

# convolutional\_codes

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The difference between **convolutional codes** and the **Viterbi algorithm** lies in their roles and purposes in a communication system:

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## 0.0.1 1. Convolutional Codes

- **What it is:**
    - A **convolutional code** is a type of error-correcting code used to encode data for transmission over noisy channels.
    - It introduces redundancy by encoding input data into output sequences using a memory-based process.
  - **Purpose:**
    - To improve the reliability of data transmission by adding redundancy, enabling the receiver to detect and correct errors.
  - **How it works:**
    - Encoding involves **shift registers** and **generator polynomials** to create output bits as a function of current and past input bits.
    - Example of an encoding process:
      - \* Input bits:  $u = [1, 0, 1]$
      - \* Output (encoded):  $v = [11, 10, 01, \dots]$
  - **Key Parameters:**
    - **Constraint Length ( $K$ ):** The memory length of the encoder.
    - **Code Rate ( $R = \frac{k}{n}$ ):** The ratio of input bits ( $k$ ) to output bits ( $n$ ).
    - **Generator Polynomials:** Define the relationships between input and output.
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## 0.0.2 2. Viterbi Algorithm

- **What it is:**
  - The **Viterbi algorithm** is a decoding algorithm used to determine the most likely sequence of transmitted bits from the received noisy encoded sequence.
  - It is specifically designed for **maximum likelihood decoding** of convolutional codes.
- **Purpose:**
  - To decode convolutionally encoded data by finding the most likely path through a **trellis diagram** that represents the encoder's state transitions.
- **How it works:**
  - Uses a **trellis structure**:
    - \* Each node in the trellis corresponds to a possible state of the encoder.

- \* Transitions between states represent possible input-output combinations.
- Computes and tracks:
  - \* **Path Metric:** A measure of how well the received sequence matches the expected sequence for a given path.
  - \* **Survivor Path:** The most likely sequence of states leading to the current state.
- Selects the path with the smallest metric as the decoded output.

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### 0.0.3 Key Differences

Aspect	Convolutional Codes	Viterbi Algorithm
<b>Type</b>	Error-correcting code (encoding mechanism).	Decoding algorithm for convolutional codes.
<b>Role</b>	Adds redundancy to the transmitted data for error correction.	Decodes the redundant data to retrieve original input.
<b>Operation</b>	Encodes input bits into output sequences.	Decodes received noisy sequences into input bits.
<b>Implementation</b>	Uses shift registers and generator polynomials.	Uses a trellis structure and dynamic programming.
<b>Output</b>	Encoded sequence (e.g., $v = [11, 10, \dots]$ ).	Most likely transmitted input sequence (e.g., $u = [1, 0, 1]$ ).
<b>Computational Complexity</b>	Encoding is simple and linear.	Decoding can be computationally intensive for large constraint lengths.

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### 0.0.4 How They Work Together

1. **Encoding with Convolutional Codes:**
    - At the transmitter, input bits are convolutionally encoded to produce a redundant sequence that can withstand noise during transmission.
  2. **Decoding with the Viterbi Algorithm:**
    - At the receiver, the Viterbi algorithm is used to decode the noisy received sequence, leveraging the redundancy introduced by the convolutional code to correct errors.
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### 0.0.5 Example

- **Convolutional Encoding:**
  - Generator polynomials:  $g_1 = 101$ ,  $g_2 = 111$  (constraint length = 3).
  - Input:  $u = [1, 0, 1, 1]$ .
  - Encoded output:  $v = [11, 01, 10, 11]$ .
- **Decoding with Viterbi:**
  - Received (noisy):  $r = [11, 00, 10, 11]$ .
  - The Viterbi algorithm reconstructs the most likely input sequence  $u = [1, 0, 1, 1]$  by analyzing the trellis and minimizing error metrics.

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### 0.0.6 Conclusion

- **Convolutional codes** are the method for encoding data with redundancy.
- The **Viterbi algorithm** is the decoding process to recover the original data from the noisy encoded transmission.

Would you like further details on how either convolutional codes or the Viterbi algorithm works, perhaps with step-by-step examples?