SIMULATION METHODS FOR STOCHASTIC SYSTEMS

COINS AND BOOTSTRAPS-PROJECT REPORT

Harish Settikere Prabhakara

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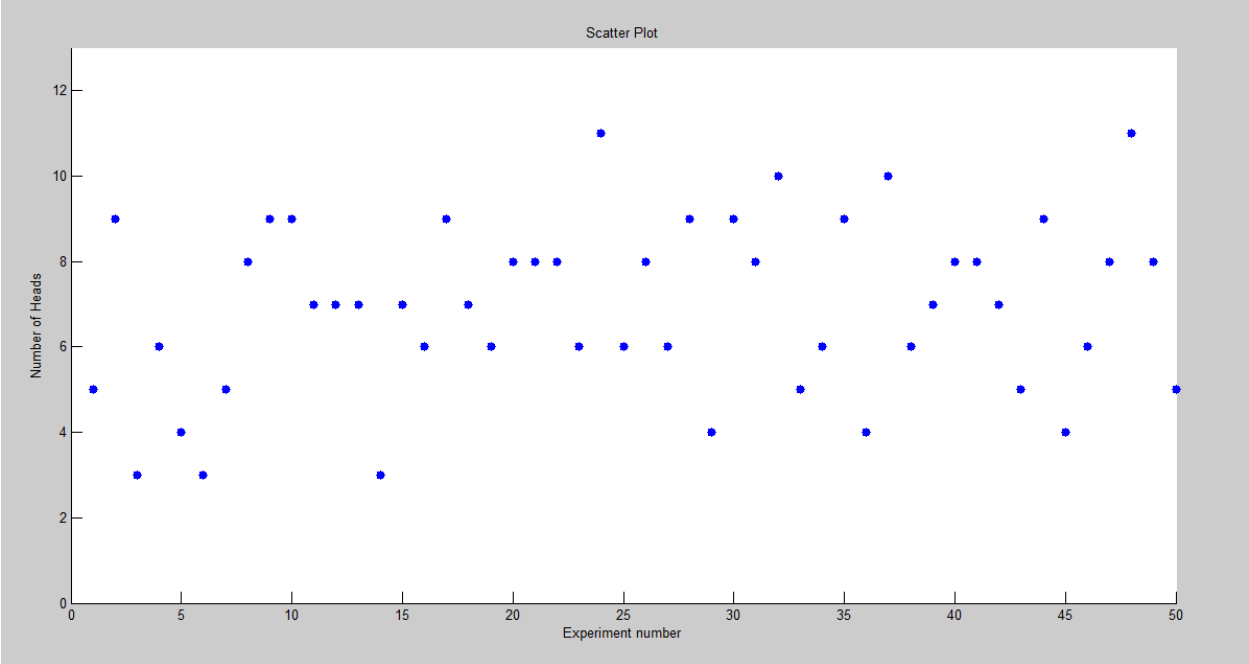
SUMMARY

