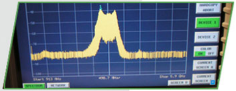


Radio Engineering

Lecture 7: Channel Models

Florian Kaltenberger

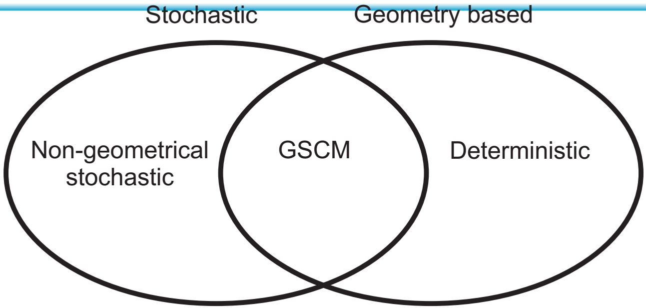
Last lecture

6. Multiple-Input Multiple-Output (MIMO) channels
    - Definitions
    - System model
    - Mutual coupling and correlation
    - Double directional channel characterization
    - Angular power spectra
7. Channel Sounding
    - Time and frequency domain sounding
    - Directionally resolved measurements
    - Parameter estimation methods

This lecture

8. Channel Models
   - Overview
   - Stochastic models
   - Geometry based models
9. Channel Simulation
   - Sampled channels
   - Correlation simulation
   - Geometry based simulation

Channel Model Overview



- Non-geometrical stochastic describe the statistics of the channel via power spectra or correlation function
- Geometry based stochastic channel models (GSCM) describe the environment (scatterer, etc.) in a stochastic way
- Deterministic channel models are either based on Maxwells equations or on stored impulse responses
- Many models contain a mix

Channel Model Overview

- Remember: channel model can be separated in 3 parts
  - Path loss
  - Large scale fading
  - Small scale fading
- Different models can be applied to different parts (but don't have to)
- Channel model depends on type of channel and needs
  - Narrowband vs wideband channels
  - Time-variant vs time-invariant
  - SISO vs MIMO

Path loss and shadow fading

EURECOM

Path loss

- Mostly modeled deterministically (as function of distance)
- Proportional to $1/d^n$, where n is the propagation exponent
- $n \in [2, 6]$, may be different at different distances
- Example: two-path model with ground reflection (and its approximation leading to $n = 2$ up to d_{break} and $n = 4$ after)

Shadow fading

- Mostly modeled stochastically
- Log-normal distribution (normal distr. in dB scale) with $\sigma \in [4, 10]$dB

Okumura's path loss measurements

Extensive measurement campaign in Japan in the 1960's.

Parameters varied during measurements:

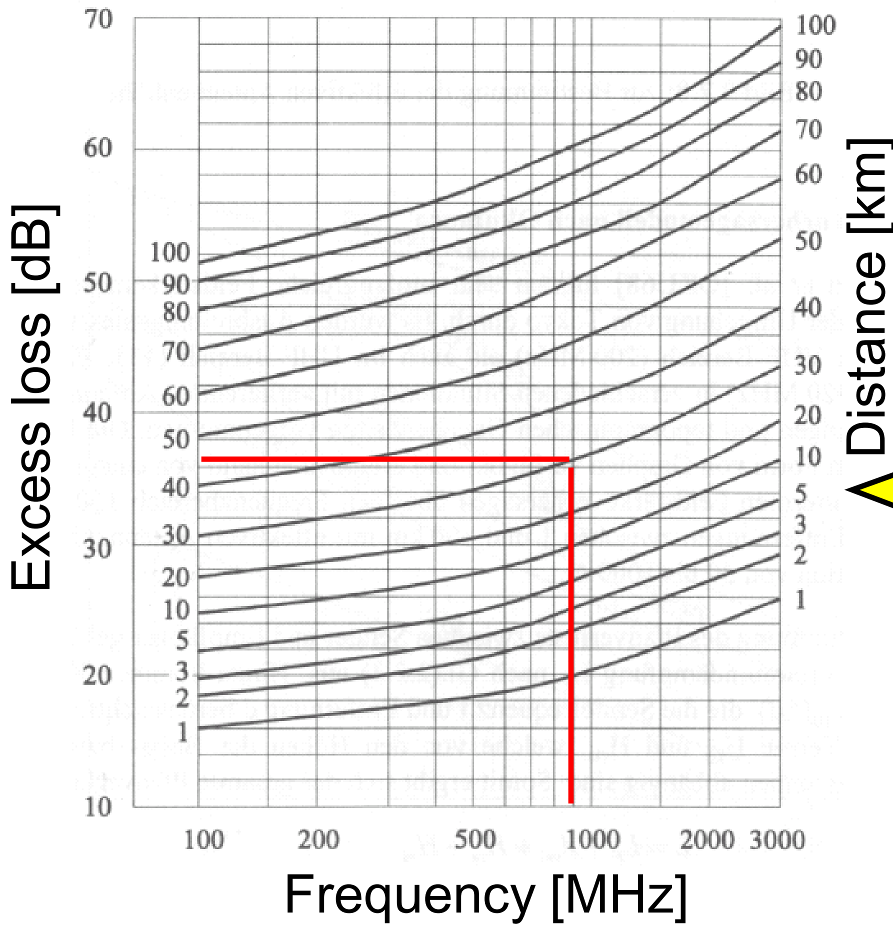
Frequency	100 – 3000 MHz
Distance	1 – 100 km
Mobile station height	1 – 10 m
Base station height	20 – 1000 m
Environment	medium-size city, large city, etc.

Propagation loss is given as **median** values (50% of the time and 50% of the area).

Okumura's measurements excess loss

Example

FIGURE 7.12



These curves are only for h_b=200 m and h_m=3 m

900 MHz and
30 km distance

From [Okumura et al.]

The Okumura-Hata model
How to calculate prop. loss

$$L_{O-H} = A + B\log\left(d_{|km}\right) + C$$

$$A = 69.55 + 26.16\log\left(f_{0|MHz}\right) - 13.82\log\left(h_b\right) - a\left(h_m\right)$$

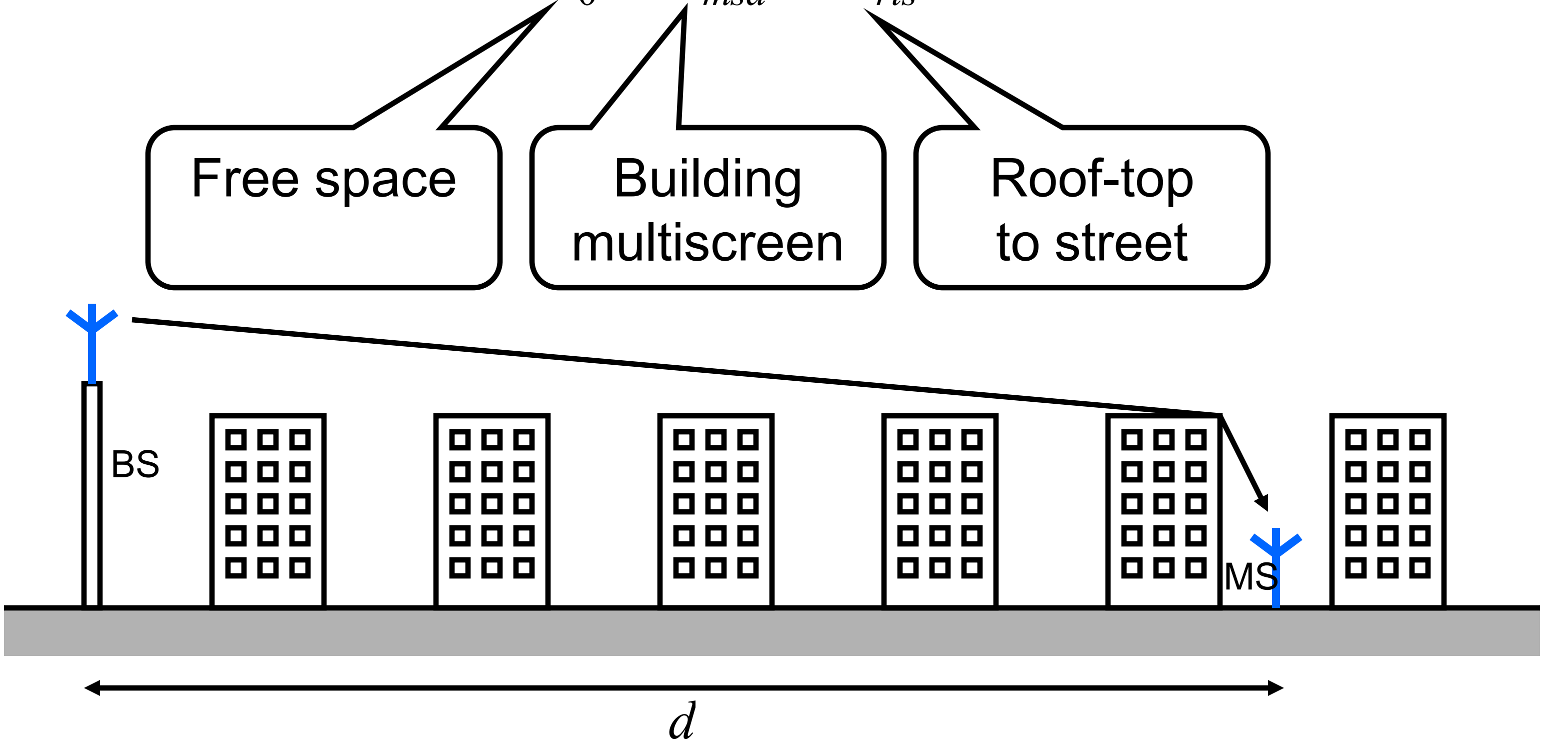
$$B = 44.9 - 6.55\log\left(h_b\right)$$

h_b and h_m in meter

	$a\left(h_m\right) =$	$C =$
Metropolitan areas	$8.29\left(\log\left(1.54h_m\right)\right)^2 - 1.1$ for $f_0 \le 200$ MHz $3.2\left(\log\left(11.75h_m\right)\right)^2 - 4.97$ for $f_0 \ge 400$ MHz	0
Small/medium-size cities	$\left(1.1\log\left(f_{0\|MHz}\right) - 0.7\right)h_m - \left(1.56\log\left(f_{0\|MHz}\right) - 0.8\right)$	0
Suburban environments	$\left(1.1\log\left(f_{0\|MHz}\right) - 0.7\right)h_m - \left(1.56\log\left(f_{0\|MHz}\right) - 0.8\right)$	$-2\left[\log\left(f_{0\|MHz}/28\right)\right]^2 - 5.4$
Rural areas	$\left(1.1\log\left(f_{0\|MHz}\right) - 0.7\right)h_m - \left(1.56\log\left(f_{0\|MHz}\right) - 0.8\right)$	$-4.78\left[\log\left(f_{0\|MHz}\right)\right]^2 + 18.33\log\left(f_{0\|MHz}\right) - 40.94$

The COST 231-Walfish-Ikegami model
How to calculate prop. loss

$$L = L_0 + L_{msd} + L_{rts}$$



Details about calculations can be found in the textbook, Section 7.6.2.

