For Online Publication

A Online Appendix: Data Sources and Sample Selection

All firm-level variables are constructed from annual business tax returns over the years 2001-2015: C-Corporations (Form 1120), S-Corporations (Form 1120-S), and Partnerships (Form 1065). Worker-level variables are constructed from annual tax returns over the years 2001-2015: Direct employees (Form W-2), independent contractors (Form 1099), and household income and taxation (Form 1040).

Variable Definitions:

- **Earnings:** Reported on W-2 box 1 for each Taxpayer Identification Number (TIN). Each TIN is de-identified in our data.
- **Gross Household Income:** We define gross household income as the sum of taxable wages and other income (line 22 on Form 1040) minus unemployment benefits (line 19 on Form 1040) minus taxable Social Security benefits (line 20a on Form 1040) plus tax-exempt interest income (line 8b on Form 1040). We at times also consider this measure when subtracting off Schedule D capital gains (line 13 on Form 1040).
- **Federal Taxes on Household Income:** This is given by the sum of two components. The first component is the sum of FICA Social Security taxes (given by 0.0620 times the minimum of the Social Security taxable earnings threshold, which varies by year, and taxable FICA earnings, which are reported on Box 3 of Form W-2) and FICA Medicare taxes (given by 0.0145 times Medicare earnings, which are reported on Box 5 of Form W-2). The second component is the sum of the amount of taxes owed (the difference between line 63 and line 74 on Form 1040, which is negative to indicate a refund) and the taxes already paid or withheld (the sum of lines 64, 65, 70, and 71 on Form 1040).
- **Net Household Income:** We construct a measure of net household income as Gross Household Income minus Federal Taxes on Household Income plus two types of benefits: unemployment benefits (line 19 of Form 1040) and Social Security benefits (line 20a of Form 1040).
- **Employer:** The Employer Identification Number (EIN) reported on W-2 for a given TIN. Each EIN is de-identified in our data.
- **Wage Bill:** Sum of Earnings for a given EIN plus the sum of 1099-MISC, box 7 nonemployee compensation for a given EIN in year t.
- **Size:** Number of FTE workers matched to an EIN in year t.
- **NAICS Code:** The NAICS code is reported on line 21 on Schedule K of Form 1120 for C-corporations, line 2a Schedule B of Form 1120S for S-corporations, and Box A of form

1065 for partnerships. We consider the first two digits to be the industry. We code invalid industries as missing.

- **Commuting Zone:** This is formed by mapping the ZIP code from the business filing address of the EIN on Form 1120, 1120S, or 1065 to its commuting zone.
- **Value Added:** Line 3 of Form 1120 for C-Corporations, Form 1120S for S-Corporations, and Form 1065 for partnerships. Line 3 is the difference between Revenues, reported on Line 1c, and the Cost of Goods Sold, reported on Line 2. We replace non-positive value added with missing values.
  - For manufacturers (NAICS Codes beginning 31, 32, or 33) and miners (NAICS Codes beginning 212), Line 3 is equal to Value Added minus Production Wages, defined as wage compensation for workers directly involved in the production process, per Schedule A, Line 3 instructions. If we had access to data from Form 1125-A, Line 3, we could directly add back in these production wages to recover value added. Without 1125-A, Line 3, we construct a measure of Production Wages as the difference between the Wage Bill and the Firm-reported Taxable Labor Compensation, defined below, as these differ conceptually only due to the inclusion of production wages in the Wage Bill.
- **Value Added Net of Depreciation:** Value Added minus Depreciation, where Depreciation is reported on Line 20 on Form 1120 for C-corporations, Line 14 on Form 1120S for S-corporations, and Line 16c on Form 1065 for partnerships.
- **EBITD:** We follow Kline et al. (2018a) in defining Earnings Before Interest, Taxes, and Depreciation (EBITD) as the difference between total income and total deductions other than interest and depreciation. Total income is reported on Line 11 on Form 1120 for C-corporations, Line 1c on Form 1120S for S-corporations, and Line 1c on Form 1065 for Partnerships. Total deductions other than interest and depreciation are computed as Line 27 minus Lines 18 and 20 on Form 1120 for C-corporations, Line 20 minus Lines 13 and 14 on Firm 1120S for S-corporations, and Line 21 minus Lines 15 and 16c on Form 1065 for partnerships.
- **Operating Profits:** We follow Kline et al. (2018a), who use a similar approach to Yagan (2015), in defining Operating Profits as the sum of Lines 1c, 18, and 20, minus the sum of Lines 2 and 27 on Form 1120 for C-corporations,, the sum of Lines 1c, 13, and 15, minus the sum of Lines 2 and 20 on Form 1120S for S-corporations, and the sum of Lines 1c, 16, and 16c, minus the sum of Lines 2 and 21 on Form 1065 for partnerships.
- **Firm-reported Taxable Labor Compensation:** This is the sum of compensation of officers and salaries and wages, reported on Lines 12 and 13 on Form 1120 for C-corporations, Lines 7 and 8 on Form 1120S for S-corporations, and Lines 9 and 10 on Form 1065 for Partnerships.

- **Firm-reported Non-taxable Labor Compensation:** This is the sum of employer pension and employee benefit program contributions, reported on Lines 17 and 18 on Form 1120 for C-corporations, Lines 17 and 18 on form 1120S for S-corporations, and Lines 18 and 19 on Form 1065 for Partnerships.

- **Multinational Firm:** We define an EIN as a multinational in year t if it reports a non-zero foreign tax credit on Schedule J, Part I, Line 5a of Form 1120 or Form 1118, Schedule B, Part III, Line 6 of Form 1118 for a C-corporation in year t, or if it reports a positive Total Foreign Taxes Amount on Schedule K, Line 16l of of Form 1065 for a partnership in year t.

- **Tenure:** For a given TIN, we define tenure at the EIN as the number of prior years in which the EIN was the highest-paying.

- **Age and Sex:** Age at t is the difference between t and birth year reported on Data Master-1 (DM-1) from the Social Security Administration, and sex is the gender reported on DM-1 (see for further details on the DM-1 link).

	Goods				Services				All
	Midwest	Northeast	South	West	Midwest	Northeast	South	West	All
Panel A.	**Full Sample**								
Observation Counts:									
Number of FTE Worker-Years	42,908,008	26,699,951	40,312,311	31,585,748	69,044,540	62,386,621	103,227,384	71,355,046	447,519,609
Number of Unique FTE Workers	9,318,707	6,088,530	10,215,128	7,712,759	17,314,497	15,167,028	26,519,284	17,949,625	89,570,480
Number of Unique Firms with FTE Workers	294,879	232,717	439,641	329,566	1,051,548	1,054,944	1,908,178	1,314,168	6,478,231
Number of Unique Markets with FTE Workers	1,508	264	1,774	910	4,092	744	4,909	2,492	16,141
Group Counts:									
Mean Number of FTE Workers per Firm	22.1	17.8	16.1	16.3	10.4	9.7	9.5	9.6	11.4
Mean Number of FTE Workers per Market	2,012.9	6,856.7	1,586.3	2,539.3	1,221.0	5,723.0	1,492.8	2,097.7	1,915.1
Mean Number of Firms per Market with FTE Workers	91.3	384.9	98.3	156.0	117.4	588.2	156.6	217.7	167.6
Outcome Variables in Log $:									
Mean Log Wage for FTE Workers	10.76	10.81	10.70	10.81	10.61	10.74	10.62	10.70	10.69
Mean Value Added for FTE Workers	17.36	16.80	16.68	16.64	16.18	16.04	15.94	16.07	16.31
Firm Aggregates in $1,000:									
Wage Bill per Worker	43.6	50.7	42.2	52.9	34.1	44.2	35.8	40.3	40.8
Value Added per Worker	91.2	107.5	85.2	91.7	90.5	111.1	94.2	92.3	95.2
Panel B.	**Movers Sample**								
Observation Counts:									
Number of FTE Mover-Years	17,455,849	11,543,303	18,066,928	15,513,020	31,643,497	28,390,782	50,052,742	35,324,301	207,990,422
Number of Unique FTE Movers	4,124,895	2,829,881	4,819,645	3,876,182	7,723,804	6,662,132	11,904,098	8,321,469	32,070,390
Number of Unique Firms with FTE Movers	188,376	144,268	265,374	215,092	571,360	549,064	1,018,957	700,618	3,559,678
Number of Unique Markets with FTE Movers	1,457	261	1,747	872	3,899	739	4,766	2,342	15,586
Group Counts:									
Mean Number of FTE Movers per Firm with FTE Movers	13.5	11.9	11.2	11.6	8.2	7.9	7.9	8.2	8.9
Mean Number of Movers per Market with FTE Movers	864.8	2,991.3	732.4	1,318.1	599.3	2,655.3	761.5	1,123.7	940.6
Mean Number of Firms per Market with FTE Movers	64.1	251.1	65.5	113.4	72.7	337.1	96.4	137.7	105.5
Outcome Variables in Log $:									
Mean Log Wage for FTE Movers	10.68	10.77	10.64	10.78	10.59	10.72	10.61	10.70	10.67
Mean Value Added for FTE Movers	16.72	16.52	16.28	16.36	16.04	16.02	15.88	16.01	16.12
Panel C.	**Stayers Sample**								
Sample Counts:									
Number of 8-year Worker-Firm Stayer Spells	2,588,628	1,777,928	1,237,821	1,150,115	2,315,238	2,527,212	2,609,997	2,207,552	16,506,865
Number of Unique FTE Stayers in Firms with 10 FTE Stayers	798,575	532,507	416,549	354,518	740,091	764,699	865,629	724,155	5,217,960
Number of Unique Firms with 10 FTE Stayers	13,884	10,896	9,409	9,767	18,083	19,475	19,626	16,185	117,698
Number of Unique Markets with 10 Firms with 10 FTE Stayers	197	111	216	104	335	213	438	219	1,826
Outcome Variables in Log $:									
Mean Log Wage for FTE Stayers	10.95	10.99	10.97	10.99	10.90	11.01	10.96	11.05	10.97
Mean Log Value Added for FTE Stayers	18.04	17.56	17.46	16.56	17.45	17.23	17.89	17.93	17.61

Table A.1: Detailed sample characteristics

Notes: This table provides a detailed examination of the full sample, movers sample, and stayers sample.

B Online Appendix: Key Features of the U.S. Labor Market

B.1 Unconditional Moment condition

Lemma 1. *Under Assumptions 1 and 2 and assuming $\gamma = \Upsilon$, we have that*

$$\mathbb{E}\left[\Delta y_{j(i)t}\left(w_{it+\tau} - w_{it-\tau'} - \gamma\left(y_{j(i),t+\tau} - y_{j(i),t-\tau'}\right)\right)|S_i\right] = 0 \text{ for } \tau \geq 2, \tau' \geq 3,$$

where we defined $S_i = 1[j(i,1) = ... = j(i,T) = j(i)]$.

Proof. We start by expressing each of the terms $\Delta y_{j(i)t}$, $w_{it+\tau} - w_{it-\tau'}$ and $y_{j(i),t+\tau} - y_{j(i),t-\tau'}$ using the assumptions on the process presented in Section 3. We get that:

$$\Delta y_{jt} = u_{jt} + \xi_{jt} - \xi_{jt-1} + \delta^y\left(\xi_{jt-1} - \xi_{jt-2}\right)$$

and

$$y_{j(i),t+\tau} - y_{j(i),t-\tau'} = \sum_{d=t-\tau'+1}^{t+\tau} u_{jd} + \xi_{jt+\tau} - \xi_{jt-\tau'} + \delta^y\left(\xi_{jt+\tau-1} - \xi_{jt-\tau'-1}\right)$$

and finally

$$w_{i,t+\tau} - w_{i,t+\tau'} = \sum_{d=t-\tau'+1}^{t+\tau} \mu_{id} + \gamma u_{j(i),d} + \nu_{it+\tau} - \nu_{it-\tau'} + \delta^w\left(\nu_{it+\tau-1} - \nu_{it-\tau'-1}\right).$$

Taking the difference using γ, the second term in the product only contains transitory firm shocks:

$$\begin{aligned} w_{it+\tau} - w_{it-\tau'} - \gamma\left(y_{j(i),t+\tau} - y_{j(i),t-\tau'}\right) = &\sum_{d=t-\tau'+1}^{t+\tau} \mu_{id} + \nu_{it+\tau} - \nu_{it-\tau'} + \delta^w\left(\nu_{it+\tau-1} - \nu_{it-\tau'-1}\right) \\ &- \gamma\left(\xi_{jt+\tau} - \xi_{jt-\tau'} + \delta^y(\xi_{jt+\tau-1} - \xi_{jt-\tau'-1})\right). \end{aligned}$$

The permanent firm shocks $\tilde{u}_{jt}$ and $\bar{u}_{jt}$ cancelled each other. Finally, when multiplying by Δy_{jt} we end up with the following list of cross-products:

$$\begin{aligned} &\mathbb{E}\left[u_{j(i)t}\mu_{id}|S_i = 1\right] \text{for } d = t-\tau'+1, ..., t+\tau \\ &\mathbb{E}\left[u_{j(i),t}\nu_{id}|S_i = 1\right] \text{for } d = t+\tau, t+\tau-1, t-\tau', t-\tau'-1 \\ &\mathbb{E}\left[\xi_{j(i),d'}\nu_{id}|S_i = 1\right] \text{for } d = t+\tau, t+\tau-1, t-\tau', t-\tau'-1 \text{ and } d' = t, t-1, t-2 \\ &\mathbb{E}\left[\xi_{j(i),d'}\mu_{id}|S_i = 1\right] \text{for } d = t-\tau'+1, ..., t+\tau \text{ and } d' = t, t-1, t-2 \end{aligned}$$

As long as $t+\tau-1 > t$ and $t-\tau' < t-2$ or in other words that $\tau \geq 2$ and $\tau' \geq 3$, all terms will be equal to 0 thanks to Assumption 2.

We also get $\mathbb{E}\left[\xi_{j(i)d}\xi_{j(i)d'}|S_i=1\right]$ which is zero by Assumption 1 when $d \neq d'$. The combination $d = d'$ does not appear when $\tau \geq 2$ and $\tau' \geq 3$ and hence we also get that all terms average to 0 giving our result:

$$\mathbb{E}\left[\Delta y_{j(i)t}\left(w_{it+\tau} - w_{it-\tau'} - \gamma\left(y_{j(i),t+\tau} - y_{j(i),t-\tau'}\right)\right)|S_i\right] = 0.$$

Finally we we also establish that the coefficient on γ is strictly positive. Indeed $\mathbb{E}\left[\Delta y_{j(i)t}\left(y_{j(i),t+\tau} - y_{j(i),t-\tau'}\right)|S_i\right]$ includes a $\mathbb{E}\left[u_{j(i)t}u_{j(i)t}|S_i=1\right]$ which is strictly positive whenever $\sigma^2_{\tilde{u}} > 0$ or $\sigma^2_{\bar{u}} > 0$. $\square$

B.2 DiD expression

Lemma 2. *Under the assumption that VA growth is $-\delta$ or δ each with probability one half, the pass-through parameters γ can be expressed as the ratio of two Difference in Differences as follows:*

$$\gamma = \frac{\mathbb{E}\left[w_{it+\tau} - w_{it-\tau'}|+\delta, S_i{=}1\right] - \mathbb{E}\left[w_{it+\tau} - w_{it-\tau'}|-\delta, S_i{=}1\right]}{\mathbb{E}\left[y_{j(i),t+\tau} - y_{j(i),t-\tau'}|+\delta, S_i{=}1\right] - \mathbb{E}\left[y_{j(i),t+\tau} - y_{j(i),t-\tau'}|-\delta, S_i{=}1\right]}.$$

Proof. We show in this section that γ can be written as the ratio of two difference-in-difference. We start with the following expression for γ where we have shown in Lemma 1 that the denominator will be strictly positive:

$$\gamma = \frac{\mathbb{E}\left[\Delta y_{j(i)t}\left(w_{it+\tau} - w_{it-\tau'}\right)|S_i{=}1\right]}{\mathbb{E}\left[\Delta y_{j(i)t}\left(y_{j(i),t+\tau} - y_{j(i),t-\tau'}\right)|S_i{=}1\right]}.$$

We rewrite the numerator under the assumption that the value added growth only takes 2 values $+\delta$ and $-\delta$, as if it was a binary instrument:

$$\begin{aligned}
\mathbb{E}\left[\Delta y_{j(i)t}\left(w_{it+\tau} - w_{it-\tau'}\right)|S_i{=}1\right] &= \delta \cdot Pr\left[\Delta y_{j(i)t} = \delta\right]\mathbb{E}\left[w_{it+\tau} - w_{it-\tau'}|S_i{=}1, \Delta y_{j(i)t} = \delta\right] \\
&\quad - \delta \cdot Pr\left[\Delta y_{j(i)t} = -\delta\right]\mathbb{E}\left[w_{it+\tau} - w_{it-\tau'}|S_i{=}1, \Delta y_{j(i)t} = -\delta\right] \\
&= \frac{\delta}{2}\mathbb{E}\left[w_{it+\tau} - w_{it-\tau'}|S_i{=}1, \Delta y_{j(i)t} = \delta\right] \\
&\quad - \frac{\delta}{2}\mathbb{E}\left[w_{it+\tau} - w_{it-\tau'}|S_i{=}1, \Delta y_{j(i)t} = -\delta\right].
\end{aligned}$$

We then rewrite the denominator in the exact same way to get:

$$\begin{aligned}
\mathbb{E}\left[\Delta y_{j(i)t}\left(y_{j(i),t+\tau} - y_{j(i),t-\tau'}\right)|S_i{=}1\right] =& \frac{\delta}{2}\mathbb{E}\left[y_{j(i),t+\tau} - y_{j(i),t-\tau'}|S_i{=}1, \Delta y_{j(i)t} = \delta\right] \\
&- \frac{\delta}{2}\mathbb{E}\left[y_{j(i),t+\tau} - y_{j(i),t-\tau'}|S_i{=}1, \Delta y_{j(i)t} = -\delta\right].
\end{aligned}$$

giving the following final expression:

$$\gamma = \frac{\mathbb{E}\left[w_{it+\tau} - w_{it-\tau'}|+\delta, S_i{=}1\right] - \mathbb{E}\left[w_{it+\tau} - w_{it-\tau'}|-\delta, S_i{=}1\right]}{\mathbb{E}\left[y_{j(i),t+\tau} - y_{j(i),t-\tau'}|+\delta, S_i{=}1\right] - \mathbb{E}\left[y_{j(i),t+\tau} - y_{j(i),t-\tau'}|-\delta, S_i{=}1\right]}.$$

The numerator is a difference-in-difference for earnings where the change between $t-\tau'$ and $t+\tau$ is the first difference and the change between $\Delta y_{j(i)t}=-\delta$ and $\Delta y_{j(i)t}=\delta$ is the second difference. The denominator is the same Diff-in-Diff applied to the firm value added. $\square$

B.3 Moment condition with market shocks

Lemma 3. *Under Assumptions 1 and 2, we have that:*

$$\mathbb{E}\left[\Delta y_{j(i)t}\left(\tilde{w}_{it+\tau}-\tilde{w}_{it-\tau}-\gamma\left(\tilde{y}_{j(i),t+\tau}-\tilde{y}_{j(i),t-\tau}\right)\right)|S_i{=}1\right]=0 \text{ for } \tau\geq 2, \tau'\geq 3.$$

where $\tilde{w}_{it}=w_{it}-\mathbb{E}\left[w_{i't}\mid j(i',t)\in J_{r(j(i,t))}\right]$ *denotes wages net of market-time dummies.*

Proof. As in Lemma 1 we express each of the three terms separately. The expression for Δy_{jt} is identical and we can split the permanent shock into market and firm specific:

$$\Delta y_{jt}=\tilde{u}_{jt}+\bar{u}_{r(j)t}+\xi_{jt}-\xi_{jt-1}+\delta^{y}\left(\xi_{jt-1}-\xi_{jt-2}\right)$$

