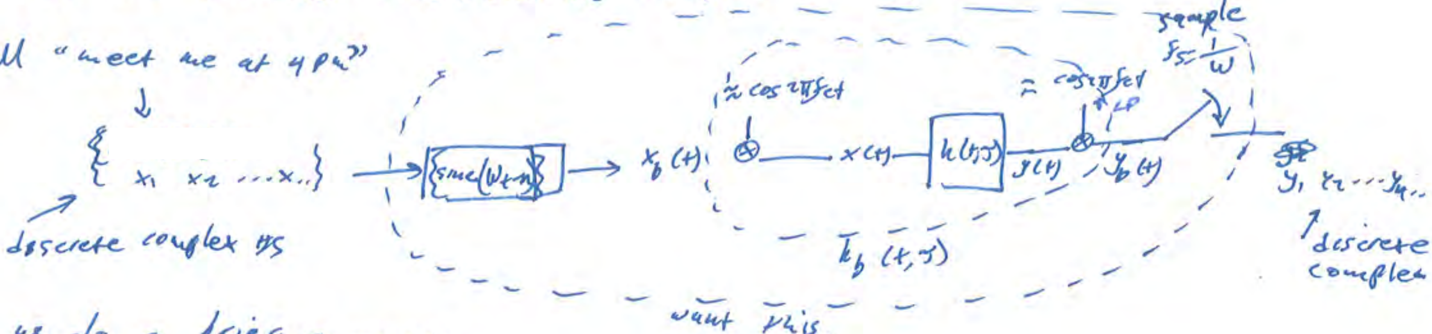
Lecture 2

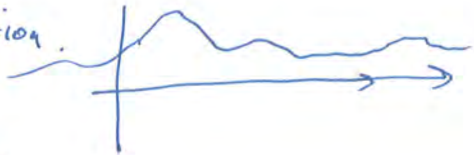
Let us now understand the input/output relationship at the discrete-time baseband complex domain.

Recall "meet me at 4pm"



Let us do a brief summary.

Consider g(t) function.



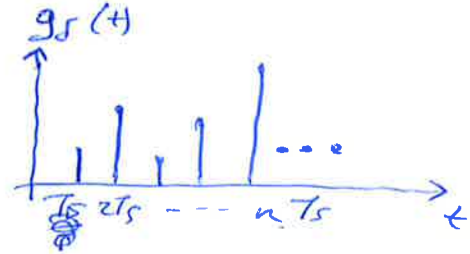
Consider sampling every T_s seconds (sampling rate $\frac{1}{T_s} = f_s$)

to get sequence $\{g(nT_s)\}$ $n \in \mathbb{Z}$. (integers).

to get $g_s(t) = \sum_{n=-\infty}^{\infty} g(nT_s) \cdot \delta(t - nT_s)$.

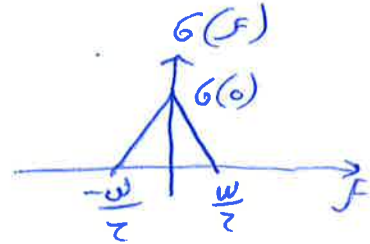
Recall that for such "comb" functions, there are two ways to write their F.T.

$$g_\delta(t) = \sum_{n=-\infty}^{\infty} g(n.T_s).\,\delta(t - n.T_s)$$



- Recall $G(f) \triangleq FT(g(t))$ is <u>band limited</u>.

$$G(f) = 0 \quad \forall f : |f| > \frac{W}{2}$$



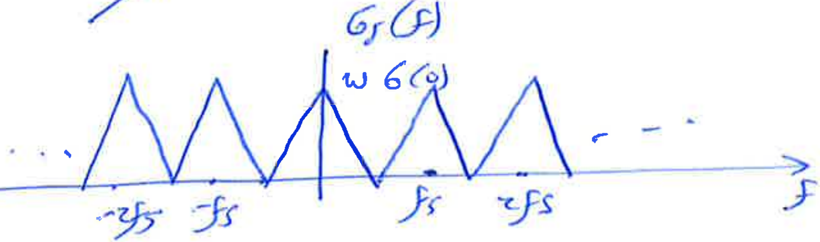
- Recall two ways of writing F.T. of combed function.

