Competition, Firm Innovation, and Growth under Imperfect Technology Spillovers*

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Abstract

We study how friction in learning others' technology, termed "imperfect technology spillovers," incentivizes firms to use different types of innovation and impacts the implications of competition through changes in innovation composition. We develop an endogenous growth model that introduces strategic innovation decisions, where multi-product firms improve their products via internal innovation and enter new product markets through external innovation. When learning others' technology takes time due to this friction, increased competitive pressure leads firms with technological advantages to intensify internal innovation to protect their markets, thereby reducing external innovation of rivals. Using the U.S. administrative firm-level data, we provide regression results supporting the model predictions. Our findings highlight the importance of strategic firm innovation choices and changes in their composition in shaping the aggregate implications of competition.

Keywords: competition, innovation, technology spillover, endogenous growth

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1 Introduction

Innovations manifest in diverse forms, impacting firm performance and economic growth differently, and firms have different incentives for using them (Akcigit and Kerr, 2018; Garcia-Macia et al., 2019; Peters, 2020; Argente et al., 2024). Although studies on the effect of competition on innovation have a long-standing history, the results remain inconclusive and provide limited guidance on assessing the implications of competition through heterogeneous innovations (Aghion et al., 2005; Gilbert, 2006; Aghion and Griffith, 2008; Bloom et al., 2016; Autor et al., 2020). How do firms use different types of innovation when faced with increasing competition? How does this bring new insights into the aggregate implications of competition?

Our paper investigates these questions when learning others' technology takes time both theoretically and empirically. Theoretically, we develop an endogenous growth model with two types of innovation and imperfect technology spillovers. This model provides a micro-foundation for the effect of competition on firm innovation, decomposed into changes in the level and composition of two innovation types. Next, we link the administrative firm-level data to the patent database in the U.S. and document new facts about the composition changes of firm innovation in response to an exogenous increase in competition. Lastly, we calibrate the model and derive the aggregate implications of competition across different economies.

In the model, multi-product firms grow through two types of innovation—internal and external—subject to imperfect technology spillovers. Internal innovation improves existing product quality, while external innovation enables firms to enter new markets by displacing incumbents.[1] Individual external innovation contributes more to product quality improvement, thus to firm and aggregate growth than internal innovation.[2,3] Imperfect technology spillovers are a new element introduced in this class of models, capturing the time needed to learn from others' technologies.

The literature widely acknowledges the time-consuming nature of learning other firms' tech-

[1] An illustrative real-world example of external innovation is Apple's entry into the cell phone industry with the introduction of the iPhone back in 2007 when its major business was computer manufacturing. An example of internal innovation is Apple's improvement and production of iPhone 16 from iPhone 15.

[2] As highlighted by Bernard et al. (2010) and Akcigit and Kerr (2018), external innovation plays a pivotal role in driving growth. It is tightly connected to creative destruction and radical innovation, which play a crucial role in enhancing existing technologies. We present empirical evidence for this in a later section and in the Online Appendix.

[3] Garcia-Macia et al. (2019) demonstrate that, at the aggregate level, internal innovation contributes more to overall growth than external innovation, as the latter tends to succeed less frequently. Our model supports their findings at the aggregate level while also distinguishing the contributions of both types of innovation at the individual level.

nology (e.g., Lucas and Moll, 2014). Accordingly, we conceptualize innovation as occurring in two stages: learning existing technology and building upon it. Internal innovation bypasses the learning phase since the firm is already familiar with the technology, whereas external innovation necessitates this learning process. Thus, entering a market through external innovation entails learning before improving on incumbents' technology, which may require significant time and resources.[4] Our model uses lagged learning as a form of imperfect technology spillovers, requiring potential rivals to spend one period learning the frontier technology of incumbents. In other words, external innovation builds on one-period lagged technology.

Our model introduces several novel features. First, spillover friction creates a technology gap between incumbents' frontier technology and the one-period lagged technology that potential rivals can only learn. Second, incumbents can exploit this technology gap and strategically use internal innovation to protect their markets, labeled as the "*market-protection effect*."[5] Third, the strategic internal innovation of incumbents endogenously prevents rivals from entering their markets and stifles the rivals' external innovation, labeled as the "*technological barrier effect*." This distinguishes our model from others in the literature on firm innovation and specialization. Thus, competition induces a shift in the composition of firm innovation, driven by the strategic choices of firms and their endogenous interactions. Lastly, as a result, the aggregate effect of competition on overall innovation depends on the relative shifts in the two types of innovation.

The strength of this framework is that it allows multi-product firms to strategically use internal innovation while generating endogenous feedback effects on external innovation and entry decisions by others. Introducing imperfect technology spillovers in a multi-product firm setup achieves this, which is our main theoretical contribution.[6] In existing models of multi-product firms growing through product scope expansion, firms cannot protect their markets because rivals can instantly learn and copy frontier technology without any friction (Klette and Kortum, 2004; Akcigit and Kerr, 2018; Peters, 2020). Step-by-step innovation models generate an escape-competition motive, but

[4]For example, external innovation may require the processes of recruiting new employees to handle new technology, reallocating resources to new projects, training workers, and preparing production facilities for new products. In the real world, Apple took three years to enter the cell phone industry, even after leveraging their previously accumulated knowledge from the iPod development and production. Moreover, Apple has been trying to enter the car industry for over seven years. We provide empirical evidence for the learning time differences in the Online Appendix.

[5]In particular, firms with a technology gap increase internal innovation to defend themselves from competitors, while those without such a gap reduce it when competition increases exogenously.

[6]In this sense, our framework brings together quality-ladder and step-by-step innovation models.

they assume single-product firms (Aghion et al., 2001, 2005; Akcigit et al., 2018). This lacks the feedback effects of incumbents' innovation choices on the innovation of rivals attempting to enter a product market and does not capture the firm-level innovation composition observed in the data.

Furthermore, our model underscores the importance of comprehending changes in innovation composition to accurately evaluate the aggregate implications of competition. Unlike earlier models, internal innovation in our framework not only improves the product quality of the owners but also impedes external innovation of others and firm entry that contribute to firm and economic growth substantially. Thus, the shifts in innovation composition are crucial for overall outcomes, and ignoring such heterogeneity in innovation may obscure the true impact of competition.

Next, to validate our model predictions, we construct a unique dataset by combining the U.S. administrative firm-level data with the USPTO patent data from 1976 to 2016.[7] This dataset provides comprehensive information for the entire population of U.S. patenting firms. We use the rise in Chinese firm entry into U.S. domestic markets following China's WTO accession in 2001 as an exogenous competition shock, and use the self-citation ratio of patents as a measure of the likelihood that patents are used for internal innovation. We find regression results consistent with the model predictions. First, heightened competition increases internal innovation among firms with existing technological advantages but decreases external innovation across all firms. Second, the positive correlation between firm patenting and employment growth diminishes by 17.1% for innovation-intensive firms following the surge in competition, as more patents are used for internal innovation.[8] Third, firm entry rate is lower in industries with higher technological barriers based on the TFPR gap measure as in Aghion et al. (2005).

Lastly, to understand the aggregate implications, we calibrate our model to the U.S. manufacturing sector and conduct two main counterfactual exercises by increasing competitive pressure exogenously by outside firms: i) in the U.S. economy, and ii) in economies where external innovation costs exceed those in the U.S.[9] Both exercises yield qualitatively similar results at the firm level: firms increase (decrease) internal innovation for products with a (no) technology gap, while external innovation drops across all firms. However, the results differ for aggregate implications. Overall

[7]We construct our own crosswalk between the two datasets with name, address matching, and the internet search-aided algorithm as in Autor et al. (2020). This improves the match rates and provides the longest and longitudinally consistent crosswalk between patent assignees and LBD firms. See Ding et al. (2022) for details.

[8]The positive correlation between patenting and product add (or TFPR growth) is also muted for internal innovation.

[9]Additional counterfactual analysis of an increasing domestic firm entry is presented in Online Appendix F.

innovation—the aggregate-level R&D to sales ratio—experiences a decline in the U.S., where firms actively engage in external innovation with lower associated costs. In contrast, this result is reversed in an economy with higher external innovation costs. This is because the initial level of external innovation is minimal even in the absence of competitive pressure, and thus, the scope for a further decline in external innovation with increased competitive pressure is limited. Also, the aggregate growth rates attributed to domestic firms fall in both economies, even though the latter has seen an increase in overall innovation. This is because heightened competition endogenously elevates technological barriers and impedes external innovation by domestic incumbents and firm entry. This result is distinctive within our framework having heterogeneous innovation types and its impact.

Our paper provides a unified framework that facilitates the comparison of the effects of competition across different countries. Notably, this framework allows for mapping non-U.S. economies, such as the European economy, into the hypothetical economy with high external innovation costs and helps reconcile the disparate findings in literature.[10] The change in innovation composition resulting from the strategic choices by firms is an important margin to understand the heterogeneous impact of competition and its aggregate implications across diverse economic landscapes.

Related Literature Our paper brings new insights and findings to the large literature linking competition, firm innovation, and technology spillovers.

First, our paper is related to an extensive body of research on competition and innovation. The empirical literature finds mixed results (Aghion et al., 2005; Bloom et al., 2016; Hombert and Matray, 2018; Shu and Steinwender, 2019; Autor et al., 2020).[11] Some explore this dynamics through the lens of a Schumpeterian growth model with step-by-step innovation, where the "Schumpeterian effect" by the laggards and the "escape-competition effect" by neck-and-neck firms arise (Aghion et al., 2001, 2004, 2005, 2009; Akcigit et al., 2018). However, this model is rooted in several assumptions lacking data support and abstracts from discussing the composition of different innovations.[12] Others introduce a trapped-factor model in which rising competition reduces the opportunity cost of innovation (Bloom et al., 2013, 2021; Medina, 2022), but this channel may not be applicable to general cases (e.g., U.S.). Alternatively, Hombert and Matray (2018) elucidate the channel of

[10]For instance, heightened frictions associated with R&D or labor mobility for external innovation.
[11]See Shu and Steinwender (2019) for further details.
[12]The model assumes single-product firms, a single innovation type, or the immediate imitation by the laggards.

product differentiation by innovative firms, while Dhingra (2013) underscores the phenomenon of firms upgrading their product production process to avoid cannibalization in response to competition. Helpman (2023) illustrates ambiguous impacts on the innovation efforts of large multi-product firms, which depend on the level of their productivity and market shares. We contribute to this literature by providing a rich theoretical framework wherein multi-product firms leverage their technological barriers, strategically use internal innovation, and endogenously affect others' external innovation, along with new data evidence. In this setting, competition changes the composition of heterogeneous innovations and creates different aggregate implications depending on the relative shifts. Our results help reconcile the prior diverging findings and enrich our understanding of the complex effects of competition on innovation.

Second, our paper adds to another growing strand of literature that explores the diverse types of innovation undertaken by firms. Aghion et al. (2004) and Akcigit et al. (2018) add an entry margin, and similarly, Atkeson and Burstein (2010) introduce the notion of product and process innovations. However, all these models assume single-product firms, in which each firm (either incumbents or entrants) can only do one type of innovation. On the other hand, Klette and Kortum (2004) build a quality-ladder model of multi-product firms, albeit under the assumption of a single type of innovation. Another set of research has expanded the study of multi-product firms: Bernard et al. (2010) highlight the role of product switching in resource allocation; Akcigit and Kerr (2018) model multi-product firms conducting internal and external innovations and illuminate the distinct importance of external innovation in economic growth; Peters (2020) highlights the importance of creative destruction (external innovation) for alleviating the misallocation that arises from the accumulation of market power by incumbent firms (via internal innovation); and Dhingra (2013) and Argente et al. (2024) document the role of cannibalization in firm innovation decisions. In addition, Garcia-Macia et al. (2019) and Atkeson and Burstein (2019) explore the impacts of varied innovation types on growth and policy implications. Our contribution arises from adding the learning friction that creates the strategic motive of firms using internal innovation and generating an endogenous feedback effect on the external innovation and entry of firms (both incumbents and entrants). Our research also broadens the empirical scope by offering insights into the strategic use of different innovations by multi-product firms and matching the observed changes in firm-level innovation composition in data.

Lastly, our paper contributes to studies on technology gap and spillovers. Previous studies have established that the technology gap between firms plays a crucial role in shaping firm innovation incentives and policy implications (Aghion et al., 2001, 2005; Dinopoulos and Syropoulos, 2007; Aghion and Griffith, 2008; Acemoglu and Akcigit, 2012; Akcigit et al., 2018); explored the trend of diminishing knowledge diffusion from market leaders to laggards (Andrews et al., 2016; Bessen et al., 2020; Akcigit and Ates, 2021; Arora et al., 2021; Akcigit and Ates, 2023); and presented various phenomena that broadly align with this trend (Shapiro, 2000; De Ridder, 2024; Olmstead-Rumsey, 2019; Argente et al., 2020; Bessen et al., 2020; Bloom et al., 2020; Aghion et al., 2023; Akcigit and Ates, 2023; Akcigit and Goldschlag, 2023). Nevertheless, these studies have not yet offered a definitive answer and mechanism for the observed shift in the diffusion process. Our contribution is uncovering an underlying endogenous force behind the decreasing technology diffusion as a consequence of firms strategically responding to increased competition due to exogenous forces such as globalization.

The rest of the paper proceeds as follows. Section 2 develops a baseline endogenous growth model. Section 3 presents empirical results about the effect of Chinese competition on the composition of firm innovation. Section 4 displays results from quantitative analysis of the baseline model. Section 5 concludes.

2 Baseline Model

We build a discrete time infinite horizon endogenous growth model with multi-product firms, two types of innovation, imperfect technological spillovers, and an exogenous source of competitive pressure.[13] The model is distinct in the following three dimensions: we i) introduce a novel friction named "imperfect technology spillovers" by assuming that firms can only learn the incumbent's technology lagged by one period in the process of external innovation; ii) generate incumbent firms' internal innovation decision as an endogenous function of the technology gap—the ratio of the current-period technology $q_{j,t}$ to the previous-period technology $q_{j,t-1}$, $\Delta_{j,t} = \frac{q_{j,t}}{q_{j,t-1}}$, due to the friction; and iii) allow for exogenous shifts of the aggregate creative destruction arrival rate to

[13]The exogenous competitive pressure emanates from firms outside the economy, which could be foreign firms or domestic incumbent firms in other sectors or states, depending on whether we consider the model economy as an aggregate economy or a specific sector or state.

analyze the effect of increasing competitive pressure on firm innovation and growth. Hereafter, the time subscript is suppressed.[14] The terms product quality and technology are used interchangeably.

