# Tutorial sheet 2.

(1)
$$E = \int_{-\infty}^{\infty} v^2(t)\,dt$$

$$= \int_0^{\infty} \left[2\exp(-3t) + 4\exp(-7t)\right]^2 dt$$

$$= \int_0^{\infty} \left[4\exp(-6t) + 16\exp(-10t) + 16\exp(-14t)\right] dt$$

$$= \left[-\frac{4}{6}\exp(-6t) - \frac{16}{10}\exp(-10t) - \frac{16}{14}\exp(-14t)\right]_0^{\infty}$$

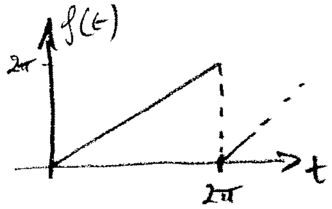
$$= 4/6 + 16/10 + 16/14 = \underline{3.410\ \mathrm{J}}$$

This is for a $1\,\Omega$ resistor thus for a $5\,\Omega$ resistor we have $3.410/5 = \underline{\underline{0.68\,\mathrm{J}}}$

(2)
$$P = \frac{1}{T}\int_0^{T} x^2(t)\,dt$$

$$= \frac{1}{1}\int_0^{0.2} 5^2\,dt = 5\,\mathrm{W}$$

③




$$a_0 = \frac{1}{\pi}\int_0^{2\pi} f(t)\,dt = \frac{1}{\pi}\int_0^{2\pi} t\,dt = \frac{1}{\pi}\left[\frac{t^2}{2}\right]_0^{2\pi} = 2\pi$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(t)\cos nt\,dt \qquad (n = 1, 2 \ldots)$$

$$= \frac{1}{\pi}\int_0^{2\pi} t\cos nt\,dt = 0 \quad \text{for all } n$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} f(t)\sin nt\,dt \qquad (n = 1, 2, \ldots)$$

$$= \frac{1}{\pi}\int_0^{2\pi} t\sin nt\,dt = \frac{1}{\pi}\left[-\frac{t}{n}\cos nt + \frac{\sin nt}{n^2}\right]_0^{2\pi}$$

$$= \frac{1}{\pi}\left(-\frac{2\pi}{n}\cos 2n\pi\right) \qquad (\text{since } \sin 2n\pi = \sin 0 = 0)$$

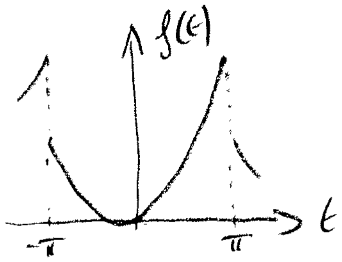
$$= -\frac{2}{n}$$

$\therefore$ The fourier series expansion of this sawtooth wave is:

$$f(t) = \pi - \sum_{n=1}^{\infty} \frac{2}{n}\sin nt$$

$$= \pi - 2\left(\sin t + \frac{\sin 2t}{2} + \frac{\sin 3t}{3} + \ldots \frac{\sin nt}{n} + \ldots\right)$$

④




$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\,dt = \frac{1}{\pi}\int_{-\pi}^{\pi} (t^2+t)\,dt = \left[\frac{t^3}{3} + \frac{t^2}{2}\right]_{-\pi}^{\pi} = \frac{2}{3}\pi^3$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\cos nt\,dt = \frac{1}{\pi}\int_{-\pi}^{\pi} (t^2+t)\cos nt\,dt$$

which on integration by parts gives

$$= \frac{1}{\pi}\left[\frac{t^2}{n}\sin nt + \frac{2t}{n^2}\cos nt - \frac{2}{n^3}\sin nt + \frac{t}{n}\sin nt + \frac{1}{n^2}\cos nt\right]_{-\pi}^{\pi}$$

$$= \frac{1}{\pi}\,\frac{4\pi}{n^2}\cos n\pi \qquad \left(\text{since } \sin n\pi = 0 \;\&\; \left[\frac{1}{n^2}\cos nt\right]_{-\pi}^{\pi} = 0\right)$$

