Supplemental Appendix:
"Workers' Job Prospects and Young Firm Dynamics"
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A Bayesian Learning

Suppose that initial prior is $\nu_j \sim N(\bar{\nu}_0, \sigma_0^2)$, and there is an observation of $\ln P_{jt} = \nu_j + \varepsilon_{jt}$ such that $\varepsilon_{jt} \sim N(0, \sigma_\varepsilon^2)$. Following the Bayes' rule, $f(\nu_j | \ln P_{jt}) \propto f(\nu_j) f(\ln P_{jt} | \nu_j)$, we have:

$$
\begin{aligned}
f(\nu_j | \ln P_{jt}) &\propto f(\nu_j) f(\ln P_{jt} | \nu_j) \\
&= \left(\frac{1}{\sqrt{2\pi\sigma_0^2}} \exp\left(-\frac{(\nu_j - \bar{\nu}_0)^2}{2\sigma_0^2} \right) \right) \left(\frac{1}{\sqrt{2\pi\sigma_\varepsilon^2}} \exp\left(-\frac{(\ln P_{jt} - \nu_j)^2}{2\sigma_\varepsilon^2} \right) \right) \\
&\propto \left(\frac{1}{\sqrt{2\pi\sigma_0^2\sigma_\varepsilon^2}} \exp\left(-\frac{\left(\nu_j - \left(\frac{\sigma_\varepsilon^2 \bar{\nu}_0 + \sigma_0^2 \ln P_{jt}}{\sigma_\varepsilon^2 + \sigma_0^2} \right) \right)^2}{2 \frac{\sigma_0^2 \sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma_0^2}} \right) \right),
\end{aligned}
$$

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$$f(\nu_j | \ln P_{jt}) \sim N\left(\frac{\sigma_\varepsilon^2 \bar{\nu}_0 + \sigma_0^2 \ln P_{jt}}{\sigma_\varepsilon^2 + \sigma_0^2}, \frac{\sigma_0^2 \sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma_0^2}\right).$$

Thus, the mean and standard deviation of the posterior distribution are given by:

$$\text{(A.1)} \qquad \bar{\nu}_{jt} = \frac{\sigma_\varepsilon^2 \bar{\nu}_{jt-1} + \sigma_{jt-1}^2 \ln P_{jt}}{\sigma_{jt-1}^2 + \sigma_\varepsilon^2} = \frac{\frac{\bar{\nu}_{jt-1}}{\sigma_{jt-1}^2} + \frac{\ln P_{jt}}{\sigma_\varepsilon^2}}{\frac{1}{\sigma_{jt-1}^2} + \frac{1}{\sigma_\varepsilon^2}},$$

$$\text{(A.2)} \qquad \sigma_{jt}^2 = \frac{\sigma_{jt-1}^2 \sigma_\varepsilon^2}{\sigma_{jt-1}^2 + \sigma_\varepsilon^2} = \frac{1}{\frac{1}{\sigma_{jt-1}^2} + \frac{1}{\sigma_\varepsilon^2}}.$$

By iterating (A.1) and (A.2) backward, (1.1) in the main text can be derived.

Note that the posterior mean in (1.1) is a weighted sum of the initial prior mean and the average observed productivity over past periods, with weights determined by firm age. The mean increases with average productivity, where higher average productivity enhances prospects about firms. On the other hand, the posterior mean increases with firm age only if the firm's average productivity is above the initial cross-sectional mean ($\tilde{P}_{jt-1} > \bar{\nu}_0$), while it decreases with firm age if the firm's average productivity is below the cross-sectional mean ($\tilde{P}_{jt-1} < \bar{\nu}_0$).[1] The posterior variance in (1.1) decreases with firm age, and the posterior converges to a degenerate distribution centered at the true type ν_j as the firm ages.

Furthermore, the following relationships between the two sufficient statis-

[1] In other words, a higher age indicates a better (worse) inferred type for the former (latter) case.

tics and the posterior mean at the beginning of each period t can be derived:

$$\frac{\partial \bar{\nu}_{jt-1}}{\partial \tilde{P}_{jt-1}} = \frac{a_{jt}\frac{1}{\sigma_\varepsilon^2}}{\frac{1}{\sigma_0^2} + a_{jt}\frac{1}{\sigma_\varepsilon^2}} > 0 \tag{A.3}$$

$$\frac{\partial \bar{\nu}_{jt-1}}{\partial a_{jt}} = \frac{(\tilde{P}_{jt-1} - \bar{\nu}_0)}{\sigma_0^2\sigma_\varepsilon^2\left(\frac{1}{\sigma_0^2} + a_{jt}\frac{1}{\sigma_\varepsilon^2}\right)^2} \begin{cases} \geq 0 & \text{if} \quad \tilde{P}_{jt-1} \geq \bar{\nu}_0 \\ < 0 & \text{if} \quad \tilde{P}_{jt-1} < \bar{\nu}_0 \end{cases}. \tag{A.4}$$

Equation (A.3) implies that the posterior mean increases with the average productivity level. As firms are observed to have higher average productivity, their prospects improve. Moreover, (A.4) shows that firm age affects job prospects differently depending on the firm's past-average productivity. Specifically, if firm j's average productivity is above the initial cross-sectional mean, a higher age implies a better inferred type, while if a firm's average productivity is below the cross-sectional mean, a higher age implies a worse inferred type.

Also, one can derive the following relationship between firm age and the posterior standard deviation:

$$\frac{\partial \sigma_{jt-1}^2}{\partial a_{jt}} = -\frac{1}{\sigma_\varepsilon^2\left(\frac{1}{\sigma_0^2} + a_{jt}\frac{1}{\sigma_\varepsilon^2}\right)^2} < 0. \tag{A.5}$$

This implies that as a firm ages, learning becomes less noisy, and the posterior converges to a degenerate distribution centered at the true type ν_j.

B Joint Surplus Maximization

As in the main text, I drop time subscripts henceforth. Solving the firms' problem, the value function (1.5) can be fully replicated by the following joint surplus maximization:

$$V^{prod}(a_j, \tilde{P}_{j,-1}, l_{j,-1}, P_j) = \max_{d'_j, s'_j, x'_j, x_j^{E\prime}, h'_j} P_j l_j^\alpha - c_f + \beta \mathbb{E}_j \Bigg[(1-\delta)(1-d'_j)\Big(V^{prod}(a'_j, \tilde{P}_j, l'_j, P'_j)$$
$$- \Big(x'_j + \frac{c}{q(\theta(x'_j))} \Big) h'_j + (1-s'_j)\lambda f(\theta(x_j^{E\prime})) x_j^{E\prime} l_j \Big) + \Big(\delta + (1-\delta)\big(d'_j + (1-d'_j)s'_j\big) \Big) U' l_j \Bigg],$$

where $V_j^{prod} \equiv J_j^{prod} + x_j h_j + \tilde{W}_j (1-s_j)(1-\lambda f\big(\theta(x_j^E)\big)) l_{j,-1}$, J_j^{prod} is the firm value function at the production stage after search and matching, and $\mathbf{\Omega_j^{-w}} = \{d'_j, s'_j, \tilde{W}'_j\}$ denotes the contract abstracting from the wage w_j^i.

Given that choice variables are contingent on future productivity, it can be transformed using the following value function, defined at the beginning of each period:

$$V_j^{init}(a_j, \tilde{P}_{j,-1}, l_{j,-1}, P_j) = \max_{d_j, s_j h_j, x_j^E} \delta U l_{j,-1} + (1-\delta)(d_j + (1-d_j)s_j) U l_{j,-1}$$
$$+ (1-\delta)(1-d_j)\Big(P_j l_j^\alpha - c^f - \kappa h_j + (1-s_j)\lambda f(\theta(x_j^E)) x_j^E l_{j,-1} + \beta \mathbb{E}_j V_j^{init}(a'_j, \tilde{P}_j, l_j, P'_j) \Big).$$

Note that the first term $\delta U l_{j,-1}$ is independent of the variables being maximized and $(1-\delta)$ in the remaining two terms simply scales the objective function. The problem can first be solved for s_j, h_j, and x_j^E, maximizing:

(B.6)
$$\max_{s_j, h_j, x_j^E} s_j U l_{j,-1} + P_j l_j^\alpha - c^f - \kappa h_j + (1-s_j)\lambda f(\theta(x_j^E)) x_j^E l_{j,-1} + \beta \mathbb{E}_j V_j^{init}(a'_j, \tilde{P}_j, l_j, P'_j).$$

And then $d_j = 1$ if $U l_{j,-1}$ is greater than the value (B.6), and $d_j = 0$, otherwise.

