SIMULATION METHODS FOR STOCHASTIC SYSTEMS

SAMPLES AND TEST SIMULATIONS PROJECT-2 REPORT

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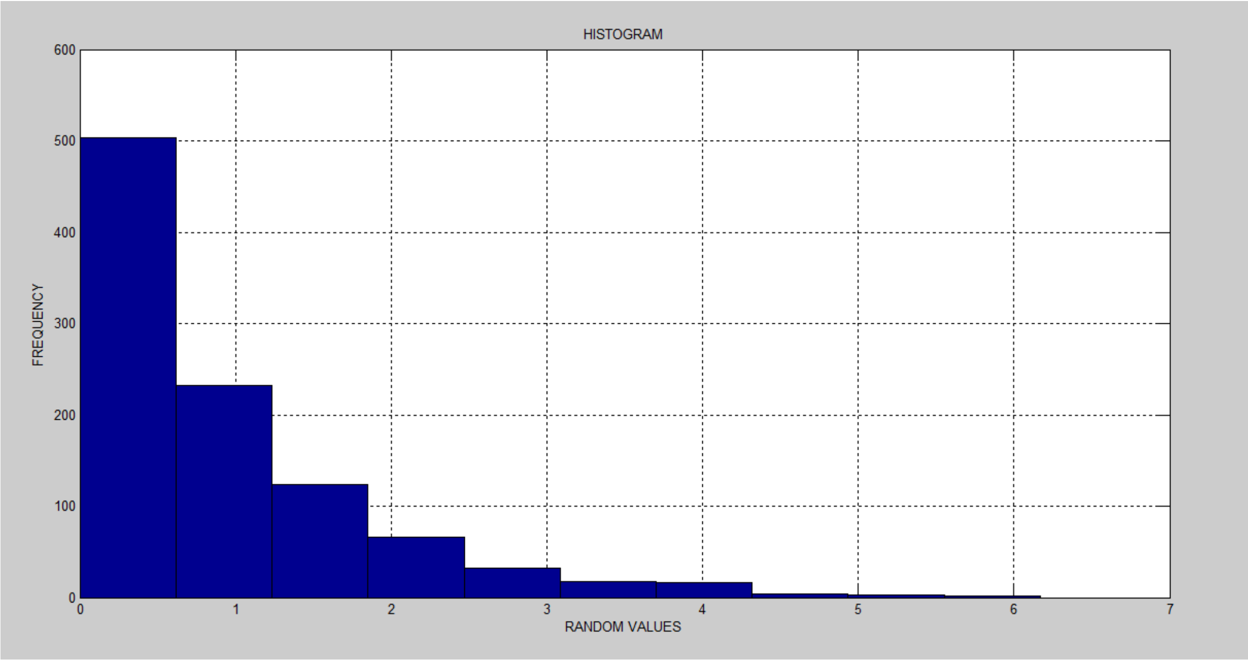
CONTENTS

SUMMARY

Question-1

- Initially, the MATLAB function “int” is used to calculate the Cumulative Distribution Function(CDF) for the Probability Density Function(PDF) of the given positive Weibull(a) distribution by integrating the PDF.
- Inverse of the CDF is calculated by using the MATLAB function “finverse”.
- 1000 iid samples of the random variable X~Weibull(a) is calculated by inverse CDF method using the MATLAB function “rand”.
- A Histogram plot of the random numbers generated is plotted using the MATLAB function “hist”.
- In order to evaluate the quality of random numbers generated, chi square test is implemented.
- A chi-square test is used to test the goodness of fit of the random number generator(RNG):
  - A very small chi square test statistic means that your observed data fits your expected data extremely well.
  - A very large chi square test statistic means that the data does not fit very well. If the chi-square value is large, you reject the null hypothesis.
- The obtained value for chi= 1.1200, which implies that the data fits the expected data very well and the quality of generated random numbers are good.

