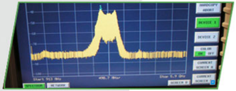


Radio Engineering

Lecture 3
Statistical Channel Characterization

Florian Kaltenberger

Last lecture

4. Antennas and Propagation
   - Maxwell equations
   - Plane waves
   - Linear and circular polarization

Last lecture

4. Antennas and Propagation
   - Maxwell equations
   - Plane waves
   - Linear and circular polarization
   - Free space loss
   - Reflection and transmission
   - Diffraction
   - Scattering

This lecture

5. Statistical description of fading
   - Equivalent baseband representation
   - Small scale fading without a dominat component
   - Small scale fading with a dominat component
   - Doppler spectra
   - Temporal dependence of fading
   - Large-scale fading

Equivalent Baseband Representation (1)

- A signal is *bandpass* if the bandwidth of the signal is small wrt the carrier frequency.
- Most signals used in wireless communication are bandpass

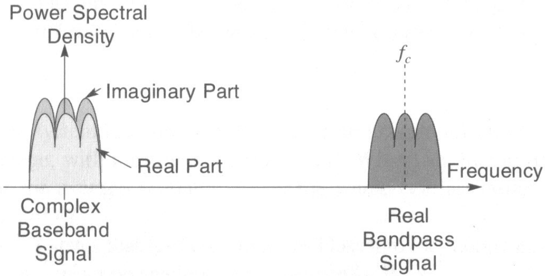


Figure 10.1: Complex baseband representation of signal spectrum

Equivalent Baseband Representation (2)

- A bandpass signal can be written as

$$s(t) = A(t)\cos(2\pi f_c t + \Phi(t))$$

- Complex baseband representation

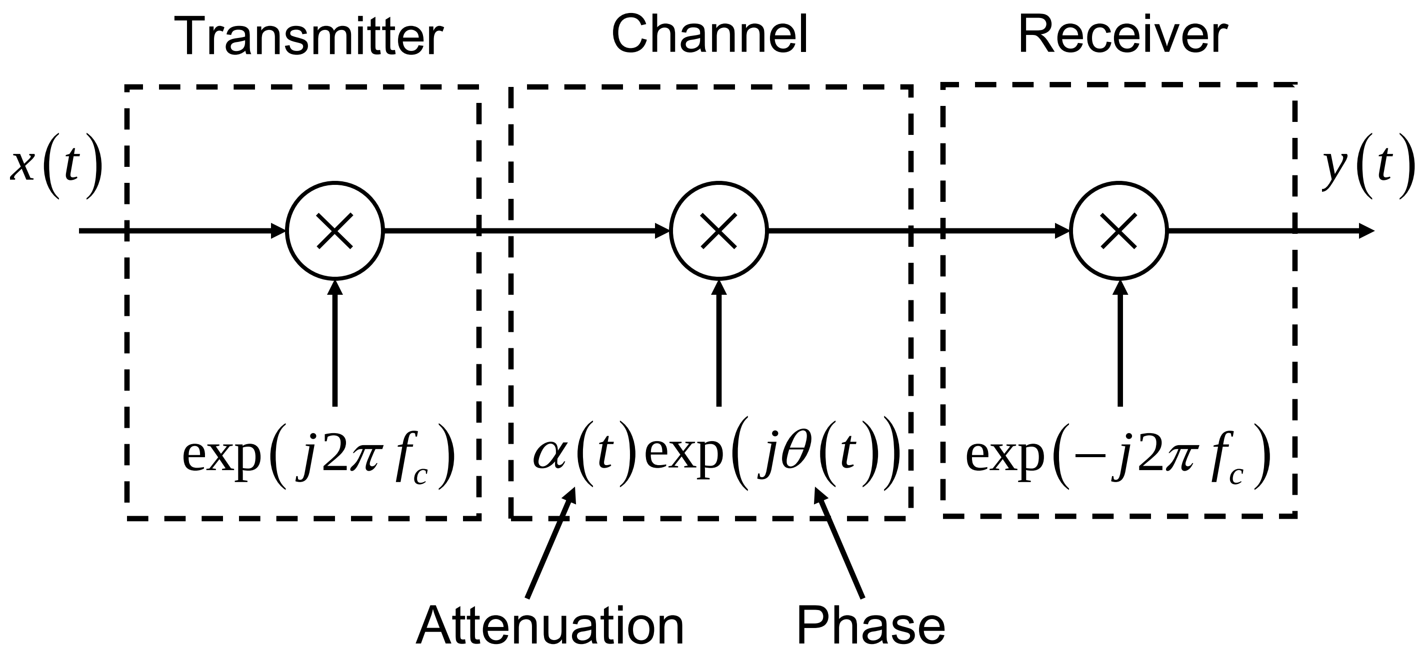
$$\begin{aligned} X(f) &= S(f + f_c) \Leftrightarrow \\ x(t) &= s(t)\exp(-j2\pi f_c t) \\ &= A(t)\exp(j\Phi(t)) \end{aligned}$$

$A(t)$ ... Amplitude, $\Phi(t)$ ... Phase

- Bandpass signal can be recovered by

$$s(t) = \Re\{x(t)\exp(j2\pi f_c t)\}$$

A narrowband system described in complex notation (noise free)



In: $x(t) = A(t)\exp(j\phi(t))$

Out: $y(t) = A(t)\exp(j\phi(t))\cancel{\exp(j2\pi f_c t)}\alpha(t)\exp(j\theta(t))\cancel{\exp(-j2\pi f_c t)}$

$$= A(t)\alpha(t)\exp\big(j(\phi(t)+\theta(t))\big)$$

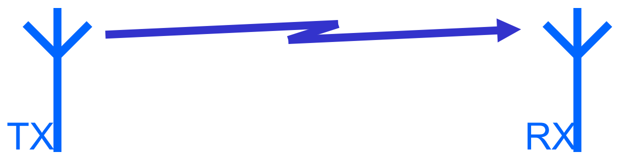
It is the behavior of the channel attenuation and phase we are going to model.

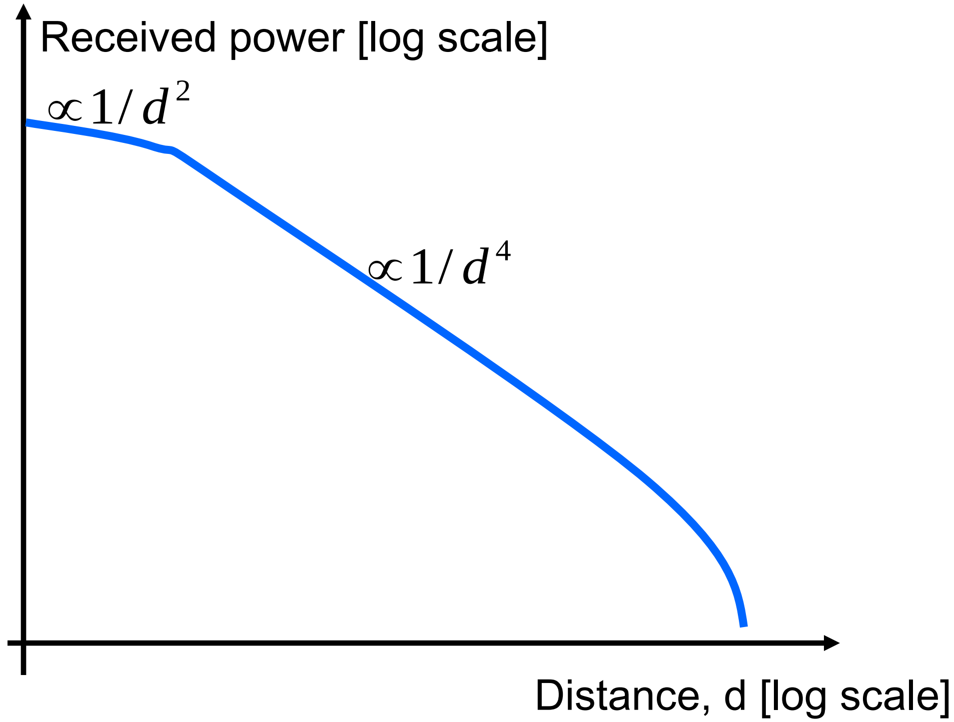
Statistical Characterization of the Radio Channel