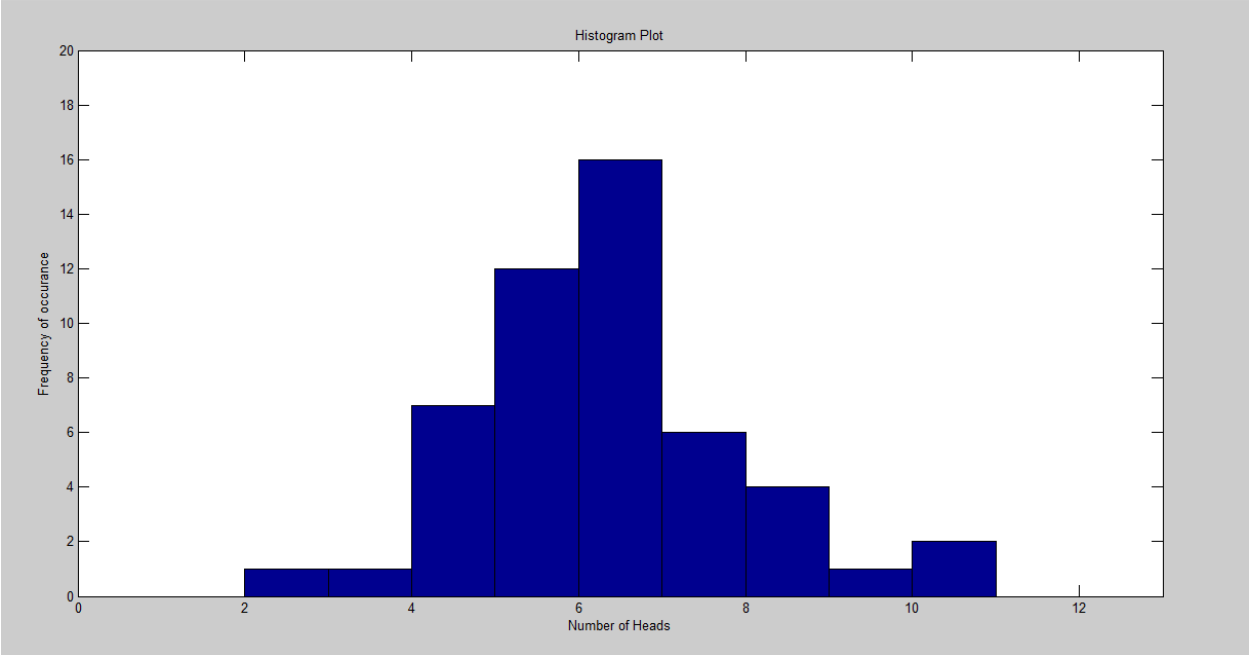
Question-1 [Coins, Coins, everywhere…]

- Initially, a MATLAB function **"rand"** is used to generate a set of {Number of heads obtained by flipping a coin 13 times and repeating the experiment for 50 times}, by defining rules so that the coin becomes **Fair**. (**The coin is made fair by making the probability of getting tails equal to the probability of getting heads** i.e. if the randomly generated number between 0 and 1 (generated using "rand" function) is greater than **0.5**, then its rounded off to 1(heads) or if it is less than **0.5**, its rounded off to 0(tails)).).
- A scatter plot of the outcomes of these experiments is plotted using the MATLAB function **"scatter"**.
- A Histogram plot of the same outcomes of the experiments is plotted using the MATLAB function "**hist**".
- A running tally (Cumulative sum) of {(Total number of Heads)/ (Total number of coin flips)} is plotted using the MATLAB function **"plot".**
- We obtain the following graphs by following the above steps:

