Essential Mathematical Methods for Engineers

Lecture 3:
Sampled data systems and the z-transform

Outline

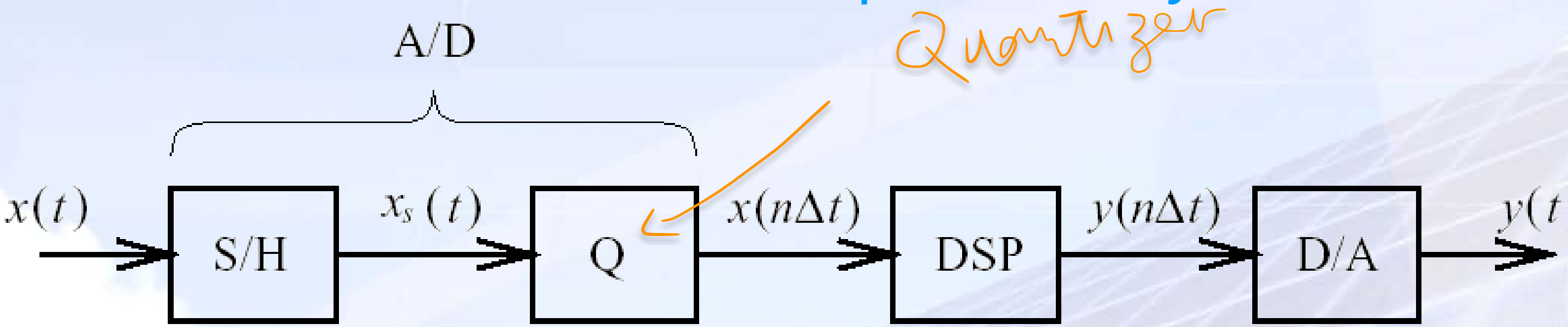
- analogue and digital processing
- sampled data systems and aliasing
  - sampling theorem – the Nyquist criterion
  - practical sampled data systems
- the z-transform
  - the inverse z-transform
  - delay theorem
- digital filters and discrete convolution
- poles and stability
- frequency response of a digital filter
- example of a complete system

Analogue and digital signal processing

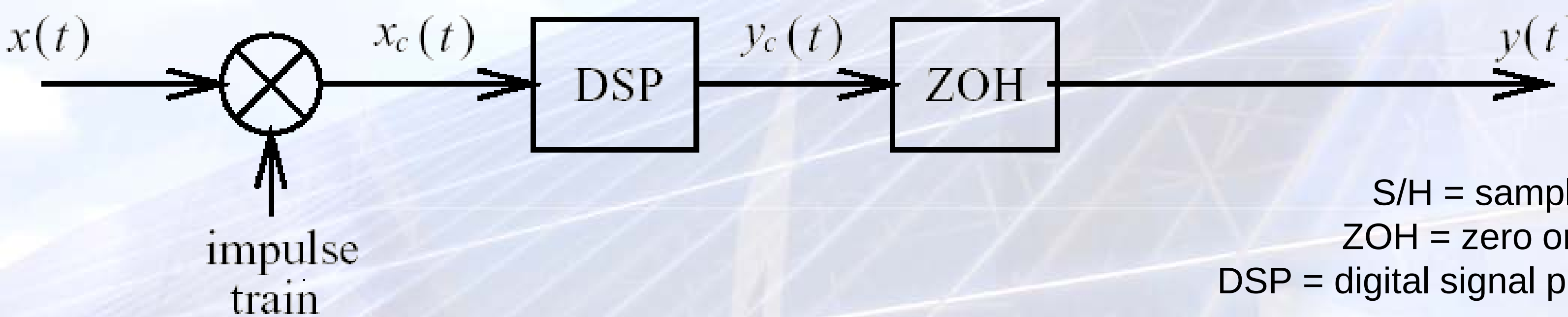
- an analogue filter

A/D = analogue to digital converter
D/A = digital to analogue converter

- an alternative discrete or sampled data system



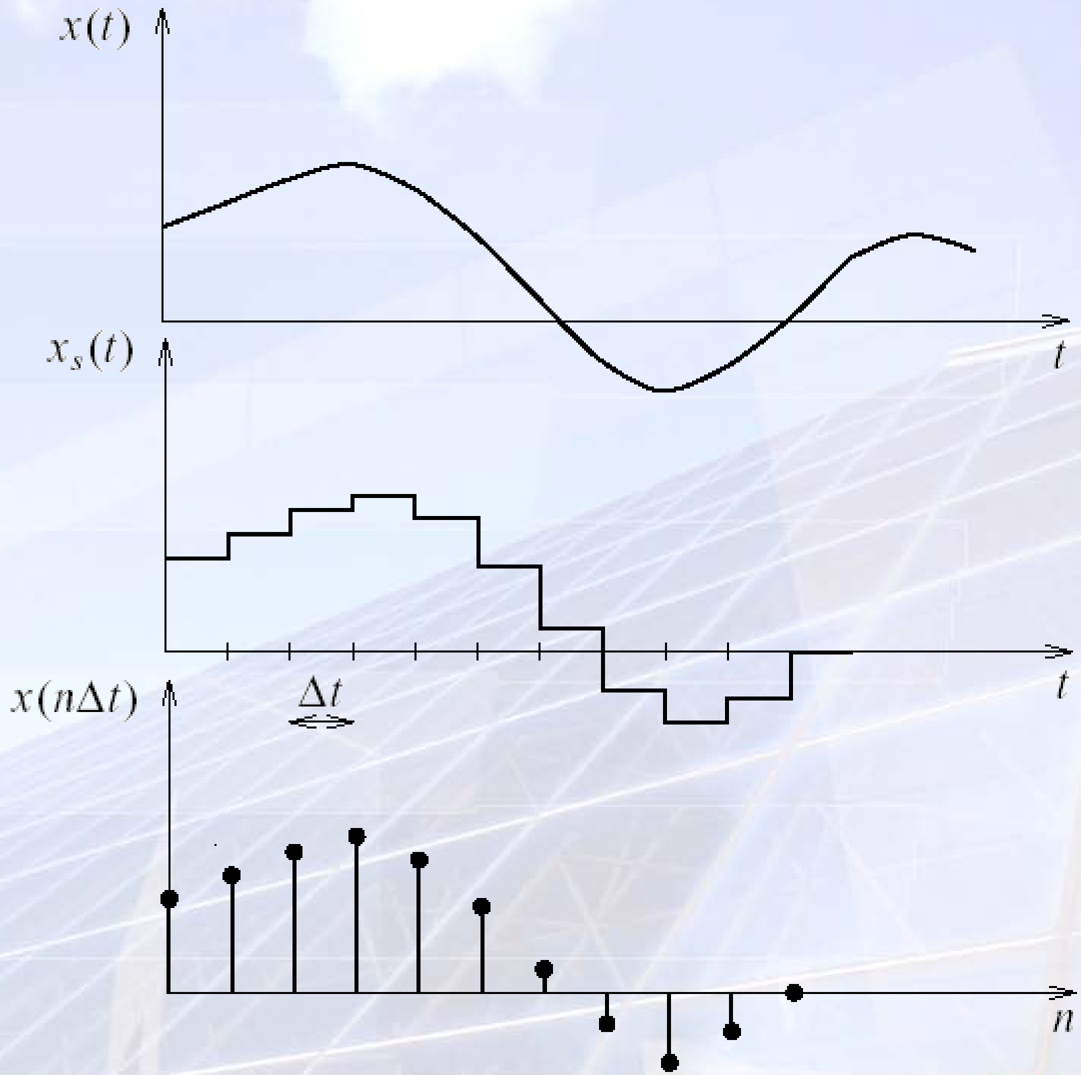
- its mathematical model



S/H = sample & hold
ZOH = zero order hold
DSP = digital signal processor

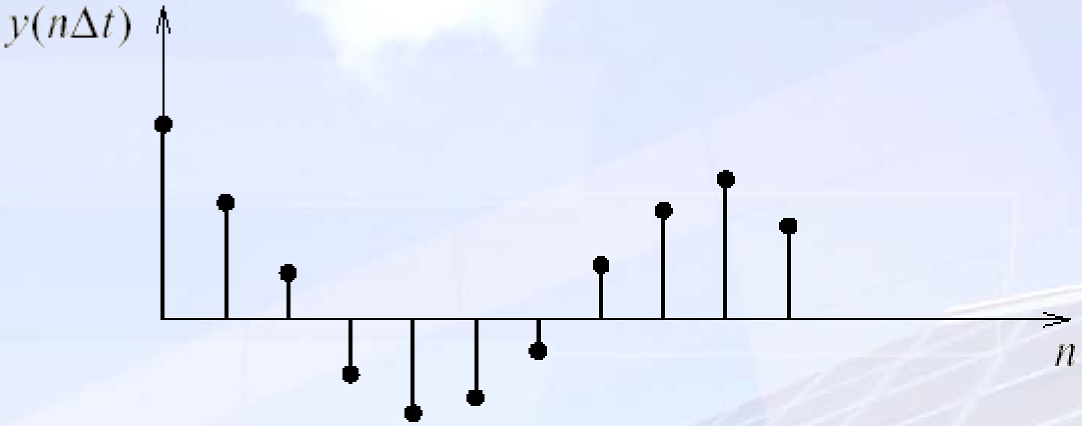
Analogue and digital signal processing

- analogue input
- output from *S/H*
- input $x(n\Delta t)$



Analogue and digital signal processing

- output $y(n\Delta t)$

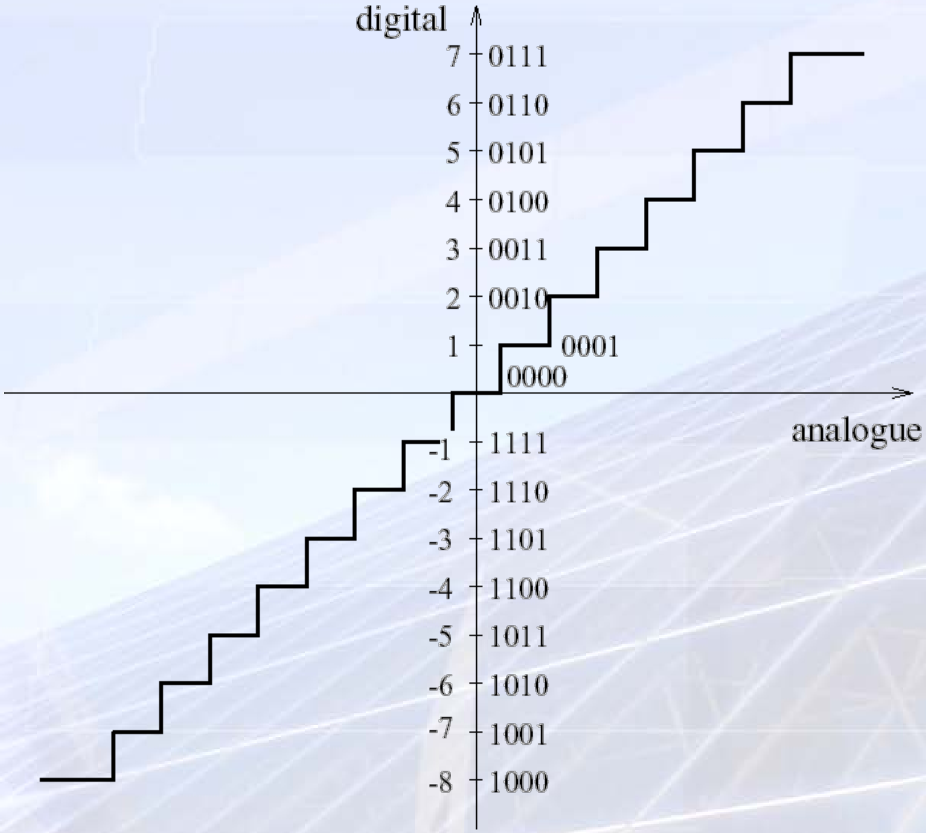


- analogue output from *DAC*

- digitisation involves
  - sampling and
  - quantisation

Analogue and digital signal processing

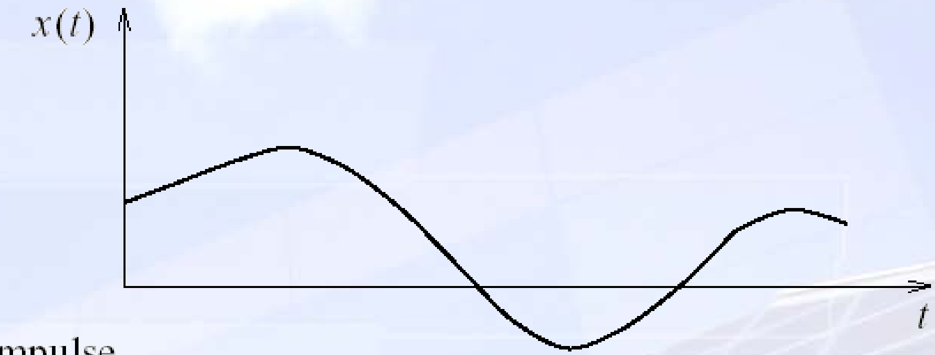
- the input / output characteristic for a 4-bit quantiser
  - quantisation noise