Motley-Keenan indoor model

For indoor environments, the attenuation is heavily affected by the building structure, walls and floors play an important rule

$$PL = PL_0 + 10n\log(d/d_0) + F_{\text{wall}} + F_{\text{floor}}$$

distance dependent path loss

sum of attenuations from walls, 1-20 dB/wall

sum of attenuation from the floors (often larger than wall attenuation)

site specific, since it is valid for a particular case

Small scale fading

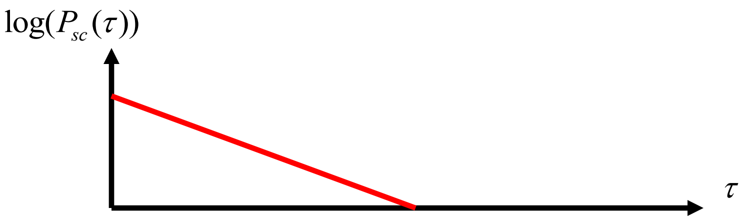
- Narrowband channels can be modeled by one single distribution
  - Rayleigh, Ricean, Nakagami, etc.
- For time-variant channel we also need a model for the Doppler Profile
  - Classical (Jakes), specular, Gauss, etc.
- For wideband channels we also need a model for the Power Delay Profile

Power delay profile

- Often described by a single exponential decay

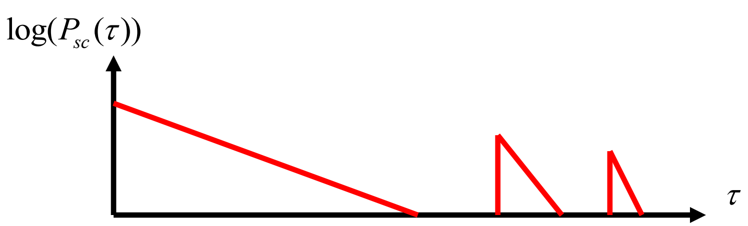
$$P_{sc}(\tau) = \begin{cases} \exp(-\tau / S_\tau) & \tau \geq 0 \\ 0 & otherwise \end{cases}$$

delay spread



- though often there is more than one "cluster"

$$P(\tau) = \begin{cases} \sum_k \frac{P_k^c}{S_{\tau,k}^c} P_{sc}(\tau - \tau_{0,k}^c) & \tau \geq 0 \\ 0 & otherwise \end{cases}$$



Wideband models
COST 207 model for GSM

The COST 207 model specifies:

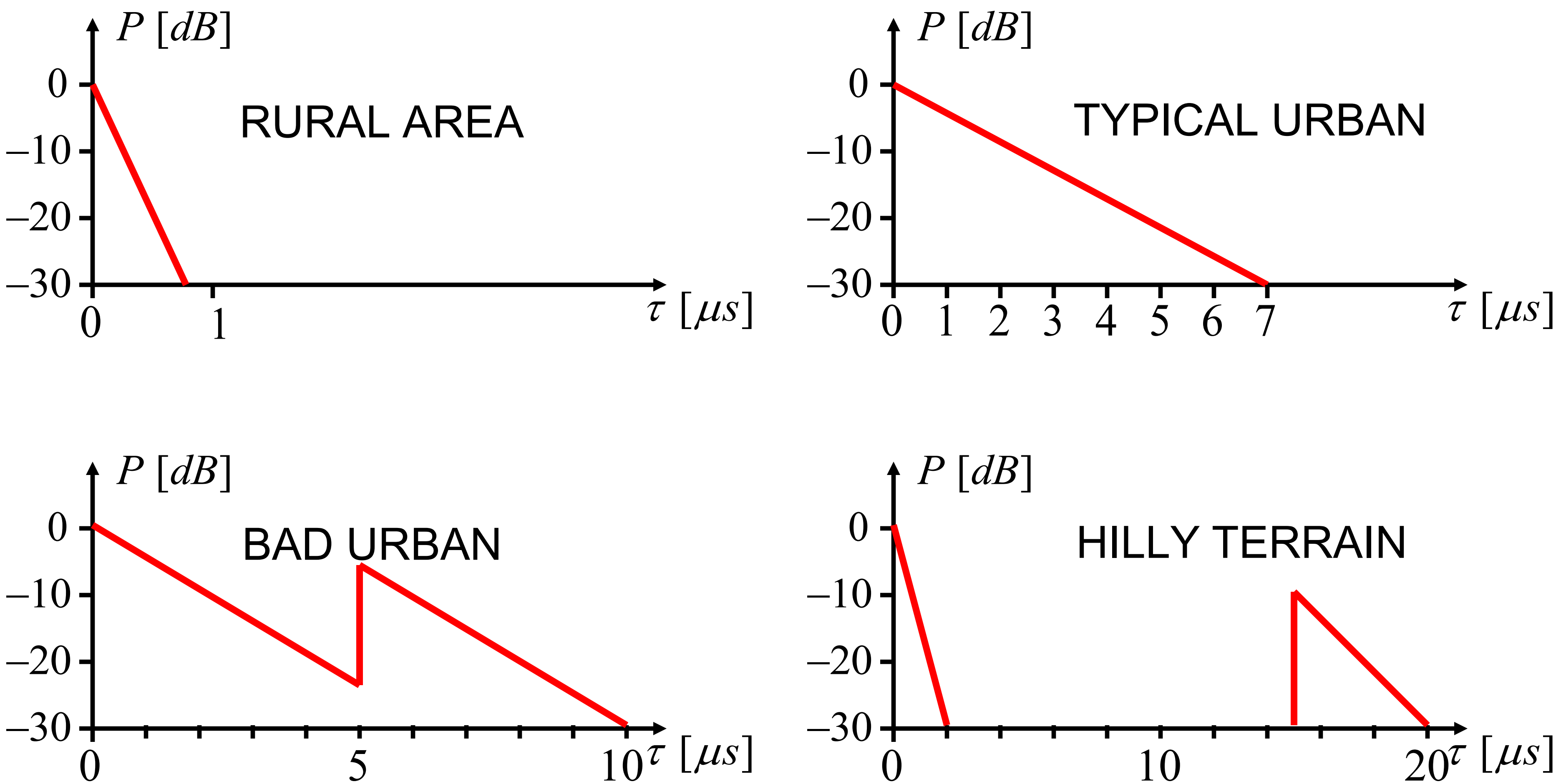
FOUR power-delay profiles for different environments.

FOUR Doppler spectra used for different delays.

IT **DOES NOT** SPECIFY PROAGATION LOSSES FOR THE DIFFERENT ENVIRONMENTS!

