Workers' Job Prospects and Young Firm Dynamics*

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Abstract

This paper studies how workers' uncertain job prospects impact the wages and growth of young firms and quantifies their aggregate implications. Building a heterogeneous-firm directed search model in which workers gradually learn about permanent firm types, I find that the learning process creates endogenous wage differentials for young firms. In the model, a high-performing young firm must pay a higher wage than that of equally high-performing old firms, while a low-performing young firm offers a lower wage than that of equally low-performing old firms. This is because workers are unsure whether the young firm's performance reflects its fundamental type or a temporary shock due to the lack of historical records. Furthermore, higher uncertainty about young firms leads to bigger wage differentials and thus hampers the overall startup rate, young firm activity, and aggregate productivity. Using employee-employer linked data from the U.S. Census Bureau, I find consistent regression results. These findings offer a new perspective on firm dynamics through the workers' job prospects channel, with important implications for business dynamism and aggregate productivity.

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1 Introduction

Acquiring workers is essential for firms to grow, especially for young firms with high growth potential. High-growth young firms account for a disproportionate share of gross job creation and productivity growth in the U.S. and have been at the center of research (Haltiwanger, 2012; Haltiwanger et al., 2013; Decker et al., 2014, 2016; Haltiwanger et al., 2016; Foster et al., 2018).[1] However, young firms are nascent and have short track records. This increases workers' uncertainty about job prospects at the firms. The inherent uncertainty about the jobs offered by young firms could be important to understanding young firm dynamics, yet this mechanism has not been much studied. This paper proposes that uncertain job prospects affect worker pay and growth of young firms, and have macroeconomic implications for overall young firm activity, resource allocation, and aggregate productivity in the economy.

Workers evaluate the value of a job by considering its expected stream of wages, the possibility of being laid off, and potential future career development. To do so, workers consider the firm's current and historical performance, build beliefs about its growth potential, and decide whether to work for that firm. However, unlike mature firms, workers are less certain about young firms' performance as an indicator of their fundamentals, due to the firms' lack of history. Thus, workers may hesitate to work for young firms with high growth potential and may require compensating wage differentials relative to otherwise similar mature firms.

This paper formalizes and examines the proposed job prospects mechanism and its implications both theoretically and empirically. I construct a heterogeneous firm directed search model with learning about firm types and test the model with two comprehensive databases from the U.S. Census Bureau; the Longitudinal Business Database (LBD) and the Longitudinal Employer-Household Dynamics (LEHD).

I build on the directed search model of Schaal (2017) and introduce symmetric learning as in Jovanovic (1982). A novel feature of the model is that workers need to learn about firms' underlying productivity types along the firm life cycle, and take jobs based on their beliefs about firm types. In the model, workers' learning and uncertain job

[1] Using the Business Dynamics Statistics, I find that young firms (aged five or less) contribute to 29.76% of job creation, whereas their share of employment is only 12.73% in the U.S. during the period of 1998-2014.

prospects create endogenous wage differentials for young firms relative to otherwise similar mature firms. Specifically, I find that young firms with high demonstrated potential, defined as those with cumulative average performance above the cross-sectional prior mean, must offer wage premia to attract workers relative to otherwise similar mature firms. This is due to the relative lack of records for young firms, so that workers are not fully convinced by their average performance. Such wage differentials can create a barrier at the hiring or retention margin of those young firms, increasing their marginal costs and hampering their growth.

At the same time, young firms with low demonstrated potential, those with cumulative average performance below the cross-sectional prior mean, can pay wage discounts compared to their otherwise similar mature counterparts. This follows the same logic, where the low performing young firms benefit from the fact that their limited history gives them some upside risk.

The model further allows me to quantify the macroeconomic implications of this job prospects channel for overall young firm activity and aggregate productivity. A counterfactual analysis suggests that an increase in the fundamental uncertainty regarding young firms' job prospects (or an increase in noise dispersion in the learning) can lead to declines in firm entry and the share of young firms. This is because higher uncertainty slows down the speed of learning about firm types, and increases gaps in workers' job prospects and the consequent wage differentials.

In particular, more uncertain prospects amplify the wage premia paid by high performing young firms and hamper the growth of those young firms with high potential. Furthermore, more uncertain prospects allow low performing firms to pay less and linger in the economy. Thus, labor markets become tighter and overall hiring costs are raised for recruiting firms. This can in turn hamper overall allocative efficiency and decrease aggregate productivity. This shows that workers' job prospects at young firms can also have important macroeconomic impacts in the economy.

Next, I use the Census datasets and confirm these model predictions. In particular, I merge the LBD with LEHD, where the LBD tracks the universe of U.S. non-farm businesses and establishments, and the LEHD tracks the earnings, jobs, and demographics of workers reported in the Unemployment Insurance (UI) systems in most U.S. states. Using the linked data, I estimate an individual-level earnings regression informed by the model. I find that controlling for worker heterogeneity and observable firm characteristics, i) young firms with high demonstrated potential (or high average productivity) pay more than their mature counterparts with the same observable characteristics, but ii) young firms with low demonstrated potential (or low average productivity) pay less relative to otherwise similar mature firms. This confirms the model's predictions about how learning and job prospects create wage differentials between young firms and their mature counterparts.

Moreover, I estimate the impact of the level of uncertainty on the earnings differentials of young firms by using industry-level variation in uncertainty (measured by the dispersion of firm-level productivity shocks) and interacting it with the earnings residuals. I find that the earnings differentials for young firms are more pronounced in industries with more dispersed firm-level productivity shocks. Lastly, I construct industry-level measures of business dynamism and examine their relationships with uncertainty. I find that higher uncertainty with more dispersed noise has a negative impact on overall business dynamism at the industry level. These findings are consistent with the model's aggregate implications.

Related Literature. This paper is related to several strands of literature. First, it contributes to a broad line of work in firm dynamics and macroeconomics that studies the post-entry dynamics and growth of young firms. Much previous research emphasizes the importance of financing constraints for entrepreneurship (Evans and Jovanovic, 1989; Holtz-Eakin et al., 1994; Cooley and Quadrini, 2001; Hurst and Lusardi, 2004; Kerr and Nanda, 2009; Robb and Robinson, 2014; Schmalz et al., 2017; Davis and Haltiwanger, 2019). Other studies including Foster et al. (2016) and Akcigit and Ates (2019) emphasize frictions related to customer base accumulation or knowledge spillovers as barriers to firm entry and the growth of young firms. This paper expands this literature by linking firm dynamics to labor market dynamics and identifying workers' job prospects as a novel friction affecting firm entry and young firm growth.

Second, this paper is also relevant to a large set of literature that studies inter-firm wage differentials and dynamics (Abowd et al., 1999, 2002, 2004; Card et al., 2013; Bloom et al., 2018; Card et al., 2018; Lopes de Melo, 2018; Song et al., 2019). Some studies mainly focus on wage differentials by firm age (Brown and Medoff, 2003; Haltiwanger et al., 2012; Burton et al., 2018; Sorenson et al., 2021; Kim, 2018; Babina et al., 2019). However, the findings exhibit disparate results across various specifications and abstract from a comprehensive theory providing a robust mechanism to explain them. This paper contributes to this literature by providing a rich structural model that guides a concrete mechanism generating earnings differentials of young firms. Guided by the model, the paper develops and estimates an empirical specification that isolates the part of inter-firm earnings differentials attributed to workers' uncertain job prospects and finds new datafacts supporting this channel.

Lastly, this paper is grounded in the directed labor search literature (Menzio and Shi, 2010, 2011). In particular, my work is closely related to Kaas and Kircher (2015) and Schaal (2017), who link directed search to standard firm dynamics models. This paper contributes to this literature by adding a firm-type learning process to the directed search framework in a tractable way. The model still obtains block recursivity with firm heterogeneity in age and size and on-the-job search. Also, the model generates endogenous wage differentials across different firm ages, even after controlling for firms' observable characteristics, and allows the quantification of their macroeconomic implications.

The remainder of this paper is structured as follows: Section 2 develops a heterogeneous firm directed search model that extends Schaal (2017) by introducing a firm-type learning process; Section 3 lays out the model's main implications and mechanisms; Section 4 describes the model calibration and counterfactual exercises; Section 5 uses the data and tests the model implications for wage differentials of young firms and aggregate outcomes; and Section 6 concludes.

2 Theoretical Model

In this section, I present a heterogeneous firm directed search model as a baseline framework, which builds on Schaal (2017) by introducing a firm-type learning process as in Jovanovic (1982).

2.1 The Environment

The model is set in discrete time and consists of a continuum of heterogeneous firms with homogeneous workers within frictional labor markets. Both firms and workers are assumed to have symmetric information. The mass of workers is normalized to one, while the mass of firms is pinned down endogenously with free entry. Both firms and workers are risk neutral and have the same discount rate β. Firms all produce an identical homogeneous good which is the numeraire.

2.2 Firm-type Learning Process

Firms are born with different productivity types ν that are time invariant and unobserved to both firms and workers. Among entrants, ν is normally distributed with mean $\bar{\nu}_0$ and standard deviation σ_0. Entrants do not know their own ν, but know that their type ν has cross-sectional distribution $N(\bar{\nu}_0, \sigma_0^2)$. Given symmetric information, workers can also only observe the cross-sectional distribution of firm type among entrants. Thus, both entrants and workers start with a belief $\nu \sim N(\bar{\nu}_0, \sigma_0^2)$ at age 0. The dispersion of firm type σ_0 indicates the signal level in the economy. The more dispersed the type distribution is, the more signal agents can gain from observing firm productivity realizations.

Observed productivity for firm j at time t, P_{jt}, follows the following log-normal process, which depends on firm type ν_j:

$$P_{jt} = e^{\nu_j + \varepsilon_{jt}}, \tag{2.1}$$

where $\varepsilon_{jt} \sim N(0, \sigma_\varepsilon^2)$ is a firm-specific shock that is independent over time and across firms. Here, the dispersion of firm-level shocks σ_ε indicates the degree of uncertainty in the economy, as higher shock dispersion generates more noise in the learning process.

Let a_{jt} denote the age of firm j at period t, which implies that the firm is born at $(t - a_{jt})$. Also, let $\bar{\nu}_{jt-1}$ and σ_{jt-1}^2 be the prior (or updated posterior) mean and variance about firm j's type at the beginning of period t, respectively. Note that $\nu_{jt-a_{jt}-1} = \bar{\nu}_0$ and $\sigma_{jt-a_{jt}-1}^2 = \sigma_0^2$ are the initial beliefs held at firm j's birth in period $(t - a_{jt})$. Upon observing the productivity level P_{jt}, both the firm and workers update their posterior beliefs about firm j's type ν_j using Bayes' rule.[2] The posterior on ν_j is

$$\nu_j|_{P_{jt}} \sim N(\bar{\nu}_{jt}, \sigma^2_{jt}), \tag{2.2}$$

where

$$\bar{\nu}_{jt} = \frac{\frac{\bar{\nu}_0}{\sigma_0^2} + \frac{\sum_{i=0}^{a_{jt}} \ln P_{jt-i}}{\sigma_\varepsilon^2}}{\frac{1}{\sigma_0^2} + (a_{jt}+1)\frac{1}{\sigma_\varepsilon^2}} = \frac{\frac{\bar{\nu}_0}{\sigma_0^2} + (a_{jt+1})\frac{\tilde{P}_{jt}}{\sigma_\varepsilon^2}}{\frac{1}{\sigma_0^2} + (a_{jt+1})\frac{1}{\sigma_\varepsilon^2}} \tag{2.3}$$

$$\sigma^2_{jt} = \frac{1}{\frac{1}{\sigma_0^2} + (a_{jt+1})\frac{1}{\sigma_\varepsilon^2}} \tag{2.4}$$

where $\tilde{P}_{jt} \equiv \frac{\sum_{i=0}^{a_{jt}} \ln P_{jt-i}}{(a_{jt+1})} = \frac{\sum_{i=0}^{a_{jt}} \ln P_{jt-i}}{(a_{jt+1})}$ is the cumulative average of log productivity up to period t. $\bar{\nu}_{jt}$ and σ^2_{jt} are key state variables for firms and workers that summarize their posterior beliefs about firm j entering period $t+1$.

Equations (2.3) and (2.4) contain several noteworthy results. First, firm age and the average log productivity $(a_{jt+1}, \tilde{P}_{jt})$ are sufficient statistics for the posterior about firm j's type at $t+1$, which one can use to track job prospects for each firm. In particular, the posterior mean is a weighted sum of the initial prior mean and the average observed productivity, and the weights depend on firm age.

Second, the following relationships between the two sufficient statistics and the posterior mean at the beginning of each period t can be derived:

$$\frac{\partial \bar{\nu}_{jt-1}}{\partial \tilde{P}_{jt-1}} = \frac{\frac{\bar{\nu}_0}{\sigma_0^2} + a_{jt}\frac{1}{\sigma_\varepsilon^2}}{\frac{1}{\sigma_0^2} + a_{jt}\frac{1}{\sigma_\varepsilon^2}} > 0 \tag{2.5}$$

$$\frac{\partial \bar{\nu}_{jt-1}}{\partial a_{jt}} = \frac{(\tilde{P}_{jt-1} - \bar{\nu}_0)}{\sigma_0^2 \sigma_\varepsilon^2 \left(\frac{1}{\sigma_0^2} + a_{jt}\frac{1}{\sigma_\varepsilon^2}\right)^2} \begin{cases} \geq 0 & \text{if} \quad \tilde{P}_{jt-1} \geq \bar{\nu}_0 \\ < 0 & \text{if} \quad \tilde{P}_{jt-1} < \bar{\nu}_0 \end{cases}. \tag{2.6}$$

Equation (2.5) implies that the posterior mean increases in the average productivity level. As firms are observed to have higher average productivity, their prospects improve. Moreover, (2.6) shows that firm age affects job prospects differently depending on the firm's cumulative average productivity. Specifically, if firm j's average productivity is above the initial cross-sectional mean, a higher age implies a better inferred

[2] See Appendix A for more details on the Bayes' rule.

type, while if a firm's average productivity is below the cross-sectional mean, a higher age implies a worse inferred type. I will refer to firms as "high performing" and "low performing" throughout the paper as follows, by the relationship between their average productivity and the initial prior mean.[3]

Definition 1. *Firms are "high performing" if their average productivity is above the cross-sectional prior mean, and "low performing" if their average productivity is below the cross-sectional prior mean.*

Lastly, one can derive the following relationship between firm age and the posterior standard deviation:

$$\frac{\partial \sigma_{jt-1}^2}{\partial a_{jt}} = -\frac{1}{\sigma_\varepsilon^2 \left(\frac{1}{\sigma_0^2} + a_{jt}\frac{1}{\sigma_\varepsilon^2}\right)^2} < 0, \tag{2.7}$$

which implies that as a firm ages, learning gets less noisy, and the posterior converges to a degenerate distribution centered at the true type ν_j.

Figure 1 summarizes the pattern of posterior beliefs across different firm ages, for a given level of average productivity (the red dashed line). The left panel shows the posteriors associated with low performing firms, and the right panel presents the counterpart for high performing firms. This clearly illustrates the properties in (2.5), (2.6), and (2.7).

2.3 Labor Market

The labor market is frictional. Following Schaal (2017), search is directed on both the worker and firm sides. Firms announce contracts to hire and retain workers each period. Following the convention in a standard directed search framework, a sufficient statistic to define labor markets is the level of promised utility that each contract delivers to workers upon matching.[4] Thus, the labor market is a continuum of submarkets indexed by the total utility x_{jt} that firms (j) promise to workers.