Next we look at the difference in earnings:

$$\begin{aligned}\tilde{w}_{it+\tau}-\tilde{w}_{it-\tau}&=\sum_{d=t-\tau'+1}^{t+\tau}\mu_{id}+\gamma\tilde{u}_{j(i),d}+\nu_{it+\tau}-\nu_{it-\tau'}+\delta^{w}\left(\nu_{it+\tau-1}-\nu_{it-\tau'-1}\right)\\&\quad-\mathbb{E}\left[\sum_{d=t-\tau'+1}^{t+\tau}\mu_{i'd}+\nu_{i't+\tau}-\nu_{i't-\tau'}+\delta^{w}(\nu_{i't+\tau-1}-\nu_{i't-\tau'-1})|j(i')\in r(j(i))\right]\\&=\sum_{d=t-\tau'+1}^{t+\tau'}\mu_{id}+\gamma\tilde{u}_{j(i),d}+\nu_{it+\tau}-\nu_{it-\tau'}+\delta^{w}\left(\nu_{it+\tau-1}-\nu_{it-\tau'-1}\right)\end{aligned}$$

where the expectation terms reduces to the common market shock since firm level permanent innovations average to zero in the market by definition and that firm specific transitory shocks average to zero by assumption. The market shock then cancel each other and the expression reduces to

$$\tilde{w}_{it+\tau}-\tilde{w}_{it-\tau}=\sum_{d=t-\tau'+1}^{t+\tau}\mu_{id}+\gamma\tilde{u}_{j(i),d}+\nu_{it+\tau}-\nu_{it-\tau'}+\delta^{w}\left(\nu_{it+\tau-1}-\nu_{it-\tau'-1}\right).$$

Similarly we can express the value added term as

$$\tilde{y}_{j(i),t+\tau}-\tilde{y}_{j(i),t-\tau}=\sum_{d=t-\tau'+1}^{t+\tau}\tilde{u}_{j(i),d}+\xi_{jt+\tau}-\xi_{jt-\tau'}+\delta^{y}\left(\xi_{jt+\tau-1}-\xi_{jt-\tau'-1}\right).$$

Combining the two gives:

$$\tilde{w}_{it+\tau} - \tilde{w}_{it-\tau} - \gamma\left(\tilde{y}_{j(i),t+\tau} - \tilde{y}_{j(i),t-\tau}\right) = \sum_{d=t-\tau'+1}^{t+\tau} \mu_{id} + \nu_{it+\tau} - \nu_{it-\tau'} + \delta^{w}\left(\nu_{it+\tau-1} - \nu_{it-\tau'-1}\right)$$
$$- \gamma\left(\xi_{jt+\tau} - \xi_{jt-\tau'} + \delta^{y}\left(\xi_{jt+\tau-1} - \xi_{jt-\tau'-1}\right)\right)$$

where the firm level shocks $\tilde{u}_{jt}$ canceled each other. For reasons identical to Lemma 1, as long as $\tau \geq 2, \tau' \geq 3$, all interaction terms will average to 0, delivering the result:

$$\mathbb{E}\left[\Delta y_{j(i)t}\left(\tilde{w}_{it+\tau} - \tilde{w}_{it-\tau} - \gamma\left(\tilde{y}_{j(i),t+\tau} - \tilde{y}_{j(i),t-\tau}\right)\right) | S_i{=}1\right] = 0.$$

Note that one can instrument with either $\Delta y_{j(i)t}$ or $\Delta \tilde{y}_{j(i)t}$ since both include $\tilde{u}_{jt}$. □

Lemma 4. *Next we establish that*

$$\mathbb{E}\left[\Delta y_{j(i),t}\left(\bar{w}_{it+\tau} - \bar{w}_{it-\tau'} - \Upsilon\left(\bar{y}_{j(i),t+\tau} - \bar{y}_{j(i),t-\tau'}\right)\right) | S_i{=}1\right] = 0$$
$$\textit{for } \tau \geq 2, \tau' \geq 3$$

where $\bar{w}_{it} = \mathbb{E}\left[w_{i't} \mid j(i',t) \in J_{r(j(i,t))}\right]$.

Proof. Similar to the previous Lemma we start by expressing each term. At the market level we get:

$$\bar{w}_{it+\tau} - \bar{w}_{it-\tau} = \Upsilon \sum_{d=t-\tau'+1}^{t+\tau} \bar{u}_{r(j(i)),d}$$
$$\bar{y}_{jt+\tau} - \bar{y}_{jt-\tau'} = \sum_{d=t-\tau'+1}^{t+\tau} \bar{u}_{r(j(i)),d}$$

and

$$\Delta y_{jt} = \tilde{u}_{jt} + \bar{u}_{r(j)t} + \xi_{jt} - \xi_{jt-1} + \delta^{y}\left(\xi_{jt-1} - \xi_{jt-2}\right).$$

This gives rise to a similar but simpler approach. as long as $\tau \geq 2, \tau' \geq 3$, all interaction terms will average to 0, delivering the result:

$$\mathbb{E}\left[\Delta y_{j(i),t}\left(\bar{w}_{it+\tau} - \bar{w}_{it-\tau'} - \Upsilon\left(\bar{y}_{j(i),t+\tau} - \bar{y}_{j(i),t-\tau'}\right)\right) | S_i{=}1\right] = 0.$$

Note that one can instrument with either $\Delta y_{j(i)t}$ or $\Delta \bar{y}_{j(i)t}$ since both include $\bar{u}_{rt}$. Also note that here we used the fact that the transitory shocks average out within market but we could have relaxed that. □

B.4 Adjusted two-way fixed effect regression

Lemma 5. *We show that*

$$\mathbb{E}[w_{it} - \gamma(y_{j(i,t),t} - y_{j(i,t),1}) - (\Upsilon - \gamma)\left(\bar{y}_{r(i,t),t} - \bar{y}_{r(i,t),1}\right)|j(i,1),...,j(i,T)] = \phi_{ij(i,t)}.$$

Proof. We assume that the initial conditions for the permanent component of earnings is $w^p_{i1} = 0$ and hence we load the initial condition into the match effect ϕ_{ij}. We then get that

$$w_{it} = \phi_{ij(i,t)} + \sum_{\tau=2}^{t} \mu_{i\tau} + \gamma \tilde{u}_{j(i,t),\tau} + \Upsilon \bar{u}_{r(j(i,t)),\tau} + \nu_{i,t} + \delta^w \nu_{i,t-1}$$

and similarly we get that

$$\begin{aligned} y_{j(i,t),t} - y_{j(i,t),1} = \sum_{\tau=2}^{t} & \tilde{u}_{j(i,t),\tau} + \bar{u}_{j(i,t),\tau} \\ & + \xi_{i,t} - \xi_{i,1} + \delta^y\left(\xi_{i,t-1} - \xi_{i,0}\right) \end{aligned}$$

and by assumption on the transitory errors and firm specific innovation we get that

$$\begin{aligned} \mathbb{E}[\bar{y}_{r(i),t} - \bar{y}_{r(i),1}|j(i,1),...,j(i,T)] &= \sum_{\tau=2}^{t} \bar{u}_{r(i),\tau} \\ \mathbb{E}[y_{j(i),t} - y_{j(i),1}|j(i,1),...,j(i,T)] &= \sum_{\tau=2}^{t} \bar{u}_{r(j(i)),\tau} + \tilde{u}_{j(i),\tau} \end{aligned}$$

and bringing all together we get that

$$\begin{aligned} \mathbb{E}[w_{it} - \gamma(y_{j(i,t),t} - y_{j(i,t),1}) - (\Upsilon - \gamma)\left(\bar{y}_{r(i,t),t} - \bar{y}_{r(i,t),1}\right)|j(i,1),...,j(i,T)] \\ = \phi_{ij(i,t)} + \mathbb{E}[\xi_{i,t} + \delta^y \xi_{i,t-1} - \xi_{i,1} - \delta^y \xi_{i,0}|j(i,1),...,j(i,T)] \\ = \phi_{ij(i,t)}. \end{aligned}$$

Using the average change at the firm and market level allows removing the time varying common part at each time t. Using $y_{j(i,t),1}$ allows removing the fixed effect ζ_j from the firms. In practice one can use the average instead of the first value in time for $y_{j(i,t),1}$ and $\bar{y}_{r(i,t),1}$. □

B.5 Estimating the rest of the process parameters

We estimate the pass-through rates in two steps. First, we estimate the parameters for the value added process. Second, we jointly estimate the pass-through parameters and the parameter of the wage process for the worker.

For the value added process, we use a GMM approach where we put the full variance-covariance in growth from a panel of 8-year spells for stayers. The matrix of moments uses the growth at $t = 3, 4, 5, 6, 7$.[1] This is in order to not use any data from the first ($t = 1$) and last ($t = 8$) years of the spell. We do this because first and last years of a spell can be partial spells, hence focusing on the middle alleviates the issue of not observing within-years dates for start and end of job.

So we construct a 5×5 matrix M_y from the data where the (p, q) element is $M_y(p, q) = Cov(\Delta y_{ip}, \Delta y_{iq})$. We can construct the same moments matrix in the model as a function of $\{\delta^y, \sigma_u, \sigma_\xi\}$ which we denote $M^*_y(p, q; \delta^y, \sigma_u, \sigma_\xi)$.We use as a matrix the diagonal matrix with variance implied by joint normality across the Δy_{it}. The weight associated with $Cov(\Delta y_{ip}, \Delta y_{iq})$ is then $W_y(p, q) = Cov(\Delta y_{ip}, \Delta y_{iq})^2 + Var(\Delta y_{ip})Var(\Delta y_{iq})$.

The estimator is the minimum distance estimator defined as:

$$\arg \min_{\delta^y, \sigma_u, \sigma_\xi} \sum_{p=3}^{7} \sum_{q=3}^{7} W_y(p, q) \left(M^*_y(p, q; \delta^y, \sigma_u, \sigma_\xi) - M_y(p, q) \right)^2 .$$

In step 2 we construct two matrices each of size 5×5:

$$\begin{aligned} M_w(p, q) &= Cov(\Delta w_{ip}, \Delta w_{iq}) \\ M_{wy}(p, q) &= Cov(\Delta w_{ip}, \Delta y_{iq}), \end{aligned}$$

and we denote $M^*_w(p, q; \delta^w, \sigma_\mu, \sigma_\nu, \gamma, \zeta)$ and $M^*_{wy}(p, q; \delta^w, \sigma_\mu, \sigma_\nu)$ as the matrices constructed from the model. These matrices are also functions of $(\delta^y, \sigma_u, \sigma_\xi)$ and we use the parameters estimated in the first step. The weighting matrix is constructed in a similar way using diagonal weights only and the joint normality assumption.

$$\begin{aligned} W_w(p, q) &= Cov(\Delta w_{ip}, \Delta w_{iq})^2 + Var(\Delta w_{ip})Var(\Delta w_{iq}) \\ W_{wy}(p, q) &= Cov(\Delta w_{ip}, \Delta y_{iq})^2 + Var(\Delta w_{ip})Var(\Delta y_{iq}) \end{aligned}$$

Finally we do the following:

$$\begin{aligned} \arg \min_{p,q;\delta^w, \sigma_\mu, \sigma_\nu, \gamma, \zeta} \sum_{p=3}^{7} \sum_{q=3}^{7} & W_w(p, q) \left(M^*_w(p, q; \delta^w, \sigma_\mu, \sigma_\nu, \gamma, \zeta) - M_w(p, q) \right)^2 + \\ & W_{wy}(p, q) \left(M^*_{wy}(p, q; \delta^w, \sigma_\mu, \sigma_\nu, \gamma, \zeta) - M_{wy}(p, q) \right)^2 . \end{aligned}$$

In practice, all of these expressions are polynomials in the parameters. We solve the minimization problem using global polynomial optimization as in Lasserre (2001). This allows us to formally certify the global optimality of the solution.

For inference, we use a joint bootstrap of M_y, M_w, M_{yw}. We computed the bootstrap by resampling at the commuting zone by industry level, representing about 2000 clusters.

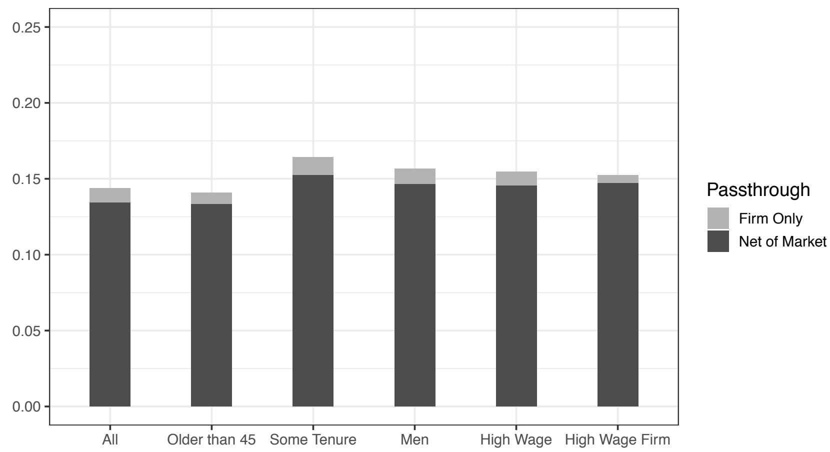
The main results are displayed in Online Appendix Table B.1. Additional heterogeneity and robustness analyses are presented in Online Appendix Figure B.1.

[1] In the case of MA(1), one can also use $t = 2$, however we wanted to test for MA(2) as a robustness.

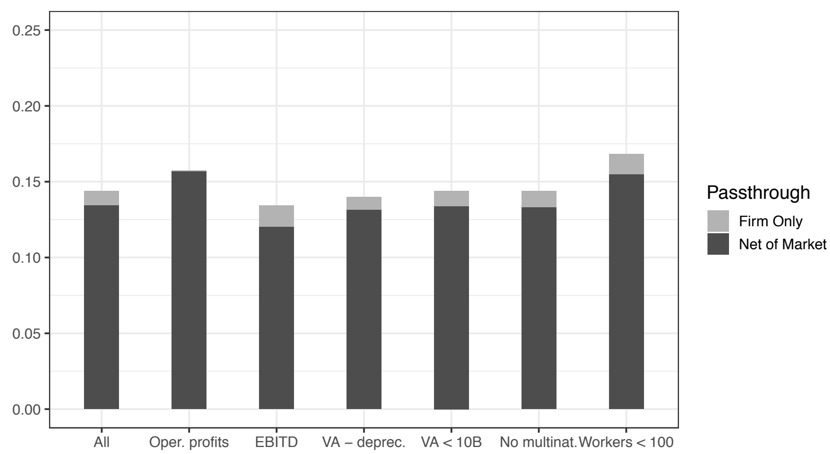
	GMM Estimates of Joint Process			
	Firm Only		Accounting for Markets	
	Log Value Added	Log Earnings	Log Value Added	Log Earnings
Panel A.	**Process: MA(1)**			
Total Growth (Std. Dev.)	0.31	0.17	0.29	0.16
	(0.01)	(0.00)	(0.01)	(0.00)
Permanent Shock (Std. Dev.)	0.20	0.10	0.17	0.10
	(0.01)	(0.00)	(0.01)	(0.00)
Transitory Shock (Std. Dev.)	0.18	0.10	0.17	0.10
	(0.01)	(0.00)	(0.01)	(0.00)
MA Coefficient, Lag 1	0.09	0.15	0.09	0.15
	(0.01)	(0.00)	(0.01)	(0.00)
MA Coefficient, Lag 2	0.00	0.00	0.00	0.00
	(0.00)	(0.00)	(0.00)	(0.00)
Permanent Passthrough Coefficient		0.14		0.13
		(0.01)		(0.01)
Transitory Passthrough Coefficient		-0.01		0.00
		(0.01)		(0.00)
Market Passthrough Coefficient				0.18
				(0.02)
Panel B.	**Process: MA(2)**			
Total Growth (Std. Dev.)	0.31	0.17	0.29	0.16
	(0.01)	(0.00)	(0.01)	(0.00)
Permanent Shock (Std. Dev.)	0.20	0.10	0.17	0.10
	(0.01)	(0.00)	(0.00)	(0.00)
Transitory Shock (Std. Dev.)	0.17	0.10	0.17	0.10
	(0.01)	(0.00)	(0.01)	(0.00)
MA Coefficient, Lag 1	0.05	0.21	0.07	0.21
	(0.05)	(0.01)	(0.04)	(0.01)
MA Coefficient, Lag 2	-0.03	0.04	-0.01	0.04
	(0.03)	(0.00)	(0.02)	(0.00)
Permanent Passthrough Coefficient		0.15		0.13
		(0.01)		(0.01)
Transitory Passthrough Coefficient		-0.02		0.00
		(0.01)		(0.00)
Market Passthrough Coefficient				0.18
				(0.03)

Table B.1: GMM estimates of the earnings and value added processes

Notes: This table displays the parameters of the joint processes of log value added and log earnings. These results come from joint estimation of the earnings and value added processes (3) and (4) using GMM. Columns 1-2 report results from the specification which imposes $\Upsilon = \gamma$ ("Firm only"), while columns 3-4 report results from the specification which allows Υ to differ from γ ("Accounting for Markets"). The top panel assumes the transitory components follow an MA(1) process. The bottom panel permits the transitory components to follow an MA(2) process. Standard errors are estimated using 40 block bootstrap draws in which the block is taken to be the market.



(a) Heterogeneity across Workers



(b) Heterogeneity across Firms and Value Added Measures

Figure B.1: Heterogeneity in pass-through rates of firm shocks

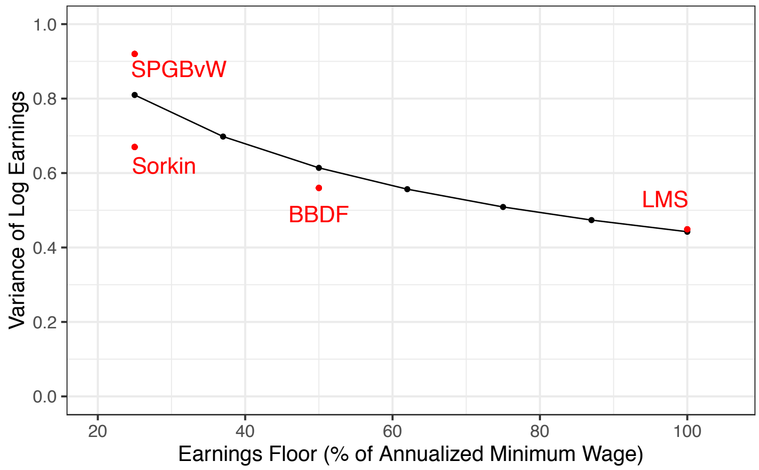
Notes: This figure displays heterogeneity in the GMM estimates of the pass-through rates of a firm shock, both for the firm only model (imposing $\Upsilon = \gamma$) and when removing market by year means (permitting $\Upsilon \neq \gamma$).

