Mobile Communication Techniques

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Final Exam

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Instructions

- Exercises fall in categories of 1-point, 2-point and 3-point exercises.
- Total of $4 \times 1 + 5 \times 2 + 3 \times 3 = 23$ points.
- The last question is for extra credit, and it accounts for 3 points.

- Each answer should be clearly written, and the solution should be developed in detail.
- Mathematical derivations need to show all steps that lead to the answer.
- Complete as many exercises as you can. Don't spend too much time on an individual question.
- Partial credit will be given for incomplete solutions.
- There is NO penalty for incorrect solutions.
- If in certain cases you are unable to provide rigorous mathematical proofs, go ahead and provide intuitive justification of your answers. Partial credit will be given.
- Calculators are allowed. Feel free to provide close form expressions *in their simplest form*.
- Open book and open class notes are allowed. No other notes are allowed.

Hints - equations - conventions:

- Notation
  - R represents the rate of communication in bits per channel use (b.p.c.u),
  - ρ represents the SNR (signal to noise ratio),
  - w will denote additive noise which will be distributed as a circularly symmetric Gaussian random variable $\mathbb{C}\mathcal{N}(0, N_0)$. If N_0 is not specified, then set $N_0 = 1$,
  - h_i will denote *independent* fading scalar coefficients which will be distributed as circularly symmetric Gaussian random variables $\mathbb{C}\mathcal{N}(0, 1)$
- Useful equations
  - For small ϵ, and for the statistical distributions of interest, the following holds:
$$P(|h_i|^2 < \epsilon) = \int_0^\epsilon p(|h_i|^2) d|h_i|^2 \approx \max_{[0,\epsilon]} \left\{ p(|h_i|^2 = \epsilon) \right\}$$
  - For small ϵ, and for the statistical distributions of interest, the following holds:
$$P(|h_i|^2 + |h_j|^2 < \epsilon) \approx p\left(\{|h_i|^2 < \epsilon\} \text{ or } \{|h_j|^2 < \epsilon\} \right)$$
  - Remember: for a given SNR, then SNR in dB is simply $10 \log_{10} SNR$
  - For a given probability of error $P(\text{error})$, the *diversity order* or *diversity gain* is defined as
$$d := - \lim_{\rho \to \infty} \frac{\log P(\text{error})}{\log \rho}.$$
- GOOD LUCK!!

EXAM PROBLEMS

1) *(1 point)*. Briefly state a disadvantage of operating in a very dense urban setting (smaller cells), where the path distances are not typically very high.

2) *(1 point)*. Consider a fast-fading channel, and let T_{coding} and T_{coh} respectively denote the coding duration and the coherence interval.
   - Which of the two holds: $T_{coding} >> T_{coh}$ or $T_{coding} << T_{coh}$?
   - What is a good measure of performance in this setting? Outage probability or capacity?

3) *(1 point)*. What is the rate, in bits per channel use, when we use the Alamouti code with 256-QAM entries?

4) *(1 point)*. Assume we have a 1×1 SISO communication system operating outdoors in the presence of unit power noise and of (flat) Rayleigh fading with a large coherence period that is larger than the maximum allowable coding duration.
   - What is the approximate power P consumed by the transmitter, if we wish to have a probability of error that is approximately $P_{err} \approx 10^{-8}$?
   - What is the approximate power savings (reduction in P) compared to the above, if we now take the same system, with the same reliability constraints, and place it indoors, in a quite library?

5) *(2 points)*. Assume that we have a 1×1 SISO communication system operating outdoors in the presence of unit power noise and of a Rayleigh fading with a large coherence period that is larger than the maximum allowable coding duration. Let the delay spread be approximately 2 times the length of a symbol duration ($T_d \approx 2/W$).
   - What is the approximate power P consumed by the transmitter, if we wish to have a probability of error that is approximately $P_{err} \approx 10^{-8}$?
   - What are the approximate power savings, if (for the same reliability) we double the signal bandwidth?
   - What are the approximate power savings, if we take the same system, with the same reliability constraints, and place it indoors, in a quite library? (please describe these savings, as a function of the channel)

6) *(2 points)*. Consider an outdoors setting where we employ a 2×1 MISO system (2 transmit antennas, and 1 receive antenna), and where the signal-to-noise ratio is approximately $SNR = P/N_o \approx 100$.
   - Describe IN DETAIL, three (non-trivially) different settings that would allow for a probability of error that is in the order of $P_{err} \approx 10^{-10}$. In describing settings, you must include the time duration, the bandwidth, the code, the rate, etc.