Multipath propagation causes *fading*, which can be categorized as

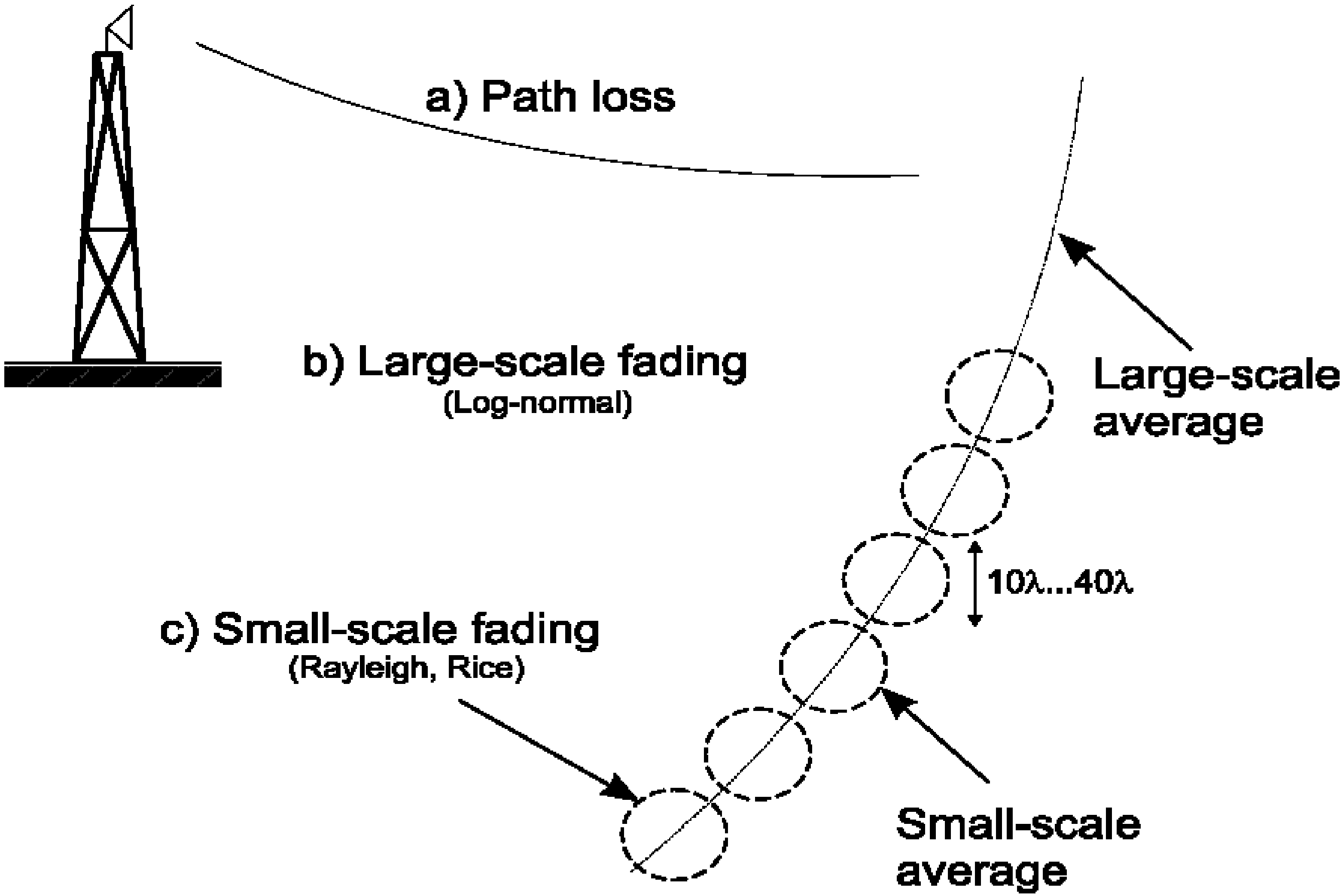
- mean path loss:
  - distance dependent loss in signal energy
  - proportional to d^{-n}, where d is the distance and n is the path loss exponent
  - typicall values $n \in [1.5, 6]$, depending on terrain and foilage
- large-scale (shadow) fading
  - Deviation of received signal energy from path loss
  - Caused by obstruction
- small scale fading
  - Rest of constructive and destructive combination of multipaths

THE RADIO CHANNEL
Path loss

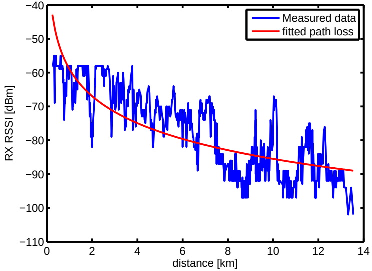


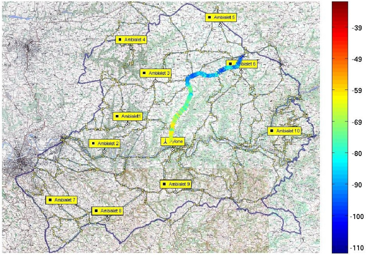


What is large scale and small scale?



Example: Path loss

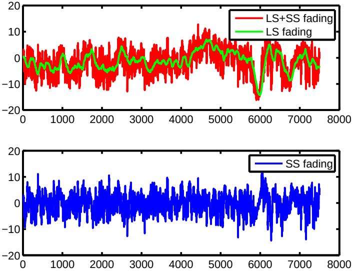




Received power of a terminal in a rural area[1].

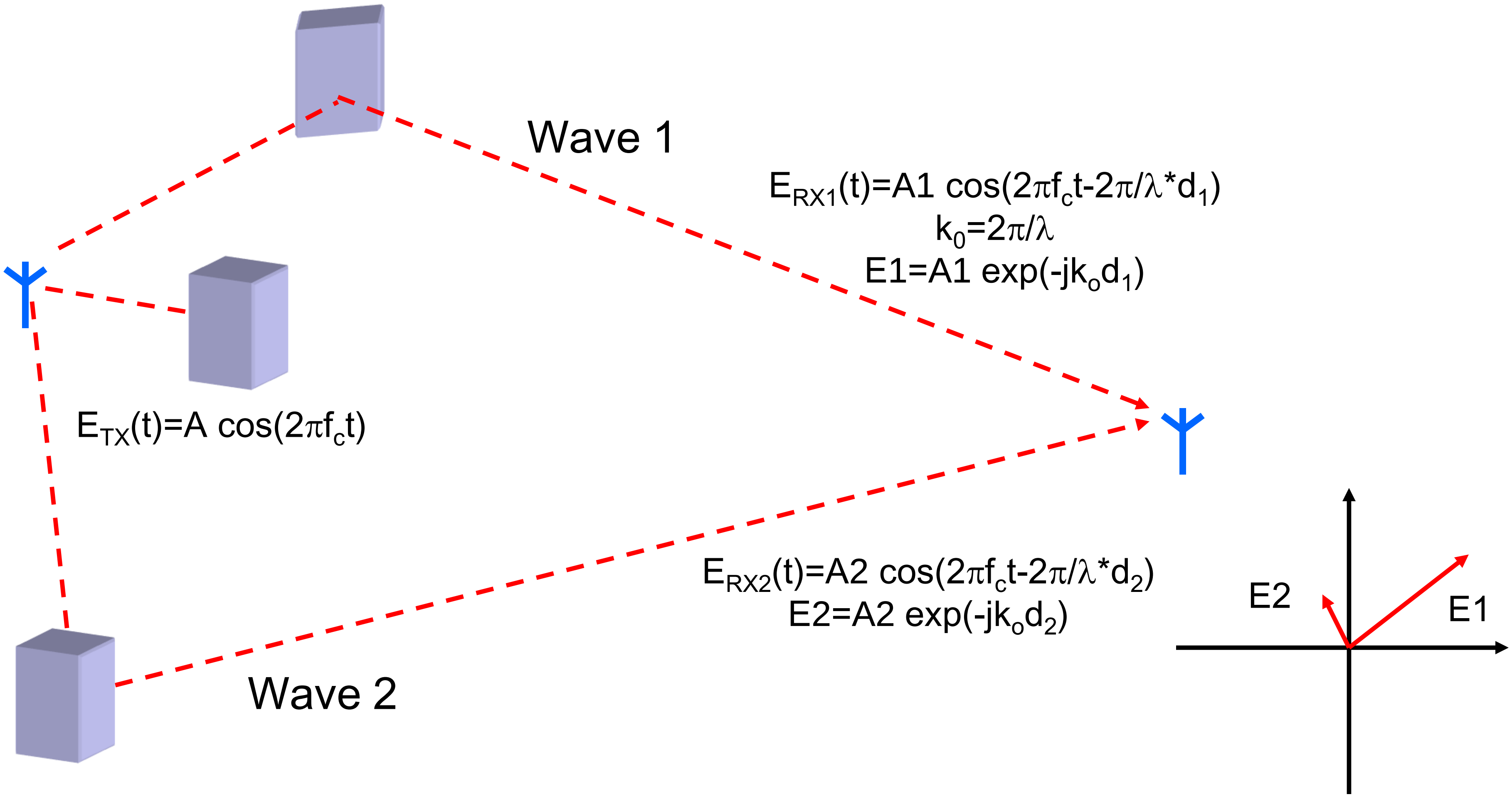
[1] Measurements were taken with OpenAirInterface.org platform at 859.5MHz close to Ambialet, France in collaboration with the CNES

