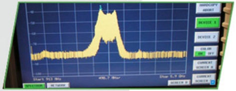


Radio Engineering

Lecture 4: Wideband channels

Florian Kaltenberger

Last lecture

5. Statistical description of fading
   - Equivalent baseband representation
   - Small scale fading without a dominat component
   - Small scale fading with a dominat component
   - Doppler spectra
   - Temporal dependence of fading
   - Large-scale fading

Repetition: Path loss

Path loss is the attenuation of the signal between transmitter and receiver.

$$P_{\mathrm{RX}} = \frac{P_{\mathrm{TX}}}{PL}$$
$$P_{\mathrm{RX}}|_{\mathrm{dB}} = P_{\mathrm{TX}}|_{\mathrm{dB}} - PL|_{\mathrm{dB}}$$

It contains the following factors

$$\mathrm{PL} = \mathrm{PL}(d) \times \mathrm{SF} \times \mathrm{SSF}$$
$$\mathrm{PL}|_{\mathrm{dB}} = \mathrm{PL}(d)|_{\mathrm{dB}} + \mathrm{SF}|_{\mathrm{dB}} + \mathrm{SSF}|_{\mathrm{dB}}$$

$\mathrm{PL}(d)$ deterministic path loss
SF large-scale (shadow) fading
SSF small scale fading

Deterministic Path Loss

$$PL(d) = \left(\frac{4\pi d}{\lambda}\right)^2 \quad 0 \le d \le d_{\text{break}}$$
$$PL(d) = PL(d_{\text{break}}) \left(\frac{d}{d_{\text{break}}}\right)^n \quad d > d_{\text{break}}$$

- distance dependent loss in signal energy
- proportional to d^n, where d is the distance and n is the path loss exponent
- typicall values $n \in [1.5, 6]$, depending on terrain and foilage

Large Scale fading

- Deviation of average received signal energy from deterministic path loss
- Averaging done over a small area (a few wavelengths)
- Is usually attributed to multiple interactions of the signal with the enviroment
- A large number of these interactions results in a log-normal distribution

$$\mathrm{pdf}(L|_{\mathrm{dB}}) = \frac{1}{\sqrt{2\pi}\sigma_F|_{\mathrm{dB}}} \exp\left(-\frac{L^2|_{\mathrm{dB}}}{2\sigma_F^2|_{\mathrm{dB}}}\right) \tag{1}$$

- This distribution matches measurements very well with $\sigma_F|_{\mathrm{dB}} \approx 4 \ldots 10$ dB.

Example: Outage probablity

Assume that, at a certain distance, we have a deterministic propagation loss of $L_0 = 127$ dB and larg-scale fading, which is log-normal distributed with $\sigma_F = 7$ dB.

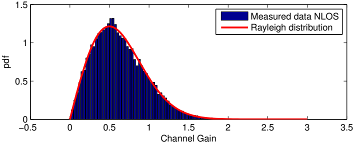
1. How large is the outage probability $P_{\text{out}} = \Pr\{L_0 + L \geq L_{\max}\}$ (due to large-scale fading) if our system is designed to handle a maximum propagation loss of $L_{\max} = 135$ dB?
2. Which of the following methods can be used to lower the outage probability? Why (not)?
   (a) Increase the transmit power
   (b) Decrease the deterministc path loss
   (c) Change the antennas
   (d) Lower σ_F
   (e) Build a better receiver

Small Scale Fading

- Small scale fading results from constructive and destructive combination of multipaths
- Isotropic scattering without dominant component $\Rightarrow$ Rayleigh Fading

$$\mathrm{pdf}(r) = \frac{r}{\sigma^2} \exp\left(-\frac{r^2}{2\sigma^2}\right), \quad 0 \leq r < \infty$$

$$\mathrm{cdf}(r) = 1 - \exp\left(-\frac{r^2}{2\sigma^2}\right)$$



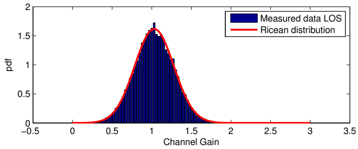
Small Scale Fading

- Dominant component (LOS) $\Rightarrow$ Ricean fading

$$\mathrm{pdf}(r) = \frac{r}{\sigma^2} \exp\left(-\frac{r^2 + A^2}{2\sigma^2}\right) I_0\left(\frac{rA}{\sigma^2}\right), \quad 0 \leq r < \infty$$

$$\mathrm{cdf}(r) = 1 - Q_M\left(\frac{A}{\sigma}, \frac{r}{\sigma}\right)$$

- The ratio of the power in the LOS component and the diffuse component $K = \frac{A^2}{2\sigma^2}$ is called the Ricean factor
- Q_M is Marcum's Q function and I_n is the modified Bessel function of the first kind, order n.



Example: Ricean Fading Margin

Compute the fading margin for a Rice distribution with $K_r = 0.3, 3,$ and 20 dB so that the outage probability is less than 5%.

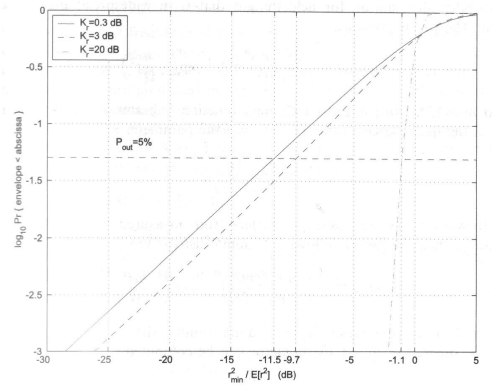


Figure 5.21 The Rice power *cdf*, $\sigma = 1$.

$$M = \frac{r_{\text{rms}}^2}{r_{\text{min}}^2} = \frac{2\sigma^2(1 + K_r)}{r_{\text{min}}^2}$$
$$= 11.5, 9.7, 1.1\text{dB}$$

Time-variant small-scale fading

- When RX (or TX) moves, channel become time-variant
- Time-variation described by autocorrelation function or Doppler Spectrum (if channel is WSS)
- Isotropic scattering without dominant component and max Doppler $\nu_{\max} \Rightarrow$ Rayleigh fading with classical Doppler spectrum (“Jakes” or “Clarkes” spectrum)

$$\rho(\Delta t) = \mathcal{E}\{h(t)h^*(t)\} = J_0(2\pi\nu_{\max}\Delta t)$$

$$S_D(\nu) = \int \rho(\Delta t)\exp(-j2\pi\nu\Delta t)\mathrm{d}t = \frac{1}{\pi\sqrt{\nu_{\max}^2 - \nu^2}}$$

This lecture

6. Wideband channels
   - Wideband vs narrowband channels
   - System theoretic description of wideband channels
   - WSSUS model
   - Condesed parameters
   - Direction channel description

