

LECTURE 1.



Communication (Simplistic view).

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Transmit $x(t) = \cos(2\pi f t)$

f is frequency. $\approx 1 \rightarrow 4$ GHz.
 t is time.

$$R_x: E(f, t, r, \theta, \varphi) = \frac{1}{r} \cdot \alpha_s(\theta, \varphi, f) \cdot \cos\left(2\pi f\left(t - \frac{r}{c}\right)\right) \quad (1)$$

α : antenna losses

\uparrow delay due to signal.

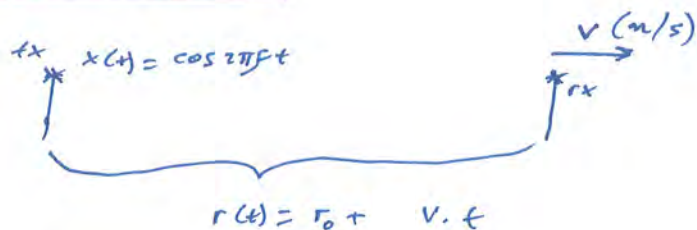
Intuition why $E \propto \frac{1}{r}$

- Tx in 3-D space.

- Power preserved in surface of sphere: (radius r , center is antenna)

\Rightarrow Fixed $\frac{\text{Total Power}}{\text{Area}} \Rightarrow R_x \text{ power} \propto \frac{1}{4\pi r^2} \propto \frac{1}{r^2}$.

Consider movement.



$$\Rightarrow E(f, t, r(t)) = \frac{A_s}{r(t)} \cdot \cos 2\pi f \left(t - \frac{r(t)}{c} \right) \quad \text{like before : } r \rightarrow r(t).$$

$$= \frac{A_s}{r_0 + v \cdot t} \cdot \cos 2\pi f \left(t - \frac{r_0}{c} - \frac{v \cdot t}{c} \right)$$

$$= \frac{A_s}{r_0 + v \cdot t} \cdot \cos 2\pi f \left[t \left(1 - \frac{v}{c} \right) - \frac{r_0}{c} \right]$$

$$\Rightarrow \text{effective frequency} \quad \text{change} \quad f t \rightarrow \underbrace{f \left(1 - \frac{v}{c} \right)}_{\dots} \cdot t \quad : f \rightarrow f \left(1 - \frac{v}{c} \right)$$

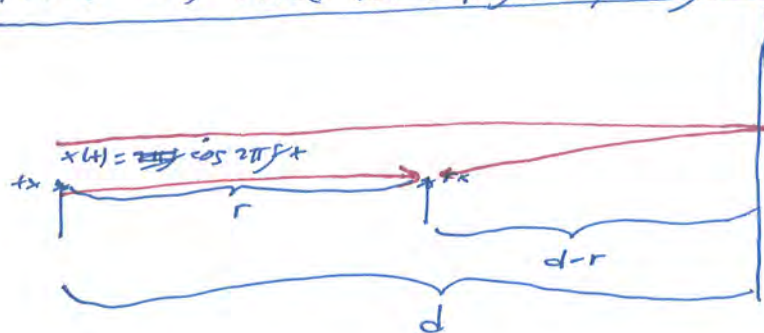
$$\Rightarrow \text{frequency reduction } f \rightarrow f \left(1 - \frac{v}{c} \right) \Rightarrow D = -\frac{fv}{c}$$

$$\text{DOPPLER shift : } D = -\frac{fv}{c}.$$

$$\Rightarrow E(f, t, r(t)) = \frac{A_s}{\underbrace{r_0 + v \cdot t}} \cdot \cos 2\pi f \left(t \left(1 - \frac{v}{c} \right) - \frac{r_0}{c} \right)$$

↑ note: not ΔT now.

Fixed Tx, Fixed Rx, perfectly reflecting wall.



Use method of "ray tracing": consider dominant (main) paths.

$$E_r(x, t) = \frac{a}{r} \cdot \cos 2\pi f \left(t - \frac{r}{c}\right) - \frac{a}{2d-r} \cos 2\pi f \left(t - \frac{2d-r}{c}\right)$$

r is distance 1. $2d-r$ = distance 2.

⇒ Superposition (addition of two sinusoids, with different phases).
possible effect of cancellation.

