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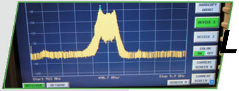


Radio Engineering

Lecture 6: MIMO Channel Characterization

Florian Kaltenberger

Last lecture

6. Wideband channels
   - Wideband vs narrowband channels
   - System theoretic description of wideband channels
   - WSSUS model
   - Condesed parameters
   - Direction channel description

Wideband channels: definition

- A communication system is narrowband if
  - The symbol duration T_s is *larger* than the maximum delay (or the delay spread) in the channel $\Delta\tau$
  - $\Rightarrow$ Receiver cannot distinguish different echos
- A communication system is wide-band if
  - The symbol duration T_s is *smaller* than the maximum delay (or the delay spread) in the channel $\Delta\tau$
  - $\Rightarrow$ One transmitted symbol can spread over more than one symbol at the receiver

Stochastic channel characterization

- The *autocorrelation function* of a stochastic process $h(t)$ is defined as

$$R_h(t, t') = \mathcal{E}\{h(t)h^*(t')\}$$

- A stochastic process $h(t)$ is *wide-sense stationary (WSS)* iff

$$R_h(t, t') = R_h(t - t') = R_h(\Delta t)$$

- The power spectrum $S_h(\nu)$ of a WSS process $h(t)$ is given by the Fourier transform of the autocorrelation function

$$S_h(\nu) = \mathcal{F}(R_h(\Delta t))$$

Stochastic channel characterization

- Autocorrelation function of $H(\nu) = \mathcal{F}(h(t))$

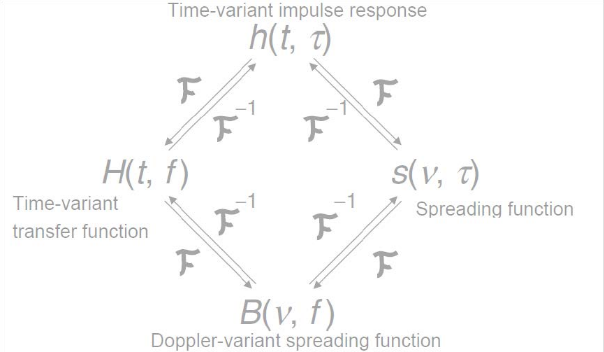
$$R_H(\nu, \nu') = \mathcal{E}\{H(\nu)H^*(\nu')\}$$

- Iff $h(t)$ is WSS then $H(\nu)$ is *uncorrelated scattering (US)*

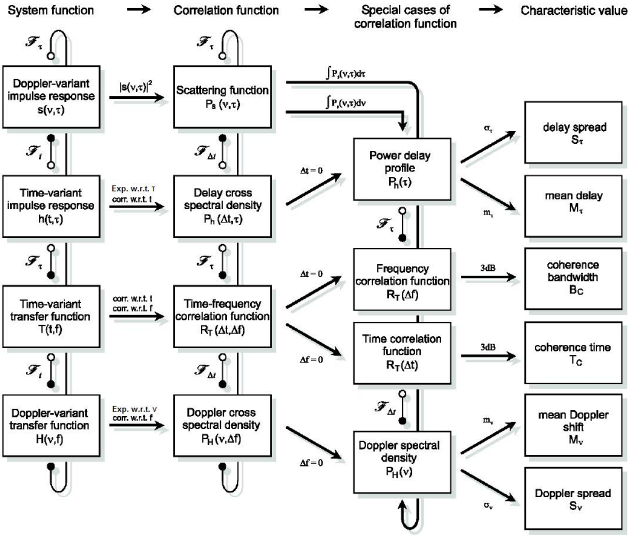
$$R_H(\nu, \nu') = \delta(\nu - \nu')S_h(\nu)$$

Linear time-variant system

- Linear time-variant systems are characterized by one of the four system functions