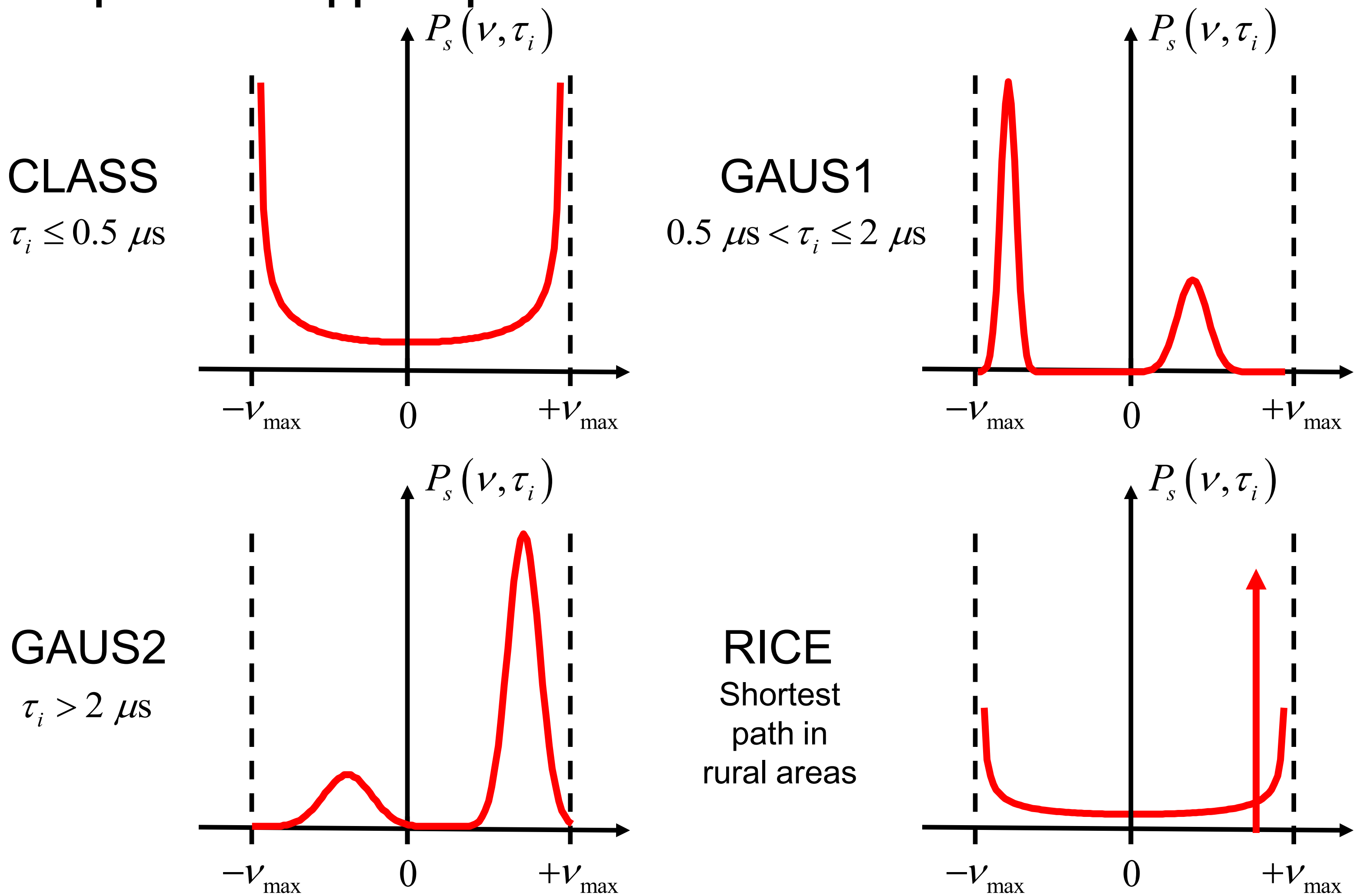
Wideband models
COST 207 model for GSM

Four specified power-delay profiles

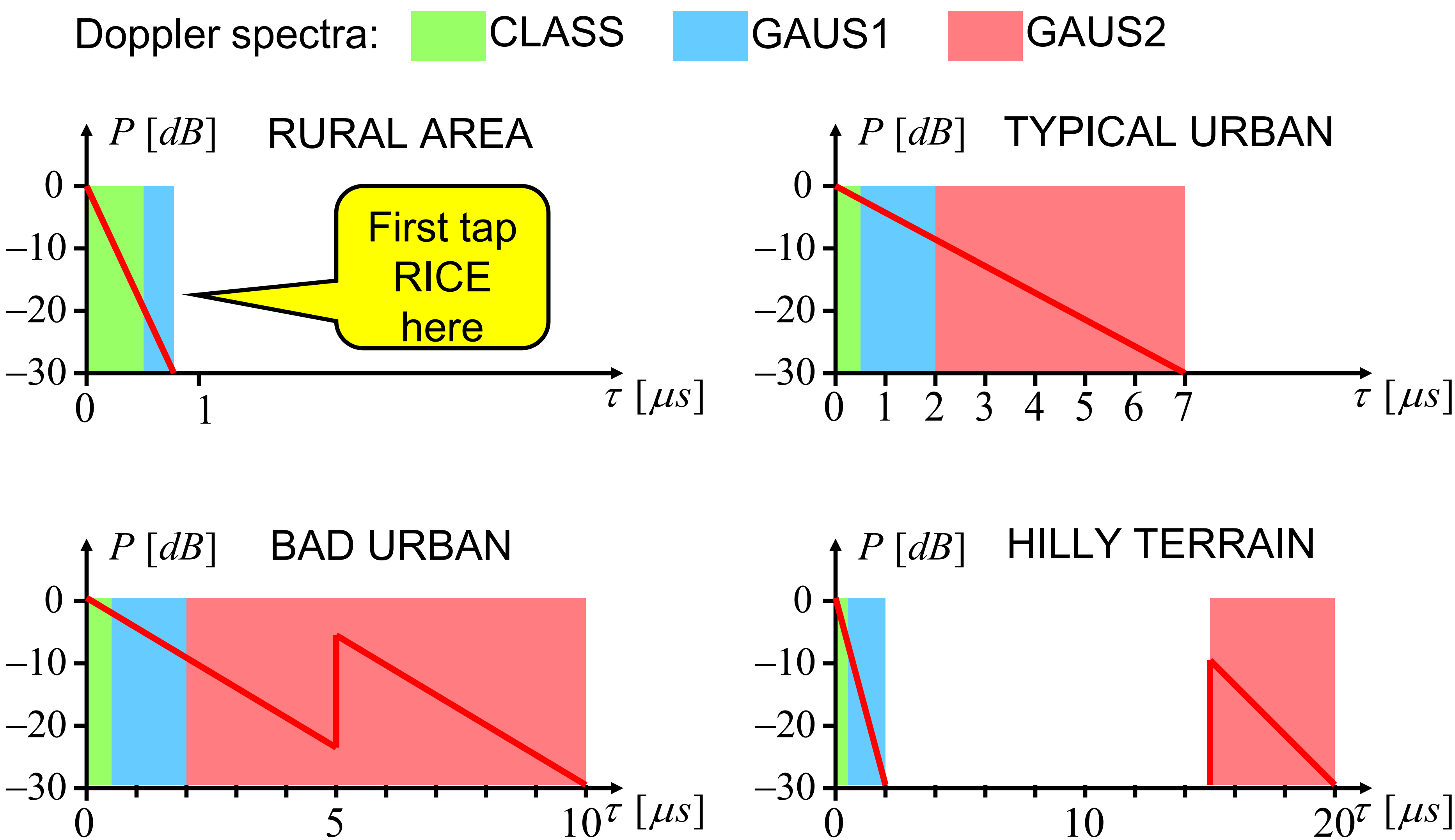


Wideband models
COST 207 model for GSM

Four specified Doppler spectra



Wideband models
COST 207 model for GSM



Tapped delay line models

- Power delay profile modeled as discrete function

$$h(t,\tau) = \sum_{i=1}^{N} \alpha_i(t) \exp\left(j\theta_i(t)\right)\delta(\tau - \tau_i)$$

- Often Rayleigh-distributed taps, but might include LOS and different distributions of the tap values
- Mean tap power determined by the power delay profile

Wideband models
ITU-R model for 3G

Tap No.	delay/ns	power/dB	delay/ns	power/dB
INDOOR	**CHANNEL A** (50%)		**CHANNEL B** (45%)	
1	0	0	0	0
2	50	-3	100	-3.6
3	110	-10	200	-7.2
4	170	-18	300	-10.8
5	290	-26	500	-18.0
6	310	-32	700	-25.2
PEDESTRIAN	**CHANNEL A** (40%)		**CHANNEL B** (55%)	
1	0	0	0	0
2	110	-9.7	200	-0.9
3	190	-19.2	800	-4.9
4	410	-22.8	1200	-8.0
5			2300	-7.8
6			3700	-23.9
VEHICULAR	**CHANNEL A** (40%)		**CHANNEL B** (55%)	
1	0	0	0	-2.5
2	310	-1	300	0
3	710	-9	8900	-12.8
4	1090	-10	12900	-10.0
5	1730	-15	17100	-25.2
6	2510	-20	20000	-16.0

Very large bandwith models

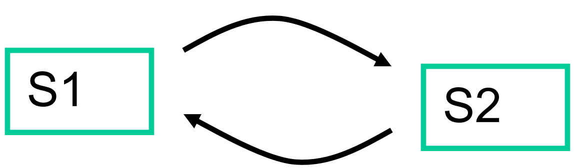
- If the bandwidth is high, the time resolution is large so we might resolve the different multipath components
- Need to model arrival time
- The Saleh-Valenzuela model:

ray arrival time (Poisson)

$$h(\tau) = \sum_{l=0}^{L}\sum_{k=0}^{K}\alpha_{k,l}(\tau)\delta(\tau - T_l - \tau_{k,l})$$

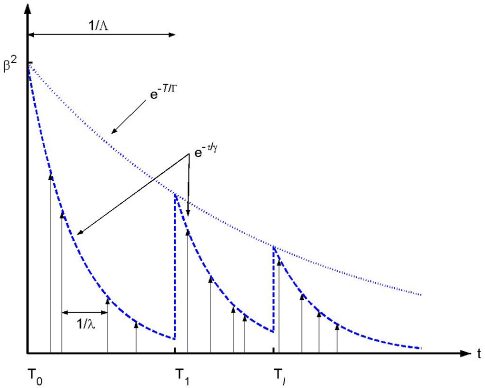
cluster arrival time (Poisson)

- The Δ-K-model:



arrival rate: $\lambda_0(t)$ $K\lambda_0(t)$

The Saleh Valenzuela Model



$$h(\tau) = \sum_{l=0}^{L} \sum_{k=0}^{K} \alpha_{k,l}(\tau)\delta(\tau - T_l - \tau_{k,l})$$

The Saleh Valenzuela Model

$$h(\tau) = \sum_{l=0}^{L} \sum_{k=0}^{K} \alpha_{k,l}(\tau)\delta(\tau - T_l - \tau_{k,l})$$

Ray arrival times $\tau_{k,l}$ and cluster arrival times T_l are Poisson distributed

$$\begin{aligned} \text{pdf}(T_l | T_{l-1}) &= \Lambda \exp\left(-\Lambda(T_l - T_{l-1})\right), \quad l > 0 \\ \text{pdf}(\tau_{k,l} | \tau_{k-1,l}) &= \lambda \exp\left(-\lambda(\tau_{k,l} - \tau_{k-1,l})\right), \quad k > 0 \end{aligned}$$

