Competition, Firm Innovation, and Growth under Imperfect Technology Spillovers*

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Abstract

We study how competition drives firms to use different types of innovation and affects the aggregate economy through changes in innovation composition. We build an endogenous growth model in which multi-product firms improve their own products through internal innovation while entering others' markets through external innovation, subject to a novel friction "imperfect technology spillovers." If learning others' technology takes time due to the friction, firms with technological advantages increase internal innovation to protect their markets from competitors ("market-protection effect"), leading to a decline in external innovation ("technological barrier effect"). Using the administrative data of the U.S. patenting firms, we find consistent regression results. The changes in innovation composition matter for the aggregate implications of competition, given the distinct roles of the two innovations in aggregate outcomes.

JEL Code: F14, L11, L25, O31, O33, O41

Keywords: competition, innovation, technology spillover, endogenous growth

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1 Introduction

Innovations manifest in diverse forms, impacting firm performance and economic growth differently, and firms have different incentives for using them (Akcigit and Kerr, 2018; Garcia-Macia et al., 2019; Argente et al., 2024). Although studies on the effect of competition on innovation have a long-standing history, the results remain inconclusive and provide limited guidance on assessing the implications of competition through heterogeneous innovations (Aghion et al., 2005; Gilbert, 2006; Aghion and Griffith, 2008; Bloom et al., 2016; Autor et al., 2020). How do firms use different types of innovation when faced with increasing competition? How does this bring new insights into the aggregate implications of competition?

Our paper investigates these questions both theoretically and empirically. On the side of theory, we develop an endogenous growth model with two types of innovation and imperfect technology spillovers. This model provides a micro-foundation for the effect of competition on firm innovation, decomposing overall changes in innovation into changes in the level and composition of two innovation types. Empirically, we link the administrative firm-level data to the patent database in the U.S. and document new facts about the composition changes of firm innovation in response to Chinese competition. Lastly, we calibrate the model to derive the aggregate implications of rising competition and compare them across different economies.

First, we build a discrete-time endogenous growth model in which multi-product firms grow through two types of innovation—internal and external—subject to imperfect technology spillovers. Internal innovation improves existing product quality, while external innovation enables firms to enter new markets by displacing incumbents.[1] External innovation contributes more to product quality improvement,

[1]An illustrative real-world example of external innovation is Apple's entry into the cell phone industry with the introduction of the iPhone back in 2007 when its major business was computer manufacturing. An example of internal innovation is Apple's improvement and production of iPhone 15 from iPhone 14.

thus to firm and aggregate growth than internal innovation.[2] Imperfect technology spillovers are a new element introduced in this model, which represent barriers to learning others' technology in the process of external innovation. When a firm attempts to enter markets through external innovation, the initial step involves acquiring and improving the technological knowledge of incumbents. Realistically, this learning takes time as external innovation entails prior processes that can demand substantial time and resources.[3] Our model uses lagged learning as a form of imperfect technology spillovers, requiring potential rivals to spend one period learning incumbents' product-specific technology. In other words, external innovation builds on one-period lagged technology.

This model creates key novel features. First, the spillover friction creates a technology gap between incumbents' frontier technology and the one-period lagged technology that potential rivals can only learn through R&D. Second, incumbents can exploit this technology gap and strategically use internal innovation to protect their markets, labeled as the "market-protection effect."[4] Third, the strategic internal innovation of incumbents prevents rivals from entering the incumbents' markets and stifles the rivals' external innovation, labeled as the "technological barrier effect." Thus, competition induces a shift in the composition of firm innovation, driven by the strategic choices made by firms and their endogenous interactions. Lastly, as a result, an exogenous increase in competition can either increase or decrease the aggregate level of innovation depending on the relative shifts in the two types of innovation.

[2]As documented in Bernard et al. (2010) and Akcigit and Kerr (2018), external innovation plays a more important role in contributing to both firm employment and aggregate growth than internal innovation. External innovation is tightly connected to creative destruction and radical innovation.

[3]For example, external innovation may require the processes of recruiting new employees to handle new technology, reallocating resources to new projects, training workers, and preparing production facilities for new products. In the real world, Apple took three years to enter the cell phone industry, even after leveraging their previously accumulated knowledge from iPod development and production. Moreover, Apple has been trying to enter the car industry for over seven years.

[4]In particular, firms with a technology gap increase internal innovation to defend themselves from competitors, while those with no such gap reduce it when competition gets heightened exogenously.

The strength of this framework is that it allows multi-product firms to strategically use different types of innovation and generates endogenous feedback effects on firm innovation decisions. We achieve this by adding imperfect technology spillovers to a multi-product firm set-up, which is our main theoretical contribution.[5] In existing models of multi-product firms growing through product scope expansion, firms cannot protect their markets because others can immediately learn and copy the frontier technology without any friction (Klette and Kortum, 2004; Akcigit and Kerr, 2018). Other step-by-step innovation models generate an escape-competition motive, but they assume single-product firms only and lack the feedback effects (Aghion et al., 2001, 2005; Akcigit et al., 2018).

Furthermore, our model underscores the importance of comprehending changes in innovation composition to accurately evaluate the aggregate implications of competition. Previous studies have mainly focused on the direction of innovation changes in response to competition with limited exploration into the implications arising from these changes. Unlike earlier models, internal innovation in our framework not only enhances the growth of product owners but also impedes external innovation and firm entry, which make a substantial contribution to firm and economic growth. Given these dynamics, the shifts in innovation composition are crucial for overall outcomes. Ignoring the heterogeneity of firm innovation and focusing solely on overall or one type of innovation may obscure the true impact of competition.

Next, to validate our model predictions, we construct a unique dataset by combining administrative firm-level data in the U.S. with patent data from the United States Patent and Trademark Office (USPTO) from 1976 to 2016.[6] This dataset provides

[5]In this sense, our framework brings together quality-ladder innovation models and step-by-step innovation models.

[6]We link the two datasets using name and address matching, along with the internet search-aided algorithm in Autor et al. (2020). This crosswalk improves the match rates and provides the longest and longitudinally consistent crosswalk between patent assignees and LBD firms. This method allows us to match 88.2% and 80.1% of all U.S. patents and assignees, respectively, from 1976 to 2016. This method also overcomes the issue of lacking longitudinal consistency from the existing crosswalks as they have covered only years before or after 2000. See Ding et al. (2022) for further details.

comprehensive information for the entire population of the U.S. patenting firms. We specifically use China's WTO accession in 2001 as an exogenous change in competitive pressure and the self-citation ratio of patents as a measure of the likelihood of patents being used for internal innovation.

With these measures, we find two main regression results consistent with our mechanism. First, heightened competition increases internal innovation among firms with existing technological advantages but decreases external innovation across all firms. Second, the positive correlation between firm patenting and employment growth diminishes by 17.1% for innovation-intensive firms following the surge in competition, as more patents are used for internal innovation. Through the lens of the model, these findings can be interpreted as evidence of firms' strategic use of internal innovation to prevent their product markets from competitors.

Lastly, to understand the aggregate implications of competition on innovation composition, we calibrate our model to the U.S. manufacturing sector from 1982 to 1997 and conduct two main counterfactual exercises by increasing competitive pressure by outside firms: i) in the U.S. economy, and ii) in an economy where external innovation costs exceed those in the U.S.[7]

Both exercises yield qualitatively the same results at the firm level responses, where internal innovation increases (decreases) for firms with a (no) technology gap, but external innovation drops across all firms. However, the results differ for aggregate implications. The overall innovation—the aggregate-level R&D to sales ratio—experiences a decline in the U.S., where firms actively engage in external innovation with lower associated costs. In contrast, this result is reversed in the other economy with higher external innovation costs. This is because the initial level of external innovation is minimal even in the absence of competitive pressure, and thus, the scope for a further decline in external innovation with increased competitive pressure is limited. It is also noteworthy that the aggregate growth rates attributed

[7] Additionally, an alternative counterfactual analysis of a reduced entry cost is presented in online Appendix **??**.

to domestic firms fall in both economies, even though the latter has seen an increase in the overall innovation. This is because heightened competition endogenously elevates technological barriers and impedes external innovation by domestic incumbents and firm entry. This result is distinctive within our framework, which considers the heterogeneity in firm innovation and its impact.

Our paper provides a unified framework that facilitates the comparison of the effects of competition across different countries. Notably, this framework allows for mapping non-U.S. economies, such as the European economy, into our hypothetical economy with high external innovation costs and helps reconcile the disparate findings in the literature.[8] The change in innovation composition resulting from firms' strategic choices is an important margin to understand the heterogeneous impact of competition and its aggregate implications across diverse economic landscapes.

Related Literature. Our paper brings new insights and findings to the large literature linking competition, firm innovation, and technology spillovers.

First, our paper is related to an extensive body of research on competition and innovation. The empirical literature finds mixed results (Aghion et al., 2005; Bloom et al., 2016; Aghion et al., 2018; Hombert and Matray, 2018; Shu and Steinwender, 2019; Autor et al., 2020).[9] Some explore this dynamics through the lens of a Schumpeterian growth model with step-by-step innovation, where the "Schumpeterian effect" by the laggards and the "escape-competition effect" by neck-and-neck firms arise (Aghion et al., 2001, 2004, 2005, 2009; Akcigit et al., 2018). However, this model is rooted in several assumptions lack of realism, and lacks explicit discussion on assessing the changes in innovation.[10] Others introduce a trapped-factor model in which rising competition reduces the opportunity cost of innovation (Bloom et al., 2013, 2021; Medina, 2022), but this channel may not be applicable to gen-

[8] For instance, heightened frictions associated with R&D or labor mobility for external innovation.

[9] See Shu and Steinwender (2019) for further details.

[10] The model assumes single-product firms, a single innovation type, or the immediate imitation by the laggards.

eral cases (e.g., the U.S.) and lacks direct data evidence. Alternatively, Hombert and Matray (2018) elucidate the channel of product differentiation by innovative firms, while Dhingra (2013) underscores the phenomenon of firms upgrading their product production process to avoid cannibalization in response to competition. Helpman (2023) illustrates ambiguous impacts on the innovation efforts of large multi-product firms, which depend on the level of their productivity and market shares. We contribute to this literature by providing a rich theoretical framework wherein multi-product firms strategically choose different types of innovation, along with new data evidence. In this setting, competition changes the composition of heterogeneous innovations and generates different aggregate implications depending on the relative shifts. Importantly, our framework enables us to derive and assess the aggregate implications of competition by considering various innovation types that make distinct contributions to economic outcomes. Our results help reconcile the prior diverging findings and enrich our understanding of the complex effects of competition on firm innovation.

Second, our paper adds to another strand of literature on technology gap and spillovers. Previous studies have established that the technology gap between firms plays a crucial role in shaping firm innovation incentives and policy implications (Aghion et al., 2001, 2005; Dinopoulos and Syropoulos, 2007; Aghion and Griffith, 2008; Acemoglu and Akcigit, 2012; Aghion et al., 2014; Akcigit et al., 2018). Our framework extends this understanding by incorporating imperfect technology spillovers, allowing firms to strategically leverage the technology gap to avoid competition. Recent studies have also explored the trend of diminishing knowledge diffusion from market leaders to laggards (Andrews et al., 2016; Bessen et al., 2020; Akcigit and Ates, 2021, 2023). Arora et al. (2021) document the decline in corporate research and attribute it to an increased sensitivity among firms to potential rivals using their research. Others present various phenomena that broadly align with this trend: the increasing use of intangible capital (Bessen et al., 2020; De Ridder, 2024), lower overhead cost associated with market leaders (Aghion et al., 2023), diminishing research productivity (Bloom et al., 2020), the concentration of patent production and reassignment to market leaders (Shapiro, 2000; Argente et al., 2020; Akcigit and Ates, 2023), the declining ability of laggard firms to make drastic improvements (Olmstead-Rumsey, 2019), and the reduced worker mobility across firms (Akcigit and Goldschlag, 2023). Nevertheless, these studies have not yet offered a definitive answer and mechanism for the observed shift in the diffusion process. Our contribution is uncovering an underlying endogenous force behind the decreasing technology diffusion as a consequence of firms strategically responding to increased competition due to exogenous forces such as globalization.

Lastly, our paper contributes to the growing literature that explores the diverse types of innovation undertaken by multi-product firms. Aghion et al. (2004) and Akcigit et al. (2018) add an entry margin, and similarly, Atkeson and Burstein (2010) introduce the notion of product and process innovations. However, all these models assume single-product firms, in which each firm (either incumbents or entrants) can only do one type of innovation (either process/incremental or product/radical). On the other hand, Klette and Kortum (2004) build a quality-ladder model of multi-product firms, albeit under the assumption of a single type of innovation. Another set of research has expanded the study of multi-product firms: Bernard et al. (2010) highlight the role of product switching in resource allocation; Akcigit and Kerr (2018) model multi-product firms engaged in both internal and external innovation and illuminate the distinct importance of external innovation in economic growth; and Dhingra (2013) and Argente et al. (2024) document the role of cannibalization in firm innovation decisions. In addition, Peters (2020) creates endogenous markup by considering different innovation types, while Garcia-Macia et al. (2019) and Atkeson and Burstein (2019) explore the impacts of varied innovation types on growth and policy implications. Our contribution to these studies lies in the development of an endogenous growth model incorporating strategic interactions among different types of innovation. A key distinction is that this strategic choice, in turn, creates an endogenous feedback effect on the innovation incentives and entry decisions of other firms. Our research also broadens the empirical scope by offering insights into the strategic use of different innovations by multi-product firms.

The rest of the paper proceeds as follows. Section 2 develops a baseline endogenous growth model. Section 3 presents empirical results about the effect of Chinese competition on the composition of firm innovation. Section 4 displays results from quantitative analysis of the baseline model. Section 5 concludes.

2 Baseline Model

In this section, we introduce a discrete time infinite horizon endogenous growth model with multi-product firms, two types of innovation, imperfect technological spillovers, and an exogenous source of competitive pressure.[11,12] The model extends Akcigit and Kerr (2018) in the following three dimensions: we i) introduce a novel friction named "imperfect technology spillovers" by assuming firms can only learn the incumbent's technology lagged by one period in the process of external innovation; ii) generate incumbent firms' internal innovation decision as an endogenous function of the technology gap—the ratio of the current-period technology $q_{j,t}$ to the last-period technology $q_{j,t-1}$, $\Delta_{j,t} = \frac{q_{j,t}}{q_{j,t-1}}$, due to the friction; and iii) allow for exogenous shifts of the aggregate creative destruction arrival rate to analyze the effect of increasing competitive pressure on firm innovation and growth. Hereafter, the

[11] In continuous-time frameworks, the probability of a product experiencing both internal innovation by its owner and external innovation by another firm is zero (Peters, 2020). Moreover, employing a continuous-time model is not feasible as we consider the imperfect technology spillover as an important friction, as will be clear in subsequent explanations. Thus, using a discrete-time model is an appropriate approach to explore our proposed mechanism.

[12] The exogenous competitive pressure may emanate from firms outside the economy. The outside firms could be foreign firms if we consider the aggregate economy or domestic incumbent firms in other sectors or states if we consider a certain sector or state as the model economy.

time subscript is suppressed whenever there is no confusion.[13] The terms product quality and technology are used interchangeably.

2.1 Representative Household

The representative household has a logarithmic utility function and is populated by a measure one continuum of individuals. Each individual supplies one unit of labor each period inelastically and consumes a portion C_t of the economy's final good. The household's lifetime utility is

$$U = \sum_{t=0}^{\infty} \beta^t \log(C_t) \, .$$

Homogeneous workers are employed in the final goods sector (L), and the labor market clears as follows in each period:

$$L = 1 \, . \tag{2.1}$$

2.2 Final Good Producer

The final good producer uses labor (L) and a continuum of differentiated intermediate goods indexed by $j \in [0, 1]$ to produce a final good. Denote $\mathcal{D}$ as an index set for intermediate goods produced by domestic firms. Intermediate goods with $j \notin \mathcal{D}$ are produced by firms outside the economy. The production function is as follows with the constant returns-to-scale technology:

$$Y = \frac{L^\theta}{1-\theta} \left[\int_0^1 q_j^\theta \, y_j^{1-\theta} \, \mathcal{I}_{\{j \in \mathcal{D}\}} \, dj \; + \; \int_0^1 q_j^\theta \, y_j^{1-\theta} \, \mathcal{I}_{\{j \notin \mathcal{D}\}} \, dj \right] ,$$

where y_j is the quantity of intermediate good j, q_j is its quality, and $\mathcal{I}_{\{\cdot\}}$ are indicator functions. The final good price is normalized to be one, and the final good is

[13] Superscript $'$ is used to denote next period variables at $(t + 1)$, and subscript -1 is used for the previous period variables at $(t - 1)$.

produced competitively with input prices taken as given.