Phase ~~$\frac{2\pi f}{c}$~~ $\Delta \phi = 2\pi f \left(\frac{2d-r}{c} - \frac{r}{c} \right) + \pi - 2\pi f \frac{t}{c}$ $\left(\begin{array}{l} (\pi \text{ phase shift in electric field of EM wave} \\ \text{when reflected from optically denser medium}). \\ \text{(essentially canceling each other out close} \\ \text{to the wall)} \end{array} \right)$

Coherence bandwidth:

Change in frequency f , so that channel (i.e. magnitude of rx signal) remains relatively the same.

$$\Delta\phi = \frac{4\pi f}{c} (d-r) + \pi \Rightarrow \text{change } f \text{ so that phase shift changes } \approx \frac{\pi}{2}$$

$$\Rightarrow \frac{4\pi f}{c} (d-r) \approx \frac{\pi}{2} \quad \text{change by}$$

$$\Rightarrow 2\pi f \left(\frac{2d-r-r}{c} \right) \approx \frac{\pi}{2} \quad \leftarrow \text{rewrite to help us later.}$$

note: $\frac{2d-r-r}{c} \triangleq T_d$ is called delay spread (difference in delay of the paths).

$$\Rightarrow 2\pi f T_d \approx \frac{\pi}{2} \Rightarrow f \approx \frac{1}{4T_d} \Rightarrow W_c = \frac{1}{4T_d}$$

$$\left(\text{note: if set } \pi f \left(\frac{2d-r-r}{c} \right) \approx \pi \Rightarrow W_c = \frac{1}{2T_d} \right.$$

↑ substantial phase shift

the constant factor does not really matter) $\left(\frac{\pi}{2} \text{ or } \pi \right)$

in fact - wiki $W_c \approx \frac{1}{T_d}$.

$$\begin{array}{c} \downarrow \\ \frac{1}{4T_d} \quad \frac{1}{2T_d} \end{array}$$

(prelude)

Coherence distance: same scenario.

$$\Delta \phi \approx 2\pi f \left(\frac{2d-r}{c} - \frac{t}{c} \right) = \frac{2\pi f}{c} (2d-2r) = \frac{4\pi f}{c} (d-r)$$

changing ⁱⁿ ~~ing~~ r prop to change in $d-r$
what change in r will result in phase shift of $\frac{\pi}{2}$

$$\frac{4\pi f}{c} \cdot x \approx \frac{\pi}{2} \Rightarrow x \approx \frac{\frac{\pi}{2}}{\frac{4\pi f}{c}} = \frac{c}{f \cdot 8} \approx \frac{\lambda}{8}$$

\Rightarrow coherence distance $\Delta x_c \approx \frac{\lambda}{8}$

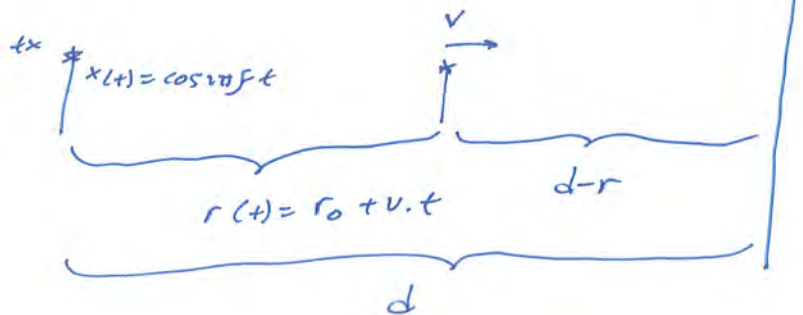
(if I set "substantial phase shift" $\Delta \phi \approx \frac{4\pi f}{c} (d-r) \approx \pi$)

$$\Rightarrow x = \frac{c}{f} \frac{1}{4} = \frac{\lambda}{4} = \Delta x_c$$

$$\Rightarrow \Delta x_c \approx \frac{\lambda}{8} \rightarrow \frac{\lambda}{4} \quad (\text{depending on convention}).$$

Reflecting wall, moving Antenna.

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- Channel strength changes as you move through constructive & destr intef. ("multipath fading").

$$E_r(f, t) = \frac{d}{r_1(t)} \cos 2\pi f \left[t - \frac{\text{distance}_1}{c} \right] + \frac{d}{r_2(t)} \cos 2\pi f \left[t - \frac{2d - r(t)}{c} + \pi \right].$$

$$= \frac{d}{r_0 + v \cdot t} \cos 2\pi f \left[t - \frac{v \cdot t}{c} - \frac{r_0}{c} \right] + \frac{d}{2d - r_0 - v \cdot t} \cdot \cos 2\pi f \left[t - \frac{(2d - r_0 - v \cdot t)}{c} + \pi \right].$$

because reflect

$$= \frac{d}{r_0 + v \cdot t} \cdot \cos 2\pi f \left[t \left(1 - \frac{v}{c} \right) - \frac{r_0}{c} \right] - \frac{d}{2d - r_0 - v \cdot t} \cdot \cos 2\pi f \left[t \left(1 + \frac{v}{c} \right) - \frac{2d - r_0}{c} \right]$$

