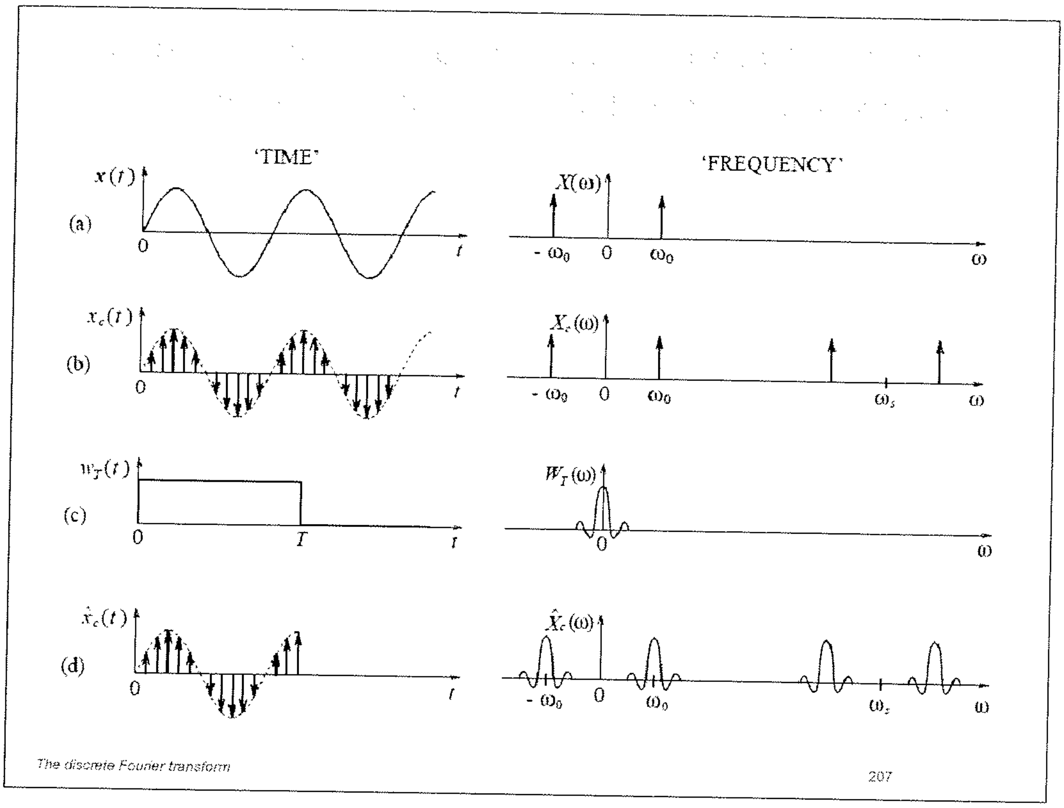
Example

Illustrate the effects of sampling and rectangular windowing on a continuous-time sine wave and its Fourier transform.

NB the difference between $X_C(\omega)$ and $\hat{X}_C(\omega)$

Solution over page



(a) In the example $x(t)$ is a sine wave with frequency ω_0 rad/s
It has a Fourier transform with impulses at $\pm\omega_0$ rad/s

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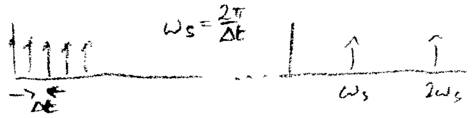
(b) Sampling at a rate of ω_s is akin to multiplying by infinite train of impulses
Multiplication in the time domain is convolution in the f domain so
to obtain the FT of $x_c(t)$ we convolve the transforms of $x(t)$ & $\delta_T(t)$



The FT of $\delta_T(t)$ is a train of impulses in the f domain at $\pm\omega_s$, $\pm 2\omega_s$.

(c) $\hat{x}_c(t)$ is obtained by multiplying $x_c(t)$ by a window of width T seconds
$W_T(t)$ has a sa(x) shape – see lecture 3.

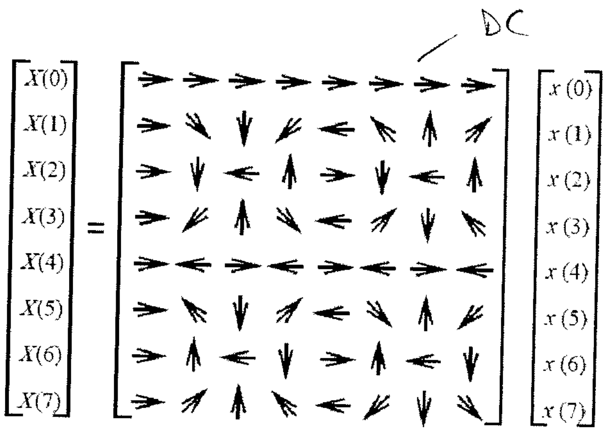
(d) We then convolve the transform of the sampled analogue signal with that of the rectangular window.