B.6 Mobility Bias and Firm and Worker Effect Estimation

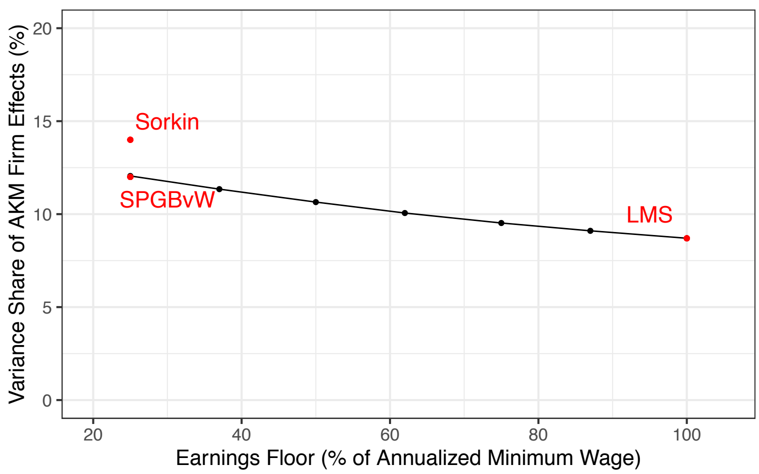
Sample:	Full Sample	≥ 2 Movers	Connected Set
Workers in 2001-2008:			
Worker-Years (Millions)	245.0	227.8	227.4
	(100.0%)	(93.0%)	(92.8%)
Unique Workers (Millions)	66.2	61.8	61.7
	(100.0%)	(93.3%)	(93.2%)
Workers in 2008-2015:			
Worker-Years (Millions)	232.9	212.4	211.9
	(100.0%)	(91.2%)	(91.0%)
Unique Workers (Millions)	64.0	58.8	58.6
	(100.0%)	(91.9%)	(91.7%)

Table B.2: Floor on Number of Movers and the Connected Set

Notes: This table demonstrates the fraction of workers kept in the sample in the AKM and BLM analysis when imposing that a firm must have at least two movers and must belong to the connected set of firms.



(a) Total Variance of Log Earnings



(b) AKM Estimates of Firm Component

Figure B.2: Earnings Variance and AKM Estimates of Firm Component by Earnings Floor

Notes: In this figure, we report estimates of the variance of log earnings (subfigure a) and AKM estimates of the firm component (subfigure b) when imposing different FTE wage floors. Literature abbreviations are BBDF for Barth et al. (2016), SPGBvW for Song et al. (2018), Sorkin for Sorkin (2018), and LMS for the baseline estimates in this paper.

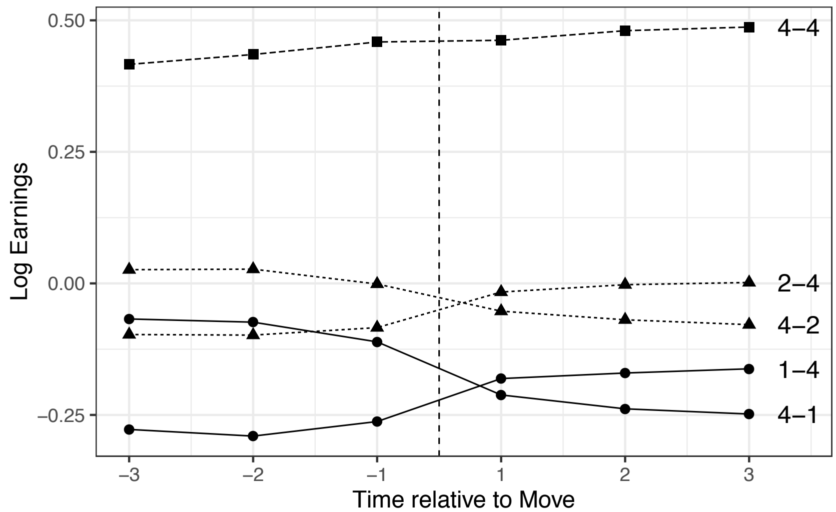


Figure B.3: Event Study of Changes in Earnings when Workers Move Between Firms

Notes: In this figure, we classify firms into four equally sized groups based on the mean earnings of stayers in the firm (with 1 and 4 being the group with the lowest and highest mean earnings, respectively). We then compute mean log earnings for the workers that move between these groups of firms in the years before and after the move. Note that the employer differs between event times -1 and 1, but we do not know exactly when the change in employer occurred. Thus, to avoid concerns over workers exiting and entering employment during these years, one might prefer to compare earnings in event years -2 and 2.

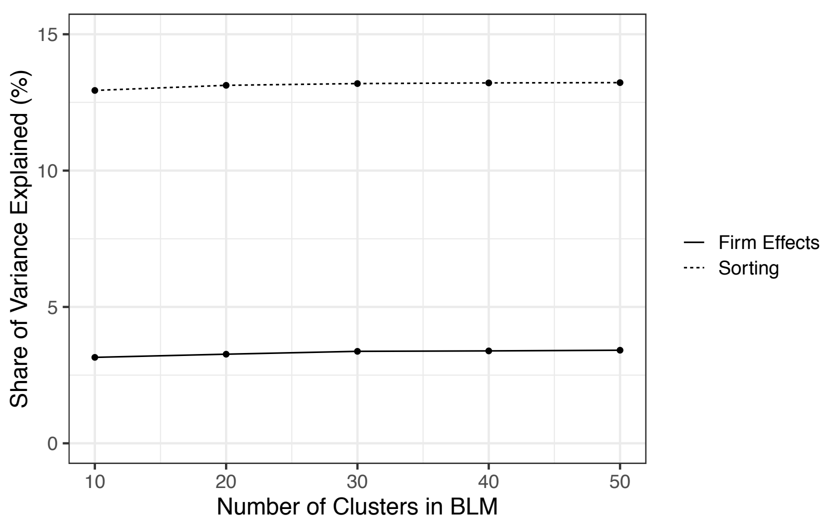


Figure B.4: BLM Decomposition by Number of Clusters

Notes: In this figure, we estimate the BLM decomposition for different numbers of firm clusters.

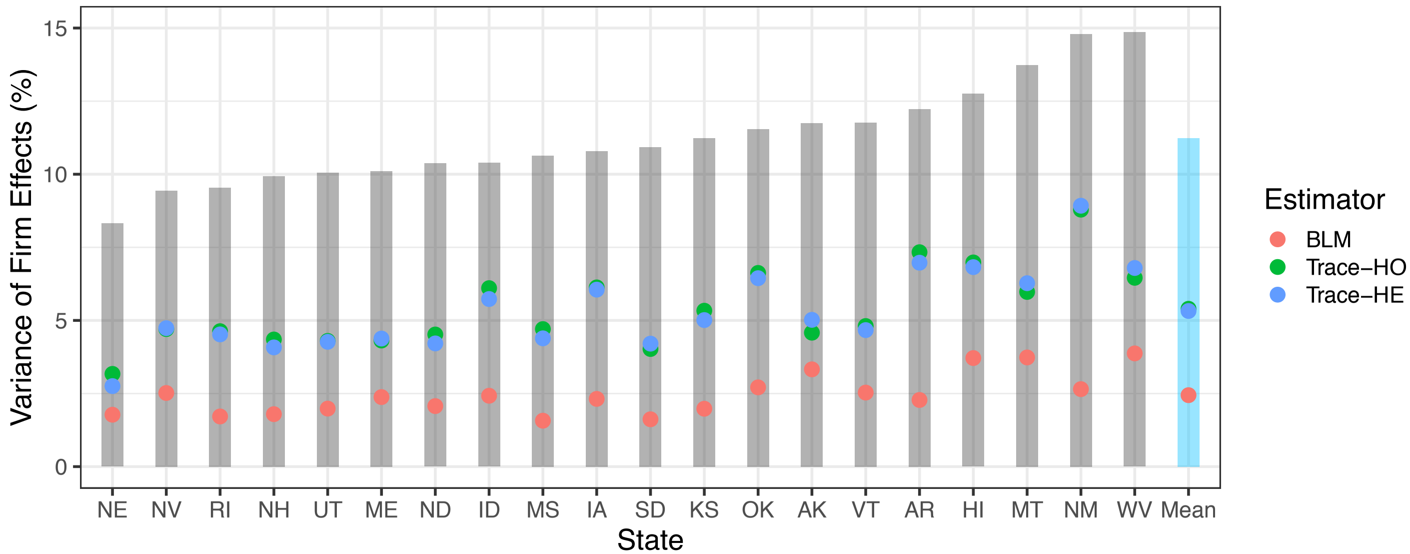


Figure B.5: Comparison of Estimators for a Subset of the Smaller States in the U.S.

Notes: This figure considers the 2001-2008 sample of workers for a number of smaller U.S. states. It compares four estimators: Abowd et al. (1999) (AKM, in the shaded bars), the Andrews et al. (2008) estimator (Trace-HO), the Kline et al. (2018b)estimator (Trace-HE), and the Bonhomme et al. (2019) estimator (BLM). All estimation is performed on the leave-one-out connected set. We also report the average of the estimated variances of firm effects across the states.

	Goods				Services			
	Midwest	Northeast	South	West	Midwest	Northeast	South	West
Panel A.	**Model Parameters**							
Idyosinctratic taste parameter (β^{-1})	0.200							
	(0.044)							
Taste correlation parameter (ρ)	0.844	0.694	0.719	0.924	0.649	0.563	0.744	0.619
	(0.179)	(0.153)	(0.160)	(0.182)	(0.141)	(0.109)	(0.246)	(0.117)
Returns to scale ($1-\alpha$)	0.746	0.764	0.863	0.949	0.753	0.740	0.814	0.752
	(0.016)	(0.013)	(0.017)	(0.019)	(0.013)	(0.015)	(0.036)	(0.015)
Panel B.	**Firm-level Rents and Rent Shares**							
Workers' Rents:								
Per-worker Dollars	6,802	6,681	5,737	8,906	4,234	4,847	5,009	4,805
	(770)	(723)	(720)	(867)	(502)	(803)	(1,295)	(684)
Share of Earnings	16%	13%	14%	17%	12%	11%	14%	12%
	(2%)	(1%)	(2%)	(2%)	(1%)	(2%)	(4%)	(2%)
Firms' Rents:								
Per-worker Dollars	4,041	4,198	7,465	20,069	3,531	3,097	6,915	3,018
	(1,243)	(1,130)	(2,681)	(6,323)	(1,004)	(1,305)	(5,650)	(1,060)
Share of Profits	8%	7%	17%	52%	6%	5%	12%	6%
	(3%)	(2%)	(6%)	(16%)	(2%)	(2%)	(10%)	(2%)
Workers' Share of Rents	63%	61%	43%	31%	55%	61%	42%	61%
	(4%)	(4%)	(5%)	(4%)	(4%)	(5%)	(9%)	(5%)
Panel C.	**Market-level Rents and Rent Shares**							
Workers' Rents:								
Per-worker Dollars	7,837	9,102	7,572	9,506	6,115	7,935	6,422	7,230
	(1,319)	(1,532)	(1,274)	(1,600)	(1,029)	(1,335)	(1,081)	(1,217)
Share of Earnings	18%	18%	18%	18%	18%	18%	18%	18%
	(3%)	(3%)	(3%)	(3%)	(3%)	(3%)	(3%)	(3%)
Firms' Rents:								
Per-worker Dollars	4,940	6,311	10,000	20,846	5,734	5,897	9,363	5,153
	(1,140)	(1,350)	(2,267)	(5,787)	(1,351)	(1,786)	(4,218)	(1,433)
Share of Profits	10%	11%	23%	54%	10%	9%	16%	10%
	(2%)	(2%)	(5%)	(15%)	(2%)	(3%)	(7%)	(3%)
Workers' Share of Rents	61%	59%	43%	31%	52%	57%	41%	58%
	(3%)	(3%)	(4%)	(5%)	(3%)	(4%)	(8%)	(4%)

Table B.3: Market Heterogeneity in Model Parameters and Rent Sharing Estimates

Notes: This table displays heterogeneity in the estimated model parameters and rents. These results correspond to the specification which allows Υ to differ from γ, and for ρ_r and α_r to vary across broad markets. Standard errors are estimated using 40 block bootstrap draws in which the block is taken to be the market.

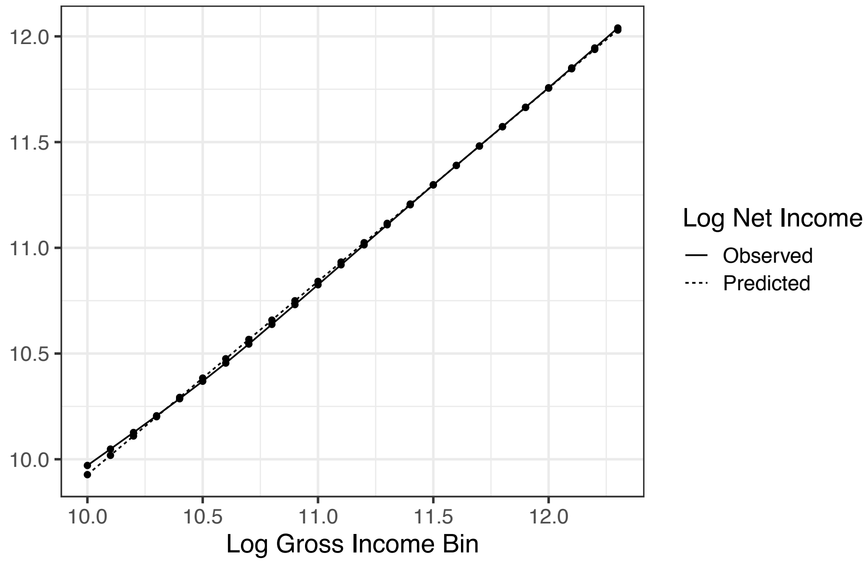


Figure B.6: Fit of the Tax Function

Notes: In this figure, we display the log net income predicted by the tax function compared to the log net income observed in the data.

C Online Appendix: Model of the Labor Market

C.1 Derivation of equilibrium wages

Given the nested Logit preferences and a given set of wages $\mathbf{W}_t = \{W_{jt}(X,V)\}_{j=1..J}$ we get that

$$
\begin{aligned}
Pr[j(i,t){=}j|X_i{=}X, V_{it}{=}V] = {} & NM_X(X)M_V(V) \\
& \times \frac{\left(\sum_{j'\in J_r} (\tau G_{j'}(X))^{\beta/\rho_r} W_{j't}(X,V)^{\lambda\beta/\rho_r}\right)^{\rho_r}}{\sum_{r'}\left(\sum_{j'\in J_{r'}} (\tau G_{j't}(X))^{\beta/\rho_{r'}} W_{j't}(X,V)^{\lambda\beta/\rho_{r'}}\right)^{\rho_{r'}}} \\
& \times \frac{(\tau G_{jt}(X))^{\beta/\rho_r} W_{jt}(X,V)^{\lambda\beta/\rho_r}}{\sum_{j'\in J_r} (\tau G_{j'}(X))^{\beta/\rho_r} W_{j't}(X,V)^{\lambda\beta/\rho_r}}
\end{aligned} \tag{33}
$$

and

$$
\mathbb{E}\left[u_{it}|X,V\right] = \frac{1}{\beta}\log\sum_r\left(\sum_{j\in J_r}(\tau G_j(X))^{\beta/\rho_r}\left(W_j(X,V)\right)^{\lambda\beta/\rho_r}\right)^{\rho_r}
$$

It is useful to introduce a few definitions before stating the lemma:

$$
\begin{aligned}
C_r &= \frac{(1-\alpha_r)\lambda\beta/\rho_r}{1+\lambda\beta/\rho_r} \\
\bar{V} &= \int V M_V(V)\mathrm{d}V
\end{aligned}
$$

Lemma 6. *Assume that firms believe they are strategically small. That is, in the firm first order condition, we impose that*

$$
\frac{\partial I_{rt}(X,V)}{\partial W_{jt}(X,V)} = 0
$$

We can then show that for firm j in market r,

$$
\begin{aligned}
W_{jt}(X,V) &= C_r X^{\theta_j} V H_{jt}^{-\alpha_r} A_{jt}^{\frac{1}{1+\alpha_r\lambda\beta/\rho_r}} & (34)\\
H_{jt} &= L_{jt} A_{jt}^{-\frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}} & (35)\\
I_{rt}(X,V) &= V\cdot I_{rt}(X) & (36)
\end{aligned}
$$

where h_{jt} is implicitly defined by

$$
H_{jt} \equiv \left(\bar{V}\int X^{\theta_j(1+\lambda\beta/\rho_r)} K_{rt}(X)\left(\tau G_j(X)\right)^{\beta/\rho_r} C_r^{\lambda\beta/\rho_r} dX\right)^{\frac{1}{1+\alpha_r\lambda\beta/\rho_r}},
$$

and we define

$$K_{rt}(X) \equiv M_X(X)\frac{(I_{rt}(X))^{\lambda\beta}}{\sum_{r'} I_{r't}(X)^{\lambda\beta}}\left(\frac{1}{I_{rt}(X)}\right)^{\lambda\beta/\rho_r}$$

$$I_{rt}(X) \equiv \left(\sum_{j'\in J_r}\left(\tau^{1/\lambda}G_{j'}(X)^{1/\lambda}C_r X^{\theta_{j'}}A_{j't}\left(\frac{Y_{j't}}{A_{j't}}\right)^{-\frac{\alpha_r}{1-\alpha_r}}\right)^{\lambda\beta/\rho_r}\right)^{\rho_r/(\lambda\beta)}$$

Proof. We start from the firm problem specified in the main text including the tax parameters. We have:

$$\max_{\{W_{jt}(X,V),D_{jt}(X,V)\}_{(X,V)}} A_{jt}\left(\iint X^{\theta_j}VD_{jt}(X,V)\mathrm{d}X\mathrm{d}V\right)^{1-\alpha_r} - \iint W_{jt}(X,V)D_{jt}(X,V)\mathrm{d}X\mathrm{d}V$$

$$\text{s.t. } D_{jt}(X,V) = M_X(X)M_V(V)\frac{(I_{rt}(X,V))^{\lambda\beta}}{\sum_{r'} I_{r't}(X,V)^{\lambda\beta}}\left(G_j(X)^{1/\lambda}\tau^{1/\lambda}\frac{W_{jt}(X,V)}{I_{rt}(X,V)}\right)^{\lambda\beta/\rho_r}$$

and defining:

$$K_{rt}(X,V) \equiv M_V(V)M_X(X)\frac{(I_{rt}(X,V))^{\lambda\beta}}{\sum_{r'} I_{r't}(X,V)^{\lambda\beta}}\left(\frac{1}{I_{rt}(X,V)}\right)^{\lambda\beta/\rho_r}$$

We substitute in the labor supply function and take the first order condition with respect to $W_{jt}(X,V)$:

$$\begin{aligned}
&(1-\alpha_r)X^{\theta_j}\left(\frac{\lambda\beta}{\rho_r}W_{jt}(X,V)^{\lambda\beta/\rho_r-1} + \frac{1}{K_{rt}(X,V)}\frac{\partial K_{rt}(X,V)}{\partial W_{jt}(X,V)}W_{jt}(X,V)^{\lambda\beta/\rho_r}\right)\tau^{\beta/\rho_r}G_j(X)^{\beta/\rho_r}VA_{jt}\left(\frac{Y_{jt}}{A_{jt}}\right)^{-\frac{\alpha_r}{1-\alpha_r}} \\
&= \tau^{\beta/\rho_r}G_j(X)^{\beta/\rho_r}\left(\left(1+\frac{\lambda\beta}{\rho_r}\right)W_{jt}(X,V)^{\lambda\beta/\rho_r} + \frac{1}{K_{rt}(X,V)}\frac{\partial K_{rt}(X,V)}{\partial W_{jt}(X,V)}W_{jt}(X,V)^{1+\lambda\beta/\rho_r}\right)
\end{aligned}$$

and under the assumption that $\frac{\partial I_{rt}(X,V)}{\partial W_{jt}(X,V)} = 0$, the FOC then simplifies to

$$\left(1+\frac{\lambda\beta}{\rho_r}\right)W_{jt}(X,V) = \frac{\lambda\beta}{\rho_r}(1-\alpha_r)X^{\theta_j}A_{jt}V\left(\frac{Y_{jt}}{A_{jt}}\right)^{-\frac{\alpha_r}{1-\alpha_r}},$$

or

$$W_{jt}(X,V) = C_r V X^{\theta_j}A_{jt}\left(\frac{Y_{jt}}{A_{jt}}\right)^{-\frac{\alpha_r}{1-\alpha_r}}$$