In a similar fashion, the free-entry condition (1.11) can be rephrased as:

(B.7)
$$\int \max_{d_j^e, l_j^e} (1-d_j^e)\Big(P_j (l_j^e)^\alpha - c^f - \kappa l_j^e + \beta \mathbb{E}_j V^{init}(1, \ln P_j, l_j^e, P'_j) \Big) dF_e(P_j) - c^e = 0.$$

C Productivity Cutoffs

Deriving the four endogeneous productivity cutoffs, $\mathcal{P}_j^h$, $\mathcal{P}_j^q$, $\mathcal{P}_j^l$, and $\mathcal{P}_j^x$, follow the following first-order conditions with respect to h_j, s_j, and x_j^E:

$$\text{(C.8)} \qquad \left[\alpha P_j l_j^{\alpha-1} + \beta \frac{\partial \mathbb{E}_j V_j^{init\prime}}{\partial l_j}\right] - \kappa = 0,$$

$$\text{(C.9)} \qquad U - \lambda f(\theta(x_j^E))x_j^E - (1 - \lambda f(\theta(x_j^E)))\left[\alpha P_j l_j^{\alpha-1} + \beta \frac{\partial \mathbb{E}_j V_j^{init\prime}}{\partial l_j}\right] = 0,$$

$$\text{(C.10)} \qquad x_j^E + \frac{f(\theta(x_j^E))}{f'(\theta(x_j^E))\theta'(x_j^E)} - \left[\alpha P_j l_j^{\alpha-1} + \beta \frac{\partial \mathbb{E}_j V_j^{init\prime}}{\partial l_j}\right] = 0.$$

First, to determine the hiring cutoff, (C.8) needs to be evaluated at $l_j = l_{j,-1}$, where firms are indifferent between hiring and not hiring. Given the state variables $(a_j, \tilde{P}_{j,-1}, l_{j,-1})$, the hiring decision depends on whether the marginal value of hiring, represented by the first term of (C.8), exceeds the cost of hiring κ or not. If P_j falls within a range where the marginal value becomes less than κ, the firm opts to stop hiring. The hiring cutoff is thus defined as the productivity level P_j at which the marginal value of hiring equals the cost, making $h_j = 0$ the optimal choice. Below this threshold, the marginal value derived from hiring new workers is insufficient to justify the cost, and firms will refrain from hiring workers.

Therefore, the hiring productivity cutoff $\mathcal{P}_j^h$ is determined by equating the marginal value of hiring to the cost of hiring κ, as expressed in the following equation:

$$\text{(C.11)} \qquad \left[\alpha \mathcal{P}_j^h l_{j,-1}^{\alpha-1} + \beta \frac{\partial \mathbb{E}_j V_j^{init\prime}}{\partial l_j} \Big|_{\tilde{P}_j = \frac{a_j \tilde{P}_{j,-1} + \mathcal{P}_j^h}{a_j + 1}, l_j = l_{j,-1}}\right] = \kappa,$$

where the expectation $\mathbb{E}_j(\cdot)$ is formed over P_j' based on the firm's and its workers' posterior beliefs at the start of the next period. The beliefs incorporate the updated firm age $a_j + 1$ and average productivity $\tilde{P}_j = \frac{a_j \tilde{P}_{j,-1} + \mathcal{P}_j^h}{a_j+1}$.

The quitting cutoff, $\mathcal{P}_j^q$, delineates the range of productivity levels where firms begin allowing workers to quit. Note that firms would not hire workers when

$$\text{(C.12)} \quad \left[\alpha P_j \Big((1-\lambda f(\theta(x_j^E)))l_{j,-1}\Big)^{\alpha-1} + \beta \frac{\partial \mathbb{E}_j V_j^{init'}}{\partial l_j}|_{l_j=(1-\lambda f(\theta(x_j^E)))l_{j,-1}}\right] < \kappa,$$

as before. At the same time, if the marginal value of x^E is still high enough, firms optimally set x^E to its upper bound. This condition arises when the marginal value of x^E is positive, which corresponds to the left-hand side of (C.10). This condition can be rephrased as:

$$\left[\alpha P_j l_j^{\alpha-1} + \beta \frac{\partial \mathbb{E}_j V_j^{init'}}{\partial l_j}\right] > x_j^E + \frac{f(\theta(x_j^E))}{f'(\theta(x_j^E))\theta'(x_j^E)},$$

given $\theta'(x_j^E) < 0$ and $f'(\theta(x_j^E)) < 0$. Also, given $x_j^E = \kappa - c$, this becomes

$$\text{(C.13)} \qquad \left[\alpha P_j l_j^{\alpha-1} + \beta \frac{\partial \mathbb{E}_j V_j^{init'}}{\partial l_j}\right] > \kappa - c.$$

Combining (C.12) and (C.13), firms would stay inactive without allowing quits in the following range:

$$\text{(C.14)} \qquad \kappa - c < \left[\alpha P_j l_{j,-1}^{\alpha-1} + \beta \frac{\partial \mathbb{E}_j V_j^{init'}}{\partial l_j}|_{l_j=l_{j,-1}}\right] < \kappa.$$

In other words, the quitting cutoff $\mathcal{P}_j^q$ is determined by the following equation:

$$\text{(C.15)} \qquad \left[\alpha \mathcal{P}_j^q l_{j,-1}^{\alpha-1} + \beta \frac{\partial \mathbb{E}_j V_j^{init'}}{\partial l_j}|_{\tilde{P}_j = \frac{a_j \tilde{P}_{j,-1} + \mathcal{P}_j^q}{a_j+1}, l_j=l_{j,-1}}\right] = \kappa - c,$$

below which firms start allowing quits. As before, the expectation $\mathbb{E}_j(\cdot)$ is formed over P'_j based on the firm's and its workers' posterior belief with a_j+1 and $\tilde{P}'_j = \frac{a_j\tilde{P}_{j,-1}+\mathcal{P}^q_j}{a_j+1}$.

Lastly, in regards to the layoff cutoff, it is determined by (C.9) evaluated at $l_j = (1-\lambda f(\theta(x^E_j)))l_{j,-1}$ where x^E_j is the root of (C.10). Similar to the hiring cutoff, given $(a_j, \tilde{P}_{j,-1}, l_{j,-1})$, if P_j lies in a range in which the marginal value of layoffs (the left-hand side of (C.9)) is lower, then firms will no longer lay off any workers. Therefore, the cutoff is determined at the point where it is optimal to set $s_j = 0$ in the separating firms' problem, above which firms would never lay off workers. The following equation thus determines the layoff productivity cutoff $\mathcal{P}^l_j$:

$$\Big[\alpha\mathcal{P}^l_j((1-\lambda f(\theta(x^E_j)))l_{j,-1})^{\alpha-1} + \beta\frac{\partial\mathbb{E}_jV_j^{init\prime}}{\partial l_j}\Big|_{\tilde{P}_j=\frac{a_j\tilde{P}_{j,-1}+\mathcal{P}^l_j}{a_j}+1,\, l_j=(1-\lambda f(\theta(x^E_j)))l_{j,-1}}\Big]$$

$$= \frac{U-\lambda x^E_j\left(\theta(x^E_j)(1+\theta(x^E_j)^\gamma)^{-\frac{1}{\gamma}}\right)}{1-\lambda\left(\theta(x^E_j)(1+\theta(x^E_j)^\gamma)^{-\frac{1}{\gamma}}\right)}, \tag{C.16}$$

where x^E_j is the root of (C.10) with the set of state variables $(a_j, \tilde{P}_{j,-1}, l_{j,-1}, \mathcal{P}^l_j)$. The expectation $\mathbb{E}_j(\cdot)$ is formed over P'_j based on the firm's and its workers' posteriors with a_j+1 and $\tilde{P}'_j = \frac{a_j\tilde{P}_{j,-1}+\mathcal{P}^l_j}{a_j+1}$ as before.

The following part shows how the productivity cutoffs vary across firms with different posteriors. To understand this, we first need to examine how the future expected marginal value of labor input, $\left(\frac{\partial\mathbb{E}V^{init}(a'_j,\tilde{P}_j,l_j,P'_j)}{\partial l_j}\right)$, varies among firms. This variation, as we will show, depends on firms' employment status, preserving the same ranking as the future expected value of workers, as discussed in the main appendix.