1.> $$G_\delta(f) \triangleq FT(g_\delta(t)) = f_s.\sum_{n=-\infty}^{\infty} G(f - n.f_s) \qquad *$$

2.> $$= \sum_{n=-\infty}^{\infty} \underbrace{g(n.T_s)}_{\text{discrete Fourier coeffs}}.\, e^{-j2\pi f.n.T_s} \qquad (**)$$

From (*) $G_\delta(f) = f_s G(f) + \cancel{\sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty} f_s G(f - m f_s)} \rightarrow 0 \rightarrow$ because band limited

Let $T_s = \frac{1}{W}$ Nyquist.



non-intersecting.

$$\Rightarrow G(f) = G_\delta(f) \cdot \frac{1}{W} \qquad f \in \left[-\frac{W}{2}, \frac{W}{2}\right]$$

$$\overset{(**)}{\Longrightarrow} G(f) = \frac{1}{W} \sum_{n=-\infty}^{\infty} g\left(\frac{n}{W}\right) \cdot e^{-j2\pi n f/W}, \qquad f \in \left[-\frac{W}{2}, \frac{W}{2}\right]$$

$\Rightarrow \left\{ g(nT_s) \right\}_{n=-\infty}^{\infty}$ completely define $G(f)$ $\Rightarrow$ $\Downarrow$ $g(t)$

Let us now see how to go from $\left\{ g\left(\frac{n}{W}\right) \right\}_{n=-\infty}^{\infty} \to g(t)$.

$$g(t) = \text{IFT}\{G(f)\} = \int_{-\infty}^{\infty} G(f)\, e^{j2\pi f t}\, df$$

(because recall that our signal is band limited) $\rightarrow$:

$$= \int_{-\frac{W}{2}}^{\frac{W}{2}} \frac{1}{W} \cdot \sum_{n=-\infty}^{\infty} g\left(\frac{n}{W}\right) e^{-2j\pi n f/W} \cdot e^{2\pi j f t}\, df$$

$$= \sum_{n=-\infty}^{\infty} g\left(\frac{n}{W}\right) \frac{1}{W} \underbrace{\int_{-\frac{W}{2}}^{\frac{W}{2}} e^{2\pi j f\left(t - \frac{n}{W}\right)}\, df}_{\text{easy to calculate ; break into } e^{jx} = \underbrace{\cos x}_{\text{odd}} + j \underbrace{\sin x}_{\text{even}}}$$

$$\Rightarrow g(t) = \sum_{n=-\infty}^{\infty} g\left(\frac{n}{W}\right) \cdot \frac{\sin(\pi W t - n\pi)}{\pi W t - n\pi}$$

$$\Rightarrow g(t) = \sum_{n=-\infty}^{\infty} g\left(\frac{n}{W}\right) \cdot \text{sinc}(Wt - n) \quad -\infty < t < \infty$$ (interpolation formula).

THE ABOVE (SUMMARY) RELATES TO US BECAUSE WHAT WE WILL DO IS: $\left\{ g\left(\frac{n}{W}\right) \right\} \leftrightarrow g(t)$.

$$\{z_n\}_{n=-\infty}^{\infty} \xrightarrow{\sum_{n=-\infty}^{\infty} z_n \cdot \text{sinc}(Wt-n)} g(t) \xrightarrow[\text{at } t = \frac{n}{W}]{\text{sampling}} z_n = g\left(\frac{n}{W}\right).$$

$G(f) = 0 \;\; \forall |f| > \frac{W}{2}$

can recover original message.

- A band limited signal, $\left[-\frac{W}{2} \to \frac{W}{2}\right]$ is completely described by $\left\{ g\left(\frac{n}{W}\right) \right\}_{n=-\infty}^{\infty}$ and may be recovered from these samples.

We adopt engineering approach that considers signals of finite bandwidth $W^{(Hz)}$, and finite duration T. (seconds).