[3]Note that in Bayesian learning, both firms and workers learn from observable performance to infer firms' fundamental types. Therefore, a firm's average observed productivity ($\tilde{P}_{jt-1}$) indicates their "potential" type in a given period t, which converges to the firm's time-invariant type ν_j in the long run.

[4]This is because firms that offer the same utility level to workers compete in the same labor market, and workers that require the same utility level search in the same market.

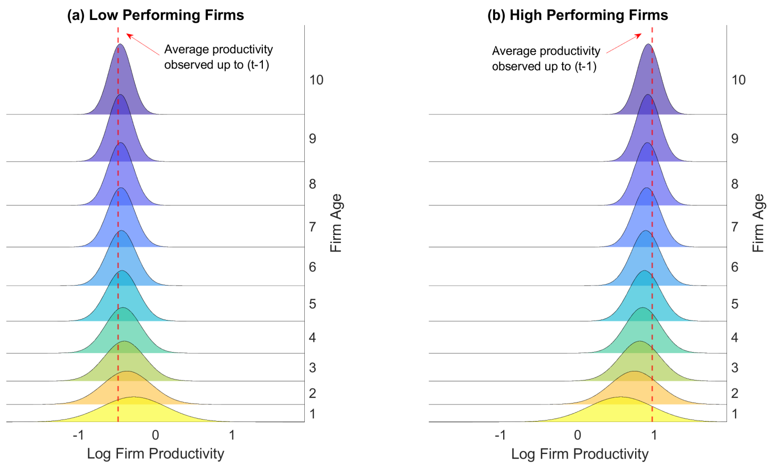


Figure 1: Posterior Distribution of Firm Type

Both firms and workers direct their search and choose a submarket to search in by taking into account a trade-off between the level of utility of a given contract and the corresponding matching probability. Matches are created within each market through a standard constant-returns-to-scale matching function. Firms post vacancies by paying a vacancy cost c.

Let $\theta(x)$ denote the market tightness, defined as the vacancy-to-searchers ratio in each submarket x.[5] Also let $f(\theta)$ and $q(\theta)$ be job finding and job filling rates for workers and firms, respectively. As is standard in the literature, I assume $f'(\theta) > 0$, $f(0) = 0$, $q'(\theta) < 0$, and $q(0) = 1$. I also assume that firms and workers can only visit one submarket at a time. Lastly, there is both on-the-job and off-the-job search, so that both unemployed and employed workers are allowed to search with the relative search efficiency λ for employed workers compared to unemployed workers.

2.4 Dynamic Contracts

Contracts are written every period after matching occurs and before production takes place. Contracts are recursive and are assumed to be state-contingent and fully committed for firms.[6] A contract for workers employed at firm j at t, $\boldsymbol{\Omega_{jt}}$, specifies the

[5] Note that searchers in a given market x are either unemployed workers or employed workers who are searching for a new job while on their current job. More details can be found in Section 2.8.

[6] Contracts are not committed for workers, which is the only distinction from Schaal (2017).

current wage w_{jt}, the next period's utility level $\mathbf{\tilde{W}_{jt+1}}$, the firm's next-period exit probability $\mathbf{d_{jt+1}}$, and the worker's next-period separation probability $\mathbf{s_{jt+1}}$, where the last three terms are contingent on the firm's next period state variables $(a_{jt+1}, \tilde{P}_{jt}, P_{jt+1}, l_{jt})$, where l_{jt} is the number of workers employed at firm j at the end of period t.

Thus, the contract can be written as

$$\mathbf{\Omega_{jt}} = \{w_{jt}, \mathbf{d_{jt+1}}, \mathbf{s_{jt+1}}, \mathbf{\tilde{W}_{jt+1}}\}, \tag{2.8}$$

where $\mathbf{d_{jt+1}} \equiv \mathbf{d}(a_{jt+1}, \tilde{P}_{jt}, P_{jt+1}, l_{jt})$, $\mathbf{s_{jt+1}} \equiv \mathbf{s}(a_{jt+1}, \tilde{P}_{jt}, P_{jt+1}, l_{jt})$, and $\mathbf{\tilde{W}_{jt+1}} \equiv \mathbf{\tilde{W}}(a_{jt+1}, \tilde{P}_{jt}, P_{jt+1}, l_{jt})$ I assume firms offer common contracts across workers with the same ex-post heterogeneity (the employment status of workers).[7] Since each firm j is committed to its contracts offered to workers each period, the firm writes new contracts at t taking as given the utility $\mathbf{\tilde{W}_{jt}}$ promised in the previous period for the remaining incumbents at t, and the promised utility x_{jt} for the new hires.

2.5 Model Timeline

Incumbent and new firms enter with the beginning-of-period priors, employment size l_{jt-1} and the contract $\mathbf{\Omega_{jt-1}}$ announced in the previous period.[8] The firms also enter with their employment level l_{jt-1} and the state-contingent contracts $\mathbf{\Omega_{jt-1}}$ that they offered in the previous period to their incumbent workers.

Next, an exogenous death shock hits incumbent firms, which drives a fraction δ of firms to exit. New firms enter afterwards by paying an entry cost c_e, where free entry is assumed. Firm productivity P_t is realized, after which firms decide whether to exit or stay, following the rule $\mathbf{d_{jt}}$. Also, they decide whether to lay off workers with probability $\mathbf{s_{jt}}$. Both $\mathbf{d_{jt}}$ and $\mathbf{s_{jt}}$ are a function of the firm state variables at t and is specified in their contract with workers at $t-1$.

[7]This means firms offer the same state-contingent next-period variables to workers as workers obtain the same ex-post heterogeneity once they join the firm in the current period. However, the current wage can vary across workers depending on the workers' previous employment status before joining the firm or being retained by the firm in a given period. Note that there is neither worker ex-ante heterogeneity nor human capital accumulated within a firm.

[8]Note that the priors are characterized by firm age and the average log productivity, a_{jt} and $\tilde{P}_{jt-1}$. The beginning-of-period priors for incumbent firms are the posteriors updated by the end of the previous period.

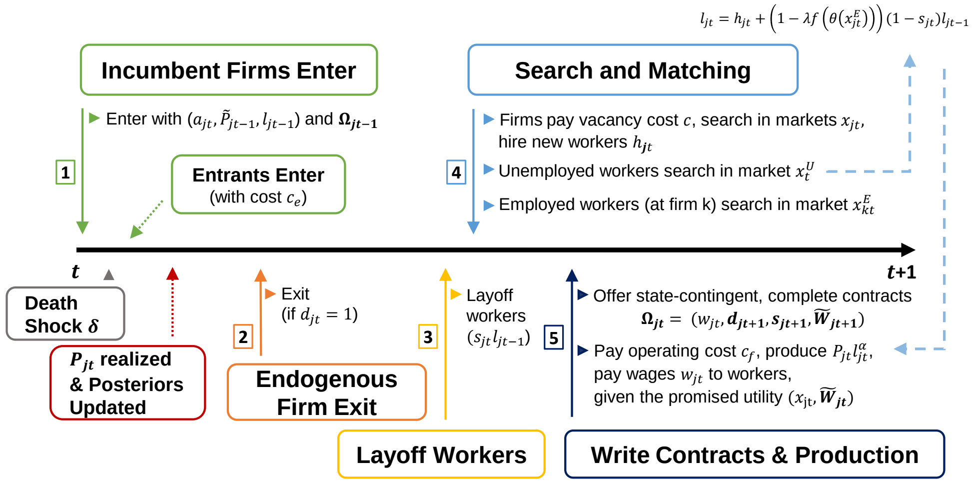


Figure 2: Timeline of the model

Search and matching follows, with new and surviving incumbent firms on one side and unemployed and employed workers on the other side. Firms choose and search in market x_{jt}, post vacancies v_{jt} by paying the per-vacancy cost c, and hire new workers h_{jt} with a job filling rate determined by market tightness $q(\theta(x_{jt}))$.[9] On the other hand, unemployed workers choose their market to search in, x_t^U, and employed workers at firm j choose search market x_{jt}^E. Unemployed and employed workers find a job with probability $f(\theta(x_t^U))$ and $f(\theta(x_{jt}^E))$, respectively.

At the end of this process, firms will end up with employment level $l_{jt} = h_{jt} + (1 - \lambda f(\theta(x_{jt}^E)))(1 - s_{jt})l_{jt-1}$, which is the sum of new hires and the remaining incumbent workers after the departure of those laid off and those moving to other jobs.

Finally, firms enter the last stage of each period, in which they write contracts to new and retained workers, and produce. They offer the workers the contract $\mathbf{\Omega_{jt}}$ as in (2.8). When writing this contract, firms are committed to providing utility $\mathbf{\tilde{W}_{jt}}$ to surviving incumbent workers from $t-1$ and x_{jt} to new hires. Lastly, firms pay a fixed operating cost c_f, produce, and pay wages w_{jt} to workers as announced in the contract $\mathbf{\Omega_{jt}}$. Figure 2 shows the timeline.

[9]Here, the number of vacancies and new hires have the relationship $h_{jt} = q(\theta(x_{jt}))v_{jt}$, and the vacancy cost per hire is $\frac{c}{q(\theta(x_{jt}))}$.

2.6 Workers' Problem

Unemployed Workers. Unemployed workers have the following value function $\mathbf{U_t}$:

$$\mathbf{U_t} = b + \beta \mathbb{E}_t \Big[\max_{x^U_{t+1}} (1 - f(\theta(x^U_{t+1}))) \mathbf{U_{t+1}} + f(\theta(x^U_{t+1})) x^U_{t+1} \Big], \tag{2.9}$$

where b is unemployment insurance and x^U_{t+1} is a market they search in, considering a trade-off between the promised utility x^U_{t+1} and the job finding probability f as a function of labor market tightness $\theta(x^U_{t+1})$. Workers do not save and are risk neutral.

Employed Workers. Employed workers at firm j under the contingent contract $\mathbf{\Omega_{jt}}$ have the following value function after the search and matching process is complete:

$$\begin{aligned} \mathbf{W}(a_{jt}, \tilde{P}_{jt-1}, l_{jt-1}, P_{jt}, \mathbf{\Omega_{jt}}) &= w_{jt} + \beta \mathbb{E}_{jt} \Bigg[\bigg(\delta + \big(1 - \delta\big) \big(\mathbf{d_{jt+1}} + (1 - \mathbf{d_{jt+1}}) \mathbf{s_{jt+1}} \big) \bigg) \mathbf{U_{t+1}} \\ &+ (1 - \delta)(1 - \mathbf{d_{jt+1}})(1 - \mathbf{s_{jt+1}}) \max_{x^E_{jt+1}} \bigg(\lambda f(\theta(x^E_{jt+1})) x^E_{jt+1} + \big(1 - \lambda f(\theta(x^E_{jt+1}))\big) \mathbf{\tilde{W}_{jt+1}} \bigg) \Bigg], \end{aligned} \tag{2.10}$$

where the firm's state variables are its age a_{jt}, average productivity $\tilde{P}_{jt-1}$ accumulated up to the beginning of t, productivity draw P_{jt} at t, and its employment level l_{jt-1} before search and matching at t, all of which determine the set of contracts $\mathbf{\Omega_{jt}} = \{w_{jt}, \mathbf{d_{jt+1}}, \mathbf{s_{jt+1}}, \mathbf{\tilde{W}_{jt+1}}\}$ for the workers employed at firm j, which the workers take as given.[10]

Equation (2.10) shows that workers employed at firm j first receive the wage w_{jt} as specified in their contracts. For the following period, they consider three possible cases: (i) they are dismissed, either because the firm exits (exogenously at rate $\delta \in [0, 1]$ or endogenously if $\mathbf{d_{jt+1}} = 1$) or because the firm lays off workers to cut back

[10]Note that we need to list the average productivity $\tilde{P}_{jt-1}$ and the current productivity draw P_{jt} separately as a part of the firm's state variables. This is because the current productivity draw P_{jt} by itself directly affects the firm's production function, and the average productivity $\tilde{P}_{jt}$ through period t (the combination of the average productivity $\tilde{P}_{jt-1}$ up to $t-1$ and the current productivity draw P_{jt}) determines the firm's posterior belief about its own type and expected future value. Therefore, knowing $\tilde{P}_{jt}$ is not sufficient to understand the firm's optimal contract choice, and we need to consider both P_{jt} and $\tilde{P}_{jt-1}$ (or $\tilde{P}_{jt}$). This will become more clear from the firm's value function (2.11) in the following subsection.

its employment level with probability $\mathbf{s_{jt+1}}$, (ii) they quit and move to other firms by successful search on the job, or (iii) they stay in the firm. In the case of firm exit or layoff, workers go to unemployment and get the value $\mathbf{U_{t+1}}$.[11]

Combining these possibilities, the first term inside the large bracket of the right-hand side of (2.10) shows the value when the worker becomes unemployed in the next period. Meanwhile, workers remain employed at $t+1$ with probability $(1-\delta)(1-\mathbf{d_{jt+1}})(1-\mathbf{s_{jt+1}})$ and are allowed to search on the job. With probability $\lambda f(\theta(x^E_{jt+1}))$ they are successful and quit, and with probability $1-\lambda f(\theta(x^E_{jt+1}))$ they remain in the firm and receive promised state-contingent utility $\tilde{\mathbf{W}}_{\mathbf{jt+1}}$ from the firm. This is summarized by the remaining terms on the right-hand side of (2.10). $\mathbb{E}_{jt}(\cdot)$ refers to the workers' expectation of P_{jt+1} based on their updated beliefs on ν_j.