2.3 Intermediate Producers

There are domestic and outside firms, which have the mass of $\mathcal{F}_d$ and $\mathcal{F}_o$, respectively, with $\mathcal{F} = \mathcal{F}_d + \mathcal{F}_o \in (0, 1)$, and are determined endogenously in equilibrium.[14] They produce and sell differentiated intermediate goods in monopolistically competitive domestic markets. Since each firm operates with at least one product line, and each product line is owned by a single firm, a firm f can be characterized by the collection of its product lines $\mathcal{J}^f = \{j : j \text{ is owned by firm } f\}$. The intermediate good is produced at a unit marginal cost in terms of the final good.

2.4 Innovation by Intermediate Producers

Intermediate producers engage in two types of R&D, internal and external, by spending expenditures in units of the final goods. Firms improve the quality of their own products through internal innovation, while taking over other markets through external innovation.[15] The R&D output manifests as improvements in product quality and is realized at the beginning of the next period.

On top of this, we introduce a novel friction named "imperfect technology spillovers," under which learning other's technology takes time in the process of external innovation. We conceptualize it in the form of lagged learning by assuming external innovation builds on the past-period technology. In other words, only the owner of a product line can observe and use the frontier level of technology $q_{j,t}$ in the market, while outsider firms can only see the lagged level $q_{j,t-1}$.[16] Following this, a product

[14] Strictly speaking, a portion of the aggregate creative destruction arrival rate accounted for by outside firms is exogenously given in this economy. Given this, along with the endogenously determined success probability of external innovation, we can retrieve the "endogenously" determined mass of outside firms attempting to take over businesses in domestic markets, as will become clear in subsequent sections.

[15] Note that the quality in this model is a marginal cost of production-adjusted measure, and can be improved through either technological advancement or cost reduction. In this sense, our concept of innovation encompasses both product and process innovation.

[16] Note that all the aggregate variables and technology gap distribution (the share of product lines

line can be sufficiently characterized by its quality q_j and technology gap between current and previous periods $\Delta_{j,t} = \frac{q_{j,t}}{q_{j,t-1}}$.[17] This friction induces incumbent firms to strategically use different types of innovation to build technological barriers and protect their markets from competitors. We name it the "market-protection effect."

When two firms' technologies are neck and neck in a particular product line, a coin-toss tiebreaker rule applies as in Acemoglu et al. (2016) to make sure each product is produced by only one firm.[18] External innovation is undirected as that the targeted product is randomly assigned among the entire set of products ($j \in [0, 1]$) with equal probability. Also, for tractability, we assume that each firm can do only one external innovation in each period regardless of its total number of products.[19]

Internal Innovation. Successful internal innovation improves the current quality $q_{j,t}$ of the product by $\lambda > 1$. The quality of good j evolves as follows, assuming the owning firm is not displaced by creative destruction:

$$\left\{q_{j,t+1}^{in}\right\} = \begin{cases} \left\{\lambda q_{j,t}\right\} & \text{with probability } z_{j,t} \\ \left\{q_{j,t}\right\} & \text{with probability } 1 - z_{j,t} \ , \end{cases} \qquad z_{j,t} = \left(\frac{R_{j,t}^{in}}{\widehat{\chi} q_{j,t}}\right)^{\frac{1}{\widehat{\psi}}} , \ \widehat{\chi} > 0, \ \widehat{\psi} > 1 \ ,$$

where the success probability of internal innovation, $z_{j,t}$, is determined by the level

with a certain level of technology gap) are publicly observable, and these are the objects individual firms need to know to make their optimal innovation decisions. In this way, an equilibrium with a stationary firm-product distribution is well defined.

[17]This technology gap summarizes the technological advantage incumbent firms have in their product market.

[18]An unused technology is assumed to depreciate by an amount sufficient to ensure that it becomes unprofitable to innovate on top of it next period. Thus, only the winning firm from the coin toss keeps the product line until it is overtaken by others through external innovation, while the losing firm never tries to enter the same market through internal innovation. This approach guarantees the undirected nature of external innovation and restricts internal innovation to the current owner of a product.

[19]Once we tie firms' external innovation decisions to the number of products firms have, as in the generalized model of Akcigit and Kerr (2018), the external innovation decisions become depending on firms' internal innovation decisions and technology gap portfolios. This linkage arises as these firm-level variables, in part, determine the number of products firms will have in the next period due to the imperfect technology spillovers. The interdependence between firms' decisions on internal and external innovation poses a challenge in isolating the constant and growing components of firms' value function, which is essential for model tractability.

of R&D expenditure $R^{in}_{j,t}$.[20]

External Innovation. Successful external innovation improves the lagged quality of the obtained product by $\eta > 1$. We assume $\lambda^2 > \eta > \lambda$, where $\eta > \lambda$ reflects the findings from Akcigit and Kerr (2018) that external innovation contributes more to both product quality and aggregate growth than internal innovation.[21] $\lambda^2 > \eta$ is based on the idea that consecutive internal innovation has a significant influence, and this assumption ensures that firms can protect their own product lines from potential rivals through internal innovation.

Existing firms with at least one product line ($n_f > 0$) invest in external innovation. As a result, the following product quality can be obtained if not pre-empted by the successful defensive innovation of the incumbent in their target market:

$$\left\{q^{ex}_{j,t+1}\right\} = \begin{cases} \left\{\eta q_{j,t-1}\right\} & \text{with probability } x_t \\ \varnothing & \text{with probability } 1 - x_t \,, \end{cases} \qquad x_t = \left(\frac{R^{ex}_t}{\widetilde{\chi}\, \bar{q}_t}\right)^{\frac{1}{\widetilde{\psi}}} \,,\ \widetilde{\chi} > 0,\ \widetilde{\psi} > 1\,.$$

The success probability of external innovation, x_t, is determined by the level of R&D expenditure R^{ex}_t and the average quality $\bar{q}_t$ in the economy.[22] With probability $1-x_t$, the external innovation fails, implying no product takeover and no quality obtained.

Illustration of Firm Innovation Decisions. To better understand firm innovation decisions between internal and external innovation, Figure 1 shows four main illustrative examples. Assume firm A owns the first three product lines and firm B owns the last four product lines in period t, where each bar represents a product and the height of the bar shows the log of product quality for each product, $\widehat{q}_{j,t} \equiv \log(q_{j,t})$.

[20]Hereafter, we represent the quality of product j as a point set. This makes it easy to describe the case where external innovation fails, and a firm does not acquire any product lines—in such cases, the product quality set is an empty set.

[21]Akcigit and Kerr (2018) and our empirical analysis in online Appendix **??** empirically show that external innovation is associated with higher firm employment growth. Furthermore, our empirical findings indicate that external innovation is associated with an increased number of products added.

[22]The average quality matters for external innovation as the target product is randomly assigned.

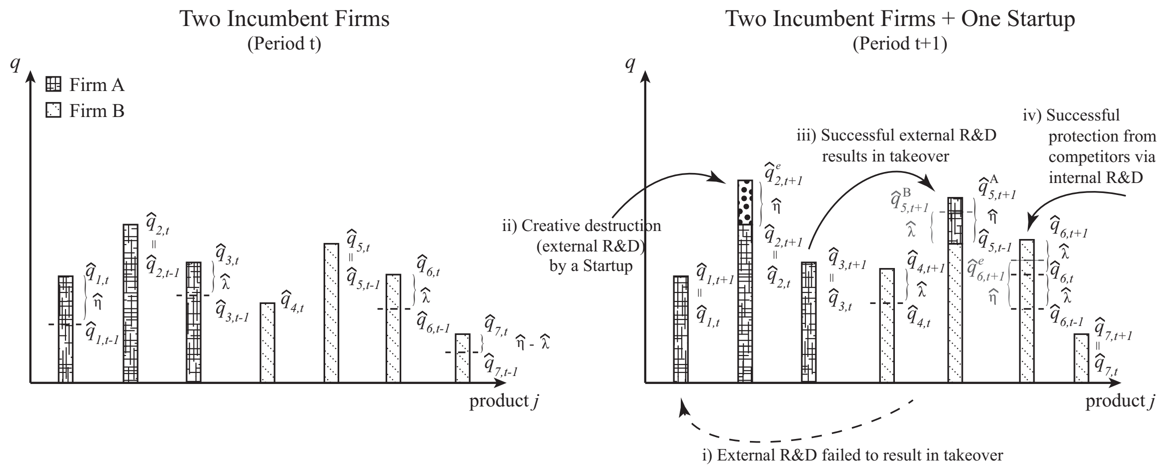

Figure 1: Firms' Innovation and Product Quality Evolution Example

- Case 1: successful external innovation failing product takeover (coin-tossing)
  Without successful internal innovation of firm A at t, it gets $q^A_{1,t+1} = \eta q_{1,t-1}$ (implying $\widehat{q}^A_{1,t+1} = \widehat{\eta} + \widehat{q}_{1,t-1}$ in logs, where $\widehat{\eta} \equiv \log(\eta)$). Meanwhile, firm B made successful external innovation at t and gets $q^B_{1,t+1} = \eta q_{1,t-1}$ at $t+1$. Thus, a coin is tossed, and firm A keeps the product by winning.

- Case 2: successful product takeover (in a market with no technological gap)
  The quality of product 2 remains as $q^A_{2,t+1} = q_{2,t-1}$ for firm A failing internal innovation at $t-1$ and t. Then, a potential startup succeeds in external innovation at t and takes over the market by achieving $q^e_{2,t+1} = \eta q_{2,t-1} > q^A_{2,t+1}$.

- Case 3: failed market protection (in a market with no technological gap)
  Firm A takes over product line 5 from firm B through successful external innovation at t, although firm B concurrently succeeds in internal innovation, because the firm obtains $q^A_{5,t+1} = \eta q_{5,t-1}$, which is greater than $q^B_{5,t+1} = \lambda q_{5,t-1}$.

- Case 4: successful market protection (in a market with a technological gap)
  Firm B achieves success in two consecutive internal innovations for product 6, starting from $t-1$, and attains $q^B_{6,t+1} = \lambda^2 q_{6,t-1}$ at $t+1$. Due to imperfect technology spillovers, competing firms can only enhance the quality up to

$q^e_{6,t+1} = \eta q_{6,t-1}$. As $\lambda^2 > \eta$, firm B successfully protects product 6 from competitors. This highlights the defensive internal innovation by incumbent firms, leveraging the spillover friction.

Product Quality Evolution. As rival firms can only learn the last period's quality of products, the gap between the current and lagged levels of product quality, $\Delta_{j,t} = \frac{q_{j,t}}{q_{j,t-1}}$, becomes a crucial factor in determining incumbent firms' ability to protect their product lines through internal innovation. This technology gap summarizes the technological advantages incumbent firms have in their respective markets. In the model, the technology gap can have the following four values.

Lemma 1. *There are four possible values for the technology gap, $\Delta^1 = 1$, $\Delta^2 = \lambda$, $\Delta^3 = \eta$, and $\Delta^4 = \frac{\eta}{\lambda}$, and Δ^3 and Δ^4 can occur only through external innovation.*
Proof: See the Appendix.

Following Lemma 1, the following describes the evolution of product quality and the implied probabilities of retaining or losing a product from the perspective of an incumbent firm, conditional on a technology gap $\Delta_{j,t} = \frac{q_{j,t}}{q_{j,t-1}}$:

$$\left\{q_{j,t+1} \mid \Delta_{j,t} = \Delta^1\right\} = \begin{cases} \varnothing & \text{, with prob. } \overline{x} \\ \{q_{j,t}\} & \text{, with prob. } (1-\overline{x})(1-z_j^1) \\ \{\lambda q_{j,t}\} & \text{, with prob. } (1-\overline{x})z_j^1 \end{cases} \tag{2.2}$$

$$\left\{q_{j,t+1} \mid \Delta_{j,t} = \Delta^2\right\} = \begin{cases} \varnothing & \text{, with prob. } \overline{x}(1-z_j^2) \\ \{q_{j,t}\} & \text{, with prob. } (1-\overline{x})(1-z_j^2) \\ \{\lambda q_{j,t}\} & \text{, with prob. } z_j^2 \end{cases} \tag{2.3}$$

$$\left\{q_{j,t+1} \mid \Delta_{j,t} = \Delta^3\right\} = \begin{cases} \varnothing & \text{, with prob. } \frac{1}{2}\overline{x}(1-z_j^3) \\ \{q_{j,t}\} & \text{, with prob. } \left(1-\frac{1}{2}\overline{x}\right)(1-z_j^3) \\ \{\lambda q_{j,t}\} & \text{, with prob. } z_j^3 \end{cases} \tag{2.4}$$

$$\left\{q_{j,t+1} \mid \Delta_{j,t} = \Delta^4\right\} = \begin{cases} \varnothing & \text{, with prob. } \overline{x}\left(1 - \frac{1}{2}z_j^4\right) \\ \{q_{j,t}\} & \text{, with prob. } (1-\overline{x})(1-z_j^4) \\ \{\lambda q_{j,t}\} & \text{, with prob. } \left(1 - \frac{1}{2}\overline{x}\right) z_j^4. \end{cases} \tag{2.5}$$

z_j^ℓ refers to the optimal choice of the firm owning product j for internal innovation when its technology gap is Δ^ℓ, $\ell \in \{1, 2, 3, 4\}$. $\overline{x}$ is the rate of creative destruction in the economy, representing the probability that an individual product market faces a rival that undergoes successful external innovation. The symbol $\varnothing$ defines an empty product quality, meaning that the firm loses its product line j in the next period, and the terms $\frac{1}{2}$ in the probabilities reflect the coin-toss tiebreaker rule in cases of a neck-and-neck scenario.

In the case of $\Delta_{j,t} = 1$, incumbents lack any technological advantage. Thus, they lose their product lines in the subsequent period if rival firms succeed in external innovation, irrespective of their success in internal innovation.[23] In contrast, for other cases where $\Delta^\ell > 1$, firms can lower the probability of losing their product lines by investing more in internal innovation. The extent of the reduction in the probability of product loss is contingent on the technology gap. Consequently, firms are more incentivized to augment their internal innovation efforts for products with technological advantages ($\Delta^\ell > 1$), when confronted with heightened competition with a higher aggregate creative destruction arrival rate $\overline{x}$.