The power of the multipath components as well as the average cluster powers are exponentially distributed

$$\begin{aligned} E\left\{|\alpha_{k,l}|^2\right\} &\propto P_l^c \exp(\tau_{k,l}/\gamma) \\ P_l^c &\propto \exp(T_l/\Gamma) \end{aligned}$$

Stochastic MIMO models

- Stochastic MIMO models model the correlation matrix $\mathbf{R}_h$
- Can be combined with models for wideband and time-variant channels
- Number of antennas and antenna geometry is predetermined
- Combined modeling of spatial correltation and mutual coupling
- Well suited for testing signal processing algorithms

Stochastic MIMO Models Overview

- iid model ("canonical model")

$$\mathbf{R}_h = \mathbf{I}$$

- Kronecker model

$$\mathbf{R}_h = \mathbf{R}_{Rx} \otimes \mathbf{R}_{Tx}$$

where $\mathbf{R}_{Rx}$ and $\mathbf{R}_{Tx}$ are the Rx and Tx correlation matrices

- Weichselberger model

$$\mathbf{R}_h = \sum_{i=1}^{N_{Tx}} \sum_{j=1}^{N_{Rx}} \omega_{ji} (\mathbf{u}_{Tx,i} \otimes \mathbf{u}_{Rx,j})(\mathbf{u}_{Tx,i} \otimes \mathbf{u}_{Rx,j})^H$$

where $\mathbf{u}_{Tx,i}$ and $\mathbf{u}_{Rx,j}$ are the eigenvectors of the Tx and the Rx correlation matrices and ω_{ij} are the elements of the coupling matrix

- Full-correlation model

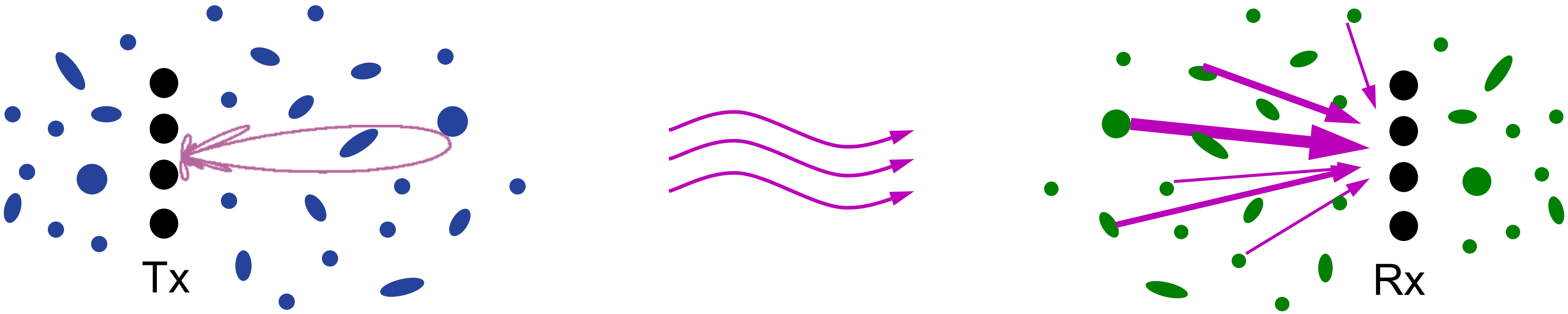
The i.i.d. model

- Simple but not realistic: assume that all elements of $\mathbf{H}(t, \tau)$ are identically and independently distributed (i.i.d)
- Any channel model can be used for the elements (Rayleigh, Ricean, etc.)
- Problems with this model:
  - ignores effects of correlation and mutual coupling
  - overestimates capacity ($rank(\mathbf{H}) = \min(N_{\mathrm{Tx}}, N_{\mathrm{Rx}})$ with probability 1)
  - not verified by measurements

The Kronecker Model

- Treats correlation independently at Tx and Rx
- Transmit correlation matrix: $\mathbf{R}_{Tx} = \mathcal{E}\left\{\mathbf{H}^H\mathbf{H}\right\}$
- Receive correlation matrix: $\mathbf{R}_{Rx} = \mathcal{E}\left\{\mathbf{H}\mathbf{H}^H\right\}$

Kronecker model



- The spatial structure of the MIMO channel is neglected.
- The MIMO channel is described by separated link ends:

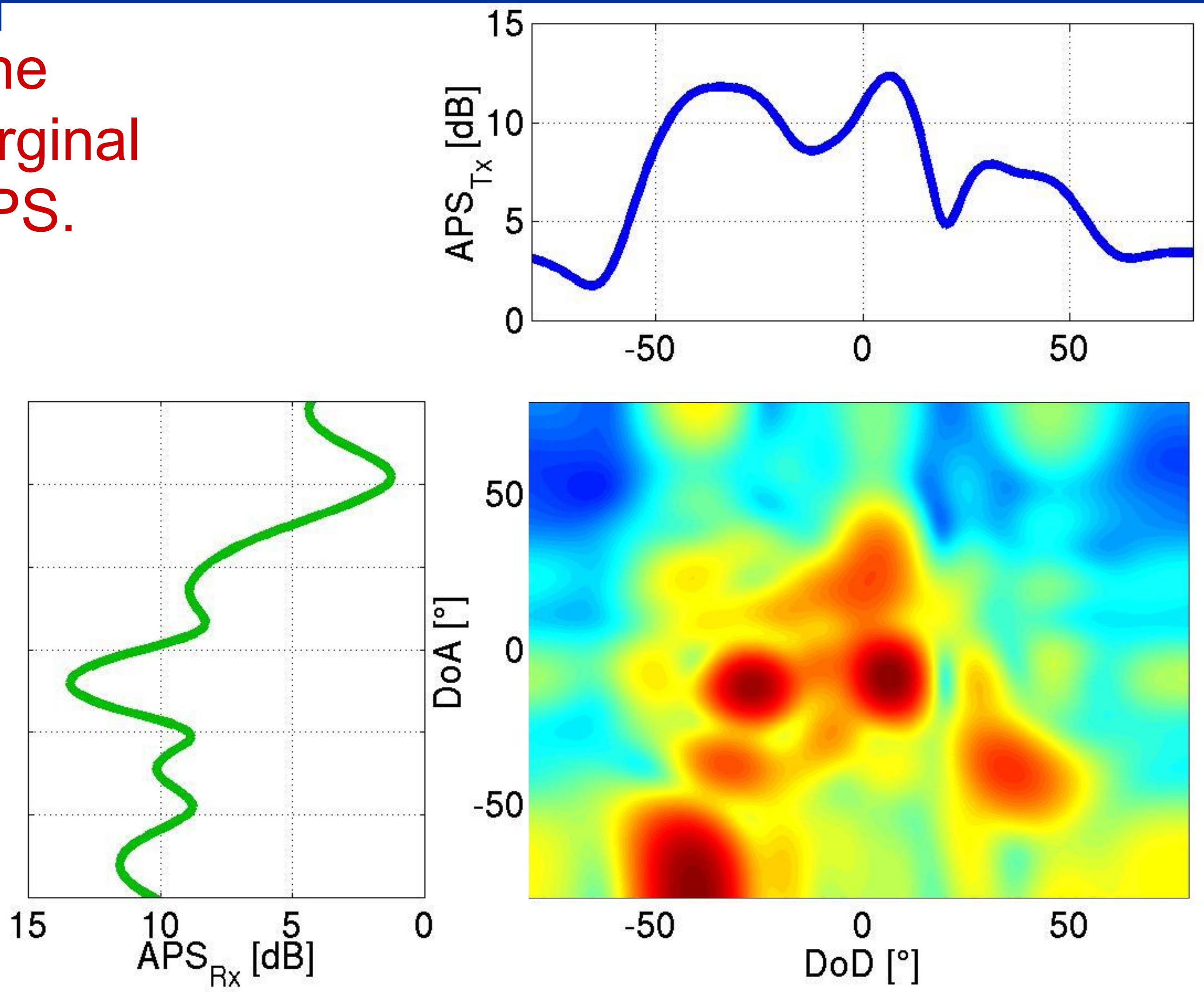
$$\boldsymbol{R}_{\boldsymbol{H}} = c \cdot \boldsymbol{R}_{\mathrm{Tx}} \otimes \boldsymbol{R}_{\mathrm{Rx}} \qquad \boldsymbol{H} = \boldsymbol{R}_{\mathrm{Rx}}^{1/2} \boldsymbol{G} \boldsymbol{R}_{\mathrm{Tx}}^{T/2}$$

Any transmit signal results in one
and the same receive correlation!

Kronecker model (cont.)

