LECTURE 1.

Communication (Simplistic view).

Transmit $x(t) = \cos(2\pi f t)$ f is frequency. $\approx 1 \rightarrow 4$ GHz.

t is time.

Rx: $E(f, t, r, \vartheta, \varphi) = \frac{1}{r} \cdot \alpha_s(\vartheta, \varphi, f) \cdot \cos\left(2\pi f\left(t - \frac{r}{c}\right)\right)$ (1)

delay due to signal.

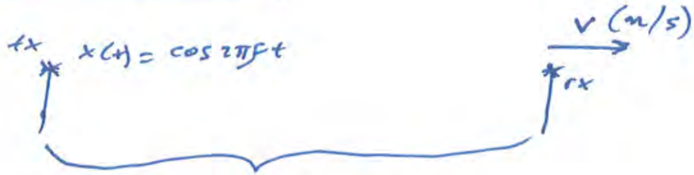
α : antenna losses

<u>Intuition why $E \propto \frac{1}{r}$</u>

- T_x in 3-D space.
- Power preserved in surface of sphere : (radius r, center is antenna)

$\Rightarrow$ Fixed $\frac{\text{~~Total~~ Power}}{\text{Area}}$ $\Rightarrow$ Rx power $\propto \frac{1}{4\pi r^2} \propto \frac{1}{r^2}$.

Consider movement.



$$r(t) = r_0 + v.t$$

$$\Rightarrow E(f, t, r(t)) = \frac{\alpha_s}{r(t)} \cdot \cos 2\pi f \left(t - \frac{r(t)}{c}\right) \quad \text{like before} : r \to r(t).$$

$$= \frac{\alpha_s}{r_0 + v.t} \cdot \cos 2\pi f \left(t - \frac{r_0}{c} - \frac{vt}{c}\right)$$

$$= \frac{\alpha_s}{r_0 + v.t} \cdot \cos 2\pi f \left[t\left(1 - \frac{v}{c}\right) - \frac{r_0}{c}\right]$$

$\Rightarrow$ effective frequency change $\quad ft \to \underbrace{f\left(1-\frac{v}{c}\right)}_{\cdots} \cdot t \quad : f \to f\left(1-\frac{v}{c}\right)$

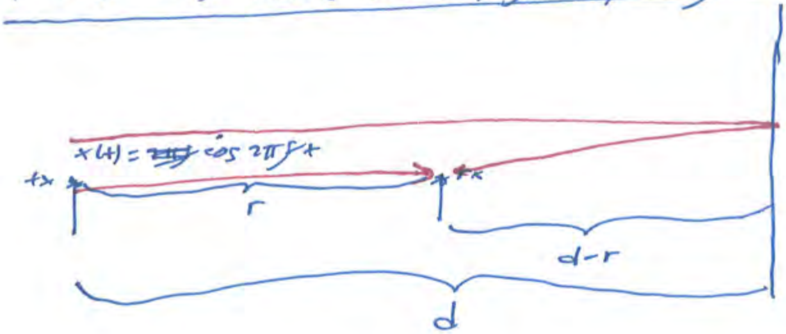
$\Rightarrow$ frequency reduction $f \to f\left(1-\frac{v}{c}\right) \Rightarrow \quad D = -\frac{fv}{c}$

DOPPLER shift : $D = -\frac{fv}{c}$.

$$\Rightarrow E(f, t, r(t)) = \underbrace{\frac{\alpha_s}{r_0 + v.t}} \cdot \cos 2\pi f \left(t\left(1 - \frac{v}{c}\right) - \frac{r}{c}\right)$$

note: not LTI now.

Fixed Tx, Fixed Rx, perfectly reflecting wall.



Use method of "ray tracing" : consider dominant (main) paths.

$$E_r(f,t) = \frac{\alpha}{r} \cdot \cos 2\pi f\left(t - \frac{r}{c}\right) - \frac{\alpha}{2d-r} \cos 2\pi f\left(t - \frac{2d-r}{c}\right)$$

r is distance 1. $2d-r$ = distance 2.

⇒ Superposition (addition of two sinusoids, with different phases).
possible effect of cancellation out.

Phase ~~diff~~ $\Delta\theta$ $\varphi_2 + \pi - \varphi_1$ → (π phase shift in electric field of EM wave when reflected from optically denser medium).

$$\Delta\theta = 2\pi f \frac{2d-r}{c} + \pi - 2\pi f \frac{r}{c}.$$

(essentially canceling each other out close to the wall)

Coherence bandwidth:

Change in frequency f, so that channel (i.e magnitude of rx signal) remains relatively the same.