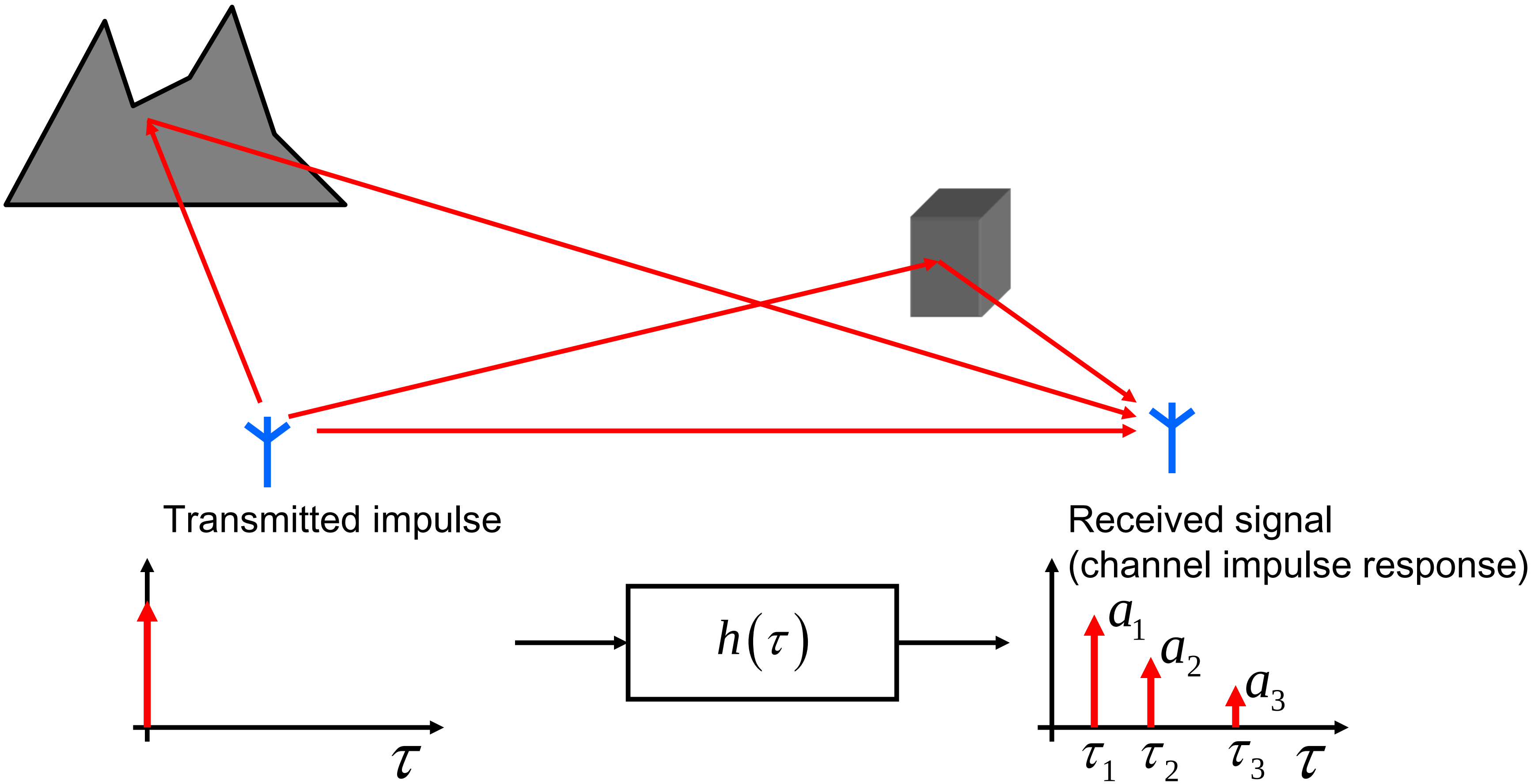
Wideband channels: definition

- So far we only looked at narrowband systems:
  - The symbol duration T_s is *larger* than the maximum delay in the channel $\Delta\tau$
  - $\Rightarrow$ Receiver cannot distinguish different echos
- A communication system is wide-band if
  - The symbol duration T_s is *smaller* than the maximum delay in the channel $\Delta\tau$
  - $\Rightarrow$ One transmitted symbol can spread over more than one symbol at the receiver

Chapter 6

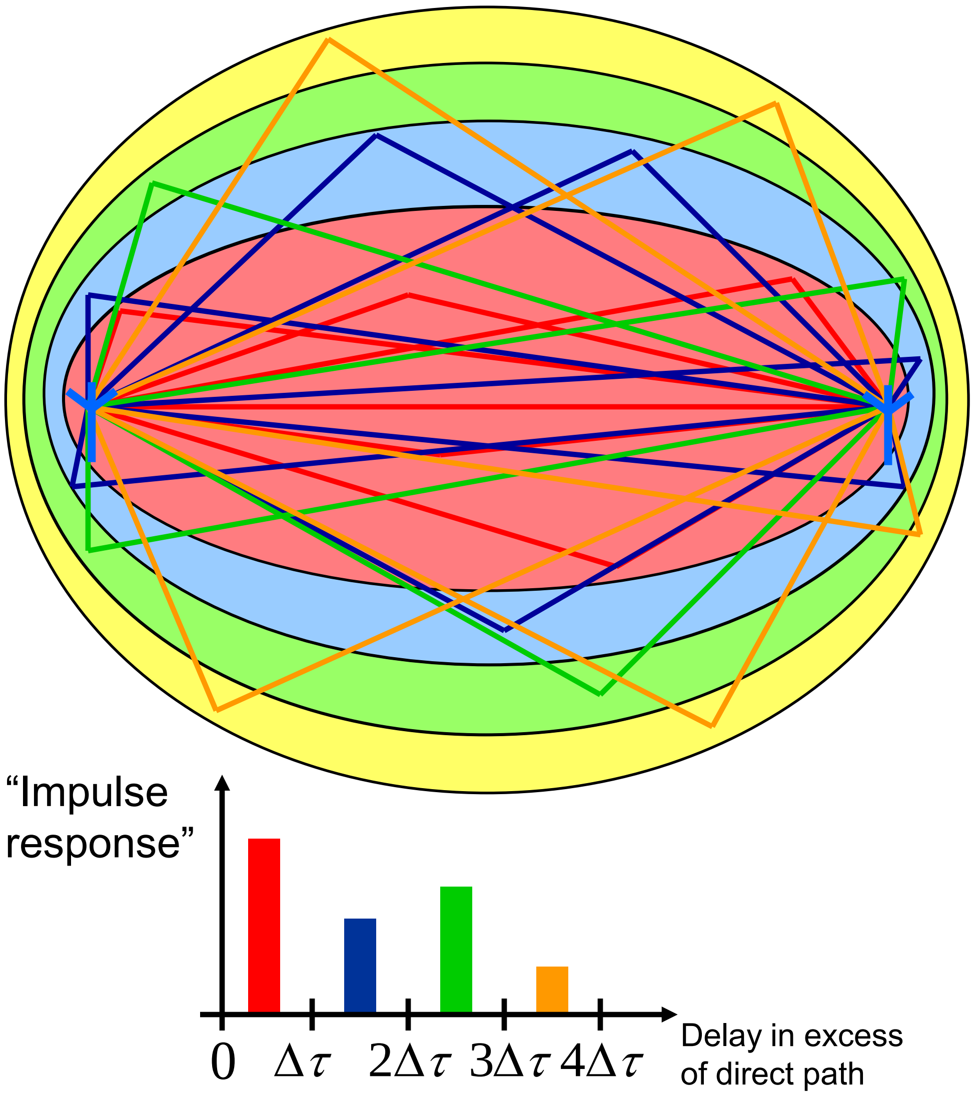
Wideband channels

Delay (time) dispersion
A simple case

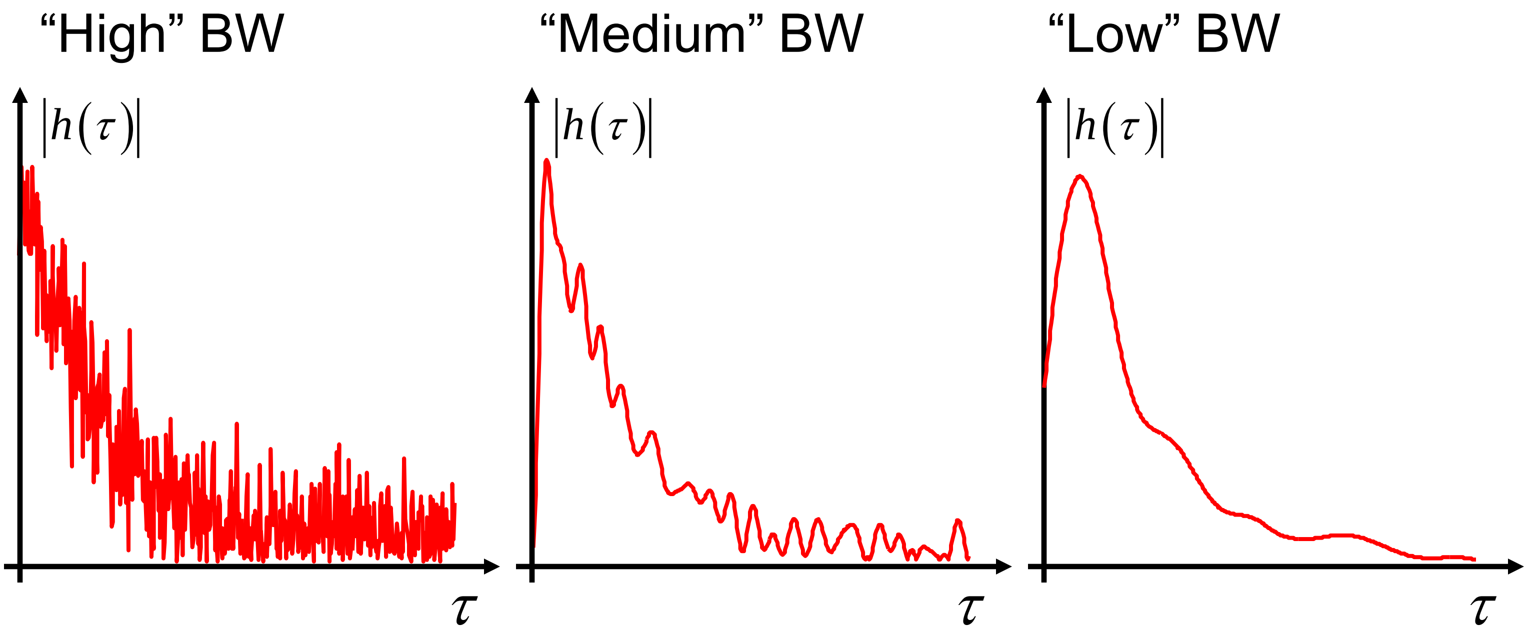


$$h(\tau) = a_1\delta(\tau - \tau_1) + a_2\delta(\tau - \tau_2) + a_3\delta(\tau - \tau_3)$$

