Model Estimation with AR(1) Endogenous Unobservable

1) Model:

The observables are y_{it} and x_{it} . The goal is to identify β in the relationship,

$$y_{it} = x'_{it}\beta + w'_{it}\gamma + \epsilon_{it} + \nu_{it}, \quad \nu_{it} \sim \text{iid}, \quad \nu_{it} \text{ is serially independent}$$
 (1)

$$x_{it} = \gamma \epsilon_{it} + (1 - \gamma) u_{it}, \quad u_{it} \sim \text{iid}, \quad u_{it} \text{ is serially dependent}$$
 (2)

$$\epsilon_{it} = \rho \epsilon_{it-1} + \eta_{it}, \quad \eta_{it} \sim \text{iid}, \quad \eta_{it} \text{ is serially independent}$$
(3)

where w_{it} includes a constant.

2) Identification:

The usual argument for identification is based on the quasi-difference expression:

$$(y_{it} - \rho y_{it-1}) = (x_{it} - \rho x_{it-1})' \beta + (w_{it} - \rho w_{it-1})' \delta + \eta_{it}$$
(4)

Panel IV Approach: One approach is to rearrange Equation (4) as a panel regression:

$$y_{it} = y_{it-1}(\rho) + x'_{it}(\beta) + x'_{it-1}(-\rho\beta) + w'_{it}(\delta) + w'_{it-1}(-\rho\delta) + \eta_{it}$$
(5)

The only source of endogeneity in this regression is that x_{it} depends on η_{it} . This implies that β is identified by a regression of y_{it} on x_{it} , controlling for $(y_{it-1}, x_{it-1}, w_{it}, w_{it-1})$, and instrumented by $z_{it} = (x_{it-2})$ or $z_{it} = (x_{it-2}, y_{it-2})$ or $z_{it} = (x_{it-2}, y_{it-2}, w_{it-2})$.

GMM Approach: For any guess $(\hat{\beta}, \hat{\delta}, \hat{\rho})$, we can define the guess of η_{it} :

$$\hat{\eta}_{it}\left(\hat{\beta},\hat{\delta},\hat{\rho}\right) \equiv \left(y_{it} - \hat{\rho}y_{it-1}\right) - \left(x_{it} - \hat{\rho}x_{it-1}\right)'\hat{\beta} - \left(w_{it} - \rho w_{it-1}\right)'\hat{\delta} \tag{6}$$

Then,

$$(\beta, \delta, \rho)$$
 solves $\mathbb{E}\left[(w_{it}, z_{it}) \ \hat{\eta}_{it} \left(\hat{\beta}, \hat{\delta}, \hat{\rho}\right)\right] = 0$ (7)

where z_{it} includes variables that are independent of η_{it} . The standard choices of instruments are $z_{it} = (x_{it-1}, y_{it-1})$ and $z_{it} = (x_{it-1}, x_{it-2})$. Note that the number of instruments z_{it} must include at least one more than the number of enodgenous variables in x_{it} .

3) Simulation Exercise:

In order to compare the estimation approaches, Figure 1 simulates the model defined above. It sets $\beta = (0.5, -0.2)$, $\delta = 1$, $\rho = 0.5$, $\gamma = 0.5$, $\eta_{it} \sim \mathcal{N}(0, 1)$, $u_{it} = u_{it-1} + \mathcal{N}(0, 1)$, and $\nu_{it} = 0$. The length of the panel is T = 3, which is the minimum required. For various choices of N, it draws 10 random samples from the model, applies the estimators, and presents the box-plot of the distribution of estimates.

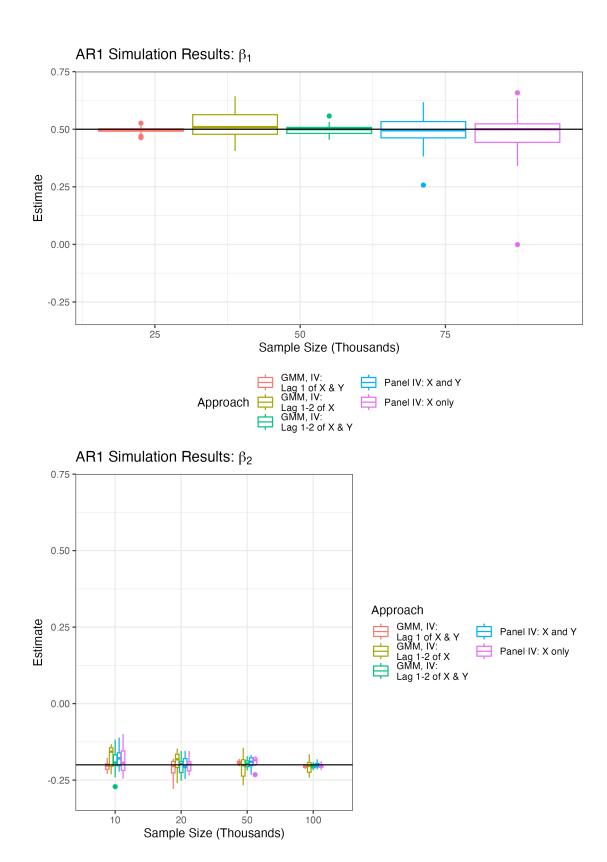


Figure 1: Simulation Exercise