Tutorial Sheet 7

(1) If A & B are mutually exclusive $P(A|B) = P(B|A) = 0$
If they are statistically independent then $P(A|B) = P(A)$ & $P(B|A) = P(B)$. The only way for both of these conditions to be true is for $P(A) = P(B) = 0$

(2) Using Bayes' rule

$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}$$

where by total probability

$$\begin{aligned} P(B) &= P(B|A)\,P(A) + P(B|\bar{A})\,P(\bar{A}) \\ &= P(B|A)\,P(A) + \left[1 - P(\bar{B}|\bar{A})\right] P(\bar{A}) \\ &= 0.9 \times 0.4 + 0.4 \times 0.6 = 0.6 \end{aligned}$$

$$\therefore\ P(A|B) = \frac{0.9 \times 0.4}{0.6} = 0.6$$

Similarly $$P(A|\bar{B}) = \frac{P(\bar{B}|A)\,P(A)}{P(\bar{B})}$$

$$\begin{aligned} P(\bar{B}) &= P(\bar{B}|A)P(A) + P(\bar{B}|\bar{A})\,P(\bar{A}) \\ &= 0.1 \times 0.4 + 0.6 \times 0.6 = 0.4 \end{aligned}$$

$$\therefore\ P(A|\bar{B}) = \frac{0.1 \times 0.4}{0.4} = 0.1$$

(3) (a) $P(A_2) = 0.3$

$P(A_2, B_1) = 0.05$

$P(A_1, B_2) = 0.05$

$P(A_3, B_3) = 0.05$

$P(B_1) = 0.15$

$P(B_2) = 0.25$

$P(B_3) = 0.6$

(b) $P(A_3|B_3) = 0.083$

$P(B_2|A_1) = 0.091$

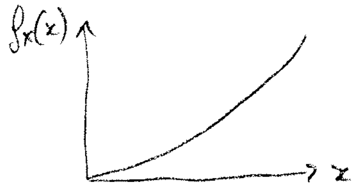
$P(B_3|A_2) = 0.333$

(4) (a) As $x \to \infty$, the cdf approaches 1. Therefore $B = 1$.
Assuming continuity of the cdf $F_X(10) = 1$ so $A \times 10^3 = 1$ or $A = 10^{-3}$.

(b) The pdf is given by

$$f_X(x) = \frac{dF_X(x)}{dx} = 3 \times 10^{-3} x^2 u(x) u(x-10)$$

The graph of the pdf is zero for $x < 0$, the quadratic $3 \times 10^{-3} x^2$ for $0 \leq x \leq 10$ & zero for $x > 0$.



(c) $P(X > 7) = 1 - F_X(0.7) = 1 - (10^{-3})(7)^3 = 0.657$

(d) $P(3 \leq X \leq 7) = F_X(7) - F_X(3) = 0.316$

(5) We can factor the pdf as

$$f_{XY}(x,y) = \sqrt{A}\, \exp(-|x|)\, \sqrt{A}\, \exp(-|y|)$$

and with proper choice of A the two separate factors are marginal pdfs. Thus X & Y are statistically independent.

(6) The constant c is determined using the normalisation integral for joint pdfs:

$$\iint_{x^2+y^2<1} c\, dx\, dy = 1 \quad \Rightarrow \quad c = \frac{1}{\pi}$$

The marginal pdf of X is found by integrating y out of the joint pdf:

$$f_X(x) = \int_{-\infty}^{\infty} f_{XY}(x,y)\, dy = \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} 1/\pi \, dy = \frac{2}{\pi}\sqrt{1-x^2} \quad \text{for } -1 \le x \le 1$$

By symmetry, the marginal pdf of Y would have the same functional form

$$f_Y(y) = \frac{2}{\pi}\sqrt{1-y^2}, \quad \text{for } -1 \le x \le 1.$$

(7) (a) To find A we evaluate the double integral

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{XY}(x,y)\,dx\,dy = 1 = \int_0^{\infty}\int_0^{\infty} Axy\,e^{-(x+y)}\,dx\,dy$$

$$= A\int_0^{\infty} x e^{-x}\,dx \int_0^{\infty} y e^{-y}\,dy$$

$$= A$$

Thus $A = 1$

(b) clearly $f_X(x) = x e^{-x} u(x)$

$f_Y(y) = y e^{-y} u(y)$

(c) Yes! The joint pdf factors into the product of the marginal pdfs.