Example: Large scale and small scale fading

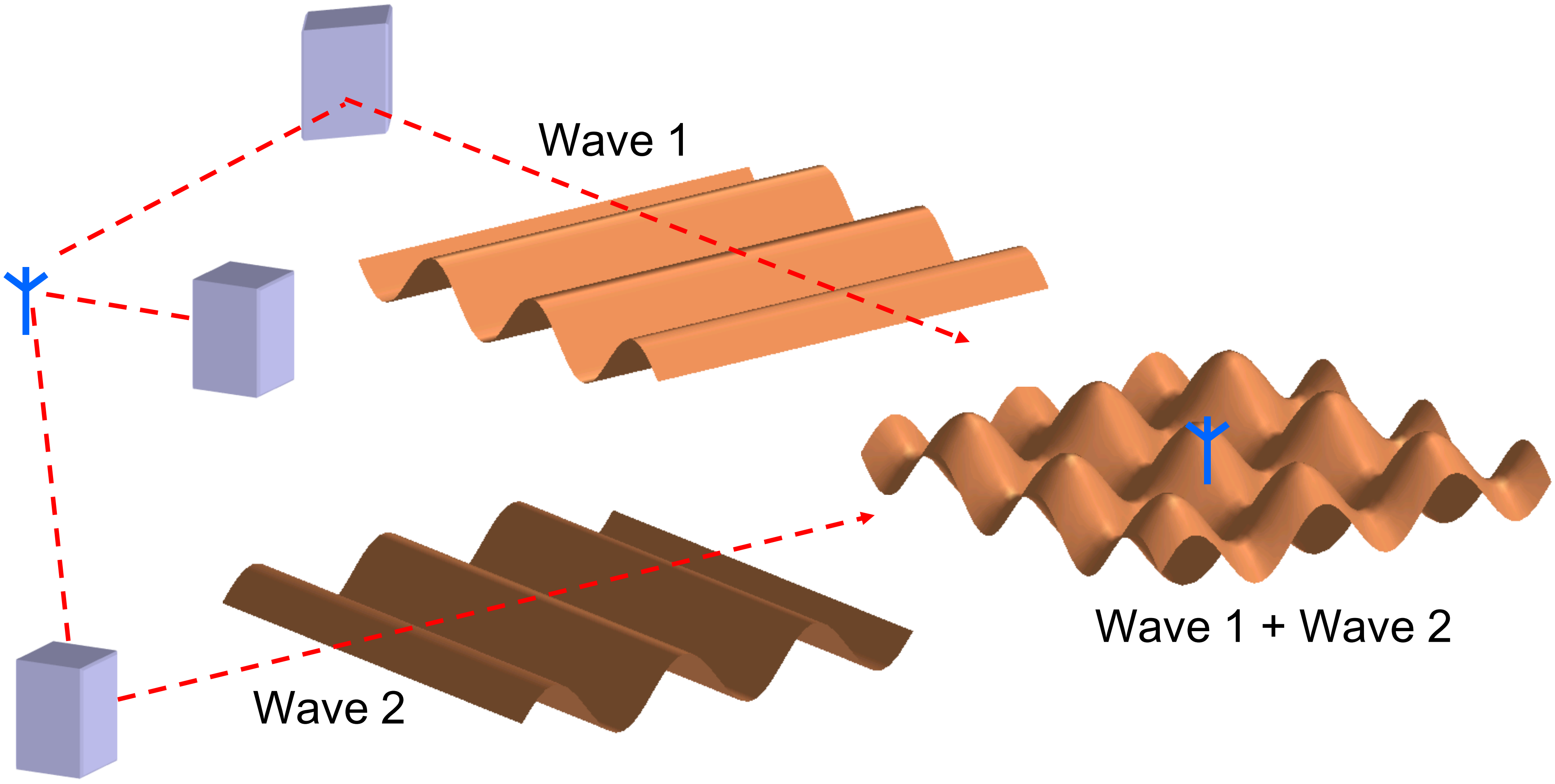


Large scale fading was obtained by applying a moving average filter over 0.25 s $\approx$ 2.5 m (at 10m/s) $\approx 8\lambda$

Small-scale fading
Two waves



Small-scale fading
Two waves



THE RADIO CHANNEL
Small-scale fading (cont.)

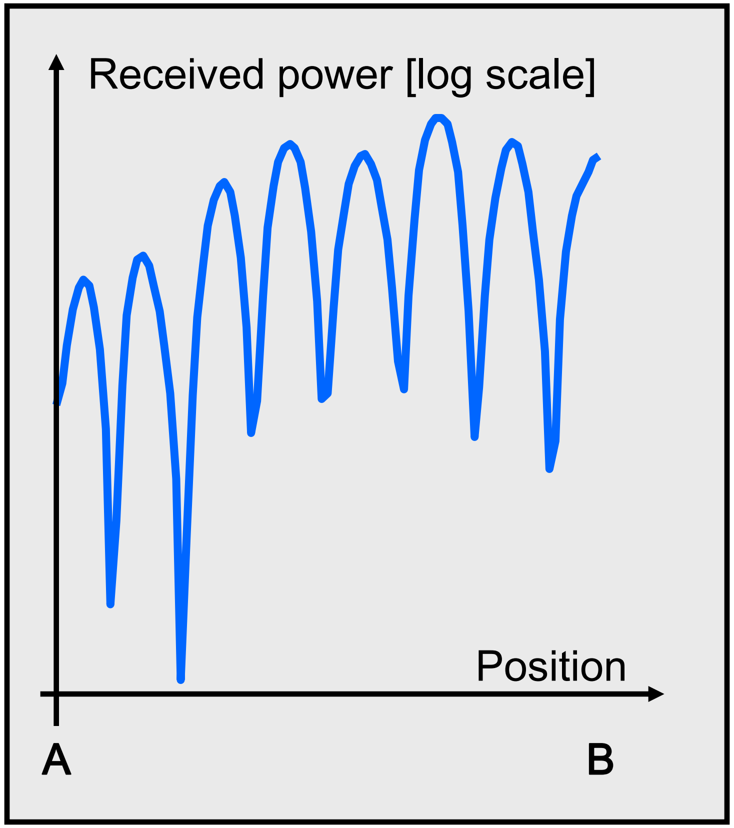
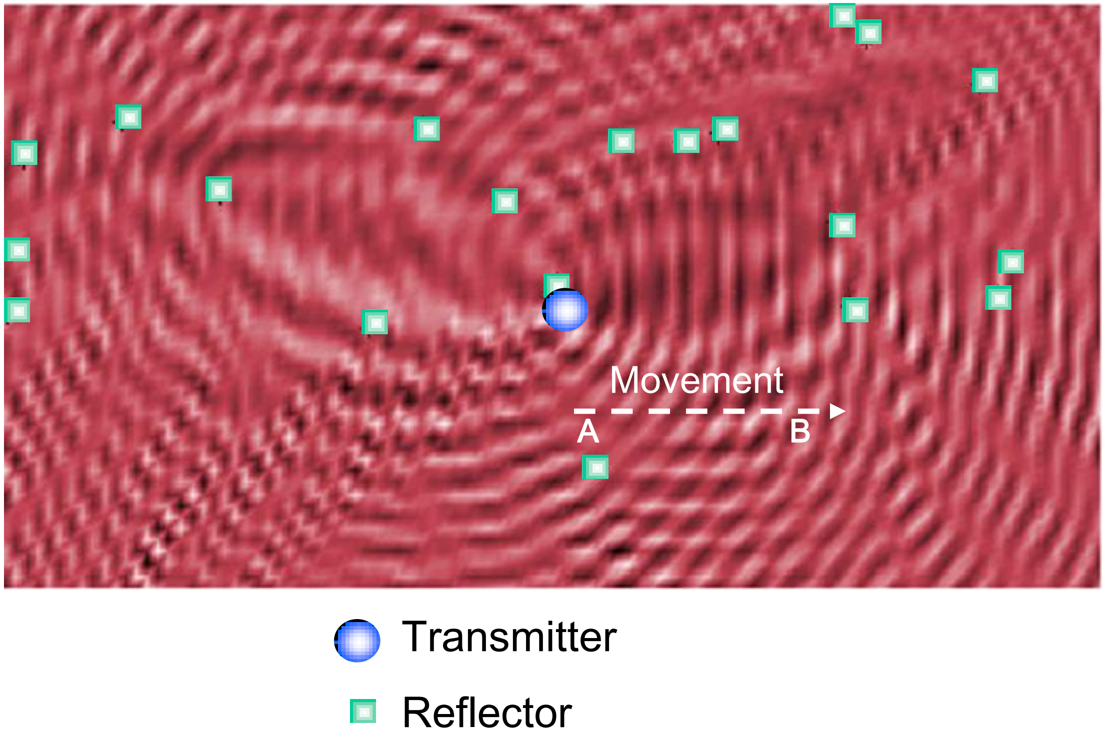
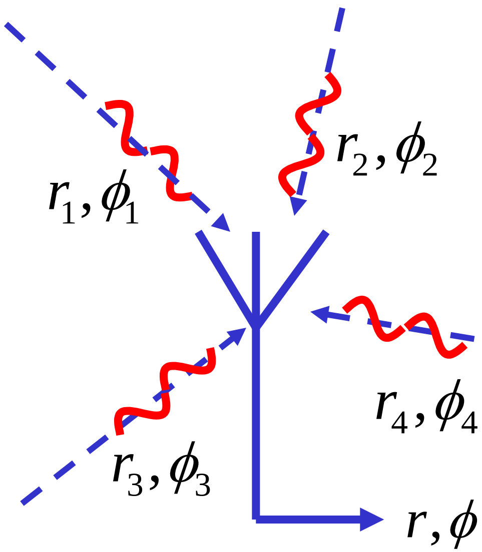