Flipping this to the side of rival firms entering into a product line, the evolution of product quality can also be defined similarly, as detailed in online Appendix **??**. Note that for firms doing external innovation as an outsider, the success probability of product takeover not just depends on the success of external innovation but also on the technology gap and the internal innovation intensity associated with the product owner (even after successful external innovation, as illustrated in Case 4 in the previous section). Thus, the success probability of product takeover can be

[23] Rivals with successful external innovation achieve $q_{j,t+1}^{rival} = \eta q_{j,t-1}$, which is greater than $\lambda q_{j,t-1}$.

decomposed into i) the success probability of external innovation x, and ii) the conditional takeover probability $\overline{x}_{takeover}$, which is the probability of product takeover given successful external innovation. The latter can be computed as follows:

$$\overline{x}_{takeover} = \mu(\Delta^1) + (1 - z^2)\mu(\Delta^2) + \frac{1}{2}(1 - z^3)\mu(\Delta^3) + \left(1 - \frac{1}{2}z^4\right)\mu(\Delta^4) \,, \quad (2.6)$$

for a given technology gap distribution $\left\{\mu(\Delta^\ell)\right\}_{\ell=1}^4$, where $\mu(\Delta^\ell)$ represents the share of product lines with technology gap Δ^ℓ.[24] Given that, the unconditional success probability of product takeover can be denoted by $x_{takeover} \equiv x\, \overline{x}_{takeover}$.[25]

Note that the higher the overall innovation intensity (both internal and external), the wider the average technology gap becomes in the economy (the mass of products with a technology gap of Δ^1 decreases). This makes it more difficult for rival firms to successfully take over product markets .[26] This is referred to as the "technological barrier effect," where increased internal innovation by incumbents or a higher $\bar{x}$ dampens the external innovation and growth of firms.[27]

Potential Startups. The economy has a fixed mass of potential domestic startups $\mathcal{E}_d$ trying to start businesses in domestic markets. To start a business, a potential startup

[24]This shows that if a firm succeeds in externally innovating a product line with a technology gap of Δ^1, then it takes over that product line with a probability of one. For a product line with a technology gap of Δ^2, this probability becomes $1 - z^2$; for a technology gap of Δ^3, it is $\frac{1}{2}(1 - z^3)$; and for technology gap Δ^4, it is $1 - \frac{1}{2}z^4$. Here it is assumed that the internal innovation intensity z depends solely on the technology gap Δ^ℓ. In the next section, we prove this assumption holds true.

[25]As shown in online Appendix **??**, the probability of a firm failing in an attempted product takeover—the probability of not winning the product line, either due to the failure of external innovation (which occurs with probability of $1 - x$) or the successful market protection by incumbent firms—is

$$(1 - x) + xz^2\mu(\Delta^2) + x\frac{1}{2}(1 + z^3)\mu(\Delta^3) + x\frac{1}{2}z^4\mu(\Delta^4),$$

which is equivalent to $1 - x_{takeover} = 1 - x\, \overline{x}_{takeover}$.

[26]Higher internal innovation intensity widens the technology gap. Simultaneously, higher external innovation intensity increases the aggregate creative destruction arrival rate, thereby incentivizing incumbent firms to engage in internal innovation more, as discussed earlier.

[27]This technological barrier effect is distinct from the well-known Schumpeterian effect, by which firms' innovation incentives decline following an increase in $\bar{x}$ due to lowered expected future profits conditional on successful innovation and business takeover.

invests in external R&D and, if successful, takes over a product line from an incumbent firm. Similar to incumbent firms, potential startups choose R&D expenditure R_e^{ex} and decide the probability of external innovation x_e:

$$x_e = \left(\frac{R_e^{ex}}{\widetilde{\chi}_e \, \bar{q}} \right)^{\frac{1}{\widetilde{\psi}_e}} ,$$

where $\widetilde{\chi}_e > 0$, and $\widetilde{\psi}_e > 1$, and $\bar{q}$ is the economy-wide average product quality.

Let $V(\{(q_j, \Delta_j)\})$ denote the value of a firm that has one product line with product quality q_j and technology gap Δ_j. Then a potential startup's expected profits from entering through R&D is

$$\Pi^e = \beta \mathbb{E}\Big[V(\{(q_j', \Delta_j')\}) \mid x_e \Big] - \widetilde{\chi}_e (x_e)^{\widetilde{\psi}_e} \overline{q} \, , \tag{2.7}$$

where the expectation conditioning on x_e is taken over the distribution of incumbents' product quality q_j and technology gap Δ_j due to the undirected nature of external innovation. Potential startups choose the probability of external innovation x_e that maximizes their expected profits from entry. Since there is no ex-ante heterogeneity among potential startups, they all choose the same optimal probability of external innovation x_e^*. Hence, the mass of potential domestic startups that succeed in external innovation and attempt to take over incumbent firms' products is $\mathcal{E}_d x_e^*$.

2.5 Exogenous Competitive Pressure and Creative Destruction

As explained before, the aggregate creative destruction arrival rate $\overline{x}$ is the probability that an incumbent faces a rival firm, who can be a domestic startup, incumbent, or a outside firm with successful external innovation.

Each firm can externally innovate at most one product line each period, and there is a continuum of unit mass of product lines (markets). Thus, the total mass of firms that succeed in external innovation is equivalent to the total mass of product markets for which an incumbent faces a rival firm. Given the undirected nature of external innovation, this implies that the probability an incumbent facing competition from other firms in each product market—the aggregate creative destruction arrival rate—is equal to the total mass of firms succeeding in external innovation.

Let $\overline{x}_d$ denote the total mass of domestic firms with successful external innovation and $\overline{x}_o$ denote the outside firm counterpart. Then the aggregate creative destruction arrival rate $\overline{x}$ is

$$\overline{x} = \overline{x}_d + \overline{x}_o \, .$$

Note that the rise in competitive pressure induced by outside firms is defined as an exogenous increase in $\overline{x}_o$ in this model economy.

2.6 Equilibrium

Optimal Production and Employment. The final good producers choose their optimal demands for labor and intermediate goods. Denote p_j as the price for differentiated product j, and w as the wage rate in the domestic economy. The inverse demand for differentiated intermediate good j is then given by:

$$p_j = q_j^{\theta} L^{\theta} y_j^{-\theta} \, . \tag{2.8}$$

Here, each product is assumed to be supplied by a single firm with the following two-stage price-bidding game.[28] This assumption ensures that only a technological leader enters the first stage and announces its price in equilibrium.

Assumption 1. *For each product line j in the economy, the current and former incumbents engage in a two-stage price-bidding game. In the first stage, each firm pays a fee*

[28] This is to avoid the case where the former market leader, having lost its leadership to the current leader in a given market, attempts to produce and sell its product through limit pricing, as the marginal cost of production is equal for every firm.

of $\varepsilon > 0$. In the second stage, all firms that have paid the fee announce their prices.

Intermediate producers take their product demand curves (2.8) as given and maximize the following operating profit for each product $j \in \mathcal{J}^f$.[29]

$$\pi(q_j) = \max_{y_j \geq 0} \left\{ L^\theta q_j^\theta y_j^{1-\theta} - y_j \right\} .$$

This along with (2.8) yields the optimal levels of the production and price as follows:

$$y_j = (1-\theta)^{\frac{1}{\theta}} q_j \quad \text{and} \quad p_j = \frac{1}{1-\theta} \tag{2.9}$$

and simplifies the equilibrium profit, wage, and final goods output to the following:

$$\pi(q_j) = \underbrace{\theta(1-\theta)^{\frac{1-\theta}{\theta}}}_{\equiv \pi} q_j \tag{2.10}$$

$$w = \theta(1-\theta)^{\frac{1-2\theta}{\theta}} \bar{q} \tag{2.11}$$

$$Y = (1-\theta)^{\frac{1-2\theta}{\theta}} \bar{q} \, , \tag{2.12}$$

where $L = 1$ is taken, and $\frac{1}{1-\theta}$ is a markup over the unit marginal cost. The wage and output grow at the same rate as the average product quality.

Optimal Internal and External Innovation. Define $\Phi^f \equiv \{(q_j, \Delta_j)\}_{j \in \mathcal{J}^f}$ as the set of product quality and technology gap pairs currently owned by intermediate producer f, where each pair (q_j, Δ_j) uniquely identifies product line j. Then, the value function of firm f is:

$$V\left(\Phi^f\right) = \max_{\substack{x, \\ \{z_j\}_{j \in \mathcal{J}^f}}} \left\{ \sum_{j \in \mathcal{J}^f} \left[\pi q_j - \hat{\chi} z_j^{\hat{\psi}} q_j \right] - \bar{q} \widetilde{\chi} x^{\widetilde{\psi}} + \widetilde{\beta}\, \mathbb{E} \left[V\left(\Phi^{f\prime} \mid \Phi^f\right) \Big| x, \{z_j\}_{j \in \mathcal{J}^f} \right] \right\} .$$

The first three terms define the current profits (revenue net of production and R&D

[29] Since each intermediate product incurs a unit marginal cost in terms of the final good, the problem is identical for both domestic firms and outside firms.

costs), and the last term accounts for the discounted expected future value. This expectation is computed over various factors, including the success probabilities of internal and external innovation, the creative destruction arrival rate, outcomes of winning or losing coin tosses, the current period product quality distribution, and the current period technology gap distribution. The stochastic discount factor is $\widetilde{\beta} = \frac{\beta C}{C'}$ as the household owns all firms.

Proposition 1. *For a given technology gap distribution $\left\{\mu(\Delta^\ell)\right\}_{\ell=1}^4$, the value function of firm f with a product quality and technology gap portfolio of $\Phi^f \equiv \left\{(q_j, \Delta_j)\right\}_{j\in\mathcal{J}^f}$ is:*

$$V(\Phi^f) = \sum_{\ell=1}^{4} A_\ell \left(\sum_{j\in\mathcal{J}^f|\Delta_j=\Delta^\ell} q_j \right) \; + \; B\,\overline{q}\,,$$

where

$$A_1 = \pi - \widehat{\chi}(z^1)^{\widehat{\psi}} + \widetilde{\beta}\Big[A_1(1-\overline{x})(1-z^1) + \lambda A_2(1-\overline{x})z^1\Big] \tag{2.13}$$

$$A_2 = \pi - \widehat{\chi}(z^2)^{\widehat{\psi}} + \widetilde{\beta}\Big[A_1(1-\overline{x})(1-z^2) + \lambda A_2 z^2\Big] \tag{2.14}$$

$$A_3 = \pi - \widehat{\chi}(z^3)^{\widehat{\psi}} + \widetilde{\beta}\left[A_1\left(1-\frac{1}{2}\overline{x}\right)(1-z^3) + \lambda A_2 z^3\right] \tag{2.15}$$

$$A_4 = \pi - \widehat{\chi}(z^4)^{\widehat{\psi}} + \widetilde{\beta}\left[A_1(1-\overline{x})(1-z^4) + \lambda A_2\left(1-\frac{1}{2}\overline{x}\right)z^4\right] \tag{2.16}$$

$$B = \frac{1}{1-\widetilde{\beta}(1+g)}\left[x\widetilde{\beta}A_{takeover} - \widetilde{\chi}x^{\widetilde{\psi}}\right]\,, \tag{2.17}$$

and the optimal innovation probabilities are

$$z^1 = \left[\frac{\widetilde{\beta}\left[(1-\overline{x})\lambda A_2 - (1-\overline{x})A_1\right]}{\widehat{\psi}\widehat{\chi}}\right]^{\frac{1}{\widehat{\psi}-1}} \tag{2.18}$$

$$z^2 = \left[\frac{\widetilde{\beta}\left[\lambda A_2 - (1-\overline{x})A_1\right]}{\widehat{\psi}\widehat{\chi}}\right]^{\frac{1}{\widehat{\psi}-1}} \tag{2.19}$$

$$z^3 = \left[\frac{\widetilde{\beta} \left[\lambda A_2 - \left(1 - \frac{1}{2}\overline{x}\right) A_1 \right]}{\widehat{\psi}\widehat{\chi}} \right]^{\frac{1}{\widehat{\psi}-1}} \tag{2.20}$$

$$z^4 = \left[\frac{\widetilde{\beta} \left[\lambda \left(1 - \frac{1}{2}\overline{x}\right) A_2 - \left(1 - \overline{x}\right) A_1 \right]}{\widehat{\psi}\widehat{\chi}} \right]^{\frac{1}{\widehat{\psi}-1}} \tag{2.21}$$

$$x = \left[\frac{\widetilde{\beta} A_{takeover}}{\widetilde{\psi}\widetilde{\chi}} \right]^{\frac{1}{\widetilde{\psi}-1}} , \tag{2.22}$$

where g in (2.17) is the average product quality growth rate in the economy, and $A_{takeover}$ in (2.17) and (2.22) is the ex-ante value of a product line obtained from successful takeover, defined as:

$$A_{takeover} \equiv \frac{1-z^3}{2} A_1 \mu(\Delta^3) + \left(1 - \frac{z^4}{2}\right) A_2 \lambda \mu(\Delta^4) + A_3 \eta \mu(\Delta^1) + (1 - z^2) A_4 \frac{\eta}{\lambda} \mu(\Delta^2) \, . \tag{2.23}$$

Proof: See the Appendix.

Note that A_ℓ is the sum of discounted expected profits from owning a product line with a technology gap of Δ^ℓ, normalized by the current period product quality. The first two terms in (2.13) through (2.16) denote the normalized instantaneous profits, net of the optimal internal R&D spending. The terms inside the brackets are the normalized future value from internal innovation. B is the sum of the discounted expected profits from owning an additional product through external innovation, normalized by the average product quality.[30]

[30] To understand this variable more clearly, we can rewrite (2.17) as

$$B\overline{q} = x\widetilde{\beta}A_{takeover}\overline{q} - \widetilde{\chi}x^{\widetilde{\psi}}\overline{q} + \widetilde{\beta}B(1+g)\overline{q} \, .$$

After investing $\widetilde{\chi}x^{\widetilde{\psi}}\overline{q}$ in external innovation in the current period, the firm receives the discounted expected profit $\widetilde{\beta}A_{takeover}\overline{q}$ next period if external innovation succeeds with a probability of x. The firm will own at least one product line next period if current period external innovation is successful. Thus, it will invest in external innovation next period and receive an expected profit of $B\overline{q}'$ two periods later, where $\overline{q}' = (1+g)\overline{q}$. Thus, (2.17) illustrates that B is the annuity value of an infinite

For the optimal internal innovation intensities (2.18)-(2.21), the first term in the brackets (after $\widetilde{\beta}$) in the numerator represents the future value from successful internal innovation, which increases the product quality by λ. The second term is the future value from no internal innovation, in which the next period's technology gap is equal to one. Thus, holding $\overline{x}$ fixed, The optimal probability of internal innovation increases in the net future value for successful internal innovation, which in turn depends on the technology gap. In other words, internal innovation becomes an endogenous function of the product's technology gap, which is a unique feature of this model due to imperfect technology spillovers. Corollary 1 further elaborates on this dependence on the technology gap.

Corollary 1. *In an equilibrium where $\{z^\ell\}_{\ell=1}^4$ are well defined, the probabilities of internal innovation satisfy $z^2 > z^3 > z^4 > z^1$. Proof: See the Appendix.*

This indicates that internal innovation increases as the technology gap is higher, as firms find it easier to protect their markets through internal innovation. However, beyond a certain point, a wider technology gap dissuades firms from investing in internal innovation. This is because it becomes less likely for them to lose the product line even without further internal innovation (as observed in the $\Delta^3 = \eta$ case).

Furthermore, the optimal internal innovation also reacts to the creative destruction arrival rate $\overline{x}$ in the economy, labeled as the "market-protection effect." The following corollary shows how the response depends on the technology gap.[31,32]

Corollary 2. *(Market-Protection Effect) With $\widetilde{\psi} \in (1, 2]$, the market-protection effect is maximized and is positive for product lines with a technology gap of Δ^2, whereas it is*

stream of constant payoffs $x\widetilde{\beta}A_{takeover} - \widetilde{\chi}x^{\widetilde{\psi}}$, evaluated at a constant discount rate $\widetilde{\beta}(1+g)$, the growth rate-adjusted stochastic discount factor.

[31]Note that as A_1 and A_2 also depend on $\overline{x}$, it is difficult to analytically sign the partial derivatives of $\{z^\ell\}_{\ell=1}^4$ with respect to $\overline{x}$. However, by holding the values for A_1 and A_2 fixed, we can explicitly determine the signs of these partial derivatives as in Corollary 2.

[32]The term A_2 in (2.18)-(2.21) reflects the well-known Schumpeterian effect—the lower the expected future profits from keeping the product line through internal innovation, the lower the incentive to invest in internal innovation.

minimized and is negative for product lines with a technology gap of Δ^1. The market-protection effect is positive for the Δ^3 case, whereas its sign is ambiguous for the Δ^4 case. Thus,

$$\left.\frac{\partial z^2}{\partial \overline{x}}\right|_{A_1,A_2} > \left.\frac{\partial z^3}{\partial \overline{x}}\right|_{A_1,A_2} > 0, \quad \left.\frac{\partial z^3}{\partial \overline{x}}\right|_{A_1,A_2} > \left.\frac{\partial z^4}{\partial \overline{x}}\right|_{A_1,A_2} \lesseqgtr 0 \ , \quad \textit{and} \quad 0 > \left.\frac{\partial z^1}{\partial \overline{x}}\right|_{A_1,A_2} .$$

Proof: See the Appendix.