C.1 Hiring Firms: $s_j = 0$ and $h_j > 0$

For hiring firms, their value function becomes:

$$V_j^{init}(a_j, \tilde{P}_{j,-1}, l_{j,-1}, P_j) = \delta U l_j + (1-\delta)\Big[P_j {l_j}^{\alpha} - c^f - \kappa h_j + \beta \mathbb{E}_j V^{init}(a'_j, \tilde{P}_j, l_j, P'_j)\Big],$$

where $h_j \equiv h(a_j, \tilde{P}_{j,-1}, l_{j,-1}, P_j)$ is the firm's hiring decision rule and $l_j \equiv h_j + l_{j,-1}$. Then, the derivative of the value function with respect to l_j is:

$$\begin{aligned}\frac{\partial V^{init}(a_j, \tilde{P}_{j,-1}, l_{j,-1}, P_j)}{\partial l_{j,-1}} &= \delta U + (1-\delta)\Big[\alpha P_j {l_j}^{\alpha-1} + \beta \frac{\partial \mathbb{E}_j V^{init}(a'_j, \tilde{P}_j, l_j, P'_j)}{\partial l_j}\Big] \\ &+ (1-\delta)\frac{\partial h_j}{\partial l_{j,-1}}\Big[\alpha P_j {l_j}^{\alpha-1} + \beta \frac{\partial \mathbb{E}_j V^{init}(a'_j, \tilde{P}_j, l_j, P'_j)}{\partial l_j} - \kappa\Big],\end{aligned}$$

where the first line represents the direct effect of $l_{j,-1}$, and the second line is an indirect effect of $l_{j,-1}$ through its optimal hiring on the value function. With the first-order condition for hiring,(C.8), the indirect effect becomes zero, consistent with the envelope theorem. This simplifies the derivative of the value function with respect to $l_{j,-1}$ as follows:

$$\frac{\partial V^{init}(a_j, \tilde{P}_{j,-1}, l_{j,-1}, P_j)}{\partial l_{j,-1}} = \delta U + (1-\delta)\kappa. \tag{C.17}$$

C.1.1 Inactive Firms: $s_j = 0$ and $h_j = 0$

Next, consider inactive firms who do not allow quits, where $h_j = 0$, $s_j = 0$, $x_j^E = 0$, and the employment size remains constant at $l_j = l_{j,-1}$. Thus, the firm's value function becomes:

$$V^{init}(a_j, \tilde{P}_{j,-1}, l_{j,-1}, P_j) = \delta U l_{j,-1} + (1-\delta)\Big[P_j l_{j,-1}^{\alpha} - c^f + \beta \mathbb{E}_j V^{init}(a'_j, \tilde{P}_j, l_j, P'_j)\Big],$$

and the first derivative of it with respect to $l_{j,-1}$ is

$$\frac{\partial V^{init}(a_j, \tilde{P}_{j,-1}, l_{j,-1}, P_j)}{\partial l_{j,-1}} = \delta U + (1-\delta)\Big[\alpha P l_{j,-1}^{\alpha-1} + \beta \frac{\partial \mathbb{E}_j V^{init}(a'_j, \tilde{P}_j, l_j, P'_j)}{\partial l_{j,-1}}\Big].$$

Note that this case can only happen with the range (C.14), and thus their marginal value of labor falls within the range of $[\kappa - c, \kappa]$ as follows:

$$\text{(C.18)} \qquad \delta U + (1-\delta)(\kappa - c) \leq \frac{\partial V^{init}(a_j, \tilde{P}_{j,-1}, l_{j,-1}, P_j)}{\partial l_{j,-1}} \leq \delta U + (1-\delta)\kappa.$$

Now, consider the case of inactive firms that allow quits. Their value function is as follows:

$$\begin{aligned} V^{init}(a_j, \tilde{P}_{j,-1}, l_{j,-1}, P_j) \;&=\; \delta U l_{j,-1} + (1-\delta)\Big[P_j {l_j}^{\alpha} - c^f + \lambda f(\theta(x_j^E)) x_j^E l_{j,-1} \\ &+\; \beta \mathbb{E}_j V^{init}(a'_j, \tilde{P}_j, l_j, P'_j)\Big], \end{aligned}$$

where $x_j^E \equiv x_j^E(a_j, \tilde{P}_{j,-1}, l_{j,-1}, P_j)$ is their optimal retention choice and $l_j \equiv (1 - \lambda f(\theta(x_j^E(a_j, \tilde{P}_{j,-1}, l_{j,-1}, P_j))))l_{j,-1}$.

Getting the derivative as before, the following can be obtained:

$$\begin{aligned} &\frac{\partial V^{init}(a_j, \tilde{P}_{j,-1}, l_{j,-1}, P_j)}{\partial l_{j,-1}} \\ =\;& \delta U + (1-\delta)\Big[(1 - \lambda f(\theta(x_j^E)))\Big(\alpha P_j {l_j}^{\alpha-1} + \beta \frac{\partial \mathbb{E}_j V^{init}(a'_j, \tilde{P}_j, l_j, P'_j)}{\partial l_j}\Big) + \lambda f(\theta(x_j^E)) x_j^E\Big] \\ +\;& (1-\delta)\frac{\partial x_j^E}{\partial l_{j,-1}}\Big[-\lambda f'(\theta)\theta'(x_j^E) l_{j,-1}\Big(\alpha P_j {l_j}^{\alpha-1} + \beta \frac{\partial \mathbb{E}_j V^{init}(a'_j, \tilde{P}_j, l_j, P'_j)}{\partial l_j}\Big) \\ +\;& \lambda f(\theta(x_j^E)) l_{j,-1} + \lambda f'(\theta)\theta'(x_j^E) x_j^E l_{j,-1}\Big]. \end{aligned}$$

Here, the first line represents the direct effect of $l_{j,-1}$, and the last two lines correspond to the indirect effect of $l_{j,-1}$ through its optimal retention on the value function. As before, the indirect effect becomes zero through the envelope theorem. Thus, the terms can be rephrased as:

$$\frac{\partial V^{init}(a_j, \tilde{P}_{j,-1}, l_{j,-1}, P_j)}{\partial l_{j,-1}} = \delta U + (1-\delta)\Big[x_j^E + \frac{(1-\lambda f(\theta(x_j^E)))f(\theta(x_j^E))}{f'(\theta)\theta'(x_j^E)}\Big].$$

Note that this term must satisfy the following range:

$$U \le \frac{\partial V^{init}(a_j, \tilde{P}_{j,-1}, l_{j,-1}, P_j)}{\partial l_{j,-1}} \le \delta U + (1-\delta)(\kappa - c). \tag{C.19}$$

The upper bound comes from $f'(\theta)\theta'(x_j^E) < 0$ and $x_j^E \le \kappa - c$. The lower bound is derived from the fact that this firm never finds $s_j > 0$ to be optimal, which is consistent to say the left-hand side of (C.9) being strictly negative for any $s_j > 0$ or zero when $s_j = 0$. Combining this with (C.10), it can be proved that:

$$U \le \Big[x_j^E + \frac{(1-\lambda f(\theta(x_j^E)))f(\theta(x_j^E))}{f'(\theta)\theta'(x_j^E)}\Big]$$

which gives the lower bound in (C.19).

C.1.2 Separating Firms with Layoffs: $s_j > 0$ and $h_j = 0$

For firms that separate workers with explicit layoffs, their value function is:

$$\begin{aligned} V^{init}(a_j, \tilde{P}_{j,-1}, l_{j,-1}, P_j) \;&=\; \delta U l_{j,-1} + (1-\delta)\Big[s_j U l_{j,-1} + P_j {l_j}^{\alpha} - c^f + (1-s_j)\lambda f(\theta(x_j^E))x_j^E l_{j,-1} \\ &+\; \beta \mathbb{E}_j V^{init}(a_j', \tilde{P}_j, l_j, P_j')\Big], \end{aligned}$$

where $s_j \equiv s(a_j, \tilde{P}_{j,-1}, l_{j,-1}, P_j)$ is their layoff decision, $x_j^E \equiv \mathbf{x^E}(a_j, \tilde{P}_{j,-1}, l_{j,-1}, P_j)$ is their retention decision, and $l_j \equiv (1-s_j)(1-\lambda f(\theta(x_j^E)))l_{j,-1}$.