Keep in mind. Strictly speaking this is physically & mathematically impossible.

since either T or W (or both) must be ∞.

But in engineering we work with thresholds.

Say $x_b(t)$ has duration T $(t=0 \to T)$ i.e, $x_b(t) = 0$ $\forall t<0, \forall t>T$.

then we say that $BW = W$ if $\sigma(f) \leq \epsilon$ $\forall |f| > \frac{W}{2}$ for some ϵ of choice.

Such signal $x_b(t)$ is then "fully" represented by $\{ \underbrace{x_b(\frac{n}{W})}_{x[n]} \}_{n=1}^{W.T} = \{ x[n] \}_{n=1}^{W.T}$

as $$x_b(t) = \sum_{n=1}^{W.T} x[n] . \operatorname{sinc}(Wt-n)$$

$$= \sum_{n=1}^{WT} x_b(\tfrac{n}{W}) \operatorname{sinc}(Wt-n).$$

- This is how you generate a Wt-dimensional signal $x_b(t)$
- We say $x_b(t)$ has WT degrees of freedom.
- Wt - dimensions. W Hz, T seconds.

1 dimension /s/Hz.

$\Rightarrow \dim\{ x_b(t) \} = W.T = \dim\{x(t)\} \approx \dim\{y(t)\}$.

the latter is because $W + D_s \approx W$ ($W \approx 1$ MHz, $D_s \approx 50 \to 100$ Hz).

Let us now figure out the I/O relationship

Recall $y_b(t) = \sum_i a_i^b(t) \cdot x_b(t - \tau_i(t))$. (& recall $x_b(t) = \sum_{n} x_b\left(\frac{n}{W}\right) \cdot \mathrm{sinc}(Wt-n)$)

$$= \sum_i a_i^b(t) \sum_n x_b\left(\frac{n}{W}\right) \mathrm{sinc}\left(W(t-\tau_i(t)) - n\right)$$

$$y_b(t) = \sum_n x[n] \sum_i a_i^b(t) \,\mathrm{sinc}\left(Wt - \tau_i(t)W - n\right)$$

Now sample output at $t = \frac{m}{W}$

$$\Rightarrow y_b\left[\frac{m}{W}\right] = \sum_n x[n] . \sum_i a_i^b\left(\frac{m}{W}\right) \mathrm{sinc}\left(\cancel{W} \cdot \frac{m}{\cancel{W}} - n - \tau_i\left(\frac{m}{W}\right)W\right)$$

Let $\ell = m - n$ i.e. $n = m - \ell$.

$$\Rightarrow y[m] = y_b\left(\frac{m}{W}\right) = \sum_\ell x[m-\ell] . \sum_i a_i^b\left(\frac{m}{W}\right) \mathrm{sinc}\left(\ell - W \tau_i\left(\frac{m}{W}\right)\right)$$

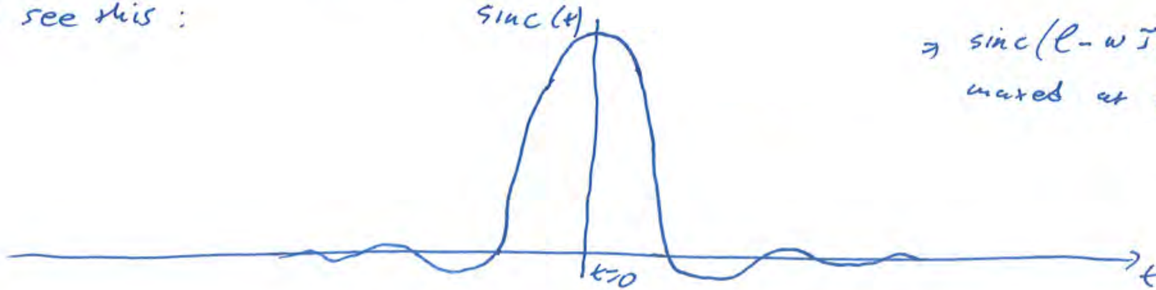
$$\Rightarrow y[m] = \sum_\ell x[m-\ell] . h_\ell[m] \quad \text{where } h_\ell[m] = \sum_i a_i^b\left[\frac{m}{W}\right] \mathrm{sinc}\left(\ell - W \cdot \tau_i\left(\frac{m}{W}\right)\right).$$