As equation (2.2) shows, a firm cannot protect its product line from takeover through internal innovation if its technology gap is equal to Δ^1. This is why z^1 is a decreasing function of the creative destruction arrival rate $\overline{x}$, other things being equal. As in equation (2.3), the impact of internal innovation on the probability of losing the product is greatest in the Δ^2 case. Thus, the market-protection effect is the highest for this case. In the Δ^3 case, a marginal increase in z^3 decreases the probability of losing the product by 50% less than in the Δ^2 case. Hence, the market-protection effect is lower, but still positive. The sign of the market-protection effect for the Δ^4 case is ambiguous, as the decrease in the probability of losing the product line resulting from higher z^4 is even smaller.

More innovation in the previous period increases the likelihood of having a high technology gap in the current period, which helps firms to protect their markets. Thus, Corollary 2 suggests that firms who have innovated intensively in the previous period are more inclined to increase internal innovation in response to heightened competition (higher $\overline{x}$) compared to firms with less recent innovation.[33] This is another important and distinctive aspect of our model, generating firms' strategic use of internal innovation to defend themselves against competitors in the presence of imperfect technology spillovers.

As a result, the optimal external innovation depends on the internal innovation intensities of incumbents, the expected value of products ($\{A_\ell\}_{\ell=1}^4$), and the tech-

[33] The simple three-period model in online Appendix **??** formalizes this observation.

nology gap distribution. Equations (2.22) and (2.23) indicate that higher overall intensities of both internal and external innovations dampen the incentive of firms for external innovation in partial equilibrium, holding $\{A_\ell\}_{\ell=1}^4$ fixed.[34] This is because, as explained previously, the increased overall innovation shifts the technology gap distribution, thereby leading to an increase in the average technology gap. The increased technology gap, in turn, makes firms difficult to take over product markets (the "technological barrier effect"). Furthermore, keeping the probability of internal innovation and the technology gap distribution constant, a decrease in the expected value of products reduces external innovation (known as the "Schumpeterian effect").[35]

In a similar manner, using the startups' value function (2.7) and Proposition 1, the optimal external innovation by potential startups x_e is determined as follows:

$$x_e = \left(\widetilde{\beta}\frac{A_{takeover} + \overline{x}_{takeover}B(1+g)}{\widetilde{\psi}_e\,\widetilde{\chi}_e}\right)^{\frac{1}{\widetilde{\psi}_e - 1}} . \tag{2.24}$$

The proof is provided in online Appendix **??**.

Aggregate Creative Destruction Arrival Rate. The aggregate creative destruction arrival rate in this economy can be characterized as follows, incorporating the optimal external innovation decisions from (2.22) and (2.24):

$$\overline{x} = \underbrace{\mathcal{F}_d\, x + \mathcal{E}_d\, x_e}_{\equiv \overline{x}_d} + \overline{x}_o\,, \tag{2.25}$$

given the undirected nature of external innovation.[36] Here, $\mathcal{F}_o = \overline{x}_o/x$.

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[34]See the simple three-period model in online Appendix **??**.

[35]The direction of the changes in the probabilities of internal and external innovation in response to changes in the aggregate creative destruction arrival rate $\overline{x}$ are ambiguous in general equilibrium. They depend on the relative magnitudes and the directions of the market-protection effect, the Schumpeterian effect, and the technological barrier effect. Nonetheless, results from the numerical exercise in Section 4.2 confirm that the partial equilibrium results for given $\{A_\ell\}_{\ell=1}^4$ and B still hold in general equilibrium in a plausible range of parameterization. Furthermore, $\{A_\ell\}_{\ell=1}^4$ and B also decrease as $\overline{x}$ increases exogenously.

[36]Note that an exogenous increase in $\overline{x}_o$ may not increase $\overline{x}$ by the same amount in equilibrium,

2.7 Balanced Growth Path

The output growth rate in this economy is the same as the product quality growth rate g as in (2.17). Proposition 2 shows the growth rate and its decomposition.

Proposition 2. *The growth rate g for aggregate variables in a Balanced Growth Path in this economy follows:*

$$\begin{aligned} g = &\Big[(1-\overline{x})(1-z^1) + \Delta^2(1-\overline{x})z^1 + \Delta^3\overline{x}\Big]\mu(\Delta^1) \\ &+ \Big[(1-\overline{x})(1-z^2) + \Delta^2 z^2 + \Delta^4\overline{x}(1-z^2)\Big]\mu(\Delta^2) \\ &+ \Big[1-z^3+\Delta^2 z^3\Big]\mu(\Delta^3) + \Big[(1-\overline{x})(1-z^4) + \Delta^2(z^4+\overline{x}(1-z^4))\Big]\mu(\Delta^4) - 1\,, \end{aligned} \tag{2.26}$$

which can be decomposed into the parts attributed to internal innovation and external innovation by domestic incumbents, startups, and outside firms.
Proof: See the Appendix.

2.8 Firm Distribution

The distribution of firms is fully characterized by the distribution of firms' technology gap portfolios. Let $\mathcal{N} = (n_f, n_f^1, n_f^2, n_f^3, n_f^4)$ represent the technology gap composition for firm f owning n_f products in total, with n_f^ℓ products having a technology gap of Δ^ℓ for each $\ell = 1,2,3,4$, and $\widetilde{\mu}(\mathcal{N})$ denote its distribution. Note that we can get the total mass of firms $\mathcal{F}$ by summing up $\widetilde{\mu}(\mathcal{N})$ across all possible $\mathcal{N}$.

Transition of the Technology Gap Portfolio Composition Distribution. Consider a firm with $n_f - k$ products having a technology gap of Δ^1, k products with a technology gap of Δ^2, and zero products with Δ^3 and Δ^4. In this case, the technology gap composition for this firm is $\widetilde{\mathcal{N}}(n_f, k) \equiv (n_f, n_f - k, k, 0, 0)$, where $k \in [0, n_f] \cap \mathbb{Z}$ and $n_f > 0$. Ignoring external innovation, the probability of the technology gap

as the mass of domestic incumbent firms $\mathcal{F}_d$ and the probabilities of external innovation x and x_e depend on $\overline{x}$. Thus, the level of $\overline{x}$ is endogenously determined, even when $\overline{x}_o$ changes exogenously.

composition changing from $\mathcal{N} = \widetilde{\mathcal{N}}(n_f, k)$ to $\mathcal{N}' = \widetilde{\mathcal{N}}(n_f, \widetilde{k})$ can be computed as:

$$\widetilde{\mathbb{P}}\Big(n_f, \widetilde{k} \mid n_f, k\Big) = \sum_{\widetilde{k}^1=\max\{0,\,\widetilde{k}-k\}}^{\min\{n_f-k,\,\widetilde{k}\}} \left(\frac{(n_f-k)!}{\widetilde{k}^1!(n_f-k-\widetilde{k}^1)!}\right)\left(\frac{k!}{(\widetilde{k}-\widetilde{k}^1)!(k-(\widetilde{k}-\widetilde{k}^1))!}\right)$$
$$\times\,(1-\overline{x})^{n_f-(\widetilde{k}-\widetilde{k}^1)}\,(1-z^1)^{n_f-k-\widetilde{k}^1}\,(z^1)^{\widetilde{k}^1}$$
$$\times\,(1-z^2)^{k-(\widetilde{k}-\widetilde{k}^1)}\,(z^2)^{\widetilde{k}-\widetilde{k}^1},\ \text{for } n_f \geq 1 \text{ and } 0 \leq \widetilde{k}, k \leq n_f, \quad (2.27)$$

and zero otherwise. Thus, changes in the technology gap composition follow a binomial process, as in Ates and Saffie (2021). Using (2.27), we can track general cases transitioning from $\mathcal{N} = (n_f, n_f^1, n_f^2, n_f^3, n_f^4)$ to $\mathcal{N}' = (n_f', n_f^{1\prime}, n_f^{2\prime}, n_f^{3\prime}, n_f^{4\prime})$ for any $n_f' \leq n_f + 1$ as products with Δ^3 or Δ^4 can only be obtained through external innovation. The whole procedure is described in detail in online Appendix **??**.

Technology Gap Distribution. Using the distribution of technology gap composition for all firms $\widetilde{\mu}(\mathcal{N})$, the aggregate distribution of technology gaps $\{\mu(\Delta^\ell)\}_{\ell=1}^4$ can be obtained as follows:

$$\mu(\Delta^\ell) = \sum_{n_f=1}^{\overline{n}_f} \sum_{n_f^\ell=0}^{n_f} \sum_{n_f^{-\ell}=0}^{n_f} n_f^\ell\, \widetilde{\mu}\big(n_f, n_f^1, n_f^2, n_f^3, n_f^4\big)\,, \quad (2.28)$$

where the third summation represents the sum over all possible values for $n_f^{-\ell}$ other than the focal ℓ. Since each product line is occupied by one incumbent and there is a unit mass of products, $\sum_{\ell=1}^4 \mu(\Delta^\ell) = 1$ should hold in equilibrium.

Aggregate Variables and Balanced Growth Path Equilibrium. Given the optimal innovation choices provided in (2.18), (2.19), (2.20), (2.21), (2.22), and (2.24), the aggregate domestic R&D expenses can be expressed as:

$$R_d = \widehat{\chi} \sum_{\ell=1}^{4} \left[\int_0^1 q_j\, \mathcal{I}_{\{\Delta_j=\Delta^\ell\,,\, j\in\mathcal{D}\}}\, dj\right] (z^\ell)^{\widehat{\psi}} + \mathcal{F}_d \widetilde{\chi}\overline{q} x^{\widetilde{\psi}} + \mathcal{E}_d \widetilde{\chi}_e (x_e)^{\widetilde{\psi}_e} \overline{q}\,, \quad (2.29)$$

where $\mathcal{I}_{\{\Delta_j=\Delta^\ell\,,\, j\in\mathcal{D}\}}$ is an indicator for product line j owned by a domestic firm with a technology gap of Δ^ℓ. Also, given (2.9), the aggregate demand for final goods by domestic intermediate producers can be written as:

$$Y_d = \int_0^1 y_j \, \mathcal{I}_{\{j \in \mathcal{D}\}} \, dj = (1-\theta)^{\frac{1}{\theta}} \int_0^1 q_j \, \mathcal{I}_{\{j \in \mathcal{D}\}} \, dj \, . \tag{2.30}$$

Given that (2.29) and (2.30) are paid with final goods, and that the production of final goods is Y, the aggregate consumption is determined by:

$$C = Y - R_d - Y_d \, . \tag{2.31}$$

Lastly, the balanced growth equilibrium is characterized by the following:

Definition 1 (Balanced Growth Path Equilibrium)**.** *A balanced growth path equilibrium of this economy consists of y_j^*, p_j^*, w^*, L^*, x^*, $\{z^{\ell *}\}_{\ell=1}^4$, $\overline{x}^*$, x_e^*, $\mathcal{F}_d^*$, R_d^*, Y^*, C^*, g^*, $\widetilde{\mu}(\mathcal{N})$, $\{\mu(\Delta^\ell)\}_{\ell=1}^4$ for every $j \in [0,1]$ with q_j such that: (i) y_j^* and p_j^* satisfy* (2.9)*; (ii) wage w^* satisfies* (2.11)*; (iii) total labor for final good production L^* satisfies* (2.1)*; (iv) the probabilities of internal innovation $\{z^{\ell *}\}_{\ell=1}^4$ satisfy* (2.18)*,* (2.19)*,* (2.20)*, and* (2.21)*, and the probability of external innovation by incumbents x^* satisfies* (2.22)*; (v) the aggregate creative destruction arrival rate $\overline{x}^*$ satisfies* (2.25)*; (vi) the probability of external innovation of potential startups x_e^* satisfies* (2.24)*; (vii) aggregate output Y^* satisfies* (2.12)*; (viii) aggregate domestic R&D expense R_d^* satisfies* (2.29)*; (ix) aggregate consumption C^* satisfies* (2.31)*; (x) the BGP growth rate g^* satisfies* (2.26)*; (xi) the invariant distribution of the technology gap portfolio composition $\widetilde{\mu}(\mathcal{N})$ and the total mass of firms $\mathcal{F}^*$ satisfy inflow($\mathcal{N}$) = outflow($\mathcal{N}$) with $\mathcal{F}_d^* = \mathcal{F}^* - \overline{x}_o / x^*$; and (xii) the invariant technology gap distribution $\{\mu(\Delta^\ell)\}_{\ell=1}^4$ follows* (2.28).

3 Empirics

In this section, we empirically test the model predictions by identifying the causal effect of competition on the composition of firm innovation. We use the China's WTO accession in 2001 as a quasi-experimental increase in competitive pressure.

3.1 Data and Measurement

To construct a comprehensive dataset of firm innovation and foregin competition, we combine the following seven sources: the USPTO PatentsView database, the Longitudinal Business Database (LBD), the Longitudinal Firm Trade Transactions Database (LFTTD), the Census of Manufactures (CMF), the UN Comtrade Database, the NBER-CES database, and the tariff data compiled by Feenstra et al. (2002).

The LBD tracks the universe of establishments and firms in the U.S. non-farm private sector with at least one paid employee annually from 1976 onward.[37] We aggregate establishment-level information into firm-level observations using these firm identifiers.[38] Firm size is measured by either total employment or total payroll. Firm age is based on the age of the oldest establishment of the firm when the firm is first observed in the data. The firm's main industry of operation is based on the six-digit North American Industry Classification System (NAICS) code associated with the establishment with the highest level of employment.[39]

The LFTTD tracks all U.S. international trade transactions at the firm level from 1992 onward. It provides information such as the U.S. dollar value of shipments, the origin and destination countries, and a related-party flag indicating whether the U.S. importer and the foreign exporter are related by ownership of at least 6 percent.[40]

The USPTO PatentsView database records all patents ultimately granted by the USPTO from 1976 onward.[41] This database provides comprehensive details for granted patents, including application and grant dates, technology class, citation,

[37] Details for the LBD and its construction can be found in Jarmin and Miranda (2002).

[38] An establishment corresponds to the physical location where business activity occurs. Establishments that are operated by the same entity, identified through the Economic Census and the Company Organization Survey, are grouped under a common firm identifier.

[39] Time-consistent NAICS codes for LBD establishments are constructed by Fort and Klimek (2018), and the 2012 NAICS codes are used throughout the entire analysis.

[40] Bernard et al. (2009) describe the LFTTD in greater detail.

[41] See https://patentsview.org/download/data-download-tables.

and the name and address of patent assignees. In the following analyses, we rely on the citation-adjusted number of utility patent applications as the main measure of firm innovation.[42] Using the patent-level information, we distinguish domestic innovation from foreign innovation, and assess the extent to which each patent represents internal innovation. The year in which a patent application is filed serves as a proxy for the innovation year.

We link the USPTO patent database to the LBD to track firm patenting activity over time. Here, the failure to match a patent assignee to its LBD firm counterpart can lead to a mismeasurement of changes in firm innovation.[43] Due to the USPTO's limitation in providing a longitudinally consistent unique firm identifier, we build our own crosswalk between the two datasets by adopting the internet search-aided algorithm as in Autor et al. (2020).[44] We pool all patents granted up to December 26, 2017 and use patent applications up to 2007 in our main analyses to avoid a right censoring issue arising from patents applied for but not yet granted. Table **??** in the online Appendix reports the summary statistics of patenting firms in 1992.

The quinquennial CMF provides detailed information about the U.S. manufacturing establishments and products they produce. It contains product-level details such as product codes and the value of shipment. We use five-digit SIC codes (for the pre-2002 years) or seven-digit NAICS codes (for 2002 onward) to define a product.