$$= \frac{4}{n^2}(-1)^n \qquad \left(\text{since } \cos n\pi = (-1)^n\right)$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\sin nt\,dt = \frac{1}{\pi}\int_{-\pi}^{\pi} (t^2+t)\sin nt\,dt$$

which on integration by parts gives

$$= \frac{1}{\pi}\left[-\frac{t^2}{n}\cos nt + \frac{2t}{n^2}\sin nt + \frac{2}{n^3}\cos nt - \frac{t}{n}\cos nt + \frac{1}{n^2}\sin nt\right]_{-\pi}^{\pi}$$

$$= \frac{-2}{n}\cos n\pi = \frac{-2}{n}(-1)^n \qquad \left(\text{since } \cos n\pi = (-1)^n\right)$$

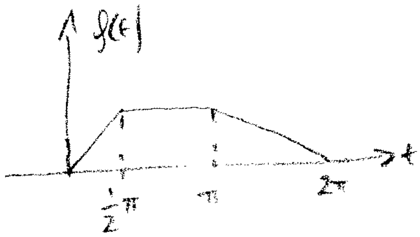
Thus the fourier series expansion of $f(t)$ is

$$f(t) = \frac{1}{3}\pi^3 + \sum_{n=1}^{\infty}\frac{4}{n^2}(-1)^n\cos nt - \sum_{n=1}^{\infty}\frac{2}{n}(-1)^n\sin nt$$

$$= \frac{1}{3}\pi^3 + 4\left(-\cos t + \frac{\cos 2t}{2^2} - \frac{\cos 3t}{3^2} + \cdots\right)$$

$$+ 2\left(\sin t - \frac{\sin 2t}{2} + \frac{\sin 3t}{3} - \cdots\right)$$

⑤




$$a_0 = \frac{1}{\pi}\int_0^{2\pi} f(t)\,dt = \frac{1}{\pi}\left[\int_0^{\pi/2} t\,dt + \int_{\pi/2}^{\pi}\frac{1}{2}\pi\,dt + \int_{\pi}^{2\pi}\left(\pi - \frac{1}{2}t\right)dt\right] = \frac{5}{8}\pi$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(t)\cos n\pi\,dt \qquad (n=1,2,\dots)$$

$$= \frac{1}{\pi}\left[\int_0^{\pi/2} t\cos nt\,dt + \int_{\pi/2}^{\pi}\frac{\pi}{2}\cos nt\,dt + \int_{\pi}^{2\pi}\left(\pi-\frac{1}{2}t\right)\cos nt\,dt\right]$$

$$= \frac{1}{\pi}\left\{\left[\frac{t}{n}\sin nt + \frac{\cos nt}{n^2}\right]_0^{\pi/2} + \left[\frac{\pi}{2n}\sin nt\right]_{\pi/2}^{\pi} + \left[\frac{2\pi-t}{2}\cdot\frac{\sin nt}{n} - \frac{\cos nt}{2n^2}\right]_{\pi}^{2\pi}\right\}$$

$$= \frac{1}{\pi}\left(\frac{\pi}{2n}\sin\frac{n\pi}{2} + \frac{1}{n^2}\cos\frac{n\pi}{2} - \frac{1}{n^2} - \frac{\pi}{2n}\sin\frac{n\pi}{2} - \frac{1}{2n^2} + \frac{1}{2n^2}\cos n\pi\right)$$

$$= \frac{1}{2\pi n^2}\left(2\cos\frac{n\pi}{2} - 3 + \cos n\pi\right) = \begin{cases} 1/\pi n^2\left[(-1)^{n/2} - 1\right] & (\text{even } n) \\ -2/\pi n^2 & (\text{odd } n)\end{cases}$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} f(t)\sin nt\,dt \qquad (n=1,2,\dots)$$

$$= \frac{1}{\pi}\left[\int_0^{\pi/2} t\sin nt\,dt + \int_{\pi/2}^{\pi}\frac{\pi}{2}\sin nt\,dt + \int_{\pi}^{2\pi}\left(\pi-\frac{1}{2}t\right)\sin nt\,dt\right]$$

