Ideas and Firm Dynamics When It Takes Two to Tango

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Abstract

Idea production is increasingly performed by teams of inventors rather than individuals. We study the two-way interaction between team idea production and firm dynamics, examining how firms shape teams and how the rise of teams impacts the size distribution of innovative firms and allocative efficiency. Using U.S. patent data from 1976-2020, we find (i) the share of patents with multiple inventors (team patents) rose from 45% to 80%; (ii) around 90% of team patents are by inventors at the same firm; (iii) team characteristics vary with firm size, productivity, and age. Motivated by these facts, we develop a tractable framework microfounding inventor team formation within firms and show that team-based production can increase returns to scale in R&D labor, raising the optimal size of innovative firms. We find empirical support for this framework in patent data, with estimated returns to scale in inventor labor at the firm level increasing over time. Embedding this framework into a Hopenhayn-style model with knowledge spillovers we study the effect of the rise of teams on the optimal allocation of R&D workers to firms. We find that the rise of teams increases innovative output but also misallocation across firms.

Keywords: innovation, R&D team, inventor allocation, firm dynamics, economic growth, organizational economics

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1 Introduction

The development of new ideas is a key driver of economic growth. Ideas can come either from individual inventors or from collaborations among teams of inventors. Teams have become increasingly important in the production of new ideas in recent years (Jones 2009; Pearce 2023), and firms play a critical role in determining collaborations between inventors. However, little is known about the implications of the rise of team invention for firm dynamics and the firm size distribution, worker (mis)allocation, and the rate and quality of innovative output. In this paper, we aim to fill this gap by studying the two-way interaction between inventor team formation and firm heterogeneity and dynamics.

We begin by using the USPTO patent database linked to Compustat and documenting stylized facts about inventor teams. First, patents developed by teams now account for 80% of all private patents in the U.S., compared to just 50% in 1980. Second, around 90% of team collaborations are by inventors at the same firm. Thus, understanding the role of firms is critical to understanding how team idea production takes place. We estimate returns to scale in inventor labor across patenting firms and precisely estimate an increase in returns to scale from 0.693 to 0.736. This motivates our approach to modeling team idea production as increasing returns to scale in R&D labor. Additionally, we observe that team characteristics vary across firm size, productivity, and age distributions: small, less productive, or young firms are more likely to patent through solo inventors or small teams, and team size rises with firm size and age.

We then propose a novel and tractable way of modeling team invention within firms. Each worker draws a bilateral productivity of collaborating with every other worker at the firm, and an individual's realized productivity is the maximum of these draws. Consequently, workers in large firms have higher expected productivity, as they have more chances to find a good match (e.g., a colleague with a complementary set of skills). With particular functional form assumptions, expected worker productivity grows in the number of employees at a diminishing rate and can easily be incorporated into a standard production function to resemble less-decreasing returns to scale in the number of R&D workers.

Building on this microfoundation of team formation, we analyze how the shift from solo to team-based invention influences firm-level decisions, the aggregate allocation of inventors across firms, and the firm size distribution. We find that team-based invention increases the optimal scale of innovative firms, resulting in a higher concentration of R&D workers in large firms and a greater share of large firms. These findings are consistent with recent empirical evidence on the growing concentration of inventors in large firms (Akcigit and Goldschlag 2023b; Akcigit and Goldschlag 2023a), which we show has been much more dramatic than the overall rise in employment concentration documented by Autor et al. (2019).

In one sense the growing concentration of inventors at large firms seems more benign through the lens of the growing importance of teams and increased returns to scale in inventor labor. But there is also evidence that small firms make disproportionate contributions to knowledge spillovers (Akcigit and Kerr 2018; Chikis, Kleinman, and Prato 2025), and we show that the rise of teams exacerbates misallocation of R&D workers across firms in the presence of knowledge spillovers.

Furthermore, given additional empirical evidence of wage differentials of inventors across firms and declining inventor mobility, inventor labor markets likely exhibit a certain degree of frictions.[1] Taking this view seriously, we embed our simple framework into a canonical model of a frictional labor market (Elsby and Michaels 2013). In this full-fledged model, a shift from solo to team-based invention generates the same results as in the frictionless model, with the exception of its effects on labor market tightness, wage differentials across firms, and inventor flows. Specifically, the transition to team production increases labor market tightness in the inventor market, raises the employment share of large firms and the wage premium paid at larger firms. This makes it

[1] Akcigit and Goldschlag (2023b) document significant wage dispersion among inventors, with otherwise similar individuals earning substantially higher wages when they join large firms. They also show that inventor mobility has declined since 2000.

harder for productive young firms to find workers and grow, with their growth rate declining from 2.74% to 2.26%. Consequently, the separation rate falls, and the hiring inaction region widens, reducing inventor mobility.

Related Literature A growing body of literature examines the match between individual inventors and firms (Akcigit and Goldschlag 2023a; Akcigit and Goldschlag 2023c; Babalievsky 2023; Manera 2022; Liu 2023). A separate strand of literature models firms as collections of coworkers or co-inventors, without incorporating additional dimensions of firm heterogeneity (Herkenhoff et al. 2024; Pearce 2023; Freund 2024). This paper bridges the gap between these two approaches by exploring how inventor team formation varies with firm-level characteristics (in particular idiosyncratic productivity, which is intimately connected in most models, including ours, to employment size) and connects to firm dynamics and inventor allocation across firms.[2]

Our paper is also related to the literature on how interactions between individuals shape economic growth (Lucas and Moll 2014; Akcigit et al. 2018; Prato 2022). Relative to these papers, we abstract from the dynamics of individual-level knowledge accumulation and instead focus on the organizational features that may determine different forms of knowledge production at the firm level (Garicano and Rossi-Hansberg 2015). Our model predicts that R&D workers will increasingly concentrate at large firms, which highlights the importance of understanding how human capital accumulation among R&D workers occurs. If variety in employers is a key source of knowledge accumulation, the trend toward greater worker concentration at large firms could pose challenges. On the other hand, the shift from solo invention to more collaborative efforts may accelerate on-the-job human capital accumulation for knowledge workers as they learn from each other through collaboration.

Several empirical papers highlight the importance of inventor teams within firms. Jaravel, Petkova, and Bell (2018) find that the death of a co-inventor negatively impacts

[2] Jarosch, Oberfield, and Rossi-Hansberg (2021) and Boerma, Tsyvinski, and Zimin (2023) study models of sorting into teams within firms, but not specifically for inventors.

subsequent inventor productivity. Bhaskarabhatla et al. (2021) show that a significant share of firm-level heterogeneity in research productivity is driven by differences in inventor quality. Kline et al. (2019) demonstrate that firms share around 30% of the rents from patenting with incumbent and high-paid employees, implying that a large share of the rents from patenting are retained by firm owners. We abstract for the most part from these issues and instead focus our attention on what determines the allocation of workers to firms and how that responds to changes over time in the idea production function that favor teams.

Our findings contribute to understanding trends in productivity differences between large and small firms, particularly in the context of research productivity (Andrews, Criscuolo, and Gal 2016; Bessen et al. 2020; Pugsley, Sedlacek, and Sterk 2021; Cavenaile, Celik, and Tian 2020; Olmstead-Rumsey 2022; Akcigit and Ates 2023). Also our findings are related to the recent trends of declining business dynamism and missing high-growth young firms (Decker et al. 2014; Decker et al. 2016; Sterk, Sedláček, and Pugsley 2021; Akcigit and Ates 2023).

Finally, our model of teamwork within firms contributes to our understanding of the boundary of the firm and optimal scale (Lucas 1978; Chandler 1977). Recent empirical work finds evidence of differences in returns to scale at the firm level (Chan et al. 2025) and evidence that the supply of managers (Engbom et al. 2025), standardization (Argente et al. 2025), and economic development (Bassi et al. 2023; Chen 2023) all influence optimal firm scale. We contribute an additional microfoundation for changing optimal firm scale: idea production in teams, and validate the existence of this explanation using firm level data on inputs (inventors) and output (patents).

2 Empirics

2.1 Data

Our primary data sources are the USPTO PatentsView and S&P's Compustat. The USPTO PatentsView tracks all patents granted by the U.S. Patent and Trademark Office (USPTO) from 1976 onward. This database provides detailed information, including application and grant dates, technology class classifications, patent inventors, citation, and the names and addresses of patent assignees. Compustat is a comprehensive database maintained by S&P Global and provides detailed firm-level data for publicly traded companies. It includes information such as firm size, industry scope, balance sheets, income and cash flow statements, and stock prices.

We collect utility patents filed between 1976 and 2018, granted by the USPTO. To identify the patents assigned to US public firms and track them by different firm characteristics, we link the USPTO assignee identifier to the Compustat GVKEY using the bridge provided by Ding, Jo, and Kim (2022) and Braguinsky et al. (2023).[3] When a patent is assigned to multiple entities, we only keep the first assignee following the sequence order recorded by USPTO.

To measure the importance of a patent, we calculate five-year forward citations with vintage fixed effects removed, and obtain real patent values from Kogan et al. (2017) (KPSS values, hereafter) for patents filed by publicly held firms. To obtain an accurate estimate of firm age, we also link PatentsView with the dataset provided by Ewens and Marx (2024) to obtain the firm's founding year. We also supplement the patent data with LinkedIn data from Revelio Labs on inventors' CVs to get a sense of where inventors are employed, including in periods when they do not successfully patent, which is one major limitation of the patent data: you only observe inventor-assignee matches at the moment of successful innovation.[4]

[3]This bridge is constructed using the standard name and address matching process, complemented by an internet search-aided algorithm following Autor et al. (2020) as well as additional manual matching. The detailed methodology is described in Ding, Jo, and Kim (2022) and Braguinsky et al. (2023).

[4]Eventually, we aim to incorporate administrative data from the U.S. Census Bureau to extend our

2.2 Stylized Facts

With this dataset, we document the following facts on team patenting, its properties, and its relationship to firms.

Fact 1: Team patents have increased from 50% in 1976 to 80% of patents in 2018

Teams have played an increasingly dominant role in innovations over the past five decades. A patent is identified as a team patent if it has multiple inventors, and the group of inventors filing the same patent is identified as a team. Among all patents in the dataset, the share of team patents has risen from below 50% in 1976 to above 80% in 2018. This trend holds either measured in patent counts, KPSS values, or five-year forward citations, as shown in Figure 1.[5]

Fact 2: Firms shape teams; 90% of team patents are by inventors at the same firm

At the same time, most team collaboration in innovation occurs within firms rather than between them, highlighting the role of firms as a critical platform for team collaboration. We identify an inventor's affiliation in a given year as the firm with the largest number of patents filed by that inventor during that year. In the event of a tie, we assign the affiliation to the firm associated with the patent having the latest filing date. In the dataset, 89.5% of the team patents are filed by inventors affiliated with the same firm.

analysis, which is currently in progress.

[5] In this figure, we restrict our sample to Compustat firms with patent filings due to the use of KPSS values and to maintain consistency of the sample across series. The result is robust when using the full set of USPTO assignees.

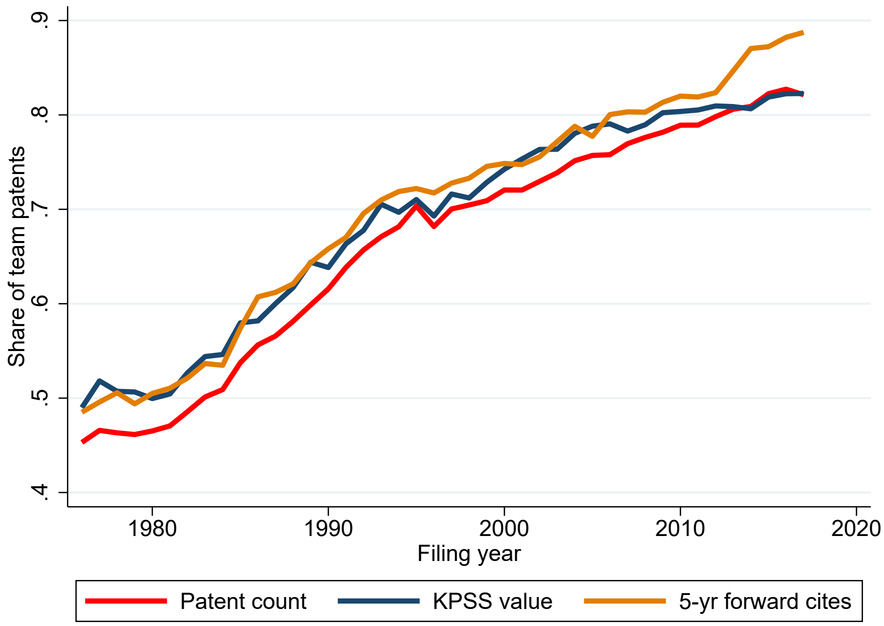

Figure 1: Team Patents Shares

Note: The figure represents the share of team patents at Compustat firms from 1976 to 2018. A team patent is defined as one assigned by multiple inventors. The number of patents is counted unweighted (red line), weighted in KPSS values (navy line) extracted from Kogan et al. (2017), and weighted in five-year forward citations with vintage fixed effects removed (orange line).

