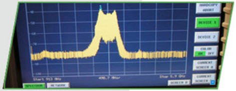

Radio Engineering

Lecture 5: Modulation and Diversity

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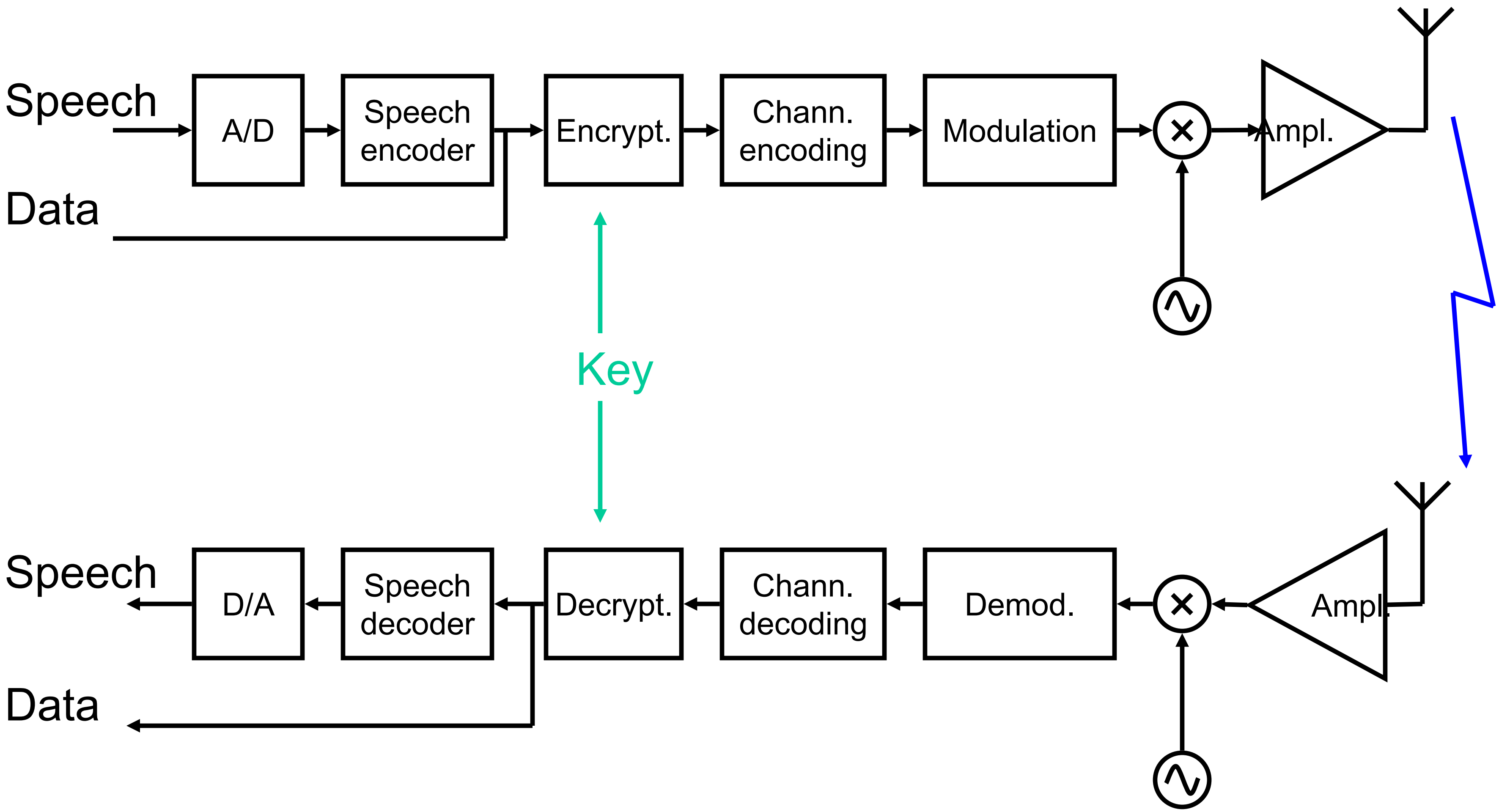
This lecture

6. Modulation and demodulation
   - Structure of a wireless link
   - Bit error rate
7. Diversity
   - Introduction
   - Macrodiversity
   - Microdiversity
     - Time
     - Freq
     - Space
   - Correlation coefficients
   - Combination of signals
     - selection combining
     - diversity combining

Block diagram



RADIO SIGNALS AND COMPLEX NOTATION

Simple model of a radio signal

- A transmitted radio signal can be written

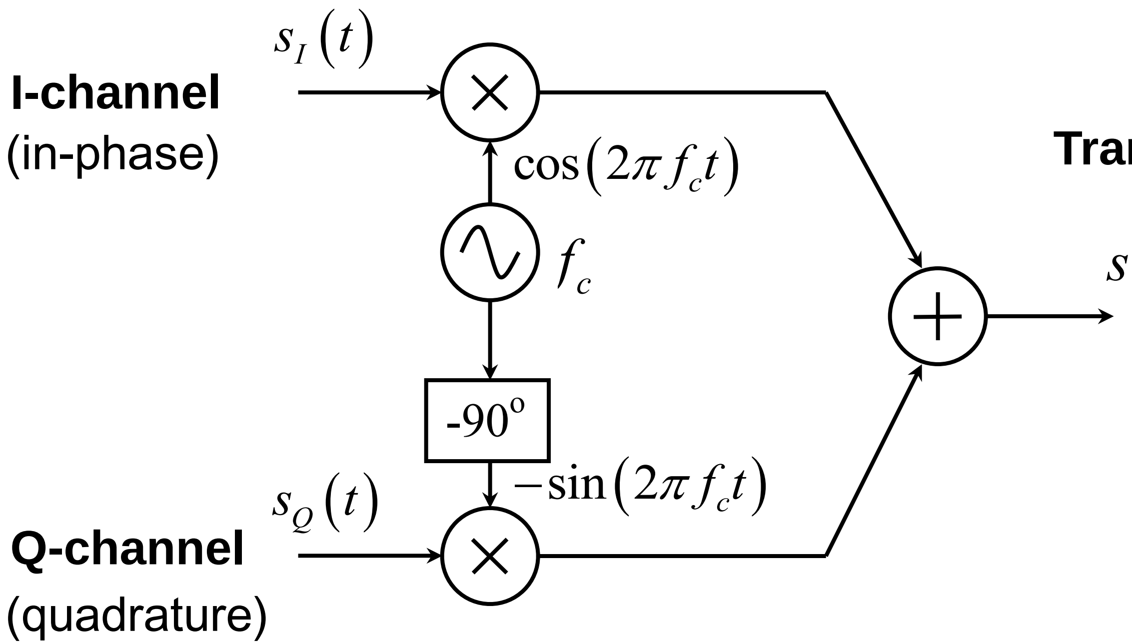
$$s(t) = \underbrace{A}_{\text{Amplitude}} \cos\left(2\pi \underbrace{f}_{\text{Frequency}} t + \underbrace{\phi}_{\text{Phase}}\right)$$

- By letting the transmitted information change the amplitude, the frequency, or the phase, we get the tree basic types of digital modulation techniques

  – **ASK** (Amplitude Shift Keying)
  – **FSK** (Frequency Shift Keying)
  – **PSK** (Phase Shift Keying)

Constant amplitude

The IQ modulator



Transmited radio signal

$$s(t) = s_I(t)\cos(2\pi f_c t) - s_Q(t)\sin(2\pi f_c t)$$

Take a step into the complex domain:

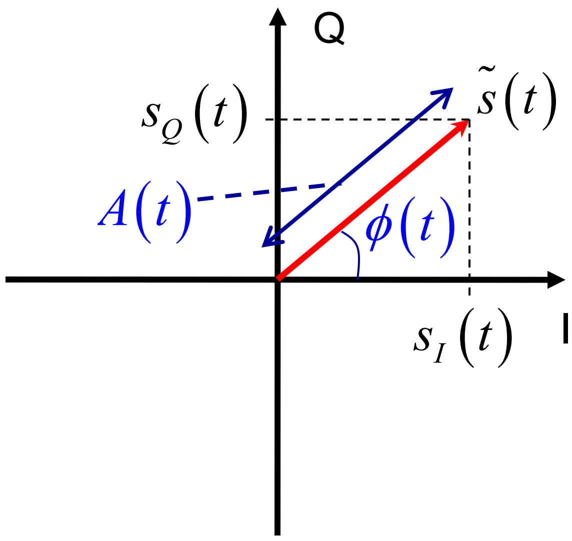
Complex envelope $\tilde{s}(t) = s_I(t) + j s_Q(t)$

Carrier factor $e^{j2\pi f_c t}$

$$\Rightarrow \quad s(t) = \mathrm{Re}\left\{\tilde{s}(t) e^{j2\pi f_c t}\right\}$$

Interpreting the complex notation

Complex envelope (phasor)



Polar coordinates:

$$\tilde{s}(t) = s_I(t) + js_Q(t) = A(t)e^{j\phi(t)}$$

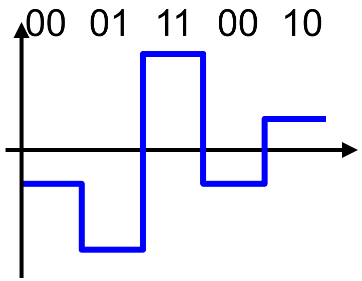
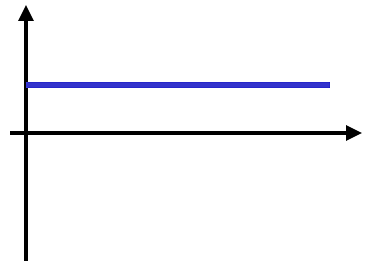
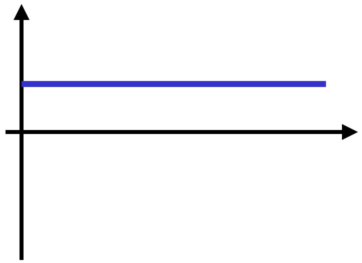
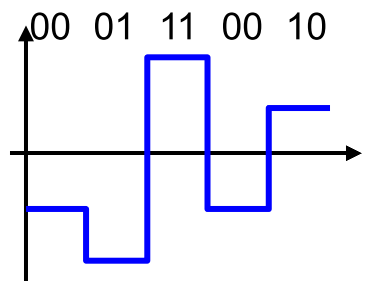
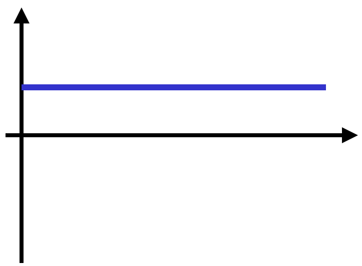
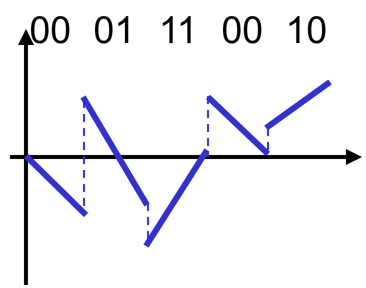
Transmitted radio signal

$$\begin{aligned} s(t) &= \mathrm{Re}\left\{\tilde{s}(t)e^{j2\pi f_c t}\right\} \\ &= \mathrm{Re}\left\{A(t)e^{j\phi(t)}e^{j2\pi f_c t}\right\} \\ &= \mathrm{Re}\left\{A(t)e^{j(2\pi f_c t+\phi(t))}\right\} \\ &= A(t)\cos\left(2\pi f_c t + \phi(t)\right) \end{aligned}$$

By manipulating the amplitude A(t) and the phase ϕ(t) of the complex envelope (phasor), we can create any type of modulation/radio signal.

Example: Amplitude, phase and frequency modulation

$$s(t) = A(t)\cos\left(2\pi f_c t + \phi(t)\right)$$

	$A(t)$	$\phi(t)$	Comment:
4ASK			- Amplitude carries information - Phase constant (arbitrary)
4PSK			- Amplitude constant (arbitrary) - Phase carries information
4FSK			- Amplitude constant (arbitrary) - Phase slope (frequency) carries information

SIGNAL SPACE DIAGRAM

Principle of signal-space diagram (1)

- Represent a continuous signal by a discrete vector
- Choice of expansion functions:
  - In passband, usually

$$\varphi_{\mathrm{BP},1}(t) = \sqrt{\frac{2}{T_{\mathrm{S}}}}\cos(2\pi f_{\mathrm{c}} t)$$

$$\varphi_{\mathrm{BP},2}(t) = \sqrt{\frac{2}{T_{\mathrm{S}}}}\sin(2\pi f_{\mathrm{c}} t) \quad .$$

  - In baseband, usually

$$\varphi_1(t) = \sqrt{\frac{1}{T_{\mathrm{S}}}} \cdot 1$$

$$\varphi_2(t) = \sqrt{\frac{1}{T_{\mathrm{S}}}} \cdot j.$$

Principle of signal-space diagram (2)

- Signal vector for m-th signal

$$s_{m,n} = \int_0^{T_S} s_m(t)\varphi_n^*(t)dt$$

- Energy contained in signal

$$E_{S,m} = \int_0^{T_S} s_{BP,m}{}^2(t)dt = \|\mathbf{s}_{BP,m}\|^2$$

$$E_{S,m} \approx \frac{1}{2}\int_0^{T_S}\|s_{LP,m}(t)\|^2 dt = \frac{1}{2}\|\mathbf{s}_{LP,m}\|^2$$

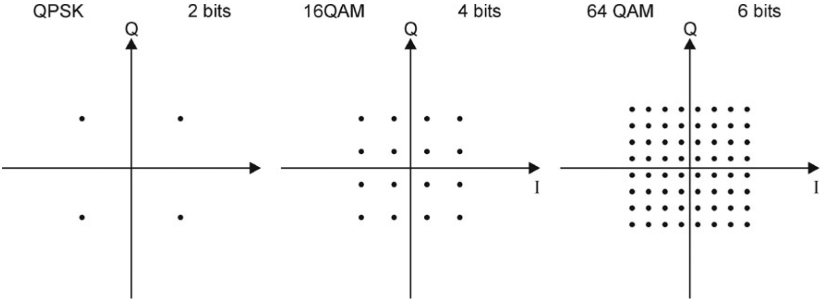
- Correlation coefficients between signals *k* and *m*

$$\mathrm{Re}\{\rho_{k,m}\} = \frac{\mathbf{s}_{BP,m}\mathbf{s}_{BP,k}}{\|\mathbf{s}_{BP,m}\|\,\|\mathbf{s}_{BP,k}\|}$$

- *Take care about normalization BP vs. LP*

Modulation example: QAM

- QAM - Quadrature Amplitude Modulation
- based on IQ modulation
- modulation oders (bits per symbol): QPSK (2), 16QAM (4), 64QAM (6), 256QAM (8) . . .
- modulation symbol = element of signal space diagram