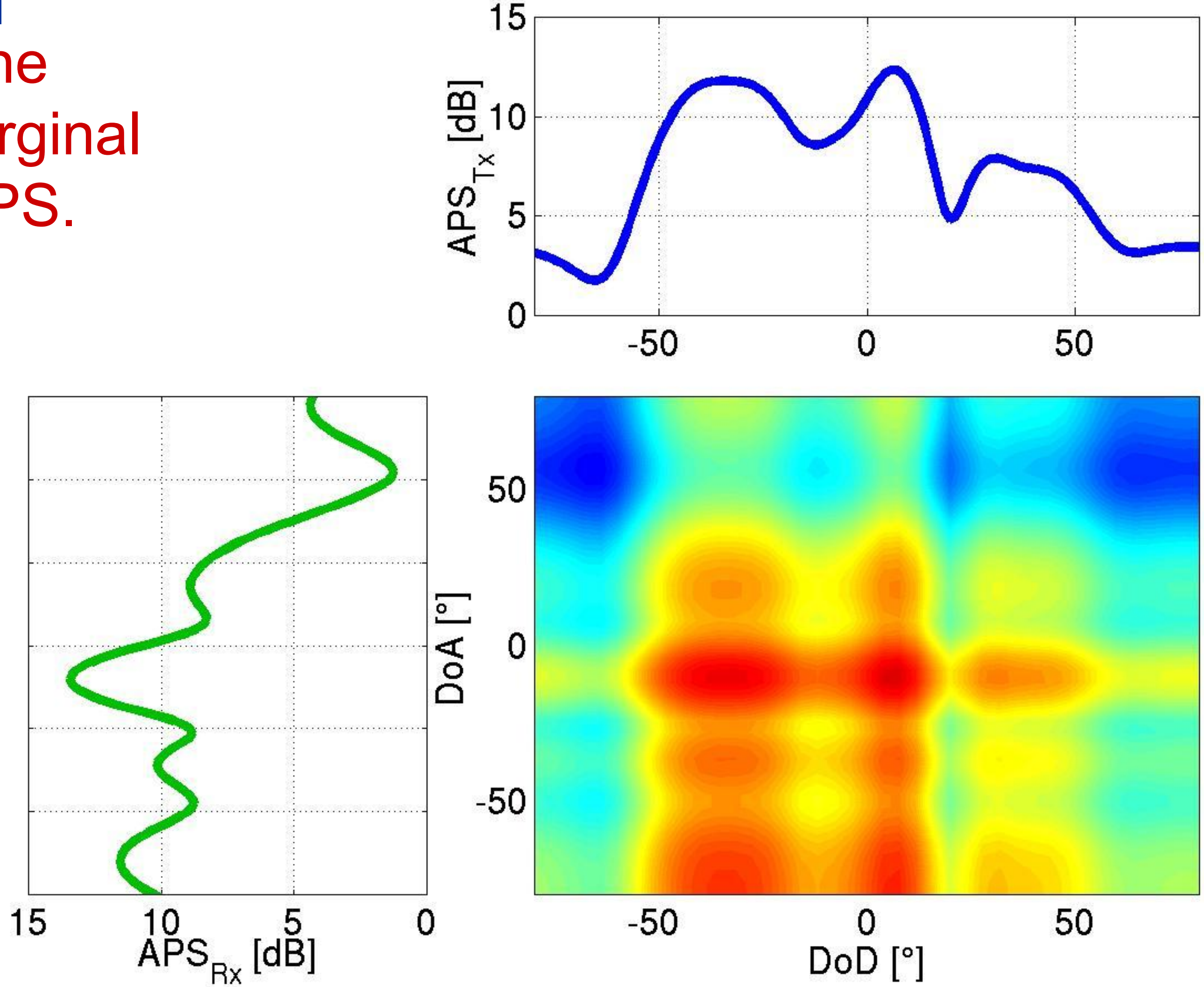
Joint APS is the product of marginal Rx- and Tx-APS.

measurement



Kronecker model (cont.)

Joint APS is the product of marginal Rx- and Tx-APS.



Kronecker approximation

Copyright: TU Vienna

Weichselberger Model – Definition

- **Relaxes assumptions of Kronecker model by using power coupling of Tx and Rx eigenmodes, defined by**

$$\mathbf{R}_{\mathsf{Tx}} = \underbrace{\mathbf{U}_{\mathsf{Tx}}}_{\text{Tx eigenmodes}} \mathbf{\Lambda}_{\mathsf{Tx}} \mathbf{U}_{\mathsf{Tx}}^{H} \qquad \mathbf{R}_{\mathsf{Rx}} = \underbrace{\mathbf{U}_{\mathsf{Rx}}}_{\text{Rx eigenmodes}} \mathbf{\Lambda}_{\mathsf{Rx}} \mathbf{U}_{\mathsf{Rx}}^{H}$$

- **Power coupling of eigenmodes is described by coupling matrix**

$$\mathbf{\Omega}_{\mathsf{WB}} = \mathrm{E}\left\{\left(\mathbf{U}_{\mathsf{Rx}}^{H}\,\mathbf{H}\,\mathbf{U}_{\mathsf{Tx}}^{*}\right) \odot \left(\mathbf{U}_{\mathsf{Rx}}^{T}\,\mathbf{H}\,\mathbf{U}_{\mathsf{Tx}}\right)\right\}$$

- **Channel correlation matrix is modelled as**

$$\mathbf{R}_{\mathbf{h}} = \sum_{i=1}^{M_{\mathrm{T}}} \sum_{j=1}^{M_{\mathrm{R}}} \omega_{ji} (\mathbf{u}_{\mathsf{Tx},i} \otimes \mathbf{u}_{\mathsf{Rx},j})(\mathbf{u}_{\mathsf{Tx},i} \otimes \mathbf{u}_{\mathsf{Rx},j})^{H}, \quad \text{with} \quad \omega_{ji} = (\mathbf{\Omega}_{\mathsf{WB}})_{j,i}$$

- **Channel realizations can be generated by**

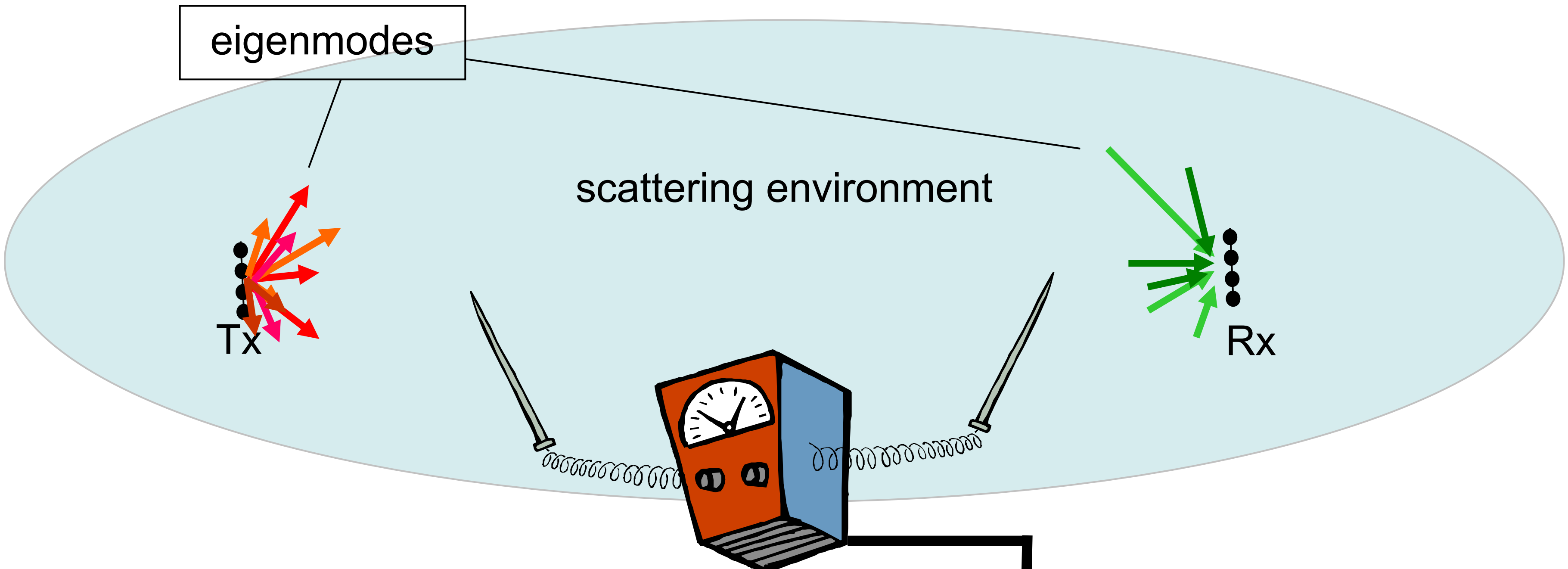
$$\mathbf{H} = \mathbf{U}_{\mathsf{Rx}}(\tilde{\mathbf{\Omega}}_{\mathsf{WB}} \odot \mathbf{G})\mathbf{U}_{\mathsf{Tx}}^{T},$$

where $\tilde{\Omega}_{\mathsf{WB}}$ is element-wise square-root of Ω_{WB}, and $\mathbf{G} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I})$

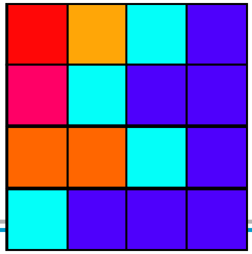
$\odot$ … element-wise matrix product (Hadamard product)

Weichselberger Model – Parameters

What are "eigenmodes" and the coupling matrix Ω_{WB}?