Fact 3: Teams increase firm-level returns to scale and effect increases over time

To test for changes in returns to scale in inventor labor due to teams over time, we run the following triple-difference regression at the firm level:

$$\log(pats_{ft}) = \mu_f + \alpha \log(n_{ft}) + \frac{1}{\gamma^{pre}}(\log n_{ft} \times \mathbb{I}(team_{ft})) + \frac{1}{\gamma^{post}}(\log n_{ft} \times \mathbb{I}(team_{ft}) \times Post_t) + \nu_{ft}, \tag{2.1}$$

where the dependent variable is the log number of patent filings by firm f in year t; n_{ft} denotes the total number of inventors filing patents assigned to firm f in year t; μ_f is a firm fixed effect capturing permanent heterogeneity in patenting; $\mathbb{I}(team_{ft})$ is a dummy equal to one if the share of team patents (those with multiple inventors listed) as a share of total firm patents in year t, $\frac{\text{team patents}_{ft}}{\text{total patents}_{ft}}$, is above the median; and $Post_t$ is an indicator for 2014–2018, the final five years of the sample. In this specification, α

captures the baseline returns to scale in inventor labor n_{ft} for firms that do not heavily use team patenting, $\frac{1}{\gamma^{pre}}$ reflects the role of using team patenting for estimated returns to scale pre-2014, and $\frac{1}{\gamma^{post}}$ captures the incremental increase in these returns for team-based firms during 2014–2018.[6] All other base effects and interactions are included in the regression, but we omit these for the sake of exposition here.

Table 1 reports the results for three different samples: the first column includes all assignees in the patent data, the second column includes all firms with at least two inventors present in year t, and the third column includes public firms only. For public firms, we additionally control for the intangible capital measure of Peters and Taylor (2017), which capitalizes past selling, general administrative expenditures, and past R&D expenses to approximate knowledge capital of firms. This adjustment is important because the first two specifications may overstate returns to scale in inventor labor if inventor labor is correlated with unobserved inputs, particularly with intangible capital.

We find evidence of decreasing returns to scale in inventor labor in the baseline period with $\alpha \in (0.755, 0.824) < 1$. Consistent with the theory, using teams increases returns to scale, with $\frac{1}{\gamma^{pre}} \in (0.0372, 0.0942)$, suggesting an initial value of γ of between 10.6 and 26.9. The positive coefficient on the triple interaction suggests that γ fell in the later period, consistent with a positive productivity shock to team idea production, at least for private firms (the coefficient is positive but not significant for public firms). Based on the coefficient associated with the dummy $Post_t$, we find a general decline in patenting output across firms in the later part of the sample period.[7] Surprisingly, the Peters and Taylor (2017) measure of knowledge capital is negatively correlated with current patenting output of the firm.

[6] We intentionally use these notations to be consistent with the model parameters in Section 3.

[7] This pattern might be related to ideas getting harder to find (Bloom et al. 2017). Whether this effect is stronger or weaker for team based firms is not clear since the coefficient on the interaction of the team indicator with the post period varies depending on the sample.

Table 1: Returns to Scale in Team Production

	(1) All	(2) $n > 1$	(3) Public Firms
$\log(n_{ft})$	0.755***	0.789***	0.824***
	(0.00165)	(0.00276)	(0.00351)
$\log(n_{ft}) \times \mathbb{I}(team_{ft})$	0.0521***	0.0372***	0.0942***
	(0.00233)	(0.00285)	(0.00345)
$\log(n_{ft}) \times \mathbb{I}(team_{ft}) \times Post_t$	0.0345***	0.0106***	0.00545
	(0.00314)	(0.00377)	(0.00575)
$\mathbb{I}(team_{ft})$	-0.696***	-0.630***	-0.742***
	(0.00276)	(0.00466)	(0.00596)
$Post_t$	-0.00678**	-0.0828***	-0.0575***
	(0.00299)	(0.00699)	(0.0111)
$\log(n_{ft}) \times Post_t$	0.0117***	0.0328***	0.0272***
	(0.00187)	(0.00283)	(0.00355)
$\mathbb{I}(team_{ft}) \times Post_t$	-0.0565***	0.0211***	0.00000563
	(0.00483)	(0.00784)	(0.0149)
$\log(k_{ft}^{intan.})$			-0.0156***
			(0.00219)
R-squared	0.704	0.720	0.817
Obs.	1105437	759085	131166
Firm FE	Yes	Yes	Yes

Notes: Column (1) includes all firm–year observations. Column (2) restricts the sample to firm–year observations with at least two inventors. Column (3) includes public firms with available intangible capital data from Peters and Taylor (2017). See equation (2.1) for details. Standard errors are reported in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Fact 4: Patent output increases with the number of potential collaborators in a firm

For each assignee in the USPTO, we count the total number of inventors each year as the number of potential collaborators in a firm. We see how the number and output of patents in a firm depend on the number of collaborators in the firm.

First, for each assignee in a given year, we calculate patents per potential collaborator and then compute the average across pairs for each value of potential collaborators. Figure 2 presents a scatterplot of the average patent count against the number of potential collaborators. The figure shows that firms with a larger pool of potential

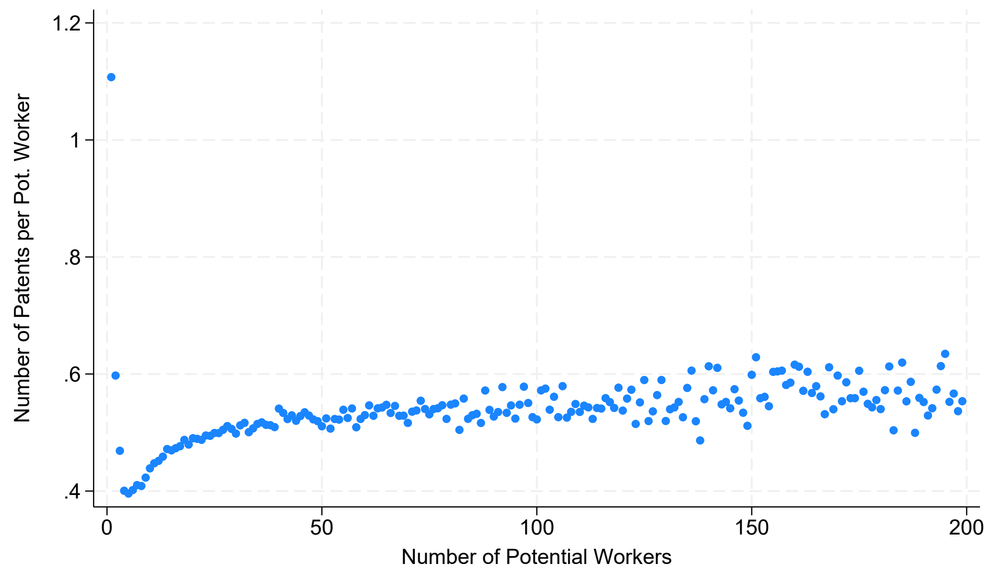

Figure 2: Patent Numbers and Potential Collaborators

Notes: The figure shows a scatterplot of the number of potential workers (x-axis) against the average number of patents per potential worker (y-axis).

collaborators tend to produce more patents, even after normalizing by the number of potential workers.

Next, we compute five-year forward citations per potential worker for each firm in a given year and then take the average across firm-year pairs for each number of potential collaborators. Figure 3 presents the corresponding scatterplot, showing that firms with more potential collaborators also receive more citations per collaborator. This pattern remains robust with using adjusted citations—filtered by CPC technology group-year fixed effects—as shown in Figure 4.[8]

Fact 5: Larger firms have a larger share of team patents

The formation of teams varies systematically with firm size. Defining firm size by the number of inventors, we estimate the following regression to examine the relationship

[8]The adjustment accounts for potential effects driven by time trends or shifts in technology composition, which may arise from differences in citation distributions across technology classes or over time.

Figure 3: Patent Citations and Potential Collaborators

Notes: The figure shows a scatterplot of the number of potential workers (x-axis) against the average citations per potential worker (y-axis).

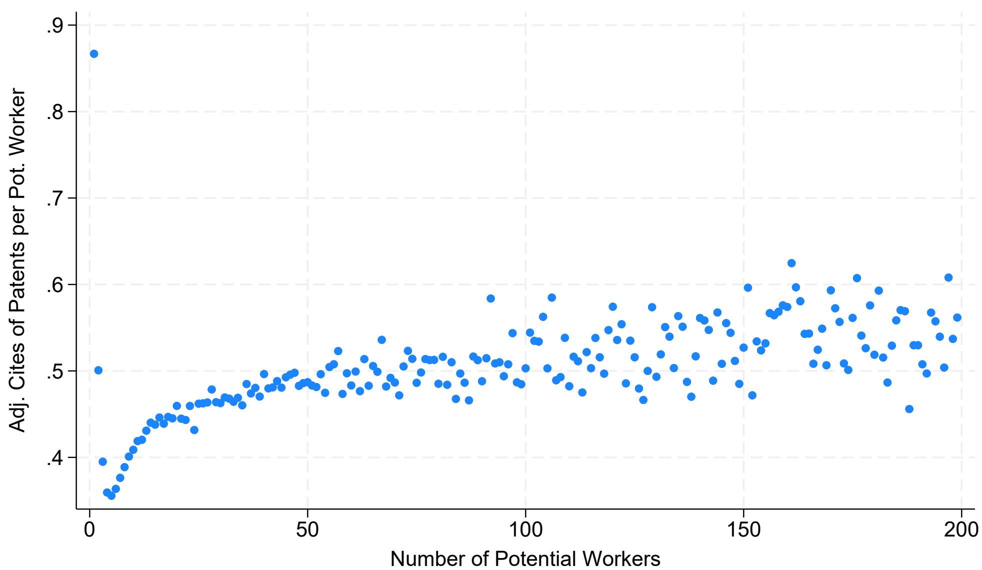

Figure 4: Patent Citations (Adjusted) and Potential Collaborators

Notes: The figure shows a scatterplot of the number of potential workers (x-axis) against the average adjusted citations per potential worker (y-axis). The adjusted citations are calculated by filtering out technology class-year fixed effects.

Table 2: Team Patent Share and Firm Size

	All Firms	Public Firms
Log num. inventors	0.204^{***}	0.122^{***}
	(282.52)	(22.46)
N of Obs.	717,111	74,501
Firm FE	Yes	Yes
Year FE	Yes	Yes
Adj. R^2	0.491	0.375

Note: The table presents the regression result from equation (2.2), where the dependent variable is the share of team patents within a firm. The coefficient of interest is on the log number of inventors in the firm. Firm and year fixed effects are included, respectively. The first column covers all USPTO assignees, and the second column includes Compustat firms only. *** p<0.01, ** p<0.05, * p<0.1

between firm size and the propensity to engage in team patenting:

$$TeamPatShare_{ft} = \eta_f + \gamma_t + \log(Num.Inventors)_{ft} + \epsilon_{ft} \tag{2.2}$$

where $TeamPatShare_{ft}$ is the share of team patents out of total patents assigned by firm f in year t; $\log(Num.Inventors)_{ft}$ is the log number of inventors within firm f in t; and η_f and γ_t are firm and year fixed effects, respectively.

Table 2 presents the results, where the first column uses all USPTO assignees, and the second column restricts the sample to public firms in Compustat. Across both samples, we find that larger firms exhibit a higher share of team patents than smaller firms. The effect is economically meaningful: a 1% increase in the number of inventors within a firm is associated with a 20.4 pp increase in the share of team patents for USPTO firms, and a 12 pp for public firms.

Fact 6: Patent output in a firm increases in team patenting

We further examine the quality of team innovation using the citation-weighted number of patents at the firm level.[9] To this end, we estimate the following regression:

$$\log(\text{CitationWgtPatents})_{ft} = \eta_f + \gamma_t + \text{TeamPatShare}_{ft} + X_{ft} + \epsilon_{ft}, \tag{2.3}$$

[9]The citations are counted for each patent within a five-year window.

Table 3: Team Patent Share and Citation-weighted Patents

Variable	All Firms	Public Firms
TeamPatShare	0.074^{***}	0.113^{***}
	(20.49)	(7.85)
Log past pats.	0.140^{***}	0.308^{***}
	(74.66)	(20.25)
Log sales	—	-0.019
		(-1.09)
Log emp.	—	0.256^{***}
		(6.64)
Log R&D	—	0.120^{***}
		(7.05)
N of Obs.	717,111	69,193
Firm FE	Yes	Yes
Year FE	Yes	Yes
Adj. R^2	0.535	0.665

Note: The table presents the regression result from equation (2.3), where the dependent variable is the log of citation-weighted patent counts within a firm. The coefficient of interest is on the share of team patents in the firm. Firm and year fixed effects are included, respectively. The first column covers all USPTO assignees, and the second column includes Compustat firms only. For Compustat firms, we include firm sales, employment size, and R&D expenditure as additional controls. *** p<0.01, ** p<0.05, * p<0.1

where $\log(\text{CitationWgtPatents})_{ft}$ is the log of the citation-weighted number of patents of firm f in year t, and TeamPatShare_{ft} is the share of team patents. The vector X_{ft} includes firm-level controls such as sales, employment, and R&D expenses, whenever available. We estimate this regression for both the full sample of USPTO assignees and the subsample of public firms.

