REPORT

December 12, 2024

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Languages used: LaTeX, Julia (in lieu of MATLAB)

Tools used: Jupyter, nbconvert (converting to PDF)

Permanent Link: https://github.com/setrar/MobCom/blob/main/Lab/REPORT.ipynb

MATLAB PROJECT for MOBCOM

EURECOM
November 21st, 2024
Class Instructor: Petros Elia
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- Read carefully the following questions, and using MATLAB, provide the answers/plots in the form of a report.
- The report should include a title page, and should be properly labeled and named. The report should be in the form of a PDF.
- Graphs should include labels, titles, and captions.
- Each graph should be accompanied with pertinent comments.
- Use optimal (maximum likelihood) decoders, unless stated otherwise.
- To compare the empirical results with the corresponding theoretical result, you should superimpose the two corresponding graphs and provide comments and intuition on the comparison.
- For each plot, describe the theoretical background that guides the proper choice of parameters for simulations (i.e., power constraint).
- You can work in groups of two or three.
- Regarding Grading:
  - All questions are weighted equally.
  - Submit your report (labeled and named) via email, to Hui Zhao (Hui.Zhao@eurecom.fr) and to myself.
  - Submission deadline is December 12th, 2024.

Enjoy!

PROBLEM 1

Consider communication over the 1×1 quasi-static fading channel, using 16-PAM. The channel model is given by

$$\overbrace{(y)}^{y} = \theta \overbrace{(h)}^{h} \overbrace{(x)}^{\text{16-PAM:}X_{\text{tr}}} + \overbrace{(w)}^{w}$$

where $h \sim \mathbb{C}N(0,1)$ (Gaussian Fading) and $w \sim \mathbb{C}N(0,2)$, and where θ is the power normalization factor that lets you regulate SNR.

Here, you are supposed to do a simulation of the action of decoding. **PROVIDE THE DETAILS OF HOW YOU SIMULATED.** Tell us which variables you change in each iteration: h, codewords, noise, and tell us how you power normalize (emphasis on θ) so that you achieve a certain signal-to-noise ratio (SNR). Naturally, in each iteration, you decode, using the maximum-likelihood (ML) rule that we learned about:

$\hat{x} = \arg\min_{x \in \mathcal{X}_{\text{tr}}} \|y - \theta h \cdot x\|^2$

going over all choices of x in the code $\mathcal{X}_{\text{tr}}$.

NOTE: Do many iterations so that your plots are "smooth." In all the above, the y-axis is the probability of error, in log scale ($\log_{10}(\text{Prob})$), and the x-axis is the SNR, in dB.

- **Plot the probability of error** on a logarithmic scale as a function of SNR (dB) by performing Monte-Carlo simulations for when x are independently chosen from 16-PAM.

For the above, use the ML decoder, and plot for SNR values — in steps of 3 dB — up to an SNR value for which your probability of error drops below 5×10^{-5}. **Again, clearly explain how you calculate θ in each case.**

Import Required Libraries

[1]:
```
using Random
using Distributions
using LinearAlgebra
using Plots, LaTeXStrings, Measures
using FFTW
```

[2]:
```
# functions and variables to increase readability
include("modules/operations.jl");
```

[17]:
```
# Define base values and offsets
base_values = [-0.00, -0.50, -1.00, -1.50, -2.00]
offsets = [-0.0, -0.02, -0.10, -0.15, -0.20, -0.30, -0.40, -0.70]
include("modules/view_helper.jl");
```

Step 2: Define Parameters

Set the simulation parameters:

[4]:
```
# Parameters (only the constants)
const M = 16  # 16-PAM
const  ² = 2.0  # Noise variance
const SNR_dB_range = 0:3:30;  # SNR range in dB
```

Step 3: Generate 16-PAM Symbol Set

Define the 16-PAM constellation:

[5]:
```
# Generate 16-PAM constellation
function generate_16pam()
    levels = -15:2:15  # PAM levels
    return collect(levels)  # Return as an array
end

X  = generate_16pam() ; @show typeof(X ), X ;  # Shows the Transmitted symbol␣
 ↪set
```

```
(typeof(X ), X ) = (Vector{Int64}, [-15, -13, -11, -9, -7, -5, -3, -1, 1, 3,
5, 7, 9, 11, 13, 15])
```

