Homework exercises set #1

Problem 1. Write each of the following complex numbers in polar form:

(1) $z_1 = -1$

(2) $z_2 = -3i/2$

(3) $z_3 = \sqrt{2}$

(4) $z_4 = 1 + i$

(5) $z_5 = -5 + 5i$

(6) $z_6 = 0$

(7) $z_7 = -\sqrt{3} + i$

(8) $z_8 = 2 - 2i\sqrt{3}$

Problem 2. Let $z_1, \ldots, z_6$ be the complex numbers defined in Problem 1.

(1) Compute $z = z_2\, z_4\, z_5$ using algebraic forms, then polar forms, and check that the results agree.

(2) Same question for $z = (z_4)^6$ and $z = (z_7)^8$.

Problem 3. Let us define the cosine and sine of a complex number z with the following formulas:

$$\cos(z) = \frac{e^{iz} + e^{-iz}}{2} \qquad \sin(z) = \frac{e^{iz} - e^{-iz}}{2i} \ .$$

(1) Compute $\cos(z)$ and $\sin(z)$ for $z = 0$, $z = i$, $z = \pi$.

(2) Prove that if z is real, then $\cos(z)$ and $\sin(z)$ agree with the standard cosine and sine for real numbers.

(3) Find all the complex numbers z such that $\sin(z) = 0$. Hint: start by denoting $Z = e^{iz}$ and express $\sin(z)$ in terms of Z.

Problem 4. Let z be a complex number such that $e^z = 2 - 2i$.

(1) Compute e^{-z}.

(2) Compute $|e^z|$, then find the real part of z.

(3) Write e^z in polar form, then find all possible values for z.