The results, reported in Table 3, show a positive association between the share of team patents and the citation-weighted number of patents. Specifically, the estimates suggest that if a firm were to shift from producing all patents solo to producing them entirely in teams, its citation-weighted patent output would increase by 7.4% among USPTO assignees and by 11.3% among public firms in Compustat.

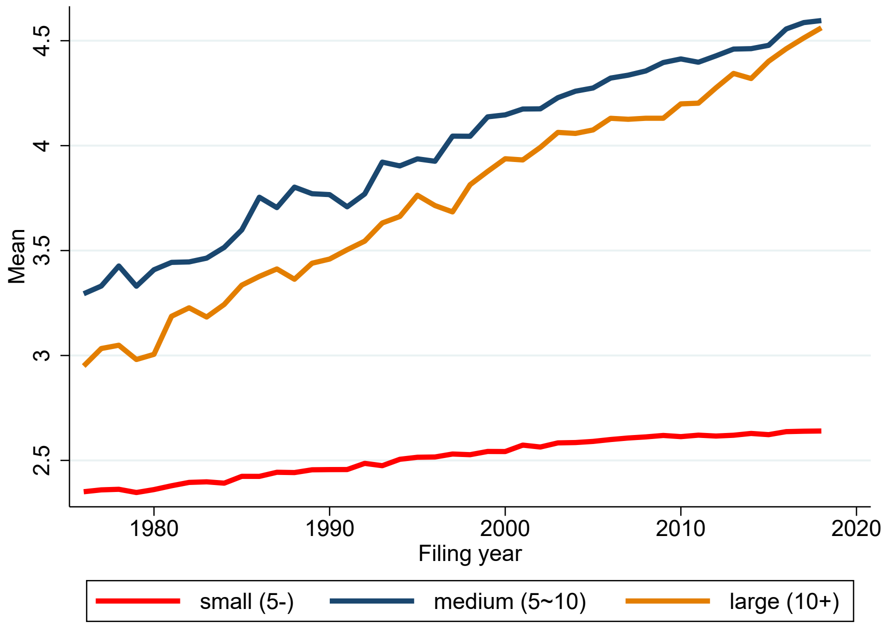

Figure 5: Average Team Sizes in a Firm (By Number of Inventors)

Note: The figure illustrates the average team size in firms, categorized by firm size groups based on the number of inventors. Small firms are those with five or fewer inventors (red line), medium-sized firms are those filing between five and ten inventors (navy line), and large firms are those with more than ten inventors (orange line) in the USPTO data.

Fact 7: Team size rises with firm size, but not one for one

Furthermore, there is heterogeneity in team size across firms of different size. To analyze this, we classify firms into three groups the number of inventors in a year as a proxy for firm size in the USPTO. We define firms as small if they have fewer than five inventors, medium-sized if they have between five and ten inventors, and large if they have more than ten inventors. For each year, we calculate the average team sizes for these three groups and examine their variations over time, as shown in Figure 5. The pattern indicates that medium-sized and large firms consistently have larger teams than small firms.

On top of these, we also investigate different firm characteristics, such as productivity and age. We find consistent results where more productive firms (with productivity defined as the number of patents in USPTO or revenue productivity in Compustat)

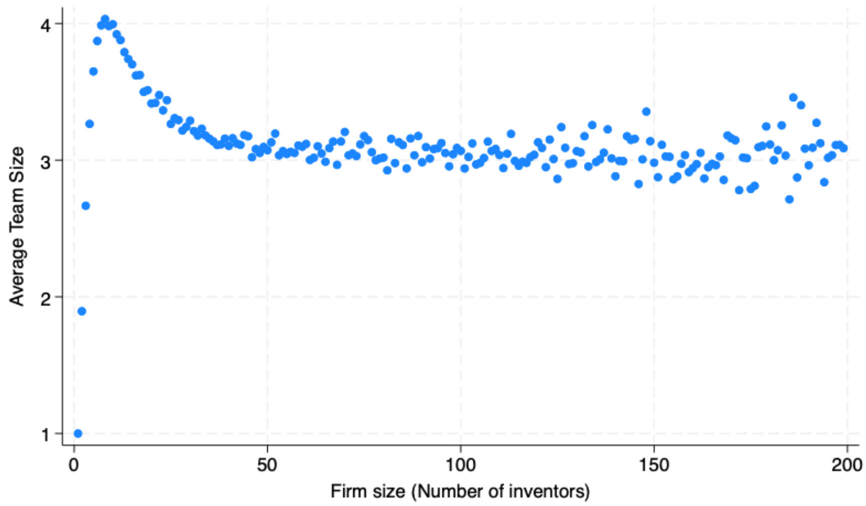

Figure 6: Average Team Size By Number of Inventors

Note: The figure illustrates the average team size within firms, as a function of the total number of patenting inventors at the firm in that year pooling all years in the USPTO data, 1976-2023.

tend to have larger teams compared to less productive firms. Also, we find a similar trend with respect to firm age, where younger firms tend to have smaller firm sizes compared to older firms. See more details in Appendix A.

However, large firms do not choose to form just one team with all the inventors. Instead, the average team size levels off as the firm's number of inventors continues to grow. Figure 6 illustrates this pattern.

Fact 8: Teams are short-lived

We also find the duration over which a team remains active in patenting. We find that most teams are short-lived: 80% of teams patent together once in our sample. This implies that serial teams are unlikely. Also, taken together with Fact 1, this finding suggests that there may be multiple variations or turnover of teams within firms.

Fact 9: Firms are less likely to be single-inventor firms now than in the past

We explore how the fraction of patenting firms with a single inventor in a given year has changed over time. Figure 7 illustrates the decreasing trend of this ratio over time.

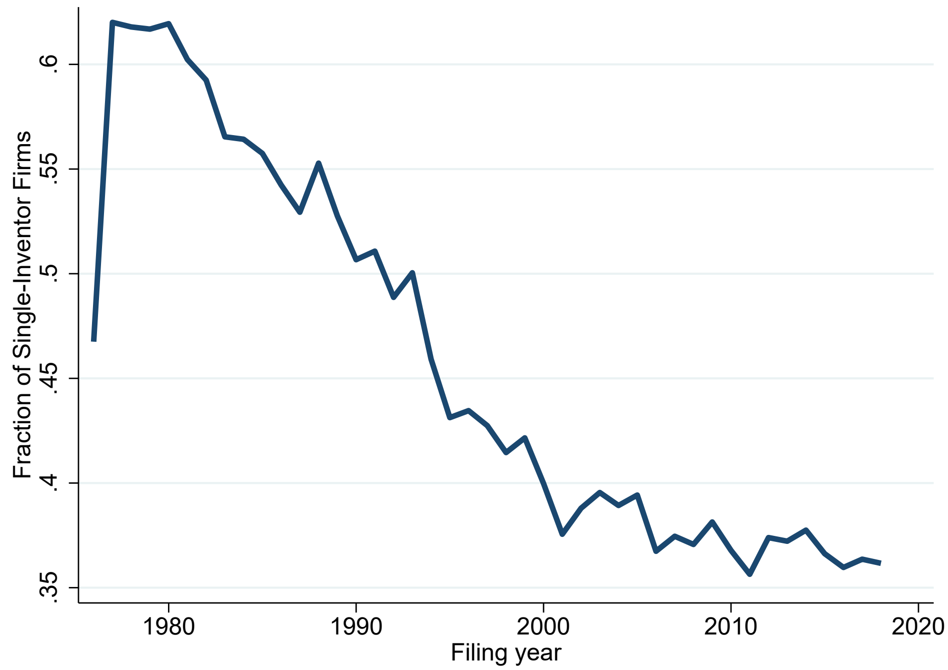

Figure 7: Fraction of Single-Inventor Firms

Note: The figure shows the fraction of patenting firms with a single inventor filing patents each year.

In addition, we find that this trend also holds amongst young firms given the fact that young firms are likely to be single-inventor firms. See more details in Appendix B.

Fact 10: Inventor employment concentration has increased much more than overall employment concentration

A large recent literature analyzes the causes and consequences of rising employment concentration in the United States (Autor et al. 2017; Rossi-Hansberg, Sarte, and Trachter 2020). We document that concentration has also increased among inventors, and to a substantially greater extent than for overall employment. Figure 8 illustrates this pattern. Our model explores how this divergence can arise from changes in optimal innovative firm size driven by an increase in the relative productivity of teams in idea production.

Finally, on top of these findings, we also document a rising trend of team diversity and

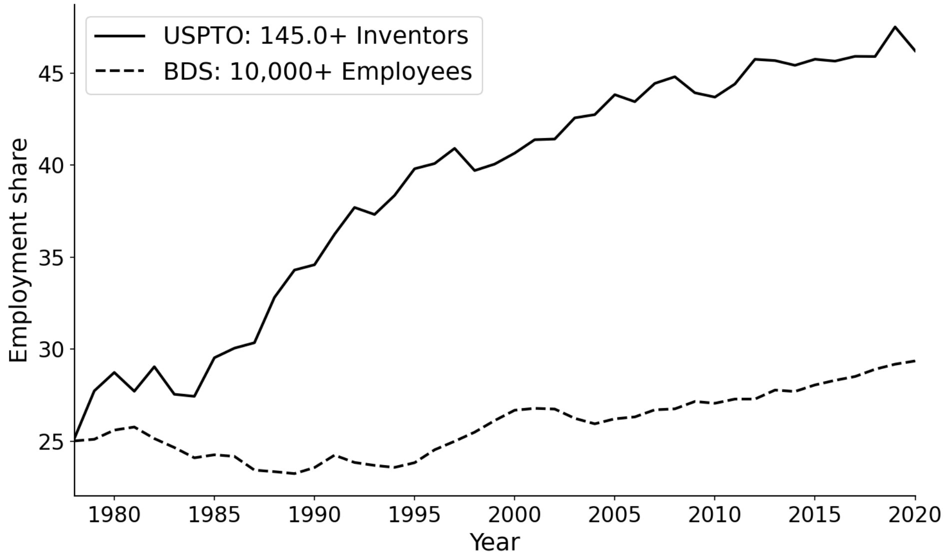

Figure 8: Employment Concentration: Inventors and Overall Employment

Note: The figure plots the employment share of inventors at the largest firms (ranked by the number of inventors), indexed to match the employment share of overall employment at the largest firms (ranked by total employment) in 1978. Inventor employment is measured using patent data in USPTO to count inventors at the firm–year level, while overall employment shares are constructed from the Census Bureau's Business Dynamics Statistics (BDS).

complementarities within teams in the Appendix. See Appendix C and D for details.

3 Simple Model

In this section, we construct a simple static model that provides a micro-foundation for team production and explains the role of firms in shaping it. The model also helps account for the stylized facts observed in the data. In this version, we prioritize parsimony to enhance expositional clarity.

3.1 Team Production

Suppose worker i has the following productivity:

$$
\begin{cases} z_{iit} = \bar{z} & \text{if solo} \\ z_{ijt} \sim P(x_{min}^{team}, \gamma^{team}) & \text{if working with another worker } j, \end{cases} \tag{3.4}
$$

where a worker can either operate independently or collaborate with another worker within a firm. When working alone, productivity is constant at $\bar{z}$. When collaborating, the workers draws a symmetric, pairwise productivity z_{ijt} from a Pareto distribution $P(x_{min}^{team}, \gamma^{team})$.[10]

Now assume that firms can observe all potential collaborations among their workers and assign each worker to the co-worker who yields the highest pairwise productivity. This defines team productivity z_{it}^{team} for worker i within a firm, which is the highest productivity the worker obtains amongst possible collaborations he can form within a firm. The team productivity for worker i in a firm of size n is given by

$$
z_{it}^{team} = \max\{z_{i1t}, z_{i2t}, ..., z_{int}\}. \tag{3.5}
$$

Based on the Pareto distribution, the expected team productivity of worker i in a firm of size n can be derived as

$$
\mathbb{E}(z_{it}^{team}) = x_{min}^{team}\Gamma(1 - \frac{1}{\gamma^{team}})n^{\frac{1}{\gamma^{team}}}. \tag{3.6}
$$

See Appendix E.1 for derivation. Note that this property also holds with a continuum of workers by interpreting n as the mass of workers rather than a discrete count. See details in Appendix E.2. This provides a tighter connection to our model, which assumes a continuum of workers.

