

Introduction to Electronics



An introduction to electronic components and a study of circuits containing such devices.

Week 3: Op Amps Part 2





Dr. Bonnie H. Ferri

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First-Order Lowpass Filters

Introduce lowpass filters



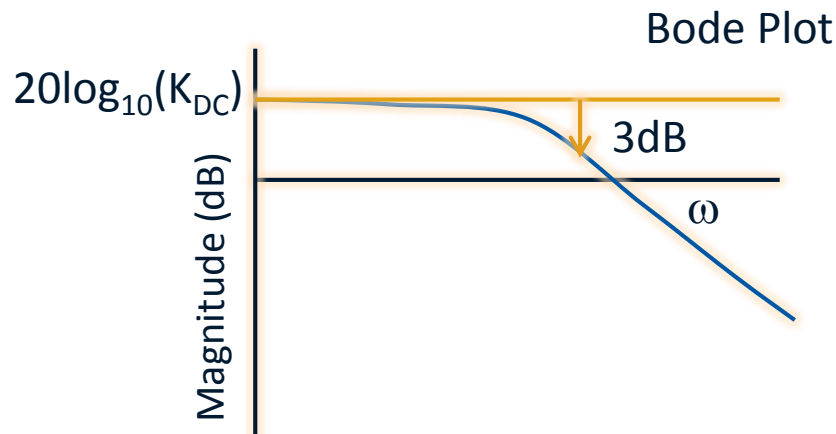
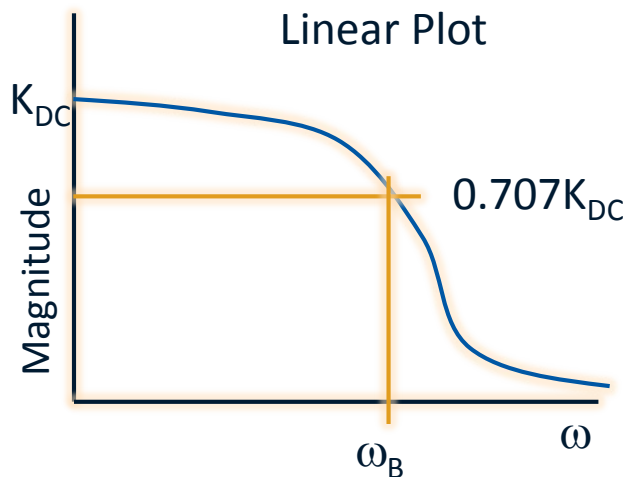
Lesson Objectives

- Introduce active lowpass filters

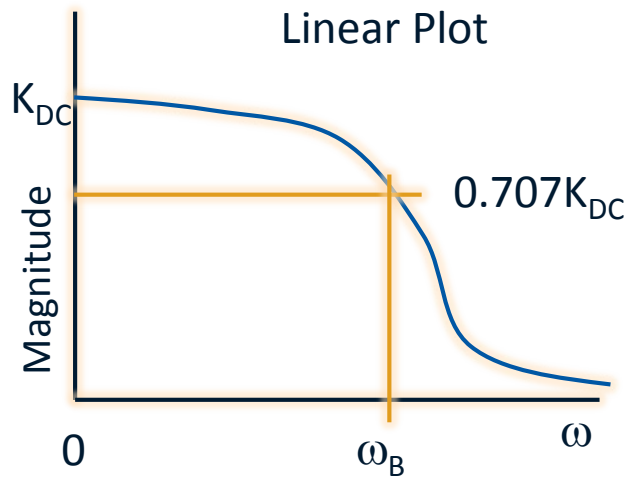
Lowpass Filters

- Lowpass filters pass low frequency components and attenuate high frequency components

Transfer Function $H(\omega)$



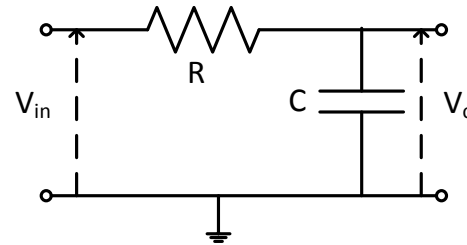
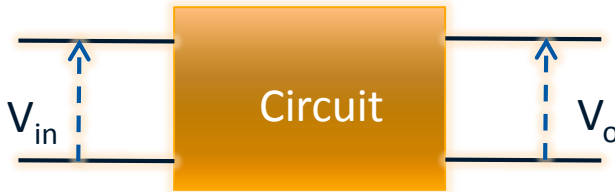
First-Order Filter



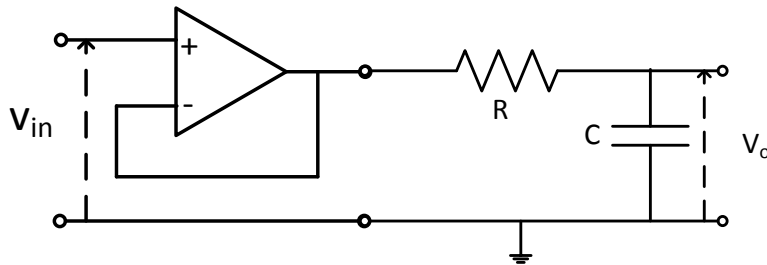
$$H(\omega) = K_{DC} \frac{1}{\tau j\omega + 1}$$

Bandwidth, $\omega_B = 1/\tau$
DC Gain = $H(0) = K_{DC}$

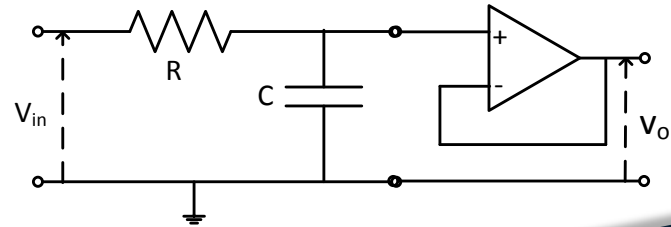
From Passive to Active Lowpass Filters



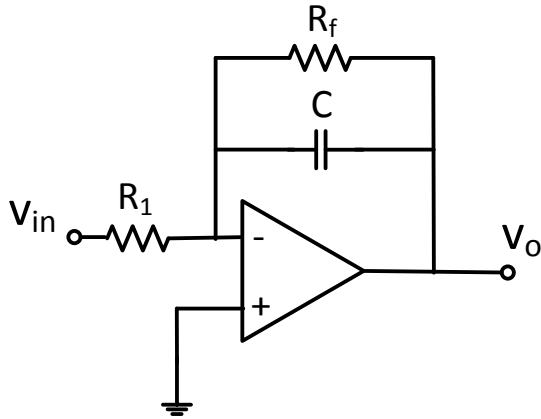
Isolation at the input:



Isolation in the output:



First-Order Inverting Lowpass Filter



$$V_o = -\frac{R_f}{R_1} \frac{1}{R_f C j\omega + 1} V_{in}$$

Frequency Characteristics of LP Filter

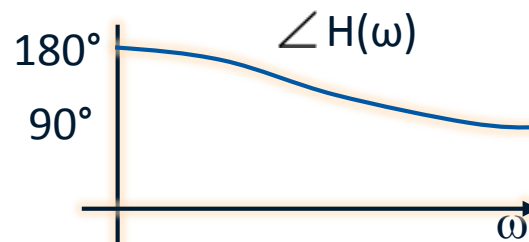
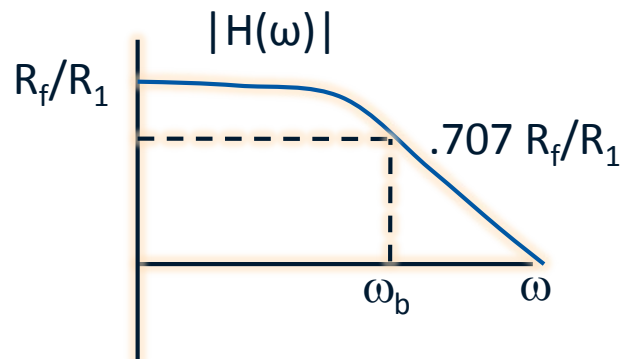
$$H(\omega) = -\frac{R_f}{R_1} \frac{1}{(R_f C_f \omega + 1)}$$

$$|H(\omega)| = \frac{R_f}{R_1} \frac{1}{\sqrt{(R_f C_f \omega)^2 + 1}}$$

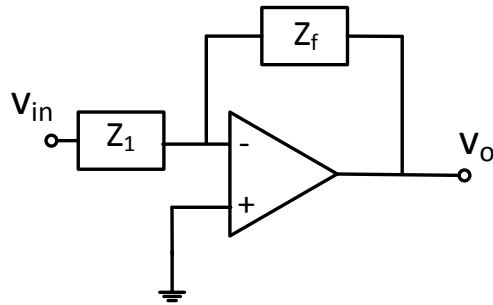
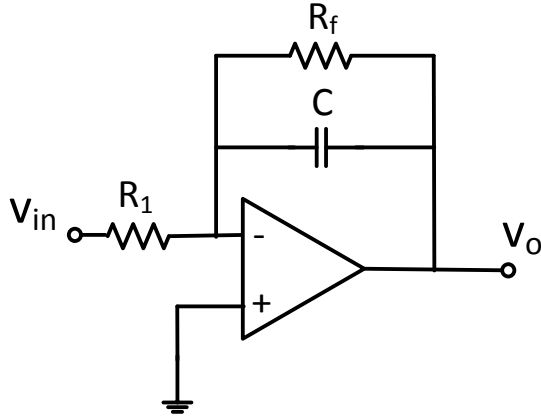
$$\angle H(\omega) = 180 - \arctan(R_f C_f \omega)$$