Illustration of interference pattern from above

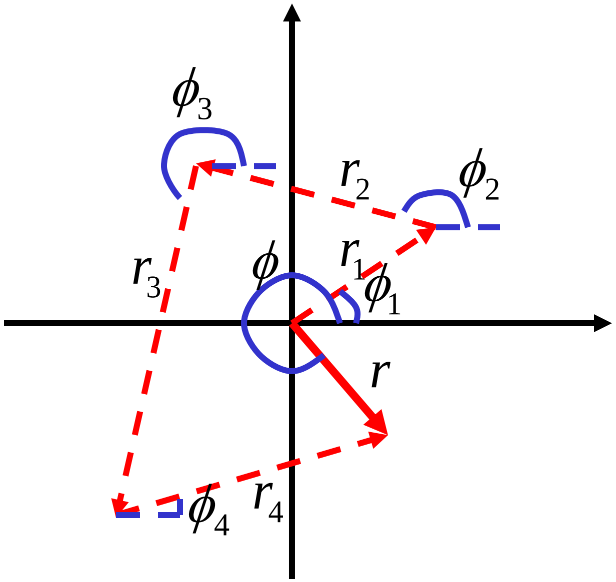


Small-scale fading
Many incoming waves

Many incoming waves with independent amplitudes and phases



Add them up as phasors

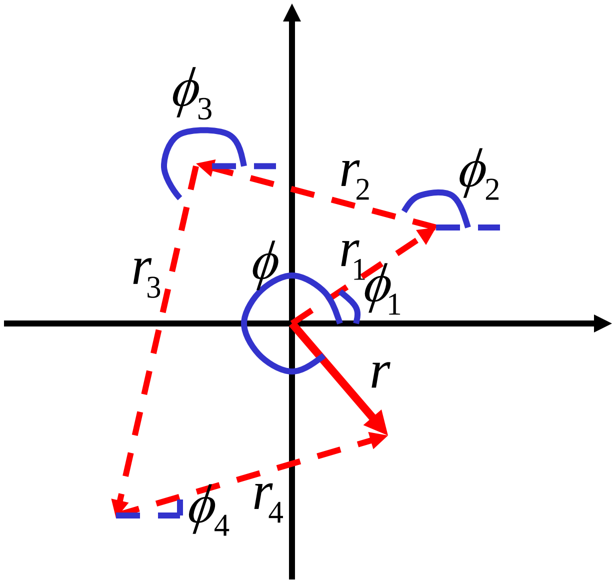


$$r\exp\left(j\phi\right)=r_1\exp\left(j\phi_1\right)+r_2\exp\left(j\phi_2\right)+r_3\exp\left(j\phi_3\right)+r_4\exp\left(j\phi_4\right)$$

Small-scale fading
Many incoming waves

Re and Im components are sums of many independent equally distributed components

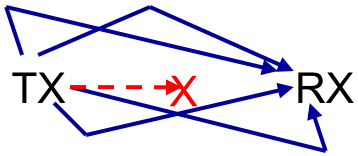
$$\mathrm{Re}(r) \in N(0,\sigma^2)$$



Re(r) and Im(r) are independent

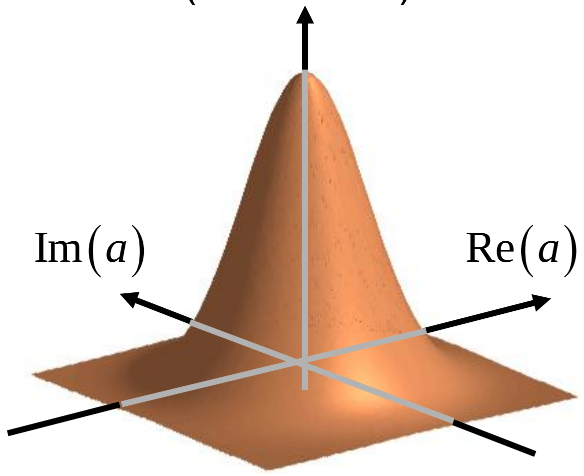
The phase of r has a uniform distribution

Small-scale fading
Rayleigh fading



No dominant component
(no line-of-sight)

Tap distribution
2D Gaussian
(zero mean)

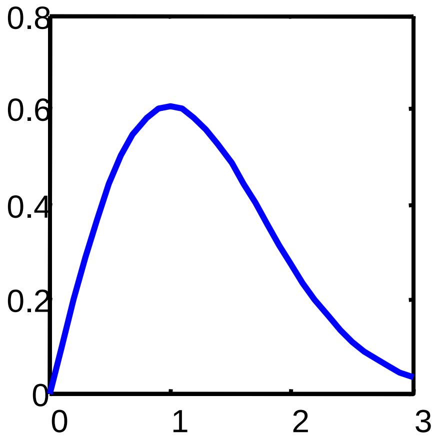


No line-of-sight component

$$r = |a|$$

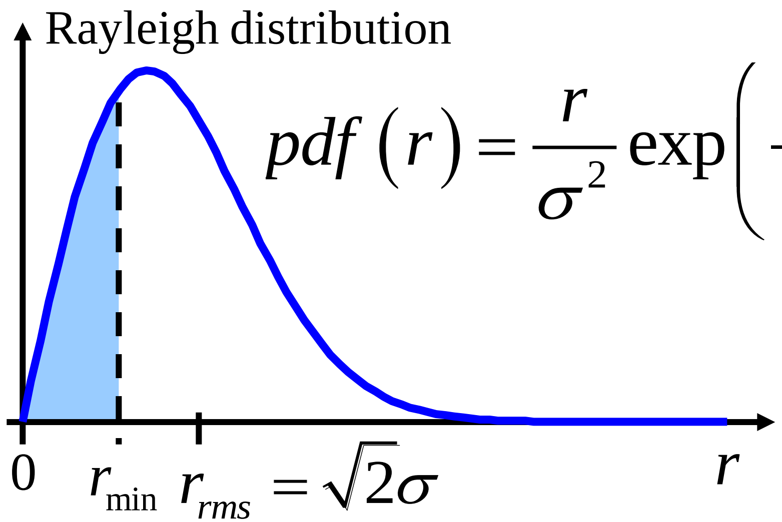
Amplitude distribution

Rayleigh



$$pdf(r) = \frac{r}{\sigma^2} \exp\left(-\frac{r^2}{2\sigma^2}\right)$$

Small-scale fading
Rayleigh fading



$$pdf(r) = \frac{r}{\sigma^2} \exp\left(-\frac{r^2}{2\sigma^2}\right)$$

$$\Pr(r < r_{\min}) = \int_0^{r_{\min}} pdf(r)\, dr = 1 - \exp\left(-\frac{r_{\min}^2}{r_{rms}^2}\right)$$

Example: Fading Margin

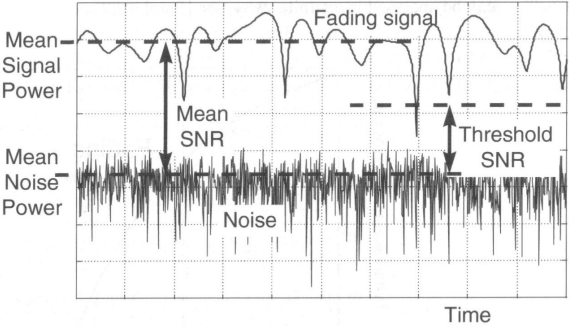
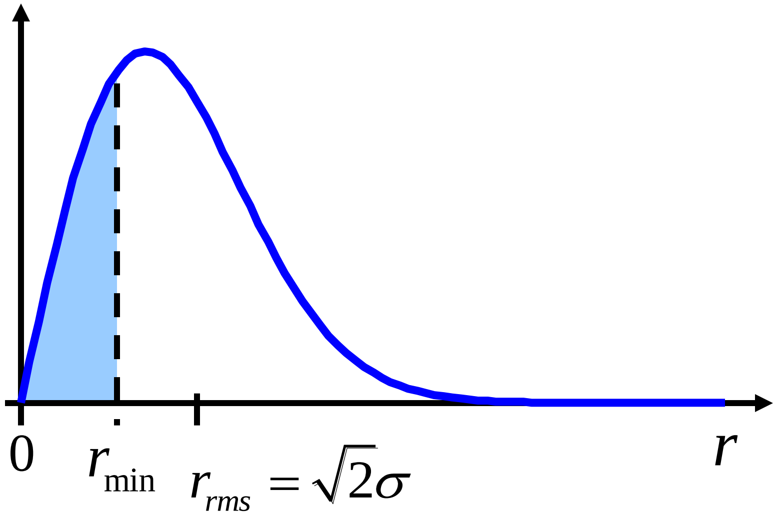


Figure 10.12: Variation of instantaneous SNR relative to mean value

Small-scale fading
Rayleigh fading – fading margin

$$M = \frac{r_{rms}^{\;2}}{r_{\min}^{\;2}}$$

$$M_{|dB} = 10\log_{10}\left(\frac{r_{rms}^{\;2}}{r_{\min}^{\;2}}\right)$$



Small-scale fading
Rayleigh fading – fading margin

How many dB fading margin, against Rayleigh fading, do we need to obtain an outage probability of 1%?

$$\Pr\left(r < r_{\min}\right) = 1 - \exp\left(-\frac{r_{\min}^2}{r_{rms}^2}\right) = 1\% = 0.01$$

Some manipulation gives

$$1 - 0.01 = \exp\left(-\frac{r_{\min}^2}{r_{rms}^2}\right) \Rightarrow \ln\left(0.99\right) = -\frac{r_{\min}^2}{r_{rms}^2}$$

$$\Rightarrow \frac{r_{\min}^2}{r_{rms}^2} = -\ln\left(0.99\right) = 0.01 \Rightarrow M = \frac{r_{rms}^2}{r_{\min}^2} = 1/0.01 = 100$$

$$\Rightarrow \boxed{M_{|dB} = 20}$$

Small-scale fading
Rayleigh fading – signal and interference

- What is the probability that the instantaneous SIR will be below 0 dB if the mean SIR is 10 dB when both the desired signal and the interferer experience Rayleigh fading?

$$\Pr\left(r < r_{\min}\right) = 1 - \frac{\overline{\sigma}^2 r_{\min}}{(\overline{\sigma}^2 + r_{\min}^2)} = 1 - \frac{10}{(10+1)} \approx 0.09$$

Small-scale fading
one dominating component

In case of Line-of-Sight (LOS) one component dominates.