$\Delta\theta = \frac{4\pi f}{c}(d-r) + \pi \Rightarrow$ change f so that phase shift changes $\approx \frac{\pi}{2}$

$$\Rightarrow \overbrace{\frac{4\pi f}{c}(d-r)}^{\text{change by}} \approx \frac{\pi}{2}$$

$$\Rightarrow 2\pi f\left(\frac{2d-r-r}{c}\right) \approx \frac{\pi}{2}$$ ← rewrite to help us later.

note: $\frac{2d-r}{c} - \frac{r}{c} \triangleq T_d$ is called <u>delay spread</u> (difference in delay of the paths).

$$\Rightarrow 2\pi f T_d \approx \pi/2 \Rightarrow \quad \Rightarrow f \approx \frac{1}{4T_d} \Rightarrow W_c = \frac{1}{4T_d}$$

(note: if set $2\pi f\left(\frac{2d-r-r}{c}\right) \approx \pi \Rightarrow W_c = \frac{1}{2T_d}$

↑ substantial phase shift

the constant factor does not really matter) ($\frac{\pi}{2}$ or π).

$\frac{\pi}{2} \downarrow \frac{1}{4T_d}$, $\pi \downarrow \frac{1}{2T_d}$

in fact. wiki: $W_c \approx \frac{1}{T_d}$.

(prelude)

Coherence distance: same scenario.

$$\Delta\phi \approx 2\pi f\left(\frac{2d-r}{c} - \frac{r}{c}\right) = \frac{2\pi f}{c}(2d-2r) = \frac{4\pi f}{c}(d-r)$$

prop to change in $d-r$

~~changing~~ in r

what change in r will result in phase shift of $\frac{\pi}{2}$

$$\frac{4\pi f}{c}\cdot x \approx \frac{\pi}{2} \Rightarrow x \approx \frac{\frac{\pi}{2}}{4\pi\frac{f}{c}} = \frac{c}{f\cdot 8} \approx \frac{\lambda}{8}.$$

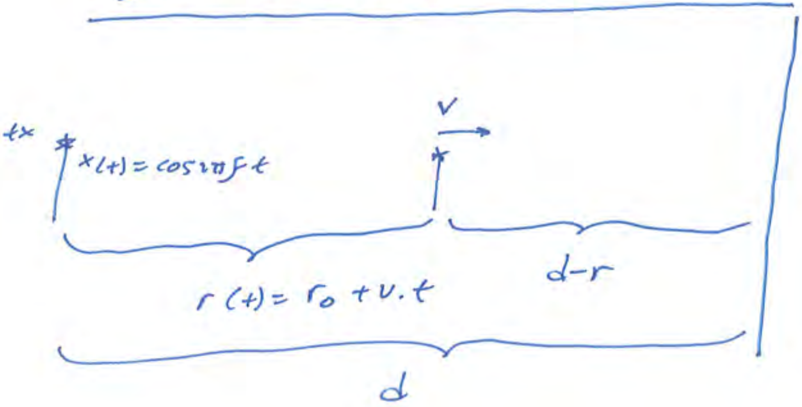
$\Rightarrow$ coherence distance $\Delta x_c \approx \frac{\lambda}{8}$

(if I set "substantial phase shift" $\Delta\phi \approx \frac{4\pi f}{c}\overbrace{(d-r)}^{x} \approx \pi$)

$\Rightarrow x = \frac{c}{f}\frac{1}{4} = \frac{\lambda}{4} = \Delta x_c$

$\Rightarrow \Delta x_c \approx \frac{\lambda}{8} \to \frac{\lambda}{4}$ (depending on convention).

Reflecting wall, moving Antenna.