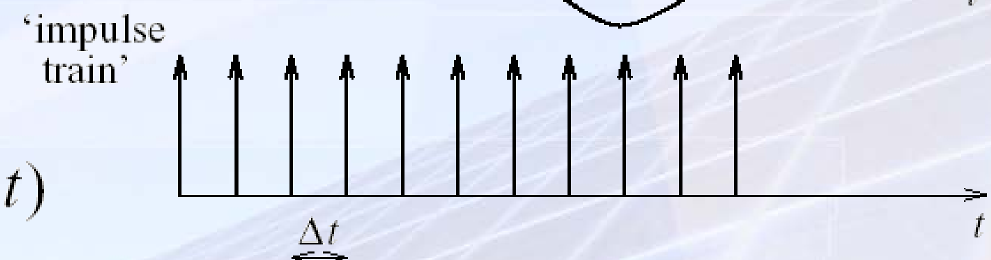
EURECOM

Sampled data systems and aliasing

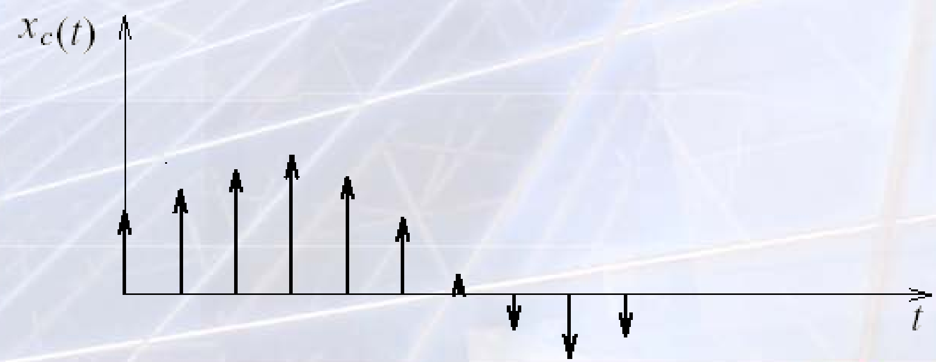
- for analysis and design we use the third model from slide 3



- $x(t)$ is multiplied by a periodic impulse train with period Δt



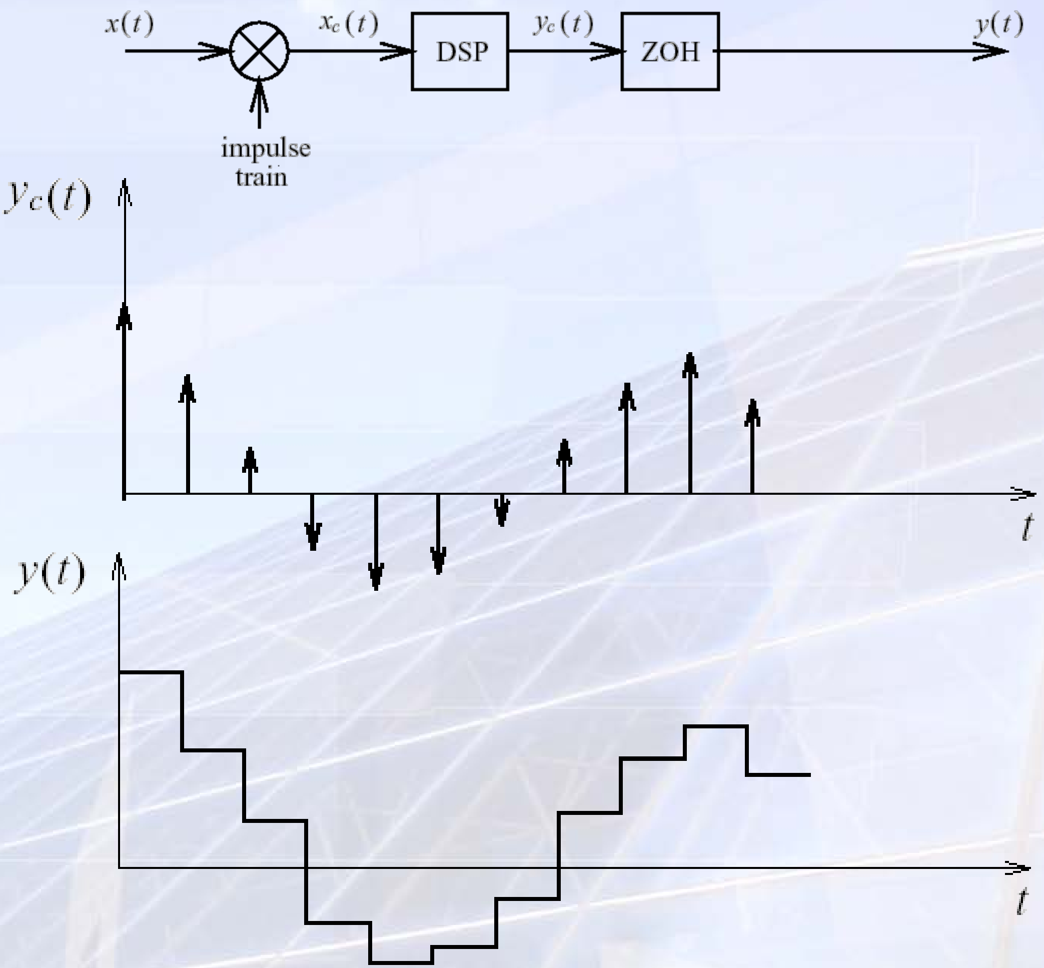
$$
\begin{aligned}
\delta_T(t) &= \cdots + \delta(t+2\Delta t) + \delta(t+\Delta t) \\
&\quad + \delta(t) + \delta(t-\Delta t) + \delta(t-2\Delta t) + \cdots \\
&= \sum_{n=-\infty}^{\infty} \delta(t - n\Delta t)
\end{aligned}
$$



EURECOM

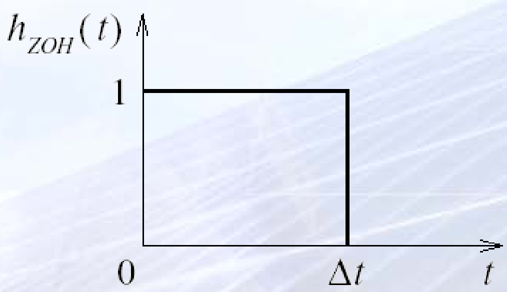
Sampled data systems and aliasing

- the output is also an impulse train



- $y(t)$ is obtained from $y_c(t)$ via a *ZOH* filter whose impulse response is

$$h_{ZOH}(t) = \begin{cases} 1, & 0 \leq t < \Delta t \\ 0, & \text{otherwise} \end{cases}$$



Sampled data systems and aliasing

- the sampled signal:

$$
\begin{aligned}
x_c(t) &= x(t)\,\delta_T(t) \\
&= x(t)\,[\,\delta(t) + \delta(t-\Delta t) + \delta(t-2\Delta t) + \cdots + \,] \\
&= x(t)\,\delta(t) \;+\; x(t)\,\delta(t-\Delta t) + x(t)\,\delta(t-2\Delta t) \cdots \\
&= \; x(0)\,\delta(t) \;+\; x(\Delta t)\,\delta(t-\Delta t) + x(2\Delta t)\,\delta(t-2\Delta t) \cdots \\
&= \sum_n \; x(n\Delta t)\,\delta(t-n\Delta t)
\end{aligned}
$$

Sampled data systems and aliasing

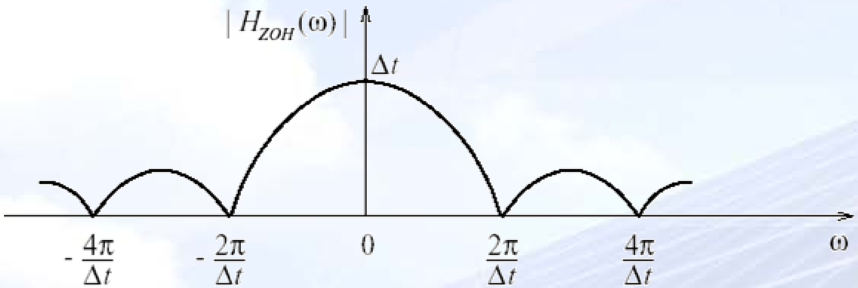
- the *ZOH* is a linear filter so the output $y(t)$ is given by

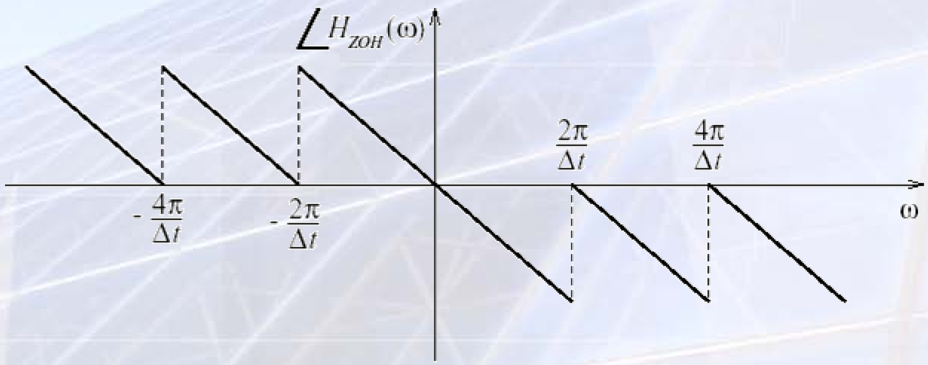
$$
\begin{aligned}
y(t) &= h_{ZOH}(t) * y_c(t) \\
&= h_{ZOH}(t) * \sum_n y(n\Delta t)\delta(t - n\Delta t) \\
&= \sum_n y(n\Delta t)\left[h_{ZOH}(t) * \delta(t - n\Delta t)\right] \\
&= \sum_n y(n\Delta t) h_{ZOH}(t - n\Delta t)
\end{aligned}
$$

Sampled data systems and aliasing

- the frequency response of the *ZOH* is obtained by taking the Fourier transform of its impulse response

$$
\begin{aligned}
H_{ZOH}(\omega) &= F[h_{ZOH}(t)] \\
&= \Delta t \exp\left(-\frac{j\omega\Delta t}{2}\right) \mathrm{sa}\left(\frac{\omega\Delta t}{2}\right)
\end{aligned}
$$





Sampled data systems and aliasing

- note that the versions of $x(t)$ and $y(t)$ on slides 4 – 5 and 7 – 8 are identical – the models are equivalent

- to analyse the effect of sampling on the signal fidelity we consider a DSP which copies data from the A/D filter to the D/A filter without modification

- the sampled signal is given by

$$x_c(t) = x(t)\ \delta_T(t)$$

and in the frequency domain

$$X_c(\omega) = F[\, x_c(t)\,] \qquad = \frac{1}{2\pi} X(\omega) * F[\delta_T(t)]$$

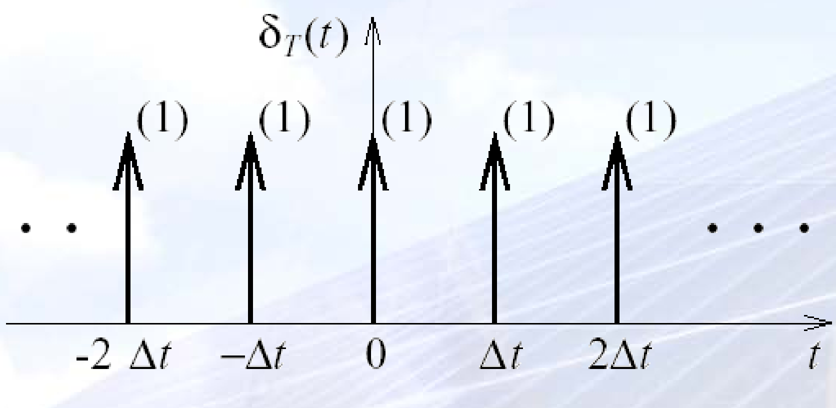
Sampled data systems and aliasing

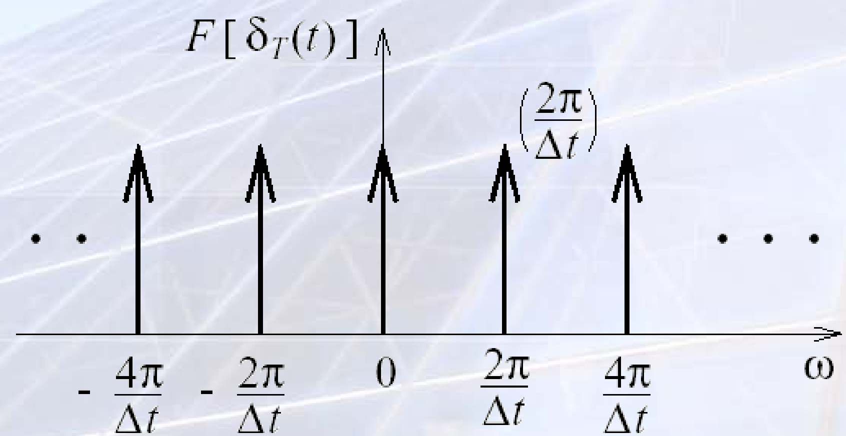
- the Fourier transform of the impulse train

$$\delta_T(t) = \sum_{n=-\infty}^{\infty} \delta(t - n\Delta t)$$

is

$$F[\delta_T(t)] = \frac{2\pi}{\Delta t} \sum_n \delta\left(\omega - \frac{2\pi n}{\Delta t}\right)$$





