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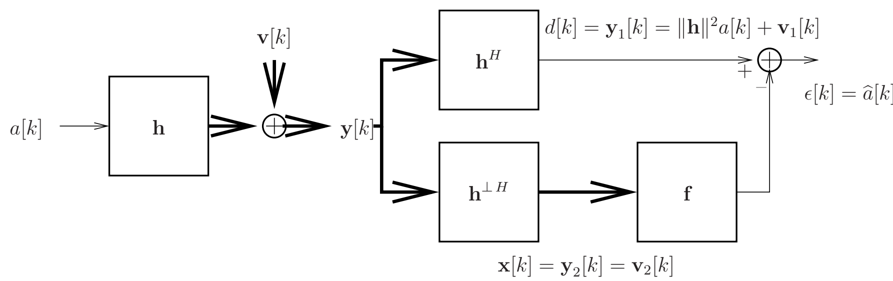
SP4COM Signal Processing for Communications

Exam

For every answer you provide, try to give it in its simplest form, while answering correctly.
If you get stuck in a certain question, do not hesitate to try the other parts of the question or continue with the next question.
Results that are available in the course notes can be used and referenced and do not need to be rederived. You can answer in French or in English.

Spatial Processing: Linear Interference Cancellation

1. Suboptimal design of the Spatial ICMF with One Interferer



Consider again adaptation of the spatial Interference Canceling Matched Filter (ICMF) depicted in the figure above and considered in the TD and the TP. The received signal $\mathbf{y}[k]$ contains m subchannels and the interference canceling filter $\mathbf{f}$ is represented as a row vector. For a generic value of $\mathbf{f}$, one obtains the error signal

$$\epsilon[k](\mathbf{f}) \;=\; d[k] - \mathbf{f}\,\mathbf{x}[k]\;. \tag{1}$$

The error signal $\epsilon[k]$ also provides an estimate $\widehat{a}[k]$ of the transmitted symbol sequence $a[k]$ (apart from a scale factor $\|\mathbf{h}\|^2$). Let the additive noise be of the form considered in the TP:

$$\mathbf{v}[k] \;=\; \mathbf{g}\,b[k] + \mathbf{u}[k] \tag{2}$$

where $\mathbf{u}[k]$ is spatiotemporally white circular complex Gaussian noise with spectrum $S_{\mathbf{uu}}(z) = \sigma_u^2\, I_m$ and $b[k]$ is the temporally white circular complex Gaussian interferer signal with spectrum $S_{bb}(z) = \sigma_b^2$, independent of the noise $\mathbf{u}[k]$. $\mathbf{g}$ is the $m \times 1$ spatial channel of the interferer.

In the case of joint detection of $a[k]$ and $b[k]$ (which is the case in which we fully exploit the structure of $\mathbf{v}[k]$), the MFB for the detection of $a[k]$, which we can also call $\text{MFB}_{\mathbf{u}}$, is $\text{MFB}_{\mathbf{u}} = \frac{\sigma_a^2}{\sigma_u^2}\|\mathbf{h}\|^2$. The interference to noise ratio is INR$= \frac{\sigma_b^2}{\sigma_u^2}\|\mathbf{g}\|^2$.

In this exercise we shall analyze the effect of suboptimal processing. We shall model the sum of interference plus noise as white noise, not only temporally but also spatially. Hence assume we model $\mathbf{v}[k]$ as $\mathbf{v}[k] = \mathbf{v}'[k]$ with $R_{\mathbf{v}'\mathbf{v}'} = \sigma_{v'}^2\, I_m$ where $\sigma_{v'}^2 = \frac{1}{m}\,\text{tr}\,\{R_{\mathbf{vv}}\}$.

So, whereas the received signal $\mathbf{y}[k]$ contains the actual interverence plus noise $\mathbf{v}[k]$, for the design of the receiver we shall act as if the model for $\mathbf{v}[k]$ is $\mathbf{v}^{'}[k]$. So the receiver coefficients will depend on $R_{\mathbf{V}'\mathbf{V}'}$ but the receiver output will be affected by $\mathbf{v}[k]$.

