Mobile Communication Techniques

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Final Exam

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Time: 09:00-11:00

Instructions

- Exercises fall in categories of 1-point, 2-point and 4-point exercises.
- Total of $10 \times 1 + 6 \times 2 + 2 \times 4 = 30$ points.
- Each answer should be clearly written, and the solution should be developed in detail.
- Mathematical derivations need to show all steps that lead to the answer.
- Complete as many exercises as you can. Don't spend too much time on an individual question.
- Partial credit will be given for incomplete solutions.
- There is NO penalty for incorrect solutions.
- If in certain cases you are unable to provide rigorous mathematical proofs, go ahead and provide intuitive justification of your answers. Partial credit will be given.
- Calculators are allowed. Feel free to provide close form expressions *in their simplest form.*
- Open book and open class notes are allowed. No other notes are allowed.

Hints - equations - conventions:

- Notation
  - R represents the rate of communication in bits per channel use (b.p.c.u),
  - ρ represents the SNR (signal to noise ratio),
  - w will denote additive noise which will be distributed as a circularly symmetric Gaussian random variable $\mathbb{C}\mathcal{N}(0, N_0)$. If N_0 is not specified, then set $N_0 = 1$,
  - h_i will denote *independent* fading scalar coefficients which will be distributed as circularly symmetric Gaussian random variables $\mathbb{C}\mathcal{N}(0, 1)$
- Useful equations
  - For small ϵ, and for the statistical distributions of interest, the following holds:

$$P(|h_i|^2 < \epsilon) = \int_0^{\epsilon} p(|h_i|^2) d|h_i|^2 \approx \max_{[0,\epsilon]} \Big\{ p(|h_i|^2 = \epsilon) \Big\}$$

  - For small ϵ, and for the statistical distributions of interest, the following holds:

$$P(|h_i|^2 + |h_j|^2 < \epsilon) \approx p\Big(\{|h_i|^2 < \epsilon\} \text{ or } \{|h_j|^2 < \epsilon\} \Big)$$

  - For a given probability of error $P(\text{error})$, the *diversity order* or *diversity gain* is defined as

$$d := -\lim_{\rho \to \infty} \frac{\log P(\text{error})}{\log \rho}.$$

- The *multiplexing gain* is defined as $r := R / \log \rho$
- The diversity multiplexing tradeoff (DMT) is defined as a diversity gain achieved, given a specific multiplexing gain
- GOOD LUCK!!

EXAM PROBLEMS

1) *(1 point).* What is the maximum and minimum wavelengths corresponding to a 25MHz signal band centered around 5.8 GHz?

2) *(1 point).* Consider a transmitter and a receiver which receives from different paths.
   - What is the maximum length difference between any two paths so that the delay spread is approximately $3\mu s$?

3) *(1 point).* At 1.9 GHz, a *narrowband* channel induces approximately how big of a coherence distance?

4) *(1 point).* What is the capacity of the AWGN channel when using power equal to the average power of 4-QAM, over noise $\sim \mathbb{C}\mathcal{N}(0,2)$?

5) *(1 point).* Define mathematically the concept of deep fade.

6) *(1 point).* What are some ways that allow for the negative effect of deep fade be reduced?

7) *(1 point).* What is the average probability of error (averaged over the fading realizations), in a $1 \times n_r$ SIMO Rayleigh fading channel (high snr approximation)?

8) *(1 point).* Describe the rate of the Alamouti code carrying QPSK elements.

9) *(1 point).* How many degrees of freedom can you have in a
   - SISO channel
   - $n_t \times 1$ MISO channel
   - $n_t \times n_r$ MIMO channel

10) *(1 point).* Describe the relation between the cardinality of the code $|\mathcal{X}|$, the rate of transmission R and the coding duration T

11) *(2 points).* Consider the general noiseless MIMO channel model

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \underbrace{[\underline{h}_1 \ \underline{h}_2 \ \underline{h}_3]}_{H} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}_{\underline{x}}$$

- What is a necessary condition on H such that decoding of all the components of $\underline{x}$ is always possible?
- What is a necessary condition on H such that decoding of any of the components of $\underline{x}$ is always possible?

12) *(2 points).* Consider the space-time (ST) code that takes the form

$$\begin{bmatrix} x_0 & x_1 & x_2 \\ x_4 & x_0 & x_2 \\ x_1 & x_3 & x_5 \end{bmatrix}$$

where the x_i are drawn independently from a 16-QAM constellation.
- What is the rate of the code, in bits per channel use (bpcu)?

13) *(2 points).* Consider the following two channel models:

$$y = h_1 x + w, \qquad \text{and} \qquad y = h_1 h_2 h_3 x + w$$

where $h_1, h_2, h_3 \sim$ i.i.d $\mathbb{C}\mathcal{N}(0,1)$.
- What is the diversity order
$$d_{\text{out}} = -\lim_{\rho\to\infty} \frac{\log P(\text{outage})}{\log \rho}$$
for the two cases? (here you can simply state the answer and the explanation - no derivation necessary)
- What is the time-diversity corresponding to channel
$$[y_1 \quad y_2 \quad y_3] = [h_1 h_2 x_1 \quad h_1 h_2 h_3 x_2 \quad h_1 h_3 x_3] + [w_1 \quad w_2 \quad w_3]$$
for x_i that optimize diversity

14) *(2 points).* Consider a space time code that takes the form

$$\begin{bmatrix} x_0 & x_1 & x_2 \\ x_4 & x_5 & x_6 \\ x_7 & x_8 & x_9 \end{bmatrix}$$

where the x_i are drawn independently from a QAM constellation. Describe the optimal diversity gain $-\lim_{\rho\to\infty} \frac{\log P(\text{error})}{\log \rho}$ provided by the code over the
- 3×1 MISO channel $\underline{h} = [h_1 \quad h_2 \quad h_3]$
- 3×5 MIMO channel.

15) *(2 points).* Let C_1 be the capacity of the AWGN channel with SNR $\rho_1 = e^{3n}$, and let C_2 be the capacity of the AWGN channel with SNR $\rho_2 = e^{-5n}$. Let $n >> 1$.
- What is the ratio $\frac{C_1}{C_2}$. (NOTE!: the final answer should NOT include logarithms!!)

16) *(2 points).* Consider the MISO 3×1 channel

$$y[m] = h_1 x_1[m] + h_2 x_2[m] + h_3 x_3[m] + w[m]$$

where the h_i coefficients are constant at $h_1 = 1, h_2 = 0.5, h_3 = 0.25$.

- What is the optimal encoding-decoding policy, and what channel state information is required at the receiver and at the transmitter, so that the corresponding capacity is achieved?
- What is the corresponding capacity?

17) *(4 points).* For the 1×3 SIMO Rayleigh fading channel

$$\underline{y} = \underline{h}x + \underline{w}$$

calculate

$$\lim_{\rho \to \infty} \frac{\log P(\text{outage})}{\log \rho}$$

as a function of the *multiplexing gain*

$$r := R / \log \rho.$$

18) *(4 points).* Consider the quasi-static Rayleigh fading channel. Let $\text{SNR} = 10^{-2}$ and $\epsilon = 10^{-3}$.

- What is the approximate ϵ-outage capacity C_ϵ for SISO?
- Show the gains in ϵ-outage capacity, brought about by adding an extra receive antenna.