$$\text{DC Gain} = -\frac{R_f}{R_1}$$

$$\text{Bandwidth, } \omega_b = \frac{1}{R_f C_f}$$

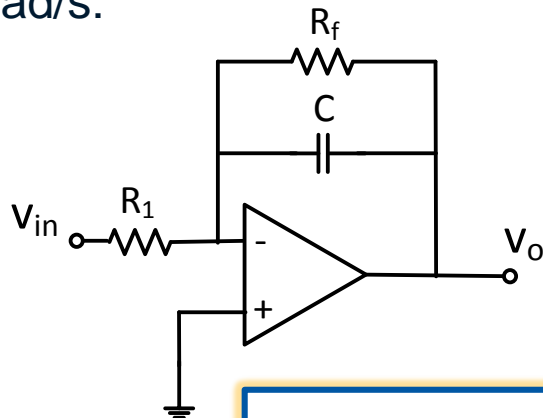


Derivation: Lowpass Filter



Example

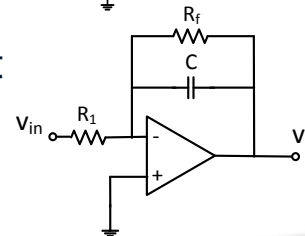
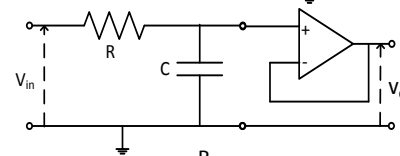
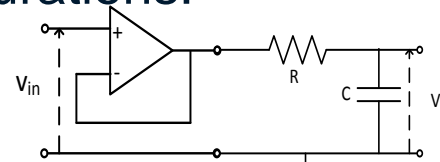
Design an inverting lowpass filter to have a DC gain of -2 and a bandwidth of 500 rad/s:



$$H(\omega) = -\frac{R_f}{R_1} \frac{1}{R_f C j\omega + 1}$$

Summary

- A **lowpass filter** passes low frequency signals and attenuates high frequency signals
- Three first-order lowpass configurations:
 - Noninverting, isolation at the input
 - Noninverting, isolation at the output
 - Inverting, isolation at input and output





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First-Order Highpass Filters

Introduce highpass filters

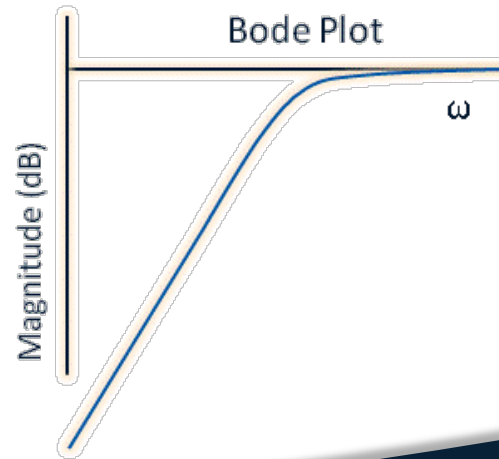
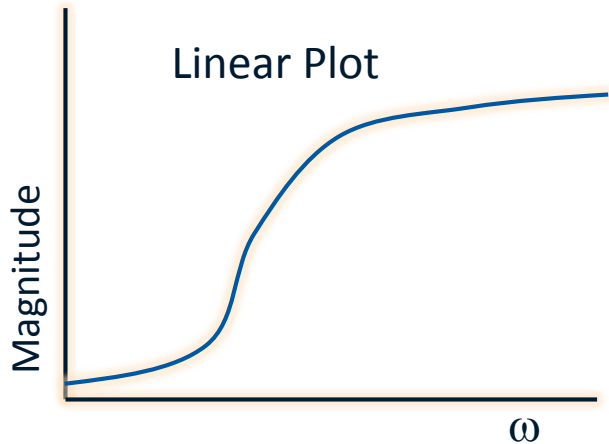


Lesson Objectives

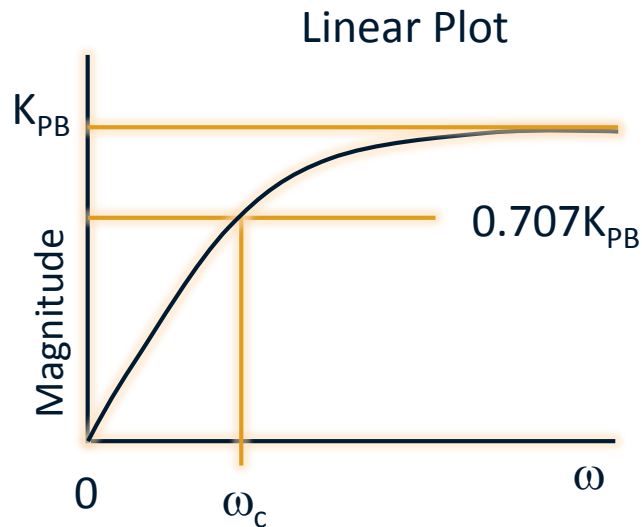
- Introduce active highpass filters

Highpass Filter

- Passes high frequency components and attenuates low frequency components



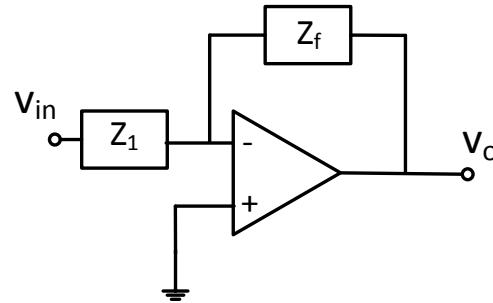
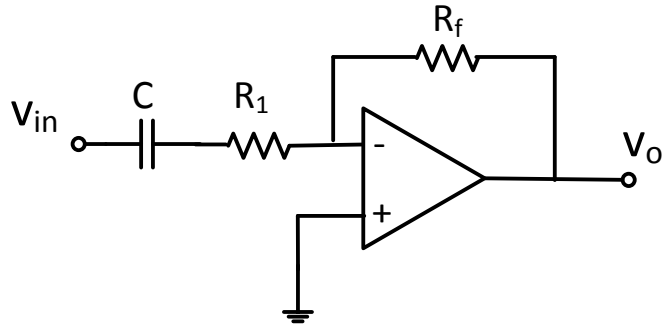
First-Order Filter



$$H(\omega) = \frac{Kj\omega}{\tau j\omega + 1}$$

Corner Frequency, $\omega_c = 1/\tau$
Passband Gain = $K_{PB} = K/\tau$

Inverting Highpass Filter Configuration



$$V_o = \frac{-R_f C j \omega}{(R_1 C j \omega + 1)} V_{in}$$

Frequency Characteristics of HP Filter

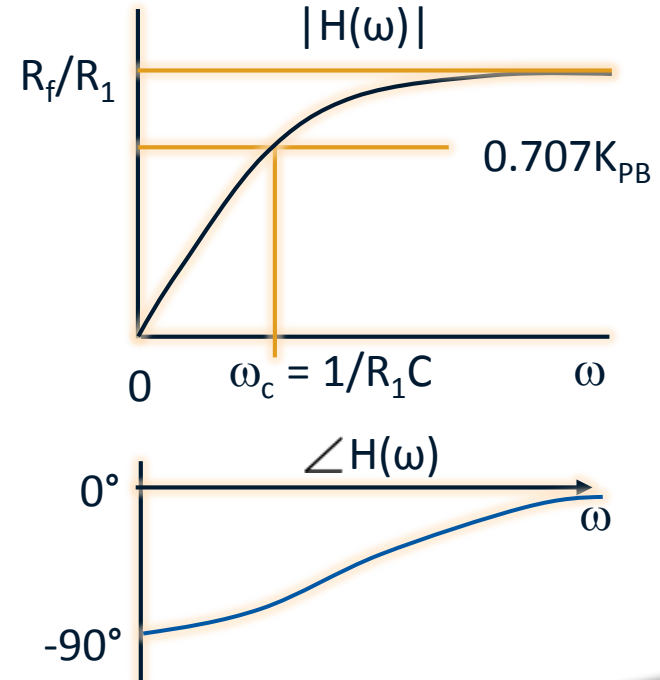
$$H(\omega) = \frac{-R_f C j \omega}{(R_1 C j \omega + 1)}$$