(π taken here)

Now assume (just for simplicity) that we are close to the wall

\Rightarrow

$$\Rightarrow r(t) \approx d \Rightarrow r(t) \approx 2d - r(t) \approx d$$

$$\Rightarrow E_r(f, t) \approx \frac{d}{r(t)} \left[\underbrace{\cos 2\pi f \left[t \left(1 - \frac{v}{c} \right) - \varphi_1 \right]}_A - \underbrace{\cos 2\pi f \left[t \left(1 + \frac{v}{c} \right) - \varphi_2 \right]}_B \right]$$

Recall $\cos A - \cos B = -2 \sin \left(\frac{A+B}{2} \right) \sin \left(\frac{A-B}{2} \right)$

$$\frac{A+B}{2} = 2\pi f \left[\frac{1}{2} \left[t \left(1 - \frac{v}{c} \right) - \frac{r_0}{c} + t \left(1 + \frac{v}{c} \right) - \frac{2d - r_0}{c} \right] \right] = 2\pi f \left[t - \frac{d}{c} \right].$$

$$\frac{A-B}{2} = 2\pi f \left[\frac{1}{2} \left[\cancel{t} - t \frac{v}{c} - \frac{r_0}{c} - \cancel{t} + t \frac{v}{c} + \frac{2d - r_0}{c} \right] \right] = 2\pi f \left[-\frac{t \cdot v}{c} + \frac{d - r_0}{c} \right]$$

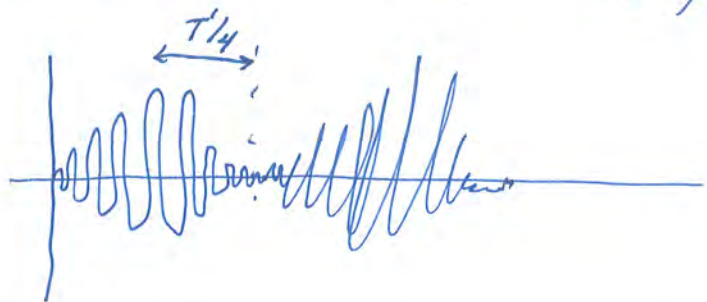
$$E_r(f, t) = \frac{2d}{d} \cdot \underbrace{\sin \left[2\pi f \left(t \frac{v}{c} - \frac{d - r_0}{c} \right) \right]}_{\text{very slow}} \cdot \underbrace{\sin \left[t - \frac{d}{c} \right]}_{\text{very fast (freq } f \text{)}}.$$

freq $f \frac{v}{c}$

⇒

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$$E_r(f, t) \propto \cos \cdot \sin 2\pi f \left(\frac{v}{c} t - \frac{d - r_0}{c} \right) \sin 2\pi f \left[t - \frac{d}{c} \right].$$



$$f' = f \cdot \frac{v}{c} = \frac{1}{T'} \quad T' = \frac{1}{f'}$$

period of slow sinusoid.
" " major ups & downs

Another way to see
Coker distance $\frac{d}{4} = \frac{v}{4} \Delta t$
can call it distance because next to it we have Δt

⇒ Coherence period: $\frac{T'}{4} = \frac{1}{4f'} = \frac{1}{4} \cdot \frac{c}{f \cdot v} = \frac{1}{4} \lambda \cdot \frac{1}{v}$ (recall $c = f \cdot \lambda$).

generally note $\frac{\lambda}{4} \ll r$. (in process, it will change many times).

↓ Also write Δf_c & T_c in terms of Doppler Spread

Doppler Spread here $D_s \triangleq f_2 - f_1 = f \frac{v}{c} - (-f \frac{v}{c}) = 2f \frac{v}{c} = D_s$

$$T_c = \frac{T}{4} = \frac{1}{4} \frac{c}{fv}$$

$$\frac{1}{2} \frac{c}{fv} = \boxed{\frac{1}{2 \cdot D_s} \approx T_c}$$

$$v = \frac{\Delta f_c}{T_c}$$

$$\Delta f_c = T_c \cdot v = \frac{v}{2 D_s}$$

Example: $v = 60 \text{ km/h}$ $f = 1.6 \text{ GHz}$ (typical freq).