- Channel strength changes as you move through constructive & destr intf. ("multipath fading").

distance 1

$$E_r(f,t) = \frac{\alpha}{r_1(t)} \cos 2\pi f\left[t - \frac{r_1(t)}{c}\right] + \frac{\alpha}{r_2(t)} \cos\left\{2\pi f\left[t - \frac{2d - r(t)}{c}\right] + \pi\right\}.$$

because reflect

$$= \frac{\alpha}{r_0+vt}\cos 2\pi f\left[t - \frac{vt}{c} - \underbrace{\frac{r_0}{c}}_{\varphi_1}\right] + \frac{\alpha}{2d-r_0-vt}\cdot\cos\left\{2\pi f\left[t - \frac{(2d-r_0-v.t)}{c}\right]+\pi\right\}.$$

π (taken here)

$$= \frac{\alpha}{r_0+v.t}\cdot\cos 2\pi f\left[t\left(1-\frac{v}{c}\right) - \underbrace{\frac{r_0}{c}}_{\varphi_1}\right] - \frac{\alpha}{2d-r_0-v.t}\cdot\cos 2\pi f\left[t\left(1+\frac{v}{c}\right) - \underbrace{\frac{2d-r_0}{c}}_{\varphi_2}\right]$$

Now assume (just for simplicity) that we are close to the wall

$\Rightarrow \quad \Rightarrow r(t) \approx d \;\Rightarrow\; r(t) \approx 2d - r(t) \approx d$

$$\Rightarrow E_r(f,t) \approx \frac{\alpha}{r(t)}\left[\cos \underbrace{2\pi f\left[t\left(1-\frac{v}{c}\right) - \varphi_1\right]}_{A} - \cos \underbrace{2\pi f\left[t\left(1+\frac{v}{c}\right) - \varphi_2\right]}_{B}\right]$$

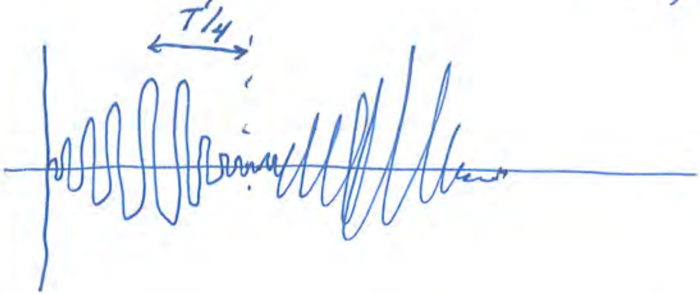
Recall $\cos A - \cos B = -2 \sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$

$$\frac{A+B}{2} = 2\pi f\left[\frac{1}{2}\left[t\left(1-\frac{v}{c}\right) - \frac{r_0}{c} + t\left(1+\frac{v}{c}\right) - \frac{2d-r_0}{c}\right]\right] = 2\pi f\left[t - \frac{d}{c}\right].$$

$$\frac{A-B}{2} = 2\pi f\left[\frac{1}{2}\left[\cancel{t} - t\frac{v}{c} - \frac{r_0}{c} - \cancel{t} - t\frac{v}{c} + \frac{2d-r_0}{c}\right]\right] = 2\pi f\left[-\frac{t.v}{c} + \frac{d-r_0}{c}\right]$$

$$E_r(f,t) = \frac{2\alpha}{d} \cdot \underbrace{\sin\left[2\pi f\left(t\frac{v}{c} - \frac{d-r_0}{c}\right)\right]}_{\substack{\text{very slow} \\ \text{freq } f\frac{v}{c}}} \cdot \underbrace{\sin 2\pi f\left[t - \frac{d}{c}\right]}_{\text{very fast (freq } f)}$$

$$E_r(f,t) \propto 2\alpha \cdot \sin 2\pi f\left(\frac{v}{c}t - \frac{r_0}{c}\right) \sin 2\pi f\left[t - \frac{d}{c}\right].$$



$f' = f \cdot \frac{v}{c} = \frac{1}{T'}$ $\qquad T' = \frac{1}{f'}$

period of slow sinusoid.
" " major ups & downs

Another way to see

Coher distance $\approx \frac{\lambda}{4} = \Delta t_c$

can call it distance because next to it we have ✓

Coherence period: $\frac{T'}{4} = \frac{1}{4f'} = \frac{1}{4} \cdot \frac{c}{f \cdot v} = \left(\frac{1}{4}\lambda\right) \cdot \frac{1}{v}$. (recall $c = f \cdot \lambda$).

Generally note $\frac{\lambda}{4} \ll r$. (in process, it will change many times).