The expected team productivity in (3.6) increases with firm size n, the scale parameter x_{min}^{team}, and the inverse of the tail parameter γ^{team} of the Pareto distribution. Figure 9 presents this feature. At the worker level, team productivity exceeds solo productivity

[10] For simplicity, we assume each potential collaboration consists of only two workers. This is consistent with empirical evidence showing that most team inventions are done by small teams.

when the worker belongs to a sufficiently large firm (with size n). Figure 10 illustrates this relationship, showing a firm-size cutoff above which workers attain higher productivity in teams than when working alone. Therefore, workers in larger firms benefit more from collaborating in teams than from working alone. This provides a potential mechanism supporting the empirical evidence of increasing inventor teams at larger firms.

3.2 Firms

Given this structure, we can now characterize the firm's problem. Let firm productivity z_{ft} be determined by the average worker productivity within the firm, combined with an idiosyncratic firm-level shock ε_{ft}. Since workers are homogeneous, the average productivity of workers in a firm coincides with the productivity of any individual worker within that firm. Equation (5.10) characterizes the firm productivity.

$$z_{ft} = \begin{cases} \bar{z}\varepsilon_{ft} & \text{w/ solo production} \\ \underbrace{x_{min}^{team}\Gamma(1-\frac{1}{\gamma^{team}})n_{ft}^{\frac{1}{\gamma^{team}}}}_{=\mathbb{E}(z_{it}^{team})}\varepsilon_{ft} & \text{w/ team production.} \end{cases} \tag{3.7}$$

Firms (f) maximize profits and decide optimal size by taking wages as given.

$$\max_{n_{ft}} z_{ft}n_{ft}^{\alpha} - w_t n_{ft},$$

where n_{ft} denotes firm size and w_t the wage. Note that, under team production, firm productivity increases with firm size, as shown in (3.7). Consequently, switching from solo to team production generates higher returns to scale for firms. This is because the probability of drawing better productivity for forming a team increases with the number of workers within a firm, which provides more potential collaborations.

The remaining steps are standard. The optimal size of firms under solo and team

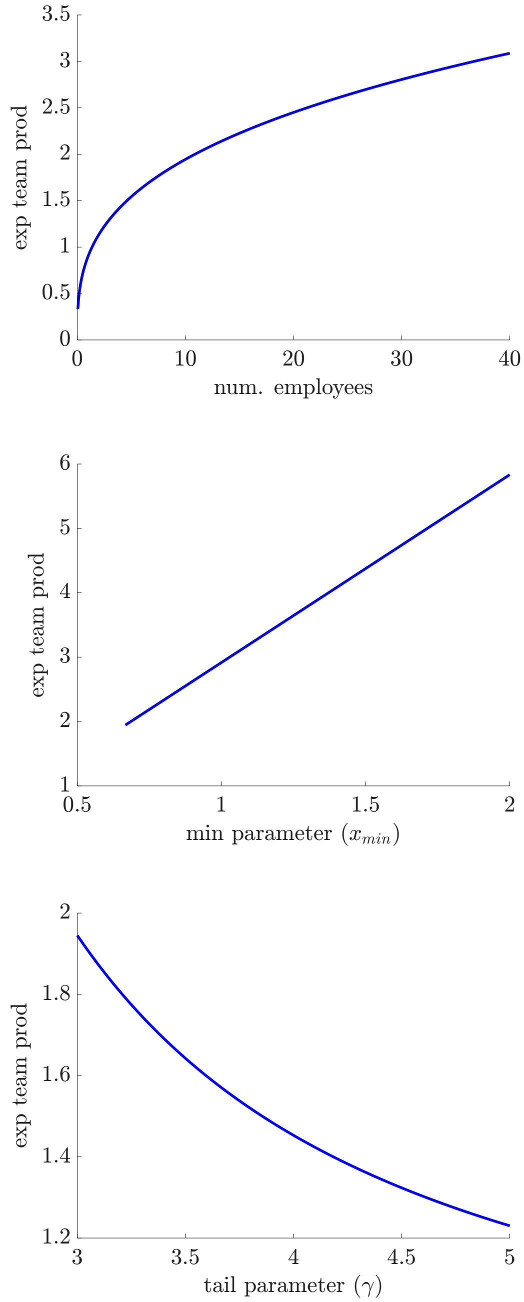

Figure 9: Expected Team Productivity $\mathbb{E}(z_{it}^{team})$

Note: The figure illustrates the expected team productivity by firm size (top) and by different values of scale (middle) and tail (bottom) parameters from the Pareto distribution.

production can be derived, respectively, as follows:

$$n_{ft}^{*} = \begin{cases} \left(\frac{\alpha\varepsilon_{ft}\bar{z}}{w_t}\right)^{\frac{1}{1-\alpha}} & \text{w/ solo production} \\ \left(\frac{(\frac{1}{\gamma^{team}}+\alpha)\varepsilon_{ft}x_{min}^{team}\Gamma(1-\frac{1}{\gamma^{team}})}{w_t}\right)^{\frac{1}{1-(\frac{1}{\gamma^{team}}+\alpha)}} & \text{w/ team production.} \end{cases} \tag{3.8}$$

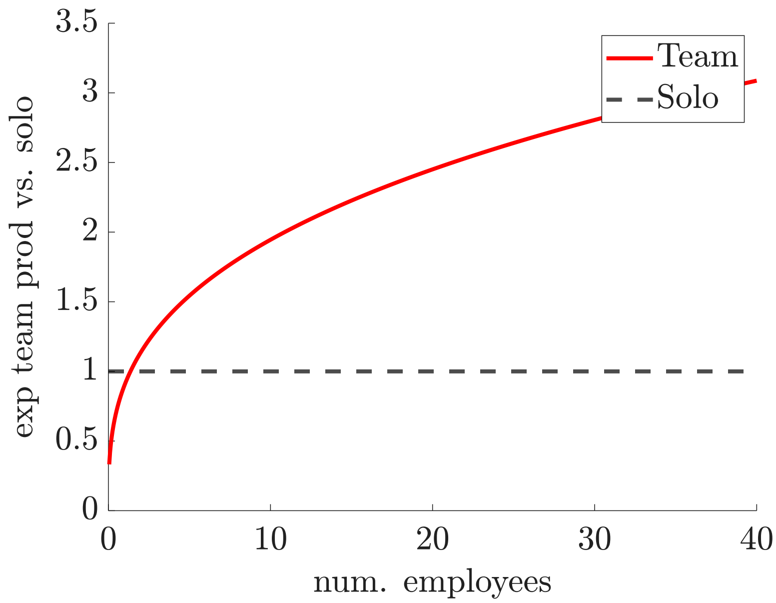

Figure 10: Team vs Solo Productivity

Note: The figure compares worker-level productivity under team and solo production. The red solid line indicates team production, while the black dashed line shows solo production.

3.3 Equilibrium Wage

For now, we assume a frictionless labor market with a fixed labor supply $\bar{N}$ to highlight the fundamental differences between solo and team production and their effects on firm labor demand and labor reallocation.[11] Under this assumption, equilibrium wages are determined by the labor market clearing condition:

$$\int n^*(z_{ft}, \bar{z}; w) dg(z_{ft}) = \bar{N},$$

where $n^*(z_{ft}, \bar{z}; w) \equiv n^*_{ft}$ in (3.8).

[11] In the next section, we extend the analysis to a frictional labor market.

3.4 Model Implications

Suppose there are two types of economies: one with only solo production and the other with team production. This is akin to considering an experiment where initially $\gamma^{team} = \infty$ and $x_{min}^{team} < 1$ so no firms choose team production, and then γ^{team} decreases sufficiently to induce all firms to choose team production. We derive the equilibrium outcomes under each type of economy to study how the rise in the relative productivity of teams affects the equilibrium wage and firm size distribution.

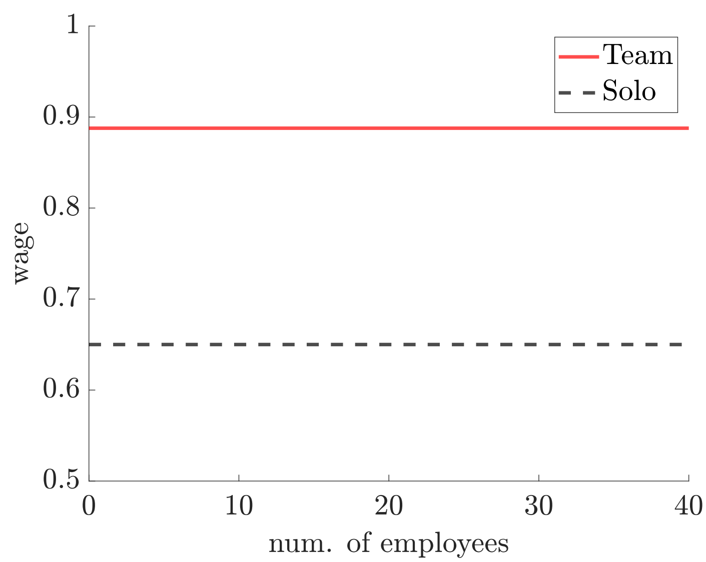

Figure 11: Equilibrium Wage under Team vs Solo Production

Note: The figure shows the equilibrium wages under team and solo production. The red solid line indicates team production, while the black dashed line shows solo production.

Figure 11 presents the equilibrium wages, and Figure 12 shows the firm size distribution, under solo and team production. The red solid line indicates the team production economy and the black dashed line the solo production economy. The equilibrium wage rises with the transition from solo to team production. This is due to having increased returns to scale with team production, which increases labor demand across firms and the equilibrium level of the wage. Second, under team production, the firm size distribution is more skewed toward larger firms. This is also intuitive, as the increased returns to scale are more pronounced for larger firms, which are better able to achieve

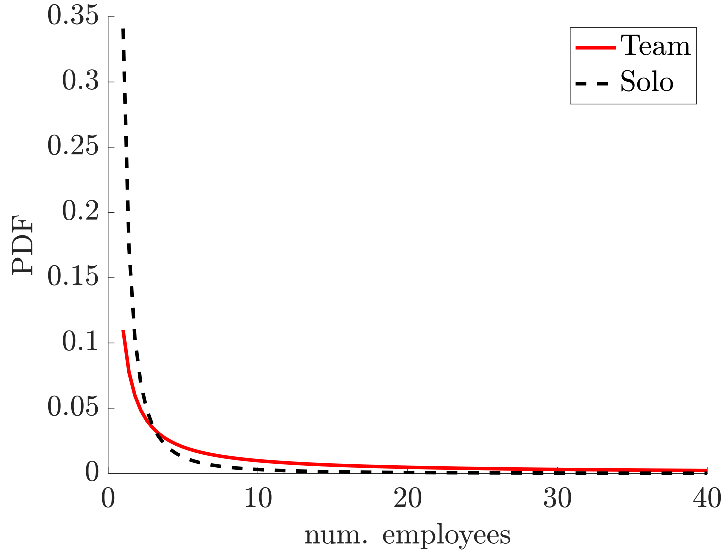

Figure 12: Size Distribution under Team vs Solo Production

Note: The figure shows the firm size distribution under team and solo production. The red solid line indicates team production, while the black dashed line shows solo production.

higher team productivity with a larger set of workers. Although higher wages tend to reduce labor demand across firms, this effect is offset by the increased returns to scale from team production. As a result, switching from solo to team production reallocates labor toward larger firms, consistent with the observed increase in inventor concentration at larger firms in the data (Akcigit and Goldschlag 2023c).

4 Adding Knowledge Spillovers

A key finding of studies of firm-level heterogeneity in research output and research productivity is that small firms are more likely to make breakthrough innovations that have spillover effects to other firms (Rosen 1991; Olmstead-Rumsey 2022; Akcigit and Kerr 2015). So far, the change in the firm size distribution in response to the rise of teams was fully efficient. We next introduce a form of knowledge spillovers across firms that capture the special role of small firms in creating knowledge spillovers, following (Chikis, Kleinman, and Prato 2025).

We modify the firm problem so that in addition to idiosyncratic productivity, each firm's output also depends on a particular average of other firms' productivities z_f and size n_f so that the firm's becomes:

$$\max_{n_f} \bar{z}^{spillover} z_f n_f^{\alpha} - w n_f$$

given:

$$\bar{z}^{spillover} = \left(\int_0^1 z_f^{1-\beta} n_f^{\beta} df \right)^{\theta}.$$

The parameter $\theta > 0$ governs the importance of knowledge spillovers for firm output. The parameter $\beta \in [0, 1]$ governs how much firm size (n_f) matters compared to firm productivity (z_f) for generating spillovers. In the extreme cases, if $\beta = 1$, only the total number of inventors matters for spillovers, not their allocation to particular firms. If $\beta = 0$, the allocation of workers also does not matter because all that matters is the average productivity of firms regardless of their size. For intermediate cases, β governs the concavity of the spillover in labor, with more concavity (lower β) making the effects of misallocation of workers across firms more pronounced.

