Essential Mathematical Methods for Engineers (MathEng)

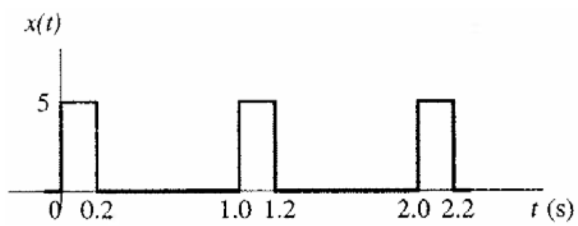
Signal representation and system response

1. A voltage signal $v(t)$ is connected across a 5 Ω resistor at $t = 0$. If the voltage is defined by:

$$v(t) = 2\exp(-3t) + 4\exp(-7t)\,.$$

What is the total energy dissipated in the resistor?

2. Calculate the power associated with the waveform shown below




3. Obtain the trigonometric Fourier series expansion of the periodic function $f(t)$ of period 2π defined by:

$$f(t) = t \quad (0 < t < 2\pi) \qquad f(t) = f(t + 2\pi)\,.$$

4. A periodic function $f(t)$ with period 2π is defined by:

$$f(t) = t^2 + t \quad (-\pi < t < \pi) \qquad f(t) = f(t + 2\pi)\,.$$

Sketch a graph of the function $f(t)$ for values of $t$ from $t = -3\pi$ to $t = +3\pi$ and obtain a trigonometric Fourier series expansion of the function.

5. A periodic function $f(t)$ of period 2π is defined within the period $0 \le t \le 2\pi$ by

$$f(t) = \begin{cases} t & (0 \le t \le \pi/2) \\ \pi/2 & (\pi/2 \le t \le \pi) \\ \pi - t/2 & (\pi \le t \le 2\pi) \end{cases}$$

Sketch a graph of $f(t)$ for $-2\pi \le t \le 3\pi$ and determine its corresponding Fourier series.

6. Develop expressions for the Fourier series representations (both trigonometric and complex forms) of the periodic signal of Q2. Plot graphs of the magnitude and phase of the complex form.

7. Determine the complex form of the Fourier series of the sawtooth function $f(t)$ defined by:

$$f(t) = \frac{2t}{T} \quad (0 < t < 2T) \quad f(t+2T) = f(t)$$

and plot the corresponding discrete magnitude and phase spectra.

8. Obtain the complex form of the Fourier series expansion of:

$$f(t) = \begin{cases} 2 & (-\pi < t < 0) \\ 1 & (0 < t < \pi) \end{cases} \quad f(t+2\pi) = f(t).$$

9. Develop an expression for the complex Fourier series of the following signals:

   a. $x(t) = 3\cos(5t) + 4\sin(10t)$, and
   b. $x(t) = \cos(\omega_0 t) + \sin^2(2\omega_0 t)$.

10. Consider first an even function where $f(t) = f(-t)$ for all $t$ and second an odd function where $f(t) = -f(-t)$ for all $t$. Comment on the values of the trigonometric Fourier series coefficients, $a_n$ and $b_n$, in each case. In addition show that, where appropriate, $a_n$ and $b_n$ may be calculated by integrating over half of a period rather than a full period of $f(t)$.

11. Determine the Fourier series expansion of the rectified sine wave:

$$f(t) = |\sin t|.$$

12. Suppose that $g(t)$ and $h(t)$ are periodic functions of period $2\pi$ and are defined within the period $-\pi \leq t \leq \pi$ by:

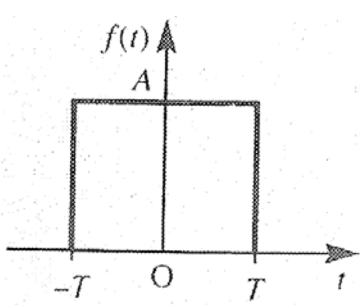
$$g(t) = t^2 \quad \text{and} \quad h(t) = t.$$

Determine the trigonometric Fourier series expansions of both $g(t)$ and $h(t)$ and comment of the comparison of this result to that of Q4 for $f(t) = t^2 + t$.

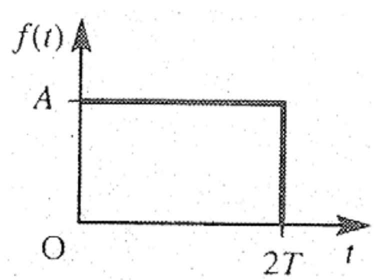
13. By applying Parseval's theorem to the function in Q7 show that:

$$\frac{1}{6}\pi^2 = \sum_{n=1}^{\infty} \frac{1}{n^2}$$

14. Determine the Fourier transforms of the following two signals:



(a)



(b)

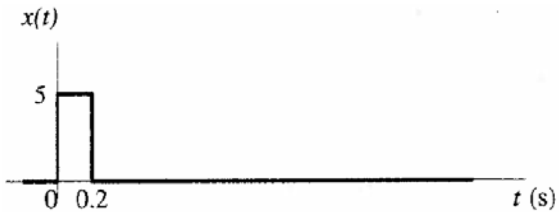
and comment on the results.

15. Determine the Fourier transform of the one-sided exponential function:

$$f(t) = H(t)\exp(-at) \quad \text{for } a > 0$$

where $H(t)$ is the Heaviside unit step function.

16. Develop an expression for the Fourier transform and the Laplace transform of the signal illustrated below. Plot graphs of the magnitude and phase of the Fourier transform.




17. Plot graphs of the amplitude and phase spectra of the causal signal in Q15.

18. Determine the frequency spectrum of the signal:

$$g(t) = f(t)\cos\omega_c t\ .$$