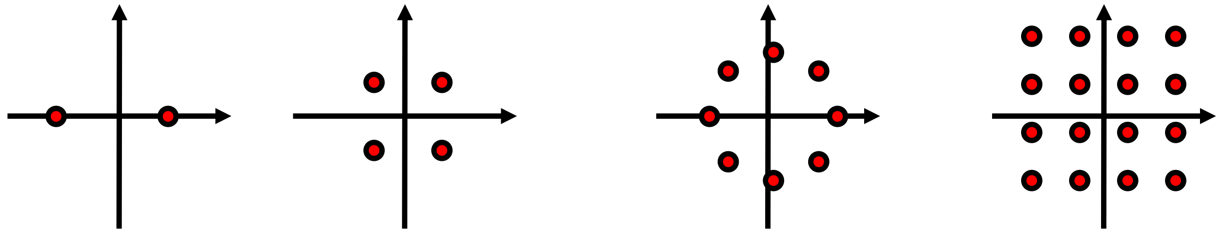


De-Modulation

- Demodulator needs to find most likely transmitted symbol (MAP - Maximum a posteriori detection)
  - in memoryless, uncoded systems with AWGN this is the closest constellation point to the received symbol
- Pairwise error probability: probability of detecting s_i instead of s_j
- Symbol error rate: average pairwise error probability
- Bit error ratio (rate): bit errors / total number of bits

Optimal receiver
Bit-error rates (BER)

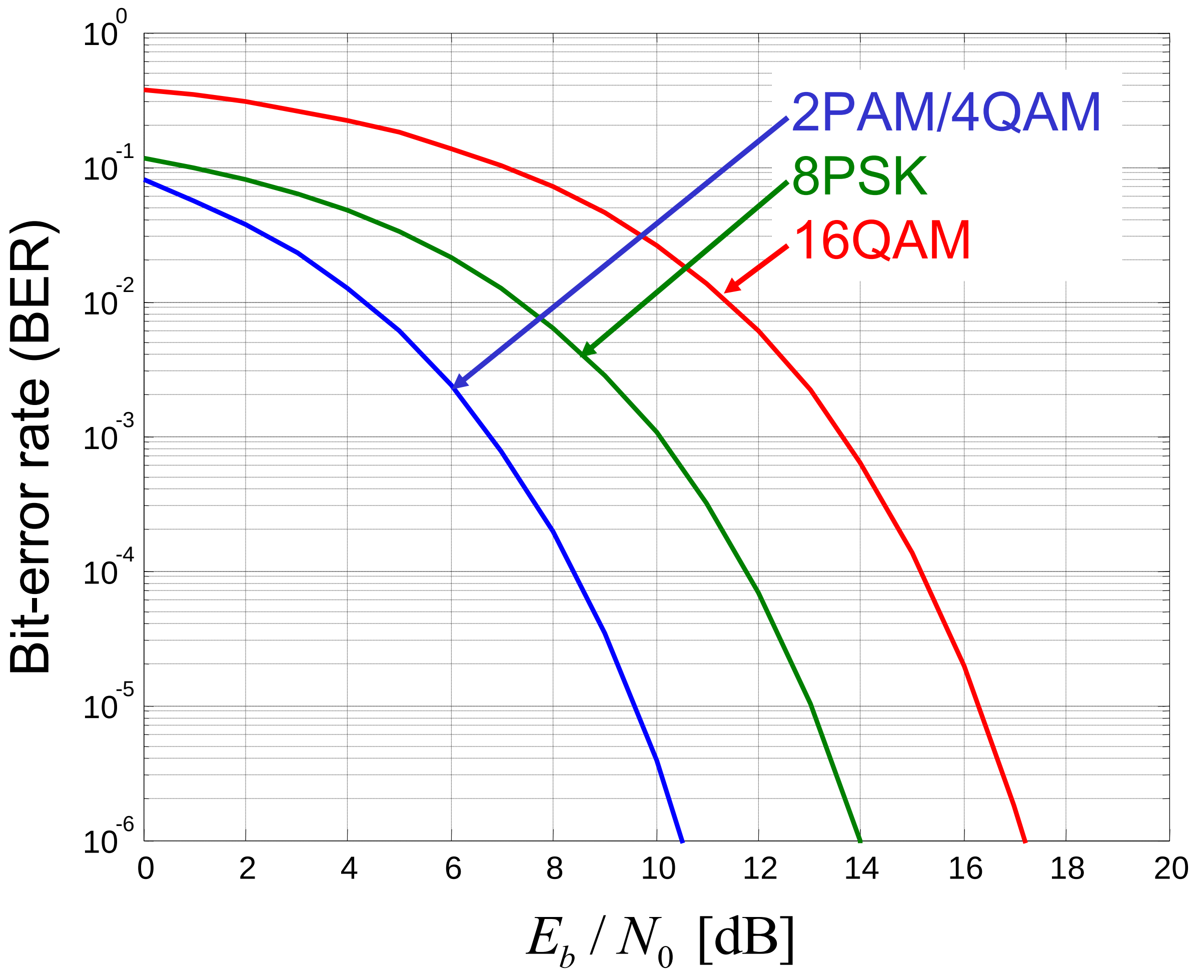
EXAMPLES: 2PAM 4QAM 8PSK 16QAM



	2PAM	4QAM	8PSK	16QAM
Bits/symbol	1	2	3	4
Symbol energy	E_b	$2E_b$	$3E_b$	$4E_b$
BER	$Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$	$Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$	$\approx \frac{2}{3} Q\left(\sqrt{0.87\frac{E_b}{N_0}}\right)$	$\approx \frac{3}{2} Q\left(\sqrt{\frac{E_{b,\max}}{2.25N_0}}\right)$

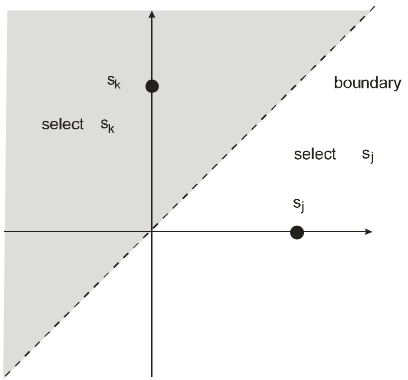
Gray coding is used when calculating these BER.

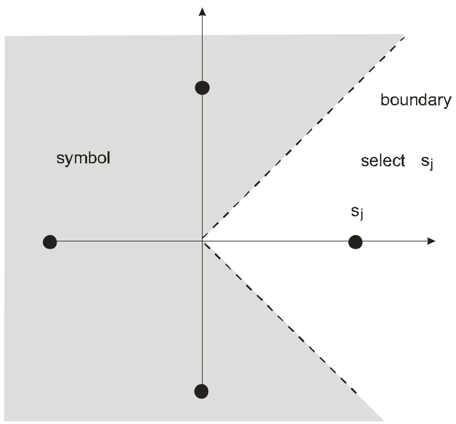
Optimal receiver
Bit-error rates (BER), cont.