(a) Express $\sigma^2_{v'}$ in terms of $\mathbf{g}$, σ^2_b and σ^2_u (see (2)).

(b) Show that with this spatially white noise model for $\mathbf{v}^{'}[k]$, $\mathbf{v}^{'}_1[k]$ and $\mathbf{v}^{'}_2[k]$ (see figure) become decorrelated: $R_{\mathbf{V}'_1\mathbf{V}'_2} = 0$.

(c) What is in this case (with $R_{\mathbf{V}'\mathbf{V}'}$) the optimal (MMSE) value of $\mathbf{f}$, namely $\mathbf{f}^o$?

(d) Let's denote $e[k] = \epsilon[k](\mathbf{f}^o)$, the optimal error signal. Derive an expression for $e[k]$ in terms of the quantities appearing in the figure (by substituting the expression found for $\mathbf{f}^o$ in(c) above), and by using the definition of $\mathbf{v}[k]$ in (2) (hence give $e[k]$ in terms of $a[k]$, $b[k]$, $\mathbf{u}[k]$, $\mathbf{h}$ and $\mathbf{g}$).

(e) For this signal $e[k] = \widehat{a}[k]$, express the SINR, also called $\text{SINR}_{\mathbf{V}'}$, in terms of the quantities you found in the expression for $e[k]$ in (d).

(f) Express this same $\text{SINR}_{\mathbf{V}'}$ in terms of $\text{MFB}_{\mathbf{V}}$, INR, and $|\cos\theta|$, where $\cos\theta = \dfrac{\mathbf{h}^H\mathbf{g}}{\|\mathbf{h}\|\,\|\mathbf{g}\|}$.

(g) What are the MFBs, by treating the interferer as colored noise, in the models $\mathbf{v}[k]$ and $\mathbf{v}^{'}[k]$ respectively?

<u>**CDMA**</u>

2. Average SINR for Single User Matched Filter (SUMF) and Decorrelator

We have derived the SINR for the Decorrelator (or MMSE-ZF) linear receiver in CDMA, namely

$$\text{SINR}^D_k = \frac{|c_k|^2\sigma^2_d}{\sigma^2_v}\,\|P^\perp_{\overline{S}_k}S_k\|^2 = \text{MFB}_k\,\|P^\perp_{\overline{S}_k}S_k\|^2 \ .$$

We shall compute here the expected value of this SINR, similarly to the analysis for the SINR of the SUMF on slide 17 of Lecture 4. So we consider the elements of spreading codes as i.i.d. constant magnitude random variables and compute the expected value of the SINR, that is random due the spreading codes. To guide the derivation, we shall proceed in two steps.

(a) In a first step show the following identity

$$\text{E}_S\{\text{SINR}^D_k\} = \text{MFB}_k\,(1 - \frac{1}{L}\,\text{E}_{\overline{S}_k}\text{tr}\{P_{\overline{S}_k}\}) \ .$$

(b) Now use the definition of projection and the properties of trace of a product of matrices to work out the expression further. You will find that $\text{tr}\{P_{\overline{S}_k}\}$ is a deterministic quantity actually. Show that you get

$$\text{E}_S\{\text{SINR}^D_k\} = \text{MFB}_k\,(1 - \frac{K-1}{L}) \ .$$

(c) Now consider the averaged SINR for the SUMF as on slide 17 of Lecture 4,

$$\mathrm{SINR}_k^{MF} = \frac{1}{\frac{K-1}{L} + \frac{1}{\mathrm{MFB}_k}}$$

As a function of SNR (MFB_k), find which receiver, among SU MF or Decorrelator, gives the highest (expected) SINR.

(d) The analysis on slide 17 of Lecture 4 assumes that $\mathrm{SNR}_k = \mathrm{MFB}_k$ is equal for all users (perfect power control). Redo this analysis for the SUMF (compute the denominator averaged SINR_k^{MF}) when all users have a possibly different $\mathrm{SNR}_k = \mathrm{MFB}_k = \frac{|c_k|^2 \sigma_d^2}{\sigma_v^2}$.