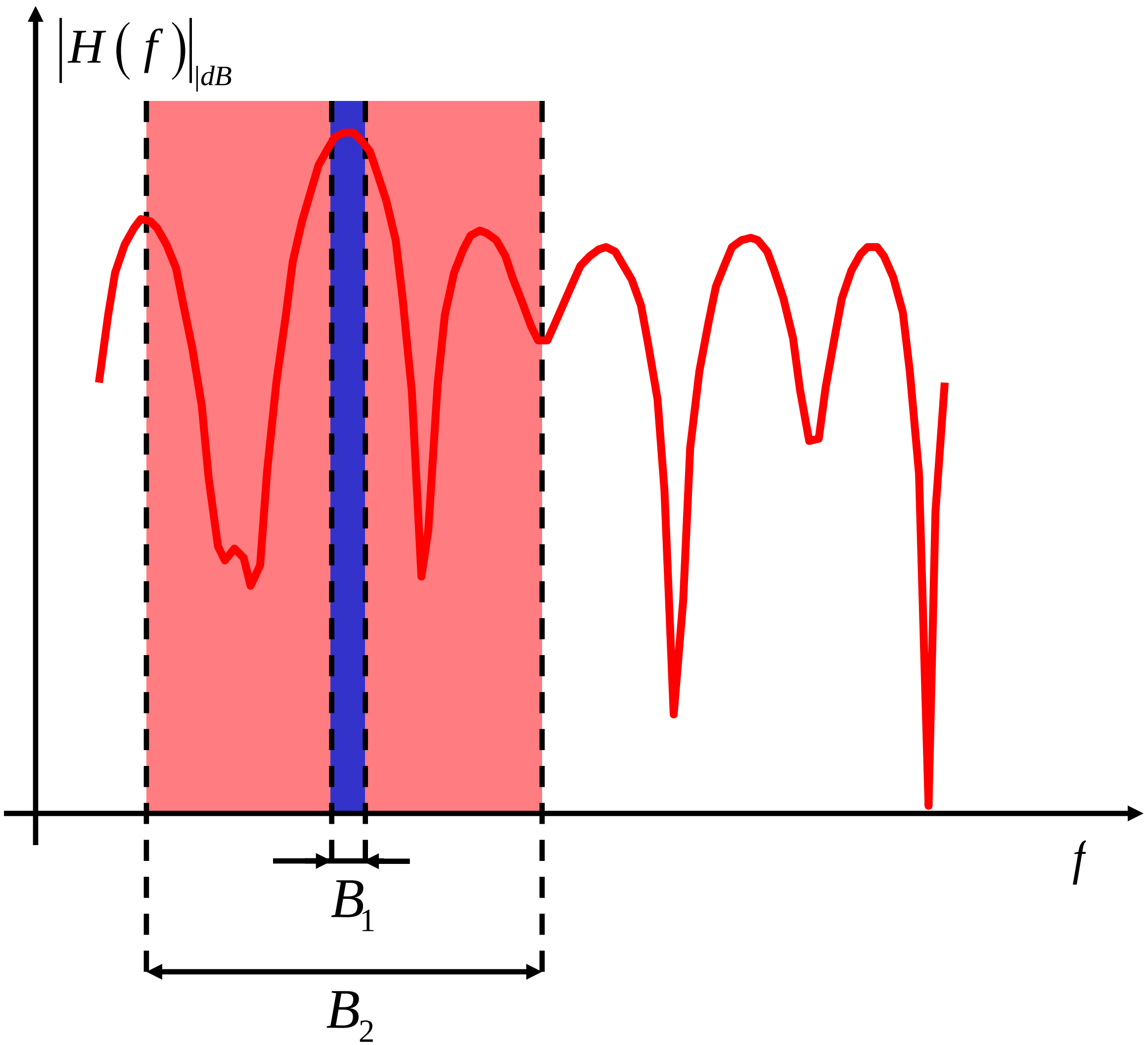
Delay (time) dispersion
One reflection/path, many paths



Narrow- versus wide-band Channel impulse response



Narrow- versus wide-band Channel frequency response



System functions (1)

- Time-variant impulse response $h(t,\tau)$
  - Due to movement, impulse response changes with time
  - Input-output relationship:

$$y(t) = \int_{-\infty}^{\infty} x(t-\tau)h(t,\tau)d\tau$$

- Time-variant transfer function *H(t,f)*
  - Perform Fourier transformation with respect to τ

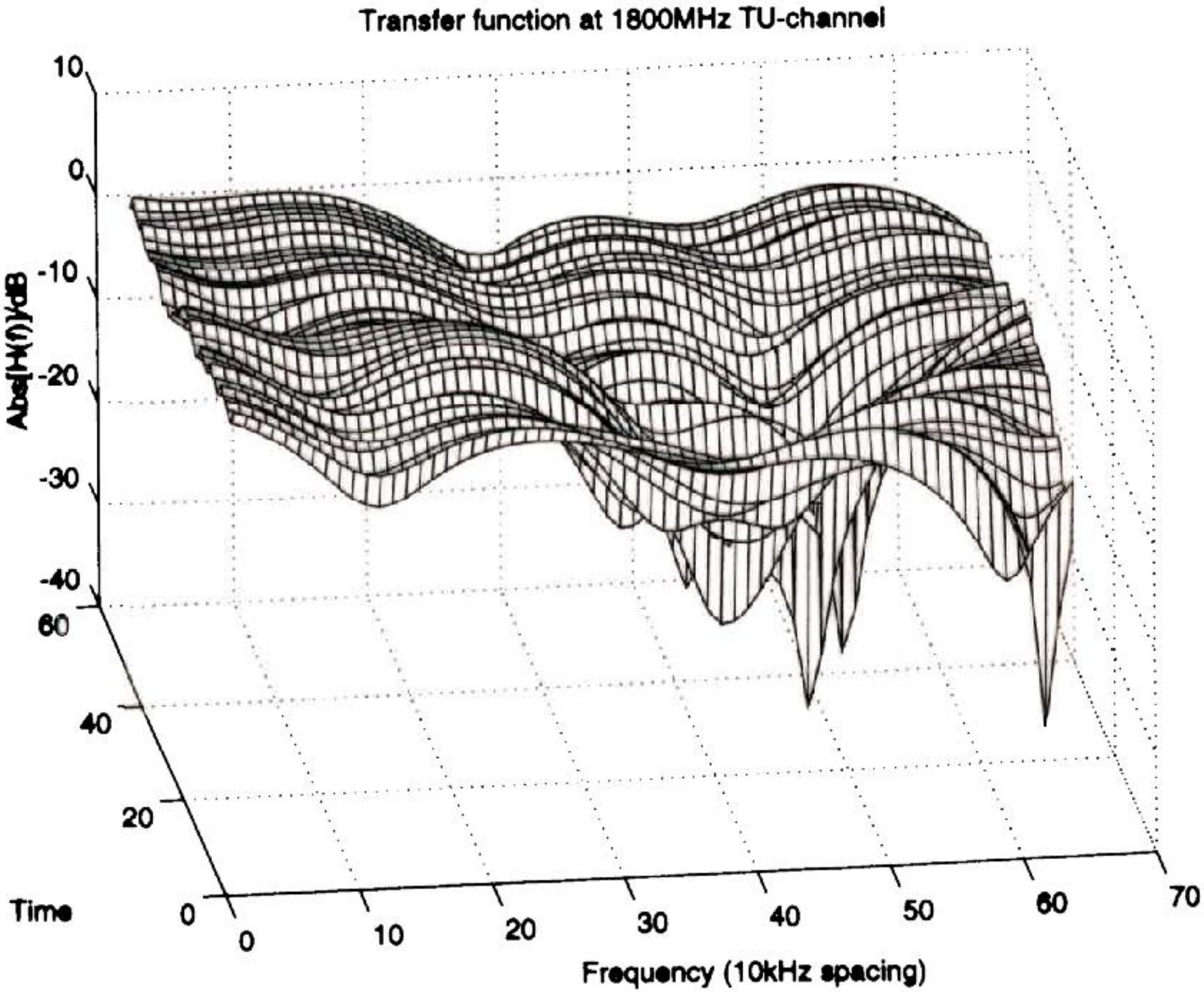
$$H(t,f) = \int_{-\infty}^{\infty} h(t,\tau)\exp(-j2\pi f\tau)d\tau$$