$$= \frac{1}{\pi}\left\{\left[-\frac{t}{n}\cos nt + \frac{1}{n^2}\sin nt\right]_0^{\pi/2} + \left[-\frac{\pi}{2n}\cos nt\right]_{\pi/2}^{\pi} + \left[\frac{t-2\pi}{2n}\cos nt - \frac{1}{2n^2}\sin nt\right]_{\pi}^{2\pi}\right\}$$

$$= \frac{1}{\pi}\left(-\frac{\pi}{2n}\cos\frac{n\pi}{2} + \frac{1}{n^2}\sin\frac{n\pi}{2} - \frac{\pi}{2n}\cos n\pi + \frac{\pi}{2n}\cos\frac{n\pi}{2} + \frac{\pi}{2n}\cos n\pi\right)$$

$$= \frac{1}{\pi n^2}\sin\frac{n\pi}{2} = \begin{cases} 0 & (\text{even } n) \\ (-1)^{(n-1)/2}/\pi n^2 & (\text{odd } n)\end{cases}$$

$\therefore$ the Fourier series expansion of $f(t)$ is given by

$$f(t) = \frac{5}{16}\pi - \frac{2}{\pi}\left(\cos t + \frac{\cos 3t}{3^2} + \frac{\cos 5t}{5^2} + \dots\right)$$
$$- \frac{2}{\pi}\left(\frac{\cos 2t}{2^2} + \frac{\cos 6t}{6^2} + \frac{\cos 10t}{10^2} + \dots\right)$$
$$+ \frac{1}{\pi}\left(\sin t - \frac{\sin 3t}{3^2} + \frac{\sin 5t}{5^2} - \frac{\sin 7t}{7^2} \dots\right)$$

(6)

$$a_n = 2\int_0^{0.2} 5\cos(n2\pi t)\,dt = 10\left[\frac{\sin(2\pi n t)}{2\pi n}\right]_0^{0.2}$$

$$= \frac{10}{2\pi n} \sin(2\pi n/5)$$

$$b_n = 2\int_0^{0.2} 5\sin(n2\pi t)\,dt = 10\left[\frac{-\cos(2\pi n t)}{2\pi n}\right]_0^{0.2}$$

$$= \frac{10}{2\pi n}\left(-\cos\left(\frac{2\pi n}{5}\right)+1\right)$$

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$$X_n = \frac{1}{T}\int_{-T/2}^{T/2} x(t)\exp(-jn\omega_0 t)\,dt$$

for $n=0$ $\quad X_0 = \int_0^{0.2} 5\,dt = 1$

$n<0$ & $n>0$ $\quad X_n = \int_0^{0.2} 5\exp(-jn2\omega t)\,dt$

$$= \frac{5}{2\pi n j}\left(1-\exp(-jn2\pi/5)\right)$$

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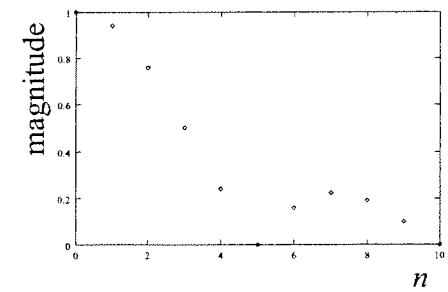
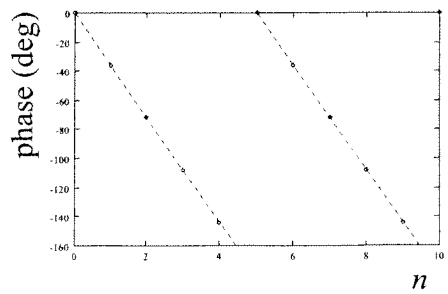
| $n$ | $X_n$ | $\lvert X_n\rvert$ | $\angle X_n$ (degrees) |
|---|---|---|---|
| 0 | 1 | 1 | 0 |
| 1 | $0.76 - j0.55$ | 0.94 | −36 |
| 2 | $0.24 - j0.72$ | 0.76 | −72 |
| 3 | $-0.16 - j0.48$ | 0.5 | −108 |
| 4 | $-0.19 - j0.14$ | 0.24 | −144 |
| 5 | 0 | 0 | – |
| 6 | $0.13 - j0.09$ | 0.16 | −36 |
| 7 | $0.07 - j0.21$ | 0.22 | −72 |
| 8 | $-0.06 - j0.18$ | 0.19 | −108 |
| 9 | $-0.08 - j0.06$ | 0.10 | −144 |
| 10 | 0 | 0 | – |