4.1 Planner's Problem

A social planner would allocate inventors across firms to maximize total output, internalizing spillovers:

$$\max_{n_{ft}} \int_0^1 \bar{z}^{spillover} z_{ft} n_{ft}^{\alpha} df$$

subject to

$$\int_0^1 n_{ft} df = \bar{N},$$

$$\bar{z}^{spillover} = \left(\int_0^1 z_f^{1-\beta} n_f^{\beta} df \right)^{\theta}.$$

We define a firm-level wedge τ_{ft} as the gap between the social marginal product of labor at firm f and the private marginal revenue product perceived by the firm:

$$MP_{ft}^{SP} = (1+\tau_{ft})MP_{ft}^{CE}.$$

The general expression for this wedge, defined in terms of the private and social marginal products, is given by:

$$\tau_{ft} = \frac{\theta\beta}{\alpha}\frac{\int z_{ft}n_{ft}^{\alpha}\,df}{\left(\int z_{ft}^{1-\beta}n_{ft}^{\beta}\,df\right)}\underbrace{z_{ft}^{-\beta}n_{ft}^{\beta-\alpha}}_{\text{Firm specific wedge}}.$$

The firm-level wedge comprises a common component that depends on parameters, aggregate output, and spillovers, as well as a firm-specific component that varies across firms. Evaluating the wedges at competitive equilibrium labor demands, in the spirit of Hsieh and Klenow (2009), allows us to express them as the sum of a common term and a firm-specific term that is a function of idiosyncratic productivity z_{ft}.

$$\tau_{ft} = \frac{\theta\beta}{\alpha}\left(\frac{\gamma^{f}}{\gamma^{f}-\frac{1-\alpha+\alpha\beta}{1-\alpha}}\right)^{\theta-1}\left(\frac{\gamma^{f}}{\gamma^{f}-\frac{1}{1-\alpha}}\right)^{1-\beta\theta} z_{ft}^{\frac{\alpha(\beta-1)}{1-\alpha}}.$$

Since $\beta < 1$ and $\alpha \in (0,1)$, the wedges are decreasing in firm productivity—and hence in employment size, given their positive relationship in competitive equilibrium. In this setting, small firms are inefficiently small, implying misallocation of inventor labor across firms. Figure 13 illustrates the shape of the wedge across firms of different sizes.

4.2 Team Production and Misallocation

We now characterize how firm-level wedges change when the economy fully switches to team production. Under team production, the wedges are given by:

$$\tau_{ft} = \frac{\theta\beta\Gamma(1-\frac{1}{\gamma^{team}})^{-\beta}}{\alpha+\frac{1}{\gamma^{team}}}\frac{\int z_{ft}n_{ft}^{\alpha+\frac{1}{\gamma^{team}}}\,df}{\left(\int z_{ft}^{1-\beta}n_{ft}^{\beta}\,df\right)}\underbrace{z_{ft}^{-\beta}n_{ft}^{\beta-\alpha-\frac{1}{\gamma^{team}}}}_{\text{Firm specific wedge}}.$$

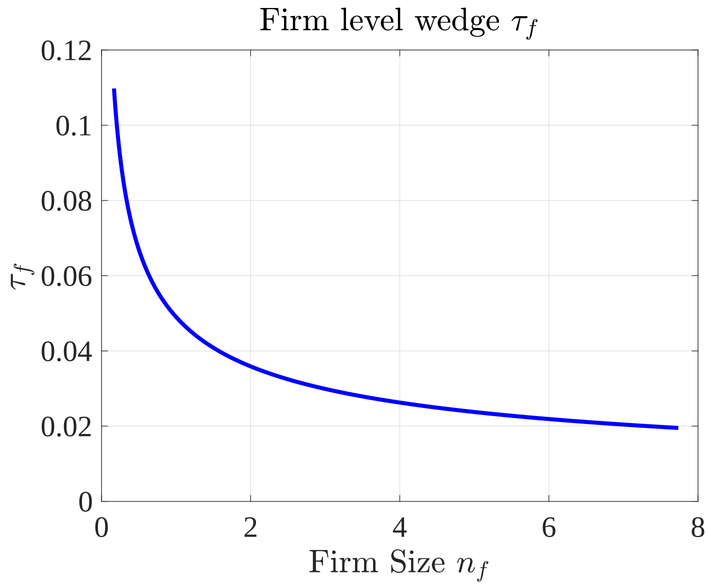

Figure 13: Firm-level Wedges under Solo Production

The increase in returns to scale raises wedges for all firms, as potential output is higher and spillovers are stronger than in the solo economy. At the same time, higher returns to scale make the firm size distribution more skewed. With a production function that is concave in inventor labor and a spillover function that is also concave, this increased skewness exacerbates misallocation across firms. Figure 14 illustrates both effects.

These results suggest an increased scope in the team production economy for size-dependent R&D subsidies favoring small firms. This may seem counterintuitive at first, but the intuition comes from general equilibrium spillovers when the economy switches to team idea production: the wage increases and small firms get smaller, as in the frictionless model. This makes them even further from their optimal employment size given the knowledge spillover function. Figure 15 illustrates the resulting gap between social and private marginal products in the two economies.

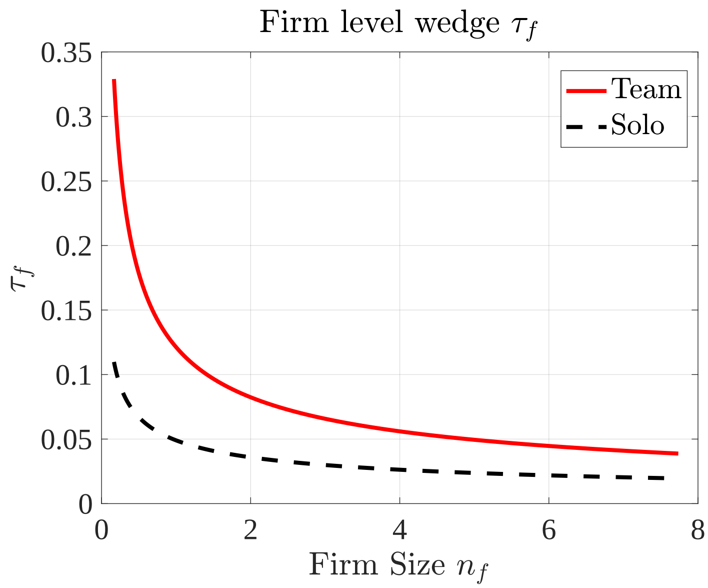

Figure 14: Firm-level Wedges: Solo vs. Team

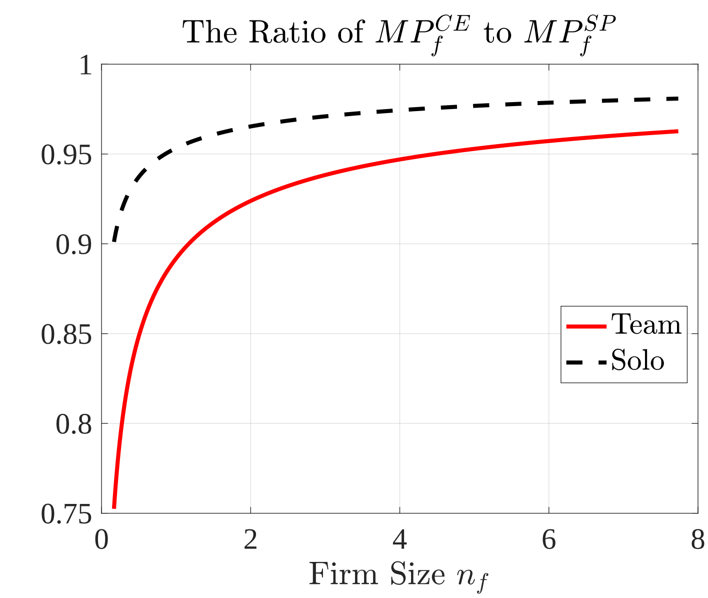

Figure 15: The Gap between Social and Private Marginal Products of Labor: Solo vs. Team

5 Full-fledged Model

In this section, we further explore how team production interacts with labor market frictions and generates additional implications for wage differentials and inventor mobility. We extend the baseline model to incorporate the frictional nature of the inventor labor market (Akcigit and Goldschlag 2023c).[12] To do so, we extend Elsby and Michaels (2013) to include the team production framework in (5.10).

5.1 Set-up

The main setup of the model follows Elsby and Michaels (2013). The model is in discrete time and has an infinite horizon, in which firms use labor to produce (innovative) output. We focus on the steady-state equilibrium of the model. To simplify notation, we omit the time subscript whenever possible, while using the superscript $'$ to denote variables in the next period $(t+1)$, and the subscript -1 to denote variables from the previous period $(t-1)$.

Workers are homogeneous and can only search from the unemployment pool. A mass of labor force is fixed to $\bar{N}$. Firms are multi-worker and face search frictions. Each firm must pay a constant vacancy cost c. A mass of firms is normalized to one, and there is no firm entry and exit.

Let U denote the total number of unemployed workers and V the total number of vacancies in the economy. Search is assumed to be random for both workers and firms. Labor market frictions are captured by a matching function, which determines the number of successful matches,

$$M(U,V) = \mu U^{\phi} V^{1-\phi}. \tag{5.9}$$

Since not all searching workers find jobs and not all vacancies are filled, the matching function implies a degree of frictions in the labor market. It also characterizes job

[12] Akcigit and Goldschlag (2023c) find that large firms pay 12% higher wages to a given inventor, and inventor mobility across firms has declined since 2000.

finding rate for workers, $f(\theta) = \frac{M}{U}$ and job filling rate for firms, $q(\theta) = \frac{M}{V}$, where labor market tightness is defined as $\theta = \frac{V}{U}$. Thus, the total number of hires in the economy is constrained by the matching function.

5.2 Firms

Firms operate with the following production function:

$$y = \varphi z n^{\alpha}, \quad z = \begin{cases} \bar{z}\varepsilon & \text{w/ solo production} \\ x_{min}^{team}\Gamma(1 - \frac{1}{\gamma^{team}})n^{\frac{1}{\gamma^{team}}}\varepsilon & \text{w/ team production} \end{cases}, \quad \varepsilon' \sim G(\varepsilon', \varepsilon), \tag{5.10}$$

where $\varphi \sim P(x_{min}^{\varphi}, \gamma^{\varphi})$ is a time-invariant productivity type, drawn from Pareto distribution, and ε' follows a persistent and mean-reverting process.[13,14] Specifically, we follow Elsby and Michaels (2013) and assume that the idiosyncratic shock ε evolves according to:

$$\varepsilon' = \begin{cases} \varepsilon & \text{with probability } 1 - \lambda, \\ \varepsilon' \sim P(x_{min}^{\varepsilon}, \gamma^{\varepsilon}) & \text{with probability } \lambda, \end{cases} \tag{5.11}$$

which specifies the process $G(\varepsilon', \varepsilon)$, where $\lambda < 1$ captures the persistence of the shock, and innovations are drawn from a Pareto distribution with scale and tail parameters $(x_{min}^{\varepsilon}, \gamma^{\varepsilon})$. We also assume $\alpha < 1$, where the production function exhibits decreasing returns to scale.

For now, we assume that firms operate either with solo or team production, which is exogenously taken as given in the economy. Under each solo or team production, firms realize their productivity shock ε each period and make decision to employ workers. They can grow by hiring workers or contract by separating workers or stay inactive given the existence of constant vacancy cost. If firms want to hire workers, they need

[13] Having a permanent firm type allows us to match the firm size distribution in the data more accurately, without materially affecting the derivation of the optimal employment decision or the equilibrium outcomes.

[14] Here, we assume that firms make an ex-ante decision based on expected team productivity with their choice of n.

to post vacancies v by paying the per-vacancy cost c. The vacancies are matched with unemployed workers through the matching function (5.9). After the matching process, firms perform production and wage setting.

A firm's value function (given type φ) is characterized as follows:

$$\Pi(n_{-1}, \varepsilon, \varphi) = \max_{n,v} \left\{ y(n, \varepsilon, \varphi) - w(n, \varepsilon, \varphi)n - cv + \beta \int \Pi(n, \varepsilon', \varphi) dG(\varepsilon'|\varepsilon) \right\}, \quad (5.12)$$

where $y(n, \varepsilon, \varphi)$ denotes the production function given in (5.10), and $w(n, \varepsilon, \varphi)$ is the wage determined through bargaining for a firm of size n and productivity ε with type φ.[15] Note that, due to search frictions in the labor market, the hiring of workers is subject to the following constraint:

$$n = n_{-1} + vq(\theta), \quad \text{if } n > n_{-1}. \quad (5.13)$$

Rephrasing (5.12) with (5.13) gives

$$\Pi(n_{-1}, \varepsilon, \varphi) = \max_{n} \left\{ y(n, \varepsilon, \varphi) - w(n, \varepsilon, \varphi)n - c\frac{(n - n_{-1})}{q(\theta)}\mathbb{I}^{+} + \beta \int \Pi(n, \varepsilon', \varphi) dG(\varepsilon'|\varepsilon) \right\}, \quad (5.14)$$

where $\mathbb{I}^{+}$ indicates a dummy for hiring firms ($n > n_{-1}$). Note that the value function exhibits a kink at $n = n_{-1}$, which generates an inaction range in which firms optimally choose to remain inactive—neither hiring nor separating workers—for a certain range of productivity ε and initial size n_{-1}.

