Lecture 3:

Spatio-Temporal Receiver Structures (TDMA)

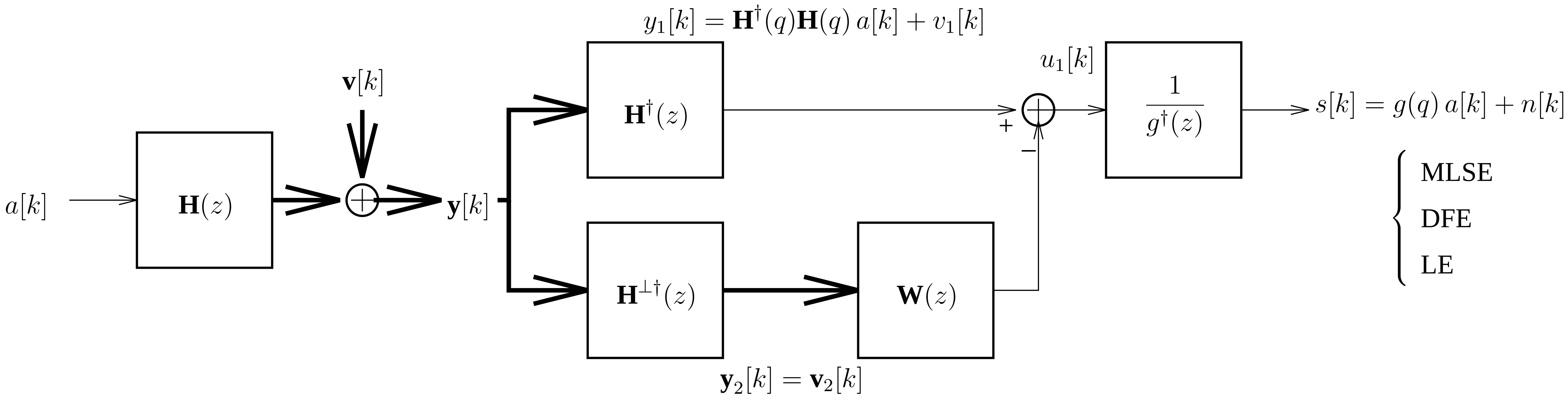
Interference Cancellation

Overview

- single-user colored-noise ST processing (equalization)
- Interference Canceling Matched Filter (ICMF)
- multi-user white noise ST processing
- Forney form MLSE for colored noise

ST SU Receivers, Colored Noise: GSC = ICMF

ICMF: Interference Canceling Matched Filter



ST SU Receivers, Colored Noise: ICMF (2)

- Generalized Sidelobe Canceler formulation allows to split reception into interference cancellation and equalization

- introduce $\mathbf{H}^{\perp}(z)$ s.t.

$$\left[\mathbf{H}\,\mathbf{H}^{\perp}\right]^{\dagger}\left[\mathbf{H}\,\mathbf{H}^{\perp}\right] = \begin{bmatrix} \mathbf{H}^{\dagger}\mathbf{H} & 0 \\ 0 & \mathbf{H}^{\perp\dagger}\mathbf{H}^{\perp} \end{bmatrix} \text{ with } \mathbf{H}^{\perp\dagger}\mathbf{H}^{\perp} \text{ nonsingular}$$

- reparameterize $\mathbf{F}(z)$ with $\mathbf{F}_{||}(z)$, $\mathbf{F}_{\perp}(z)$ or $\mathbf{F}_{||}(z)$, $\mathbf{W}(z)$:

$$\mathbf{F} = \left[\mathbf{F}_{||}\ \ -\mathbf{F}_{\perp}\right]\underbrace{\begin{bmatrix} \mathbf{H}^{\dagger} \\ \mathbf{H}^{\perp\dagger} \end{bmatrix}}_{\text{invertible transformation}} = \underbrace{\mathbf{F}_{||}[1\ \ -\mathbf{W}]\begin{bmatrix} \mathbf{H}^{\dagger} \\ \mathbf{H}^{\perp\dagger} \end{bmatrix}}_{\text{interference canceling MF}}$$

with $\mathbf{W}(z) = \mathbf{F}_{||}^{-1}(z)\,\mathbf{F}_{\perp}(z)$ ICMF

- role split: $\mathbf{F}_{||}$: equalization, $\mathbf{W}$: interference cancellation

