

Essential Mathematical Methods for Engineers

Lecture 2:
Transfer function and system characterisation

Outline

- transfer function, poles and zeros
- transfer function and frequency response
 - frequency response from pole/zero diagram
 - Fourier transform of periodic signals
 - measurement of frequency response
 - Bode plots
 - Fourier and Laplace
- transfer function and impulse response
- time-domain response
 - first order systems
 - second order systems
- rise time and bandwidth

Transfer function and system characterisation

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Transfer function, poles and zeros

- for zero initial conditions a linear system can be described by:

$$a_0 y + a_1 \frac{dy}{dt} + \cdots + a_n \frac{d^n y}{dt^n} = b_0 x + b_1 \frac{dx}{dt} + \cdots + b_m \frac{d^m x}{dt^m}$$

$$a_0 Y(s) + a_1 sY(s) + \cdots + a_n s^n Y(s) = b_0 X(s) + b_1 sX(s) + \cdots + b_m s^m X(s)$$

$$A(s) Y(s) = B(s) X(s)$$

where $A(s)$ and $B(s)$ are polynomials in s

- therefore $H(s) = \frac{Y(s)}{X(s)} = \frac{B(s)}{A(s)} = \frac{b_0 + b_1 s + \cdots + b_m s^m}{a_0 + a_1 s + \cdots + a_n s^n}$
- poles correspond to values of s for which $H(s)$ is equal to infinity
- zeros correspond to values of s for which $H(s)$ is equal to zero

Transfer function and system characterisation

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Transfer function, poles and zeros

Example

A typical transfer function that might be obtained by applying Kirchhoff's laws to a circuit is:

$$H(s) = \frac{s^2 + 2s + 2}{s^2 + 4s + 13}$$

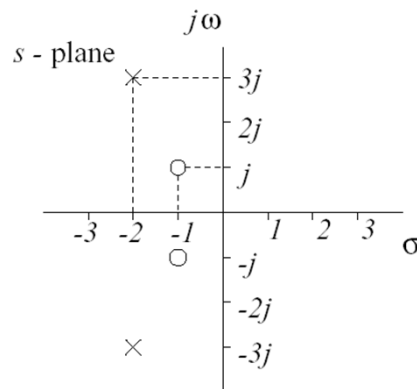
Determine the positions of its poles and zeros.

Transfer function and system characterisation

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Transfer function, poles and zeros

A pole/zero diagram illustrating a pair of complex conjugate poles and zeros:

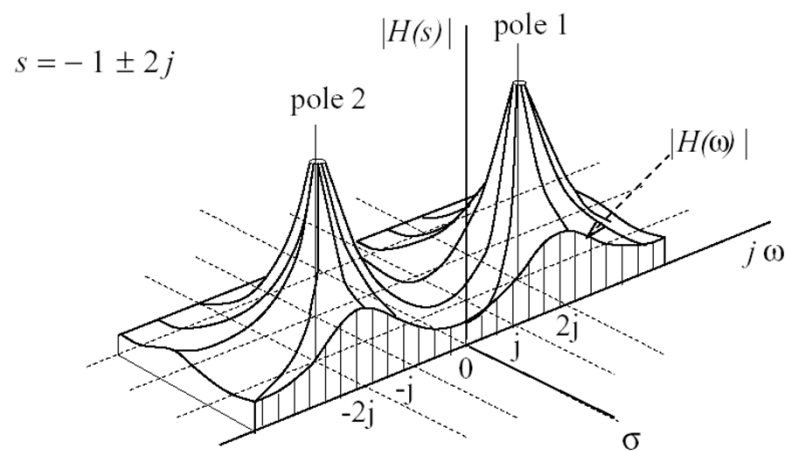


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Transfer function, poles and zeros

A plot of $|H(s)|$ against s for transfer function $H(s) = 1/(s^2 + 2s + 5)$



Transfer function and system characterisation

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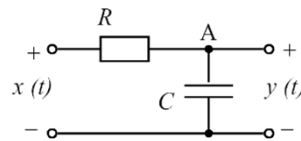
Transfer function and frequency response

- the frequency response of a system is defined as:

$$H(\omega) = \frac{F[\text{output}]}{F[\text{input}]} = \frac{Y(\omega)}{X(\omega)}$$

- given an input $\exp(j\omega t)$ the output is $H(\omega)\exp(j\omega t)$
- for the simple RC circuit the frequency response, or Fourier transfer function is given by:

$$H(\omega) = \frac{1}{1 + j\omega RC}$$



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Transfer function and frequency response

- note the similarity to the Laplace transform

$$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{(1 + RCs)} \qquad H(\omega) = \frac{1}{1 + j\omega RC}$$

- for $\sigma = 0$ the Laplace and Fourier basis functions are identical:

$$\exp(st) = \exp(\sigma t + j\omega t)$$

$$\exp(st) = \exp(j\omega t)$$

- we can observe $|H(\omega)|$ in the plot of $|H(s)|$ by setting $\sigma = 0$
- Q: what if the poles moved closer to the axis?