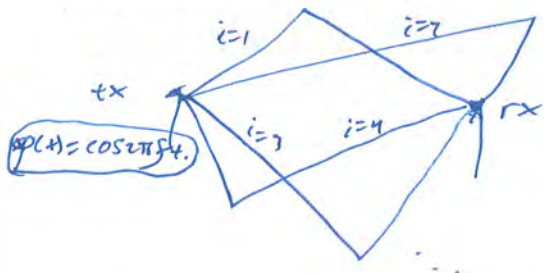
$$v = \frac{60 \cdot 10^3 \text{ m}}{3600 \text{ s}} = \frac{50}{3} \text{ m s}^{-1} \Rightarrow D_s \approx 2 \cdot f \cdot \frac{v}{c} = \frac{2 \cdot 10^9 \cdot 50}{3 \cdot 3 \cdot 10^8} \approx \frac{2 \cdot 5 \cdot 100}{9} \approx 110 \text{ Hz}$$

$$\Rightarrow T_c \approx \frac{1}{2 D_s} = \frac{1}{2 \cdot 110} \text{ s} \approx 5 \text{ ms} ; \Delta f_c \approx v \cdot T_c = \frac{50}{3} \cdot 5 \cdot 10^{-3} \approx 75 \text{ ^{cm} } \approx \text{few cm}$$

Input-output Model.

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Consider many paths (many reflectors).



different paths i (many...).

Input $x(t) = \cos 2\pi f_c t$

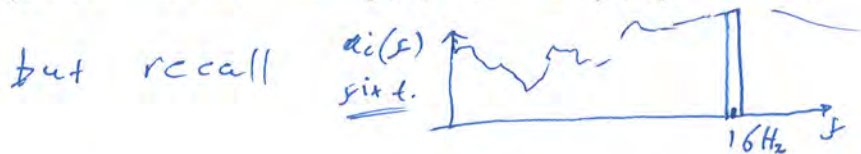
Recall our attenuation examples before:

$$y(t) = \sum_i a_i(f, t) \cdot x(t - \tau_i(f, t)).$$

propagation delays $\tau_i(f, t)$: how long it takes for each path to travel from $t \rightarrow t + \tau$.

path attenuations $a_i(f, t)$.

Note that $\tau_i(f, t)$ & $a_i(f, t)$ are functions of both f, t .



Bandwidth is small compared to $f_c = 16 \text{ Hz}$ (central frequency).

So all frequencies that we use are (in relative terms) very close to $f_c = 16 \text{ Hz}$. (BW typically 1 kHz or so)

$$\Rightarrow f \in [0.9996 \text{ Hz} \rightarrow 1.0016 \text{ Hz}] \sim 16 \text{ Hz}$$

We can nicely

\Rightarrow Assume τ_i & a_i are indep of f

$$\Rightarrow \tau_i(t), a_i(t)$$

$$\Rightarrow y(t) = \sum_i a_i(t) \cdot x(t - \tau_i(t))$$

Be careful: channel response still a function of f

just like in previous examples.

Example: Perfectly reflecting wall, with movement.

Recall : $E_r(f, t) = \frac{\alpha}{r_1(t)} \cos 2\pi f(t - \tau_1(t)) + \frac{\alpha}{r_2(t)} \cos 2\pi f(t - \tau_2(t) - \pi).$

$$= \frac{\alpha}{r_0 + v(t)} \cdot \cos 2\pi f\left(t - \underbrace{\frac{(vt + r_0)}{c}}_{\tau_1(t)}\right) + \frac{\alpha}{2d - r_0 - vt} \cos 2\pi f\left(t - \underbrace{\frac{2d - r_0 - vt}{c}}_{\tau_2(t)} - \pi\right)$$

Recall: we have accepted linearity.

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→ I/O relationship via convolution.

$y(t) = h(t, \tau) * x(t)$: For $h(t, \tau)$ a T.V. channel impulse response.

$$y(t) = \int_{-\infty}^{\infty} h(t, \tau) \cdot x(t - \tau) d\tau$$

but also (from before).

$$y(t) = \sum_i a_i(t) \cdot x(t - \tau_i(t))$$

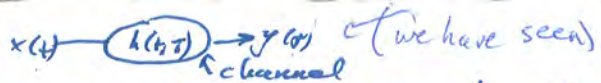
Put two together.

