Mathematical Methods for Engineers (MathEng) EXAM

16th February 2011

Duration: 2 hrs, calculators permitted, no documents

This exam paper contains 8 questions and 70 marks.

ATTEMPT ALL QUESTIONS – ANSWER IN ENGLISH

EURECOM
Sophia Antipolis

---

1. Identify the pivot and free variables of the following matrices. Find a special solution for each free variable and, by combining the special solutions, describe every solution to $Ax = 0$ and $Bx = 0$.

$$A = \begin{bmatrix} 1 & 2 & 2 & 4 & 6 \\ 1 & 2 & 3 & 6 & 9 \\ 0 & 0 & 1 & 2 & 3 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 2 & 4 & 2 \\ 0 & 4 & 4 \\ 0 & 8 & 8 \end{bmatrix}$$

[8 marks]

---

2. Show how QR decompositions can be useful for solving least squares approximations.

[6 marks]

---

3. Determine an expression in the form of $g(t) = c \sin t$ to approximate the square signal $f(t)$ over the interval $0 \le t \le 2\pi$ such that the error signal $e(t) = f(t) - g(t)$ is minimised.

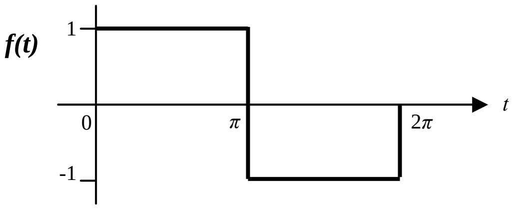


Figure Q3

[HINT: one approach involves minimising the energy in $e(t)$, or just a simple projection!]

[10 marks]

---

4. Sketch the functions $f(t) = \exp(-t)\,u(t)$ and $h(t) = \exp(-2t)\,u(t)$, where $u(t)$ is the unit step function, and then sketch **and** determine an expression for $y(t) = f(t) * h(t)$.

[8 marks]

---

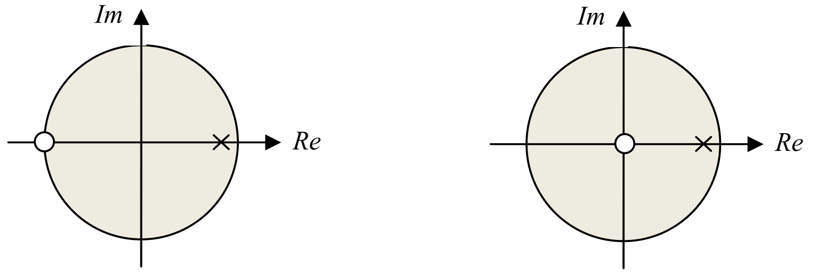
5. Sketch Bode plots of the magnitude and phase responses for a system with transfer function:

$$H(s) = \frac{10^4(s+1)}{s^2 + 110s + 1000}$$

[12 marks]

---

6. Sketch the magnitude and phase frequency responses for discrete-time systems with pole and zero positions as illustrated in the z-planes of Figure Q6 (a) and (b).



(a) (b)

Figure Q6

[6 marks]

7. Assuming zero initial conditions determine the response of a system with difference equation:

$$y(n+2) + y(n+1) + 0.16y(n) = x(n+1) + 0.32x(n)$$

to an input $x(n) = (-2)^{-n}u(n)$. State whether or not the system is stable and explain why.

[14 marks]

8. Suppose a probabilistic model involves 5 continuous random variables X_1, X_2, X_3, X_4, X_5 where only X_4 is observed. Suppose also that X_4 depends on X_1 and X_2, and that X_5 depends on X_3 and X_4. Sketch a directed graph model and determine whether, given X_4, pairs X_1, X_2 and X_1, X_3 are dependent or independent.

[6 marks]

Table of selected Laplace transforms

$x(t) \quad (t \ge 0)$	$X(s)$
$\delta(t)$	1
$\delta(t-\alpha)$	$\exp(-\alpha s)$
1 (unit step)	$\frac{1}{s}$
t (unit ramp)	$\frac{1}{s^2}$
$\exp(-\alpha t)$	$\frac{1}{s+\alpha}$
$t\exp(-\alpha t)$	$\frac{1}{(s+\alpha)^2}$
$\sin(\alpha t)$	$\frac{1}{s^2+\alpha^2}$
$\cos(\alpha t)$	$\frac{s}{s^2+\alpha^2}$
$e^{-\alpha t}\sin(\omega t)$	$\frac{\omega}{(s+\alpha)^2+\omega^2}$
$e^{-\alpha t}\cos(\omega t)$	$\frac{s+\alpha}{(s+\alpha)^2+\omega^2}$

Table of selected z-transforms

$x(n) \quad (n \geq 0)$	$X(z)$
$\delta(n)$ unit pulse	1
$\delta(n-m)$	z^{-m}
1 (unit step)	$\dfrac{z}{z-1}$
n (unit ramp)	$\dfrac{z}{(z-1)^2}$
$\exp(-\alpha n)$	$\dfrac{z}{(z-e^{-\alpha})}$
$n\exp(-\alpha n)$	$\dfrac{e^{-\alpha}z}{(z-e^{-\alpha})^2}$
$\sin(\beta n)$	$\dfrac{z\sin(\beta)}{z^2-2z\cos(\beta)+1}$
$\cos(\beta n)$	$\dfrac{z^2-z\cos(\beta)}{z^2-2z\cos(\beta)+1}$
$e^{-\alpha n}\sin(\beta n)$	$\dfrac{ze^{-\alpha}\sin(\beta)}{z^2-2ze^{-\alpha}\cos(\beta)+e^{-2\alpha}}$
$e^{-\alpha n}\cos(\beta n)$	$\dfrac{z^2-ze^{-\alpha}\cos(\beta)}{z^2-2ze^{-\alpha}\cos(\beta)+e^{-2\alpha}}$

Fourier series and transforms

Fourier series – periodic and continuous in time, discrete in frequency	
$x(t) = \sum_{n=-\infty}^{+\infty} X_n \exp(jn\omega_0 t)$	$X_n = \frac{1}{T} \int_{-T/2}^{T/2} x(t)\exp(-jn\omega_0 t)\, dt$
Fourier transform – continuous in time, continuous in frequency	
$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega)\exp(j\omega t)\, d\omega$	$X(\omega) = \int_{-\infty}^{\infty} x(t)\exp(-j\omega t)\, dt$
Discrete-time Fourier transform – discrete in time, continuous and periodic in frequency	
$x(n\Delta t) = \frac{\Delta t}{2\pi} \int_{-\pi/\Delta t}^{\pi/\Delta t} X(\omega)\exp(jn\Delta t\omega)\, d\omega$	$X(\omega) = \sum_{n=-\infty}^{\infty} x(n\Delta t)\exp(-j\omega n\Delta t)$
Discrete Fourier transform – discrete and periodic in time and in frequency	
$x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k) \exp\left(\frac{jnk2\pi}{N}\right)$	$X(k) = \sum_{n=0}^{N-1} x(n)\exp\left(\frac{-jnk2\pi}{N}\right)$

 [END OF PAPER]