2.7 Firms' Problem

Incumbent Firms. Incumbent firm j ($a_{jt} \geq 1$) has the following problem at the search and matching stage in period t:

$$\mathbf{J}(a_{jt}, \tilde{P}_{jt-1}, l_{jt-1}, P_{jt}, \{\mathbf{\Omega^i_{jt-1}}\}_{i\in[0,l_{jt-1}]}) = \max_{\substack{\{\mathbf{\Omega^i_{jt}}\}_{i\in[0,l_{jt}]},\\ h_{jt}, x_{jt}}} P_{jt} l^\alpha_{jt} - \int_0^{l_{jt}} w^i_{jt} di - c_f - \frac{c}{q(\theta(x_{jt}))} h_{jt}$$

$$+ \beta(1-\delta)\mathbb{E}_{jt}\left[(1-d_{jt+1})\mathbf{J}(a_{jt+1}, \tilde{P}_{jt}, l_{jt}, P_{jt+1}, \{\mathbf{\Omega^i_{jt}}\}_{i\in[0,l_{jt}]})\right], \tag{2.11}$$

subject to the following constraints:

$$l_{jt} = h_{jt} + (1-s_{jt})\big(1-\lambda f\big(\theta(x^E_{jt})\big)\big) l_{jt-1} \tag{2.12}$$

$$\lambda f(\theta(x^E_{jt+1}))x^E_{jt+1} + (1-\lambda f(\theta(x^E_{jt+1})))\tilde{\mathbf{W}}_{\mathbf{jt+1}} \geq U_{t+1} \tag{2.13}$$

$$x^E_{jt+1} = \mathbf{x^E}(\tilde{\mathbf{W}}_{\mathbf{jt+1}}) \equiv \operatorname*{argmax}_x f(\theta(x))\big(x - \tilde{\mathbf{W}}_{\mathbf{jt+1}}\big) \tag{2.14}$$

$$\mathbf{W}(a_{jt}, \tilde{P}_{jt-1}, l_{jt-1}, P_{jt}, \mathbf{\Omega^i_{jt}}) \geq x_{jt} \quad \text{for new hires } i \in [0, h_{jt}] \tag{2.15}$$

$$\mathbf{W}(a_{jt}, \tilde{P}_{jt-1}, l_{jt-1}, P_{jt}, \mathbf{\Omega^i_{jt}}) \geq \tilde{\mathbf{W}}_{\mathbf{jt}} \quad \text{for incumbent workers } i \in [h_{jt}, l_{jt}], \tag{2.16}$$

[11] Here, layoffs are i.i.d. across incumbent workers. Note that both $\mathbf{d_{jt+1}}$ and $\mathbf{s_{jt+1}}$ are contingent on the firm's state at $t+1$, depending on its productivity draw P_{jt+1}.

where the firm produces with labor using the decreasing returns-to-scale technology $(\alpha < 1)$, w^i_{jt} refers to the wage paid to worker $i \in [0, l_{jt}]$ as a component of the contract $\mathbf{\Omega^i_{jt}} \equiv \{w^i_{jt}, \mathbf{d_{jt+1}}, \mathbf{s_{jt+1}}, \mathbf{\tilde{W}_{jt+1}}\}$, h_{jt} is the new hires by firm j, x_{jt} is the market firm j searches in at t, and $q(\theta(x_{jt}))$ is the job filling probability within the market as a function of labor market tightness.[12]

Note that (2.12) is the employment law of motion, which shows that total employment is the sum of new hires and incumbent workers remaining after firm layoffs and workers' successful on-the-job search. (2.13) is a participation constraint, which prevents workers' return to unemployment unless separations take place, and (2.14) is an incentive constraint based on incumbent workers' optimal on-the-job search. The firm takes into account their workers' incentive to move to other firms and internalizes the impact of their utility promises on workers' on-the-job search behavior.[13] In addition, (2.15) and (2.16) are promise-keeping constraints for new hires at t and surviving incumbent workers from the previous period, respectively.[14]

After search and matching is complete, the firm enjoys an instantaneous profit equal to revenue $P_{jt} l^{\alpha}_{jt}$ minus the sum of the wage bill to its workers $\int_0^{l_{jt}} w^i_{jt} di$, the operating fixed cost c_f, and the vacancy cost $\frac{c}{q(\theta(x_{jt}))} h_{jt}$, as specified in the first line in (2.11). In the following period, conditional on surviving the exogenous death shock with probability $(1 - \delta)$ and the state-contingent decision rule $\mathbf{d_{jt+1}} = 0$, the firm enters the search and matching process again and obtains the next period value $\mathbf{J_{jt+1}} = \mathbf{J}(a_{jt+1}, \tilde{P}_{jt}, l_{jt}, P_{jt+1}, \{\mathbf{\Omega_{jt}}\}_i)$.

Entrants. New firms enter each period by paying entry cost c_e after the death shock hits incumbent firms, but before the entrants' initial productivity is realized. Entrants have initial beliefs about their types, characterized by the cross-sectional mean $\bar{\nu}_0$ and

[12]Note that firms offer the common values for $\mathbf{d^i_{jt+1}} = \mathbf{d_{jt+1}}, \mathbf{s^i_{jt+1}} = \mathbf{s_{jt+1}}, \mathbf{\tilde{W}^i_{jt+1}} = \mathbf{\tilde{W}_{jt+1}}$ to workers as they become incumbents and no longer have ex-post heterogeneity in the next period.

[13]In other words, firms' choice of promised utility to remaining incumbent workers $\mathbf{\tilde{W}_{jt+1}}$ determines incumbent workers' choice of submarket for on-the-job search x^E_{jt+1} by the incentive condition. Therefore, the number of workers who quit upon successful on-the-job search, $\lambda f(\theta(x^E_{jt})) l_{jt-1}$, is predetermined by the state-contingent utility level $\mathbf{\tilde{W}_{jt}}$ that the firm announced in the preceding period and is committed to in the current period. Furthermore, the firm optimally chooses the state-contingent utility level $\mathbf{\tilde{W}_{jt+1}}$ to deliver in the next period as a component of the contract $\mathbf{\Omega_{jt}}$, taking into account the workers' incentive constraint (2.14) in the next period.

[14]Because of the commitment assumption, the firm needs to announce contracts at t that deliver at least x_{jt} and $\mathbf{\tilde{W}_{jt}}$ to their newly hired and incumbent workers, respectively.

standard deviation σ_0. Based on their priors, they calculate the expected value from entry and keep entering until the expected value equals the entry cost, following the free-entry assumption. After entering and observing their initial productivity, new firms decide whether to exit or stay, and in the latter case they search and hire workers to produce as incumbents. They pay c for each vacancy they post and hire new workers with probability $q(\theta(x_t^e))$ as a function of the market tightness within the market x_t^e they choose to search in.

The entry mass is endogenously pinned down by the following free entry condition, which must hold when there is a positive entry mass M_t^e:

$$\int \max_{\substack{\mathbf{\Omega^e_{jt}}=\{w^e_{jt},\mathbf{d_{jt+1}},\mathbf{s_{jt+1}},\mathbf{\tilde{W}_{jt+1}}\},\\ d^e_{jt},l^e_{jt},x^e_{jt}}} (1-d^e_{jt})\Bigg(P_{jt}(l^e_{jt})^\alpha - w^e_{jt}l^e_{jt} - c_f - \frac{c}{q(\theta(x^e_{jt}))}l^e_{jt}$$
$$+\beta(1-\delta)\mathbb{E}_{jt}\Big[(1-\mathbf{d_{jt+1}})\mathbf{J}(1,P_{jt},l^e_{jt},P_{jt+1},\mathbf{\Omega^e_{jt}})\Big]\Bigg)dF_e(P_{jt}) - c_e = 0, \tag{2.17}$$

where $\mathbf{\Omega^e_{jt}}$ is entrant firm j's contract decision, which consists of the four components in (2.8), w^e_{jt}, d^e_{jt}, l^e_{jt}, and x^e_{jt} stand for entrant firm j's wage paid to workers, exit, hiring, and search decisions, respectively, after the firm's initial productivity P_{jt} is observed at t.[15] Also, the distribution $F_e(P_{jt})$ of productivity is based on the entrant's initial prior about its own type ν_j, and $\mathbb{E}_{jt}(\cdot)$ stands for the firm's updated posterior after observing P_{jt}. Lastly, the firm is subject to the participation and incentive constraints (2.13) and (2.14) for retaining incumbent workers in the next period, and the following promise-keeping constraint for new hires in the current period:

$$\mathbf{W}(0,0,0,P_{jt},\mathbf{\Omega^e_{jt}}) \geq x^e_{jt} \quad \text{for all workers } l^e_{jt}. \tag{2.18}$$

2.8 Labor Market Equilibrium

Equilibrium in each labor market is determined by workers' and firms' optimal search. First, workers trade off the utility level of a given contract and the corresponding proba-

[15]Note that these terms are a function only of the initial productivity P_{jt} as the entrant does not have any previous history. On the other hand, the last three terms in $\mathbf{\Omega^e_{jt}}$ depend on the entrant's next-period state variables $(1, P_{jt}, l^e_{jt}, P_{jt+1})$ after drawing productivity P_{jt+1}.

bility of being matched. The trade-off depends on workers' current employment status, which determines their outside option of finding a job. In particular, unemployed workers choose a labor market x_t^U to search in by solving

$$x_t^U = \underset{x}{\operatorname{argmax}}\, f(\theta(x))(x - \mathbf{U_t}), \tag{2.19}$$

where the outside option $\mathbf{U_t}$ is given by (2.9). In a similar fashion, employed incumbent workers at firm j solve

$$\mathbf{x^E}(a_{jt}, \tilde{P}_{jt-1}, l_{jt-1}, P_{jt}) = \underset{x}{\operatorname{argmax}}\, f(\theta(x))(x - \tilde{\mathbf{W}}(a_{jt}, \tilde{P}_{jt-1}, l_{jt-1}, P_{jt})), \tag{2.20}$$

taking into account their outside option $\tilde{\mathbf{W}}_{\mathbf{jt}}$ provided by the current employer j. Equations (2.19) and (2.20) determine the optimal labor submarkets in which unemployed and employed workers choose to search.[16]

On firms' side, (2.11), (2.15), (2.17), and (2.18) imply that all firms face the following same problem when choosing their optimal submarket x_{jt} to search in:

$$x_{jt} = \underset{x}{\operatorname{argmin}}\, \frac{c}{q(\theta(x))} + x, \tag{2.21}$$

independent of their state variables. This means that all firms are indifferent across the various submarkets x_{jt} that are solutions to (2.21).

Labor market equilibrium is pinned down by the (possibly multiple) intersection points between the workers' and firms' choices in (2.19), (2.20), and (2.21). These equilibria are computed as follows. Starting with the firms' problem, only submarkets that satisfy (2.21) are searched by firms. This implies that in equilibrium, the following complementary slackness condition should hold for any active labor submarket x_t:

$$\theta(x_t)\Big(\frac{c}{q(\theta(x_t))} + x_t - \kappa\Big) = 0, \tag{2.22}$$

[16]Note that there exists ex-post heterogeneity among workers depending on their current employment status, although there is no ex-ante worker heterogeneity. This means that workers' choices and offers will be the same for all workers of a given employment status, being either unemployed or employed at a particular firm j with a given set of state variables $(a_{jt}, \tilde{P}_{jt-1}, l_{jt-1}, P_{jt})$.

where κ is the minimized cost value

$$\kappa \equiv \min\Big(\frac{c}{q(\theta(x_t))} + x_t\Big). \tag{2.23}$$

I assume a CES matching function

$$M(S(x_t), V(x_t)) = (S(x_t)^{-\gamma} + V(x_t)^{-\gamma})^{-\frac{1}{\gamma}}, \tag{2.24}$$

which is common across labor submarkets x_t. $S(x_t)$ and $V(x_t)$ are the total number of searching workers and vacancies, respectively, in each labor submarket x_t.[17] This gives the equilibrium labor market tightness in different submarkets as follows:

$$\theta(x_t) = \begin{cases} \left(\left(\frac{\kappa - x_t}{c}\right)^{\gamma} - 1\right)^{\frac{1}{\gamma}} & \text{if} \quad x_t < \kappa - c \\ 0 \quad \text{if} \quad x_t \geq \kappa - c, \end{cases} \tag{2.25}$$

implying that $\theta(\cdot)$ is decreasing in x_t, and if x_t is greater or equal to $\kappa - c$, there are no firms posting vacancies, so that the market becomes inactive, i.e. $\theta(x_t) = 0$.[18]

2.9 Firm Distribution and Labor Market Clearing

Since the model is solved at the steady state in a recursive form, I drop time subscripts onward.[19] Let $\mathbf{G}(a, \tilde{P}, l)$ be the steady state mass of firms aged a with average log-productivity $\tilde{P}$ and employment size l at the beginning of each period. This distribution

[17]Note that the job searchers $S(x_t)$ are workers searching either from the unemployment pool (if x_t is the optimal market for unemployed workers to search in) or on the job (if x_t is the optimal market for workers employed at j to search in).

[18]Proof is provided in Appendix B.1. With (2.19), (2.20), (2.23), and (2.24), the solutions for x_t^U and x_{jt}^E can be derived, which is shown in Appendix B.2.

[19]To be clear, I use x to denote state variables at the beginning of each period and use x' to express the next period value of x. To avoid confusion, let me clarify that $\tilde{P}$ is the average productivity and l is the employment level that firms take as given when they enter the period, before observing their new productivity draw P. Thus, firm state variables are $(a, \tilde{P}, l, P)$. Also, $\tilde{\mathbf{W}}(a, \tilde{P}, l, P)$ is the utility level promised to incumbent workers by firms with $(a, \tilde{P}, l)$ at the beginning of each period and with P drawn subsequently.

satisfies the following law of motion for all $a \geq 1$, $\tilde{P}$, and l:

$$\mathbf{G}(a+1, \tilde{P}', l') = (1-\delta) \int_l \int_{\tilde{P}} \Big(1 - \mathbf{d}\big(a, \tilde{P}, l, P'\big)\Big) \mathbb{I}_{l'} \mathbf{G}(a, \tilde{P}, l) f_P(P') d\tilde{P} dl \tag{2.26}$$

subject to

$$P' = \mathrm{e}^{(a+1)\tilde{P}' - a\tilde{P}},$$

where $\mathbb{I}_{l'}$ denotes an indicator function for firms choosing l' (i.e. $\mathbf{l}(a, \tilde{P}, l, P') = l'$), and P' is the next period productivity draw. Note that $(\bar{\nu}, \sigma^2)$ are the mean and variance of the posterior distribution for a firm with age a and average log-productivity $\tilde{P}$ at the beginning of each period, given by:

$$\bar{\nu} \equiv \frac{\frac{\bar{\nu}_0}{\sigma_0^2} + \frac{a\tilde{P}}{\sigma_\varepsilon^2}}{\frac{1}{\sigma_0^2} + \frac{a}{\sigma_\varepsilon^2}}, \; \sigma^2 \equiv \frac{1}{\frac{1}{\sigma_0^2} + \frac{a}{\sigma_\varepsilon^2}},$$

and $f_P(\cdot)$ is the log-normal probability density function of productivity P, with the corresponding mean $\bar{\nu}$ and variance $\sigma^2 + \sigma_\varepsilon^2$.[20]

We can track the stationary firm mass by iterating on the law of motion along with the following initial condition:

$$\mathbf{G}(1, \tilde{P}, l) = \begin{cases} M^e(1 - d^e(\mathrm{e}^{\tilde{P}})) f_e(\mathrm{e}^{\tilde{P}}) & \text{if} \quad l^e(\mathrm{e}^{\tilde{P}}) = l, d^e(\mathrm{e}^{\tilde{P}}) \neq 1 \\ 0 \quad \text{otherwise.} \end{cases}$$

Here, M^e is the firm entry mass, $f_e(\cdot)$ is the initial prior density of P, i.e. $\ln P \sim N(\bar{\nu}_0, \sigma_0^2 + \sigma_\epsilon^2)$, and d^e and l^e are derived from (2.17).[21]

To close the model, I impose the following labor market clearing condition:

$$f\big(\theta(x^U)\big)\Big(N - \sum_{a\geq 1} \int_{\tilde{P}} \int_l l\mathbf{G}(a, \tilde{P}, l) dl d\tilde{P}\Big) = \sum_{a \geq 1} \int_{\tilde{P}} \int_l \int_P \Big\{ \Big(\delta + (1-\delta)\big(\mathbf{d}(a, \tilde{P}, l, P)$$

[20](2.26) defines the next period mass of firms with age $(a+1)$, average log-productivity $\tilde{P}'$, and employment size l' as the sum of the surviving incumbents of age a that end up having the average log-productivity $\tilde{P}$, productivity draw P', and size $\mathbf{l}(a, \tilde{P}, l, P') = l'$.