ST SU Receivers, Colored Noise: ICMF (3)

- Consider LMMSE estimation of $y_1[k]$ in terms of $\mathbf{y}_2[k]$
  = LMMSE estimation of $v_1[k]$ in terms of $\mathbf{v}_2[k]$:
  $\mathbf{W}(z) = S_{y_1\mathbf{y}_2}(z) S^{-1}_{\mathbf{y}_2\mathbf{y}_2}(z) = S_{v_1\mathbf{v}_2}(z) S^{-1}_{\mathbf{v}_2\mathbf{v}_2}(z)$
  Only need to know $\mathbf{H}(z)$ to start estimating $\mathbf{W}(z)$ from Rx signal.
- $u_1[k] = \mathbf{H}^\dagger(q)\mathbf{H}(q)\, a[k] + \widetilde{v}_1[k]$, $\widetilde{v}_1[k] = v_1[k] - \mathbf{W}(q)\, \mathbf{v}_2[k]$
  $\perp$ property of LMMSE: $S_{\widetilde{v}_1\mathbf{v}_2}(z) = 0$: uncorrelated
- If $\mathbf{v}[.]$ is Gaussian, then $\widetilde{v}_1[.]$ and hence $u_1[.]$ are independent of $\mathbf{y}_2[.] = \mathbf{v}_2[.]$ and only $u_1[.]$ contains symbols $a[.]$. Hence $u_1[.]$ constitutes a sufficient statistics signal for the detection of $a[k]$.
  The vector Rx signal $\mathbf{y}[.]$ has been compressed into a scalar signal $u_1[.]$ without any loss of information (in the Gaussian noise case) by a linear interference canceling step.

ST SU Receivers, Colored Noise: ICMF (4)

- Introduce white noise rewhitening filter $g^{-\dagger}(z)$ to reduce the channel length back to N. $g(z) = (\mathbf{H}^{\dagger}(z)\mathbf{H}(z))^{\frac{1}{2}}$.
- Obtain $g^{-\dagger}(q)\, u_1[k] = s[k] = g(q)\, a[k] + n[k]$ with
$$S_{nn}(z) = \frac{\mathbf{H}^{\dagger} S_{\mathbf{vv}} \mathbf{H} - \mathbf{H}^{\dagger} S_{\mathbf{vv}} \mathbf{H}^{\perp} \left(\mathbf{H}^{\perp\dagger} S_{\mathbf{vv}} \mathbf{H}^{\perp}\right)^{-1} \mathbf{H}^{\perp\dagger} S_{\mathbf{vv}} \mathbf{H}}{\mathbf{H}^{\dagger}\mathbf{H}}$$
- Conservation of MFB in $s[k]$ w.r.t. $\mathbf{y}[k]$:
$$\text{MFB} \;=\; \frac{\sigma_a^2}{2\pi j} \oint \frac{dz}{z} \mathbf{H}^{\dagger}(z) S_{\mathbf{vv}}^{-1}(z) \mathbf{H}(z) \;=\; \frac{\sigma_a^2}{2\pi j} \oint \frac{dz}{z} \frac{g^{\dagger} g}{S_{nn}} \;.$$
- To $s[k]$, can apply any equalization technique: MLSE (Viterbi algorithm, neglecting color in S_{nn} or not) or DFE.
  Or LE, according to MMSE, UMMSE or (MMSE) ZF: determines the $\mathbf{F}_{||}(z)$ part of the linear Rx.