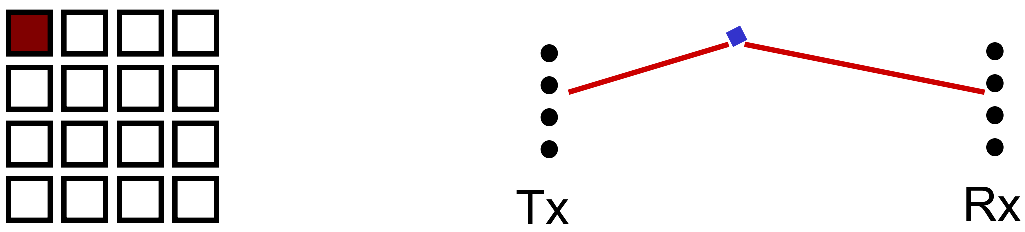
Coupling matrix $\Omega_{WB} =$



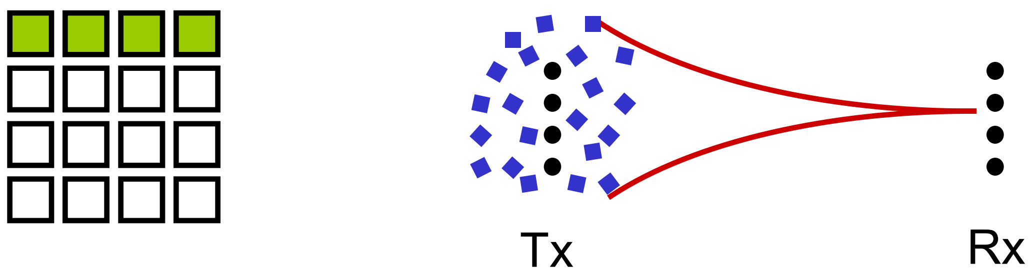
hot … strong power
cold .. weak power

Weichselberger Model – Coupling Matrix (1)

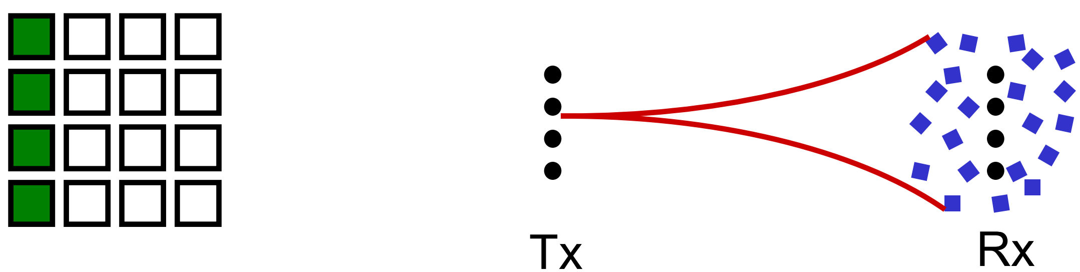
Structure of Ω_{WB} strongly depends on the environment:

$\Omega_{WB} =$ 

low-rank channel

$\Omega_{WB} =$ 

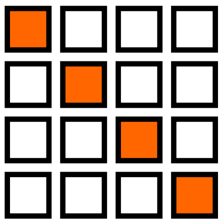
low rank, Tx diversity

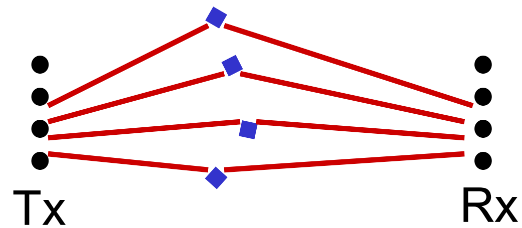
$\Omega_{WB} =$ 

low rank, Rx diversity

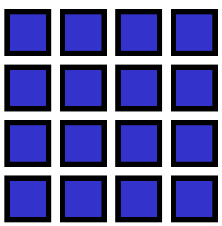
Weichselberger Model – Coupling Matrix (2)

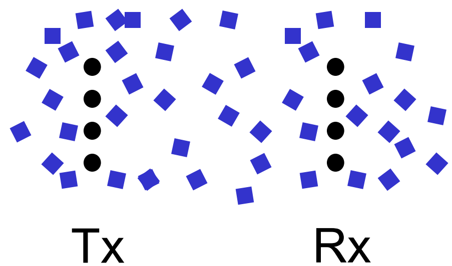
Structure of Ω_{WB} strongly depends on the environment:

$\Omega_{WB} =$ 

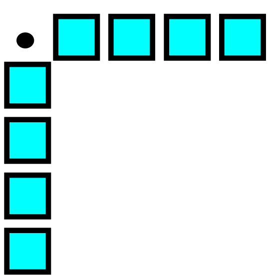


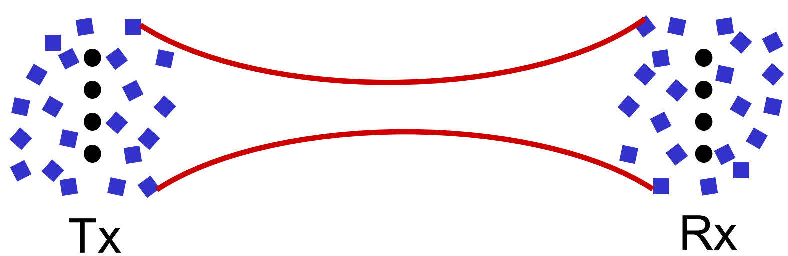
full rank,
no diversity

$\Omega_{WB} =$ 



full rank,
full diversity

$\Omega_{WB} =$ 



full rank,
Kronecker

Stochastic MIMO models summary

- Full-correlation model
  - Very complex
  - Most accurate
- Weichselberger model
  - Good approximation
  - Good performance-complexity compromise
- Kronecker model
  - Separates channel into Tx and Rx sides
  - Very popular
  - Very limited validity
- iid model
  - Most simple
  - No physical relevance

Stochastic MIMO model example

- 3GPP specifies four simplified Spatial Channel Models (SCM) for link level evaluations
- See 3GPP TR 25.814 V7.1.0, Section A.1.3

Geometry Based Models

- Geometry-based models use the theory of electromagnetic wave propagation (Maxwell equations) to characterize wireless channels.
- Solutions can be written as a sum of plane waves

$$h(t, f, \vec{x}, \vec{y}) = \sum_p \beta_p e^{2\pi j(\phi_p + \langle \vec{\zeta}_p, \vec{x} \rangle - \langle \vec{\xi}_p, \vec{y} \rangle - f\tau_p + t\omega_p)},$$

where

- β_p and ϕ_p are the amplitude and phase
- $\vec{\zeta}_p$ and $\vec{\xi}_p$ are the Tx and Rx wavevectors
- τ_p and ω_p are the delay and the Doppler shift

of path p.

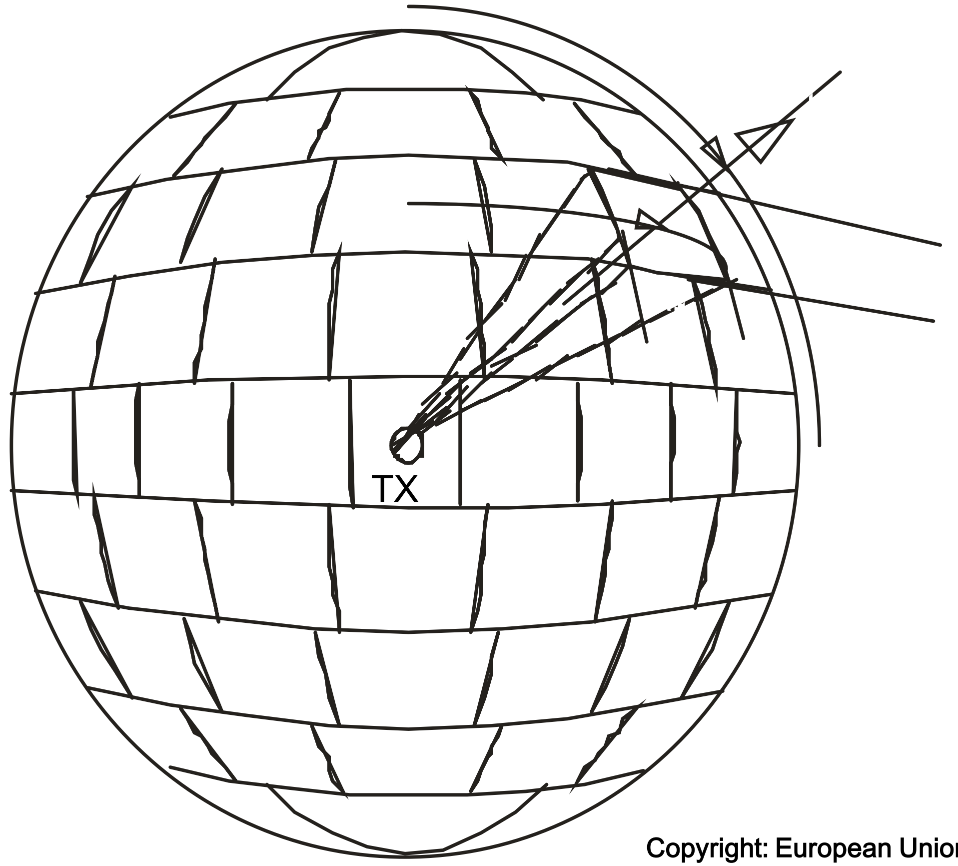
Deterministic modeling methods

- Solve Maxwell's equations with boundary conditions
- Problems:
  - Data base for environment
  - Computation time
- "Exact" solutions
  - Method of moments
  - Finite element method
  - Finite-difference time domain (FDTD)
- High frequency approximation
  - All waves modeled as rays that behave as in geometrical optics
  - Refinements include approximation to diffraction, diffuse scattering, etc.

Ray launching

Rays are launched from TX into discrete directions

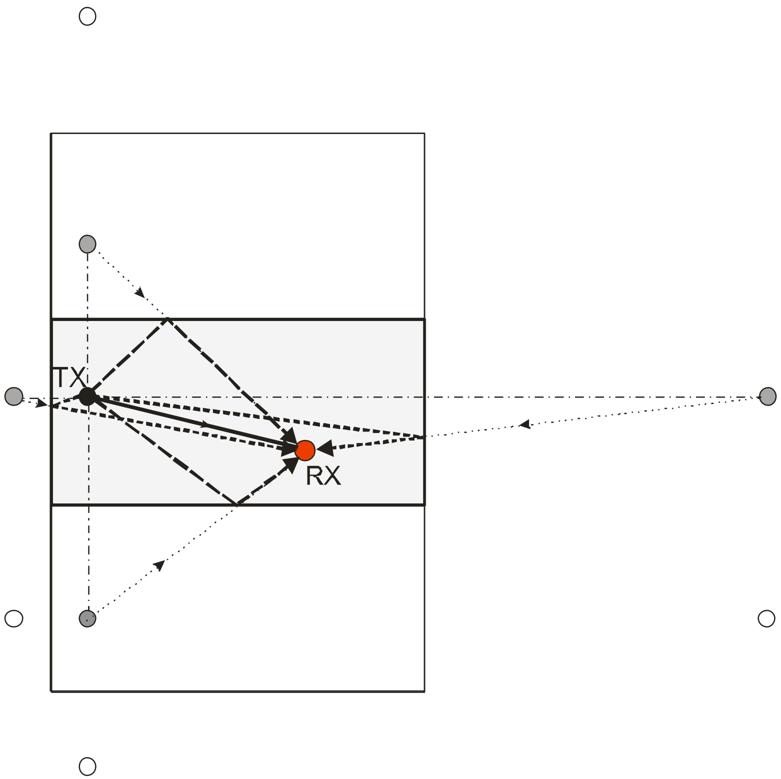
Rays are followed until their energy is below a threshold or if they get in the vicinity of the receiver.



