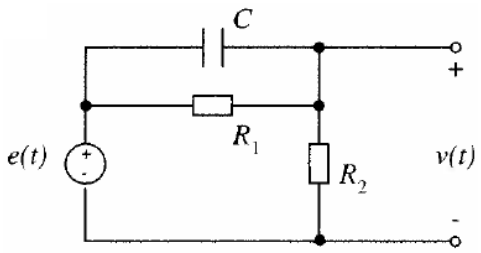
EURECOM
Sophia Antipolis

Essential Mathematical Methods for Engineers (MathEng)

Transfer function and system characterisation

1. Determine the transfer function and sketch the pole/zero map for the following circuit:



where the corresponding differential equation is given by:

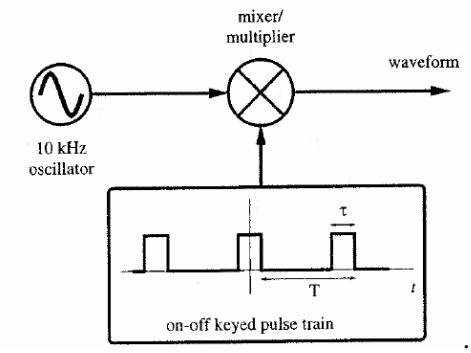
$$\frac{e-v}{R_1}+C\frac{d}{dt}(e-v)=\frac{v}{R_2}.$$

2. Draw the pole/zero maps and hence sketch the frequency responses of a system with the following transfer function:

$$H(s)=\frac{s^2-0.4s+400.04}{(s^2+2s+101)(s^2+2s+901)}.$$

3. Given that the Fourier transform of the complex phasor signal $\exp(j\omega_a t)/(2\pi)$ is $\delta(\omega-\omega_a)$, determine the Fourier transform of the signal $A\sin(\omega_a t)$.

4. A radar waveform is generated using the mechanism shown below. Sketch the three time-domain waveforms and the corresponding magnitude Fourier transforms. The frequency of the sine wave (carrier frequency) is 10 kHz and the pulse duration τ = 1 ms. The pulse repetition period T used in the system is 10 ms.



5. The transfer function of a linear analogue system is given by the following expression:

$$H(s) = \frac{2500(s+10)}{(s^2+2s)(s^2+30s+2500)}.$$

Draw the magnitude and phase Bode plots for the system.

6. Is the following transfer function stable or unstable? Make a sketch of its impulse response clearly indicating the time constant and the period of oscillation.

$$H(s) = \frac{1}{s^2+0.2s+1.01}$$

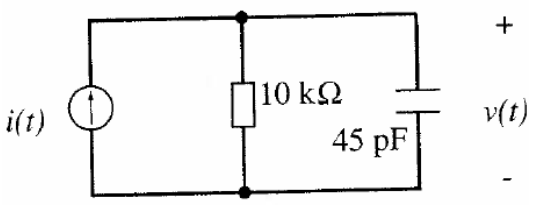
7. What is the damping factor and undamped natural frequency of a system with transfer function:

$$H(s) = \frac{1}{10s^2+20s+40}?$$

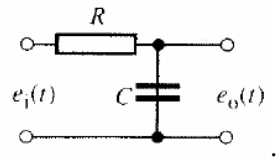
8. What is the bandwidth of a system with transfer function:

$$H(s) = \frac{4\times10^7 s}{(s+15)(s^2+2100s+2250000)}?$$

9. For the circuit shown calculate the rise time and determine the relationship between the rise time and the bandwidth for circuits of this type.



10. Determine the frequency response of the following *RC* filter circuit:



Sketch the amplitude and phase-shift plots.

11. Draw the approximate Bode plots corresponding to the transfer function:

$$G(s) = \frac{4\times10^3(5+s)}{s(100+s)(20+s)}.$$