(8) We first note that

$$P(Y=0) = P(X \leq 0) = 0.5$$

For $y > 0$, transformation of variables gives

$$f_Y(y) = f_X(x) \left| \frac{dg^{-1}(y)}{dy} \right|_{x=g^{-1}(y)}$$

Since $y = g(x) = ax$, we have $g^{-1}(y) = y/a$. Therefore

$$f_Y(y) = \frac{\exp\left(\frac{-y^2}{2a^2\sigma^2}\right)}{\sqrt{2\pi a^2\sigma^2}}, \quad y > 0$$

For $y = 0$, we need to add $0.5\,\delta(y)$ to reflect the fact that Y takes on the value 0 with probability 0.5. Hence for all y the result is

$$f_Y(y) = \frac{1}{2}\delta(y) + \frac{\exp\left(\frac{-y^2}{2a^2\sigma^2}\right)}{\sqrt{2\pi a^2\sigma^2}}\, u(y)$$

where $u(y)$ is the unit step function.

(9) (a) The normalisation of the pdf to unity provides the relationship

$$A\int_{-\infty}^{\infty} e^{-b|x|}\,dx = 2A\int_{0}^{\infty} e^{-bx}\,dx = 2A/b = 1$$

where the second integral follows because of the evenness of the integrand. Thus $A = b/2$.

(b) $E[x] = 0$ because the pdf is an even function of x.

(c) Since the expectation of X is zero

$$\sigma_x^2 = E\{X^2\} = \int_{-\infty}^{\infty} \frac{b}{2} x^2 e^{-b|x|}\,dx = b\int_{0}^{\infty} x^2 e^{-bx}\,dx = \frac{2}{b^2}$$

where evenness of the integrand has again been used, and the last integral can be found in an integral table.

(10) $$E[X] = \int_{-\infty}^{\infty} x\left\{\frac{1}{2}\delta(x-4) + \frac{1}{8}\left[u(x-3) - u(x-7)\right]\right\}dx$$

$$= \frac{1}{2}\int_{-\infty}^{\infty} x\,\delta(x-4)\,dx + \frac{1}{8}\int_{3}^{7} x\,dx = \frac{9}{2}$$

$$E[X^2] = \int_{-\infty}^{\infty} x^2\left\{\frac{1}{2}\delta(x-4) + \frac{1}{8}\left[u(x-3) - u(x-7)\right]\right\}dx = \frac{127}{6}$$

$$\therefore\ \sigma_X^2 = E[X^2] - E^2[X] = \frac{127}{6} - \frac{81}{4} = \frac{11}{12}$$

(11) Consider independent Gaussian random variables U & V and define a transformation

$$g_1(u,v) = x = \rho u + \sqrt{1-\rho^2}\,u$$

and

$$g_2(u,v) = y = u$$

The inverse transformation is $u = y$ & $v = (x-\rho y)/(1-\rho^2)^{1/2}$

Since the transformation is linear, the new random variables are Gaussian.

The Jacobian is $(1-\rho^2)^{-1/2}$. Thus the joint pdf of the new random variables X & Y is

$$f_{XY}(x,y) = \frac{1}{\sqrt{1-\rho^2}} \left. \frac{e^{-\frac{(u^2+v^2)}{2\sigma^2}}}{2\pi\sigma^2} \right|_{\substack{u=g_1^{-1}(x,y) \\ v=g_2^{-1}(x,y)}} = \frac{\exp\left[-\frac{x^2-2\rho xy+y^2}{2\sigma^2(1-\rho^2)}\right]}{2\pi\sigma^2(1-\rho^2)}$$

Thus X & Y are equivalent to the random variables in the problem statement which proves the desired result.

To see this, note that

$$E(XY) = E\{(\rho U + \sqrt{1-\rho^2}\,V)\,U\} = \rho\sigma^2$$

which proves the desired result.

(12) (a) The characteristic function is given by

$$M_X(jv) = \frac{a}{a - jv}$$

(b) The mean & mean-square values are

$$E[X] = \frac{1}{a} \quad \text{and} \quad E[X^2] = \frac{2}{a^2}$$

respectively.

