Mobile Communication Techniques

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Midterm Exam

November 21 - 2024

Time: 9:00-10:00

Instructions

- Exercises fall in categories of 1-point and 2-point exercises.
- Total of $11 \times 1 + 2 \times 2 = 15$ points.
- NOTE!!! The exam will be evaluated, out of 13 points. Any points you get beyond 13 points, will be offered as extra bonus.

- Each answer should be clearly written, and the solution should be developed in detail.
- Mathematical derivations need to show all steps that lead to the answer.
- Complete as many exercises as you can. Don't spend too much time on an individual question.
- There is NO penalty for incorrect solutions.
- If in certain cases you are unable to provide rigorous mathematical proofs, go ahead and provide intuitive justification of your answers. Partial credit will be given.
- Calculators are not allowed.
- You are allowed your class notes and class book.

Hints - equations - conventions:

- Notation
  - SISO = single-input single-output, MISO = multiple-input single-output, SIMO = single-input multiple-output, MIMO = single-input multiple-output,
  - R represents the rate of communication in bits per channel use (b.p.c.u),
  - ρ represents the SNR (signal to noise ratio),
  - w will denote additive noise which will be distributed as a circularly symmetric Gaussian random variable $\mathbb{C}\mathcal{N}(0, N_0)$. If N_0 is not specified, then set $N_0 = 1$,
  - h_i will denote *independent* fading scalar coefficients which will be distributed as circularly symmetric Gaussian random variables $\mathbb{C}\mathcal{N}(0, 1)$
- GOOD LUCK!!

MIDTERM EXAM PROBLEMS

1) *(1 point).* In a multi-path fading scenario with delay spread $6\mu s$ and $L = 3$ channel taps, what is the operational bandwidth W?

2) *(1 point).* Imagine a given SNR equal to ρ, and imagine that we are operating over a (quasi-static) Rayleigh fading SISO channel. Can you describe a code that achieves probability of error approximately equal to $P_e \approx \rho^{-4}$, and rate equal to $R = 2$ bpcu.

3) *(1 point).* How much time diversity will we get with the following SISO (time-diversity) channel model

$$[y_1 \quad y_2 \quad y_3] = [h_1 u_1 \quad h_2(u_1 + u_2) \quad h_3 u_2] + [w_1 \quad w_2 \quad w_3]$$

where the u_1, u_2, u_3 are independent PAM elements. Justify your answer.

4) *(1 point).* In a SISO case, what is the degrees of freedom (DOF) if we have a time-diversity code (spanning three channel uses) of the form

$$\mathcal{X} = \{[u_1 + u_2 \quad u_1 + u_3 \quad u_2 + u_3]\}$$

where the u_1, u_2, u_3 are independent 16-PAM elements?

5) *(1 point).* For the case of time diversity in the SISO (quasi-static) fading channel, what is the advantage and the disadvantage of the repetition code, compared to uncoded transmission.

6) *(1 point).* In a SISO case, what is the DOF and the rate (in bpcu), of the following time-diversity code (three channel uses) that takes the form

$$\mathcal{X} = \{[u_1 + u_4 \quad u_2 \quad u_1 + u_2 + u_3]\}$$

where the u_1, u_2, u_3, u_4 are independent 64-QAM elements?

7) *(1 point).* Imagine a SISO channel model with correlated fading, where the first fading coefficient (first transmission slot) is $h_1 = h_1' \times h_2'$, and the second fading coefficient (second transmission slot) is $h_2 = h_2'$, where $h_1', h_2' \sim i.i.d\ \mathbb{CN}(0, 1)$. What is the maximum diversity we can achieve here?

8) *(1 point).* Describe the steps of converting a binary vector detection problem over a time diversity fading channel, into a scalar detection problem. Imagine that you are sending BPSK symbols using a repetition code, and consider $\mathbb{CN}(0, N_0)$ noise.

9) *(1 point).* Consider a deep-space communications scenario, where the received SNR is equal to 20dB. If you assume low rate communications, what do you expect the probability of error to be?

10) *(1 point).* What is the approximate coherence time T_c in a typical urban wireless network if you are driving approximately 20 kilometers per hour?

11) *(1 point).* Consider communication over a SISO fading channel with a delay spread of $T_d = 3\mu$s and a signal bandwidth of $W = 1$ MHz.
    - Write all the received signals, if we only send $x[0]$ and then we stop transmitting.

12) *(2 points).* What is the optimal diversity order over a 2×1 MISO channel $\mathbf{h} = [h_1 \quad h_2]$, $h_i \sim$ i.i.d $\mathbb{CN}(0, 1)$?

- In the same channel as above (again with no time diversity), consider a space time code whose matrices take the form
$$\begin{bmatrix} x_0 & x_1 \\ x_1 & x_0 \end{bmatrix}$$
where the x_i are drawn independently from a QAM constellation. Will this code achieve optimal diversity order? (argue why or why not)
- What is the diversity order achieved by the Alamouti code, over this 2×1 MISO channel? (again, you can just argue in words)

13) *(EXTRA CREDIT: 2 points).* Consider a setting where the transmit antenna array has length of 50 cm, the received antenna array has size 20cm, the transmission frequency is 1000 MHz, the signal bandwidth is 1 MHz, the channel coherence time is $T_c = 21$ ms, and the coding duration is $T_{coding} = 7$ms.
    - How much diversity can you get, in total?