4.6.1 Appendix 4.A: Derivation of the d^{-4} Law

In this appendix, we derive the d^{-4} law for the received power. Consider the geometry in Fig. 4.15. The transmit antenna is placed at height h_{TX} over ground, where the ground is assumed to be at least partially conducting. The receive antenna is at height h_{RX} above ground. The distance to the transmit antenna is d. Two rays reach the receiver: the direct (line-of-sight) path, and the ground reflection. The angle of incidence for the ground-reflected ray is usually close to 90° (grazing angle of incidence), as the antenna heights (1.5m for the MS, 10 – 100m for the BS), are much smaller than the distance between MS and BS. We saw in Sec. 4.2.1 that in this case, the reflection coefficient is approximately -1, irrespective of the actual conductivity of the ground

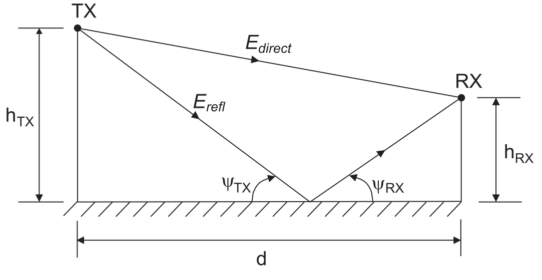

Figure 4.15 Geometry for the derivation of the d^{-4} law.

The field strength for the direct path is thus

$$E_{\mathrm{direct}}(d_{\mathrm{direct}}) = E(1\mathrm{m}) \left(\frac{1}{d_{\mathrm{direct}}|_{\mathrm{m}}} \right) \exp\left[j \left(2\pi f_{\mathrm{c}} t - 2\pi f_{\mathrm{c}} \frac{d_{\mathrm{direct}}}{c_0} \right) \right] \tag{4.50}$$

which follows from Eq. 4.9 (though note the phase term, as we are considering field strength now). Here, c_0 is the speed of light. The field strength of the reflected path is

$$E_{\mathrm{refl}}(d_{\mathrm{refl}}) = (-1) \cdot E(1\mathrm{m}) \left(\frac{1}{d_{\mathrm{refl}}|_{\mathrm{m}}} \right) \exp\left[j \left(2\pi f_{\mathrm{c}} t - 2\pi f_{\mathrm{c}} \frac{d_{\mathrm{refl}}}{c_0} \right) \right] \tag{4.51}$$

where the factor -1 is due to the reflection coefficient of the ground. The pathlengths d_{direct} and d_{refl} can be computed from simple geometric considerations as

$$d_{\mathrm{direct}} = \sqrt{(h_{\mathrm{TX}} - h_{\mathrm{RX}})^2 + d^2} \qquad d_{\mathrm{refl}} = \sqrt{(h_{\mathrm{TX}} + h_{\mathrm{RX}})^2 + d^2} \tag{4.52}$$

The total fieldstrength is obtained by coherent addition of the direct and the reflected ray.

Additional simplifications can be made due to the fact that the height of the antennas is much smaller than the distance between them. The impact of the longer distance for the ground reflected path on the amplitudes of the field strengths is negligible; only the resulting phase shift plays a role. This allows to write the total field as

$$\begin{aligned} E_{\mathrm{tot}}(d) &= E(1m) \left(\frac{1}{d|_m} \right) \left\{ \exp\left[j \left(2\pi f_{\mathrm{c}} t - 2\pi f_{\mathrm{c}} \frac{d_{\mathrm{direct}}}{c_0} \right) \right] - \exp\left[j \left(2\pi f_{\mathrm{c}} t - 2\pi f_{\mathrm{c}} \frac{d_{\mathrm{refl}}}{c_0} \right) \right] \right\} \\ &= E(1m) \left(\frac{1}{d|_m} \right) \exp\left[j \left(2\pi f_{\mathrm{c}} t - 2\pi f_{\mathrm{c}} \frac{d_{\mathrm{direct}}}{c_0} \right) \right] \left\{ 1 - \exp\left[-j \left(2\pi f_{\mathrm{c}} \frac{d_{\mathrm{refl}} - d_{\mathrm{direct}}}{c_0} \right) \right] \right\} \end{aligned} \tag{4.53}$$