(c) The variance $\text{var}[X] = \frac{1}{a^2}$

(13) (a)

$$f_X(x) = \int_{-\infty}^{\infty} f_{XY}(x,y)\,dy$$

We have for the joint pdf

$$f_{XY}(x,y) = \frac{1}{2\pi\sigma_X\sigma_Y(1-\rho^2)^{1/2}} \exp\left(-\frac{1}{2}q(x,y)\right)$$

where $$q(x,y) = \frac{1}{1-\rho^2}\left[\left(\frac{x-m_X}{\sigma_X}\right)^2 - 2\rho\left(\frac{x-m_X}{\sigma_X}\right)\left(\frac{y-m_Y}{\sigma_Y}\right) + \left(\frac{y-m_Y}{\sigma_Y}\right)^2\right]$$

We may rewrite $q(x,y)$ as

$$q(x,y) = \frac{1}{1-\rho^2}\left[\left(\frac{y-m_Y}{\sigma_Y}\right) - \rho\left(\frac{x-m_X}{\sigma_X}\right)\right]^2 + \left(\frac{x-m_X}{\sigma_X}\right)^2$$

$$= \frac{1}{(1-\rho^2)\sigma_Y^2}\left[y - m_X - \rho\frac{\sigma_Y}{\sigma_X}(x-m_X)\right]^2 + \left(\frac{x-m_X}{\sigma_X}\right)^2$$

Thus
$$f_{XY}(x,y) = \frac{\exp\left(-\frac{1}{2}\left(\frac{x-m_X}{\sigma_X}\right)^2\right)}{\sqrt{2\pi}\,\sigma_X} \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\,\sigma_Y(1-\rho^2)^{1/2}} \exp\left(-\frac{1}{2}q_1(x,y)\right)dy$$

where $$q_1(x,y) = \frac{1}{(1-\rho^2)\sigma_Y^2}\left(y - m_Y - \rho\frac{\sigma_Y}{\sigma_X}(x-m_X)\right)^2$$

The integrand is a normal pdf with mean $y+\rho\frac{\sigma_Y}{\sigma_X}(x-m_X)$ and variance $(1-\rho^2)\sigma_Y^2$ – thus the integral must be unity and we obtain

$$f_X(x) = \frac{1}{\sqrt{2\pi}\,\sigma_X}\exp\left(-\frac{(x-m_X)^2}{2\sigma_X^2}\right)$$

In a similar manner

$$f_Y(y) = \frac{1}{\sqrt{2\pi}\,\sigma_Y}\exp\left(-\frac{(x-m_Y)^2}{2\sigma_Y^2}\right)$$

(b) First find the characteristic function

$$M_X(jv) = E\{e^{jvX}\} = \int_{-\infty}^{\infty} e^{jvX} \frac{e^{-\frac{(x-m_X)^2}{2\sigma_X^2}}}{\sqrt{2\pi\sigma_X^2}}\, dx$$

Putting the exponents together and completing the square we obtain

$$M_X(jv) = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma_X^2}} \exp\left[-\frac{1}{2}\sigma_X^2 v^2 + jm_X v\right] \exp\left[-\frac{1}{2\sigma_X^2}(x - m_X - jv\sigma_X^2)^2\right] dx$$

$$= \exp\left(jvm_X - \frac{1}{2}\sigma_X^2 v^2\right)$$

It is then easy to differentiate to get

$$E[X] = -jM_X'(jv)\Big|_{v=0} = m_X$$

and

$$E[X^2] = (-j)^2 M_X''(jv)\Big|_{v=0} = \sigma_X^2 + m_X^2$$

from which it follows that the variance is σ_X^2.