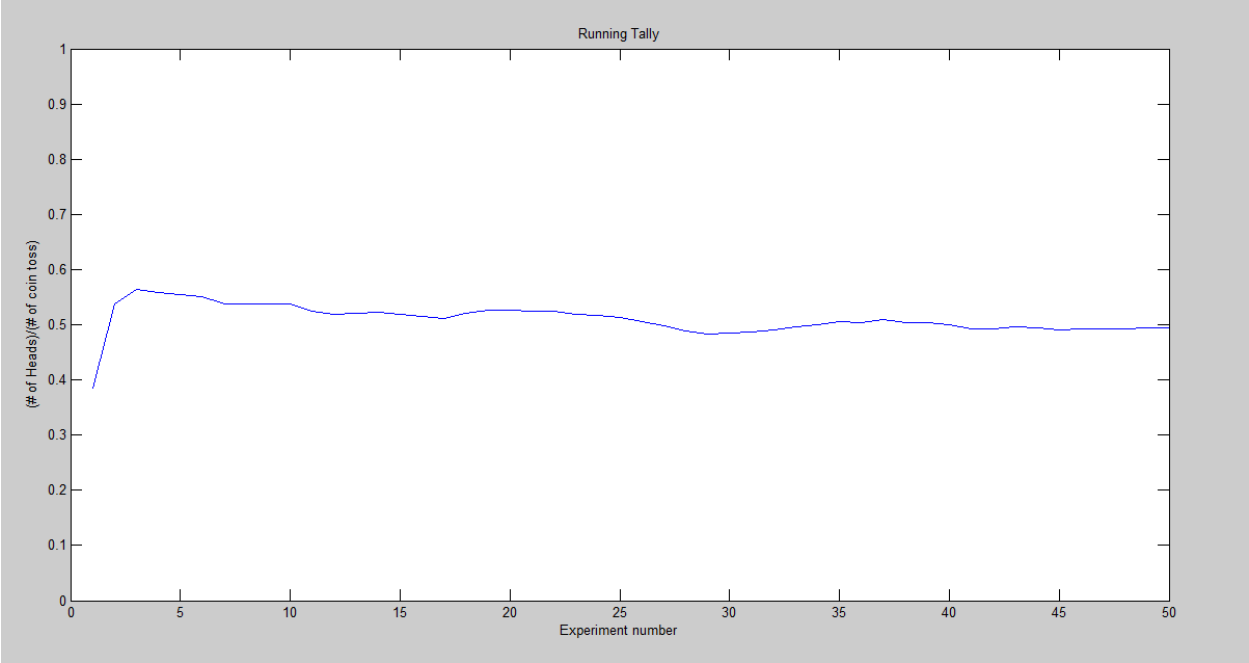
Scatter Plot



Histogram Plot



Running Tally

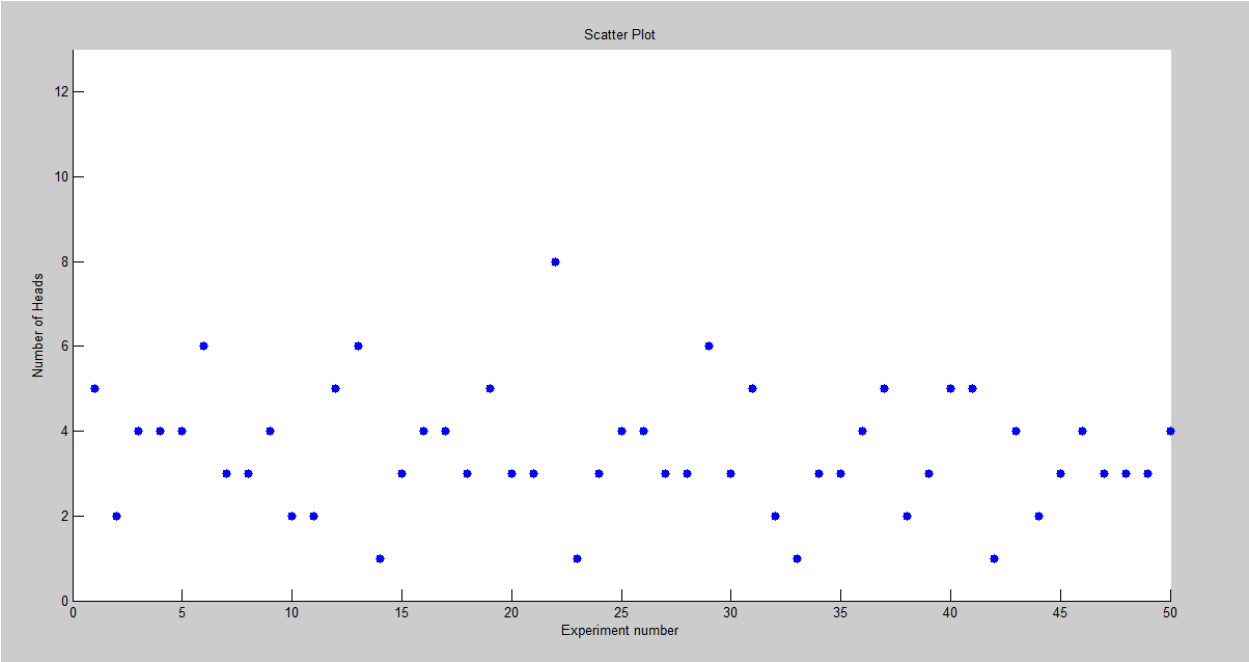


- From the above graphs, we observe that the probability of number of heads is almost equal to half in most of the experiments, which means that the Probability {Number of heads} is almost equal to Probability {Number of tails}. Therefore, based on visual inspection of graphs, we conclude that the coin is FAIR.

