Lecture 5:

CDMA: UL Multi-User Detection,

Rake receiver, DL receivers

Overview

- CDMA Background
- Synchronous Flat Channel case
  - Optimum Detection
  - Linear and Non-Linear Multi-User Detectors

Successive Interference Cancellation (SIC)

- Consider the SUMF outputs

$$\boldsymbol{x}[n] \;=\; S^H \boldsymbol{y}[n] = S^H S\, C\, \boldsymbol{d}[n] + S^H \boldsymbol{v}[n] \tag{1}$$

The SUMF concentrates on the diagonal elements only of the matrix $S^H S$ and decides on $\widehat{d}_k[n] = c_k^{-1}\, S_k^H \boldsymbol{y}[n] = c_k^{-1}\, x_k[n]$ (as if $Q = 0$)

- For SIC, we assume that the users have been ordered in order of decreasing $\text{SNR} = |c_k|^2 \sigma_d^2 / \sigma_v^2$ at the SUMF outputs. The first (strongest) user gets detected as in the SUMF. For the detection of user k (see line k in equation (1)), the contributions of the previously detected $k{-}1$ stronger users get subtracted first: (dec = decision)

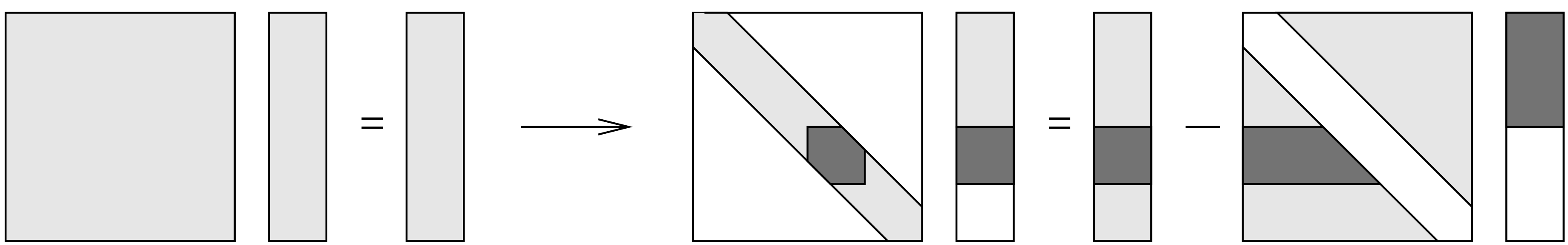
$$\begin{aligned}
\widehat{d}_k[n] &= dec\left\{ c_k^{-1} \left(S_k^H \boldsymbol{y}[n] - \sum_{i=1}^{k-1} S_k^H S_i c_i\, \widehat{d}_i[n] \right) \right\} \\
&= dec\left\{ c_k^{-1}\, S_k^H \left[\boldsymbol{y}[n] - \sum_{i=1}^{k-1} S_i c_i\, \widehat{d}_i[n] \right] \right\}
\end{aligned}$$

Successive Interference Cancellation (SIC) (2)

- We shall solve successively for $d_k[n]$ from

$$\underbrace{(S^H S)\,(C}_{R}\;\boldsymbol{d}[n]) \;=\; \boldsymbol{x}[n]$$

by separating the diagonal and off-diagonal elements in R.
$S^H S\, C = R = C + Q\, C$



Successive Interference Cancellation (SIC) (3)

- This allows the following interpretation: the previously detected symbols get remodulated (respread) to reproduce their contribution to the received signal $\boldsymbol{y}[n]$ and their contribution gets subtracted from $\boldsymbol{y}[n]$ (expression in square brackets); the thus partially cleaned received signal gets passed to the usual SUMF for decision making.
- Twofold rationale for doing successive cancellation in order of decreasing signal strength:
  - canceling the strongest user (= interferer) yields the most benefit for detecting the following users
  - the strongest user will yield the most reliable decisions and cancellations
- SIC works especially well if the user powers are well separated.

Successive Interference Cancellation (SIC) (4)

- Wrong decisions can have desastrous effects: in the BPSK case, a wrong decision leads to a quadruple increase in the corresponding interferer's power: $|d_k - \widehat{d}_k| = |\widetilde{d}_k| = 2|d_k| \Rightarrow \sigma^2_{\widetilde{d}} = 4\sigma^2_d$

$$\boldsymbol{y}[n] - \sum_{i=1}^{k-1} S_i c_i \, \widehat{d}_i[n]$$

$$= \underbrace{S_k c_k \, d_k[n]}_{\text{user of interest}} + \underbrace{\sum_{i=k+1}^{K} S_i c_i \, d_i[n]}_{\text{uncanceled MAI}} + \underbrace{\sum_{i=1}^{k-1} S_i c_i \, \widetilde{d}_i[n]}_{\text{IC errors}} + \underbrace{\boldsymbol{v}[n]}_{\text{noise}}$$

- SIC requires a symbol delay per cancellation (if implemented via respreading-despreading)
- The complexity of SIC can be controlled by limiting the number of users to be cancelled to the strongest ones.

Multistage SIC

- The procedure can be reiterated when all users have been detected. In that case, not only the decisions for the stronger users of the current iteration get subtracted but also the decisions for the weaker users of the previous iteration. We get in iteration m:

$$\widehat{d}_k^{(m)} = dec\left\{c_k^{-1}\,S_k^H\left[\boldsymbol{y}[n] - \sum_{i=1}^{k-1} S_i c_i\,\widehat{d}_i^{(m)} - \sum_{i=k+1}^{K} S_i c_i\,\widehat{d}_i^{(m-1)}\right]\right\}$$

with initialization $\widehat{\boldsymbol{d}}^{(0)} = 0$ ($\Rightarrow$ $\widehat{d}_k^{(1)} = \widehat{d}_k^{SUMF}$).

Parallel Interference Cancellation (PIC)

- PIC corresponds to MLSE for a particular symbol, assuming that all other symbols have been detected. This is done for all symbols in parallel. The decisions for the other symbols are provided by the SUMF. This process can be repeated, leading to multistage PIC. One stage (iteration) of PIC can be written in matrix form

$$\widehat{\boldsymbol{d}}^{(m)}[n] = dec\Big\{C^{-1}\Big[S^H\,\boldsymbol{y}[n] - S^H S\,C\,\widehat{\boldsymbol{d}}^{(m-1)}[n]\Big] + \widehat{\boldsymbol{d}}^{(m-1)}[n]\Big\}$$

- PIC works particularly well with power control. In that case and with BPSK, the probability of symbol error should be at most 0.2 for an iteration to yield improvement.
- A linear MUD can be used to provide the initial decisions.
- The parallel approach can be combined with the serial one: use the already detected symbols in the current stage for the detection of the remaining symbols in that stage.

Decision-Feedback MUD

- The linear receiver output satisfies

$$R\,C\,\widehat{\boldsymbol{d}}[n] \;=\; S^H\,\boldsymbol{y}[n] \;=\; \boldsymbol{x}[n]$$

where

$$R \;=\; \begin{cases} S^H\,S & \text{(MMSE) ZF case} \\ S^H\,S + \sigma_v^2\,C^{-H}R_{\boldsymbol{dd}}^{-1}C^{-1} & \text{MMSE case} \end{cases}$$

Assume again that the users are ordered according to decreasing SNR at the SUMF outputs.

- Consider the Upper-Diagonal-Lower triangular factorization $R = F^H\,D\,F$ where F is unit-diagonal lower triangular. Then we get

$$C^{-1}F\,C\,\widehat{\boldsymbol{d}}[n] \;=\; C^{-1}D^{-1}F^{-H}S^H\,\boldsymbol{y}[n]$$

where $C^{-1}F\,C$ is again unit-diagonal lower triangular.

