Essential Mathematical Methods for Engineers (MathEng)
EXAM
February 2021
Duration: 2 hrs, all documents and calculators permitted
ATTEMPT ALL QUESTIONS – ANSWER IN ENGLISH

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1. (a) Find the discrete Fourier transform (DFT) of the 3-point signal $f(n)$ illustrated in solid black lines in Figure Q5(a) and plot the magnitude and phase spectrums.

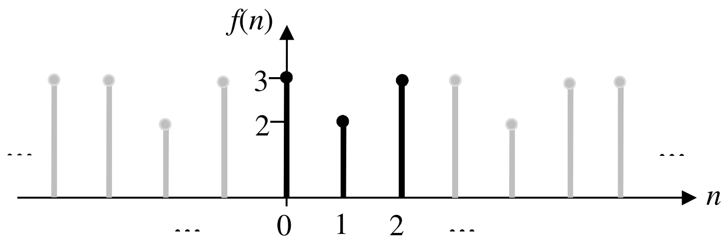

Figure Q5(a)

(b) Repeat part (a) by padding three zeros to $f(n)$ as illustrated in Figure Q5(b). Compare and comment on the result.

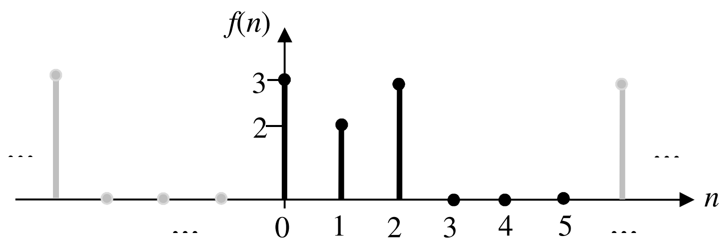

Figure Q5(b)

[15 marks]

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2. Match each of the five pole/zero configurations illustrated in the z-planes of Figures Q2(a)-(e) (on page 2) to one of the frequency responses in Figures Q2(1)-(5). Justify your answers in each case.

[5 marks]

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3. The output $y[n]$ of a discrete-time linear time-invariant system is found to be $2(\frac{1}{3})^n u[n]$ when the input $x[n]$ is $u[n]$ (the unit step sequence).
(a) Find the impulse response $h[n]$ of the system.
(b) Find the output $y[n]$ when the input $x[n]$ is $(\frac{1}{2})^n u[n]$.

[10 marks]

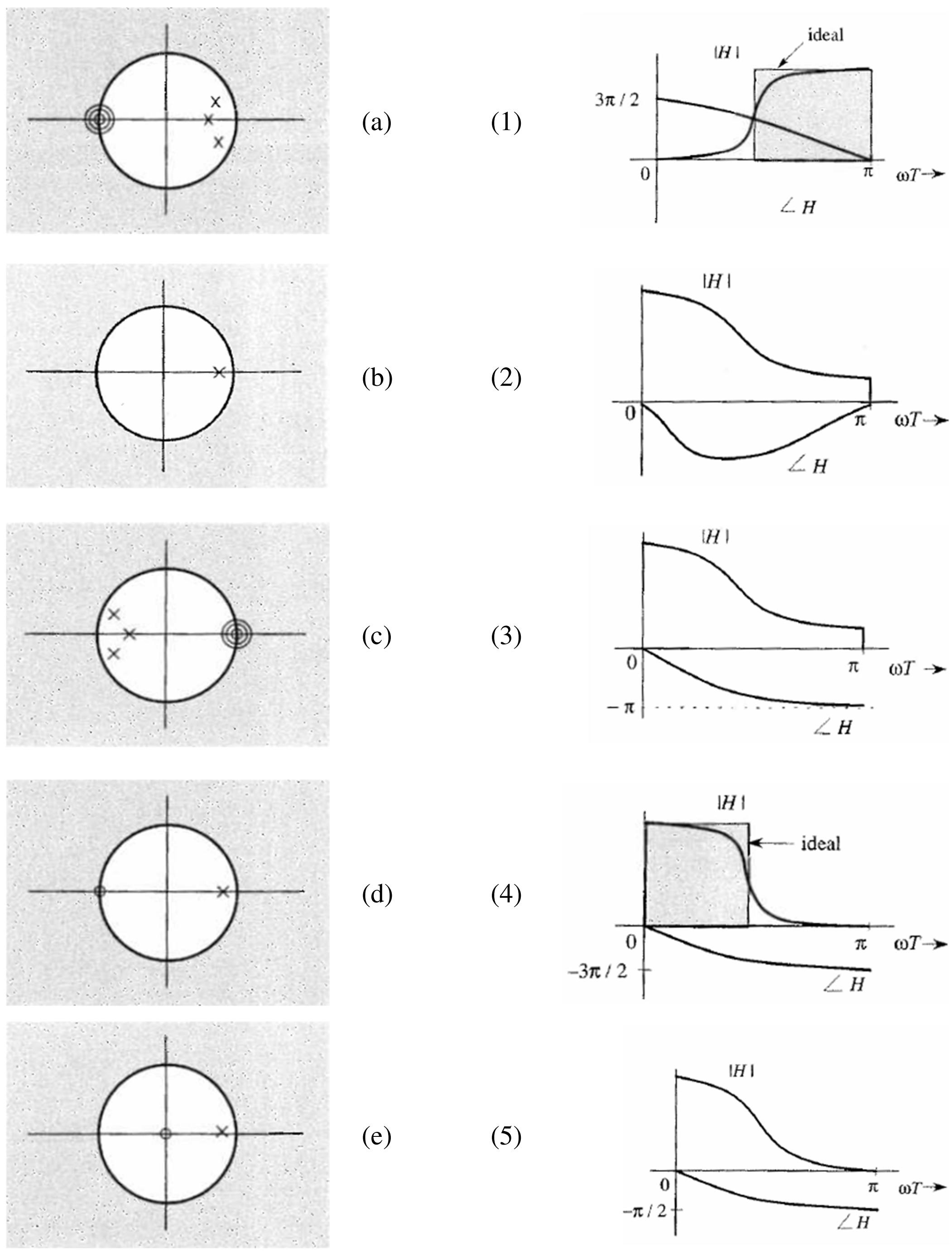

Figure Q2.