Fourier Transform

Transfer function and system characterisation

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Transfer function and frequency response

Example

If the Laplace transfer function of a system is:

$$H(s) = \frac{1}{s + 7}$$

what is its frequency response?

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Transfer function and frequency response

Frequency response from pole/zero diagram

- a transfer function with m zeros and n poles

$$H(s) = \frac{A(s - z_1)(s - z_2) \cdots (s - z_m)}{(s - p_1)(s - p_2) \cdots (s - p_n)}$$

- replacing s with $j\omega$

$$H(\omega) = \frac{A(j\omega - z_1)(j\omega - z_2) \cdots (j\omega - z_m)}{(j\omega - p_1)(j\omega - p_2) \cdots (j\omega - p_n)}$$

- amplitude response

$$|H(\omega)| = \frac{A|j\omega - z_1||j\omega - z_2| \cdots |j\omega - z_m|}{|j\omega - p_1||j\omega - p_2| \cdots |j\omega - p_n|}$$

- phase response

$$\begin{aligned} \angle H(\omega) = & \angle(j\omega - z_1) + \angle(j\omega - z_2) + \cdots + \angle(j\omega - z_m) \\ & - \angle(j\omega - p_1) - \angle(j\omega - p_2) \cdots - \angle(j\omega - p_n) \end{aligned}$$

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Transfer function and frequency response

Frequency response from pole/zero diagram

- we can thus obtain the amplitude and phase responses for any ω by evaluating the modulus and argument
- it is useful to visualise the term $(j\omega_0 - p_1)$

- consider a simple example of a single pole system

$$H(s) = \frac{1}{(s - p_1)}$$

- the frequency response is given by

$$H(\omega) = \frac{1}{(j\omega - p_1)}$$

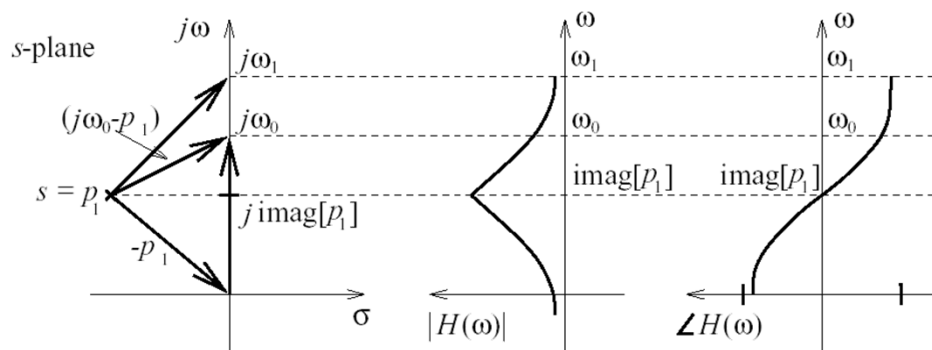
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Transfer function and frequency response

Frequency response from pole/zero diagram

- evaluating the amplitude and phase response through consideration of $(j\omega_0 - p_1)$ as a vector



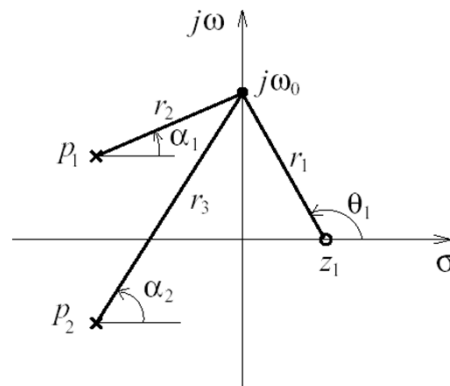
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Frequency response from pole/zero diagram

- evaluating the phase response



$$|H(\omega_0)| = \frac{r_1}{r_2 r_3} \quad \angle H(\omega_0) = \theta_1 - \alpha_1 - \alpha_2$$

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Transfer function and frequency response

Frequency response from pole/zero diagram

Example

Sketch the amplitude frequency response of each of the following 2 systems:

(a) $(s^2 + 9)/(s^2 + 0.6s + 4.09)$

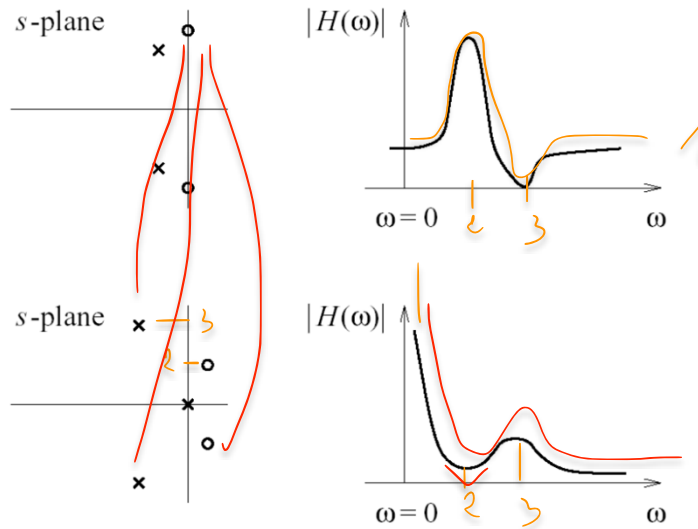
(b) $(s^2 + 0.6s + 4.09)/(s^3 + 0.8s^2 + 9.16s)$