Decision-Feedback MUD (2)

- In the decorrelating (MMSE ZF) case, the covariance matrix of $D^{-1/2}F^{-H}S^H\boldsymbol{v}[n]$ is $\sigma_v^2\,I_K$. Since noise components are uncorrelated, the following symbol by symbol detection scheme is close to optimal.
- In the first equation, only the first (strongest) user appears without MAI. So we can detect it. We can subtract its contribution from the second equation to remove the MAI for the second user etc. We get

$$\widehat{d}_k[n] = dec\left\{\left(C^{-1}D^{-1}F^{-H}S^H\boldsymbol{y}[n]\right)_k - \sum_{i=1}^{k-1}\left(C^{-1}FC\right)_{ki}\,\widehat{d}_i[n]\right\}$$

  Anticausal MAI gets canceled linearly, causal MAI nonlinearly.
- The performance of the ZF DF MUD is that of the decorrelator for the strongest user and gradually approaches the single user MFB as the user's power decreases relative to the (previously detected) interferers. So the DF benefits are mainly for the weaker users.
- (U)MMSE still better than MMSE-ZF

Recap: MUD $\leftrightarrow$ Equalization

- duality: time index $\leftrightarrow$ user index
- (ZF & MMSE) linear equalization $\leftrightarrow$ (ZF & MMSE) L-MUD
- DF-MUD $\leftrightarrow$ DFE
- PIC $\leftrightarrow$ non-causal DFE
- SIC = PIC with future decisions = 0 : SIC does not make sense for equalization

Soft Decisions (SDs)

- In absence of channel decoding, soft decision means making no decision (omitting $dec\{.\}$).
- A SD version of a DF MUD immediately gives the corresponding L-MUD. SD versions of SIC and PIC yield iterative techniques to approach the decorrelator or LMMSE solutions while avoiding inversion of $R = S^H S(+\frac{\sigma_v^2}{\sigma_a^2} C^{-H} C^{-1})$.
- *Clipped SDs* (BPSK): hard decisions (HDs) can be replaced by tanh(.) (nonlinear MMSE) or its piecewise linear approximation corresponding to clipped (saturated) soft decisions. Clipped SDs correspond to HDs if the soft decision magnitude exceeds a threshold and to (scaled) SD otherwise.
- A related notion is scaled HD, in which HDs are scaled down according to a decision reliability measure. Scaled HDs have been successfully applied to multistage PIC.

Soft Decisions (2)

- These last two techniques allow to improve the performance of SIC and PIC dramatically: initially, only reliable decisions are actually made so that error propagation gets much limited (unreliable symbol estimates are left undecided $\Rightarrow$ linear cancellation only).
- MUD combined with channel decoding: Turbo MUD
  - soft decisions provide reliability information to the channel decoder
  - the MUD uses decoded symbols in the IC
  - iterate between MUD and channel decoding

CDMA Overview Cont'd

- UMTS-FDD uplink: General Asynchronous Case with Delay Spread

General Asynchronous Case with Delay Spread

$$
\begin{bmatrix} y_1[0] \\ \vdots \\ y_L[0] \\ y_1[1] \\ \vdots \\ y_L[1] \\ \vdots \\ y_1[N+2] \\ \vdots \\ y_L[N+2] \end{bmatrix}
= \sum_{k=1}^{K}
\begin{bmatrix} & 0_{t_k \times L(N+1)} & \\ c_k[0] & \cdots & 0 \\ \vdots & \ddots & \vdots \\ c_k[P_k] & \ddots & \\ & \ddots & c_k[0] \\ \vdots & \ddots & \vdots \\ 0 & \ddots & c_k[P_k] \\ \vdots & & 0 \\ & & \vdots \end{bmatrix}
\begin{bmatrix} s_k[0] & & & \\ \vdots & & & \\ s_k[L-1] & & & \\ & s_k[0] & & \\ & \vdots & & \\ & s_k[L-1] & & \\ & & \ddots & \\ & & & s_k[0] \\ & & & \vdots \\ & & & s_k[L-1] \end{bmatrix}
\begin{bmatrix} d_k[0] \\ d_k[1] \\ \vdots \\ d_k[N-1] \\ d_k[N] \end{bmatrix}
+
\begin{bmatrix} v_1[0] \\ \vdots \\ v_L[0] \\ v_1[1] \\ \vdots \\ v_L[1] \\ \vdots \\ v_1[N+2] \\ \vdots \\ v_L[N+2] \end{bmatrix}
$$

General Asynchronous Case with Delay Spread (2)

- this representation assumes delay spread range < guard interval between slots
- slot length $(N{+}1)T$
- $t_k = \lfloor \frac{\tau_k}{T_c} \rfloor \in \{0, 1, \ldots, L{-}1\}$, τ_k delay (modulo T) of user k
- $c_k[0 : P_k]$: convolution of pulse shaping filter, actual propagation channel and receiver filter, sampled at the chip rate.
  If oversampling w.r.t. the chip rate and/or multiple sensors are used, then $c_k[n]$ is a column vector of dimension the oversampling factor times the number of sensors (vector channel at the chip rate with spread sequence as discrete-time input).
- compact representation: $H = [H_1 \cdots H_K]$, $D = [D_1^T \cdots D_K^T]^T$

$$Y = \sum_{k=1}^{K} C_k \, S_k \, D_k + V = \sum_{k=1}^{K} H_k \, D_k + V = H \, D + V$$

Optimal MUD in the General Case

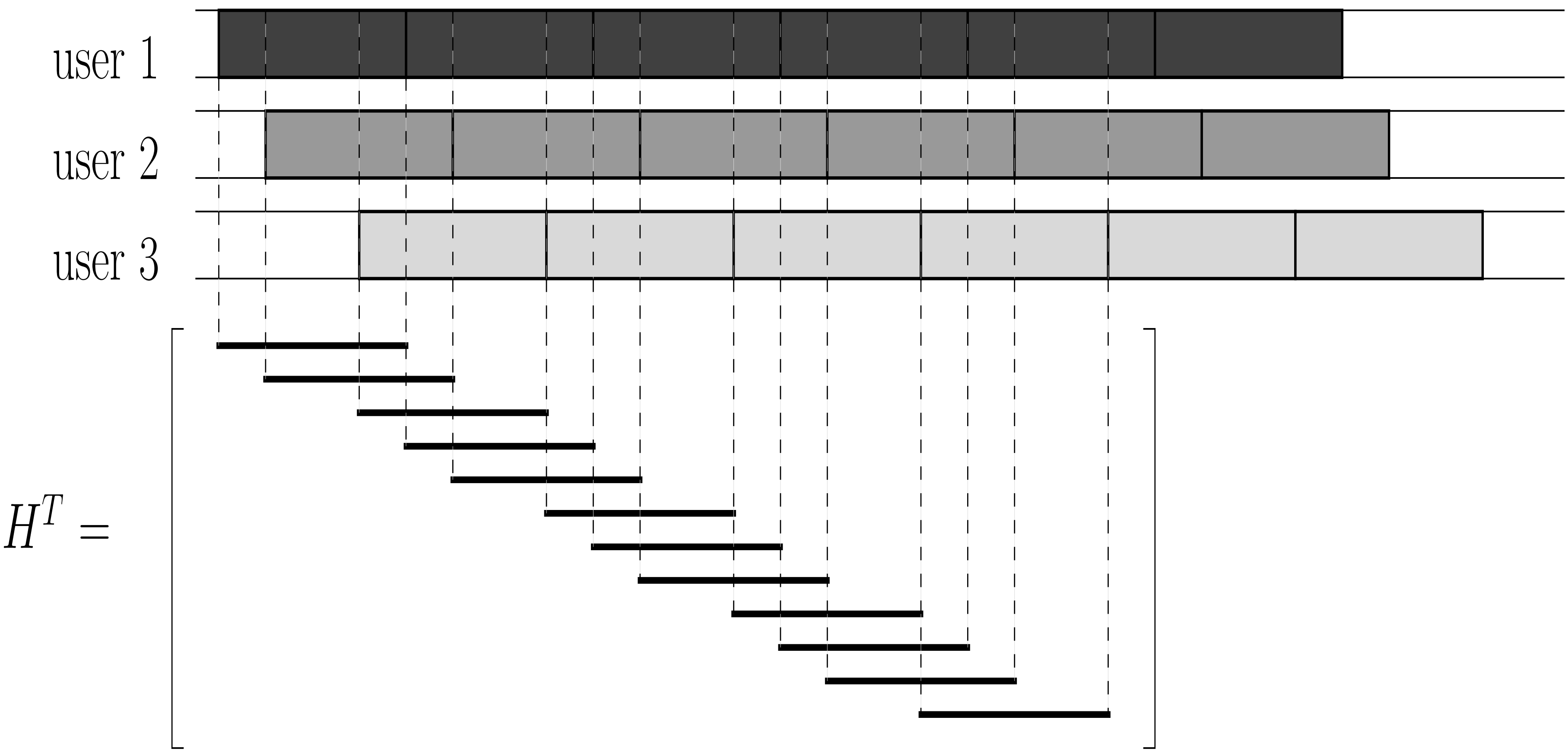
- Consider the asynchronous flat channel case first.
- Consider a grouping of symbols per symbol period in order of increasing delay, instead of per user as on the previous slide. With this reordering of symbols, H is "irregularly banded".
- Consider MLSE in white Gaussian noise. Then the ML criterion can be rewritten as

$$\min_{D \in \mathcal{A}^{K(N+1)}} \|Y - H\,D\|^2 \Leftrightarrow \min_{D \in \mathcal{A}^{K(N+1)}} \left\|U^{-1} H^H Y - U^H\,D\right\|^2$$

where $H^H H$ is a banded matrix with $K{-}1$ non-zero diagonals above and below the main diagonal, and $H^H H = U U^H$ is the Cholesky (triangular) factorization of $H^H H$ with U upper triangular (with positive real main diagonal) and banded with K non-zero super-diagonals.