Also write Δx_c & T_c in terms of Doppler Spread (prelude)

DEF Doppler Spread here $D_s \triangleq D_2 - D_1 = f\frac{v}{c} - \left(-f\frac{v}{c}\right) = \frac{2fv}{c} = D_s$

$$T_c = \frac{T'}{4} = \frac{1}{4}\frac{c}{fv} = \frac{1}{2}\frac{c}{2fv} = \boxed{\frac{1}{2 \cdot D_s} \approx T_c}$$

$$v = \frac{\Delta x_c}{T_c} \qquad \Delta x_c = T_c \cdot v = \frac{v}{2D_s}.$$

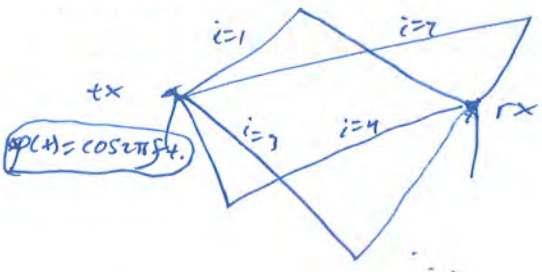
Example: $v = 60\,\mathrm{Km/h}$ $f = 1\,\mathrm{GHz}$ (typical freq).

$$v = \frac{60 \cdot 10^3\,\mathrm{m}}{3600\,\mathrm{s}} = \frac{50}{3}\,\mathrm{m\,s^{-1}} \Rightarrow D_s \approx 2 \cdot f \cdot \frac{v}{c} = \frac{2 \cdot 10^9 \cdot 50}{3 \cdot 3 \cdot 10^8} \approx \frac{2 \cdot 5 \cdot 100}{9} \approx 110\,\mathrm{Hz}.$$

$$\Rightarrow T_c \approx \frac{1}{2D_s} = \frac{1}{2 \cdot 110}\,\mathrm{s} \approx 5\,\mathrm{mS}\ ; \quad \Delta x_c \approx v \cdot T_c = \frac{50}{3} \cdot 5 \cdot 10^{-3} \approx 75\,\mathrm{ms} \approx \text{few c}$$

Input - output Model.

Consider many paths (many reflectors).



different paths i (many ...).

Input $x(t) = \cos 2\pi f t$

Recall our attenuation examples before:

$$\boxed{y(t) = \sum_i a_i(f,t) \cdot x(t - \tau_i(f,t))}.$$

probagation delays $\tau_i(f,t)$: how long it takes for each path to travel from start.

Path attenuations $a_i(f,t)$.

Note that $\tau_i(f,t)$ & $a_i(f,t)$ are functions of both f, t.

but recall

Bandwidth is small compared to $f_c = 1\,GHz$ (central frequency).
So all frequencies that we use are (in relative terms) very close to $f_c = 1\,GHz$. (BW typically 1 MHz or so)

$\Rightarrow f \in [0.999\,GHz \to 1.001\,GHz] \approx 1\,GHz$

We can nicely
$\Rightarrow$ Assume τ_i & a_i are indep of f

$\Rightarrow \tau_i(t)\ ,\ a_i(t)$

$$\Rightarrow \boxed{y(t) = \sum_i a_i(t) \cdot x(t - \tau_i(t))}.$$

(Be careful: channel response still a function of F).

Just like in previous examples.

Example: Perfectly reflecting wall, with movement.

Recall : $E_r(f,t) = \frac{\alpha}{r_1(t)} \cos 2\pi f\left(t - \tau_1(t)\right) + \frac{\alpha}{r_2(t)} \cos 2\pi f\left(t - \tau_2(t) - \pi\right).$

$$= \overbrace{\frac{\alpha}{r_0 + v(t)}}^{a_1(t)} \cdot \cos 2\pi f\left(t - \underbrace{\frac{(vt + r_0)}{c}}_{t_1(t)}\right) + \overbrace{\frac{\alpha}{2d - r_0 - vt}}^{a_2(t)} \cos 2\pi f\left(t - \underbrace{\frac{2d - r_0 - v}{c}}_{\tau_2(t)} \ldots\right)$$

Recall: we have accepted linearity.

$\Rightarrow$ I/O relationship via convolution.

$y(t) = h(t,\tau) * x(t)$: For $h(t,\tau)$ a T.V. channel impulse response.

$$y(t) = \int_{-\infty}^{\infty} h(t,\tau) \cdot \cancel{x}(t-\tau)\, d\tau$$

but also (from before).

$$y(t) = \sum_i a_i(t) \cdot \cancel{x}(\cancel{t} - \tau_i(t))$$

Put two together.