Sampled data systems and aliasing

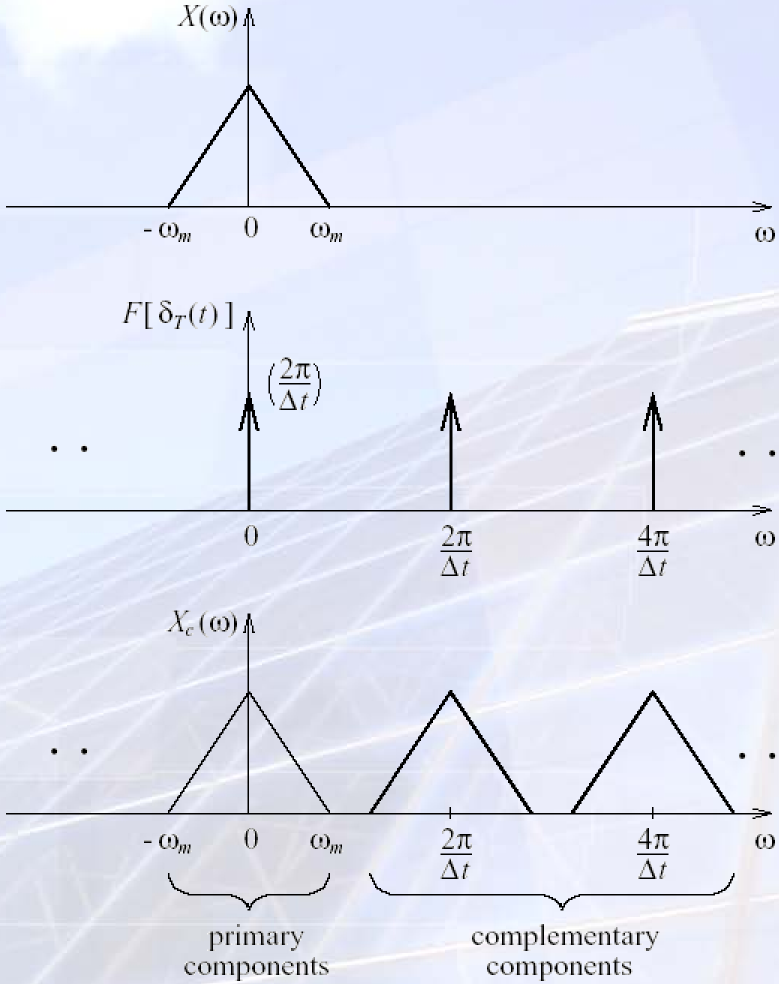
- moving the pulses closer together in the time domain increases the separation in the frequency domain

- now $$X_c(\omega) = \frac{1}{\Delta t}\, X(\omega) * \sum_n \delta\left(\omega - \frac{2\pi n}{\Delta t}\right)$$

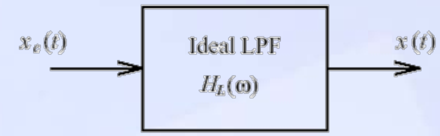
$$= \frac{1}{\Delta t} \sum_n X(\omega)\;\; \delta\left(\omega - \frac{2\pi n}{\Delta t}\right) = \frac{1}{\Delta t} \sum_n X\left(\omega - \frac{2\pi n}{\Delta t}\right)$$

Sampled data systems and aliasing

- the Fourier transform of the analogue signal is assumed to be triangular with maximum frequency component ω_m
- the Fourier transform of the sampled signal is obtained by convolving the impulse at $\omega = 0$ to form the primary component at $\omega = 0$ in $X_c(\omega)$
- $X(\omega)$ is convolved with the impulse at the sampling frequency $\omega=2\pi/\Delta t$ rad/s to produce a secondary component at $\omega = 2\pi/\Delta t$ rad/s
- this is repeated for all impulses



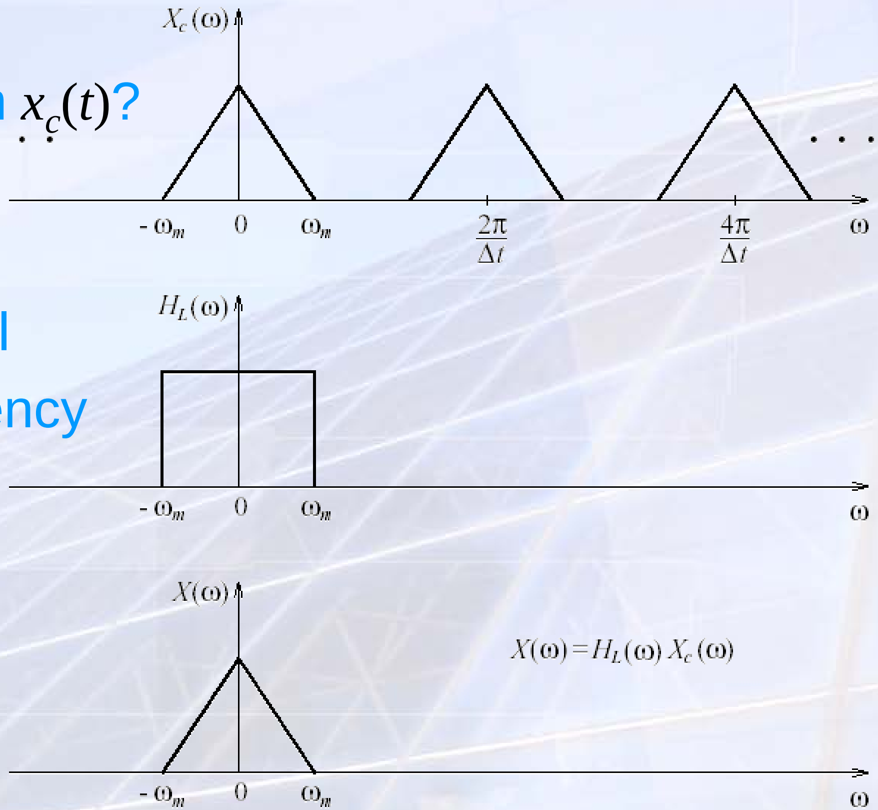
Sampled data systems and aliasing



- $X_c(\omega)$ is a periodic function of ω with period $2\pi/\Delta t$
- for this example the sampling rate in rad/s $(2\pi/\Delta t) >> \omega_m$
- is it possible to reconstruct $x(t)$ from $x_c(t)$?
  - the original signal is recovered by passing $x_c(t)$ through an ideal low-pass filter (LPF) with frequency response $H_L(\omega)$, i.e.

$$X(\omega) = H_L(\omega)\, X_c(\omega)$$

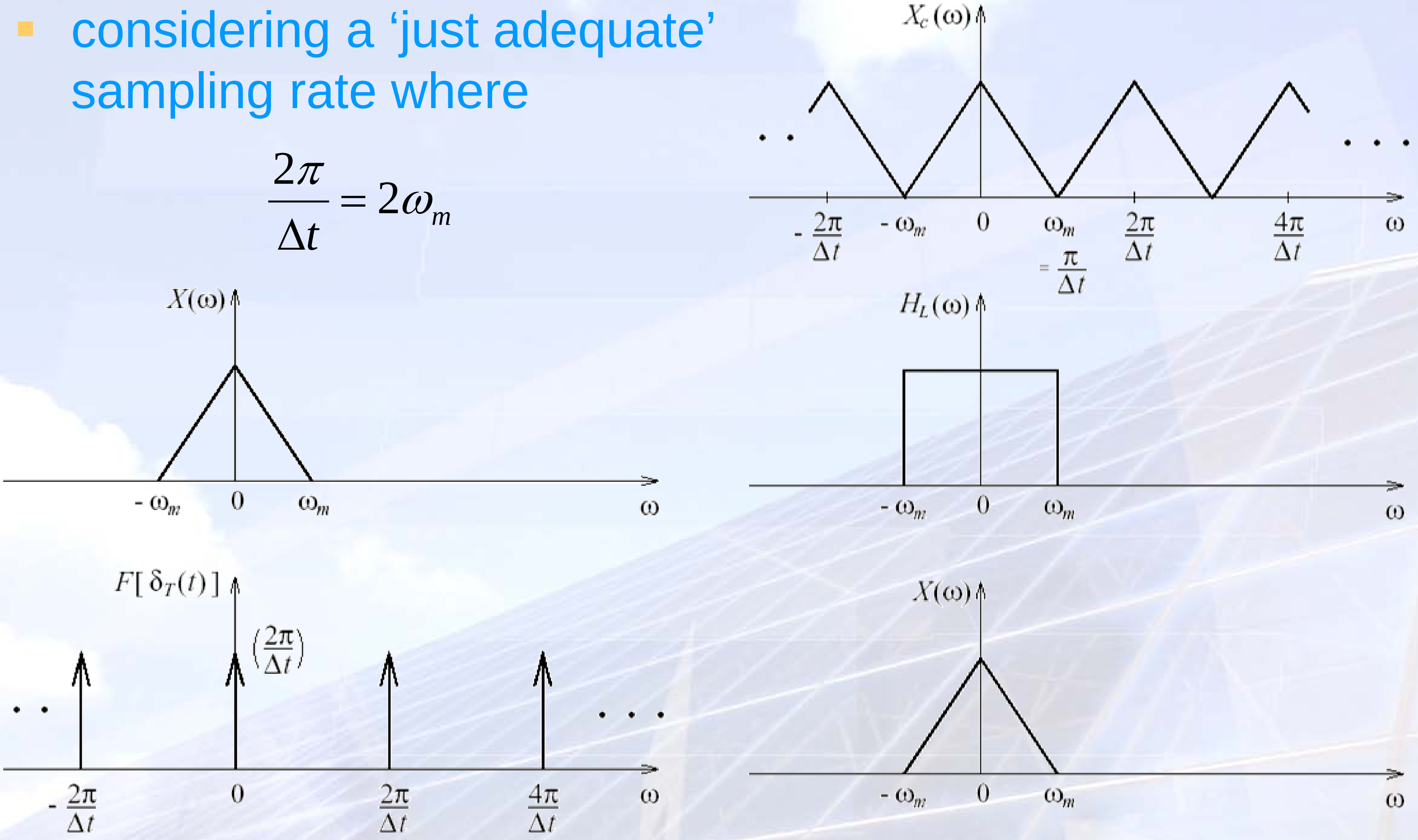
  - the LPF isolates the primary component



Sampled data systems and aliasing

- considering a 'just adequate' sampling rate where

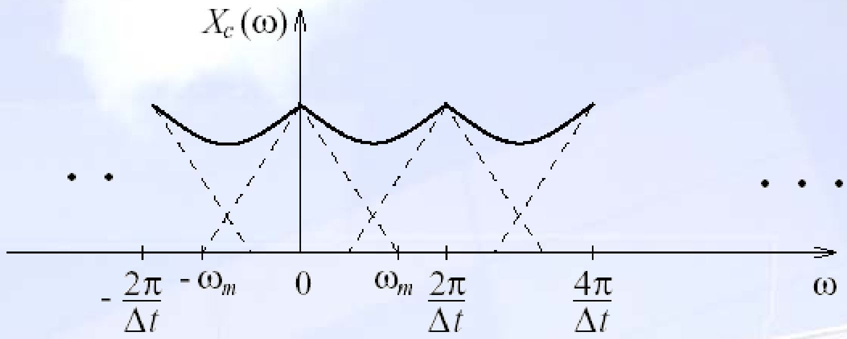
$$\frac{2\pi}{\Delta t} = 2\omega_m$$

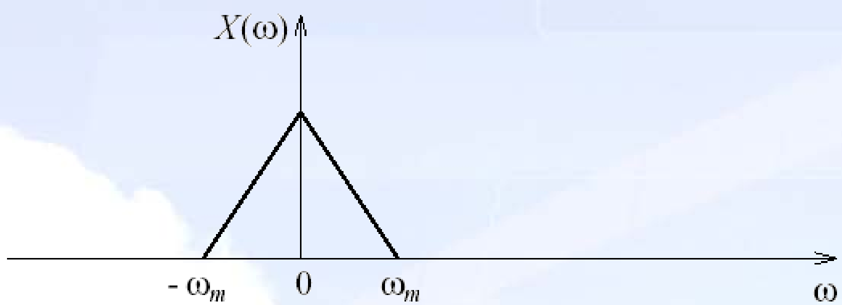


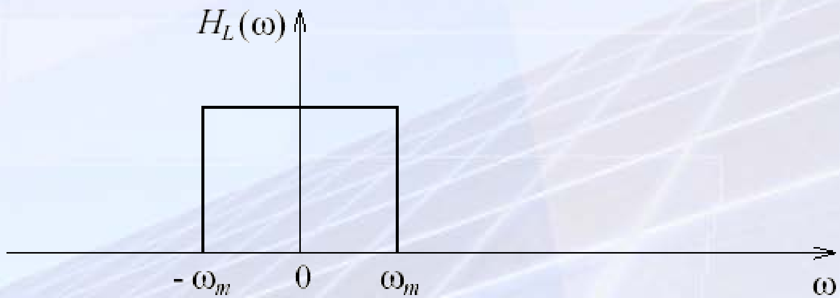
EURECOM

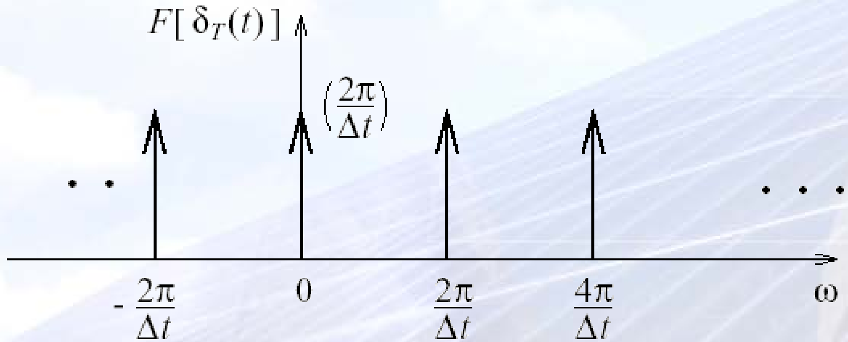
Sampled data systems and aliasing

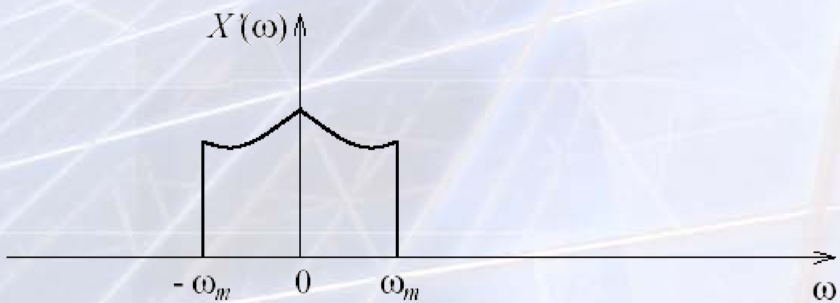
- when the sampling rate is too low the images of $X(\omega)$ and $X_c(\omega)$ overlap and we obtain a distorted version of $X(\omega)$ at the LPF output
  - » **ALIASING**











