Samples and test simulations

Problem 1

The positive Weibull(a) distribution has the density:

$$f(x) = ax^{a-1}e^{-x^a}$$

where a and x are strictly positive. Use the inverse CDF method to generate 1000 iid samples of the random variable X~Weibull(a). Argue for or against the quality of your random number generator (RNG) using (e.g. using histograms and goodness-of-fit tests).

Problem 2

The random variable X has a bimodal distribution made up of an equally weighted, convex summation of a beta and a triangle distribution:

$$f(x) = \begin{cases} 0.5\times Beta(8,5), & 0 < x \le 1 \\ 0.5\times(x-4), & 4 < x \le 5 \\ -0.5\times(x-6), & 5 < x \le 6 \\ 0, & else \end{cases}$$

Implement rejection sampling routines for X using two different envelope functions:

- a single uniform envelope over the full range of X (0,6)
- a convex combination of two separate uniform envelopes over the two peaks {(0,1), (4,6)} Generate 1000 samples of the random variable using each envelope. Track the rejection rate of your rejection sampling RNGs for each envelope function. The rejection rate is the average number of rejected candidates per sample. This is a measure of the efficiency of your RNG. Discuss the efficiency of the two envelopes.

Problem 3

Implement rejection sampling routines for the standard half-normal random variable X1~ HN(1) and the beta random variable X2~Beta(1/2, 5). You will need a suitably adapted infinite-support envelope function for X1 (e.g. exp(1)). Generate 1000 samples of each random variable. Should the two sets of samples be independent? How would you verify that?