Optimal MUD in the General Case (2)

- H^H is of the following form:



Optimal MUD in the General Case (3)

- H^H represents matched filtering, U^{-1} anti-casual noise whitening and hence $U^{-1}H^H$ is a whitened matched filter (WMF):

$$U^{-1}H^H(\sigma_v^2 I)H\,U^{-H} = \sigma_v^2\,U^{-1}H^H H\,U^{-H} = \sigma_v^2\,U^{-1}U\,U^H\,U^{-H} = \sigma_v^2 I$$

- U^H represents causal filtering with a filter of memory $K-1$. The MLSE can hence be performed by the Viterbi algorithm with a state dimension of $|\mathcal{A}|^{K-1}$. U^H becomes asymptotically (in N) block Toeplitz with $K \times K$ blocks, hence the system, apart from initial conditions, is periodically time-varying with period K.
- In the case of delay spread, the memory increases (PhD of Verdu).

Suboptimal MUD

- All techniques we have seen for the synchronous flat channel case apply fairly straightforwardly by just considering the larger set of equations (with banded matrices).
- This formulation applies even if the channels are time-varying (in a known fashion) and the spreading sequences are aperiodic. Even with aperiodic spreading sequences, the contributions from the interfering users can be perfectly separated with a decorrelating detector for instance. However, the whole system is time-varying and filter coefficients need to be recomputed at every time instant.
- In the aperiodic spreading case, if a spreading sequence is unknown, it is reasonable to model it as stationary white noise. In that case, the spread signal (discrete-time at the chip rate) is stationary white noise and hence the corresponding received component is cyclostationary with chip period T_c ($\Rightarrow$ stationary if sampled at the chip rate).

Periodic Spreading Case (PSC)

- In the time-invariant, periodic spreading case, it is advantageous to formulate the system in the frequency domain.
- Consider $N = 0$ in the previous formulas (1 symbol sent). Then we find the composite channel response

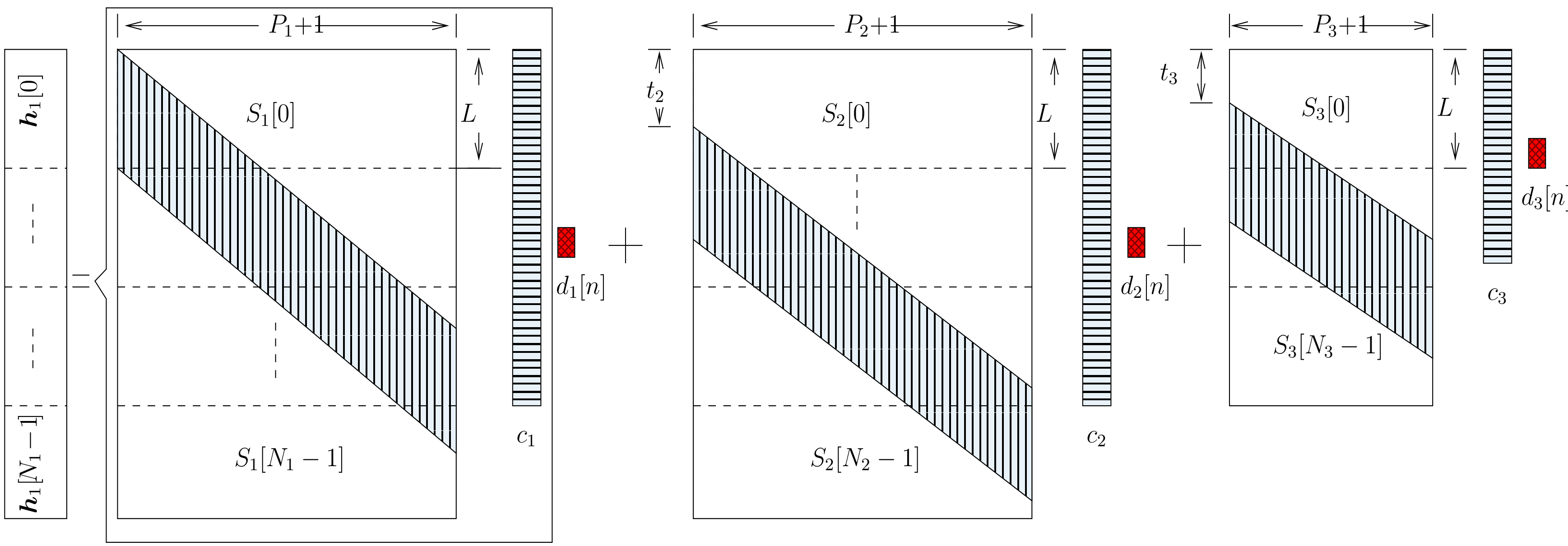
$$\begin{bmatrix} \boldsymbol{h}_k[0] \\ \boldsymbol{h}_k[1] \\ \vdots \end{bmatrix} = C_k\, S_k = C_k \begin{bmatrix} s_k[0] \\ \vdots \\ s_k[L-1] \end{bmatrix}$$

- If $P_k < L$ (assumed), then only $\boldsymbol{h}_k[0], \boldsymbol{h}_k[1]$ are non-zero for a synchronized user, and possibly also $\boldsymbol{h}_k[2]$ for a non-synchronized user.
- Introducing the delay operator q^{-1} (corresponding to z^{-1}), we get

$$\boldsymbol{y}[n]-\boldsymbol{v}[n] = \sum_{k=1}^{K} \boldsymbol{H}_k(q)\, d_k[n] = \sum_{k=1}^{K} (\sum_{m=0}^{2} \boldsymbol{h}_k[m]\, q^{-m})\, d_k[n] = \sum_{k=1}^{K} \sum_{m=0}^{2} \boldsymbol{h}_k[m]\, d_k[n-m]$$

PSC (2)

- case without oversampling, $R = 1$, and one antenna, $I = 1$
  $\Rightarrow$ can commute channel (scalar) and spreading
- $K = 3$ asynchronous users



- $\boldsymbol{h}_k[n] = S_k[n]\, c_k$ In general: $\boldsymbol{h}_k[n] = (S_k[n] \otimes I_{RI})\, c_k$ ($\otimes$ = Kronecker product)

PSC (3)

- We get a K input L output MIMO system:

$$\begin{aligned}\underbrace{\boldsymbol{y}[n]}_{L\times 1} &= \underbrace{\boldsymbol{H}(q)}_{L\times K}\,\underbrace{\boldsymbol{d}[n]}_{K\times 1} + \underbrace{\boldsymbol{v}[n]}_{L\times 1} \\ &= \sum_{m=0}^{2} \boldsymbol{h}[m]\,\boldsymbol{d}[n-m] + \boldsymbol{v}[n]\end{aligned} \quad , \quad \boldsymbol{H}(q) = [\boldsymbol{H}_1(q) \cdots \boldsymbol{H}_K(q)]\,,\ \boldsymbol{d}[n] = \begin{bmatrix} d_1[n] \\ \vdots \\ d_K[n] \end{bmatrix}$$

- Could do Viterbi on this!
- If the $d_k[.]$ and $\boldsymbol{v}[.]$ are stationary, then $\boldsymbol{y}[.]$ is a stationary vector process; the continuous-time $y(t)$ is cyclo-stationary with period T.
- Power spectral density matrix:

$$S_{\boldsymbol{yy}}(z) = \boldsymbol{H}(z)\,S_{\boldsymbol{dd}}(z)\,\boldsymbol{H}^{\dagger}(z) + S_{\boldsymbol{vv}}(z)\,, \quad \boldsymbol{H}^{\dagger}(z) = \boldsymbol{H}^{H}(1/z^{*})$$

Signal part is low rank if $K < L$.