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Furthermore, the differences in the pathlength, d_{refl} - d_{direct}, can be expanded into a Taylor series, so that

$$d_{\text{refl}} - d_{\text{direct}} = 2\frac{h_{\text{TX}}h_{\text{RX}}}{d} \tag{4.54}$$

The magnitude of the field strength can thus be written as

$$|E_{\text{tot}}(d)| = E(1m)\frac{1}{d|_m}\sqrt{(1-\cos(\Delta\varphi))^2 + \sin^2(\Delta\varphi)} \tag{4.55}$$

where

$$\Delta\varphi = 2\frac{h_{\text{TX}}h_{\text{RX}}}{d}\frac{2\pi f_{\text{c}}}{c_0} \tag{4.56}$$

Assume now that $\Delta\varphi$ is much smaller than $\pi/2$ (an assumption that is consistent with our basic assumption that d is large and the antenna heights are small). This is fulfilled for

$$d_{\text{limit}} \gg \frac{8h_{\text{TX}}h_{\text{RX}}}{\lambda} \tag{4.57}$$

In that case, $\sin(\Delta\varphi) \approx \Delta\varphi$, and $1 - \cos(\Delta\varphi)$ is on the order of $(\Delta\varphi)^2$, i.e., negligible. Inserting this into the equation for E_{tot} results in

$$|E_{\text{tot}}(d)| = E(1m)\frac{1}{d}2\frac{h_{\text{TX}}h_{\text{RX}}}{d}\frac{2\pi}{\lambda} \tag{4.58}$$

According to this equation, the field strength decreases with the square of the distance, and the received power thus decreases with d^4. Friis' law is then replaced with

$$P_{\text{RX}}(d) \approx P_{\text{TX}}G_{\text{TX}}G_{\text{RX}}\left(\frac{h_{\text{TX}}h_{\text{RX}}}{d^2}\right)^2 \tag{4.59}$$

From Eq. (4.53), it follows that this behavior occurs when $\Delta\varphi \leq \pi$, i.e., for a distance $d \geq d_{\text{break}}$ where

$$d_{\text{break}} = \frac{4h_{\text{TX}}h_{\text{RX}}}{\lambda} \tag{4.60}$$

4.6.2 Appendix 4.B: Diffraction Coefficients for Diffraction by a Wedge or Cylinder

The diffraction coefficient D can be obtained from the *Uniform Theory of Diffraction* [Kouyoumjian and Pathak 1974], which in turn is a generalization of the *Geometrical Theory of Diffraction* [Keller 1962]

$$D(\phi_{\mathrm{TX}}, \phi_{\mathrm{RX}}) = -\frac{\exp(-j\pi/4)}{2\psi\sqrt{2\pi k_0}}\left[D_1 + D_2 + \rho_{\mathrm{TX}} D_3 + \rho_{\mathrm{RX}} D_4\right] \tag{4.61}$$

where $\pi \cdot \psi$ defines the outer angle of the wedge according to Fig. 4.8, and