- Assume it is aligned with the real axis

$$\mathrm{Re}(r) \in N(A, \sigma^2) \quad \mathrm{Im}(r) \in N(0, \sigma^2)$$

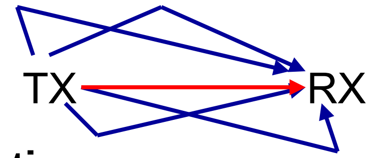
- The received amplitude has now a Ricean distribution instead of a Rayleigh

- The ratio between the power of the LOS component and the diffuse components is called Ricean K-factor

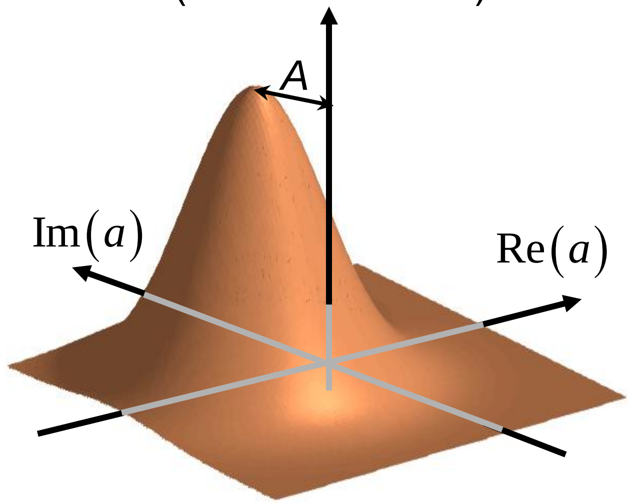
$$k = \frac{\text{Power in LOS component}}{\text{Power in random components}} = \frac{A^2}{2\sigma^2}$$

Small-scale fading
Rice fading

A dominant component (line of sight)

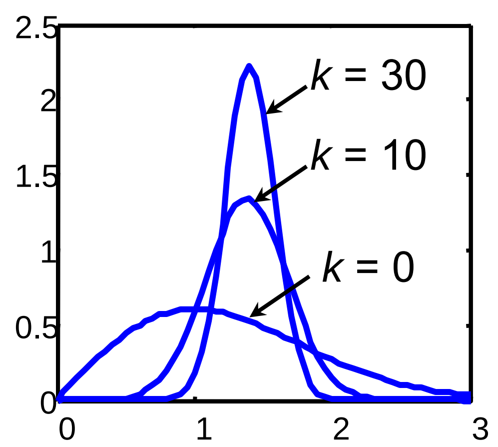


Tap distribution
2D Gaussian
(non-zero mean)



$$r = |\alpha|$$

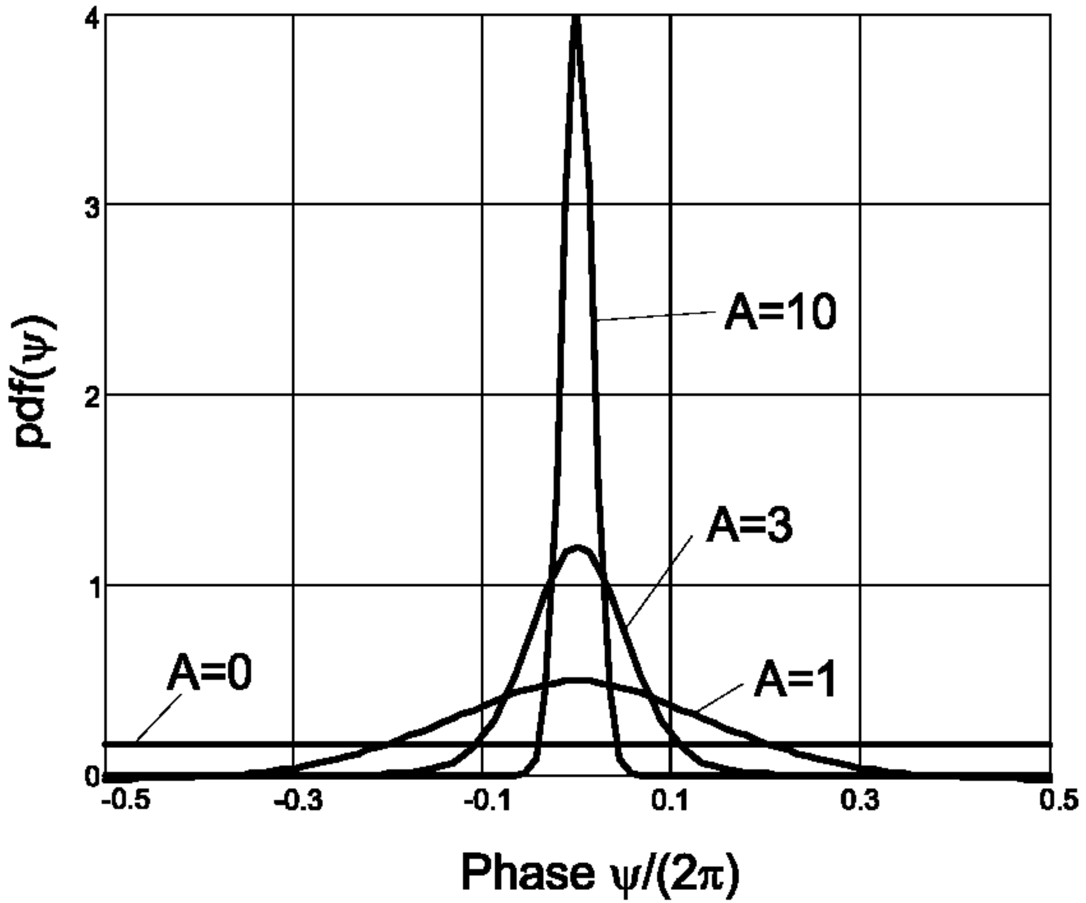
Amplitude distribution



Line-of-sight (LOS) component with amplitude A.

$$pdf(r) = \frac{r}{\sigma^2} \exp\left(-\frac{r^2 + A^2}{2\sigma^2}\right) I_0\left(\frac{rA}{\sigma^2}\right)$$

Small-scale fading
Rice fading, phase distribution



Small Scale Fading: Rice fading

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- Probability density function, cumulative distribution function, and mean square value of Ricean distribution

$$\mathrm{pdf}(r) = \frac{r}{\sigma^2} \exp - \frac{r^2 + A^2}{2\sigma^2} I_0 \left(\frac{rA}{\sigma^2} \right), \quad 0 \leq r < \infty,$$

$$\mathrm{cdf}(r) = 1 - Q_M \left(\frac{A}{\sigma}, \frac{r}{\sigma} \right)$$

$$\bar{r^2} = 2\sigma^2 + A^2$$

where I_0 is the modified Bessel function of the first kind, order 0 and Q_M is Marcum's Q function

Example: Ricean Fading Margin

Compute the fading margin for a Rice distribution with $\sigma = 1$ and $K_r = 0.3, 3$, and 20 dB so that the outage probability is less than 5%.

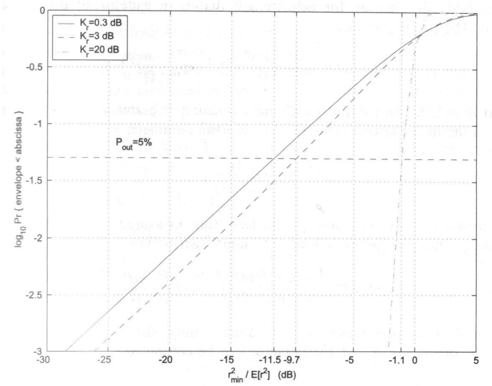


Figure 5.21 The Rice power *cdf*, $\sigma = 1$.

$$M = \frac{r_{\text{rms}}^2}{r_{\text{min}}^2} = \frac{2\sigma^2(1+K_r)}{r_{\text{min}}^2}$$
$$= 11.5, 9.7, 1.1\text{dB}$$

Small-scale fading
Nakagami distribution

- In many cases the received signal can not be described as a pure LOS + diffuse components
- The Nakagami distribution is often used in such cases

$$pdf(r) = \frac{2}{\Gamma(m)}\left(\frac{m}{\Omega}\right)^m r^{2m-1} \exp\left(-\frac{m}{\Omega} r^2\right)$$

$\Gamma(m)$ is the gamma function

$$\Omega = \overline{r^2}$$

$$m = \frac{\Omega^2}{(r^2 - \Omega)^2}$$

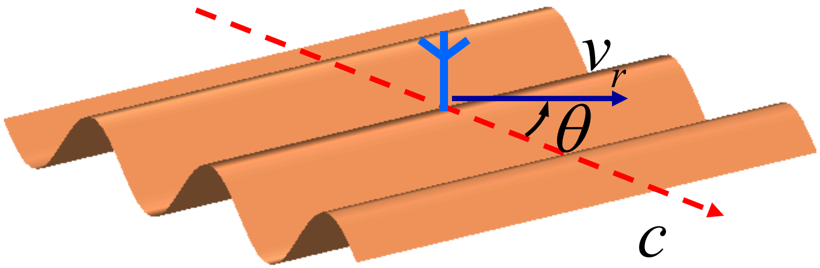
with m it is possible to adjust the dominating power

Second-order fading statistics

- Capture dynamic effects of the channel (evolution over time, rate of change)
- Let $x(t)$ be a stochastic process, then the *autocorrelation of x* is defined as
$$R_{xx}(t_1, t_2) = \mathcal{E}\{x(t_1)x^*(t_2)\}$$
- Iff $R_{xx}(t_1, t_2) = R_{xx}(t_1 - t_2)$, x is *wide sense stationary* (WSS)
- The *power spectrum* of a WSS process is given by

$$S(f) = \mathcal{F}\{R_{xx}(\tau)\} = \int R_{xx}(\tau)e^{-j2\pi f \tau}\mathrm{d}\tau$$

Small-scale fading
Doppler shifts



Receiving antenna moves with speed v_r at an angle θ relative to the propagation direction of the incoming wave, which has frequency f_0.