PSC (4)

- Time domain window processing as opposed to filtering formulation. Consider signal over M symbol periods:

$$\boldsymbol{Y}_M[n] = \begin{bmatrix} \boldsymbol{y}[n] \\ \boldsymbol{y}[n-1] \\ \vdots \\ \boldsymbol{y}[n-M+1] \end{bmatrix}$$

from which we want to extract $\boldsymbol{d}[n-\Delta]$ for some delay $\Delta \approx \frac{M-1}{2}$.

- Due to delay spread and asynchronicity $\boldsymbol{Y}_M[n]$ will contain interference, not of MK symbols,, but of up to $(M+2)K$ symbols (in previous limited delay spread example).
- For small M this leads to quite a suboptimal treatment compared to the filtering approach proposed here.

PSC – MUD: RAKE

- RAKE = MF w.r.t. overall channel. Joint RAKE outputs:

$$\widehat{\boldsymbol{d}}[n] = \boldsymbol{x}[n] = \boldsymbol{H}^{\dagger}(q)\, \boldsymbol{y}[n] \;=\; \boldsymbol{H}^{\dagger}(q)\boldsymbol{H}(q)\, \boldsymbol{d}[n] + \boldsymbol{H}^{\dagger}(q)\, \boldsymbol{v}[n]$$

- 2 ways for computing RAKE outputs:
  1. as above: MF with overall channel
  2. MF with spreading sequence (=correlator, spreading sequence binary valued, possibly special hardware), followed by MF with propagation channel (possibly sparse ⇒ fingers ⇒ RAKE)
- $\boldsymbol{H}^{\dagger}(q)\boldsymbol{H}(q)$:
  - diagonal elements: user signal energies (if $\sigma_d^2 = 1$)
  - off-diagonal elements $\neq 0 \Rightarrow$ MAI
- if $S_{\boldsymbol{vv}}(z) = \sigma_v^2 I_L$, then RAKE = optimal preprocessing for all MUD (projects received signal on signal subspace)

PSC – L-MUD

- ZF L-MUD (MIMO Equalizer) $\boldsymbol{F}(z)$:

$$\widehat{\boldsymbol{d}}[n] \;=\; \boldsymbol{F}(q)\,\boldsymbol{y}[n]\;,\;\;\boldsymbol{F}(z)\boldsymbol{H}(z) = I_K$$

not unique! If $L > K$, FIR ZF L-MUD exist in general.

- Among all ZF L-MUD, one has minimal noise enhancement:
decorrelator = MMSE ZF L-MUD

$$\widehat{\boldsymbol{d}}[n] \;=\; (\boldsymbol{H}^{\dagger}(q)\boldsymbol{H}(q))^{-1}\,\boldsymbol{x}[n]$$

IIR in causal and anti-causal directions

- MMSE L-MUD:

$$\widehat{\boldsymbol{d}}[n] = S_{\boldsymbol{dy}}(q)S_{\boldsymbol{yy}}^{-1}(q)\boldsymbol{y}[n] = S_{\boldsymbol{dd}}\boldsymbol{H}^{\dagger}(\boldsymbol{H}S_{\boldsymbol{dd}}\boldsymbol{H}^{\dagger}+S_{\boldsymbol{vv}})^{-1}\boldsymbol{y}[n] = (\boldsymbol{H}^{\dagger}\boldsymbol{H}+\frac{\sigma_v^2}{\sigma_d^2}I_K)^{-1}\,\boldsymbol{x}[n]$$

assuming $S_{\boldsymbol{dd}}(z) = \sigma_d^2 I_K$, $S_{\boldsymbol{vv}}(z) = \sigma_v^2 I_L$.

- Polynomial Expansion approximation possible
(contribution at delay 0 of diagonal of $\boldsymbol{H}^{\dagger}\boldsymbol{H}$ dominating)

PSC – MUD: Subtractive IC

- introduce $\boldsymbol{g}[-m] = \boldsymbol{g}^H[m]$

$$\underbrace{\boldsymbol{G}(z)}_{K\times K} = \sum_{m=-2}^{2} \underbrace{\boldsymbol{g}[m]}_{K\times K} z^{-m} = \boldsymbol{H}^\dagger(z)\boldsymbol{H}(z) + \begin{cases} 0 & \text{, MMSE ZF} \\ \frac{\sigma_v^2}{\sigma_d^2} I_K & \text{, MMSE} \end{cases}$$

- we need to solve $\widehat{\boldsymbol{d}}[n]$ from

$$\boldsymbol{G}(q)\,\widehat{\boldsymbol{d}}[n] = \boldsymbol{g}[2]\widehat{\boldsymbol{d}}[n-2]+\boldsymbol{g}[1]\widehat{\boldsymbol{d}}[n-1]+\boldsymbol{g}[0]\underline{\underline{\widehat{\boldsymbol{d}}[n]}}+\boldsymbol{g}[-1]\widehat{\boldsymbol{d}}[n+1]+\boldsymbol{g}[-2]\widehat{\boldsymbol{d}}[n+2] \;=\; \boldsymbol{x}[n]$$

- Subtractive IC: solve for 1 symbol at a time and use decisions for values of other symbols.

- Noniterative receivers needs to perform: $\widehat{\boldsymbol{d}}[n] = \boldsymbol{G}^{-1}(q)\,\boldsymbol{x}[n]$: matrix IIR (in causal and anticausal directions), iterative receivers filter with $\boldsymbol{G}(q)$: matrix FIR, in every iteration.

PSC – MUD: SIC & PIC

- SIC: assume future decisions = 0

$$\boldsymbol{g}[0]\,\widehat{\boldsymbol{d}}[n] \;=\; \boldsymbol{x}[n] - \boldsymbol{g}[1]\,\widehat{\boldsymbol{d}}[n{-}1] - \boldsymbol{g}[2]\,\widehat{\boldsymbol{d}}[n{-}2]$$

  order d_k along k in order of decreasing power, solve as for sync. static channel case and take decisions.

- PIC: take past and future decision from previous iteration $(i{-}1)$:

$$\boldsymbol{g}[0]\,\widehat{\boldsymbol{d}}^{(i)}[n] \;=\; \boldsymbol{x}[n] - \sum_{\substack{m=-2\\ m\neq 0}}^{2} \boldsymbol{g}[m]\,\widehat{\boldsymbol{d}}^{(i-1)}[n{-}m]$$

  solve as for sync. static channel case and take decisions.

- Can combine SIC & PIC, do multistage etc.