$h_\ell[m]$ is referred to as the ℓ^{th} complex channel filter tap (at time $t = \frac{m}{W}$) (we will call it, the ℓ^{th} tap).

$$\boxed{h_\ell[m] = \sum_i a_i\left(\frac{m}{W}\right) e^{-j2\pi f_c \tau_i\left(\frac{m}{W}\right)} \mathrm{sinc}\left(\ell - W \cdot \tau_i\left(\frac{m}{W}\right)\right)}$$

Intuition: at time $t = \frac{m}{W}$ (m discrete time slot), $h_\ell[m]$ mainly consists of the gains $a_i^b(t)$ from paths of delay $\tau_i(\frac{m}{W}) \approx \frac{\ell}{W}$

To see this:



$\Rightarrow \operatorname{sinc}(\ell - W\tau_i(\frac{m}{W}))$
maxed at $\ell - W\tau_i(\frac{m}{W}) \approx 0$
$\tau_i(\frac{m}{W}) \approx \frac{\ell}{W}$

- In reality engineers consider finite summation $y[m] = \sum_{\ell=0}^{L-1} h_\ell[m]\, x[m-\ell]$.

what is L "# of important taps"?

$L = \dfrac{\text{memory of channel}}{\text{duration of channel use}}$:

channel use: Same at rate W samples/second $\Rightarrow$ $T_s \approx \frac{1}{W}$.

time slot

$\Rightarrow L = \dfrac{T_d}{T_s}$.

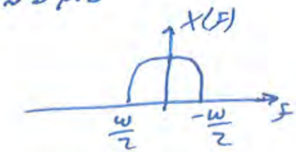
Example:

Assume a standard cellular (urban) setting, where there are many paths i.

Among the important (strong) paths,

$$\max_{i,j} |d_i - d_j| \approx 1\,km$$

$$\Rightarrow T_d = \max_{i,j} |\tau_i - \tau_j| = \max_{i,j} \left| \frac{d_i}{c} - \frac{d_j}{c} \right| \approx \frac{10^3\, m}{3 \cdot 10^8\, m s^{-1}} \approx 3\,\mu S$$



Assume a signal bandwidth of $W = 1\,MHz$

$\Rightarrow$ sampling period of $T_s = \frac{1}{W} = 1\,\mu S$.

This means that you sample every $T_s \approx 1\,\mu S$.

This defines the duration of a $\underbrace{\text{(discrete) time-slot}}$ to be $T_s \approx 1\,\mu S$

This means that the delay spread T_d is approximately 3 time-slots.

This means that a signal that is sent at $t=0$ will be received at the rx, from different paths, at different times, but the great majority of the energy originating from $t=0$, will be received before $t = T_d = 3\,\mu S$.

→ If you send $x[m]$ (at time $t = \frac{m}{W}$, or discrete time m)

h_0 $y[m]$ will get part of $x[m]$ from the short paths that will carry the signal there before $m+1$ ie $t: \frac{m}{W} \to \frac{m+1}{W}$

h_1 $y[m+1]$ will get part of $x[m]$ from other paths (of a bit longer length), that will carry $x[m]$ to rx between

$t = \frac{m+1}{W} \to \frac{m+2}{W}$

h_2 $y[m+2]$ will get part of $x[m]$ from other (longer paths) that will deliver $x[m]$ to rx between $t = \frac{m+2}{W} \to t = \frac{m+3}{W}$

$y[m+3]$ will not rx much from $x[m]$

FOR NOW

assume only $x[m]$ is sent.