  - Input-output relationship

$$Y(\tilde{f}) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} X(f)H(t,f)\exp(j2\pi ft)\exp(-j2\pi\tilde{f}t)dfdt$$

becomes *Y(f)=X(f)H(f)* only in *slowly* time-varying channels

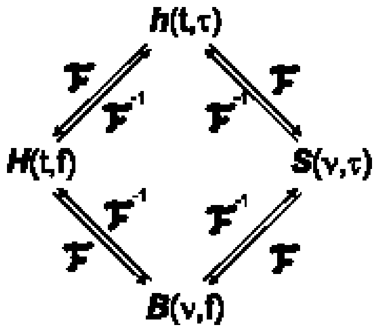
Transfer function, Typical urban



System functions (2)

- Further equivalent system functions:
  - Since impulse response depends on two variables, Fourier transformation can be done w.r.t. each of them

  -> *four* equivalent system descriptions are possible:
    - Impulse response
    - Time-variant transfer function
    - Spreading function $$S(\nu,\tau) = \int_{-\infty}^{\infty} h(t,\tau)\exp(-j2\pi\nu t)dt$$
    - Doppler-variant spreading function $$B(\nu,f) = \int_{-\infty}^{\infty} S(\nu,\tau)\exp(-j2\pi f\tau)d\tau$$



Stochastic system functions

- autocorrelation function (second-order statistics)

$$R_h(t,t',\tau,\tau') = E\{h^*(t,\tau)h(t',\tau')\}$$

- Input-output relationship:

$$R_{yy}(t,t') = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} R_{xx}(t-\tau,t'-\tau')R_h(t,t',\tau,\tau')d\tau d\tau'$$

The WSSUS model: mathematics

- If WSSUS is valid, ACF depends only on two variables (instead of four)
- ACF of impulse response becomes

$$R_h(t, t+\Delta t, \tau, \tau') = \delta(\tau - \tau') P_h(\Delta t, \tau)$$

$$P_h(\Delta t, \tau) \text{....} \textit{Delay cross power spectral density}$$

- ACF of transfer function

$$R_H(t+\Delta t, f+\Delta f) = R_H(\Delta t, \Delta f)$$

- ACF of spreading function

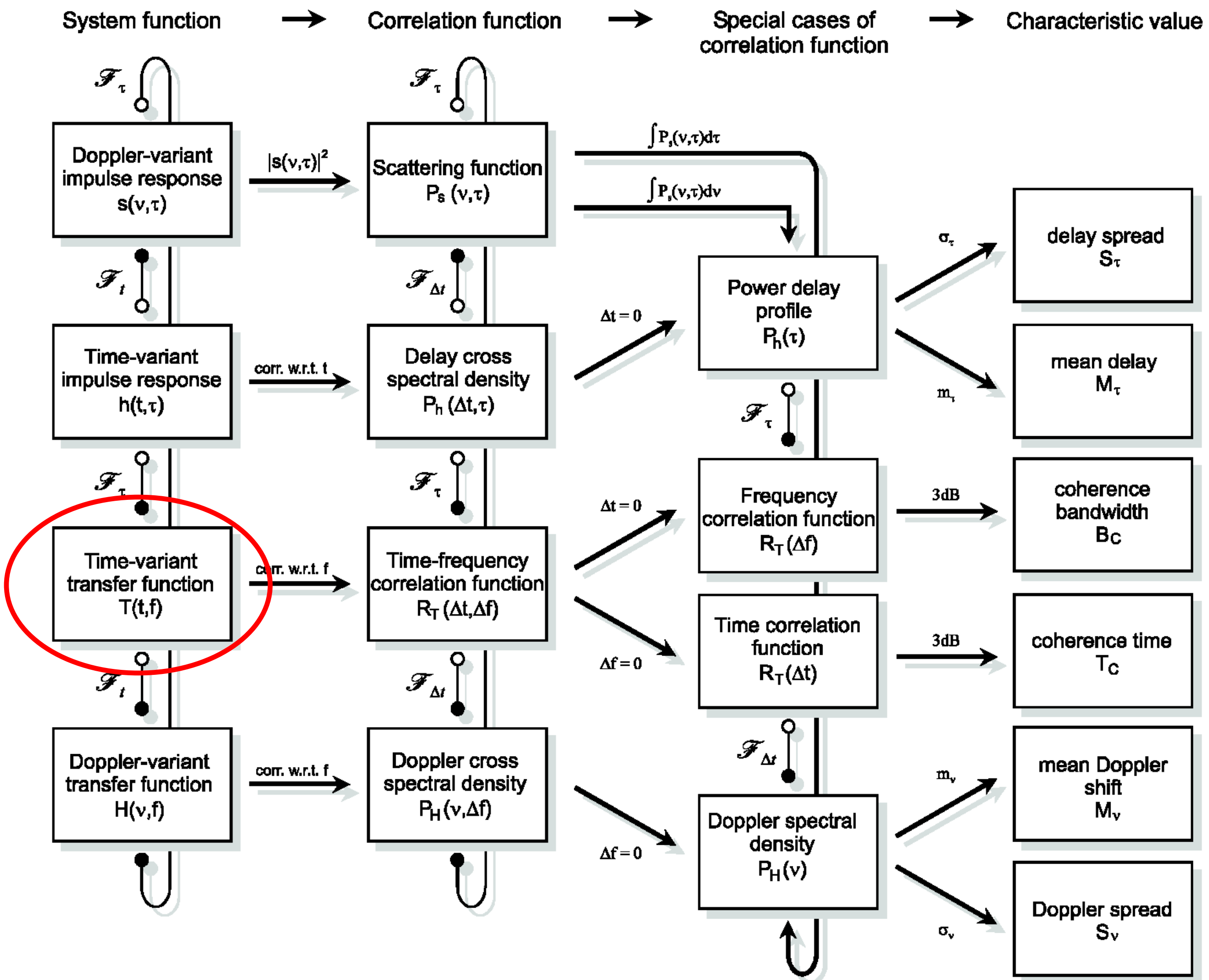
$$R_s(\nu, \nu', \tau, \tau') = \delta(\nu - \nu')\delta(\tau - \tau') P_s(\nu, \tau)$$