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Transfer function and frequency response

Frequency response from pole/zero diagram



Transfer function and system characterisation

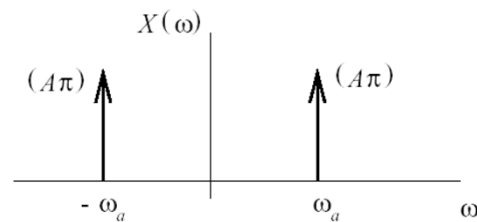
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Transfer function and frequency response

Fourier transform of periodic signals

- a cosine wave $x(t) = A \cos(\omega_a t)$

can be represented by $X(\omega) = A\pi [\delta(\omega - \omega_a) + \delta(\omega + \omega_a)]$



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Transfer function and frequency response

Fourier transform of periodic signals

- as justification consider a signal with Fourier transform

$$X(\omega) = \delta(\omega - \omega_a)$$

- its time domain representation is given by

$$\begin{aligned}
 x(t) &= F^{-1}[X(\omega)] \\
 &= \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) \exp(j\omega t) d\omega \\
 &= \frac{1}{2\pi} \int_{-\infty}^{\infty} \delta(\omega - \omega_a) \exp(j\omega t) d\omega \\
 &= \frac{1}{2\pi} \exp(j\omega_a t) \int_{-\infty}^{\infty} \delta(\omega - \omega_a) d\omega \\
 &= \frac{1}{2\pi} \exp(j\omega_a t)
 \end{aligned}$$

thus $F[\exp(j\omega_a t)] = 2\pi\delta(\omega - \omega_a)$

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Transfer function and frequency response

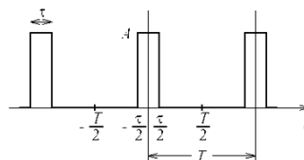
Fourier transform of periodic signals

Example

Determine the Fourier transform of a sine wave $A \sin(\omega_a t)$

Example

Derive an expression for the Fourier transform of the following signal:



Note the similarity to the example in the previous lecture

This example shows how the Fourier transform of a periodic signal is easily derived from the Fourier series.

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Transfer function and frequency response

Measurement of frequency response

- we can't generate $\exp(j\omega t)$ in order to measure $H(\omega)\exp(j\omega t)$ and deduce $H(\omega)$
- however we can generate a cosine or sine wave from the addition of two complex phasors

$$\cos(\omega t) = \frac{\exp(j\omega t) + \exp(-j\omega t)}{2} = \Re[\exp(j\omega t)]$$

- using superposition the response to a cosine wave is

$$\begin{aligned} & \frac{1}{2} H(\omega) \exp(j\omega t) + \frac{1}{2} H(-\omega) \exp(-j\omega t) \\ &= \frac{1}{2} H(\omega) \exp(j\omega t) + \frac{1}{2} H^*(\omega) \exp(-j\omega t) \\ &= \frac{1}{2} H(\omega) \exp(j\omega t) + \frac{1}{2} (H(\omega) \exp(j\omega t))^* \\ &= \Re[H(\omega) \exp(j\omega t)] \end{aligned}$$

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Transfer function and frequency response

Measurement of frequency response

- but $H(\omega)$ is complex and can be expressed in terms of magnitude and phase, thus

$$\begin{aligned} & \Re[|H(\omega)| \exp(j\angle H(\omega)) \exp(j\omega t)] \\ &= |H(\omega)| \Re[\exp(j\angle H(\omega)) \exp(j\omega t)] \\ &= |H(\omega)| \Re[\exp(j\omega t + j\angle H(\omega))] \\ &= |H(\omega)| \cos(\omega t + \angle H(\omega)) \end{aligned}$$

- the output is a cosine wave of identical frequency but amplified by $|H(\omega)|$ and phase shifted by $\angle H(\omega)$
- the complex frequency response is thus given by:

$$H(\omega) = |H(\omega)| \cos(\angle H(\omega)) + j|H(\omega)| \sin(\angle H(\omega))$$

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Transfer function and frequency response

Bode plots

- a more powerful graphical technique of frequency response analysis
- with any transfer function $H(s)$ there are four general factors
 - a constant gain, K ;
 - poles or zeros at the origin, s ;
 - poles or zeros on the real axis – terms of the form

$$(s + a) = a \left(\frac{s}{a} + 1 \right) = \frac{1}{\tau} (\tau s + 1)$$

- complex conjugate poles or zeros, $(s^2 + As + B)$
- the quadratic is usually expressed as

$$(s^2 + As + B) = (s^2 + 2\zeta\omega_0 s + \omega_0^2)$$

where ζ is the damping factor and ω_0 is the natural, undamped frequency

Transfer function and system characterisation

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Transfer function and frequency response

Bode plots

- scaling by ω_0 :

$$(s^2 + As + B) = \omega_0^2 \left((s/\omega_0)^2 + 2\zeta(s/\omega_0) + 1 \right)$$

for which the roots are complex if $\zeta < 1$.