EURECOM

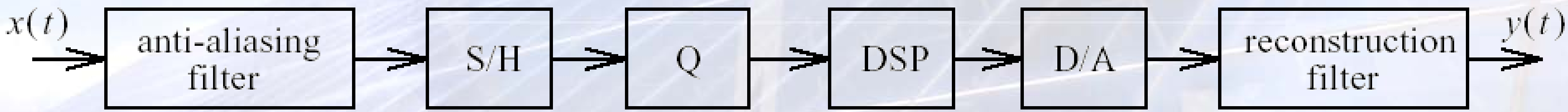
Sampling theorem – the Nyquist criterion

- in the last example the maximum frequency present in the analogue signal was greater than half the sampling frequency

- the sampling theorem states that to prevent aliasing distortion, a bandlimited signal of bandwidth ω_B rad/s must be sampled at a rate of at least $2\omega_B$ rad/s

- a perfect reconstruction is not possible in practice
  - it is impossible to construct an ideal *LPF* from a finite number of components
  - the frequency response $H_{ZOH}(\omega)$ of the *D/A* converter is far from ideal

- in practice
  - reconstruction filters with nominal cut-off $1/(2\Delta t)$ Hz
  - anti-aliasing filter to attenuate components above half the sampling frequency



EURECOM

The z-transform

- in the analysis and design of discrete-time digital systems the *z*-transform replaces the Laplace transform

Δt

- starting with $$x_c(t) = \sum_{n=0}^{\infty} x(n\Delta t)\,\delta(t - n\Delta t) \quad \exp(-st)\,\cancel{dt}$$

and taking Laplace transforms

$$X_c(s) = L[\,x_c(t)\,]$$

$$= \sum_{n=0}^{\infty} x(n\Delta t)\,\exp(-n\Delta t s)$$

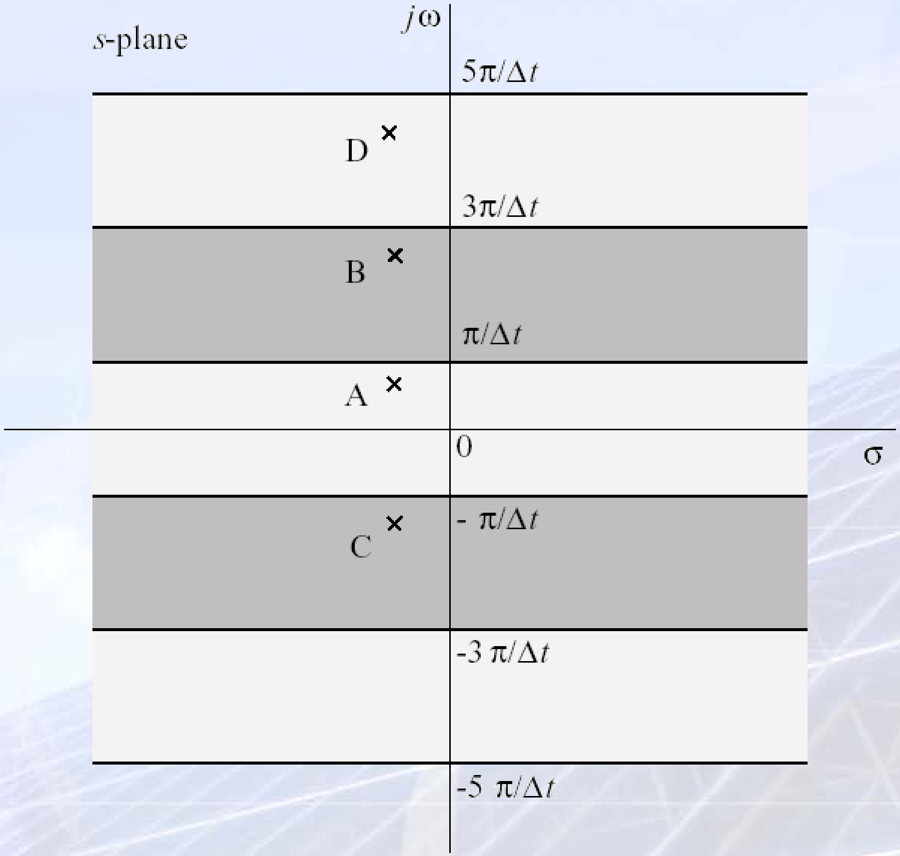
- problems arise for the inverse transform because $X_c(s)$ is periodic

$$X_c(s) = X_c\left(s + \frac{2\pi j}{\Delta t}\right)$$

EURECOM

The z-transform

- $X_c(s)$ cannot have an infinite number of poles

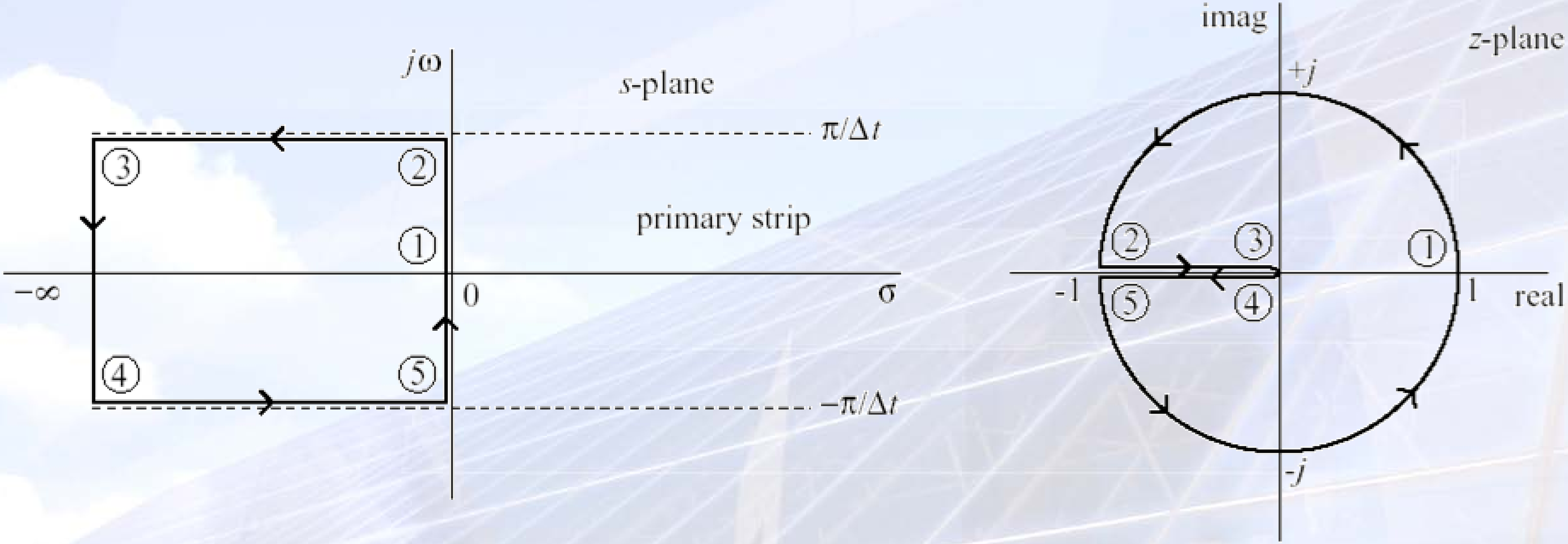


The z-transform

- the solution is to make a simple substitution

$$z = \exp(\Delta t \; s)$$

  and can be viewed as a
  - mapping of all points in the *s*-plane to points in a *z*-plane
  - short-hand notation to save writing many exp terms



The z-transform

- defining the z-transform
$$X(z) = X_c(s)\,\big|_{\exp(\Delta t\, s) = z} = \sum_{n=0}^{\infty} x(\,n\Delta t\,)\, z^{-n}$$
- it can be summarised as
  - the analogue signal, $x(t)$, is sampled by multiplying it by the impulse train $\delta_T(t)$ to give $x_c(t)$
$$x_c(t) = x(t)\ \delta_T(t)$$
  - take the Laplace transform of $x_c(t)$ to give
$$X_c(s) = L[\ x_c(t)\ ]$$
  - replace $\exp(\Delta ts)$ by z in $X_c(s)$ to get
$$X(z) = X_c(s)\,\big|_{\exp(\Delta t\, s) = z}$$
- alternatively if only a sequence of numbers $\{x(n\Delta t)\}$ exists where $n = 0, 1, 2, \ldots$ the z-transform is defined as
$$X(z) = Z[\ \{\ x(n\Delta t)\ \}\ ] = \sum_{n=0}^{\infty} x(n\Delta t)\ z^{-n}$$

EURECOM

The z-transform

Example

Evaluate the z-transform of a unit pulse $\delta(n\Delta t)$: The unit pulse is a discrete sequence with a single sample of value one at time zero.

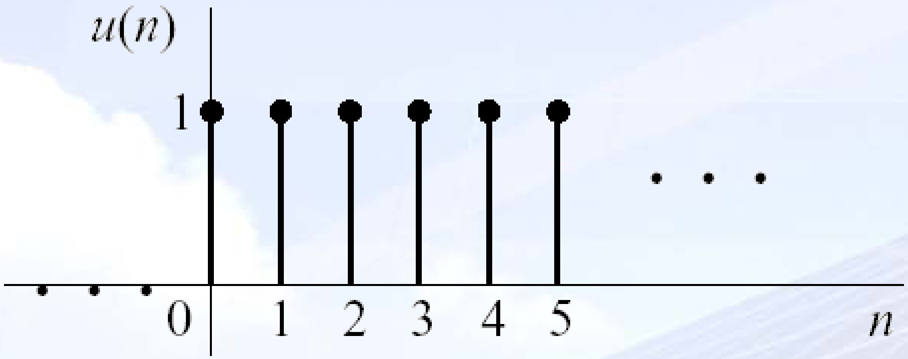
$$x(n\Delta t) = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$

EURECOM

The z-transform

Example

Evaluate the z-transform of a unit step. A discrete step is a sampled version of the analogue or continuous step.



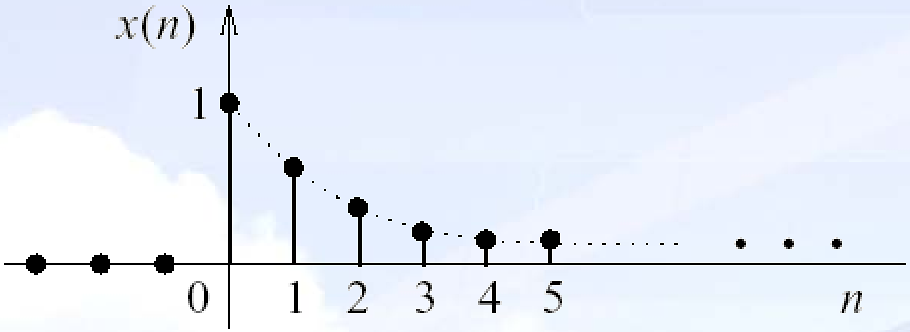
$$x(n\Delta t) = 1; \quad n \geq 0$$
$$= 0; \quad n < 0$$

$$\sum_{n=0}^{\infty} c^n = \frac{1}{1-c} \quad \text{provided} \quad |c| < 1$$

The z-transform

Example

Evaluate the z-transform of a sampled exponential.



$$x(n\Delta t) = \exp(-\alpha n \Delta t) \; ; \quad n \geq 0$$

The inverse z-transform

- from the forward transform

$$X(z) = Z\left[\left\{x(n)\right\}\right]$$

$$= \sum_{n=0}^{\infty} x(n) z^{-n}$$

where Δt has been removed to simplify the notation since it does not affect the summation

the inverse transform may be obtained by taking partial fraction expansions and using a look-up table

$$\{x(n)\} = Z^{-1}[\ X(z)\ ]$$

The inverse z-transform

Example

The following is a typical example of the type of transform which might be encountered in analysing or designing a sampled data system:

$$X(z) = \frac{3 - \frac{5}{2} z^{-1}}{1 - \frac{3}{2} z^{-1} + \frac{z^{-2}}{2}}$$

$$= \frac{3z^2 - \frac{5}{2} z}{\left(z^2 - \frac{3}{2} z + \frac{1}{2} \right)}$$

Develop an expression for the *n*th time sample $x(n)$.