$$P_s(\nu, \tau) \text{.......} \textit{Scattering function}$$

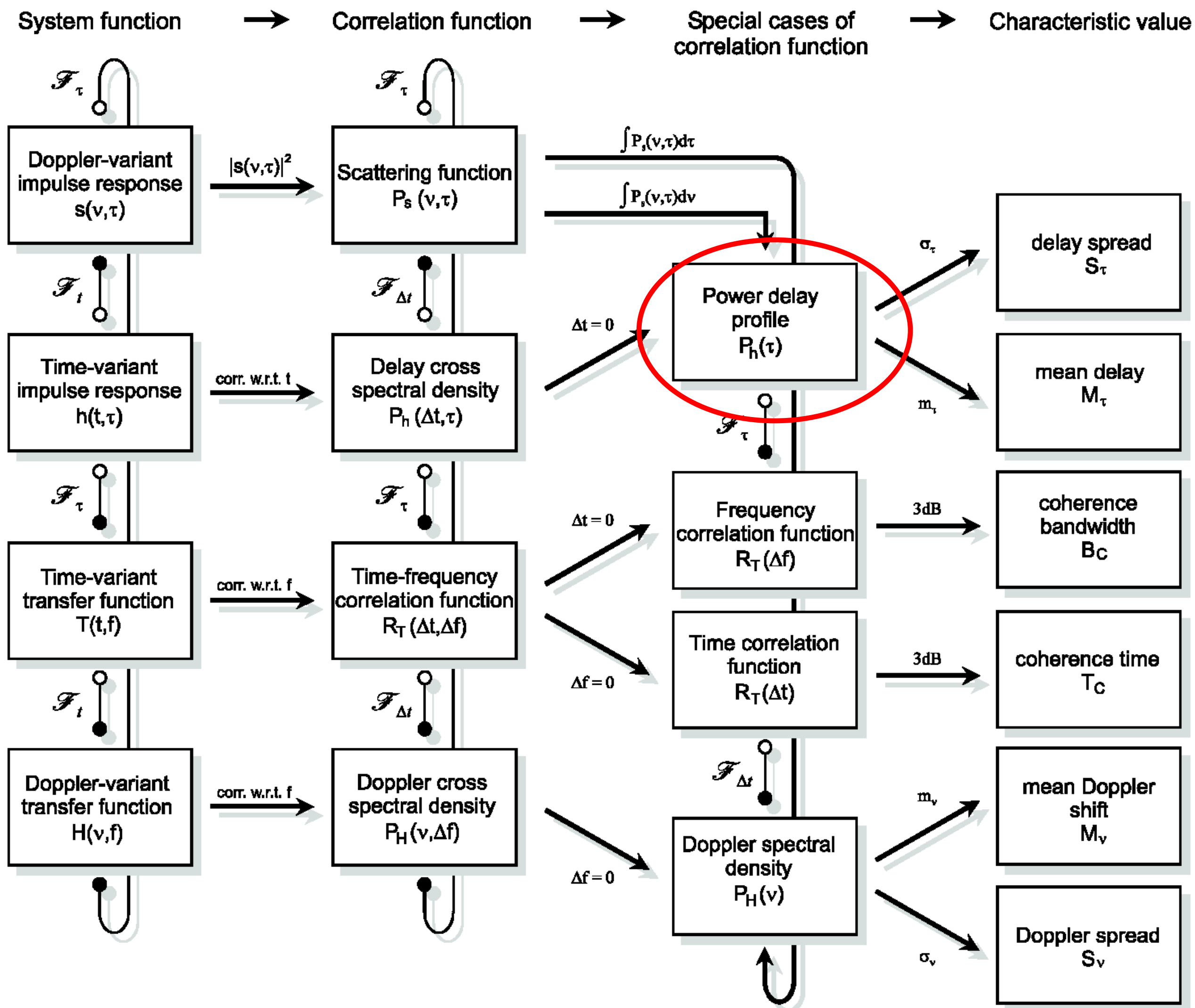
Condensed parameters

- Correlation functions depend on two variables
- For concise characterization of channel, we desire
  - A function depending on one variable or
  - A single (scalar) parameter
- Most common condensed parameters
  - Power delay profile
  - Rms delay spread
  - Coherence bandwidth
  - Doppler spread
  - Coherence time

Channel measures



Channel measures



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Condensed parameters Power-delay profile

One interesting channel property is the **power-delay profile** (PDP), which is the expected value of the received power at a certain delay:

$$P(\tau) = \mathrm{E}_t\left[\left|h(t,\tau)\right|^2\right]$$

E_t denotes expectation over time.

For our tapped-delay line we get:

$$P(\tau) = \mathrm{E}_t\left[\left|\sum_{i=1}^{N} \alpha_i(t) \qquad \delta(\tau-\tau_i)\right|^2\right]$$

Rayleigh distributed

$$= \sum_{i=1}^{N} \mathrm{E}_t\left[\alpha_i^2(t)\right]\delta(\tau-\tau_i) = \sum_{i=1}^{N} 2\sigma_i^2\delta(\tau-\tau_i)$$

Average power of tap i.

Condensed parameters
Power-delay profile (cont.)

We can “reduce” the PDP into more compact descriptions of the channel:

Total power (time integrated):

$$P_m = \int_{-\infty}^{\infty} P(\tau)\, d\tau$$

Average mean delay:

$$T_m = \frac{\int_{-\infty}^{\infty} \tau P(\tau)\, d\tau}{P_m}$$

Average rms delay spread:

$$S = \sqrt{\frac{\int_{-\infty}^{\infty} \tau^2 P(\tau)\, d\tau}{P_m} - T_m^2}$$

For our tapped-delay line channel:

$$P_m = \sum_{i=1}^{N} 2\sigma_i^2$$

$$T_m = \frac{\sum_{i=1}^{N} \tau_i 2\sigma_i^2}{P_m}$$

$$S = \sqrt{\frac{\sum_{i=1}^{N} \tau_i^2 2\sigma_i^2}{P_m} - T_m^2}$$

Condensed parameters
Frequency correlation

A property closely related to the power-delay profile (PDP) is the **frequency correlation** of the channel. It is in fact the Fourier transform of the PDP:

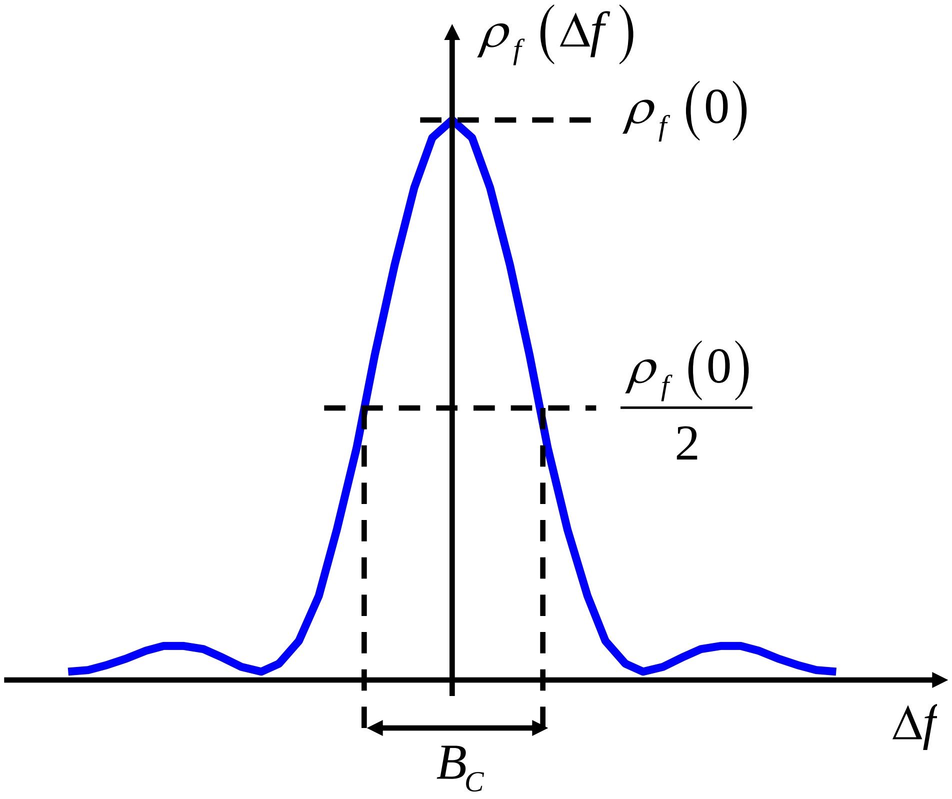
$$\rho_f(\Delta f) = \int_{-\infty}^{\infty} P(\tau)\exp(-j2\pi\Delta f\,\tau)\,d\tau$$

For our tapped delay-line channel we get:

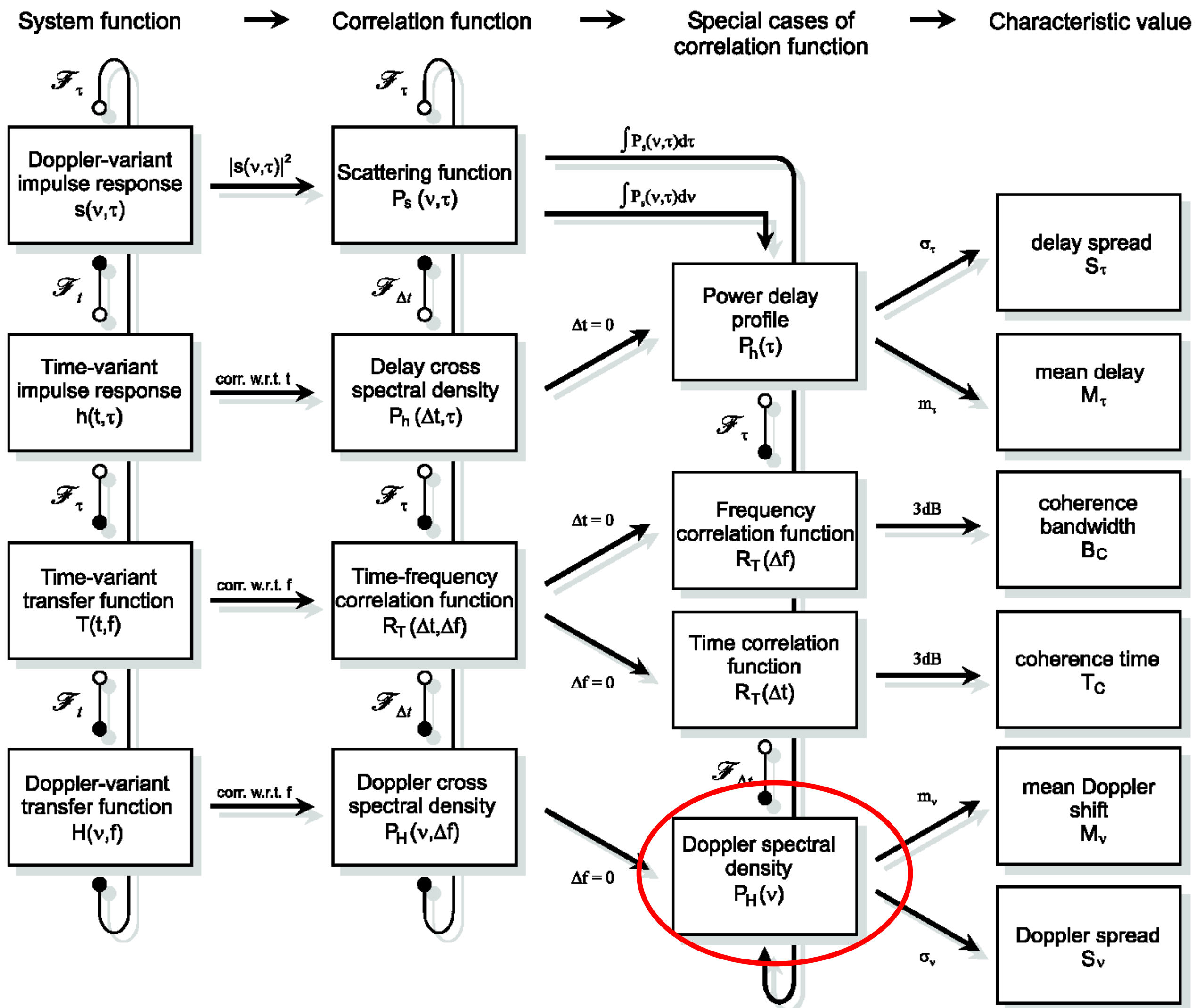
$$\rho_f(\Delta f) = \int_{-\infty}^{\infty}\left(\sum_{i=1}^{N} 2\sigma_i^2\delta(\tau-\tau_i)\right)\exp(-j2\pi\Delta f\,\tau)\,d\tau$$