$0\ 0\ 0\ 0\ \underbrace{x[m]}_{t=\frac{m}{W}}\ 0\ 0\ 0\ 0$

$y[m+3] \approx 0$

$h_3[m+3] \approx 0.$

$y[m] = h_0[m]\, x[m]$ $y[m+1] = h_1[m+1]\, x[m]$ $y[m+2] = h_2[m+2] \cdot x[m].$ $L = 3$

Another point of view. – Example

$$\begin{array}{cccc} m=0 & 1 & 2 & 3 \\ \hline t=\frac{0}{W}, & \frac{1}{W}, & \frac{2}{W}, & \frac{3}{W} \end{array}$$

Assume tx of $x[0]$ $x[1]$ $x[2]$

and consider reception at $t=\frac{3}{W}$ $(m=3)$.

i.e consider $y(t=\frac{3}{W})$ i.e consider $y[m=3]$. $(y[3])$.

$\Rightarrow$ As before $W \approx 1\,MHz$ $\Rightarrow$ $T_s \approx 1\mu S$ & $T_d \approx 3\mu S$ $\Rightarrow$ $\boxed{L = \frac{T_d}{T_s} \approx 3}$

$\Rightarrow$ $y[3]$ does not see $x[0]$ because (as in previous example) the signals that left at $t=\frac{0}{W}$ $(m=0)$ were delivered to the rx at time interval $t \in [\frac{0}{W}, \frac{L}{W}] = [\frac{0}{W}, \frac{3}{W}]$

(note $\frac{0}{W}$ $\frac{1}{W}$ $2/W$ $3/W$; h_0 h_1 h_2 h_3 ; $\underline{\underline{h_3 = 0.}}$

Also assume that taps stay fixed over duration of transmission (which is a pretty good assumption as we will see soon).

$m=0$: $y[0] = \underset{\sim}{h_0} \cdot x[0]$ direct, as before.

$m=1$ $y[1] = \underbrace{h_0 \cdot x[1]}_{\text{direct}} + \underbrace{h_1 \cdot x[0]}_{\text{with 1 delay}}$

$m=2$ $y[2] = h_0\, x[2] + h_1 \cdot x[2-1] + h_2\, x[2-2]$

$= \underbrace{h_0\, x[2]}_{\text{direct}} + \underbrace{h_1\, x[1]}_{\text{delay 1}} + \underbrace{h_2\, x[0]}_{\text{delay 2.}}$

$L = 3 \left(\frac{T_d}{T_s} \approx 2 \right)$

$m=3$

$y[3] = \underbrace{h_0\, x[3-0]}_{0} + h_1\, x[3-1] + h_2\, x[3-2] + \overbrace{\cancel{h_3}\, x[3-3]}^{0}$

$= h_1\, x[2] + h_2\, x[1]$

$y[4] = h_2\, x[4-2] = h_2\, x[2].$

$y[5] = 0.$

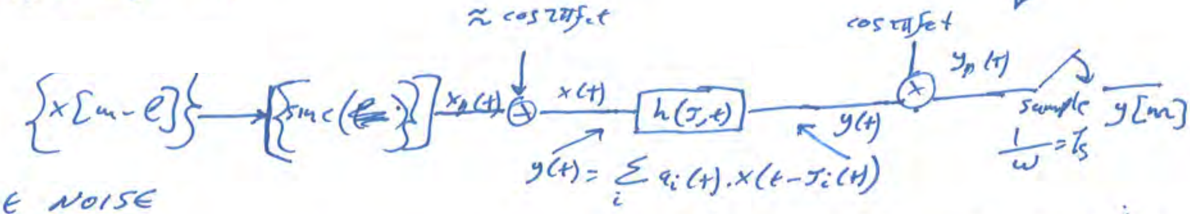
In summary, we have a relationship (which starts off from $y(t) = \sum_i a_i(t)\, x(t - \tau_i(t))$ (*)

$$y[m] = \sum_{\ell=0}^{L-1} h_\ell[m]\; x[m-\ell]$$

$$h_\ell[m] = \sum_i a_i^b\left[\tfrac{m}{W}\right] \cdot \operatorname{sinc}\left(\ell - W \cdot \tau_i\left(\tfrac{m}{W}\right)\right).$$

Corresponding to

here we also have low pass filters.