The inverse z-transform

Example

What is the inverse *z*-transform of $X(z) = 1 + 2z^{-1} + 3z^{-2}$?

The delay theorem

- if we know the z-transform of a sequence

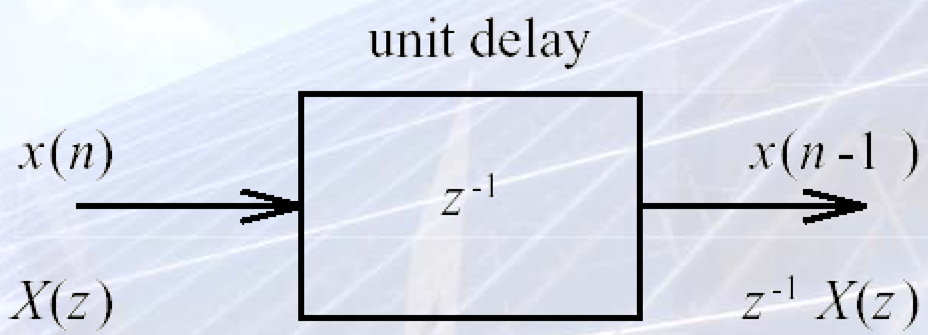
$$Z[\ \{\ x(n)\ \}\ ] = X(z)$$

  then the z-transform of the sequence delayed by one sample is

$$Z[\ \{\ x(n-1)\ \}\ ] = z^{-1}\ X(z)$$

- easy to implement on a microprocessor with clock period Δt
  - z^{-1} is often called the delay operator
    - can be extended to two samples etc…

$$Z[\ \{\ x(n-2)\ \}\ ] = z^{-2}\ X(z)$$



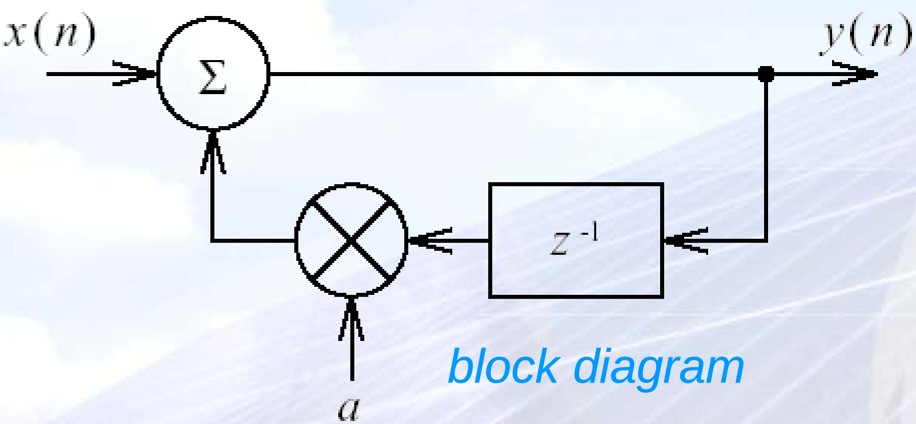
EURECOM

Digital filters and discrete convolution

- easily implemented on a DSP
- possible to describe in one of three ways:

$$y(n) = a\ y(n-1) + x(n)$$

difference equation



block diagram

```
1    read x
     y = a * y + x
     write y
     go to line 1
```

pseudo code

Digital filters and discrete convolution

- the *z*-transform can be used for digital filter analysis
  - e.g. a filter with difference equation

$$y(n) = a_o\ x(n) + a_1\ x(n-1) + a_2\ x(n-2) + b_1\ y(n-1) + b_2\ y(n-2)$$

  - taking *z*-transforms of both sides we have

$$Y(z) \ = Z\,[\ \{\ y(n)\ \}\ ]$$

$$= \ a_0\ X(z) + a_1\ z^{-1}\ X(z) + a_2\ z^{-2}\ X(z) + b_1\ z^{-1}\ Y(z) + b_2\ z^{-2}\ Y(z)$$

  - and by collecting terms in $Y(z)$ and $X(z)$

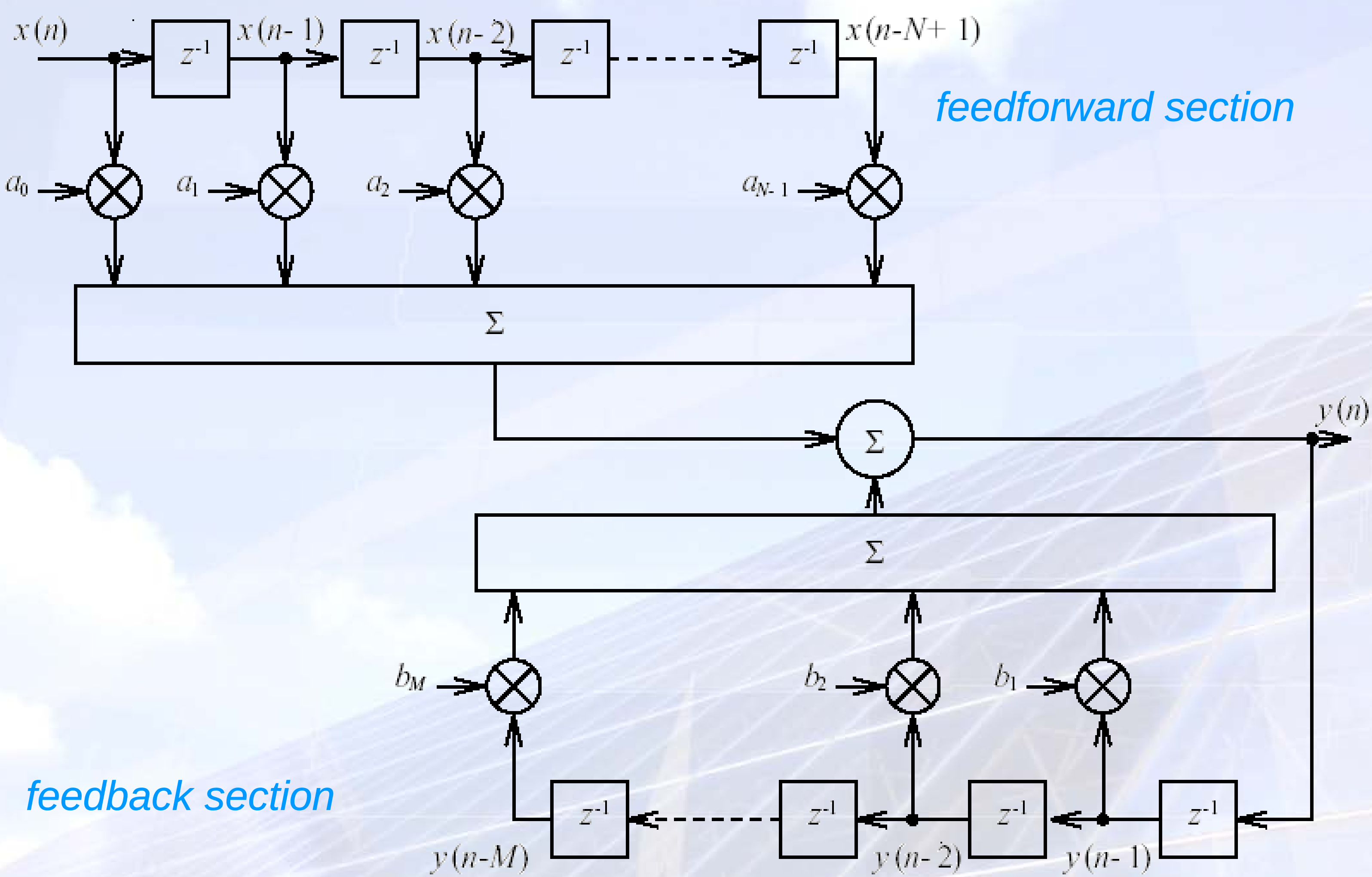
$$Y(z)\,[\ 1 - b_1\ z^{-1} - b_2\ z^{-2}\ ] = X(z)\,[\ a_0 + a_1\ z^{1} + a_2\ z^{-2}\ ]$$

  - hence the transfer function is given by

$$H(z) = \frac{Y(z)}{X(z)} = \frac{a_o + a_1\ z^{-1} + a_2\ z^{-2}}{1 - b_1\ z^{-1} - b_2\ z^{-2}}$$

EURECOM

Digital filters and discrete convolution



Digital filters and discrete convolution

- the general form of the difference equation is given by

$$y(n) = \sum_{i=0}^{N-1} a_i \, x(n-i) + \sum_{i=1}^{M} b_i \, y(n-i)$$

- and again taking *z*-transforms leads to the general form of the transfer function

$$H(z) = \frac{\sum_{i=0}^{N-1} a_i \, z^{-i}}{1 - \sum_{i=1}^{M} b_i \, z^{-i}}$$

- the feedforward and feedback sections influence the impulse response
  - stability issues

Digital filters and discrete convolution

- in summary:
  - if $b_i = 0$ for all values of i, the filter is the finite impulse response (**FIR**)
  - if any $b_i \neq 0$, the filter is the infinite impulse response (**IIR**)

- to calculate the output of a filter with a given transfer function to an input sequence $\{x(n)\}$
  - take the *z*-transform of the input sequence

$$X(z) = Z[\ \{\ x(n)\ \}\ ]$$

  - multiply by the transfer function

$$Y(z) = H(z)\ X(z)$$

  - take the inverse *z*-transform

$$\{\ y(n)\ \} = Z^{-1}[\ Y(z)\ ]$$

Digital filters and discrete convolution

Example

A sequence where $x(n) = 0.2^n,\ n \geq 0$ is applied to a digital filter with difference equation $y(n) = 0.5y(n\text{-}1) + x(n)$. Use transform techniques to develop an expression for the output $y(n)$.

Discrete convolution

- the time-domain description of a discrete-time linear system (digital filter) has two equivalent forms

$$y(n) = \sum_{m=0}^{\infty} x(m)\, h(n-m)$$

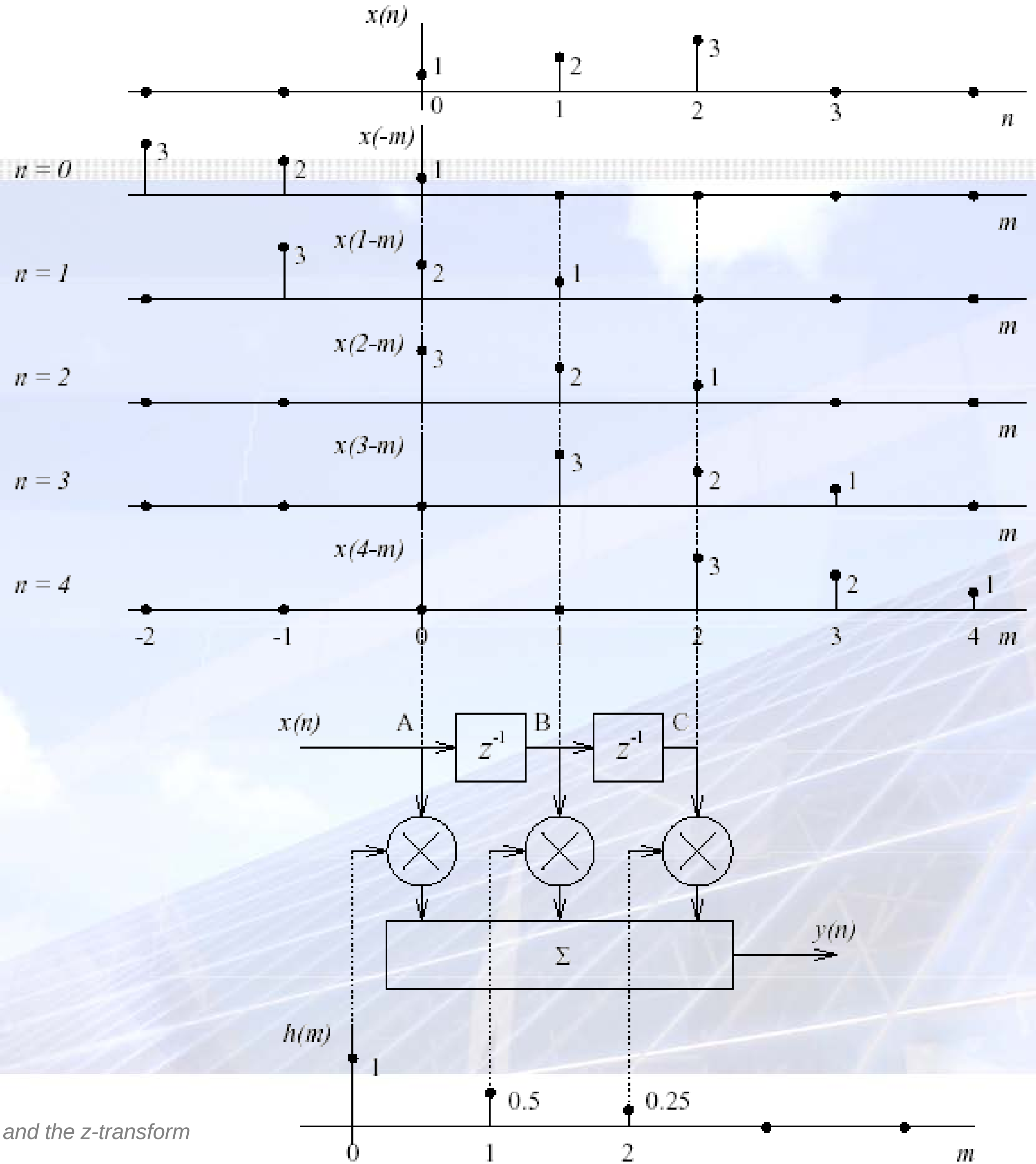
$$y(n) = \sum_{m=0}^{\infty} h(m)\, x(n-m)$$

$$\{\, y(n) \,\} = \{\, h(n) \,\} * \{\, x(n) \,\}$$

  – this is discrete-time convolution

Example

A sequence {1, 2, 3} is applied to an FIR filter with transfer function $1 + 0.5z^{-1} + 0.25z^{-2}$. Use discrete convolution to evaluate the output.