(7) Here the period is $2T$ so $\omega = \frac{\pi}{T}$, thus

$$X_n = \frac{1}{2T}\int_0^{2T} x(t)\exp(-jn\omega_0 t)\,dt = \frac{1}{2T}\int_0^{2T}\frac{2}{T}te^{-jn\pi t/T}dt$$

$$= \frac{1}{T^2}\left[\frac{Tt}{-jn\pi}\exp(-jn\pi t/T) - \frac{T^2}{(jn\pi)^2}\exp(-jn\pi t/T)\right]_0^{2T} \quad n \neq 0$$

but $\exp(-jn2\pi) = 1$ so

$$X_n = \frac{1}{T^2}\left[\frac{2T^2}{-jn\pi} + \frac{T^2}{(n\pi)^2} - \frac{T^2}{(n\pi)^2}\right] = \frac{j2}{n\pi} \quad (n \neq 0)$$

where $n = 0$

$$X_0 = \frac{1}{2T}\int_0^{2T} f(t)\,dt = \frac{1}{2T}\int_0^{2T}\frac{2t}{T}\,dt = \frac{1}{T^2}\left[\frac{t^2}{2}\right]_0^{2T} = 2$$

so the complex form of the fourier series for the sawtooth wave is given by

$$f(t) = 2 + \sum_{\substack{n=-\infty \\ n\neq 0}}^{n=\infty}\frac{j2}{n\pi}\exp(jn\pi t/T)$$

& noting that $j = \exp(j\pi/2)$

$$= 2 + \frac{2}{\pi}\sum_{\substack{n=-\infty \\ n\neq 0}}^{n=\infty}\frac{1}{n}\exp\left[j\left(\frac{n\pi t}{T} + \frac{\pi}{2}\right)\right]$$

Compare this to the answer for Q3.

Q8

$$X_n = \frac{1}{2\pi}\left[\int_{-\pi}^{0} 2e^{-jnt}\,dt + \int_{0}^{\pi} 1 e^{-jnt}\,dt\right]$$

$$= \frac{1}{2\pi}\left\{\left[-\frac{2}{jn}e^{-jnt}\right]_{-\pi}^{0} + \left[-\frac{1}{jn}e^{-jnt}\right]_{0}^{\pi}\right\}$$

$$= \frac{-1}{2jn\pi}\left[2 - 2e^{jn\pi} + e^{-jn\pi} - 1\right]$$

$$= \frac{j}{2n\pi}\left[1-(-1)^n\right], \quad n \neq 0$$

$$X_0 = \frac{1}{2\pi}\left[\int_{-\pi}^{0} 2\,dt + \int_{0}^{\pi} 1\,dt\right] = 3/2$$

Thus the complex fourier series for the function $f(t)$ is

$$f(t) = \frac{3}{2} + \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \frac{j}{2n\pi}\left[1-(-1)^n\right]e^{jnt}$$

Q9

(a) From Euler's identity

$$x(t) = 3\frac{(e^{j5t} + e^{-j5t})}{2} + 4\left(\frac{e^{j10t} - e^{-j10t}}{2j}\right)$$

$$= 2je^{-j10t} + \frac{3}{2}e^{-j5t} + \frac{3}{2}e^{j5t} - 2je^{j10t}$$

thus $X_{-2} = 2j$, $X_{-1} = 3/2$, $X_1 = 3/2$, $X_2 = -2j$

(b) $$x(t) = \frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2} + \left(\frac{(e^{j2\omega_0 t} - e^{-j2\omega_0 t})}{2j}\right)^2$$

$$= -\frac{1}{4}e^{j4\omega_0 t} + \frac{1}{2}e^{j\omega_0 t} + \frac{1}{2} + \frac{1}{2}e^{-j\omega_0 t} - \frac{1}{4}e^{-j4\omega_0 t}$$

thus $X_{-4} = -1/4$, $X_{-1} = 1/2$, $X_0 = 1/2$, $X_1 = 1/2$ & $X_4 = -1/4$

(10)

An example of an even function, for which $f(t) = f(-t)$, is a cosine wave. An example of an odd function, for which $f(t) = -f(-t)$, is a sine wave.