ADDITIVE NOISE

Before we try to get a complete picture, we must consider effect of additive noise.

Instead of (*), we actually have

$$y(t) = \sum_i a_i(t)\, x(t - \tau_i(t)) + w(t)$$

where generally, $w(t)$ is nicely approximated to be a Gaussian process.

Going back through the steps of

- demodulation
-
- low pass filtering
- sampling.

we get $y[m] = \sum_{\ell} h_\ell[m]\, x[m-\ell] + w[m]$.

where ... essentially (we will skip details).

$$w[m] = \int_{-\infty}^{\infty} w(t)\, \psi_m(t)\, dt$$

where $\{\psi_m(t)\}_m$ is a set of orthogonal functions (i.e., an orthogonal basis).

Due to property that projections of standard Gaussian random vectors ($w(t)$) onto $\perp$ dimensions preserve the distributions we conclude that $w[m]$ is also Gaussian (iid) random variables (also circularly symmetric).

$$w[m] \sim \mathcal{CN}(0, N_0).$$

Time & Frequency coherence.

How $h_\ell[m]$ changes?

Let us go back to understanding how the channel changes (in time (also space), & frequency). recall $y[m] = \sum_{\ell=0}^{L-1} h_\ell[m]\, x[m-\ell] + \cancel{w[m]}$

Recall

$$h_\ell[m] = \sum_i \underbrace{a_i(t)}_{\text{slow}}\, e^{\overbrace{-j2\pi f_c \cdot \tau_i\left(\frac{m}{W}\right)}^{2^{\text{nd}}\text{ term (phase)}}}\ \underbrace{\text{sync}\left(\ell - W\tau_i\left(\frac{m}{W}\right)\right)}_{3^{\text{rd}}\text{ term.}}.$$

(recall: this is ℓ^{th}: tap, mainly corresponding to paths i with delay $\tau_i\left(\frac{m}{W}\right) \approx \frac{\ell}{W}$.

To track changes of entire summation, we focus on a single path i.

- Changes in $a_i\left(\frac{m}{W}\right)$ occur in order of seconds
  - e.g due to car moving slowly away from BS, resulting in path-loss.

- <u>2nd term</u>: Substantial changes in $e^{-j2\pi f_c \tau_i\left(\frac{m}{\omega}\right)}$ correspond to phase changes $\approx \frac{\pi}{2}$.

Have done this before

$$2\pi f_c \tau_i\left(\tfrac{m}{\omega}\right) \approx \frac{\pi}{2} \Rightarrow \text{path } \overset{\text{duration}}{\text{change}} \text{ s.t. } \delta\tau_i\left(\tfrac{m}{\omega}\right) \approx \frac{1}{4f_c}$$

& path length change $\approx c \cdot \delta\tau_i\left(\frac{m}{\omega}\right) \approx \frac{c}{4f_c} = \frac{\lambda}{4}$

$$\left(\text{say } \frac{3.10^8 \, m\,s^{-1}}{4.10^9 \, s^{-1}} \approx \cancel{6}\text{-}10 \text{ cm}\right)$$

- How long does it take for the "car" to cover this distance (i.e for rx & tx to move so that this distance change happens.
(still focus on second term)

$$\approx \frac{\lambda}{4v} = \frac{c}{4f_c v}$$

recall $\frac{c}{f_c v} \approx D_i$ (doppler shift) $\Rightarrow \Delta T_c \approx \frac{1}{4D_s}$

$$\approx D_s \triangleq \max_{i,j} |D_i - D_j|$$

(note: in some books, they simply say $\Delta T_c \propto \frac{1}{D_s}$.

(say: (continuing from above $\Delta X_c \approx \cancel{6}\text{-}10$ cm $\approx \cancel{6}.10^{-2}$ m)

say car at $V \approx 100$ Km/h $= \frac{100.10^3 \, m}{3600 \, s} \approx 30 \, m s^{-1} \Rightarrow \frac{30 \, m}{6.10^{-2} \, m} \quad \frac{1 \, s}{T_c?}$

$\Rightarrow T_c = \frac{6.10^{-2}}{30} = 2.10^{-3} \approx 2$ ms (few ms).