4. Figure Q4 illustrates three bivariate Gaussian probability density functions (PDFs) and their corresponding contour plots (left and right columns respectively) for random variables *X* and *Y*. Describe the differences between the PDFs in terms of the mean and variance of each component and their correlation.

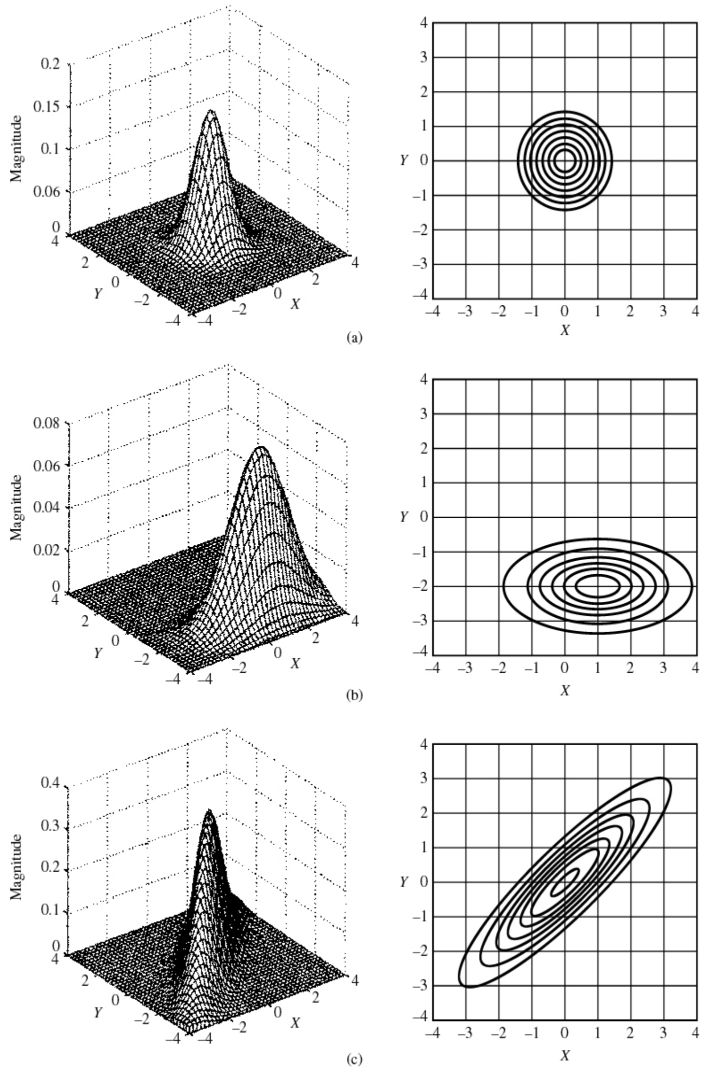

Figure Q4

[7 marks]

5. Determine the energy or power in the following signals (if defined) and hence or otherwise state whether they are energy or power signals.

$$x_1(t) = \sin(2\pi t) \qquad \text{and} \qquad x_2(t) = \exp(-t)$$

[5 marks]

6. By graphical time convolution, sketch the system output corresponding to the input signal $x(t)$ and the system impulse response $h(t)$ illustrated in Figure Q6.

[5 marks]

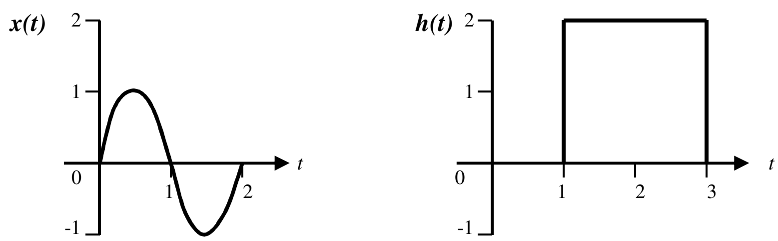

Figure Q7

7. Identify the pivot and free variables of the following matrices. Find a special solution for each free variable and, by combining the special solutions, describe every solution to $Ax = 0$ and $Bx = 0$.

$$A = \begin{bmatrix} 1 & 2 & 2 & 4 & 6 \\ 1 & 2 & 3 & 6 & 9 \\ 0 & 0 & 1 & 2 & 3 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 2 & 4 & 2 \\ 0 & 4 & 4 \\ 0 & 8 & 8 \end{bmatrix}$$

[8 marks]

8. Perform an LDU decomposition for the following matrix A and **use it** to solve $Ax = b$ where:

$$A = \begin{bmatrix} 2 & 2 \\ 4 & 9 \end{bmatrix} \qquad \text{and} \qquad b = \begin{bmatrix} 8 \\ 21 \end{bmatrix}.$$

What could be the advantage of solving the system in this way rather than by Gaussian elimination?

[5 marks]