$$\Rightarrow h(t,\tau) = \sum_i a_i(t) \cdot \delta(\tau - \tau_i(t)).$$

$$\left(\text{verify}: y(t) = \int_{-\infty}^{\infty} h(t,\tau)\, x(t-\tau)\, d\tau = \int_{-\infty}^{\infty} \sum_i a_i(t) \cdot \delta(\tau - \tau_i(t)) \cdot x(t-\tau)\, d\tau \right.$$

$$\left. = \sum_i a_i(t) \cdot \underbrace{\int_{-\infty}^{\infty} \delta(\tau - \tau_i(t))\, x(t-\tau)\, d\tau}_{x(t-\tau_i(t))} = \sum_i a_i(t) \cdot x(t - \tau_i(t)) \right).$$

$\Rightarrow$ LTV impulse response of channel. $\boxed{y(t) = h(t,\tau) * x(t).}$

$x(t) \longrightarrow \boxed{h(t,\tau)} \longrightarrow y(t)$ (we have seen)
channel

Our task now will be to extend the concept of this "channel" to include important processes of communication.

Let us thus sketch this process

Communication process in a glimpse: From #'s to #'s.

message "meet me at 4pm" $= m_1$

$\rightarrow$ $m_1 \rightarrow 1101110100 \quad = b_1$

$b_1 \rightarrow (1-i,\ 1+i,\ 1-i,\ -1+i,\ 1+i) = [x[1]\ x[2] \ldots x[5]] = \underline{x}$

$Re\{\underline{x}\} = [1\ -1\ 1\ -1\ 1]$, $Im[\underline{x}] = [-1\ 1\ -1\ 1\ 1]$

This requires infinite bandwidth

$Re\,x[1] = 1 \quad Re[x[2]] = -1$

$\Rightarrow \infty$ BW (because FT of step is sinc)

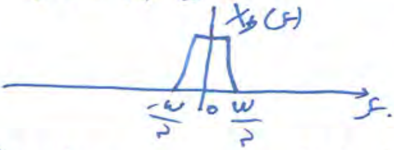
$\Rightarrow$ need to smoothen signals.

$\Rightarrow$ modulate with sequence of smooth sync functions

$\underline{x} * \{sinc\{Wt - n\}\}_n$

to give $x_b(t)$ that has finite BW

D.A conv.

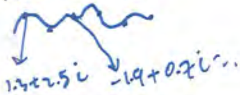


- High Frequency modulation

Low frequency signals suffer from rapid attenuation (easily absorbed by walls, etc).

$\Rightarrow$ $x_b(t) \longrightarrow \otimes \longrightarrow x(t)$, $\approx \cos 2\pi f_c t$, $f_c \approx 1-2\,GHz$