Correlation functions and condensed parameters



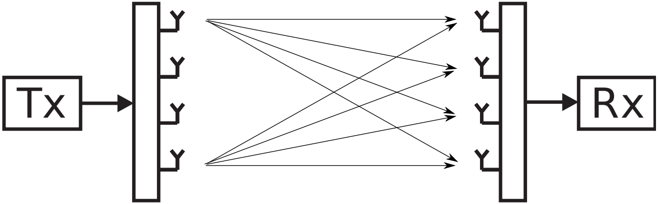
Summary

- In wideband communication systems, multipath propagation results in inter-symbol interference (ISI)
- Wideband channels fade also in frequency, causing *frequency-selective* fading
- Wideband channels can be mathematically described as linear time-variant systems (LTV)
- Wideband channels are wide-sense stationary (WSS) with uncorrelated scattering (US) iff
    1. their second order statistics (autocorrelation function) do not change over time
    2. contributions with different delays are uncorrelated

This lecture

1. Multiple-Input Multiple-Output (MIMO) channels
   - Definitions
   - System model
   - Mutual coupling and correlation
   - Double directional channel characterization
   - Angular power spectra
2. Channel Sounding
   - Time and frequency domain sounding
   - Directionally resolved measurements
   - Parameter estimation methods

Definitions



- SISO: Single-Input Single-Output
- SIMO: Single-Input Multiple-Output
- MISO: Multiple-Input Single-Output
- MIMO: Multiple-Input Multiple-Output

System Model

MIMO input-output realtion

$$\mathbf{y}(t) = \int_{-\infty}^{\infty} \mathbf{H}(t,\tau)\mathbf{x}(t-\tau)\mathrm{d}\tau + \mathbf{n}(t),$$

where

- $\mathbf{x}(t) = [x_0(t), \ldots, x_{N_{\mathrm{TX}}-1}(t)]^T$ is the transmitted signal
- $\mathbf{n}(t) = [n_0(t), \ldots, n_{N_{\mathrm{RX}}-1}(t)]^T$ is the AWGN (i.i.d.)
- $\mathbf{y}(t) = [r_0(t), \ldots, r_{N_{\mathrm{RX}}-1}(t)]^T$ is the received signal
- $\mathbf{H}(t,\tau) = \begin{bmatrix} h_{0,0}(t,\tau) & \ldots & h_{0,N_{\mathrm{TX}}-1}(t,\tau) \\ \vdots & \ddots & \vdots \\ h_{N_{\mathrm{RX}},0}(t,\tau) & \ldots & h_{N_{\mathrm{RX}}-1,N_{\mathrm{TX}}-1}(t,\tau) \end{bmatrix}$
  is the MIMO channel response

Benefits of MIMO

- Array Gain
  - Increase Power (RX)
  - Beamforming (TX)
- Diversity
  - Mitigate Fading
  - Space-Time Coding
- Spatial Multiplexing
  - Multiply Data Rates
  - Spatially Orthogonal Codes

Capacity of a MIMO channel

- Capacity of a MIMO channel

$$C = \log_2\left[\det\left(\mathbf{I}_{N_{\text{Tx}}} + \frac{\bar{\gamma}}{N_{\text{Tx}}}\mathbf{H}\mathbf{R}_x\mathbf{H}^H\right)\right],$$

where $\bar{\gamma}$ is the mean SNR and $\mathbf{R}_x$ is the correlation matrix of the transmitted data

- If channel is know at transmitter, $\mathbf{R}_x$ can be matched to the channel and

$$C = \sum_{k=1}^{\min(N_{\text{Tx}},N_{\text{Rx}})} \log_2\left[1 + \frac{P_k}{\sigma_n^2}\sigma_k^2\right],$$

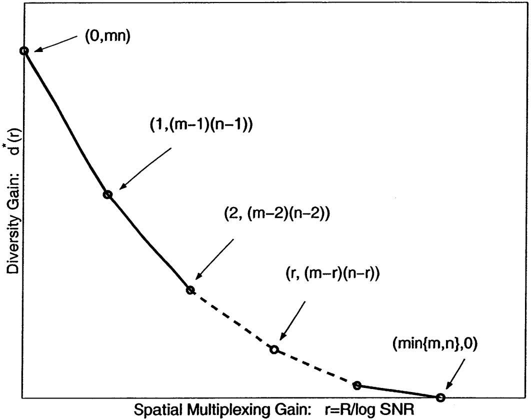
where P_k is the results of the power allocation (waterfilling), σ_n^2 is the noise variance and σ_k are the singular values of the channel $\mathbf{H}$

Capacity of a MIMO channel (2)

- Capacity C of the channel is proportional to the rank (=number of non-zero singular values) of the channel matrix $\mathbf{H}$
- In the ideal case (full channel rank), capacity C thus scales with $\min(N_{\mathrm{Tx}}, N_{\mathrm{Rx}})$
- The factor $r = C/\log(\mathrm{SNR})$ is also known as *multiplexing gain*.