- considering the transfer function

$$H(s) = \frac{K}{s (\tau s + 1) ((s/\omega_0)^2 + 2\zeta(s/\omega_0) + 1)}$$

for which there is a

- constant gain, K
- pole at the origin
- pole on the real axis at $s = -1/\tau$
- pair of complex poles

Transfer function and system characterisation

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Transfer function and frequency response Bode plots

- to obtain the frequency response

$$H(\omega) = \frac{K}{j\omega(j\omega\tau + 1) ((j\omega/\omega_0)^2 + 2\zeta(j\omega/\omega_0) + 1)}$$

- the logarithmic magnitude:

$$20 \log_{10} |H(\omega)| = 20 \log_{10}(K) - 20 \log_{10}(|j\omega|) - 20 \log_{10}(|j\omega\tau + 1|) \\ - 20 \log_{10}(|1 + (2\zeta/\omega_0)j\omega + (j\omega/\omega_0)^2|)$$

- the phase:

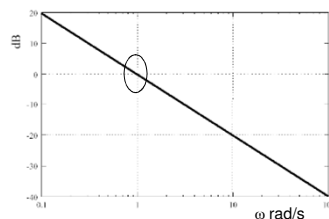
$$\angle H(\omega) = -90^\circ - \tan^{-1}(\omega\tau) - \tan^{-1}\left(\frac{2\zeta\omega_0\omega}{\omega_0^2 - \omega^2}\right)$$

Transfer function and system characterisation

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Transfer function and frequency response Bode plots

- constant gain term:**
 - a gain of $20\log_{10}(K)$ at all frequencies
- pole at origin:**
 - gives rise to a $j\omega$ term in denominator
 - gain of $20\log_{10}(|j\omega|)$ – a straight line with slope -20 dB/decade
 - 0 dB at 1 rad/s
 - phase shift of -90° due to presence of j



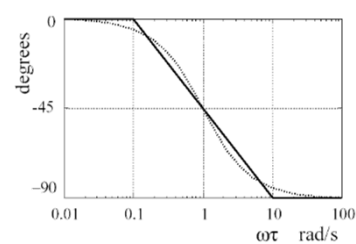
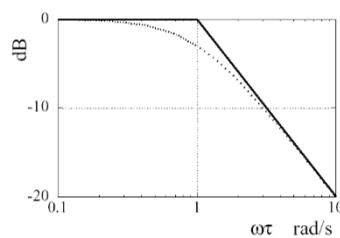
Transfer function and system characterisation

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Transfer function and frequency response Bode plots

■ pole on real axis:

- a pole at $s = -1/\tau$ gives rise to the term $j\omega\tau + 1$ in the denominator
- at the cut-off frequency $\omega = 1/\tau$, the gain is -3 dB
- above cut-off the gain has a slope of -20 dB/decade
- phase of -45° at cut-off,
 - $\sim 0^\circ$ 1 decade below, $\sim -90^\circ$ 1 decade above



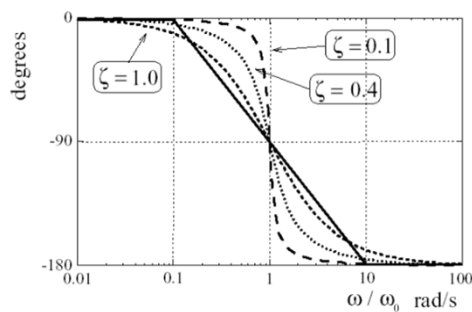
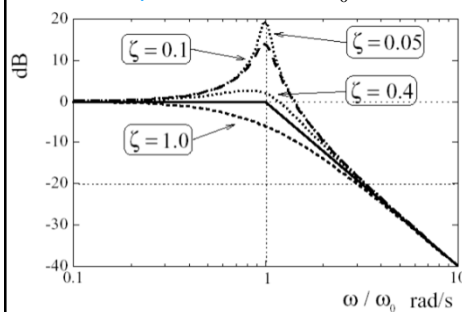
Transfer function and system characterisation

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Transfer function and frequency response Bode plots