$$\begin{aligned}
D_1 &= \cot\left(\frac{\pi + (\phi_{\mathrm{RX}} - \phi_{\mathrm{TX}})}{2\psi}\right) F_{\mathrm{T}}\left(k_0 L a_{\mathrm{p}}(\phi_{\mathrm{RX}} - \phi_{\mathrm{TX}})\right)\\
D_2 &= \cot\left(\frac{\pi - (\phi_{\mathrm{RX}} - \phi_{\mathrm{TX}})}{2\psi}\right) F_{\mathrm{T}}\left(k_0 L a_{\mathrm{m}}(\phi_{\mathrm{RX}} - \phi_{\mathrm{TX}})\right)\\
D_3 &= \cot\left(\frac{\pi - (\phi_{\mathrm{RX}} + \phi_{\mathrm{TX}})}{2\psi}\right) F_{\mathrm{T}}\left(k_0 L a_{\mathrm{m}}(\phi_{\mathrm{RX}} + \phi_{\mathrm{TX}})\right)\\
D_4 &= \cot\left(\frac{\pi + (\phi_{\mathrm{RX}} + \phi_{\mathrm{TX}})}{2\psi}\right) F_{\mathrm{T}}\left(k_0 L a_{\mathrm{p}}(\phi_{\mathrm{RX}} + \phi_{\mathrm{TX}})\right)
\end{aligned} \tag{4.62}$$

where L is

$$L = \frac{d_{\mathrm{TX}} d_{\mathrm{RX}}}{d_{\mathrm{TX}} + d_{\mathrm{RX}}} \tag{4.63}$$

The function

$$F_T(x) = 2j\sqrt{x}\exp(jx)\int_{\sqrt{x}}^{\infty} \exp\left(-ju^2\right) du \tag{4.64}$$

is related to the Fresnel integral. The parameters a_{p} and a_{m} are given by the equations

$$a_{\mathrm{p}} = 2\cos^2\left(\frac{2\psi\pi N_{\mathrm{p}} - (\phi_{\mathrm{TX}} + \phi_{\mathrm{RX}})}{2}\right) \tag{4.65}$$

$$a_{\mathrm{m}} = 2\cos^2\left(\frac{2\psi\pi N_{\mathrm{m}} - (\phi_{\mathrm{RX}} - \phi_{\mathrm{TX}})}{2}\right) \tag{4.66}$$

where N_{p} and N_{m} are the integers that best approximate the solution to the equations

$$2\pi\psi N_{\mathrm{p}} - (\phi_{\mathrm{RX}} + \phi_{\mathrm{TX}}) = \pi \tag{4.67}$$

$$2\pi\psi N_{\mathrm{m}} - (\phi_{\mathrm{RX}} - \phi_{\mathrm{TX}}) = -\pi \tag{4.68}$$

The exact solution for a perfectly conducting wedge is given in [Bowman et al. 1987]. For a dielectric wedge, [Bergljung 1994] derives approximate equations, which, however, still need numerical evaluations of a single integral.

The wedge is a structure that frequently occurs in man-made structures. Natural terrain obstacles have more gentle changes in their shapes. A hill might be approximated, for example, by a cylinder. The received field strength for a diffraction by a cylinder is derived both for the TE and TM case in [Bowman et al. 1987]; however, these solutions are very complicated. Since a cylinder is only

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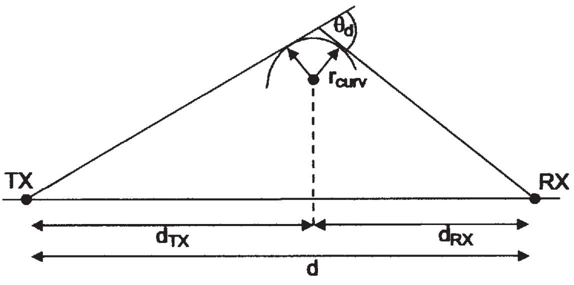

Figure 4.16 Diffraction by a cylinder.

an approximate description of naturally occuring obstacles, an exact treatment is not necessary. A simple approximate solution was proposed by Hacking (quoted in [Parsons 1992]): the total field is given by the attenuation of an "equivalent" screen (see Fig. 4.16), plus an excess loss

$$L_{\mathrm{ex}}|_{dB} = 11.7\theta_{\mathrm{d}}\sqrt{\pi\frac{r_{\mathrm{curv}}}{\lambda}} \tag{4.69}$$

that is created by the curvature of the cylinder.