PB2-REPORT

December 12, 2024

Student: Brice Robert, ***Track:*** ICS

Document: REPORT.pdf, ***Type:*** Laboratory

Languages used: LaTeX, Julia (in lieu of MATLAB)

Tools used: Jupyter, nbconvert (converting to PDF)

Permanent Link: https://github.com/setrar/MobCom/blob/main/Lab/REPORT.ipynb

MATLAB PROJECT for MOBCOM

EURECOM
November 21st, 2024
Class Instructor: Petros Elia
elia@eurecom.fr

- Read carefully the following questions, and using MATLAB, provide the answers/plots in the form of a report.
- The report should include a title page, and should be properly labeled and named. The report should be in the form of a PDF.
- Graphs should include labels, titles, and captions.
- Each graph should be accompanied with pertinent comments.
- Use optimal (maximum likelihood) decoders, unless stated otherwise.
- To compare the empirical results with the corresponding theoretical result, you should superimpose the two corresponding graphs and provide comments and intuition on the comparison.
- For each plot, describe the theoretical background that guides the proper choice of parameters for simulations (i.e., power constraint).
- You can work in groups of two or three.
- Regarding Grading:
  - All questions are weighted equally.
  - Submit your report (labeled and named) via email, to Hui Zhao (Hui.Zhao@eurecom.fr) and to myself.
  - Submission deadline is December 12th, 2024.

Enjoy!

PROBLEM 2

- **Use simulations to establish the probability of deep fade**

$$P(\|h\|^2 < \text{SNR}^{-1})$$

for the random fading model:

$$y = h \cdot x + w$$

where $w \sim \mathbb{C}N(0,1)$, and where h is a Rician random variable, where you can choose the parameters of this distribution.

- **Now do the same when h is now a 3-length vector with i.i.d. Rician elements.**

In all the above, the y-axis is the probability of deep fade, in log scale ($\log_{10}(\text{Prob})$), and the x-axis is the SNR, in dB.

Step-by-step implementation in `Julia`, including simulations for both a single `Rician fading coefficient` and a `3-length vector` of i.i.d. Rician fading elements.

1 Mathematical Model

- The random fading model is: $y = h \cdot x + w$, where:
  - h: Rician fading random variable.
  - x: Transmitted signal (can be any constant since it's irrelevant for this computation).
  - $w \sim \mathbb{C}N(0,1)$: Complex Gaussian noise.
- Deep fade probability: $P(\|h\|^2 < \text{SNR}^{-1})$, where:
  - $\|h\|^2$ is the power of the fading channel.
  - $\text{SNR} = 10^{\text{SNR}_{\text{dB}}/10}$.

For a 3-length vector of i.i.d. Rician fading elements: $\|h\|^2 = \sum_{i=1}^{3} |h_i|^2$, where h_i are i.i.d. Rician variables.

2 Simulation Steps

Import Required Libraries

[1]:
```julia
using Random
using Distributions
using LinearAlgebra
using Plots, LaTeXStrings, Measures
using FFTW
```

```
[2]: # functions and variables to increase readability
include("modules/operations.jl");
```

```
[3]: # Define base values and offsets
base_values = [-0.00, -0.50, -1.00, -1.50, -2.00]
offsets = [-0.0, -0.02, -0.10, -0.15, -0.20, -0.30, -0.40, -0.70]
include("modules/view_helper.jl");
```

Step 1: Generate Rician Fading Coefficients A Rician fading random variable \$ h \$ is parameterized by: - \$ K \$: The Rician \$ K \$-factor (ratio of LOS to NLOS power). - \$ \$: The standard deviation of the NLOS component.

The Rician fading can be generated as: $h = v + z$, where: - v: Deterministic LOS component ($v = \sqrt{K/(K+1)}$). - $z \sim \mathbb{C}N(0, \sigma^2/2)$: Complex Gaussian NLOS component.

```
[4]: # Generate Rician fading
function generate_rician(n, K,  )
      = sqrt(K / (K + 1))  # LOS component
     _r =   / sqrt(2 * (K + 1))  # NLOS component
    real = rand(Normal( ,  _r), n)
    imag = rand(Normal(0,  _r), n)
    return real .+ j .* imag  # Complex Rician fading
end;
```

Step 2: Compute $\|h\|^2$ For the single random variable h:

```
[5]: # Compute magnitude squared for single Rician variable
function compute_magnitude_squared(h)
    return abs2.(h)  # Compute |h|^2 for all samples
end;
```

For the 3-length vector:

```
[6]: # Compute magnitude squared for a 3-length Rician vector
function compute_vector_magnitude_squared(h_vector::Matrix{ComplexF64})
    return sum(abs2, h_vector, dims=1)  # Sum squared magnitudes along rows
end;
```

Step 3: Compute Probability of Deep Fade Evaluate the probability: $P(\|h\|^2 < \mathrm{SNR}^{-1})$ for a range of SNR values.

```
[7]: # Compute deep fade probability
function deep_fade_probability(h, SNR_range)
    probabilities = Float64[]
    for SNR_dB in SNR_range
        SNR_linear = 10^(SNR_dB / 10)  # Convert dB to linear scale
        threshold = 1 / SNR_linear
        fade_count = count(x -> x < threshold, compute_magnitude_squared(h))
```

```julia
        push!(probabilities, fade_count / length(h))
    end
    return probabilities
end

# Compute deep fade probability for vector
function deep_fade_probability_vector(h_vector, SNR_range)
    probabilities = Float64[]
    for SNR_dB in SNR_range
        SNR_linear = 10^(SNR_dB / 10)  # Convert dB to linear scale
        threshold = 1 / SNR_linear
        fade_count = count(x -> x < threshold
            , compute_vector_magnitude_squared(h_vector)
        )
        # Use second dimension for vectors
        push!(probabilities, fade_count / size(h_vector, 2))
    end
    return probabilities
end;
```

Step 4: Perform Monte Carlo Simulation Simulate h for both cases.