ST Multi-User Receivers, MLSE White Noise

- $\mathbf{y}[k] = [\mathbf{H}_1(q) \cdots \mathbf{H}_p(q)] \begin{bmatrix} a_1[k] \\ \vdots \\ a_p[k] \end{bmatrix} + \mathbf{v}[k] = \mathbf{H}(q)\,\mathbf{a}[k] + \mathbf{v}[k] = \sum_{i=1}^{p} \mathbf{H}_i(q)\,a_i[k] + \mathbf{v}[k] =$

  $\left[\mathbf{H}_1(q)\ \overline{\mathbf{H}}_1(q)\right] \begin{bmatrix} a_1[k] \\ \overline{\mathbf{a}}_1[k] \end{bmatrix} + \mathbf{v}[k] = \mathbf{H}_1(q)\ a_1[k]\ +\ \overline{\mathbf{H}}_1(q)\ \overline{\mathbf{a}}_1[k]\ +\ \mathbf{v}[k]$

- assume white Gaussian noise: $\mathbf{v}[k] \sim \mathcal{CN}(0, \sigma_v^2 I_m)$ i.i.d.

- Maximum Likelihood Sequence Estimation (MLSE):

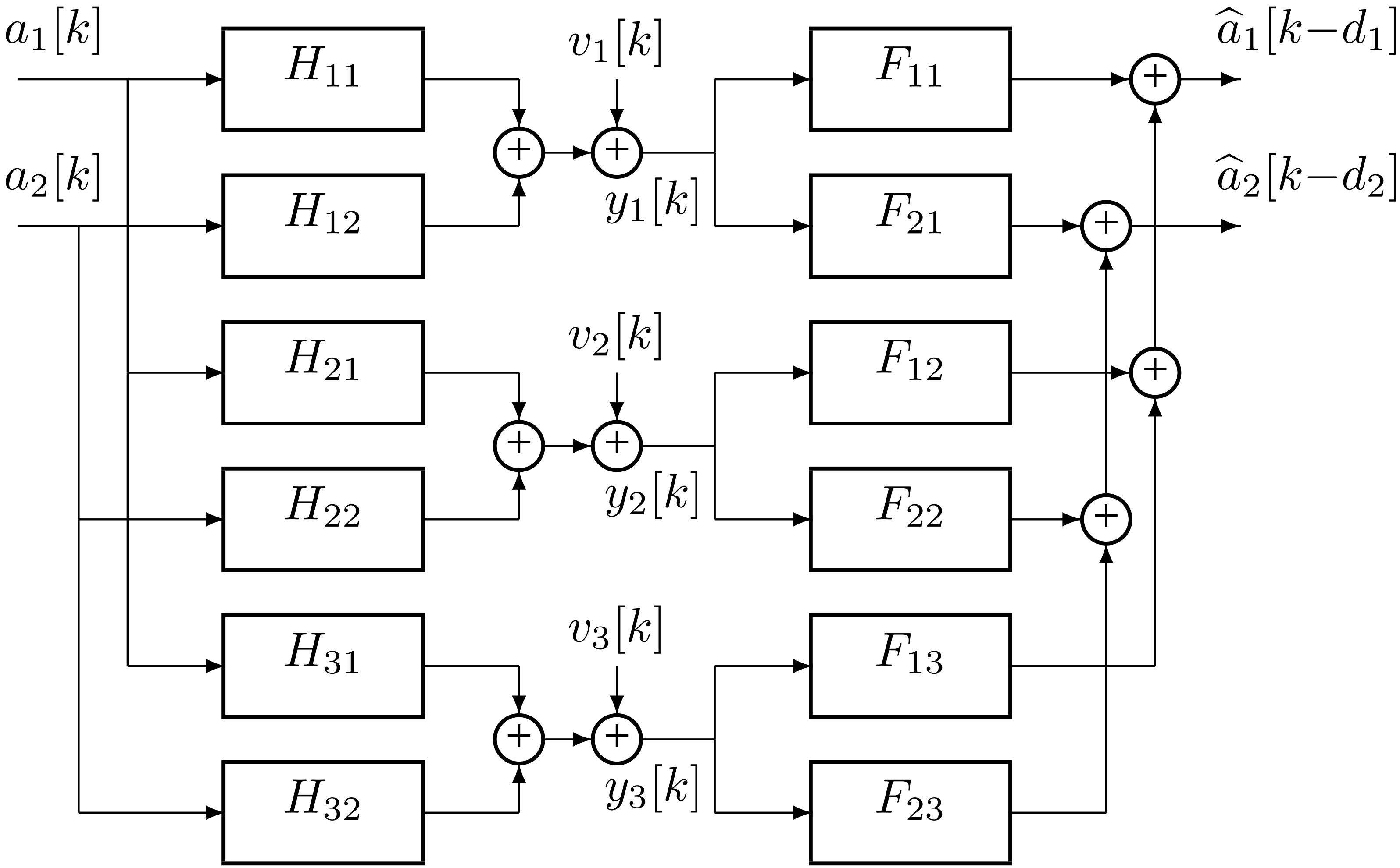
$$\min_{\mathbf{a}[k] \in \mathcal{A}^p} \sum_k \|\mathbf{y}[k] - \mathbf{H}(q)\,\mathbf{a}[k]\|^2$$