(14) We can express the received signal $e_r(t)$ as the sum of N delayed components:-

$$e_r(t) = \sum_{i=1}^{N} a_i \, p(t - t_i)$$

where a_i is the amplitude of the scattered component, $p(t)$ is the transmitted pulse shape and t_i is the time taken by the pulse to reach the receiver. Using a phasor notation

$$e_r(t) = \sum_{i=1}^{N} a_i \cos(2\pi f_0 t + \phi_i)$$

where f_0 is the carrier frequency (we assume a single carrier).
In terms of in-phase & quadrature notation

$$e_r(t) = \cos(2\pi f_0 t) \sum_{i=1}^{N} a_i \cos(\phi_i) - \sin(2\pi f_0 t) \sum_{i=1}^{N} a_i \sin(\phi_i)$$

We can assume that the phase terms (ϕ_i) are uniformly distributed then under conditions of large N

$$e_r(t) = X\cos(2\pi f_0 t) - Y\sin(2\pi f_0 t)$$

where

$$X = \sum_{i=1}^{N} a_i \cos\phi_i \quad \& \quad Y = \sum_{i=1}^{N} a_i \sin\phi_i$$

X & Y will be independent, identically distributed Gaussian random variables by virtue of the Central Limit Theorem.

The envelope of the received signal is $R = (X^2 + Y^2)^{1/2}$ and the power P is $X^2 + Y^2$.

Defining two new random variables

$$R = \sqrt{X^2 + Y^2} \quad \& \quad \Theta = \tan^{-1}\frac{Y}{X}$$

To find $f_{R\Theta}(r,\theta)$ in terms of $f_{XY}(x,y)$ we assume $r \geq 0$ and $0 \leq \Theta < 2\pi$. With this assumption the transformation

$$\sqrt{x^2+y^2} = r \qquad \tan^{-1}\frac{y}{x} = \theta$$

has the inverse transform

$$x = r\cos\theta \qquad y = r\sin\theta$$

thus

$$J(x,y) = \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} \end{vmatrix} = \begin{vmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{vmatrix} = r$$

and

$$f_{R\Theta}(r,\theta) = r f_{XY}(r\cos\theta, r\sin\theta) = \frac{r}{2\pi\sigma^2}\exp\left(\frac{-r^2}{2\sigma^2}\right)$$

now $$f_R(r) = \int_0^{2\pi} f_{R\Theta}(r,\theta)\,d\theta = \frac{r}{2\pi\sigma^2}\exp\left(\frac{-r^2}{2\sigma^2}\right)\int_0^{2\pi} d\theta = \frac{r}{\sigma^2}\exp\left(\frac{-r^2}{2\sigma^2}\right)$$

& $$f_\Theta(\theta) = \int_0^{\infty} f_{R\Theta}(r,\theta)\,dr = \frac{1}{2\pi\sigma^2}\int_0^{\infty} r\exp\left(\frac{-r^2}{2\sigma^2}\right)dr = \frac{1}{2\pi}$$

R is a Rayleigh random variable & Θ is a uniform random over $(0,2\pi)$.

The power is just $P = R^2$ thus

$$f_P(p) = f_R(r)\left|\frac{dr}{dp}\right| \qquad \& \qquad \begin{array}{l} p = r^2 \\ r = \pm p^{1/2} \end{array}$$

$$= f_R(g^{-1}(p))\left|\frac{dg^{-1}(p)}{dp}\right|$$

$$= \frac{1}{2\sigma^2}\exp\left(\frac{-p}{2\sigma^2}\right)$$

Thus the envelope follows a Rayleigh distribution & the power follows a negative exponential distribution.

The probability that the received power is below 50μW is given by

$$\int_0^{P_{thr}} f_P(p)\,dp = \int_0^{P_{thr}} \frac{1}{P_0} \exp\left(-p/P_0\right) dp = 1 - \exp\left(-P/P_0\right)$$

where P_0 is the average power, given by $2\sigma^2$.

$$= 1 - \exp\left(-\frac{50\times10^{-3}}{100\times10^{-3}}\right) = \underline{\underline{0.394}}$$

(15) The power at the input is given by the product of the PSD and the bandwidth of the signal: PB_s. The power at the output is given by the product of the PSD, the filter gain and the bandwidth of the filter PGB_f. Since the gain is unity, the power at the output of the filter will be less than at the input by a factor of 10.