Step 4: Define Channel Model and Noise

1. Gaussian Fading Channel ($\tilde{h} \sim \mathcal{CN}(0,1)$):

[6]:
```
# Generate Gaussian fading channel
function generate_gaussian_fading(n)
    real_part = rand(Normal(0, 1), n)  # Real part
    imag_part = rand(Normal(0, 1), n)  # Imaginary part
    return real_part .+ im .* imag_part  # Complex Gaussian
end;
```

2. Additive Noise ($\tilde{w} \sim \mathcal{CN}(0,\sigma^2)$):

[7]:
```
# Generate complex Gaussian noise
function generate_noise(n,  ²)
    real_part = rand(Normal(0, sqrt( ² / 2)), n)
    imag_part = rand(Normal(0, sqrt( ² / 2)), n)
    return real_part .+ im .* imag_part
end;
```

Step 5: Power Normalization

Compute the normalization factor θ based on the SNR:

[8]:
```
# Compute power normalization factor
function compute_ (SNR_dB,  ², X )
    SNR    = 10^(SNR_dB / 10)  # Convert SNR from dB to linear scale
    P  = mean(abs2.(X ))  # Average power of 16-PAM symbols
    return sqrt((SNR    *  ²) / P )  # Calculate
```

```
end;
```

Step 6: ML Decoding Rule

Implement the ML decoding rule:

[9]:
```
# ML decoding
function ml_decode(ŷ, h,  , X )
    distances = abs2.(ŷ .-   .* h .* X )  # Compute distances for all symbols
    idx = argmin(distances)  # Find the index of the minimum distance
    return X [idx]  # Return the estimated symbol
end;
```

Step 7: Monte Carlo Simulation

Simulate the system and calculate the probability of error:

[10]:
```
# Monte Carlo simulation
function monte_carlo_simulation(SNR_dB_range, n_samples,  ², X_tr)
    P_error = Float64[]

    for SNR_dB in SNR_dB_range
          = compute_ (SNR_dB,  ², X_tr)  # Compute normalization factor
        h = generate_gaussian_fading(n_samples)  # Generate fading coefficients
        x = rand(X_tr, n_samples)  # Randomly transmit symbols
        w = generate_noise(n_samples,  ²)  # Generate noise
        ŷ =   .* h .* x .+ w  # Received signal

        # Perform decoding
        x̂ = [ml_decode(ŷ[i], h[i],  , X_tr) for i in 1:n_samples]

        # Compute error probability
        error_count = count(x .!= x̂)
        push!(P_error, error_count / n_samples)
    end

    return P_error
end;
```

Step 8: Plot Results

Plot the probability of error vs. SNR (logarithmic scale):

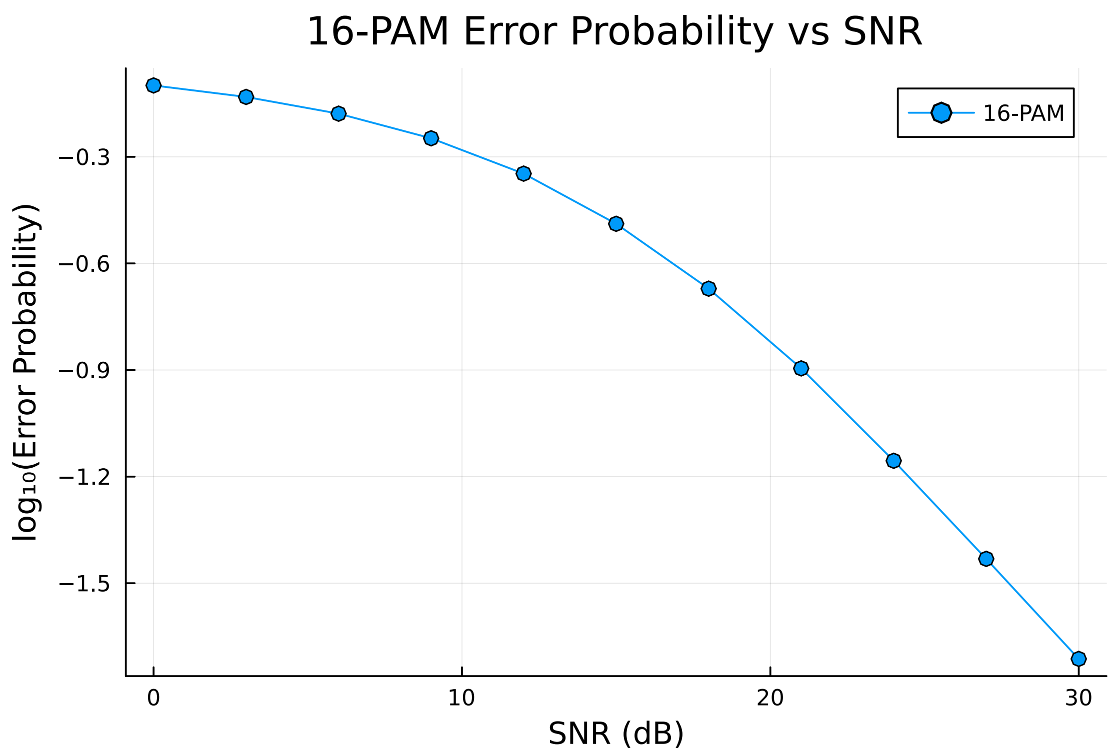
[19]:
```
# Parameters
n_samples = 10^6  # Number of Monte Carlo samples

# Run the simulation
P_error = monte_carlo_simulation(SNR_dB_range, n_samples,  ², X ); @show␣
↪P_error;
```

```
# Plot results
plot(SNR_dB_range, log10.(P_error)
    , marker=:o, label="16-PAM"
    , xlabel="SNR (dB)", ylabel="log (Error Probability)"
    , title="16-PAM Error Probability vs SNR"
    , grid=true
)
# add_combined_hlines!(offsets, base_values, linestyle=:dash, lw=1, color=:
 ↪gray, alpha=0.3)
```

```
P_error = [0.795997, 0.738788, 0.662925, 0.565179, 0.449493, 0.32472, 0.213301,
0.127217, 0.06992, 0.037029, 0.019362]
```