PSC – MUD: DF

- spectral factorization: $\boldsymbol{G}(z) = \boldsymbol{F}^{\dagger}(z)\, D\, \boldsymbol{F}(z)$
  $\underbrace{\boldsymbol{F}(z)}_{K\times K} = \boldsymbol{f}[0] + \boldsymbol{f}[1]\, z^{-1} + \boldsymbol{f}[2]\, z^{-2}$ minimum phase (det$\boldsymbol{F}(z)$ min. phase)
  D diagonal, real and positive; $\boldsymbol{f}[0]$ unit-diagonal lower triangular
- how? $\boldsymbol{F}^{-\dagger}(z)$ = backward prediction filter for $\boldsymbol{G}(z)$
- solve for $\widehat{\boldsymbol{d}}[n]$ from $\boldsymbol{x}[n]$ by removing anticausal MAI & ISI linearly, causal MAI & ISI non-linearly:

$$\boldsymbol{F}(q)\, \widehat{\boldsymbol{d}}[n] = D^{-1} \boldsymbol{F}^{-\dagger}(q)\, \boldsymbol{x}[n] = \boldsymbol{z}[n] \quad \rightarrow \quad \boldsymbol{F}^{\dagger}(q) D\, \boldsymbol{z}[n] = \boldsymbol{x}[n]$$

$$\boldsymbol{x}[n] \rightarrow \boldsymbol{z}[n] : \quad \boldsymbol{f}^{H}[0]\, D\boldsymbol{z}[n] = \boldsymbol{x}[n] - \boldsymbol{f}^{H}[1]\, D\boldsymbol{z}[n{+}1] - \boldsymbol{f}^{H}[2]\, D\boldsymbol{z}[n{+}2]$$

backwards in time, backsubstitution

$$\boldsymbol{z}[n] \rightarrow \widehat{\boldsymbol{d}}[n] : \quad \boldsymbol{f}[0]\, \widehat{\boldsymbol{d}}[n] = \boldsymbol{z}[n] - \boldsymbol{f}[1]\, \widehat{\boldsymbol{d}}[n{-}1] - \boldsymbol{f}[2]\, \widehat{\boldsymbol{d}}[n{-}2]$$

forward in time, backsubstitution

if take decisions $\Rightarrow$ DF-MUD, if not $\Rightarrow$ L-MUD

PSC – MUD Complexity

- The complexity of the MUD processing of the RAKE outputs increase by at most a factor 5 compared to the synchronous flat channel case (delay spread < symbol period).
- As before, the complexity of SIC is about half that of PIC, DF or L.
- Add the complexity of the RAKE (2 solutions).
- This MUD/IC does not take undetected intercell interference into account. Can combine PIC for intracell MAI with LMMSE for intercell MAI.
- Diversity: if oversample with factor R (assuming sufficient excess bandwidth) and receive with I antennas, then $c_k[l]$ is $RI \times 1$ and $\boldsymbol{y}[n]$ is $LRI \times 1$. Can handle now $K < LRI$ users (decorrelation possible).

Multirate Case

- The period of cyclostationarity corresponds to the smallest common multiple of the various symbol periods (in the periodic spreading case).
- Consider e.g. user k having $T/2$ as symbol period (double rate):

$$\begin{bmatrix} s_k[0] \\ \vdots \\ \vdots \\ s_k[L-1] \end{bmatrix} d_k[0] \rightarrow \begin{bmatrix} s_k[0] & 0 \\ \vdots & \vdots \\ s_k[\frac{L}{2}-1] & 0 \\ 0 & s_k[0] \\ \vdots & \vdots \\ 0 & s_k[\frac{L}{2}-1] \end{bmatrix} \begin{bmatrix} d_k[0] \\ d_k[1] \end{bmatrix} = \begin{bmatrix} s_k[0] \\ \vdots \\ s_k[\frac{L}{2}-1] \\ 0 \\ \vdots \\ 0 \end{bmatrix} d_k[0] + \begin{bmatrix} 0 \\ \vdots \\ 0 \\ s_k[0] \\ \vdots \\ s_k[\frac{L}{2}-1] \end{bmatrix} d_k[1]$$

can be considered as two users at the original rate with appropriately modified spreading sequence definition.

Colored Noise Case

- as in the spatiotemporal case ...
- colored noise: $\sigma_v^2 \, I_m \;\; \rightarrow \;\; S_{\mathbf{VV}}(z)$ $\qquad S(z) = \underbrace{S^{1/2}(z)}_{\text{min. phase}} \; \underbrace{S^{\dagger/2}(z)}_{\text{max. phase}}$
- reduce colored noise case to white noise case by

$$\begin{aligned} \mathbf{y}^{'}[k] &= S_{\mathbf{VV}}^{-1/2}(q)\,\mathbf{y}[k] \\ \mathbf{H}^{'}(q) &= S_{\mathbf{VV}}^{-1/2}(q)\,\mathbf{H}(q) \\ \sigma^2_{v'} &= 1 \end{aligned}$$

  yielding $\mathbf{y}^{'}[k] = \mathbf{H}^{'}(q)\, a[k] + \mathbf{v}^{'}[k]$

  and $S_{\mathbf{VV}}(z) = S_{\mathbf{VV}}^{1/2}(z)\, S_{\mathbf{VV}}^{\dagger/2}(z)$
- in particular, colored noise MF

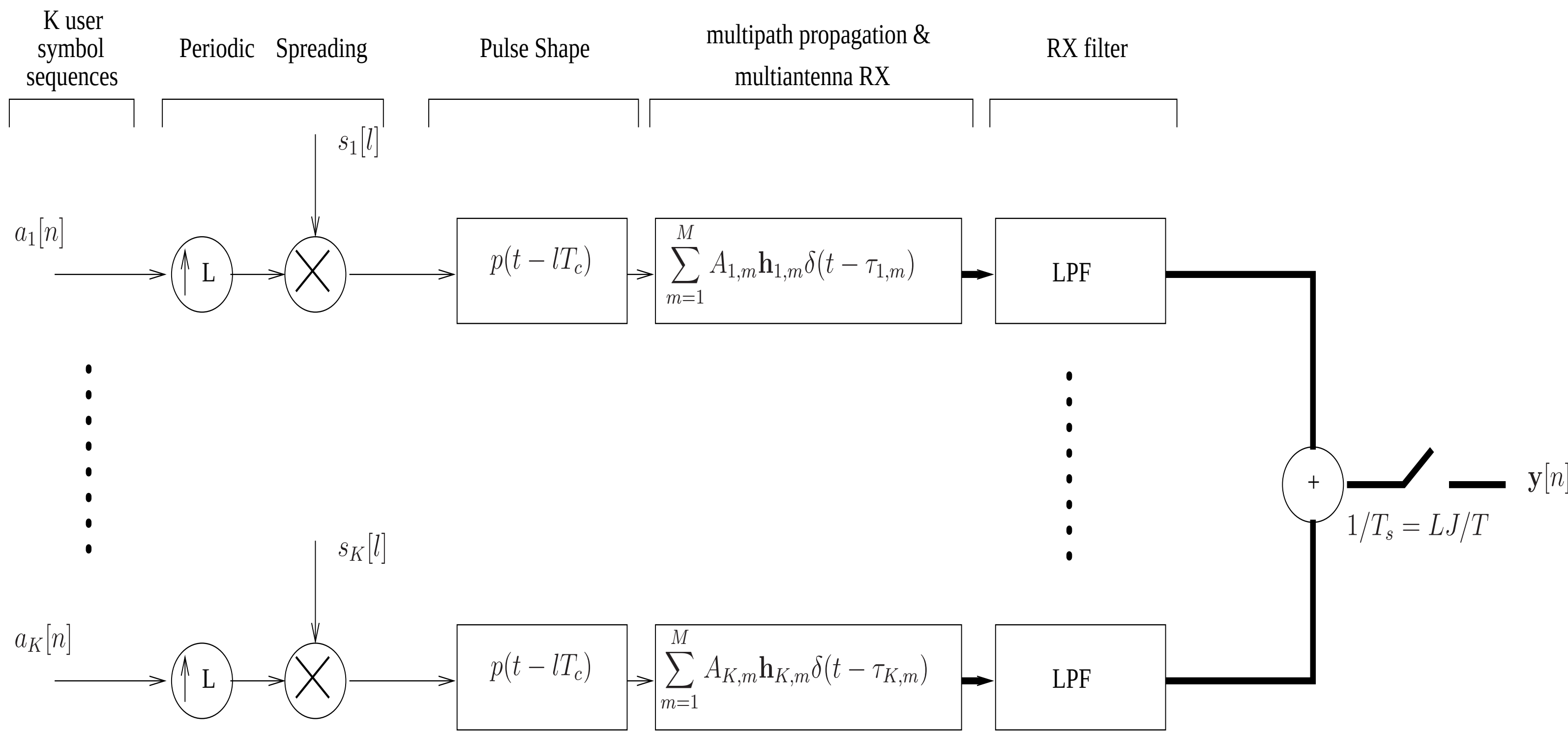
$$\mathbf{F}_{MF}(q) \; = \; \mathbf{H}^{\dagger}(q)\, S_{\mathbf{VV}}^{-1}(q) \;\; \text{since} \;\; \mathbf{F}\,\mathbf{y} = \mathbf{H}^{'\dagger}\mathbf{y}^{'}$$

- from the moment the noise is temporally colored, even in the frequency-flat channel synchronous case, symbol period wise processing is suboptimal