Noting these properties, for an even function $f(t)$

$$a_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\cos n\omega t \, dt = \frac{4}{T}\int_{0}^{T/2} f(t)\cos n\omega t \, dt$$

$$b_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\sin n\omega t \, dt = 0$$

Thus the Fourier series expansion of an even, periodic function $f(t)$ with period $T$ consists of cosine terms only.

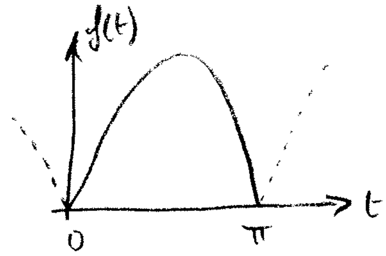
Similarly for an odd function $f(t)$

$$a_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\cos n\omega t \, dt = 0$$

$$b_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\sin n\omega t \, dt = \frac{4}{T}\int_{-T/2}^{T/2} f(t)\sin n\omega t \, dt$$

and the Fourier series expansion of an odd, periodic function $f(t)$ with period $T$ consists of sine terms only.

(11)




From the sketch we see that

$$f(t+\pi) = f(t)$$

thus only even harmonics are present in the Fourier series expansion. $f(t)$ is also an even function of $t$ so the Fourier series expansion will consist of even harmonic cosine terms.

Taking $T = 2\pi$ we have $\omega = 1$

$$a_n = \frac{2}{\pi}\int_0^{\pi} f(t)\cos nt \quad (\text{even } n) \quad = \frac{2}{\pi}\int_0^{\pi} \sin t \cos nt \, dt$$

$$= \frac{1}{\pi}\int_0^{\pi} [\sin(n+1)t - \sin(n-1)t]\, dt$$

$$= \frac{1}{\pi}\left[-\frac{\cos(n+1)t}{n+1} + \frac{\cos(n-1)t}{n-1}\right]_0^{\pi}$$

& since both $n+1$ & $n-1$ are odd when $n$ is even

$$\cos(n+1)\pi = \cos(n-1)\pi = -1$$

so

$$a_n = \frac{1}{\pi}\left[\left(\frac{1}{n+1} - \frac{1}{n-1}\right) - \left(-\frac{1}{n+1} + \frac{1}{n-1}\right)\right] = -\frac{4}{\pi}\frac{1}{n^2-1}$$

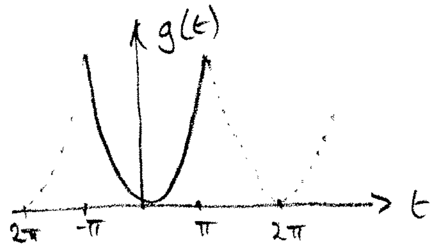
Thus the Fourier series expansion of $f(t)$ is

$$f(t) = \frac{1}{2}a_0 + \sum_{\substack{n=2 \\ (n \text{ even})}}^{\infty} a_n \cos nt = \frac{2}{\pi} - \frac{4}{\pi}\sum_{\substack{n=2 \\ (n \text{ even})}}^{\infty} \frac{1}{n^2-1}\cos nt$$

$$= \frac{2}{\pi} - \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{1}{4n^2-1}\cos 2nt$$

$$= \frac{2}{\pi} - \frac{4}{\pi}\left(\frac{1}{3}\cos 2t + \frac{1}{15}\cos 4t + \frac{1}{35}\cos 6t + \dots\right)$$

(12)




$g(t)$ is an even function of $t$ so its Fourier series expansion consist of cosine terms only.