■ complex conjugate pair of poles

- gives rise to the term $((j\omega/\omega_0)^2 + 2\zeta(j\omega/\omega_0) + 1)$ in denominator
- for $\omega < \omega_0$ the gain is ~ 0 dB
- above cut-off the gain has a slope of -40 dB/decade
- at ω_0 the gain is dependent on ζ
- phase $\sim -90^\circ$ at ω_0 , $\sim 0^\circ$ 1 decade below, $\sim -180^\circ$ 1 decade above



Transfer function and system characterisation

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Transfer function and frequency response Bode plots

Example

Sketch the Bode plot of a system with the following transfer function:

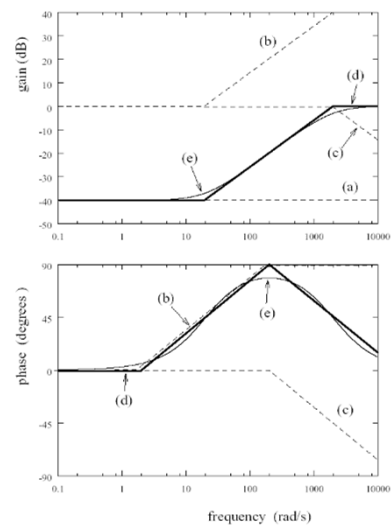
$$H(s) = \frac{s + 20}{s + 2000}$$

Transfer function and system characterisation

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Transfer function and frequency response Bode plots

$$H(\omega) = \frac{j\omega + 20}{j\omega + 2000} = \frac{j\omega/20 + 1}{(j\omega/2000 + 1)100}$$



Transfer function and system characterisation

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Transfer function and impulse response

- for a system with poles at

$$s = p_1 = \sigma_a + j\omega_a$$

$$s = p_2 = \sigma_a - j\omega_a$$

the transfer function is

$$H(s) = \frac{1}{(s - \sigma_a - j\omega_a)_{p_1} (s - \sigma_a + j\omega_a)_{p_2}}$$

taking a partial fraction expansion

$$H(s) = \frac{A_1}{(s - \sigma_a - j\omega_a)_{p_1}} + \frac{A_2}{(s - \sigma_a + j\omega_a)_{p_2}}$$

$$H(s) = \frac{\frac{1}{2j\omega_a}}{(s - \sigma_a - j\omega_a)_{p_1}} + \frac{\frac{-1}{2j\omega_a}}{(s - \sigma_a + j\omega_a)_{p_2}}$$

Transfer function and system characterisation

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Transfer function and impulse response

- the impulse response is

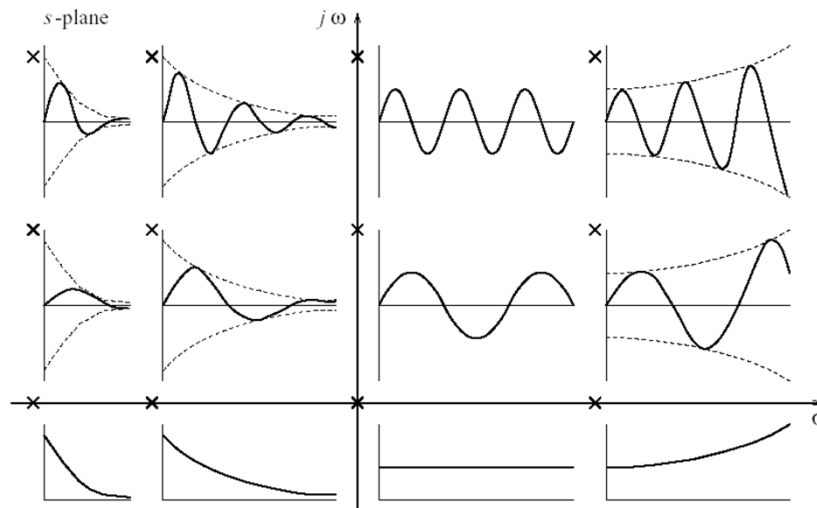
$$\begin{aligned} h(t) &= \left[\frac{1}{2j\omega_a} \exp(\sigma_a + j\omega_a)t \right]_{p_1} - \left[\frac{1}{2j\omega_a} \exp(\sigma_a - j\omega_a)t \right]_{p_2} \\ &= \frac{1}{2j\omega_a} \exp(\sigma_a t) [\exp(j\omega_a t) - \exp(-j\omega_a t)] \\ &= \frac{1}{\omega_a} \exp(\sigma_a t) \sin(\omega_a t) \end{aligned}$$

- σ_a controls the decay rate and ω_a the frequency of oscillation
- there is a strong relation between the impulse response and pole location

Transfer function and system characterisation

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Transfer function and impulse response



Transfer function and system characterisation

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Transfer function and impulse response

- this relationship tells us about the stability of the system
- if a pole has $\sigma > 0$ the output will grow exponentially
 - such a system is said to be unstable
- if a pole has $\sigma < 0$ the response will decay exponentially
 - such a system is said to be stable
- most systems have several poles and zeros

$$H(s) = \frac{A (s - z_1)(s - z_2) \cdots (s - z_m)}{(s - p_1)(s - p_2) \cdots (s - p_n)}$$