[19]:



[12]:

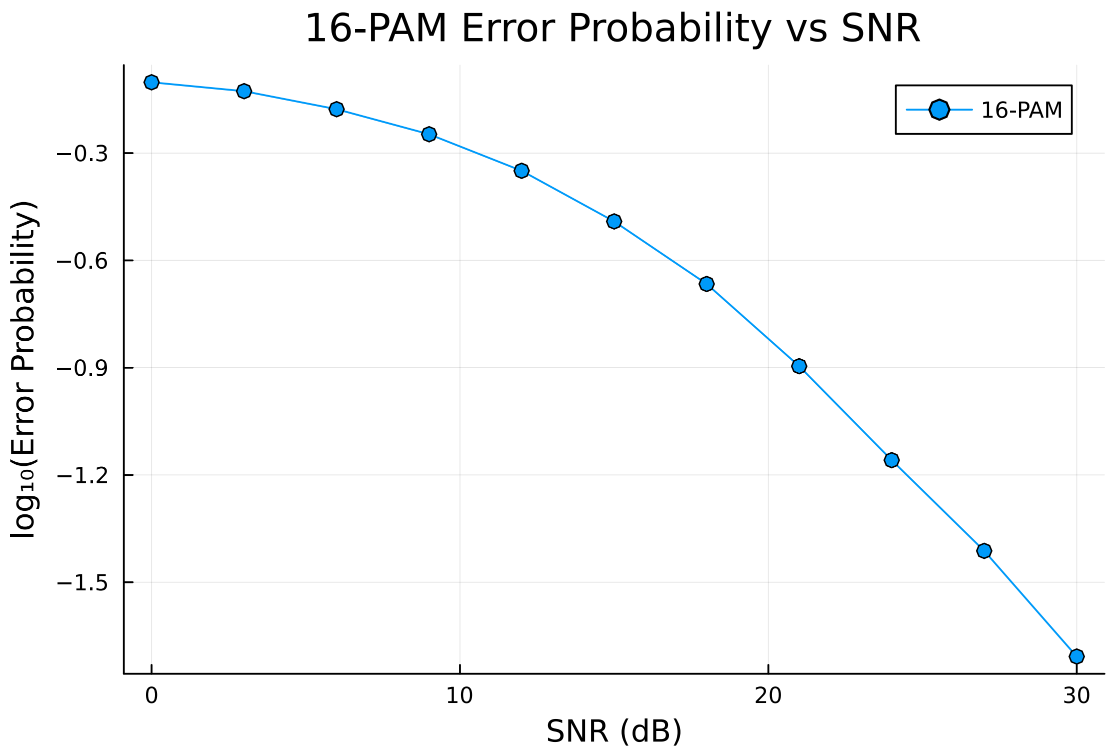
```
# Parameters
n_samples = 10^4  # Number of Monte Carlo samples

# Run the simulation
P_error = monte_carlo_simulation(SNR_dB_range, n_samples,  ², X )

# Plot results
plot(SNR_dB_range, log10.(P_error)
    , marker=:o, label="16-PAM"
    , xlabel="SNR (dB)", ylabel="log (Error Probability)"
    , title="16-PAM Error Probability vs SNR"
```

```
    , grid=true
)
```