$$= \sum_{i=1}^{N} 2\sigma_i^2 \exp(-j2\pi\Delta f\,\tau_i)$$

Condensed parameters
Coherence bandwidth



Channel measures



Condensed parameters
The Doppler spectrum

Given the scattering function P_s (doppler spectrum as function of delay) we can calculate a total **Doppler spectrum** of the channel as:

$$P_B(\nu) = \int P_S(\nu, \tau)\, d\tau$$

For our tapped delay-line channel, we have:

Doppler spectrum of tap *i*.

$$P_S(\nu, \tau) = \frac{2\sigma_i^2}{\pi\sqrt{\nu_{i,\max}^2 - \nu^2}} \delta(\tau - \tau_i)$$

$$P_B(\nu) = \int_{-\infty}^{\infty} \frac{2\sigma_i^2}{\pi\sqrt{\nu_{i,\max}^2 - \nu^2}} \delta(\tau - \tau_i)\, d\tau$$

$$= \sum_{i=1}^{N} \frac{2\sigma_i^2}{\pi\sqrt{\nu_{i,\max}^2 - \nu^2}}$$

Condensed parameters
The Doppler spectrum (cont.)

We can "reduce" the Doppler spectrum into more compact descriptions of the channel:

Total power (frequency integrated):

$$P_{B,m} = \int_{-\infty}^{\infty} P_B(\nu)\, d\nu$$

Average mean Doppler shift:

$$T_{B,m} = \frac{\int_{-\infty}^{\infty} \nu P_B(\nu)\, d\nu}{P_{B,m}}$$

Average rms Doppler spread:

$$S_B = \sqrt{\frac{\int_{-\infty}^{\infty} \nu^2 P(\nu)\, d\nu}{P_{B,m}} - T_{B,m}^2}$$

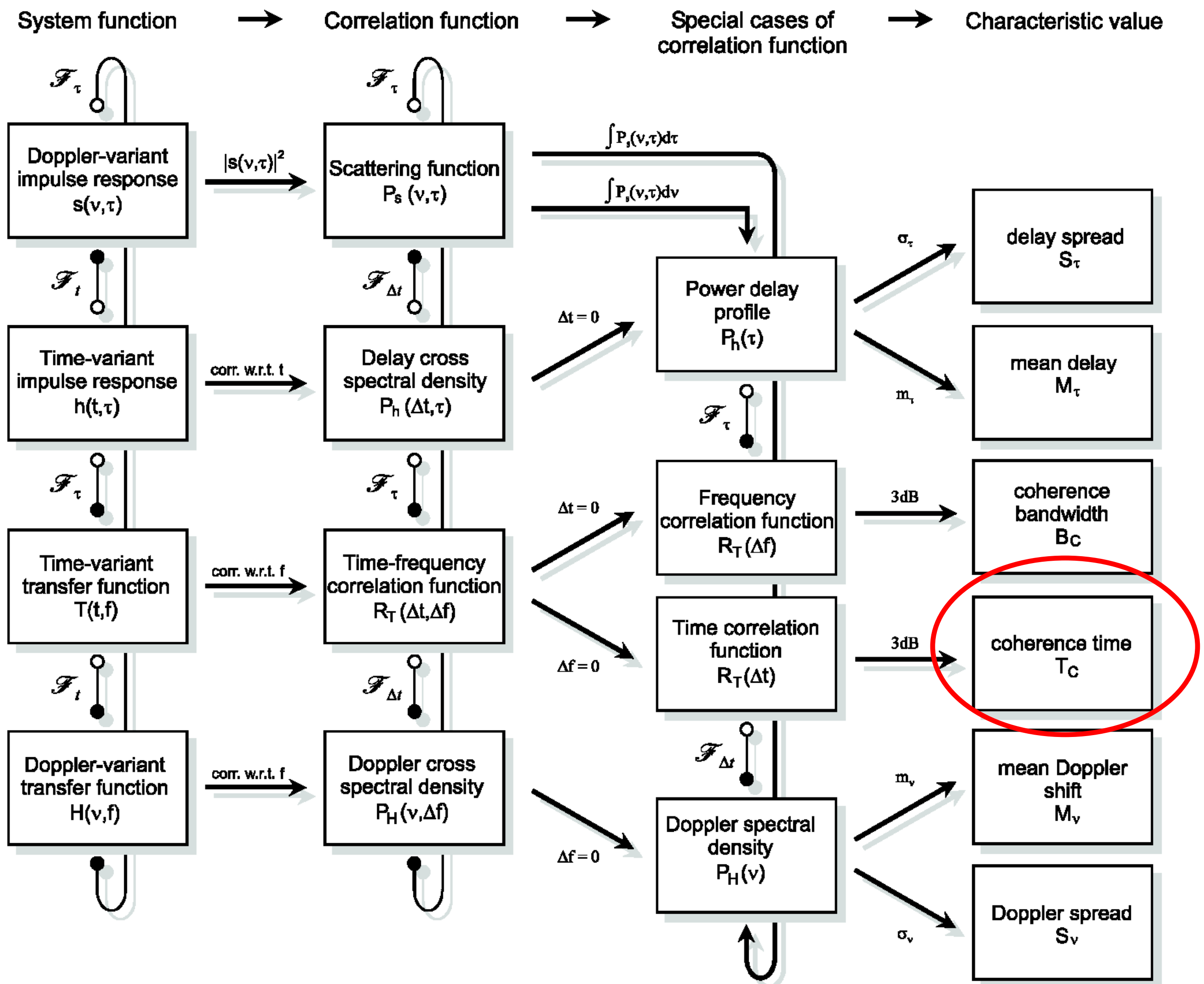
For our tapped-delay line channel:

$$P_{B,m} = \sum_{i=1}^{N} 2\sigma_i^{\,2}$$

$$T_{B,m} = 0$$

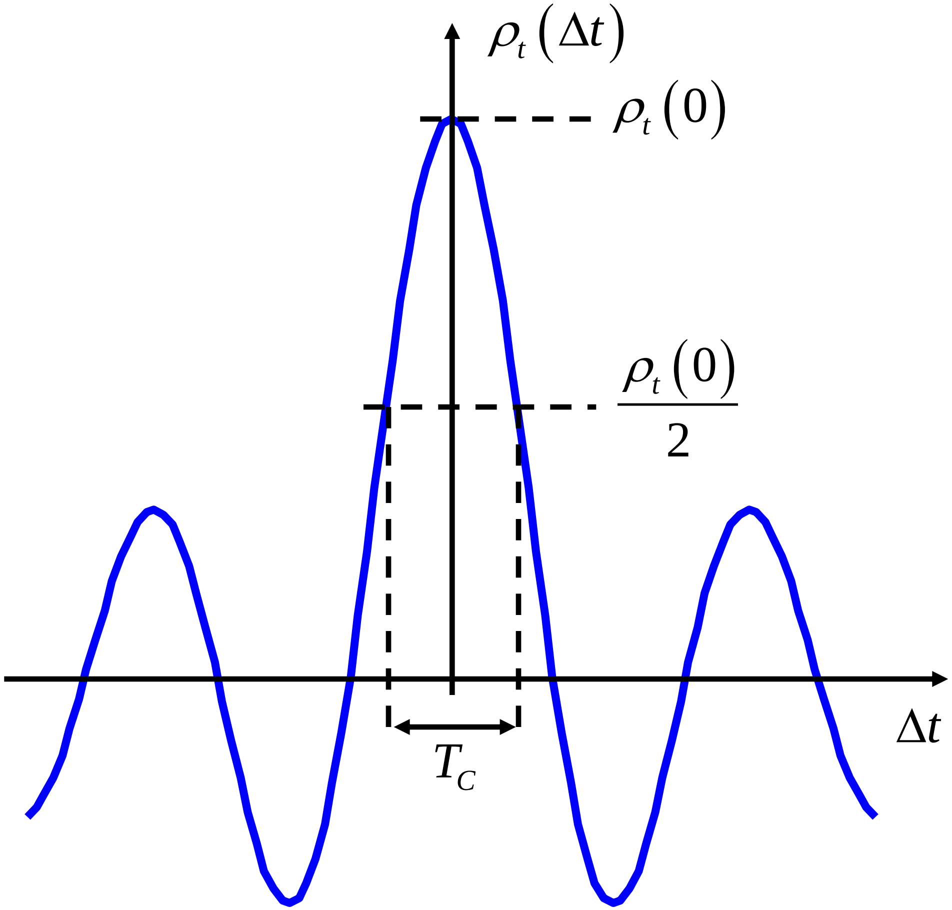
$$S_B = \sqrt{\frac{\sum_{i=1}^{N} \sigma_i^{\,2} \nu_{i,\max}^{\;\;2}}{P_{B,m}}}$$

Channel measures



Condensed parameters
Coherence time

Given the time correlation of a channel, we can define the **coherence time** T_C:



Condensed parameters
The time correlation

A property closely related to the Doppler spectrun is the **time correlation** of the channel. It is in fact the inverse Fourier transform of the Doppler spectrum:

$$\rho_t(\Delta t) = \int_{-\infty}^{\infty} P_B(\nu) \exp(j2\pi\nu\Delta t)\, d\nu$$

For our tapped-delay line channel we get

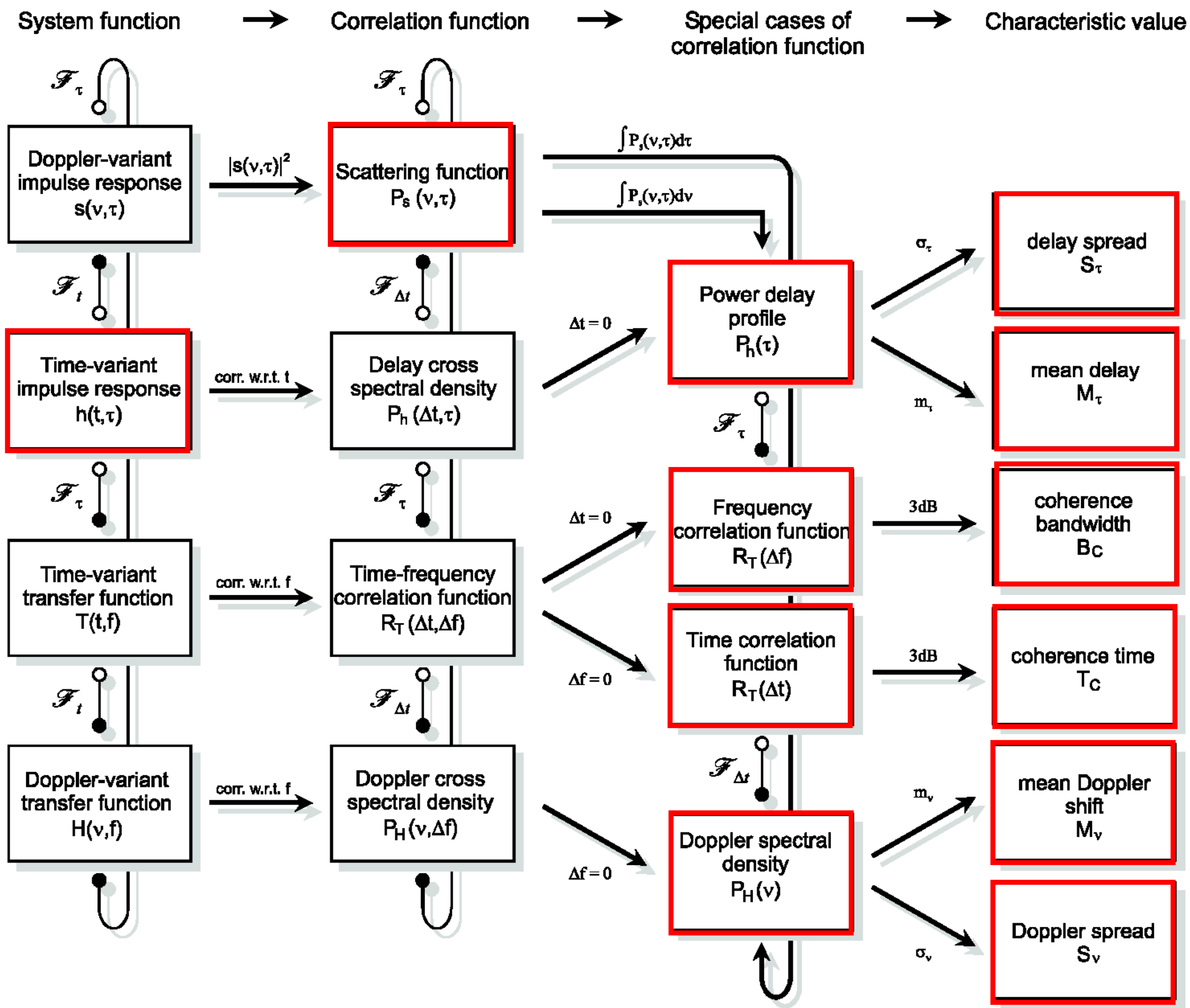
$$\rho_t(\Delta t) = \int_{-\infty}^{\infty} \sum_{i=1}^{N} \frac{2\sigma_i^{\,2}}{\pi\sqrt{\nu_{i,\max}^{\,2} - \nu^2}} \exp(j2\pi\nu\Delta t)\, d\nu$$

$$= \sum_{i=1}^{N} \int_{-\infty}^{\infty} \frac{2\sigma_i^{\,2}}{\pi\sqrt{\nu_{i,\max}^{\,2} - \nu^2}} \exp(j2\pi\nu\Delta t)\, d\nu$$

$$= \sum_{i=1}^{N} 2\sigma_i^{\,2} J_0(2\pi\nu_{i,\max}\Delta t)$$

Sum of time correlations for each tap.

It's much more complicated than what we have discussed!



Properties of Wideband Channels

- In wideband communication systems, multipath propagation results in inter-symbol interference (ISI)
- Wideband channels fade also in frequency, causing *frequency-selective* fading
- Wideband channels can be mathematically described as linear time-variant systems (LTV)
- Wideband channels are wide-sense stationary (WSS) with uncorrelated scattering (US) iff
  1. their second order statistics (autocorrelation function) do not change over time
  2. contributions with different delays are uncorrelated

WSS-US channels: discussion

- The WSSUS assumption greatly simplifies many calculations, and is thus very popular
- However, it should be used with great care and applicability should always be checked
- In order to practically compute statistics (expectations), channels need to be *ergodic* (i.e., ensemble average can be interchanged with time average)
- However, in theory ergodic channels are always WSS
- Can be resolved in practice by defining stationarity regions, or quasi-stationarity

Double directional impulse response

TX position

RX position

number of multipath components
for these positions

$$h(t,\vec{r}_{\mathrm{TX}},\vec{r}_{\mathrm{RX}},\tau,\Omega,\Psi) = \sum_{\ell=1}^{N(\vec{r})} h_\ell(t,\vec{r}_{\mathrm{TX}},\vec{r}_{\mathrm{RX}},\tau,\Omega,\Psi)$$

