SIMULATION METHODS FOR STOCHASTIC SYSTEMS

MONTE CARLO (MC) AND MARKOV CHAIN MONTE CARLO (MCMC) SIMULATIONS -PROJECT REPORT

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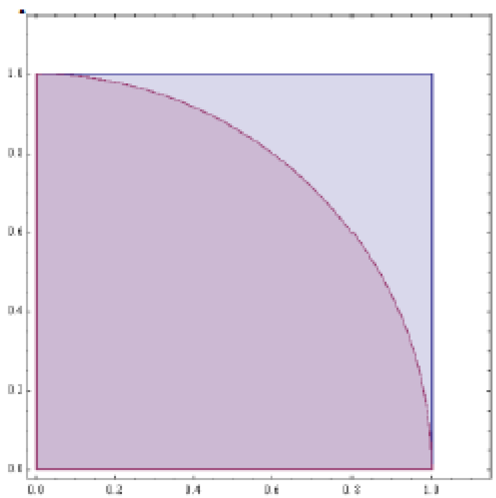
CONTENTS

SUMMARY

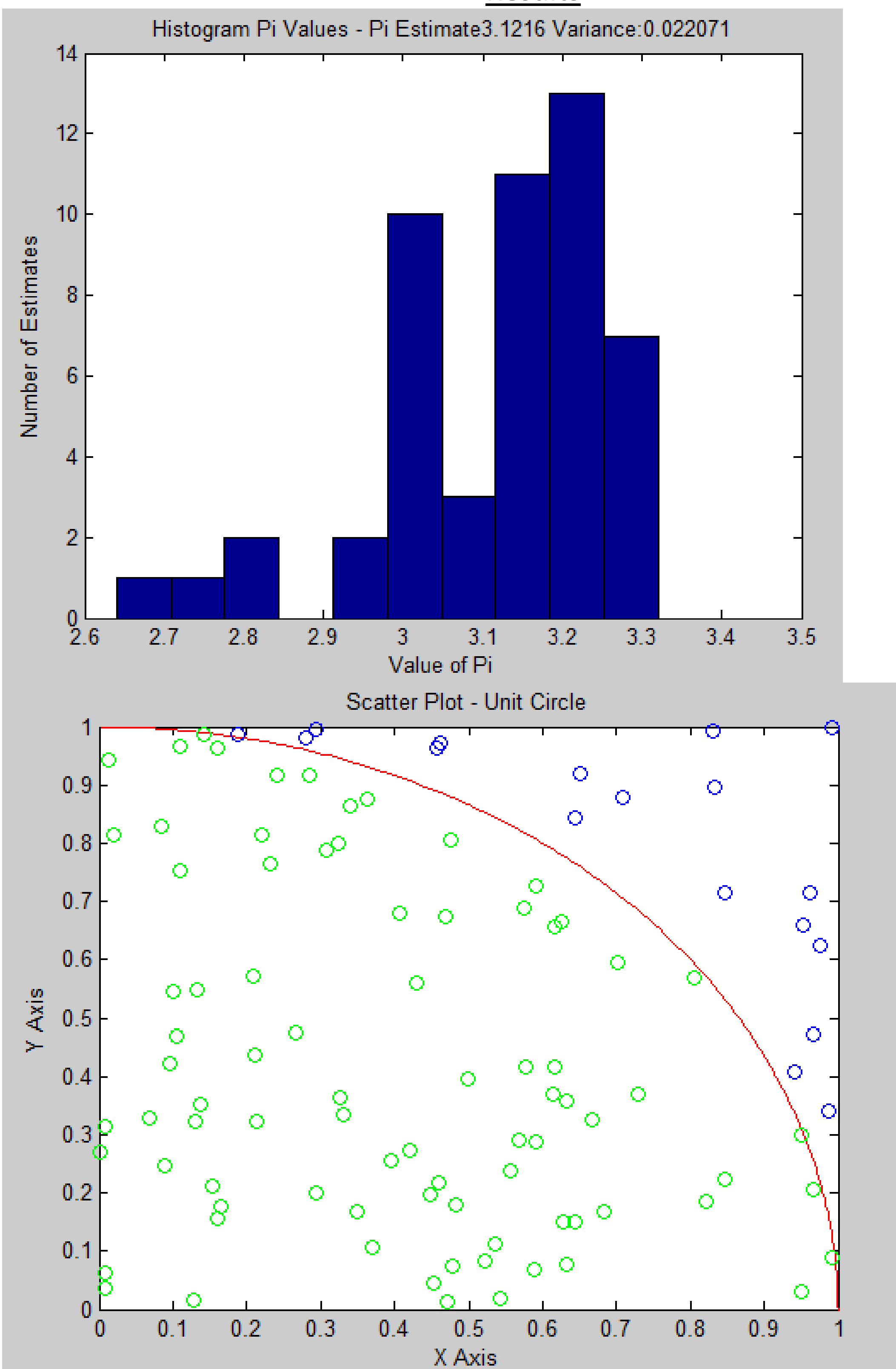
Problem 1 [Monte Carlo]