5.3 Workers

The value function of unemployed workers is as follows

$$\Gamma = b + \beta \mathbb{E}_{(n', \varepsilon', \varphi)}[(1 - f(\theta))\Gamma' + f(\theta)W(n', \varepsilon', \varphi)], \quad (5.15)$$

[15] The bargaining protocol will be specified in the next subsection.

where b is unemployment benefits or leisure value to a worker, $W(n', \varepsilon', \varphi)$ is the value of being employed at a firm of size n' and productivity ε' with type φ, and $\mathbb{E}_{(n',\varepsilon',\varphi)}$ denotes the expectation over the distribution of $(n', \varepsilon', \varphi)$ among firms posting vacancies.

The value of employed workers at a firm of size n, productivity ε, and type φ is given by

$$W(n, \varepsilon, \varphi) = w(n, \varepsilon, \varphi) + \beta \mathbb{E}_{\varepsilon'|\varepsilon}[(1 - s)W(n', \varepsilon', \varphi) + s\Gamma'], \tag{5.16}$$

where $w(n, \varepsilon, \varphi)$ is the bargained wage at the firm, s is separation probability, which is endogenously determined by the firm's problem (5.14), and $\mathbb{E}_{\varepsilon'|\varepsilon}$ denotes the expectation over ε' given ε. Note that if the worker is laid off from the firm, she returns to the unemployment pool and receives the value Γ'.

5.4 Wage Setting

We follow Stole and Zwiebel (1996) and Elsby and Michaels (2013) to specify a wage bargaining protocol in the context of multi-worker firms with decreasing returns to scale. As emphasized by Elsby and Michaels (2013), the marginal surplus of a worker within a firm differs between the marginal worker and infra-marginal workers when production exhibits decreasing returns to scale. To account for this, we adopt the bargaining protocol from Stole and Zwiebel (1996), which implements Nash bargaining over the marginal surplus of a match.[16]

The marginal surplus of a worker to a firm (after the firm's employment decision has been made) is given by:

$$J(n, \varepsilon, \varphi) = y(n, \varepsilon, \varphi) - w(n, \varepsilon, \varphi) - w_n(n, \varepsilon, \varphi)n + \beta \int \frac{\partial \Pi(n, \varepsilon', \varphi)}{\partial n} dG(\varepsilon'|\varepsilon), \tag{5.17}$$

where the vacancy cost is already sunk at this point.

Similarly, the marginal surplus for a worker employed at a firm of size n and produc-

[16] Conceptually, this corresponds to the firm negotiating with each worker in turn, where the breakdown of a negotiation with any individual worker triggers the renegotiation of wages with all remaining workers.

tivity ε is

$$W(n, \varepsilon, \varphi) - \Gamma,$$

where $W(n, \varepsilon, \varphi)$ and Γ are defined in (5.16) and (5.15), respectively.

Wages are determined through the Nash bargaining between a firm and its workers over the marginal surplus as follows:

$$(1-\eta)(W(n, \varepsilon, \varphi) - \Gamma) = \eta J(n, \varepsilon, \varphi), \tag{5.18}$$

where η denotes the worker's bargaining power. Thus, the bargained wage between a firm of size n, productivity ε, and type φ and workers is the solution of the following differential equation:

$$w(n, \varepsilon, \varphi) = \eta\Big[y(n, \varepsilon, \varphi) - w_n(n, \varepsilon, \varphi)n + \beta f(\theta)\frac{c}{q(\theta)}\Big] + (1-\eta)b. \tag{5.19}$$

5.5 Optimal Employment Decision

To obtain the firm's optimal employment decision, we revisit (5.14) and get the first-order condition with respect to n. Combining it with the wage equation (5.19) yields the firm's optimal employment decision as follows:

$$n(n_{-1}, \varepsilon, \varphi) = \begin{cases} R_v^{-1}(\varepsilon|\varphi), & \text{if } \varepsilon > R_v(n_{-1}, \varphi) \\ n_{-1}, & \text{if } \varepsilon \in [R(n_{-1}, \varphi), R_v(n_{-1}, \varphi)] \\ R^{-1}(\varepsilon|\varphi), & \text{if } \varepsilon < R(n_{-1}, \varphi), \end{cases} \tag{5.20}$$

where the functions $R_v(\cdot)$ and $R(\cdot)$ are determined by the followings; if firms operate in solo production,

$$(1-\eta)\Big[\frac{\varphi \bar{z} R_v(n, \varphi)\alpha n^{\alpha-1}}{1-\eta(1-\alpha)} - b\Big] - \eta\beta f(\theta)\frac{c}{q(\theta)} + \beta\int\frac{\partial\Pi(n, \varepsilon', \varphi)}{\partial n}dG(\varepsilon'|R_v(n, \varphi)) \equiv \frac{c}{q(\theta)}, \tag{5.21}$$

$$(1-\eta)\Big[\frac{\varphi\bar{z}R(n,\varphi)\alpha n^{\alpha-1}}{1-\eta(1-\alpha)}-b\Big]-\eta\beta f(\theta)\frac{c}{q(\theta)}+\beta\int\frac{\partial\Pi(n,\varepsilon',\varphi)}{\partial n}dG(\varepsilon'|R(n,\varphi))\equiv 0; \tag{5.22}$$

and if firms work with team production,

$$\begin{aligned}
&(1-\eta)\Big[\frac{\varphi x_{min}^{team}\Gamma(1-\frac{1}{\gamma^{team}})R_v(n,\varphi)(\alpha+\frac{1}{\gamma^{team}})n^{\alpha+\frac{1}{\gamma^{team}}-1}}{1-\eta(1-\alpha-\frac{1}{\gamma^{team}})}-b\Big]-\eta\beta f(\theta)\frac{c}{q(\theta)}\\
&+\beta\int\frac{\partial\Pi(n,\varepsilon',\varphi)}{\partial n}dG(\varepsilon'|R_v(n,\varphi))\equiv\frac{c}{q(\theta)}, \qquad (5.23)\\
&(1-\eta)\Big[\frac{\varphi x_{min}^{team}\Gamma(1-\frac{1}{\gamma^{team}})R(n,\varphi)(\alpha+\frac{1}{\gamma^{team}})n^{\alpha+\frac{1}{\gamma^{team}}-1}}{1-\eta(1-\alpha-\frac{1}{\gamma^{team}})}-b\Big]-\eta\beta f(\theta)\frac{c}{q(\theta)}\\
&+\beta\int\frac{\partial\Pi(n,\varepsilon',\varphi)}{\partial n}dG(\varepsilon'|R(n,\varphi))\equiv 0. \qquad (5.24)
\end{aligned}$$

There are several things to note from the decision rule (5.20). First, there exist two productivity cutoffs $R_v(n_{-1},\varphi)$ and $R(n_{-1},\varphi)$, given initial size n_{-1} and type φ. The first cutoff, $R_v(n_{-1},\varphi)$, marks the productivity level above which firms start posting vacancies and hiring workers. The second cutoff, $R(n_{-1},\varphi)$, identifies the productivity below which firms start separating workers to reduce their size. For productivity levels between these two cutoffs, firms stay inactive without hiring or separating workers.

5.6 Aggregation

We define aggregate steady state equilibrium and pin down aggregate variables such as aggregate unemployment U, vacancies V, and labor market tightness θ.

First, based on the shock process in (5.11), we define the steady-state cumulative distribution of employment. Let $\tilde{G}(\varepsilon)$ denote the cumulative distribution function of the Pareto distribution of innovations in (5.11). In steady-state equilibrium, for a given level of employment n, the inflow into this state must equal the outflow from it. Denoting by $H(n,\varphi)$ the steady-state cumulative distribution function of employment conditional on type φ, this condition is expressed as

$$H(n,\varphi)\big(1-\tilde{G}(R_v(n,\varphi))\big) = \big(1-H(n,\varphi)\big)\tilde{G}(R(n,\varphi)), \tag{5.25}$$

where the left-hand side represents the inflow into size n, and the right-hand side represents the outflow from size n, given firm type φ. Rearranging (5.25), we can express the steady-state cumulative distribution function of employment as

$$H(n,\varphi) = \frac{\tilde{G}(R(n,\varphi))}{1-\tilde{G}(R_v(n,\varphi))+\tilde{G}(R(n,\varphi))}. \tag{5.26}$$

Next, integrating (5.26) across φ, we obtain the steady-state unconditional cumulative distribution function of employment $\tilde{H}(n)$ as follows:

$$\tilde{H}(n) = \int H(n,\varphi)dF(\varphi), \tag{5.27}$$

where $F(\varphi)$ denotes the cumulative distribution function of the Pareto distribution of firm type.

Based on the steady-state distribution, we define the Job Creation condition as follows:

$$U(\theta) + \int n d\tilde{H}(n;\theta) = \bar{N}, \tag{5.28}$$

which ensures that the total number of unemployed and employed workers equals the total workforce $\bar{N}$.[17]

Furthermore, we can derive the Beveridge Curve from the evolution of unemployment over time, where the inflow into the unemployment pool (the total number of separations) subtracting the outflow from the unemployment pool (the total hires of unemployed workers) characterizes the change in the unemployment stock between t and $t+1$. This follows

$$U' - U = S(\theta) - f(\theta)U,$$

[17]This is the analog of the job creation condition in the standard Mortensen and Pissarides (1994) model.

and $U' = U$ in the steady-state equilibrium. As a result the Beveridge Curve is given by

$$U(\theta) = \frac{S(\theta)}{f(\theta)}, \tag{5.29}$$

where $S(\theta)$ is the total number of separations, which can be obtained by

$$S(\theta) = \lambda \int \int (1 - H(n, \varphi))\tilde{G}(R(n, \varphi))dndF(\varphi).$$

6 Quantitative Analysis

6.1 Calibration

We set the baseline economy to solo production and calibrate the model following Elsby and Michaels (2013). The time period is set to one week.

First, we set the matching elasticity, ϕ, to 0.6, following the estimates in Petrongolo and Pissarides (2001). The matching efficiency parameter, μ, is set to 0.129 by targeting a mean labor market tightness of $\theta = 0.72$, as reported in Pissarides (2009). We target a labor share of 0.72 to pin down α, based on Gomme and Rupert (2007), and use the quarterly interest rate to set β to 0.999. The leisure value, b, is chosen to match the weekly unemployment inflow rate of $s = 0.0078$, consistent with Shimer (2012).

Worker hiring costs are targeted to be 14% of the quarterly worker wage, following Silva and Toledo (2013), to calibrate the vacancy cost c. We use the elasticity of the wage with respect to output per worker from Pissarides (2009) to set $\eta = 0.443$. The size of the labor force, $\bar{N}$, is chosen to match a mean unemployment rate of 6.5%, which corresponds to a weekly job-finding rate of 0.1125. We set the arrival rate of innovations in productivity shock, λ, to match the share of inactive firms in the Longitudinal Business Database (LBD, henceforth). Inactive firms are defined as those whose log employment changes by less than 5% in a year.

Lastly, we set the lower bound $x^{\varepsilon}_{\min}$ of firm productivity, ε, to normalize the mean pro-

Table 4: Parameterization

Parameter	Definition	Value	Source
ϕ	Matching elasticity	0.600	Petrongolo and Pissarides, 2009
μ	Matching efficiency	0.129	Pissarides, 2009
α	$F(n) = n^{\alpha}$	0.601	Labor share = 0.72
β	Discount factor	0.999	Quarterly interest rate = 0.012
b	Value of leisure	0.387	Mean inflow rate = 0.0078
c	Flow vacancy cost	0.133	Hiring cost = 14% quarterly wage
η	Worker bargaining power	0.443	Cyclicality of wage in baseline MP
$\bar{N}$	Labor force	18.65	Mean job-finding rate = 0.1125
λ	Arrival rate of x	0.043	LBD data: $\Pr(\Delta \ln n < \|0.05\|) = 0.415$
x^{ε}_{min}	Lower bound of ε	1.000	Normalization
γ^{ε}	Shape parameter of the dist. of ε	0.250	LBD data: $\sigma(\Delta \ln n) = 0.416$
x^{φ}_{min}	Lower bound of φ	1.218	Mean employment = 17.38
γ^{φ}	Shape parameter of the dist. of φ	1.009	Minimum employment = 1

Table 5: Baseline Economy (under Solo Production)

Output	11.16
Tightness θ	0.70
Unemployment rate, %	6.55
Wage rate	0.46
Separation rate, weekly %	0.78
$\mathbb{E}(n)$	17.25
Emp. share, small firms, %	9.59
Emp. share, large firms, %	41.07
Growth rate, small prod. firms, %	2.74
Share of firms with 0 growth, %	39.02
Wage prem., large firms, %	0.88

ductivity to one, and the shape parameter, γ^{ε}, to match the dispersion in employment growth in LBD, following Elsby and Michaels (2013). Also, the minimum value $x^{\varphi}_{\min}$ of the firm time-invariant type, φ, is set to match the minimum establishment-level employment of one worker, and its shape parameter, γ^{φ}, is chosen to match a mean establishment size of 17.38, based on Small Business Administration (SBA) data.