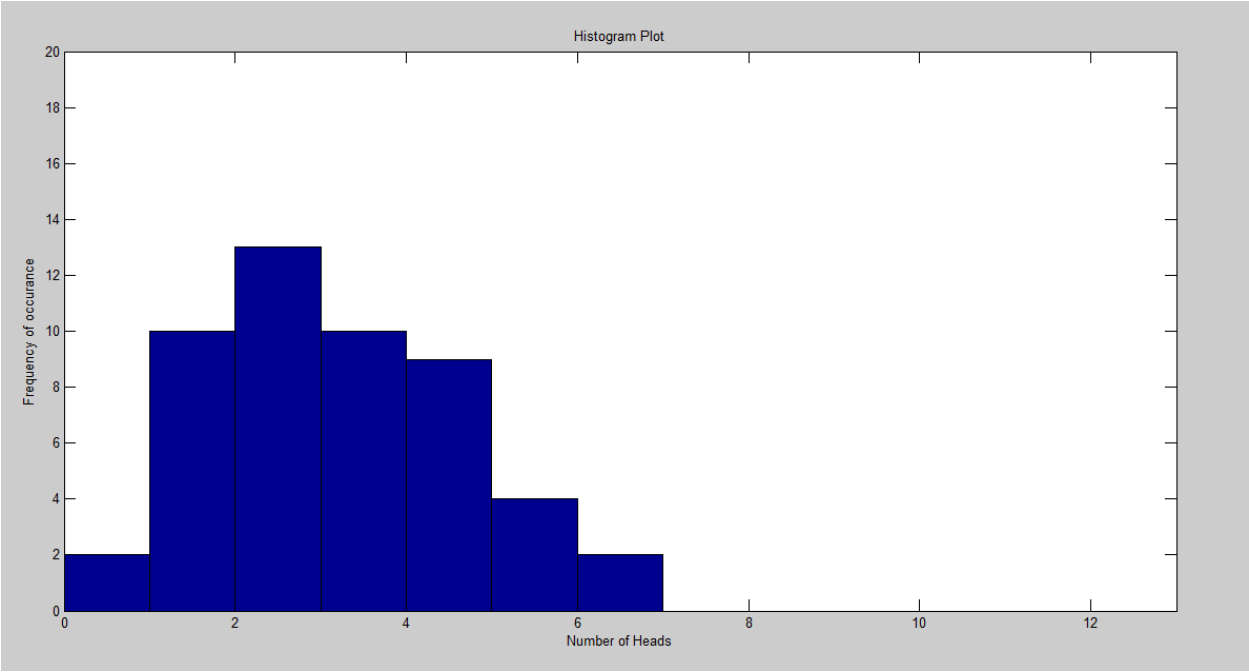
Question-2 [So Unfair]

- Initially, a MATLAB function **"rand"** is used to generate a set of {Number of heads obtained by flipping a coin 13 times and repeating the experiment for 50 times}, by defining rules so that the coin becomes **Unfair** (**The coin is made Unfair by making the probability of getting tails greater than the probability of getting heads** i.e. if the randomly generated number between 0 and 1 (generated using "rand" function) is greater than **0.7**, then its rounded off to 1(heads) otherwise its rounded off to 0(tails)).
- A scatter plot of the outcomes of these experiments is plotted using the MATLAB function **"scatter".**
- A Histogram plot of the same outcomes of the experiments is plotted using the MATLAB function **"hist"**.
- A running tally (Cumulative sum) of {(Total number of Heads)/ (Total number of coin flips)} is plotted using the MATLAB function **"plot".**
- We obtain the following graphs by following the above steps:

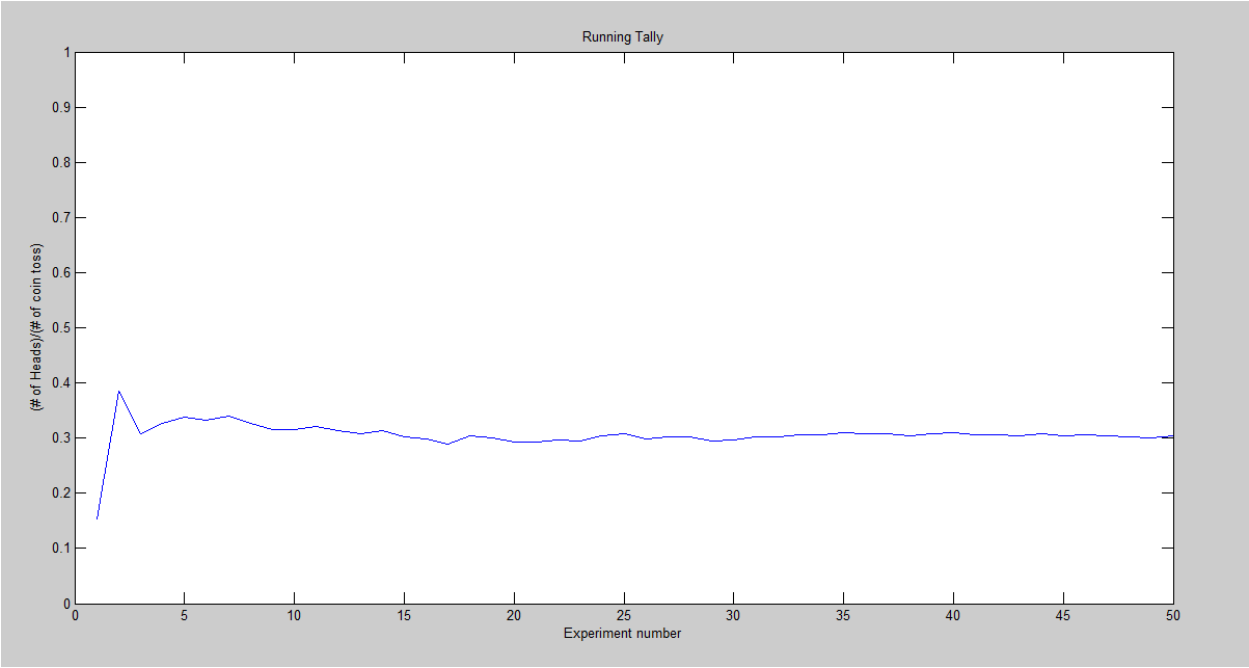
Scatter Plot



Histogram Plot



Running Tally



- The coin was made unfair by making the probability of heads less than the probability of tails. Due to the above defined rule for making a coin unfair, we can clearly observe from the graphs that the probability of heads in most of the experiments is less than half. Therefore, the above graphs clearly justify our claim that the coin is UNFAIR.

Question-3 [Gas Bootstrap Confidence Intervals]

- NJGAS data set, which has 12 values: 150, 367, 38, 12, 11, 134, 12, 251, 63, 8, 13, 107.
  is downloaded from Blackboard. The data set has
- The given data set is resampled with replacement for 1000 times using the MATLAB function **"datasample",** and the mean is calculated in each of these cases using the MATLAB function **"mean"**.
- Means of the dataset obtained are sorted in ascending order using the MATLAB function **"sort"**.
- 95% bootstrap confidence interval for the sorted means of the given data set is found by selecting the **2.5 percentile and 97.5 percentile** of the sorted mean values using the MATLAB function "**prctile**".
- We obtain the following results:

  2.5 percentile= 42.1667

  97.5 percentile= 162.5000

  95% Bootstrap confidence interval is given by: [42.1667, 162.5000].