$$|H(\omega)| = \frac{R_f C \omega}{\sqrt{(R_1 C \omega)^2 + 1}}$$

$$\angle H(\omega) = -90^\circ - \arctan(R_1 C \omega)$$

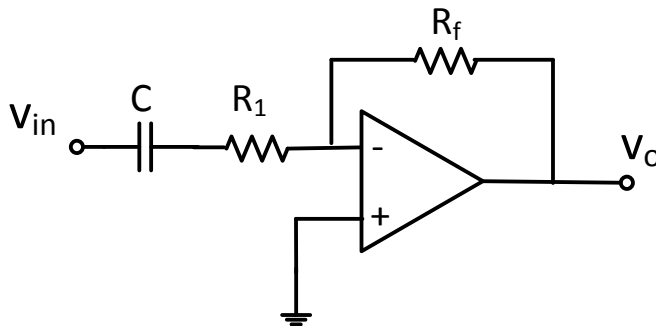
$$\text{Passband Gain } (\omega \rightarrow \infty) = -\frac{R_f}{R_1}$$

$$\text{Corner Freq., } \omega_c = \frac{1}{R_1 C}$$



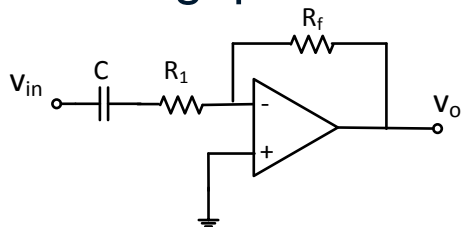
Example

Design a highpass filter to have a passband gain of 2 and a corner frequency of 1k rad/s:



Summary

- A **highpass filter** passes high frequency components in signals and attenuates low frequency components
- First-order highpass filter



$$H(\omega) = \frac{-R_f C j \omega}{(R_1 C j \omega + 1)}$$


- Design based on
 - Corner frequency of the passband, ω_c
 - Passband gain, K_{PB}



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Cascaded First-Order Filters

Introduce cascaded first-order op-amp filters



Previous Lesson

- Introduced op-amp first-order highpass filters

Lesson Objectives

- Introduce cascaded filters
- Introduce bandpass filter characteristics

Transfer Functions in Hertz f

Lowpass

$$H(\omega) = K_{DC} \frac{1}{\tau j\omega + 1}$$

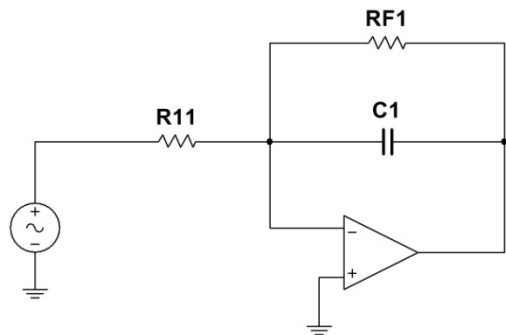
Bandwidth, $\omega_B = 1/\tau$
DC Gain = $H(0) = K_{DC}$

Highpass

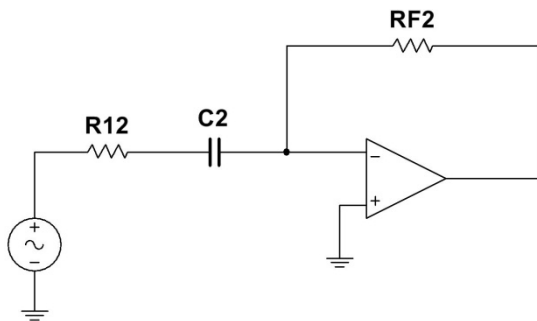
$$H(\omega) = \frac{Kj\omega}{\tau j\omega + 1}$$

Corner Frequency, $\omega_c = 1/\tau$
Passband Gain = $K_{PB} = K/\tau$

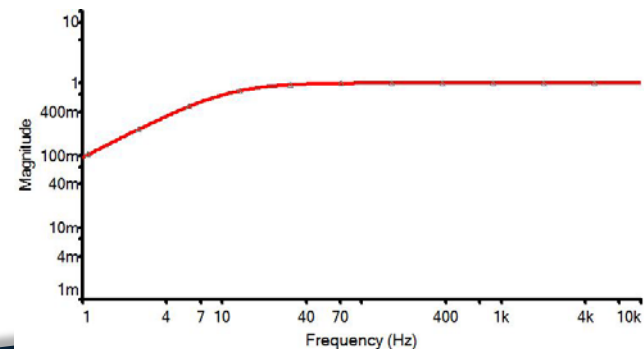
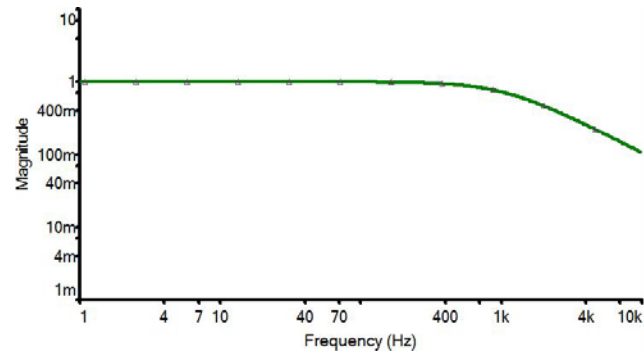
First-Order LPF and HPF



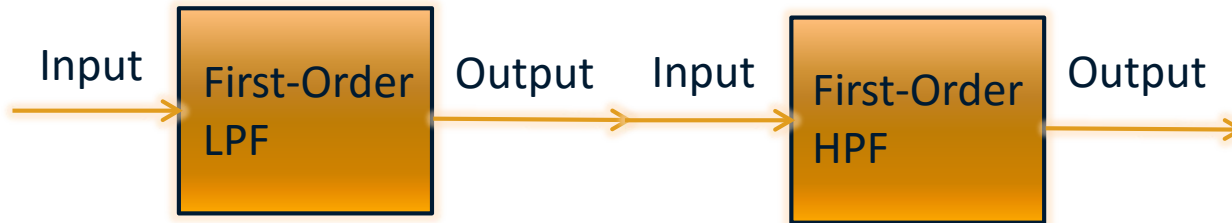
$$H_{LP}(f) = K_{DC} \frac{1}{\frac{jf}{f_0} + 1}$$



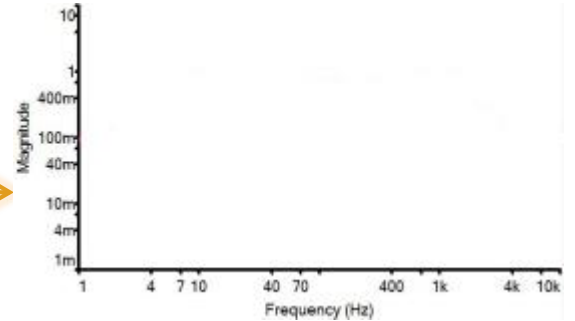
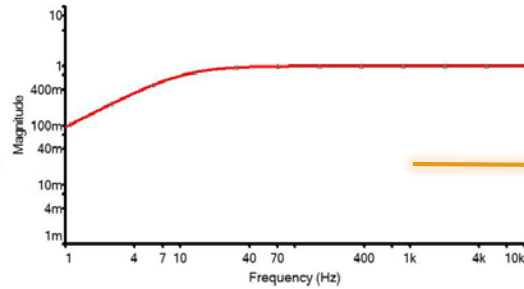
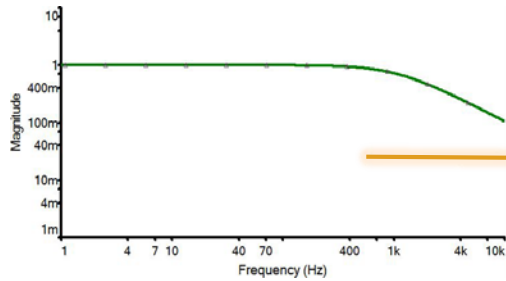
$$H_{HP}(f) = K_{PB} \frac{\frac{jf}{f_0}}{\frac{jf}{f_0} + 1}$$



Cascaded Filter



Cascaded Filter



Bandpass Filter Characteristics



Cascaded Filter Transfer Function

$$H_{BP}(f) = H_{LP}(f)H_{HP}(f) = K_{DC} \frac{1}{\frac{jf}{f_{lp}} + 1} K_{PB} \frac{\frac{jf}{f_{hp}}}{\frac{jf}{f_{hp}} + 1}$$

$$K = K_{DC}K_{PB} \left(\frac{f_{lp}}{f_{lp} + f_{hp}} \right)$$

$$f_0 = \sqrt{f_{lp}f_{hp}}$$

$$Q = \frac{\sqrt{f_{lp}f_{hp}}}{f_{lp} + f_{hp}}$$

$$BW = f_{lp} + f_{hp}$$

Summary

- Cascaded Lowpass and Highpass Filters
- Bandpass Filter Characteristics

Next Lesson


- Second-Order Transfer Functions



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Second-Order Transfer Functions