[8]:
```julia
# Parameters
n_samples = 10^6  # Number of samples
K = 2  # Rician K-factor
  = 1.0  # Standard deviation
SNR_dB_range = 0:3:30  # SNR range in dB

# Single Rician random variable
h = generate_rician(n_samples, K,  )
prob_single = deep_fade_probability(h, SNR_dB_range)

# 3-length Rician vector
h_vector = reshape(
            reduce(vcat
                  # Generate 3 independent Rician variables
                , [generate_rician(n_samples, K,  ) for _ in 1:3])
            , (3, :)
        ) # Reshape to (3, n_samples)
prob_vector = deep_fade_probability_vector(h_vector, SNR_dB_range);
```

Step 5: Plot the Results Plot the deep fade probabilities on a logarithmic scale ($\log_{10}$(Prob)).

[9]:
```julia
println("Single Rician Variable (log): ", log10.(prob_single))
println("3-Length Rician Vector (log): ", log10.(prob_vector))
```

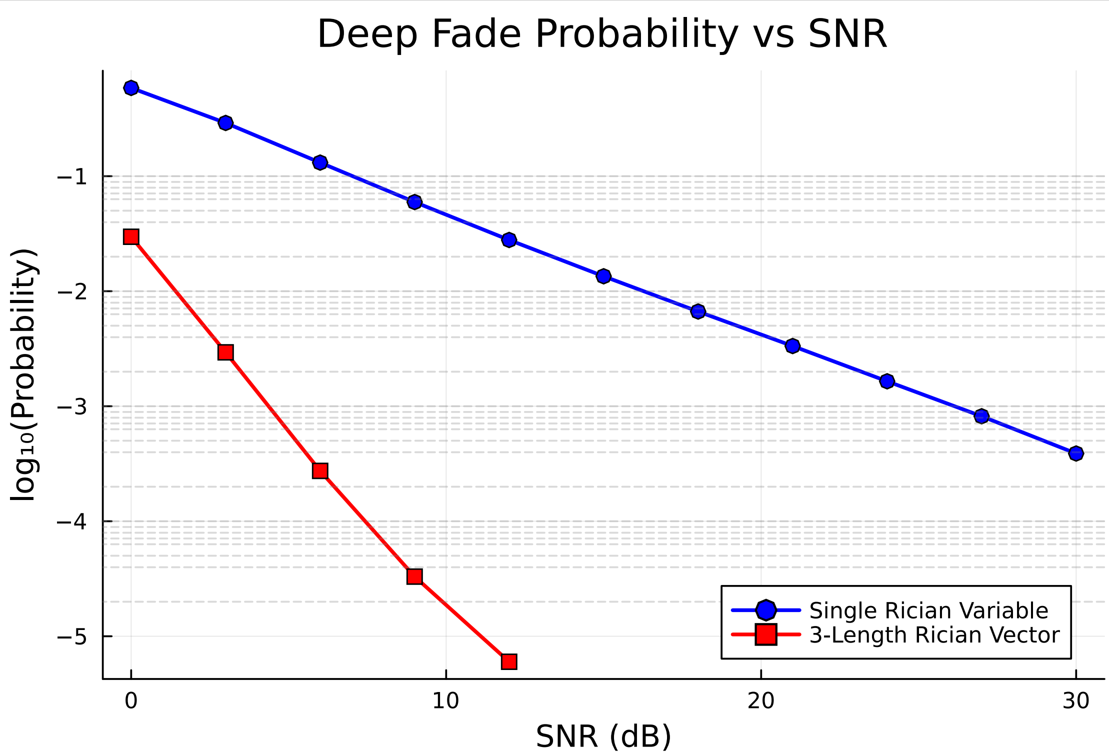
```
Single Rician Variable (log): [-0.23255915208786465, -0.5371948547366029,
-0.8818973790242725, -1.2249213694702286, -1.5545514857339502,
```

```
-1.87014904921109, -2.1767214430483293, -2.476643793345207, -2.7833064008302455,
-3.087246696328677, -3.4111682744057927]
3-Length Rician Vector (log): [-1.526586026739016, -2.5316526695878427,
-3.562249437179612, -4.481486060122113, -5.221848749616356, -Inf, -Inf, -Inf,
-Inf, -Inf, -Inf]
```

[10]:

```julia
# Plot results with labels, title, and grid in the first plot call
plot(SNR_dB_range, log10.(prob_single)
    , label="Single Rician Variable", marker=:o, lw=2, color=:blue
    , xlabel="SNR (dB)", ylabel="log (Probability)"
    , title="Deep Fade Probability vs SNR"
    , grid=true, legend = :bottomright)
plot!(SNR_dB_range, log10.(prob_vector)
    , label="3-Length Rician Vector", marker=:square, lw=2, color=:red)
add_combined_hlines!(offsets, base_values
    , linestyle=:dash, lw=1, color=:gray, alpha=0.3)
```

[10]:



3 Expected Results

1. **Single Rician Variable:**
   - At low SNR, the deep fade probability is high ($\log_{10}(P) \approx 0$).
   - At high SNR, the probability drops exponentially ($\log_{10}(P) < -6$).
2. **3-Length Rician Vector:**

- The deep fade probability is lower than for a single variable due to the diversity gain.
- The curve decreases faster with increasing SNR compared to the single variable case.

```
[ ]:
```