The UN Comtrade Database offers information on global trade flows at the six-digit HS product-level, which can be concorded to the six-digit 2012 NAICS codes using the crosswalks provided by Pierce and Schott (2009, 2012).[45] Using this data, we construct industry-level imports and exports. Also, we obtain the U.S. tariff schedules from Feenstra et al. (2002) to measure industry-level Trade Policy Uncertainty (TPU)

[42] See Cohen (2010) for a comprehensive review of the literature on the determination of firm/industry innovative activity and related patent measures.

[43] As the USPTO assigns patent applications to self-reported firm names, there is vulnerability to misspelling of firm names.

[44] This algorithm utilizes the machine-learning capacities of internet search engines. The entire matching methodology is outlined in our accompanying paper Ding et al. (2022).

[45] https://comtrade.un.org/db/default.aspx.

as a proxy for foreign competitive pressure. The construction of this measure is discussed in detail in the following section.

Lastly, all nominal values are converted to 1997 U.S. dollars, using the industry-level deflator from the NBER CES Manufacturing Industry Database for manufacturing industries and the Consumer Price Index from the BEA for other industries.[46]

Following this, for our main analyses, we use the USPTO patents matched to LBD firms and industry-level trade data spanning from 1982 to 2007.[47]

Measure of Internal Innovation. We follow Akcigit and Kerr (2018) and use the self-citation ratio, the ratio of self-citations to total citations, as a measure of the likelihood a patent is used for internal innovation.[48] A higher self-citation ratio implies a greater probability that a patent reflects internal innovation.[49] This is because the more an idea is based on the firm's internal knowledge stock (self-citation), the more likely the innovation is used to improve the firm's existing products (internal innovation).

Measures of Foreign Competition. Following Pierce and Schott (2016) and Handley and Limão (2017), we use the removal of trade policy uncertainty as a measure of an exogenous competitive pressure shock. Specifically, we use the following industry-level tariff rate gaps between WTO members and non-market economies in the year 1999 as a proxy for the industry-level competitive pressure shock from

[46] The NBER CES data are complied by Becker et al. (2013) and can be accessed from http://www.nber.org/nberces/.

[47] Our procedure links patents to the firms initially reported by the USPTO as owners and does not track ownership changes resulting from, for instance, M&A activities. We expect our analysis not to be contaminated by firms substituting their innovation activities with the acquisition of other firms, particularly given the fact that M&A activities in the U.S. have started declining since around 2000 and did not fully recover by 2007 as demonstrated in Phillips and Zhdanov (2023).

[48] Each granted patent is required to cite all prior patents on which it builds itself. When a cited patent belongs to the owner of the citing patent, these citations are called self-citations.

[49] Thus, 100% self-citation means the patent is used for internal innovation with a 100% probability, and 0% self-citation means the patent is used for external innovation with a 100% probability.

China occurring in 2001.[50,51]

$$NTRGap_j = Non\ NTR\ Rate_j \ - \ NTR\ Rate_j \text{ (for industry } j) \ .$$

If a firm operates in multiple six-digit NAICS industries, we use the employment-weighted average of $NTRGap_j$.

3.2 Empirical Strategy Testing the Market-Protection Effect

Main Regression and Results. Our model predicts the market-protection effect: firms that have accumulated technological advantage increase internal innovation when confronted with higher competition.

Following Pierce and Schott (2016), we use the following Difference-in-Difference (DD) specification to identify the effect of the Chinese competitive pressure shock on U.S. firm innovation for two periods, $p \in \{1992-1999\ ,\ \ 2000-2007\}$:

$$\begin{aligned}\Delta y_{ijp} \ &= \beta_1 Post_p \times NTRGap_{ijp0} \times InnovIntens_{ijp0} \ + \ \beta_2 Post_p \times NTRGap_{ijp0} \\ &+ \ \beta_3 Post_p \times InnovIntens_{ijp0} \ + \ \beta_4 NTRGap_{ijp0} \times InnovIntens_{ijp0} \\ &+ \ \beta_5 NTRGap_{ijp0} + \ \beta_6 InnovIntens_{ijp0} \\ &+ \ \mathbf{X}_{ijp0}\,\gamma_1 \ + \ \mathbf{X}_{jp0}\,\gamma_2 \ + \ \delta_j \ + \ \delta_p \ + \ \alpha \ + \ \varepsilon_{ijp}\ , \end{aligned} \tag{3.32}$$

where y_{ijp} is either i) the total citation-adjusted number of patents, or ii) the citation-

[50] We can consider the NTR gap as a first-order Taylor approximation of model-based TPU measures, such as those proposed by Handley and Limão (2017). This approximation is also positively related to the non-NTR rate and negatively related to the NTR rate.

[51] Nonmarket economies such as China are by default subject to relatively high tariff rates, known as non-Normal Trade Relations (non-NTR) or column 2 tariffs, when they export to the U.S. On the other hand, the U.S. offers WTO member countries NTR or column 1 tariffs, which are substantially lower than non-NTR tariffs. Although U.S. Despite the temporary NTR status granted to China from 1980, Congress voted on a bill to revoke China's temporary NTR status every year from 1990 to 2001 after the Tiananmen Square protests in 1989. This caused uncertainty about whether the low tariffs would revert to non-NTR rates. Following an agreement on China's entry into the WTO, the U.S. Congress passed a bill granting China permanent NTR (PNTR), and PNTR was implemented on January 1, 2002. The PNTR has reduced trade policy uncertainty (TPU), more for industries with a large prior gap between NTR and non-NTR tariff rates. See Pierce and Schott (2016) for details.

weighted average self-citation ratio for firm i in industry j, and Δy_{ijp} is the DHS (Davis et al., 1996) growth rate of y between the start-year and end-year for each period $p \in \{1992 - 1999\ ,\ \ 2000 - 2007\}$.[52] Note that an increase in the self-citation ratio indicates that the firm's innovation has become more internal.

To maximize the sample size, we include firms that applied for at least one patent in the start-year and at least one patent in or before the end-year for each period. We compute the DHS growth rates for the longest available span of years. We also require firms to have at least one patent before the start-year of each period, or to have an age greater than 0, to avoid the impact of firm entry. The sample comprises all patenting LBD firms meeting these three criteria and excludes FIRE (finance, insurance, and real estate) industries.

$Post_p$ is a dummy variable for the post-treatment period 2000-2007, which captures changes in firm innovation after China's WTO accession. $\mathbf{X}_{ijp0}$ and $\mathbf{X}_{jp0}$ is a vector of firm and industry controls, respectively, measured at the start-year for each period p (1992 or 2000).[53] δ_j is an industry fixed effect (six-digit NAICS), and δ_p is a period fixed effect. The regression is unweighted, and standard errors are clustered on the six-digit NAICS industries.

$InnovIntens_{ijp0}$ is the lagged five-year average of the ratio of the number of patent applications to total employment for firm i. This proxies the technological advantage the firm has. It is measured in the start year for each period p and is normalized by its time-average at the two-digit NAICS level to control for industry effects. The model predicts $\beta_1 > 0$ when the dependent variable is the changes in self-citation ratio.

In these specifications, firms in low TPU industries are the control group, whereas firms in high TPU industries are the treatment group. We use the 1992 and 2000 cohorts of firms to gauge firm innovation before and after the policy change in De-

[52]The long-difference regression specification is standard in settings with a slow-moving process, such as innovation or technological progress (Acemoglu and Restrepo, 2020; Babina et al., 2024).

[53]Baseline firm controls include: firm employment, firm age, the past five-year growth of U.S. patents in the CPC technology classes in which the firm operates, and a dummy variable for publicly traded firms. Industry control variables include NTR rates measured at the start of each period.

cember 2001. In this way, the composition of firms in terms of their innovation is minimally affected by the policy change.

The second row of Table 1 shows the estimates of β_1.[54] The results are consistent with our model prediction in several dimensions. First, the first two columns show that the Chinese competitive pressure shock has no statistically significant effect on firms' overall innovation, regardless of the set of firm controls included. According to our model, as competition intensifies, firms increase or decrease internal innovation based on the technological advantages accumulated within their markets. However, the model predicts a universal decrease in external innovation for all firms. Considering both internal and external innovation changes, the overall effect of competition on firm innovation need not be statistically significant.

However, when examining the effect on internal innovation by substituting the dependent variable with the growth rate of the self-citation ratio, the effect becomes positive and statistically significant, as indicated in the last two columns.[55] This supports the model prediction of the market-protection effect. The estimated coefficient implies a 4.2 percentage points increase in the growth rate of the average self-citation ratio during the period 2000-2007 for a firm with an average lagged innovation intensity (0.18) in an industry with an average NTR gap (0.291). Given that the average value of the seven-year growth rate of the average self-citation ratio between 2000 and 2007 is 28.2 percentage points, this effect represents a 15.0% increase in internal innovation by firms with accumulated technological advantages.

The estimated effect is economically important as well. Table **??** in online Appendix **??** shows that for average firms, creating one more patent is associated with a 1.32 percentage points increase in employment growth. However, the magnitude of this association diminishes if the new patent has a higher self-citation ratio. Com-

[54] To conserve space, Table 1 reports the main coefficient estimates for the triple interaction and the DD-term only. The full results are presented in Table **??** in the online Appendix.

[55] Note that because firms do not change their overall innovation, the increasing self-citation ratio implies that innovative firms (those with above-average innovation intensity) increase their internal innovation while decreasing their external innovation.

Table 1: Market-Protection Effect

	ΔPatents	ΔPatents	ΔSelf-cite	ΔSelf-cite
	(1)	(2)	(3)	(4)
NTR gap × Post	0.238	0.071	-0.075	-0.062
	(0.237)	(0.283)	(0.257)	(0.291)
× Innovation intensity	0.077	-0.054	0.732**	0.795***
	(0.231)	(0.242)	(0.299)	(0.277)
Observations	6,500	6,500	6,500	6,500
Fixed effects	j, p	j, p	j, p	j, p
Controls	no	baseline	no	baseline

Notes: The baseline controls include the past five-year U.S. patent growth in firms' own technology fields, log employment, firm age, NTR rate, and a dummy for publicly traded firms. The estimates for industry (j) and the period (p) fixed effects, along with the coefficient associated with the binary indicator, are suppressed due to disclosure restrictions. The constant is also omitted. Robust standard errors, adjusted for clustering at the level of the firms' major industries, are displayed below each coefficient. Observations are unweighted, and observation counts are rounded due to the Census Bureau disclosure avoidance procedures. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

bining this with the main result indicates that the association between patenting and employment growth decreases by 17.1% (from 1.32pp to 1.10pp) for innovation-intensive firms following the competitive pressure shock from China.[56]

Validity of the Identification Strategy and Robustness Tests. In this section, we confirm the validity of our identification strategy and corroborate our main results across various dimensions. First, we verify the parallel pre-trends assumption, which is a key identifying assumption for the Diff-in-Diff model. We estimate (3.32) for the two seven-year periods preceding the policy change, 1984-1991 and 1992-1999. The results in Table **??** in the online Appendix confirm the validity of the assumption.

Furthermore, we perform several robustness checks. First, we include additional controls (such as the cumulative number of patents, firm payroll, the number of industries in which firms operate, industry-level skill and capital intensities, as well as dummies for importers and exporters) beyond the baseline set to eliminate potential alternative interpretations. Second, to control the effect of trade shocks through firms' I-O network, we include upstream and downstream competitive pres-

[56]Innovation-intensive firms are those with innovation intensity one standard deviation above the average.

sure shocks as covariates in model (3.32).[57,58] The third test replaces the baseline firm-level NTR gaps with the industry-level NTR gaps based on the primary industry (with the largest employment size) in which firms operate.[59] The fourth test addresses a potential sampling bias using the inverse propensity score weights.[60,61] The fifth test adjusts the level of standard error clustering to the firm level.[62] Finally, we test the robustness by replacing the dependent variable with the number of products added as an alternative measure for external innovation (the inverse of internal innovation). Table **??- ??** in the online Appendix present the results for each test, all of which confirm the robustness of the main regression results.[63]

4 Quantitative Analysis

In this section, we calibrate the model to the U.S. manufacturing sector spanning from 1982 to 1997. We then perform counterfactual exercises to draw out the aggregate implications of increasing competition and the compositional shifts in firm

[57]Using the 1992 BEA input-output table, we construct upstream and downstream competitive pressure shocks as the weighted averages of industry-level trade shocks. Following the approach in Pierce and Schott (2016), we assign I-O weights to zero for both upstream and downstream industries within the same three-digit NAICS broad industries for each six-digit NAICS industry. We assume that the shocks within the same three-digit industries capture direct effects rather than indirect effects through the I-O linkage. This is grounded in empirical evidence presented in Bernard et al. (2010), documenting that U.S. manufacturing establishments commonly manufacture clusters of products within the same three-digit NAICS industry.

[58]The upstream measure captures the effect of trade shocks propagating upstream from an industry's buyers, and the downstream measure shows the effect of trade shocks propagating downstream from its suppliers.

[59]The baseline measure uses the employment-share weighted average of the industry-level NTR gaps, where the employment share is measured at the start year of each period and averaged across the firm's operating industries.

[60]This issue can potentially arise from the selection of samples with a positive number of patents granted in the start year and in any of the last four years of each period in the regression analysis, which is inevitable to compute the self-citation ratio over two years for each period.

[61]To formulate the weights, we employ a logit regression on the entire universe of the LBD. The dependent variable is set to one if the firm belongs to the regression sample and zero otherwise. The independent variables include firm characteristics such as firm size, age, employment growth rate, industry, and a multi-unit status indicator.

[62]In our baseline analysis, we cluster the standard errors at the six-digit NAICS level as most variations in the firm-level NTR gap occur at the industry-level.

[63]Note that Table **??** in the online Appendix is consistent with our model prediction: higher competitive pressure reduces the number of new products added (external innovation) by innovative firms.

Table 2: Parameter Estimates

Parameter	Description	Value	Identification
β	time discount rate	0.9615	annual interest rate of 4%
$\widehat{\psi}$	curvature of internal R&D	2.000	Akcigit and Kerr (2018)
$\widetilde{\psi}$	curvature of external R&D	2.000	Akcigit and Kerr (2018)
$\widetilde{\psi}^e$	curvature of external R&D, startup	2.000	Akcigit and Kerr (2018)
θ	quality share in final goods production	0.1090	data
$\widehat{\chi}$	scale of internal R&D	0.0426	indirect inference
$\widetilde{\chi}$	scale of external R&D	0.9340	indirect inference
$\widetilde{\chi}^e$	scale of external R&D, startup	10.5860	indirect inference
λ	quality multiplier of internal innovation	1.0210	indirect inference
η	quality multiplier of external innovation	1.0416	indirect inference
$\overline{x}_o$	exogenous c.d. arrival rate by outside firms	0.0125	indirect inference

innovation. To have a better match of the model to the data, we extend our baseline model in Section 2 and allow firms to take over at most two product lines at random once their external innovation succeeds. Online Appendix **??** displays the computational algorithm used to solve the model.

4.1 Calibration

There are eleven structural parameters to calibrate in the model, listed in Table 2. The first group of five parameters is externally calibrated according to the literature and the data. The second group of six parameters is internally calibrated to match target moments associated with firm level variables and import penetration ratio in the U.S. manufacturing sector from 1982 to 1997.[64] The total mass of potential domestic startups ($\mathcal{E}_d$) is normalized to be one.

[64]The firm-related target moments are based on innovative manufacturing firms from 1982 to 1997, where innovative firms are defined as those with positive R&D expenditure or a positive number of patent filings. The R&D to sales ratio, firm entry rate, and average growth rate are sourced from Akcigit and Kerr (2018), whose sample period aligns with ours. The average number of products is obtained from the 1992 Census of Manufactures, and the high-growth firm growth rate is extracted from the LBD. For the latter, the hp-trended value in 1992 is estimated after kernel density approximation. The import penetration ratio in the manufacturing sector is defined as the ratio between manufacturing imports and manufacturing value of shipments net of exports plus imports. Data for manufacturing imports and exports are sourced from Schott (2008), and manufacturing value added is drawn from the NBER-CES Manufacturing Industry Database. We calculate the import penetration ratio (hp-trend) as the average value from 1992 to 1999, given our use of the 1999 value for the trade policy uncertainty measure in the empirical analysis.