Introduce second-order filter transfer functions



Previous Lesson

- Introduced cascaded first-order op-amp filters

Lesson Objectives

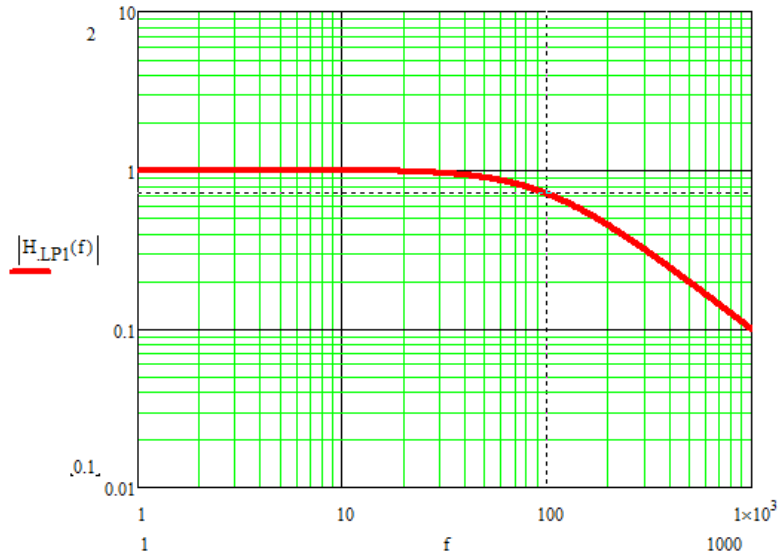
- Introduce second-order filter transfer functions
- Examine features of transfer functions

Filter Transfer Function

- Ratio of output voltage to input voltage as a function of frequency
- For any frequency, the transfer function is a complex number that indicates how the filter modifies the magnitude and phase of the input to produce the output

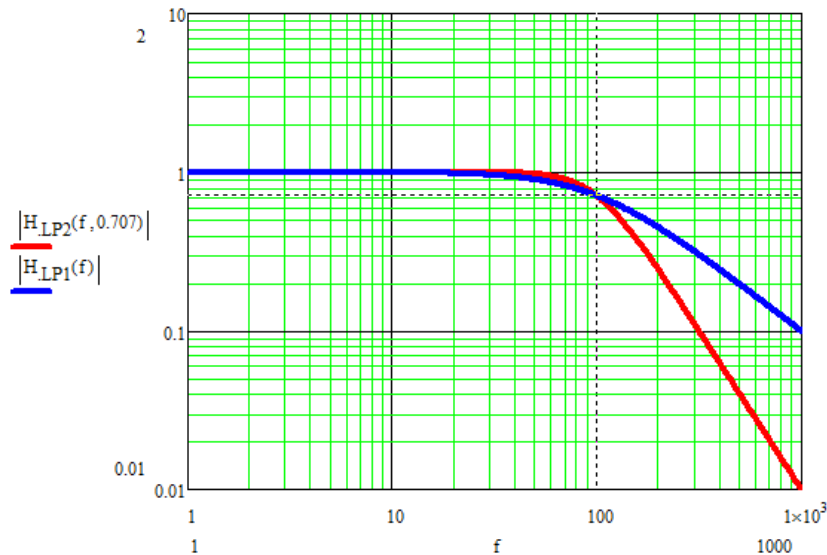
$$H(f) = \frac{V_{out}(f)}{V_{in}(f)}$$

First-Order Low-Pass Filter



$$H_{LP1}(f) = K \frac{1}{\frac{jf}{f_0} + 1}$$

Second-Order Low-Pass Filter

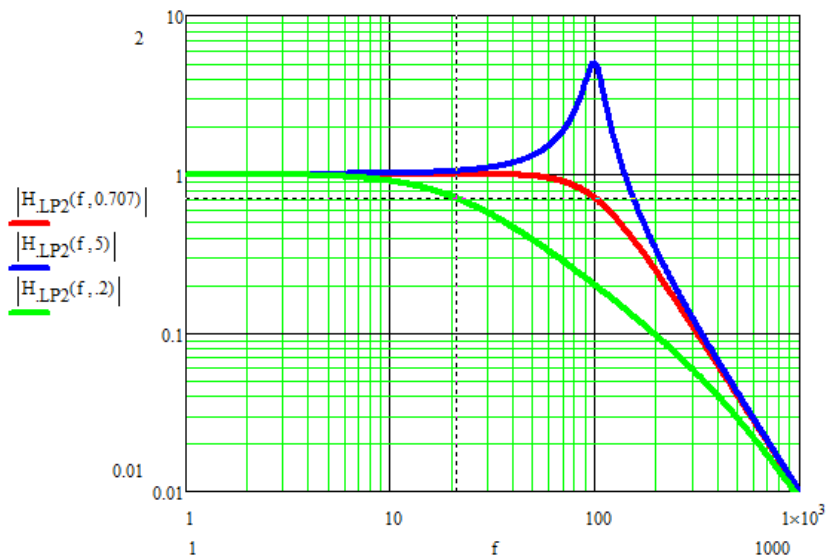


$$H_{LP1}(f) = K \frac{1}{\frac{jf}{f_0} + 1}$$

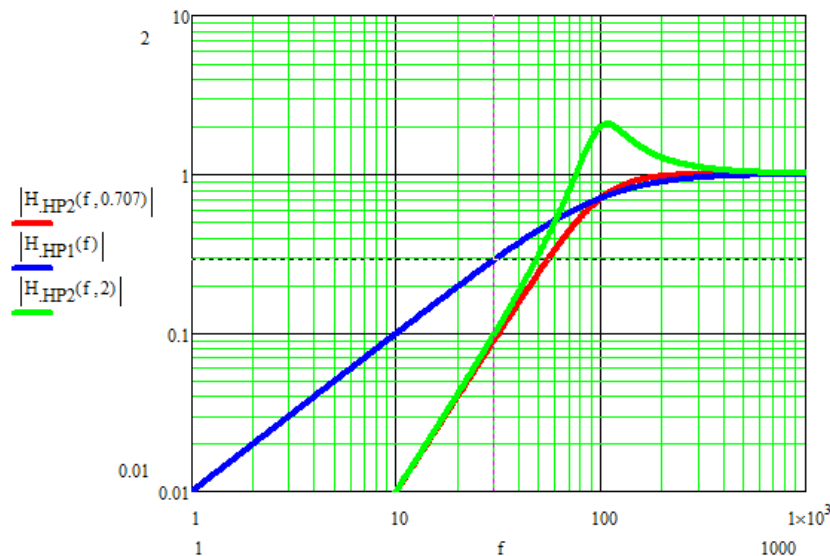
$$H_{LP2}(f) = K \frac{1}{\left(\frac{jf}{f_0}\right)^2 + \frac{jf}{f_0} \frac{1}{Q} + 1}$$

Effect of Quality Factor (Q)

$$H_{LP2}(f) = K \frac{1}{\left(\frac{jf}{f_0}\right)^2 + \frac{jf}{f_0} \frac{1}{Q} + 1}$$



High-Pass Filters



$$H_{HP1}(f) = K \frac{j\frac{f}{f_0}}{j\frac{f}{f_0} + 1}$$

$$H_{LP1}(f) = K \frac{1}{j\frac{f}{f_0} + 1}$$

$$H_{HP2}(f) = K \frac{\left(j\frac{f}{f_0}\right)^2}{\left(j\frac{f}{f_0}\right)^2 + j\frac{f}{f_0} \frac{1}{Q} + 1}$$

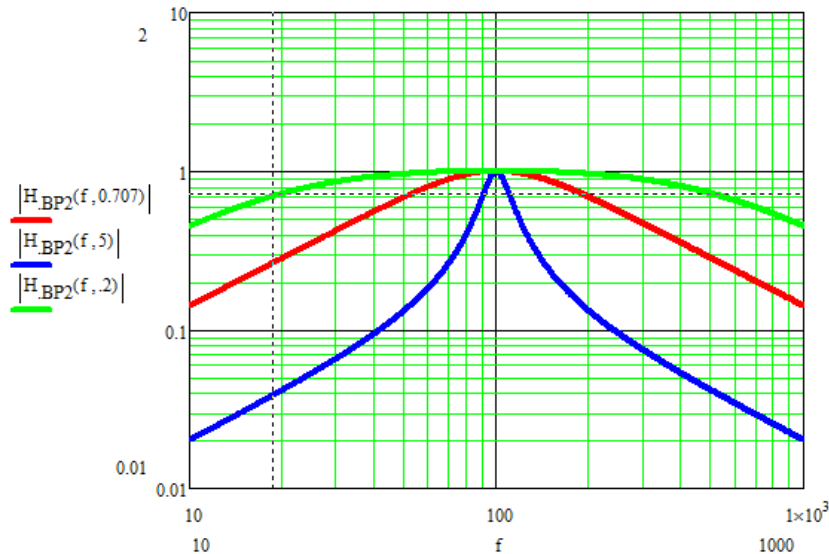
$$H_{LP2}(f) = K \frac{1}{\left(j\frac{f}{f_0}\right)^2 + j\frac{f}{f_0} \frac{1}{Q} + 1}$$