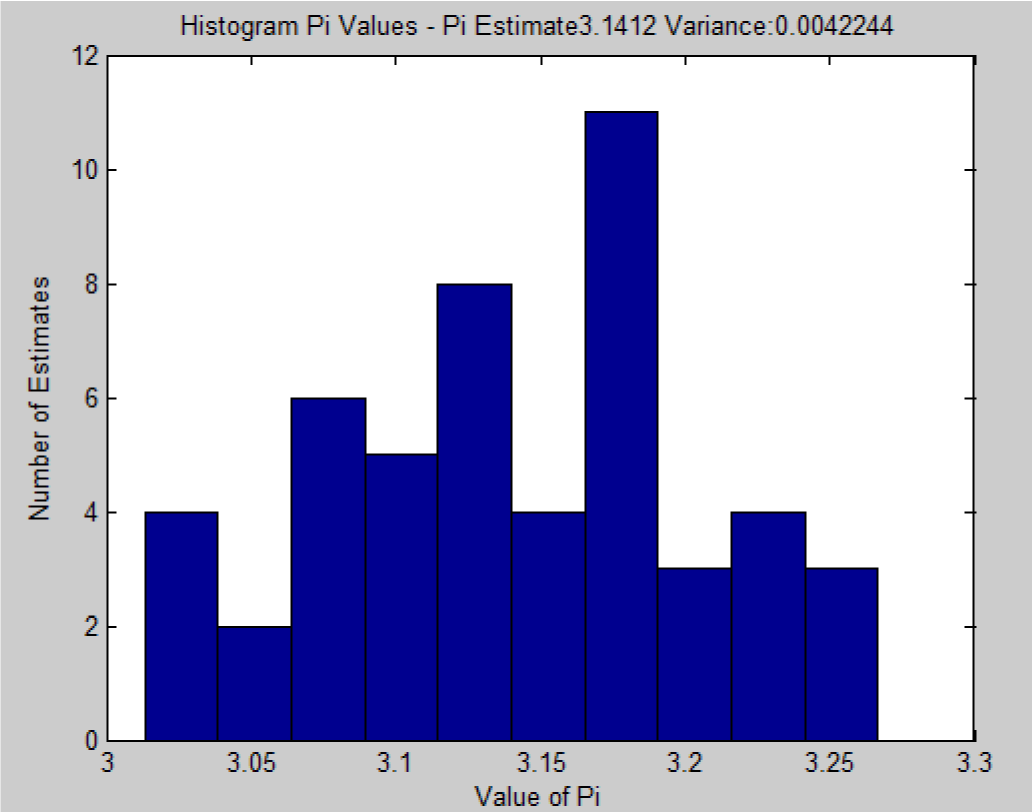
- 100 samples of i.i.d 2-dimensional uniform random variables are generated in the unit-square and the samples which fall within the quarter unit-circle centered at the origin are counted. The quarter circle inscribes the unit square as shown below:

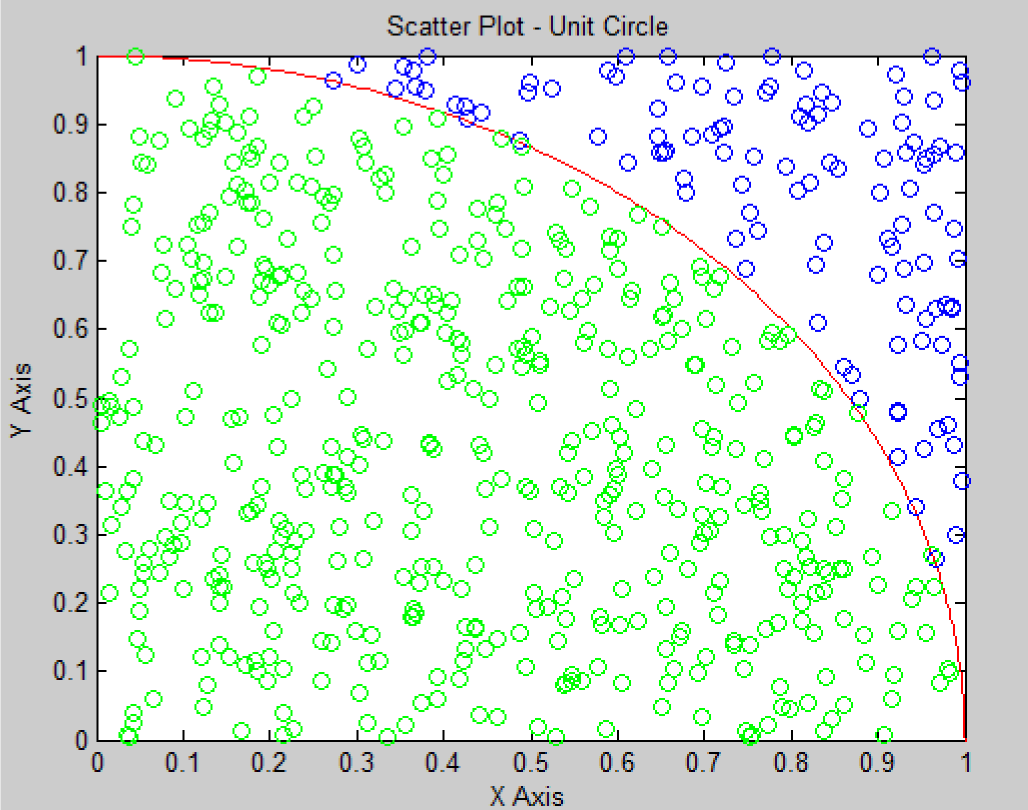


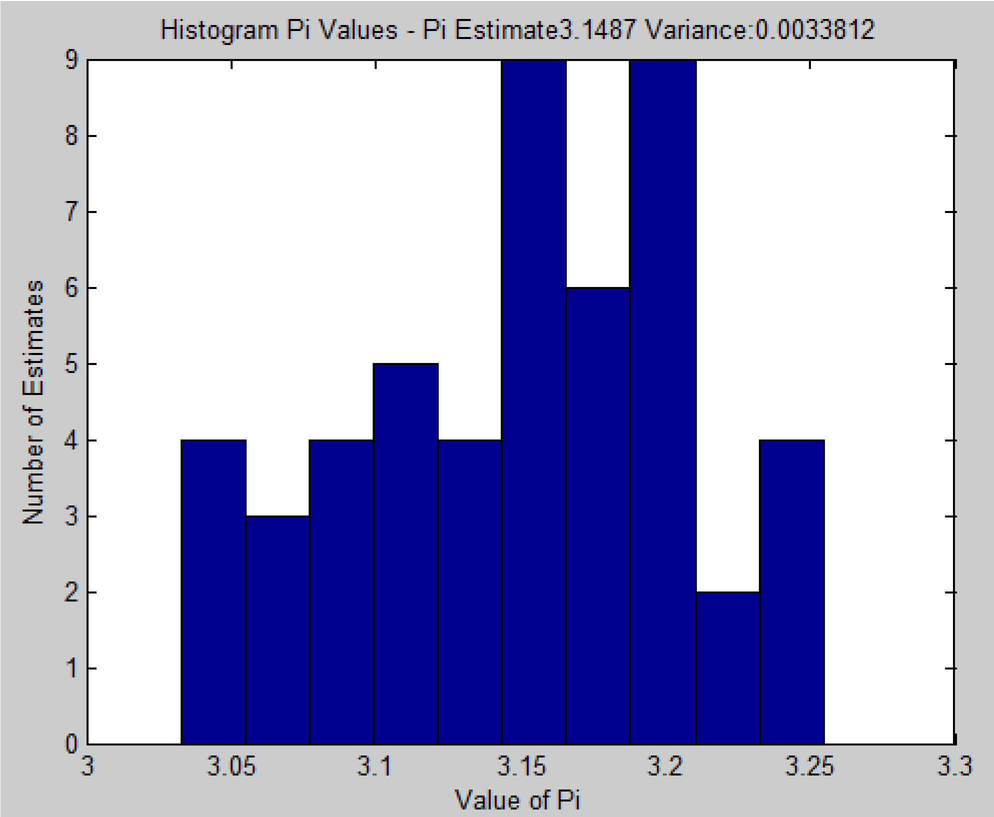
- Using the random samples generated, we estimate the area of the inscribed quarter circle and use this area estimate to estimate the value of pi. The same experiment is repeated for k=50 runs of these pi-estimations and the histogram of the 50 pi-estimates is plotted.
- The experiment is repeated with different numbers of uniform samples, n (using k=50 for all these runs) and the sample variance of the pi-estimates for these different values of n is plotted. Then, we find the line of best-fit for the relationship between our estimate variance and Monte Carlo sample size.
- Also, our Monte Carlo solution is adapted to provide integral and error estimates for the function: $g(x,y)=(x-1)2+100(y-x2)2$ (x, y) in [-1, 1] for g(x,y)

Results


Histogram Pi Values - Pi Estimate3.1216 Variance:0.022071
Number of Estimates
Value of Pi
Scatter Plot - Unit Circle
Y Axis
X Axis


Histogram Pi Values - Pi Estimate3.1412 Variance:0.0042244
Number of Estimates
Value of Pi
0
2
4
6
8
10
12
3
3.05
3.1
3.15
3.2
3.25
3.3


Scatter Plot - Unit Circle
Y Axis
X Axis
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1


