1.i.(a)

$$\begin{array}{c} \longrightarrow \\ \\ \\ \end{array} \begin{vmatrix} 1 & -2 & -3 \\ 0 & 4 & 0 \\ 5 & 0 & 6 \end{vmatrix}$$

$$= a_{11} C_{11} + a_{12} C_{12} + a_{13} C_{13}$$

$$= 1 \; (-1)^{1+1} \begin{vmatrix} 4 & 0 \\ 0 & 6 \end{vmatrix} + (-2) \; (-1)^{1+2} \begin{vmatrix} 0 & 0 \\ 5 & 6 \end{vmatrix} + (-3) \; (-1)^{1+3} \begin{vmatrix} 0 & 4 \\ 5 & 0 \end{vmatrix}$$

$$= 1 \; (24-0) + 2 \; (0-0) - 3 \; (0-20)$$

$$= 24 + 0 + 60$$

$$= 84$$

1.i.(b)

$$\begin{array}{c} \\ \longrightarrow \\ \\ \end{array} \begin{vmatrix} -1 & 4 & 0 \\ -2 & -3 & 0 \\ -4 & 1 & 5 \end{vmatrix}$$

$$= a_{21} C_{21} + a_{22} C_{22} + a_{23} C_{23}$$

$$= -2 \; (-1)^{2+1} \begin{vmatrix} 4 & 0 \\ 1 & 5 \end{vmatrix} + (-3) \; (-1)^{2+2} \begin{vmatrix} -1 & 0 \\ -4 & 5 \end{vmatrix} + 0 \; (-1)^{2+3} \begin{vmatrix} -1 & 4 \\ -4 & 1 \end{vmatrix}$$

$$= 2 \; (20-0) + (-3) \; (-5+0) + 0$$

$$= 40 + 15$$

$$= 55$$

1.i.(c)

$$\begin{array}{ccc} & \downarrow & \\ \end{array}$$
$$\begin{vmatrix} 1/2 & \pi & 2 \\ -1 & 0 & -4 \\ 1/2 & -5 & 6 \end{vmatrix}$$

$$= a_{12} C_{12} + a_{22} C_{22} + a_{32} C_{32}$$

$$= \pi \; (-1)^{1+2} \begin{vmatrix} -1 & -4 \\ 1/2 & 6 \end{vmatrix} + 0 \; (-1)^{2+2} \begin{vmatrix} 1/2 & 2 \\ 1/2 & 6 \end{vmatrix} + (-5) \; (-1)^{3+2} \begin{vmatrix} 1/2 & 2 \\ -1 & -4 \end{vmatrix}$$

$$= -3.142 \; (-6+2) + 0 + 5 \; (-2+2)$$

$$= 12.568 + 0$$

$$= 12.568$$

1.ii Let, k=2, n = 4, A is a 4 by 4 matrix.

$$A = \begin{pmatrix} -2 & 0 & -1 & 0 \\ 6 & 1 & 0 & -4 \\ 0 & 0 & -1 & 3 \\ 0 & 2 & 4 & 5 \end{pmatrix}$$

$$kA = 2 \begin{pmatrix} -2 & 0 & -1 & 0 \\ 6 & 1 & 0 & -4 \\ 0 & 0 & -1 & 3 \\ 0 & 2 & 4 & 5 \end{pmatrix} = \begin{pmatrix} -4 & 0 & -2 & 0 \\ 12 & 2 & 0 & -8 \\ 0 & 0 & -2 & 6 \\ 0 & 4 & 8 & 10 \end{pmatrix}$$

$$\det(kA) = \begin{vmatrix} -4 & 0 & -2 & 0 \\ 12 & 2 & 0 & -8 \\ 0 & 0 & -2 & 6 \\ 0 & 4 & 8 & 10 \end{vmatrix}$$

$$\xrightarrow{R_2+3R_1} \quad = \begin{vmatrix} -4 & 0 & -2 & 0 \\ 0 & 2 & -6 & -8 \\ 0 & 0 & -2 & 6 \\ 0 & 4 & 8 & 10 \end{vmatrix}$$

$$= (-4) \begin{vmatrix} 2 & -6 & -8 \\ 0 & -2 & 6 \\ 4 & 8 & 10 \end{vmatrix}$$

$$= (-4)(2) \begin{vmatrix} 2 & -6 & -8 \\ 0 & -2 & 6 \\ 2 & 4 & 5 \end{vmatrix}$$