- then demodulate (mult by $\approx \cos 2\pi f_c t$) $\Rightarrow$ then sample



- then sample

meet me at 4pm

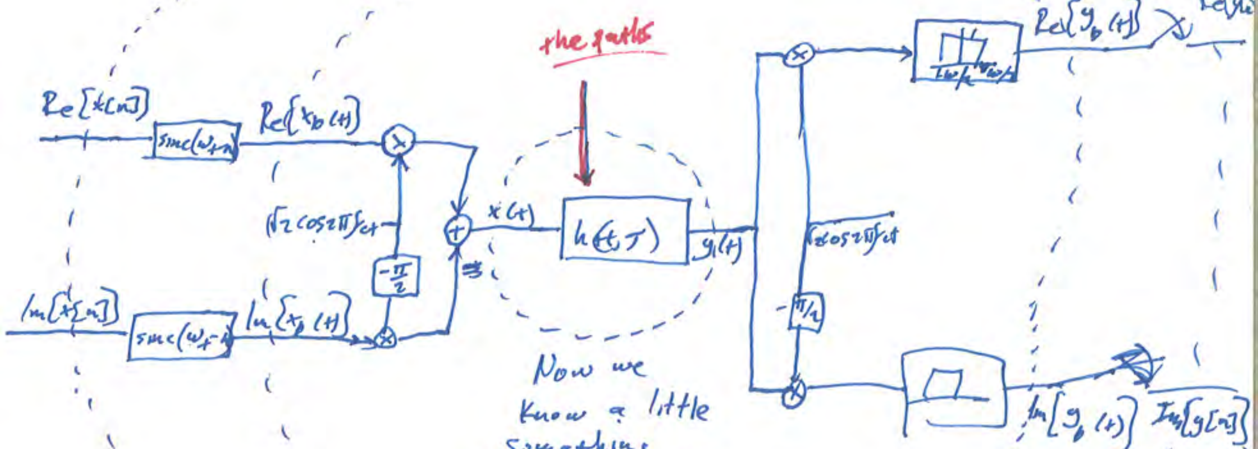
↓

11 01 11 01 00

↓

$[x[1] \; x[2] \dots x[B]] = \underline{x}$

$Re\{\underline{x}\}$, $Im[\underline{x}]$



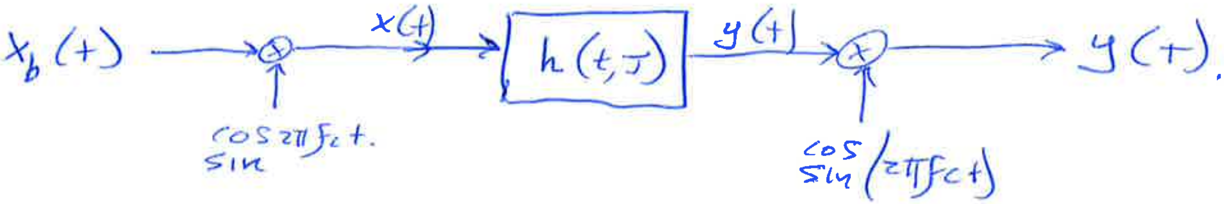
Now we know a little something about this relationship.

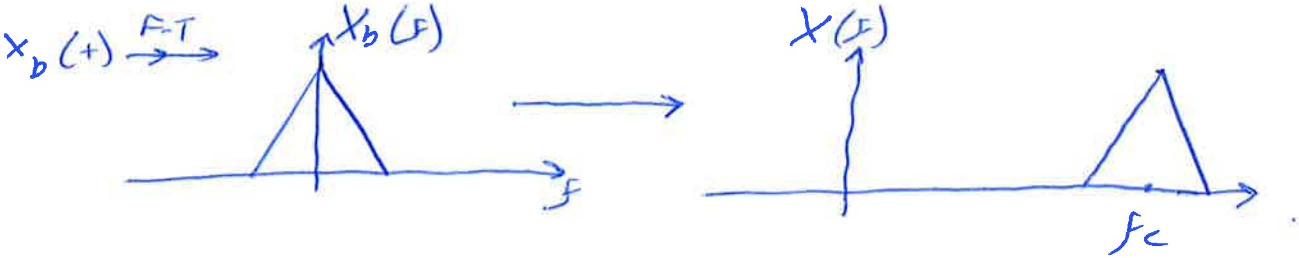
then: "continuous time baseband I/O"

then "discrete-time baseband I/O"

then add noise.

Establish relationship between <u>Baseband</u> input & output



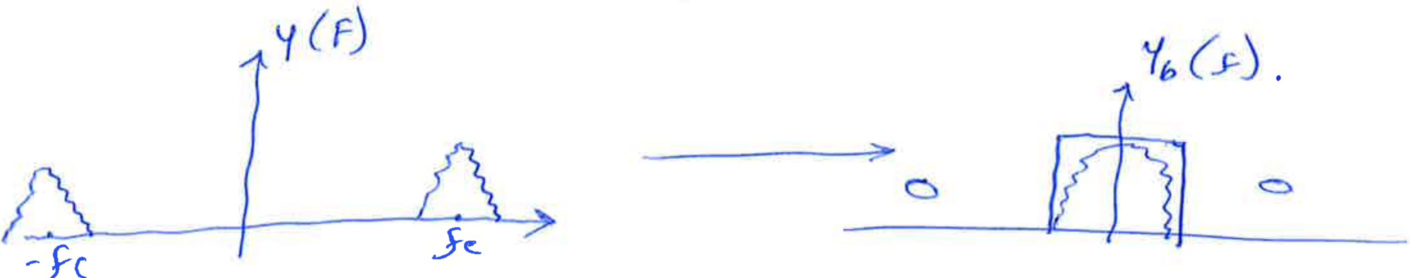


in time domain

$$x(t) = Re\{ x_b(t) \cdot e^{j2\pi f_c \cdot t} \}$$

Similarly $y(t) = Re\{ y_b(t) \cdot e^{j2\pi f_c t} \}$.