2.1 Representative Household

A representative household has a logarithmic utility and is populated by a measure one continuum of individuals. Each individual supplies one unit of labor inelastically and consumes a portion C_t of the economy's final goods each period. The household's lifetime utility is

$$U = \sum_{t=0}^{\infty} \beta^t \log(C_t).$$

Workers are employed in the final goods sector (L), and the labor market clears as follows:

$$L = 1. \tag{1}$$

2.2 Final Goods Producer

The final goods producer produces a final good with labor (L) and a continuum of differentiated intermediate goods indexed by $j \in [0, 1]$ (produced by either domestic firms $j \in \mathcal{D}$ or outsiders $j \notin \mathcal{D}$). The production function has the constant returns-to-scale technology:

$$Y = \frac{L^\theta}{1-\theta} \left[\int_0^1 q_j^\theta y_j^{1-\theta} \mathcal{I}_{\{j \in \mathcal{D}\}} dj + \int_0^1 q_j^\theta y_j^{1-\theta} \mathcal{I}_{\{j \notin \mathcal{D}\}} dj \right],$$

where y_j and q_j are the quantity and quality of good j, and $\mathcal{I}_{\{\cdot\}}$ are indicator functions. The market is competitive with the price normalized to one and input prices taken as given.

2.3 Intermediate Producers

Domestic and outside firms have the mass of $\mathcal{F}_d$ and $\mathcal{F}_o$, respectively, with $\mathcal{F} = \mathcal{F}_d + \mathcal{F}_o \in (0, 1)$. They produce and sell differentiated intermediate goods in monopolistically competitive domestic markets. Each firm operates with at least one product, and each product is owned by a single firm.

[14] Superscript $'$ denotes the forward-period variables at $(t + 1)$, and subscript -1 is used for the previous-period variables at $(t - 1)$.

Thus, firm f can be characterized by the collection of its products $\mathcal{J}^f = \{j : j \text{ is owned by firm } f\}$. The intermediate good is produced at a unit marginal cost in terms of final goods.

2.4 Innovation by Intermediate Producers

Intermediate producers engage in two types of R&D, internal and external, by spending expenditures in units of final goods. Firms improve the quality of their own products through internal innovation, while taking over other markets through external innovation.[15] The R&D output manifests as improving product quality and is realized in next period.

On top of this, we introduce a novel friction named "imperfect technology spillovers," under which learning others' technology takes time in the process of external innovation.[16] We conceptualize it in the form of lagged learning by assuming external innovation builds on the past-period technology. Thus, only the owner of a product line can observe the frontier level of technology $q_{j,t}$ in the market, while outsider firms can only see the lagged level $q_{j,t-1}$.[17] Following this, a product line can be sufficiently characterized by its quality q_j and technology gap between current and previous periods $\Delta_{j,t} = \frac{q_{j,t}}{q_{j,t-1}}$.[18] This friction induces incumbent firms to strategically use internal innovation to build technological barriers and protect their markets from competitors. We name it the "market-protection effect."

When two firms are neck and neck in a particular product line, a coin-toss tiebreaker rule applies as in Acemoglu et al. (2016) to make sure each product is produced by only one firm.[19] External innovation is undirected and the targeted product is randomly assigned among the entire set of products with equal probability. Also, for now, we assume that firms can only attempt one external innovation each period, which helps us derive analytic expressions for firm decision rules

[15] Note that the quality in this model is a marginal cost of production-adjusted measure, and can be improved through either technological advancement or cost reduction. In this sense, our concept of innovation encompasses both product and process innovations. Thus, firms with lower technology can compete with firms with advanced technology if they have a sufficiently lower marginal cost of production.

[16] Our empirical analysis in Online Appendix G.2 suggests that external innovation takes longer to develop than internal innovation. We quantify this time difference by measuring the gap between the application year of a patent and the application years of the patents it backward cites.

[17] All the aggregate variables and technology gap distribution are publicly observable, and firms make optimal innovation decisions by considering them. In this way, a stationary firm-product distribution is well defined.

[18] This technology gap summarizes the technological advantage incumbents have in their market.

[19] An unused technology is assumed to depreciate sufficiently to make it unprofitable for external innovators to build upon it next period. This approach guarantees the undirected nature of external innovation and restricts internal innovation to the current owner only.

and distributions with minimal assumptions. In quantitative analysis, we allow multiple external innovations as in Klette and Kortum (2004).

Internal Innovation Successful internal innovation improves the current quality $q_{j,t}$ of the product by $\lambda > 1$. The quality of good j evolves as follows, conditional on the owning firm not displaced by creative destruction:

$$\left\{q^{\text{in}}_{j,t+1}\right\} = \begin{cases} \left\{\lambda q_{j,t}\right\} & \text{with probability } z_{j,t} \\ \left\{q_{j,t}\right\} & \text{with probability } 1 - z_{j,t}, \end{cases} \qquad z_{j,t} = \left(\frac{R^{\text{in}}_{j,t}}{\widehat{\chi} q_{j,t}}\right)^{\frac{1}{\widehat{\psi}}}, \ \widehat{\chi} > 0, \ \widehat{\psi} > 1.$$

The success probability of internal innovation, $z_{j,t}$, depends on R&D investment $R^{\text{in}}_{j,t}$.[20]

External Innovation Successful external innovation improves the lagged quality of the obtained product by $\eta > 1$. We assume $\lambda^2 > \eta > \lambda$, where $\eta > \lambda$ reflects the findings from Akcigit and Kerr (2018) and our own that external innovation contributes more to both firm and aggregate growth than internal innovation.[21] $\lambda^2 > \eta$ is based on the idea that consecutive internal innovation has a significant influence, and this assumption ensures that firms can protect their own product lines from potential rivals through internal innovation.[22]

Firms invest in external innovation. As a result, the following product quality can be obtained if not pre-empted by the successful internal innovation of the incumbent in their target market:

$$\left\{q^{\text{ex}}_{j,t+1}\right\} = \begin{cases} \left\{\eta q_{j,t-1}\right\} & \text{with probability } x_t \\ \emptyset & \text{with probability } 1 - x_t, \end{cases} \qquad x_t = \left(\frac{R^{\text{ex}}_{t}}{\widetilde{\chi} \bar{q}_t}\right)^{\frac{1}{\widetilde{\psi}}}, \ \widetilde{\chi} > 0, \ \widetilde{\psi} > 1.$$

The success probability of external innovation, x_t, is determined by the amount of R&D expenditure

[20]Hereafter, we represent the quality of product j as a point set. This makes it easy to describe the case where a firm fails to acquire any product lines—in such cases, the product quality set is an empty set.

[21]Our empirical analysis in Section 3.4 shows that external innovation is of higher quality than internal innovation, both in terms of market value (Kogan et al., 2017) and the number of forward citations received. Furthermore, Akcigit and Kerr (2018) and our empirical analysis in Online Appendix G.3 show that external innovation is associated with higher firm employment growth compared to internal innovation. We also find that external innovation is positively associated with TFPR growth and the number of products added at the firm level.

[22]We discuss this in more detail in our quantitative analysis shown in Section 4.

R_t^{ex} and the average quality $\bar{q}_t$ in the economy.[23] With probability $1-x_t$, the external innovation fails, implying no product takeover and no quality obtained. See Online Appendix A.1 for illustrative examples of how firms choose internal and external innovations.

Product Quality Evolution Due to imperfect technology spillovers, the gap between the current and lagged levels of product quality, $\Delta_{j,t} = \frac{q_{j,t}}{q_{j,t-1}}$, reflects the technological advantage that incumbent firms possess in their markets and enables them to protect their product lines through internal innovation. The technology gap can have the following four values.

Lemma 1. *There are four possible values for a technology gap, $\Delta^1 = 1$, $\Delta^2 = \lambda$, $\Delta^3 = \eta$, and $\Delta^4 = \frac{\eta}{\lambda}$, where Δ^3 and Δ^4 can occur only through external innovation.*
Proof: See the Appendix.

Then, incumbents' product quality (conditional on a technology gap) evolves as follows:

$$\left\{q_{j,t+1} \middle| \Delta_{j,t} = \Delta^1\right\} = \begin{cases} \emptyset & \text{, with prob. } \overline{x} \\ \{q_{j,t}\} & \text{, with prob. } (1-\overline{x})(1-z_j^1) \\ \{\lambda q_{j,t}\} & \text{, with prob. } (1-\overline{x})z_j^1 \end{cases} \tag{2}$$

$$\left\{q_{j,t+1} \middle| \Delta_{j,t} = \Delta^2\right\} = \begin{cases} \emptyset & \text{, with prob. } \overline{x}(1-z_j^2) \\ \{q_{j,t}\} & \text{, with prob. } (1-\overline{x})(1-z_j^2) \\ \{\lambda q_{j,t}\} & \text{, with prob. } z_j^2 \end{cases} \tag{3}$$

$$\left\{q_{j,t+1} \middle| \Delta_{j,t} = \Delta^3\right\} = \begin{cases} \emptyset & \text{, with prob. } \frac{1}{2}\overline{x}(1-z_j^3) \\ \{q_{j,t}\} & \text{, with prob. } \left(1-\frac{1}{2}\overline{x}\right)(1-z_j^3) \\ \{\lambda q_{j,t}\} & \text{, with prob. } z_j^3 \end{cases} \tag{4}$$

$$\left\{q_{j,t+1} \middle| \Delta_{j,t} = \Delta^4\right\} = \begin{cases} \emptyset & \text{, with prob. } \overline{x}\left(1-\frac{1}{2}z_j^4\right) \\ \{q_{j,t}\} & \text{, with prob. } (1-\overline{x})(1-z_j^4) \\ \{\lambda q_{j,t}\} & \text{, with prob. } \left(1-\frac{1}{2}\overline{x}\right)z_j^4. \end{cases} \tag{5}$$

Note that z_j^ℓ is the optimal internal innovation of the firm owning product j with its technology gap Δ^ℓ, $\ell \in \{1,2,3,4\}$. $\overline{x}$ is the aggregate creative destruction arrival rate, representing the probability that a product market faces a rival firm with successful external innovation. The symbol $\emptyset$ indicates

[23] The average quality matters for external innovation as the target product is randomly assigned.

that the firm loses product line j in the next period, and the term $\frac{1}{2}$ in the probabilities reflects a coin-toss tiebreaker in neck-and-neck scenarios.

In the case of $\Delta_{j,t} = 1$, incumbents lack any technological advantage and lose their product lines if a rival firm arrives with successful external innovation, irrespective of their success in internal innovation.[24] In contrast, for other cases where $\Delta^\ell > 1$, firms can lower the probability of losing their product lines by investing more in internal innovation.[25] Hence, firms are more incentivized to augment their internal innovation efforts for products with technological advantages ($\Delta^\ell > 1$) if competitive pressure increases (with higher $\overline{x}$).[26]

For rival firms entering into a market, the success probability of product takeover not just depends on the success of their external innovation but also on the technology gap and the internal innovation intensity associated with the product owner (even after successful external innovation). Thus, the success probability of product takeover x_{takeover} ($\equiv x\overline{x}_{\text{takeover}}$) can be decomposed into i) the success probability of external innovation x, and ii) conditional takeover probability $\overline{x}_{\text{takeover}}$, which is defined as

$$\overline{x}_{\text{takeover}} = \mu(\Delta^1) + (1 - z^2)\mu(\Delta^2) + \frac{1}{2}(1 - z^3)\mu(\Delta^3) + \left(1 - \frac{1}{2}z^4\right)\mu(\Delta^4), \tag{6}$$

with technology gap distribution $\left\{\mu(\Delta^\ell)\right\}_{\ell=1}^4$ (the mass of products with a gap Δ^ℓ).[27]

Note that the higher the overall innovation intensity (both internal and external), the wider the average technology gap becomes in the economy (the mass of products with Δ^1 decreases). This makes it difficult for firms to successfully take over product markets.[28] This is referred to as the "technological barrier effect," where increased internal innovation by incumbents or higher $\bar{x}$ dampens the external innovation and growth of firms.[29]

[24] Rivals with successful external innovation achieve $q^{\text{rival}}_{j,t+1} = \eta q_{j,t-1}$, which is greater than $\lambda q_{j,t-1}$.

[25] The extent of the reduction in the probability of product loss is contingent on the technology gap.

[26] The evolution of product quality is defined for rival firms entering into a market in Online Appendix A.2.

[27] This shows that if a firm succeeds in externally innovating a product line with a technology gap of Δ^1, then it takes over that product line with a probability of one. For a product line with Δ^2, this probability becomes $1 - z^2$; for Δ^3, it is $\frac{1}{2}(1 - z^3)$; and for Δ^4, it is $1 - \frac{1}{2}z^4$. It is assumed that internal innovation intensity z depends solely on technology gap Δ^ℓ. In the next section, we prove this assumption holds true.

[28] Higher internal innovation intensity widens the technology gap. Simultaneously, higher external innovation intensity increases the aggregate creative destruction arrival rate, thereby incentivizing incumbent firms to engage in internal innovation more endogenously, as discussed earlier.

[29] This technological barrier effect is a novel feature of our model, which is distinct from the well-known Schumpeterian effect. The Schumpeterian effect is that firm innovation incentives decline following an increase in $\bar{x}$ due to

Potential Startups There is a fixed mass of potential domestic startups $\mathcal{E}_d$. To start a business, they invest in external R&D and attempt to take over a product line from an incumbent firm. Potential startups choose R&D expenditure R_e^{ex} and decide the probability of external innovation $x_e = \left(R_e^{\text{ex}} / \left(\widetilde{\chi}_e \overline{q}\right)\right)^{\frac{1}{\widetilde{\psi}_e}}$, where $\widetilde{\chi}_e > 0$ and $\widetilde{\psi}_e > 1$. Let $V(\{(q_j, \Delta_j)\})$ denote the value of a firm that has a product with quality q_j and a technology gap of Δ_j. Then a potential startup's expected profits from entering through R&D is

$$\Pi^e = \widetilde{\beta}\mathbb{E}\left[V(\{(q_j', \Delta_j')\})\middle|x_e\right] - \widetilde{\chi}_e(x_e)^{\widetilde{\psi}_e}\overline{q}, \tag{7}$$

where $\widetilde{\beta}$ is the stochastic discount factor, and the expectation conditioning on x_e is taken over the distribution of incumbents' product quality q_j and technology gap Δ_j due to the undirected nature of external innovation.[30] Potential startups choose the probability of external innovation x_e to maximize expected profits from entry. Since potential startups are ex-ante homogeneous, they all choose the same level of external innovation intensity x_e^*. Hence, the mass of potential domestic startups that succeed in external innovation and attempt to take over product markets is $\mathcal{E}_d x_e^*$.

