Given

$$\pi_0 = \begin{bmatrix} 0.1 & 0.3 & 0.6 \end{bmatrix}$$

$$\mathbf{A} = \begin{bmatrix} 0.2 & 0.2 & 0.4 \\ 0.3 & 0.1 & 0.6 \\ 0.5 & 0.2 & 0.3 \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} 0.2 & 0.5 & 0.1 & 0.2 \\ 0.3 & 0.2 & 0.3 & 0.1 \\ 0.4 & 0.4 & 0.1 & 0.1 \end{bmatrix}$$

1. Calculate $\alpha_2(2), \alpha_3(2)$ using forward algorithm given $\mathbf{V}^T = \{1, 4, 1\}$
2. Calculate $\beta_2(2)$ given $\mathbf{V}^T = \{1, 4, 1\}$
3. find $\omega^* = \underset{\omega}{^{argmax}} p(\theta|\mathbf{V}^T)$ given $\mathbf{V}^T = \{1, 4, 1\}$