Band-Pass Filters

$$H_{BP2}(f) = K \frac{jf \frac{1}{Q}}{\left(\frac{jf}{f_0}\right)^2 + jf \frac{1}{Q} + 1}$$

$$Q = \frac{f_0}{BW}$$

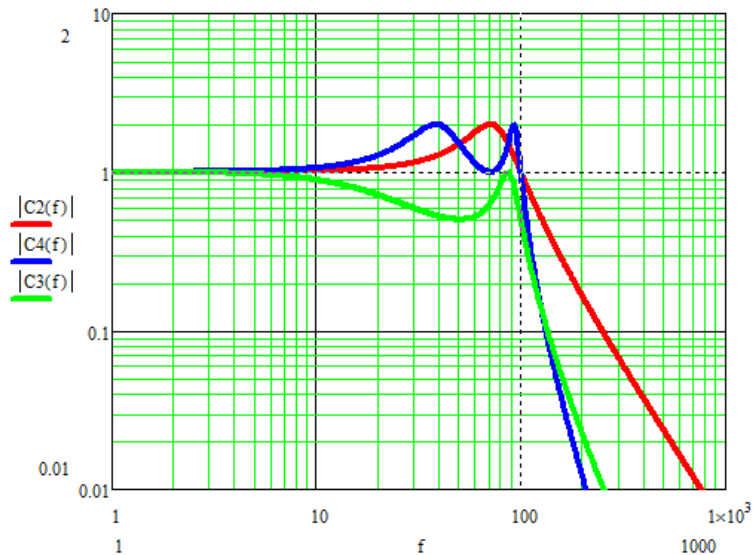
$$BW = \text{Bandwidth} = f_u - f_l$$



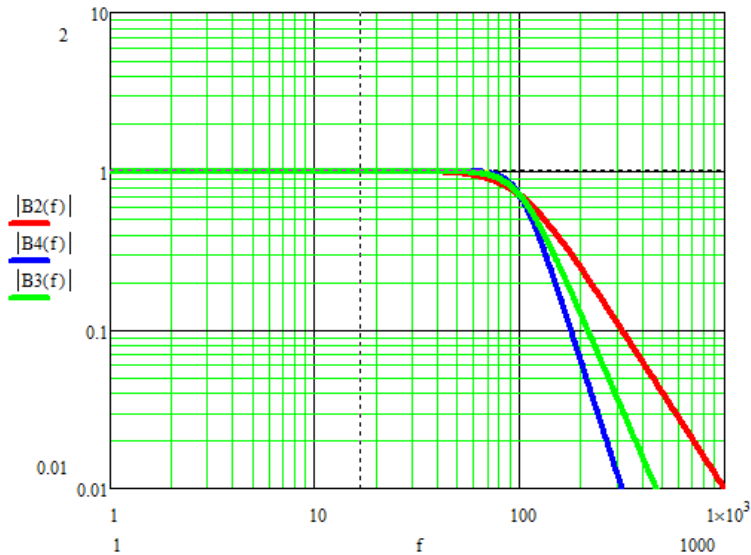
Butterworth and Chebyshev

- Types of transfer functions
- For second-order filters, the type is determined by the Q value
- $Q = 1/\sqrt{2}$ Butterworth (Maximally Flat)
- $Q > 1/\sqrt{2}$ Chebyshev

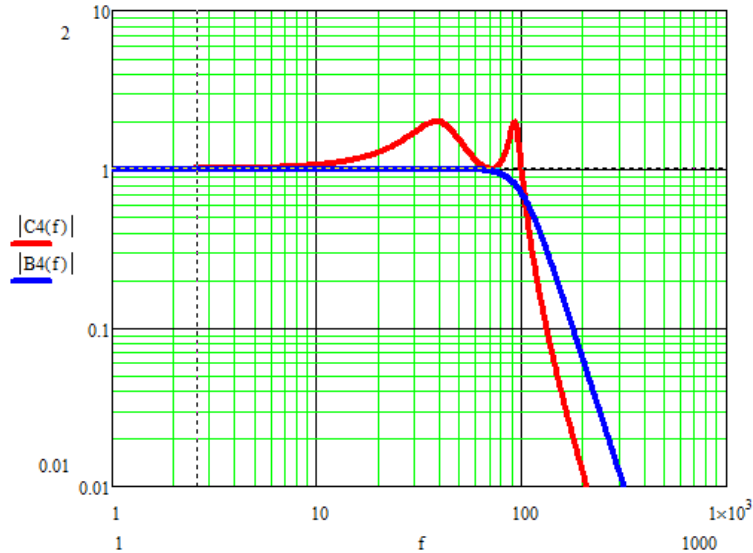
Chebyshev Filters



Butterworth Filters



Fourth-Order Butterworth vs. Chebyshev



Summary

- Introduced second-order transfer functions
- Examined features of transfer functions

Next Lesson


- Op-Amp Second-Order Filter Circuits



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Second-Order Filter Circuits

Introduce second-order Sallen-Key filter circuits



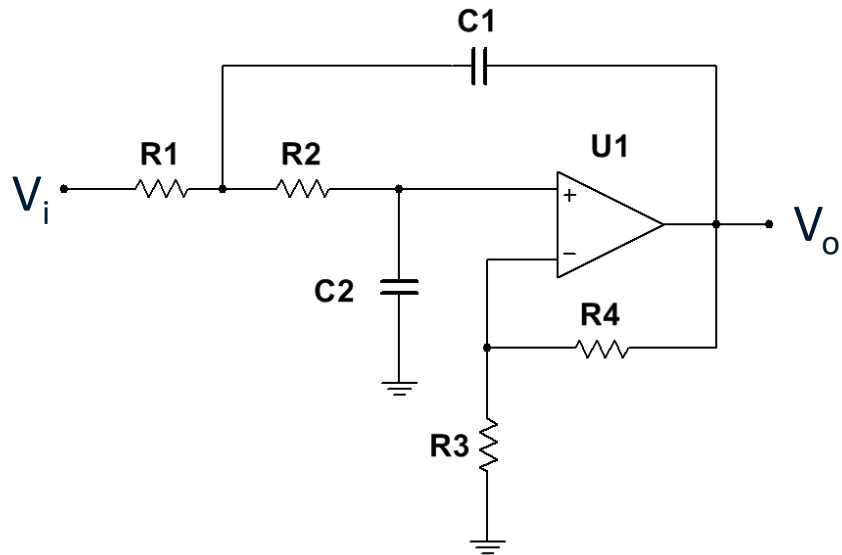
Previous Lesson

- Introduced second-order transfer functions

Lesson Objectives

- Introduce second-order filter circuits
- Design second-order filters

Sallen-Key Low-Pass Filter



$$H_{LP2}(f) = K \frac{1}{\left(\frac{jf}{f_0}\right)^2 + \frac{jf}{f_0} \frac{1}{Q} + 1}$$

$$K = 1 + \frac{R_4}{R_3}$$

$$f_0 = \frac{1}{2\pi\sqrt{R_1 R_2 C_1 C_2}}$$

$$Q = \frac{\sqrt{R_1 R_2 C_1 C_2}}{(1 - K)R_1 C_1 + (R_1 + R_2)C_2}$$

Lowpass Design Equations

Special Case 1
($K = 1$, Solve for C's)

$$K = 1 \quad (R_3 = \infty, R_4 = 0)$$

$$C_1 = \frac{Q}{\omega_o} \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$$

$$C_2 = \frac{1}{Q\omega_o (R_1 + R_2)}$$

Can simplify with $R_1 = R_2$

Special Case 2
($K = 1$, Solve for R's)

$$K = 1 \quad (R_3 = \infty, R_4 = 0)$$

$$R_1, R_2 = \frac{1}{2Q\omega_o C_2} \left(1 \pm \sqrt{1 - 4Q^2 \frac{C_2}{C_1}} \right)$$

$$4Q^2 C_2 / C_1 \leq 1$$

R_1 and R_2 are interchangeable

Special Case 3
(R's equal and C's equal)