Making the first derivative of it with respect to $l_{j,-1}$, it can be obtained that:

$$
\begin{aligned}
&\frac{\partial V^{init}(a_j, \tilde{P}_{j,-1}, l_{j,-1}, P_j)}{\partial l_{j,-1}} = \delta U + (1-\delta)\Big[s_j U \\
+ \quad & (1-s_j)(1-\lambda f(\theta(x_j^E)))\Big(\alpha P_j {l_j}^{\alpha-1} + \beta \frac{\partial \mathbb{E}_j V^{init}(a_j', \tilde{P}_j, l_j, P_j')}{\partial l_j}\Big)(1-s_j)\lambda f(\theta(x_j^E))x_j^E\Big] \\
+ \quad & (1-\delta)\frac{\partial s_j}{\partial l_{j,-1}}\Big[U l_{j,-1} - (1-\lambda f(\theta(x_j^E)))l_{j,-1}\Big(\alpha P_j {l_j}^{\alpha-1} + \beta \frac{\partial \mathbb{E}_j V^{init}(a_j', \tilde{P}_j, l_j, P_j')}{\partial l_j}\Big) \\
- \quad & \lambda f(\theta(x_j^E))x_j^E l_{j,-1}\Big] \\
+ \quad & (1-\delta)(1-s_j)\frac{\partial x_j^E}{\partial l_{j,-1}}\Big[-\lambda f'(\theta)\theta'(x_j^E)l_{j,-1}\Big(\alpha P_j {l_j}^{\alpha-1} + \beta \frac{\partial \mathbb{E}_j V^{init}(a_j', \tilde{P}_j, l_j, P_j')}{\partial l_j}\Big) \\
+ \quad & \lambda f(\theta(x_j^E))l_{j,-1} + \lambda f'(\theta)\theta'(x_j^E)x_j^E l_{j,-1}\Big],
\end{aligned}
$$

where the first two lines represent the direct effect of $l_{j,-1}$, the third and fourth lines correspond to the indirect effect of $l_{j,-1}$ through its optimal layoffs, and the last two lines are the indirect effect of $l_{j,-1}$ through its optimal retention on the value function. Note that, using the optimal conditions (which again implies the envelope theorem), (C.9) and (C.10) make the indirect effects zero, and the first line simplifies further. Ultimately, the derivative becomes:

$$
\frac{\partial V^{init}(a_j, \tilde{P}_{j,-1}, l_{j,-1}, P_j)}{\partial l_{j,-1}} = U. \tag{C.20}
$$

C.1.3 Exiting firms: $d_j = 1$

Lastly, for exiting firms, their value function is:

$$
V^{init}(a_j, \tilde{P}_{j,-1}, l_{j,-1}, P_j) = U l_{j,-1},
$$

and the derivative with respect to $l_{j,-1}$ is:

$$\frac{\partial V^{init}(a_j, \tilde{P}_{j,-1}, l_{j,-1}, P_j)}{\partial l_{j,-1}} = U. \tag{C.21}$$

Combining (C.17), (C.18), (C.19), (C.20), and (C.21), it can be proved that for $\frac{\partial V^{init}(a_j, \tilde{P}_{j,-1}, l_{j,-1}, P_j)}{\partial l_{j,-1}}$, hiring firms have the highest value, inactive firms without quits have the second highest value, quitting firms have the third highest value, and firms laying off workers or exiting have the lowest value.

(C.22)

$$\frac{\partial V^{init}(a_j, \tilde{P}_{j,-1}, l_{j,-1}, P_j)}{\partial l_{j,-1}} = \begin{cases} \delta U + (1-\delta)\kappa \text{ if } P_j > \mathcal{P}_j^h \\ \delta U + (1-\delta)(\kappa - c) \text{ if } \mathcal{P}_j^q < P_j < \mathcal{P}_j^h \\ \delta U + (1-\delta)\left[x_j^E + \frac{(1-\lambda f(\theta(x_j^E)))f(\theta(x_j^E))}{f'(\theta)\theta'(x_j^E)}\right] \text{ if } \mathcal{P}_j^l < P_j < \mathcal{P}_j^q \\ U \text{ if } P_j < \mathcal{P}_j^l, \end{cases}$$

From (C.10), it can be derived that higher P_j, holding all else constant, should increase the optimal x_j^E for firms experiencing worker quits in the range $\mathcal{P}_j^l < P_j < \mathcal{P}_j^q$. This conclusion arises because higher P_j increases both the marginal revenue and the posterior mean $\tilde{P} = \frac{a_j P_{j,-1} + \ln P_j}{a_j + 1}$, which firms take into account when forming the expected marginal value of a labor input. To accommodate these changes, x_j^E must be adjusted upward as the first two terms $x_j^E + \frac{f(\theta(x_j^E))}{f'(\theta(x_j^E))\theta'(x_j^E)}$ in (C.10) need to increase, and this expression is an increasing function of x_j^E. Furthermore, in (C.22), it can be shown that: $\frac{\partial\left[x_j^E + \frac{(1-\lambda f(\theta(x_j^E)))f(\theta(x_j^E))}{f'(\theta)\theta'(x_j^E)}\right]}{\partial x_j^E} = 1 + (1-\lambda)\gamma c^{-\gamma}(\kappa - x_j^E)^{\gamma-1} > 0$, for firms in the range $\mathcal{P}_j^l < P_j < \mathcal{P}_j^q$.

Altogether, this implies that $\frac{\partial V^{init}(a_j, \tilde{P}_{j,-1}, l_{j,-1}, P_j)}{\partial l_{j,-1}}$ is a weakly increasing function of P_j, all else equal. Therefore, firms that are more likely to draw higher

P_j' and expand in the next period will obtain a higher expected future marginal value of a labor input, $\frac{\partial \mathbb{E}_j V_j{}^{init}(a_j', \tilde{P}_j, l_j, P_j')}{\partial l_j}$.

Next, to analyze how the cutoffs vary with firm age analytically, we can use the first-order stochastic dominance of the posterior distribution across different firm ages.[2] Given (1.1) and the log normality assumption, there is a point $\hat{P} \equiv \frac{\bar{\nu}^{old}\sigma^{young} - \bar{\nu}^{young}\sigma^{old}}{\sigma^{young} - \sigma^{old}}$ for $\ln P$, with which the cumulative distribution functions F for the posteriors of young and old firms follow: $F^{old}(\ln P) \geq (\leq) F^{young}(\ln P)$ if $\ln P \geq (\leq) \hat{P}$.

Suppose the productivity cutoffs $\mathcal{P}^{h,q,l}$ are given as constant. Given $\frac{\partial \hat{P}}{\partial \tilde{P}} = \frac{\sigma^{young}\sigma^{old}(a^{old} - a^{young})}{\sigma^{young} - \sigma^{old}} > 0$, as $\tilde{P}$ increases, there will be a point after which the middle cutoff $\mathcal{P}^q$ (for worker quits) goes below $\hat{P}$. Let $\bar{\tilde{P}}^H$ denote this point of $\tilde{P}$. Conversely, as $\tilde{P}$ decreases, there will be another point after which the lower cutoff $\mathcal{P}^l$ (for layoffs) goes above $\hat{P}$, which is denoted by $\bar{\tilde{P}}^L$.[3] Given (C.22), the marginal value of a labor input increases in P (with the constant productivity cutoffs $\mathcal{P}^{h,q,l}$ assumed as before). Thus, the condition $\mathbb{E}_j^{old}[\frac{\partial V^{init}(a_j', \tilde{P}_j, l_j, P_j')}{\partial l_j}] \geq (\leq) \mathbb{E}_j^{young}[\frac{\partial V^{init}(a_j', \tilde{P}_j, l_j, P_j')}{\partial l_j}]$ holds if $\tilde{P}_j \geq \bar{\tilde{P}}^H$ ($\tilde{P}_j \leq \bar{\tilde{P}}^L$). This implies, from (C.11), (C.15), (C.16), that the productivity cutoffs need to differ between young and old firms. In particular, they are adjusted to be lower for older firms if $\tilde{P}_j \geq \bar{\tilde{P}}^H$ and for younger firms if $\tilde{P}_j \leq \bar{\tilde{P}}^L$.

[2]The exact productivity cutoffs can only be determined numerically as an endogenous function of firm state variables.

[3]The exact values of $\bar{\tilde{P}}^H$ and $\bar{\tilde{P}}^L$ can be determined numerically.