Table 3: Target Moments

Moment	Data	Model	Moment	Data	Model
R&D to sales ratio (%)	4.1	4.2	avg. growth rate (%)	1.0	1.0
avg. number of products	2.3	1.8	high-growth firm growth rate (%)	57.0	57.3
firm entry rate (%)	5.8	5.9	import penetration rate (%)	18.1	18.3

Externally Calibrated Parameters. The time discount factor (β) is set to 0.9615, which corresponds to an annual interest rate of 4%. The curvature parameters of the three R&D cost functions ($\widehat{\psi}$, $\widetilde{\psi}$, $\widetilde{\psi}^e$) are taken from Akcigit and Kerr (2018).[65] Given the average profit-to-sales ratio in the model ($\int_f \frac{profit_f}{sales_f} df = \theta$), we set θ to match with the quality share in final goods production in the data over the 1982-1997 period, which is 10.9% according to Akcigit and Kerr (2018).

Internally Calibrated Parameters. The remaining six parameters are estimated using an indirect inference approach. For a given set of six parameter values, we compute six model-generated moments, compare them to the data counterparts, and find a set of parameter values that minimizes the following objective function:

$$\min \sum_{i=1}^{6} \frac{\left|\text{model moment}_i - \text{data moment}_i\right|}{\frac{1}{2}\left|\text{model moment}_i\right| + \frac{1}{2}\left|\text{data moment}_i\right|} .$$

The six moments are shown in Table 3, which are chosen by considering their importance in answering the main research question of this paper and their relevance to the parameters and functional forms in the model. Although the parameters are jointly calibrated, the most relevant moments for a set of parameters can be noted.

The scales of internal R&D ($\widehat{\chi}$) and external R&D ($\widetilde{\chi}$) are set to match the aggregate R&D to sales ratio of incumbents and the average number of products per firm, respectively. The scale of external R&D for startups ($\widetilde{\chi}^e$) is used to match firm entry rate. We target the average growth rate and the employment growth rate of high-

[65] Akcigit and Kerr (2018) base their parameterization on two lines of literature: one evaluating the empirical relationship between patents and R&D expenditure, and the other evaluating the impact of R&D tax credits on the R&D expenditure of firms.

growth firms (the 90th percentile of the firm employment growth distribution) to pin down the quality multipliers of internal innovation (λ) and external innovation (η), respectively.[66] Lastly, the exogenous creative destruction arrival rate $\overline{x}_o$ by outside firms is used to target the import penetration ratio in the manufacturing sector. The calibration results are presented in Table 3.

4.2 Counterfactual Exercises

In this section, we assess the impact of heightened competition on overall firm innovation, the composition of firm innovation, and aggregate growth. We use the calibrated model and increase the creative destruction arrival rate $\overline{x}_o$ by outside firms by 50% (from 0.013 to 0.019). This corresponds to the rise in the import penetration ratio in the U.S. manufacturing sector from 18.3% to 25.5% (similar to the observed level of 25.1% in 2007).[67]

Increasing Competitive Pressure from Outside Firms. Table 4 shows the results, where the exogenous increase in creative destruction arrival rate $\overline{x}_o$ leads to an increase in the aggregate creative destruction arrival rate $\overline{x}$ and gives rise to the following three moving forces: i) the elevation in $\overline{x}$ results in reduced expected profits for firms involved in internal ($\{A_\ell\}_{\ell=1}^4$) or external innovation (B), as presented in Table 5, which creates the well-known Schumpeterian effect; ii) incumbent firms attempt to protect their existing product lines by intensifying internal innovation, which is particularly pronounced and dominant for product lines with a technology gap of $\Delta^\ell > 1$ (the market-protection effect); iii) as a consequence of both increase in $\overline{x}$ and market-protection effects, average technology gap increases as reported in Table 6, which makes it difficult for firms to take over other product markets via

[66]Note that in the model, intermediate producers use the final good for production. We use the number of workers hired by the final good producer to produce the final goods used by intermediate producers to compute their (implied) employment growth rates.

[67]In online Appendix **??**, we explore an additional counterfactual analysis involving an increase in the creative destruction arrival rate by domestic startups. This comparison allows us to assess the results in light of varying sources of increasing competitive pressure.

Table 4: Innovation Intensities Changes

Description	Variables	Before	After	% Change
creative destruction arrival rate by outside firms	$\overline{x}_o$	0.013	0.019	50.00%
aggregate creative destruction arrival rate	$\overline{x}$	0.144	0.146	1.52%
prob. of internal innovation ($\Delta^1 = 1$)	z^1	0.178	0.177	-0.22%
prob. of internal innovation ($\Delta^2 = \lambda$)	z^2	0.587	0.588	0.21%
prob. of internal innovation ($\Delta^3 = \eta$)	z^3	0.397	0.398	0.16%
prob. of internal innovation ($\Delta^4 = \frac{\eta}{\lambda}$)	z^4	0.382	0.383	0.11%
prob. of external innovation, incumbents	x	0.137	0.135	-1.59%
prob. of external innovation, potential startups	x_e	0.033	0.032	-2.87%
conditional takeover probability	$\overline{x}_{takeover}$	0.754	0.750	-0.47%

external innovation (the technological barrier effect).

Our novel mechanism comes through ii) and iii). At the general equilibrium, these effects come into play together by interacting with each other. For instance, the technological barrier effect in iii) additionally influences the aggregate creative destruction arrival rate $\overline{x}$, causing a recurrence of i) to iii).

The technological barrier effect works in the following way. As a result of increased external innovation by outside firms and successful internal innovation, the technology gap distribution shifts as in Table 6. In particular, the density of Δ^2, Δ^3 and Δ^4 increases, and as per (2.6) and (2.23), this leads to a decrease in the value of the conditional takeover probability as well as the ex-ante value of successful product takeover.[68] Thus, the probability of external innovation x falls according to (2.22). Similarly, potential startups' incentives for external innovation decrease. This effect contributes to the decline observed in x and x^e in Table 4.[69] These all align with our analytical findings in the previous section.

Table 7 reports changes in aggregate moments. Importantly, the aggregate R&D

[68]Note that the increased share of product lines with a technology gap of Δ^3 or Δ^4 is entirely driven by an increasing number of outside firms engaging in external innovation in the domestic market. The increased density of Δ^2 captures the increased internal innovation by the market-protection effect and the contributions from external innovation by outside firms.

[69]Recall that the total decline in x and x^e results from the combined impact of the Schumpeterian and technological barrier effects. A decomposition reveals that 21.4% of the total change in x and 11.2% of the total change in x^e are ascribed to the technological barrier effect (due to the shifts in the technology gap distribution $\mu(\Delta^\ell)$, given all else equal).

Table 5: Firm Value Change

Description	Variables	Before	After	% Change
Firm Values	A_1	0.236	0.234	-1.18%
	A_2	0.250	0.247	-1.09%
	A_3	0.258	0.255	-1.06%
	A_4	0.241	0.239	-1.15%
	B	0.457	0.443	-3.15%

Table 6: Technology Gap Distribution Change

Description	Variables	Before	After	% Change
Technology gap distribution (shares)	$\Delta^1 = 1$	0.582	0.580	-0.34%
	$\Delta^2 = \lambda$	0.316	0.317	0.24%
	$\Delta^3 = \eta$	0.084	0.085	1.16%
	$\Delta^4 = \frac{\eta}{\lambda}$	0.019	0.019	1.48%

Table 7: Aggregate Moment Change

Description	Before	After	% Change
R&D to sales ratio (%)	4.171	4.143	-0.67%
external R&D intensity (%)	61.349	60.824	-0.86%
total mass of domestic firms	0.462	0.429	-7.32%
total mass of domestic startups	0.025	0.024	-3.33%
domestic firm entry rate (%)	5.851	6.118	4.56%
average number of products	1.767	1.738	-1.65%
average growth rate (%)	1.049	1.055	0.58%

Table 8: Aggregate Growth Rate Decomposition

Description	Before	After	% Change
average growth rate (g, %)	1.049	1.055	0.58%
growth rate by outside firms (%)	0.155	0.220	42.00%
growth rate by domestic firms (%)	0.894	0.823	-8.03%
growth rate from domestic internal innovation (%)	0.544	0.497	-8.59%
growth rate from domestic external innovation (%)	0.230	0.209	-9.19%
growth rate from domestic startups (%)	0.121	0.117	-3.29%

to sales ratio of domestic incumbents drops as a result of increasing competitive pressure from outside firms.[70] This is because external innovation falls by more than the increase in internal innovation. Consequently, the external R&D intensity,

[70]The aggregate R&D to sales ratio is defined as the ratio of total R&D expenses (the sum of internal and external R&D expenses) of domestic incumbents to their total sales.

measured as the ratio of total domestic R&D expenses for external innovation to total domestic R&D expenses for all innovation, also decreases. The total masses of both domestic firms and startups decline. However, the reduction in the total mass of domestic firms is more pronounced, so that the domestic firm entry rate increases. The average number of products per firm decreases after heightened competitive pressure from outside firms, in line with the empirical findings of Bernard et al. (2011).[71] The average growth rate g increases in response to increased competitive pressure from outside firms. However, this increase is solely driven by outside firms. Table 8 provides a breakdown of the change in the average growth rate (g). After subtracting the contribution attributed to outside firms, the average growth rate accounted for by domestic firms (both incumbents and startups) falls by 8.03%.[72]

Comparison: Economy with High External Innovation Costs. To compare how the implications vary across environments having different innovation structure, we run the same counterfactual exercise in a hypothetical economy with lower creativity (a lower external innovation intensity due to higher frictions) than the U.S. To see this, we re-calibrate the model to an economy having higher external innovation costs (with 50 times higher $\widetilde{\chi}$ than the baseline calibration of 0.934 and the rest of the parameters remaining the same) and do the same counterfactual analysis by increasing the creative destruction arrival rate by outside firms $\overline{x}_o$ by 50% as before.

Table 9 shows the results for the changes in firm-level internal and external innovation. In comparison to the U.S. case in Table 4, the direction of the changes remains consistent, although the magnitude of each variable is lower in this economy.

Furthermore, Table 10 compares the aggregate target moments across the two

[71] Using datasets from the Census Bureau, they find that firms experiencing greater tariff reductions after the Canada-U.S. Free Trade Agreement tend to reduce the number of products they produce relative to those with smaller tariff reductions.

[72] Note that the changes in the growth rate stem from changes in both firm-level innovation intensities and the mass of firms. Keeping the mass of domestic incumbents constant, approximately 22.0% of the decline in the growth rate resulting from external innovation by domestic incumbents can be ascribed to changes in firm-level external innovation intensity (-9.19% vs. -2.0%). A detailed decomposition is presented in Table **??** in online Appendix **??**.

Table 9: Innov. Intensities Change in an Economy w/ High Ext. Innov. Costs

Description	Variables	Before	After	% Change
creative destruction arrival rate by outside firms	$\overline{x}_o$	0.013	0.019	50.00%
aggregate creative destruction arrival rate	$\overline{x}$	0.054	0.058	7.67%
prob. of internal innovation ($\Delta^1 = 1$)	z^1	0.188	0.187	-0.44%
prob. of internal innovation ($\Delta^2 = \lambda$)	z^2	0.473	0.482	1.93%
prob. of internal innovation ($\Delta^3 = \eta$)	z^3	0.336	0.340	1.36%
prob. of internal innovation ($\Delta^4 = \frac{\eta}{\lambda}$)	z^4	0.330	0.334	1.26%
prob. of external innovation, incumbents	x	0.006	0.005	-4.43%
prob. of external innovation, potential startups	x_e	0.027	0.026	-4.79%
conditional takeover probability	$\overline{x}_{takeover}$	0.849	0.845	-0.55%

economies (in the first two columns) and illustrates their response to the competition shock in the low creativity economy (in the last two columns). The former highlights that the low creativity economy exhibits lower dynamism than the U.S., characterized by less R&D, a smaller number of startups, and lower average and high-growth firm growth. The latter indicates that the response of the moments to increasing foreign competition follows the same direction in the low creativity economy, with the exception of the R&D to sales ratio. In contrast to the U.S. case, the low creativity economy experiences an increase in the aggregate R&D to sales ratio. This is because, in this economy, firms allocate minimal effort to external innovation, and the initial level of external innovation is inherently low, even without heightened competitive pressure. Thus, while external innovation decreases following the rise in foreign competitive pressure, the reduction is marginal in absolute terms and is outweighed by the concurrent increase in internal innovation.

Despite the increased domestic overall innovation (the R&D to sales ratio), the growth attributable to innovation by domestic firms has decreased in this economy, as displayed in Table 11. This reduction in growth is associated with decreases in external innovation by domestic incumbents and startups, along with a decline in the mass of domestic incumbents.[73]

[73] As before, another version of the growth decomposition, holding the mass of firms fixed, is presented in Table **??** in online Appendix **??**. It is confirmed that this pattern persists even when excluding the effect of the changes in firm mass.

Table 10: Aggregate Moment Comparison: U.S. vs. High Ext. Innov. Costs

Moment	Baseline	High Ext. Costs	After Shock	% Change
R&D to sales ratio (%)	4.171	1.295	1.303	0.60%
average number of products	1.767	1.206	1.198	-0.64%
total mass of domestic firms	0.462	0.661	0.596	-9.87%
total mass of domestic startups	0.025	0.023	0.022	-5.31%
average growth rate (%)	1.049	0.722	0.740	2.46%
p90 emp. growth rate (%)	57.344	1.355	1.341	-1.02%

Table 11: Aggregate Growth Rate Decomposition, High Ext. Innov. Costs

Description	Before	After	% Change
average growth rate (g, %)	0.722	0.740	2.46%
growth rate by outside firms (%)	0.151	0.218	44.20%
growth rate by domestic firms (%)	0.571	0.521	-8.81%
growth rate from domestic internal innovation (%)	0.438	0.396	-9.50%
growth rate from domestic external innovation (%)	0.016	0.014	-14.46%
growth rate from domestic startups (%)	0.117	0.110	-5.44%

These results underscore the importance of examining changes in the composition of innovation along with the changes in overall innovation. Understanding the compositional shifts in firm innovation can reconcile the disparate findings from previous studies. For instance, if the European economy corresponds to the economy with lower creativity with higher external innovation costs (such as frictions related to R&D or labor mobility for external innovation) than the U.S., our mechanism can explain why rising foreign competition has increased overall innovation in Europe, in contrast to the U.S., through the innovation composition changes.

Moreover, understanding the compositional changes in innovation is essential to properly evaluate the aggregate implications of competition. Increasing overall innovation in response to heightened competition might not necessarily be beneficial if it is mainly driven by increased defensive internal innovation, which contributes less to economic growth than external innovation and hampers new firm entry. Our findings indicate that competition does not effectively help resolving the inherent problem of having a low stock of external innovation in the low-creativity economy, but rather exacerbate this issue.

Discussion: Relationship with Inverted-U Theory. In our model, the initial level of competition is determined by the cost of external innovation and an exogenous source of competition. The two counterfactual exercises illustrate that, holding the level of exogenous competition fixed, an economy with a low initial level of competition due to the high cost of external innovation exhibits a lower level of overall innovation than the U.S., where the initial level of competition is high. With an increase in competition by outside firms, the former experiences an increase in overall innovation, while the latter observes the opposite pattern. This aligns precisely with the inverted U-shape relationship between competition and firms' overall innovation, documented in Aghion et al. (2005). We extend and explain their findings as a result of changes in the composition of heterogeneous innovations in response to competition emanating from an entry margin.

5 Conclusion

In this paper, we explore the impact of competition on firm innovation by examining the shifts in innovation composition driven by firms' strategic choices, employing both theoretical and empirical analyses. We find that heightened competition prompts firms to increase internal innovation within product lines with technological advantages, while dampening external innovation. Moreover, the overall impact of escalating competition on innovation is contingent upon the innovation cost structure, leading to changes in innovation composition with different magnitudes and shaping distinct economic implications. This channel contributes to bridging gaps in the existing literature, reconciling previous inconclusive results, and advancing our understanding of the intricate role of competition on firm innovation.