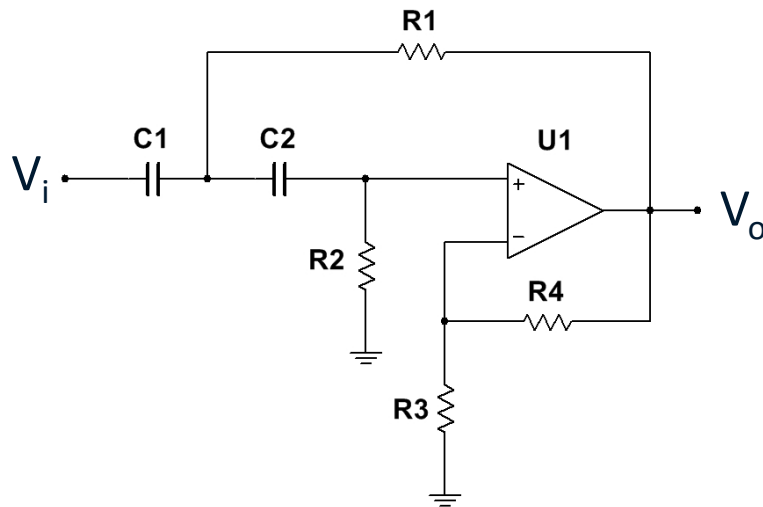
$$R_1 = R_2 = R \quad K = 1 + \frac{R_4}{R_3}$$

$$C_1 = C_2 = C$$

$$K = 3 - \frac{1}{Q}$$

$$R = \frac{1}{\omega_o C}$$

Sallen-Key Highpass Filter



$$H_{HP2}(f) = K \frac{\left(\frac{jf}{f_0}\right)^2}{\left(\frac{jf}{f_0}\right)^2 + \frac{jf}{f_0} \frac{1}{Q} + 1}$$

$$K = 1 + \frac{R_4}{R_3}$$

$$f_0 = \frac{1}{2\pi\sqrt{R_1 R_2 C_1 C_2}}$$

$$Q = \frac{\sqrt{R_1 R_2 C_1 C_2}}{(1 - K)R_2 C_2 + (C_1 + C_2)R_1}$$

Highpass Design Equations

Special Case 1

($K = 1, C_1 = C_2 = C$)

$$K = 1 \quad (R_3 = \infty, R_4 = 0)$$

$$C_1 = C_2 = C$$

$$R_1 = \frac{1}{2Q\omega_o C}$$

$$R_2 = \frac{2Q}{\omega_o C}$$

Special Case 2

(R 's equal and C 's equal)

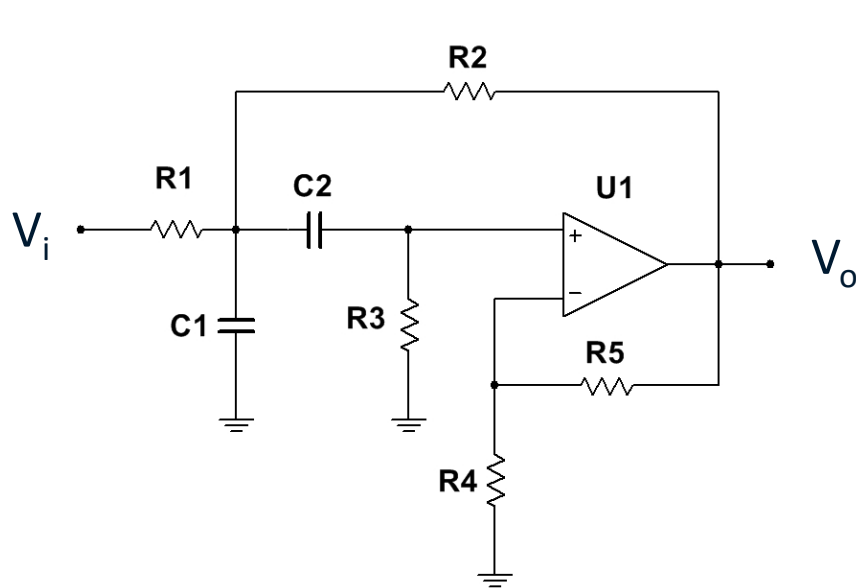
$$R_1 = R_2 = R \quad K = 1 + \frac{R_4}{R_3}$$

$$C_1 = C_2 = C$$

$$K = 3 - \frac{1}{Q}$$

$$C = \frac{1}{\omega_o R}$$

Sallen-Key Bandpass Filter



$$H_{BP2}(f) = K \frac{\frac{jf}{f_0} \frac{1}{Q}}{\left(\frac{jf}{f_0}\right)^2 + \frac{jf}{f_0} \frac{1}{Q} + 1}$$

$$K = \frac{R_2}{R_1 + R_2} \frac{K_0 R_3 C_2}{(R_1 \parallel R_2)(C_1 + C_2) + R_3 C_2 \left[1 - \frac{K_0 R_1}{(R_1 + R_2)}\right]}$$

$$f_0 = \frac{1}{2\pi \sqrt{(R_1 \parallel R_2) R_3 C_1 C_2}}$$

$$Q = \frac{\sqrt{(R_1 \parallel R_2) R_3 C_1 C_2}}{(R_1 \parallel R_2)(C_1 + C_2) + R_3 C_2 \left[1 - \frac{K_0 R_1}{(R_1 + R_2)}\right]}$$

$$K_0 = 1 + \frac{R_5}{R_4}$$

Bandpass Design Equations

Special Case

(R's equal and C's equal)

$$R_1 = R_2 = R_3 = R \quad K_0 = 1 + \frac{R_5}{R_4}$$

$$C_1 = C_2 = C$$

$$R = \frac{\sqrt{2}}{2\pi f_0 C}$$

$$K_0 = 4 - \frac{1}{Q^2}$$

$$K = 4Q^2 - 1$$

Notch Filters

$$H_{BR2}(f) = K \frac{\left(\frac{jf}{f_0}\right)^2 + 1}{\left(\frac{jf}{f_0}\right)^2 + \frac{jf}{f_0} \frac{1}{Q} + 1}$$

Summary

- Introduced second-order filter circuits

Next Lesson

- Filter Design Example



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Lowpass Filter Design Example

Design a second-order Sallen-Key lowpass filter circuit



Example Design

⦿ Butterworth 2nd Order LPF

Special Case 1

(K = 1, Solve for C's)

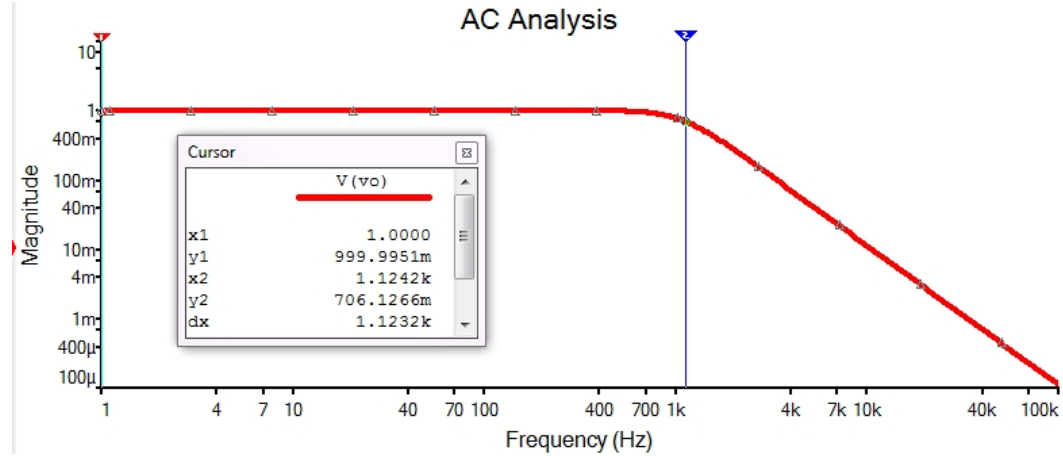
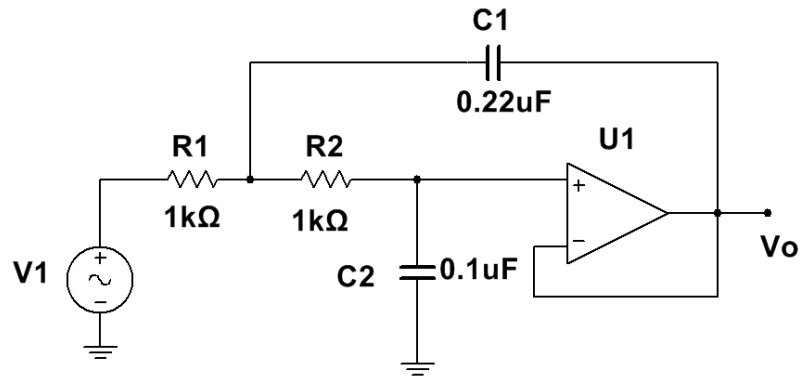
$$K = 1 \quad (R_3 = \infty, R_4 = 0)$$

$$C_1 = \frac{Q}{\omega_o} \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$$

$$C_2 = \frac{1}{Q\omega_o (R_1 + R_2)}$$

Can simplify with $R_1 = R_2$

Example Design



Summary

- Designed a second-order lowpass filter

Next Lesson

- Filter Demonstration



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A large, ornate building with a prominent copper-roofed steeple. The word "TECH" is displayed in large, white, three-dimensional block letters on the building's facade.