Let's then turn to the output of the firm,

$$\begin{aligned}
Y_{jt}/A_{jt} &= \left(\iint X^{\theta_j}V\cdot K_{rt}(X,V)\left(\tau G_j(X)\right)^{\beta/\rho_r}W_{jt}(X,V)^{\lambda\beta/\rho_r}\mathrm{d}X\mathrm{d}V\right)^{1-\alpha_r} \\
&= \left(\iint \left(X^{\theta_j}V\right)^{1+\lambda\beta/\rho_r}K_{rt}(X,V)\left(\tau G_j(X)\right)^{\beta/\rho_r}\left(C_rA_{jt}\right)^{\lambda\beta/\rho_r}\left(\frac{Y_{jt}}{A_{jt}}\right)^{-\frac{\alpha_r\lambda\beta/\rho_r}{1-\alpha_r}}\mathrm{d}X\mathrm{d}V\right)^{1-\alpha_r}
\end{aligned}$$

and so,

$$(Y_{jt}/A_{jt})^{1+\alpha_r\lambda\beta/\rho_r} = \left(\iint \left(X^{\theta_j}V\right)^{1+\lambda\beta/\rho_r} K_{rt}(X,V)\left(\tau G_j(X)\right)^{\beta/\rho_r} C_r^{\lambda\beta/\rho_r}\mathrm{d}X\mathrm{d}V\right)^{1-\alpha_r} \times \left(A_{jt}\right)^{(1-\alpha_r)\lambda\beta/\rho_r}.$$

We then note that

$$\begin{aligned} I_{rt}(X,V) &= \left(\sum_{j'\in J_r}\left(\tau^{1/\lambda}G_{j'}(X)^{1/\lambda}W_{j't}(X,V)\right)^{\lambda\beta/\rho_r}\right)^{\rho_r/(\lambda\beta)} \\ &= V\left(\sum_{j'\in J_r}\left(\tau^{1/\lambda}G_{j'}(X)^{1/\lambda}C_rX^{\theta_{j'}}A_{j't}\left(\frac{Y_{j't}}{A_{j't}}\right)^{-\frac{\alpha_r}{1-\alpha_r}}\right)^{\lambda\beta/\rho_r}\right)^{\rho_r/(\lambda\beta)} \\ &= V\cdot I_{rt}(X). \end{aligned}$$

Next we define $K_{rt}(X)\equiv M_X(X)\frac{(I_{rt}(X))^{\lambda\beta}}{\sum_{r'}I_{r't}(X)^{\lambda\beta}}\left(\frac{1}{I_{rt}(X)}\right)^{\lambda\beta/\rho_r}$, we replace the expression for the wage to get:

$$\begin{aligned} K_{rt}(X,V) &= M_V(V)M_X(X)\frac{(I_{rt}(X,V))^{\lambda\beta}}{\sum_{r'}I_{r't}(X,V)^{\lambda\beta}}\left(\frac{1}{I_{rt}(X,V)}\right)^{\lambda\beta/\rho_r} \\ &= M_V(V)K_{rt}(X)V^{-\lambda\beta/\rho_r}. \end{aligned}$$

Using $\bar{V}$, we have:

$$(Y_{jt}/A_{jt})^{1+\alpha_r\lambda\beta/\rho_r} = \left(\bar{V}\int X^{\theta_j(1+\lambda\beta/\rho_r)}K_{rt}(X)\left(\tau G_j(X)\right)^{\beta/\rho_r}C_r^{\lambda\beta/\rho_r}\mathrm{d}X\right)^{1-\alpha_r}\left(A_{jt}\right)^{(1-\alpha_r)\lambda\beta/\rho_r},$$

and for the wage, introducing

$$H_{jt}\equiv\left(\bar{V}\int X^{\theta_j(1+\lambda\beta/\rho_r)}K_{rt}(X)\left(\tau G_j(X)\right)^{\beta/\rho_r}C_r^{\lambda\beta/\rho_r}\mathrm{d}X\right)^{\frac{1}{1+\alpha_r\lambda\beta/\rho_r}},$$

then

$$(Y_{jt}/A_{jt})^{1+\alpha_r\lambda\beta/\rho_r} = \left(H_{jt}\right)^{(1-\alpha_r)(1+\alpha_r\lambda\beta/\rho_r)}\left(A_{jt}\right)^{(1-\alpha_r)\lambda\beta/\rho_r},$$

we get:

$$\begin{aligned} W_{jt}(X,V) &= C_rX^{\theta_j}VA_{jt}\left(\frac{Y_{jt}}{A_{jt}}\right)^{-\frac{\alpha_r}{1-\alpha_r}} \\ &= C_rX^{\theta_j}VH_{jt}^{-\alpha}A_{jt}^{\frac{1}{(1+\alpha_r\lambda\beta/\rho_r)}} \end{aligned}$$

In addition we have

$$\begin{aligned}
L_{jt} &= \int\int X^{\theta_j} V \cdot K_{rt}(X,V)\left(\tau G_j(X)\right)^{\beta/\rho_r} W_{jt}(X,V)^{\lambda\beta/\rho_r} \mathrm{d}X\mathrm{d}V \\
&= \bar{V}\int X^{\theta_j(1+\lambda\beta/\rho_r)} K_{rt}(X)\left(\tau G_j(X)\right)^{\beta/\rho_r}\left(C_r H_{jt}^{-\alpha_r}\right)^{\lambda\beta/\rho_r}\left(A_{jt}\right)^{\frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}} \mathrm{d}X \\
&= H_{jt}^{1+\alpha_r\lambda\beta/\rho_r-\alpha_r\lambda\beta/\rho_r} A_{jt}^{\frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}} \\
&= H_{jt} A_{jt}^{\frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}}
\end{aligned}$$

hence

$$H_{jt} = L_{jt} A_{jt}^{-\frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}}$$

□

Lemma 7 (Uniqueness of H_{jt})**.** *The firm and time-specific equilibrium constants H_{jt} are uniquely defined.*

Proof. As we have established in Lemma 6, for firm j in market r, H_{jt} solves the following system:

$$\begin{aligned}
H_{jt} = \Bigg[\bar{V}\int &\left(\sum_{r'}\left(\sum_{j'\in J_{r'}}\left(X^{\lambda\theta_{j'}}\tau G_{j'}(X) C_{r'}^{\lambda} H_{j't}^{-\alpha_{r'}\lambda}\right)^{\beta/\rho_{r'}} A_{j't}^{\frac{\lambda\beta/\rho_{r'}}{1+\alpha_{r'}\lambda\beta/\rho_{r'}}}\right)^{\rho_{r'}}\right)^{-1} \\
&\times\left(\sum_{j'\in J_r}\left(X^{\lambda\theta_{j'}}\tau G_{j'}(X) C_r^{\lambda} H_{j't}^{-\alpha_r\lambda}\right)^{\beta/\rho_r} A_{j't}^{\frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}}\right)^{\rho_r-1} \\
&\times X^{\theta_j(1+\lambda\beta/\rho_r)}\left(\tau G_j(X) C_r^{\lambda}\right)^{\beta/\rho_r} M_X(X)\mathrm{d}X\Bigg]^{\frac{1}{1+\alpha_r\lambda\beta/\rho_r}},
\end{aligned}$$

Where we replaced the $K_{rt}(X)$ and then $I_{rt}(X)$ and finally $W_j(X,V)$ with their definitions in the expression for H_{jt}. To show uniqueness we are going to show that $\tilde{H}_{jt} \equiv (H_{jt})^{\alpha_r}$ is unique. Using $\vec{H}_t = (\tilde{H}_{1t}, ..., \tilde{H}_{Jt})$, it solves the following fixed point expression:

$$\begin{aligned}
\tilde{H}_{jt} = \Bigg[\bar{V}\int &\left(\sum_{r'}\left(\sum_{j'\in J_{r'}}\left(\tau X^{\lambda\theta_{j'}}\tau G_{j'}(X) C_{r'}^{\lambda}\tilde{H}_{j't}^{-\lambda}\right)^{\beta/\rho_{r'}} A_{j't}^{\frac{\lambda\beta/\rho_{r'}}{1+\alpha_{r'}\lambda\beta/\rho_{r'}}}\right)^{\rho_{r'}}\right)^{-1} \\
&\times\left(\sum_{j'\in J_r}\left(X^{\lambda\theta_{j'}}\tau G_{j'}(X) C_r^{\lambda}\tilde{H}_{j't}^{-\lambda}\right)^{\beta/\rho_r} A_{j't}^{\frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}}\right)^{\rho_r-1} \\
&\times X^{\theta_j(1+\lambda\beta/\rho_r)}\left(\tau G_j(X) C_r^{\lambda}\right)^{\beta/\rho_r} M_X(X)\mathrm{d}X\Bigg]^{\frac{\alpha_r}{1+\alpha_r\lambda\beta/\rho_r}} \\
= \Gamma_{jt}(\vec{H}_t) &
\end{aligned} \tag{37}$$

We show that this expression satisfies the two conditions required to apply Theorem 1 of Kennan

(2000). We first look at the common part to all j terms given by

$$\bar{\Gamma}_t(X,\vec{H}_t) \equiv \left(\sum_{r'} \left(\sum_{j'\in J_{r'}} \left(X^{\lambda\theta_{j'}} \tau G_{j'}(X) C^{\lambda}_{r'} \tilde{H}^{-\lambda}_{j't} \right)^{\beta/\rho_{r'}} A_{j't}^{\frac{\lambda\beta/\rho_{r'}}{1+\alpha_{r'}\lambda\beta/\rho_{r'}}} \right)^{\rho_{r'}} \right)^{-1}$$

and we see that

$$\begin{aligned}\bar{\Gamma}_t(X,\mu\cdot\vec{H}_t) &= \left(\sum_{r'} \mu^{-\lambda\beta} \left(\sum_{j'\in J_{r'}} \left(X^{\lambda\theta_{j'}} \tau G_{j'}(X) C^{\lambda}_{r'} \tilde{H}^{-\lambda}_{j't} \right)^{\beta/\rho_{r'}} A_{j't}^{\frac{\lambda\beta/\rho_{r'}}{1+\alpha_{r'}\lambda\beta/\rho_{r'}}} \right)^{\rho_{r'}} \right)^{-1} \\ &= \mu^{\lambda\beta}\bar{\Gamma}_t(X,\vec{H}_t)\end{aligned}$$

Hence we get that

$$\begin{aligned}\Gamma_{jt}(\mu\cdot\vec{H}_t) = &\left[\bar{V}\int X^{\theta_j(1+\lambda\beta/\rho_r)}\bar{\Gamma}_t(X,\mu\cdot\vec{H}_t)\left(\tau G_j(X)C^{\lambda}_r\right)^{\beta/\rho_r}\right. \\ &\left.\times\left(\sum_{j'\in J_r}\left(X^{\lambda\theta_{j'}}\tau G_{j'}(X)\mu^{-\lambda}C^{\lambda}_r\tilde{H}^{-\lambda}_{j't}\right)^{\beta/\rho_r} A_{j't}^{\frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}}\right)^{\rho_r-1} M_X(X)\mathrm{d}X\right]^{\frac{\alpha_r}{1+\alpha_r\lambda\beta/\rho_r}} \\ =&\mu^{\frac{(1-\rho_r)\alpha_r\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}}\left[\bar{V}\int X^{\theta_j(1+\beta/\rho_r)}\bar{\Gamma}_t(X,\mu\cdot\vec{H}_t)\left(\tau G_j(X)C^{\lambda}_r\right)^{\beta/\rho_r}\right. \\ &\left.\times\left(\sum_{j'\in J_r}\left(X^{\lambda\theta_{j'}}\tau G_{j'}(X)C^{\lambda}_r\tilde{H}^{-\lambda}_{j't}\right)^{\beta/\rho_r} A_{j't}^{\frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}}\right)^{\rho_r-1} M_X(X)\mathrm{d}X\right]^{\frac{\alpha_r}{1+\alpha_r\lambda\beta/\rho_r}} \\ =&\mu^{\frac{(1-\rho_r)\alpha_r\lambda\beta/\rho_r+\lambda\beta\alpha_r}{1+\alpha_r\lambda\beta/\rho_r}}\Gamma_{jt}(\vec{H}_t) \\ =&\mu^{\frac{\alpha_r\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}}\Gamma_{jt}(\vec{H}_t)\end{aligned}$$

Then, given $\vec{H}_t > 0$ such that $\Gamma_t(\vec{H}_t) = \vec{H}_t$ then for any $0 < \mu < 1$, any r and any $j \in J_r$, we have

$$\begin{aligned}\Gamma_{jt}(\mu\cdot\vec{H}_t) - \mu\cdot\tilde{H}_{jt} &= \mu^{\frac{\alpha_r\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}}\cdot\Gamma_{jt}(\vec{H}_t) - \mu\cdot\tilde{H}_{jt} \\ &= \mu^{\frac{\alpha_r\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}}\cdot\tilde{H}_{jt} - \mu\cdot\tilde{H}_{jt} \\ &= \mu\underbrace{\left(\mu^{\frac{\alpha_r\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}-1} - 1\right)}_{>0}\cdot\tilde{H}_{jt} \\ &> 0\end{aligned}$$

which means that we have shown that $\Gamma_t(\vec{H}_t) - \vec{H}_t$ is strictly "radially quasi-concave". The next step is to show monotonicity. Consider $\vec{H}_{1t}$ and $\vec{H}_{2t}$ such that for a given j we have $\tilde{H}_{1jt} = \tilde{H}_{2jt}$ and $\tilde{H}_{1j't} \leq \tilde{H}_{2j't}$ for all other. Then we have that for all j',t and X:

$$\tilde{H}_{1j't} \leq \tilde{H}_{2j't}$$

hence for $r' = r(j')$,

$$\left(X^{\lambda\theta_{j'}}\tau G_{j'}(X)C_{r'}^{\lambda}\tilde{H}_{1j't}^{-\lambda}\right)^{\beta/\rho_{r'}} A_{j't}^{\frac{\lambda\beta/\rho_{r'}}{1+\alpha_{r'}\lambda\beta/\rho_{r'}}} \geq \left(X^{\lambda\theta_{j'}}\tau G_{j'}(X)C_{r'}^{\lambda}\tilde{H}_{2j't}^{-\lambda}\right)^{\beta/\rho_{r'}} A_{j't}^{\frac{\lambda\beta/\rho_{r'}}{1+\alpha_{r'}\lambda\beta/\rho_{r'}}}$$

and for any r' and X,

$$\sum_{j'\in J_{r'}}\left(X^{\lambda\theta_{j'}}\tau G_{j'}(X)C_{r'}^{\lambda}\tilde{H}_{1j't}^{-\lambda}\right)^{\beta/\rho_{r'}} A_{j't}^{\frac{\lambda\beta/\rho_{r'}}{1+\alpha_{r'}\lambda\beta/\rho_{r'}}} \geq \sum_{j'\in J_{r'}}\left(X^{\lambda\theta_{j'}}\tau G_{j'}(X)C_{r'}^{\lambda}\tilde{H}_{2j't}^{-\lambda}\right)^{\beta/\rho_{r'}} A_{j't}^{\frac{\lambda\beta/\rho_{r'}}{1+\alpha_{r'}\lambda\beta/\rho_{r'}}}$$

and taken to the power minus one implies that $\bar{\Gamma}_t(X, \vec{H}_{1t}) \leq \bar{\Gamma}_t(X, \vec{H}_{2t})$. Then, since $\rho_r \leq 1$ we also get that:

$$\left(\sum_{j'\in J_r}\left(X^{\lambda\theta_{j'}}\tau G_{j'}(X)C_r^{\lambda}\tilde{H}_{1j't}^{-\lambda}\right)^{\beta/\rho_r} A_{j't}^{\frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}}\right)^{\rho_r-1}$$
$$\leq \left(\sum_{j'\in J_r}\left(X^{\lambda\theta_{j'}}\tau G_{j'}(X)C_r^{\lambda}\tilde{H}_{2j't}^{-\lambda}\right)^{\beta/\rho_r} A_{j't}^{\frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}}\right)^{\rho_r-1}$$

Combining the last two results gives us that

$$\Gamma_{jt}(\vec{H}_{1t}) \leq \Gamma_{jt}(\vec{H}_{2t})$$

The fact that the function is "radially quasi-concave" together with the monotonicity gives uniqueness of the fixed point by the theorem in Kennan (2001). This means that $\vec{H}_t$ is unique, and hence that $\tilde{H}_{jt}$ is unique and finally that H_{jt} is unique. $\square$

Definition 2. We consider a sequence of increasingly larger economies indexed by an increasing $n^{\mathrm{r}}$where $n_r^f = \kappa_r n^r$ for some fixed κ_r. In this sequence of economies we assume that the amenities scale according to $G_j(X) = \mathring{G}_j(X)\left(n_{r(j)}^{\mathrm{f}}\right)^{-\rho_{r(j)}/\beta}$ for some fixed $\mathring{G}_j(X)$. We also assume that the mass of workers grows according to $N = n^{\mathrm{r}} \cdot \bar{n}^{\mathrm{f}} \cdot \mathring{N}$.

Lemma 8. *Here we establish that the unique solution for H_{jt} in the limit of a sequence of growing economies is given by*

$$H_{jt} = H_j \cdot \bar{A}_{rt}^{\frac{\lambda\beta/\rho_r}{(1+\alpha_r\lambda\beta)}\frac{(\rho_r-1)}{(1+\alpha_r\lambda\beta/\rho_r)}}$$

where H_j solves the following fixed point:

$$H_j = \left(\bar{V} \int X^{\theta_j} \left(\frac{I_{r0}(X)}{I_0(X)} \right)^{\lambda\beta} \left(\frac{1}{I_{r0}(X)} \right)^{\lambda\beta/\rho_r} \left(X^{\lambda\theta_j} \tau \mathring{G}_j(X) C_r^{\lambda} \right)^{\beta/\rho_r} \frac{\bar{\kappa}}{\kappa_r} \mathring{N} M_X(X) dX \right)^{\frac{1}{1+\alpha_r\lambda\beta/\rho_r}}$$

$$I_{r0}(X)^{\lambda\beta/\rho_r} = \mathbb{E}_j \left[\left(X^{\lambda\theta_j} \tau \mathring{G}_j(X) C_r^{\lambda} H_j^{-\lambda\alpha_r} \right)^{\beta/\rho_r} \tilde{A}_{jt}^{\frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}} \right]$$

$$I_0(X)^{\lambda\beta} = \mathbb{E}_r \left[I_{r0}(X)^{\lambda\beta} \bar{A}_{rt}^{\frac{\lambda\beta}{1+\alpha_r\lambda\beta}} \right]$$