From reverse action of demodulation



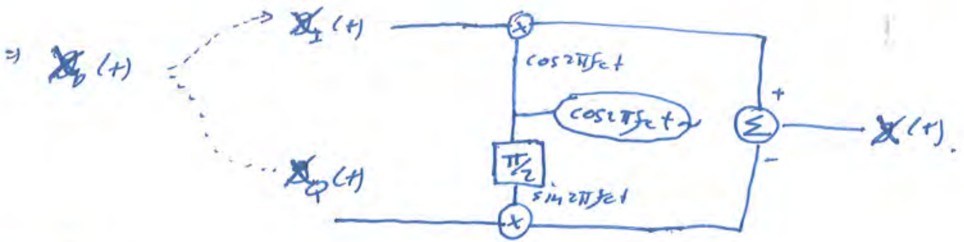
<u>In practice</u>: how is this achieved?

$$x(t) = \mathrm{Re}\left\{ \underbrace{x_b(t)}_{\substack{\uparrow \\ \text{complex.}}} \cdot e^{j2\pi f_c t} \right\}$$

$\Rightarrow$ write $x_b(t) = x_I(t) + j x_Q(t)$ & recall $e^{j2\pi f_c t} = \cos 2\pi f_c t + j \sin 2\pi f_c t$

$$\Rightarrow x(t) = \mathrm{Re}\left\{ x_I(t)\left(\cos 2\pi f_c t + j \sin 2\pi f_c t\right) + j x_Q(t)\left(\cos 2\pi f_c t + j \sin 2\pi f_c t\right) \right\}.$$

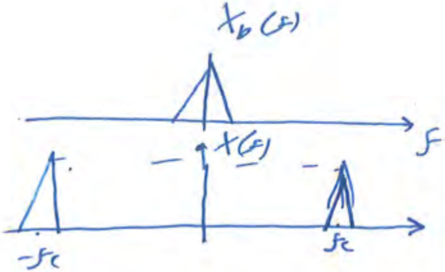
$$\Rightarrow \boxed{x(t) = x_I(t) \cdot \cos 2\pi f_c t - x_Q(t) \sin 2\pi f_c t} \quad (***)$$



Recap

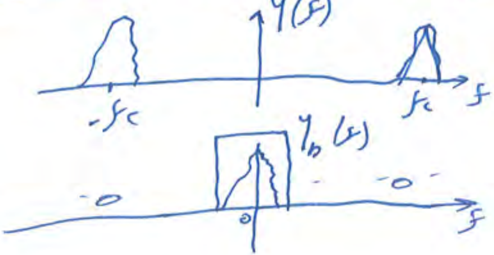
⇒ For our signals. $x_b(t)$ & $x(t)$

⇒
$$\boxed{\begin{aligned} x(t) &= \mathrm{Re}\left\{ x_b(t)\, e^{j2\pi f_c t} \right\} \\ &= x_I(t)\cos 2\pi f_c t - x_Q(t)\cos 2\pi f_c t \end{aligned}}$$



similarly $y(t) = \mathrm{Re}\left\{ y_b\, e^{j2\pi f_c t} \right\}$

From the reverse action of demodulation



Now ready to establish relationship

$$x_b(t) \longrightarrow \boxed{h_b(t,\tau)} \longrightarrow y_b(t)$$

Recall:

$$y(t) = \sum_i a_i(t)\cdot x(t-\tau_i(t))$$

$a_i(t)$ path attenuation
$\tau_i(t)$ " delay.

(*4) $$= \mathrm{Re}\left\{ y_b(t)\, e^{j2\pi f_c t} \right\}, \quad x(t) = \mathrm{Re}\left\{ x_b(t)\, e^{j2\pi f_c t} \right\}.$$ (from before).

$$\Rightarrow \mathrm{Re}\left\{ y_b(t)\, \cancel{e^{j2\pi f_c t}} \right\} = \sum_i a_i(t)\, \mathrm{Re}\left\{ x_b(t-\tau_i(t)).\, e^{j2\pi f_c (t-\tau_i(t))} \right\} = \mathrm{Re}\left\{ \sum_i \underbrace{a_i(t)}_{(\text{real})} x_b(t-\tau_i(t))\, \cancel{e^{j2\pi f_c t}}\, e^{-j2\pi f_c \tau_i(t)} \right\}$$

$$\Rightarrow \mathrm{Re}\left\{ y_b(t) \right\} = \mathrm{Re}\left\{ \sum_i a_i(t)\, x_b(t-\tau_i(t))\, e^{-j2\pi f_c \tau_i(t)} \right\}.$$

(easy exercise to show)