D Wage Differentials: Baseline vs. Counterfactual

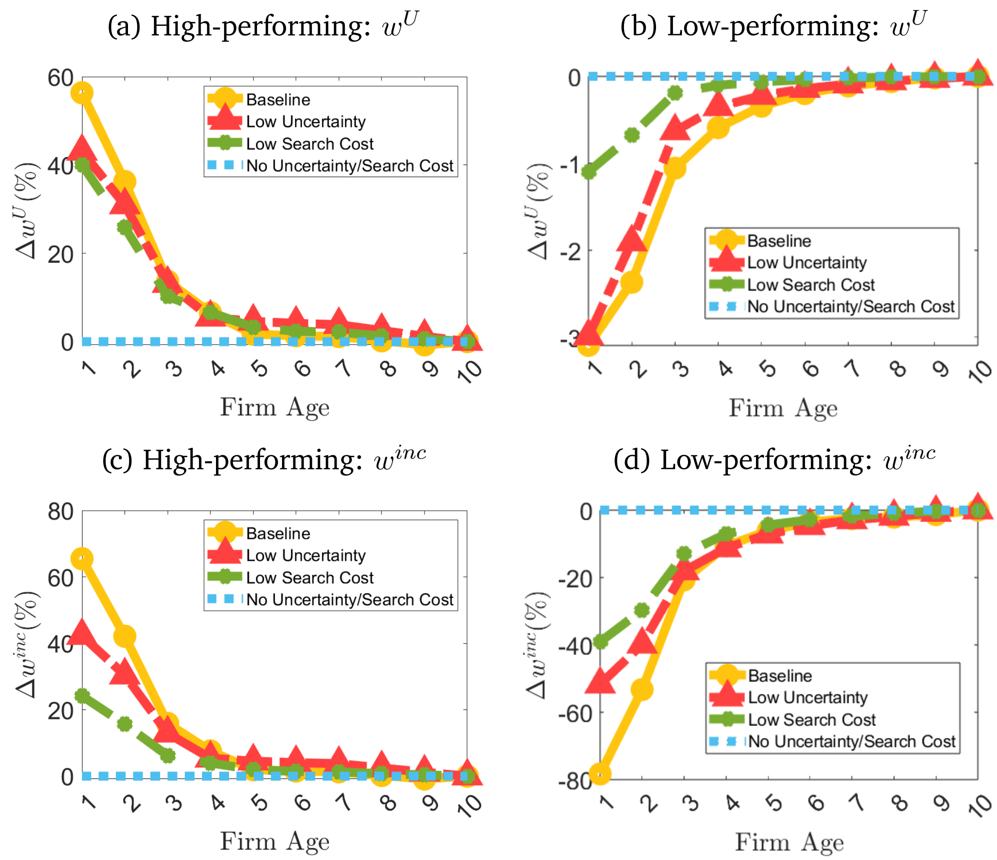

Figure D.1: Baseline vs. Counterfactual (Low Uncertainty, Search Cost)

Figure D.1 compares the wage differentials for young firms between the baseline economy and two counterfactual scenarios. It illustrates the wages of unemployed workers (top) and incumbent workers (bottom) at high-performing firms (left) and low-performing firms (right) with the same $(\tilde{P}_{j,-1}, l_{j,-1}, P_j)$

[3]This figure shows wages for unemployed workers (top) and incumbent workers (bottom) at high-performing firms (left) and low-performing firms (right) with the same $(\tilde{P}_{j,-1}, P_j, l_{j,-1})$ across different ages in the counterfactual economy, under conditions of low uncertainty (red), low search friction (green), and no uncertainty or search friction (blue). Wage differentials are presented as percentage differences relative to those paid by the oldest firm (age 10).

across different firm ages in the counterfactual economy. The analysis is conducted under three conditions: low uncertainty (red), low search frictions (green), and no uncertainty or search frictions (blue). Wage differentials are expressed as percentage differences relative to those paid by the oldest firms (age 10) in this figure.

Consistent with the main findings, wage differentials are reduced in both counterfactuals with either lower uncertainty or search frictions. Note that if there is no uncertainty or no search friction, the wage differentials across firm age become zero.

E Data Appendix

E.1 Longitudinal Business Database (LBD)

The LBD tracks the universe of U.S. business establishments and firms that have at least one paid employee, annually from 1976 onward. Establishments that are owned by a parent firm are grouped under a common firm identifier, which allows me to aggregate establishment-level activities to the firm level. The LBD contains basic information such as employment, payroll, revenue, NAICS codes, employer identification numbers, business name, and location, which enables me to measure firm size, age, entry, exit, productivity, and employment growth.[4]

E.1.1 Longitudinal Firm Identifiers

One limitation of the LBD is the lack of longitudinally consistent firm identifiers.[5] However, longitudinal consistency of firm identifiers is necessary for my analysis to track firms' history of performance as well as to estimate noise components in firm type learning process. Therefore, I construct and use longitudinal firm identifiers following Dent et al. (2018). Henceforth, I will use the term "firm identifier" to refer to the longitudinal firm identifiers constructed using this method.

[4] Jarmin and Miranda (2002), Haltiwanger et al. (2016), and Chow et al. (2021) contain more detailed information about the LBD. Fort and Klimek (2018) construct time-consistent NAICS codes for LBD establishments after the implementation of a change from the SIC to NAICS in 1997.

[5] Although the redesigned LBD has a new firm identifier that links firms across time by correcting previous firm identifiers that are recycled in the old LBD, it is still not yet a true longitudinal identifier and has not yet resolved firm reorganization issues. See more discussion in Chow et al. (2021).

E.2 Longitudinal Employer Household Dynamics (LEHD)

The LEHD is constructed from quarterly Unemployment Insurance (UI) system wage reports of states participating in the program, which collect quarterly earnings and employment information, along with demographic information.[6] The data cover over 95 percent of private sector workers, and the length of time series varies across states covered by the LEHD. I have access to 29 states covering over 60 percent of U.S. private sector employment.[7] The data enable me to identify worker heterogeneity, employment history, and job mobility. Linking the LEHD to the LBD with a crosswalk between employer identification numbers (EINs) and state-level employer identification numbers (SEINs), I track employer information for each job. The UI data, the main source of the LEHD, assign firms a state-level employer identification number (SEIN) that captures the activity of a firm within a state.

E.2.1 Main Jobs

The LEHD defines a job as the presence of an individual-employer match, with earnings defined as the amount earned from that job during the quarter. However, it does not record the start and end dates of a job, which makes the total number of weeks during that quarter unknown. To avoid potential bias from this, I follow the literature and restrict my analysis to full-quarter main jobs that give the highest earnings in a given quarter and are present for the quarter prior to and the quarter after the focal quarter. For any worker-quarter pairs that are associated with multiple jobs paying the same earnings,

[6] The earnings data in the LEHD are reported on a quarterly basis, which include all forms of compensation that are taxable.

[7] The 29 states are AL, AZ, CA, CO, CT, DE, ID, IN, KS, MD, ME, ND, NE, NJ, NM, NV, NY, OH, OK, OR, PA, SD, TN, TX, UT, VA, WA, WI, and WY.

I pick the job that shows up the most frequently in the worker's job history. This leaves one main job observation for each worker-quarter pair.

E.2.2 Previous Employment Status

Following Haltiwanger et al. (2018), I can identify workers' previous job using a within/adjacent quarter approach, which allows for a brief nonemployment period between workers' last day on the previous job and their first day on the contemporaneous job. Therefore, workers are identified as previously employed if they had at least one full-quarter job within the most recent three quarters before t, and as non-employed if they had no full-quarter jobs within those three quarters.

Note that restricting the sample to full-quarter main jobs makes use of the three-quarter duration to define previous jobs. For notational convenience, let $(t - q1)$ denote the quarter prior to t, and $(t - q2)$ denote two quarters prior to t, and so on. If a worker had any full-quarter jobs at either $(t - q1)$ or $(t-q2)$, this implies that the worker must have moved to the contemporaneous job within quarter $(t - q1)$. The latter could happen if the worker had some overlapping period between $(t - q1)$ and t in job transition. If a worker had any full-quarter jobs at $(t - q3)$, this means that the worker must have left the job at $(t - q2)$, had a brief nonemployment period between $(t - q2)$ and $(t - q1)$, and joined the contemporaneous job at $(t - q1)$. Alternatively, the within quarter approach identifies workers as previously employed if they had at least one full-quarter job within the latest two quarters before t, where the previous job is defined by the most recent main full-quarter job within the most recent two quarters before t.

In the LEHD, I identify workers who had no employment in any states during the previous period, i.e., those who had no earnings from any states in any of the three most recent quarters before time t, as unemployed. For this group, I set their previous employer fixed effect to zero and introduce a dummy variable indicating their non-employment status. Additionally, for workers employed in states beyond the scope of my data in the previous period, where I lack information about their previous employer and earnings, I set the previous employer fixed effect to zero and include a dummy variable for their employment status.