2.5 Exogenous Competitive Pressure and Creative Destruction

As explained before, the aggregate creative destruction arrival rate $\overline{x}$ is the probability that an incumbent faces a rival firm (either a domestic startup, incumbent, or an outside firm) with successful external innovation. The aggregate creative destruction arrival rate is equal to the total mass of firms succeeding in external innovation given the undirected nature of external innovation and the continuum of unit mass of product lines.[31] Let $\overline{x}_d$ denote the total mass of domestic firms with successful external innovation and $\overline{x}_o$ denote the outside firms' counterpart. The creative destruction arrival rate is defined as

$$\overline{x} = \overline{x}_d + \overline{x}_o.$$

lowered expected future profits conditional on successful innovation and business takeover.

[30] $\widetilde{\beta} = \frac{\beta C}{C'}$ as the household owns all firms.

[31] This follows along with the assumption that each firm can externally innovate at most one product line each period, which makes the total mass of firms with successful external innovation equivalent to the total mass of product markets for which an incumbent faces a rival firm. This assumption is extended in our full-fledged version, and this result still holds with additional aggregation across products within successful firms.

Competitive pressure from outside firms is captured by an exogenous increase in $\overline{x}_o$, resulting from either increased external innovation intensity or a larger mass of outside firms.[32]

2.6 Equilibrium

Optimal Production and Employment The final goods producers choose labor and intermediate goods inputs. Let p_j denote the price of differentiated product j, and w denote the wage in the domestic economy. The inverse demand for intermediate good j is:

$$p_j = q_j^{\theta} L^{\theta} y_j^{-\theta}. \tag{8}$$

Each product is assumed to be supplied by a single firm. We follow Acemoglu et al. (2012) and Acemoglu et al. (2018) and assume that the current and former incumbents engage in the following two-stage price-bidding game for each product line j: i) each firm pays a fee of ε (> 0), and ii) those that have paid the fee announce their prices.[33]

Intermediate producers take (8) as given and maximize their operating profits $\pi(q_j)$ for each product $j \in \mathcal{J}^f$.[34] The optimal production and price are derived as follows:

$$y_j = (1-\theta)^{\frac{1}{\theta}} q_j \quad \text{and} \quad p_j = \frac{1}{1-\theta}, \tag{9}$$

which simplify the equilibrium profit, wage, and final goods output to the following:

$$\pi(q_j) = \underbrace{\theta(1-\theta)^{\frac{1-\theta}{\theta}}}_{\equiv \pi} q_j, \quad w = \theta(1-\theta)^{\frac{1-2\theta}{\theta}} \overline{q}, \quad \text{and} \quad Y = (1-\theta)^{\frac{1-2\theta}{\theta}} \overline{q}. \tag{10}$$

[32] Jo (2024) extends our baseline model to two-country framework and endogenizes the changes in $\overline{x}_o$ using the changes in bilateral tariff rates.

[33] This is to avoid the case where the former market leader, having lost its leadership to the current leader in a market, attempts to produce and sell its product through limit pricing. This ensures that only the firm with the leading-edge technology enters the first stage and announces its price in equilibrium.

[34] Since each intermediate product incurs a unit marginal cost in terms of final goods, the problem is identical for both domestic and outside firms.

Optimal internal and external innovations Let $\Phi^f \equiv \{(q_j, \Delta_j)\}_{j\in\mathcal{J}^f}$ denote the set of product quality and technology gap for intermediate goods producer f. The firm value is:

$$V(\Phi^f) = \max_{\substack{x,\\ \{z_j\}_{j\in\mathcal{J}^f}}} \left\{ \sum_{j\in\mathcal{J}^f} \left[\pi q_j - \hat{\chi} z_j^{\hat{\psi}} q_j\right] - \bar{q}\widetilde{\chi} x^{\widetilde{\psi}} + \widetilde{\beta}\mathbb{E}\left[V(\Phi^{f\prime} \middle| \Phi^f)\middle| x, \{z_j\}_{j\in\mathcal{J}^f}\right]\right\}.$$

The first three terms define the current profits (revenue net of production and R&D costs), and the last term is the discounted future expected value. This expectation is computed over various factors, including the success probabilities of internal and external innovations, the creative destruction arrival rate, the outcomes of winning or losing coin tosses, the current-period product quality distribution, and the current-period technology gap distribution.

Proposition 1. *The firm value function and optimal innovation choice are derived as:*

$$V(\Phi^f) = \sum_{\ell=1}^{4} A_\ell \left(\sum_{j\in\mathcal{J}^f|\Delta_j=\Delta^\ell} q_j\right) + B\overline{q} \tag{11}$$

$$z^1 = \left[\widetilde{\beta}\left((1-\overline{x})\lambda A_2 - (1-\overline{x})A_1\right) / \left(\widehat{\psi}\widehat{\chi}\right)\right]^{\frac{1}{\widehat{\psi}-1}} \tag{12}$$

$$z^2 = \left[\widetilde{\beta}\left(\lambda A_2 - (1-\overline{x})A_1\right) / \left(\widehat{\psi}\widehat{\chi}\right)\right]^{\frac{1}{\widehat{\psi}-1}} \tag{13}$$

$$z^3 = \left[\widetilde{\beta}\left(\lambda A_2 - \left(1-\overline{x}/2\right)A_1\right) / \left(\widehat{\psi}\widehat{\chi}\right)\right]^{\frac{1}{\widehat{\psi}-1}} \tag{14}$$

$$z^4 = \left[\widetilde{\beta}\left(\lambda\left(1-\overline{x}/2\right)A_2 - \left(1-\overline{x}\right)A_1\right) / \left(\widehat{\psi}\widehat{\chi}\right)\right]^{\frac{1}{\widehat{\psi}-1}} \tag{15}$$

$$x = \left[\widetilde{\beta}A_{\text{takeover}} / \left(\widetilde{\psi}\widetilde{\chi}\right)\right]^{\frac{1}{\widetilde{\psi}-1}}, \tag{16}$$

where

$$A_1 = \pi - \widehat{\chi}(z^1)^{\widehat{\psi}} + \widetilde{\beta}\left[A_1(1-\overline{x})(1-z^1) + \lambda A_2(1-\overline{x})z^1\right] \tag{17}$$

$$A_2 = \pi - \widehat{\chi}(z^2)^{\widehat{\psi}} + \widetilde{\beta}\left[A_1(1-\overline{x})(1-z^2) + \lambda A_2 z^2\right] \tag{18}$$

$$A_3 = \pi - \widehat{\chi}(z^3)^{\widehat{\psi}} + \widetilde{\beta}\left[A_1\left(1-\overline{x}/2\right)(1-z^3) + \lambda A_2 z^3\right] \tag{19}$$

$$A_4 = \pi - \widehat{\chi}(z^4)^{\widehat{\psi}} + \widetilde{\beta}\left[A_1(1-\overline{x})(1-z^4) + \lambda A_2\left(1-\overline{x}/2\right)z^4\right] \tag{20}$$

$$B = \left(x\widetilde{\beta}A_{\text{takeover}} - \widetilde{\chi}x^{\widetilde{\psi}}\right) / \left(1-\widetilde{\beta}(1+g)\right), \tag{21}$$

g is the growth rate of the average product quality, and A_{takeover} is the ex-ante value of a successful takeover of a product line as follows:

$$A_{\text{takeover}} \equiv \frac{1-z^3}{2}A_1\mu(\Delta^3) + \left(1-\frac{z^4}{2}\right)A_2\lambda\mu(\Delta^4) + A_3\eta\mu(\Delta^1) + (1-z^2)A_4\frac{\eta}{\lambda}\mu(\Delta^2). \quad (22)$$

Proof: See the Appendix.

Note that A_ℓ is the sum of discounted expected profits from owning a product with a technology gap of Δ^ℓ, normalized by the current-period product quality. The first two terms in (17) through (20) denote the normalized instantaneous profits, net of the optimal internal R&D spending. The terms inside the brackets are the normalized future value from internal innovation. B is the sum of the discounted expected profits from owning an additional product through external innovation, normalized by the average product quality.[35]

For the optimal internal innovation (12)-(15), the first term in the brackets (after $\widetilde{\beta}$) in the numerator represents the future value from successful internal innovation with the quality increased by λ, and the second term is the counterpart from no successful internal innovation. Thus, holding $\overline{x}$ fixed, the net future value of successful internal innovation depends on the firm's technology gap, pinning down its optimal choice. Consequently, internal innovation becomes an endogenous function of the technology gap, which is a unique feature of this model due to imperfect technology spillovers. Corollary 1 details this further.

Corollary 1. *In an equilibrium where $\{z^\ell\}_{\ell=1}^4$ are well defined, the probabilities of internal innovation satisfy $z^2 > z^3 > z^4 > z^1$. Proof: See the Appendix.*

This shows that internal innovation increases with the technology gap, which helps firms protect their markets. However, beyond a certain point, a wider technology gap can discourage further investment in internal innovation. This occurs because firms are less likely to lose their product line even without doing additional internal innovation.

[35] To understand this variable clearly, we can rewrite (21) as $B\overline{q} = x\widetilde{\beta}A_{\text{takeover}}\overline{q} - \widetilde{\chi}x^{\widetilde{\psi}}\overline{q} + \widetilde{\beta}B(1+g)\overline{q}$. After investing $\widetilde{\chi}x^{\widetilde{\psi}}\overline{q}$ in external innovation in the current period, the firm receives the discounted expected profit $\widetilde{\beta}A_{\text{takeover}}\overline{q}$ in the next period if the external innovation succeeds with probability x. Then, the firm plans to invest in external innovation next period and receive an expected profit of $B\overline{q}'$ in the following period, where $\overline{q}' = (1+g)\overline{q}$. Thus, (21) illustrates that B is the annuity value of an infinite stream of constant payoffs $x\widetilde{\beta}A_{\text{takeover}} - \widetilde{\chi}x^{\widetilde{\psi}}$, evaluated at a constant discount rate of $\widetilde{\beta}(1+g)$, which is the growth rate-adjusted stochastic discount factor.

Furthermore, the optimal internal innovation also depends on creative destruction arrival rate $\overline{x}$ as shown in Corollary 2, which we label the "market-protection effect."[36,37]

Corollary 2 (Market-Protection Effect)**.** *With $\widetilde{\psi} \in (1, 2]$, the market-protection effect is maximized and is positive for product lines with a technology gap of Δ^2, whereas it is minimized and is negative for product lines with Δ^1. The market-protection effect is positive for the Δ^3 case, whereas its sign is ambiguous for the Δ^4 case. Thus,*

$$\left.\frac{\partial z^2}{\partial \overline{x}}\right|_{A_1,A_2} > \left.\frac{\partial z^3}{\partial \overline{x}}\right|_{A_1,A_2} > 0, \quad \left.\frac{\partial z^3}{\partial \overline{x}}\right|_{A_1,A_2} > \left.\frac{\partial z^4}{\partial \overline{x}}\right|_{A_1,A_2} \lesseqqgtr 0, \quad \textit{and} \quad 0 > \left.\frac{\partial z^1}{\partial \overline{x}}\right|_{A_1,A_2}.$$

Proof: See the Appendix.

In the Δ^1 case, internal innovation fails to effectively protect the firm's product, as shown in (2). Consequently, z^1 decreases as the rate of creative destruction $\overline{x}$ increases. In contrast, the Δ^2 case has the strongest market-protection effect, exerting the highest impact on reducing the probability of losing a product as in (3). In the Δ^3 case, increasing z^3 lowers the probability of product loss, though less than in the Δ^2 case. This results in a positive but diminished market-protection effect. The effect in the Δ^4 case remains ambiguous, where higher z^4 leads to a smaller decrease in the probability of losing the product line. This suggests firms that have innovated intensively previously (and thus larger technology gaps) are more likely to intensify internal innovation in response to increased competition (higher $\overline{x}$) than those with less recent innovation. This highlights another crucial and unique aspect of our model: firms strategically employ internal innovation to defend against competitors, leveraging imperfect technology spillovers.

As a result, optimal external innovation depends on internal innovation, the technology gap distribution among incumbents, and the expected value of products ($\{A_\ell\}_{\ell=1}^4$). Equations (16) and (22) show that higher overall internal and external innovation intensities reduce firms' incentive for external innovation in partial equilibrium, with $\{A_\ell\}_{\ell=1}^4$ held constant. This is because increased overall innovation shifts the technology gap distribution, which raises the average technology gap,

[36] Note that as A_1 and A_2 also depend on $\overline{x}$, it is difficult to analytically determine the signs of the partial derivatives of $\{z^\ell\}_{\ell=1}^4$ with respect to $\overline{x}$. However, by holding the values of A_1 and A_2 fixed, we can explicitly ascertain these signs as in Corollary 2.

[37] The term A_2 in (12)-(15) reflects the well-known Schumpeterian effect—the lower the expected future profits from keeping the product line through internal innovation, the lower the incentive to invest in internal innovation.

and hampers firms' market takeover (the "technological barrier effect"). Furthermore, keeping the probabilities of internal innovation and the technology gap distribution constant, a decrease in the expected product values reduces external innovation (the "Schumpeterian effect").[38] Our simple three-period model in Online Appendix B formally proves these predictions.

Similarly, the optimal external innovation by potential startups x_e is derived as follows:

$$x_e = \left[\widetilde{\beta}\left(A_{\text{takeover}} + \overline{x}_{\text{takeover}} B(1+g)\right) / \left(\widetilde{\psi}_e \widetilde{\chi}_e\right)\right]^{\frac{1}{\widetilde{\psi}_e - 1}}, \tag{23}$$

and the proof is provided in the Appendix.