Diversity and Multiplexing Tradeoff

- Fundamental Tradeof in MIMO sytems [1]



What is included in **H**?

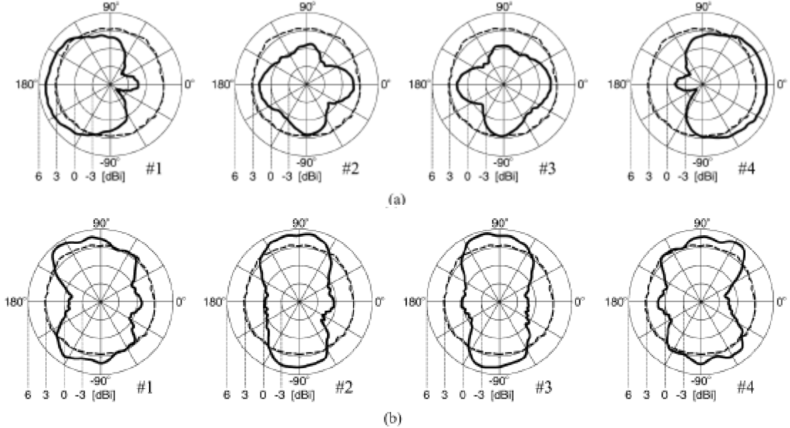
- Depending on the application/ scenario, **H** might include the effects of the antenna array or not.
- In other words, **H** models either (1) the *physical* channel between antennas or (2) the *composite* channel between the antenna ports.
- In case (2), we must include the effect of *mutual coupling*!

Mutual coupling (1)

- Radiation pattern of each single antenna is influenced by neighboring antennas [2]
- Changes in the input impedance of the individual antenna elements in an array
- Property of the antenna array only (antenna spacing and layout, antenna design)
- Can be modeled using mutual impedance, S-parameters, coupling matrix, or element pattern

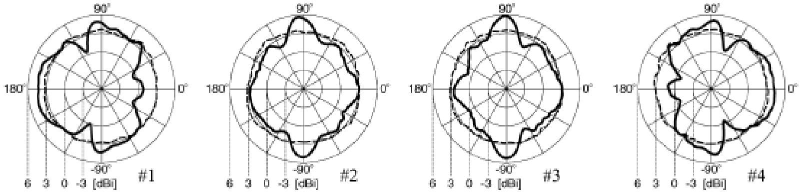
Mutual coupling (2)

Individual antenna patterns influenced by mutual coupling for antenna spacings $d = \lambda/4$ and $d = \lambda/2$ [2]

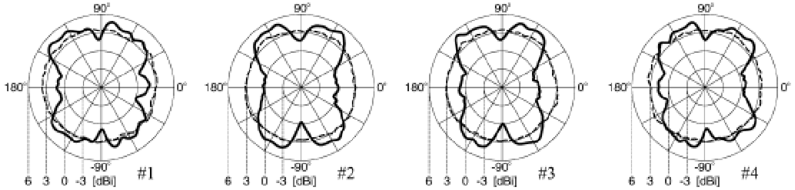


Mutual coupling (3)

Individual antenna patterns influenced by mutual coupling for antenna spacings $d = 3\lambda/4$ and $d = \lambda$ [2]



(c)



(d)

Correlation

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Definition (Correlation Coefficient)

For two complex random variables x and y the correlation coefficient $\rho_{x,y}$ is defined as

$$\rho_{x,y} = \frac{\mathcal{E}\{xy^*\} - \mathcal{E}\{x\}\mathcal{E}\{y^*\}}{\sqrt{\left(\mathcal{E}\{x^2\} - \mathcal{E}\{x\}^2\right)\left(\mathcal{E}\{y^2\} - \mathcal{E}\{y\}^2\right)}}$$

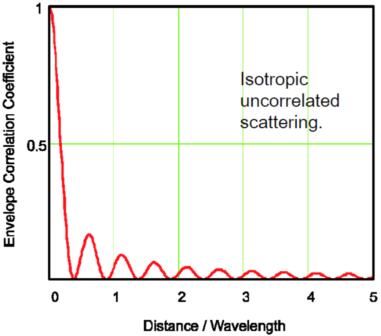
- If x and y have zero mean and unit variance $\rho_{x,y} = \mathcal{E}\{xy^*\}$
- x and y are uncorrelated if $\rho_{x,y} = 0$. Practically $\rho_{x,y} < 0.5$ is sufficient.