Optimal receiver – BER of QPSK

- Compute via union bound
- Pairwise error probability $Q\left(\sqrt{2\gamma_{\mathrm{B}}}\right)$
- Symbol error probability $SER \approx 2Q(\sqrt{2\gamma_{\mathrm{B}}})$
- Bit error probability $BER = Q(\sqrt{2\gamma_{\mathrm{B}}})$





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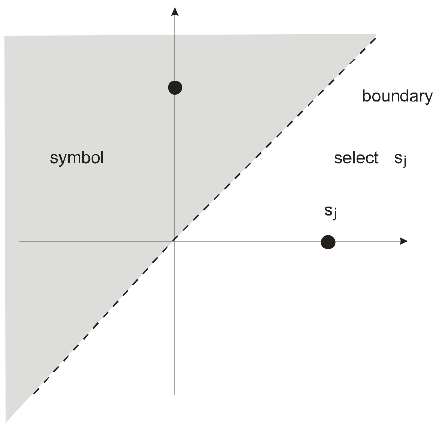

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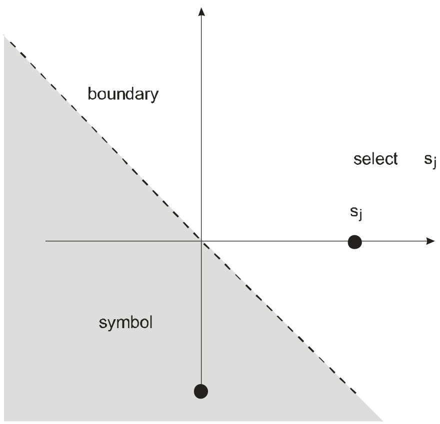


Optimal receiver
Where do we get E_b and N_0?

Where do those magic numbers E_b and N_0 come from?

The noise power spectral density N_0 is calculated according to

$$N_0 = kT_0F_0 \Leftrightarrow N_{0|dB} = -204 + F_{0|dB}$$

where F_0 is the noise factor of the "equivalent" receiver noise source.

The bit energy E_b can be calculated from the received power C (at the same reference point as N_0). Given a certain data-rate d_b [bits per second], we have the relation

$$E_b = C / d_b \Leftrightarrow E_{b|dB} = C_{|dB} - d_{b|dB}$$

THESE ARE THE EQUATIONS THAT RELATE DETECTOR PERFORMANCE ANALYSIS TO LINK BUDGET CALCULATIONS!

BER in fading channels

- In an AWGN channel the BER decreases exponentially with SNR
- Example: QPSK

$$\text{BER} = Q(\sqrt{2\gamma})$$

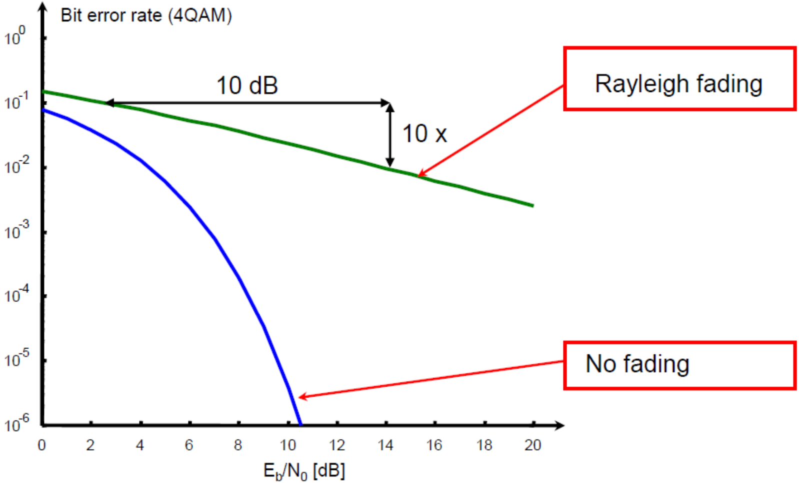
- In a fading channel with SNR distribution pdf(γ) and mean $\bar{\gamma}$

$$\text{BER}_{\text{fading}}(\bar{\gamma}) = \int_0^{\infty} \text{BER}_{\text{AWGN}}(\gamma)\text{pdf}(\gamma)d\gamma$$

- Example: Rayleigh fading channel

$$\text{pdf}(\gamma) = \frac{1}{\gamma}\exp\left(\frac{-\gamma}{\bar{\gamma}}\right)$$

BER in fading channels



Fading is one of the biggest challenges in wireless communications!

Diversity

- Fading can also be used as an advantage if exploited properly
- If the transmitted signal is available on two or more channels (known as diversity branches), the probability that this signal is affected by a deep fade, occurring simultaneously in all branches, is very low.
- With a convenient algorithm (known as combining method) it is possible to obtain a resulting signal where the effects of fading are minimized.

Diversity: Example

- Consider a fading channel with 2 states:
  - SNR=13.5dB for 90% of the time $\Rightarrow$ BER = 10^{-10}
  - SNR=0dB for 10% of the time $\Rightarrow$ BER = 0.5
- Average BER is $0.9 \cdot 10^{-10} + 0.1 \cdot 0.5 = 0.05$
- For a two antenna receiver employing selection combining
  - SNR=0dB at both chains for 1% of the time
  - SNR=13.5dB at at least one chain for 99% of the time
- Average BER is $0.99 \cdot 10^{-10} + 0.01 \cdot 0.5 = 0.005$

Diversity exponent

More generally we can define

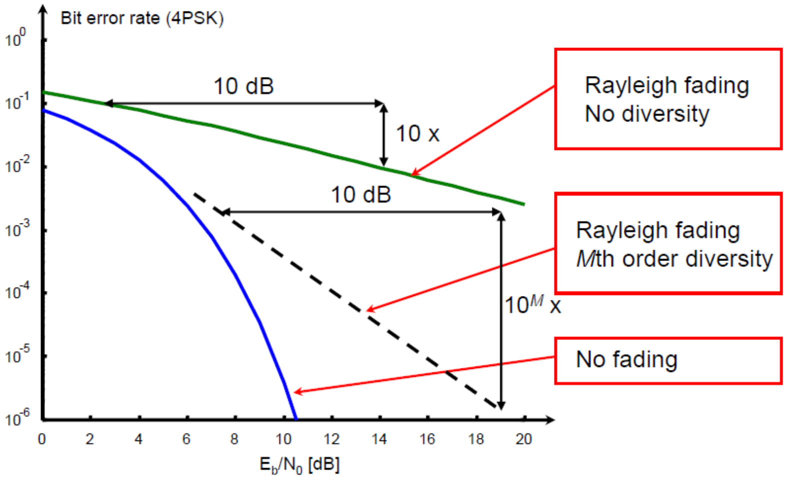
Definition (Diversity exponent)

The diversity exponent is the slope of the Bit Error Rate (BER) for large SNR on a log-log scale

$$d_{\text{div}} = - \lim_{\bar{\gamma} \to \infty} \frac{\log \mathrm{BER}(\bar{\gamma})}{\log \bar{\gamma}}$$

It depends on the transmission method and the receiver used.