$$\Rightarrow h(t, \tau) = \sum_i a_i(t) \cdot \delta(\tau - \tau_i(t)).$$

$$\left(\begin{aligned} \text{verify: } y(t) &= \int_{-\infty}^{\infty} h(t, \tau) x(t - \tau) d\tau = \int_{-\infty}^{\infty} \sum_i a_i(t) \cdot \delta(\tau - \tau_i(t)) \cdot x(t - \tau) d\tau \\ &= \sum_i a_i(t) \cdot \underbrace{\int_{-\infty}^{\infty} \delta(\tau - \tau_i(t)) x(t - \tau) d\tau}_{x(t - \tau_i(t))} = \sum_i a_i(t) \cdot x(t - \tau_i(t)) \end{aligned} \right).$$

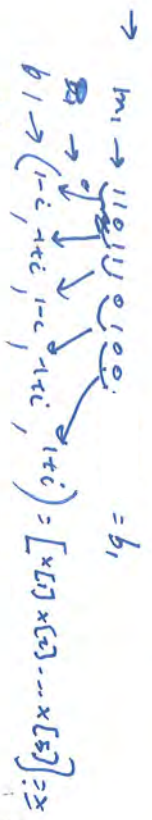
→ LTV impulse response of channel.

$$y(t) = h(t, \tau) * x(t).$$



Communication process in a simple: from this to this

message "meet me at 4pm" = m_1



$$\text{Re}\{z\} = [1 \ 1 \ 1 \ 1], \text{Im}\{z\} = [-1 \ 1 \ -1 \ 1]$$

This means infinite bandwidth

$$\text{Re}\{x_L\} = 1 \quad \text{Re}\{x_U\} = -1$$

→ as $B \rightarrow \infty$
(because $M \rightarrow \infty$)

→ need to smooth signals.

→ modulate with sequence of smooth sync functions

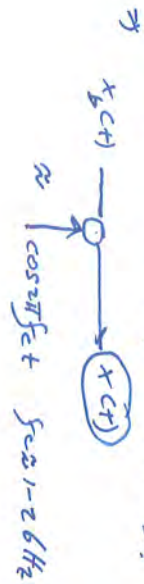
$$z \approx \pm * \{ \text{sync}\{w_t - n\} \}_n$$

to give $x_b(t)$ that has finite BW



- High frequency modulation

low frequency signals suffer from rapid attenuation (easily absorbed by walls, etc).



- then demodulate

Recall by $\approx \cos 2\pi f_c t$

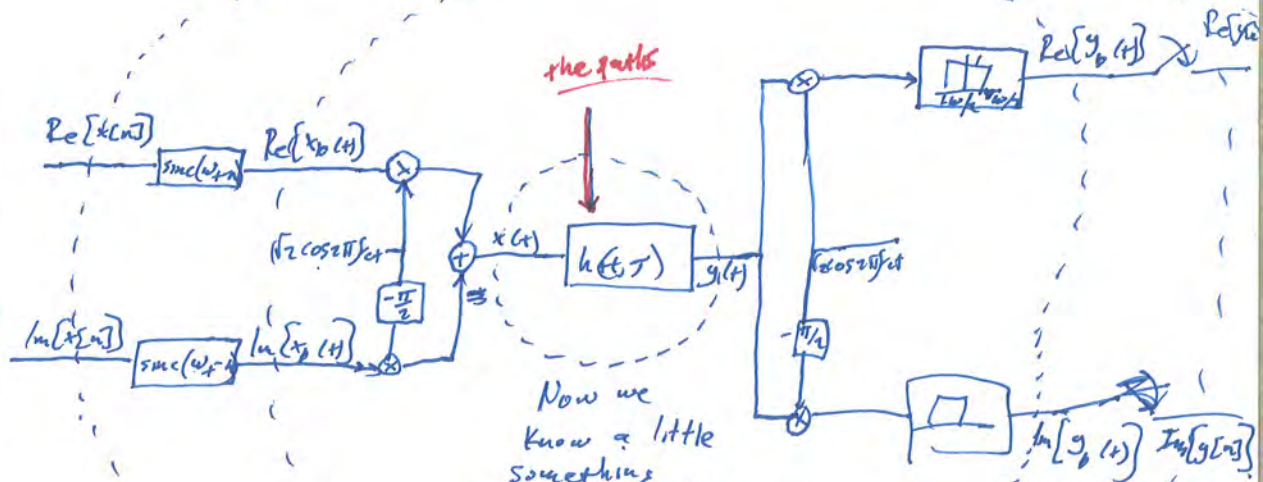
→ then sample
interval = $1/(4 + 0.2 \times 10^9)$