$$f(t) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos nt$$

$$a_0 = \frac{2}{\pi}\int_0^{\pi} f(t)\,dt = \frac{2}{\pi}\int_0^{\pi} t^2\,dt = \frac{2}{3}\pi^2$$

$$a_n = \frac{2}{\pi}\int_0^{\pi} f(t)\cos nt\,dt \qquad (n = 1, 2, 3, \dots)$$

$$= \frac{2}{\pi}\int_0^{\pi} t^2 \cos nt\,dt$$

$$= \frac{2}{\pi}\left[\frac{t^2}{n}\sin nt + \frac{2t}{n^2}\cos nt - \frac{2}{n^3}\sin nt\right]_0^{\pi} = \frac{2}{\pi}\left(\frac{2\pi}{n^2}\cos n\pi\right) = \frac{4}{n^2}(-1)^n$$

since $\sin n\pi = 0$ & $\cos n\pi = (-1)^n$. Thus the Fourier series expansion of $t^2$ is

$$g(t) = \frac{1}{3}\pi^2 + 4\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}\cos nt = \frac{1}{3}\pi^2 - 4\cos t + \cos 2t - \frac{4}{9}\cos 3t + \dots$$

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$h(t)$ is an odd function of $t$ thus $h(t) = \sum_{n=1}^{\infty} b_n \sin nt$

where $b_n = \frac{2}{\pi}\int_0^{\pi} h(t)\sin nt\,dt \quad (n = 1, 2, \dots)$

$$= \frac{2}{\pi}\int_0^{\pi} t\sin nt\,dt = \frac{2}{\pi}\left[\frac{-t}{n}\cos nt + \frac{\sin nt}{n^2}\right]_0^{\pi} = \frac{-2}{n}(-1)^n$$

$$\therefore\ h(t) = -2\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\sin nt$$

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From Q4 we have for $f(t) = t^2 + t$, $-\pi < t < \pi$, $f(t) = f(t + 2\pi)$

$$f(t) = \frac{1}{3}\pi^2 + \sum_{n=1}^{\infty}\frac{4}{n^2}(-1)^n\cos nt - \sum_{n=1}^{\infty}\frac{2}{n}(-1)^n\sin nt$$

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We see that the Fourier series expansion of $f(t)$ has coefficients equal to the sums of the coefficients in the Fourier series expansions of $g(t)$ & $h(t)$. This illustrates the linearity property of the Fourier series.

(13) From Q7 $X_0 = 2$ &

$$X_n = \frac{j2}{n\pi} \quad (n \neq 0)$$

Parseval's theorem states that the power in a signal may be calculated from the trigonometric or complex Fourier series coefficient, thus

$$P = \frac{1}{2T}\int_0^{2T} [f(t)]^2\, dt = C_0^2 + \sum_{n=-\infty}^{-1} |C_n|^2 + \sum_{n=1}^{\infty} |C_n|^2$$

$$\frac{1}{2T}\int_0^{2T} \frac{4t^2}{T^2}\, dt = 4 + 2\sum_{n=1}^{\infty} \left(\frac{2}{n\pi}\right)^2$$

$$\frac{16}{3} = 4 + \sum_{n=1}^{\infty} \frac{8}{n^2\pi^2}$$

which gives $\frac{1}{6}\pi^2 = \sum_{n=1}^{\infty} \frac{1}{n^2}$

Q14

(a) $$F\{f(t)\} = \int_{-T}^{T} Ae^{-j\omega t}\,dt = \begin{cases} \left[-(A/j\omega)\,e^{-j\omega t}\right]_{-T}^{T} & \omega \neq 0 \\ 2A & \omega = 0 \end{cases}$$

$$= \frac{2A}{\omega}\sin\omega T = 2AT\,\mathrm{sinc}\,\omega T$$

(b) $$F\{f(t)\} = e^{-j\omega T}\,2AT\,\mathrm{sinc}\,\omega T = 2AT\,e^{-j\omega T}\,\mathrm{sinc}\,\omega T$$

as a consequence of the shift property.