Diversity exponent



Types of diversity

Macrodiversity

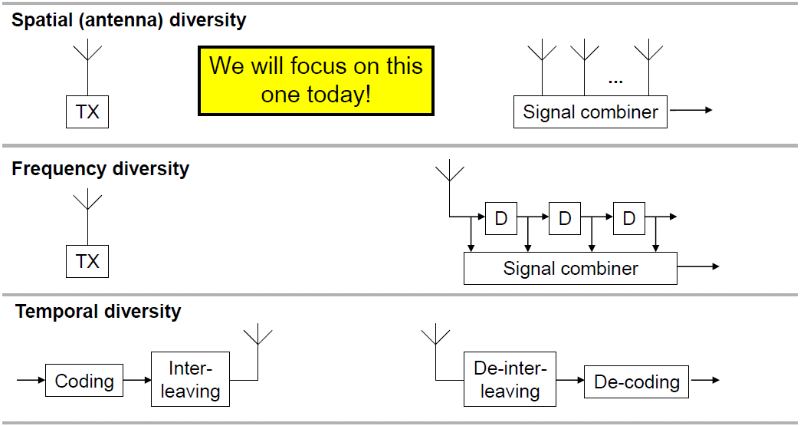
- Macrodiversity tries to counteract large scale fading caused by obstruction etc.
- Use of more than one base station strategically positioned so that the mobiles always have a clear radio path to at least one base station
- Examples: Soft handover, simulcast, coordinated multipoint transmission

Types of diversity

Microscopic diversity

- Microscopic diversity tries to conteract small scale fading caused by multipath propagation
- RX exploits multiple independent copies of the same signal (diversity branches)
- Several methods are availiable
  - Spatial diversity
  - Temporal diversity
  - Frequency diversity
  - Polarization diversity

Microscopic diversity



(We also have angular and polarization diversity)

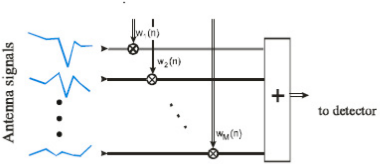
Spatial Diversity: RX

- Signal model:

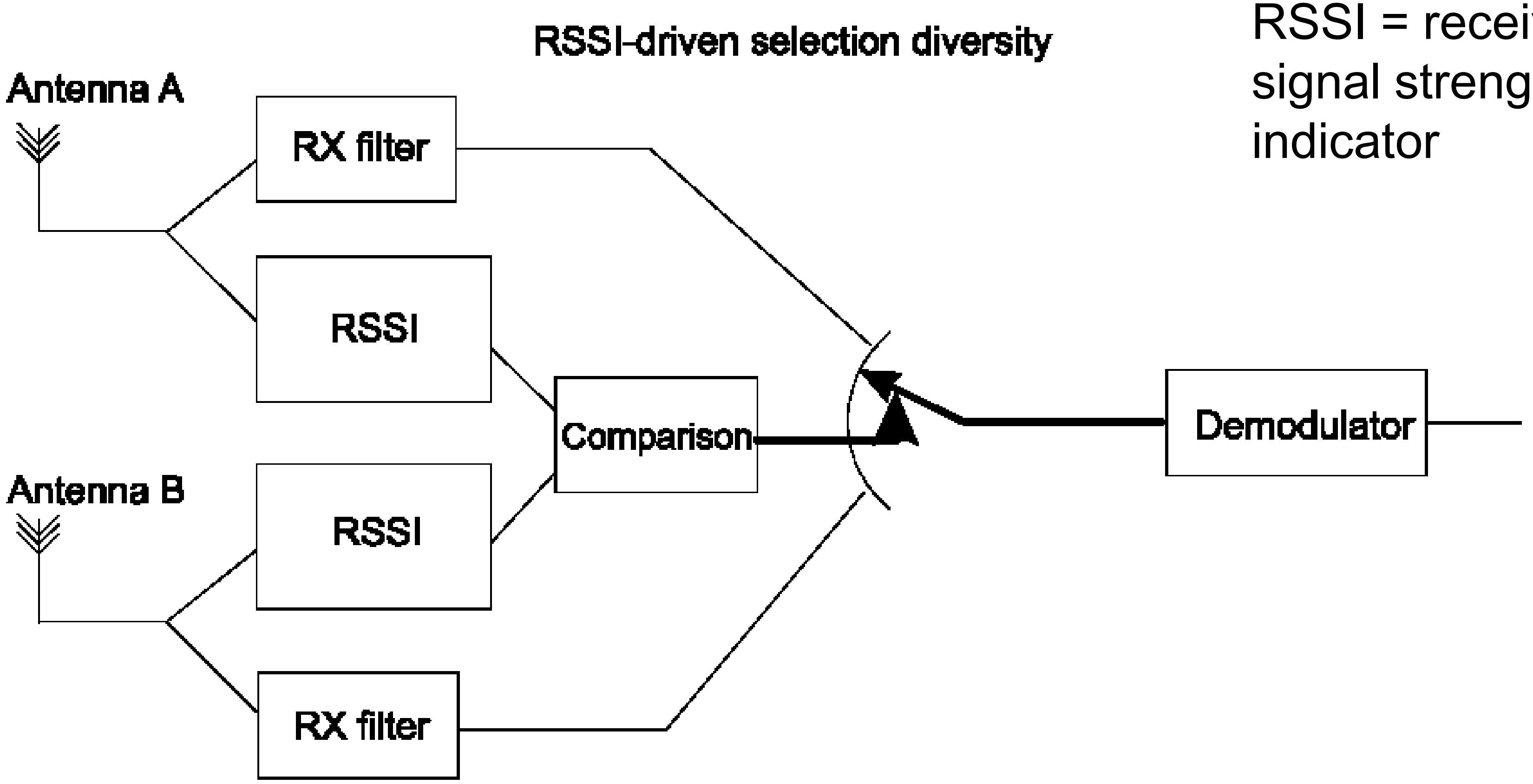
$$\mathbf{r} = \mathbf{h}s + \mathbf{n},$$

  where
  - s is the transmitted signal,
  - $\mathbf{h} = [h_0, \ldots, h_{N-1}]^T$ is the (narrowband) channel response at antenna elements $0, \ldots, N-1$,
  - $\mathbf{n} = [n_0, \ldots, n_{N-1}]^T$ is the i.i.d. noise (AWGN) with variance σ_n^2 and
  - $\mathbf{r} = [r_0, \ldots, r_{N-1}]^T$ is the received signal.
- Basic principle of diversity combining