Transfer function and system characterisation

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Transfer function and impulse response

- the transfer function may be written as

$$H(s) = \frac{A_1}{(s - p_1)} + \frac{A_2}{(s - p_2)} + \cdots + \frac{A_n}{(s - p_n)}$$

and taking inverse Laplace transforms

$$h(t) = A_1 e^{p_1 t} + A_2 e^{p_2 t} + \cdots + A_n e^{p_n t} +$$

- the impulse response is the sum of complex phasors such as $e^{p_n t}$
 - these phasors are called the modes of the system
 - all must have $\sigma < 0$ for the system to be stable
 - all poles thus lie in the left-half of the s-plane

Transfer function and system characterisation

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Transfer function and impulse response

Example

Is the transfer function $H(s) = s/(s^2 + 4s + 68)$ stable or unstable? What is the period of oscillation and the time constant of its impulse response?

Transfer function and system characterisation

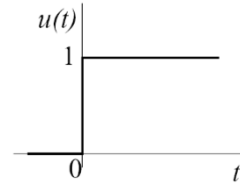
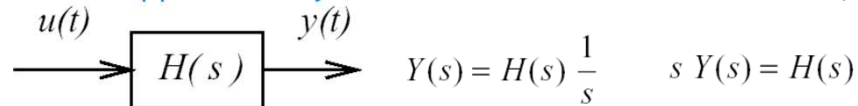
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Time domain response

- time-domain or transient response

- unit step function $L[u(t)] = \frac{1}{s}$

- when applied to a system



and taking inverse Laplace transforms of both sides

$$\frac{dy}{dt} = h(t)$$

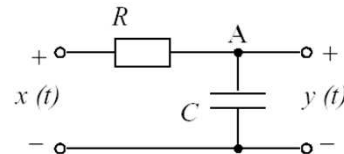
the impulse response is the derivative of the step response

Transfer function and system characterisation

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Time domain response First order systems

- a simple first order, RC circuit



with transfer function $H(s) = \frac{1}{(1 + sRC)}$

thus there is one real pole at $s = -1/RC$

- to obtain the step response: $Y(s) = H(s) X(s) = \frac{1}{(1 + sRC)} \frac{1}{s}$

$$= \frac{1}{s} - \frac{1}{\left(s + \frac{1}{RC}\right)}$$

Transfer function and system characterisation

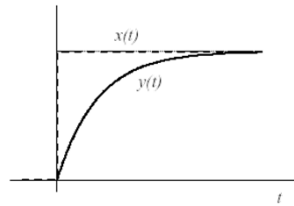
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Time domain response First order systems

- taking inverse Laplace transforms

$$y(t) = 1 - \exp\left(-\frac{t}{RC}\right); \quad t \geq 0.$$

- we obtain the response to the step input

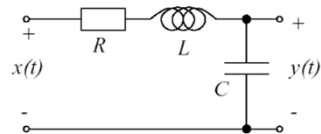


Transfer function and system characterisation

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Time domain response Second order systems

- repeating for an *RLC* circuit



$$Y(s) = \frac{\frac{1}{sC}}{R + sL + \frac{1}{sC}} X(s)$$

therefore

$$H(s) = \frac{1}{s^2 LC + sCR + 1}$$

$$= \frac{\frac{1}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

Transfer function and system characterisation

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Time domain response Second order systems

- normalising to the form $H(s) = \frac{\omega_0^2}{s^2 + 2\zeta\omega_0s + \omega_0^2}$
- we have $\omega_0^2 = \frac{1}{LC}$ (omega)
 $\zeta = \frac{R}{2\sqrt{\frac{L}{C}}}$ (zeta)
- the poles of $H(s)$ are $s_1 = -\zeta\omega_0 + \omega_0\sqrt{\zeta^2 - 1}$
 $s_2 = -\zeta\omega_0 - \omega_0\sqrt{\zeta^2 - 1}$

Transfer function and system characterisation

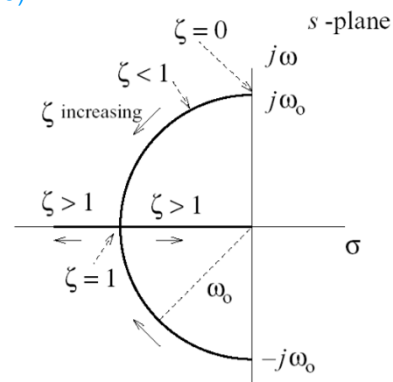
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Time domain response Second order systems

- three possibilities
 - $\zeta < 1 \Rightarrow$ two complex roots (underdamped)
 - $\zeta = 1 \Rightarrow$ two real roots at $s = -\omega_0$ (critically damped)
 - $\zeta > 1 \Rightarrow$ two real roots (overdamped)

$$s_1 = -\zeta\omega_0 + \omega_0\sqrt{\zeta^2 - 1}$$

$$s_2 = -\zeta\omega_0 - \omega_0\sqrt{\zeta^2 - 1}$$



Transfer function and system characterisation

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Time domain response Second order systems