Correlation Coefficient: Example

What is the minimum distance so that the signals received by two isotropic antennas are uncorrelated?

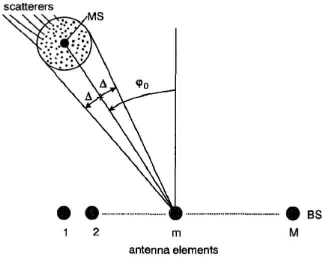
$x(t)$ Rayleigh fading random variable, $y(t) = x(t + \Delta t) = x(t + \frac{d}{v})$

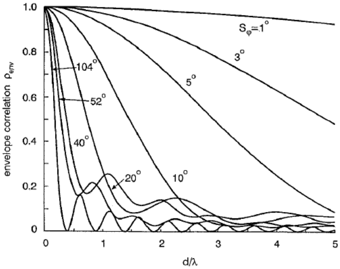
$$\rho_{x,y} = \mathcal{E}\{x(t)y^*(t)\} = J_0^2(2\pi\nu_{\max}\Delta t) = J_0^2(2\pi d/\lambda)$$



Correlation Coefficient: Example

Correlation between antenna elements for non-isotropic power distribution with $\varphi_0 = 60^\circ$ and linear antenna array [3]





Correlation Matrices

- Autocorrelation function of a SISO channel

$$R_h(t, t', \tau, \tau') = \mathcal{E}\left\{h(t, \tau)h^*(t', \tau')\right\}$$

- Autocorrelation function of a MIMO channel

$$R_h(t, t', \tau, \tau', n, n', m, m') = \mathcal{E}\left\{h_{n,m}(t, \tau)h^*_{n',m'}(t', \tau')\right\}$$

- Can also be written as a *correlation matrix*

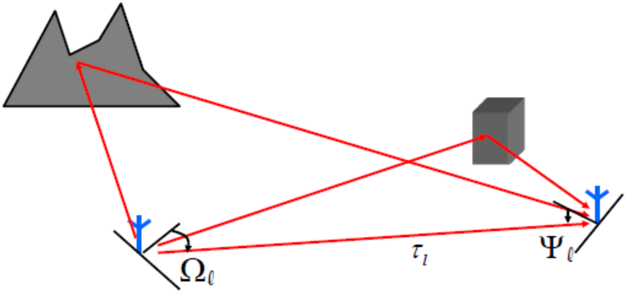
$$\mathbf{R}(t, t', \tau, \tau') = \mathcal{E}\left\{vec(\mathbf{H}(t, \tau))\, vec(\mathbf{H}(t', \tau'))^H\right\}$$

- Size of $\mathbf{R}(t, t', \tau, \tau')$ is $N_{TX}N_{RX} \times N_{TX}N_{RX}$

The double directional channel description (1)

- Correlation matrices fully describe the second order statistics of the channels
- However, they are dependent on the antenna geometry (number and layout of antennas)
- It is desirable to have an antenna-independent description of the channel

The double-directional channel description (2)



$$h(t,\tau,\Omega,\Psi) = \sum_{l=0}^{N-1} h_l(t,\tau,\Omega,\Psi)$$

$$h_l(t,\tau,\Omega,\Psi) = |a_l| e^{j\varphi_l} \delta(\tau - \tau_l)\delta(\Omega - \Omega_l)\delta(\Psi - \Psi_l)$$

where Ω is the angle of departure and Ψ is the angle of arrival

The double-directional channel description (3)

- The MIMO channel matrix can be calculated from the double-directional channel description
- First include the antenna patterns (including mutual coupling)

$$\bar{h}_{n,m}(t,\tau,\varphi,\psi) = G_{\mathrm{Tx}}^{(m)}(\varphi)h(t,\tau,\varphi,\psi)G_{\mathrm{Rx}}^{(n)}(\psi),$$

where

- $G_{\mathrm{Tx}}^{(m)}(\varphi)$ is the antenna pattern of the *n*-th transmit antenna and
- $G_{\mathrm{Rx}}^{(n)}(\psi)$ is the antenna pattern of the *m*-th receive antenna.