Aggregate Creative Destruction Arrival Rate With (16) and (23), the aggregate creative destruction arrival rate in this economy is defined as follows:

$$\overline{x} = \underbrace{\mathcal{F}_d x + \mathcal{E}_d x_e}_{\equiv \overline{x}_d} + \underbrace{\mathcal{F}_o x + \mathcal{E}_o}_{\equiv \overline{x}_o}, \tag{24}$$

where $\mathcal{E}_o$ is the total mass of outside entrants with successful external innovation, which is exogenously determined.[39,40]

2.7 Balanced Growth Path

Proposition 2. *The aggregate growth rate g in a Balanced Growth Path is:*

$$\begin{aligned} g = &\left[(1-\overline{x})(1-z^1) + \Delta^2(1-\overline{x})z^1 + \Delta^3\overline{x}\right]\mu(\Delta^1) \\ &+ \left[(1-\overline{x})(1-z^2) + \Delta^2 z^2 + \Delta^4\overline{x}(1-z^2)\right]\mu(\Delta^2) \\ &+ \left[1 - z^3 + \Delta^2 z^3\right]\mu(\Delta^3) + \left[(1-\overline{x})(1-z^4) + \Delta^2(z^4 + \overline{x}(1-z^4))\right]\mu(\Delta^4) - 1, \end{aligned} \tag{25}$$

[38] The direction of the changes in the probabilities of internal and external innovation in response to changes in the aggregate creative destruction arrival rate $\overline{x}$ are ambiguous in general equilibrium. They depend on the relative magnitudes and the directions of the market-protection effect, the technological barrier effect, and the Schumpeterian effect. Nonetheless, results from the numerical exercise in Section 4.2 confirm that the partial equilibrium results, given $\{A_\ell\}_{\ell=1}^4$ and B, still hold in general equilibrium within plausible parameter ranges. Furthermore, $\{A_\ell\}_{\ell=1}^4$ and B decrease with an exogenous increase in $\overline{x}$.

[39] Note that an exogenous increase in $\mathcal{E}_o$ may not increase $\overline{x}$ by the same amount in equilibrium, as the mass of domestic incumbent firms $\mathcal{F}_d$ and the probabilities of external innovation x and x_e depend on $\overline{x}$. Thus, the level of $\overline{x}$ is endogenously determined, even when $\mathcal{E}_o$ changes exogenously.

[40] The outside firms in domestic markets make the same innovation decisions as the domestic firms.

which can be decomposed into the parts attributed to internal innovation and external innovation by domestic incumbents and startups (g_d), as well as outside firms (g_o).

Proof: See the Appendix.

2.8 Firm Distribution

Let $\mathcal{N} = (n_f, n_f^1, n_f^2, n_f^3, n_f^4)$ denote the technology gap composition of firm f, where n_f is the total number of products and n_f^ℓ is the count of products with a technology gap of Δ^ℓ . Let $\widetilde{\mu}(\mathcal{N})$ denote its distribution. Summing $\widetilde{\mu}(\mathcal{N})$ over all possible $\mathcal{N}$ gives the total mass of firms $\mathcal{F}$.

Transition of Technology Gap Portfolio Consider a firm with technology gap composition given by $\widetilde{\mathcal{N}}(n_f, k) \equiv (n_f, n_f - k, k, 0, 0)$, where $k \in [0, n_f] \cap \mathbb{Z}$ and $n_f > 0$. Ignoring external innovation, the probability of technology gap composition changing from $\mathcal{N} = \widetilde{\mathcal{N}}(n_f, k)$ to $\mathcal{N}' = \widetilde{\mathcal{N}}(n_f, \widetilde{k})$ is

$$\begin{aligned}\widetilde{\mathbb{P}}\Big(n_f, \widetilde{k} | n_f, k\Big) = &\sum_{\widetilde{k}^1 = \max\{0, \widetilde{k} - k\}}^{\min\{n_f - k, \widetilde{k}\}} \left(\frac{(n_f - k)!}{\widetilde{k}^1!(n_f - k - \widetilde{k}^1)!}\right)\left(\frac{k!}{(\widetilde{k} - \widetilde{k}^1)!(k - (\widetilde{k} - \widetilde{k}^1))!}\right)\\ &\times (1 - \overline{x})^{n_f - (\widetilde{k} - \widetilde{k}^1)}(1 - z^1)^{n_f - k - \widetilde{k}^1}(z^1)^{\widetilde{k}^1}(1 - z^2)^{k - (\widetilde{k} - \widetilde{k}^1)}(z^2)^{\widetilde{k} - \widetilde{k}^1},\end{aligned}$$

for $n_f \geq 1$ and $0 \leq \widetilde{k}, k \leq n_f$, and zero, otherwise. This follows a binomial process as in Ates and Saffie (2021).

Using the above, we can track general cases transitioning from $\mathcal{N} = (n_f, n_f^1, n_f^2, n_f^3, n_f^4)$ to $\mathcal{N}' = (n_f', n_f^{1\prime}, n_f^{2\prime}, n_f^{3\prime}, n_f^{4\prime})$ for any $n_f' \leq n_f + 1$ as products with Δ^3 or Δ^4 can only be obtained through external innovation. Details can be found in Online Appendix A.4.

Technology Gap Distribution The aggregate distribution of technology gaps is

$$\mu\big(\Delta^\ell\big) = \sum_{n_f=1}^{\overline{n}_f} \sum_{n_f^\ell=0}^{n_f} \sum_{n_f^{-\ell}=0}^{n_f} n_f^\ell \widetilde{\mu}\big(n_f, n_f^1, n_f^2, n_f^3, n_f^4\big), \text{ for } \ell = 1, 2, 3, 4 \tag{26}$$

where the third summation represents the sum over all possible values for $n_f^{-\ell}$ other than the focal ℓ. Note $\sum_{\ell=1}^4 \mu\big(\Delta^\ell\big) = 1$ holds in equilibrium.[41]

[41]This is because each product line is occupied by one incumbent and there is a unit mass of products.

Aggregate Variables and Balanced Growth Path Given the optimal innovation choices (12), (13), (14), (15), (16), and (23), the aggregate domestic R&D expenses becomes

$$R_d = \widehat{\chi} \sum_{\ell=1}^{4} \left[\int_0^1 q_j \mathcal{I}_{\{\Delta_j=\Delta^\ell, j\in\mathcal{D}\}} dj \right] (z^\ell)^{\widehat{\psi}} + \mathcal{F}_d \widetilde{\chi} \overline{q} x^{\widetilde{\psi}} + \mathcal{E}_d \widetilde{\chi}_e (x_e)^{\widetilde{\psi}_e} \overline{q}, \tag{27}$$

where $\mathcal{I}_{\{\Delta_j=\Delta^\ell, j\in\mathcal{D}\}}$ is an indicator for product line j owned by a domestic firm with Δ^ℓ. With (9), the aggregate demand for final goods by domestic intermediate producers is

$$Y_d = \int_0^1 y_j \mathcal{I}_{\{j\in\mathcal{D}\}} dj = (1-\theta)^{\frac{1}{\theta}} \int_0^1 q_j \mathcal{I}_{\{j\in\mathcal{D}\}} dj, \tag{28}$$

and the aggregate consumption is determined by

$$C = Y - \int_{j\notin\mathcal{D}} p_j y_j dj - Y_d - R_d, \tag{29}$$

where the second term is the payments to outside intermediate producers.[42] Lastly, the balanced growth path (BGP) equilibrium is characterized by the following:

Definition 1. *A balanced growth path equilibrium consists of y_j^*, p_j^*, w^*, L^*, x^*, $\{z^{\ell*}\}_{\ell=1}^4$, $\overline{x}^*$, x_e^*, $\mathcal{F}^*$, R_d^*, Y^*, C^*, g^*, $\widetilde{\mu}(\mathcal{N})$, $\{\mu(\Delta^\ell)\}_{\ell=1}^4$ for $j \in [0,1]$ with q_j such that: (i) y_j^* and p_j^* satisfy* (9)*; (ii) w^* satisfies* (10)*; (iii) L^* satisfies* (1)*; (iv) $\{z^{\ell*}\}_{\ell=1}^4$ satisfy* (12)-(15)*, and x^* satisfies* (16)*; (v) $\overline{x}^*$ satisfies* (24)*; (vi) x_e^* satisfies* (23)*; (vii) Y^* satisfies* (10)*; (viii) R_d^* satisfies* (27)*; (ix) C^* satisfies* (29)*; (x) the BGP growth rate g^* satisfies* (25)*; (xi) the distribution of technology gap portfolio composition $\widetilde{\mu}(\mathcal{N})$ and $\mathcal{F}^*$ satisfy $inflow(\mathcal{N}) = outflow(\mathcal{N})$; and (xii) the technology gap distribution $\{\mu(\Delta^\ell)\}_{\ell=1}^4$ follows* (26).

3 Empirics

In this section, we empirically test the model predictions by identifying the causal effect of competition on the composition of firm innovation and analyzing the industry-level association between technological barriers and firm entry. We use the rise in Chinese firm entry into U.S. domestic

[42]We assume outside firms use final goods from their economy for production and R&D.

markets following China's WTO accession in 2001 as a quasi-experimental increase in competitive pressure. We then test the quality improvement differential between the two types of innovation.

3.1 Data and Measurement

To compile comprehensive data on firm innovation and a foreign competition shock, we combine the USPTO PatentsView database, the Longitudinal Business Database (LBD), the Longitudinal Firm Trade Transactions Database (LFTTD), the Census of Manufactures (CMF), the NBER-CES database, and the tariff data in Feenstra et al. (2002).

The LBD tracks the universe of establishments and firms in the U.S. non-farm private sector with at least one paid employee annually from 1976 onward.[43] We aggregate establishment-level data into firm-level using firm identifiers.[44] Firm size is measured by total employment or payroll, and firm age by the age of the oldest establishment of the firm when the firm is first observed in the data. The firm's main industry of operation is based on the six-digit North American Industry Classification System (NAICS) code of the establishment with the highest employment.[45]

The LFTTD tracks all U.S. international trade transactions at the firm level from 1992 onward. It provides information such as the U.S. dollar value of shipments, the origin and destination countries, and a related-party flag indicating whether the U.S. importer and the foreign exporter are related by ownership of at least 6 percent.[46]

The USPTO PatentsView database records all patents ultimately granted by the USPTO from 1976 onward.[47] This database provides comprehensive details for patents, including application and grant dates, technology class, citation, and the name and address of patent assignees. In our analyses, we rely on the citation-adjusted number of utility patents as the main measure of firm innovation.[48] Using the patent-level information, we distinguish domestic innovation from foreign innovation, and assess the extent to which each patent represents internal innovation. The patent

[43] Details for the LBD and its construction can be found in Jarmin and Miranda (2002).

[44] An establishment corresponds to the physical location where business activity occurs. Establishments that are operated by the same entity, identified through the Economic Census and the Company Organization Survey, are grouped under a common firm identifier.

[45] Time-consistent NAICS codes for LBD establishments are constructed by Fort and Klimek (2018), and the 2012 NAICS codes are used throughout the entire analysis.

[46] Bernard et al. (2009) describe the LFTTD in greater detail.

[47] See https://patentsview.org/download/data-download-tables.

[48] See Cohen (2010) for a comprehensive review of the literature on the determination of firm/industry innovative activity and related patent measures.

application year is used for the innovation year.

We link the USPTO patent database to the LBD to track firm patenting over time. Failure to match a patent assignee with its LBD firm counterpart can mismeasure firm innovation changes.[49] Since the USPTO patent data lacks a longitudinally consistent firm identifier, we build our own crosswalk between the two datasets by adopting the internet search-aided algorithm as in Autor et al. (2020).[50] We pool all patents granted up to December 26, 2017, and use patent applications up to 2007 in our main analyses to avoid a right censoring issue arising from patents applied for but not yet granted. Table G1 in the Online Appendix reports summary statistics.

The quinquennial CMF provides detailed information about the U.S. manufacturing establishments and products they produce. It contains product-level details such as product codes and the value of shipment. We use five-digit SIC codes (for the pre-2002 years) or seven-digit NAICS codes (for 2002 onward) to define a product. We obtain the U.S. tariff schedules from Feenstra et al. (2002) to measure the industry-level Trade Policy Uncertainty (TPU) as a proxy for foreign competitive pressure. Lastly, all nominal values are converted to 1997 U.S. dollars, using the industry-level deflator from the NBER CES Manufacturing Industry Database for manufacturing industries and the Consumer Price Index from the BEA for other industries.[51]

Following this, for our main analyses, we use the USPTO patents matched to LBD firms and industry-level trade data spanning from 1982 to 2007.[52]

Measure of Internal vs. External Innovation We follow Akcigit and Kerr (2018) and use the self-citation ratio, the ratio of self-citations to total citations, as a measure of the likelihood a patent is used for internal vs. external innovation.[53] A higher (lower) self-citation ratio implies a greater probability that a patent reflects internal (external) innovation.[54] This is because the more an idea is

[49]The USPTO assigns patent applications to self-reported firm names, which are frequently misspelled.

[50]This algorithm utilizes the machine-learning capacities of internet search engines. The entire matching methodology is outlined in our accompanying paper Ding et al. (2022).

[51]The NBER CES data are complied by Becker et al. (2013) (http://www.nber.org/nberces/).

[52]Our procedure links patents to the firms initially reported by the USPTO as owners and does not track ownership changes resulting from, for instance, M&A activities. We expect our analysis not to be contaminated by firms substituting innovation with acquisitions of other firms, particularly given that U.S. M&A activities began declining around 2000 and did not fully recover by 2007, as shown in Phillips and Zhdanov (2023).

[53]Each granted patent is required to cite all prior patents on which it builds itself. When a cited patent belongs to the owner of the citing patent, these citations are called self-citations.

[54]Thus, 100% self-citation means the patent is used for internal innovation with a 100% probability, and 0% self-citation means the patent is used for external innovation with a 100% probability.

based on the firm's internal knowledge stock (self-citation), the more likely the innovation is used to improve the firm's existing products (internal innovation). Alternatively, we measure internal innovation by the number of patents with a self-citation ratio above a certain threshold (0% or 10%) and within-firm product sales concentration. Additionally, we measure external innovation by the number of new products added. We discuss the properties of the two innovations in a later section.

