Essential Mathematical Methods for Engineers (MathEng)
EXAM
December 2021
Duration: 2 hrs, all documents and calculators permitted
ATTEMPT ALL QUESTIONS – ANSWER IN ENGLISH

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1. Determine an expression for the complex Fourier series of the periodic sawtooth waveform illustrated in Figure Q1.

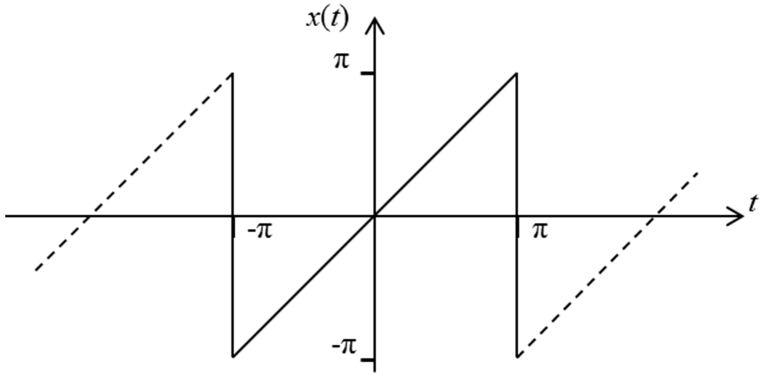

Figure Q1

[10 marks]

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2. By graphical time convolution, sketch the system output corresponding to the input signal $x(t)$ and the system impulse response $h(t)$ illustrated in Figure Q2.

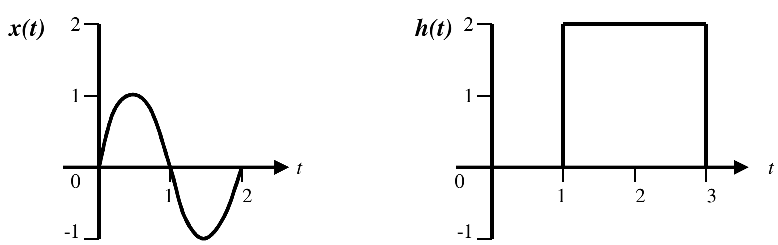

Figure Q2

[8 marks]

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3. Consider a system with the following transfer function:

$$H(s) = \frac{s^2 - 0.4s + 400.04}{(s^2 + 2s + 101)(s^2 + 2s + 901)}$$

(a) Determine the positions of any poles and zeros
(b) Sketch the pole/zero positions in the s-plane
(c) Sketch the magnitude frequency response
(d) Determine the gain at 0 rad/s and the asymptotic gain at high frequencies

[10 marks]

4. Find the system transfer function $H(z)$ and the unit impulse response $h(n)$ of the filter illustrated in Figure Q4 in which all symbols have their usual meaning.

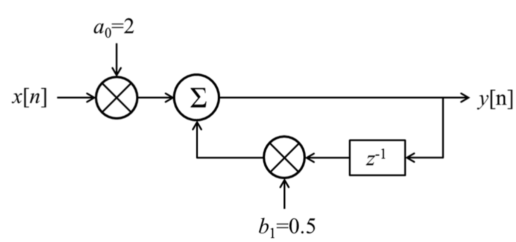

Figure Q4

[8 marks]

5. A pair of signals $x_1(t)$ and $x_2(t)$ illustrated in Figure Q5 (a) have the normalised magnitude spectrums illustrated on a decibel scale in Figure Q5 (b). Both spectrums are derived using 64-point discrete Fourier transforms. Despite both $x_1(t)$ and $x_2(t)$ containing a single sine wave, there are marked differences in the two magnitude spectrums. Stating any assumptions you make, explain what accounts for these differences.

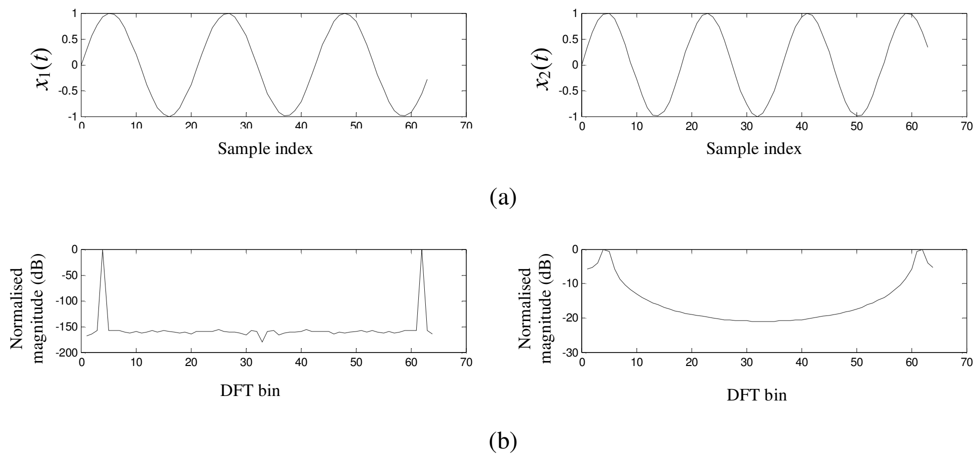

Figure Q5

[8 marks]

6. The joint probability density function of a pair of random variables X and Y is given by:

$$f_{X,Y}(x,y) = Axy\exp(-[x+y]), \quad x \geq 0 \text{ and } y \geq 0$$

(a) Determine the constant A
(b) Write down expressions for the marginal probability density functions $f_X(x)$ and $f_Y(y)$
(c) Justifying your answer, state whether or not the random variables X and Y are statistically independent

[9 marks]

7. Determine bases for the column, row and null spaces of

$$A = LU = E^{-1}R = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 5 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 & 0 & 5 \\ 0 & 0 & 1 & 6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

and state the dimensions of each.

[7 marks]