



Mathematics Colloquium

Surface group representations in higher-rank complex Lie groups

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Abstract

The classical uniformization theorem asserts that every Riemann surface has a complete, constant-curvature conformal metric which is unique up to scale. In this way, the Teichmüller space of complex structures on a compact surface S of higher genus can be identified with a space of geometric structures. These are in turn described by Fuchsian representations –faithful, discrete homomorphisms from the fundamental group $\pi_1(S)$ to the real Lie group $PSL(2, \mathbb{R})$. A generalization of this picture, due to Bers, identifies a natural family of deformations of these representations into the complex Lie group $PSL(2, \mathbb{C})$, the quasi-Fuchsian representations, with a product of Teichmüller spaces.

After recalling this rank one theory, we will survey some attempts at developing a corresponding geometric picture for families of representations of $\pi_1(S)$ into other real and complex Lie groups. For representations in $PSL_n(\mathbb{R})$, the Hitchin component provides a natural analogue of the Teichmüller space, and this “higher Teichmüller theory” has been the subject of extensive investigation over the last 30 years. The corresponding “higher quasi-Fuchsian theory” in $PSL_n(\mathbb{C})$ is less fully developed, though we will touch on recent joint work with Andrew Sanders that establishes rigidity and flexibility phenomena in this case that are analogous to the local versions of Bers’ results for $PSL(2, \mathbb{C})$.

Wednesday, 23 March 2016, 4pm

Smith Hall 204

Tea and refreshments will be served at 3:45pm.