Measure of Increasing Competition Following Pierce and Schott (2016) and Handley and Limão (2017), we use the removal of trade policy uncertainty (TPU) as a measure of an exogenous competitive pressure shock. Specifically, we use the following industry-level tariff rate gaps between WTO members and non-market economies in the year 1999 as a proxy for the industry-level competitive pressure shock from China occurring in 2001.[55]

$$NTRGap_j = Non\ NTR\ Rate_j\ -\ NTR\ Rate_j \text{ (for industry } j\text{)}.$$

For multi-industry firms, we use the employment-weighted average of $NTRGap_j$.[56]

The removal of TPU encouraged Chinese firms to enter U.S. markets and export their products (Pierce and Schott, 2016), which captures an exogenous increase in competitive pressure by foreign firms and directly maps into an increase in $\overline{x}_o$ in our model.

3.2 Empirical Strategy Testing the Market-Protection Effect

Regression Results To test the market-protection effect in our model, we follow Pierce and Schott (2016) and use the following Difference-in-Difference (DD) specification to identify the effect of

[55] Nonmarket economies such as China are by default subject to relatively high tariff rates, known as non-Normal Trade Relations (non-NTR) or column 2 tariffs, when they export to the U.S. On the other hand, the U.S. offers WTO member countries NTR or column 1 tariffs, which are substantially lower than non-NTR tariffs. Although the U.S. granted temporary NTR status to China from 1980, the U.S. Congress voted on a bill to revoke China's temporary NTR status every year from 1990 to 2001 after the Tiananmen Square protests in 1989. This caused uncertainty about whether the low tariffs would revert to non-NTR rates. Following an agreement on China's entry into the WTO, the U.S. Congress passed a bill granting China permanent NTR (PNTR), and PNTR was implemented on January 1, 2002. The PNTR has reduced trade policy uncertainty, more for industries with a large prior gap between NTR and non-NTR tariff rates. See Pierce and Schott (2016) for details.

[56] Table G2 in the Online Appendix reports summary statistics of the NTR-related measures.

the Chinese competitive pressure shock on the U.S. firm innovation:

$$\begin{aligned}\Delta y_{ijp} =& \beta_1 Post_p \times NTRGap_{ijp0} \times InnovIntens_{ijp0} + \beta_2 Post_p \times NTRGap_{ijp0} \\ &+ \beta_3 Post_p \times InnovIntens_{ijp0} + \beta_4 NTRGap_{ijp0} \times InnovIntens_{ijp0} \\ &+ \beta_5 NTRGap_{ijp0} + \beta_6 InnovIntens_{ijp0} \\ &+ \mathbf{X}_{ijp0}\gamma_1 + \mathbf{X}_{jp0}\gamma_2 + \delta_j + \delta_p + \alpha + \varepsilon_{ijp}, \end{aligned} \tag{30}$$

where $p \in \{1992 - 1999,\ 2000 - 2007\}$, y_{ijp} is either i) the total citation-adjusted number of patents (overall innovation), or ii) the citation-weighted average self-citation ratio (internalness of innovation) for firm i in industry j, and Δy_{ijp} is the DHS (Davis et al., 1996) growth rate of y between the start-year and end-year for each period p.[57]

To maximize the sample size, we include firms that applied for at least one patent in the start-year and at least one patent in or before the end-year for each period. We compute the DHS growth rates for the longest available span of years. We also require firms to have at least one patent before the start-year of each period, or to have an age greater than 0, to avoid the impact of firm entry. The sample comprises all patenting LBD firms meeting these three criteria and excludes FIRE (finance, insurance, and real estate) industries.

$Post_p$ is a dummy for the post-treatment period (2000-2007). $\mathbf{X}_{ijp0}$ and $\mathbf{X}_{jp0}$ are vectors of firm and industry controls, respectively, measured at the start-year for each period p.[58] δ_j is an industry fixed effect (six-digit NAICS), and δ_p is a period fixed effect. The regression is unweighted, and standard errors are clustered by industry (six-digit NAICS). Firms in low TPU industries are the control group, whereas high TPU industry firms are the treatment group. We use the 1992 and 2000 cohorts of firms to gauge firm innovation before and after the policy change in December 2001, minimizing policy-driven changes in firm composition.

$InnovIntens_{ijp0}$ is the lagged five-year average of the ratio of the number of patent applications to total employment for firm i. This proxies the technological advantage the firm has. It is measured in the start year for each period p and is normalized by its time-average at the two-digit NAICS

[57] The long-difference regression specification is standard in settings with a slow-moving process, such as innovation or technological progress (e.g., Acemoglu and Restrepo, 2020). This specification takes out the firm-fixed effect.

[58] Baseline firm controls include: firm employment, firm age, the past five-year growth of U.S. patents in the CPC technology classes in which the firm operates, and a dummy variable for publicly traded firms. Industry control variables include NTR rates measured at the start of each period.

level to control for industry effects. The model predicts $\beta_1 > 0$ when the dependent variable is the changes in self-citation ratio.

The second row of Table 1 shows the estimates of β_1.[59] The results are consistent with our model prediction in several dimensions. First, the first two columns show that the Chinese competitive pressure shock has no statistically significant effect on firms' overall innovation, regardless of the set of firm controls included. According to our model, as competition intensifies, firms increase or decrease internal innovation based on the technological advantages accumulated within their markets. However, firms universally decrease their external innovation. Considering both internal and external innovation changes, the overall effect of competition on firm innovation need not be statistically significant.

However, when examining the effect on internal innovation by substituting the dependent variable with the growth rate of the self-citation ratio, the effect becomes positive and statistically significant, as indicated in the last two columns.[60] This supports the model prediction of the market-protection effect. The estimated coefficient implies a 4.2 percentage points increase in the growth rate of the average self-citation ratio during the period 2000-2007 for a firm with an average lagged innovation intensity (0.18) in an industry with an average NTR gap (0.291). Given that the average value of the seven-year growth rate of the average self-citation ratio between 2000 and 2007 is 28.2 percentage points, this effect represents a 15.0% increase in internal innovation by firms with technological advantages.

The estimated effect is economically important as well. Table G4 in the Online Appendix shows that for average firms, creating one more patent is associated with a 1.32 percentage points increase in employment growth. However, this association weakens if the new patent has a higher self-citation ratio. Combined with the main result, this indicates that the association between patenting and employment growth decreases by 17.1% (from 1.32pp to 1.10pp) for innovation-intensive firms following the increased competitive pressure.[61] Furthermore, Table G5 in the Online Appendix shows that patents in general exhibit a positive association with both the number of products added

[59]To conserve space, Table 1 reports the main coefficient estimates for the triple interaction and the DD-term only. The full results are available on request.

[60]Note that because firms do not change their overall innovation, the increasing self-citation ratio implies that innovative firms (those above the average innovation intensity) increase their internal innovation while decreasing their external innovation.

[61]Innovation-intensive firms are those with innovation intensity one standard deviation above the average.

Table 1: Market-Protection Effect

	ΔPatents	ΔPatents	ΔSelf-cite	ΔSelf-cite
NTR gap $\times$ Post	0.238	0.071	-0.075	-0.062
	(0.237)	(0.283)	(0.257)	(0.291)
$\times$ Innovation intensity	0.077	-0.054	0.732**	0.795***
	(0.231)	(0.242)	(0.299)	(0.277)
Observations	6,500	6,500	6,500	6,500
Fixed effects	j, p	j, p	j, p	j, p
Controls	no	baseline	no	baseline

Notes: The baseline controls include the past five-year U.S. patent growth in firms' own technology fields, log employment, firm age, NTR rate, and a dummy for publicly traded firms. The estimates for industry (j) and the period (p) fixed effects, along with the coefficient associated with the binary indicator, are suppressed due to disclosure restrictions. The constant is also omitted. Robust standard errors, adjusted for clustering at the level of the firms' major industries, are displayed below each coefficient. Observations are unweighted, and observation counts are rounded due to the Census Bureau disclosure avoidance procedures. For the sake of space, only the main coefficients are presented. Full results are available on request. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

and productivity growth at the firm level. However, this effect gets muted if patents have a higher self-citation ratio. Additionally, patents with a higher self-citation ratio increase the growth of product sales concentration within firms. The result is robust with alternative measures of external and internal innovations, as shown in the Online Appendix Table G6.

Validity of the Identification Strategy and Robustness Tests We also confirm the validity of our identification and main results across various dimensions. First, we test the parallel pre-trends assumption, a key identifying assumption for the Diff-in-Diff model. We estimate (30) for the two seven-year periods preceding the policy change, 1984-1991 and 1992-1999. Table G7 in the Online Appendix supports the validity of the assumption.

Furthermore, we perform several robustness checks. First, we replace the baseline firm-level NTR gaps with the industry-level NTR gaps based on the primary industry (with the largest employment size) in which firms operate.[62] Second, we include upstream and downstream competitive pressure shocks as covariates to control the effect of trade shocks through firms' I-O networks.[63] The

[62]The baseline measure uses the employment-share weighted average of the industry-level NTR gaps, where the employment share is measured at the start year of each period and averaged across the firm's operating industries.

[63]The upstream (downstream) measure captures the effect of trade shocks propagating upstream (downstream) from an industry's buyers (suppliers). Using the 1992 BEA input-output table, we construct upstream and downstream competitive pressure shocks as the weighted averages of industry-level trade shocks. Following the approach in Pierce and Schott (2016), we assign I-O weights to zero for both upstream and downstream industries within the same

third test addresses a potential sampling bias using the inverse propensity score weights.[64,65] The fourth test adjusts the level of standard error clustering to the firm level.[66] The fifth test considers the potential correlation between the innovation intensity measure and firm size or age (e.g., Acemoglu et al., 2018), which may blur the effect of technological barriers. To address this concern, we control additional terms that interact innovation intensity with firm age and size. Moreover, we use an alternative measure based on the inverse of the innovation intensity gap relative to the industry frontier, averaged over the past five years, as the level of technological advantage. The sixth test confirms the robustness of alternative measures for external and internal innovations. External innovation is directly measured by the number of new product added, and internal innovation is directly measured by the number of patents with a self-citation ratio above 0% or 10%. Also, we examine the impact on within-firm product market concentration. Lastly, we include additional controls (such as the cumulative number of patents, firm payroll, the number of industries or products, industry-level skill and capital intensities, as well as dummies for importers and exporters) beyond the baseline set to eliminate potential alternative interpretations. Table G8-G18 in the Online Appendix present the results for each test, all of which confirm the robustness.[67]

3.3 Technological Barrier Effect

To test the technological barrier effect, we run the following industry-level regression for the four census years during the pre-shock period (1982-1999):

$$FirmEntry_{jt} = \beta TechBarrier_{jt} + \delta_j + \delta_t + \alpha + \varepsilon_{jt}. \tag{31}$$

three-digit NAICS broad industries for each six-digit NAICS industry.

[64] This issue can potentially arise from the selection of samples with a positive number of patents granted in the start year and in any of the last four years of each period in the regression analysis, which is inevitable to compute the self-citation ratio over two years for each period.

[65] To formulate the weights, we employ a logit regression on the entire universe of the LBD. The dependent variable is set to one if the firm belongs to the regression sample and zero otherwise. The independent variables include firm size, age, employment growth rate, industry, and a multi-unit status indicator.

[66] In our baseline analysis, we cluster the standard errors at the six-digit NAICS level as most variations in the firm-level NTR gap occur at the industry level.

[67] Tables G14-G16 in the Online Appendix are consistent with our model prediction: higher competitive pressure reduces the number of new products added (external innovation) for all firms; increases the number of patents with a high level of self-citation (internal innovation) for innovative firms; and increases product market concentration.

Table 2: Technological Barrier Effect

	Firm entry	Firm entry
Technological barriers	-0.012**	-0.016**
	(0.006)	(0.007)
Observation	1,300	1,300
Fixed effects	j, t	j, t
Tech. barrier thresholds	Top 5%	Top 10%

Note: Industries are NAICS6 defined in the Census of Manufacturers. Technological barriers are measured by either the top 5th or top 10th percentile level of firm-level TFPR (normalized by the frontier level) within each industry. The first column uses the top 5th percentile, and the second column uses the top 10th percentile. Estimates for industry (j) and year (t) fixed effects as well as the constant are suppressed. Observations are unweighted. Observation counts are rounded due to Census Bureau disclosure avoidance procedures. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

$FirmEntry_{jt}$ is the firm entry rate, and $TechBarrier_{jt}$ is the technological barrier in industry j at year t. We measure the industry-level technological barrier using the skewness of the firm-level TFPR distribution (normalized by the industry frontier level). A right-skewed distribution indicates more firms near the frontier, implying intensive innovation in that industry. We normalize firm-level TFPR by the industry frontier level and use the top 5th or 10th percentile value.[68]

Table 2 indicates that firm entry is lower in industries with higher technological barriers, which supports the technological barrier effect predicted in our model.

3.4 Quality of Innovation: Internal vs. External

We provide empirical evidence consistent with our theoretical assumption regarding the quality improvement differential between internal and external innovation ($\eta > \lambda$). We measure innovation quality in two ways: the number of forward citations each patent receives as a proxy for the scientific value of innovation, and the stock market response to news about patents as a measure of the market value of innovation (Kogan et al., 2017).[69] Since the market value of innovation measure is available only for publicly traded firms, we restrict this analysis to patenting firms in Compustat from the pre-shock period, specifically 1982 to 1999.

To test whether patent quality varies across the two types of innovation, we estimate the

[68]Note that this is the inverse of the TFPR gap in Aghion et al. (2005) (i.e., 1-TFPR gap).

[69]Data is sourced from the paper's website (https://github.com/KPSS2017/Technological-Innovation-Resource-Allocation-and-Growth-Extended-Data), which is available through 2023.