- then sample

meet me at 4 pm

↓
11 01 11 01 00

↓
 $\{x[1] \ x[2] \ \dots \ x[5]\} \mapsto x$
 $\text{Re}\{x\}, \text{Im}\{x\}$



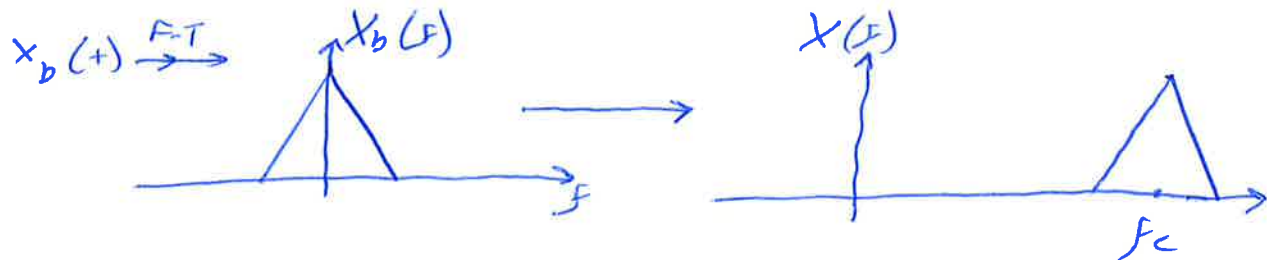
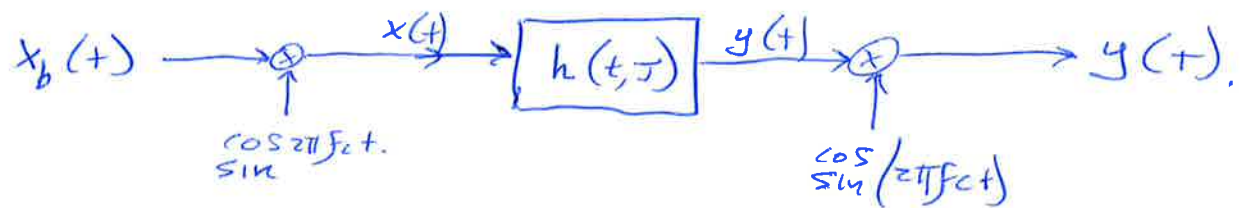
then "continuous time baseband I/O"

then "discrete-time baseband I/O"

then add noise.

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Establish relationship between Baseband input & output

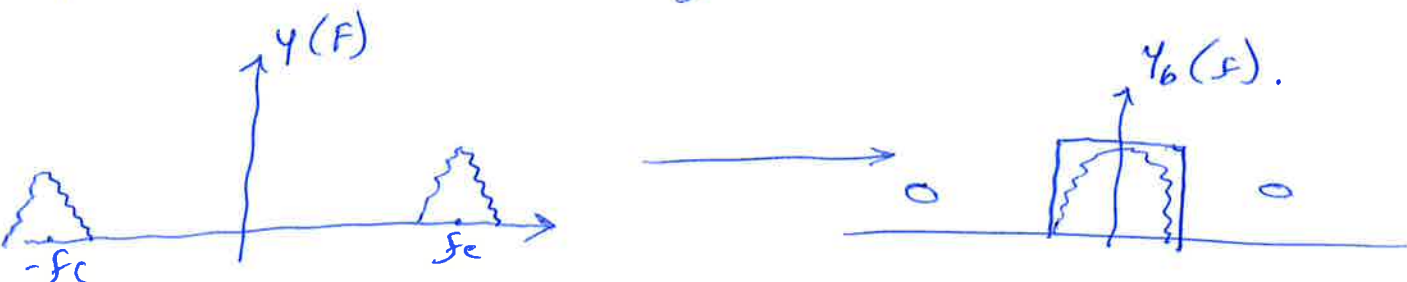


in time domain

$$x(t) = \text{Re} \left\{ X_b(t) \cdot e^{j2\pi f_c t} \right\}$$

Similarly $y(t) = \text{Re} \left\{ y_b(t) \cdot e^{j2\pi f_c t} \right\}$

From reverse action of demodulation



In practice: how is this achieved?