  Viterbi complexity $\sim$ number of possibilities in collective channel memory =

  $|\mathcal{A}|^{\sum_{i=1}^{p} N_i - p}$

- MFB$_1$ = SU MFB = $\|\mathbf{H}_1\|^2 \sigma_a^2 / \sigma_v^2$

ST MU Receivers, White Noise: Linear Equalizers



ST MU Receivers, White Noise: Linear Equalizers (2)

- MIMO linear equalizer $\mathbf{F}(q)$: m inputs p outputs
- ZF condition: $\mathbf{F}(z)\,\mathbf{H}(z) = \text{diag}\{z^{-d_1}, \ldots, z^{-d_p}\}$
- FIR equalizers exist for FIR channels iff $\mathbf{H}$ is *irreducible*: $\mathbf{H}(z)$ has full rank $(= p)$ $\forall\, z$ (no zeros)
- $\mathbf{H}$ is *column reduced* if $[\mathbf{h}_1[N_1{-}1] \cdots \mathbf{h}_p[N_p{-}1]]$ has full (column) rank
- if $\mathbf{H}$ is irreducible and column reduced, then the block Toeplitz block channel convolution matrix $[\mathcal{T}_L(\mathbf{H}_1) \cdots \mathcal{T}_L(\mathbf{H}_p)]$ has full column rank iff

  $$mL \geq \sum_{i=1}^{p}(N_i{+}L{-}1) \;\Rightarrow\; L \geq \frac{\sum_{i=1}^{p} N_i \; - p}{m-p}$$ which means that FIR ZF equalizers exist that *remove ISI and IUI* ($\mathbf{F}_i(q)\mathbf{H}_k(q) = \delta_{ik}\, q^{-d_i}$) as long as more subchannels than users are available

ST MU Receivers, White Noise: ICMF again

- Interference Canceling Matched Filter (ICMF = philosophy of Linear MMSE): focuses on user 1, treating other users as colored noise $\Rightarrow$ single-user case (MFB) with $S_{\mathbf{vv}}(z) = \sigma_a^2\,\overline{\mathbf{H}}_1(z)\overline{\mathbf{H}}_1^\dagger(z) + \sigma_v^2\,I_m$
  With optimal equalization (e.g. MLSE) for user 1.