Table 3: Patent Quality Comparison

	M-value	M-value	M-value	S-value	S-value	S-value
Self-citation	0.192***	-0.289***	-0.027***	-0.110***	-0.082***	-0.047***
	(0.008)	(0.006)	(0.005)	(0.008)	(0.008)	(0.008)
Market cap$_{-1}$		0.431***	0.289***		-0.025***	-0.043***
		(0.001)	(0.003)		(0.001)	(0.005)
Observations	360,750	360,750	360,750	360,750	360,750	360,750
Fixed effects	ct	ct	i, ct	ct	ct	i, ct

Notes: The regression sample consists of patenting firms in Compustat from 1982 to 1999. The estimates for firm (i), CPC technology-year (ct) fixed effects, and the constant are suppressed. Robust standard errors are displayed below each coefficient. Observations are unweighted. The mean (standard deviation) of the market value, scientific value, and self-citation ratio are 2.07 (1.32), 2.88 (1.22), and 0.17 (0.24), respectively. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

following regression model:

$$Quality_{ipct} = \alpha + \beta_1 SelfCite_{ipct} + \beta_2 X_{it-1} + \delta_{ct} + \varepsilon_{ipct}. \tag{32}$$

$Quality_{ipct}$ is either the log of market value (M-value) or the log of one plus the number of forward citations (S-value) for patent p, created by firm i in year t, within CPC subsection c. $SelfCite_{ipct}$ is a self-citation ratio of firm i's patent p. X_{it-1} is the size of firm i in period $t-1$. In the baseline analysis, we use the logged value of market capitalization as the measure of firm size.[70] δ_{ct} is a CPC technology-year fixed effect that allows us to compare patent quality within each market while controlling for varying forward citation trends across different technologies and years.[71]

Table 3 presents the estimation results. Because a patent's market value is the product of its estimated stock return and the firm's market capitalization (Kogan et al., 2017), it is positively correlated with market capitalization (pairwise correlation = 0.67 in our sample). Large firms also tend to produce patents with higher self-citation ratios (correlation = 0.16). Thus, this leads to a positive association between market value and the internalness of innovation within each product market without any controls (column 1). After controlling for these factors, we find a negative relationship (column 2) that continues to persist within firms (column 3, with firm fixed effects δ_i).

[70] We calculate market capitalization by multiplying the closing market price (PRCC_F) by the number of common shares outstanding (CSHO).

[71] Since we do not have information on the specific product market for each patent, we use the main CPC subsection as a proxy for the product markets.

Table 4: Parameter Estimates

External calibration			Internal calibration		
Param.	Description	Value	Param.	Description	Value
β	Time discount rate	0.947	$\widehat{\chi}$	Scale of internal R&D	0.044
$\widehat{\psi}$	Curvature of internal R&D	2.000	$\widetilde{\chi}$	Scale of external R&D	0.405
$\widetilde{\psi}$	Curvature of external R&D	2.000	$\widetilde{\chi}^e$	Scale of startup R&D	1.689
$\widetilde{\psi}^e$	Curvature of startup R&D	2.000	λ	Step size, internal innovation	1.040
θ	Quality share, final goods	0.109	η	Step size, external innovation	1.075
			$\mathcal{E}_o$	Mass of outside entrants	0.007

Similarly, the scientific value and the internalness of innovation are negatively correlated (columns 4-6). These results support the notion that external innovations are of higher quality than internal innovations (with higher self-citation ratios).[72]

In the Online Appendix, we present additional regression results highlighting differences between internal and external innovation, including learning time. Especially, the analysis in Online Appendix G.2 shows that external innovation takes longer to develop than internal innovation, as indicated by the gap between the application year of a patent and those of the patents it cites. Appendix G.3 further compares the impacts of the two types of innovation on various firm performance metrics, as discussed earlier. These results align with our model predictions.

4 Quantitative Analysis

In this section, we calibrate the model to the U.S. manufacturing sector in 1992 and conduct counterfactual exercises to analyze the aggregate effects of increased competition and shifts in firm innovation composition. To better align the model with data, we expand the baseline model in Section 2 by allowing firms to undertake multiple external innovations simultaneously, depending on their number of products, as in Klette and Kortum (2004).[73]

Table 5: Target Moments

Moment	Data	Model	Moment	Data	Model
Number of products	2.3	2.3	Avg. productivity growth (%)	1.9	1.9
Number of products added	0.3	0.3	High-growth firm growth (%)	22.5	22.3
Firm entry rate (%)	7.6	7.6	Import penetration rate (%)	15.3	15.3

4.1 Calibration

There are eleven structural parameters in the model, listed in Table 4. The first group of five parameters is externally calibrated, and the second group of six parameters is internally calibrated to match moments associated with firm-level variables and the import penetration ratio in the U.S. manufacturing sector.[74,75]

Externally Calibrated Parameters The time discount factor (β) is set to 0.947, which corresponds to an annual interest rate of 5.6%. The curvature parameters of the three R&D cost functions ($\widehat{\psi}$, $\widetilde{\psi}$, $\widetilde{\psi}^e$) are taken from Acemoglu et al. (2018) and Akcigit and Kerr (2018). We set the average profit-to-sales ratio θ $(= \int_f \frac{profit_f}{sales_f} df)$ to match the quality share in final goods production (10.9%) reported in Akcigit and Kerr (2018). We normalize the mass of potential domestic startups.

Internally Calibrated Parameters The remaining six parameters are internally calibrated to minimize the objective function, $\min \sum_{i=1}^{6} \frac{\left|\text{model moment}_i - \text{data moment}_i\right|}{\frac{1}{2}\left|\text{model moment}_i\right| + \frac{1}{2}\left|\text{data moment}_i\right|}$, with the six target moments in Table 5. Although the parameters are jointly calibrated, the most relevant moments for each set of parameters can be noted. Internal and external R&D scales ($\widehat{\chi}$, $\widetilde{\chi}$) are set to match the average number of products and the number of products added per firm. The startup external R&D scale ($\widetilde{\chi}^e$) matches the firm entry rate. We target the average productivity growth rate and the employment growth rate of high-growth firms (90th percentile) to determine the quality multipliers for internal (λ) and external innovations (η). Lastly, the mass of potential outside

[72]The findings remain robust across different measures of firm size, even when incorporating firm-year fixed effects.

[73]See Online Appendix C and D for further details and computational algorithms used to solve the model.

[74]The average number of products and the number of products added are from the 1992 CMF. The high-growth firm growth rate is sourced from the LBD (Decker et al., 2016). Data on manufacturing imports and exports for the import penetration ratio come from Schott (2008), while data on manufacturing value added and productivity are from the NBER-CES Manufacturing Industry Database. The firm entry rate is taken from the BDS.

[75]We use the import penetration ratio because it is an observable moment that partially reflects exogenous changes in competitive pressure.

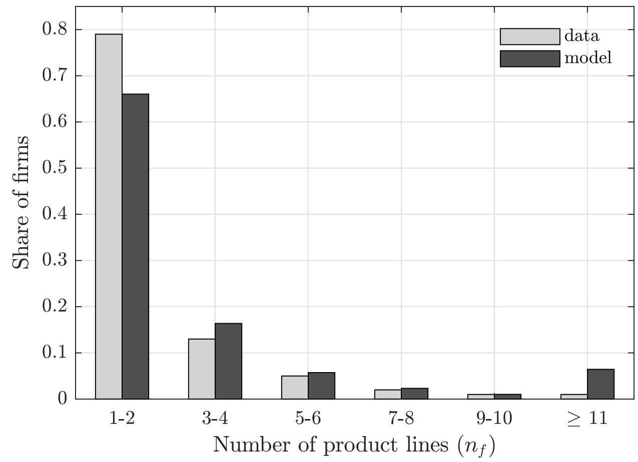

Figure 1: Firm Size Distribution: Theory and Data

entrants (ε_o) targets the import penetration ratio in the manufacturing sector.

Model Properties The calibration results are presented in Table 5, where our model performs well in matching the target moments overall. In particular, it matches well the number of products and products added. Conditional on the number of products, the number of products added reflects both innovation intensity and the duration required to learn the frontier technology in the data. These variables may vary across products or technologies, and the learning time may not be strictly annual. By assuming a fixed annual learning duration, the calibration adjusts the R&D cost parameters (innovation intensity) to align the data with the annual frequency of the model.

Also, note that targeting the growth of high-growth firms helps us pin down the relative size of the two step sizes λ and η, since external innovation has a greater impact on the right tail of the firm growth distribution. Our model aligns well with these moments, and the estimated parameter values suggest that external innovation contributes 1.88 ($\frac{0.075}{0.04}$) times more to growth compared to internal innovation. Also, the estimates satisfy the assumption $\lambda^2 > \eta > \lambda$, even without imposing any parameter restrictions such as $\lambda^2 > \eta$.

As an external validation of our model, Figure 1 compares the firm size distribution (in terms of the number of products) between the model and the data, which is untargeted. While the model exhibits a thicker right tail, indicating a greater number of firms with 11 or more products, it generally aligns closely with the data. Another important untargeted moment is the aggregate

R&D to sales ratio.[76] Our model estimate is 4.6%, which closely matches the data estimate of 4.1% in Akcigit and Kerr (2018). Our model incorporates all resources used for product quality improvement and product scope expansion into R&D expenses, some of which might not be fully captured in the data.

4.2 Counterfactual Exercises

In this section, we assess the impact of heightened competition on overall firm innovation, the composition of firm innovation, and aggregate growth. In the model, we increase the mass of potential outside entrants $\mathcal{E}_o$ by 83%, corresponding to the rise in import penetration ratio in the U.S. manufacturing sector from 1992 to 2007 (from 15.3% to 25.1%).[77,78]

Increasing Competitive Pressure from Outside Firms Table 6 shows that an exogenous increase in outside firm entry leads to a rise in the aggregate creative destruction arrival rate $\overline{x}$ and results in three key effects: i) the expected profits of both internal and external innovation ($\{A_\ell\}_{\ell=1}^{4}$ and B) decrease, known as the Schumpeterian effect (Panel B); ii) incumbents intensify internal innovation to protect their existing product lines, especially those with a technology gap of $\Delta^\ell > 1$, referred to as the market-protection effect; iii) the market-protection efforts along with the increase in $\overline{x}$ raise the average technology gap, making it harder for firms to take over product markets via external innovation, labeled as the technological barrier effect (Panel C).[79]

Our novel mechanism comes through ii) and iii). In the general equilibrium, these effects come into play together and interact. For instance, the technological barrier effect in iii) additionally influences the aggregate creative destruction arrival rate $\overline{x}$, causing a feedback loop involving i) to iii). This effect arises as external innovation by outside firms and successful internal innovation shift the technology gap distribution (Panel C). Specifically, the density of Δ^2, Δ^3, and Δ^4 increases,

[76] The aggregate R&D to sales ratio is defined as the ratio of total R&D expenses (the sum of internal and external R&D expenses) of domestic incumbents to their total sales.

[77] Although we use a trade-related moment in our analysis, we do not intend to assess the effect of trade. We borrow the competition aspect embedded in the trade. For a detailed analysis of the effect of trade on innovation, see Jo (2024), which extends our framework to a two-country model to explore the impacts of globalization on business dynamism.

[78] In Online Appendix F, we explore an additional counterfactual analysis involving an increase in the creative destruction arrival rate by domestic startups. This comparison allows us to assess the results in light of varying sources of increased competitive pressure.

[79] $\overline{A} \equiv \sum_{\ell=1}^{4} A_\ell \mu(\Delta^\ell)$. Table E1 in Online Appendix E presents details for the changes in $\{A_\ell\}_{\ell=1}^{4}$.

Table 6: Counterfactual: Increasing Competitive Pressure in the U.S.

Description	Variables	Before	After	% Change
Panel A: Changes in Firm Innovation				
creative destruction arrival rate by outside firms (%)	$\overline{x}_o$	3.3	5.5	66.4%
aggregate creative destruction arrival rate (%)	$\overline{x}$	21.5	21.9	1.51%
prob. of internal innovation ($\Delta^1 = 1$, %)	z^1	16.9	16.8	-0.43%
prob. of internal innovation ($\Delta^2 = \lambda$, %)	z^2	57.8	57.9	0.19%
prob. of internal innovation ($\Delta^3 = \eta$, %)	z^3	39.7	39.7	0.13%
prob. of internal innovation (%) ($\Delta^4 = \frac{\eta}{\lambda}$, %)	z^4	37.3	37.4	0.05%
prob. of external innovation, incumbents (%)	x	16.8	16.5	-1.33%
prob. of external innovation, potential startups (%)	x_e	4.02	3.97	-1.33%
Panel B: Changes in Innovation Values				
Average of internal innovation values	$\overline{A}$	0.167	0.165	-1.04%
External innovation value	B	0.011	0.011	-2.6%
Panel C: Changes in Technology Gap Distribution				
Technology gap distribution (shares)	$\Delta^1 = 1$	0.541	0.539	-0.4%
	$\Delta^2 = \lambda$	0.314	0.314	0.2%
	$\Delta^3 = \eta$	0.116	0.118	1.1%
	$\Delta^4 = \frac{\eta}{\lambda}$	0.028	0.029	1.4%

reducing the conditional takeover probability and the ex-ante value of successful product takeover.[80] Consequently, firm incentives for external innovation and domestic firm entry get reduced. This effect contributes to the decline observed in x and x_e in Table 6.[81]

Finally, Table 7 summarizes how aggregate variables change in response to the increased competitive pressure from outside firms. The aggregate R&D to sales ratio of domestic incumbents drops, indicating that the decrease in external innovation outweighs the increase in internal innovation. Thus, external R&D intensity—the ratio of domestic R&D expenses for external innovation to total R&D expenses—decreases. The average number of products per firm declines, aligning with the empirical findings of Bernard et al. (2011). Furthermore, the total number of domestic firms falls.

In addition, Panel B shows that the average productivity growth of domestic firms (g_d) declines. This decrease is attributed to shifts in firm-level innovation intensities and the mass of firms. Keeping

[80] $\overline{x}_{\text{takeover}}$ decreases from 73.2% to 73.0%. The increase in densities $\mu(\Delta^3)$ and $\mu(\Delta^4)$ is solely attributed to increased external innovation by outside firms. The higher density of Δ^2 reflects both increased internal innovation driven by the market-protection effect and external innovation by outside firms.

[81] Recall that the total decline in x and x_e results from the combined impact of the Schumpeterian and technological barrier effects. A decomposition reveals that 17.0% and 15.0% of the total change in x and x_e are ascribed to the technological barrier effect (due to the shifts in $\mu(\Delta^\ell)$, given all else equal).