Histogram Plot



SUMMARY

Question-2

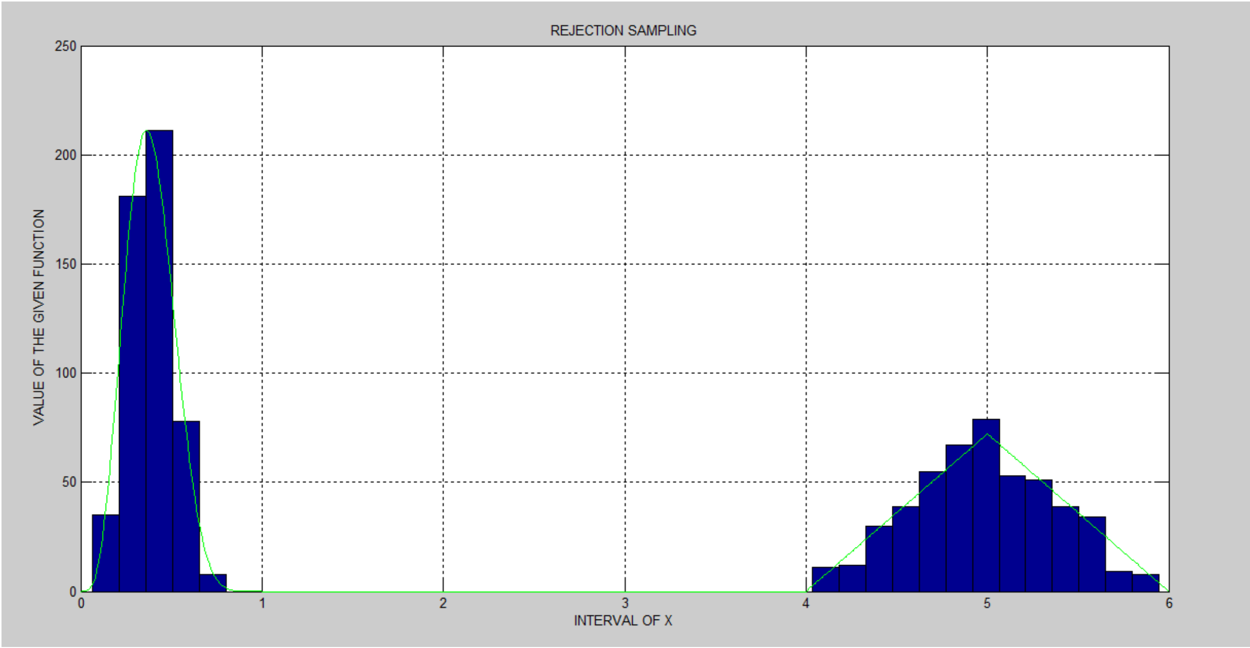
- Initially, a MATLAB function **"our_function"** is created using MATLAB command **"function"** to implement the function f(x) given by

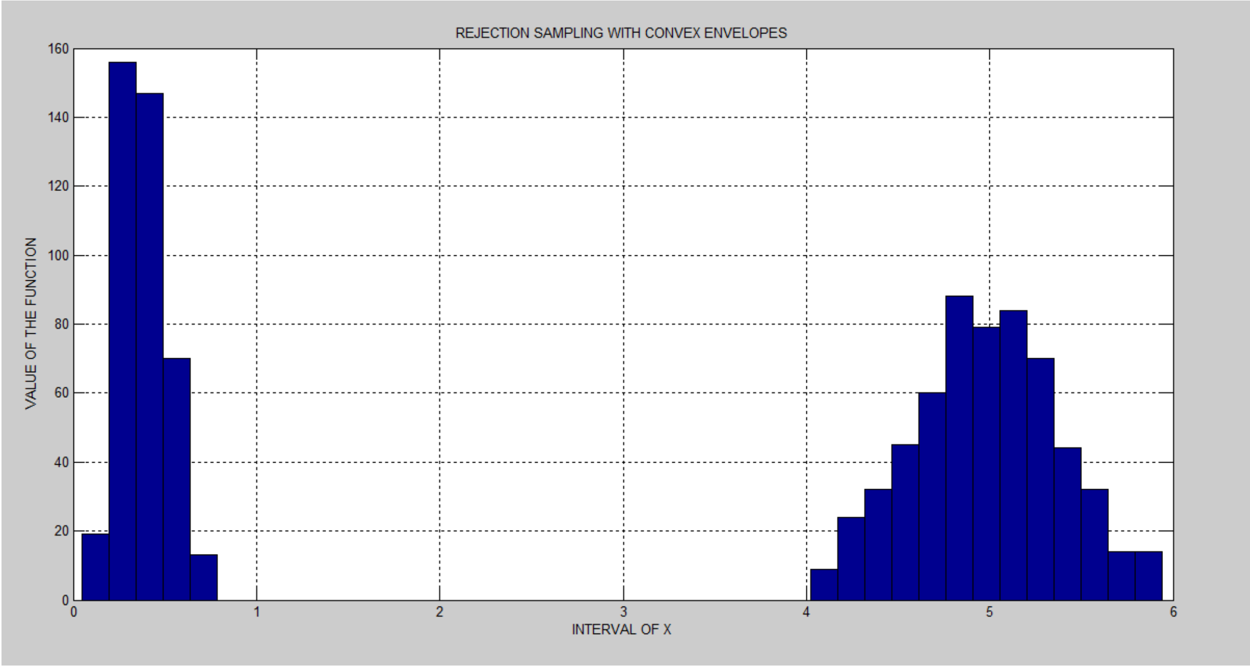
$$f(x)=\begin{cases}0.5\times Beta(8,5), & 0<x\leq 1\\ 0.5\times(x-4), & 4<x\leq 5\\ -0.5\times(x-6), & 5<x\leq 6\\ 0, & else\end{cases}$$