Q15

$$F\{f(t)\} = \int_{-\infty}^{\infty} H(t)\,e^{-at}e^{-j\omega t}\,dt \qquad (a > 0)$$

$$= \int_{0}^{\infty} e^{-(a+j\omega)t}\,dt = \left[-\frac{e^{-(a+j\omega)t}}{a+j\omega}\right]_{0}^{\infty}$$

so that $$F\{H(t)e^{-at}\} = \frac{1}{a+j\omega}$$

Fourier

$$X(\omega) = \int_{-\infty}^{\infty} x(t)\exp(-j\omega t)\,dt = \int_0^{0.2} 5\exp(-j\omega t)\,dt = \frac{5}{j\omega}\left(1-\exp\left(\frac{-j\omega}{5}\right)\right)$$

& to plot the magnitude & phase spectra we would calculate the modulus & argument for all $\omega$. However manipulating the expression as

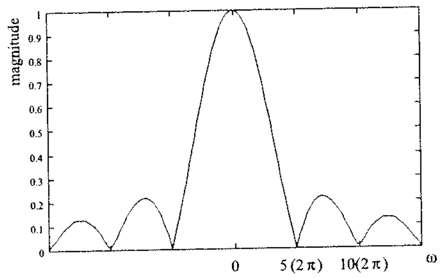
$$X(\omega) = \frac{5}{j\omega} e^{-j\omega/10}\left(e^{j\omega/10} - e^{-j\omega/10}\right)$$

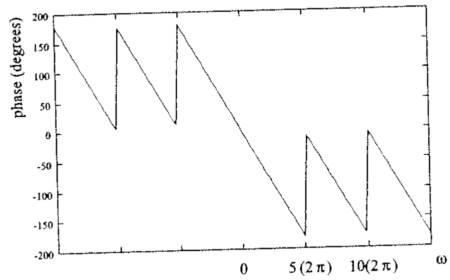
$$= \frac{5}{j\omega} e^{-j\omega/10} 2j\sin(\omega/10) = e^{-j\omega/10}\frac{\sin(\omega/10)}{\omega/10}$$

$$= e^{-j\omega/10}\,\mathrm{sinc}(\omega/10)$$

The magnitude is thus $|X(\omega)| = |e^{-j\omega/10}||\mathrm{sinc}(\omega/10)| = |\mathrm{sinc}(\omega/10)|$

phase is $\angle X(\omega) = \angle e^{-j\omega/10} + \angle\,\mathrm{sinc}(\omega/10) = -\omega/10 + \angle\,\mathrm{sinc}(\omega/10)$




Laplace

$$X(s) = \int_{0^-}^{\infty} x(t)\exp(-st)\,dt = \int_0^{0.2} 5\exp(-st)\,dt = \frac{5}{s}\left(1-\exp(-s/5)\right)$$

Note the similarity between the Fourier & Laplace transforms.

Q17 We have that $\mathcal{F}\{f(t)\} = F(j\omega) = \frac{1}{a + j\omega}$

Therefore the amplitude & argument of $F(j\omega)$ are

$$|F(j\omega)| = \frac{1}{\sqrt{a^2+\omega^2}}$$

$$\arg F(j\omega) = \tan^{-1}(1) - \tan^{-1}\left(\frac{\omega}{a}\right) = -\tan^{-1}\left(\frac{\omega}{a}\right)$$

Q18 Since $\cos\omega_c t = \frac{1}{2}(e^{j\omega_c t} + e^{-j\omega_c t})$ from the linearity property $\mathcal{F}\{g(t)\} = \mathcal{F}\{\frac{1}{2} f(t)(e^{j\omega_c t} + e^{-j\omega_c t})\}$

$$= \frac{1}{2}\mathcal{F}\{f(t)e^{j\omega_c t}\} + \frac{1}{2}\mathcal{F}\{f(t)e^{-j\omega_c t}\}$$

If $\mathcal{F}\{f(t)\} = F(j\omega)$ then

$$\mathcal{F}\{f(t)\cos\omega_c t\} = \mathcal{F}\{g(t)\} = \frac{1}{2}F(j(\omega-\omega_c)) + \frac{1}{2}F(j(\omega+\omega_c))$$

& the effect of multiplying the signal $f(t)$ by the carrier signal $\cos\omega_c t$ is thus to produce a signal whose spectrum consists of two (scaled) versions of $F(j\omega)$. This is modulation.