E.3 Summary Statistics

Table E1: Summary Statistics

A. Worker-year Sample	Mean (sd)	B. Firm-year Sample	Mean (sd)
Worker Age	40.05	Firm Size	10.42
	(14.67)		(50.2)
Earnings (2009$)	9,670	Firm Age	5.492
	(27,830)		(3.347)
Earnings (log, 2009$)	8.697	Revenue (thousands, 2009$)	1,633
	(1.027)		(7,736)
Job Tenure (years)	3.66	Revenue Prod. (log, 2009$)	4.764
	(2.6)		(1.041)
Education	2.68	Employment Growth	0.0174
	(1.025)		(0.382)
Observations	50,170,000	Observations	6,959,000

Note: The table presents summary statistics for the main regression samples. Panel A displays statistics for the worker-year level sample, while Panel B presents statistics for the firm-year level sample. The first row of each variable indicates the mean, and the second row (in brackets) displays the standard deviation. Jobs are defined by the full-quarter main job in the first quarter of each year. Education categorizes workers based on their highest level of education attainment (1 - Less than high school, 2 - High school, 3 - Some college, 4 - Bachelor's degree or higher). All nominal variables are adjusted to 2009 dollars. Observation counts are rounded to the nearest 10,000 to avoid potential disclosure risks.

F Full Tables

Table F2: Wage Differentials for Young Firms

	Earnings Residuals	Earnings Residuals
Young	-0.002***	-0.003***
	(0.001)	(0.001)
Young $\times$ High performing	0.015***	0.016***
	(0.001)	(0.001)
High performing	0.002	0.002
	(0.001)	(0.001)
Average Firm Prod. (up to $t-1$)	0.009***	0.012***
	(0.001)	(0.001)
Current Prod. (at t)	0.020***	0.015***
	(0.001)	(0.001)
Firm Size (at t)	0.017***	
	(0.001)	
Firm Size (at $t-1$)		0.013***
		(0.001)
Previous Employer (AKM)	0.267***	0.270***
	(0.001)	(0.001)
Observations	50,170,000	50,170,000
Fixed effects	g, s	g, s
Controls	Full (current size)	Full (lagged size)

Note: The table reports the full results for the main earnings regression. Firm controls include past-average productivity, current productivity, and log employment size. Controls associated with worker's previous employment status are the AKM firm fixed effect associated with the previous employer and a dummy for non-employed workers in the previous period. Observation counts are rounded to the nearest 10,000 to avoid potential disclosure risks. Estimates for constant, industry (g), state (s) fixed effects, the coefficient of the indicator for worker's previous non-employment status are suppressed. Observations are unweighted.

Table F3: The Effect of Uncertainty on Young Firms' Wage Differentials

	Earnings Residuals	Earnings Residuals
Young	-0.001	-0.001
	(0.001)	(0.001)
$\times$ Uncertainty (at t)	-0.004	-0.004
	(0.002)	(0.002)
Young $\times$ High performing	0.012***	0.012***
	(0.002)	(0.002)
$\times$ Uncertainty (at t)	0.006***	0.006***
	(0.002)	(0.002)
High performing	-0.022***	-0.022***
	(0.001)	(0.001)
Uncertainty	-0.033***	-0.033***
	(0.001)	(0.001)
Uncertainty $\times$ High performing	0.028***	0.028***
	(0.001)	(0.001)
Average Firm Prod. (up to $t-1$)	0.009***	0.011***
	(0.000)	(0.000)
Current Prod. (at t)	0.020***	0.016***
	(0.000)	(0.000)
Firm Size (at t)	0.012***	
	(0.000)	
Firm Size (at $t-1$)		0.010***
		(0.000)
Previous Employer (AKM)	0.269***	0.271***
	(0.000)	(0.000)
Observations	50,170,000	50,170,000
Fixed effects	g, s	g, s
Controls	Full (current size)	Full (lagged size)

Note: The table reports the full results for the earnings regression interacted with industry-level uncertainty. The set of controls and fixed effects remain the same as in the baseline Table F2. Each column uses either current or lagged firm size. Observation counts are rounded to the nearest 10,000 to avoid potential disclosure risks. Estimates for constant, fixed effects, the coefficient of the indicator for worker's previous non-employment status are suppressed. Observations are unweighted.

G Robustness Test for the Baseline Regression

Table G4: Wage Differentials for Young Firms (excluding firm size)

	Earnings Residuals	Earnings Residuals
Young	-0.006***	-0.007***
	(0.001)	(0.001)
Young $\times$ High performing	0.013***	0.015***
	(0.001)	(0.001)
High performing	0.005***	0.004***
	(0.001)	(0.001)
Average Firm Prod. (up to $t-1$)	0.016***	0.006***
	(0.001)	(0.001)
Current Prod. (at t)		0.015***
		(0.001)
Previous Employer (AKM)	0.283***	0.281***
	(0.001)	(0.001)
Observations	50,170,000	50,170,000
Fixed effects	g, s	g, s

Note: The table reports the earnings regression results. Firm controls include past-average productivity and current productivity (but not log employment size). Controls associated with worker's previous employment status are AKM firm fixed effect associated with the previous employer and a dummy for non-employed workers in the previous period. Observation counts are rounded to the nearest 10,000 to avoid potential disclosure risks. Estimates for constant, industry (g), state (s) fixed effects, the coefficient of the indicator for worker's previous non-employment status are suppressed. Observations are unweighted.

Table G5: Wage Differentials for Young Firms (propensity score weighted)

	Earnings Residuals	Earnings Residuals	Earnings Residuals	Earnings Residuals
Young	-0.007***	-0.008***	-0.003***	-0.003***
	(0.001)	(0.001)	(0.001)	(0.001)
Young $\times$ High performing	0.015***	0.018***	0.019***	0.019***
	(0.001)	(0.001)	(0.001)	(0.001)
High performing	0.004***	0.002*	-0.000	0.000
	(0.001)	(0.001)	(0.001)	(0.001)
Avg. Firm Prod (up to $t-1$)	0.017***	0.003***	0.005***	0.009***
	(0.001)	(0.001)	(0.001)	(0.001)
Current Prod. (at t)		0.021***	0.027***	0.021***
		(0.001)	(0.001)	(0.001)
Firm Size			0.020***	
			(0.000)	
Firm Size (at $t-1$)				0.015***
				(0.000)
Previous Employer (AKM)	0.281***	0.278***	0.266***	0.269***
	(0.001)	(0.001)	(0.001)	(0.001)
Observations	50,170,000	50,170,000	50,170,000	50,170,000
Fixed effects	g, s	g, s	g, s	g, s

Note: The table reports results for regression of earning residuals on young firm and High performing indicators. Firm controls include past-average productivity level, current productivity level, and log employment size. Controls associated with worker's previous employment status are AKM firm fixed effect associated with the previous employer and a dummy for non-employed workers in the previous period. Observation counts are rounded to the nearest 10,000 to avoid potential disclosure risks. Estimates for constant, industry, state fixed effects, the coefficient of the indicator for worker's previous non-employment status are suppressed. Observations are weighted with inverse propensity score weights of author's own construction.

Table G6: Wage Differentials for Young Firms (bootstrapped standard errors)

	Earnings Residuals	Earnings Residuals	Earnings Residuals
Young	-0.006***	-0.007***	-0.002***
	(0.001)	(0.001)	(0.001)
Young $\times$ High performing	0.013***	0.015***	0.015***
	(0.002)	(0.002)	(0.002)
High performing	0.005***	0.004*	0.002
	(0.002)	(0.002)	(0.002)
Average Firm Prod. (up to $t-1$)	0.016***	0.006***	0.009***
	(0.000)	(0.001)	(0.001)
Current Prod. (at t)		0.015***	0.020***
		(0.001)	(0.001)
Firm Size			0.017***
			(0.000)
Previous Employer (AKM)	0.283***	0.281***	0.267***
	(0.001)	(0.001)	(0.001)
Observations	50,170,000	50,170,000	50,170,000
Fixed effects	g, s	g, s	g, s

Note: The table reports the earnings regression results. Firm controls include past-average productivity, current productivity, and log employment size. Controls associated with worker's previous employment status are the AKM firm fixed effect associated with the previous employer and a dummy for non-employed workers in the previous period. Note that the only difference from the main table is the standard errors. Observation counts are rounded to the nearest 10,000 to avoid potential disclosure risks. Estimates for constant, industry (g), state (s) fixed effects, the coefficient of the indicator for worker's previous non-employment status are suppressed. Observations are unweighted.