Histogram Pi Values - Pi Estimate3.1487 Variance:0.0033812
Number of Estimates
Value of Pi
0
1
2
3
4
5
6
7
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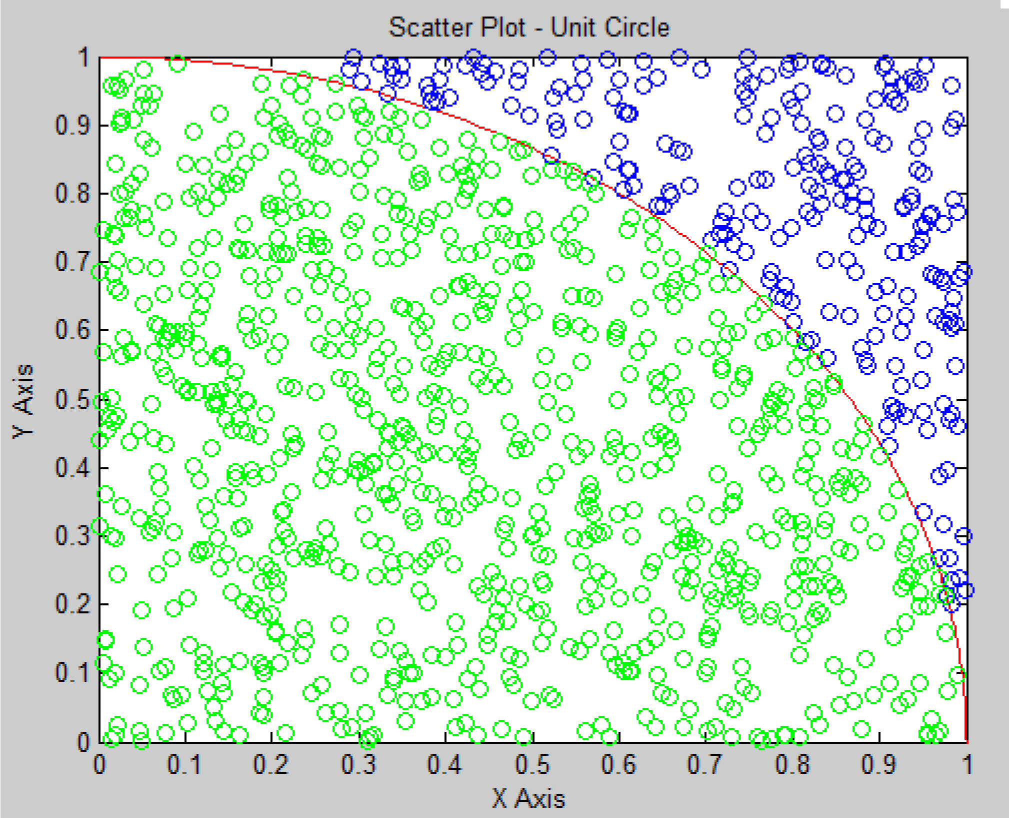
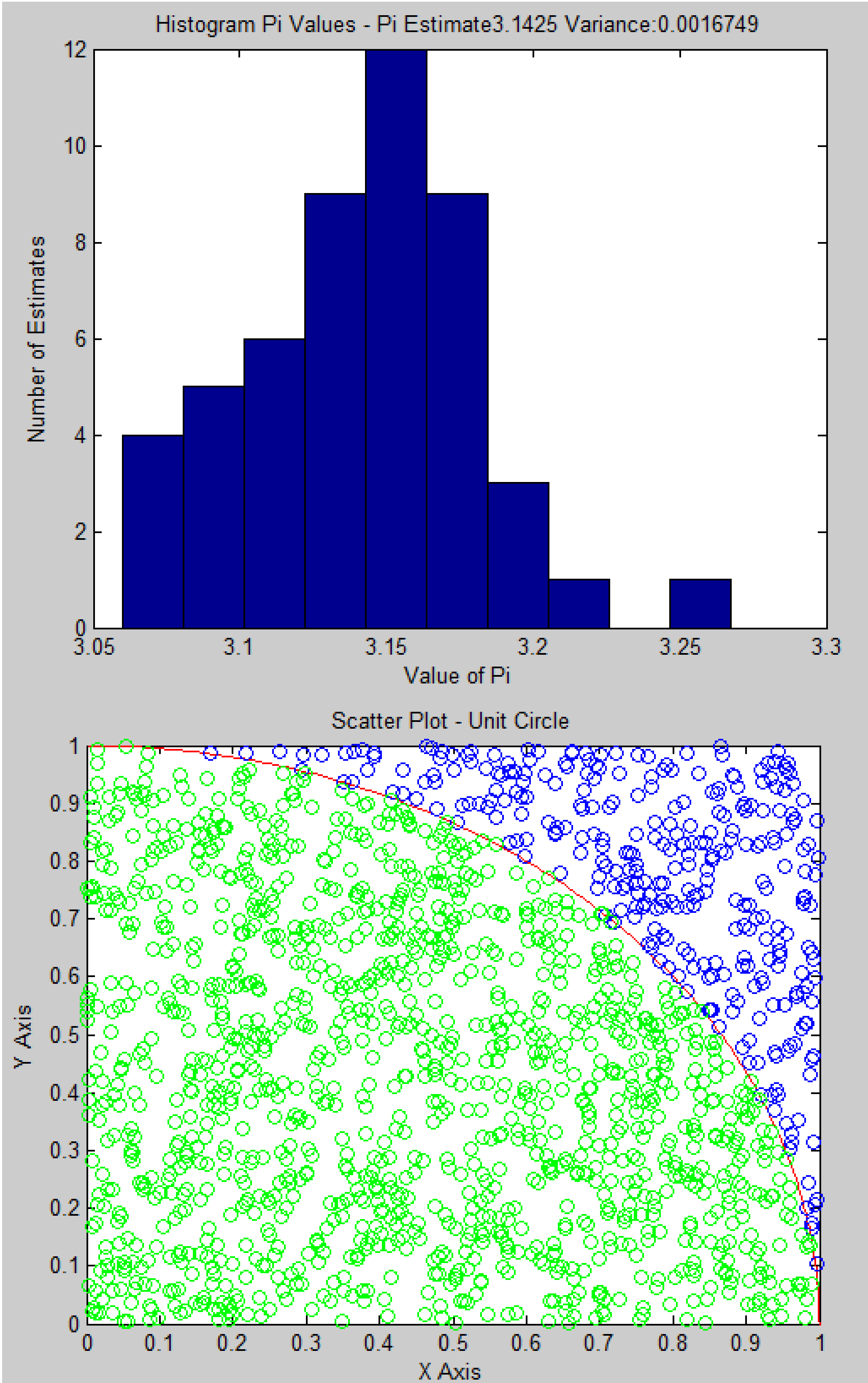
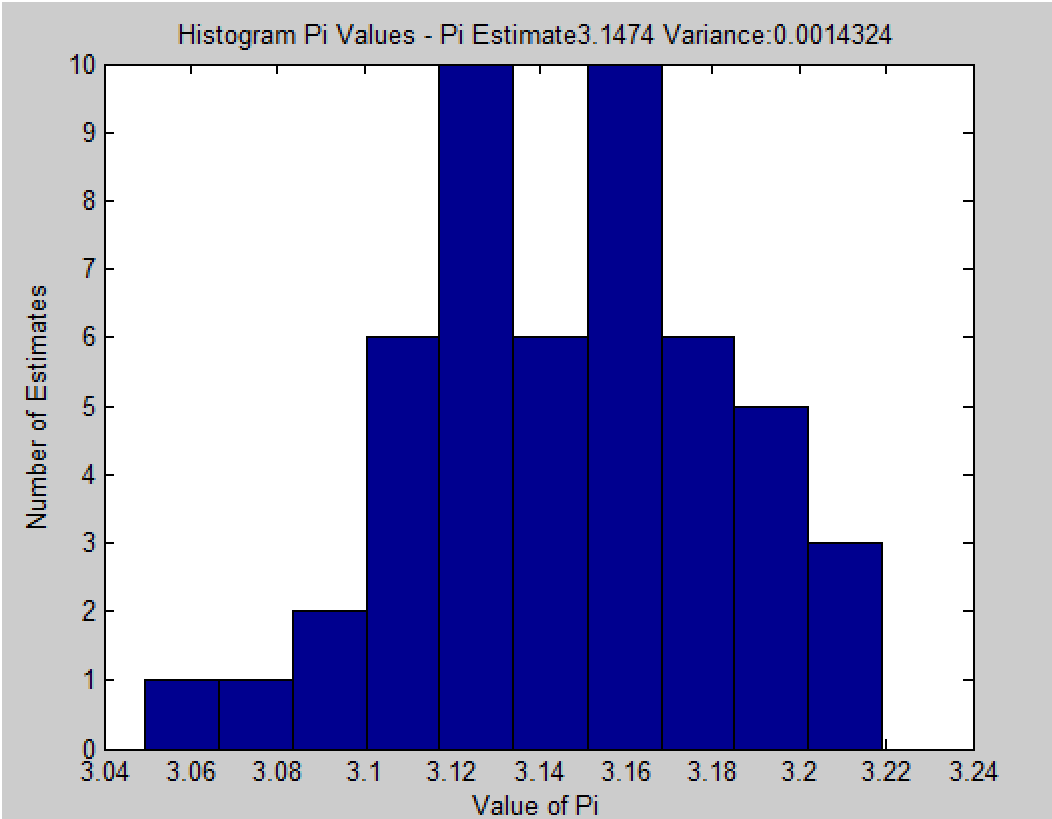
9
3
3.05
3.1
3.15
3.2
3.25
3.3