x(n)
1
2
3
0
1
2
3
n
x(-m)
n = 0
3
2
1
m
x(1-m)
n = 1
3
2
1
m
x(2-m)
n = 2
3
2
1
m
x(3-m)
n = 3
3
2
1
m
x(4-m)
n = 4
3
2
1
-2
-1
0
1
2
3
4
m
x(n)
A
B
C
z^{-1}
z^{-1}
Σ
y(n)
h(m)
1
0.5
0.25
0
1
2
m

Poles and stability

- the position of the poles and zeros of a digital filter influence the stability

- consider a system with two poles, $p_1 = r \exp(j\text{ø})$ and $p_2 = r \exp(-j\text{ø})$ where the transfer function is given by

$$H(z) = \frac{z}{(z - r\,e^{j\phi})(z - r\,e^{-j\phi})}$$

- making a partial fraction expansion

$$\frac{H(z)}{z} = \frac{A}{(z - r\,e^{j\phi})} + \frac{B}{(z - r\,e^{-j\phi})}$$

$$H(z) = \frac{Az}{(z - r\,e^{j\phi})} + \frac{Bz}{(z - r\,e^{-j\phi})}$$

Poles and stability

- and taking inverse *z*-transforms

$$A = \frac{1}{2jr\sin\phi}$$

$$B = \frac{-1}{2jr\sin\phi}$$

$$\begin{aligned} h(n) &= A\,(\,r\,e^{j\phi}\,)^n + B\,(\,r\,e^{-j\phi}\,)^n \\ &= \frac{r^n\,e^{j\phi n}}{2j\,r\,\sin\phi} - \frac{r^n\,e^{-j\phi n}}{2j\,r\,\sin\phi} \\ &= \frac{r^{n-1}}{\sin\phi}\left\{\frac{e^{j\phi n} - e^{-j\phi n}}{2j}\right\} = \frac{r^{n-1}}{\sin\phi}\,\sin(\,\phi n\,) \end{aligned}$$

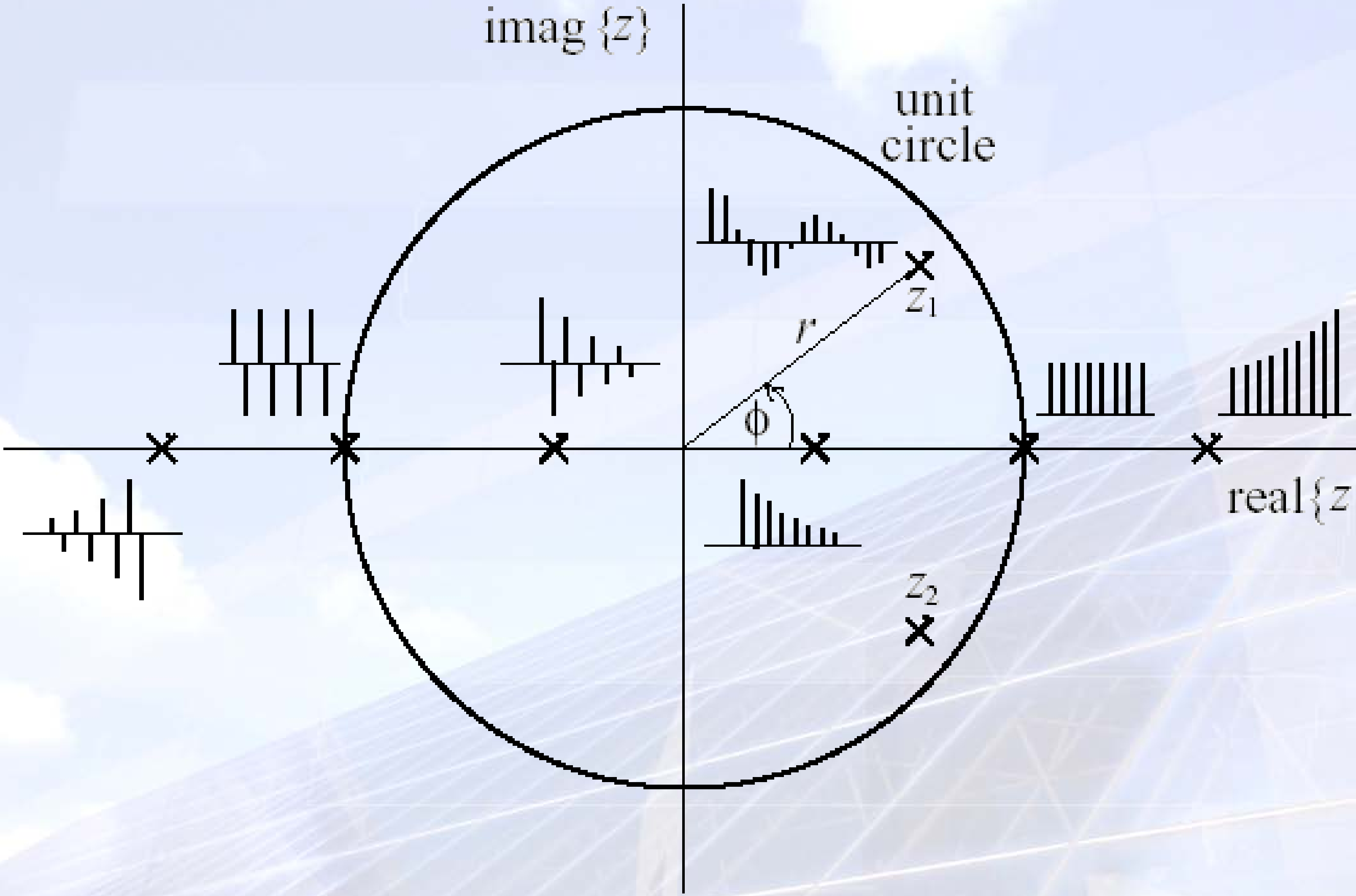
- thus the unit pulse or impulse response is given by

$$h(n) = \frac{r^{n-1}}{\sin\phi}\,\sin(\,\phi n\,)$$

- the response will decay to zero if $r < 1$
  - r is just the distance of the pole from the origin in the *z*-plane

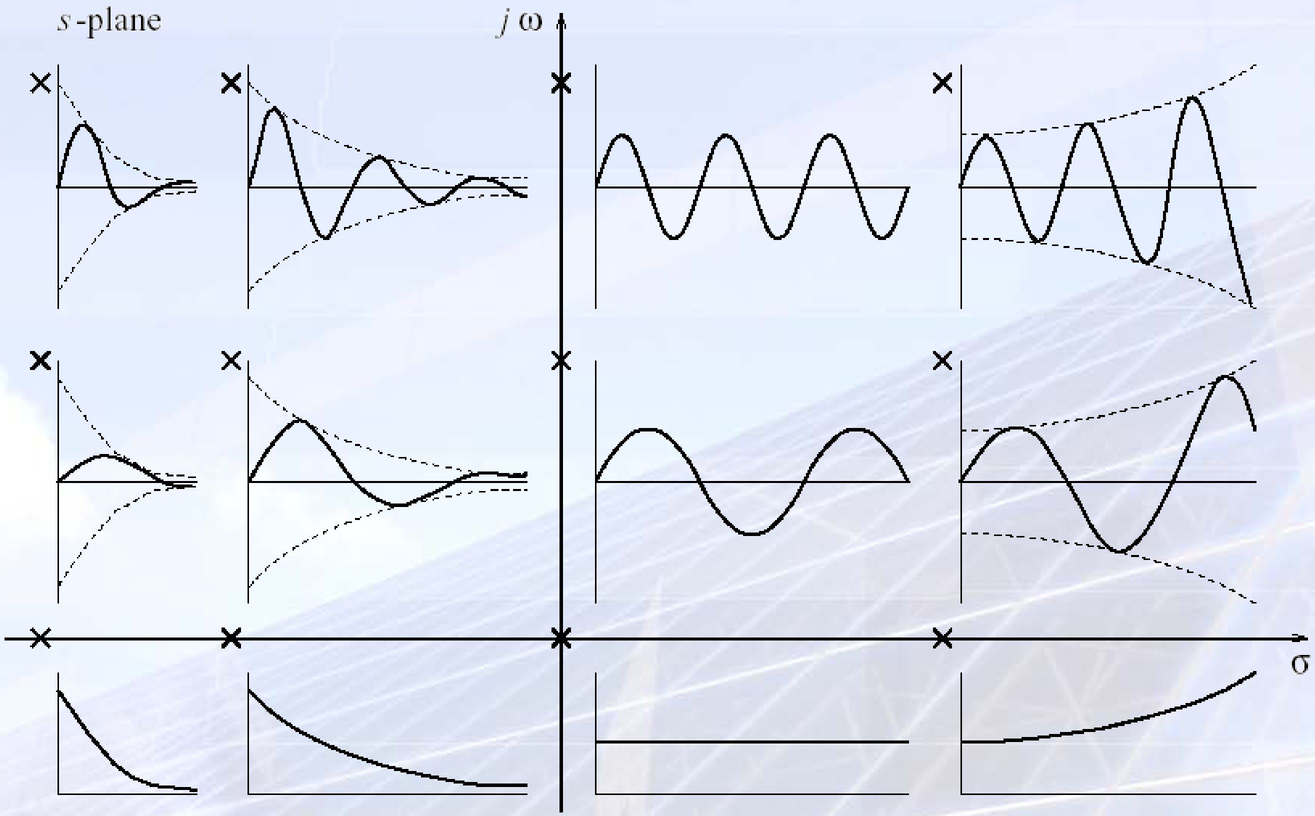
EURECOM

Poles and stability



Recap

- for continuous signals, the Laplace transform and the *s*-plane we had the following responses

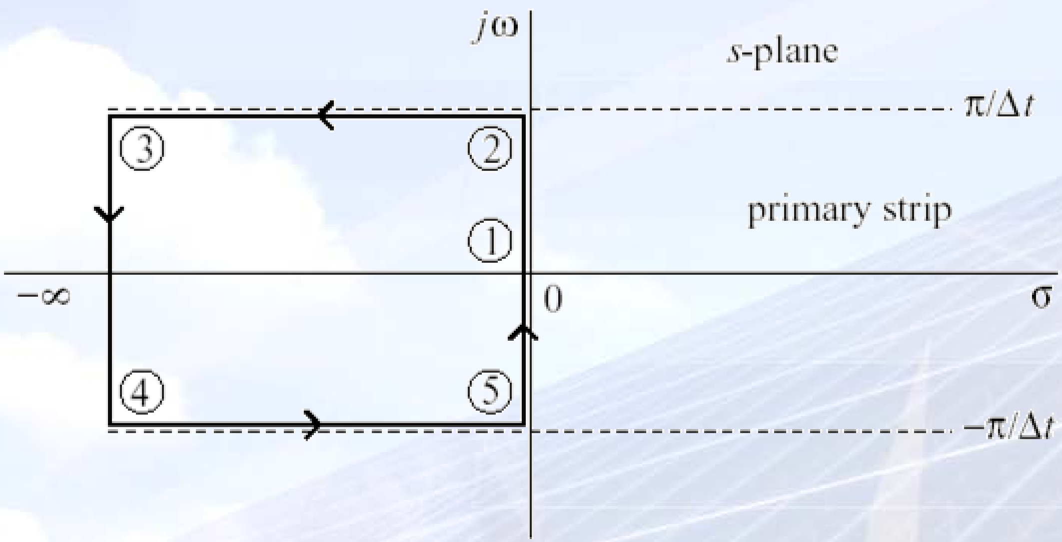


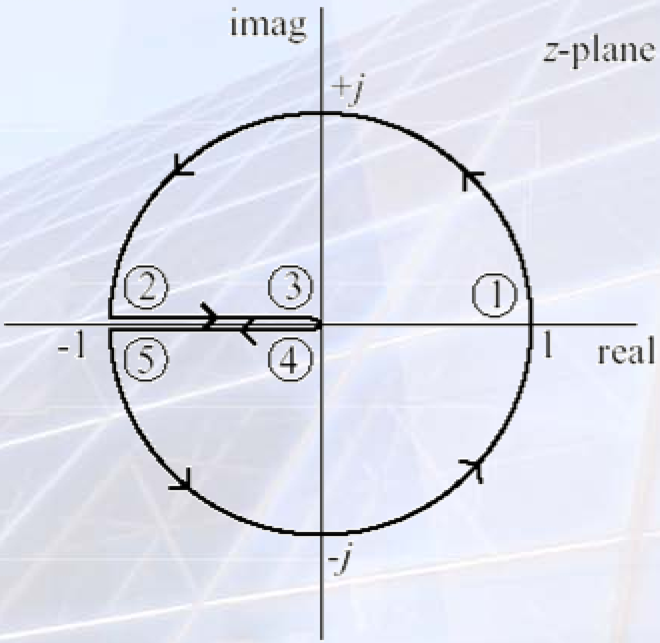
Recap

- the simple substitution

$$z = \exp(\Delta t\ s)$$

  and can be viewed as a mapping of all points in the *s*-plane to points in the *z*-plane





Frequency response of a digital filter

- the relationship between the Laplace transform of a sampled signal and the z-transform:

$$X_c(s) = X(z)\big|_{z\,=\,e^{\Delta t s}}$$

- the Fourier transform is obtained from the Laplace transform by replacing s by $j\omega$:

$$X_c(\omega) = X_c(s)\big|_{s\,=\,j\omega}$$

- in a similar way the Fourier transform is obtained from the *z*-transform according to:

$$X(\omega) = X(z)\,\big|_{z\,=\,e^{\Delta t j\omega}}$$

and we can obtain the frequency response from the transfer function:

$$H(\omega) = H(z)\,\big|_{z\,=\,\exp(\Delta t\; j\omega)}$$

Frequency response of a digital filter

Example

The transfer function of a digital filter is

$$H(z) = \frac{z}{z - a}$$

What is its frequency response?