The result is that $\hat{X}_c(\omega)$ is a similar but distorted version of $X_c(\omega)$

This raises the question of whether or not there are better windows.

Example

Explain the operation of the 8-point DFT by a complex input phasor with an integer number of cycles within the 8-sample input data sequence.



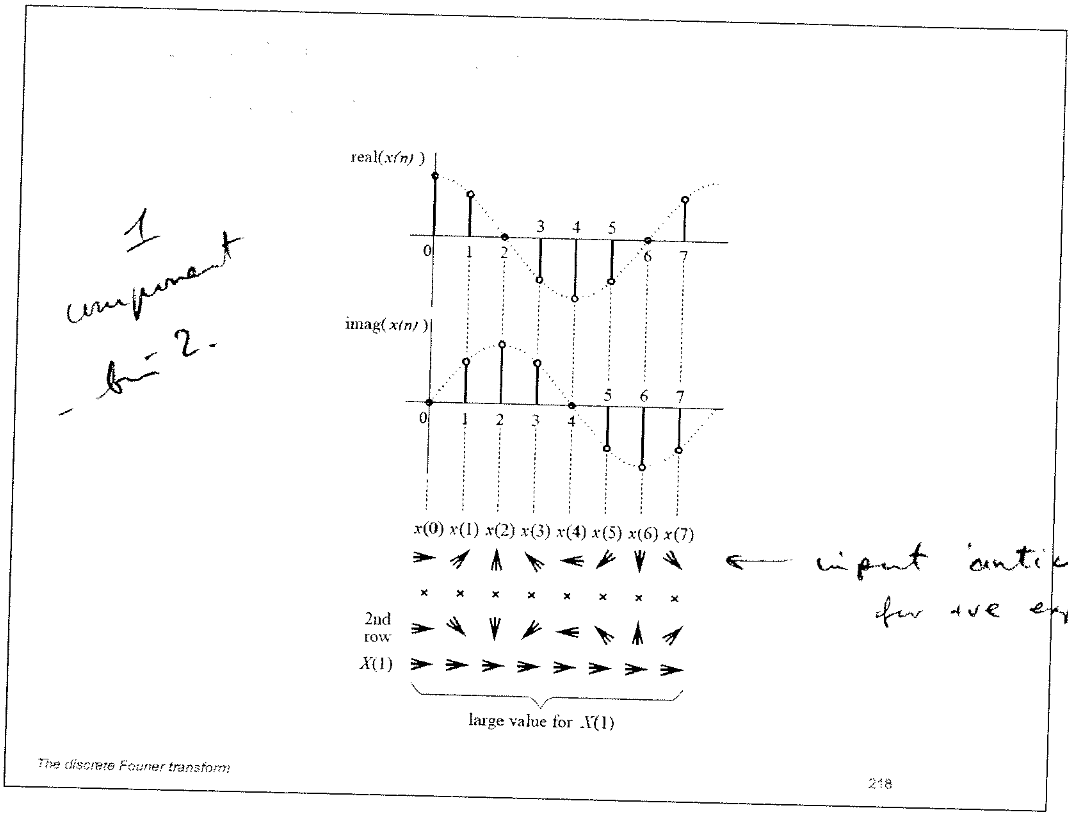
The discrete Fourier transform 217

again notice how the speed of rotation increases going down the matrix.

the phasors rotate clockwise
input rotates anticlockwise.

Can you explain the symmetry about ω_s, ie. aliasing

NB that the last row appears to rotate anticlockwise
– explains aliasing about ω_s



1 component for 2.

← input 'anticlockwise'! for +ve exponential.

The second row finds the component at 1 cycle per block length

eg for a simple input signal a sampled complex phasor with a period of 8 samples

$$x(n) = \exp(j 2\pi n/8) = \cos(2\pi n/8) + j \sin(2\pi n/8)$$

The second row of the matrix represents a filter ~~in~~ matched to a centre frequency corresponding to a component at one rotational cycle of the phasor per block length.

All values of $x(\cdot)$ are rotated as shown to give a large value to $X(1)$

What happens for $X(2) \ldots X(7)$?

What happens for $\theta \neq 0$