CDMA Overview Cont'd

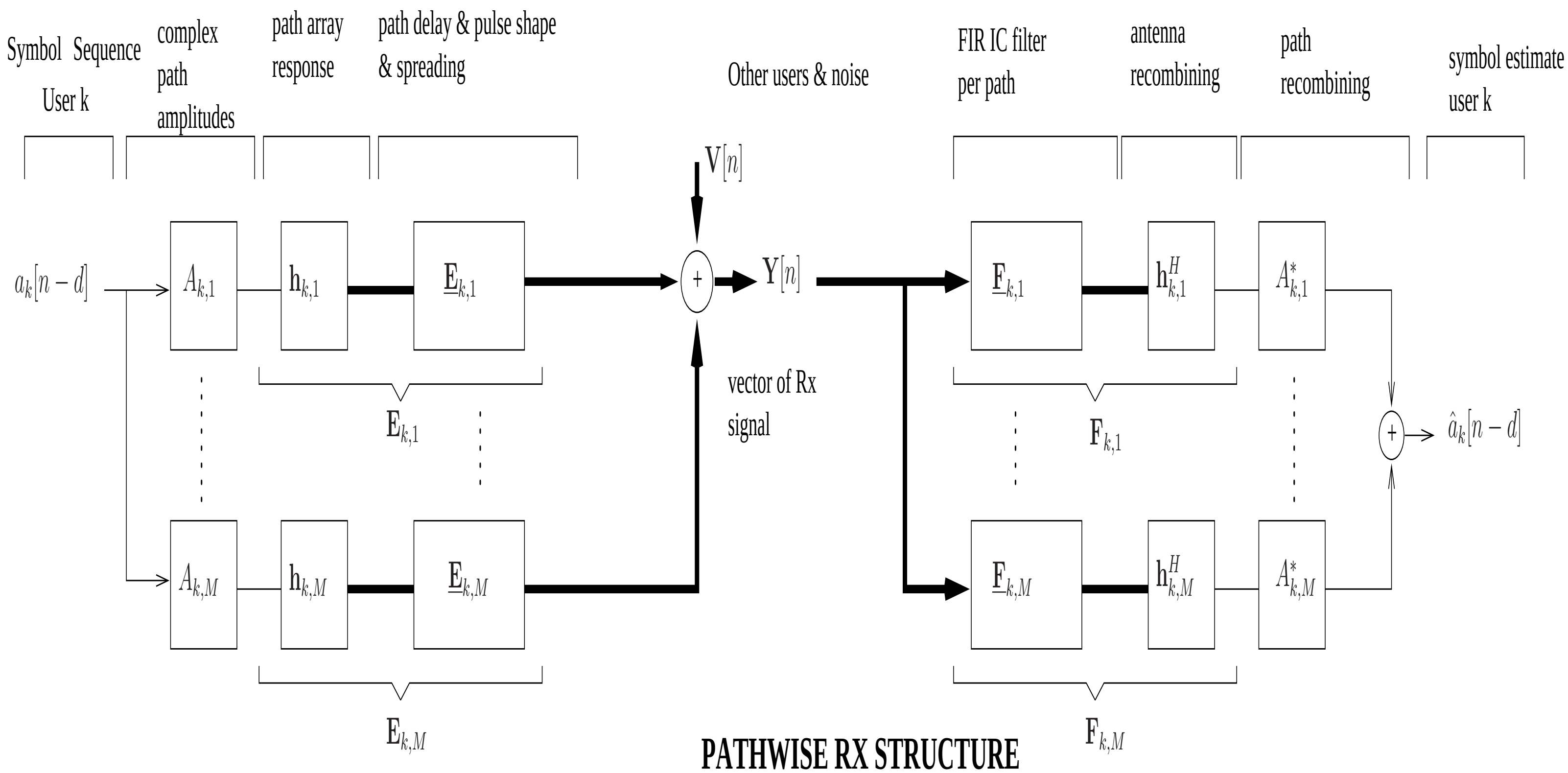
- UMTS-FDD uplink: RAKE and pathwise processing
- UMTS-FDD downlink: Max SINR Equalizer-Correlator receiver

UMTS Uplink Signal Model



UMTS FDD UPLINK

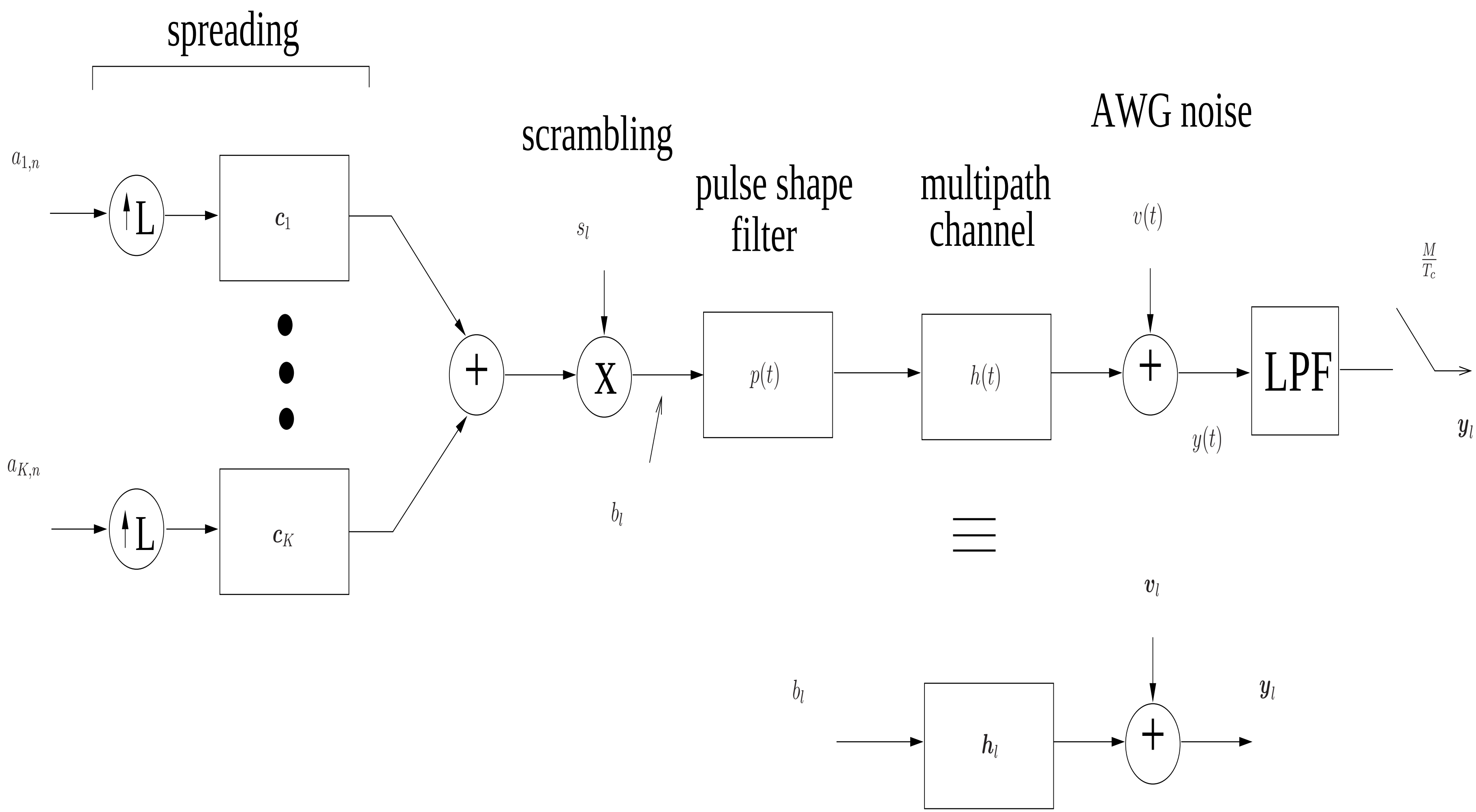
UMTS Uplink RAKE Receiver Structure



UMTS Uplink

- BS with multiple RX antennas:
  can replace spatial matched filters in 2D RAKE by optimized spatial filters: 2D GRAKE
- having multiple antennas allows to
  - increase loading: # users > spreading factor (combination with SDMA)
  - and/or to handle intercell interference
- antenna array at BS:
  - fixed beams/sectoring: K & L large, limit to a sector to limit interference
  - adaptive antennas: if $\exists$ a few high-rate users, reduce their interference