Frequency response of a digital filter

- if we apply a sampled cosine wave $x(n)$ of frequency ω_0 rad/s, where

$$x(n) = \cos(\ \omega_0\ \Delta t\ n\ )$$

to a digital filter with frequency response $H(\omega)$ then the steady-state output $y(n)$ will be

$$y(n) = |H(\omega_0)|\ \cos\ (\omega_0\ \Delta t\ n + \angle H(\omega_0)\ )$$

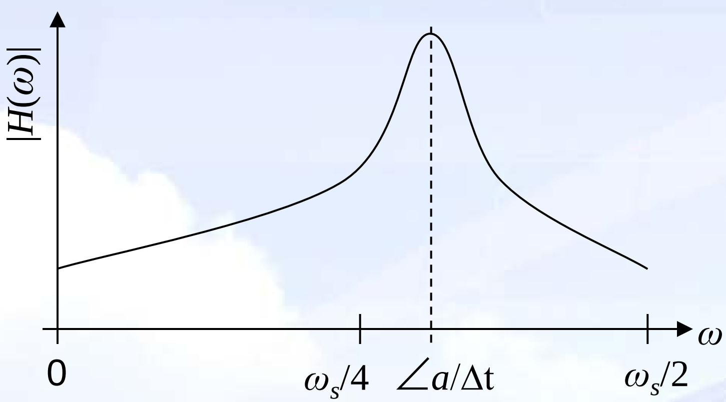
and in the same way as before the positions of the poles and zeros influence the shape of the frequency response

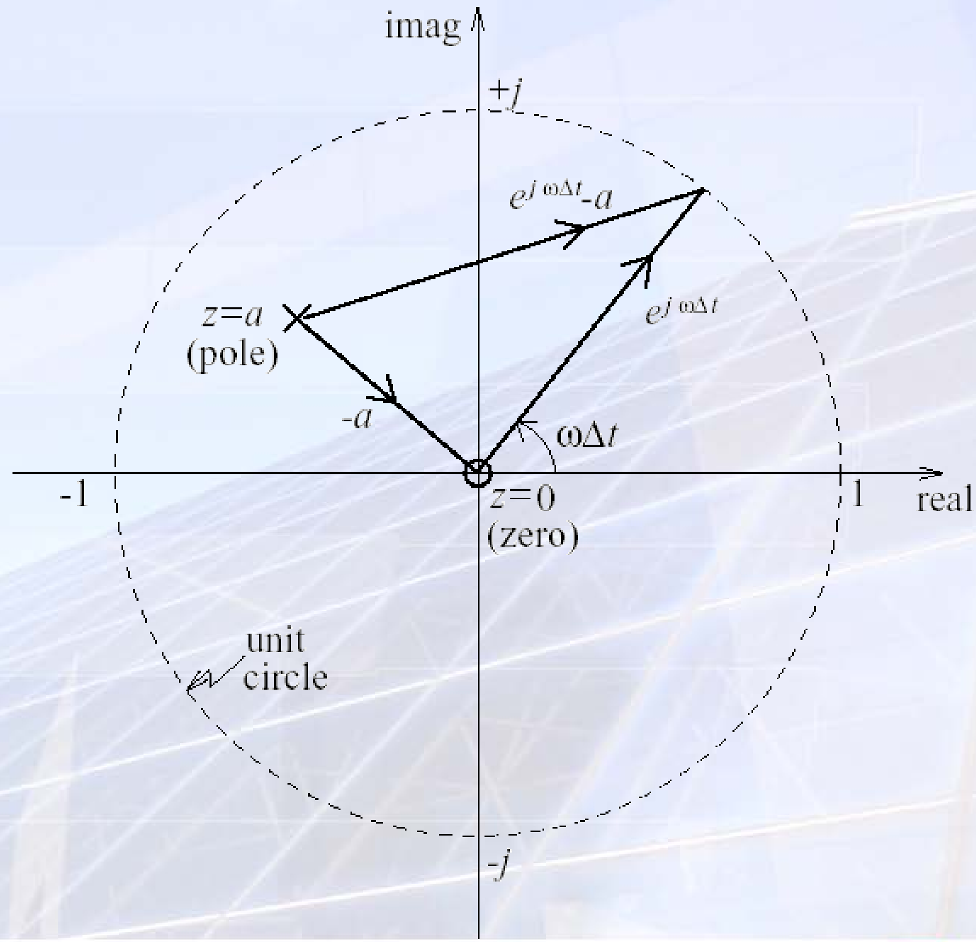
- for the previous example the amplitude and phase responses are given by

$$|H(\omega)| = \frac{1}{|\exp(j\omega\Delta t) - a|} \qquad \angle H(\omega) = \omega\Delta t - \angle(\exp(j\omega\Delta t) - a)$$

Frequency response of a digital filter

- we are interested in how the magnitude and phase change with ω
  - again a vector interpretation is useful





Frequency response of a digital filter

- extending to the case where $H(z)$ has many poles and zeros

$$H(z) = \frac{(z - z_1)(z - z_2) \cdots (z - z_m)}{(z - p_1)(z - p_2) \cdots (z - p_n)}$$

and replacing z with $\exp(j\omega\Delta t)$

$$H(\omega) = \frac{A(e^{j\omega\Delta t} - z_1)(e^{j\omega\Delta t} - z_2) \cdots (e^{j\omega\Delta t} - z_m)}{(e^{j\omega\Delta t} - p_1)(e^{j\omega\Delta t} - p_2) \cdots (e^{j\omega\Delta t} - p_n)}$$

we can calculate the amplitude and phase response for any ω

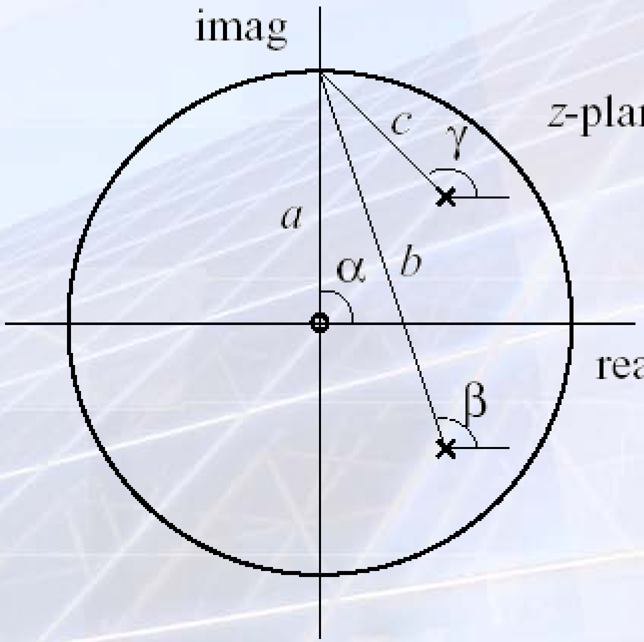
$$|\, H(\omega) \,| = \frac{A|e^{j\omega\Delta t} - z_1||e^{j\omega\Delta t} - z_2| \cdots |e^{j\omega\Delta t} - z_m|}{|e^{j\omega\Delta t} - p_1||e^{j\omega\Delta t} - p_2| \cdots |e^{j\omega\Delta t} - p_n|}$$

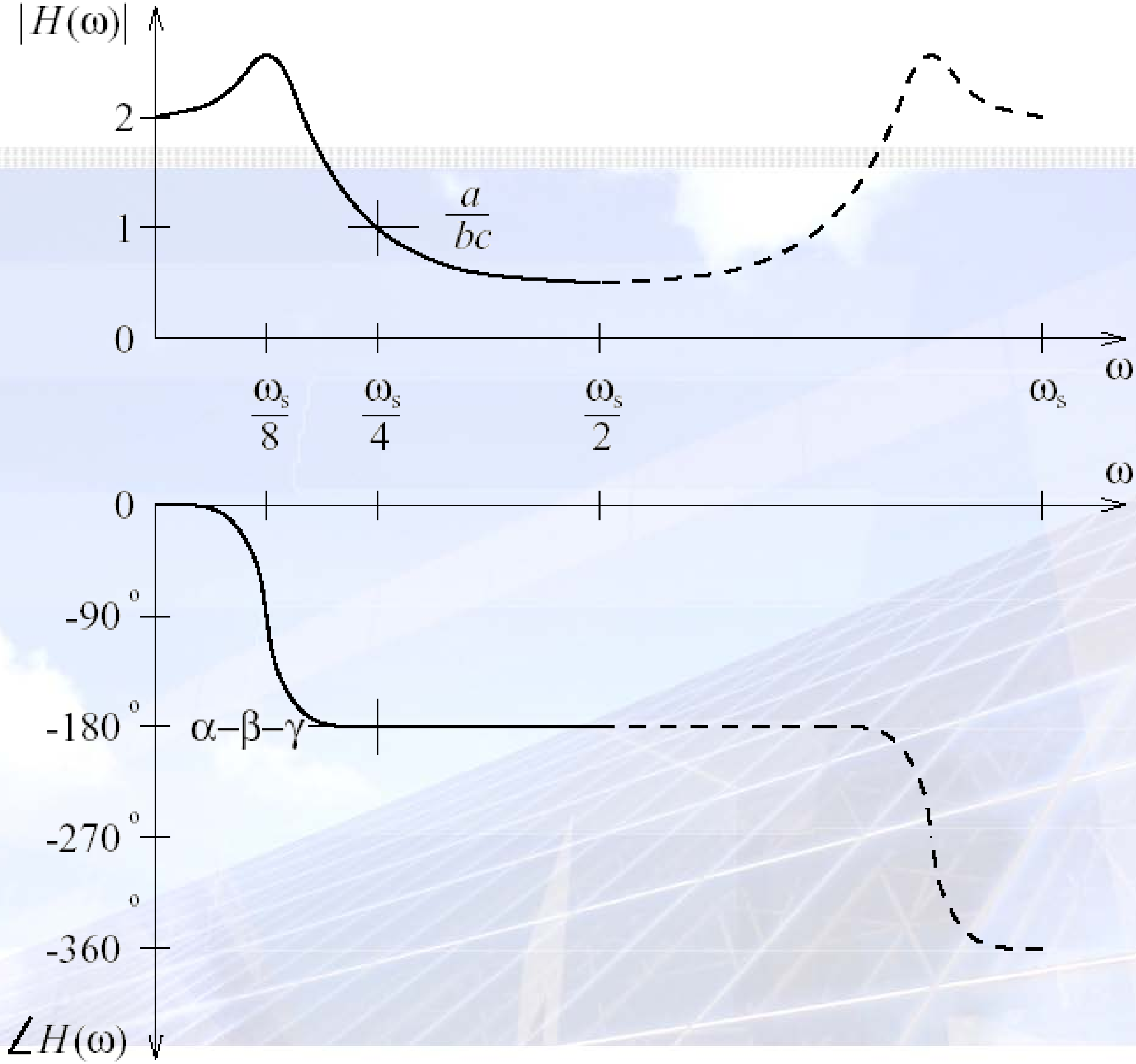
$$\begin{aligned}\angle H(\omega) = {} & \angle(e^{j\omega\Delta t} - z_1) + \angle(e^{j\omega\Delta t} - z_2) + \cdots + \angle(e^{j\omega\Delta t} - z_m) \\ & - \angle(e^{j\omega\Delta t} - p_1) - \angle(e^{j\omega\Delta t} - p_2) \cdots - \angle(e^{j\omega\Delta t} - p_n)\end{aligned}$$