The double-directional channel description (3)

- Then we transform from the angular to the spatial domain

$$\begin{aligned} h_{n,m}(t,\tau) &= h_{n,m}(t,\tau,\vec{x}_m,\vec{y}_n) \\ &= \iint \bar{h}_{n,m}(t,\tau,\varphi,\psi) e^{2\pi j/\lambda \langle \vec{\zeta},\vec{x}_m \rangle} e^{2\pi j/\lambda \langle \vec{\xi},\vec{y}_m \rangle} \,\mathrm{d}\varphi \,\mathrm{d}\psi, \end{aligned}$$

where

- $\vec{\zeta} = \begin{pmatrix} \cos\varphi \\ \sin\varphi \end{pmatrix}$, and $\vec{\xi} = \begin{pmatrix} \cos\psi \\ \sin\psi \end{pmatrix}$,
- $\vec{x}_0, \ldots, \vec{x}_{N_{\mathrm{TX}}-1}$ are the transmit antenna locations,
- $\vec{y}_0, \ldots, \vec{y}_{N_{\mathrm{RX}}-1}$ are the receive antenna locations, and
- $\langle .,. \rangle$ denotes the scalar product.

Angular Power Spectra

- The full autocorrelation function of a double-direction channel is given by

$$S(t,\tau,\Omega,\Psi,t',\tau',\Omega',\Psi') = \mathcal{E}\left\{h(t,\tau,\Omega,\Psi)h(t',\tau',\Omega',\Psi')^*\right\}$$

- If the channel is WSS-US and also contributions from different directions are uncorrelated[1]

$$S(t,\tau,\Omega,\Psi,t',\tau',\Omega',\Psi') = P(\Delta t,\tau,\Omega,\Psi)\delta(\tau-\tau')\delta(\Omega-\Omega')\delta(\Psi-\Psi')$$

- Equivalently, in this case the correlation function of the MIMO channel (including the antenna array) is WSS in the antenna domain

$$R_h(t,t',\tau,\tau',n,n',m,m') = R_h(\Delta t,\tau,n-n',m-m')\delta(\tau-\tau')$$

[1] This assumption is sometimes also called homogeneous

Angular Power Spectra

- For $\Delta t = 0$ we get the double directional delay power spectrum

$$DDDPS(\tau, \Omega, \Psi) = P(0, \tau, \Omega, \Psi)$$

- Integrating over τ gives the double directional power spectrum

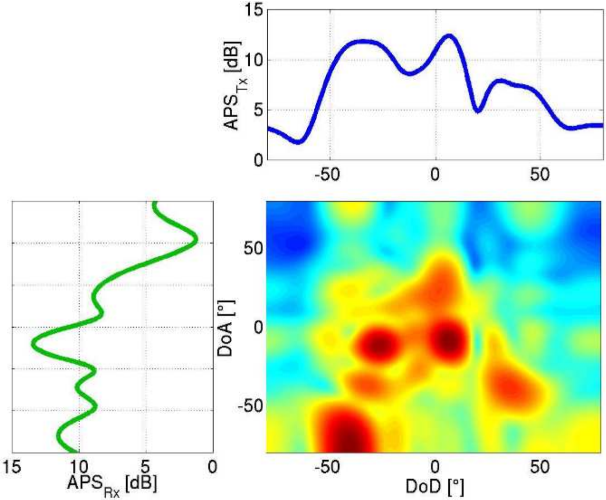
$$DDPS(\Omega, \Psi) = \int DDDPS(\tau, \Omega, \Psi) \mathrm{d}\tau$$

- Integrating over Ω or Ψ gives the angular power spectrum at TX or RX

$$APS_{RX}(\Omega) = \int DDPS(\Omega, \Psi) \mathrm{d}\Psi$$

$$APS_{TX}(\Psi) = \int DDPS(\Omega, \Psi) \mathrm{d}\Omega$$

Angular Power Spectra: Example



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