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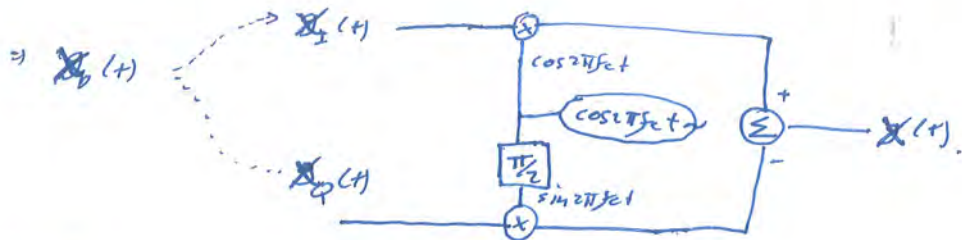
$$x(t) = \text{Re} \left\{ \underbrace{x(t)}_{\text{complex}} \cdot e^{j2\pi f_c t} \right\}$$

complex.

⇒ write $x(t) = x_I(t) + jx_Q(t)$ & recall $e^{j2\pi f_c t} = \cos 2\pi f_c t + j \sin 2\pi f_c t$.

⇒ $x(t) = \text{Re} \left\{ x_I(t) (\cos 2\pi f_c t + j \sin 2\pi f_c t) + j x_Q(t) (\cos 2\pi f_c t + j \sin 2\pi f_c t) \right\}$.

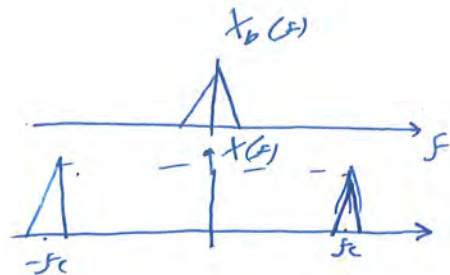
$$x(t) = x_I(t) \cdot \cos 2\pi f_c t - x_Q(t) \sin 2\pi f_c t \quad (***)$$



Recall

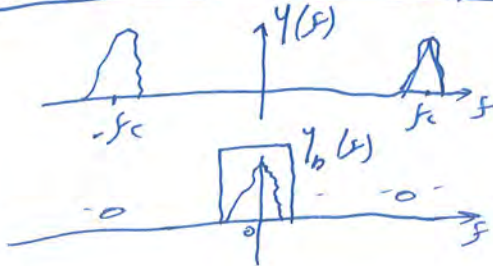
⇒ For our signals. $x_b(t)$ & $x(t)$

$$\Rightarrow \boxed{\begin{aligned} x(t) &= \operatorname{Re} \left\{ x_b(t) e^{j2\pi f_c t} \right\} \\ &= x_I(t) \cos 2\pi f_c t - x_Q(t) \sin 2\pi f_c t \end{aligned}}$$



similarly $y(t) = \operatorname{Re} \left\{ y_b(t) e^{j2\pi f_c t} \right\}$

From the reverse action of demodulators



Now ready to establish relationship

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$$x_b(t) \longrightarrow \boxed{h_b(t, \tau)} \longrightarrow y_b(t)$$

Recall:

$$y(t) = \sum_i a_i(t) \cdot x(t - \tau_i(t))$$

$a_i(t)$ path attenuation
 $\tau_i(t)$ " delay.

$$(*) \quad y_b(t) = \operatorname{Re} \left\{ y_b(t) e^{j2\pi f_c t} \right\}, \quad x_b(t) = \operatorname{Re} \left\{ x_b(t) e^{j2\pi f_c t} \right\} \quad (\text{from before}).$$

$$\Rightarrow \operatorname{Re} \left\{ y_b(t) e^{j2\pi f_c t} \right\} = \sum_i a_i(t) \operatorname{Re} \left\{ x_b(t - \tau_i(t)) \cdot e^{j2\pi f_c (t - \tau_i(t))} \right\} = \operatorname{Re} \left\{ \sum_i \underbrace{a_i(t)}_{\downarrow \text{ (real)}} x_b(t - \tau_i(t)) e^{j2\pi f_c t} e^{-j2\pi f_c \tau_i(t)} \right\}$$

$$\Rightarrow \operatorname{Re} \{ y_b(t) \} = \operatorname{Re} \left\{ \sum_i a_i(t) x_b(t - \tau_i(t)) e^{-j2\pi f_c \tau_i(t)} \right\}.$$

(easy exercise to show)

Similarly

$$\operatorname{Im} \left(y_b(t) e^{j2\pi f_c t} \right) = \operatorname{Im} \left\{ \sum_i a_i(t) x_b(t - \tau_i(t)) \cdot e^{j2\pi f_c t} \cdot e^{-j2\pi f_c \tau_i(t)} \right\}.$$

$$\Rightarrow \boxed{y_b(t) = \sum_i a_i(t) e^{-j2\pi f_c \tau_i(t)} \cdot x_b(t - \tau_i(t))} \quad (*) \quad \Rightarrow y_b(t) = \sum_i a_i^b(t) x_b(t - \tau_i(t))$$

$$\text{where } a_i^b(t) = \underbrace{a_i(t)}_{\text{slow}} \cdot e^{-j2\pi f_c \underbrace{\tau_i(t)}_{\text{Fast}}}.$$