- what happens to the impulse response as ζ changes

- underdamped ($\zeta < 1$) $p_1, p_2 = -\zeta \omega_0 \pm j \omega_0 \sqrt{1 - \zeta^2}$

- the impulse response is given by

$$h(t) = \frac{\omega_0}{\sqrt{1 - \zeta^2}} e^{-\zeta \omega_0 t} \sin(\sqrt{1 - \zeta^2} \omega_0 t)$$

- regarding the poles
 - the real part determines the time constant associated with the exponential decay
 - the imaginary part determines the frequency of oscillation

Transfer function and system characterisation

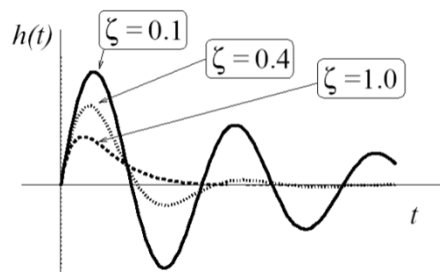
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Time domain response Second order systems

- critically damped ($\zeta = 1$)

- two identical real poles $p_1 = p_2 = -\omega_0$
 - for real poles there is no oscillation

- the impulse response is $h(t) = \omega_0 t e^{-\omega_0 t}$



Transfer function and system characterisation

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Time domain response Second order systems

- overdamped ($\zeta > 1$)
 - equivalent to two first order systems with two different real poles

$$p_1, p_2 = -\zeta\omega_0 \pm \omega_0\sqrt{\zeta^2 - 1}$$

- the impulse response is

$$h(t) = \frac{\omega_0}{\sqrt{\zeta^2 - 1}} e^{-\zeta\omega_0 t} \sinh\left(\sqrt{\zeta^2 - 1} \omega_0 t\right)$$

- again there is no oscillation due to real poles

Transfer function and system characterisation

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Time domain response Second order systems

- for the step response

$$\begin{aligned} Y(s) &= H(s) L[u(t)] \\ &= \frac{\omega_0}{(s^2 + 2\zeta\omega_0 s + \omega_0^2)} \frac{1}{s} \end{aligned}$$

and again taking inverse Laplace transforms

$$\begin{aligned} y(t) &= L^{-1}[Y(s)] \\ &= 1 - \frac{\exp(-\zeta\omega_0 t)}{\sqrt{1 - \zeta^2}} \sin\left(\sqrt{1 - \zeta^2} \omega_0 t + \theta\right) \end{aligned}$$

where

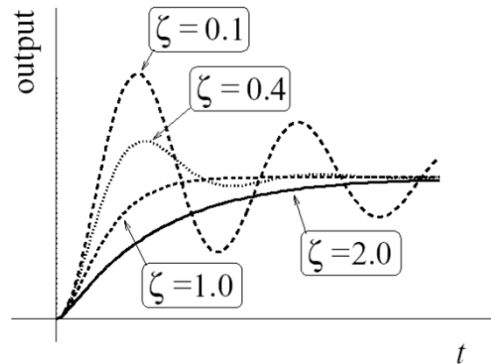
$$\theta = \tan^{-1}\left(\frac{\sqrt{1 - \zeta^2}}{\zeta}\right)$$

Transfer function and system characterisation

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Time domain response Second order systems

- the last result applies to the underdamped case
- for other damping factor values



Transfer function and system characterisation

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Time domain response Second order systems

Example

What is the damping factor and undamped natural frequency of a system with transfer function:

$$H(s) = 7s/(12s^2 + 118.8s + 2700)?$$

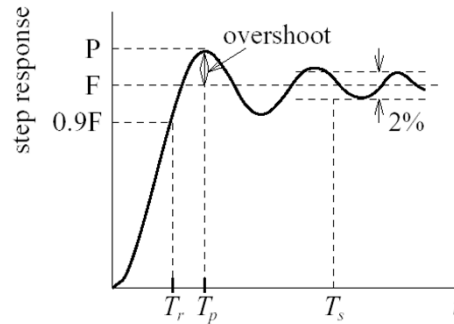
Transfer function and system characterisation

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Rise time and bandwidth

- the rise time measures the speed of response
 - rise time, T_r : time taken to reach 90% of final value, F
 - peak time, T_p : time to first peak, P , for $\zeta < 1$
 - settling time, T_s : time to get to within 2% of F
 - percentage overshoot

$$100 \frac{(P - F)}{F}$$

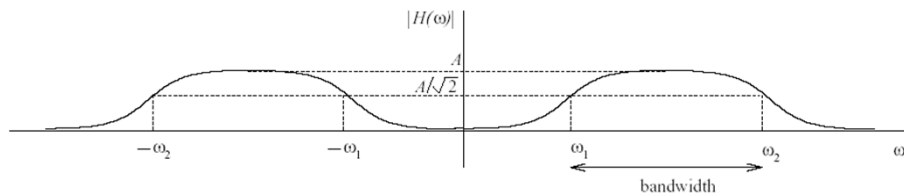
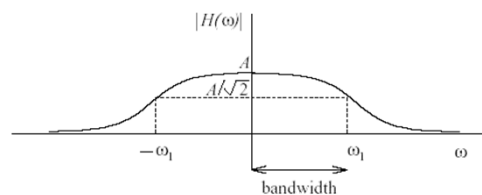