Supplementary Materials. Additional supplementary materials can be accessed in the Online Appendix of this article.

Appendix: Proofs of Propositions

Proof of Lemma 1 . To make the argument clearer, let's consider two cases when 1) there is no ownership change between $t-1$ and t, and 2) there is an ownership change between $t-1$ and t:
1) If there is no ownership change between $t-1$ and t, $q_{j,t} = \Delta_{j,t} q_{j,t-1}$ should hold, where only $\Delta_{j,t} \in \{\Delta^1 = 1, \Delta^2 = \lambda\}$ is possible due to $\Delta_{j,t}$ being an outcome of internal innovation.
2) If ownership changes between $t-1$ and t, $q_{j,t} = \eta q_{j,t-2}$ should hold. Let's consider all potentially possible cases where i. $\Delta_{j,t} = 1$, ii. $\Delta_{j,t} = \lambda$, iii. $\Delta_{j,t} = \eta$, iv. $\Delta_{j,t} = \frac{\eta}{\lambda}$, v. $\Delta_{j,t} = \frac{\eta^n}{\lambda^m}$ with $n \geq m > 0$, and vi. $\Delta_{j,t} = \frac{\lambda^n}{\eta^m}$ with $n > m > 0$. These are the only potentially possible values Δ can assume, as there are only three step sizes (1, λ, and η) product quality can be adjusted by between two periods and there is no technology regression ($q_t < q_{t-1}$). In the end, only the first four cases are possible.

case 2)-i. $\Delta_{j,t} = 1$: For this to be true, $q_{j,t} = q_{j,t-1}$ should hold. Since $q_{j,t} = \eta q_{j,t-2}$, this implies $q_{j,t-1} = \eta q_{j,t-2}$. This is possible if there was external innovation between $t-2$ and $t-1$, and no internal innovation between $t-3$ and $t-1$, leading to $q_{j,t-2} = q_{j,t-3}$. Thus $\Delta_{j,t} = 1$ is possible with an ownership change between $t-1$ and t.

case 2)-ii. $\Delta_{j,t} = \lambda$: For this to be true, $\Delta_{j,t-1} = \frac{\eta}{\lambda}$ should hold, as $\Delta_{j,t} = \frac{q_{j,t}}{q_{j,t-1}} = \frac{\eta q_{j,t-2}}{\Delta_{j,t-1} q_{j,t-2}}$. This can be possible if there were internal innovation between $t-3$ and $t-2$, and external innovation between $t-2$ and $t-1$, but no internal innovation between $t-2$ and $t-1$. In this case, $q_{j,t-2} = \lambda q_{j,t-3}$, and $q_{j,t-1} = \eta q_{j,t-3}$, and thus $\Delta_{j,t-1} = \frac{q_{j,t-1}}{q_{j,t-2}} = \frac{\eta q_{j,t-3}}{\lambda q_{j,t-3}} = \frac{\eta}{\lambda}$. So we proved both $\Delta_{j,t} = \lambda$ and $\Delta_{j,t} = \frac{\eta}{\lambda}$ are possible and $\Delta_{j,t} = \frac{\eta}{\lambda}$ can be realized only through external innovation between $t-1$ and t.

case 2)-iii. $\Delta_{j,t} = \eta$: For this to be true, $q_{j,t-1} = q_{j,t-2}$ should hold. This is possible if there was neither ownership change nor internal innovation between $t-1$ and $t-2$. Thus $\Delta_{j,t} = \eta$ is possible.

case 2)-iv. $\Delta_{j,t} = \frac{\eta}{\lambda}$: The possibility of this case is shown in case 2)-ii.

case 2)-v. $\Delta_{j,t} = \frac{\eta^n}{\lambda^m}$ with $n \geq m > 0$: Let's suppose this is the case. Since $\Delta_{j,t} \notin \{\Delta^1 = 1, \Delta^2 = \lambda\}$ there should be an ownership change between $t-1$ and t. Thus $q_{j,t} = \eta q_{j,t-2}$ should hold, and this implies $q_{j,t-1} = \frac{\lambda^m}{\eta^{n-1}} q_{j,t-2}$. $m \leq n-1$ (implying technology regression) is not possible. Let's suppose $m > n-1$. Since $n \geq m > 0$, this implies $m = n$ should hold. Suppose this is the case, and $g_{j,t-2} = \frac{\lambda^m}{\eta^{m-1}} q_{j.t-1}$ is obtained. If the values for λ, η, and m are such that $\frac{\lambda^m}{\eta^{m-1}} < 1$, then this means technology regression, which can be ruled out. Let's suppose $\frac{\lambda^m}{\eta^{m-1}} > 1$ is true.

If $m = 1$, we are back to the cases 2)-ii and 2)-iv. Let's suppose $m > 1$. Since $\frac{\lambda^m}{\eta^{m-1}} \neq 1$ or λ, there should be an ownership change between $t-2$ and $t-1$. Thus $q_{j,t-1} = \eta q_{j,t-3}$, implying $q_{j,t-2} = \frac{\eta^m}{\lambda^m} q_{j,t-3}$. Thus if $\Delta_{j,t} = \frac{\eta^n}{\lambda^m}$ is possible, then

$$q_{j,t-s} = \begin{cases} \frac{\eta^m}{\lambda^m} q_{j,t-s-1} & , s\text{: even number} \\ \\ \frac{\lambda^m}{\eta^{m-1}} q_{j,t-s-1} & , s\text{: odd number .} \end{cases}$$

Thus in this case, either $q_{j,1} = \frac{\eta^m}{\lambda^m} q_{j,0}$ or $q_{j,1} = \frac{\lambda^m}{\eta^{m-1}} q_{j,0}$ should hold, which can be ruled out (or we assume this case out). Thus $\Delta_{j,t} = \frac{\eta^n}{\lambda^m}$ with $n \geq m > 0$ is not possible.

case 2)-vi. $\Delta_{j,t} = \frac{\lambda^n}{\eta^m}$ with $n > m > 0$: With a similar argument, this case is not possible.

Therefore $\Delta_{j,t}$ can assume only four values, $\{1, \lambda, \eta, \frac{\eta}{\lambda}\}$. □

Proof of Proposition 1. Due to the linearity of expectation, $\sum_{\ell=1}^{4} A_\ell \sum_{j \in \mathcal{J}^f | \Delta_j = \Delta^\ell} q_j$ portion of the conjectured value function from $\mathbb{E}\left[V\left(\Phi^{f\prime} \mid \Phi^f\right) \middle| \{z_j\}_{j\in\mathcal{J}^f}, x\right]$ can be written as:

$$\mathbb{E}\left[\sum_{\ell=1}^{4} A_\ell \sum_{j\in\mathcal{J}^{f\prime}|\Delta'_j=\Delta^\ell} q'_j\right] = \mathbb{E}\left[\sum_{\ell=1}^{2} A_\ell \sum_{j\in\mathcal{J}^{f}|(\Delta'_j|\Delta_j)=\Delta^\ell} \Delta^\ell q_j\right] + \mathbb{E}\left[\sum_{\ell=1}^{4} A_\ell\, I_{\left\{\frac{\eta}{\Delta_j}=\Delta^\ell\right\}} \frac{\eta}{\Delta_j}\, q_j\right],$$

where the first term is the expected value of existing product lines, and the second term is the expected value of a new product line added through external innovation.

Since the realization of internal innovation outcomes and the creative destruction shock are independent of the realization of external innovation outcome, the expected value of a new product line can be phrased as:

$$\begin{aligned}
\mathbb{E}\left[\sum_{\ell=1}^{4} A_\ell\, I_{\left\{\frac{\eta}{\Delta_j}=\Delta^\ell\right\}} \frac{\eta}{\Delta_j}\, q_j\right] &= \sum_{I^x=0}^{1} x^{I^x}(1-x)^{1-I^x}\, \mathbb{E}_{q_j,\Delta_j}\left[\sum_{\ell=1}^{4} A_\ell\, I_{\left\{\frac{\eta}{\Delta_j}=\Delta^\ell\right\}}\, I^x\, \frac{\eta}{\Delta_j}\, q_j\right] \\
&= x\, \mathbb{E}_{q_j}\left[\frac{1}{2}(1-z^3)A_1\mu(\Delta^3) + \left(1-\frac{1}{2}z^4\right)A_2\lambda\mu(\Delta^4) + A_3\eta\mu(\Delta^1) + (1-z^2)A_4\frac{\eta}{\lambda}\mu(\Delta^2)\right] q_j \\
&= x\left[\frac{1}{2}(1-z^3)A_1\mu(\Delta^3) + \left(1-\frac{1}{2}z^4\right)A_2\lambda\mu(\Delta^4) + A_3\eta\mu(\Delta^1) + (1-z^2)A_4\frac{\eta}{\lambda}\mu(\Delta^2)\right] \overline{q}\,.
\end{aligned}$$

The second equality follows from the random property of external innovation. The assigned product can have a technology gap of Δ^ℓ with a probability of $\mu(\Delta^\ell)$, and the probability of taking over this product line depends on its technology gap. The third equality integrates it over the entire set of available products.[74]

The first expectation can further be decomposed into the following four cases, depending on the current period technology gap Δ:

$$\mathbb{E}\left[\sum_{\ell=1}^{2} A_\ell \sum_{j\in\mathcal{J}^f|(\Delta'_j|\Delta_j=\Delta^\ell)} \Delta^\ell q_j\right] = \sum_{\widetilde{\ell}=1}^{4} \mathbb{E}\left[\sum_{\ell=1}^{2} A_\ell \sum_{j\in\mathcal{J}^f|(\Delta'_j|\Delta_j=\Delta^{\widetilde{\ell}})=\Delta^\ell} \Delta^\ell q_j\right].$$

For the sake of expositional simplicity, let's rearrange the product quality portfolio q_j based on the technology gap Δ^ℓ and renumber them accordingly:

$$q^f = \Big\{ \underbrace{q_{j_1},\, q_{j_2},\, \ldots,\, q_{j_{n_f^1}}}_{\Delta^1},\, \underbrace{q_{j_{n_f^1+1}},\, \ldots,\, q_{j_{n_f^1+n_f^2}}}_{\Delta^2},\, \underbrace{q_{j_{n_f^1+n_f^2+1}},\, \ldots,\, q_{j_{n_f^1+n_f^2+n_f^3}}}_{\Delta^3},\, \underbrace{q_{j_{n_f^1+n_f^2+n_f^3+1}},\, \ldots,\, q_{j_{n_f^1+n_f^2+n_f^3+n_f^4}}}_{\Delta^4} \Big\}.$$

Then for $i = 1, 2, \ldots, n_f^1$ ($\Delta_{j_i} = \Delta^1 = 1$),

$$\mathbb{E}\left[\sum_{\ell=1}^{2} A_\ell \sum_{j_i\in\mathcal{J}^f|(\Delta'_{j_i}|\Delta_{j_i}=\Delta^1)=\Delta^\ell} \Delta^\ell q_{j_i}\right] = \sum_{i=1}^{n_f^1} \Big[A_1(1-\overline{x})(1-z_i^1) + \lambda A_2(1-\overline{x})z_i^1\Big] q_{j_i},$$

for $i = n_f^1 + 1, \ldots, n_f^1 + n_f^2$ ($\Delta_{j_i} = \Delta^2 = \lambda$),

$$\mathbb{E}\left[\sum_{\ell=1}^{2} A_\ell \sum_{j_i\in\mathcal{J}^f|(\Delta'_{j_i}|\Delta_{j_i}=\Delta^2)=\Delta^\ell} \Delta^\ell q_{j_i}\right] = \sum_{i=n_f^1+1}^{n_f^1+n_f^2} \Big[A_1(1-\overline{x})(1-z_i^2) + \lambda A_2 z_i^2\Big] q_{j_i},$$

for $i = n_f^1 + n_f^2 + 1, \ldots, n_f - n_f^4$ ($\Delta_{j_i} = \Delta^3 = \eta$),

$$\mathbb{E}\left[\sum_{\ell=1}^{2} A_\ell \sum_{j_i\in\mathcal{J}^f|(\Delta'_{j_i}|\Delta_{j_i}=\Delta^3)=\Delta^\ell} \Delta^\ell q_{j_i}\right] = \sum_{i=n_f^1+n_f^2+1}^{n_f-n_f^4} \left[A_1\left(1-\frac{1}{2}\overline{x}\right)(1-z_i^3) + \lambda A_2 z_i^3\right] q_{j_i},$$

and for $i = n_f - n_f^4 + 1, \ldots, n_f$ ($\Delta_{j_i} = \Delta^4 = \frac{\eta}{\lambda}$),

$$\mathbb{E}\left[\sum_{\ell=1}^{2} A_\ell \sum_{j_i\in\mathcal{J}^f|(\Delta'_{j_i}|\Delta_{j_i}=\Delta^4)=\Delta^\ell} \Delta^\ell q_{j_i}\right] = \sum_{i=n_f-n_f^4}^{n_f} \left[A_1(1-\overline{x})(1-z_i^4) + \lambda A_2\left(1-\frac{1}{2}\overline{x}\right) z_i^4\right] q_{j_i}.$$

[74] Note that individual firms only have information about the distribution of technology gap $\{\mu(\Delta^\ell)\}_{\ell=1}^4$ and the average quality level $\overline{q}$. That is, as an individual firm, a technology gap and product quality are independent.

The $B\overline{q}$ portion of the conjectured value function from $\mathbb{E}\left[V\left(\Phi^{f\prime} \mid \Phi^{f}\right) \middle| \{z_j\}_{j\in\mathcal{J}^f}, x\right]$ can be written as:

$$\mathbb{E}B\overline{q}' = B(1+g)\overline{q}\,,$$

where g is the growth rate of product quality in a balanced growth path (BGP) equilibrium. Plugging in the conjectured value function, the original value function can be rephrased as:

$$\begin{aligned}
&\sum_{i=1}^{n_f^1} A_1 q_{j_i} + \sum_{i=n_f^1+1}^{n_f^1+n_f^2} A_2 q_{j_i} + \sum_{i=n_f^1+n_f^2+1}^{n_f-n_f^4} A_3 q_{j_i} + \sum_{i=n_f-n_f^4+1}^{n_f} A_4 q_{j_i} + B\overline{q} = \\
&\max_{\substack{x\in[0,\bar{x}],\\ \{z_i\in[0,\bar{z}]\}_{i=1}^{n_f}}} \left\{
\begin{array}{l}
\sum_{i=1}^{n_f}\left[\pi q_{j_i} - \hat{\chi} z_i^{\hat{\psi}} q_{j_i}\right] - \bar{q}\widetilde{\chi}x^{\widetilde{\psi}} \\
+\widetilde{\beta}\sum_{i=1}^{n_f^1}\left[A_1(1-\overline{x})(1-z_i^1) + \lambda A_2(1-\overline{x})z_i^1\right]q_{j_i} \\
+\widetilde{\beta}\sum_{i=n_f^1+1}^{n_f^1+n_f^2}\left[A_1(1-\overline{x})(1-z_i^2) + \lambda A_2 z_i^2\right]q_{j_i} \\
+\widetilde{\beta}\sum_{i=n_f^1+n_f^2+1}^{n_f-n_f^4}\left[A_1\left(1-\frac{1}{2}\overline{x}\right)(1-z_i^3) + \lambda A_2 z_i^3\right]q_{j_i} \\
+\widetilde{\beta}\sum_{i=n_f-n_f^4}^{n_f}\left[A_1(1-\overline{x})(1-z_i^4) + \lambda A_2\left(1-\frac{1}{2}\overline{x}\right)z_i^4\right]q_{j_i} \\
+\widetilde{\beta}x\left[\frac{1}{2}(1-z^3)A_1\mu(\Delta^3) + \left(1-\frac{1}{2}z^4\right)A_2\lambda\mu(\Delta^4)\right. \\
\qquad\qquad \left. + A_3\eta\mu(\Delta^1) + (1-z^2)A_4\frac{\eta}{\lambda}\mu(\Delta^2)\right]\overline{q} \\
+\widetilde{\beta}B(1+g)\overline{q}
\end{array}
\right\},
\end{aligned}$$