Scatter Plot - Unit Circle
Y Axis
X Axis
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1


Histogram Pi Values - Pi Estimate3.1425 Variance:0.0016749
Number of Estimates
Value of Pi
Scatter Plot - Unit Circle
Y Axis
X Axis


Histogram Pi Values - Pi Estimate3.1474 Variance:0.0014324
Number of Estimates
Value of Pi
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1
2
3
4
5
6
7
8
9
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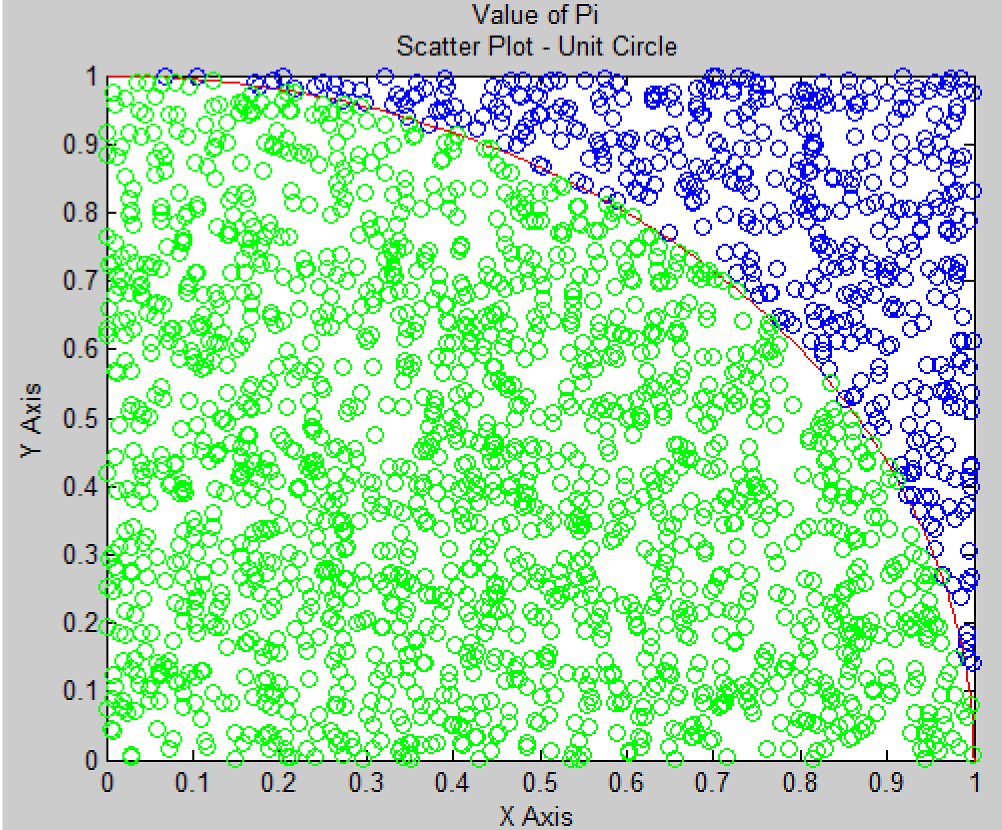
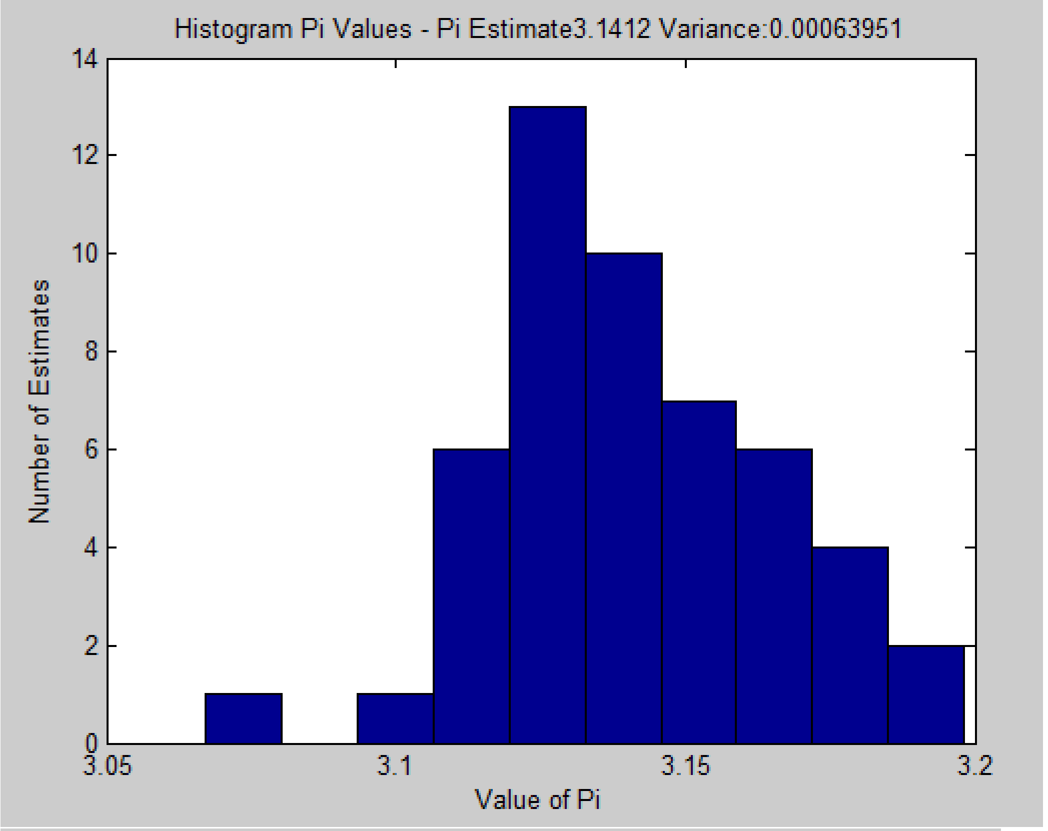
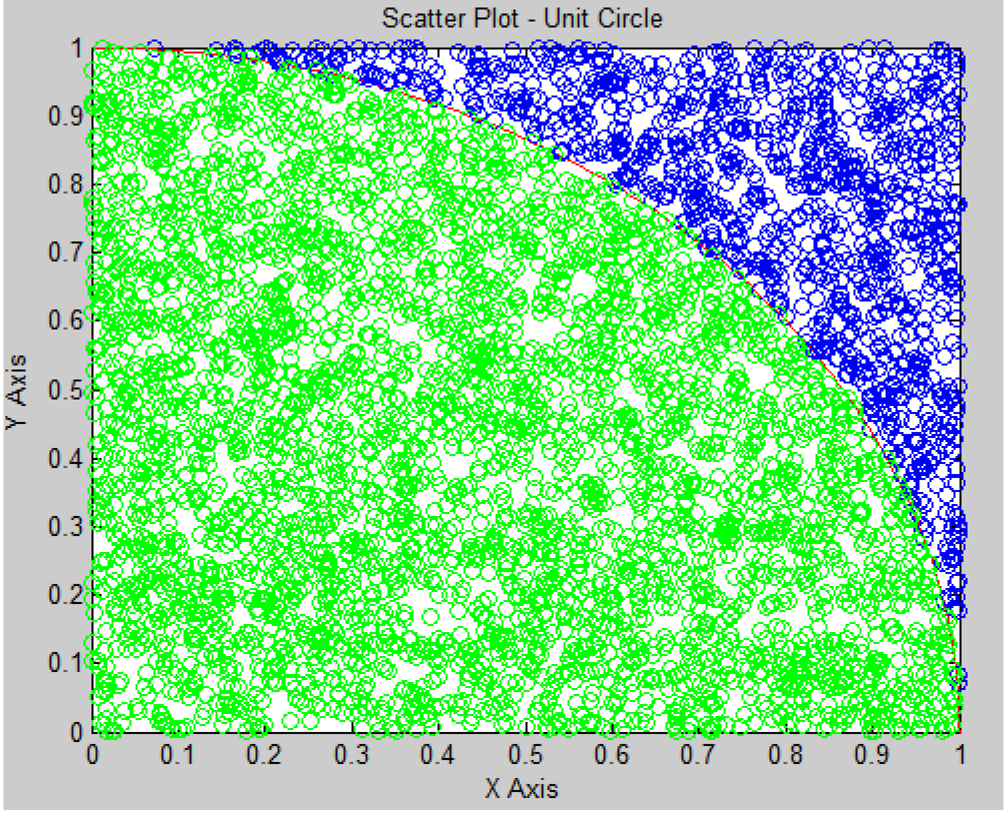
3.06
3.08
3.1
3.12
3.14
3.16
3.18
3.2
3.22
3.24


Scatter Plot - Unit Circle
Y Axis
X Axis
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