$$y = \mathbf{w}^T\mathbf{r}$$

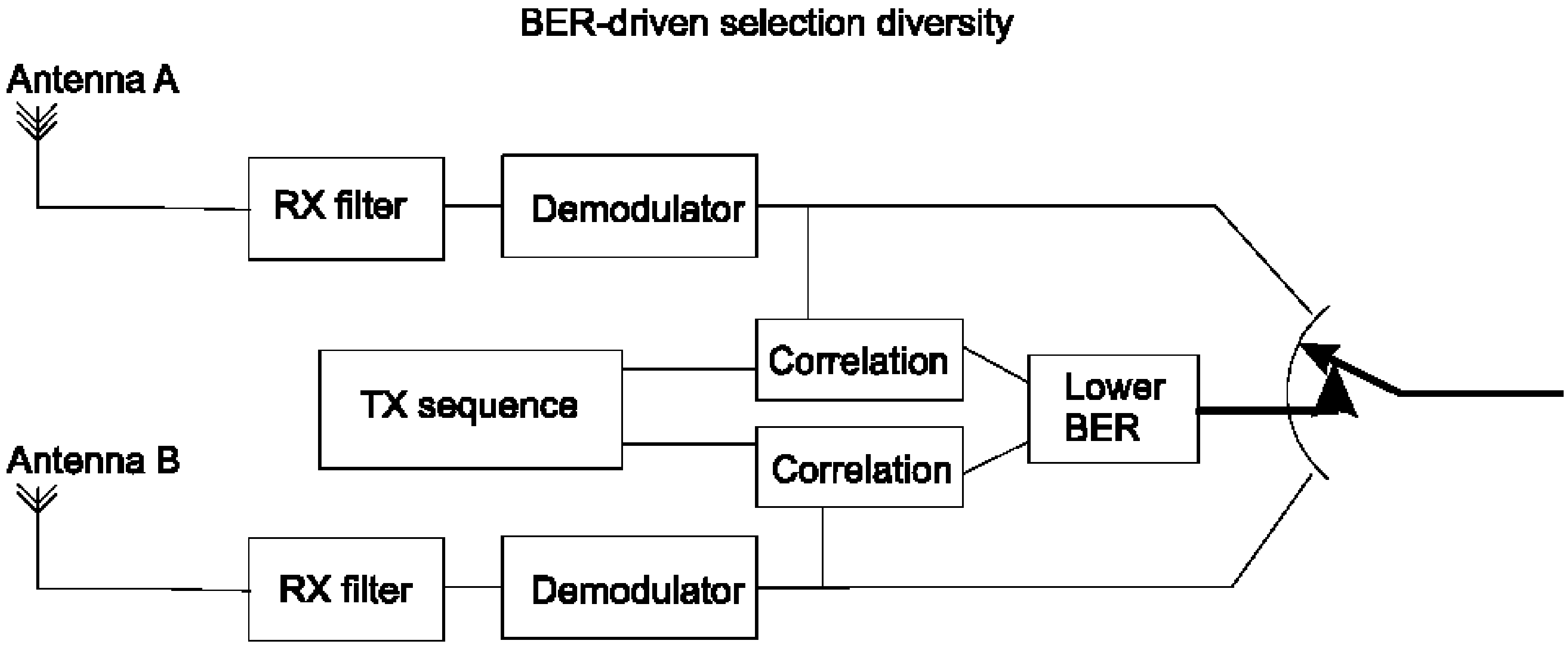


Spatial (antenna) diversity
Selection diversity

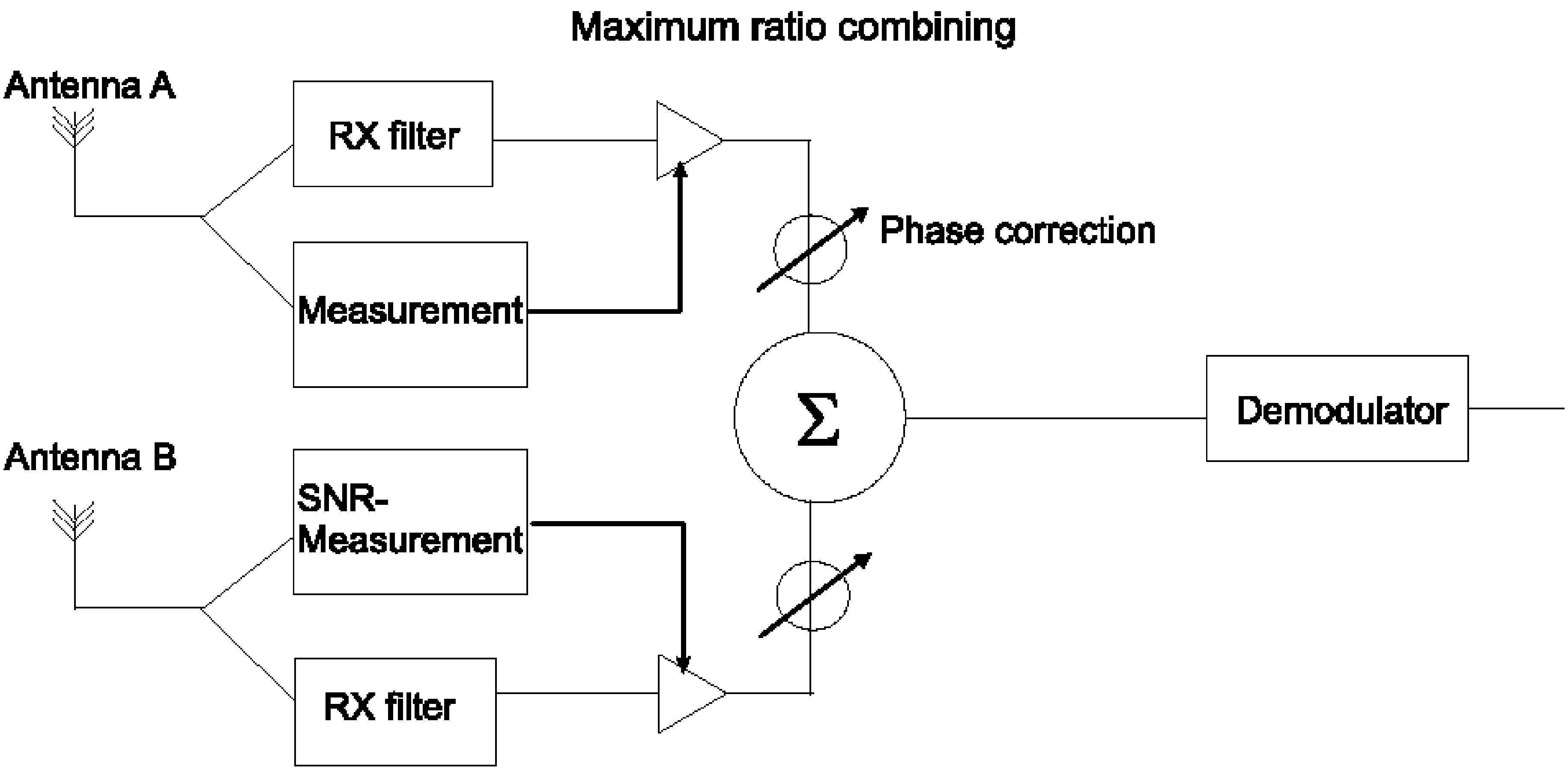


RSSI = received signal strength indicator

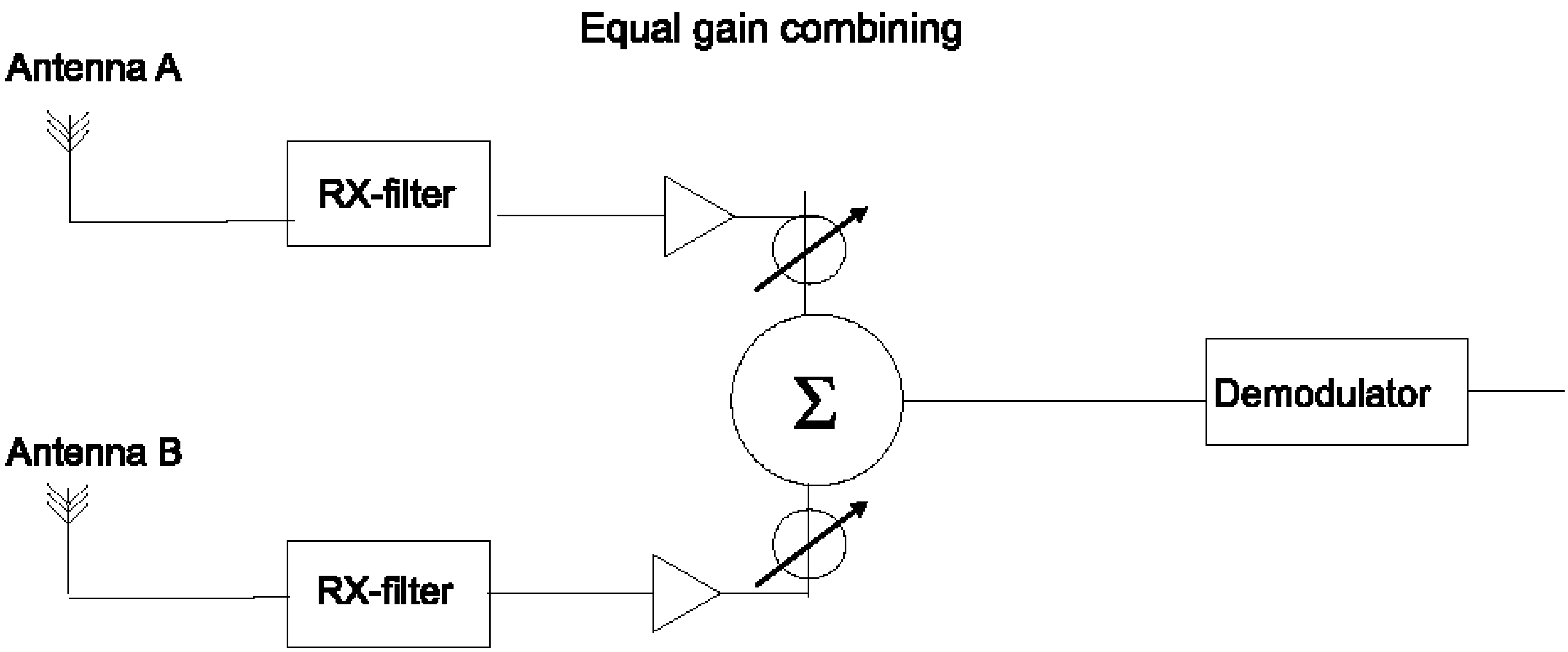
Spatial (antenna) diversity
Selection diversity, cont.