[21]This shows that the mass of firms with age 1, average log-productivity $\tilde{P}$, and employment size l consists of surviving entrants whose initial productivity is $P = \mathrm{e}^{\tilde{P}}$ and who choose initial employment size $l^e(\mathrm{e}^{\tilde{P}}) = l$. Note that the entrant's log productivity $\ln P$ equals its average log productivity $\tilde{P}$ at the beginning of the next period when they become age 1

$$+\Big(1-\mathbf{d}(a,\tilde{P},l,P)\big)\mathbf{s}(a,\tilde{P},l,P)\Big)\Big)lf_P(P)\mathbf{G}(a,\tilde{P},l)\Bigg\}dPdld\tilde{P}, \tag{2.27}$$

where $N=1$ given the normalization of worker mass. This implies that in the steady state equilibrium, the inflow to the unemployment pool is equal to the outflow from the unemployment pool.[22,23]

2.10 Stationary Recursive Competitive Equilibrium

Definition 2. *A stationary recursive competitive equilibrium consists of: (i) the posteriors on types $\{\bar{\nu},\sigma^2\}$; (ii) a set of value functions $\mathbf{U},\mathbf{W}(a,\tilde{P},l,P,\mathbf{\Omega})$, and $\mathbf{J}(a,\tilde{P},l,P,\mathbf{\Omega})$ for workers and firms; (iii) a decision rule for unemployed workers x^U, for employed workers $\{\mathbf{x^E}\}$, for incumbent firms $\{\mathbf{\Omega}=\{w,\{\mathbf{d}',\mathbf{s}',\tilde{\mathbf{W}}'\}\},h,l,x\}$, and for entrants $\{\mathbf{\Omega^e}=\{w^e,\{\mathbf{d}',\mathbf{s}',\tilde{\mathbf{W}}'\}\},d^e,l^e,x^e\}$; (iv) κ characterizing the firms' indifference curve; (v) the labor market tightness $\{\theta(x)\}$ for all active markets x; (vi) the stationary firm distribution $\mathbf{G}(a,\tilde{P},l)$; (vii) the mass of entrants M^e; such that equations (2.3)-(2.4),(2.9)-(2.11),*

[22] To be specific, the left-hand side of (2.27) is the number of unemployed workers finding a job, which is the total outflow from the unemployment pool. The number of unemployed workers equals the total population of workers minus the number of employees before firm exit and layoffs. This is because of the timing assumption that workers laid off in period t cannot search until period $t+1$. The right-hand side of (2.27) is the sum of the number of workers that lose their jobs because of firm exit (both exogenous δ and endogenous $\mathbf{d}$) or layoff $\mathbf{s}$ from their current employer with age a, average log-productivity $\tilde{P}$, employment size l and current productivity P, which characterizes the total inflow to the unemployment pool. Note that there is no loss of workers when entrant firms decide to exit, since entrants that immediately exit never hire workers.

[23] Furthermore, in a steady state equilibrium, total job creation by firms needs to be equal to total job finding by workers. For notational convenience, let $\tilde{\mathbf{G}}(a,\tilde{P},l,P)$ be the mass of firms who survive after observing the death shock and their productivity P, i.e. $\tilde{\mathbf{G}}(a,\tilde{P},l,P)\equiv(1-\delta)(1-\mathbf{d}(a,\tilde{P},l,P))f_P(P)\mathbf{G}(a,\tilde{P},l)$. Then, the following equation holds:

$$\begin{aligned}&M^e\int_P l^e(P)(1-d^e(P))f_e(P)dP+\sum_{a\geq 1}\int_{\tilde{P}}\int_l\int_P\Bigg\{\mathbf{h}(a,\tilde{P},l,P)\mathbb{I}_{\mathbf{h}>0}\tilde{\mathbf{G}}(a,\tilde{P},l,P)\Bigg\}dPdld\tilde{P}\\
=&f\big(\theta(x^U)\big)\Big(N-\sum_{a\geq 1}\int_{\tilde{P}}\int_l l\mathbf{G}(a,\tilde{P},l)dld\tilde{P}\Big)\\
&+\sum_{a\geq 1}\int_{\tilde{P}}\int_l\int_P\Bigg\{\lambda f(\theta(\mathbf{x^E}(a,\tilde{P},l,P)))(1-\mathbf{s}(a,\tilde{P},l,P))l\tilde{\mathbf{G}}(a,\tilde{P},l,P)\Bigg\}dPdld\tilde{P}\end{aligned}$$

where the left-hand side is the sum of new jobs created by new entrants and recruiting incumbent firms, and the right-hand side is total job finding, which is the sum of newly hired unemployed and poached workers.

(2.17), (2.19)-(2.20), (2.25)-(2.27) are satisfied, given the exogenous process for P, initial conditions $(\bar{\nu}_0, \sigma_0^2)$ and $\mathbf{G}(1, \tilde{P}, l)$, and the total number of workers, normalized as $N = 1$.[24]

3 Model Implications

In this section, I discuss several implications of the model, which are the foundation of the quantitative analysis in Section 4.

3.1 Equilibrium Wages and Workers' Job Prospects

The propositions in this section discuss the determinants of equilibrium wages offered by firms to workers.

Lemma 1. *Firm promise-keeping constraints (2.15) and (2.16) bind.*

Proof. From (2.10), (2.11), (2.15), and (2.16), each firm j optimally chooses the lowest possible $\{w_{jt}^i\}_i$ that complies with the promise-keeping constraints. This does not change any incentive structure, and the promise-keeping constraints bind. □

Proposition 1. *Equilibrium current wages are determined by workers' outside options and their expected future value (job prospects) at a given firm.*

Proof. With Lemma 1, the promise-keeping constraints (2.15) and (2.16) can be rephrased in terms of the current wage w for each new hires and incumbent workers:

$$\mathbf{w} = \mathbf{x} - \beta \mathbb{E}\Bigg[\Big(\delta + (1-\delta)\big(\mathbf{d}' + (1-\mathbf{d}')\mathbf{s}'\big)\Big)\mathbf{U} \tag{3.28}$$

$$+ (1-\delta)(1-\mathbf{d}')(1-\mathbf{s}')\Big(\lambda f(\theta(\mathbf{x}^{\mathbf{E}'}))\mathbf{x}^{\mathbf{E}'} + \big(1 - \lambda f(\theta(\mathbf{x}^{\mathbf{E}'}))\big)\tilde{\mathbf{W}}'\Big)\Bigg] \text{ for new hires,}$$

$$\mathbf{w} = \tilde{\mathbf{W}} - \beta \mathbb{E}\Bigg[\Big(\delta + (1-\delta)\big(\mathbf{d}' + (1-\mathbf{d}')\mathbf{s}'\big)\Big)\mathbf{U} \tag{3.29}$$

[24] The derivation of the equilibrium is provided in Appendix B, and the computation algorithm is described in Appendix F.

$$+ (1-\delta)(1-\mathbf{d}')(1-\mathbf{s}')\left(\lambda f(\theta(\mathbf{x}^{\mathbf{E}'}))\mathbf{x}^{\mathbf{E}'} + \big(1-\lambda f(\theta(\mathbf{x}^{\mathbf{E}'}))\big)\tilde{\mathbf{W}}'\right)\Bigg] \text{ for incumbents,}$$

where the first term on the right hand side of (3.28) and (3.29) shows the promised utility level for each type of worker, which in equilibrium is determined by the worker's outside options and depends on the worker's previous employment status.[25] The term in large brackets on the right hand side refers to workers' expected future value at a given firm, which depends on their posterior beliefs about firm type. Note that workers' expected future value is identical across all workers a given firm, as they share the same information about the firm. □

Proposition 2. *The equilibrium current wage varies by firm age, controlling for workers' previous employment status, firm average and current productivity, as well as size.*

Proof. Following Proposition 1, the state continceny of contracts, the worker optimality condition (2.20), and the posterior beliefs (2.3) and (2.4), given the worker's previous employment status, the wage is a function of firm state variable $(a, \tilde{P}, l, P)$ and varies by firm age even after controlling for firm average and current productivity $(\tilde{P}, P)$, as well as firm size l.[26] □

Next, workers' expected future value (job prospects) varies across firms as follows.

Proposition 3. *Workers expect future values at a firm in the following descending order: hiring or inactive (without quits) firms, quitting firms, firms laying off workers, and exiting firms.*

Proof. See Appendix B.4 and C.1. □

The intuition behind this result is as follows. After observing firm productivity, the remaining incumbent workers' value is determined by the state-contingent continuation utility $\tilde{\mathbf{W}}$ promised by their employer and the workers' target utility in on-the-job

[25] $\mathbf{x}$ is pinned down by the equilibrium submarket choices (B.42) and (B.43) for each unemployed and poached worker, as discussed in Appendix B.2, and $\tilde{\mathbf{W}}$ is the total utility level firms promise to incumbent workers in equilibrium, as shown in Appendix B.4.

[26] The contract is contingent on firm state variables $(a, \tilde{P}, l, P)$, and the posterior beliefs are sufficiently characterized by firm age and average productivity $(a, \tilde{P})$. Through the optimality condition (2.20), workers' on-the-job search choice $\mathbf{x}^{\mathbf{E}'}$ is indeed a function of the promised utility $\tilde{W}'$ in the contract.

search $\mathbf{x^E}$. Taking into account (2.20), firms' choice of $\tilde{\mathbf{W}}$ depends on their desire to retain workers in the face of potential poaching by other firms.[27] Therefore, expanding firms with more willingness to retain workers offer higher value and deter poaching more successfully than contracting firms.[28] Lastly, following (2.13), workers' value in unemployment is lower than the value of being employed.

Then workers expect higher future value at firms that are more likely to hire or stay inactive without allowing quits in the next period, which guarantees higher stability as well as better career options to workers. This is because these firms would not only offer higher continuation value to workers but also make workers more ambitious when targeting their on-the-job search options. On the other hand, if firms are expected to lose workers in the next period, either by poaching or layoffs, workers anticipate lower future value, as these are seen as less stable and less willing to retain workers with strong continuation utility. Therefore, workers' future expected value is higher for firms with better posteriors and less likelihood of losing workers.

Result 1 (Worker's Expected Future Value across Firm Age).

$$
\begin{aligned}
&\mathbb{E}[*'](a_y, \tilde{P}, l, P) \leq \mathbb{E}[*'](a_o, \tilde{P}, l, P) \quad \textit{if} \quad \tilde{P} > \bar{\nu}_0 \\
&\mathbb{E}[*'](a_y, \tilde{P}, l, P) \geq \mathbb{E}[*'](a_o, \tilde{P}, l, P) \quad \textit{if} \quad \tilde{P} < \bar{\nu}_0, \quad \forall a_o > a_y \geq 0.
\end{aligned}
$$

Result 1 further shows that when comparing two firms with the same observable characteristics ($\tilde{P}$, l, P) but different ages, workers have lower (higher) expected future values at younger firms if their cumulative average productivity is above (below) the cross-sectional mean.[29] In other words, for high performing firms with the same set of observable characteristics, workers' expected future value is lower at younger firms, while the opposite is true for low performing firms. This is due to the limited information available about younger firms, which makes workers pessimistic about job prospects at younger firms with high average performance, but optimistic at younger

[27] In Appendix B.2, I show that workers' target utility in on-the-job search $\mathbf{x^E}$ is increasing in their promised utility $\tilde{\mathbf{W}}$ in the current employer. In other words, the higher utility $\tilde{\mathbf{W}}$ workers obtain from their current employer, the higher utility $\mathbf{x^E}$ an outsider firm needs to provide to poach them.

[28] This is due to the existence of vacancy costs as it is more costly to lose incumbent workers and hire new workers.

[29] Note that the equality holds when both firms are mature enough as the posterior converges to the firms' actual type.

firms with low average performance.

This result holds over a broad parameter space, and the main intuition is as follows. Note that the workers' expected future value is rooted in their posterior beliefs about firm type, defined by (ν_{jt}, σ_{jt}), and in particular their beliefs about the next-period productivity cutoffs and the workers' values (contingent on firms' hiring status) as defined in Lemma 2. The likelihood of drawing better productivity and expanding next period is higher for firms with better posterior beliefs, while the probability of laying off workers or exiting is higher for firms with worse posterior beliefs. Furthermore, as discussed in Appendix D.2, productivity cutoffs are (weakly) lower for firms with better prospects. This suggests that workers should generally perceive higher (lower) expected value at firms having better (worse) posterior beliefs.

Applying this insight to the firm age dimension, we know from (2.5) that younger firms have a lower (higher) posterior mean than their mature counterparts, if they are high (low) performing. This is because the posterior mean is a weighted sum of average performance and the initial prior mean, and a higher weight is put on average performance for older firms, given their longer track record. Thus, the posterior mean of older firms gets closer to the firms' observed performance. Therefore, if two firms have equally good performance, the posterior beliefs about the younger firm are relatively worse than for their mature counterpart. The opposite holds for two firms having the same low average performance.

Connecting this result with Proposition 1, firms can pay lower wages to workers all else equal if they are more likely to hire or stay inactive in the next period, whereas they need to pay higher wages if they have higher likelihood of losing workers by poaching or layoffs in the next period. These wage differentials are based on differences in expected future value due to differences in posterior beliefs. The following result shows how this insight applies to the wages paid by young firms:

Result 2 (Wage Differentials across Firm Age). *Given the firms' state variables $(\tilde{P}, l, P)$, equilibrium current wages offered to a given type of newly hired worker (unemployed or poached from a given firm) satisfy the following relationship across firm age:*

$$\mathbf{w}^{\mathbf{type}}(a_y, \tilde{P}, l, P) \geq \mathbf{w}^{\mathbf{type}}(a_o, \tilde{P}, l, P) \quad \textit{if} \quad \tilde{P} > \bar{\nu}_0$$

$$\mathbf{w}^{\mathbf{type}}(a_y, \tilde{P}, l, P) \leq \mathbf{w}^{\mathbf{type}}(a_o, \tilde{P}, l, P) \quad \textit{if} \quad \tilde{P} < \bar{\nu}_0, \quad \forall a_o > a_y \geq 0,$$

where $type \in \{U, E\}$ for unemployed and poached workers, respectively. Also, given the firms' state variables $(\tilde{P}, l, P)$ and the number of incumbent workers the firm wants to retain (or equivalently, the promised utility $\tilde{\mathbf{W}}$ to incumbent workers), equilibrium current wages offered to incumbent workers satisfy:

$$\mathbf{w}^{\mathbf{inc}}(a_y, \tilde{P}, l, P) \geq \mathbf{w}^{\mathbf{inc}}(a_o, \tilde{P}, l, P) \quad \textit{if} \quad \tilde{P} > \bar{\nu}_0$$
$$\mathbf{w}^{\mathbf{inc}}(a_y, \tilde{P}, l, P) \leq \mathbf{w}^{\mathbf{inc}}(a_o, \tilde{P}, l, P) \quad \textit{if} \quad \tilde{P} < \bar{\nu}_0, \quad \forall a_o > a_y \geq 0.$$

This result implies that high performing younger firms need to pay higher current wages than otherwise similar mature firms to hire or retain workers. On the other hand, low performing younger firms can pay lower current wages than otherwise similar mature firms.[30] These age gaps are due to different job prospects across firms with different ages and history of performance, conditional on the promised future utility $\mathbf{x}$ or $\tilde{\mathbf{W}}$.

Figure 1 displays workers' expected future value (the top left panel) and the equilibrium current wage to hire unemployed workers (in the top right panel), to poach workers from a median firm (in the bottom left panel), and to retain incumbent workers (in the bottom right panel). The figure shows the wage differentials across firms of different ages, controlling for the workers' previous employment status and the firms' observable characteristics (equally-sized firms that have equal above-average productivity). This confirms that wages decline with firm age for high performing firms. The counterparts for firms having low average productivity are displayed in Figure G.1 in

[30] Note that since firms are indifferent across the various labor submarkets along their indifference curve characterized by (2.23), there can be multiple active labor submarkets in equilibrium, although there is no systematic linkage between firm characteristics and the specific submarkets they choose. In other words, there is no systematic pattern of sorting between firms (with heterogeneous characteristics) and workers (with different origins from the previous period) across submarkets. The labor market equilibrium is defined as a continuum of such submarkets, indexed by the promised utility level offered by firms. The wage relationships discussed above hold within each submarket, implying that on average, high performing young firms pay wage premia, while low performing young firms pay wage discounts.