Recall: linearity still holds since

all we did is multiply linear function.

$$y_b(t) = \int_{-\infty}^{\infty} h_b(\tau, t) x_b(t-\tau) d\tau$$

\Rightarrow

$$h_b(\tau, t) = \sum_i \underbrace{a_i(t)}_{\text{slow (minutes)}} \underbrace{\delta(\tau - \tau_i(t))}_{\text{fast}}$$

Note: here there is a notion of having one path.

LTV complex continuous-time baseband. 1/0 channel representation.

$a_i(t) \cdot e^{-j2\pi f_c \tau_i(t)}$
slow (minutes) fast

rate of change define rate of change of $a_i(t)$ thus of $h(\tau, t)$.

- Say not so big change in $a_i(t)$ (coherence in $a_i(t)$) corresponding to shift phase - shift of $\pi/2$

$$2\pi f_c \tau_i(t) \approx \frac{\pi}{2} \Rightarrow \tau_i \approx \frac{1}{4 f_c} \quad \begin{array}{l} \text{change in phase when delay of path changes} \\ \text{by } \approx \frac{1}{4 f_c} \end{array}$$

at speed of light $\Rightarrow \Delta X_c \approx \frac{1}{4 f_c}$ (recall $c = \lambda \cdot f$). (make note that $f \approx f_c$)

$$\Rightarrow T_c = \frac{\Delta X_c}{v} = \frac{1}{4v} \approx \frac{c}{4 \cdot f_c \cdot v} \approx \frac{1}{4 D_s} \quad \left(\text{recall doppler spread } D_s \approx \frac{f \cdot v}{c} \right)$$

$\Rightarrow h_b(t, \tau)$ changes $\Delta x_c \approx \frac{1}{4}$

$$\left(\frac{1}{4} \approx \frac{1}{4} \frac{c}{f} \approx \frac{1}{4} \frac{3 \cdot 10^8 \text{ m/s}}{10^9 \text{ Hz}} \right) \approx 7.5 \text{ cm (few cm)}$$

& at ^{reasonable} speeds ($v \approx 60 \text{ km/s}$).

$$\frac{c}{4 \cdot f \cdot v} \approx \frac{3 \cdot 10^8}{4 \cdot 10^9 \cdot 60 \cdot 10^3} = \frac{3 \cdot 10^8}{240 \cdot 10^{12}} = \frac{3}{2400} = \frac{1}{800}$$

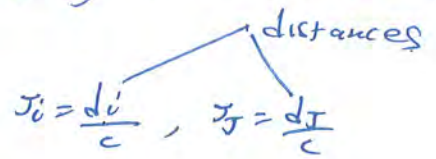
$$\frac{3 \cdot 10^8}{4 \cdot 10^9 \cdot 60 \cdot 10^3} = \frac{9}{1000} \approx 9 \text{ } \mu\text{s} \text{ (few } \mu\text{s)}$$

- interesting observation on frequency response of $h_b(t, \tau)$.
 $T_c \gg$ memory of channel $\approx T_d$
 (a bit trickier).

by definition $\delta(t) \rightarrow \boxed{h_b(t, \tau)} \rightarrow y_b(t) = h_b(t, \tau)$
 of impulse response.

- send an impulse through channel
- All signals will be collected (from all guests) in the order of what we will call

"delay spread" $= T_d \triangleq \max_{i,j} |\tau_i - \tau_j|$



- In cellular nets (mainly urban), typical distance - differences
 $d_i - d_j \approx$ few hundred meters

$$\Rightarrow \tau_i - \tau_j = \frac{d_i - d_j}{c} \approx \frac{\text{few hundred m}}{3 \cdot 10^8 \text{ m/s}} \approx \text{few } \mu\text{s} \quad (\approx 1 \rightarrow 10 \cdot 10^{-6} \text{ s})$$

\Rightarrow "memory of channel" $h_b(t, \tau) \approx \text{few } \mu\text{s} = T_d$

But $T_d \ll T_c \approx \text{few ms}$.

\Rightarrow

can consider channel to be almost ^{impulse response} time-invariant
(since it is only after many impulses that the channel will change)

(but clarify what is meant by this.)