Proof. Consider the expression for H_{jt} from the beginning of Lemma 7:

$$H_{jt} = \Bigg[\bar{V} \int \left(\sum_{r'} \left(\sum_{j' \in J_{r'}} \left(X^{\lambda\theta_{j'}} \tau G_{j'}(X) C_{r'}^{\lambda} H_{j't}^{-\alpha_{r'}\lambda} \right)^{\beta/\rho_{r'}} A_{j't}^{\frac{\lambda\beta/\rho_{r'}}{1+\alpha_{r'}\lambda\beta/\rho_{r'}}} \right)^{\rho_{r'}} \right)^{-1}$$
$$\times \left(\sum_{j' \in J_r} \left(X^{\lambda\theta_{j'}} \tau G_{j'}(X) C_r^{\lambda} H_{j't}^{-\alpha_r\lambda} \right)^{\beta/\rho_r} A_{j't}^{\frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}} \right)^{\rho_r - 1}$$
$$\times X^{\theta_j(1+\lambda\beta/\rho_r)} \left(\tau G_j(X) C_r^{\lambda} \right)^{\beta/\rho_r} N M_X(X) \mathrm{d}X \Bigg]^{\frac{1}{1+\alpha_r\lambda\beta/\rho_r}},$$

and let's introduce n^{r} and n_r^{f}, $\mathring{G}_j(X) = \left(n_{r(j)}^{\mathrm{f}} \right)^{\rho_{r(j)}/\beta} G_j(X)$ and $\mathring{N} = (n^{\mathrm{r}}\ n^{\mathrm{r}}\ \bar{\kappa})^{-1} N$

$$H_{jt} = \Bigg[\bar{V} \int \left(\frac{1}{n^{\mathrm{r}}} \sum_{r'} \left(\frac{1}{n_{r'}^{\mathrm{f}}} \sum_{j' \in J_{r'}} \left(X^{\lambda\theta_{j'}} \tau \mathring{G}_j(X) C_{r'}^{\lambda} H_{j't}^{-\alpha_{r'}\lambda} \right)^{\beta/\rho_{r'}} A_{j't}^{\frac{\lambda\beta/\rho_{r'}}{1+\alpha_{r'}\lambda\beta/\rho_{r'}}} \right)^{\rho_{r'}} \right)^{-1}$$
$$\times \left(\frac{1}{n_{r'}^{\mathrm{f}}} \sum_{j' \in J_r} \left(X^{\lambda\theta_{j'}} \tau \mathring{G}_j(X) C_r^{\lambda} H_{j't}^{-\alpha_r\lambda} \right)^{\beta/\rho_r} A_{j't}^{\frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}} \right)^{\rho_r - 1}$$
$$\times X^{\theta_j(1+\lambda\beta/\rho_r)} \left(\tau \mathring{G}_j(X)(X) C_r^{\lambda} \right)^{\beta/\rho_r} \frac{\bar{\kappa}}{\kappa_r} \mathring{N} M_X(X) \mathrm{d}X \Bigg]^{\frac{1}{1+\alpha_r\lambda\beta/\rho_r}},$$

As the economy grows large, ie has n^{r} grows to infinity, we end up with the following expression

$$H_{jt} = \Bigg[\bar{V} \int \left(\mathbb{E}_r \left(\mathbb{E}_{j'} \left(X^{\lambda\theta_{j'}} \tau \mathring{G}_{j'}(X) C_{r'}^{\lambda} H_{j't}^{-\alpha_{r'}\lambda} \right)^{\beta/\rho_{r'}} A_{j't}^{\frac{\lambda\beta/\rho_{r'}}{1+\alpha_{r'}\lambda\beta/\rho_{r'}}} \right)^{\rho_{r'}} \right)^{-1}$$
$$\times \left(\mathbb{E}_j \left(X^{\lambda\theta_{j'}} \tau \mathring{G}_j(X) C_r^{\lambda} H_{j't}^{-\alpha_r\lambda} \right)^{\beta/\rho_r} A_{j't}^{\frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}} \right)^{\rho_r - 1}$$
$$\times X^{\theta_j(1+\lambda\beta/\rho_r)} \left(\tau \mathring{G}_j(X)(X) C_r^{\lambda} \right)^{\beta/\rho_r} \frac{\bar{\kappa}}{\kappa_r} \mathring{N} M_X(X) \mathrm{d}X \Bigg]^{\frac{1}{1+\alpha_r\lambda\beta/\rho_r}},$$

next we show that H_{jt} can indeed be expressed as stated in the lemma. Let's assume that

$H_{jt} = H_j \cdot \bar{A}_{rt}^{\frac{\lambda\beta/\rho_r}{(1+\alpha_r\lambda\beta)}\frac{(\rho_r-1)}{(1+\alpha_r\lambda\beta/\rho_r)}}$ and show that it solves the problem. We first note that

$$
\begin{aligned}
\mathbb{E}_{j'\in J_r}\left(X^{\lambda\theta_{j'}}\tau\mathring{G}_j(X)C_{r'}^{\lambda}H_{j't}^{-\alpha_{r'}\lambda}\right)^{\beta/\rho_{r'}} A_{j't}^{\frac{\lambda\beta/\rho_{r'}}{1+\alpha_{r'}\lambda\beta/\rho_{r'}}} &= \mathbb{E}_{j'}\left(X^{\lambda\theta_{j'}}\tau\mathring{G}_{j'}(X)C_{r'}^{\lambda}H_{j'}^{-\alpha_{r'}\lambda}\right)^{\beta/\rho_{r'}} \bar{A}_{rt}^{-\alpha_{r'}\lambda\beta/\rho_{r'}\frac{\lambda\beta/\rho_r}{(1+\alpha_r\lambda\beta)}\frac{(\rho_r-1)}{(1+\alpha_r\lambda\beta/\rho_r)}} A_{j't}^{\frac{\lambda\beta/\rho_{r'}}{1+\alpha_{r'}\lambda\beta/\rho_{r'}}} \\
&= \bar{A}_{rt}^{\frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta}}\mathbb{E}_{j'}\left(X^{\lambda\theta_{j'}}\tau\mathring{G}_j(X)C_{r'}^{\lambda}H_{j'}^{-\alpha_{r'}\lambda}\right)^{\beta/\rho_{r'}} \tilde{A}_{j't}^{\frac{\lambda\beta/\rho_{r'}}{1+\alpha_{r'}\lambda\beta/\rho_{r'}}} \\
&= \bar{A}_{rt}^{\frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta}} I_{r0}(X)^{\lambda\beta/\rho_r}
\end{aligned}
$$

hence

$$
\begin{aligned}
H_{jt} = \Bigg[\bar{V}\int\left(\mathbb{E}_{r'}\bar{A}_{r't}^{\frac{\lambda\beta}{1+\alpha_{r'}\lambda\beta}} I_{r'0}(X)^{\lambda\beta}\right)^{-1} \\
\times\left(\bar{A}_{rt}^{\frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta}} I_{r0}(X)^{\lambda\beta/\rho_r}\right)^{\rho_r-1} \\
\times X^{\theta_j(1+\lambda\beta/\rho_r)}\left(\tau\mathring{G}_j(X)C_r^{\lambda}\right)^{\beta/\rho_r}\kappa_r\mathring{N}M_X(X)\mathrm{d}X\Bigg]^{\frac{1}{1+\alpha_r\lambda\beta/\rho_r}} \\
\Bigg[\bar{V}\int\left(\mathbb{E}_{r'}\bar{A}_{r't}^{\frac{\lambda\beta}{1+\alpha_{r'}\lambda\beta}} I_{r'0}(X)^{\lambda\beta}\right)^{-1}\times\left(I_{r0}(X)^{\lambda\beta/\rho_r}\right)^{\rho_r-1} \\
\times X^{\theta_j(1+\lambda\beta/\rho_r)}\left(\tau\mathring{G}_j(X)C_r^{\lambda}\right)^{\beta/\rho_r}\kappa_r\mathring{N}M_X(X)\mathrm{d}X\Bigg]^{\frac{1}{1+\alpha_r\lambda\beta/\rho_r}}
\end{aligned}
$$

where a sufficient condition is that H_j solves

$$
H_j = \left[\bar{V}\int X^{\theta_j}\left(\frac{I_{r0}(X)}{I_0(X)}\right)^{\lambda\beta}\left(\frac{1}{I_{r0}(X)}\right)^{\lambda\beta/\rho_r}\left(X^{\lambda\theta_j}\tau\mathring{G}_j(X)C_r^{\lambda}\right)^{\beta/\rho_r}\mathring{N}M_X(X)\mathrm{d}X\right]^{\frac{1}{1+\alpha_r\lambda\beta/\rho_r}}
$$

where

$$
I_0(X) = \left(\mathbb{E}_r\bar{A}_{rt}^{\frac{\lambda\beta}{1+\alpha_r\lambda\beta}} I_{r0}(X)^{\lambda\beta}\right)^{1/(\lambda\beta)}
$$

□

We can then establish the final result.

Proposition 1. *The wage equation is given by*

$$
w_j(x, v, \bar{a}, \tilde{a}) = c_r + \theta_j x + v - \alpha_r h_j + \frac{1}{1+\alpha_r\lambda\beta/\rho_r}\tilde{a} + \frac{1}{1+\alpha_r\lambda\beta}\bar{a}
$$

where

$$
h_j = \ell_{jt} - \frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}\tilde{a}_{jt} - \frac{\lambda\beta}{1+\alpha_r\lambda\beta}\bar{a}_{rt}
$$

Recall that

$$
\begin{aligned}
h_{jt} &= \ell_{jt} - \frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r} a_{jt} \\
&= \frac{(\rho_r-1)\lambda\beta/\rho_r}{(1+\alpha_r\lambda\beta)\left(1+\alpha_r\lambda\beta/\rho_r\right)} \bar{a}_{rt} + h_j
\end{aligned}
$$

hence we get

$$
\begin{aligned}
h_j &= \ell_{jt} - \frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r} \tilde{a}_{jt} - \frac{\lambda\beta}{1+\alpha_r\lambda\beta} \bar{a}_{rt} \\
h_j &= \ell_j(\bar{a}=0, \tilde{a}=0)
\end{aligned}
$$

Next, we replace the expression in the wage to get

$$
\begin{aligned}
w_{jt}(x,v) &= c_r + \theta_j x + v - \alpha_r h_j + \frac{1}{1+\alpha_r\lambda\beta/\rho_r} \tilde{a}_{jt} + \frac{1}{1+\alpha_r\lambda\beta} \bar{a}_{rt} \\
&= w_j(x, v, \bar{a}_{rt}, \tilde{a}_{jt}).
\end{aligned}
$$

where we defined

$$
w_j(x, v, \bar{a}, \tilde{a}) \equiv c_r + \theta_j x + v - \alpha_r h_j + \frac{1}{1+\alpha_r\lambda\beta/\rho_r} \tilde{a} + \frac{1}{1+\alpha_r\lambda\beta} \bar{a}.
$$

Note that $w_{jt}(x,v)$ depends on time only through $\bar{a}_{rt}$ and $\tilde{a}_{jt}$.

Corollary 1. *The firm demand is given by:*

$$
D_{jt}(X,V) = M_X(X) \left(\frac{I_{r0}(X) \bar{A}_{rt}^{\frac{1}{1+\alpha_r\lambda\beta/\rho_r}}}{I_0(X)} \right)^{\lambda\beta} \left(\frac{\tau^{1/\lambda} G_j(X)^{1/\lambda} W_j(X)}{I_{r0}(X) \bar{A}_{rt}^{\frac{1}{1+\alpha_r\lambda\beta/\rho_r}}} \right)^{\lambda\beta/\rho_r}
$$

We also derive the other quantities of the model.

Corollary 2. *The value added and wage bills are given by*

$$
\begin{aligned}
y_j(\bar{a}, \tilde{a}) =& (1-\alpha_r) h_j + \frac{1+\lambda\beta}{(1+\alpha_r\lambda\beta)} \bar{a} + \frac{1+\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r} \tilde{a} \\
b_j(\bar{a}, \tilde{a}) =& c_r + (1-\alpha_r) h_j + \frac{1+\lambda\beta}{(1+\alpha_r\lambda\beta)} \bar{a} + \frac{1+\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r} \tilde{a}
\end{aligned}
$$

We turn to expressing the value added at the firm:

$$
(Y_{jt}/A_{jt})^{1+\alpha_r\lambda\beta/\rho_r} = \left(\int \bar{V} X^{\gamma_j(1+\lambda\beta/\rho_r)} \cdot G_j(X)^{\beta/\rho_r} \mathrm{d}X \right)^{1-\alpha_r} (C_r \cdot A_{jt})^{(1-\alpha_r)\lambda\beta/\rho_r}
$$

$$
(1+\alpha_r\lambda\beta/\rho_r)(y_{jt} - a_{jt}) = (1-\alpha_r) \left(h_j + \frac{(\rho_r-1)\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta} a_{rt} \right) + (1-\alpha_r)\lambda\beta/\rho_r (c_r + a_{jt})
$$

and for the wage bill:

$$
\begin{aligned}
B_{jt} &= \iint W(X,V) D_{jt}(X,V) \mathrm{d}X \mathrm{d}V \\
&= \iint W(X,V) m_X(X,V) \left(\frac{I_{r0}(X) \bar{A}_{rt}^{\frac{1}{1+\alpha_r \lambda\beta}}}{I_0(X)} \right)^{\lambda\beta} \left(\frac{\tau^{1/\lambda} G_j(X)^{1/\lambda}}{I_{r0}(X) \bar{A}_{rt}^{\frac{1}{1+\alpha_r \lambda\beta}}} \right)^{\lambda\beta/\rho_r} (W(X,V))^{\lambda\beta/\rho_{r(j)}} \mathrm{d}X \mathrm{d}V \\
&= \iint m_X(X,V) \left(\frac{I_{r0}(X) \bar{A}_{rt}^{\frac{1}{1+\alpha_r \lambda\beta}}}{I_0(X)} \right)^{\lambda\beta} \left(\frac{\tau^{1/\lambda} G_j(X)^{1/\lambda}}{I_{r0}(X) \bar{A}_{rt}^{\frac{1}{1+\alpha_r \lambda\beta}}} \right)^{\lambda\beta/\rho_r} \\
&\qquad \times \left(C_r H_j^{-\alpha} \right)^{\lambda\beta/\rho_{r(j)}} \left(\tilde{A}_{jt} \right)^{\frac{1+\lambda\beta/\rho_{r(j)}}{1+\lambda\alpha_{r(j)}\beta/\rho_{r(j)}}} \bar{A}_{rt}^{\frac{1+\lambda\beta/\rho_{r(j)}}{1+\lambda\alpha_{r(j)}\beta}} \mathrm{d}X \mathrm{d}V \\
b_{jt} &= c_r + (1-\alpha_r) h_j + \frac{1+\lambda\beta}{(1+\alpha\beta)} \bar{a} + \frac{1+\lambda\beta/\rho_r}{1+\alpha_r \lambda\beta/\rho_r} \tilde{a}
\end{aligned}
$$

and so we get that

$$
y_j(\bar{a}, \tilde{a}) - b_j(\bar{a}, \tilde{a}) = c_r
$$

Note that this gives us that the structural pass-through rate of the firm level shock is (with abuse of notation):

$$
\begin{aligned}
\frac{\partial \log w_j(\bar{a}, \tilde{a})}{\partial \log \bar{a}} \cdot \frac{\partial \log \bar{a}}{\partial \log y_j(\bar{a}, \tilde{a})} &= \frac{1}{1+\lambda\beta} \\
\frac{\partial \log w_j(\bar{a}, \tilde{a})}{\partial \log \tilde{a}} \cdot \frac{\partial \log \tilde{a}}{\partial \log y_j(\bar{a}, \tilde{a})} &= \frac{\rho_r}{\rho_r + \lambda\beta}.
\end{aligned}
$$

Corollary 3. *Firm j worker composition does not depend on $\bar{a}$ and $\tilde{a}$.*

Proof. Consider $Pr[X|j,t]$:

$$
\begin{aligned}
\Pr[X|j,t] &= \Pr[X,j|t] / \Pr[j|t] \\
&= \frac{\left(\frac{I_{r0}(X)}{I_0(X)} \right)^{\lambda\beta} \left(\frac{1}{I_{r0}(X)} \right)^{\lambda\beta/\rho_r} \left(\tau \mathring{G}_j(X) W_{jt}(X)^{\lambda} \right)^{\beta/\rho_r} \mathring{N} M_X(X)}{\int \left(\frac{I_{r0}(X')}{I_0(X')} \right)^{\lambda\beta} \left(\frac{1}{I_{r0}(X')} \right)^{\lambda\beta/\rho_r} \left(\tau \mathring{G}_j(X') W_{jt}(X')^{\lambda} \right)^{\beta/\rho_r} \mathring{N} M_X(X') \mathrm{d}X'} \\
&= \frac{\left(\frac{I_{r0}(X)}{I_0(X)} \right)^{\lambda\beta} \left(\frac{1}{I_{r0}(X)} \right)^{\lambda\beta/\rho_r} \left(X^{\lambda\theta_j} \tau \mathring{G}_j(X) C_r^{\lambda} \right)^{\beta/\rho_r} \mathring{N} M_X(X)}{\int \left(\frac{I_{r0}(X')}{I_0(X')} \right)^{\lambda\beta} \left(\frac{1}{I_{r0}(X')} \right)^{\lambda\beta/\rho_r} \left(X'^{\lambda\theta_j} \tau \mathring{G}_j(X') C_r^{\lambda} \right)^{\beta/\rho_r} \mathring{N} M_X(X') \mathrm{d}X'} \\
&= \Pr[X|j]
\end{aligned}
$$

□

C.2 Worker rents

Lemma 9. *We establish that for workers of type (X,V) working at firm j in market r at time t, the average firm level rent is given by $\frac{W_{jt}(X,V)}{1+\lambda\beta/\rho_r}$ and the average market level rent is given by $\frac{W_{jt}(X,V)}{1+\lambda\beta}$.*

Proof. The average rents at the firm is defined as the difference between the worker's willingness to pay W and the wage they actually get at firm j at time t denoted $W_{jt}(X,V)$. The supply curve $S_{jt}(X,V,W)$ exactly defines the number of people willing work at firm j at some given wage W. Hence the density of willingness to pay among workers in firm j at time t at wage $W_{jt}(X,V)$ is given by

$$\frac{1}{S_{jt}(X,V,W_{jt}(X,V))}\frac{\partial S_{jt}(X,V,W)}{\partial W}$$

and so we get the average value of the rent by taking the expectation with respect to that density:

$$\begin{aligned}
R^w_{jt}(X,V) &\equiv \mathbb{E}\left[R^w_{it}\,|\,j(i,t)=j, X_i=X, V_{it}=V\right]\\
&= \int_0^{W_{jt}(X,V)} \left(W_{jt}(X,V)-W\right)\frac{1}{S_{jt}(X,V,W_{jt}(X,V))}\frac{\partial S_{jt}(X,V,W)}{\partial W}\mathrm{d}W\\
&= W_{jt}(X,V)\int_0^1 (1-\omega)\frac{1}{S_{jt}(X,V,W_{jt}(X,V))}\frac{\partial S_{jt}(X,V,\omega W_{jt}(X,V))}{\partial \omega}\mathrm{d}\omega\\
&= W_{jt}(X,V)\int_0^1 (1-\omega)\frac{\partial \omega^{\lambda\beta/\rho_r}}{\partial\omega}\mathrm{d}\omega\\
&= \frac{W_{jt}(X,V)}{1+\lambda\beta/\rho_r}
\end{aligned}$$

where the second to last step relies on the definition of $S_{jt}(X,V,W)$ and the fact that we assume the presence of many firms in each market to show that $S_{jt}(X,V,\omega W)=\omega^{\lambda\beta/\rho_r}S_{jt}(X,V,W)$. We can then take the average over (X_i,V_{it}) the workers in the firm $j\in J_r$ at time t to get

$$\begin{aligned}
\mathbb{E}\left[R^w_{it}\,|\,j(i,t)=j\right] &= \mathbb{E}\left[R^w_{jt}(X_i,V_{it})\,|\,j(i,t)=j\right]\\
&= \frac{1}{1+\lambda\beta/\rho_r}\mathbb{E}\left[W_{jt}(X_i,V_{it})\,|\,j(i,t)=j\right]
\end{aligned}$$