Figure 1: High Performing Firms (average size)

the Appendix.[31,32]

3.2 Equilibrium Employment Size

Given the wage differentials between young and mature firms, the following relationship between employment levels of firms at different ages can be derived:

[31] For this level of performance in Figure G.1, firms above age 4 no longer operate in the economy, while firms aged 4 and below operate and hire workers. Upon survival, younger firms pay lower wages to either newly hired or incumbent workers. Also, the dotted grey line indicates counterfactual wages that firms would have to pay if they continued operating, which shows that mature firms with the same observable characteristics would have to pay higher wages to hire or retain workers. Note that this pattern only applies to firms with low average performance.

[32] These figures are drawn for the baseline set of parameters calibrated in Tables 1 and 2 in the following Section 5. As discussed earlier, these patterns are robust across different sets of parameter values.

Result 3 (Employment Levels across Firm Age). *Given the firms' state variables $(\tilde{P}, l, P)$,*

$$\begin{aligned} &l(a_y, \tilde{P}, l, P) \leq l(a_o, \tilde{P}, l, P) \quad \textit{if} \quad \tilde{P} > \bar{\nu}_0 \\ &l(a_y, \tilde{P}, l, P) \geq l(a_o, \tilde{P}, l, P) \quad \textit{if} \quad \tilde{P} < \bar{\nu}_0, \quad \forall a_o > a_y \geq 0 \end{aligned}$$

This result shows that high performing younger firms have weakly lower employment levels (and growth) than older firms, all else equal, while low performing younger firms have weakly higher employment (and growth) than older firms. This result comes through the workers' learning and job prospects mechanism and the consequent wage differentials associated with firm age, which directly affect the hiring and retention margins of firms. Since high performing young firms pay higher wages relative to otherwise similar mature firms, they have a lower net marginal value of employment, as the wage premia increase the marginal cost of hiring or retaining a worker. High performing young firms also find it more beneficial to lay off workers relative to their mature counterparts, due to the increased wages needed to retain them. On the contrary, low performing young firms have a higher marginal value of hiring or retaining a worker as well as a lower marginal value of laying off a worker, compared to otherwise similar mature firms. I discuss details in Appendix D.3.

3.3 Uncertainty and Job Prospects

In this section, I discuss how the degree of uncertainty in the economy affects model outcomes. The following proposition shows how the learning process depends on the degree of productivity noise, σ_ε.[33]

Proposition 4. *If productivity noise σ_ε increases, high performing firms have a relatively lower posterior mean, while low performing firms have a relatively higher posterior mean, for any given age and average observed performance. Furthermore, higher noise increases the posterior variance for all firms.*

Proof. See Appendix D.4.1. □

[33] Recall that the dispersion of shock σ_ε refers to the degree of noise in the economy, while the dispersion of firm types σ_0 indicates the signal level. Thus, for a given level of signal σ_0, the dispersion σ_ε measures the degree of uncertainty in the economy. In the empirical section below, I directly estimate the noise-to-signal ratio $\frac{\sigma_\varepsilon}{\sigma_0}$ to proxy the level of uncertainty in different industries over time.

Proposition 4 implies that higher noise reduces the prospects at high performing firms, while improving the prospects of low performing firms, all else equal. This is because agents are less certain about firms' actual type.

Proposition 5. *As productivity noise σ_ε rises, firms' average observed productivity becomes less informative about firms' actual type.*

Proof. See Appendix D.4.2. □

Proposition 5 shows that the positive relationship in (2.5) between the average productivity level and the posterior mean is dampened as productivity noise rises in the economy. Both Propositions 4 and 5 imply that slow learning harms the prospects of high performing firms.

Proposition 6. *For $\frac{\sigma_\varepsilon}{\sigma_0} < 1$, the effect of firm age on the speed of updating posteriors is more pronounced as noise increases.*[34]

Proof. See Appendix D.4.3. □

Proposition 6 shows how the degree of noise affects the learning process at different firm ages. As in (2.6), firm age affects learning about firm type in a different way depending on firms' observed performance. Specifically, firms with high average performance have better prospects due to a higher posterior mean when they are older, while firms with low average performance have better prospects due to a higher posterior mean when they are younger. Furthermore, the posterior variance decreases monotonically in firm age as seen in (2.7). Proposition 6 shows that as the noise level rises, such age effects get more pronounced for $\frac{\sigma_\varepsilon}{\sigma_0} < 1$.

Corollary 1. *For $\frac{\sigma_\varepsilon}{\sigma_0} < 1$, the difference in job prospects between otherwise similar firms of different ages increases in the degree of noise.*

Proof. See Appendix D.4.4. □

[34] In Section 4, I externally calibrate both σ_ε and σ_0 using estimated values from the Census data. These estimates are consistent with the assumption that $\frac{\sigma_\varepsilon}{\sigma_0} < 1$.

Overall, higher noise particularly harms the job prospects of young firms with high performance. Although higher noise generally harms firms with high performance, as shown in Propositions 4 and 5, the damage is more pronounced to young firms, following Proposition 6 and Corollary 1. This is because the speed of updating over the firm life cycle is dragged out as noise increases, widening the gap in job prospects between young and mature firms.

3.4 Welfare Implications

Lastly, I discuss welfare implications of the model. I prove that the model's decentralized block-recursive allocation given the level of uncertainty is constrained efficient. However, the decentralized allocation is distorted relative to the social optimum if the planner could eliminate uncertainty about firm type. More discussion can be found in Appendix E.

4 Quantitative Analysis

I calibrate model to quarterly data for the U.S. economy from 1998Q1 to 2014Q4. There are thirteen model parameters, where the first six are externally calibrated and the remaining seven are internally calibrated.

External Calibration. I externally calibrate the parameters $\{\beta, \alpha, N, \bar{\nu}_0, \sigma_0, \sigma_\varepsilon\}$. I set the discount rate β to 0.99 to match a quarterly interest rate of 1.2%. I set the curvature of the revenue function α to be 0.65 as in Cooper et al. (2007). I normalize the total number of workers $N = 1$ and the initial prior mean $\bar{\nu}_0 = 0$. I estimate σ_0 and σ_ε using the LBD data described below. These are shown in Table 1.

Internal Calibration. I internally calibrate the remaining parameters $\{b, \lambda, c, \gamma, c_e, c_f, \delta\}$ to jointly match the following target data moments in the model's steady state: (i) the unemployment rate, (ii) the Employment-Employment (EE) job transition rate, (iii) the Unemployment-Employment (UE) rate, (iv) the elasticity of the UE rate with respect to the vacancy-employment ratio, (v) the firm entry rate, (vi) average firm size, and (vii) the young firm rate.[35]

[35]The EE rate is defined as the share of employed workers who transition to a new job in the next

Table 1: Externally Calibrated Parameters

Parameter	Definition	Value	From
β	Discount factor	0.99	Interest rate ($\beta = \frac{1}{(1+r)}$)
α	Revenue curvature	0.65	Cooper et. al. (2007)
N	Total number of workers	1	Normalization
ν_0	Initial prior on firm type mean	0	Normalization
σ_0	Initial prior on firm type dispersion	0.65	LBD
σ_ε	Idiosyncratic shock dispersion	0.47	LBD

I apply the simulated method of moments (SMM) which minimizes the following objective function over the parameter space Θ:

$$\min_{\Theta} \sum_{i=1}^{7} \left(\frac{M_i^{model}(\Theta) - M_i^{data}(\Theta)}{0.5(M_i^{model}(\Theta) + M_i^{data}(\Theta))} \right)^2,$$

which is the sum of squared percentage distances between the model-simulated moments $\{M_i^{model}(\Theta)\}_{i=1}^7$ and their counterpart moments in data $\{M_i^{data}(\Theta)\}_{i=1}^7$.

Although the parameters are jointly calibrated, in the following I discuss the most relevant moment for each parameter. The unemployment insurance b is set to match the average BLS quarterly unemployment rate. The relative on-the-job search efficiency λ is used to match the Employment-Employment (EE) rate as measured using the Census Job to Job flows database (J2J, a public version of the LEHD).[36] The vacancy cost c is used to target the Unemployment-Employment (UE) rate in a quarter (the UE rate), which is calculated from BLS data as the average ratio of unemployment-to-employment flows relative to total unemployment. The CES matching function parameter γ is set to target an elasticity of unemployed workers' job-finding rate with respect to labor market tightness of 0.72, following Shimer (2005). The firm entry rate, average employment size, and the young firm rate are calculated from the Business Dynamics Statistics (BDS, a public version of the LBD) and are targeted to calibrate the entry cost

period, the UE rate is defined as the share of unemployed workers who find a job in the next period, and the young firm rate is the share of firms aged five year or less in total firms.

[36]To be consistent with the model, only hires with no observed interim nonemployment spell (so-called within-quarter job-to-job transitions) are used to define the EE rate. This variable is named "EEHire" in the J2J database. Note that the J2J data only begins in 2000Q2. I target the average of "EEHire" between 2000Q2 and 2014Q4.

Table 2: Internally Calibrated Parameters

Parameter	Definition	Value	Targets	Data	Model
b	Unemployment insurance	0.50	Unemployment rate	0.061	0.069
λ	Relative on-the-job search efficiency	0.90	EE rate	0.033	0.032
c	Vacancy cost	0.54	UE rate	0.244	0.296
γ	CES matching function parameter	0.78	Elasticity of UE rate w.r.t. θ	0.720	0.674
c_e	Entry cost	18.57	Firm entry rate	0.089	0.089
c_f	Fixed operating cost	0.78	Average employment size	23.04	22.40
δ	Exogenous death shock	0.01	Share of young firms	0.365	0.332

Notes: Target moments are based on literature and the author's calculation with the BLS, BDS, and J2J data.

c_e, the operating fixed cost c_f, and the exogenous death shock δ, respectively.[37]

Aggregate Implications. I conduct a counterfactual analysis to draw out the aggregate implications of the job prospects mechanism, by changing the variance of productivity shocks σ_ε. From the baseline economy in which $\sigma_\varepsilon = 0.47$, I increase σ_ε to 0.58 (a one standard deviation increase) in the counterfactual economy.[38] Having a higher σ_ε implies slower learning and higher noise surrounding young firms.

First, as uncertainty rises, the wages offered by high performing firms to both unemployed and employed workers increase, while those offered by low performing firms decline. This reduces exit of firms with low average performance. Second, as uncertainty increases, the compensating wage differentials that high performing young firms pay relative to their mature counterparts also increase, provided the mature firms are old enough. This implies the age effects on job prospects are amplified with higher uncertainty.

Table 3 shows how changes in σ_ε affect macroeconomic variables. With higher uncertainty about firm type, the firm entry rate and the startup share of employment decrease. Furthermore, resources are reallocated toward low performing firms and away from high performing firms, as indicated by the lowered covariance between firm size and productivity as in Olley and Pakes (1996). Therefore, aggregate productivity is decreased.

The intuition behind this result is simple. As the speed of learning about firm type

[37] Note that the target moments have mixed frequency in the data. The job flow moments and unemployment rate are measured using quarterly data, while the moments regarding firm dynamics are estimated using annual data. I calculate model moments using model data at the same frequency as the data counterparts.

[38] The standard deviation of σ_ε estimated in the LBD is approximately 0.11.

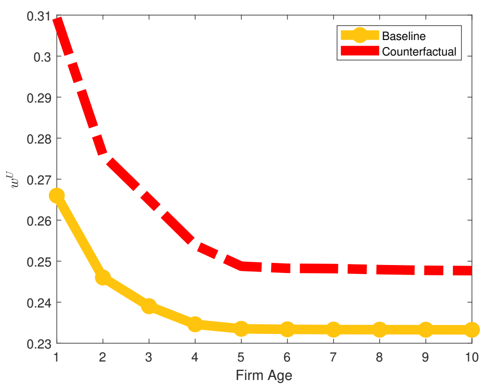
Table 3: Implications of Uncertainty

Description	Baseline ($\sigma_\epsilon = 0.47$)	High Uncertainty ($\sigma_\epsilon = 0.58$)	% Changes
Firm entry rate (%)	8.93	8.19	-8.29%
Share of young firms (%)	33.22	32.54	-2.05%
Olley-Pakes covariance	0.50	0.46	-7.84%
Aggregate productivity	1.07	0.95	-11.21%
Low performing firm share (%)	13.89	23.44	+68.75%
Average job filling rate (%)	72.10	70.89	-1.68%
Welfare	72.33	69.93	-3.32%

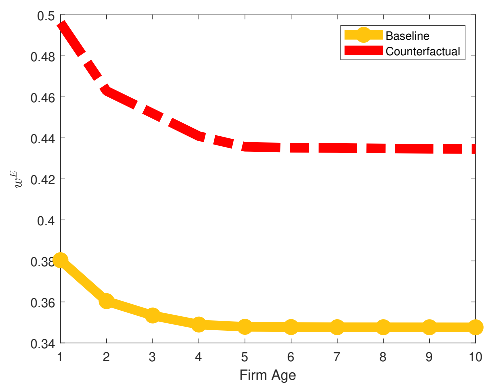
slows down, the gap in job prospects between young and mature firms becomes larger. The current wage premia that high performing young firms need at both the hiring and retention margins increase relative to otherwise similar established firms increases. Similarly, the wage discounts of low performing young firms compared to their mature counterparts also persist longer in the counterfactual economy. Figure 2 compares wage differentials for high performing young firms between the baseline and counterfactual economies. The counterpart wages for low performing firms are shown in Figure G.2 in the Appendix. This makes mature low performing firms no longer exit and continue operating in the counterfactual economy.

Thus, the growth of high performing young firms is dampened, while low performing young firms absorb more workers. This increases the mass of surviving firms with low productivity. Total unemployment goes down, because more firms survive, including potentially bad types, and this induces higher labor market tightness and hiring costs. Consequently, the firm entry rate declines and the activity of young firms with high growth potential is muted. These results suggest that magnified uncertainty about job prospects can be a source of declining business dynamism and lowered allocative efficiency in the economy.

(a) Wages to Unemployed Workers



(b) Wages to Poached Workers



(c) Wages to Incumbent Workers

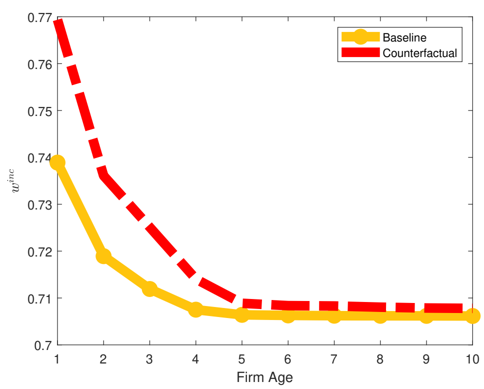


Figure 2: High Performing Firms: Baseline vs. Counterfactual (higher uncertainty)

5 Empirical Analysis

5.1 Data and Measures

Data Source. To test the model predictions, I construct a comprehensive dataset containing firm-level measures, worker characteristics, employment records, and earnings using the Longitudinal Business Database (LBD) and Longitudinal Employer Household Dynamics (LEHD) from 1998 through 2014 for the main analysis.