  {Since the number of heads is obtained using random function (rand), we don't get the same values as above each time the code is run. The values are different, but close to the values quoted above.}

Question-4 [Unfair Confidence…]

- The dataset (number of heads) as in the case of Question-2(Unfair coin case) is generated in a similar manner by using **"rand"** function. This dataset is resampled with replacement for 1000 times using the MATLAB function **"datasample",** and the mean is calculated in each of these cases using the MATLAB function **"mean"**.
- Means of the dataset obtained are sorted in ascending order using the MATLAB function **"sort"**.
- 95% bootstrap confidence interval for the sorted means of the given data set is found by selecting the **2.5 percentile and 97.5 percentile** of the sorted mean values using the MATLAB function "prctile".
- We obtain the following results:

  2.5 percentile= 3.0833

  97.5 percentile= 4.7500

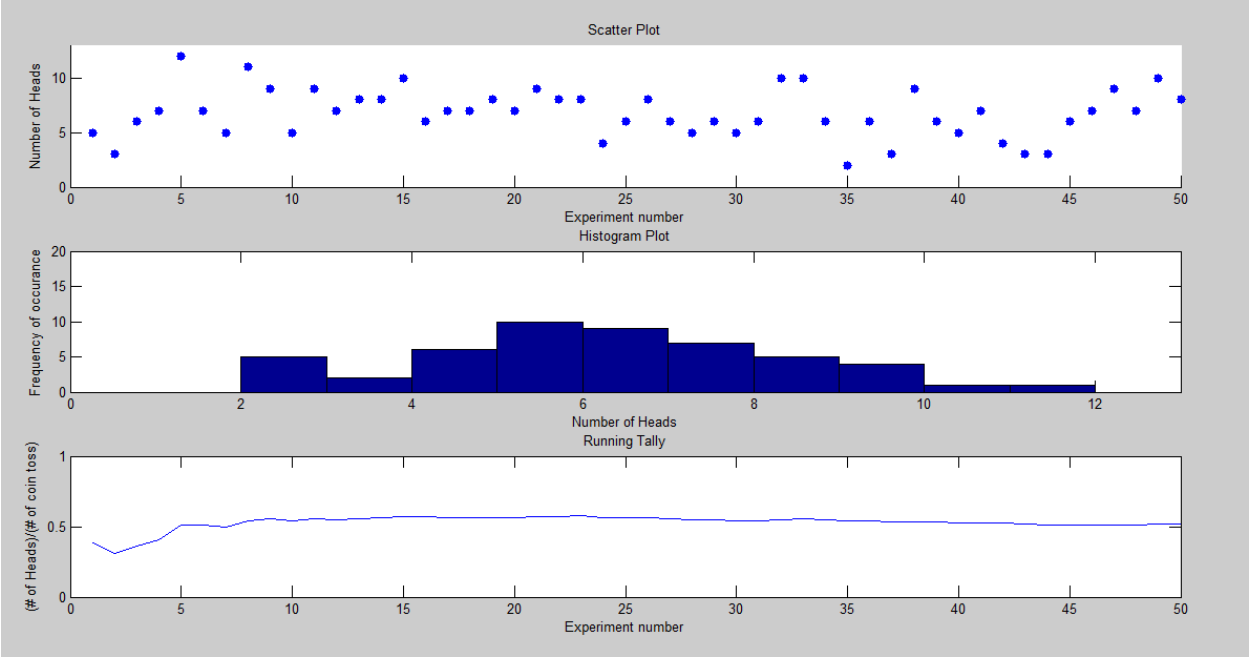
  95% Bootstrap confidence interval is given by: [3.0833, 4.7500].

  From the above values obtained for 95% Bootstrap confidence interval, our claim that the coin now is UNFAIR is justified.

  {Since the number of heads is obtained using random function (rand), we don't get the same values as above each time the code is run. The values are different, but close to the values quoted above.}

Question-1 [Coins]

Graphs



Question-2 [So Unfair]

Graphs