Spatial (antenna) diversity
Maximum ratio combining



Spatial (antenna) diversity
Equal gain combining



Selection combining (SC)

- Select diversity branch which is “better”
- Possilbe metrics: BER, RSSI
- Combined SNR $\gamma_{\mathrm{SC}} = \max \gamma_n$

If each branch has Rayleigh distribution with mean $\bar{\gamma}$,

- Combined cdf

$$\mathrm{cdf}_{\mathrm{SC}}(\gamma) = \left(1 - \exp\left(-\frac{\gamma^2}{\bar{\gamma}^2}\right)\right)^N$$

Maximum Ratio Combining (MRC)

- Optimal combining when only disturbance is AWGN
- Each branch is phase corrected and weighted by amplitude
- Combiner weights $\mathbf{w}_{\text{MRC}} = \mathbf{h}^*$
- Combined SNR $\gamma_{\text{MRC}} = \sum_{n=0}^{N-1} \gamma_n$

If each branch has Rayleigh distribution with mean $\bar{\gamma}$,

- Combined pdf

$$\text{pdf}_{\text{MRC}}(\gamma) = \frac{1}{(N-1)!} \frac{\gamma^{N-1}}{\bar{\gamma}^N} \exp\left(-\frac{\gamma}{\bar{\gamma}}\right)$$

- Combined cdf

$$\text{cdf}(\gamma) = 1 - \exp\left(-\frac{\gamma}{2\bar{\gamma}}\right) \sum_{i=0}^{N-1} \frac{1}{i!} \left(\frac{\gamma}{2\bar{\gamma}}\right)^i$$

- Combined mean SNR $\bar{\gamma}_{\text{MRC}} = N\bar{\gamma}$

Equal gain combining (EGC)

- Like MRC, but each branch is only phase corrected (not weighted)
- Combiner weights $\mathbf{w}_{\text{EGC}} = [h_0^*/|h_0|, \ldots, h_{N-1}^*/|h_{N-1}|]$
- Combined SNR $\gamma_{\text{EGC}} = \frac{1}{N}\left(\sum_{n=0}^{N-1}\sqrt{\gamma_n}\right)^2$

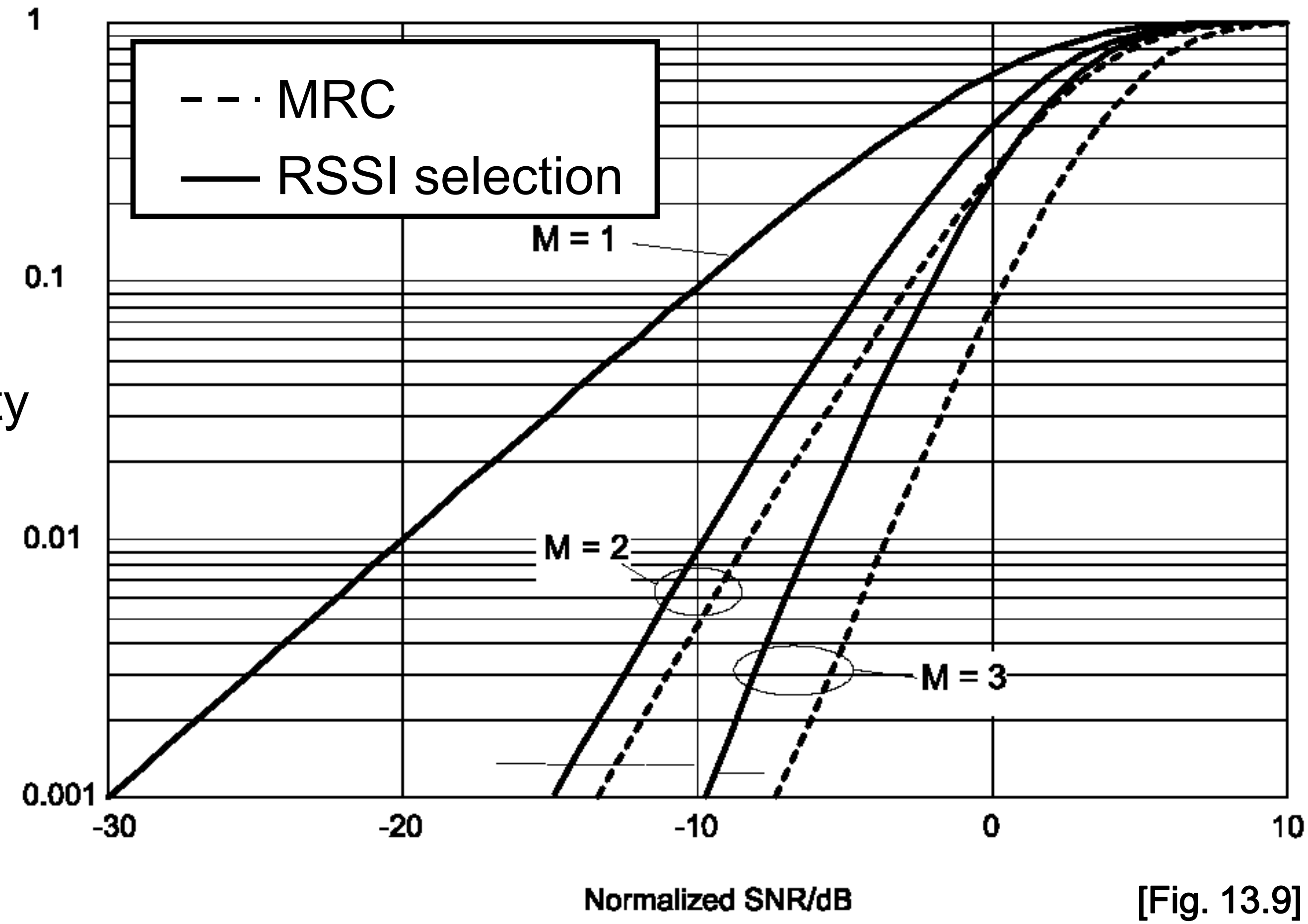
If each branch has Rayleigh distribution with mean $\bar{\gamma}$,

- Combined mean SNR $\bar{\gamma}_{\text{EGC}} = \bar{\gamma}\left(1 + (N-1)\frac{\pi}{4}\right)$

Spatial (antenna) diversity Performance comparison

Comparison of SNR distribution for different number of antennas *M* and two different diversity techniques.