The LBD tracks the universe of U.S. business establishments and firms that have at least one paid employee, annually from 1976 onward. Establishments that are owned by a parent firm are grouped under a common firm identifier, which allows me to aggregate establishment-level activities to the firm level. The LBD contains basic information such as employment, payroll, revenue, NAICS codes, employer identification numbers, business name, and location, which enables me to measure firm size, age, entry, exit, productivity, and employment growth.[39,40]

The LEHD is constructed from quarterly Unemployment Insurance (UI) system wage reports of states participating in the program, which collect quarterly earnings and employment information, along with demographic information.[41] The data cover over 95 percent of private sector workers, and the length of time series varies across states covered by the LEHD. I have access to 29 states covering over 60 percent of U.S. private sector employment.[42] The data enable me to identify worker heterogeneity, employment history, and job mobility. The LEHD can be linked to the LBD through a crosswalk between employer identification numbers (EINs) and state-level employer identification numbers (SEINs), which allows me to track employer information for each job.[43]

Firm Characteristics and Identifiers. Following Haltiwanger et al. (2013), I define

[39] Jarmin and Miranda (2002), Haltiwanger et al. (2016), and Chow et al. (2021) contain more detailed information about the LBD.

[40] Fort and Klimek (2018) construct time-consistent NAICS codes for LBD establishments after the implementation of a change from the SIC to NAICS in 1997 in the U.S.

[41] The earnings data in the LEHD are reported on a quarterly basis, which include all forms of compensation that are taxable.

[42] The 29 states are AL, AZ, CA, CO, CT, DE, ID, IN, KS, MD, ME, ND, NE, NJ, NM, NV, NY, OH, OK, OR, PA, SD, TN, TX, UT, VA, WA, WI, and WY.

[43] The UI data, the main source of the LEHD, assigns firms a state-level employer identification number (SEIN) that captures the activity of a firm within a state.

firm age as the age of the oldest establishment that the firm owns when the firm is first observed in the data.[44] I label firms aged five years or below as young firms. Firm size is measured as total employment.[45] Lastly, I measure firm-level productivity as the log of real revenue per worker (normalized to 2009 U.S. dollars).[46]

One limitation of the LBD is the lack of longitudinally consistent firm identifiers.[47] However, longitudinal consistency of firm identifiers is necessary for my analysis to track firms' history of performance as well as to estimate noise components in firm type learning process. Therefore, I construct and use longitudinal firm identifiers following Dent et al. (2018). Henceforth, I will use the term "firm identifier" to refer to the longitudinal firm identifiers constructed using this method.

Firm Type Learning Process. Using firm-level revenue productivity, I estimate a firm type learning process in my data. First, I take the deviation of firm-level log revenue productivity from its industry-year mean, and project the demeaned log productivity on its own lag. Thus, I estimate the following regression:

$$\ln P_{jt} = \rho \ln P_{jt-1} + \nu_j + \varepsilon_{jt}, \tag{5.30}$$

where $\ln P_{jt}$ refers to the log real revenue productivity for firm j demeaned at the industry-year level, and ν_j is a firm-level fixed effect. I include the lag term $\ln P_{jt-1}$ to factor out the productivity persistence observed in the data.[48] Removing industry-year means controls for the effects of fundamental industry-specific differences in technology or production processes as well as time trends or cyclical shocks.

The underlying assumption is that firms and workers can observe the industry-by-time means as well as the persistence in the firm-level productivity process, and filter these out when estimating the firm's fundamental. Therefore, they infer a firm's type

[44]To be precise, I use firm age variables constructed by the Census using the method of Haltiwanger et al. (2013), which enables me to obtain firm age for the whole sample period and avoid any left-censoring issue.

[45]As a robustness check, I measure size using payroll.

[46]The revenue per worker is highly correlated with TFPQ within industries.

[47]Although the redesigned LBD has a new firm identifier that links firms across time by correcting previous firm identifiers that are recycled in the old LBD, it is still not yet a true longitudinal identifier and has not yet resolved firm reorganization issues. See more discussion in Chow et al. (2021).

[48]This is a dynamic panel data model with the lagged dependent variable. To resolve the potential endogeneity bias, I adopt the Generalized Method of Moments (GMM) estimator in Blundell and Bond (1998).

using the remaining terms, which reflect the firm-level fixed effect ν_j and the residual ε_{jt}. This is the term that I map into the model productivity estimates, which I denote henceforth as $\ln \hat{P}_{jt}$, i.e. $\ln \hat{P}_{jt} \equiv \hat{\nu}_j + \hat{\varepsilon}_{jt}$. Then, I define noise in the learning process as the variance of the estimated residual $\hat{\varepsilon}_{jt}$ from (5.30).

Next, I construct average productivity ($\tilde{P}_{jt-1}$) over the firm life-cycle for each firm using the productivity estimates ($\hat{P}_{jt}$) and longitudinal firm identifiers. To do so, I restrict the sample to firms that have consecutively non-missing observations of $\ln \hat{P}_{jt}$ from their birth. I define $\tilde{P}_{jt-1}$ as follows:

$$\tilde{P}_{jt-1} \equiv \frac{\sum_{\tau=t-a_{jt}}^{t-1} \ln \hat{P}_{j\tau}}{a_{jt}}, \tag{5.31}$$

where a_{jt} is the age of firm j in year t. I use $\ln \hat{P}_{jt}$ and $\tilde{P}_{jt-1}$ in my regression below as measures representing the current and average productivity levels, respectively.

I indicate high performing firms as those having average productivity above the within-industry cross-sectional mean of firm-level estimated prior mean productivity:

$$\mathbb{I}^H_{jt} \equiv \begin{cases} 1 & \text{if} \quad \tilde{P}_{jt-1} > \frac{\sum_{j \in G(j,t)} \hat{\nu}_j}{N_{G(j,t)}} \\ 0 & \text{otherwise} \end{cases}, \tag{5.32}$$

where $N_{G(j,t)}$ is the number of firms in industry $G(j,t)$ in a given year t.[49]

Uncertainty Measure. Using the estimated parameters from (5.30), I estimate the within-industry cross-sectional dispersion of $\hat{\varepsilon}_{jt}$ and the fixed effect estimates $\hat{\nu}_j$, respectively, on a yearly basis. I denote these estimates by $\hat{\sigma}_{\varepsilon gt}$ and $\hat{\sigma}_{0gt}$, respectively, for each industry g. I use the ratio of the former to the latter to measure industry-level uncertainty in period t, as follows:

$$Uncertainty_{gt} \equiv \frac{\hat{\sigma}_{\varepsilon gt}}{\hat{\sigma}_{0gt}}. \tag{5.33}$$

This measure is known as the "noise-to-signal" ratio in the literature. Note that the

[49] As a robustness check, I also use different thresholds to define high performing firms, such as the within-industry cross-sectional median or the within-industry-cohort mean of the estimated prior mean productivity.

denominator can be translated into the initial dispersion of firm fundamentals, which captures the informativeness of signals in each industry. This indicates the degree of uncertainty conditional on this fundamental dispersion, to take into account the differences of the informativeness of signals across industries observed in the data.

Jobs and Earnings. The LEHD is defines a job as the presence of an individual-employer match, with earnings defined as the amount earned from that job during the quarter. However, it does not record the start and end dates of a job, which makes the total number of weeks during that quarter unknown. To avoid potential bias from this, I follow the literature and restrict my analysis to full-quarter main jobs that give the highest earnings in a given quarter and are present for the quarter prior to and the quarter after the focal quarter.[50] All of my analysis is based on the log real quarterly earnings associated with these jobs, normalized to 2009 U.S. dollars.

5.2 Baseline Two-stage Earnings Regression

To test the cross-sectional implications of the job prospects channel, I regress earnings on a young firm indicator, a high performing firm indicator, and their interaction, controlling for worker fixed effects along with time-varying worker characteristics, a measure of workers' previous employment status (workers' outside options), firm-level observable characteristics, and fixed effects for time, state, and industry.[51] This enables me to estimate how wages vary by firm ages and depend on workers' job prospects at the firm (tracked by firm age and average productivity), all else equal.

I operationalize my empirical strategy using a two-stage regression at the individual level. In the first stage, I use workers' full-quarter earnings and estimate worker and year fixed effects, controlling for time-varying worker characteristics. I get earnings

[50] For any worker-quarter pairs that are associated with multiple jobs paying the same earnings, I pick the job that shows up the most frequently in the worker's job history. This leaves one main job observation for each worker-quarter pair.

[51] Note that the theoretical model abstracts from ex-ante worker heterogeneity, although ex-post heterogeneity still exists in the model depending on workers' previous employment status, which affects wage offers provided by potential employers. In other words, whether the worker was hired from unemployment or poached from an existing job matters for their current wage, as does how much they were paid at the previous job. Prior job status matters for current wages regardless of the current firm's unobserved fundamentals or observed performance. Thus, I control for either the previous employer's firm fixed effect or the worker's previous earnings at the previous employer. I also control for an indicator for whether a worker was unemployed in the previous period.

residuals subtracting these fixed effects and the effects of the controls.[52] In the second stage, I regress the earnings residuals on the young firm indicator, the high performing firm indicator and their interaction, controlling for the worker's previous employment status, the current firm's time-varying characteristics, as well as the fixed effects of industry and state, respectively.[53]

Stage 1: Estimating Earnings Residuals. In the first stage, I estimate earnings residuals controlling for worker age, and worker and year fixed effects, as follows:

$$y_{it} = \delta_i + \eta_t + X_{it}\gamma + \epsilon_{it}, \tag{5.34}$$

where y_{it} is the logarithm of the Q1 earnings of individual i in year t, δ_i is a time-invariant individual effect, η_t is a year effect, and X_{it} is a vector of controls for individual age, using quadratic and cubic polynomials centered around age 40.[54,55]

Stage 2: Wage Differentials across Firm Age and Performance. In the second stage, I use the estimated earnings residuals $\hat{\epsilon}_{it}$ from (5.34) and regress it on the young firm dummy, the high performing firm dummy in (5.32), and their interaction.

Following the discussion above, I control for workers' previous employment status by controlling for the fixed effect for the firm where each worker was employed in the previous period. For those workers previously employed before period t, their previous job is identified as the most recent full-quarter main job within the three most recent quarters before t, and the employer of that job is denoted by $j(i, t-1)$.[56,57] Next, I

[52] Following the literature, I use the first quarter value of the full-quarter main jobs as a baseline. As robustness checks, I also use the second, third, and fourth quarter values.

[53] As a baseline, I control for the worker's previous employment status, using the AKM firm fixed effect estimate for the previous employer and a dummy indicating if the worker was not employed in the previous period. The AKM firm fixed effect is the firm fixed effect obtained from estimating the standard two-way fixed-effect framework in my data, following Abowd et al. (1999).

[54] This follows Card et al. (2016), Crane et al. (2018), and Haltiwanger et al. (2021).

[55] In order to estimate the fixed effects, I implement the iterative algorithm proposed by Guimaraes and Portugal (2010), which helps to estimate a model with high-dimensional fixed effects without explicitly using dummy variables to account for the fixed effects.

[56] Following Haltiwanger et al. (2018), I can identify workers' previous job using a within/adjacent quarter approach, which allows for a brief nonemployment period between workers' last day on the previous job and their first day on the contemporaneous job. Therefore, workers are identified as previously employed if they had at least one full-quarter job within the most recent three quarters before t, while they are identified as non-employed if they had no full-quarter jobs within those three quarters.

[57] Note that restricting the sample to full-quarter main jobs makes use of the three-quarter duration to define previous jobs. For notational convenience, let $(t - q1)$ denote the quarter prior to t, and $(t - q2)$

estimate the fixed effect for $j(i,t-1)$ following Abowd et al. (1999).[58,59]

Equation (5.35) presents the second stage regression, where the main coefficients of interest are β_1 and β_2, which capture the earnings differentials associated with young firms depending on their average performance.

$$\hat{\epsilon}_{it} = \beta_1 Young_{j(i,t)t} + \beta_2 Young_{j(i,t)t} \times \mathbb{I}^H_{j(i,t)t} + \beta_3 \mathbb{I}^H_{j(i,t)t} + Z_{j(i,t)t}\gamma_1 + Z_{j(i,t-1)}\gamma_2 + \mu_{G(j(i,t))} + \mu_{S(j(i,t))} + \alpha + \xi_{it} \tag{5.35}$$

The regression is at the worker-year level, where $\hat{\epsilon}_{it}$ is the earnings residual of worker i in a given year t, $j(i,t)$ is the employer where worker i is employed at t, $Young_{j(i,t)t}$ is the young firm indicator for firm $j(i,t)$, $\mathbb{I}^H_{j(i,t)t}$ is the high performing firm indicator for firm $j(i,t)$, $Z_{j(i,t)t}$ is a vector of controls for time-varying properties of firm $j(i,t)$, and $Z_{j(i,t-1)}$ is a vector of controls for the worker's employer in the previous period. To be consistent with the model, I include average productivity, current productivity, and employment size of firm $j(i,t)$ in $Z_{j(i,t)t}$. For $Z_{j(i,t-1)}$, as a baseline, I use the AKM firm fixed effect associated with the worker's previous employer along with the non-employment indicator. Lastly, the regression includes industry fixed effects $\mu_{G(j(i,t))}$ and state fixed effects $\mu_{S(j(i,t))}$, where $G(j(i,t))$ is the industry that the firm belongs to and $S(j(i,t))$ is the state where the firm is located.

Note that the firm variables have the same values across all workers employed at that firm at t (i.e., workers employed at the SEINs associated with the same firm identifier). The novelty in (5.35) comes from the coefficients β_1 and β_2, which capture how firms with a given set of observable characteristics pay differently by firm age, and how the

denote two quarters prior to t, and so on. If a worker had any full-quarter jobs at either $(t-q1)$ or $(t-q2)$, this implies that the worker must have moved to the contemporaneous job within quarter $(t-q1)$. The latter could happen if the worker had some overlapping period between $(t-q1)$ and t in job transition. If a worker had any full-quarter jobs at $(t-q3)$, this means that the worker must have left the job at $(t-q2)$, had a brief nonemployment period between $(t-q2)$ and $(t-q1)$, and joined the contemporaneous job at $(t-q1)$. Alternatively, the within quarter approach identifies workers as previously employed if they had at least one full-quarter job within the latest two quarters before t, where the previous job is defined by the most recent main full-quarter job within the most recent two quarters before t.

[58] Note that the baseline fixed effect is estimated at the SEIN level. As a robustness check, I also use the fixed effects estimated at the (longitudinally consistent) firm identifier level.

[59] For workers who were non-employed in the previous period, i.e. those who had no full-quarter earnings in any of the most recent three quarters before t, I set their previous employer fixed effect to zero and include a dummy variable for nonemployment.

Table 4: Wage Differentials for Young Firms

	(1) Earnings Residuals	(2) Earnings Residuals
Young firm	-0.002***	-0.003***
	(0.001)	(0.001)
Young firm $\times$ High performing firm	0.015***	0.016***
	(0.001)	(0.001)
High performing firm	0.002	0.002
	(0.001)	(0.001)
Observations	50,170,000	50,170,000
Fixed effects	Industry, State	Industry, State
Controls	Full (current size)	Full (lagged size)

Notes: The table reports results for regression of earning residuals on young firm and high performing firm indicators. Firm controls include cross-time average productivity level, current productivity level, and log employment size. Controls associated with worker's previous employment status are AKM firm fixed effect associated with the previous employer and a dummy for non-employed workers in the previous period. Observation counts are rounded to the nearest 10,000 to avoid potential disclosure risks. Estimates for constant, industry, state fixed effects, the coefficient of the indicator for worker's previous non-employment status are suppressed. Observations are unweighted. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

age effect depends on the firm's history of performance.