The parameterization is summarized in Table 4, and the resulting calibrated economy is presented in Table 5.

Table 6: Counterfactual Exercise

	Solo Production (Baseline)	Team Production
Output	11.16	11.64
Tightness θ	0.70	0.89
Unemployment rate, %	6.55	5.81
Wage rate	0.46	0.48
Separation rate, weekly %	0.78	0.76
$\mathbb{E}(n)$	17.25	17.36
Emp. share, small firms, %	9.59	9.51
Emp. share, large firms, %	41.07	43.31
Growth rate, small prod. firms, %	2.74	2.26
Share of firms with 0 growth, %	39.02	41.35
Wage prem., large firms, %	0.88	0.90

6.2 Counterfactual Exercise: from Solo to Team Production

In this section, we perform a counterfactual exercise in which the production function shifts from solo to team-based production. This exercise captures the rise in team patenting and the declining trend of solo innovation in recent periods. This may reflect a structural change in technology development, potentially arising from the increasing complexity of knowledge—making innovation more difficult for individuals to achieve—or from gains in team productivity. By shifting the production function from solo to team production, we model how these structural changes in technology influence firm behavior and aggregate outcomes in the economy.

The results are summarized in Table 6. The first column reports the baseline economy with solo production, while the second column presents the counterfactual economy with a transition to team production. The top panel displays the aggregate labor market outcomes. Moving from solo to team production raises total output and lowers the unemployment rate. Labor market tightness and the average wage also increase, reflecting the higher returns to scale under team production, which boosts labor demand and market tightness. The bottom panel reports the firm-level outcomes. In the counterfactual economy, average firm size rises, accompanied by a reallocation of labor across firms. In particular, team production leads to a higher concentration of inventors at larger firms. The employment share of large firms—defined as those with 1,000 or more employees—increases, while the share of small firms with 5 or fewer employees declines. Given the rise of labor market tightness, which offsets the effect of increased returns to scale under team production, the average growth rate at small firms with high productivity declines as well. Furthermore, the rise in labor market tightness widens the inaction range, as reflected in the higher share of firms exhibiting zero employment growth. Lastly, the size–wage premium increases with team production. The higher returns to scale mitigate differences in the marginal product of labor across firms of different sizes, reducing the lower marginal product typically observed at larger firms. Also, the rise in returns to scale disproportionately increases labor demand at larger firms, which increases their willingness to share a larger portion of the marginal surplus from a match.

7 Conclusion

Understanding inventor teams has become increasingly important as a growing share of ideas is produced by teams rather than individuals. Inventors do not collaborate randomly; instead, they disproportionately form teams with colleagues within the same firm. We document new empirical evidence showing that team production is associated with less-decreasing returns to scale in inventor inputs, and that larger, more productive, and older firms are more likely to form inventor teams, assemble larger teams, and create teams with more diverse skill sets.

Our novel theoretical framework explains these patterns through a simple mechanism: when the set of potential coworkers is larger, inventors are more likely to find a productive match, and team production raises the returns to scale in inventor labor. Our data estimates based on firm-level patent output and inventor counts are consistent with this mechanism. In our baseline frictionless model, this alone can account for the rising concentration of inventors in large firms. We then show that the shift toward team production increases labor demand at the top of the firm productivity distribution, which squeezes less productive firms and leads them to hire fewer inventors. Furthermore, we find additional implications of team production when it interacts with knowledge spillovers and labor market frictions. In a model calibrated with realistic spillovers, the resulting reallocation from team production amplifies misallocation across firms. In a frictional labor market, the shift toward team production further squeezes small, productive firms searching for new workers and generates more pronounced wage premiums paid by larger firms. The incumbent workforce effectively becomes an asset that reinforces the advantage of large firms.

In future work, we plan to combine the model with microdata on inventors and firms to identify the underlying sources of the rise of teams—how much reflects changes in the idea-production technology versus shifts in the firm-productivity distribution, which offers a novel explanation for the growth of team innovation. These distinct forces interact differently with the misallocation of inventors across firms, making it essential to understand them for designing size-dependent R&D policies. Another promising direction is to examine how the transition toward team-based idea production, particularly within specific firms, shapes the long-run dynamics of knowledge accumulation for R&D workers.

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Appendix

A Team Size, Firm Productivity, and Age

We measure firm productivity by the number of patents in USPTO or revenue productivity (TFPR) in Compustat as proxy. Using each measure, we classify firms into three groups. For the number of patents, we categorize firms filing a single patent as small (or low-productivity) firms, those filing between two and five patents as moderately sized (or moderately productive) firms, and firms filing more than five patents as large (or highly productive) firms. With TFPR, firms grouped into three categories based on their percentiles in the within-industry (NAICS4) cross-sectional productivity distribution for each year. Figure A16 and A17 display that team size increases in more productive firms, in terms of both patent counts and TFPR.

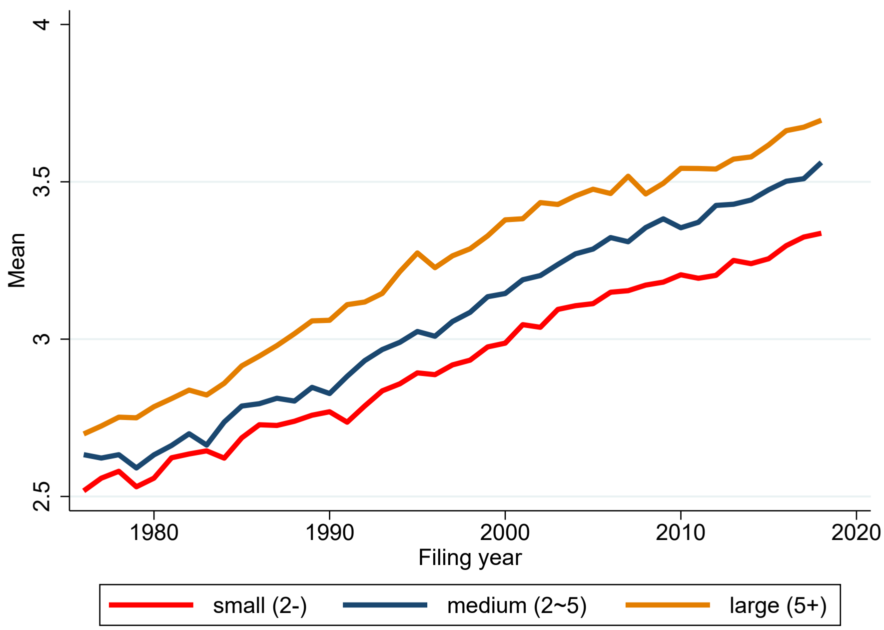

Figure A16: Average Team Sizes in a Firm (By Number of Patents)

Note: The figure illustrates the average team size in firms, categorized by firm size groups based on the number of patents filed. Small firms are those with one patent (red line), medium-sized firms are those filing between two and five patents (navy line), and large firms are those filing more than five patents (orange line) in the USPTO data.

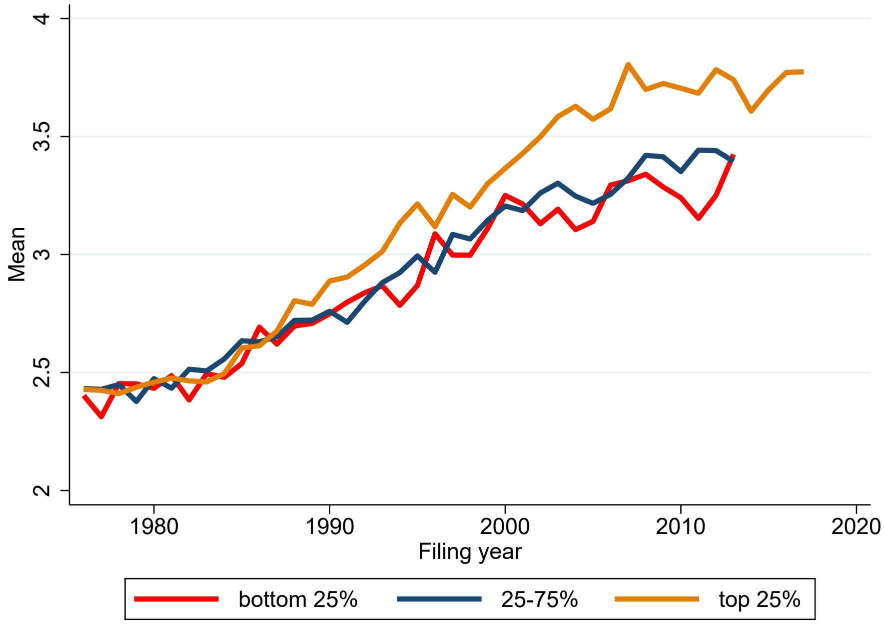

Figure A17: Average Team Sizes in a Firm over Years (By Firm TFPR)

Note: The figure illustrates the average team size in firms, categorized by firm productivity. Firms are classified by their percentile of the cross-sectional productivity distribution within a pair of industry (NAICS4) and year. Top 25% (red line) indicates the top 25 percentile firms, 25-75% (navy line) indicates firms between 25 and 75 percentiles, and bottom 25% (orange line) indicates those in the bottom 25 percentile in productivity distribution. Firm productivity is measured by the revenue productivity in Compustat.

In addition, firm age is calculated by subtracting the founding year from Ewens and Marx (2024) from the current year. With firm age, those younger than six years are considered young firms, those between six and twenty years are medium-aged firms, and those older than twenty years are categorized as old firms. A similar trend is observed with respect to firm age. The trends in team sizes across these age groups are plotted in Figure A18. After the year 2000, young firms tend to have smaller team sizes compared to older firms.

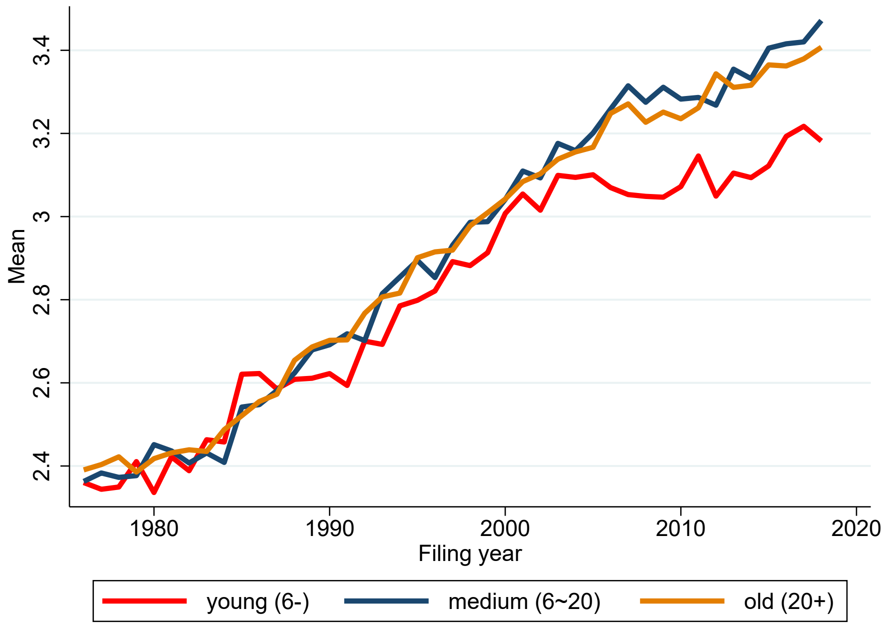

Figure A18: Average Team Sizes in a Firm over Years (By Firm Age)

Note: The figure illustrates the average team size in firms, categorized by firm age groups. Firm age data is sourced from Ewens and Marx (2024). Firms are classified as young if their age is five years or less (red line), medium-aged if their age is between six and twenty years (navy line), and old if their age is above twenty years (orange line).

B Single Inventor Share of Young Firms

Young firms are particularly likely to be single-inventor firms. To see the trend of the likelihood for young firms over time, we calculate the proportion of single-inventor firms among all young firms that have ever filed a patent in a given year. This is plotted in Figure A19, showing a declining trend of the fraction of single-inventor young firms over time.