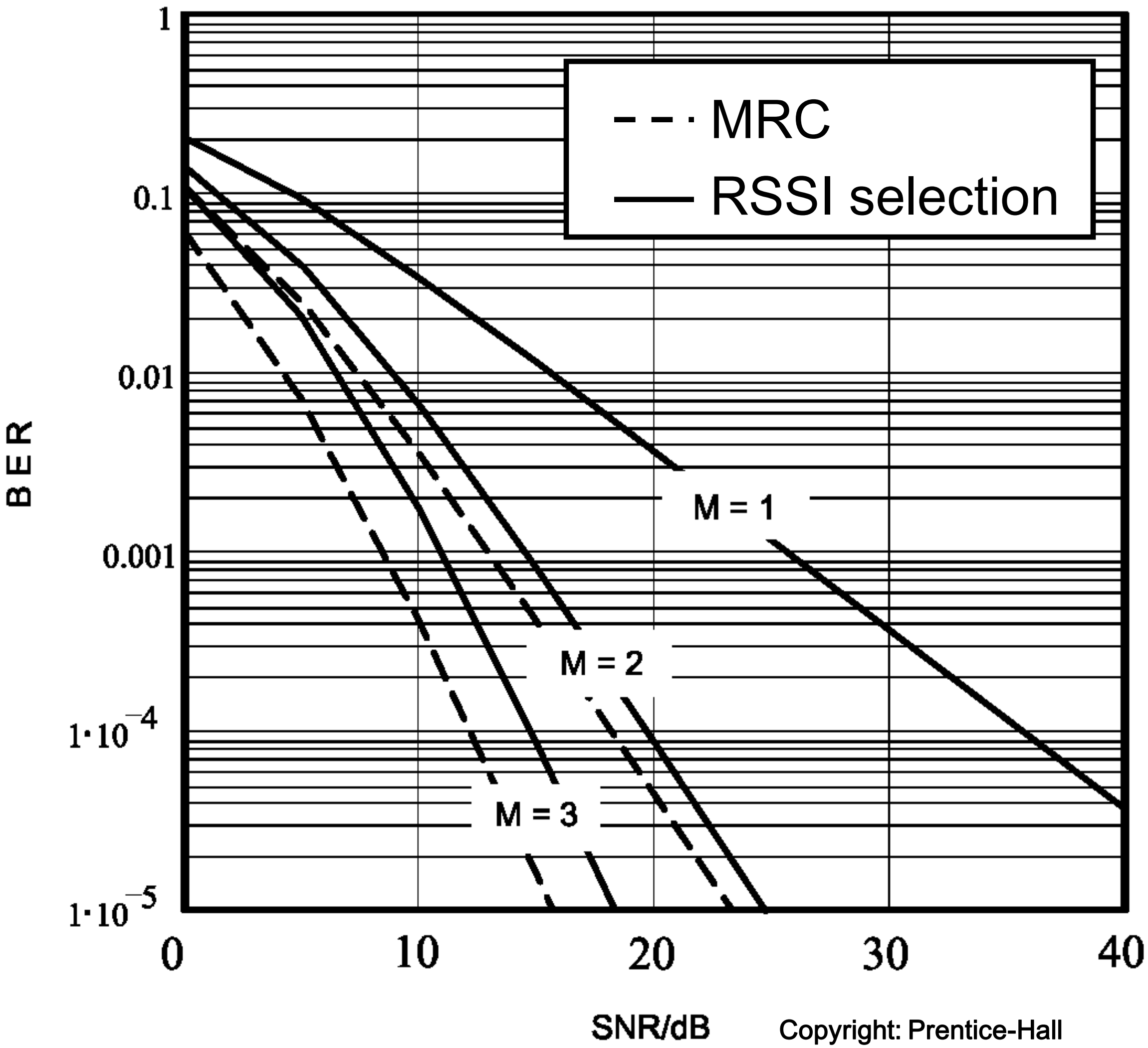
Cumulative distribution of SNR



[Fig. 13.9]

Spatial (antenna) diversity
Performance comparison, cont.

Comparison of 2ASK/2PSK BER for different number of antennas *M* and two different diversity techniques.



Example: Fading margin with SC and MRC

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- Consider a receiver with N receive antennas that uses either selection combining or maximum ratio combining. Compute the fading margin for an outage probability of 1% in an uncorrelated Rayleigh fading environment for $N = 1, 2, 3$.

Optimum combining (OC)

- Most systems interference limited
- OC reduces not only fading but also interference
- Each antenna can eliminate one interferer or give one diversity degree for fading reduction (zero-forcing)
- Signal model

$$\mathbf{y}(t) = \mathbf{h}_0 x_0(t) + \sum_{k=1}^{K} \mathbf{h}_k x_k(t) + \mathbf{n}$$

with $\mathcal{E}\{\mathbf{n}\mathbf{n}^H\} = \sigma^2 \mathbf{I}$

- Computation of weights for combining

$$\mathbf{w}_{\mathrm{opt}} = \mathbf{R}^{-1}\mathbf{h}_0, \; \mathbf{R} = \sigma_n \mathbf{I} + \sum_{k=1}^{K} \mathcal{E}\left\{\mathbf{h}_k \mathbf{h}_k^H\right\},$$

Spatial diversity: TX

- Method depends on Channel State Information at the Transmitter (CSIT)
- If available, equivalent methods as for RX can be used
  - Antenna selection
  - Beamforming (Equal gain combining)
  - Maximum Ratio Combining
- If no CSIT is available,
  - Alamouti precoding
  - Cyclic delay diversity

Alamouti precoding

- Alamouti code: transmission of two symbols s_1, s_2 over two transmit antennas and two time instances (rate 1 code)

$$\mathbf{x}_1 = \begin{pmatrix} s_1 \\ s_2 \end{pmatrix}, \quad \mathbf{x}_2 = \begin{pmatrix} -s_2^* \\ s_1^* \end{pmatrix}$$

- Received signal

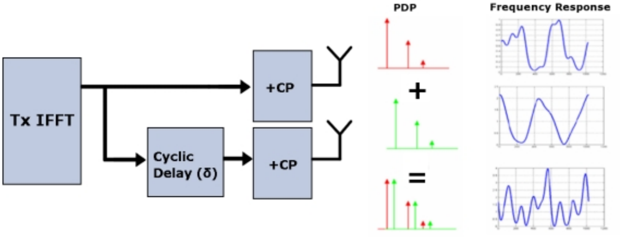
$$y_1 = \mathbf{h}\mathbf{x}_1 + n_1 = s_1 h_1 + s_2 h_2 + n_1, \quad y_2 = \mathbf{h}\mathbf{x}_2 + n_2 = -s_2^* h_1 + s_1^* h_2 + n_2$$

- Diversity combiner

$$\hat{s}_1 = (h_1^* y_1 + h_2 y_2^*)/2, \quad \hat{s}_2 = (h_2^* y_1 - h_1 y_2^*)/2$$

Cyclic Delay Diversity

- Requires OFDM with cyclic prefix
- Transforms spatial diversity into frequency diversity



Temporal and frequency diversity

- Main idea: use coding and interleaving to spread information over multiple symbols
- The symbols can be distributed in time and/or frequency
- Classical frequency diversity methods
  - Spreading (Used in CDMA systems)
  - Frequency hopping (Used in FDMA/TDMA systems)

Temporal diversity

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- Transmission over different (uncorrelated) time instances
- Repetition coding: bandwidth inefficient
- Automatic repeat request (ARQ):
  - RX informs TX about (un-)successful reception (ACK/NACK); TX repeats if necessary
  - Requires feedback channel
  - Introduces delay (however, several ARQ processes can be pipelined)
- Hybrid ARQ: exploit redundancy in channel code for different retransmissions
- Can also be combined with frequency diversity schemes

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