Table G7: Wage Differentials for Young Firms (with previous earnings)

	(1) Earnings Residuals	(2) Earnings Residuals	(3) Earnings Residuals	(4) Earnings Residuals	(5) Earnings Residuals	(6) Earnings Residuals	(7) Earnings Residuals
Young	-0.003***	-0.003***	-0.004***	-0.005***	-0.005***	-0.000	-0.001
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
Young $\times$ High performing	0.014***	0.014***	0.016***	0.018***	0.018***	0.019***	0.019***
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
High performing	0.001***	0.001***	-0.004	-0.010***	-0.010***	-0.012***	-0.012***
	(0.002)	(0.002)	(0.001)	(0.002)	(0.002)	(0.001)	(0.001)
Average Prod. (up to $t-1$)	0.006***	0.003***	0.006***	-0.009***	-0.007***	-0.005***	-0.002***
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
Current Prod. (at t)		0.005***	0.012***		-0.003***	0.003***	0.000
		(0.001)	(0.001)		(0.001)	(0.001)	(0.001)
Firm Size (at t)			0.028***			0.018***	
			(0.001)			(0.000)	
Firm Size (at $t-1$)							0.014***
							(0.000)
Previous Employer (AKM)				0.155***	0.155***	0.141***	0.160***
				(0.001)	(0.001)	(0.001)	(0.001)
Previous Earnings	0.194***	0.194***	0.190***	0.167***	0.167***	0.167***	0.165***
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
Observations	50,170,000	50,170,000	50,170,000	50,170,000	50,170,000	50,170,000	50,170,000
Fixed effects	g, s	g, s	g, s	g, s	g, s	g, s	g, s

Note: The table reports the earnings regression results. Firm controls include past-average productivity, current productivity, and log employment size. Controls associated with worker's previous employment status are previous earning level (in all columns) along with AKM firm fixed effect associated with the previous employer (in the last four columns) and a dummy for non-employed workers in the previous period. Observation counts are rounded to the nearest 10,000 to avoid potential disclosure risks. Estimates for constant, industry, state fixed effects, the coefficient of the indicator for worker's previous non-employment status are suppressed. Observations are unweighted.

Table G8: Wage Differentials for Young Firms (worker skill controlled)

	Earnings Residuals	Earnings Residuals	Earnings Residuals
Young	-0.008***	-0.009***	-0.004***
	(0.001)	(0.001)	(0.001)
Young $\times$ High performing	0.016***	0.017***	0.017***
	(0.001)	(0.001)	(0.001)
High performing	0.004***	0.002*	0.002
	(0.001)	(0.001)	(0.001)
Average Firm Prod. (up to $t-1$)	0.015***	0.005***	0.008***
	(0.001)	(0.001)	(0.001)
Current Prod. (at t)		0.014***	0.020***
		(0.001)	(0.001)
Firm Size			0.017
			(0.000)
Previous Employer (AKM)	0.281***	0.279***	0.265***
	(0.001)	(0.001)	(0.001)
Observations	50,170,000	50,170,000	50,170,000
Fixed effects	g, s	g, s	g, s

Note: The table reports the earnings regression results. Firm controls include past-average productivity, current productivity, and log employment size. Controls associated with worker's previous employment status are AKM firm fixed effect associated with the previous employer and a dummy for non-employed workers in the previous period. Note that the only difference from the main table is the earnings residuals, which are computed after additionally controlling for worker skills in the first stage. Observation counts are rounded to the nearest 10,000 to avoid potential disclosure risks. Estimates for constant, industry (g), state (s) fixed effects, the coefficient of the indicator for worker's previous non-employment status are suppressed. Observations are unweighted.

Table G9: Wage Differentials for Young Firms (with young firm risks)

	Earnings Residuals	Earnings Residuals	Earnings Residuals
Young	-0.006***	-0.006***	-0.003***
	(0.001)	(0.001)	(0.001)
Young $\times$ High performing	0.013***	0.015***	0.015***
	(0.001)	(0.001)	(0.001)
High performing	0.005***	0.004***	0.002
	(0.001)	(0.001)	(0.001)
Average Firm Prod. (up to $t-1$)	0.016***	0.006***	0.009***
	(0.001)	(0.001)	(0.001)
Firm Prod. (at t)		0.015***	0.020***
		(0.001)	(0.001)
Firm Size			0.017***
			(0.000)
Previous Employer (AKM)	0.283***	0.281***	0.267***
	(0.001)	(0.001)	(0.001)
Young Firm Risks	-0.009	-0.005	0.005
	(0.002)	(0.002)	(0.002)
Observations	50,170,000	50,170,000	50,170,000
Fixed effects	g, s	g, s	g, s

Note: The table reports the earnings regression results. Firm controls include past-average productivity, current productivity, and log employment size. Controls associated with worker's previous employment status are AKM firm fixed effect associated with the previous employer and a dummy for non-employed workers in the previous period. In addition, the dispersion of productivity shocks for young firms is included to control for the level of unobserved risks associated with them. Observation counts are rounded to the nearest 10,000 to avoid potential disclosure risks. Estimates for constant, industry (g), state (s) fixed effects, the coefficient of the indicator for worker's previous non-employment status are suppressed. Observations are unweighted.

Table G10: Wage Differentials for Young Firms (firm-level previous employment)

	Earnings Residuals	Earnings Residuals	Earnings Residuals
Young	-0.004***	-0.005***	-0.000
	(0.001)	(0.001)	(0.001)
Young $\times$ High performing	0.013***	0.014***	0.015***
	(0.001)	(0.001)	(0.001)
High performing	0.007***	0.006***	0.003**
	(0.001)	(0.001)	(0.001)
Average Firm Prod. (up to $t-1$)	0.022***	0.010***	0.013***
	(0.001)	(0.001)	(0.001)
Firm Prod. (at t)		0.017***	0.023***
		(0.001)	(0.001)
Firm Size			0.020***
			(0.000)
Previous Employer (AKM)	0.281***	0.279***	0.264***
	(0.001)	(0.001)	(0.001)
Observations	50,170,000	50,170,000	50,170,000
Fixed effects	g, s	g, s	g, s

Note: The table reports the earnings regression results. Firm controls include past-average productivity, current productivity, and log employment size. Controls associated with worker's previous employment status are AKM firm fixed effect associated with the previous employer (estimated at the firm level, rather than the SEIN level) and a dummy for non-employed workers in the previous period. Observation counts are rounded to the nearest 10,000 to avoid potential disclosure risks. Estimates for constant, industry (g), state (s) fixed effects, the coefficient of the indicator for worker's previous non-employment status are suppressed. Observations are unweighted.

Table G11: Wage Differentials for Young Firms (firm-level regression)

	Earnings Residuals	Earnings Residuals	Earnings Residuals	Earnings Residuals
Young	-0.010***	-0.010***	-0.010***	-0.010***
	(0.001)	(0.001)	(0.001)	(0.001)
Young $\times$ High performing	0.016***	0.017***	0.019***	0.020***
	(0.001)	(0.001)	(0.001)	(0.001)
High performing	0.018***	0.018***	0.016***	0.017***
	(0.001)	(0.001)	(0.001)	(0.001)
Average Firm Prod. (up to $t-1$)	0.033***	0.049***	0.029***	0.043***
	(0.001)	(0.001)	(0.001)	(0.001)
Current Prod. (at t)	0.072***	0.055***	0.0746***	0.0586***
	(0.001)	(0.001)	(0.001)	(0.001)
Firm Size (at t)	0.067***		0.067***	
	(0.000)		(0.001)	
Firm Size (at $t-1$)		0.0576***		0.0562***
		(0.000)		(0.001)
Observations	6,959,000	6,959,000	6,959,000	6,959,000
Fixed effects	g, s	g, s	g, s	g, s
Weighted	No	No	Yes	Yes

Note: The table reports the firm-level earnings regression results. The dependent variable is the average earnings residuals across workers within each firm. As before, firm-level characteristics are controlled, including past-average productivity, current productivity, and log employment size. Observation counts are rounded to the nearest 10,000 to avoid potential disclosure risks. Estimates for constant, industry (g), state (s) fixed effects. Observations are unweighted in the first two columns, and are weighted by inverse propensity score weights in the last two columns.