UMTS FDD Downlink Data Model



UMTS FDD Downlink

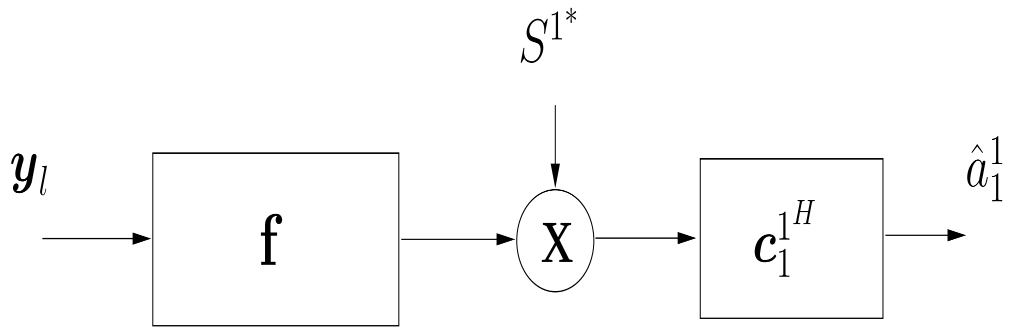
- downlink synchronous $\Rightarrow$ orthogonal codes
- if BS does not employ beamforming, then all intracell user signals pass through the same downlink channel, the delay spread of which destroys the orthogonality of the codes
- orthogonality can be restored by an equalizer
- a MMSE equalizer maximizes the SINR at the output of the overall equalizer-correlator RX

$$\begin{aligned}\text{SINR} &= \frac{\sigma_1^2 |\, \mathbf{f}(q)\mathbf{h}(q)|_{[0]}\,|^2}{\frac{1}{2\pi j}\oint \frac{dz}{z}\mathbf{f}(z)S_{\mathbf{vv}}(z)\mathbf{f}^\dagger(z) + \sigma_{tot}^2\left(\|\mathbf{f}(z)\mathbf{h}(z)\|^2 - |\,\mathbf{f}(q)\mathbf{h}(q)|_{[0]}\,|^2\right)} \\ &= \frac{\sigma_1^2 |\, \mathbf{f}(q)\mathbf{h}(q)|_{[0]}\,|^2}{\frac{1}{2\pi j}\oint \frac{dz}{z}\mathbf{f}(z)S_{\mathbf{yy}}(z)\mathbf{f}^\dagger(z) - \sigma_{tot}^2\,|\,\mathbf{f}(q)\mathbf{h}(q)|_{[0]}\,|^2}\end{aligned}$$

where $\sigma_{tot}^2 = \frac{1}{L}\sum_{k=1}^{K}\sigma_k^2$.

- spatiotemporal equalizers perform better and/or allow furthermore intercell interference reduction

Equalizer-Correlator Rx



- structure: filter (equalizer) + descrambler + correlator
- RAKE receiver:
  f = channel matched filter $\Rightarrow$ max SNR (multipath $\Rightarrow$ fingers)
- **f** = ZF equalizer $\Rightarrow$ max SIR ($= \infty$) (Anja Kleijn)
  (I = intracell interference, intercell interference in noise N)
- **f** = MMSE equalizer $\Rightarrow$ max SINR
  multichannel aspect $\Rightarrow$ + intercell interference reduction (N)

Filter/Descrambler/Correlator vs True Max SINR

- SINR maximized by MMSE receiver (within class of linear RXs)
- True MMSE RX:

$$\widehat{a} = F\,Y = R_{aY}\,R_{YY}^{-1}\,Y = (E_{A,V}\,aY^H)\,(E_{A,V}\,Y\,Y^H)^{-1}\,Y$$

  F time-varying since scrambler considered deterministic here (since it is known)

- Approximate MMSE RX:

$$\begin{aligned}\widehat{a} &= (E_{A,V}\,aY^H)\,(E_{A,V,S}Y\,Y^H)^{-1}\,Y \\ &= \sigma_a^2 \underbrace{c^H}_{\text{correlator}} \underbrace{S^H}_{\text{descrambler}} \underbrace{\mathcal{T}(h)^H\,(\underbrace{E_{A,V,S}Y\,Y^H}_{\text{(block) Toeplitz}})^{-1}}_{\text{LTI chip rate MMSE equalizer}}\,Y\end{aligned}$$

Filter/Descrambler/Correlator vs True MSINR (2)

- Walsh-Hadamard spreading codes:

$$C_n = W_n \; , \; W_{2n} = \begin{bmatrix} W_n & W_n \\ W_n & -W_n \end{bmatrix} , \; W_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

- True LMMSE

$$\begin{aligned} E_{A,V} Y\, Y^H &= E_{A,V} \mathcal{T}(h)\, S\, C\, A\, A^H\, C^H\, S^H\, \mathcal{T}^H(h) + E_{A,V} V\, V^H \\ &= \sigma_a^2\, \mathcal{T}(h)\, S \underbrace{C\, C^H}_{\text{subspace}} S^H\, \mathcal{T}^H(h) + \sigma_v^2\, I \end{aligned}$$

- Approximate LMMSE

$$\begin{aligned} E_{A,V,S} Y\, Y^H &= E_{A,V,S} \mathcal{T}(h)\, S\, C\, A\, A^H\, C^H\, S^H\, \mathcal{T}^H(h) + E_{A,V} V\, V^H \\ &= \sigma_a^2\, \mathcal{T}(h) \underbrace{E_S\{S\, C\, C^H\, S^H\}}_{=\text{diag}\{C\, C^H\} = \frac{K}{L} I} \mathcal{T}^H(h) + \sigma_v^2\, I \\ &= \frac{\sigma_a^2\, K}{L}\, \mathcal{T}(h)\, \mathcal{T}^H(h) + \sigma_v^2\, I \end{aligned}$$

Filter/Descrambler/Correlator vs True MSINR (3)

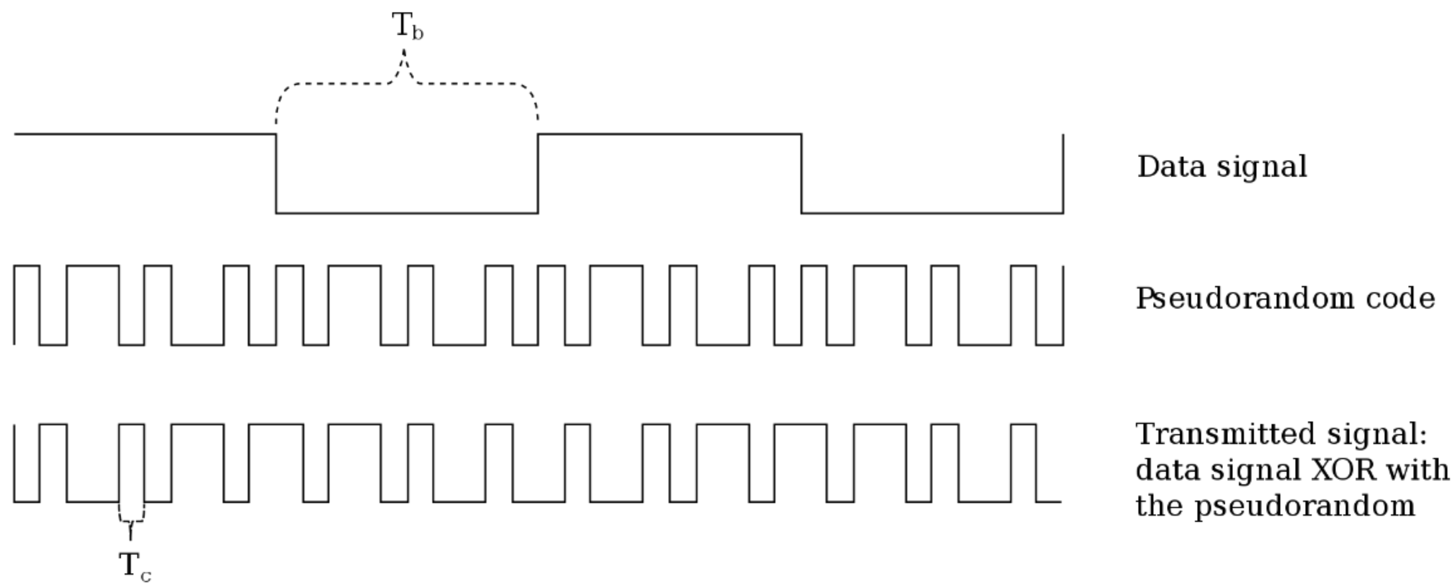
- all time variation now limited to descrambler
- the constrained RX structure consisting of a filter-descrambler-correlator that maximizes the RX output SINR has filter = MMSE equalizer
- chip rate MMSE equalizer = MMSE RX for extracting the chip rate cyclostationary desired user chip sequence from the chip rate cyclostationary RX signal (assume scrambler random at this point in the RX)
- descrambler and correlator = optimal RX for extracting the desired user symbol sequence from the desired user chip sequence