$$\xrightarrow{R_3-R_1} \quad = (-8) \begin{vmatrix} 2 & -6 & -8 \\ 0 & -2 & 6 \\ 0 & 10 & 13 \end{vmatrix}$$

$$= (-8)(2) \begin{vmatrix} -2 & 6 \\ 10 & 13 \end{vmatrix}$$

$$= (-16) \quad (-26-60)$$

$$= 1376$$

$$\det(A) = \begin{vmatrix} -2 & 0 & -1 & 0 \\ 6 & 1 & 0 & -4 \\ 0 & 0 & -1 & 3 \\ 0 & 2 & 4 & 5 \end{vmatrix}$$

$$= \xrightarrow{R_2+3R_1} \begin{vmatrix} -2 & 0 & -1 & 0 \\ 0 & 1 & -3 & -4 \\ 0 & 0 & -1 & 3 \\ 0 & 2 & 4 & 5 \end{vmatrix}$$

$$= -2 \begin{vmatrix} 1 & -3 & -4 \\ 0 & -1 & 3 \\ 2 & 4 & 5 \end{vmatrix}$$

$$= \xrightarrow{R_3+2R_1} (-2) \begin{vmatrix} 1 & -3 & -4 \\ 0 & -1 & 3 \\ 0 & 10 & 13 \end{vmatrix}$$

$$= (-2)(1) \begin{vmatrix} -1 & 3 \\ 10 & 13 \end{vmatrix}$$

$$= (-2)(-13-30)$$

$$= 86$$

$$k^n.\det(A) = 2^4 \times 86 = 1376$$

So, $\det(kA) = k^n.\det(A)$

1.iii.(a)

$$\begin{vmatrix} -4 & 3 & 1 \\ 2 & 6 & -1 \\ 6 & -3 & -4 \end{vmatrix} = \xrightarrow{R_2+R_1} \begin{vmatrix} -4 & 3 & 1 \\ -2 & 9 & 0 \\ 6 & -3 & -4 \end{vmatrix}$$

$$= \xrightarrow{R_3+4R_1} \begin{vmatrix} -4 & 3 & 1 \\ -2 & 9 & 0 \\ -10 & 9 & 0 \end{vmatrix}$$

$$= 1 \begin{vmatrix} -2 & 9 \\ -10 & 9 \end{vmatrix}$$

$$= -18-(-90)$$

$$= 72$$

1.iii.(b)

$$\begin{vmatrix} -2 & 0 & -1 & 0 \\ 6 & 1 & 0 & -4 \\ 0 & 0 & -1 & 3 \\ 0 & 2 & 4 & 5 \end{vmatrix}$$

$$= \xrightarrow{R_2+3R_1} \begin{vmatrix} -2 & 0 & -1 & 0 \\ 0 & 1 & -3 & -4 \\ 0 & 0 & -1 & 3 \\ 0 & 2 & 4 & 5 \end{vmatrix}$$

$$= -2 \begin{vmatrix} 1 & -3 & -4 \\ 0 & -1 & 3 \\ 2 & 4 & 5 \end{vmatrix}$$

$$= \xrightarrow{R_3+2R_1} (-2) \begin{vmatrix} 1 & -3 & -4 \\ 0 & -1 & 3 \\ 0 & 10 & 13 \end{vmatrix}$$

$$= (-2)(1) \begin{vmatrix} -1 & 3 \\ 10 & 13 \end{vmatrix}$$

$$= (-2)(-13-30)$$

$$= 86$$

1.iii.(c)

$$\begin{vmatrix} -1 & -3 & 7 \\ 2 & 4 & -20 \\ 5 & 17 & -27 \end{vmatrix}$$

$$= \xrightarrow{R_2+2R_1} \begin{vmatrix} -1 & -3 & 7 \\ 0 & -2 & -6 \\ 5 & 17 & -27 \end{vmatrix}$$

$$= \xrightarrow{R_3+5R_1} \begin{vmatrix} -1 & -3 & 7 \\ 0 & -2 & -6 \\ 0 & 2 & 8 \end{vmatrix}$$

$$= (-1) \begin{vmatrix} -2 & -6 \\ 2 & 8 \end{vmatrix}$$

$$= (-1) \quad (-16)-(-12)$$

$$= (-1) \quad (-4)$$

$$= 4$$