(16) The mean is given by

$$\frac{(-1.6129 - 1.2091 - 0.4379 - 2.0639 - 0.6484)}{5} = {}^{-}1.1944$$

After subtracting the mean from each element the variance is

$$\frac{(0.4185^2 + 0.0147^2 + 0.7565^2 + 0.8695^2 + 0.5460^2)}{5} = 0.3604$$

The autocorrelation is given by

$$\phi_{xx}(0) = (1.6129^2 + 1.2091^2 + 0.4379^2 + 2.0639^2 + 0.6484^2)/5 = 1.787$$

$$\phi_{xx}(1) = ({}^{-}1.6129(-1.2091) - 1.2091(-0.4379) - 0.4379(-2.0639) - 2.0639(-0.648))/4 = 1.1804$$

$$\phi_{xx}(2) = (-1.6129(-0.4379) - 1.2091(-2.0639) - 0.4379(-0.6484))/3 = 1.1619$$

For the autocovariance

$$\gamma_{xx}(0) = 0.3604$$

$$\gamma_{xx}(1) = ({}^{-}0.4185(-0.0147) - 0.0147(0.7565) + 0.7565({}^{-}0.8695) - 0.8695(0.5460))/4 = {}^{-}0.2844$$

$$\gamma_{xx}(2) = (-0.4185(0.7565) - 0.0147(-0.8695) + 0.7565(0.5460))/3 = 0.0364$$

For a zero mean process

$$S_{xx}(\omega) = \sum_{m=-\infty}^{\infty} \gamma_{xx}(m) \exp(-j\omega m \Delta t)$$

and at a quarter of the sampling frequency $\omega = \frac{\pi}{(2\Delta t)}$

$$S_{xx}(\omega) = \sum_{m=-2}^{2} \gamma_{xx}(m) \exp(-jm\pi/2)$$

$$= \gamma_{xx}(0) + 2\gamma_{xx}(1)\cos \pi/2 + 2\gamma_{xx}(2)\cos(\pi)$$

$$= 0.2875$$

$$= {}^{-}5.41\,dB$$

(17)

$$S_{yy}(z) = H(z)H(z^{-1})S_{xx}(z) = (0.5+0.75z^{-1})(0.5+0.75z)\,\sigma_x^2$$

$$= (0.375z + 0.8125 + 0.375z^{-1})\,\sigma_x^2$$

The autocorrelation at the output is the inverse z-transform of this:

$$\phi_{yy}(-1) = 0.375\sigma_x^2 \qquad \phi_{yy}(0) = 0.8125\sigma_x^2 \qquad \phi_{yy}(1) = 0.375\sigma_x^2$$

The variance of the output is $0.8125\sigma_x^2$ with a corresponding RMS value of $\sqrt{0.8125}\,\sigma_x$. The PSD

$$S_{yy}(\omega) = (0.375e^{-j\omega\Delta t} + 0.8125 + 0.375e^{j\omega\Delta t})\,\sigma_x^2$$

$$= (0.8125 + 0.75\cos(\omega\Delta t))\,\sigma_x^2$$

(18) At the output of the filter

$$S_{yy}(z) = (0.1-0.8z^{-1})(0.1-0.8z)$$

with zeros at $z=8$ and $1/8$. The term $(0.1-0.8z)$ is minimum since it has a root at $1/8$ but it is non-causal because of the positive power of z. Rewriting we have:

$$(0.1-0.8z) = z(0.1z^{-1}-0.8) \text{ and } (0.1-0.8z^{-1}) = (0.1z-0.8)z^{-1}$$

thus:

$$S_{yy}(z) = (0.1z-0.8)(0.1z^{-1}-0.8)$$

The minimum phase causal filter is $(0.1z^{-1}-0.8)$. The whitening filter is the inverse of this, ie. $1/(0.1z^{-1}-0.8)$. This is not the inverse of $H(z)$.

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Filter #1

$$H_1(z) = 1 - 2.75z^{-1} - 0.75z^{-2.}$$

z-transform of autocorrelation at output

$$S_{y_1y_1}(z) = H_1(z)\, H_1(z^{-1})\, \sigma_x^2$$
$$= (1 - 2.75z^{-1} - 0.75z^{-2})\,(1 - 2.75z^{1} - 0.75z^{2})\; 2$$
$$= -1.5z^2 - 1.375z^1 + 18.25 - 1.375z^{-1} - 1.5z^{-2}$$