- if assume $\#$ of interferers $p{-}1 \leq m{-}1 \;\Rightarrow$ ZF of interference possible

- and $S_{nn}(z) = \sigma_v^2 \left(1 + \mathrm{tr}\left\{\overline{\mathbf{H}}_1^\dagger P_{\mathbf{H}_1}\overline{\mathbf{H}}_1\left(\overline{\mathbf{H}}_1^\dagger P_{\mathbf{H}_1^\perp}\overline{\mathbf{H}}_1 + \frac{\sigma_v^2}{\sigma_a^2} I_{p-1}\right)^{-1}\right\}\right)$
  where $P_{\mathbf{H}(z)} = \mathbf{H}(z)\left(\mathbf{H}^\dagger(z)\mathbf{H}(z)\right)^{-1}\mathbf{H}^\dagger(z)$ (projection) $P_{\mathbf{H}}^\perp = I - P_{\mathbf{H}} = P_{\mathbf{H}^\perp}$

- issue: orientation of $\overline{\mathbf{H}}_1$ w.r.t. $\mathbf{H}_1$: 2 extreme cases:

(i) $\overline{\mathbf{H}}_1 \perp \mathbf{H}_1$: $\overline{\mathbf{H}}_1^\dagger P_{\mathbf{H}_1}\overline{\mathbf{H}}_1 = 0$, $\overline{\mathbf{H}}_1^\dagger P_{\mathbf{H}_1^\perp}\overline{\mathbf{H}}_1 = \overline{\mathbf{H}}_1^\dagger\overline{\mathbf{H}}_1$

$$\mathrm{MFB}_1 = \mathrm{MFB}_{JD} = \frac{\sigma_a^2}{2\pi j\sigma_v^2}\oint\frac{dz}{z}\mathbf{H}_1^\dagger\mathbf{H}_1$$

the Joint Detection MFB = SU MFB

ST MU Receivers, White Noise: ICMF again (2)

(ii) $\overline{\mathbf{H}}_1 \parallel \mathbf{H}_1$: $\overline{\mathbf{H}}_1^\dagger P_{\mathbf{H}_1} \overline{\mathbf{H}}_1 = \overline{\mathbf{H}}_1^\dagger \overline{\mathbf{H}}_1, \overline{\mathbf{H}}_1^\dagger P_{\mathbf{H}_1^\perp} \overline{\mathbf{H}}_1 = 0$

$$\begin{aligned} \mathrm{MFB}_1 = \mathrm{MFB}_{\|} &= \frac{1}{2\pi j} \oint \frac{dz}{z} \frac{\sigma_a^2\, \mathbf{H}_1^\dagger \mathbf{H}_1}{\sigma_v^2 + \sigma_a^2 \mathrm{tr}\left\{\overline{\mathbf{H}}_1^\dagger \overline{\mathbf{H}}_1\right\}} \\ &= \underbrace{\frac{\sigma_a^2}{\sigma_v^2} \frac{1}{2\pi j} \oint \frac{dz}{z}\, \mathbf{H}_1^\dagger \mathbf{H}_1}_{\mathrm{MFB}_{JD}} \frac{1}{1+\frac{\sigma_a^2}{\sigma_v^2}\mathrm{tr}\left\{\overline{\mathbf{H}}_1^\dagger \overline{\mathbf{H}}_1\right\}} \end{aligned}$$

which is the integrated frequency-dependent SINR.

At high INR, $\mathrm{MFB}_{\|}$ can be $\ll \quad \mathrm{MFB}_{JD}$.

- Can rewrite

$$\mathrm{MFB}_1 = \underbrace{\oint \frac{dz}{2\pi j\, z} \frac{\sigma_a^2 \mathbf{H}_1^\dagger \mathbf{H}_1}{\sigma_v^2}}_{\mathrm{MFB}_{JD}} \left(1 - \mathrm{tr}\left\{ \left(\overline{\mathbf{H}}_1^\dagger \overline{\mathbf{H}}_1 + \frac{\sigma_v^2}{\sigma_a^2} I_{p-1}\right)^{-1} \overline{\mathbf{H}}_1^\dagger P_{\mathbf{H}_1} \overline{\mathbf{H}}_1 \right\}\right)$$