Table 4 presents the regression results with the full set of controls, which are consistent with the model predictions.[60] The first column controls for the current value of firm size and the second column uses the lagged value of firm size. Both columns include industry (NAICS6) and state fixed effects.

In the regression, the impact of being a young firm on earnings depends on β_1 and $\beta_2 \mathbb{I}^H_{j(i,t)t}$, and the total impact depends on whether the observed average productivity $\tilde{P}_{j(i,t)}$ is below or above the industry mean. For low performing firms, the wage differential for young firms is given by β_1. For high performing firms, the wage differential for young firms is given by $\beta_1 + \beta_2$. Table 4 shows that $\hat{\beta}_1 < 0$, $\hat{\beta}_2 > 0$, and $\hat{\beta}_1 + \hat{\beta}_2 > 0$, where all of these point estimates are statistically significant. The results indicate that high performing young firms pay more than their otherwise similar mature counterparts, while low performing young firms pay less. This is consistent with the model

[60] For the sake of space, I only present the main coefficients. The full results can be found in Table H7 in Appendix.

prediction about young firms' wage differentials through the channel of learning and workers' uncertain job prospects.

5.3 Robustness Checks

To further confirm the validity of the baseline results, I perform several robustness checks, with results reported in Appendix I.

Firm Size Effects. The baseline regression controls for firm size. However, firm size is highly correlated with firm age, and the firm size distribution varies by different firm age.[61] This may cause the size covariate to absorb firm age effects. To check this possibility, I run regressions without controlling for firm size and using different sets of firm controls. Results stay robust as shown in Appendix Table I9.

Correcting Sample Selection Bias. Another potential source of bias is sample selection. The current sample is drawn from the population of U.S. firms having consecutively non-missing observations of revenue data, and workers matched with these firms. Therefore, the sample drops firms with missing revenue data. To mitigate the possible resulting selection bias, I estimate a propensity score model using logistic regressions and weight the baseline regression sample with inverse propensity score weights.[62] The results are similar and documented in Appendix Table I10.

Standard Error Bootstrapping. The high performing firm indicator as well as firm control variables in the second-stage regression are constructed based on estimates from the regression in (5.30). This might cause the reported standard errors in Table 4 to be incorrect. To address this, I estimate the standard errors with bootstrapping and check the robustness of the results.[63] The statistical significance of the coefficient estimates stays robust across all columns, as presented in Appendix Table I11.

[61] For instance, most young firms are small in the U.S. economy.

[62] Following Haltiwanger et al. (2017), I use logistic regressions with a dependent variable equal to one if the firm belongs to the current sample and zero otherwise, along with firm characteristics such as firm size, age, employment growth rate, industry, and a multi-unit status indicator from the universe of the LBD. Using inverse probability weights calculated from the predicted values from the logistic regression, I weight the sample and rerun the regressions.

[63] To do so, I draw 5000 random samples with replacement repeatedly from the main dataset, estimate the main coefficients corresponding to these bootstrap samples, form the sampling distribution of the coefficients, and calculate the standard deviation of the sampling distribution for each coefficient.

Unobserved Worker Characteristics. In the current specification, I control for the effect of worker age and their previous employment status, along with worker fixed effects. However, alternative interpretations of the main results may arise from other potential sources, specifically related to unobserved time-varying worker characteristics. For instance, high performing young firms might demand workers with more experience or longer tenure than their mature counterparts given the burden of training costs. This scenario may result in the earnings premia paid by high performing young firms, independent of the uncertain job prospects provided by the firms.

To rule out such cases, I control for earnings on the previous job as a proxy of worker tenure or experience. The previous earnings can also measure the workers' place on the job ladder in alignment with the model.[64] The results controlling for earnings on the previous job are shown in Appendix Table I12, where the first three columns replace the AKM firm fixed effect with the worker's previous earnings, and the next three columns use both variables to control for the worker's previous employment status properly. The baseline results stay robust in all cases.

5.4 The Impact of Uncertainty on Wages and Aggregate Outcomes

Cross-sectional Implications on Wage Differentials. In the model, higher uncertainty drags out the speed of learning for young firms and pronounces the wage differentials. To test this implication, I add additional interaction terms involving the industry-level uncertainty measure (5.33) to the baseline regression, as follows:

$$\begin{aligned}\hat{\epsilon}_{it} =\ & \beta_1 Young_{j(i,t)t} + \beta_2 Young_{j(i,t)t} \times \mathbb{I}^H_{j(i,t)t} + \beta_3 Young_{j(i,t)t} \times Uncertainty_{gt-1} \\ & + \beta_4 Young_{j(i,t)t} \times \mathbb{I}^H_{j(i,t)t} \times Uncertainty_{G(j,t)t-1} + \beta_5 Uncertainty_{gt-1} \\ & + \beta_6 \mathbb{I}^H_{j(i,t)t} \times Uncertainty_{gt-1} + \beta_7 \mathbb{I}^H_{j(i,t)t} + Z_{j(i,t)t}\gamma_1 + Z_{j(i,t-1)}\gamma_2 \\ & + \mu_{G(j(i,t))} + \mu_{S(j(i,t))} + \alpha + \xi_{it}, \end{aligned} \tag{5.36}$$

[64]Based on the model, using the AKM fixed effect might be conservative, as the equilibrium wages at both the hiring and retention margins (eventually) only depend on whether workers came from the unemployment pool, from an employer that wanted to expand or stay inactive, or from an employer that cut back on their size. Thus, another potential proxy to control for the worker's previous job and their place on the job ladder in the previous period would be the earnings associated with the previous employer (still also controlling for the non-employment status indicator).

Table 5: The Effect of Uncertainty on Young Firms' Wage Differentials

	(1) Earnings Residuals	(2) Earnings Residuals
Young firm	-0.001	-0.002
	(0.002)	(0.002)
Young firm $\times$ High performing firm	0.003	0.005**
	(0.002)	(0.002)
Young firm $\times$ Uncertainty	-0.005**	-0.004*
	(0.002)	(0.002)
Young firm $\times$ High performing firm $\times$ Uncertainty (at $t-1$)	0.016***	0.015***
	(0.003)	(0.003)
Observations	50,170,000	50,170,000
Fixed effects	Sector, State	Sector, State
Controls	Full (current size)	Full (lagged size)

Notes: The table reports results for regression of earning residuals on young firm, high performing firm indicators, and the uncertainty measure. Firm controls include cross-time average productivity level, current productivity level, and log employment size. Controls associated with worker's previous employment status are AKM firm fixed effect and a dummy for non-employed workers in the previous period, associated with the previous employer to capture time-varying components. Observation counts are rounded to the nearest 10,000 to avoid potential disclosure risks. Estimates for constant, sector and state fixed effects, and the coefficient of the indicator for worker's previous non-employment status are suppressed. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

where $Uncertainty_{G(j,t)t-1}$ is the lagged value of the uncertainty measure in (5.33) for the main industry that firm $j(i,t)$ is associated with in year t. Note that I use lagged values to avoid any potential reverse causality. In this case, I define $\mu_{G(j(i,t))}$ as sectoral rather than industry fixed effects, where each sector is defined at the NAICS2 level. The regression captures how the wage differentials associated with young firms vary across different industries with different levels of uncertainty.

Table 5 displays the results, using the current and lagged value of firm size in the first and second columns, respectively.[65] The table shows that the coefficient estimate of the triple interaction term between the young firm indicator, the high performing firm indicator and the uncertainty measure is positive, i.e. $\beta_4 > 0$, which is consistent with the model prediction that higher uncertainty increases the wage premium that high performing young firms need to pay. Furthermore, the coefficient estimate for the

[65] As before, the main coefficients are presented in the main text to conserve space, and the full results are provided in Appendix Table H8.

Table 6: Aggregate Implications of Uncertainty

	(1) Entry rate	(2) Young firm share	(3) Young firm emp. share
Uncertainty (at $t-1$)	-0.011***	-0.050***	-0.021***
	(0.004)	(0.008)	(0.008)
Observations	4,300	4,300	4,300
Fixed effects	Industry, Year	Industry, Year	Industry, Year
	(4) HG young firm share	(5) HG young firm emp.share	(6) HG young firm avg. emp. growth
Uncertainty (at $t-1$)	-0.028***	-0.012***	-0.029***
	(0.005)	(0.003)	(0.008)
Observations	4,300	4,300	4,300
Fixed effects	Industry, Year	Industry, Year	Industry, Year

Notes: The table reports results for regression of young firm activities in each column on the lagged value of the uncertainty at the industry level. Observation counts are rounded to the nearest 100 to avoid potential disclosure risks. Estimates for constant, industry and year fixed effects are suppressed. Observations are unweighted. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

interaction between the young firm indicator and the uncertainty measure is negative, i.e. $\beta_3 < 0$, which is also in line with the model result that the wage discounts for low performing young firms gets larger as uncertainty rises.

Macroeconomic Implications. Next, I test the aggregate implications of uncertain job prospects in the model. The calibrated model predicts that more uncertainty will reduce firm entry and young firm activity by slowing down learning and selection. To test these predictions, I estimate the following industry-level regression:

$$BusinessDynamism_{gt} = \beta Uncertainty_{gt-1} + \delta_g + \delta_t + \epsilon_{gt}, \tag{5.37}$$

where $BusinessDynamism_{gt}$ is either the firm entry rate, the share of young firms, the share of high-growth young firms, the employment share of young firms, the employment share of high-growth young firms, or the average employment growth rate of high-growth young firms at the industry level (industry g) in a given year t. Here, high-growth firms are those above the 90th percentile of the industry employment growth distribution, and high-growth young firms are high-growth firms aged five years or less. Industry is defined at the NAICS4 level. I use lagged values of uncertainty and take out industry and year fixed effects, δ_g and δ_t, respectively.

Table 6 displays the results. This shows a negative association between uncertainty and business dynamism at the industry level, which holds across different measures of young firm activity. Uncertainty has a negative and significant impact on the firm entry rate, as well as the share and employment share of young firms and high-growth young firms. The employment growth rate of high-growth young firms is also dampened when uncertainty rises. This along with the previous result indicates less business dynamism in more uncertain industries, in which the wage differentials for young firms are more amplified. This aligns with the macroeconomic implications of the job prospects channel in the model.

6 Conclusion

In this paper, I study how workers' job prospects impact the wage and growth of young firms, as well as aggregate outcomes in the economy. The paper develops a rich theoretical framework linking firm dynamics to labor market frictions and leverages micro-level administrative data to test the model's predictions. The following set of implications are found in the model and supported in the data: i) the uncertain job prospects of workers result in wage premia for high-performing young firms and wage discounts for low-performing young firms, relative to their observationally identical mature counterparts; ii) increasing uncertainty about young firms amplifies both types of wage differentials for young firms; and iii) heightened uncertainty dampens the growth of high performing young firms, redirects labor inputs to low performing young firms, and diminishes overall business dynamism in the economy. This mechanism operates in part through higher market tightness and overall hiring costs caused by the increase in uncertain prospects about young firms. In summary, this paper provides a foundation for understanding the dynamics of wages and growth among young firms, driven by the inherent nascency and uncertainty surrounding young firms.

To the best of my knowledge, this paper is the first to offer a theory as well as evidence about how workers' uncertain job prospects generate wage differentials for young firms relative to otherwise similar mature firms. The paper develops a rich structural model that provides a novel channel that endogenously generates wage differentials for young firms that depend on firms' average productivity. Furthermore, the paper quantifies and highlights the importance of this channel by drawing out its implications for aggregate business dynamism and productivity.

There are several directions in which the current work can be extended. First, one could incorporate aggregate shocks to explore how the job prospects channel affects the cyclical dynamics of young firms. It is well documented that young firm activity is procyclical and that young firms are hit harder than mature firms during recessions. The main mechanism of my paper can potentially provide an explanation for the differential cyclicality of young firm activity. If the economy is in a downturn and there is higher uncertainty about finding a job, the costs of job loss should increase. Thus, workers may value stability more in recessions, which would reduce the value of working at young firms in general. In addition, if the degree of noise rises in recessions, learning would slow down and the uncertain prospects at young firms could be magnified. Thus, the wage differentials of young firms could be more pronounced in recessions. I find preliminary empirical evidence that supports this hypothesis.

Furthermore, one could introduce risk aversion and worker heterogeneity to the model. The current model abstracts from risk aversion components for tractability. Once risk aversion is added, there will be another source of wage differentials for young firms attributed to risk premia. Also, workers may have different outside options or opportunity costs that would affect their willingness to work at young firms. These differences could be correlated with worker age or skill.[66] Considering worker heterogeneity could enrich our understanding of the job prospects channel and its impact on potential young firm growth and business dynamism through the effects of sorting between firms and workers. This mechanism could potentially create a linkage between workforce composition and business dynamism in the economy. These extensions are left as interesting avenues for future research.

[66] For instance, younger workers have more potential job opportunities in the event that their current job is destroyed and can be more risk tolerant of taking a job.

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Appendix A Bayesian Learning

Suppose that initial prior is $\nu_j \sim N(\bar{\nu}_0, \sigma_0^2)$, and there is an observation of $\ln P_{jt} = \nu_j + \varepsilon_{jt}$ such that $\varepsilon_{jt} \sim N(0, \sigma_\varepsilon^2)$, $\ln P_{jt}|_{\nu_j} \sim N(\bar{\nu}_0, \sigma_0^2 + \sigma_\varepsilon^2)$. Following the Bayes' rule,

$$f(\nu_j | \ln P_{jt}) \propto f(\nu_j) f(\ln P_{jt} | \nu_j),$$

we have:

$$\begin{aligned} f(\nu_j | \ln P_{jt}) \propto f(\nu_j) f(\ln P_{jt} | \nu_j) &= \left(\frac{1}{\sqrt{2\pi\sigma_0^2}} \exp\left(-\frac{(\nu_j - \bar{\nu}_0)^2}{2\sigma_0^2} \right) \right) \left(\frac{1}{\sqrt{2\pi\sigma_\varepsilon^2}} \exp\left(-\frac{(\ln P_{jt} - \nu_j)^2}{2\sigma_\varepsilon^2} \right) \right) \\ &\propto \left(\frac{1}{\sqrt{2\pi\sigma_0^2\sigma_\varepsilon^2}} \exp\left(-\frac{\left(\nu_j - \left(\frac{\sigma_\varepsilon^2 \bar{\nu}_0 + \sigma_0^2 \ln P_{jt}}{\sigma_\varepsilon^2 + \sigma_0^2} \right) \right)^2}{2 \frac{\sigma_0^2 \sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma_0^2}} \right) \right), \end{aligned}$$

which implies that

$$f(\nu_j | \ln P_{jt}) \sim N\left(\frac{\sigma_\varepsilon^2 \bar{\nu}_0 + \sigma_0^2 \ln P_{jt}}{\sigma_\varepsilon^2 + \sigma_0^2}, \frac{\sigma_0^2 \sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma_0^2} \right).$$

Thus, the mean and standard deviation of the posterior distribution are

$$\bar{\nu}_{jt} = \frac{\sigma_\varepsilon^2 \bar{\nu}_{jt-1} + \sigma_{jt-1}^2 \ln P_{jt}}{\sigma_{jt-1}^2 + \sigma_\varepsilon^2} = \frac{\frac{\bar{\nu}_{jt-1}}{\sigma_{jt-1}^2} + \frac{\ln P_{jt}}{\sigma_\varepsilon^2}}{\frac{1}{\sigma_{jt-1}^2} + \frac{1}{\sigma_\varepsilon^2}}, \tag{A.38}$$

$$\sigma_{jt}^2 = \frac{\sigma_{jt-1}^2 \sigma_\varepsilon^2}{\sigma_{jt-1}^2 + \sigma_\varepsilon^2} = \frac{1}{\frac{1}{\sigma_{jt-1}^2} + \frac{1}{\sigma_\varepsilon^2}}. \tag{A.39}$$

By iterating (A.38) and (A.39) backward and using $a_{jt} + 1 = a_{jt+1}$, I can rewrite them as (2.3) and (2.4) in the main text.

