

## Exam #2

Monday, November 13 2017

Duration: 1H20	
NAME:	
Please write clearly and properly. Ju	stify your answers carefully.

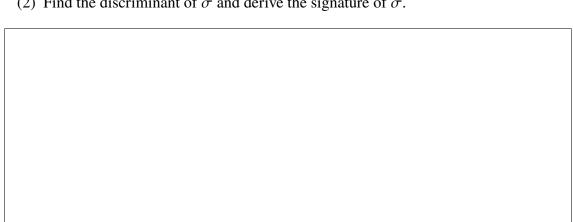
Problem	Grade
1	
2	
3	
Total	

**Problem 1** ( $\sim$  8 points).

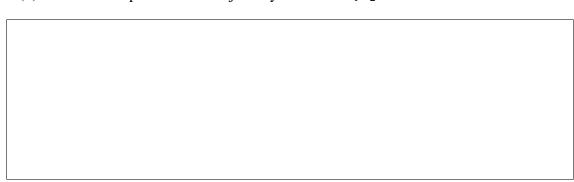
Let  $S_{10}$  denote the symmetric group on 10 letters. Consider the following permutation  $\sigma \in S_{10}$ :

(1)	What	are the	orbits	of $\alpha$	r?

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(2)	Find the	discriminant	of $\sigma$	and derive	the signat	ure of $\sigma$ .



(3) Write  $\sigma$  as a product of 2 disjoint cycles:  $\sigma = C_1C_2$ .



	Show that for any $k \in \mathbb{N}$ , $\sigma^k = C_1^k C_2^k$ .
(5)	Derive from the two previous questions that $\sigma^k$ is the identity permutation if and
	only if $k \in m_1 \mathbb{Z} \cap m_2 \mathbb{Z}$ , where $m_1$ is the length of $C_1$ and $m_2$ is the length of $C_2$ .
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(7) Find $\sigma^{18}$ .						

(8) Fin	nd $\sigma^{2017}$ .			
Hir	nt: 2017 = 12	$\times$ 168 + 1.		

Problem 2 (~ 6 points).
Let $G$ be a finite group with identity element $e$ . Denote by $N$ the cardinality of $G$ .
We recall that the order of an element $x \in G$ is the smallest positive integer $k \in \mathbb{N}$ such that $x^k = e$ .
NB: If you get stuck on a question, you may skip it and still use the result in the nex questions.
(1) Show that the order of any element $x \in G$ exists and is a divisor of $N$ . Hint: Consider the subgroup generated by $x$ and use the theorem of Lagrange.
(2) Show that if there exists an element of order $N$ , then $G$ is a cyclic group.

(3) Show that if $N$ is a prime number, then $G$ is a cyclic group.	
<ul> <li>(4) Show that if G is a cyclic group, then:</li> <li>If N is odd, there exists no element of order 2.</li> <li>If N is even, there exists exactly one element of order 2.</li> </ul>	

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(6) Show	that $S_n$ is not a c	yclic group unle	ess n = 2.		

<b>Problem 3</b> (~ 2 points).
Consider the group $\mathbb{Z}=(\mathbb{Z},+)$ and the direct product $\mathbb{Z}^2=\mathbb{Z}\oplus\mathbb{Z}$ . Consider the map
$\varphi\colon \mathbb{Z}^2 \to \mathbb{Z}$
$(x,y)\mapsto 2x-3y$ .
Show that $\varphi$ is a group homomorphism. Bonus question: is $\varphi$ a group isomorphism?