$$
\begin{aligned}
2x_1 &+ 3x_2 + 5x_3 = 5 \\
3x_1 &+ 4x_2 + 7x_3 = 6 \\
x_1 &+ 3x_2 + 2x_3 = 5
\end{aligned}
$$

By Crumer's Rule,

$$
\left| A \right| = \begin{vmatrix} 2 & 3 & 5 \\ 3 & 4 & 7 \\ 1 & 3 & 2 \end{vmatrix}
$$

$$= 2 (8 - 21) - 3 (6 - 7) + 5 (9 - 4)$$

$$= 2 \neq 0$$

$$
\left| A_1 \right| = \begin{vmatrix} 5 & 3 & 5 \\ 6 & 4 & 7 \\ 1 & 3 & 2 \end{vmatrix}
$$

$$= 5 (8 - 21) - 3 (12 - 35) + 5 (18 - 20)$$

$$= -6$$

$$
\left| A_2 \right| = \begin{vmatrix} 2 & 5 & 5 \\ 3 & 6 & 7 \\ 1 & 5 & 2 \end{vmatrix}
$$

$$= 2 (12 - 35) - 5 (6 - 7) + 5 (15 - 6)$$

$$= 4$$

$$
\left| A_3 \right| = \begin{vmatrix} 2 & 3 & 5 \\ 3 & 4 & 6 \\ 1 & 3 & 5 \end{vmatrix}
$$

$$= 2 (20 - 18) - 3 (15 - 6) + 5 (9 - 4)$$

$$= 2$$

$$x_1 = \frac{|A_1|}{|A|} = \frac{-6}{2} = -3$$

$$x_2 = \frac{|A_2|}{|A|} = \frac{4}{2} = 2$$

$$x_3 = \frac{|A_3|}{|A|} = \frac{2}{2} = 1$$

By Adjoint Method

Let,

$$A = \begin{pmatrix} 2 & 3 & 5 \\ 3 & 4 & 7 \\ 1 & 3 & 2 \end{pmatrix}, \quad B = \begin{pmatrix} 5 \\ 6 \\ 5 \end{pmatrix}, \quad X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

$$AX = B$$

$$X = A^{-1}B = \frac{\text{adj}(A) \cdot B}{|A|}$$

$$|A| = 2$$

$$\text{Cofactor of } A = \begin{pmatrix} -13 & 1 & 5 \\ 9 & -1 & -3 \\ 1 & 1 & -1 \end{pmatrix}$$

$$\text{Adj}(A) = \begin{pmatrix} -13 & 9 & 1 \\ 1 & -1 & 1 \\ 5 & -3 & -1 \end{pmatrix}$$

$$X = \frac{\text{adj}(A) \cdot B}{|A|}$$

$$= \frac{1}{2}\begin{pmatrix} -13 & 9 & 1 \\ 1 & -1 & 1 \\ 5 & -3 & -1 \end{pmatrix}\begin{pmatrix} 5 \\ 6 \\ 5 \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} -6 \\ 4 \\ 2 \end{pmatrix}$$

$$= \begin{pmatrix} -3 \\ 2 \\ 1 \end{pmatrix}$$

$x_1 = -3$, $x_2 = 2$, $x_3 = 1$

By Gauss-Jordan Elimination

Let,

$$A = \begin{pmatrix} 2 & 3 & 5 \\ 3 & 4 & 7 \\ 1 & 3 & 2 \end{pmatrix}, \quad B = \begin{pmatrix} 5 \\ 6 \\ 5 \end{pmatrix}, \quad X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

$$\left[A \mid B \right]$$

$$= \left(\begin{array}{ccc|c} 2 & 3 & 5 & 5 \\ 3 & 4 & 7 & 6 \\ 1 & 3 & 2 & 5 \end{array}\right) \xrightarrow[R_3 - 1/2\,R_1]{R_2 - 3/2\,R_1}$$

$$= \left(\begin{array}{ccc|c} 2 & 3 & 5 & 5 \\ 0 & -1/2 & -1/2 & -3/2 \\ 0 & 3/2 & -1/2 & 5/2 \end{array}\right) \xrightarrow{R_3 + 3\,R_2}$$

$$= \left(\begin{array}{ccc|c} 2 & 3 & 5 & 5 \\ 0 & -1/2 & -1/2 & -3/2 \\ 0 & 0 & -2 & -2 \end{array}\right) \xrightarrow[\substack{R_2 \times -2 \\ R_3 \times -1/2}]{1/2\,R_1}$$

$$= \left(\begin{array}{ccc|c} 1 & 3/2 & 5/2 & 5/2 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 1 & 1 \end{array}\right) \xrightarrow[R_2 - R_1]{R_3 - 3/2\,R_2}$$

$$= \left(\begin{array}{ccc|c} 1 & 0 & 1 & -2 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{array}\right) \xrightarrow{R_1 - R_3}$$

$$= \left(\begin{array}{ccc|c} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{array}\right)$$

$x_1 = -3$, $x_2 = 2$, $x_3 = 1$