7) *(2 points)*. Consider the quasi-static Rayleigh fading channel. Let $SNR = 10^{-3}$ and $\epsilon = 10^{-2}$.
   - What is the approximate ϵ-outage capacity for SISO.

- What are the gains in ϵ-outage capacity, brought about by adding two extra receive antennas?

8) *(2 points).* Consider communication in a SISO AWGN environment. Assume that we have a total power constraint of $\bar{P} \approx 10^8$ Watts, and a specific application that only allows for total bandwidth of about $W \approx 1$ KHz. Per dimension, the noise has unit power (Standard noise assumption).

   - What is the APPROXIMATE gain in the total capacity (over the entire bandwidth), if we double bandwidth (fix $\bar{P}$), vs. if we double power (fix W)? I.e., what is best: to double bandwidth, or to double power? (Hint: think of convexity)

9) *(2 points).* Assume that we have a 4×3 MIMO channel (4 transmit and 3 receive antennas) in the presence of Rayleigh fading (outdoors), where the coefficients are i.i.d. in space. Assume that we use a 4×2 space-time code $\mathcal{X}_1$ that spans 4 spatial dimensions (4 transmit antennas) and $T = 2$ time slots.
   - What is the maximum diversity that we can achieve with $\mathcal{X}_1$?
   - How many independent QAM symbols must $\mathcal{X}_1$ carry, so that we can achieve full degrees of freedom? (Hint: count number of variables)
   - If $\mathcal{X}_1$ does not achieve the maximum possible diversity, what is the first thing you must change about it?

10) *(3 points).* Consider communication in a SISO AWGN sensor environment. Assume that we have a total power constraint of $\bar{P} \approx 100$ Watts (over all frequencies), and a total bandwidth constraint of about $W \approx 1$ MHz. Per dimension, the noise has unit power (Standard noise assumption). Let the desired probability of error be approximately $P_{err} \approx 10^{-2}$.
How long (in seconds) will it take to send (over the entire bandwidth), 20 bits of information?

11) *(3 points).* Assume that we are operating over a 2×1 MISO channel in the presence of *Raleigh fading* and in the presence of additive noise of power $N_o = 1$. Consider a code:
$$\mathcal{X} = \{X = \theta \cdot \begin{vmatrix} f_0 & -f_1^* \\ f_1 & f_0 \end{vmatrix}\}$$
where θ is the power normalizer, and f_0, f_1 are *independently* drawn from a 4-QAM constellation.
a) Assume that we want a signal-to-noise ratio of about 40 dB. What must θ be?
b) Given an SNR of about 40dB, what is the approximate probability of error (averaged over noise and averaged over fading)? Hint: argue what is the maximum possible diversity, and then if the code achieves it.
c) Does the code allow for the maximum degrees of freedom?
d) Assume now that the code is drawing f_0, f_1, not from a 4-QAM constellation, but from a 64-QAM constellation. Does the code now satisfy the full rank criterion? Does it achieve the full degrees of freedom?

12) *(3 points - extra credit).* Consider communication in a SISO Rayleigh flat-fading environment (outdoors), where the coherence period of the channel is 10 milliseconds ($T_{coh} \approx 10ms$), and it is bigger than the coding duration $T_{coding} < T_{coh}$. Assume that we want a rate of approximately $R = 1$ bit per channel use.

- What is the SNR (call it $SNR = \rho_1$) needed to achieve a probability of outage that is approximately $P_{outage} \approx 10^{-5}$?

For this same $SNR = \rho_1$, let us consider now the case where we actually have perfect channel state information at the transmitter and the receiver (perfect CSITR). In this latter CSITR case:

- What is the AVERAGE rate that we can achieve for the same $SNR = \rho_1$?
  Hint: emphasis on 'CSIT' and the word 'AVERAGE'.
  Hint - two words: R - - -   A - - - - - - - - N