Frequency response of a digital filter

- the **amplitude** response is obtained by taking the products of the lengths of the various vectors drawn from the zeros to the point on the unit circle $e^{j\omega\Delta t}$ associated with the frequency ω, and dividing this by the product of the lengths of the vectors from the poles
- the **phase** response is found by summing the phases of the individual vectors from the zeros minus the sum of the phases of the individual vectors from the poles

e.g. for a transfer function with a complex pair of poles and a zero at the origin we have the following amplitude and phase responses





$|H(\omega)|$
2
1
0
$\frac{a}{bc}$
ω
$\frac{\omega_s}{8}$
$\frac{\omega_s}{4}$
$\frac{\omega_s}{2}$
ω_s
ω
0
-90°
-180°
$\alpha-\beta-\gamma$
-270°
-360°
$\angle H(\omega)$

Example of a complete system

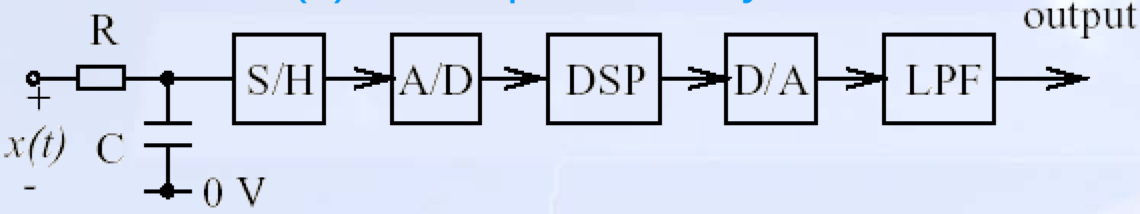
An analogue signal, $x(t)$, is applied to the sampled data system illustrated in (a) on the next slide, which has a sampling rate of 10 kHz. The sample and hold (S/H) has a high impedance input stage and hence does not draw any current from the *RC* circuit (*R* = 21 kΩ and *C* = 3 nF). There are two sine waves present in the analogue signal: one has an amplitude of A and frequency of 1 kHz, and the other has a amplitude of $A/2$ and a frequency of 7.5 kHz. What frequencies are present at the output of the analogue-to-digital (A/D) converter and what are their amplitudes?

The digital signal processing (DSP) unit implements a digital filter which is also illustrated in (b) on the next slide. What frequencies are present at the output of the digital signal processor and what are their amplitudes? It may be assumed that the low-pass filter (LPF) is ideal and has a cut-off frequency of 5 kHz.

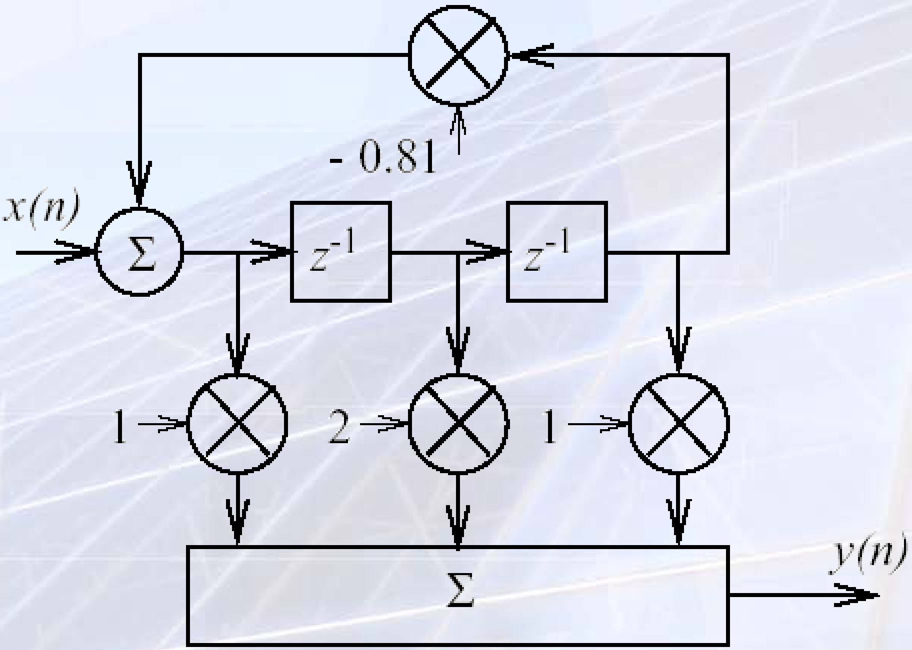
What frequencies are present at the output of the system and what are their amplitudes?

Example of a complete system

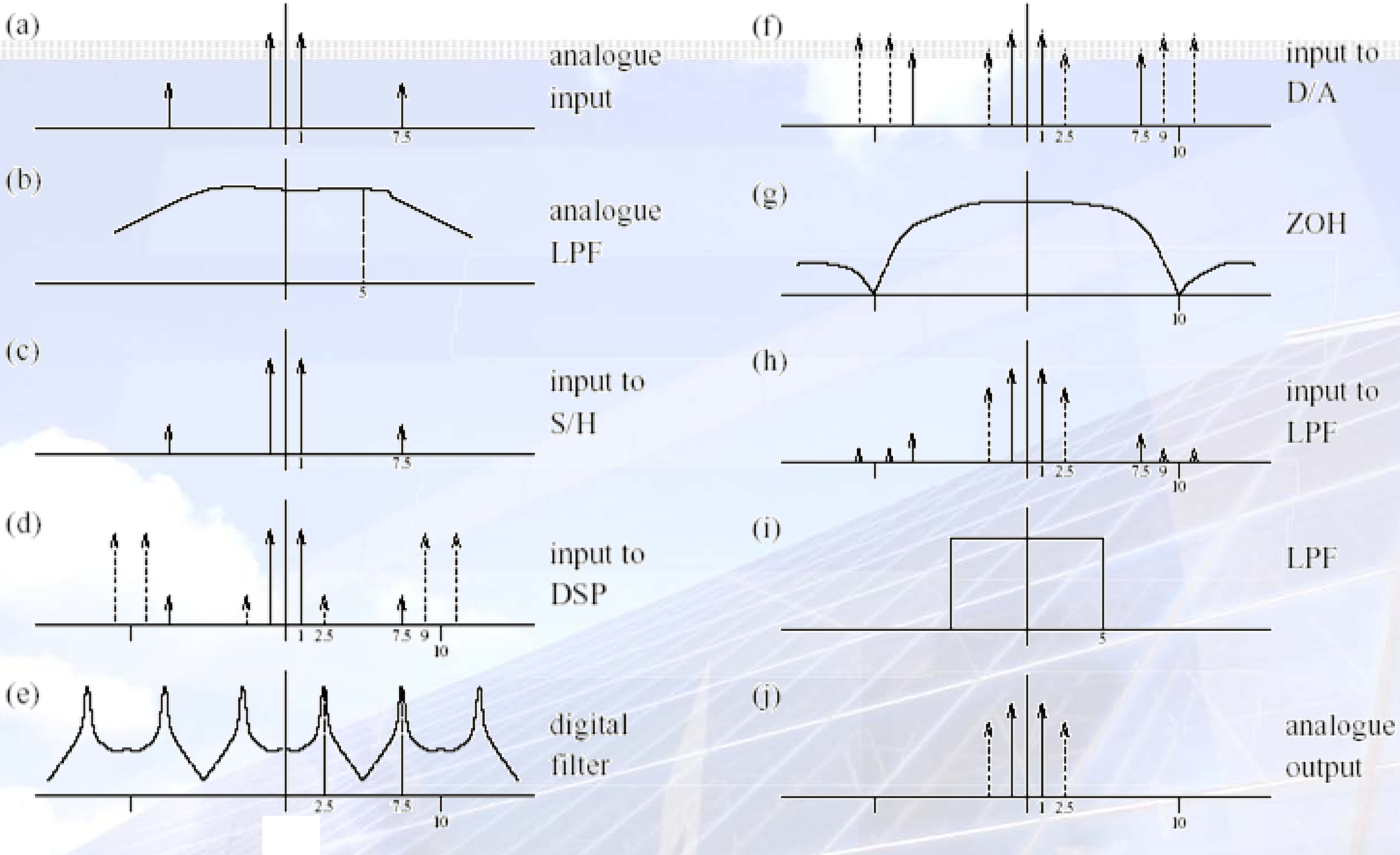
(a) A sampled data system



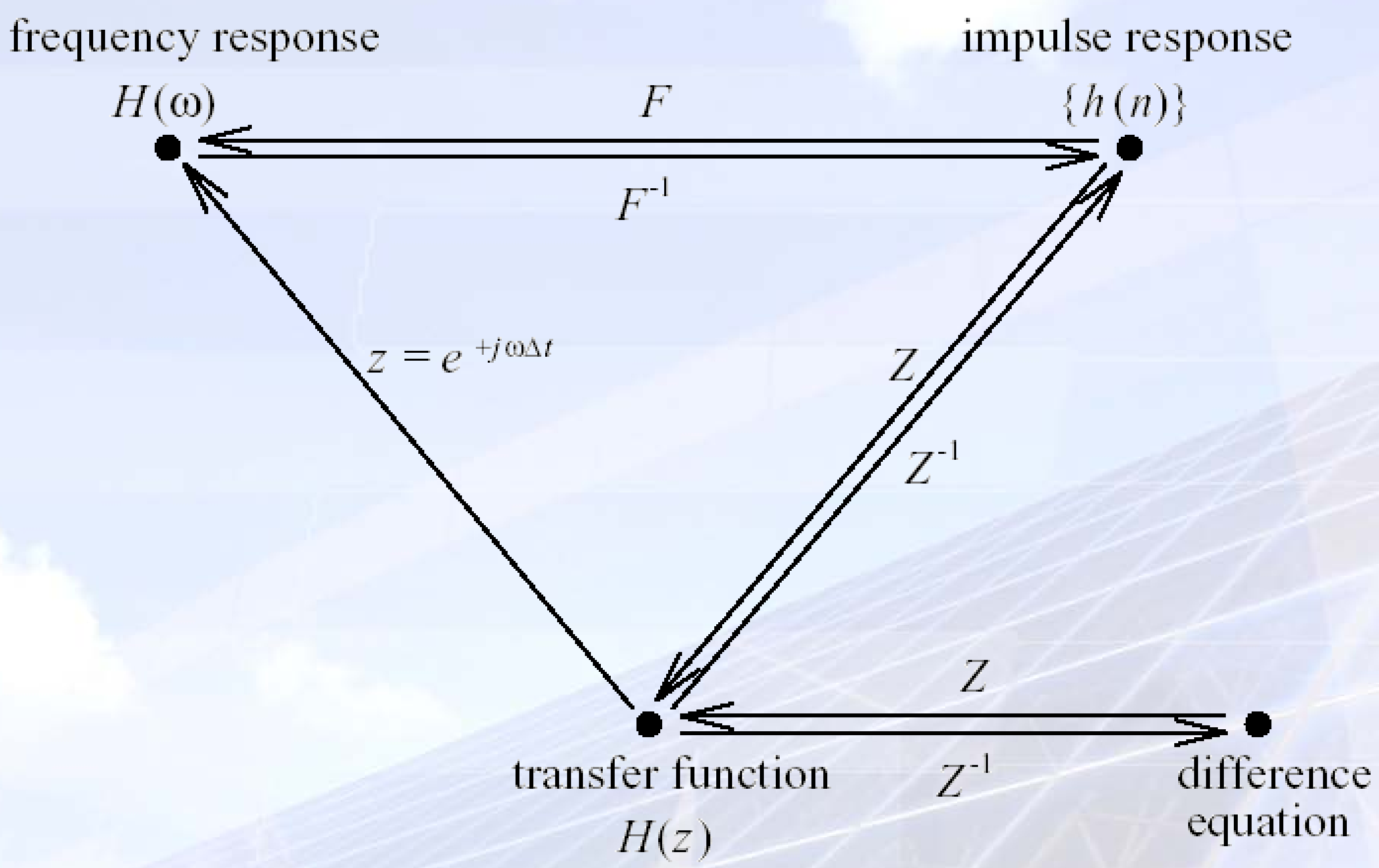
(b) Digital filter block diagram



EURECOM


(a)
analogue input
1
7.5
(b)
analogue LPF
5
(c)
input to S/H
1
7.5
(d)
input to DSP
1
2.5
7.5
9
10
(e)
digital filter
2.5
7.5
10
(f)
input to D/A
1
2.5
7.5
9
10
(g)
ZOH
10
(h)
input to LPF
1
2.5
7.5
9
10
(i)
LPF
5
(j)
analogue output
1
2.5
10

Summary



frequency response
$H(\omega)$
impulse response
$\{h(n)\}$
F
F^{-1}
$z = e^{+j\omega\Delta t}$
Z
Z^{-1}
Z
Z^{-1}
transfer function
$H(z)$
difference
equation

Summary

You should be able to:

- analyse the effect of sampling on the frequency context of a signal;
- evaluate the *z*-transform of simple sequences;
- evaluate the inverse *z*-transform using partial fraction expansion and tables;
- move between the difference equation, black diagram and software descriptions of a signal filter;
- identify if a filter is a finite or infinite impulse response;
- evaluate the output of a digital filter using discrete convolution;
- sketch the frequency response of a digital filter from its pole/zero map;
- understand the relationship between the position of the poles and the impulse response.