This corresponds to Doppler spread of $D_i \approx f_c V \approx \frac{1}{\Delta T_c}$ $\approx$ few hundred Hz.

Third term

$$h_\ell[m] = \sum_i \underbrace{a_i(t)}_{\text{few second}} \; \underbrace{e^{-j2\pi f_c \cdot \tau_i\left(\frac{m}{W}\right)}}_{\text{few ms.}} \cdot \underbrace{\text{sinc}\left(\ell - W\cdot \tau_i\left(\frac{m}{W}\right)\right)}_{\text{third term}}$$

Let us compare speed of change of 2nd & 3rd term

Note $\text{sinc}\left(\ell - W\tau_i\left(\frac{m}{W}\right)\right) \propto \sin\left(\ell - W\tau_i\left(\frac{m}{W}\right)\right) \propto e^{-j\left(\ell - W\tau_i\left(\frac{m}{W}\right)\right)} \propto e^{jW\tau_i\left(\frac{m}{W}\right)}$.

So compare $e^{-j2\pi f_c \cdot \tau_i\left(\frac{m}{W}\right)}$ v.s. $e^{jW\cdot \tau_i\left(\frac{m}{W}\right)}$

how fast the terms change, with a change of τ_i (due to, let's say movement).

Clearly the first term changes much faster since $\underset{\text{GHz}}{f_c} \gg \underset{\text{MHz}}{W}$.

⇒ Second term $e^{-j2\pi f_c \tau_i(t)}$ dominates rate of change of $h_\ell[m]$.
" " " " of channel.

⇒

$$\boxed{\begin{array}{l} \text{Coherence distance } \Delta X_c \approx \dfrac{\lambda}{4} \approx \text{few cm} \\ \text{Coherence period } \Delta T_c \approx \dfrac{\Delta X_c}{V} \approx \dfrac{\lambda}{4V} \approx \dfrac{c}{4 f_c \cdot V} \approx \dfrac{1}{4 D_s} \propto \dfrac{1}{D_s} \quad (\text{few ms}) \end{array}}$$

↓ this corresponds to $y[m] = \sum_{\ell=0}^{L-1} h_\ell[m]\, x[m-\ell] + w[m]$

What is typical # of taps L? (have seen a bit of that before).

turns out, these is related to questions.

What is coherence BW?

Recall

$$y(t) = \sum_i a_i(t) \cdot x(t - \tau_i(t)) \quad \& \quad y(t) = \int_{-\infty}^{\infty} x(t-\tau) h(t,\tau)\, d\tau$$

$$\Rightarrow h(t,\tau) = \sum_i a_i(t) \underbrace{\delta(\tau - \tau_i(t))}_{F.T.} \quad (\text{recall that } h(t,\tau) \approx h_t(\tau)).$$

$$\Rightarrow H_t(f) = \sum_i a_i(t)\, e^{-j2\pi f \cdot \tau_i(t)}$$

Pick two paths: $i \& j$

phase difference of the form $2\pi f\,(\tau_i(t) - \tau_j(t))$

set this $\approx \frac{\pi}{2}$ $\Rightarrow 2\pi f\,|\tau_i(t) - \tau_j(t)| \approx \frac{\pi}{2} \Rightarrow f \approx \frac{1}{4|\tau_i - \tau_j|}$

$\Rightarrow$ if only those two paths were there, then $\frac{1}{4|\tau_i - \tau_j|}$

but other important paths could give a higher $\tau_i - \tau_j \Rightarrow$ smaller f

$$\Rightarrow W_c \approx \frac{1}{4 \max_{i,j} |\tau_i - \tau_j|} \propto \frac{1}{T_d}. \quad (**)$$

$\Rightarrow$ Coherence BW $\propto \frac{1}{T_d}$.

- Relationship between W, W_c, T_d, L.

Recall $y[m] = \sum_{\ell=0}^{L-1} h_\ell[m]\, x[m-\ell] + w[m]$. but what is L?