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Filtering Demonstration

Demonstrate filtering of signals



Previous Lesson

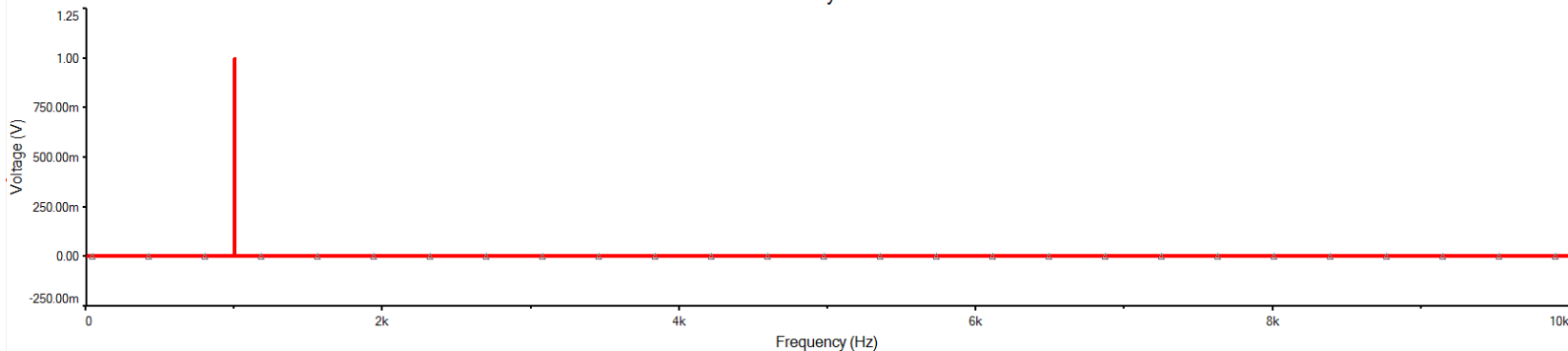
- Introduced second-order filter circuits

Lesson Objectives

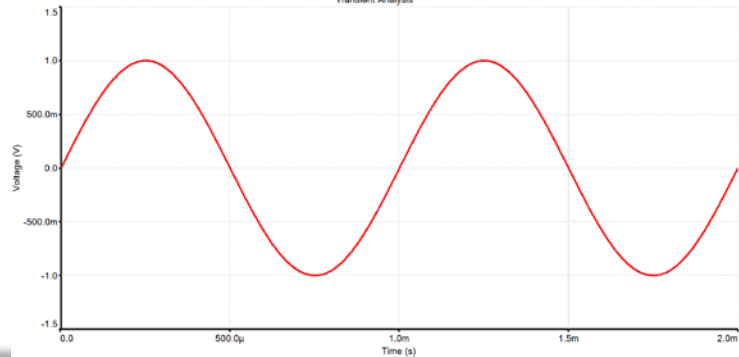
- Examine frequency spectra of signals
- Demonstrate filtering by a second-order filter circuit

Spectrum of Sine Wave

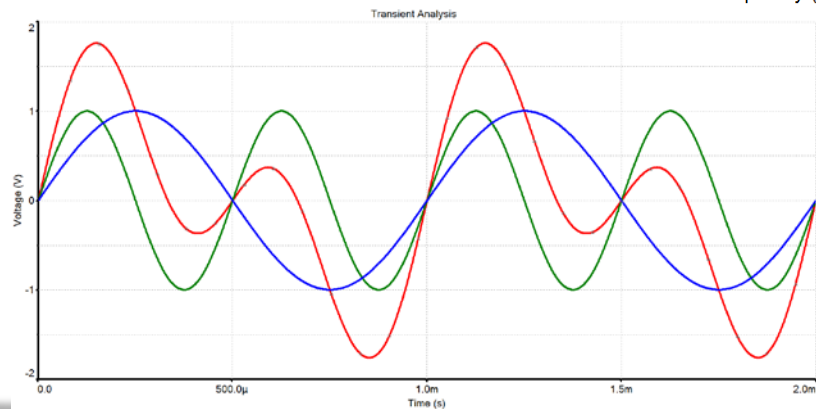
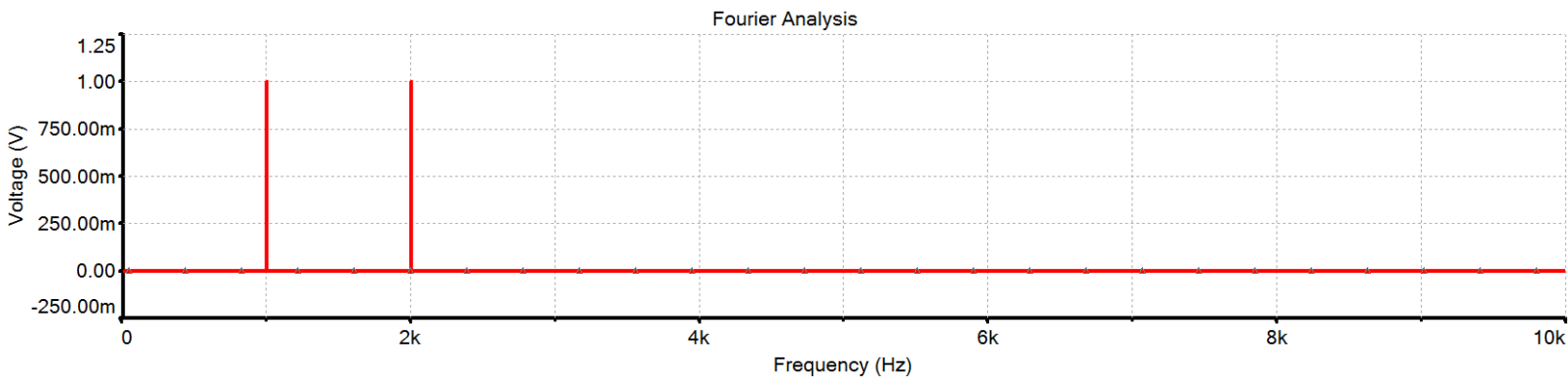
Fourier Analysis