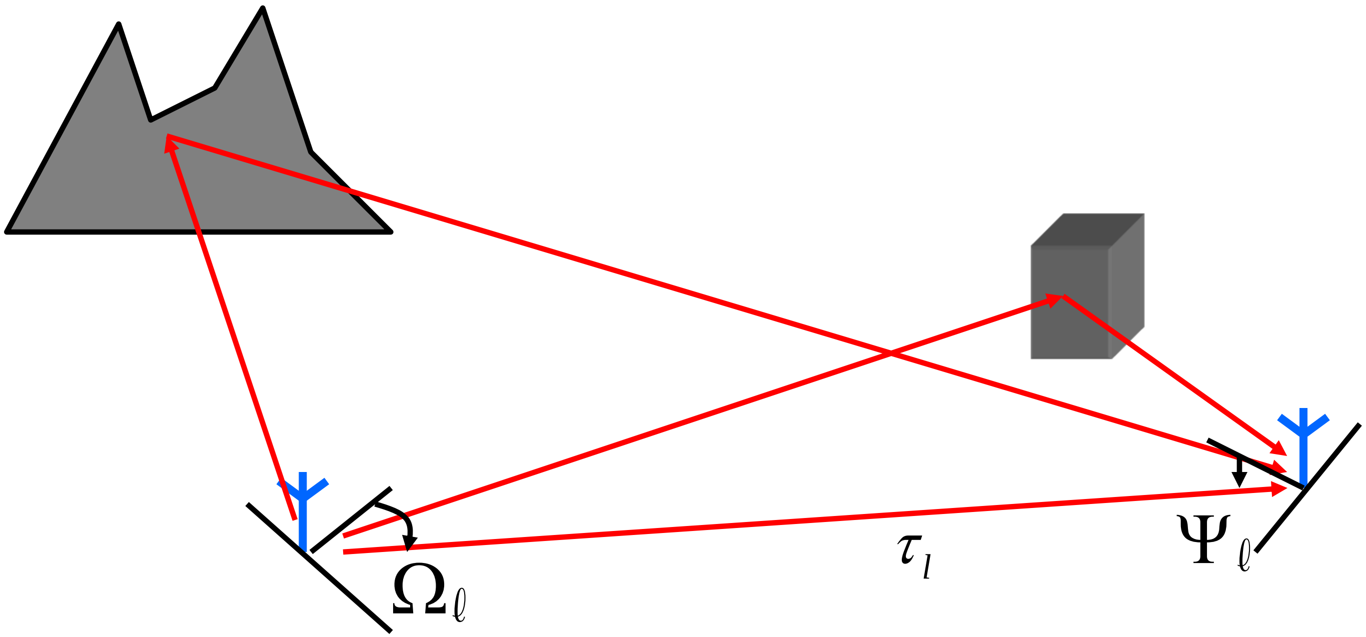
direction-of-arrival

delay

direction-of-departure

$$h_\ell(t,\vec{r}_{\mathrm{TX}},\vec{r}_{\mathrm{RX}},\tau,\Omega,\Psi) = |a_\ell| e^{j\varphi_\ell}\delta(\tau-\tau_\ell)\delta(\Omega-\Omega_\ell)\delta(\Psi-\Psi_\ell)$$

Physical interpretation



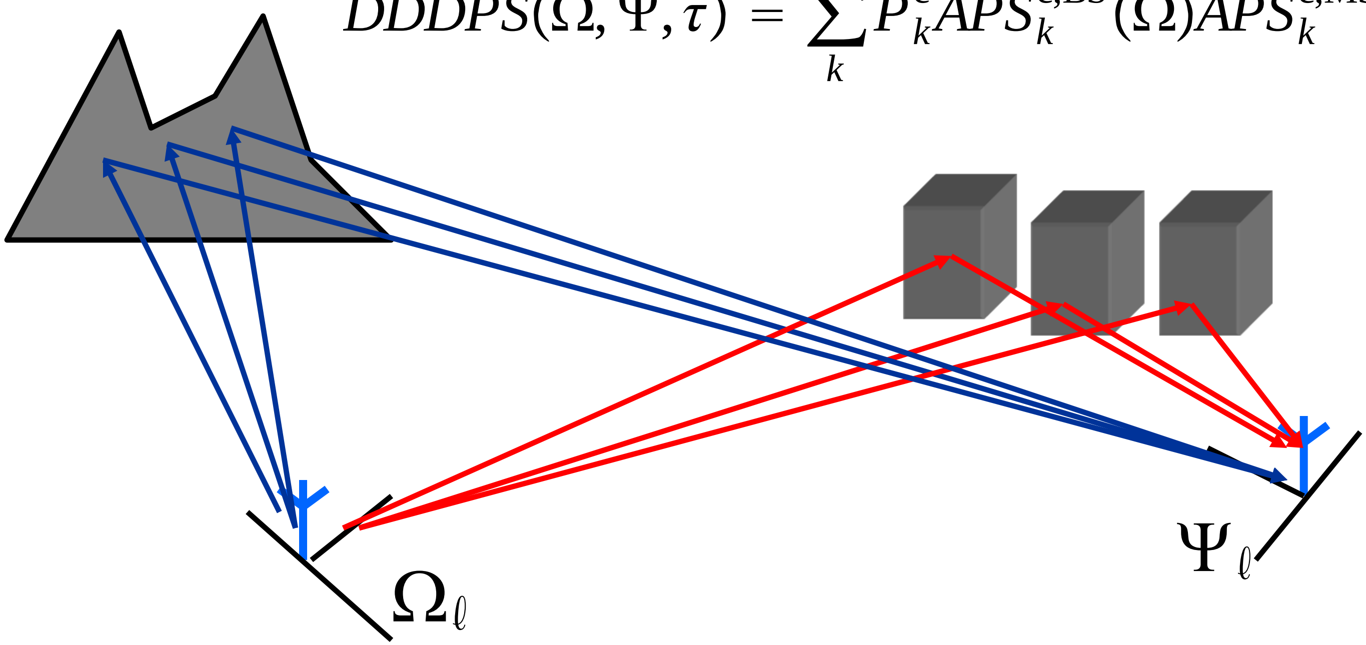
Directional models

- The double directional delay power spectrum is sometimes factorized w.r.t. DoD, DoA and delay.

$$DDDPS(\Omega, \Psi, \tau) = APS^{\mathrm{BS}}(\Omega) APS^{\mathrm{MS}}(\Psi) PDP(\tau)$$

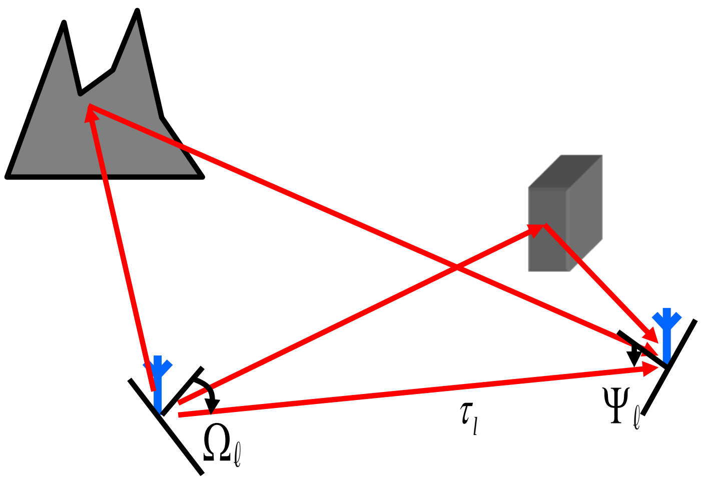
- Often in reality there are groups of scatterers with similar DoD and DoA – clusters

$$DDDPS(\Omega, \Psi, \tau) = \sum_k P_k^{\mathrm{c}} APS_k^{\mathrm{c,BS}}(\Omega) APS_k^{\mathrm{c,MS}}(\Psi) PDP_k^{\mathrm{c}}(\tau)$$



Angular spread

$$E\{s^*(\Omega,\Psi,\tau,v)s(\Omega',\Psi',\tau',v')\} = P_s(\Omega,\Psi,\tau,v)\delta(\Omega-\Omega')\delta(\Psi-\Psi')\delta(\tau-\tau')\delta(v-v')$$



double directional delay power spectrum

$$DDDPS(\Omega,\Psi,\tau) = \int \mathrm{P_s}(\Psi,\Omega,\tau,\nu)\mathrm{d}\nu$$

angular delay power spectrum

$$ADPS(\Omega,\tau) = \int DDDPS(\Psi,\Omega,\tau)G_{\mathrm{MS}}(\Psi)d\Psi$$

angular power spectrum

$$APS(\Omega) = \int \mathrm{APDS}(\Omega,\tau)\mathrm{d}\tau$$

power

$$P = \int \mathrm{APS}(\Omega)\mathrm{d}\Omega$$

References

P. Bello,
"Characterization of randomly time-variant linear channels,"
Communications Systems, IEEE Transactions on, vol. 11, no. 4, pp. 360 –393, 1963.