ST MU Receivers, White Noise: ICMF again (3)

- Average behavior? Take $\mathbf{H}_1 = \mathbf{U}(\mathbf{H}_1^\dagger \mathbf{H}_1)^{\frac{\dagger}{2}}$, $\overline{\mathbf{H}}_1 = \mathbf{V}(\overline{\mathbf{H}}_1^\dagger \overline{\mathbf{H}}_1)^{\frac{\dagger}{2}}$, where $\mathbf{U}$ and $\mathbf{V}$ are paraunitary, together p vectors that we consider random, uniformly distributed and i.i.d. at any frequency.
  The normalizing factors $(\mathbf{H}_1^\dagger \mathbf{H}_1)^{\frac{1}{2}}$, $(\overline{\mathbf{H}}_1^\dagger \overline{\mathbf{H}}_1)^{\frac{1}{2}}$ are still deterministic.
- Can show: $\mathrm{E}\, \mathbf{V}^\dagger \mathbf{U}\mathbf{U}^\dagger \mathbf{V} = \frac{1}{p} I_{p-1}$.
- This leads to
  $$\mathrm{E}\,\mathrm{MFB} = \frac{p-1}{p}\mathrm{MFB}_{JD} + \frac{1}{p}\mathrm{MFB}_{\|} \ .$$
  Depending on the number of users, the average performance in the suboptimal ICMF approach (treating interference as colored Gaussian noise) can be close to optimal.

Linear IUI Cancellation: Limited MFB Loss (example: GSM)

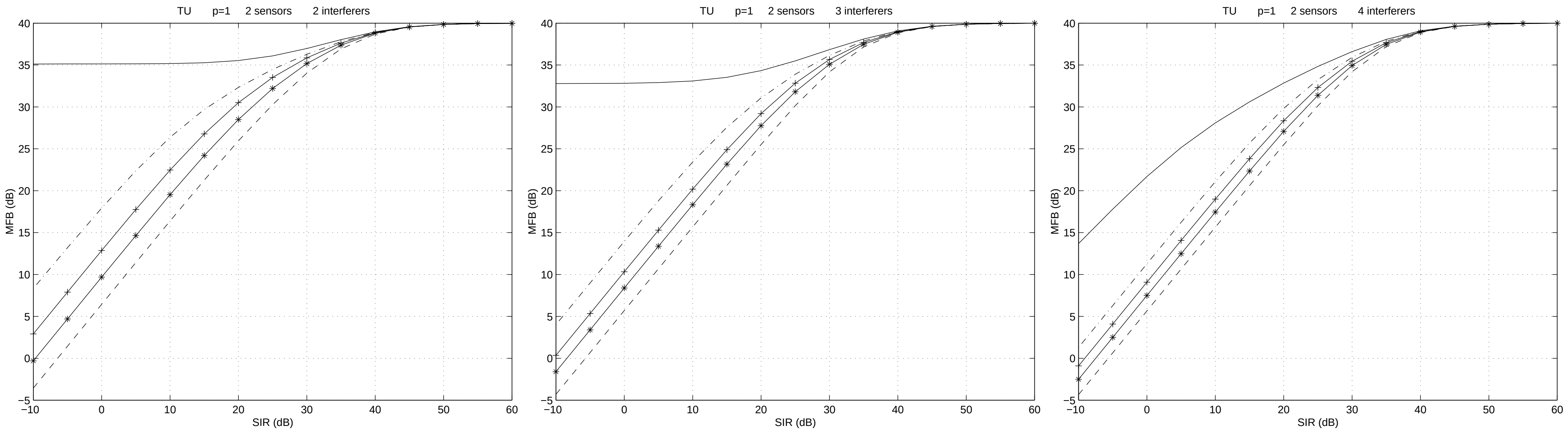


Figure 1: MFB vs SIR for SNR=20dB, 2 sensors, I & Q channels, no oversampling, the Typical Urban (TU) channel model, for 2, 3 and 4 interferers.