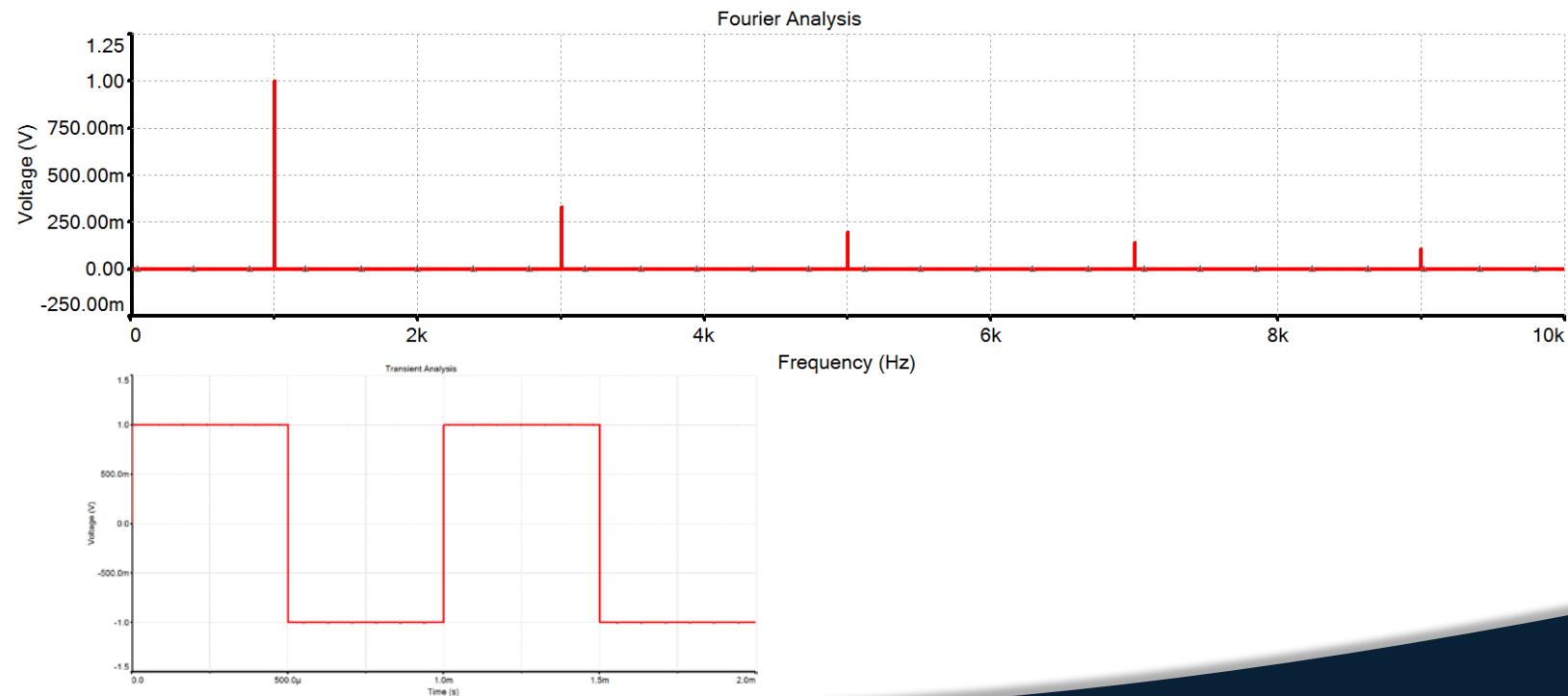
Transient Analysis



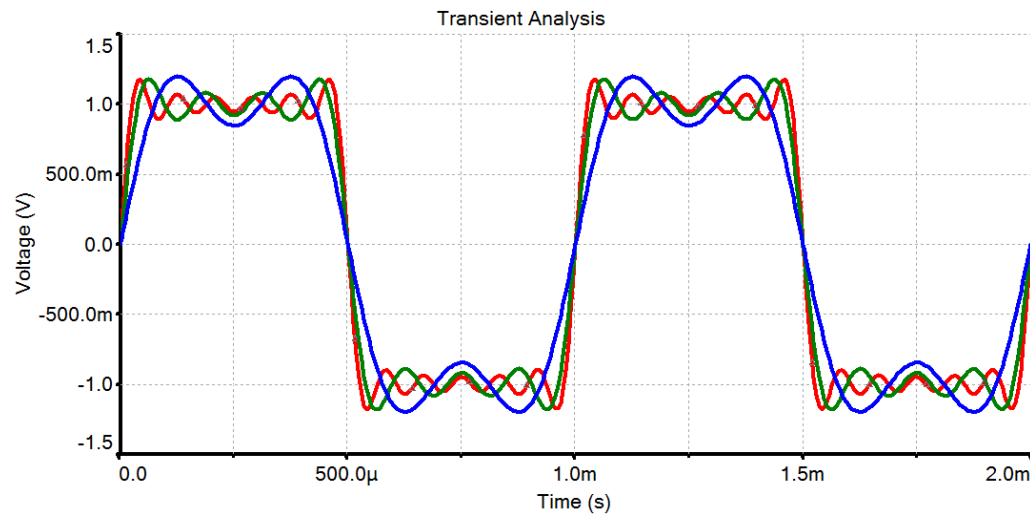
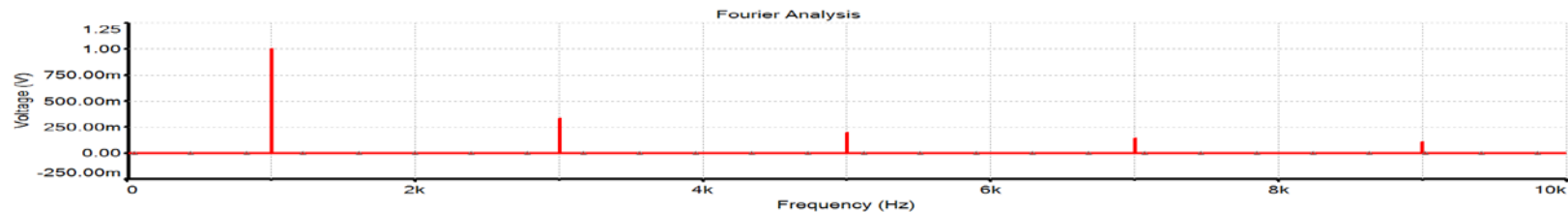
Spectrum of Sum of Two Sine Waves



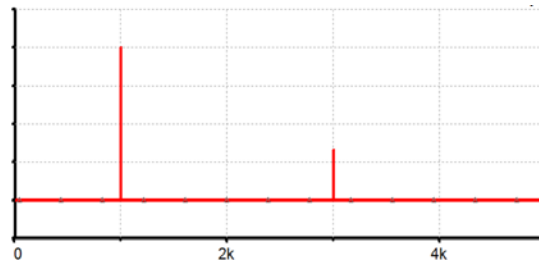
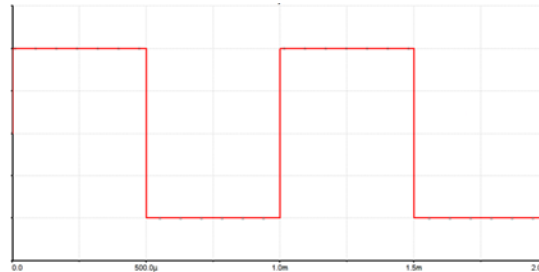
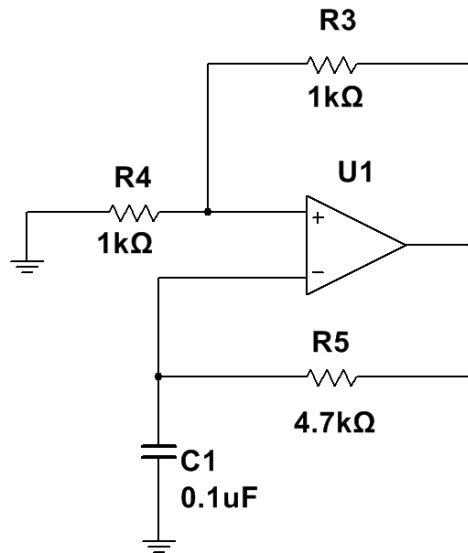
Spectrum of Square Wave



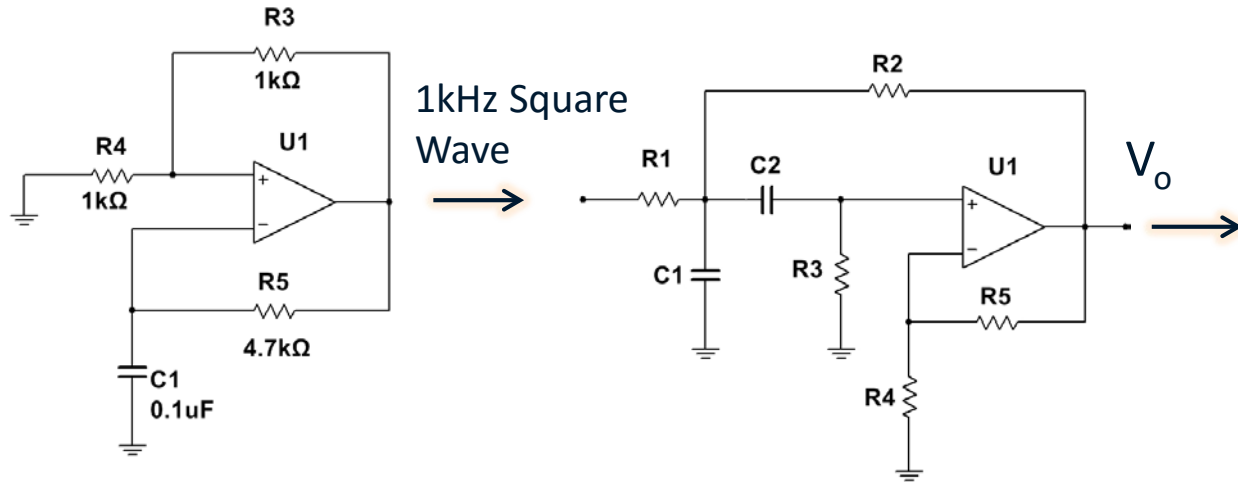
Spectrum of Square Wave



Relaxation Oscillator



Measurements

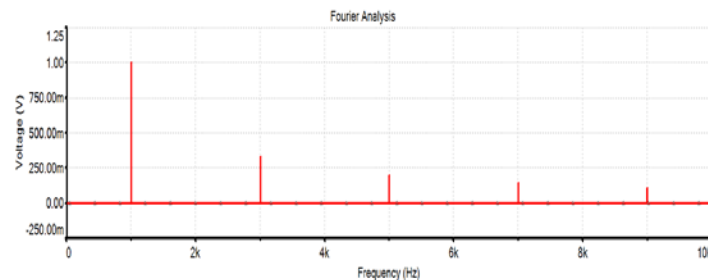
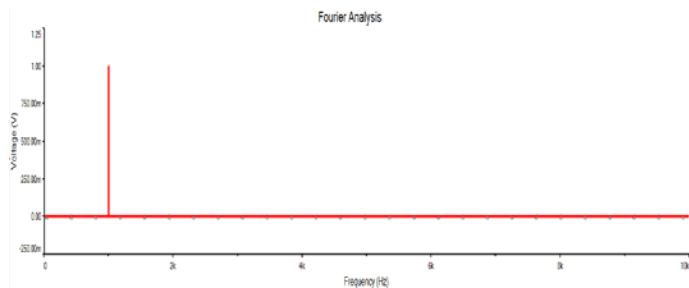


Relaxation Oscillator

$f_0 = 1\text{kHz}$ $Q=5$ Sallen-Key BPF

Total Harmonic Distortion (THD)

$$THD = \frac{\sqrt{v_2^2 + v_3^2 + v_4^2 + \dots}}{v_1} * 100\%$$



Summary

- Introduced frequency spectra
- Examined physical circuit filtering performance