Table G12: The Effect of Uncertainty on Young Firms' Wage Differentials (lagged value)

	Earnings Residuals	Earnings Residuals
Young	-0.001	-0.002
	(0.002)	(0.002)
$\times$ Uncertainty (at $t-1$)	-0.005**	-0.004*
	(0.002)	(0.002)
Young $\times$ High performing	0.003	0.005**
	(0.002)	(0.002)
$\times$ Uncertainty (at $t-1$)	0.016***	0.015***
	(0.003)	(0.003)
Observations	50,170,000	50,170,000
Fixed effects	g, s	g, s
Controls	Full (current size)	Full (lagged size)

Note: The table reports the earnings regression interacted with industry-level uncertainty (lagged value). The set of controls remains the same as in the baseline Table F2. The first column uses the current size, and the second column uses the lagged value. Observation counts are rounded to the nearest 10,000 to avoid potential disclosure risks. Estimates for constant, industry, state fixed effects, the coefficient of the indicator for worker's previous non-employment status are suppressed. Observations are unweighted.

H Impact of Earnings Differentials on Firm Outcomes

In this section, I examine the relationship between earnings differentials and firm outcomes (hiring or employment growth) as follows:

$$Y_{jt} = \beta\hat{\epsilon}_{jt} + Z_{jt}\gamma + \mu_{g(j,t)} + \mu_{s(j,t)} + \alpha + \xi_{jt}, \tag{H.23}$$

where Y_{jt} is either the number of new hires or employment growth of firm j, $\hat{\epsilon}_{jt}$ denotes the average earnings residuals, averaging $\hat{\epsilon}_{it}$ across workers i at firm $j(i,t)$, Z_{jt} is a vector of firm controls (age, size, and productivity), and $\mu_{g(j,t)}$ and $\mu_{s(j,t)}$ are industry and state fixed effects, respectively. The top panel (A) in Table H13 shows the results, indicating a negative association between earnings residuals and both firm hiring and employment growth, independent of firm age, size, and productivity effects. This supports interpreting earnings differentials as stemming from uncertain job prospects, ruling out other hypotheses such as performance pay or surplus sharing. The results are robust using $\hat{P}_{jt}$ estimated in (**??**) as shown in the bottom panel (B) of the table. Furthermore, the results are robust to applying inverse propensity score weights, as presented in the following Table H14.

Table H13: The Effect of Wage Differentials on Firm Outcomes

A. Productivity (P)	Hire (firm)	Hire (SEIN)	ΔEmp (Δlog)	ΔEmp (DHS)
Earnings Residuals	-0.520***	-0.387***	-0.015***	-0.018***
	(0.020)	(0.024)	(0.000)	(0.000)
Firm Productivity	0.588***	0.302***	0.092***	0.102***
	(0.033)	(0.035)	(0.000)	(0.000)
Firm Size	7.964***	6.230***	-0.040***	-0.048***
	(0.133)	(0.068)	(0.000)	(0.000)
Firm Age	0.039***	0.007	-0.001***	-0.001***
	(0.008)	(0.008)	(0.000)	(0.000)
Observations	6,959,000	6,959,000	6,959,000	6,959,000
Fixed effects	g, s	g, s	g, s	g, s
Controls	P, size, age	P, size, age	P, size, age	P, size, age

B. Productivity ($\hat{P}$)	Hire (firm)	Hire (SEIN)	ΔEmp (Δlog)	ΔEmp (DHS)
Earnings Residuals	-0.498***	-0.369***	-0.012***	-0.015***
	(0.020)	(0.024)	(0.000)	(0.000)
Average Firm Prod.	-0.904***	-0.845***	-0.095***	-0.108**
	(0.035)	(0.050)	(0.000)	(0.001)
Current Firm Prod.	1.31***	0.924***	0.176***	0.197***
	(0.039)	(0.044)	(0.000)	(0.001)
Firm Size	7.998***	6.259***	-0.035***	-0.043***
	(0.134)	(0.068)	(0.000)	(0.000)
Firm Age	0.042***	0.009	-0.001***	-0.001***
	(0.008)	(0.008)	(0.000)	(0.000)
Observations	6,959,000	6,959,000	6,959,000	6,959,000
Fixed effects	g, s	g, s	g, s	g, s
Controls	$\hat{P}$, $\tilde{P}$, size, age	$\hat{P}$, $\tilde{P}$, size, age	$\hat{P}$, $\tilde{P}$, size, age	$\hat{P}$, $\tilde{P}$, size, age

Note: The table reports the effect of earnings residuals on firm-level outcomes. Firm controls include firm productivity, log employment size, and age in the top panel (A), and the past-average productivity level ($\tilde{P}_{j,-1}$, up to $t-1$) and the current value (P_j, at t) of the estimated firm productivity ($\hat{P}_j$) is used for firm productivity in the bottom panel (B). New hires are either the firm-level total new hire (first column) or the average of the SEIN-level new hires (second column). Employment growth is either the log-difference (third column) or the DHS growth (last column) of firm employment size. Observation counts are rounded to the nearest 10,000 to avoid potential disclosure risks. Estimates for constant, and industry (g), state (s) fixed effects are suppressed. Observations are unweighted.

Table H14: The Effect of Wage Differentials on Firm Outcomes (propensity score weighted)

A. Productivity (P)	Hire (firm)	Hire (SEIN)	ΔEmp (Δlog)	ΔEmp (DHS)
Earnings Residuals	-0.285***	-0.275***	-0.016***	-0.019***
	(0.010)	(0.041)	(0.000)	(0.000)
Firm Prod.	0.370***	0.254***	0.086***	0.095***
	(0.014)	(0.030)	(0.000)	(0.000)
Firm Size	5.426***	4.839***	-0.055***	-0.064***
	(0.071)	(0.058)	(0.000)	(0.000)
Firm Age	0.009**	-0.014*	-0.001***	-0.001***
	(0.004)	(0.007)	(0.000)	(0.000)
Observations	6,959,000	6,959,000	6,959,000	6,959,000
Fixed effects	g, s	g, s	g, s	g, s
Controls	P, size, age	P, size, age	P, size, age	P, size, age

B. Productivity ($\hat{P}$)	Hire (firm)	Hire (SEIN)	ΔEmp (Δlog)	ΔEmp (DHS)
Earnings Residuals	-0.274***	-0.266***	-0.014***	-0.016***
	(0.010)	(0.042)	(0.000)	(0.000)
Average Prod.	-0.515***	-0.504***	-0.092***	-0.103***
	(0.022)	(0.052)	(0.001)	(0.001)
Current Prod.	0.793***	0.646***	0.168***	0.187***
	(0.021)	(0.043)	(0.001)	(0.001)
Firm Size	5.452***	4.864***	-0.049***	-0.058***
	(0.071)	(0.059)	(0.000)	(0.000)
Firm Age	0.009**	-0.014*	-0.001***	-0.001***
	(0.004)	(0.007)	(0.000)	(0.000)
Observations	6,959,000	6,959,000	6,959,000	6,959,000
Fixed effects	g, s	g, s	g, s	g, s
Controls	$\hat{P}$, $\tilde{P}$, size, age	$\hat{P}$, $\tilde{P}$, size, age	$\hat{P}$, $\tilde{P}$, size, age	$\hat{P}$, $\tilde{P}$, size, age

Note: All remains the same as in Table H13, except for the observations weighted with inverse propensity score weights of author's own construction.

Table I15: Aggregate Implications of Uncertainty (long run)

Industry FE	(1) Entry	(2) Young firm share	(3) HG young firm share	(4) HG young avg. growth	(5) Prod.
Uncertainty	-0.126***	-0.372***	-0.183***	-0.279***	-2.06***
	(0.020)	(0.071)	(0.026)	(0.046)	(0.288)
Observations	250	250	250	250	250
Fixed effects	g, t	g, t	g, t	g, t	g, t

Note: The table reports results from regressions of the long-run value (industry fixed effects) of firm entry, the share of young firms, and the share and growth of high-growth young firms, and aggregate productivity in each column. Each measure is regressed on the counterpart for uncertainty at the industry level. Observation counts are rounded to the nearest 50 to mitigate potential disclosure risks. Estimates for the constant term are suppressed. Observations are unweighted.

I Long-Run Relationship

I further examine the long-run relationship in the steady-state economy of the model by estimating the industry fixed effects of the variables, which proxy the steady-state level for each industry. I then run the following cross-sectional regression:

$$\hat{\delta}_g^{Y} = \beta\hat{\delta}_g^{Uncertainty} + \alpha + \epsilon_g, \tag{I.24}$$

where δ_g^Y and $\delta_g^{Uncertainty}$ represent the industry fixed effects of Y and uncertainty, respectively.[8]

The result is displayed in Table I15. This confirms a negative and statistically significant correlation between uncertainty and the aggregate variables, even in the long run.

[8] The industry fixed effects of a variable X are estimated as follows: $X_{gt} = \delta_g^X + \delta_t^X + \alpha^X + \varepsilon_{gt}^X$, with year fixed effects δ_t^X controlled.

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