and the optimal innovation intensities are derived as follows:

$$\frac{\partial}{\partial z_i^1} : -\widehat{\psi}\widehat{\chi}(z_i^1)^{\widehat{\psi}-1}q_{j_i} + \widetilde{\beta}(1-\overline{x})\left[\lambda A_2 - A_1\right]q_{j_i} = 0 \Rightarrow z^1 = \left[\frac{\widetilde{\beta}(1-\overline{x})\left[\lambda A_2 - A_1\right]}{\widehat{\psi}\widehat{\chi}}\right]^{\frac{1}{\widehat{\psi}-1}}$$

$$\frac{\partial}{\partial z_i^2} : -\widehat{\psi}\widehat{\chi}(z_i^2)^{\widehat{\psi}-1}q_{j_i} + \widetilde{\beta}\left[\lambda A_2 - (1-\overline{x})A_1\right]q_{j_i} = 0 \Rightarrow z^2 = \left[\frac{\widetilde{\beta}\left[\lambda A_2 - (1-\overline{x})A_1\right]}{\widehat{\psi}\widehat{\chi}}\right]^{\frac{1}{\widehat{\psi}-1}}$$

$$\frac{\partial}{\partial z_i^3} : -\widehat{\psi}\widehat{\chi}(z_i^3)^{\widehat{\psi}-1}q_{j_i} + \widetilde{\beta}\left[\lambda A_2 - \left(1-\frac{1}{2}\overline{x}\right)A_1\right]q_{j_i} = 0 \Rightarrow z^3 = \left[\frac{\widetilde{\beta}\left[\lambda A_2 - \left(1-\frac{1}{2}\overline{x}\right)A_1\right]}{\widehat{\psi}\widehat{\chi}}\right]^{\frac{1}{\widehat{\psi}-1}}$$

$$\frac{\partial}{\partial z_i^4}: -\widehat{\psi}\widehat{\chi}(z_i^4)^{\widehat{\psi}-1}q_{j_i} + \widetilde{\beta}\left[\lambda\left(1-\frac{1}{2}\overline{x}\right)A_2 - (1-\overline{x})A_1\right]q_{j_i} = 0 \Rightarrow z^4 = \left[\frac{\widetilde{\beta}\left[\lambda\left(1-\frac{1}{2}\overline{x}\right)A_2 - (1-\overline{x})A_1\right]}{\widehat{\psi}\widehat{\chi}}\right]^{\frac{1}{\widehat{\psi}-1}}$$

$$\frac{\partial}{\partial x}: -\widetilde{\psi}\widetilde{\chi}\overline{q}x^{\widetilde{\psi}-1} + \widetilde{\beta}\left[\frac{1}{2}(1-z^3)A_1\mu(\Delta^3) + \left(1-\frac{1}{2}z^4\right)A_2\lambda\mu(\Delta^4) + A_3\eta\mu(\Delta^1) + (1-z^2)A_4\frac{\eta}{\lambda}\mu(\Delta^2)\right]\overline{q} = 0$$

$$\Rightarrow x = \left[\frac{\widetilde{\beta}\left[\frac{(1-z^3)A_1\mu(\Delta^3)}{2} + \left(1-\frac{z^4}{2}\right)A_2\lambda\mu(\Delta^4) + A_3\eta\mu(\Delta^1) + (1-z^2)A_4\frac{\eta}{\lambda}\mu(\Delta^2)\right]}{\widetilde{\psi}\widetilde{\chi}}\right]^{\frac{1}{\widetilde{\psi}-1}}$$

By plugging in the optimal innovation intensities and equating the LHS to the RHS, we get the five coefficients of the conjectured value function as follows:

$$A_1 = \pi - \widehat{\chi}(z^1)^{\widehat{\psi}} + \widetilde{\beta}\Big[A_1(1-\overline{x})(1-z^1) + \lambda A_2(1-\overline{x})z^1\Big]$$

$$A_2 = \pi - \widehat{\chi}(z^2)^{\widehat{\psi}} + \widetilde{\beta}\Big[A_1(1-\overline{x})(1-z^2) + \lambda A_2 z^2\Big]$$

$$A_3 = \pi - \widehat{\chi}(z^3)^{\widehat{\psi}} + \widetilde{\beta}\left[A_1\left(1-\frac{1}{2}\overline{x}\right)(1-z^3) + \lambda A_2 z^3\right]$$

$$A_4 = \pi - \widehat{\chi}(z^4)^{\widehat{\psi}} + \widetilde{\beta}\left[A_1(1-\overline{x})(1-z^4) + \lambda A_2\left(1-\frac{1}{2}\overline{x}\right)z^4\right]$$

$$\begin{aligned} B &= \frac{1}{1-\widetilde{\beta}(1+g)}\left[\widetilde{\beta}x\left[\frac{1}{2}(1-z^3)A_1\mu(\Delta^3) + \left(1-\frac{1}{2}z^4\right)A_2\lambda\mu(\Delta^4) + A_3\eta\mu(\Delta^1) + (1-z^2)A_4\frac{\eta}{\lambda}\mu(\Delta^2)\right] - \widetilde{\chi}(x)^{\widetilde{\psi}}\right] \\ &= \frac{\left(\widetilde{\psi}\widetilde{\chi}\right)^{-\frac{1}{\widetilde{\psi}-1}}\left(1-\frac{1}{\widetilde{\psi}}\right)}{1-\widetilde{\beta}(1+g)}\left[\widetilde{\beta}\left[\frac{1}{2}(1-z^3)A_1\mu(\Delta^3) + \left(1-\frac{1}{2}z^4\right)A_2\lambda\mu(\Delta^4) + A_3\eta\mu(\Delta^1) + (1-z^2)A_4\frac{\eta}{\lambda}\mu(\Delta^2)\right]\right]^{\frac{\widetilde{\psi}}{\widetilde{\psi}-1}}. \end{aligned}$$

□

Proof of Corollary 1. Define $\widetilde{z^\ell} = \frac{\widehat{\psi}\widehat{\chi}}{\widetilde{\beta}}\left(z^\ell\right)^{(\widehat{\psi}-1)}$. Then $z^\ell > z^{\ell'} \Leftrightarrow \widetilde{z^\ell} > \widetilde{z^{\ell'}}$ for $\ell, \ell' \in [1,4] \cap \mathbb{Z}$ with $\widehat{\psi} > 1$. Since $\widetilde{z^2} - \widetilde{z^3} = \frac{1}{2}\overline{x}A_1 > 0$, $\widetilde{z^2} - \widetilde{z^1} = \overline{x}\lambda A_2 > 0$, $\widetilde{z^2} - \widetilde{z^4} = \frac{1}{2}\overline{x}\lambda A_2 > 0$, and $\widetilde{z^4} - \widetilde{z^1} = \frac{1}{2}\overline{x}\lambda A_2 > 0$, we have $z^2 > z^3$, $z^2 > z^1$, $z^2 > z^4$, and $z^4 > z^1$. Now, if we know the sign for $\widetilde{z^3} - \widetilde{z^4} = \frac{1}{2}\overline{x}\left[\lambda A_2 - A_1\right]$ then we know the entire relationships between $\{z^\ell\}_{\ell=1}^4$. But in an equilibrium, $\widetilde{z^1} = (1-\overline{x})\left[\lambda A_2 - A_1\right] > 0$ should hold, which implies $\lambda A_2 - A_1 > 0$. Thus $\widetilde{z^3} > \widetilde{z^4} \Leftrightarrow z^3 > z^4$, followed by $z^2 > z^3 > z^4 > z^1$. □

Proof of Corollary 2. The partial derivatives of $\{z^\ell\}_{\ell=1}^4$ with respect to $\overline{x}$ are

$$\begin{aligned} \left.\frac{\partial z^1}{\partial \overline{x}}\right|_{A_1,A_2} &: \quad -\frac{\widetilde{\beta}}{\widehat{\psi}\widehat{\chi}}(z^1)^{2-\widehat{\psi}}\left[\lambda A_2 - A_1\right] < 0 \\ \left.\frac{\partial z^2}{\partial \overline{x}}\right|_{A_1,A_2} &: \quad \frac{\widetilde{\beta}}{\widehat{\psi}\widehat{\chi}}(z^2)^{2-\widehat{\psi}}A_1 > 0 \end{aligned}$$

$$\left.\frac{\partial z^3}{\partial \overline{x}}\right|_{A_1,A_2} \quad : \quad \frac{\widetilde{\beta}}{\widehat{\psi}\widehat{\chi}}\,(z^3)^{2-\widehat{\psi}}\,\frac{1}{2}A_1 > 0$$

$$\left.\frac{\partial z^4}{\partial \overline{x}}\right|_{A_1,A_2} \quad : \quad -\frac{\widetilde{\beta}}{\widehat{\psi}\widehat{\chi}}\,(z^4)^{2-\widehat{\psi}}\left[\,\frac{1}{2}\lambda A_2 - A_1\,\right] \gtrless 0\,,$$

holding A_1 and A_2 fixed. Since we know $\lambda A_2 - A_1 > 0$, $\left.\frac{\partial z^1}{\partial \overline{x}}\right|_{A_1,A_2}$ should be negative. Also, since $z^2 > z^3$, we have $\left.\frac{\partial z^2}{\partial \overline{x}}\right|_{A_1,A_2} > \left.\frac{\partial z^3}{\partial \overline{x}}\right|_{A_1,A_2}$. Since $z^3 > z^4$ and $A_1 > A_1 - \frac{1}{2}\lambda A_2$, we obtain $\left.\frac{\partial z^3}{\partial \overline{x}}\right|_{A_1,A_2} > \left.\frac{\partial z^4}{\partial \overline{x}}\right|_{A_1,A_2}$, however, the sign for $\frac{1}{2}\lambda A_2 - A_1$ remains ambiguous. □

Proof of Proposition 2. In this model economy, the output growth rate is equal to the product quality growth rate. Pick any q_j. Then its technology gap is equal to $\Delta_j = \Delta^\ell$ with a probability of $\mu(\Delta^\ell)$, and the probability of Δ_j' becoming a certain level of the technology gap follows:

$$\begin{array}{llll}
\text{If } \Delta_j = \Delta^1, & q_j' = \Delta^1 q_j & \text{w/ prob.} & (1-\overline{x})(1-z^1) \\
 & q_j' = \Delta^2 q_j & \text{w/ prob.} & (1-\overline{x})z^1 \\
 & q_j' = \Delta^3 q_j & \text{w/ prob.} & \overline{x} \\
 & q_j' = \Delta^4 q_j & \text{w/ prob.} & 0 \\
\text{If } \Delta_j = \Delta^2, & q_j' = \Delta^1 q_j & \text{w/ prob.} & (1-\overline{x})(1-z^2) \\
 & q_j' = \Delta^2 q_j & \text{w/ prob.} & z^2 \\
 & q_j' = \Delta^3 q_j & \text{w/ prob.} & 0 \\
 & q_j' = \Delta^4 q_j & \text{w/ prob.} & \overline{x}(1-z^2) \\
\text{If } \Delta_j = \Delta^3, & q_j' = \Delta^1 q_j & \text{w/ prob.} & 1-z^3 \\
 & q_j' = \Delta^2 q_j & \text{w/ prob.} & z^3 \\
 & q_j' = \Delta^3 q_j & \text{w/ prob.} & 0 \\
 & q_j' = \Delta^4 q_j & \text{w/ prob.} & 0 \\
\text{If } \Delta_j = \Delta^4, & q_j' = \Delta^1 q_j & \text{w/ prob.} & (1-\overline{x})(1-z^4) \\
 & q_j' = \Delta^2 q_j & \text{w/ prob.} & z^4 + \overline{x}(1-z^4) \\
 & q_j' = \Delta^3 q_j & \text{w/ prob.} & 0 \\
 & q_j' = \Delta^4 q_j & \text{w/ prob.} & 0
\end{array}$$

Following this, we can derive:

$$\begin{aligned}
\mathbb{E}\big[q_j' \mid q_j\big] = \Bigg[& \Big[(1-\overline{x})(1-z^1) + \lambda(1-\overline{x})z^1 + \eta\overline{x}\Big]\mu(\Delta^1) + \Big[(1-\overline{x})(1-z^2) + \lambda z^2 + \frac{\eta}{\lambda}\overline{x}(1-z^2)\Big]\mu(\Delta^2) \\
& + \Big[1 - z^3 + \lambda z^3\Big]\mu(\Delta^3) + \Big[(1-\overline{x})(1-z^4) + \lambda(z^4 + \overline{x}(1-z^4))\Big]\mu(\Delta^4)\Bigg] q_j\,,
\end{aligned}$$

and thus,

$$1+g=\Bigg[\Big[(1-\overline{x})(1-z^1)+\lambda(1-\overline{x})z^1+\eta\overline{x}\Big]\mu(\Delta^1)+\Big[(1-\overline{x})(1-z^2)+\lambda z^2+\frac{\eta}{\lambda}\overline{x}(1-z^2)\Big]\mu(\Delta^2)$$
$$+\Big[1-z^3+\lambda z^3\Big]\mu(\Delta^3)+\Big[(1-\overline{x})(1-z^4)+\lambda(z^4+\overline{x}(1-z^4))\Big]\mu(\Delta^4)\Bigg].$$

Using the share of products owned by domestic incumbents $s_d=\mathcal{F}_d/\mathcal{F}$, the definition of $\overline{x}$, and the evolution of product quality, the growth rate can be decomposed into the following components:

$$g=\underbrace{\left(\Delta^2-1\right)s_d\left[(1-\overline{x})z^1\mu(\Delta^1)+z^2\mu(\Delta^2)+z^3\mu(\Delta^3)+\left(1-\frac{1}{2}\overline{x}\right)z^4\mu(\Delta^4)\right]}_{\text{internal innovation by domestic incumbent firms}}$$
$$+\underbrace{\left(\Delta^2-1\right)(1-s_d)\left[(1-\overline{x})z^1\mu(\Delta^1)+z^2\mu(\Delta^2)+z^3\mu(\Delta^3)+\left(1-\frac{1}{2}\overline{x}\right)z^4\mu(\Delta^4)\right]}_{\text{internal innovation by foreign firms}}$$
$$+\underbrace{\left(\overline{\Delta^{ex}}-1\right)\ \mathcal{F}_d\,x\,\mu(\overline{\Delta^{ex}})}_{\text{external innovation by domestic incumbent firms}}+\underbrace{\left(\overline{\Delta^{ex}}-1\right)\ \mathcal{E}_d\,x_e\,\mu(\overline{\Delta^{ex}})}_{\text{external innovation by domestic startups}}+\underbrace{\left(\overline{\Delta^{ex}}-1\right)\ \overline{x}_o\,\mu(\overline{\Delta^{ex}})}_{\text{external innovation by foreign firms}},$$

where

$$\overline{\Delta^{ex}}\equiv\frac{\Delta^3\mu(\Delta^1)+\Delta^4(1-z^2)\mu(\Delta^2)+\frac{1}{2}(1-z^3)\mu(\Delta^3)+\Delta^2\left(1-\frac{1}{2}z^4\right)\mu(\Delta^4)}{\mu(\Delta^1)+(1-z^2)\mu(\Delta^2)+\frac{1}{2}(1-z^3)\mu(\Delta^3)+\left(1-\frac{1}{2}z^4\right)\mu(\Delta^4)}$$

is an increase in the average product quality due to external innovation (with successful business takeover by both incumbents and startups), and

$$\mu(\overline{\Delta^{ex}})\equiv\mu(\Delta^1)+(1-z^2)\mu(\Delta^2)+\frac{1}{2}(1-z^3)\mu(\Delta^3)+\left(1-\frac{1}{2}z^4\right)\mu(\Delta^4)$$

is the share of product lines affected by external innovation. $\square$

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