Next we want to compute the integral of the market level supply curve for each worker of type (X,V). In contrast to the worker rent at the firm level, we want to shift the wages of all firms in the market for a given individual in a given market. This means that we want to shift both the current firm j but also all other firms j' in market r. Given the labor supply curve to a firm j given by equation 33, we integrate by scaling all wages in market r by ω in $[0,1]$. More precisely we consider the demand realized by the set of wages $\left\{\omega^{\mathbf{1}[j\in J_r]}W_{jt}(X,V)\right\}_{jt}$ for a given market r. The supply curve at firm j in this market as a function of the scaling ω is then given:

$$
\begin{aligned}
M \cdot M_X(X) M_V(V) & \frac{\left(\sum_{j' \in J_r} \left(\tau^{1/\lambda} G_{j'}(X)^{1/\lambda} \omega W_{j't}(X,V)\right)^{\lambda\beta/\rho_r}\right)^{\rho_r}}{\sum_{r'} \left(\sum_{j' \in J_{r'}} \left(\tau^{1/\lambda} G_{j'}(X)^{1/\lambda} W_{j't}(X,V)\right)^{\lambda\beta/\rho_{r'}}\right)^{\rho_{r'}}} \\
& \times \frac{\left(\tau^{1/\lambda} G_{j'}(X)^{1/\lambda} \omega W_{jt}(X,V)\right)^{\lambda\beta/\rho_r}}{\sum_{j' \in J_r} \left(\tau^{1/\lambda} G_{j'}(X)^{1/\lambda} \omega W_{j't}(X,V)\right)^{\lambda\beta/\rho_r}} \\
= \omega^{\lambda\beta} S_{jt}(X, V, W_{jt}(X,V))
\end{aligned}
$$

where we also used the assumption that there are many markets in the first denominator. Hence the density of willingness to pay is given by

$$
\frac{1}{S_{jt}(X,V,W_{jt}(X,V))} \frac{\partial}{\partial \omega} \left[\omega^{\lambda\beta} S_{jt}(X,V,W_{jt}(X,V))\right]
$$

and by the same logic as as the firm level we get the following:

$$
\begin{aligned}
R_{jt}^{wm}(X,V) &\equiv \mathbb{E}\left[R_{it}^{wm} \,|\, j(i,t) = j, X_i = X, V_{it} = V\right] \\
&= \frac{W_{jt}(X,V)}{1+\lambda\beta},
\end{aligned}
$$

and we get that:

$$
\begin{aligned}
\mathbb{E}\left[R_{it}^{wm} \,|\, j(i,t) = j\right] &= \mathbb{E}\left[R_{jt}^{wm}(X_i, V_{it}) \,|\, j(i,t) = j\right] \\
&= \frac{1}{1+\lambda\beta/\rho_r} \mathbb{E}\left[W_{jt}(X_i, V_{it}) \,|\, j(i,t) = j\right]
\end{aligned}
$$

□

C.3 Firm-specific rent

Lemma 10. *We establish that the firm rent is given by*

$$
\Pi_{jt} - \Pi_{jt}^{pt} \;= \left(1 - \frac{\alpha_r \left(1 + \lambda\beta/\rho_r\right)}{1 + \alpha_r \lambda\beta/\rho_r} \left(\frac{\lambda\beta/\rho_r}{1+\lambda\beta/\rho_r}\right)^{-\frac{(1-\alpha_r)\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}}\right) \Pi_{jt}
$$

Proof. In the following proof we abstract from V to keep the derivation simpler. The firm rent is defined as the difference between the profit that firm would make if it were a price taker and the profit in equilibrium. To get the price-taker profit we maximize profit,

$$
A_{jt} \left(\int X^{\theta_j} \cdot D_{jt}^{\mathrm{pt}}(X) \mathrm{d}X\right)^{1-\alpha_r} - \int W_{jt}^{\mathrm{pt}}(X) \cdot D_{jt}^{\mathrm{pt}}(X) \mathrm{d}X
$$

taking the wage as given, and then equate with the supply equation. The first order condition is

$$\underbrace{(1-\alpha_r)}_{C^{\mathrm{pt}}} A_{jt} X^{\theta_j} \left(\frac{Y^{\mathrm{pt}}}{A_{jt}}\right)^{-\frac{\alpha_r}{1-\alpha_r}} = W_{jt}^{\mathrm{pt}}\,(X)$$

and the realized demand is given by:

$$D_{jt}^{\mathrm{pt}}(X) = M \cdot M_X(X) \left(\frac{I_{rt}(X)}{I(X)}\right)^{\lambda\beta} \left(\tau^{1/\lambda} G_j(X)^{1/\lambda} \frac{W_{jt}^{\mathrm{pt}}\,(X)}{I_{rt}(X)}\right)^{\lambda\beta/\rho_r}$$

where we use $I(X)^{\lambda\beta} = \sum_r I_{rt}(X)^{\lambda\beta}$, assumed constant due to the large number of markets. We then get that

$$\begin{aligned}
\frac{Y^{\mathrm{pt}}}{A_{jt}} &= \left(\int X^{\theta_j} \cdot D_{jt}^{\mathrm{pt}}(X)\mathrm{d}X\right)^{1-\alpha_r} \\
&= \left(\int X^{\theta_j} \cdot \left(\frac{I_{rt}(X)}{I(X)}\right)^{\lambda\beta} (\tau G_j(X))^{\beta/\rho_r} \left(\frac{W_{jt}^{\mathrm{pt}}\,(X)}{I_{rt}(X)}\right)^{\lambda\beta/\rho_r} N M_X(X)\mathrm{d}X\right)^{1-\alpha_r} \\
&= \left(\int X^{\theta_j} \cdot \left(\frac{I_{rt}(X)}{I(X)}\right)^{\lambda\beta} (\tau G_j(X))^{\beta/\rho_r} \left(\frac{C^{\mathrm{pt}} A_{jt} X^{\theta_j}}{I_{rt}(X)} \left(\frac{Y_{jt}^{\mathrm{pt}}}{A_{jt}}\right)^{-\frac{\alpha_r}{1-\alpha_r}}\right)^{\lambda\beta/\rho_r} N M_X(X)\mathrm{d}X\right)^{1-\alpha_r} \\
&= (A_{jt})^{(1-\alpha_r)\lambda\beta/\rho_r} \left(\frac{Y_{jt}^{\mathrm{pt}}}{A_{jt}}\right)^{-\alpha_r\lambda\beta/\rho_r} \\
&\quad \times \left(\int X^{\theta_j} \left(\frac{I_{rt}(X)}{I(X)}\right)^{\lambda\beta} (\tau G_j(X))^{\beta/\rho_r} \left(\frac{X^{\theta_j} C^{\mathrm{pt}}}{I_{rt}(X)}\right)^{\lambda\beta/\rho_r} N M_X(X)\mathrm{d}X\right)^{1-\alpha_r} \\
&= \left(\frac{C_r^{\mathrm{pt}}}{C_r} A_{jt}\right)^{(1-\alpha_r)\lambda\beta/\rho_r} \left(\frac{Y_{jt}^{\mathrm{pt}}}{A_{jt}}\right)^{-\alpha_r\lambda\beta/\rho_r} \\
&\quad \times \left(\int X^{\theta_j} \left(\frac{I_{rt}(X)}{I(X)}\right)^{\lambda\beta} (\tau G_j(X))^{\beta/\rho_r} \left(\frac{X^{\theta_j} C_r}{I_{rt}(X)}\right)^{\lambda\beta/\rho_r} N M_X(X)\mathrm{d}X\right)^{1-\alpha_r}
\end{aligned}$$

and

$$\begin{aligned}
\left(\frac{Y_{jt}^{\mathrm{pt}}}{A_{jt}}\right)^{1+\alpha_r\lambda\beta/\rho_r} &= \left(\frac{C^{\mathrm{pt}}}{C_r} A_{jt}\right)^{(1-\alpha_r)\lambda\beta/\rho_r} H_{jt}^{(1-\alpha_r)(1+\alpha_r\lambda\beta/\rho_r)} \\
Y_{jt}^{\mathrm{pt}} &= \left(\frac{C_r^{\mathrm{pt}}}{C_r}\right)^{\frac{\lambda\beta/\rho_r \cdot (1-\alpha_r)}{1+\alpha_r\lambda\beta/\rho_r}} Y_{jt}
\end{aligned}$$

which we replace to get the wage

$$
\begin{aligned}
W_{jt}^{\mathrm{pt}}(X) &= C_r^{\mathrm{pt}} A_{jt} X^{\theta_j} \left(\frac{Y^{\mathrm{pt}}}{A_{jt}}\right)^{-\frac{\alpha_r}{1-\alpha_r}} \\
&= C_r^{\mathrm{pt}} A_{jt} X^{\theta_j} \left(\frac{C^{\mathrm{pt}}}{C_r} A_{jt}\right)^{-\alpha_r \frac{\lambda\beta/\rho_r}{(1+\alpha_r\lambda\beta/\rho_r)}} H_{jt}^{-\alpha_r} \\
&= \left(\frac{C_r^{\mathrm{pt}}}{C_r}\right)^{\frac{1}{1+\alpha_r\lambda\beta/\rho_r}} \cdot C_r A_{jt} X^{\theta_j} \left(\frac{C^{\mathrm{pt}}}{C_r} A_{jt}\right)^{-\alpha_r \frac{\lambda\beta/\rho_r}{(1+\alpha_r\lambda\beta/\rho_r)}} H_{jt}^{-\alpha_r} \\
&= \left(\frac{C_r^{\mathrm{pt}}}{C_r}\right)^{\frac{1}{1+\alpha_r\lambda\beta/\rho_r}} W_{jt}(X)
\end{aligned}
$$

and hence similarly for demand

$$
D_{jt}^{\mathrm{pt}}(X) = \left(\frac{C_r^{\mathrm{pt}}}{C_r}\right)^{\frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}} D_{jt}(X)
$$

and wage bill

$$
\begin{aligned}
B_{jt}^{\mathrm{pt}} &= \int W_{jt}^{\mathrm{pt}}(X) \cdot D_{jt}^{\mathrm{pt}}(X) \mathrm{d}X \\
&= \int \left(\frac{C_r^{\mathrm{pt}}}{C_r}\right)^{\frac{1}{1+\alpha_r\lambda\beta/\rho_r}} W_{jt}(X) \cdot \left(\frac{C_r^{\mathrm{pt}}}{C_r}\right)^{\frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}} D_{jt}(X) \mathrm{d}X \\
&= \left(\frac{C_r^{\mathrm{pt}}}{C_r}\right)^{\frac{1+\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}} B_{jt}
\end{aligned}
$$

We finally note that

$$
\begin{aligned}
Y_{jt} &= A_{jt} \left(A_{jt}^{(1-\alpha_r)\lambda\beta/\rho_r} H_{jt}^{(1-\alpha_r)(1+\alpha_r\lambda\beta/\rho_r)}\right)^{\frac{1}{1+\alpha_r\lambda\beta/\rho_r}} \\
&= A_{jt}^{\frac{1+\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}} H_{jt}^{1-\alpha_r}
\end{aligned}
$$

And similarly we get that

$$
\begin{aligned}
B_{jt} &= \int W_{jt}(X) \cdot D_{jt}(X) \mathrm{d}X \\
&= \int X^{\theta_j} C_r H_{jt}^{-\alpha_r} (A_{jt})^{\frac{1}{1+\alpha_r\lambda\beta/\rho_r}} \cdot D_{jt}(X) \mathrm{d}X \\
&= C_r H_{jt}^{-\alpha_r} (A_{jt})^{\frac{1}{1+\alpha_r\lambda\beta/\rho_r}} \left(\frac{Y_{jt}}{A_{jt}}\right)^{\frac{1}{1-\alpha_r}} \\
&= C_r H_{jt}^{-\alpha_r} (A_{jt})^{\frac{1}{1+\alpha_r\lambda\beta/\rho_r}} H_{jt} (A_{jt})^{\frac{\lambda\beta/\rho_r}{(1+\alpha_r\lambda\beta/\rho_r)}} \\
&= C_r Y_{jt}
\end{aligned}
$$

And similarly we get that $B_{jt}^{\mathrm{pt}} = C_r^{\mathrm{pt}} Y_{jt}^{\mathrm{pt}}$. We get that

$$\begin{aligned}
\frac{\Pi_{jt} - \Pi_{jt}^{\mathrm{pt}}}{\Pi_{jt}} &= 1 - \frac{Y_{jt}^{\mathrm{pt}} - B_{jt}^{\mathrm{pt}}}{Y_{jt} - B_{jt}} \\
&= 1 - \frac{1 - C_r^{\mathrm{pt}}}{1 - C_r} \left(\frac{C_r^{\mathrm{pt}}}{C_r} \right)^{\frac{\lambda\beta/\rho_r \cdot (1-\alpha_r)}{1+\alpha_r \lambda\beta/\rho_r}} \\
&= 1 - \frac{\alpha_r \left(1 + \lambda\beta/\rho_r\right)}{1 + \alpha_r \lambda\beta/\rho_r} \left(\frac{\lambda\beta/\rho_r}{1 + \lambda\beta/\rho_r} \right)^{-\frac{(1-\alpha_r)\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}}
\end{aligned}$$

$$\Pi_{jt} - \Pi_{jt}^{\mathrm{pt}} = \left(1 - \frac{\alpha_r \left(1 + \lambda\beta/\rho_r\right)}{1 + \alpha_r \lambda\beta/\rho_r} \left(\frac{\lambda\beta/\rho_r}{1 + \lambda\beta/\rho_r} \right)^{-\frac{(1-\alpha_r)\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}} \right) \Pi_{jt}$$

□

C.4 Firms market rent

Lemma 11. *We establish that the market level rents for firm $j \in J_r$ is given by*

$$\Pi - \Pi^{ptm} = \left(1 - \frac{\alpha_r \left(1 + \lambda\beta/\rho_r\right)}{1 + \alpha_r \lambda\beta/\rho_r} \left(\frac{\lambda\beta/\rho_r}{1 + \lambda\beta/\rho_r} \right)^{-\frac{(1-\alpha_r)\lambda\beta}{1+\alpha_r\lambda\beta}} \right) \Pi$$

Proof. Here we consider the case where all firm in a given market are price taker. In this case we also get that the $I_r(X)$ changes. Firm wage is still given by

$$(1-\alpha_r) A_{jt} X^{\theta_j} \left(\frac{Y^{\mathrm{ptm}}}{A_{jt}} \right)^{-\frac{\alpha_r}{1-\alpha_r}} = W_{jt}^{\mathrm{ptm}}(X)$$

However the labor supply curve is not the same as in equilibrium anymore since all firms change their demand.

$$S_{jt}(X, W) = \left(\frac{I_{rt}^{\mathrm{ptm}}(X)}{I(X)} \right)^{\lambda\beta} \left(G_j(X)^{1/\lambda} \frac{\tau^{1/\lambda} W}{I_{rt}^{\mathrm{ptm}}(X)} \right)^{\lambda\beta/\rho_r}$$

where

$$I_{rt}^{\mathrm{ptm}}(X) = \left(\sum_{j' \in J_r} \left(\tau G_{j'}(X)\right)^{\beta/\rho_r} \left(W_{j't}^{\mathrm{ptm}}(X) \right)^{\lambda\beta/\rho_r} \right)^{\rho_r/\lambda\beta}$$

so similarly, we want to sub this definitions into Y_{jt}^{ptm}

$$
\begin{aligned}
\frac{Y_{jt}^{\text{ptm}}}{A_{jt}} &= \left(\int X^{\theta_j} \cdot D_j(X) \mathrm{d}X\right)^{1-\alpha_r} \\
&= \left(\int X^{\theta_j} \cdot \left(\frac{I_{rt}^{\text{ptm}}(X)}{I(X)}\right)^{\lambda\beta} (\tau G_{j'}(X))^{\beta/\rho_r} \left(\frac{W_j^{\text{ptm}}\ (X)}{I_{rt}^{\text{ptm}}(X)}\right)^{\lambda\beta/\rho_r} \mathrm{d}X\right)^{1-\alpha_r} \\
&\left(\int X^{\theta_j} \cdot \left(\frac{I_{rt}^{\text{ptm}}(X)}{I(X)}\right)^{\lambda\beta} (\tau G_{j'}(X))^{\beta/\rho_r} \left(\frac{(1-\alpha_r) A_{jt} X^{\theta_j}}{I_{rt}^{\text{ptm}}(X)} \left(\frac{Y_{jt}^{\text{ptm}}}{A_j}\right)^{-\frac{\alpha_r}{1-\alpha_r}}\right)^{\lambda\beta/\rho_r} \mathrm{d}X\right)^{1-\alpha_r} \\
&= A_{jt}^{(1-\alpha_r)\beta/\rho_r} \left(\int X^{\theta_j} \cdot \left(\frac{I_{rt}^{\text{ptm}}(X)}{I(X)}\right)^{\lambda\beta} (\tau G_{j'}(X))^{\beta/\rho_r} \left(\frac{C^{\text{pt}}\ X^{\theta_j}}{I_{rt}^{\text{ptm}}(X)}\right)^{\lambda\beta/\rho_r} \mathrm{d}X\right)^{1-\alpha_r} \left(\frac{Y_{jt}^{\text{ptm}}}{A_{jt}}\right)^{-\alpha_r\lambda\beta/\rho_r} \\
&= A_{jt}^{(1-\alpha_r)\lambda\beta/\rho_r} \left(\underbrace{\int X^{\theta_j} \cdot \left(\frac{I_{rt}^{\text{ptm}}(X)}{I(X)}\right)^{\lambda\beta} (\tau G_{j'}(X))^{\beta/\rho_r} \left(\frac{C^{\text{pt}}\ X^{\theta_j}}{I_{rt}^{\text{ptm}}(X)}\right)^{\lambda\beta/\rho_r} \mathrm{d}X}_{\left(H_{jt}^{\text{ptm}}\ \right)^{1+\alpha_r\lambda\beta/\rho_r}}\right)^{1-\alpha_r} \left(\frac{Y_{jt}^{\text{ptm}}}{A_{jt}}\right)^{-\alpha_r\lambda\beta/\rho_r} \\
&= A_{jt}^{(1-\alpha_r)\lambda\beta/\rho_r} \left(H_{jt}^{\text{ptm}}\ \right)^{(1-\alpha_r)(1+\alpha_r\lambda\beta/\rho_r)} \left(\frac{Y^{\text{ptm}}}{A_{jt}}\right)^{-\alpha_r\lambda\beta/\rho_r}
\end{aligned}
$$

and we get a wage equation

$$
W_j^{\text{ptm}}(X) = C^{\text{pt}}\ X^{\theta_j} \left(H_{jt}^{\text{ptm}}\right)^{-\alpha_r} A_{jt}^{\frac{1}{1+\alpha_r\lambda\beta/\rho_r}}
$$

and as in the usual equilibrium we are left with finding H_{rt}^{ptm} as a function of the market TFP and amenities

$$
H_{jt}^{\text{ptm}} = \left(\int X^{\theta_j} \cdot \left(\frac{I_{rt}^{\text{ptm}}(X)}{I(X)}\right)^{\lambda\beta} (\tau G_{j'}(X))^{\beta/\rho_r} \left(\frac{C_r^{\text{pt}}\ X^{\theta_j}}{I_{rt}^{\text{ptm}}(X)}\right)^{\lambda\beta/\rho_r} N M_X(X) \mathrm{d}X\right)^{\frac{1}{1+\alpha_r\lambda\beta/\rho_r}}
$$

and note first that

$$
\begin{aligned}
I_{rt}^{\text{ptm}}(X) &= \left(\sum_{j' \in J_r} (\tau G_{j'}(X))^{\beta/\rho_r} \left(W_{j't}^{\text{ptm}}(X)\right)^{\lambda\beta/\rho_r}\right)^{\rho_r/\lambda\beta} \\
&= \left(\sum_{j' \in J_r} (\tau G_{j'}(X))^{\beta/\rho_r} \left(C_r^{\text{pt}}\ \left(H_{jt}^{\text{ptm}}\right)^{-\alpha_r}\right)^{\lambda\beta/\rho_r} (A_{jt})^{\frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}}\right)^{\rho_r/\lambda\beta} \\
&= \left(\frac{C_r^{\text{pt}}}{C_r}\right) \left(\sum_{j' \in J_r} (\tau G_{j'}(X))^{\beta/\rho_r} \left(C_r \left(H_{jt}^{\text{ptm}}\right)^{-\alpha_r}\right)^{\lambda\beta/\rho_r} (A_{jt})^{\frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}}\right)^{\rho_r/\lambda\beta}
\end{aligned}
$$

and next we want to show that $H_{jt}^{\text{ptm}} = \left(\frac{C^{\text{pt}}}{C}\right)^{\frac{\lambda\beta}{1+\alpha_r\lambda\beta}1[j\in J_r]} H_{jt}$. To see this we note that it solves a very similar fixed point. Indeed

$$\begin{aligned}
H_{jt}^{\text{ptm}} &= \left(\int X^{\theta_j}\cdot\left(\frac{I_{rt}^{\text{ptm}}(X)}{I(X)}\right)^{\lambda\beta}(\tau G_j(X))^{\beta/\rho_r}\left(\frac{C_r^{\text{pt}}\ X^{\theta_j}}{I_{rt}^{\text{ptm}}(X)}\right)^{\lambda\beta/\rho_r} NM_X(X)\mathrm{d}X\right)^{\frac{1}{1+\alpha_r\lambda\beta/\rho_r}} \\
&= \left(\frac{C_r^{\text{pt}}}{C_r}\right)^{\frac{\lambda\beta}{1+\alpha_r\lambda\beta/\rho_r}1[j\in J_r]}\Gamma_{jt}^{*}\left[\vec{H}_{jt}^{\text{ptm}}\right]
\end{aligned}$$

where $\Gamma_{jt}^{*}(\cdot)$ is the operator defined in Lemma 7, equation 37 that defines H_{jt} as a fixed point. For this operator we know that $\Gamma_{jt}^{*}(\vec{H}_{jt}) = H_{jt}$ is the unique fixed point. The next step is to check that $\vec{H}_t'$, defined such that its j component is $H_{jt}' = \left(\frac{C_r^{\text{pt}}}{C_r}\right)^{-\frac{\lambda\beta}{1+\alpha_r\lambda\beta}1[j\in J_r]} H_{jt}^{\text{ptm}}$, is a fixed point of the same operator Γ^*.