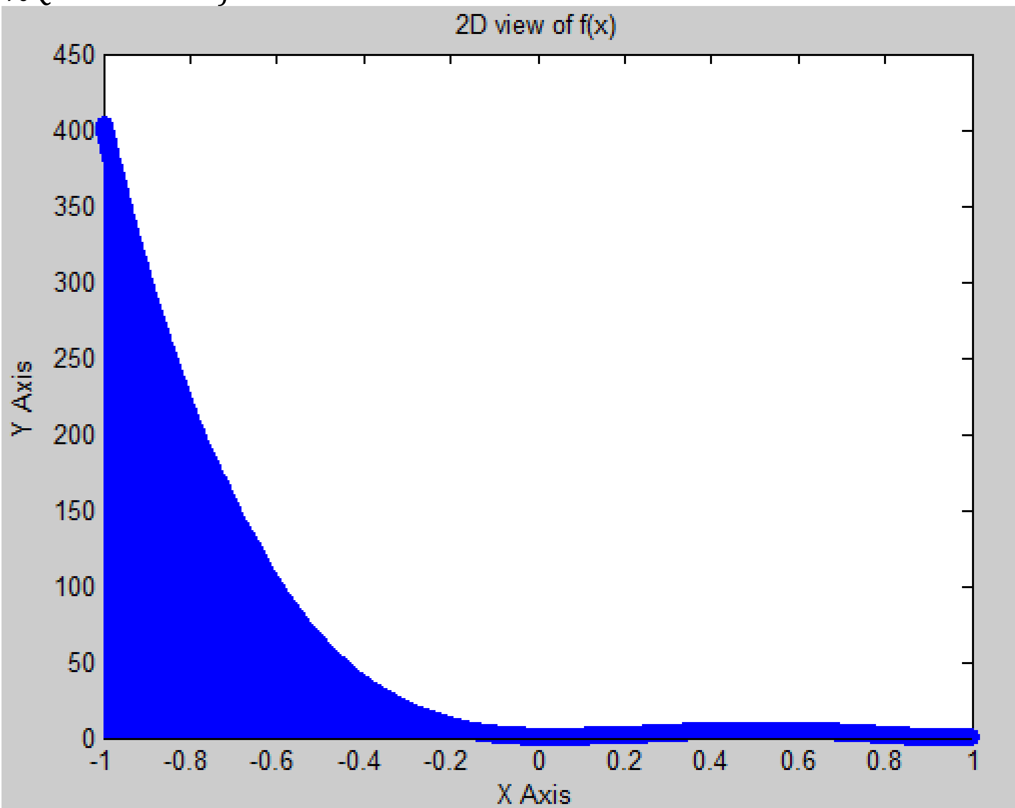
And so on…

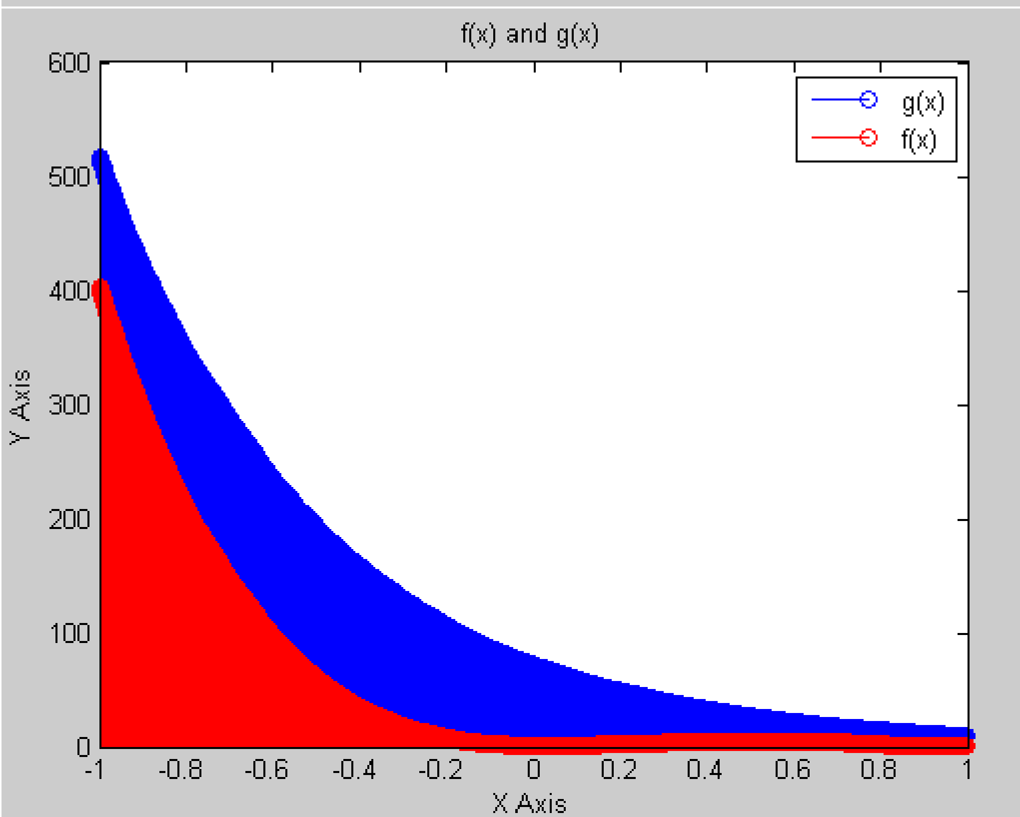
Final iteration





%Question 1.iii.)





Problem 2 [Variance Reduction Methods for Monte Carlo]

- Simple Monte Carlo Technique:

For the given functions, we first generate 1000 random samples and calculate the mean and variance using simple Monte Carlo technique. Expected value is calculated by taking mean of 1000 samples generated using above equations. Similarly variances are also calculated.

- Stratified Sampling Technique

We take the 1000 random samples generated and divide the given function into two 'stratas'. Then we calculate the expected value and variance in each of the two 'stratas'. Finally we sum the obtained expectations to get the final expectation. Same technique is used for variance.