Transfer function and system characterisation

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Rise time and bandwidth

- the half power bandwidth ω_B is the interval of frequencies where the gain varies by less than 3 dB



Transfer function and system characterisation

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Rise time and bandwidth

- the step response of a first order system is given by

$$y(t) = 1 - \exp\left(-\frac{t}{RC}\right)$$

- since the final value $F = y(\infty) = 1$, T_r can be found from

$$0.9F = 1 - \exp\left(-\frac{T_r}{RC}\right)$$

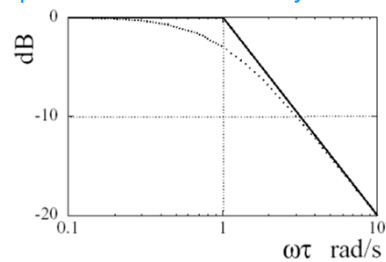
- thus $T_r = RC \ln(10)$ s

Transfer function and system characterisation

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Rise time and bandwidth

- the frequency response for a first order system:



- the bandwidth is $1/\tau = 1/RC$ rad/s for the simple RC circuit
- thus the rise time is the reciprocal of the bandwidth
- a faster response would require more bandwidth

Transfer function and system characterisation

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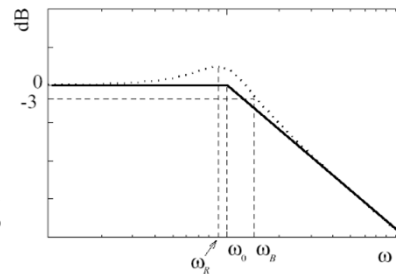
Rise time and bandwidth

- for an underdamped second order system

$$T_p = \frac{\pi}{\omega_0 \sqrt{1 - \zeta^2}} \text{ s}$$

and the percentage overshoot, $PO = 100 \exp\left(\frac{-\zeta\pi}{\sqrt{1 - \zeta^2}}\right)$

- a typical response:
 - the peak is associated with ω_0 and ω_R – the resonant frequency
 - the bandwidth depends on ζ



Transfer function and system characterisation

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Rise time and bandwidth

- two important trade-offs
 - case (i): ζ constant
 - if ω_B is increased then so is ω_0 which reduces T_p and in turn T_r
 - thus the rise time is inversely proportional to bandwidth
 - a faster response requires more bandwidth
 - case (ii): ω_B constant
 - can be achieved by holding ω_0 constant
 - T_p can be reduced by reducing ζ which would also reduce T_r
 - thus a faster response can be achieved just by reducing ζ
 - greater overshoot in the time domain
 - higher resonant peak in the frequency domain

Transfer function and system characterisation

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Rise time and bandwidth

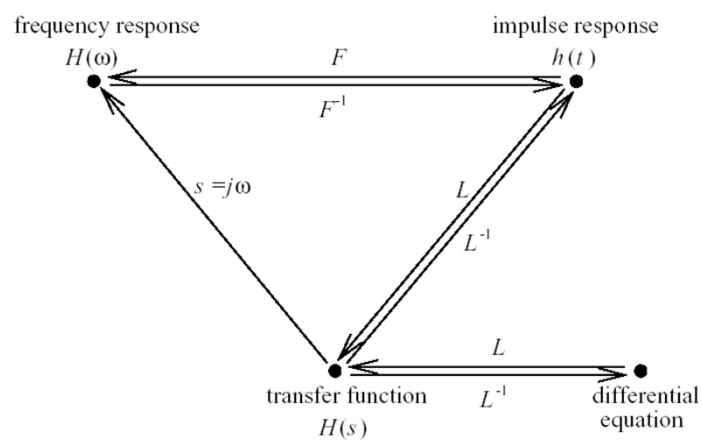
Example

A low pass, second order system has a peak time of 2 s and an overshoot of 10%. Estimate its bandwidth.

Transfer function and system characterisation

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Summary



Transfer function and system characterisation

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Summary

- you should be able to:
 - calculate the poles and zeros of first and second order systems and plot them in the s-plane;
 - sketch the frequency response of a system from its pole/zero map;
 - draw the Bode plots of first, second and third order systems;
 - sketch the impulse response of first and second order systems from their transfer function;
 - identify if a system is stable or unstable;
 - sketch the step response of first and second order systems from their transfer function;
 - describe how the rise time of a second order system is related to both overshoot and bandwidth.