C Team Diversity

Another trend over the past decades is rising knowledge diversity within teams. To measure knowledge diversity, we first identify the "core" CPC subgroup associated with

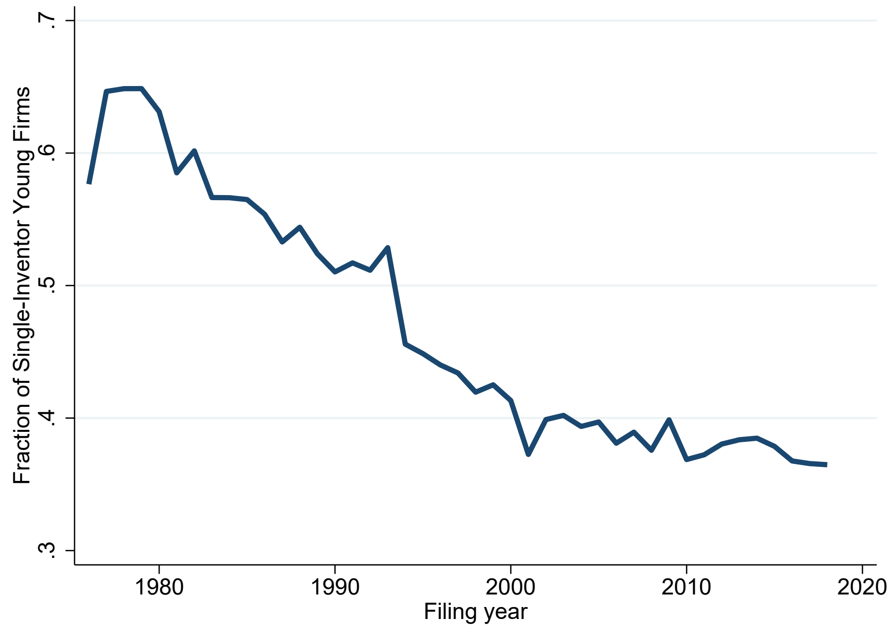

Figure A19: Fraction of Single-Inventor Young Firms

Note: The figure shows the fraction of young firms (aged five or less) with a single inventor in a given year. Firm age is sourced from Ewens and Marx (2024).

an inventor in a given year as a proxy for the inventor's type or area of expertise.[18,19] The core CPC subgroup of an inventor is defined as the subgroup with the highest number of patents filed by that inventor up to that year.[20] For each team, we calculate the number of core CPC subgroups associated with its inventors, which serves as a measure of the team's knowledge diversity in that year. We then average this knowledge diversity measure across all teams within a firm for that year. Finally, we compute the average of these firm-level measures across all firms in a given year.

The result is plotted in Figure A20, which shows an upward trend and an increasing

[18]For example, G06N 20/00 is "Machine learning" or A23B 11/00 is "Preservation of milk or dairy products."

[19]For a patent associated with multiple CPC subgroups, we pick the main CPC subgroups based on the sequence variable provided by the USPTO.

[20]Eventually, we aim to leverage the Longitudinal Employer-Household Dynamics (LEHD) dataset, which contains detailed demographic and educational information for workers covered by the U.S. Unemployment Insurance (UI) system. By linking it to the Longitudinal Business Database (LBD) and USPTO inventor data, we plan to track workers' patenting activities within and across firms. This is in progress.

likelihood of the average team working with inventors with different specialties.

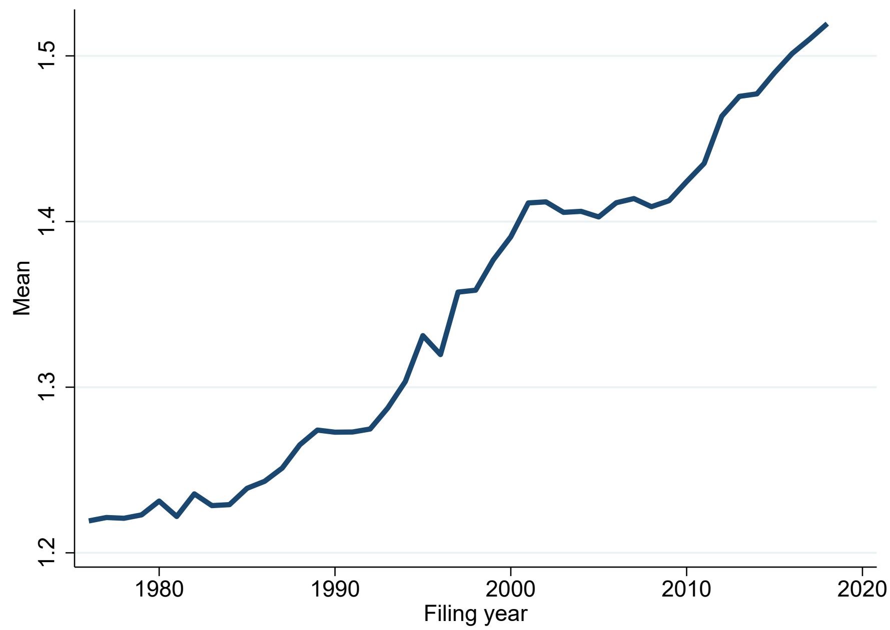

Figure A20: Average Number of Inventor CPC Subgroups in Teams in a Firm

Note: The figure illustrates the average number of core CPC subgroups associated with inventors on team patents in USPTO. A core CPC subgroup is defined as the modal CPC subgroup in an inventor's patenting history up to a given year. For each firm, we calculate the average number of core CPC subgroups for inventors on team patents and then compute the mean across all firms in a given year.

D Team Complementarities

We also aim to directly assess differences in complementarities among patenting team members across firms of varying sizes and ages, and to explore how these differences have evolved over time.

To do this, we estimate the linear model of team complementarities proposed by Bonhomme (2021):

$$Y_j = \lambda_n(\alpha_{i1(j)} + \cdots + \alpha_{in(j)}) + \epsilon_j,$$

where Y_{nj} is the five-year forward citations of patents produced by team j of size n (including single inventor patents), α_i is individual i's fixed effect, and λ_n is the team-size scaling factor for size n, which takes value one for single inventor teams.

The procedure of model estimation follows Section 3 in Bonhomme (2021). First, we use the identification condition to iteratively determine the subset of inventors for which the inventor fixed effects can be determined, and restrict the sample to this subset. Next, we estimate the complementarity coefficients from a linear moment condition. Last, the inventor fixed effects can be estimated in a standard OLS regression.

The interpretation is as follows. Output (citations) is a linear function of the team members' fixed effects. λ_n captures the effects of working in a team of size n. If $\lambda_n = 1$ for $n > 1$, the citations produced from this collaboration are the same as the sum of what the team members could produce individually. If $\lambda_n > 1$ it indicates that the team produces more citations than the members could generate on their own. An increase in λ_n over time suggests increasing complementarities for teams of size n. However, this model does not account for selection into teams or the impact of teams on the probability of securing a patent, among other factors.

As shown by the coefficients listed in Table A1, firms in the post-2000 period usually exhibit larger team complementarities than firms in the pre-2000 period, especially for teams of size two. Notably, complementarities for teams of two grew significantly for older firms after 2000.

This exercise also allows us to estimate inventor fixed effects that control for participation in teams of different sizes (say, selection into solo invention). Solo inventors, those with above median share (20%) of their patents filed alone, are slightly positively selected but this gaps has narrowed over time (Figure A21). Surprisingly, we find evidence of negative rather than positive assortative matching into teams of two based on the fixed effects we estimate (Figure A22). Formally, the exercise computes the a histogram and the mean of the absolute difference between team members' estimated fixed effects in the data versus 1000 random draws of two person teams the same size as the data with sampling weights for inventors the same in the bootstrap as that inventor's share of patents in the data. The mean expected difference in the fixed effects is 6% higher in the data, mean quality differences are larger in the data than patenting teams drawn at random (negative assortative matching).

Table A1: Complementarity estimates for periods and subgroups of firms

Period	Group	λ_1	λ_2	λ_3	$\lambda_{\geq 4}$	# of Patents	# of Inventors
≤ 2000	All	1.0000	0.5674	0.4147	0.3116	1,022,209	206,849
≤ 2000	Small	1.0000	0.5699	0.4089	0.3209	48,392	10,697
≤ 2000	Large	1.0000	0.5627	0.4116	0.3088	885,695	181,694
≤ 2000	Young	1.0000	0.5919	0.4424	0.3581	12,982	3,234
≤ 2000	Old	1.0000	0.5587	0.4173	0.3149	323,371	65,928
> 2000	All	1.0000	0.6607	0.4803	0.3543	3,103,493	590,872
> 2000	Small	1.0000	0.6447	0.5027	0.3949	308,769	71,193
> 2000	Large	1.0000	0.6569	0.4750	0.3492	2,652,074	503,741
> 2000	Young	1.0000	0.5870	0.4526	0.3489	49,209	12,803
> 2000	Old	1.0000	0.6417	0.4631	0.3384	906,673	181,569

Note: This table lists the complementarity estimates for various periods and subgroups of firms. The dataset consists of all USPTO patents filed between 1976 and 2018. The sample of patents is divided into two periods based on their filing years: the first period includes patents filed no later than 2000, and the second period includes patents filed after 2000. Within each period, the sample is further divided by firm size and firm age. The group of small firms consists of those classified as small or micro entities in the USPTO dataset, while the group of large firms includes the remaining firms. The group of young firms includes those with an age of five years or less, where firm age is defined as the difference between the current year and the founding year from Ewens and Marx (2024). The group of old firms includes those with an age greater than five years.

E Derivation of Team Productivity

E.1 Discrete Firm Size

Recall that bilateral productivity is:

$$z_{ijt} \sim P(x_{min}^{team}, \gamma^{team}), \quad P(z \leq x) = 1 - (\frac{x_{min}^{team}}{x})^{\gamma^{team}} \quad \forall x \geq x_{min}^{team}$$

Let the max of n draws be

$$M_n \equiv \max\{z_{1jt}, z_{2jt}, .., z_{njt}\}$$

Figure A21: Mean estimated inventor fixed effect α_i. The median share of solo patents for inventors is 20%. Above = solo inventor, below = team inventor.

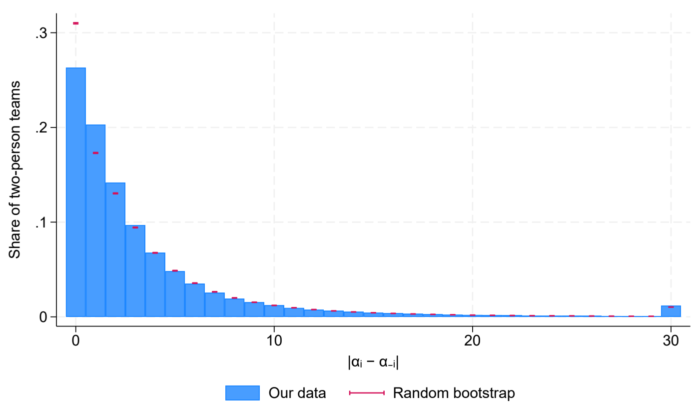

Figure A22: Histogram of the absolute difference in two-person team member fixed effects $|\alpha_i - \alpha_{-i}|$ in the data vs. 1000 random samples of two person teams with the same number of teams and sampling weights on inventors as in the data. Mean difference: 4.293 in the data and 4.041 in random bootstrap.

then

$$F_{M_n}(x) \equiv P(M_n \leq x) = \Big[1 - (\frac{x_{min}^{team}}{x})^{\gamma^{team}}\Big]^n \quad \forall x \geq x_{min}^{team}$$

Then, we can derive the pdf as follows:

$$f_{M_n}(x) = n\gamma^{team}(\frac{(x_{min}^{team})^{\gamma^{team}}}{x^{\gamma^{team}+1}})\Big[1 - (\frac{x_{min}^{team}}{x})^{\gamma^{team}}\Big]^{n-1} \quad \forall x \geq x_{min}^{team}.$$

Thus, the expected value is

$$\mathbb{E}(M_n) = \int_{x_{min}^{team}} x f_{M_n}(x)dx = x_{min}^{team} nB(n, 1 - \frac{1}{\gamma^{team}}),$$

where B is Beta function.[21]

Note that for large n, the Beta function asymptotically becomes:

$$B(n, 1 - \frac{1}{\gamma^{team}}) = \frac{\Gamma(n)\Gamma(1 - \frac{1}{\gamma^{team}})}{\Gamma(n + 1 - \frac{1}{\gamma^{team}})} \simeq \Gamma(1 - \frac{1}{\gamma^{team}})n^{-1+\frac{1}{\gamma^{team}}}.$$

Therefore, the expected productivity of working in a firm with n team members becomes:

$$\mathbb{E}(M_n) = x_{min}^{team}\Gamma(1 - \frac{1}{\gamma^{team}})n^{\frac{1}{\gamma^{team}}} \tag{E.30}$$

which increases in n.

E.2 Continuous Firm Size

Assume that worker bilateral productivity is drawn from an inhomogeneous Poisson point process with intensity function within firm of size $n \in [0, \infty)$.

$$\lambda(x) = n\frac{f(x)}{F(x)}$$

[21]It can be derived with a transformation of variable with $t = 1 - (x_{min}^{team}/x)^{\gamma^{team}}$. So that it can be rephrased as $\mathbb{E}(M_n) = nx_{min}^{team} \int_0^1 t^{n-1}(1-t)^{-\frac{1}{\gamma^{team}}} dt$.

Then, the distribution function of the maximum talent is

$$\begin{aligned} F_{z_{it}^{team}}(x) &= \mathbb{P}[N(x, +\infty) = 0] \\ &= \exp\left(-\int_x^{+\infty} \lambda(u) du\right) \\ &= F_{z_{it}}(x)^n, \end{aligned}$$

and thus, everything follows the same as before.