$$\begin{aligned}
\Gamma_{jt}^{*}\left[\vec{H}_t'\right] &= \left(\frac{C_r^{\text{pt}}}{C_r}\right)^{-\frac{\lambda\beta}{1+\alpha_r\lambda\beta}\frac{(1-\rho_r)\alpha_r\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}1[j\in J_r]}\Gamma_{jt}^{*}\left[\vec{H}_{jt}^{\text{ptm}}\right] \\
&= \left(\frac{C_r^{\text{pt}}}{C_r}\right)^{-\frac{\lambda\beta}{1+\alpha_r\lambda\beta}\frac{(1-\rho_r)\alpha_r\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}1[j\in J_r]} H_{jt}^{\text{ptm}}\left(\frac{C_r^{\text{pt}}}{C_r}\right)^{-\frac{\lambda\beta}{1+\alpha_r\lambda\beta/\rho_r}1[j\in J_r]} \\
&= \left(\frac{C_r^{\text{pt}}}{C_r}\right)^{-\frac{\lambda\beta/\rho}{1+\alpha_r\lambda\beta}\frac{(1-\rho_r)\alpha_r\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}1[j\in J_r]-\frac{\lambda\beta}{1+\alpha_r\lambda\beta/\rho_r}1[j\in J_r]} H_{jt}^{\text{ptm}} \\
&= \left(\frac{C_r^{\text{pt}}}{C_r}\right)^{-\frac{\lambda\beta/\rho}{1+\alpha_r\lambda\beta}1[j\in J_r]} H_{jt}^{\text{ptm}} \\
&= H_{jt}'
\end{aligned}$$

hence $H_{jt}' = H_{jt}$ for all j and so we get that

$$H_{jt}^{\text{ptm}} = \left(\frac{C_r^{\text{pt}}}{C_r}\right)^{\frac{\lambda\beta}{1+\alpha_r\lambda\beta}} H_{jt}$$

and so we get that

$$\begin{aligned}
W_{jt}^{\text{ptm}}(X) &= C_r^{\text{pt}}\ X^{\theta_j}\left(H_{jt}^{\text{ptm}}\right)^{-\alpha_r} A_{jt}^{\frac{1}{1+\alpha_r\lambda\beta/\rho_r}} \\
&= C_r^{\text{pt}}\ X^{\theta_j} H_{jt}^{-\alpha_r} A_{jt}^{\frac{1}{1+\alpha_r\lambda\beta/\rho_r}} \\
&= \left(\frac{C_r^{\text{pt}}}{C_r}\right)^{\frac{1}{1+\alpha_r\lambda\beta}} W_{jt}(X)
\end{aligned}$$

and note that them

$$
\begin{aligned}
I_{rt}^{\mathrm{ptm}}(X) &= \left(\sum_{j' \in J_r} (\tau G_{j'}(X))^{\beta/\rho_r} \left(W_{j't}^{\mathrm{ptm}}(X) \right)^{\lambda\beta/\rho_r} \right)^{\rho_r/\lambda\beta} \\
&= \left(\frac{C_r^{\mathrm{pt}}}{C_r} \right)^{\frac{1}{1+\alpha_r\lambda\beta}} I_{rt}(X)
\end{aligned}
$$

Next, let us rewrite the realized demand:

$$
\begin{aligned}
D_{jt}^{\mathrm{ptm}}(X) &= \left(\frac{I_{rt}^{\mathrm{ptm}}(X)}{I(X)} \right)^{\lambda\beta} (\tau G_{j'}(X))^{\beta/\rho_r} \left(\frac{W_j^{\mathrm{ptm}}(X)}{I_{rt}^{\mathrm{ptm}}(X)} \right)^{\lambda\beta/\rho_r} \\
&= \left(\frac{C_r^{\mathrm{pt}}}{C_r} \right)^{\frac{\lambda\beta}{1+\alpha_r\lambda\beta}} \left(\frac{I_{rt}(X)}{I(X)} \right)^{\lambda\beta} (\tau G_{j'}(X))^{\beta/\rho_r} \left(\frac{W_j(X)}{I_{rt}(X)} \right)^{\lambda\beta/\rho_r} \\
&= \left(\frac{C_r^{\mathrm{pt}}}{C_r} \right)^{\frac{\lambda\beta}{1+\alpha_r\lambda\beta}} D_{jt}(X)
\end{aligned}
$$

We then go back and compute output and wage bills.

$$
\begin{aligned}
Y_{jt}^{\mathrm{ptm}} &= A_{jt} \left(\int X^{\theta_j} \cdot D_{jt}^{\mathrm{ptm}}(X) \mathrm{d}X \right)^{1-\alpha_r} \\
&= \left(\frac{C_r^{\mathrm{pt}}}{C_r} \right)^{\frac{(1-\alpha_r)\lambda\beta}{1+\alpha_r\lambda\beta}} Y_{jt}
\end{aligned}
$$

$$
\begin{aligned}
B_{jt}^{\mathrm{ptm}} &= \int W_{jt}^{\mathrm{ptm}}(X) \cdot D_{jt}^{\mathrm{ptm}}(X) \mathrm{d}X \\
&= \left(\frac{C_r^{\mathrm{pt}}}{C_r} \right)^{\frac{1+\lambda\beta}{1+\alpha_r\lambda\beta}} B_{jt}
\end{aligned}
$$

As before we get that

$$
\begin{aligned}
\frac{\Pi_{jt} - \Pi_{jt}^{\mathrm{ptm}}}{\Pi_{jt}} &= 1 - \frac{Y_{jt}^{\mathrm{ptm}} - B_{jt}^{\mathrm{ptm}}}{Y_{jt} - B_{jt}} \\
&= 1 - \frac{1 - C_r^{\mathrm{pt}}}{1 - C_r} \left(\frac{C_r^{\mathrm{pt}}}{C_r} \right)^{\frac{(1-\alpha_r)\lambda\beta}{1+\alpha_r\lambda\beta}} \\
&= 1 - \frac{\alpha_r (1 + \lambda\beta/\rho_r)}{1 + \alpha_r\lambda\beta/\rho_r} \left(\frac{\lambda\beta/\rho_r}{1 + \lambda\beta/\rho_r} \right)^{-\frac{(1-\alpha_r)\lambda\beta}{1+\alpha_r\lambda\beta}}
\end{aligned}
$$

$$
\Pi_{jt} - \Pi_{jt}^{\mathrm{ptm}} = \left(1 - \frac{\alpha_r (1 + \lambda\beta/\rho_r)}{1 + \alpha_r\lambda\beta/\rho_r} \left(\frac{\lambda\beta/\rho_r}{1 + \lambda\beta/\rho_r} \right)^{-\frac{(1-\alpha_r)\lambda\beta}{1+\alpha_r\lambda\beta}} \right) \Pi_{jt}
$$

□

C.5 A model with capital and monopolistic competition in the product market

We develop here a simple extension of the model with capital and monopolistic competition in the product market. Without loss of generality, we derive the results here in the case of homogenous labor.

Consider a firm with production function $Q = AK^{\rho}L^{1-\alpha}$, access to a local monopolistic market with revenue curve $Y = Q^{1-\epsilon}$, hiring labor from a local labor supply curve $L(W) = W^{\beta}$ and renting capital at price r. Profit is given by:

$$Q^{1-\epsilon} - LW - rK.$$

We first note that we can replace Q with the production function and get

$$\left(AK^{\rho}L^{1-\alpha}\right)^{1-\epsilon} - LW - rK.$$

Hence, considering perfect or monopolistic competition in the product market give rise to the same revenue function. We will focus directly on the revenue function parametrized as:

$$Y = AK^{\rho}L^{1-\alpha},$$

where $\rho = \tilde{\rho}(1-\epsilon)$ with $\tilde{\rho}$ being production and ϵ being product market. We then have the following Lagrangian for our problem:

$$AK^{\rho}L^{1-\alpha} - LW - rK - \mu(L - W^{\beta}).$$

We take the first order condition for K and get:

$$K = \left(\frac{r}{\rho AL^{1-\alpha}}\right)^{\frac{1}{\rho-1}},$$

which we then replace in

$$\begin{aligned}
AK^{\rho}L^{1-\alpha} - LW - rK &= A\left(\frac{r}{\rho AL^{1-\alpha}}\right)^{\frac{\rho}{\rho-1}} L^{1-\alpha} - LW - r\left(\frac{r}{\rho AL^{1-\alpha}}\right)^{\frac{1}{\rho-1}} \\
&= (1+A)\left(\frac{r}{\rho AL^{1-\alpha}}\right)^{\frac{\rho}{\rho-1}} L^{1-\alpha} - LW \\
&= (1+A)\left(\frac{r}{\rho A}\right)^{\frac{\rho}{\rho-1}} L^{1-\frac{1+\alpha\rho}{1-\rho}} - LW
\end{aligned}$$

which is just a re-interpretation of the original problem with $\tilde{A} = (1+A)\left(\frac{r}{\rho A}\right)^{\frac{\rho}{\rho-1}}, \alpha = \frac{1+\alpha\rho}{1-\rho}$.

C.6 Welfare

We start by defining a measure of welfare given a set of wages and tax parameters. As presented in the previous section the average utility that worker enjoy for a given set of wages is given by

$$\mathbb{E}u_{ijt} = \int M(X) \log \sum_r \left(\sum_{j \in J_r} \left(\tau W_{jt}(X)^{\lambda} G_j(X) \right)^{\beta/\rho_r} \right)^{\rho_r} \mathrm{d}X$$

and the total tax revenue and total firm profits are given by

$$\begin{aligned}
R_t &= \int \sum_r \sum_{j \in J_r} D_{jt}(X) \left(W_{jt}(X) - \tau W_{jt}(X)^{\lambda} \right) \mathrm{d}X \\
&= \int \sum_r \sum_{j \in J_r} D_{jt}(X) W_{jt}(X) \mathrm{d}X - \int \sum_r \sum_{j \in J_r} D_{jt}(X) \tau W_{jt}(X)^{\lambda} \mathrm{d}X \\
&= B - B^{\text{net}} \\
\Pi_t &= \sum_r \sum_{j \in J_r} A_{jt} \left(\int X^{\theta_j} \cdot D_{jt}(X) \mathrm{d}X \right)^{1-\alpha} - \int W_{jt}(X) \cdot D_{jt}(X) \mathrm{d}X \\
&= Y_t - B_t
\end{aligned}$$

To take into account changes in tax revenue and firm profit across counterfactuals, we redistribute Π_t and R_t to workers in the form of a non-distortionary payment proportional to net wages, governed by ϕ_t. This means that each worker receives $\phi_t \tau W_{jt}(X)^{\lambda}$ in transfers. This total transfer equates $\Pi + R$ and is given by

$$\begin{aligned}
\int \phi_t \tau W_{jt}(X)^{\lambda} \cdot D_{jt}(X) \mathrm{d}X &= \Pi_t + R_t \\
\phi_t B^{\text{net}} &= \Pi_t + R_t
\end{aligned}$$

and hence

$$\begin{aligned}
1 + \phi_t &= \frac{\Pi + R + B^{\text{net}}}{B^{\text{net}}} \\
&= \frac{\Pi + B}{B^{\text{net}}} \\
&= \frac{Y}{B^{\text{net}}}
\end{aligned}$$

Thus, the welfare is given by

$$
\begin{aligned}
\mathbb{W}_t = \frac{1}{\beta}\mathbb{E}u_{ijt} + \log(1+\phi_t) \\
= \underbrace{\frac{1}{\beta}\int M_X(X)\log\sum_r\left(\sum_{j\in J_r}\left(\tau W_{jt}(X)^\lambda G_j(X)\right)^{\beta/\rho_r}\right)^{\rho_r}\mathrm{d}X}_{\text{utility from net-wages and amenities}} \\
+ \underbrace{\log\sum_r\sum_{j\in J_r}A_{jt}\left(\int X^{\theta_j}\cdot D_{jt}(X)\mathrm{d}X\right)^{1-\alpha}}_{\text{utility from profits}} \\
- \underbrace{\log\int\sum_r\sum_{j\in J_r}D_{jt}(X)\tau W_{jt}(X)^\lambda}_{\text{extra cost because paid proportionaly}}
\end{aligned}
$$

C.7 Walrasian Equilibrium

We consider an equilibrium as defined by a set of wages $W^c_{jt}(X)$ such that worker choose where to work optimally given these wages, and firm choose labor demand optimally also taking these wages as given. In this equilibrium we make the tax system neutral $\lambda = \tau = 1$.

$$
\max_{\{D_{jt}(X)\}_{(X)}} A_{jt}\left(\int X^{\theta_j}D_{jt}(X)\mathrm{d}X\right)^{1-\alpha_r} - \iint W^c_{jt}(X)D_{jt}(X)\mathrm{d}X
$$

which gives the first order condition

$$
(1-\alpha_r)X^{\theta_j}A_j\left(\int X^{\theta_j}\cdot D_{jt}(X)\mathrm{d}X\right)^{-\alpha} = W^c_{jt}(X)
$$

or

$$
W^c_{jt}(X) = \underbrace{(1-\alpha_r)}_{C^{\mathrm{pt}}}X^{\theta_j}A_{jt}\left(\frac{Y^c_{jt}}{A_{jt}}\right)^{-\frac{\alpha}{1-\alpha}}
$$

Finally we can replace into the expression for output:

$$
\begin{aligned}
\left(\frac{Y^c_{jt}}{A_{jt}}\right)^{\frac{1}{1-\alpha_r}} &= \int X^{\theta_j}\cdot D_{jt}(X)\mathrm{d}X \\
&= \int X^{\theta_j} NM(X)\cdot \frac{\left(\sum_{j\in J_r}\left(W^{\mathrm{c}}_{jt}(X)G_j(X)\right)^{\beta/\rho_r}\right)^{\rho_r}}{\sum_r \left(\sum_{j\in J_r}\left(W^{\mathrm{c}}_{jt}(X)G_j(X)\right)^{\beta/\rho_r}\right)^{\rho_r}}\cdot \frac{\left(W^{\mathrm{c}}_{jt}(X)G_j(X)\right)^{\beta/\rho_r}}{\sum_{j\in J_r}\left(W^{\mathrm{c}}_{jt}(X)G_j(X)\right)^{\beta/\rho_r}}\mathrm{d}X \\
&= A_{jt}^{\beta/\rho_r}\left(\frac{Y_{jt}}{A_{jt}}\right)^{-\frac{\alpha_r\beta/\rho_r}{1-\alpha}} \int X^{\theta_j+\beta/\rho_r} M(X)\cdot \frac{\left(\sum_{j\in J_r}\left(W^{\mathrm{c}}_{jt}(X)G_j(X)\right)^{\beta/\rho_r}\right)^{\rho_r}}{\sum_r \left(\sum_{j\in J_r}\left(W^{\mathrm{c}}_{jt}(X)G_j(X)\right)^{\beta/\rho_r}\right)^{\rho_r}}\cdot \frac{(C^{\mathrm{pt}}G_j(X)}{\sum_{j\in J_r}(W^{\mathrm{c}}_{jt}(X)G} \\
&= A_{jt}^{\beta/\rho_r}\left(\frac{Y_{jt}}{A_{jt}}\right)^{-\frac{\alpha\beta/\rho_r}{1-\alpha}} \left(H^{\mathrm{c}}_{jt}\right)^{1+\alpha_r\beta/\rho_r} \\
\left(\frac{Y_{jt}}{A_{jt}}\right)^{\frac{1+\alpha_r\beta/\rho_r}{1-\alpha_r}} &= \left(H^{\mathrm{c}}_{jt}\right)^{1+\alpha\beta/\rho_r} A_{jt}^{\beta/\rho_r}
\end{aligned}
$$

where we defined

$$
\left(H^{\mathrm{c}}_{jt}\right)^{1+\alpha\beta/\rho_r} \equiv \int X^{\theta_j+\beta/\rho_r} NM(X)\cdot \frac{\left(\sum_{j\in J_r}\left(W^{\mathrm{c}}_{jt}(X)G_j(X)\right)^{\beta/\rho_r}\right)^{\rho_r}}{\sum_r \left(\sum_{j\in J_r}\left(W^{\mathrm{c}}_{jt}(X)G_j(X)\right)^{\beta/\rho_r}\right)^{\rho_r}}\cdot \frac{\left(C^{\mathrm{pt}}G_j(X)\right)^{\beta/\rho_r}}{\sum_{j\in J_r}\left(W^{\mathrm{c}}_{jt}(X)G_j(X)\right)^{\beta/\rho_r}}\mathrm{d}X
$$

giving

$$
\begin{aligned}
W^{\mathrm{c}}_{jt}(X) &= C^{\mathrm{pt}} X^{\theta_j} A_{jt}\left(\left(H^{\mathrm{c}}_{jt}\right)^{1+\alpha\beta/\rho_r} A_{jt}^{\beta/\rho_r}\right)^{\frac{-\alpha}{1+\alpha\beta/\rho_r}} \\
&= C^{\mathrm{pt}} X^{\theta_j}\left(H^{\mathrm{c}}_{jt}\right)^{-\alpha_r}\left(A_{jt}\right)^{\frac{1}{1+\alpha\beta/\rho_r}}
\end{aligned}
$$

next, defining $H^{\mathrm{c}}_{jt} = H^{\mathrm{c}}_j \bar{A}^{\frac{(\rho-1)\beta/\rho_r}{(1+\alpha\beta)(1+\alpha\beta/\rho_r)}}$ following a similar proof to the main proposition we find that

$$
w^{\mathrm{c}}_j(x,v,\bar{a},\tilde{a}) = c^{\mathrm{pt}} + \theta_j x - \alpha h^{\mathrm{c}}_j + \frac{1}{1+\alpha_r\beta/\rho_r}\tilde{a} + \frac{1}{1+\alpha\beta}\bar{a}
$$

where

$$
\begin{aligned}
H^{\mathrm{c}}_j &= \left[\int X^{\theta_j(1+\beta/\rho_r)}\left(\frac{I^{\mathrm{c}}_{r0}(X)}{I^{\mathrm{c}}_0(X)}\right)^{\beta}\left(\frac{1}{I^{\mathrm{c}}_{r0}(X)}\right)^{\beta/\rho_r}\mathring{G}_j(X)^{\beta/\rho_r} C^{\mathrm{pt}}_r \mathring{N} M_X(X)\mathrm{d}X\right]^{\frac{1}{1+\alpha_r\beta/\rho_r}} \\
I^{\mathrm{c}}_{r0}(X) &= \left(\mathbb{E}_{j\in J_r}\left(\mathring{G}_j(X)X^{\theta_j}C^{\mathrm{pt}}_r\left(H^{\mathrm{c}}_j\right)^{-\alpha_r}\right)^{\beta/\rho_r}\left(\tilde{A}_{jt}\right)^{\frac{\beta/\rho_r}{1+\alpha_r\beta/\rho_r}}\right)^{\rho_r/\beta}
\end{aligned}
$$

which we can then replace to get the allocation to each firm given by

$$
\text{for } j\in J_r \qquad D^{\mathrm{c}}_{jt}(X) = \mathring{N} M_X(X)\frac{\left(I^{\mathrm{c}}_{r0}(X)\bar{A}_{rt}^{\frac{1}{1+\alpha_r\beta}}\right)^{\beta}}{\sum_{r'}\left(I^{\mathrm{c}}_{r'0}(X)\bar{A}_{r't}^{\frac{1}{1+\alpha_{r'}\beta}}\right)^{\beta}}\left(\frac{G_j W^c_{jt}(X)}{I^{\mathrm{c}}_{r0}(X)\bar{A}_{rt}^{\frac{1}{1+\alpha_r\beta}}}\right)^{\beta/\rho_r}
$$

where we note that τ does not enter the allocation.