(e) Now consider the following problem

$$\max_{\{\mathrm{MFB}_{i,i=1,\ldots,K}\}} \quad \min_{k \in \{1,\ldots,K\}} \mathrm{SINR}_k^{MF}$$

Show that the solution to this problem is that all SINR_k^{MF} are equal. To do this, argue that when the SINR_k^{MF} are not all equal, it is always possible to increase $\min_{k \in \{1,\ldots,K\}} \mathrm{SINR}_k^{MF}$.

Relatively Short Questions

In case the questions below admit a simple yes/no type of answer, you should add a bit of explanation.

3. Spatial Processing / Diversity / MIMO

(a) What is a MMSE-ZF linear receiver (Minimum Mean Squared Error Zero Forcing), compared to a basic ZF receiver?
Is there a relation between the number of antennas m and the number of users p for MMSE-ZF receivers to exist?

(b) In a multi-user (MU) setting, focusing on a given user of interest, can a linear MMSE receiver be determined/estimated if only the channel of the user of interest is known?

(c) Is spatial processing relevant in the context of OFDM?

(d) (Single-User SIMO.) Does receive antenna selection achieve the full multi-antenna receive diversity order m?

(e) (Single-User SIMO.) Assume that at the output of a certain receiver, we get

$$\mathrm{SNR} = \frac{\sigma_a^2}{\sigma_v^2} \left| \sum_{i=1}^{m} i\, h_i \right| . \tag{3}$$

Does this receiver achieve full diversity order m?

(f) (Single-User MIMO.) Consider now a MIMO channel with N_t transmit antennas and N_r receive antennas. In an attempt to exploit the rich diversity of a MIMO channel, we are going to transmit a single stream from transmit antenna i to receive antenna j such that the channel gain $|h_{ij}|$ is maximum over all entries in the $N_r \times N_t$ MIMO channel response.

(i) Does this approach allow to reach maximum diversity order?

(ii) What is the maximum diversity order in this MIMO channel?

(iii) Does this scheme enjoy spatial multiplexing gain?

(g) (Single-User MIMO.) Consider still the same $N_r \times N_t$ MIMO channel, with $N_t > N_r$.

(i) How many streams can pass through this channel simultaneously? In other words, what is the maximum spatial multiplexing gain?

(ii) In order to simplify transmission a bit, we are going to send N_r streams by using only the $N_r < N_t$ best transmit antennas. Does this scheme still allow full spatial multiplexing?

(iii) How should the N_r "best" transmit antennas be chosen?

(iv) Does this require (any) channel state information at the transmitter (CSIT)?

(v) Does this approach suffer any diversity loss compared to an optimal approach using all N_t transmit antennas?

4. CDMA Uplink (UL)

(a) What is the MMSE-ZF receiver called in the context of CDMA? Why?

(b) Why is Polynomial Expansion (PE) well-suited for CDMA (at least in the flat channel case)?

(c) Which receiver structure(s) benefit from unequal user powers? Why?

(d) In the multipath propagation channel case, what is the matched filter receiver called in the context of CDMA? Why?

5. CDMA Downlink (DL)

(a) Why are orthogonal codes used in the downlink and not in the uplink?

(b) If channel transfer function $h(z) = \alpha\, z^{-d}$, what would the optimal receiver look like?

(c) Why is a scrambler used?

(d) Does the scrambler affect the orthogonality of the Walsh-Hadamard codes?

(e) The "chip equalizer" receiver is the cascade "channel equalizer + descrambler + correlator". Is the MMSE chip equalizer an optimal (MMSE) overall linear receiver structure?

(f) In lecture 4, slide 21, we find for the MMSE receiver:

$$\mathrm{SINR}_k = \left(\frac{1}{S_k^H R_{\boldsymbol{yy}}^{-1} S_k \, |c_k|^2 \sigma_d^2} - 1 \right)^{-1} = \frac{\sigma_d^2}{\mathrm{MMSE}_k} - 1 \, . \tag{4}$$

Show the second equality.
Hints: for an MMSE estimator the MMSE appears on slide 36 of Lecture 1.
Also, show that $S_k^H R_{\boldsymbol{yy}}^{-1} S_k \, |c_k|^2 \sigma_d^4$ is the power at the output of the MMSE receiver.