Frequency of received signal:

$$f = f_0 + \nu$$

where the Doppler shift is

$$\nu = -f_0 \frac{v_r}{c} \cos(\theta)$$

The maximal Doppler shift is

$$\nu_{\max} = f_0 \frac{v}{c}$$

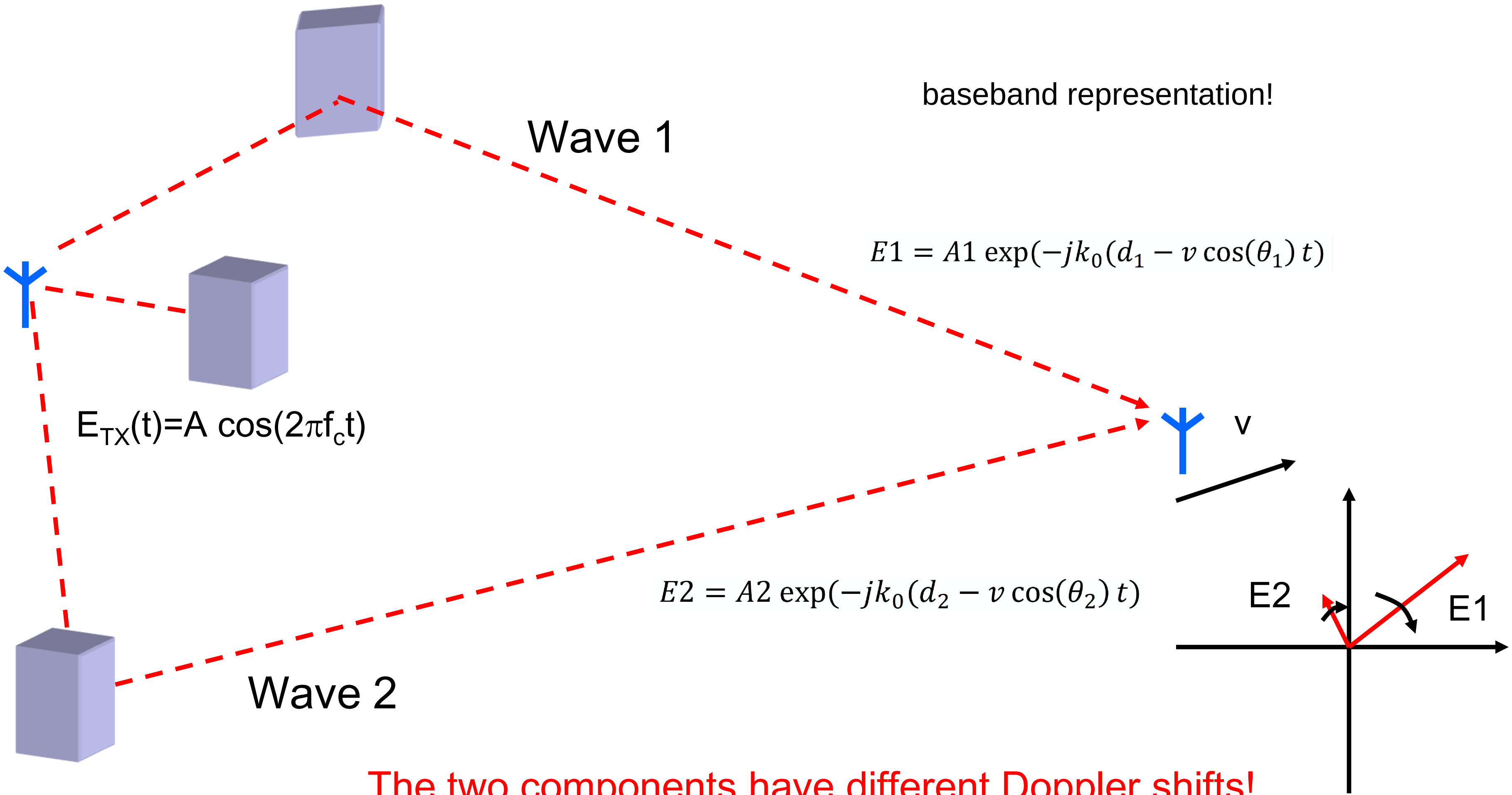
Small-scale fading
Doppler shifts

How large is the maximum Doppler frequency at pedestrian speeds for 5.2 GHz WLAN and at highway speeds using GSM 900?

$$\nu_{\max} = f_0 \frac{v}{c}$$

- f_0=5.2 10^9 Hz, v=5 km/h, (1.4 m/s) ⇒ 24 Hz
- f_0=900 10^6 Hz, v=110 km/h, (30.6 m/s) ⇒ 92 Hz

Small-scale fading
Doppler spectra



baseband representation!

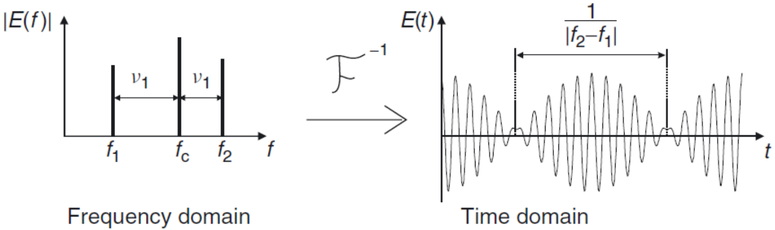
$$E1 = A1\exp(-jk_0(d_1 - v\cos(\theta_1)\,t)$$

$$E2 = A2\exp(-jk_0(d_2 - v\cos(\theta_2)\,t)$$

The two components have different Doppler shifts!
The Doppler shifts will cause a random frequency modulation

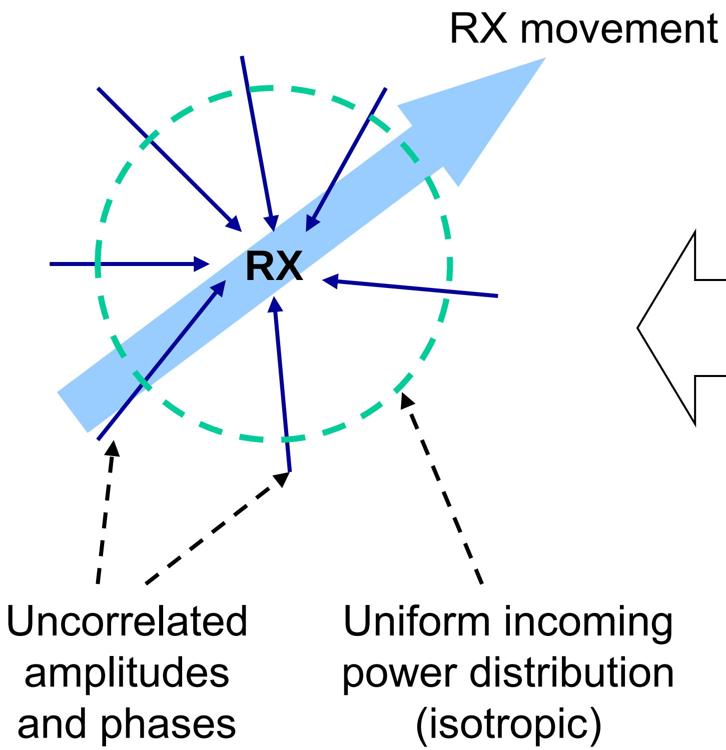
Small-scale fading: Doppler shifts

- Superposition of waves with different Doppler shifts creates "beating" effect

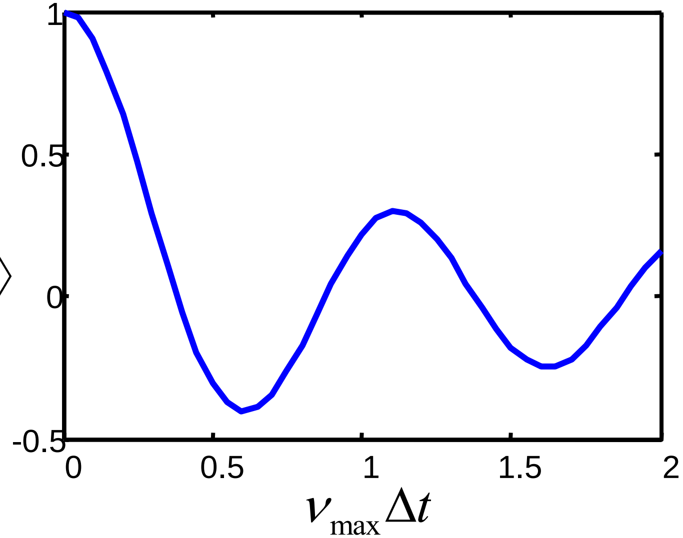


Small-scale fading
Doppler spectrum

Isotropic uncorrelated scattering



Time correlation



Small-scale fading
Doppler spectrum

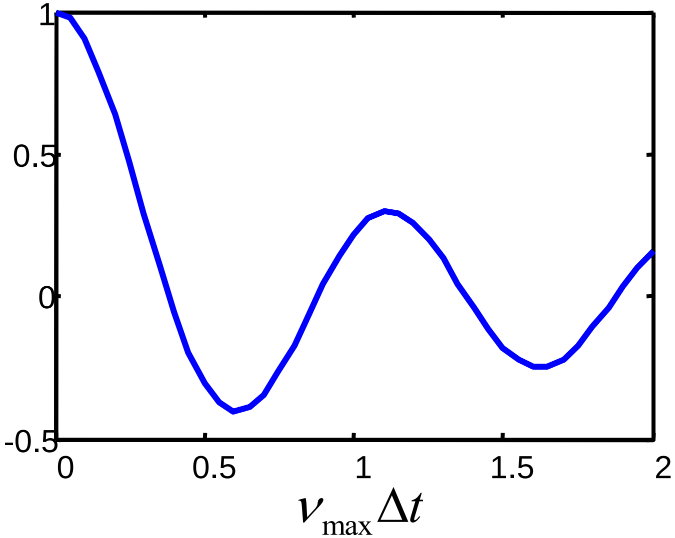
How static is the channel?