Similarly

$$\mathrm{Im}\left(y_b(t)\, \cancel{e^{j2\pi f_c t}} \right) = \mathrm{Im}\left\{ \sum_i a_i(t)\, x_b(t-\tau_i(t)).\, \cancel{e^{j2\pi f_c t}} \cdot e^{-j2\pi f_c \tau_i(t)} \right\}.$$

$$\Rightarrow \boxed{y_b(t) = \sum_i a_i(t)\, e^{-j2\pi f_c .\tau_i(t)} \cdot x_b(t-\tau_i(t))}$$ **4

$$\Rightarrow y_b(t) = \sum_i a_i^b(t)\, x(t-\tau_i(t))$$

$$\boxed{\text{where } a_i^b(t) = \underbrace{a_i(t)}_{\text{slow}}.\, e^{-j2\pi f_c \underbrace{\tau_i(t)}_{\text{fast}}}}$$

Recall: linearity still holds since all we did is multiply (linear function).

$$\Rightarrow y_b(t) = \int_{-\infty}^{\infty} h_b(\tau, t)\, x_b(t-\tau)\, d\tau \overset{**}{\Longrightarrow} \boxed{h_b(\tau, t) = \sum_i \underbrace{a_i^b(t)}_{} \delta(\tau - \tau_i(t))}$$

$$a_i(t) \cdot e^{-j2\pi f_c \cdot \tau_i(t)}$$

$a_i(t)$: slow (minutes); $\tau_i(t)$: fast

↓ rate of change define rate of change of $a_i^b(t)$ & thus of $h(\tau, t)$.

Note: here there's a notion of having one path.

LTV complex continuous-time baseband. I/O channel representation.

- Say not so big change in $a_i^b(t)$ (coherence in $a_i^b(t)$) corresponding to ~~shift~~ phase-shift of $\pi/2$

$$2\pi f_c \cdot \tau_i(t) \approx \frac{\pi}{2} \Rightarrow \tau_i \approx \frac{1}{4 f_c}$$

$\frac{\pi}{2}$ change in phase when delay of path changes by $\approx \frac{1}{4 f_c}$

at speed of light $\Rightarrow \Delta X_c \approx \frac{\lambda}{4}$ (recall $c = \lambda \cdot f$). (make note that $f \approx f_c$)

$$\Rightarrow T_c = \frac{\Delta X_c}{V} = \frac{\lambda}{4v} \approx \frac{c}{4 \cdot f_c \cdot V} \approx \frac{1}{4 D_s}$$

(recall doppler spread $D_s \approx \frac{f \cdot v}{c}$).

$\Rightarrow h_b(t,\tau)$ changes $\Delta X_c \approx \frac{\lambda}{4}$ $\left(\frac{\lambda}{4} \approx \frac{1}{4}\frac{c}{f} \approx \frac{1}{4}\frac{3\cdot 10^8\,m s^{-1}}{10^9\,s^{-1}} \approx 7\text{ cm (few cm)}\right)$.

& at reasonable ↑ speeds (say ≈ 60 km/s).

$$\frac{c}{4\cdot f_c \cdot V} \approx \frac{3\cdot 10^8}{4\cdot 10^9 \cdot \frac{60\cdot 10^3}{3600}} = \frac{3\cdot 3600}{40\cdot 60\cdot 10^3} = \frac{9}{2000} \approx 4{-}5\ \mu S$$

(few μS).

- Interesting observation on frequency response of $h_b(t,\tau)$.
- $T_c \gg$ memory of channel $\approx T_d$ (a bit tricky).

by definition of impulse response. $\delta(t) \longrightarrow \boxed{h_b(t,\tau)} \longrightarrow y_b(t) = h_b(t,\tau)$

- send an impulse though channel
- All signals will be collected (from all paths) in the order of what we will call

"delay spread" $= T_d \triangleq \max\limits_{i,j} |\tau_i - \tau_j|$ $\quad \tau_i = \frac{d_i}{c}$, $\tau_j = \frac{d_j}{c}$ distances

- In cellular nets (mainly urban), typical distance-differences

$d_i - d_j \approx$ few hundred meters

$\Rightarrow \tau_i - \tau_j = \frac{d_i - d_j}{c} \approx \frac{\text{few hundred m}}{3 \cdot 10^8 \text{ m s}^{-1}} \approx$ few μS $(\approx 1 \to 10 \cdot 10^{-6} s)$.

$\Rightarrow$ "memory of channel" $h_b(t, \tau) \approx$ few $\mu S. = T_d$

But $T_d \ll T_c \approx$ few mS.

$\Rightarrow$

(but clarify what is meant by this).

$\Rightarrow$ can consider channel impulse response to be almost time-invariant
(since it is only after many impulses that the channel will change)