Table 7: Changes in Aggregate Moments

Description	Before	After	% Change
Panel A: Changes in the Aggregate Moments			
R&D to sales ratio (%)	4.6	4.5	-1.6%
External R&D intensity (%)	63.9	63.1	-1.2%
Average number of products	2.3	2.2	-5.5%
Total mass of domestic firms	0.386	0.361	-6.4%
Panel B: Changes in the Aggregate Growth and Decomposition			
Average productivity growth by domestic firms (%)	1.9	1.7	-11.0%
Growth from domestic internal innovation (%)	1.0	0.9	-11.4%
Growth from domestic external innovation (%)	0.7	0.6	-13.0%
Growth from domestic startups (%)	0.2	0.2	-1.7%

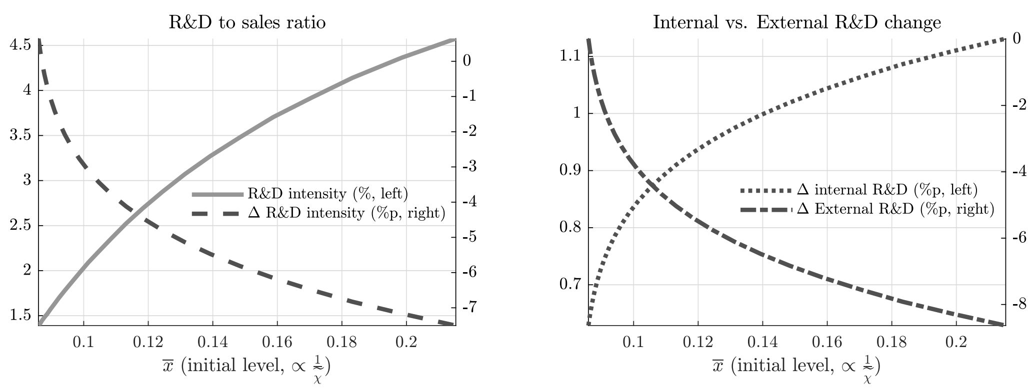

Figure 2: Decomposition of Innovation Change Across different $\widetilde{\chi}$

the mass of domestic incumbents constant, 12.7% of this decline in growth can be attributed to changes in firm-level external innovation.[82]

Comparison: Economy with High External Innovation Costs To compare implications across environments with different innovation structures, we re-calibrate the model to hypothetical economies characterized by lower creativity (less external innovation due to higher frictions) compared to the U.S. Specifically, we increase the parameter associated with external innovation costs ($\widetilde{\chi}$) up to 80 times higher than the baseline value of 0.405, while keeping other parameters unchanged. We then perform the same counterfactual analysis.

Figure 2 shows the results across different initial levels of $\overline{x}$ (reflecting different degrees of

[82]For more detailed breakdowns, refer to Tables E2 and E3 in Online Appendix E. Note that if we take into account the contribution of outside firms, the aggregate growth rate increases.

Table 8: Aggregate Moment Comparison: U.S. vs. High External Innovation Cost Economy

Moment	Baseline	High Ext. Costs	After Shock	% Change
R&D to sales ratio (%)	4.58	1.39	1.41	1.0%
External R&D intensity (%)	63.9	8.6	7.8	-9.8%
Average number of products	2.3	1.0	1.0	-0.2%
Avg. growth by domestic firms (%)	1.9	1.4	1.3	-9.7%

initial competitive pressure) corresponding to varied values of $\widetilde{\chi}$ (that negatively affects $\overline{x}$). The U.S. economy represents the highest $\overline{x}$ level in the figures. The left panel shows the initial R&D to sales ratios and their changes following a competitive pressure shock, and the right panel breaks down the latter into the changes in internal and external innovations.

Internal R&D expenses rise as competitive pressure intensifies, while external R&D expenses decline. However, the decrease in external R&D is more pronounced when its cost $\widetilde{\chi}$ is low (high initial $\overline{x}$). While both types of innovation shift similarly across different economies, internal innovation increases more than external innovation in high-cost external innovation (low initial $\overline{x}$) environments, whereas the reverse holds for low-cost (high initial $\overline{x}$) environments.[83] Thus, in economies where external innovation costs are high, aggregate R&D increases in response to competitive pressure, contrasting with the U.S.

Table 8 compares aggregate moments between the U.S. and an economy with high external innovation costs ($\widetilde{\chi} \times 80$), as well as the response of the latter economy to a competition shock. The first two columns show that the low creativity economy exhibits lower dynamism than the U.S. with less R&D, fewer products, and lower average productivity growth. The last two columns indicate that both economies respond similarly to increased foreign competition, except for the R&D to sales ratio, where the difference arises from the initially lower level of external innovation in the low creativity economy. Despite the increased domestic innovation, the growth attributable to domestic innovation drops in this economy. The reduction is associated with decreases in external innovation by domestic incumbents and startups, coupled with a decline in the mass of domestic incumbents.[84]

[83] See Table E4 in Online Appendix E for detailed results when $\widetilde{\chi}$ is 80 times higher than the U.S.

[84] The version with the mass of firms fixed is presented in Table E5 in Online Appendix E. Note that this pattern holds even without the effect of the changes in firm mass.

Discussion Our results underscore the importance of examining changes in innovation composition, which can further help reconcile disparate findings from previous studies. For instance, if the European economy faces higher external innovation costs due to barriers like complex approval processes or labor regulation (e.g., Peters, 2020; Aghion et al., 2023), our model suggests that increased foreign competition may raise overall innovation in Europe compared to the U.S. through the changes in innovation composition. This extends Aghion et al. (2005) by integrating multiple strands of literature and highlighting compositional changes as a mechanism determining the aggregate impacts of competition.

Understanding the compositional changes in innovation is also important to properly evaluate the aggregate implications of competition. Increased overall innovation from heightened competition may not be beneficial if driven by defensive internal innovation, which contributes less to economic growth than external innovation and limits firm entry. Our findings suggest that competition does not effectively address the challenge of low external innovation in low-creativity economies but could rather exacerbate this issue.[85]

5 Conclusion

In this paper, we explore the impact of competition on firm innovation under imperfect technology spillovers by examining the shifts in innovation composition driven by firms' strategic choices, both theoretically and empirically. We find that heightened competition prompts firms to increase internal innovation within products with technological advantages, while dampening external innovation. Moreover, the overall impact of competition on innovation depends on innovation cost structures, leading to changes in innovation composition of varying magnitudes and shaping distinct aggregate implications. This channel helps fill gaps in existing literature, reconcile previous findings, and advance our understanding of the intricate role of competition in firm innovation.

[85] Peters (2020) also documents that external innovation reduces misallocation in the economy by limiting the market power accumulation of incumbents. This provides further evidence supporting our claim that increasing overall innovation does not always benefit the economy.

Appendix: Proofs of Propositions

Proof of Lemma 1 . Consider the following two cases: 1) no ownership change between $t-1$ and t, and 2) ownership change happens between $t-1$ and t. In scenario 1), $q_{j,t} = \Delta_{j,t} q_{j,t-1}$ with only $\Delta_{j,t} \in \{\Delta^1 = 1, \Delta^2 = \lambda\}$ as a result of internal innovation. In scenario 2), $q_{j,t} = \eta q_{j,t-2}$ holds. Let's consider all possible cases where i) $\Delta_{j,t} = 1$, ii) $\Delta_{j,t} = \lambda$, iii) $\Delta_{j,t} = \eta$, iv) $\Delta_{j,t} = \frac{\eta}{\lambda}$, v) $\Delta_{j,t} = \frac{\eta^n}{\lambda^m}$ with $n \geq m > 0$, and vi) $\Delta_{j,t} = \frac{\lambda^n}{\eta^m}$ with $n > m > 0$. These are the only possible values Δ can assume, given that product quality can only be adjusted by three step sizes (1, λ, and η) between two periods without technology regression ($q_t < q_{t-1}$).

- i) $\Delta_{j,t} = 1$: For this to be true, $q_{j,t} = q_{j,t-1}$ should hold. Since $q_{j,t} = \eta q_{j,t-2}$, we need $q_{j,t-1} = \eta q_{j,t-2}$. This is possible if there was external innovation between $t-2$ and $t-1$, and no internal innovation between $t-3$ and $t-1$, leading to $q_{j,t-2} = q_{j,t-3}$.

- ii) $\Delta_{j,t} = \lambda$: For this to be true, $\Delta_{j,t-1} = \frac{\eta}{\lambda}$ should hold, as $\Delta_{j,t} = \frac{q_{j,t}}{q_{j,t-1}} = \frac{\eta q_{j,t-2}}{\Delta_{j,t-1} q_{j,t-2}}$. This can be possible if there were internal innovation between $t-3$ and $t-2$, and external innovation between $t-2$ and $t-1$, but no internal innovation between $t-2$ and $t-1$. In this case, $q_{j,t-2} = \lambda q_{j,t-3}$ and $q_{j,t-1} = \eta q_{j,t-3}$ holds, and thus $\Delta_{j,t-1} = \frac{q_{j,t-1}}{q_{j,t-2}} = \frac{\eta q_{j,t-3}}{\lambda q_{j,t-3}} = \frac{\eta}{\lambda}$ follows. So we have shown that both $\Delta_{j,t} = \lambda$ and $\Delta_{j,t} = \frac{\eta}{\lambda}$ are possible, and $\Delta_{j,t} = \frac{\eta}{\lambda}$ can be realized only through external innovation between $t-1$ and t.

- iii) $\Delta_{j,t} = \eta$: For this to be true, $q_{j,t-1} = q_{j,t-2}$ should hold. This is possible if there was neither ownership change nor internal innovation between $t-1$ and $t-2$.

- iv) $\Delta_{j,t} = \frac{\eta}{\lambda}$: This follows the illustration in case ii)

- v) $\Delta_{j,t} = \frac{\eta^n}{\lambda^m}$ with $n \geq m > 0$: Suppose this is the case. As $\Delta_{j,t} \notin \{\Delta^1 = 1, \Delta^2 = \lambda\}$, there should be an ownership change between $t-1$ and t. Thus $q_{j,t} = \eta q_{j,t-2}$ holds, implying $q_{j,t-1} = \frac{\lambda^m}{\eta^{n-1}} q_{j,t-2}$. Note that $m \leq n-1$ is not possible without technology regression. Thus, $m = n$ (as $m > n-1$ and $n \geq m > 0$). If $\frac{\lambda^m}{\eta^{m-1}} < 1$, this implies technology regression and can be ruled out. Suppose $\frac{\lambda^m}{\eta^{m-1}} > 1$. If $m = 1$, we are back to the cases ii) and iv). Suppose $m > 1$. As $\frac{\lambda^m}{\eta^{m-1}} \neq 1$ or λ, there should be an ownership change between $t-2$ and $t-1$. Thus, $q_{j,t-1} = \eta q_{j,t-3}$ holds, implying $q_{j,t-2} = \frac{\eta^m}{\lambda^m} q_{j,t-3}$. If $\Delta_{j,t} = \frac{\eta^n}{\lambda^m}$ is possible,

$q_{j,t-s} = \frac{\eta^m}{\lambda^m} q_{j,t-s-1}$ holds for even numbers s, and $\frac{\lambda^m}{\eta^{m-1}} q_{j,t-s-1}$ holds for odd numbers s. Thus, in this case, either $q_{j,1} = \frac{\eta^m}{\lambda^m} q_{j,0}$ or $q_{j,1} = \frac{\lambda^m}{\eta^{m-1}} q_{j,0}$ must hold, which can be ruled out (or we assume this case does not occur). Thus, $\Delta_{j,t} = \frac{\eta^n}{\lambda^m}$ with $n \geq m > 0$ is not possible.

- vi) $\Delta_{j,t} = \frac{\lambda^n}{\eta^m}$ with $n > m > 0$: Following the same argument, this case is not possible.

Therefore $\Delta_{j,t}$ can assume only the four values of $\{1, \lambda, \eta, \frac{\eta}{\lambda}\}$. $\square$

Proof of Proposition 1. Using the conjectured value function, we can decompose the expected value into two parts with the linearity of expectation: the expected value of existing product lines $\mathbb{E}\left[\sum_{\ell=1}^{2} A_\ell \sum_{j\in\mathcal{J}^f|(\Delta_j'|\Delta_j)=\Delta^\ell} \Delta^\ell q_j\right]$ and the expected value for the new product line added through external innovation $\mathbb{E}\left[\sum_{\ell=1}^{4} A_\ell \, I_{\{\eta/\Delta_j=\Delta^\ell\}} \, \frac{\eta}{\Delta_j} \, q_j\right]$. As the realization of internal innovation outcomes and the creative destruction are independent of the realization of external innovation, the expected value of a new product line becomes:

$$\begin{aligned}\mathbb{E}\left[\sum_{\ell=1}^{4} A_\ell \, I_{\left\{\frac{\eta}{\Delta_j}=\Delta^\ell\right\}} \, \frac{\eta}{\Delta_j} \, q_j\right] &= \sum_{I^x=0}^{1} x^{I^x}(1-x)^{1-I^x} \mathbb{E}_{q_j,\Delta_j}\left[\sum_{\ell=1}^{4} A_\ell \, I_{\left\{\frac{\eta}{\Delta_j}=\Delta^\ell\right\}} \, I^x \, \frac{\eta}{\Delta_j} \, q_j\right] \\ &= x\left[\frac{1-z^3}{2} A_1 \mu(\Delta^3) + \left(1-\frac{z^4}{2}\right) A_2 \lambda \mu(\Delta^4) + A_3 \eta \mu(\Delta^1) + (1-z^2) A_4 \frac{\eta}{\lambda} \mu(\Delta^2)\right] \overline{q}\,.\end{aligned}$$

The terms in the bracket arise from the random property of external innovation. The assigned product can have a technology gap of Δ^ℓ with a probability of $\mu(\Delta^\ell)$, and the probability of taking over this product line depends on its technology gap. Integrating over all possible qualities q_j over the entire set of available products gives us $\overline{q}$.[86]

The expected value of existing product lines can further be broken down into the four cases of Δ and integrated as $\sum_{\widetilde{\ell}=1}^{4} \mathbb{E}\left[\sum_{\ell=1}^{2} A_\ell \sum_{j\in\mathcal{J}^f|(\Delta_j'|\Delta_j=\Delta^{\widetilde{\ell}})=\Delta^\ell} \Delta^\ell q_j\right]$. To simplify the derivation, we reorder product quality q_j by its technology gap Δ_j and categorize it into the following four groups: $q_1^f = \{q_{j_1}, q_{j_2}, \ldots, q_{j_{n_f^1}}\}$; $q_2^f = \{q_{j_{n_f^1+1}}, \ldots, q_{j_{n_f^1+n_f^2}}\}$; $q_3^f = \{q_{j_{n_f^1+n_f^2+1}}, \ldots, q_{j_{n_f^1+n_f^2+n_f^3}}\}$; and $q_4^f = \{q_{j_{n_f^1+n_f^2+n_f^3+1}}, \ldots, q_{j_{n_f^1+n_f^2+n_f^3+n_f^4}}\}$, $q^f = \cup_{\widetilde{\ell}=1}^{4} q_{\widetilde{\ell}}^f$.

