Mathematical Methods for Engineers (MathEng)
EXAM
20th February 2008
Duration: 2 hrs, all documents and calculators permitted
ATTEMPT ALL QUESTIONS – ANSWER IN ENGLISH

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1. Determine the Cholesky decomposition of the following matrix:

$$A = \begin{bmatrix} 9 & 0 & 6 \\ 0 & 16 & 4 \\ 6 & 4 & 30 \end{bmatrix}$$

and use it to solve $Ax = b$ when $b = [3\ {-36}\ {-32}]^T$.

[5 marks]

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2. Match each of the five pole/zero configurations illustrated in the z-planes of Figures Q2(a)-(e) (on page 2) to one unique frequency responses in Figures Q2(1)-(5). Justify your answers in each case.

[5 marks]

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3. The output $y[n]$ of a discrete-time linear time-invariant system is found to be $2(1/3)^n u[n]$ when the input $x[n]$ is $u[n]$ (the unit step sequence).
   a. Find the impulse response $h[n]$ of the system.
   b. Find the output $y[n]$ when the input $x[n]$ is $(1/2)^n u[n]$.

[10 marks]

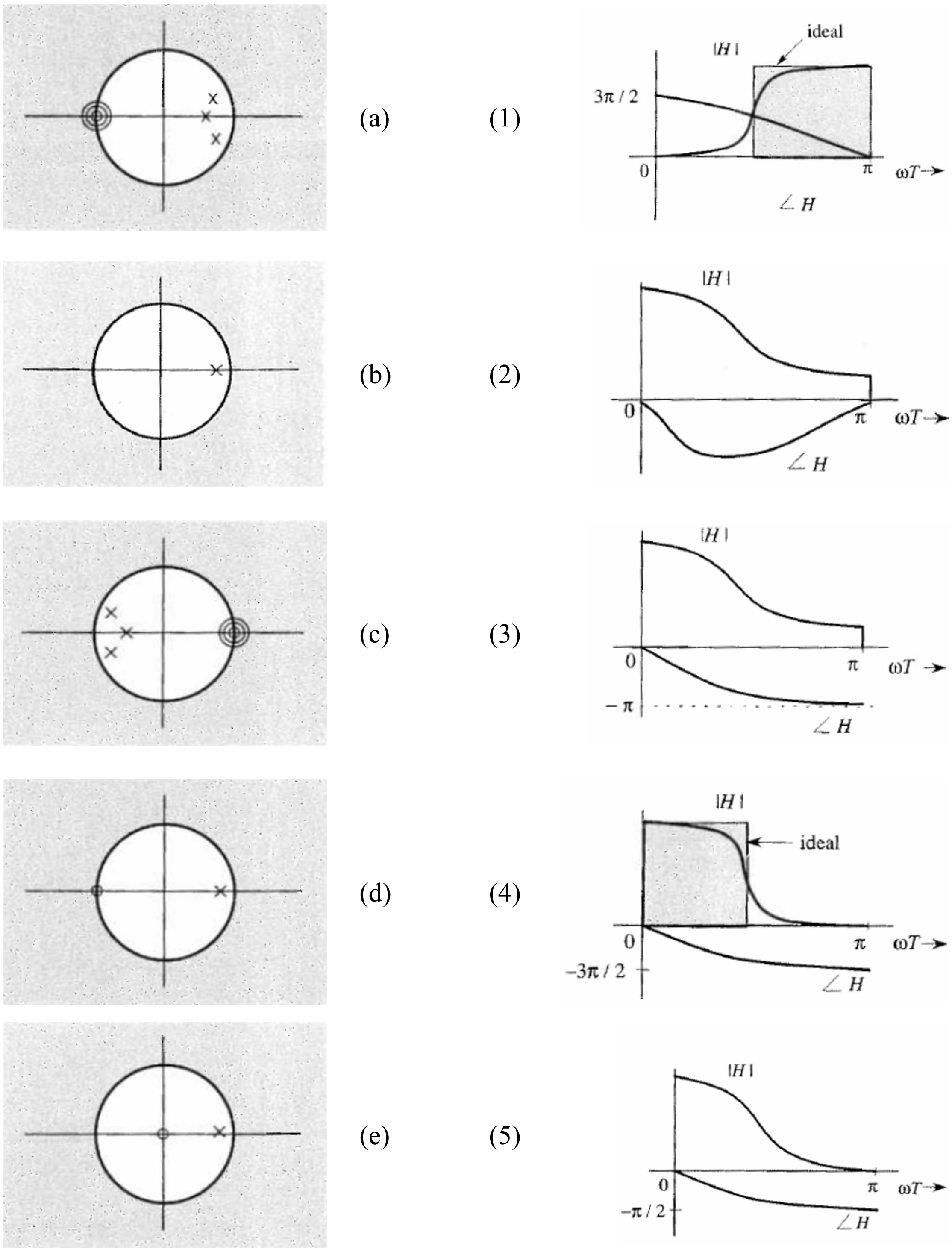

Figure Q2.

4. The circuit below:

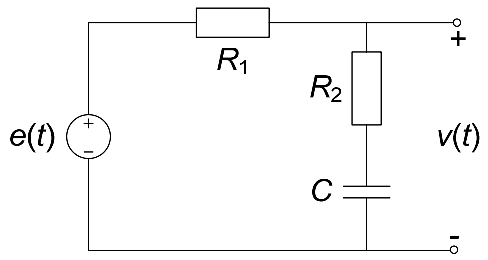

Figure Q4.

has the following differential equation:

$$\frac{e}{R_1} + C\frac{R_2}{R_1}\frac{de}{dt} = \frac{v}{R_1} + CR_2\left(\frac{1}{R_2} + \frac{1}{R_1}\right)\frac{dv}{dt}:$$

a. Determine the transfer function by taking Laplace transforms.
b. Sketch the corresponding pole / zero map in the s-plane.
c. Determine the gain of the system at $\omega = 0$ rad/s and an expression for the gain at high frequencies in terms of C, R_1, and R_2. Hence state the nature of the frequency response.