We first study the effect of market power on the allocation. This goes through the market specific constant $\frac{\beta/\rho_r}{1+\beta/\rho_r}$. In the case where $\rho = \rho_r$, the markdown only acts as an overall economy markdown. This means it does not actually affect allocation. To see that, we note that when $\rho = \rho_r$ the fixed point for h_j reduces to the fixed-point of h_j^{p} time a constant. Hence we end up with only a scale in the equilibrium equation and no distortion in the MRS for workers across firms.

In the case where ρ_r are not the same across regions, we end up with a wedge in the wage equation. This creates in terms a wedge in the allocation. Correcting such miss-allocation can be achieved by having region specific taxes τ_r set precisely to the inverse of the wedge. This brings us back to the planner allocation.

We can also use the planner solution to evaluate the effect of tax-policies. The parameter τ alone, does not creates a wedge on in the wedge equation, however it does not affect the allocation as all options are equally affected.

C.8 Policy counterfactual

Lemma 12. *Setting a tax policy to* $\tau_r = \frac{1+\beta/\rho_r}{\beta/\rho_r}$ *and* $\lambda = 1$ *achieves the competitive allocation of workers to firms.*

Proof. We plug in $\tau_r = \frac{1+\beta/\rho_r}{\beta/\rho_r}$ into the firm problem and show that it actually achieves the planner solution in this context. Using the fix point equation for h_j we get that

$$\begin{aligned} H_j &= \bar{V} \int X^{\theta_j(1+\lambda\beta/\rho_r)} \left(\frac{I_{r0}(X)}{I_0(X)}\right)^{\lambda\beta} \left(\frac{1}{I_{r0}(X)}\right)^{\lambda\beta/\rho_r} \left(\tau \mathring{G}_j(X) C_r^{\lambda} H_j^{-\lambda\alpha_r}\right)^{\beta/\rho_r} \mathring{N} M(X) \mathrm{d}X \\ I_{r0}(X)^{\lambda\beta} &= \mathbb{E}_j \left[\left(\tau \mathring{G}_j(X) X^{\lambda\theta_j} C_r^{\lambda} H_j^{-\lambda\alpha_r}\right)^{\beta/\rho_r} \tilde{A}_{jt}^{\frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}} \right] \\ I_0(X)^{\lambda\beta} &= \mathbb{E}_r \left[I_{r0}(X)^{\lambda\beta} \bar{A}_{rt}^{\frac{\lambda\beta}{1+\alpha_r\lambda\beta}} \right] \end{aligned}$$

where we notice that τC_r^{λ} always appear together and under this particular policy we get that $\tau_r C_r^{\lambda} = (1-\alpha_r)$ and hence the h_j coincides exactly with h_j^{c}. This also applies to $I_{r0}(X)$ and $I_0(X)$. We then see that this implies that $D_j(X) = D_j^{\mathrm{c}}(X)$. In other words such policy achieves exactly the planner allocation. □

D Model identification and estimation

D.1 Moment condition in the dynamic model

We now derive the estimating equations and show how to consistently estimate the parameters of interest using a sample of workers that stay within the firm over time. The derivation follows closely the derivations of Section 3 of the paper. We seek to establish that the following moment condition holds in the model:

$$\mathbb{E}\left[\Delta\tilde{y}_{j(i),t}\left(\tilde{w}_{it+\tau}-\tilde{w}_{it-\tau'}-\frac{1}{1+\lambda\beta/\rho_r}\left(\tilde{y}_{j(i),t+\tau}-\tilde{y}_{j(i),t-\tau'}\right)\right)|S_i{=}1\right]=0$$
$$\mathbb{E}\left[\Delta\bar{y}_{j(i),t}\left(\bar{w}_{it+\tau}-\bar{w}_{it-\tau'}-\frac{1}{1+\lambda\beta}\left(\bar{y}_{j(i),t+\tau}-\bar{y}_{j(i),t-\tau'}\right)\right)|S_i{=}1\right]=0$$
$$\text{for } \tau\geq 2, \tau'\geq 3$$

which are the conditions of equation (6).

We start by looking at each quantity using the structure of the model.

$$\tilde{y}_{it+\tau}-\tilde{y}_{j,t-\tau'}=\frac{1+\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}\sum_{t'=t-\tau'+1}^{t+\tau}\tilde{u}_{jt'}+\xi_{j,t+\tau}-\xi^y_{j,t-\tau'}+\delta^y\left(\xi^y_{j,t+\tau-1}-\xi^y_{j,t-\tau'-1}\right)$$
$$\tilde{w}_{it+\tau}-\tilde{w}_{it-\tau'}=\sum_{t'=t-\tau'+1}^{t+\tau}\mu_{it'}+v_{it+\tau}-v_{i,t-\tau'}+\xi^x_{i,t-\tau'}-\xi^x_{i,t+\tau'}+\delta^x\left(\xi^x_{i,t-\tau'-1}-\xi^x_{i,t+\tau'-1}\right)$$
$$+\frac{1}{1+\alpha_r\lambda\beta/\rho_r}\sum_{t'=t-\tau'+1}^{t+\tau}\tilde{u}_{jt'}$$

and hence we get that for i such that $S_i{=}1$

$$\tilde{w}_{it+\tau}-\tilde{w}_{it-\tau'}-\frac{1}{1+\lambda\beta/\rho_r}\left(\tilde{y}_{j(i),t+\tau}-\tilde{y}_{j(i),t-\tau'}\right)=-\frac{1}{1+\lambda\beta/\rho_r}\left(\xi^y_{j,t+\tau}-\xi^y_{j,t-\tau'}+\delta^y\left(\xi^y_{j,t+\tau-1}-\xi^y_{j,t-\tau'-1}\right)\right)$$
$$+\xi^x_{i,t-\tau'}-\xi^x_{i,t+\tau'}+\delta^x\left(\xi^x_{i,t-\tau'-1}-\xi^x_{i,t+\tau'-1}\right)$$

and

$$\Delta\tilde{y}_{j(i),t}=\frac{1}{1+\alpha_r\lambda\beta/\rho_r}\tilde{u}_{j(i),t}+\xi^y_{j,t}-\xi^y_{j,t-1}+\delta^y\left(\xi^y_{j,t}-\xi^y_{j,t-1}\right).$$

We combine it all to get construct $\mathbb{E}\left[\Delta\tilde{y}_{j(i),t}\left(\tilde{w}_{it+\tau}-\tilde{w}_{it-\tau'}-\gamma\left(\tilde{y}_{j(i),t+\tau}-\tilde{y}_{j(i),t-\tau'}\right)\right)|S_i{=}1\right]$. The MA(1) measurement error in y_{jt} is fully exogenous and doesn't drive any decision as long as use $\tau\geq 2$ or $\tau'\geq 3$ we can ignore them. We then focus on the terms left in the expression and get:

$$\mathbb{E}\left[\frac{1}{1+\alpha_r\lambda\beta/\rho_r}\cdot\frac{1}{1+\beta/\rho_r}\tilde{u}_{jt}\left(\sum_{t'=t-\tau'}^{\tau}\mu_{it'}+\xi^x_{i,t-\tau'}-\xi^x_{i,t+\tau'}+\delta^x\left(\xi^x_{i,t-\tau'-1}-\xi^x_{i,t+\tau'-1}\right)\right)|S_i{=}1\right]$$

and given that the transitory worker shocks are also drawn iid and do not affect mobility decisions (they are rewarded in the same way everywhere), we also drop them and finally focus on the following term:

$$\mathbb{E}\left[\tilde{u}_{j(i),t}\sum_{t'=t-\tau'}^{\tau}\mu_{it'}|S_i{=}1\right]=\sum_{t'=t-\tau'}^{\tau}\mathbb{E}\left[\tilde{u}_{j(i),t}\mu_{it'}|S_i{=}1\right]=0,$$

where we rely simply on the sequential independence assumption of Assumption 2: for $t' \geq t$, $\mu_{it'}$ is drawn independently of past $\tilde{u}_{j(i),t}$ and for $t' < t$, $\tilde{u}_{j(i),t}$ is drawn independently of the past $\mu_{it'}$. This finally gives that provided that $\tau \geq 2$ or $\tau' \geq 3$, then

$$\mathbb{E}\left[\Delta\tilde{y}_{j(i),t}\left(\tilde{w}_{it+\tau} - \tilde{w}_{it-\tau'} - \frac{1}{1+\lambda\beta/\rho_r}\left(\tilde{y}_{j(i),t+\tau} - \tilde{y}_{j(i),t-\tau'}\right)\right)|S_i{=}1\right] = 0$$

A similar result can be established when replacing $\Delta\tilde{y}_{j(i),t}$ with simply $\Delta y_{j(i),t}$. The derivation of the second moment condition involving Υ follows from an identical derivation at the market level. We get following the derivation of Lemma 4:

$$\begin{aligned}
\bar{w}_{it+\tau} - \bar{w}_{it-\tau} =& \frac{1}{1+\lambda\alpha_{r(j(i))}\beta}\sum_{d=t-\tau'+1}^{t+\tau}\bar{u}_{r(j(i)),d} \\
\bar{y}_{jt+\tau} - \bar{y}_{jt-\tau'} =& \frac{1+\lambda\beta}{1+\lambda\alpha_{r(j)}\beta}\sum_{d=t-\tau'+1}^{t+\tau}\bar{u}_{r(j),d}
\end{aligned}$$

which cancel out in the difference to get the moment condition:

$$\mathbb{E}\left[\Delta\bar{y}_{j(i),t}\left(\bar{w}_{it+\tau} - \bar{w}_{it-\tau'} - \frac{1}{1+\lambda\beta}\left(\bar{y}_{j(i),t+\tau} - \bar{y}_{j(i),t-\tau'}\right)\right)|S_i{=}1\right] = 0.$$

A similar result can be established when replacing $\Delta\bar{y}_{j(i),t}$ with simply $\Delta y_{j(i),t}$. Finally the fact that $\frac{1}{1+\lambda\beta/\rho_r}$ and $\frac{1}{1+\lambda\beta}$ satisfy the same moment conditions as γ and Υ establishes the fact that

$$\begin{aligned}
\gamma &= \frac{1}{1+\lambda\beta/\rho_r} \\
\Upsilon &= \frac{1}{1+\lambda\beta}
\end{aligned}$$

D.2 Firm specific TFP a_{jt} and amenities h_j

We look the equation in the text given by

$$\mathbb{E}\left[w_{it} - \frac{1}{1+\lambda\beta}\bar{y}_{r,t} - \frac{\rho_r}{\rho_r+\lambda\beta}(y_{j,t} - \bar{y}_{r,t})\middle| \begin{array}{c} j(i,t){=}j \\ j \in J_r \end{array}\right].$$

We follow closely the steps of Online Appendix (B.4). We assume that the initial condition for value added permanent component is $\tilde{u}_{j1} = \bar{u}_{r(j)1} = 0$ and similarly for wages. We then get that

$$\begin{aligned}
w_{it} &= \theta_j x_i + v_{it} + c_r - \alpha_r h_{j(i,t)} + \frac{1}{1+\lambda\alpha_r\beta/\rho_r}\tilde{a}_{j(i,t),t} + \frac{1}{1+\lambda\alpha_r\beta}\bar{a}_{r(j(i,t)),t} \\
y^*_{j,t} &= (1-\alpha_r)h_j + \frac{1+\lambda\beta/\rho_r}{1+\lambda\alpha_r\beta/\rho_r}\tilde{a}_{jt} + \frac{1+\lambda\beta}{1+\lambda\alpha_r\beta}\bar{a}_{rt} \\
\bar{y}^*_{r,t} &= \frac{1+\lambda\beta}{1+\lambda\alpha_r\beta}\bar{a}_{rt} + (1-\alpha_r)\bar{h}_r
\end{aligned}$$

where we defined $\bar{h}_r = \mathbb{E}\left[h_j \,|\, j \in J_r\right]$. Given that the measurement error in y_{jt} is mean 0 and the same applies to v_{it} even conditional on mobility, we get that:

$$\mathbb{E}\left[w_{it} - \frac{1}{1+\lambda\beta}\bar{y}_{r,t} - \frac{\rho_r}{\rho_r+\lambda\beta}(y_{j,t}-\bar{y}_{r,t}) \middle| \begin{array}{c} j(i,t){=}j \\ j \in J_r \end{array} \right] = \theta_j x_i + \psi_j,$$

where we define

$$\psi_j \equiv c_r - \alpha_r h_j - \frac{\lambda\beta(\rho_r - 1)(1-\alpha_r)}{(1+\lambda\beta)(\rho_r+\beta)}\bar{h}_r.$$

D.3 Identification and estimation of $G_j(X)$

Lemma 13. *We show that for all $t, j \in J_r, r, X$ we have:*

$$\tau \exp(\lambda\psi_{jt}) X^{\lambda\theta_j} G_j(X) = \left(Pr[j(i,t) \in J_r \,|\, X]\right)^{1/\beta} \left(Pr[j(i,t) = j \,|\, X, j(i,t) \in J_r]\right)^{\rho_r/\beta}.$$

Proof. We have that:

$$Pr[j(i,t) = j \,|\, X, j(i,t) \in J_r] = \frac{\left(\tau^{1/\lambda} G_j(X)^{1/\lambda} \exp(\psi_{jt}) X^{\theta_j}\right)^{\lambda\beta/\rho_r}}{\sum_{j' \in J_r}\left(\tau^{1/\lambda} G_{j'}(X)^{1/\lambda} \exp(\psi_{j't}) X^{\theta_{j'}}\right)^{\lambda\beta/\rho_r}}$$

$$Pr[j(i,t) \in J_r \,|\, X] = \frac{\left(\sum_{j' \in J_r}\left(\tau^{1/\lambda} G_{j'}(X)^{1/\lambda} \exp(\psi_{j't}) X^{\theta_{j'}}\right)^{\lambda\beta/\rho_r}\right)^{\rho_r}}{\sum_{r'}\left(\sum_{j' \in J_{r'}}\left(\tau^{1/\lambda} G_{j'}(X)^{1/\lambda} \exp(\psi_{j't}) X^{\theta_{j'}}\right)^{\lambda\beta/\rho_{r'}}\right)^{\rho_{r'}}}$$

let's fix a given t and let's write $G_j(X) = \bar{G}_r(X)\tilde{G}_j(X)$ where we impose the normalization that

$$\sum_{j' \in J_r}\left(\tau^{1/\lambda} \tilde{G}_{j'}(X)^{\frac{1}{\lambda}} \exp(\psi_{j't}) X^{\theta_{j'}}\right)^{\lambda\beta/\rho_r} = 1$$

$$\sum_r \bar{G}_r(X)^{\beta} = 1$$

then we get that

$$Pr[j(i,t) = j \,|\, X, j(i,t) \in J_r] = \left(\tau^{1/\lambda} \tilde{G}_j(X)^{\frac{1}{\lambda}} \exp(\psi_{jt}) X^{\theta_j}\right)^{\lambda\beta/\rho_r}$$

$$Pr[j(i,t) \in J_r \,|\, X] = \left(\bar{G}_r(X)\right)^{\beta}$$

and hence

$$\tau \exp(\lambda\psi_{jt}) X^{\lambda\theta_j} G_j(X) = \left(Pr[j(i,t) \in J_r \,|\, X]\right)^{1/\beta} \left(Pr[j(i,t) = j \,|\, X, j(i,t) \in J_r]\right)^{\rho_r/\beta}$$

and since this is independent of the normalization, we get that this is true for all t.

□

Next we explain the estimation procedure that relies on the expression that we just derived.

For estimation we are going to use a group structure both at the firm and at the market level. For the firm grouping we use the one we obtained by classifying based on firm specific empirical distribution of wages called $k(j)$. We follow a similar structure at the market level and group based on the market level empirical distribution of earnings. We denote such classification by $m(r)$. At this point we think of the firm class $k(j)$ to be within market type m, hence when using the classification for Section 3, we interact it with the market grouping.

Using these two classifications we are going to rely on the fact that worker composition can be estimated at the group level instead of try ing to estimate a distribution for each individual firm and market:

$$\begin{aligned} Pr[X|j] &= Pr[X|k(j)] \\ Pr[X|r] &= Pr[X|m(r)]. \end{aligned}$$

Similarly to the Lemma we define $G_j(X) = \bar{G}_r \tilde{G}_j G_{k(j)}(X)$. Following the lemma we impose the following constraints on $\bar{G}_r$ and $\tilde{G}_j$:

$$\begin{aligned} \sum_{j' \in J_r} \left(\tau^{1/\lambda} \left(\tilde{G}_{j'} G_{k(j')}(X) \right)^{\frac{1}{\lambda}} \exp(\lambda \psi_{j't}) X^{\lambda \theta_{j'}} \right)^{\lambda\beta/\rho_r} &= 1 \\ \sum_r \bar{G}_r^{\beta} &= 1 \end{aligned}$$

We then directly apply the formula for $G_j(X)$ at the firm group level $k(j)$ within market $m(r(j))$):

$$G_k(X) = X^{-\lambda\theta_k} \left(\frac{Pr[X|m]}{Pr[X]} \right)^{1/\beta} \left(\frac{Pr[X|k]}{Pr[X|m]} \right)^{\rho_r/\beta}.$$

Next we recover the j specific part by matching the size of each firm within its market:

$$Pr[j(i,t) = j \,|\, j(i,t) \in J_r] = \tilde{G}_j^{\beta/\rho_r} \int \frac{\left(\tau G_{k(j)}(X) \exp(\lambda \psi_{jt}) X^{\lambda\theta_j}\right)^{\beta/\rho_r}}{\sum_{j' \in J_r} \left(\tau \tilde{G}_{j'} G_{k(j')}(X) \exp(\lambda \psi_{j't}) X^{\lambda \theta_{j'}} \right)^{\beta/\rho_r}} Pr[X|m(r)] \mathrm{d}X$$

And similarly we get the market level constant by matching the market level size:

$$Pr[j(i,t) \in J_r \,|\, X] = \bar{G}_r^{\beta} \int \frac{\left(\sum_{j' \in J_r} \left(\tau \tilde{G}_j G_{k(j)}(X) \exp(\lambda \psi_{j't}) X^{\lambda\theta_{j'}} \right)^{\beta/\rho_r} \right)^{\rho_r}}{\sum_{r'} \left(\sum_{j' \in J_{r'}} \left(\tau \bar{G}_{r'} \tilde{G}_{j'} G_{k(j')}(X) \exp(\lambda \psi_{j't}) X^{\lambda \theta_{j'}} \right)^{\beta/\rho_{r'}} \right)^{\rho_{r'}}} N M_X(X) \mathrm{d}X$$