- Importance Sampling Technique

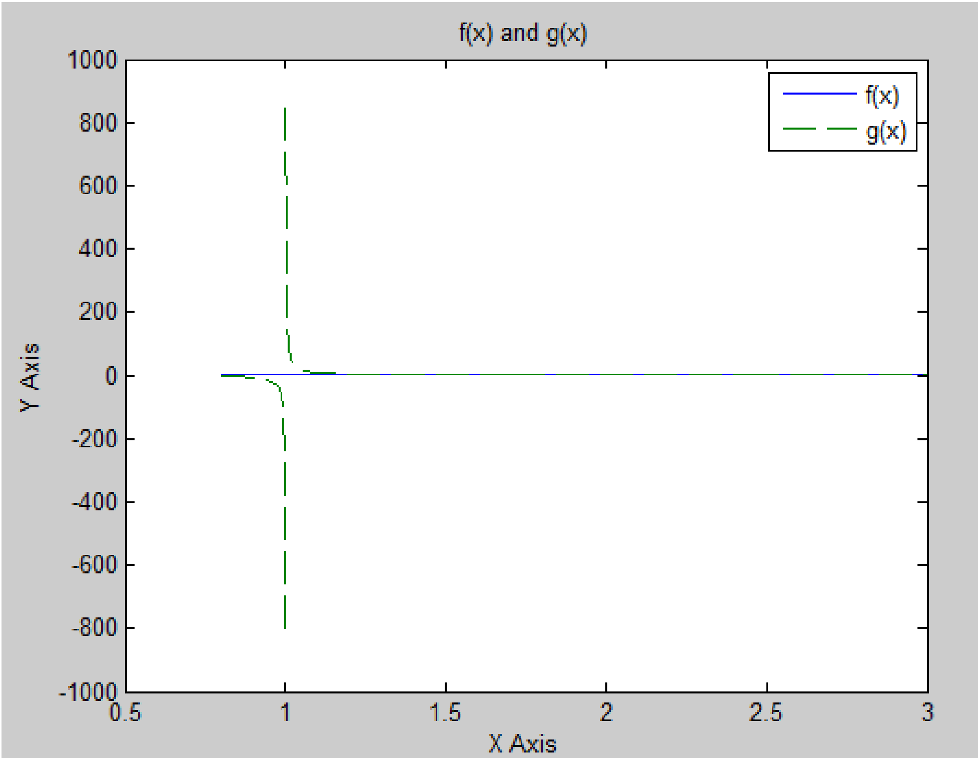
Importance sampling is a variance reduction technique. The idea behind importance sampling is that certain random variables which are taken as input have more impact than others. So here we generate random variables using random variable generator. We try to generate a function which is very close to the given function using the intervals of the given function.

- Comparison:

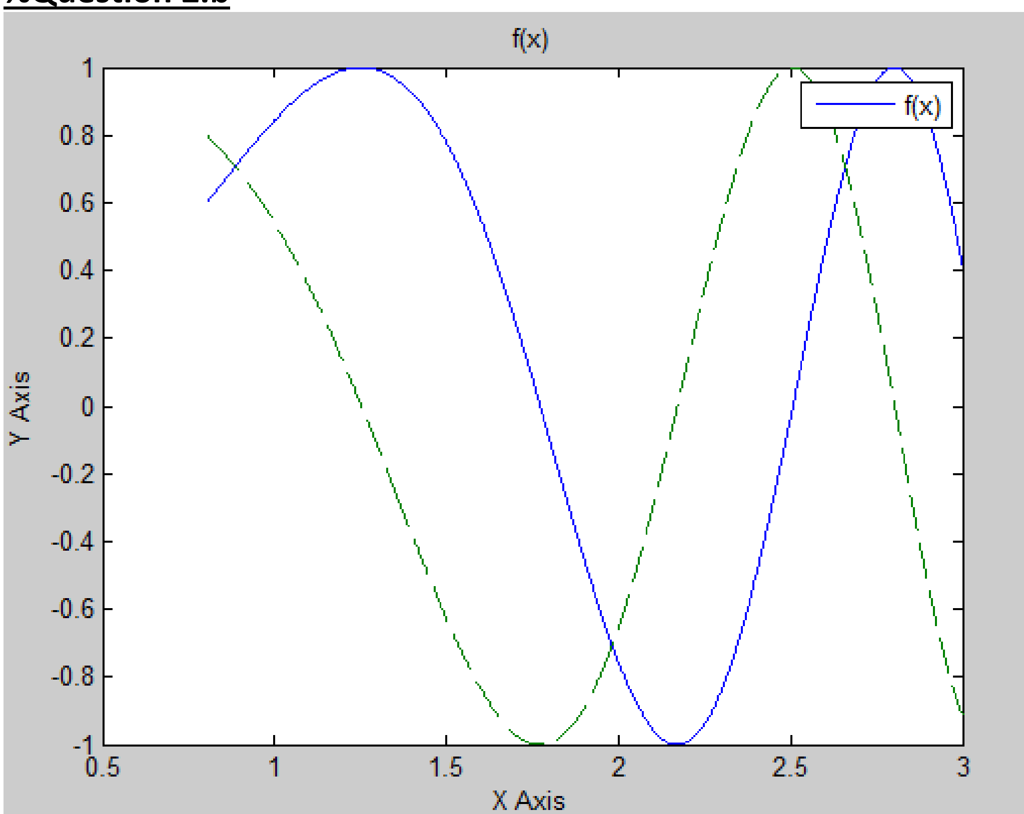
  - A stratified sample can provide greater precision than a simple random sample of the same size.
  - Because it provides greater precision, a stratified sample often requires a smaller sample, which saves money.
  - A stratified sample can guard against an unrepresentative sample.
  - One main disadvantage of stratified random sampling is that is can be difficult to identify appropriate strata for a study.
  - Second disadvantage is that is more complex to organize and analyze the results compared to simple random sampling.
  - Importance sampling is an unbiased sampling method used to sample.

Results:

%Question 2.a



%Question 2.b



Problem 3 [Markov Chain Monte Carlo]

- In this question, Metropolis-Hastings algorithm is implemented to generate 1000 samples from the following given distribution: 0.6*Beta (1, 8) +0.4*Beta (9, 1)
- By visual inspection of the curves, we can say that the samples generated above by the algorithm are from target distribution r(x), and the algorithm has successfully converged to the equilibrium distribution.
- If the variance of the proposal distribution is made high or low, we observe that the algorithm does not converge to the equilibrium distribution, and the generated samples have a high discrepancy when compared to the target distribution given.