- Time correlation of in-phase and quadrature components*

$$\rho(\Delta t) = E\{a(t)a^*(t+\Delta t)\} \propto J_0(2\pi\nu_{\max}\Delta t)$$

- The time correlation for the amplitude is

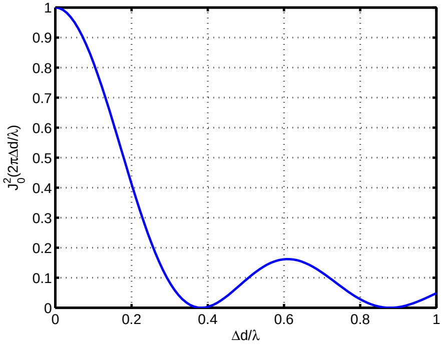
$$\rho(\Delta t) \propto J_0^2(2\pi\nu_{\max}\Delta t)$$



* correlation between in-phase and quadrature is 0!

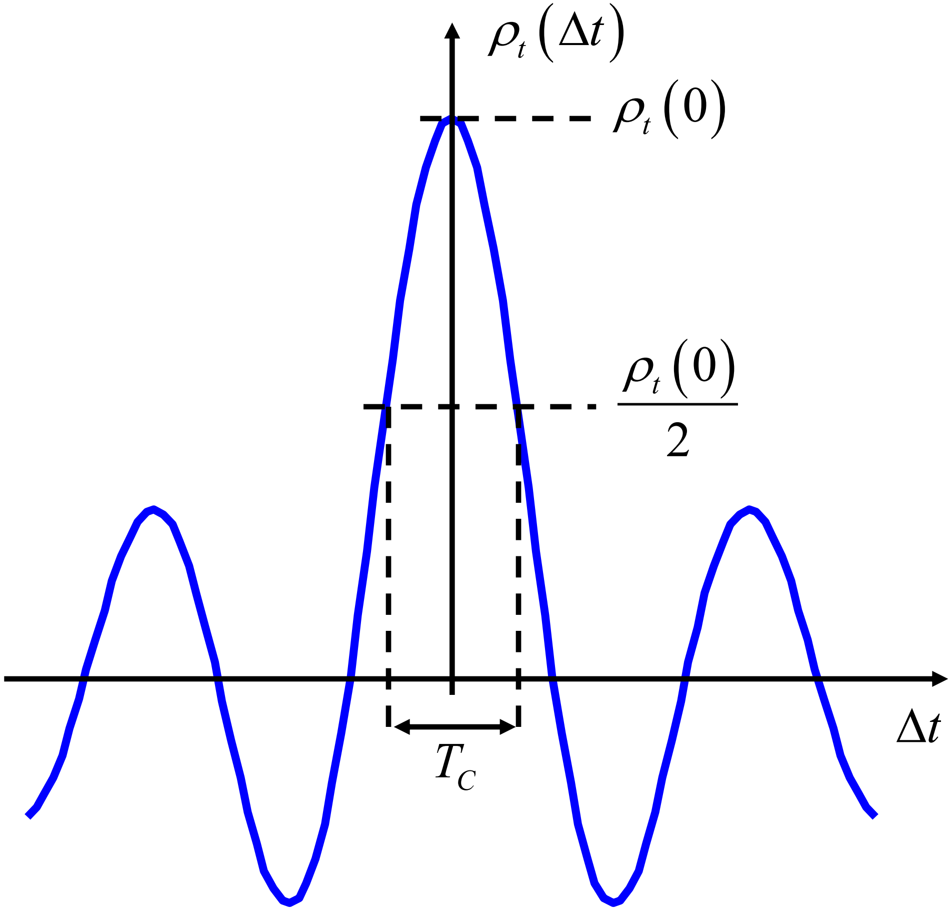
Example: Autocorrelation

Assume that the mobile is in a fading dip. On average, what minimum distance should the user move, so that it is no longer influenced by this fading dip?



Condensed parameters
Coherence time

Given the time correlation of a channel, we can define the **coherence time** T_C:



Small-scale fading
Doppler spectrum

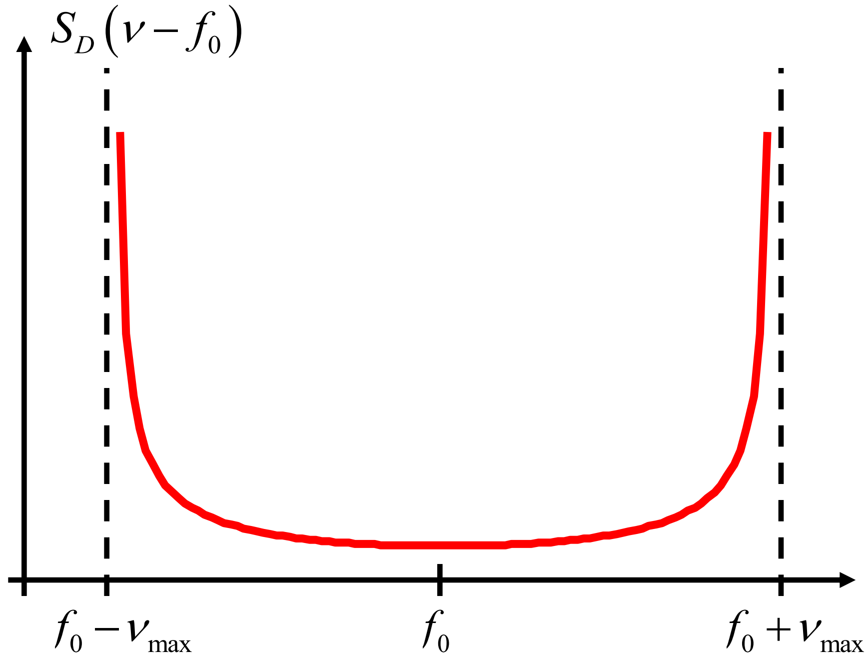
AoA are uniformly distributed

$$S_D(\nu) = \int \rho(\Delta\tau) e^{-j2\pi\nu\Delta\tau} d\Delta\tau$$

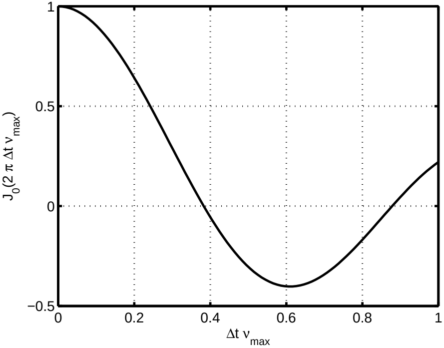
$$\propto \frac{1}{\pi\sqrt{{\nu_{\max}}^2 - \nu^2}}$$

for $-\nu_{\max} < \nu < \nu_{\max}$

Doppler spectrum
at center frequency f_0.



Coherence time and Doppler bandwidth



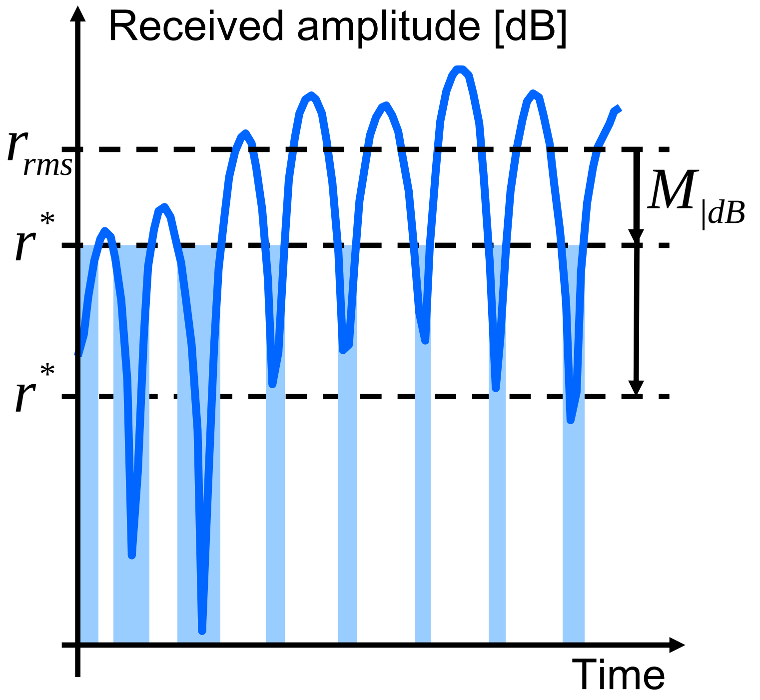
$$T_c \nu_{\max} \approx 0.25$$

Small-scale fading
Fading dips

What about the length and the frequency of fading dips ?