- To implement rejection sampling routines for X using a single uniform envelope over the full range of X (0,6), we generate 1000 random numbers in the range (0,6) and enter these generated random values in the function f(x). We then multiply each of the generated random number by the maximum value(peak) of f(x) and compare this value with the value obtained by entering each random number into f(x). If the product is less than or equal to the value generated using f(x), then we accept that sample(random number value) otherwise we reject it. Thus, using Acceptance Rejection method, we implement rejection sampling routines for X.
- The above procedure of Acceptance Rejection method is repeated for a convex combination of two separate uniform envelopes over the two peaks {(0,1), (4,6)}.
- Histograms for Rejection sampling with and without convex envelopes are plotted.
- We obtain the following results for rejection rates:
    - The rejection rate of the rejection sampling RNG in the case of a single uniform envelope over the full range of X (0,6) is 7.688
    - The rejection rate of the rejection sampling RNG in the case of a convex combination of two separate uniform envelopes over the two peaks {(0,1), (4,6)} is 1.432

From the above obtained results, we conclude that using the efficiency of RNG for a convex combination of two separate uniform envelopes over the two peaks {(0,1), (4,6)} is more than the efficiency of RNG using a single uniform envelope over the full range of X (0,6).

Histogram Plots



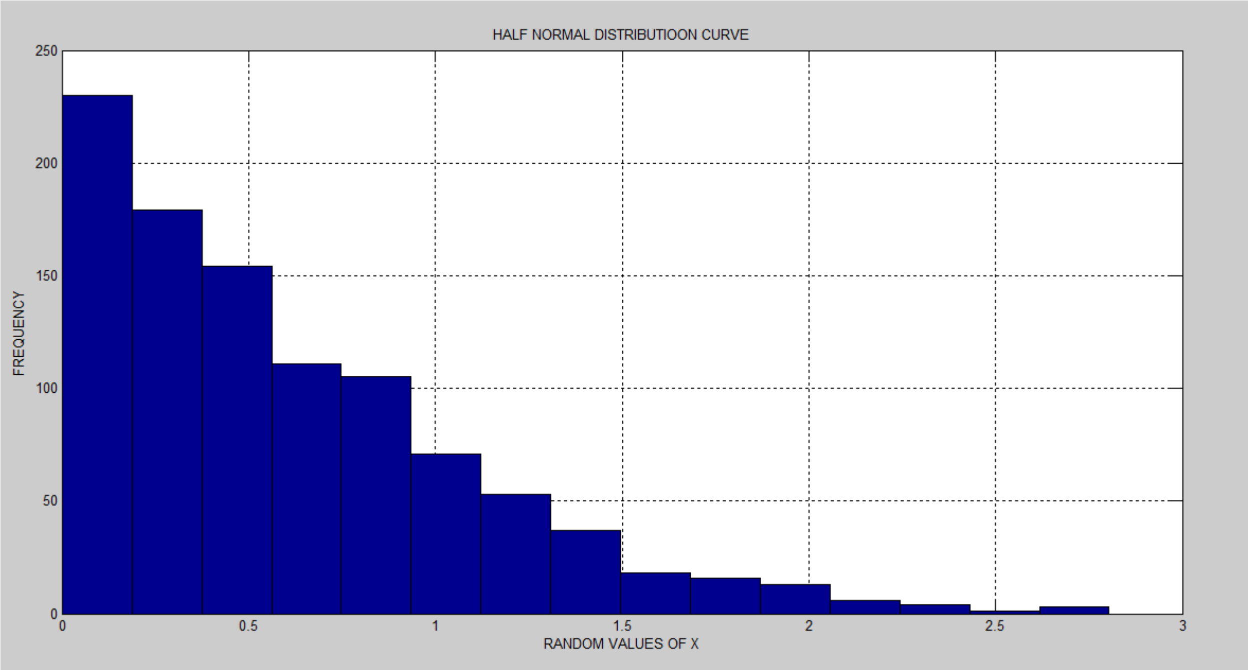


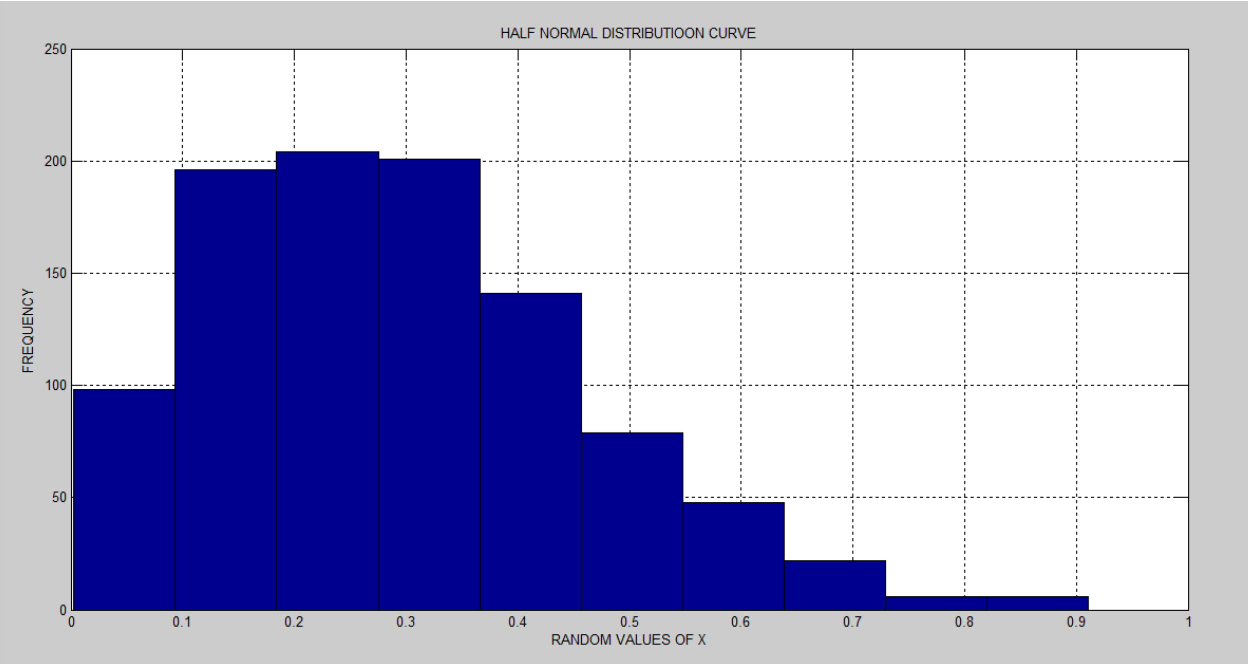
SUMMARY

Question-3

Rejection sampling routines for the standard half-normal random variable X1~ HN(1) and the beta random variable X2~Beta(2, 5) are implemented by using a suitably adapted infinite-support envelope function for X1 (e.g. exp(1)). We perform Contingency test and chi square tests, results of which verify that the samples generated are independent.

Histogram Plots





Results

table =

1	0
0	1

chisquare_stat =

2

val_p =

0.1573