d. Sketch a Bode plot for the system specifying any cut-in or cut-off points in terms of the values of C, R_1, and R_2.
e. Obtain an expression, in terms of C, R_1, and R_2 for the step response and rise time of the system.

[25 marks]

5. A linear system with the impulse response shown in Figure Q5(a) is driven by the input signal shown in Figure Q5(b). Determine:
   a. the Fourier transform of the input signal,
   b. the frequency response of the system,
   c. an expression for the Fourier transform of the output signal, and
   d. derive, via any appropriate method, an expression for the time-domain output.

(a) (b)

Figure Q5.

[25 marks]

6. The probability density function of two random variables, X and Y, is given by:

$$f_{XY}(x,y) = C(1-x-y), \quad \text{where} \quad 0 \le x \le 1\text{-}y \quad \text{and} \quad 0 \le y \le 1\,.$$

a. Derive the probability density functions $f_X(x)$ and $f_Y(y)$, and
b. determine whether or not X and Y are independent.

[10 marks]

7. A design team is tasked with developing a digital ‘black box’ speech enhancement device for in-car use as illustrated in Figure Q7. There are two inputs. Input 1 (I_1) is connected to a headset microphone. Input 2 (I_2) is connected to a noise reference microphone situated some distance away from the speaker (so that it captures the noise signal only and does not capture the speech signal). The device output (O_1) is connected to the analogue audio input of the mobile telephone. The idea is to subtract an estimate of the noise (obtained using the noise reference microphone) from the noisy speech (obtained using the lapel microphone). It is decided to use a sampling frequency of 8 kHz and that the noise subtraction is to occur in the frequency domain.