Level crossing rate: how often does the signal cross the level r*?

Average duration of fade: how long does the signal stay below r*?



Level crossing rate and avarage duration of fades

- Level crossing rate

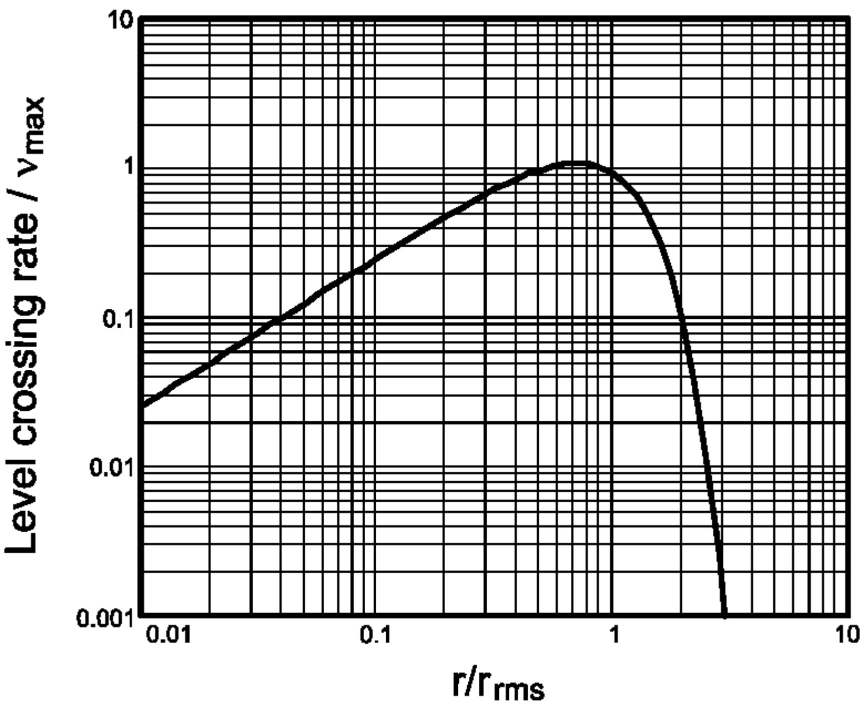
$$
\begin{aligned}
N_R(r) &= \int_0^\infty \dot{r} \cdot \mathrm{pdf}(r, \dot{r}) d\dot{r} \\
&= \sqrt{\frac{\Omega_2}{\pi\Omega_0}} \frac{r}{\sqrt{2\Omega_0}} \exp\left(-\frac{r^2}{2\Omega_0}\right)
\end{aligned}
$$

  where Ω_n is the n-th moment of the Doppler power spectrum ($r_{rms} = \sqrt{\Omega_0}$)

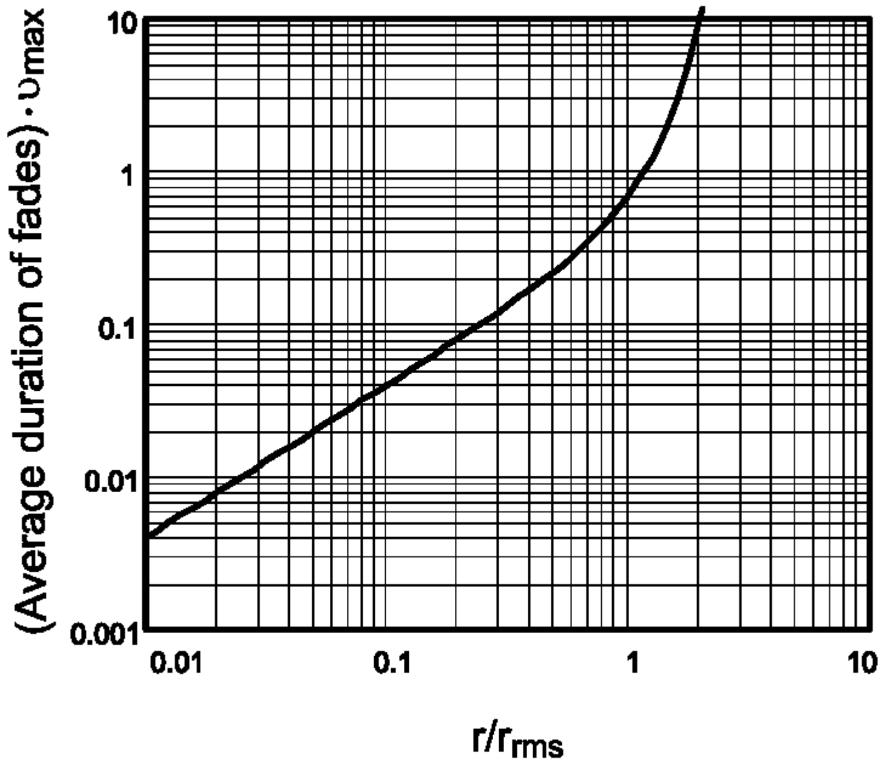
- Average duration of fade

$$
ADF(r) = \frac{\mathrm{cdf}(r)}{N_R(r)}
$$

Frequency of the fading dips
(normalized dips/second)



Length of fading dips
(normalized dip-length)



Example: GSM system

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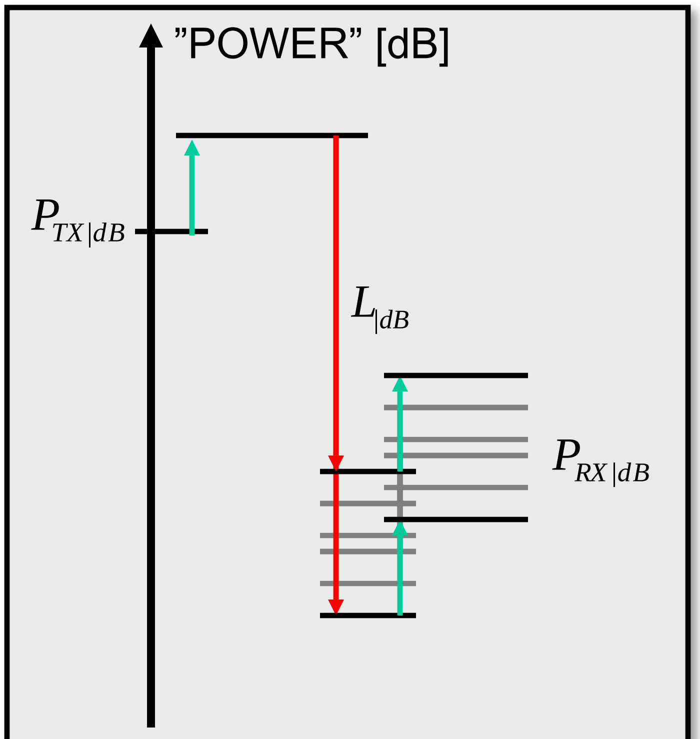
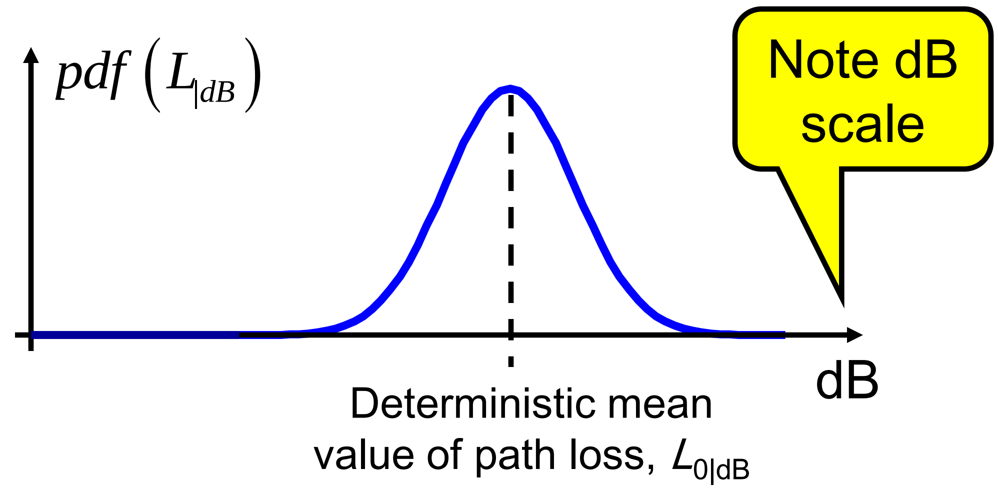
Consider a GSM system at $f_c = 900$ MHz and a maximum user speed of $v_{\max} = 100$ km/h. Assume that the channel has a classical Doppler spectrum. What is the Doppler bandwidth and the coherence time? What is the level crossing rate and the average fade duration given a fading margin of 10 and 20 dB respectively.
Discuss the implication of the finding under the consideration that GSM has a burst duration of 0.5 ms.

Large-scale fading
Log-normal distribution





$$pdf\left(L_{|dB}\right) = \frac{1}{\sqrt{2\pi}\sigma_{F|dB}} \exp\left(-\frac{\left(L_{|dB} - L_{0|dB}\right)^2}{2\sigma_{F|dB}^{\ 2}} \right)$$

Standard deviation $\sigma_{F|dB} \approx 4 \ldots 10 \text{ dB}$

Large-scale fading
Basic principle