Chip Equalizer vs. G-RAKE

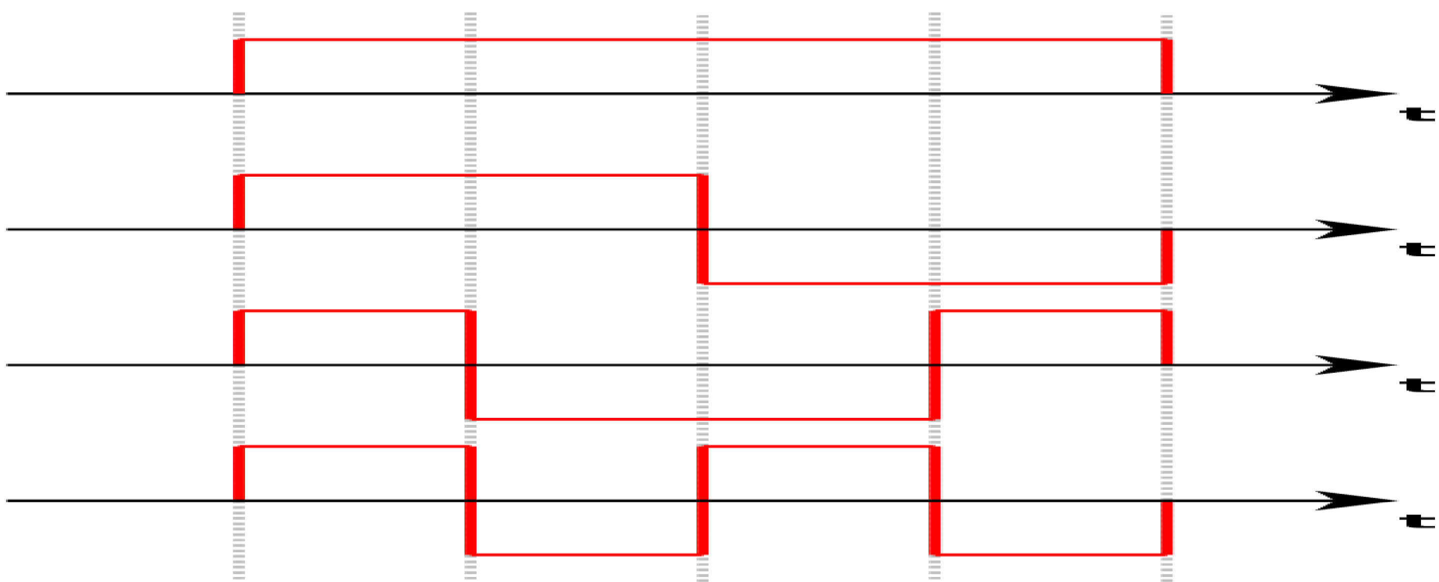
- G-RAKE (Generalized): similar sparse structure to RAKE, but tap coefficients optimized for SINR (not SNR)
- L-chip FIR equalizer, M-tap (delays) G-RAKE, N spreading factor (=16 for HSDPA), m subchannels, n multi-codes
  $\beta = \dfrac{n}{N}$: loading fraction, $\alpha = \dfrac{M}{L}$: sparsification fraction
  complexity comparison:

$$\left\{ \begin{array}{rcll} \text{EQUAL} & : & m\,L\,N + n\,N & = & m\,L\,N + \beta\,N^2 \\ \text{G-RAKE} & : & m\,M\,n + m\,M\,n & = & m\,L\,N\,\alpha\beta + \beta\,N^2\,\dfrac{m\,\alpha\,L}{N} \end{array} \right.$$

  first term: complex multiply-accumulate
  second term: code/scrambler chip multiply-accumulate



Walsh-Hadamard codes length 4



$$P_k \underbrace{P_k^H}_{= P_k^{-1}} = I$$

$$P_k = \begin{array}{c} \begin{array}{ccc} & k & \end{array} \\ \left[\begin{array}{c|c|c} 0 - 0 & 1 & 0 - 0 \\ \hline I_{k-1} & 0 & 0 \\ \hline 0 & 0 & I_{K-k} \end{array}\right] \end{array}$$

$$\left[(S^H S)^{-1}\right]_{k,k} = \left[P_k (S^H S)^{-1} P_k^H\right]_{1,1}$$

$$= \left[P_k^{-H} (S^H S)^{-1} P_k^{-1}\right]_{1,1} = \left[\left(P_k S^H S P_k^{H}\right)^{-1}\right]_{1,1}$$

$$E\{\bar{S}_k \bar{S}_k^H\} = \sum_{\substack{i=1 \\ \neq k}}^{K} \underbrace{E\{s_i s_i^H\}}_{\frac{1}{L} I_L}$$

$$= \frac{K-1}{L} I_L$$

$$E|F_k^H \bar{S}_k \bar{C}_k \underline{\bar{d}}_k|^2 = E\, F_k^H \bar{S}_k \bar{C}_k \underbrace{\underline{\bar{d}}_k \underline{\bar{d}}_k^H}_{\sigma_d^2 I_{K-1}} \bar{C}_k^H \bar{S}_k^H F_k$$

$$\min_A \; (V-XA)^H(V-XA) = V^HV - V^HXA - A^HX^HV + A^HX^HXA$$

$$\frac{\partial}{\partial A^*} = -X^HV + X^HXA = 0$$

$$A = (X^HX)^{-1}X^HV$$

$$XA = \underbrace{X(X^HX)^{-1}X^H}_{P_X} V$$

$$P_X y = X\underbrace{(X^H X)^{-1} X^H y}_{\alpha}$$

$$(C^H S^H S C)^{-1} C^H S^H$$

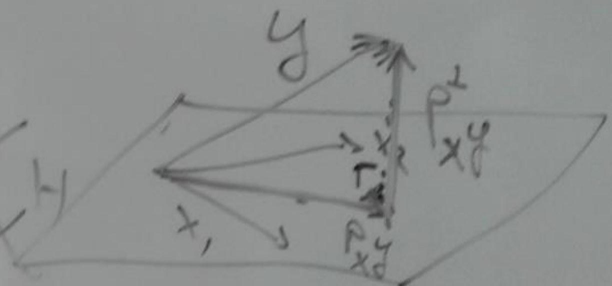
$$\min_{\alpha} \| y - X\alpha \|^2$$

$$C^{-1}(S^H S)^{-1} \cancel{C^{-H}} \cancel{C^H} S^H$$

$SINR_k =$

$$P_X P_X = X(X^H X)^{-1} \cancel{X^H X (X^H X)^{-1}} X^H$$

$$= P_X$$



Polynomial Expansion:

$$\frac{1}{1-x} = \sum_{k=0}^{\infty} x^k = 1 + x + x^2 + \cdots = \overbrace{(1+x)(1+x^2)(1+x^4)(1+x^8)}^{= 1+x+x^2+\cdots+x^{15}}\cdots$$

$|x|<1$

matrices: $T = (I-R)^{-1} = I + R + R^2 + \cdots$

$(I-R)T = I \Rightarrow T_k = I + R\,T_{k-1}$ "Jacobi iteration"

converges if $|\lambda_i(R)| < 1$.

$T_{-1} = 0$, $T_0 = I$, $T_1 = I + R$, $T_2 = I + R + R^2$, $T_k = \sum_{i=0}^{k} R^i$.