If $\Delta = \Delta^1$ ($\widetilde{\ell} = 1$), the expected value can be rephrased as $\sum_{i=1}^{n_f^1} \left[A_1(1-\overline{x})(1-z_i^1) + \lambda A_2(1-\overline{x})z_i^1\right] q_{j_i}$; if $\Delta = \Delta^2$ ($\widetilde{\ell} = 2$), it becomes $\sum_{i=n_f^1+1}^{n_f^1+n_f^2} \left[A_1(1-\overline{x})(1-z_i^2) + \lambda A_2 z_i^2\right] q_{j_i}$; if $\Delta = \Delta^3$

[86] Note that individual firms only have information about the distribution of technology gaps $\{\mu(\Delta^\ell)\}_{\ell=1}^4$ and the average quality level $\overline{q}$. That is, for an individual firm, a technology gap and product quality are independent considerations.

($\widetilde{\ell}=3$), it is $\sum_{i=n_f^1+n_f^2+1}^{n_f-n_f^4}\left[A_1\left(1-\frac{1}{2}\overline{x}\right)(1-z_i^3)+\lambda A_2 z_i^3\right]q_{j_i}$; and if $\Delta=\Delta^4$ ($\widetilde{\ell}=4$), it is $\sum_{i=n_f-n_f^4}^{n_f}\left[A_1(1-\overline{x})(1-z_i^4)+\lambda A_2\left(1-\frac{1}{2}\overline{x}\right)z_i^4\right]q_{j_i}$.

The $B\overline{q}$ portion of the conjectured value function in $\mathbb{E}\left[V\left(\Phi^{f\prime}\mid\Phi^f\right)\Big|\{z_j\}_{j\in\mathcal{J}^f},x\right]$ can be expressed as $\mathbb{E}B\overline{q}'=B(1+g)\overline{q}$, where g denotes the growth rate of product quality in a balanced growth path (BGP) equilibrium. Plugging this into the conjectured value function, we can rephrase the original value function as:

$$
\begin{aligned}
&\sum_{i=1}^{n_f^1}A_1q_{j_i}+\sum_{i=n_f^1+1}^{n_f^1+n_f^2}A_2q_{j_i}+\sum_{i=n_f^1+n_f^2+1}^{n_f-n_f^4}A_3q_{j_i}+\sum_{i=n_f-n_f^4+1}^{n_f}A_4q_{j_i}+B\overline{q}=\\
&\max_{\substack{x\in[0,\bar{x}],\\ \{z_i\in[0,\bar{z}]\}_{i=1}^{n_f}}}\left\{\begin{array}{l}
\sum_{i=1}^{n_f}\left[\pi q_{j_i}-\hat{\chi}z_i^{\hat{\psi}}q_{j_i}\right]-\overline{q}\widetilde{\chi}x^{\widetilde{\psi}}\\
+\widetilde{\beta}\sum_{i=1}^{n_f^1}\left[A_1(1-\overline{x})(1-z_i^1)+\lambda A_2(1-\overline{x})z_i^1\right]q_{j_i}\\
+\widetilde{\beta}\sum_{i=n_f^1+1}^{n_f^1+n_f^2}\left[A_1(1-\overline{x})(1-z_i^2)+\lambda A_2z_i^2\right]q_{j_i}\\
+\widetilde{\beta}\sum_{i=n_f^1+n_f^2+1}^{n_f-n_f^4}\left[A_1\left(1-\frac{1}{2}\overline{x}\right)(1-z_i^3)+\lambda A_2z_i^3\right]q_{j_i}\\
+\widetilde{\beta}\sum_{i=n_f-n_f^4}^{n_f}\left[A_1(1-\overline{x})(1-z_i^4)+\lambda A_2\left(1-\frac{1}{2}\overline{x}\right)z_i^4\right]q_{j_i}\\
+\widetilde{\beta}x\left[\frac{1}{2}(1-z^3)A_1\mu(\Delta^3)+\left(1-\frac{1}{2}z^4\right)A_2\lambda\mu(\Delta^4)\right.\\
\qquad\qquad\left.+A_3\eta\mu(\Delta^1)+(1-z^2)A_4\frac{\eta}{\lambda}\mu(\Delta^2)\right]\overline{q}\\
+\widetilde{\beta}B(1+g)\overline{q}
\end{array}\right\}.
\end{aligned}
$$

By taking the first-order conditions with respect to each innovation intensity, we get the optimal innovation decision rules, which depend solely on technology gaps. Substituting these optimal innovation intensities into the value function, equating the left-hand side (LHS) to the right-hand side (RHS), and collecting terms, we obtain the five coefficients of the conjectured value function. □

Proof of Corollary 1. Define $\widetilde{z}^\ell=\frac{\widehat{\psi}\widehat{\chi}}{\widetilde{\beta}}\left(z^\ell\right)^{(\widehat{\psi}-1)}$. Then $z^\ell>z^{\ell'}\Leftrightarrow\widetilde{z}^\ell>\widetilde{z}^{\ell'}$ for all $\ell,\ell'\in[1,4]\cap\mathbb{Z}$ under the condition $\widehat{\psi}>1$. Given $\widetilde{z}^2-\widetilde{z}^3=\frac{1}{2}\overline{x}A_1>0$, $\widetilde{z}^2-\widetilde{z}^1=\overline{x}\lambda A_2>0$, $\widetilde{z}^2-\widetilde{z}^4=\frac{1}{2}\overline{x}\lambda A_2>0$, and $\widetilde{z}^4-\widetilde{z}^1=\frac{1}{2}\overline{x}\lambda A_2>0$, we can obtain the following relationships: $z^2>z^3$, $z^2>z^1$, $z^2>z^4$, and $z^4 > z^1$. Given $\widetilde{z}^1 = (1-\overline{x})\left[\lambda A_2 - A_1\right] > 0$ in equilibrium, $\lambda A_2 - A_1 > 0$ holds, and $\widetilde{z}^3 > \widetilde{z}^4 \Leftrightarrow z^3 > z^4$ is derived. Thus, the order of $\{z^\ell\}_{\ell=1}^4$ in equilibrium is $z^2 > z^3 > z^4 > z^1$. $\square$

Proof of Corollary 2. The partial derivatives of $\{z^\ell\}_{\ell=1}^4$ with respect to $\overline{x}$ are (after removing the common terms) $\frac{\partial z^1}{\partial \overline{x}}\Big|_{A_1,A_2} : -(z^1)^{2-\widehat{\psi}}\big[\lambda A_2 - A_1\big] < 0$; $\frac{\partial z^2}{\partial \overline{x}}\Big|_{A_1,A_2} : (z^2)^{2-\widehat{\psi}} A_1 > 0$; $\frac{\partial z^3}{\partial \overline{x}}\Big|_{A_1,A_2} : (z^3)^{2-\widehat{\psi}}\frac{1}{2}A_1 > 0$; and $\frac{\partial z^4}{\partial \overline{x}}\Big|_{A_1,A_2} : -(z^4)^{2-\widehat{\psi}}\big[\frac{1}{2}\lambda A_2 - A_1\big] \gtreqless 0$, with A_1 and A_2 fixed. As $\lambda A_2 - A_1 > 0$, it follows that $\frac{\partial z^1}{\partial \overline{x}}\Big|_{A_1,A_2} < 0$. Similarly, $\frac{\partial z^2}{\partial \overline{x}}\Big|_{A_1,A_2} > \frac{\partial z^3}{\partial \overline{x}}\Big|_{A_1,A_2}$ holds with $z^2 > z^3$, and $\frac{\partial z^3}{\partial \overline{x}}\Big|_{A_1,A_2} > \frac{\partial z^4}{\partial \overline{x}}\Big|_{A_1,A_2}$ holds with $z^3 > z^4$ and $\lambda A_2 - A_1 > 0$. However, the sign for $\frac{1}{2}\lambda A_2 - A_1$ remains ambiguous. $\square$

Proof of Potential Startups' Problem. With the value function defined for incumbents, we have $\mathbb{E}V(\{(q_j', \Delta_j')\}) = x_e\big[\frac{1}{2}(1-z^3)A_1\mu(\Delta^3) + \big(1-\frac{1}{2}z^4\big)A_2\lambda\mu(\Delta^4) + A_3\eta\mu(\Delta^1) + (1-z^2)A_4\frac{\eta}{\lambda}\mu(\Delta^2)\big]\overline{q} + x_e\big[\frac{1}{2}(1-z^3)\mu(\Delta^3) + \big(1-\frac{1}{2}z^4\big)\mu(\Delta^4) + \mu(\Delta^1) + (1-z^2)\mu(\Delta^2)\big]B(1+g)\overline{q}$, from which the optimal external innovation choice for potential startups can be derived. $\square$

Proof of Proposition 2. In this model, the output growth rate is the same as the product quality growth rate. For product j with quality q_j and a technology gap of $\Delta_j = \Delta^\ell$, we can derive the following law of motion of q_j:

$$
\begin{array}{llll|llll}
\Delta^1: & q_j' = \Delta^1 q_j & \text{prob.} & (1-\overline{x})(1-z^1) & \Delta^2: & q_j' = \Delta^1 q_j & \text{prob.} & (1-\overline{x})(1-z^2) \\
 & q_j' = \Delta^2 q_j & \text{prob.} & (1-\overline{x})z^1 & & q_j' = \Delta^2 q_j & \text{prob.} & z^2 \\
 & q_j' = \Delta^3 q_j & \text{prob.} & \overline{x} & & q_j' = \Delta^3 q_j & \text{prob.} & 0 \\
 & q_j' = \Delta^4 q_j & \text{prob.} & 0 & & q_j' = \Delta^4 q_j & \text{prob.} & \overline{x}(1-z^2) \\
\Delta^3: & q_j' = \Delta^1 q_j & \text{prob.} & 1-z^3 & \Delta^4: & q_j' = \Delta^1 q_j & \text{prob.} & (1-\overline{x})(1-z^4) \\
 & q_j' = \Delta^2 q_j & \text{prob.} & z^3 & & q_j' = \Delta^2 q_j & \text{prob.} & z^4 + \overline{x}(1-z^4) \\
 & q_j' = \Delta^3 q_j & \text{prob.} & 0 & & q_j' = \Delta^3 q_j & \text{prob.} & 0 \\
 & q_j' = \Delta^4 q_j & \text{prob.} & 0 & & q_j' = \Delta^4 q_j & \text{prob.} & 0.
\end{array}
$$

Following this, we can compute the expected growth rate of q_j ($\mathbb{E}\big[q_j' \mid q_j\big]/q_j - 1$) and the aggregate growth rate in (25) by taking the expectation across all product lines.

Using the share of products owned by domestic incumbents ($s_d = \mathcal{F}_d/\mathcal{F}$), the definition of $\overline{x}$, and the evolution of product quality, the growth rate can be decomposed as follows:

$$
\begin{aligned}
g = & \underbrace{\left(\Delta^2 - 1\right) s_d \left[(1-\overline{x})z^1\mu(\Delta^1) + z^2\mu(\Delta^2) + z^3\mu(\Delta^3) + \left(1 - \frac{1}{2}\overline{x}\right) z^4\mu(\Delta^4)\right]}_{\text{internal innovation by domestic incumbent firms}} \\
& + \underbrace{\left(\Delta^2 - 1\right)(1 - s_d) \left[(1-\overline{x})z^1\mu(\Delta^1) + z^2\mu(\Delta^2) + z^3\mu(\Delta^3) + \left(1 - \frac{1}{2}\overline{x}\right) z^4\mu(\Delta^4)\right]}_{\text{internal innovation by foreign firms}} \\
& + \underbrace{\left(\overline{\Delta^{ex}} - 1\right) \mathcal{F}_d x \mu(\overline{\Delta^{ex}})}_{\text{external innov. by domestic incumbents}} + \underbrace{\left(\overline{\Delta^{ex}} - 1\right) \mathcal{E}_d x_e \mu(\overline{\Delta^{ex}})}_{\text{external innov. by domestic startups}} + \underbrace{\left(\overline{\Delta^{ex}} - 1\right) \overline{x}_o \mu(\overline{\Delta^{ex}})}_{\text{external innov. by foreign firms}},
\end{aligned}
$$

where $\overline{\Delta^{ex}} \equiv \frac{\Delta^3\mu(\Delta^1)+\Delta^4(1-z^2)\mu(\Delta^2)+\frac{1}{2}(1-z^3)\mu(\Delta^3)+\Delta^2\left(1-\frac{1}{2}z^4\right)\mu(\Delta^4)}{\mu(\Delta^1)+(1-z^2)\mu(\Delta^2)+\frac{1}{2}(1-z^3)\mu(\Delta^3)+\left(1-\frac{1}{2}z^4\right)\mu(\Delta^4)}$ is an increase in the average product quality due to external innovation and successful business takeover, and $\mu(\overline{\Delta^{ex}}) \equiv \mu(\Delta^1) + (1-z^2)\mu(\Delta^2) + \frac{1}{2}(1-z^3)\mu(\Delta^3) + \left(1 - \frac{1}{2}z^4\right)\mu(\Delta^4)$ is the share of product lines affected by external innovation. □

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Supplementary Materials The Online Appendix contains supplementary materials.

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