Ray tracing

- Determines rays that can go from one TX position to one RX position
  - Uses imagining principle
  - Similar to techniques known from computer science

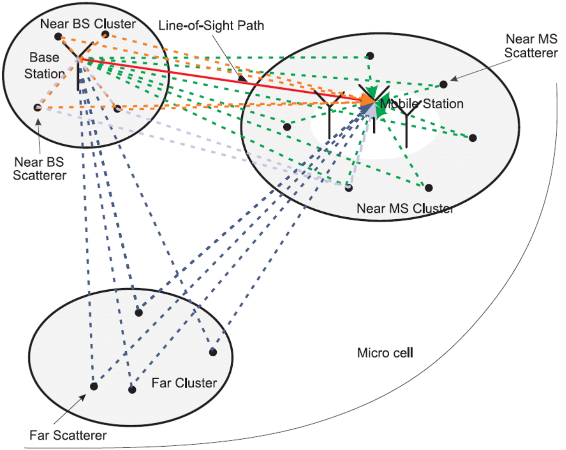
- Then determine attenuation of all those possible paths



Geometry Based Stochasic Models (GSCM)

- GSCM model the environment by placing clusters of scatterers in space
- Distribution of clusters and scatterers within clusters modeled stochastically
- Losses and phase shifts of scatterers are modeled stochastically
- Use simplified ray tracing to compute the path parameters (power, delay, phase, AoA, AoD)
- Single bounce or multiple bounce

Geometry Based Stochasic Models (GSCM)



Geometry Based Stochasic Models (GSCM)

Advantages of GSCM

- Simpler than fully deterministic models
- More realistic than stochastic models
- Implicitly models mobility, MIMO, multi-user, etc
- Valid for regions

Disadvantages of GSCM

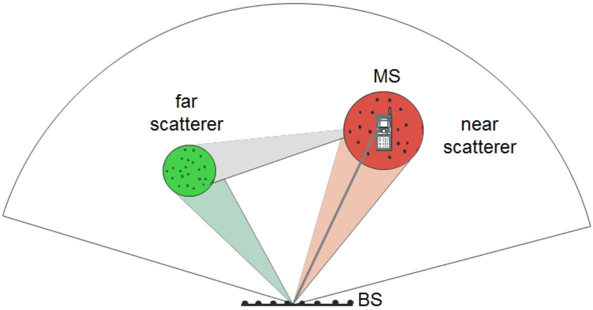
- Very hard to parameterize
- Can still be computationally expensive

Geometry Based Stochasic Models (GSCM)

- For every GSCM statistics can be computed
- Some GSCMs and stochastic models are equivalent
- Example: One ring model
  - Scatterers uniformly distributed on a ring around Rx
  - Equivalent to Rayleigh fading
- More sophisticated models (including MIMO)
  - COST 259, 273, and 2100
  - WINNER models
  - 3GPP spatial channel model

The COST 259 Approach

- Single-bounce model, no scatterers around BS
- Fixed relationship between AOD, AOA, and delay
- Well suited for smart antenna systems, but not MIMO



AOA: angle of arrival; AOD: angle of departure; BS/MS: base/mobile station

COST 259 DCM - Philosophy

- Parametric approach, WSSUS not required
- No statement about implementation method (stochastic or GSCM)
- Based on clustering approach
- Multi-layer approach:
  - Radio environments
  - Large-scale effects
  - Small-scale effects

Radio environments

- COST 259
  - Macrocell
    - GTU
    - GRA
    - GBU
    - GHT
  - Microcell
    - GSN
    - GSL
    - GSX
    - GOP
  - Picocell
    - GOL
    - GON
    - GCL
    - GCN
    - GFH

GTU	Generalized Typical Urban
GRA	Generalized Rural Area
GBU	Generalized Bad Urban
GHT	Generalized Hilly Terrain

GSN	Generalized Street NLOS
GSL	Generalized Street Canyon LOS
GSX	Generalized Street Crossing
GOP	Generalized Open Place

GOL	Generalized Office LOS
GON	Generalized Office NLOS
GCL	Generalized Corridor LOS
GCN	Generalized Corridor NLOS
GFH	Generalized Factory Hall

COST 259 DCM - Simulation procedure

Simulation steps:

1) select scenario

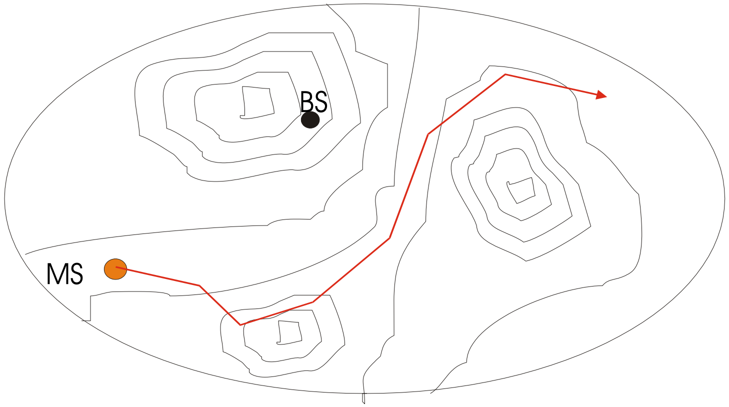
2) select global parameters (number of clusters, mean Rice factor,....)

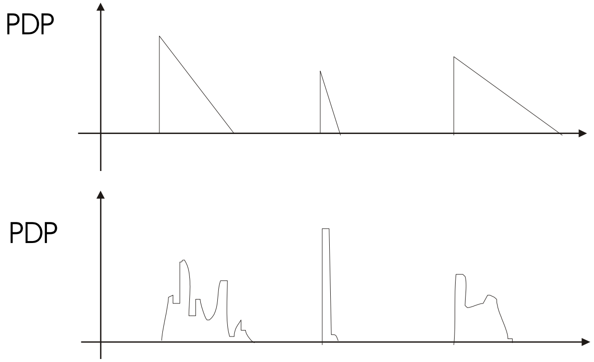
3) REPEAT

compute one realization of global parameters. This realization prescribes small-scale averaged power profiles (ADPS)

create many instantaneous complex impulse responses from this average ADPS

Generalized Hilly Terrain (GHT)





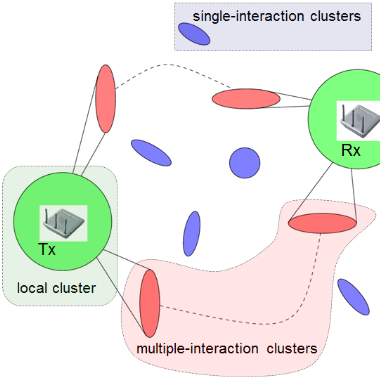
realization of
global parameters:
3 clusters, Rice factor of first: 3.2,
Rice factor of later clusters: 0
delay spread of clusters: 1,0.3,2s µ

COST 259 DCM - Important features

- Very realistic !
- Distinguishes 13 different radio environments
- Treats large-scale and small-scale variations
- Far scatterer clusters included, with birth/death process
- Delay spread and angular spread treated as (correlated) random variables
- Angular spectra are functions of delay
- Azimuth and elevation

The COST 273/2100 Approach (1)

- Introduction of “multiple interaction clusters” and local clusters around BS to model MIMO channels better

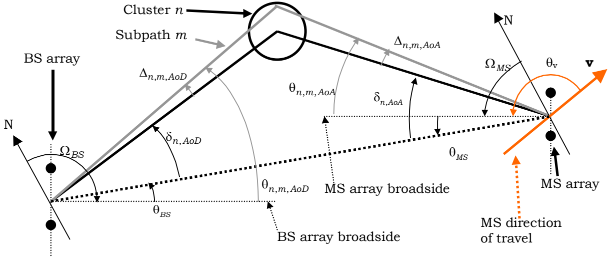


The COST 273/2100 Approach (2)

- Any combination of delay and angles can be modeled, not limited to double scattering
- All parameters are given per cluster; there are no global spreads
- Direct coupling between AOAs and AODs; no “Kronecker” structure
- Cluster “visibility regions” for smooth transition between non-stationarity regions
- The COST 2100 channel model extends the COST 273 model to cover MIMO systems at large, including multi-user, multicellular, and cooperative aspects

The 3GPP Approach

- The 3GPP models were mainly influenced by the WINNER projects
- Introduces the concepts of “drops”, which corresponds to one instance of the cluster/scatterer distribution.
- Model non-stationary channel by introducing correlation between large scale parameters
- Easier to parameterize from channel measurements than the COST models



The 3GPP Spatial Channel Model (SCM)