- ICMF spatio-temporal processing vs. optimized (max SINR) spatial (only) processing
- near-far resistance if # subchannels $>$ # interferers

Spatial vs Space-Time IUI Cancellation

to achieve zero-forcing

- spatiotemporally: number of antennas needs to exceed the number of interferers
- spatially: $\mathbf{f}\,\mathbf{h}_i[n] = 0,\ n = 0,\ldots,N_i-1,\ \ i = 2,\ldots,p$
  number of antennas needs to exceed the number of temporally resolvable spatial signatures of the interferers : $m \geq \sum_{i=2}^{p} N_i$
- spatial signature = superposition of spatial responses of temporally irresolvable paths

ST SU, Colored Noise MLSE: Forney form

- optimal (∞ length) prediction error filter (= whitener) of colored noise:

$$\widetilde{\mathbf{v}}[k] = \mathbf{P}(q)\,\mathbf{v}[k]\;, \quad S_{\widetilde{\mathbf{v}}\widetilde{\mathbf{v}}}(z) = \mathbf{P}(z)\,S_{\mathbf{v}\mathbf{v}}(z)\,\mathbf{P}^{\dagger}(z) = R_{\widetilde{\mathbf{v}}\widetilde{\mathbf{v}}}$$

monic, causal, min. phase $\mathbf{P}(q) = I_m - \overline{\mathbf{P}}(q)$, strictly causal $\overline{\mathbf{P}}$

Square MIMO minimum phase: $\mathbf{P}(z)$ min. phase if $\det(\mathbf{P}(z))$ min. phase.

Leads to a possible matrix spectrum factorization:

$$S_{\mathbf{v}\mathbf{v}}(z) = \mathbf{P}^{-1}(z)\,R^{1/2}_{\widetilde{\mathbf{v}}\widetilde{\mathbf{v}}}\,R^{1/2}_{\widetilde{\mathbf{v}}\widetilde{\mathbf{v}}}\,\mathbf{P}^{-\dagger}(z) = S^{1/2}_{\mathbf{v}\mathbf{v}}(z)\,S^{\dagger/2}_{\mathbf{v}\mathbf{v}}(z)$$

- $\widetilde{\mathbf{v}}[k] = \mathbf{v}[k] - \widehat{\mathbf{v}}[k] = \mathbf{v}[k] - \overline{\mathbf{P}}(q)\,\mathbf{v}[k]$ i.i.d. $\sim \mathcal{CN}(0, R_{\widetilde{\mathbf{v}}\widetilde{\mathbf{v}}})$

- likelihood $\sim \sum_k \|\widetilde{\mathbf{v}}[k]\|^2_{R^{-1}_{\widetilde{\mathbf{v}}\widetilde{\mathbf{v}}}}$

- MLSE: *approximate* Viterbi: from memory of $\mathbf{P}(q)\mathbf{H}(q)$ to that of $\mathbf{H}(q)$ only

$$\min_{\{a[k]\in\mathcal{A}\}} \sum_k \|\mathbf{P}(q)(\mathbf{y}[k]-\mathbf{H}(q)\,a[k])\|^2_{R^{-1}_{\widetilde{\mathbf{v}}\widetilde{\mathbf{v}}}} = \min_{\{a[k]\in\mathcal{A}\}} \sum_k \|\mathbf{y}[k]-\overline{\mathbf{P}}(q)\,\mathbf{v}[k]-\mathbf{H}(q)\,a[k]\|^2_{R^{-1}_{\widetilde{\mathbf{v}}\widetilde{\mathbf{v}}}}$$

where $\mathbf{v}[k] = \mathbf{y}[k] - \mathbf{H}(q)\,a[k]$ computed along every survivor path in the Viterbi algorithm trellis