Inverse z-transform by inspection to give autocorrelation sequence:

$$\phi_{y_1y_1}(m) = Z^{-1}[\, S_{y_1y_1}(z)\,]$$

The autocorrelation sequence is :

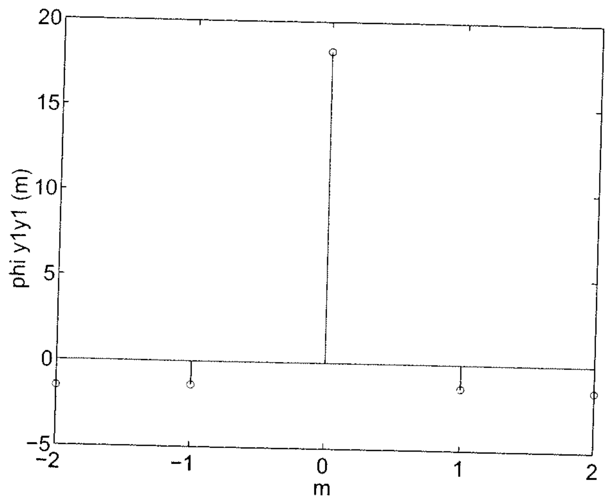
$\{\phi_{y_1y_1}(-2) = -1.5,\ \phi_{y_1y_1}(-1) = -1.375,\ \phi_{y_1y_1}(0) = 18.25,\ \phi_{y_1y_1}(1) = -1.375,\ \phi_{y_1y_1}(2) = -1.5\}$

Filter #2

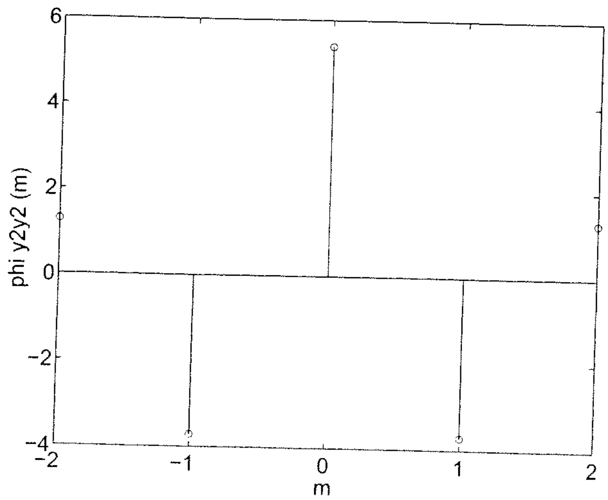
$$H_2(z) = 1 - 1.1314z^{-1} + 0.64z^{-2.}$$

The calculation proceeds in a similar manner using $H_2(z)$ instead of $H_1(z)$. The autocorrelation sequence associated with the output is:

$\{\phi_{y_2y_2}(-2) = 2.28,\ \phi_{y_2y_2}(-2) = -3.722,\ \phi_{y_2y_2}(0) = 5.379,\ \phi_{y_2y_2}(1) = -3.711,\ \phi_{y_2y_2}(2) = 1.28\}$



Autocorrelation sequence $\phi_{y_1y_1}(m)$



Autocorrelation sequence $\phi_{y_2y_2}(m)$

Cross-correlation sequence $\phi_{y_1y_2}(m) = E[\ y_1(n)\ y_2(n+m)\ ]$. Start with the z-transform using equation (7.15).

$$\begin{aligned} S_{y_1y_2}(z) &= H_1(z^{-1})\ H_2(z)\ \sigma_x^2 \\ &= (1 - 2.75z^1 - 0.75z^2)\ (1 - 1.1314z^{-1} + 0.64z^{-2})\ 2 \\ &= -1.5z^2 - 3.803z^1 + 7.263 - 5.783z^{-1} + 1.28z^{-2} \end{aligned}$$

Inverse z-transform yields:

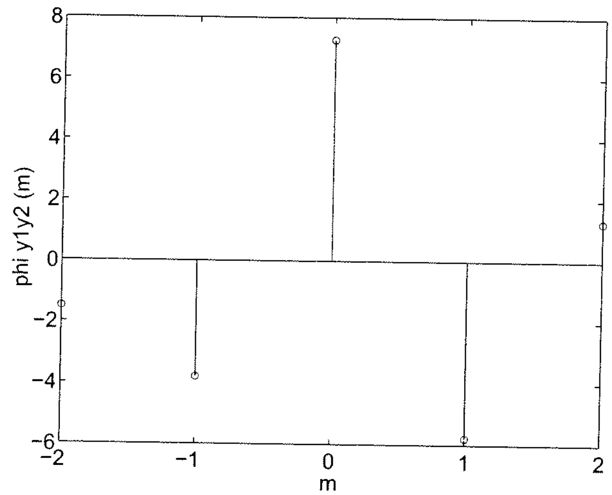
$\{\phi_{y_1y_2}(-2) = -1.5,\ \phi_{y_1y_2}(-1) = -3.803,\ \phi_{y_1y_2}(0) = 7.263,\ \phi_{y_1y_2}(1) = -5.783,\ \phi_{y_1y_2}(2) = 1.28\}$

The second cross-correlation is most easily obtained by using the property that $\phi_{xy}(m) = \phi_{yx}(-m)$ i.e.

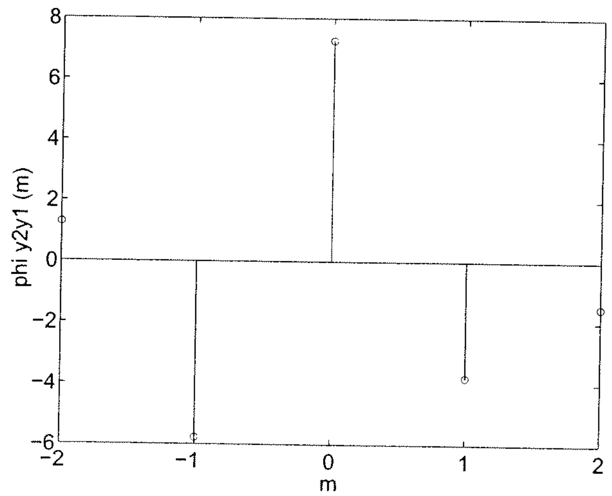
$$\phi_{y_2y_1}(m) = \phi_{y_1y_2}(-m)$$

to give the sequence:

$\{\phi_{y_1y_2}(2) = -1.5,\ \phi_{y_1y_2}(1) = -3.803,\ \phi_{y_1y_2}(0) = 7.263,\ \phi_{y_1y_2}(-1) = -5.783,\ \phi_{y_1y_2}(-2) = 1.28\}$



Cross-correlation sequence $\phi_{y_1y_2}(m)$



Cross-correlation sequence $\phi_{y_2y_1}(m)$

To design the whitening filter we need to find the zeros of the the two filters. Starting with filter 1.

$$z^2 - 2.75z - 0.75 = 0$$

zeros at $z = 3$ and $z = -0.25$. Hence the transfer function can be written as:

$$H_1(z) = \frac{(z-3)\,(z+0.25)}{z^2}$$

This is a non-minimum phase filter as one of its zeros ($z = 3$) is outside the unit circle. To form the whitening filter we must reflect the back inside the unit circle to $z = \frac{1}{3}$ to form a minimum phase filter.

$$H'_1(z) = \frac{(z-\frac{1}{3})\,(z+0.25)}{z^2}$$

The signal at the output of thin minimum phase filter will have the same autocorrelation sequence and power spectral density as the non-minimum phase filter. The whitening filter is the inverse of the minimum phase filter.

$$\begin{aligned} W_1(z) &= \frac{1}{H'_1(z)} \\ &= \frac{z^2}{(z-\frac{1}{3})\,(z+0.25)} \\ &= \frac{1}{1-0.0833z^{-1}-0.0833z^{-2}} \end{aligned}$$

We proceed in a similar way for filter 2.

$$z^2 - 1.1314z + 0.64 = 0$$

This filter has zeros at $z = 0.5657 \pm j0.5657$ and hence is a minimum phase filter. The whitening filter is the inverse of $H_2(z)$.

$$\begin{aligned} W_2(z) &= \frac{1}{H_2(z)} \\ &= \frac{1}{1-1.1314z^{-1}+0.64z^{-2}} \end{aligned}$$

Only $W_2(z)$ is both a whitening filter and an inverse filter.