[12]:



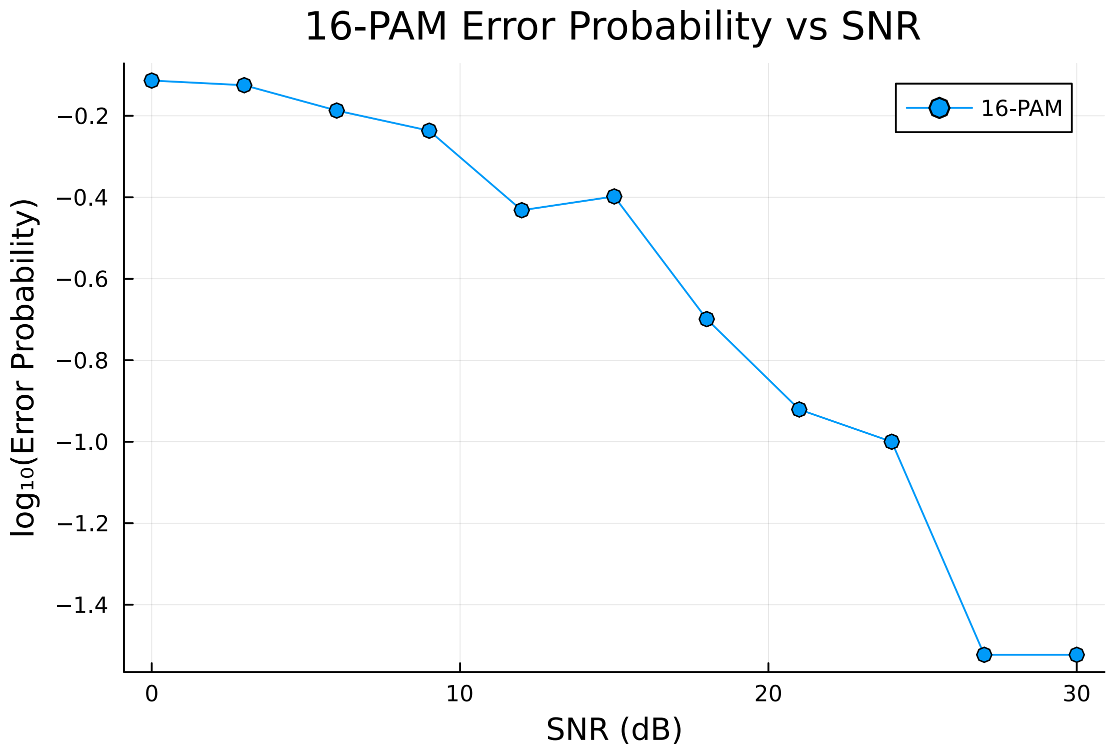
[13]:

```
n_samples = 10^2  # Number of Monte Carlo samples

# Run the simulation
P_error = monte_carlo_simulation(SNR_dB_range, n_samples,  ², X )

# Plot results
plot(SNR_dB_range, log10.(P_error)
    , marker=:o, label="16-PAM"
    , xlabel="SNR (dB)", ylabel="log (Error Probability)"
    , title="16-PAM Error Probability vs SNR"
    , grid=true
)
```

[13]:

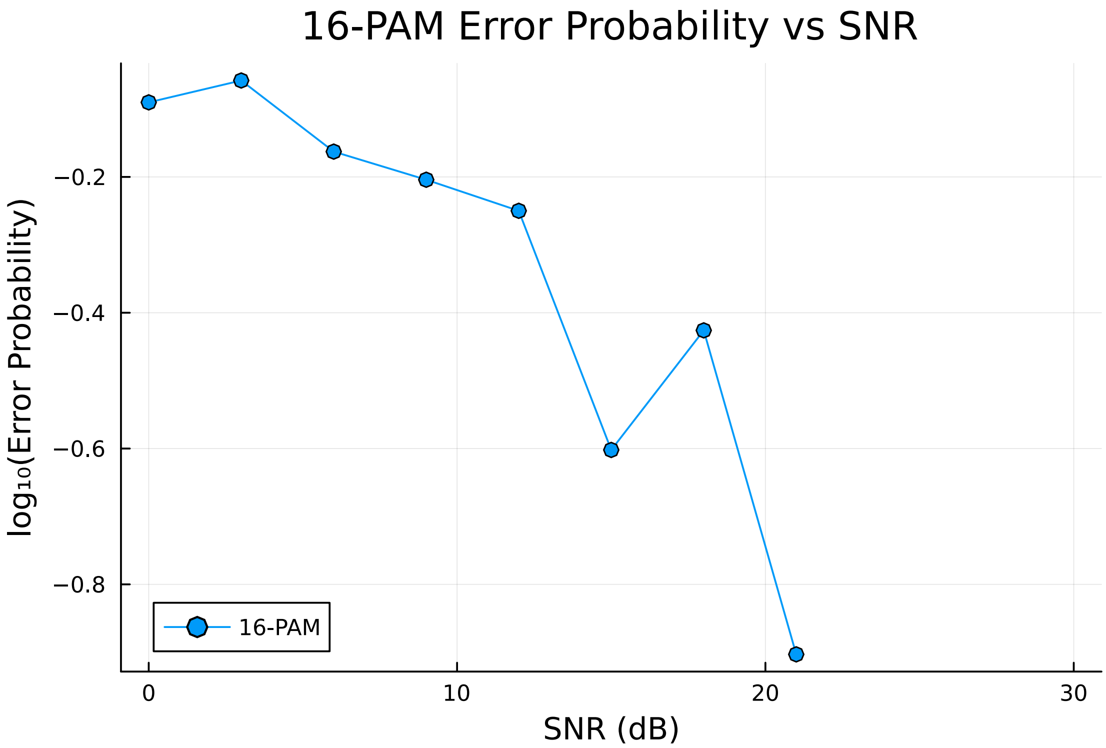


[14]:
```
n_samples = 16  # Number of Monte Carlo samples

# Run the simulation
P_error = monte_carlo_simulation(SNR_dB_range, n_samples,  ², X )

# Plot results
plot(SNR_dB_range, log10.(P_error)
    , marker=:o, label="16-PAM"
    , xlabel="SNR (dB)", ylabel="log (Error Probability)"
    , title="16-PAM Error Probability vs SNR"
    , grid=true
)
```

[14]:



PROBLEM 2

- **Use simulations to establish the probability of deep fade**

$$P(\|h\|^2 < \mathrm{SNR}^{-1})$$

for the random fading model:

$$y = h \cdot x + w$$

where $w \sim \mathbb{C}N(0, 1)$, and where h is a Rician random variable, where you can choose the parameters of this distribution.

- **Now do the same when h is now a 3-length vector with i.i.d. Rician elements.**

In all the above, the y-axis is the probability of deep fade, in log scale ($\log_{10}(\text{Prob})$), and the x-axis is the SNR, in dB.

```
[ ]:
```

PROBLEM 3

Use simulations to establish the probability of deep fade

$P(\|\tilde{h}\|^2 < \mathrm{SNR}^{-1})$

where $\|\tilde{h}\|^2$ now comes from the χ^2-squared fading distribution with $2 \times 3 = 6$ degrees of freedom.

- **What do you observe compared to the previous two problems?**

In all the above, the y-axis is the probability of deep fade, in log scale ($\log_{10}(\mathrm{Prob})$), and the x-axis is the SNR, in dB.

[ ]:

PROBLEM 4

Create different experiments to check the validity of the following:

- For Gaussian random variables $h_r \sim \mathcal{N}(0, \sigma)$, the far tail is approximated by an exponential, i.e., \$ Q( ) e{- 2 / 2 z}. \$ Identify what is z in this case.
- For $h \sim \mathbb{C}\mathcal{N}(0, 1)$, the near-zero behavior is approximated as follows:

$$P(\|h\|^2 < \epsilon) \approx \epsilon.$$

- Same as the above, but for $h \sim CN(0, 5)$. Show how the near-zero behavior is approximated.

NOTE: The important thing in the above exercise is to describe **IN DETAIL** the way you perform the different experiments, as well as the results.

NOTE: We need statistical experiments, i.e., experiments that involve the generation of random variables, and the measuring of their behavior using — if you wish — histograms.

[15]:

```
# '/.
```