Recall "memory" of channel $\approx T_d$ seconds $\approx \frac{T_d}{T_s}$ channel uses (timeslots)

$\Rightarrow$

$\approx T_d \cdot W$ time slots

$\Rightarrow L \approx T_d \cdot W$ from (**).

Also $L \approx \frac{W}{W_c}$ (from 3 lines above).

- What is typical L?

Typical $T_d = \max_{i,j} |\tau_i - \tau_j| \approx \frac{300\,m}{c}$ (cell size $\approx$ 1-2 km).

$\Rightarrow$ few $\mu s \approx$

$T_s \approx 1\,\mu s$ ($W \approx 1$ MHz) $\Rightarrow L: 2 \rightarrow 5$ typically nicely represent entire chan

- $W_c \approx \frac{W}{L}$ (few hundred kHz $\rightarrow$ 1 MHz or so).

- We now know that we can focus on $\{h_\ell[m]\}_{\ell=0}^{L-1}$ to describe I/O discrete, baseband.

$$y[m] = \sum_{\ell=0}^{L-1} h_\ell[m]\, x[m-\ell] + w[m]$$

$$h_\ell[m] = \sum_i a_i(t)\, e^{-j2\pi f_c \cdot \tau_i\left(\frac{m}{W}\right)} \operatorname{sinc}\left(\ell - W\tau_i\left(\frac{m}{W}\right)\right).$$

- We know that channel taps change every ΔT_c secs (few ms)
- We know that $L \approx$ few taps — $h_0\ h_1\ h_2\ h_3\ h_4$; $h_5\ h_6 \rightarrow$ small

ΔX_c m (few cm)

W_c Hz ($\approx$ hundred kHz $\rightarrow$ MHz)

- But how do they change?
- How often are they big & how often small?

Statistical characterization of channel (i.e. of channel taps).

Good news. There are many paths, $\Rightarrow$ law of averages helps simplify analysis.

One very well known statistical channel model:

Rayleigh fading.

- Large # of stat. indep. paths that "land" (that contribute) in a tap.
- Uniform phase of the i^{th} path $2\pi f \tau_i$ (modulo 2π) helps to calculate summation statistics.
  - Consider a race between me & Usain Bold (in seconds)

(in looking only at the last digit of hundredth seconds)

Usain Bolt	9.58	9.61	....
Me	12.53	12.96	

(modulo $0.01 \ll 9.58$)
(" $2\pi \ll 2\pi f \tau_i$).

at the end we will have "won" an equal # of times.

Same as above since $f \cdot \tau_i \approx \frac{d_i}{c} \gg 2\pi$

This "phase uniformity" allows us to model each phase's contribution

$$a_i\left(\frac{m}{w}\right) e^{-j2\pi f_c \tau_i\left(\frac{m}{w}\right)} \cdot \operatorname{sinc}\left[\ell - w\tau_i\left(\frac{m}{w}\right)\right]$$

as a circularly symmetric, complex random variable
(from uniformity).

- Each tap is the sum of a large # of such random variables

$\Rightarrow$ $\mathrm{Re}[h_\ell[m]]$ can be modeled (from CLT) as a zero mean Gaussian RV.

Similarly $\mathrm{Re}[h_\ell[m] \cdot e^{j\varphi}] \sim$ Gaussian $(\forall \varphi)$.

$\Rightarrow h_e[m] \sim \mathcal{CN}(0, \sigma_e^2)$

$\Rightarrow |h_e[m]| \sim$ Rayleigh

$|h_e[m]|^2 \sim$ Exponential $\frac{1}{\sigma_e^2} e^{-|h_e[m]|^2/\sigma_e^2}$

- Model good for environments with many small reflectors.

> There are many other models.

<u>Rician channel model</u> $h_e[m] = \sqrt{\frac{K}{K+1}} \sigma_e \cdot e^{j\theta} + \frac{1}{\sqrt{K+1}} \mathcal{CN}(0, \sigma_e^2).$

> Other models developed right here.