Stating any assumptions that you make:

a. describe, with suitable block diagrams and/or equations, the analogue-to-digital conversion (ADC);
b. describe a suitable approach with equations with which a frequency domain description of each input signal may be obtained;
c. giving suitable equations show how the noise reference signal may be subtracted from the noisy speech signal in the frequency domain;
d. giving suitable equations show how a sampled time domain descriptions of the enhanced speech may be synthesised, and
e. describe, with suitable block diagrams and/or equations, the digital-to-analogue conversion (DAC).

[20 marks]

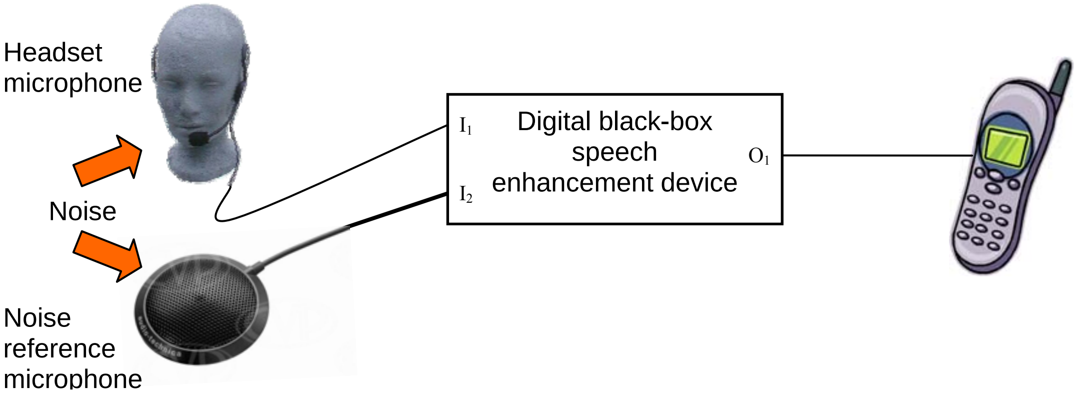

Figure Q7

Table of selected Laplace transforms

$x(t)\quad (t \geq 0)$	$X(s)$
$\delta(t)$	1
$\delta(t-\alpha)$	$\exp(-\alpha s)$
1 (unit step)	$\frac{1}{s}$
t (unit ramp)	$\frac{1}{s^2}$
$\exp(-\alpha t)$	$\frac{1}{s+\alpha}$
$t\exp(-\alpha t)$	$\frac{1}{(s+\alpha)^2}$
$\sin(\alpha t)$	$\frac{1}{s^2+\alpha^2}$
$\cos(\alpha t)$	$\frac{s}{s^2+\alpha^2}$
$e^{-\alpha t}\sin(\omega t)$	$\frac{\omega}{(s+\alpha)^2+\omega^2}$
$e^{-\alpha t}\cos(\omega t)$	$\frac{s+\alpha}{(s+\alpha)^2+\omega^2}$

Table of selected z-transforms

$x(n) \quad (n \geq 0)$	$X(z)$
$\delta(n)$ unit pulse	1
$\delta(n-m)$	z^{-m}
1 (unit step)	$\dfrac{z}{z-1}$
n (unit ramp)	$\dfrac{z}{(z-1)^2}$
$\exp(-\alpha n)$	$\dfrac{z}{(z-e^{-\alpha})}$
$n\exp(-\alpha n)$	$\dfrac{e^{-\alpha}z}{(z-e^{-\alpha})^2}$
$\sin(\beta n)$	$\dfrac{z\sin(\beta)}{z^2-2z\cos(\beta)+1}$
$\cos(\beta n)$	$\dfrac{z^2-z\cos(\beta)}{z^2-2z\cos(\beta)+1}$
$e^{-\alpha n}\sin(\beta n)$	$\dfrac{ze^{-\alpha}\sin(\beta)}{z^2-2ze^{-\alpha}\cos(\beta)+e^{-2\alpha}}$
$e^{-\alpha n}\cos(\beta n)$	$\dfrac{z^2-ze^{-\alpha}\cos(\beta)}{z^2-2ze^{-\alpha}\cos(\beta)+e^{-2\alpha}}$