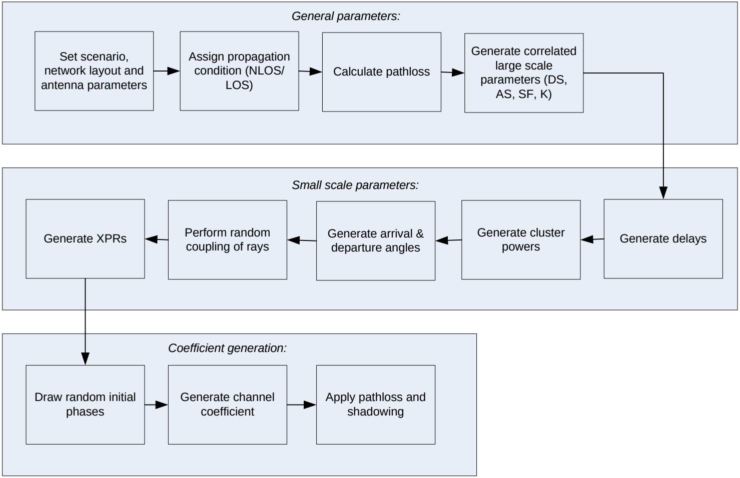
- Original SCM developed for 3G (UMTS) [TR 25.996]
- Extended to larger bandwidth (20MHz) for 4G (LTE Rel 8): SCME
- Extended to 3D for LTE Rel 12 [TR 36.873]
- Extended to frequency ranges up to 100GHz and bandwidth of up to 2GHz in 5G New Radio [TR 38.901]

The 3GPP 5G Channel Model

EURECOM

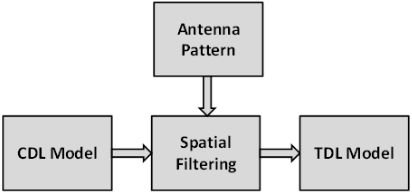
- For frequencies from 0.5 to 100 GHz
- For system level simulations, supported scenarios are urban microcell street canyon, urban macrocell, indoor office, and rural macrocell.
- Bandwidth is supported up to 10% of the center frequency but no larger than 2GHz.
- Mobility of one end of the link is supported
- For the stochastic model, spatial consistency is supported by correlation of LSPs and SSPs as well as LOS/NLOS state.
- Large array support is based on far field assumption and stationary channel over the size of the array.

Fast Fading coefficient generation



Channel models for link-level evaluations

- For link level simulations, simplified cluster delay line (CDL) and tapped delay line (TDL) are defined.
- CDL models specify for each scenario a number of clusters with corresponding AOA and AOD spread
- TDL models assume a pre-defined antenna pattern
- MIMO extensions:
  - CDL extension: Scaling of angles
  - TDL extension: Applying a correlation matrix



Channel Simulation

- Sampled channels
- Correlation-based simulation
- Geometry based simulation

Sampling

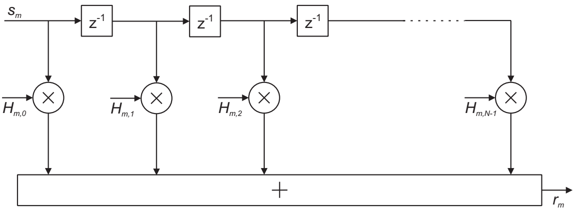
- Only sampled channels can be simulated (T_S is the sampling rate):

$$h[m,n] = h(mT_S, nT_S)$$

- Input-output relation

$$r[m] = \sum_{n=0}^{N-1} h[m,n]s[m-n] + n[m]$$

- Can be implemented as a tapped delay line



Sampling

- Special case: narrowband channel: $h[m, n] = 0$ for $n \neq 0$
- Special case: static channel $h[m, n] = h[n]$

Generation of correlated time series

- We wish to generate samples of a WSS process $h[m]$ with autocorrelation function (ACF)

$$R_h(m-m') = \mathcal{E}\{h[m]h^*[m']\}$$

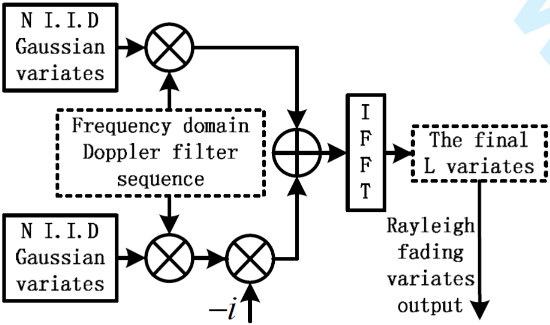
or equivalently with power spectrum desnity (PSD)

$$S_h(\nu) = \mathrm{DFT}\{R_h(\Delta m)\}$$

- Two main methods exist
  - Frequency-domain filtering
  - Time-domain filtering
  - Sum-of-sinusoids

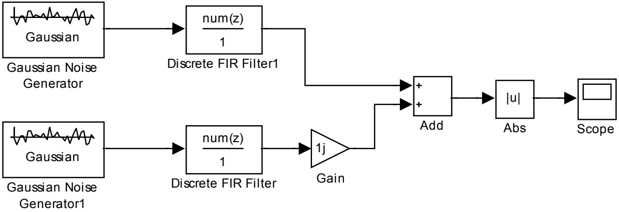
Frequency-domain filtering

- First generate N i.i.d. Gaussian random variates (real and imaginary)
- Shape them with a frequency domain filter corresponding to the desired PSD
- Apply an IFFT



Time-domain filtering

- First generate i.i.d. Gaussian random variates (real and imaginary)
- Pass them through a time-domain filter corresponding to the desired ACF
- Advantage: non-block based, no discontinuities



Problems

- The Doppler fading process is usually highly oversampled.
- Example: Sampling rate 7.68MHz, max. Doppler 500Hz
- For the frequency-domain method
  - $\Rightarrow$ Only a small part of the PSD is non-zero
  - $\Rightarrow$ large number of samples N required for accuracy
  - $\Rightarrow$ Large IFFT has high memory and complexity requirements
- For the time-domain method
  - $\Rightarrow$ A large number of filter coefficients required
  - $\Rightarrow$ High complexity
- Both methods can be improved by generating a correlated process with a lower sampling rate and then using interpolation

Generation of wideband impulse responses

- We wish to generate an US process $h[n]$, with a certain power delay profile (PDP = PSD)
- Same problem as before, but simpler since
  - Samples can be generated directly in the delay domain (n)
  - Process is sampled at lower rate
- However, interpolation might be necessary if the delays of taps are not multiples of the sampling rate

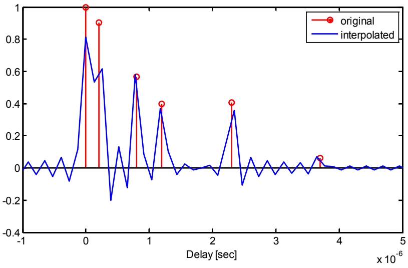
Example: sinc interpolation

- Impulse response

$$h(\tau) = \sum_{p=0}^{P-1} a_p \delta(\tau - \tau_p)$$

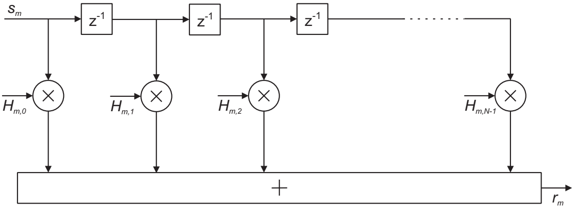
- Sampled at rate T_S:

$$h[n] = \sum_{p=0}^{P-1} a_p\, sinc(n - \frac{\tau_p}{T_S})$$



Generation of time-variant impulse responses

- Each tap of the tapped delay line is a time-correlated sequences multiplied with the weight of the tap



- Example: 3GPP channel model in Maltab

Stochastic MIMO models

- We wish to generate a MIMO channel $\mathbf{H}$ with a certain correlation matrix $\mathbf{R} = \mathcal{E}\left\{ vec(\mathbf{H})\, vec(\mathbf{H})^H \right\}$
- Since the number of correlated samples is finite and small, we can use the direct method
- Let $\mathbf{R}^{\frac{1}{2}}$ be the matrix square root of $\mathbf{R}$ and $\mathbf{G}$ a matrix of i.i.d. Gaussian random variates. Then

$$\mathbf{H} = unvec\left(\mathbf{R}^{\frac{1}{2}}\, vec(\mathbf{G})\right)$$

Stochastic MIMO models (2)

Simplified MIMO models

- Weichselberger model

$$\mathbf{H} = \mathbf{U}_{Rx}(\mathbf{\Omega} \odot \mathbf{G})\mathbf{U}_{Tx}^{T}$$

- Kronecker model

$$\mathbf{H} = \mathbf{R}_{Rx}^{\frac{1}{2}}\mathbf{G}(\mathbf{R}_{Tx}^{\frac{1}{2}})^{T}$$

- i.i.d. model

$$\mathbf{H} = \mathbf{G}$$

Geometry based simulation

- Based on and used for geometry based model

$$h(t, f, \vec{x}, \vec{y}) = \sum_{p} \beta_p e^{2\pi j(\phi_p + \langle \vec{\zeta}_p, \vec{x} \rangle - \langle \vec{\xi}_p, \vec{y} \rangle - f\tau_p + t\omega_p)},$$

- Parameters for each path are either taken from a random distribution or from geometrical calculations
- In general more realistic (especially for MIMO) but also the most computationally complex

Example: Generation of time-correlated series using geometry based simulation

- For a narrowband SISO channel

$$h_m = \sum_{p=0}^{P-1} \beta_p e^{2\pi j(\phi_p + m\nu_p)},$$

  where $\nu_p = \omega_p T_S$ is the normalized Doppler shift of path p
- If
  - $\beta_p = 1/\sqrt{P}$,
  - $\nu_p = \nu_{\max} \cos \psi_p$ where $\nu_{\max}$ is the maximum Doppler shift and ψ_p is the AoA of path p
  - ϕ_p and ψ_p are mutually independent and uniformly distributed in $[-\pi, \pi)$
- Then, as $P \to \infty$, the spectrum of h_m approaches

$$S_h(\nu) = \frac{1}{\pi \nu_{\max} \sqrt{1 - \left(\frac{\nu}{\nu_{\max}}\right)^2}}$$