Appendix B Derivation of the Stationary Recursive Competitive Equilibrium

B.1 Matching Function and Labor Market Tightness

Using the matching function in (2.24), the job finding rate $f(\cdot)$ and filling rate $q(\cdot)$ for each submarket x_t are given by:

$$f(\theta(x_t)) = \theta(x_t)(1+\theta(x_t)^\gamma)^{-\frac{1}{\gamma}} \tag{B.40}$$

$$q(\theta(x_t)) = (1+\theta(x_t)^\gamma)^{-\frac{1}{\gamma}}, \tag{B.41}$$

where $\theta(x_t)$ is the ratio of total vacancies to searching workers, $\frac{V(x_t)}{S(x_t)}$, in each submarket x_t. Based on this, the firm's complementary slackness condition (2.22) can be rewritten as follows:

$$\theta(x)\left(\frac{c}{(1+\theta(x)^\gamma)^{-\frac{1}{\gamma}}} + x - \kappa\right) = 0,$$

which proves (2.25).

B.2 Workers' Problem

B.2.1 Unemployed Workers

Using the job finding rate (B.40), the unemployed workers' problem can be simplified as follows:

$$\max_{x^U} \theta(x^U)(1+\theta(x^U)^\gamma)^{-\frac{1}{\gamma}}\big(x^U - \mathbf{U}\big).$$

The first-order condition with respect to x^U gives

$$\theta(x^U) + \frac{1}{(1+\theta(x^U)^\gamma)}\theta'(x^U)\big(x^U - \mathbf{U}\big) = 0.$$

Using (2.25), if $x^U < \kappa - c$, the following holds:

$$\theta(x^U) = \Big(\big(\frac{\kappa - x^{\mathbf{U}}}{c}\big)^\gamma - 1\Big)^{\frac{1}{\gamma}}$$

Plugging it back to the unemployed workers' first-order condition, the following can be derived:

$$x^U = \kappa - (c^\gamma(\kappa - \mathbf{U}))^{\frac{1}{1+\gamma}}. \tag{B.42}$$

The result shows that x^U is constant with respect to firms' state variables. This is because unemployed workers have no heterogeneity (both ex-ante and ex-post) and thus all choose the same market to search.

Thus,

$$\theta(x^U) = \begin{cases} \left(c^{-\frac{\gamma}{1+\gamma}}(\kappa - \mathbf{U})^{\frac{\gamma}{1+\gamma}} - 1\right)^{\frac{1}{\gamma}} & \text{if} \quad x^U < \kappa - c \\ 0 \quad \text{if} \quad x^U \geq \kappa - c, \end{cases}$$

and

$$f(\theta(x^U)) = \begin{cases} \left(c^{-\frac{\gamma}{1+\gamma}}(\kappa - \mathbf{U})^{\frac{\gamma}{1+\gamma}} - 1\right)^{\frac{1}{\gamma}}\left(c^{-\frac{1}{1+\gamma}}(\kappa - \mathbf{U})^{\frac{1}{1+\gamma}}\right) & \text{if} \quad x^U < \kappa - c \\ 0 \quad \text{if} \quad x^U \geq \kappa - c, \end{cases}$$

which implies that if $x^U \geq \kappa - c$, the market x^U is inactive and workers remain unemployed.

Furthermore, $\mathbf{U}$ is a fixed point of the following equation:

$$\mathbf{U} = b + \beta\left(\mathbf{U} + \max\left[0, \left(c^{-\frac{\gamma}{1+\gamma}}(\kappa - \mathbf{U})^{\frac{\gamma}{1+\gamma}} - 1\right)^{\frac{1}{\gamma}}\left(c^{-\frac{1}{1+\gamma}}(\kappa - \mathbf{U})^{\frac{1}{1+\gamma}}\right)\left(\kappa - (c^\gamma(\kappa - \mathbf{U}))^{\frac{1}{1+\gamma}} - \mathbf{U}\right)\right]\right).$$

B.2.2 Employed Workers

In a similar fashion, the employed workers' problem can be solved, and a similar solution for $x^E(a, \tilde{P}, l, P)$ can be obtained for workers employed at a firm having $(a, \tilde{P}, l, P)$. That is, given the promised utility $\tilde{\mathbf{W}}(a, \tilde{P}, l, P)$ offered by the firm, the workers will direct their on-the-job search to:

$$x^E(a, \tilde{P}, l, P) = \kappa - (c^\gamma(\kappa - \tilde{\mathbf{W}}(a, \tilde{P}, l, P)))^{\frac{1}{1+\gamma}}, \tag{B.43}$$

as far as the market is active, i.e. $\mathbf{x^E}(a, \tilde{P}_{a-1}, l, P) < \kappa - c$. This depends on workers' opportunity cost of moving to other firms, which is a function of the current employer's state variables. x^E is increasing in the workers' opportunity cost $\tilde{\mathbf{W}}$, which means that the higher utility $\tilde{\mathbf{W}}$ workers receive from their current employer, the higher utility x^E another firm needs to deliver to poach them successfully. In other words, workers only climb up to a labor market that provides higher utility than what they currently have, which captures the standard job ladder property in a directed search framework.

Notably from the solutions (B.42) and (B.43), firms' promised utility to both unemployed and employed workers in the search market does not depend on recruiting firms' characteristics, but rather only on workers' employment status. In other words, workers are not indifferent across active submarkets, and search in a specific submarket that provides a certain promised utility (at least equal to or above their outside options) upon successful job match, while firms are indifferent across active submarkets in equilibrium.

Also, the equilibrium market tightness and job finding rate for the market x^E are derived as follows:

$$\theta(x^E(a,\tilde{P}_{a-1},l,P)) = \begin{cases} \left(c^{-\frac{\gamma}{1+\gamma}}(\kappa - \tilde{\mathbf{W}}(a,\tilde{P},l,P))^{\frac{\gamma}{1+\gamma}} - 1\right)^{\frac{1}{\gamma}} & \text{if} \quad x^E < \kappa - c \\ 0 \quad \text{otherwise,} \end{cases}$$

and

$$f(\theta(x^E)) = \begin{cases} \left(c^{-\frac{\gamma}{1+\gamma}}(\kappa - \tilde{\mathbf{W}}(a,\tilde{P},l,P))^{\frac{\gamma}{1+\gamma}} - 1\right)^{\frac{1}{\gamma}} \left(c^{-\frac{1}{1+\gamma}}(\kappa - \tilde{\mathbf{W}}(a,\tilde{P},l,P))^{\frac{1}{1+\gamma}}\right) & \text{if} \quad x^E < \kappa - c \\ 0 \quad \text{if} \quad x^E \geq \kappa - c. \end{cases}$$

B.3 Joint Surplus Maximization

Using Lemma 1 and substituting out $\{w^i_{jt}\}_i$ in (2.11), I have:

$$\begin{aligned}
&\mathbf{J}(a_{jt},\tilde{P}_{jt-1},l_{jt-1},P_{jt},\Omega^{-w}_{jt-1}) \\
&= \max_{\substack{\Omega^{-w}_{jt}=\{d_{jt+1},s_{jt+1},\tilde{\mathbf{W}}_{\mathbf{jt+1}}\},\\ x_{jt},h_{jt}}} P_{jt}l^{\alpha}_{jt} - x_{jt}h_{jt} - \frac{c}{q(\theta(x_{jt}))}h_{jt} - \tilde{\mathbf{W}}_{\mathbf{jt}}(1-s_{jt})(1-\lambda f(\theta(x^E_{jt})))l_{jt-1} - c_f \\
&\quad + \beta\mathbb{E}_{jt}\Bigg[(1-\delta)(1-d_{jt+1})\Big(\mathbf{J}(a_{jt+1},\tilde{P}_{jt},l_{jt},P_{jt+1},\Omega^{-w}_{jt}) + (1-s_{jt+1})\lambda f(\theta(x^E_{jt+1}))x^E_{jt+1}l_{jt} \\
&\quad + \tilde{\mathbf{W}}_{\mathbf{jt+1}}(1-s_{jt+1})(1-\lambda f(\theta(x^E_{jt+1})))l_{jt}\Big) + \Big(\delta + (1-\delta)(d_{jt+1} + (1-d_{jt+1})s_{jt+1})\Big)\mathbf{U_{t+1}}l_{jt}\Bigg],
\end{aligned} \tag{B.44}$$

subject to (2.12), (2.13), and (2.14). For notation, I use Ω^{-w}_{jt} to denote the contract abstracting from the wage $\{w^i_{jt}\}_i$. Note here that the problem can be solved without the participation constraint first, and one can prove that the solution satisfies the participation constraint. Also, using the incentive constraint, the problem can be rephrased as the firm choosing x^E_{jt+1} and pinning down $\tilde{\mathbf{W}}_{\mathbf{jt+1}}$ indirectly. In other words, the firm indirectly controls the job-hopping rate $\lambda f(\theta(x^E_{jt+1}))$ by taking into account the workers' optimal job search behavior and offers

$\tilde{\mathbf{W}}_{\mathbf{jt+1}}$ backed out from the worker's incentive constraint. Following this, once the solution is obtained, I prove in section C.1 that the participation constraint holds.

Reformatting (B.44) to be at the production stage after search and matching, the firm value function can be rewritten as:

$$\begin{aligned}
&\mathbf{J}^{prod}(a_{jt}, \tilde{P}_{jt-1}, l_{jt}, P_{jt}, \Omega_{jt-1}^{-w}) \\
&= \max_{\substack{\Omega_{jt}^{-w}=\{d_{jt+1}, s_{jt+1}, \tilde{\mathbf{W}}_{\mathbf{jt+1}}\}, \\ x_{jt+1}, h_{jt+1}}} P_{jt} l_{jt}^{\alpha} - x_{jt} h_{jt} - \tilde{\mathbf{W}}_{\mathbf{jt}}(1-s_{jt})(1-\lambda f(\theta(x_{jt}^{E}))) l_{jt-1} - c_f \\
&\quad + \beta \mathbb{E}_{jt}\Bigg[(1-\delta)(1-d_{jt+1})\Big(\mathbf{J}^{prod}(a_{jt+1}, \tilde{P}_{jt}, l_{jt}, P_{jt+1}, \Omega_{jt}^{-w}) - \Big(x_{jt+1} + \frac{c}{q(\theta(x_{jt+1}))}\Big) h_{jt+1} \\
&\quad + (1-s_{jt+1})\lambda f(\theta(x_{jt+1}^{E})) x_{jt+1}^{E} l_{jt} + \tilde{\mathbf{W}}_{\mathbf{jt+1}}(1-s_{jt+1})(1-\lambda f(\theta(x_{jt+1}^{E}))) l_{jt}\Big) \\
&\quad + \Big(\delta + (1-\delta)(d_{jt+1} + (1-d_{jt+1}) s_{jt+1})\Big) \mathbf{U}_{\mathbf{t+1}} l_{jt}\Bigg],
\end{aligned} \tag{B.45}$$

Let $\mathbf{V}_{jt}^{prod} \equiv \mathbf{J}_{jt}^{prod} + x_{jt} h_{jt} + \tilde{\mathbf{W}}_{\mathbf{jt}}(1-s_{jt})(1-\lambda f\big(\theta(x_{jt}^{E})\big)) l_{jt-1}$ be the joint surplus of the firm and its workers at the production stage. Using this and rewriting (B.44), I obtain:

$$\begin{aligned}
\mathbf{V}^{prod}(a_{jt}, \tilde{P}_{jt-1}, l_{jt}, P_{jt}) &= \max_{d_{jt+1}, s_{jt+1}, x_{jt+1}, x_{jt+1}^{E}, h_{jt+1}} P_{jt} l_{jt}^{\alpha} - c_f \\
&\quad + \beta \mathbb{E}_{jt}\Bigg[(1-\delta)(1-d_{jt+1})\Big(\mathbf{V}^{prod}(a_{jt+1}, \tilde{P}_{jt}, l_{jt+1}, P_{jt+1}) - \Big(x_{jt+1} + \frac{c}{q(\theta(x_{jt+1}))}\Big) h_{jt+1} \\
&\quad + (1-s_{jt+1})\lambda f(\theta(x_{jt+1}^{E})) x_{jt+1}^{E} l_{jt}\Big) + \Big(\delta + (1-\delta)\big(d_{jt+1} + (1-d_{jt+1}) s_{jt+1}\big)\Big) \mathbf{U}_{\mathbf{t+1}} l_{jt}\Bigg],
\end{aligned} \tag{B.46}$$

subject to (2.12), (2.13) and (2.14). The firm's original profit maximization can be fully replicated by the joint surplus maximization in (B.46), given that the last two terms defining $\mathbf{V}^{prod}$, $x_{jt} h_{jt}$ and $\tilde{\mathbf{W}}_{\mathbf{jt}}(1-s_{jt})(1-\lambda f(\theta(x_{jt}^{E}))) l_{jt-1}$, are predetermined so that maximizing $\mathbf{V}^{prod}$ gives the same results as maximizing $\mathbf{J}_{jt}^{prod}$ and thus $\mathbf{J}_{jt}$. Furthermore, using (B.46) simplifies the set of state variables in $\mathbf{J}_{jt}$ and increases tractability. Lastly, (2.15) and (2.16) (assumed to hold with equality) characterize the equilibrium wages that the firm needs to pay $\{w_{jt}^{i}\}_i$.

In a similar fashion, the free-entry condition (2.17) can be rephrased as follows:

$$\int \max_{d_{jt}^{e}, l_{jt}^{e}, x_{jt}^{e}} \Bigg[(1-d_{jt}^{e})\Big(\mathbf{V}^{prod}(0, 0, l_{jt}^{e}, P_{jt}) - x_{jt}^{e} l_{jt}^{e} - \frac{c}{q(\theta(x_{jt}^{e}))} l_{jt}^{e}\Big)\Bigg] dF_e(P_{jt}) - c_e = 0. \tag{B.47}$$

B.4 Firms' Decision Rules

As discussed in the previous section, the firm profit maximization can be replicated by the following joint surplus maximization problem:

$$
\begin{aligned}
&\mathbf{V}^{prod}(a_{jt}, \tilde{P}_{jt-1}, l_{jt}, P_{jt}) \\
&\quad = \max_{d_{jt+1}, s_{jt+1}, h_{jt+1}, x^E_{jt+1}} P_{jt} l_{jt}^{\alpha} - c^f + \beta \mathbb{E}_{jt}\Big[\delta \mathbf{U_{t+1}} l_{jt} + (1-\delta)\big(d_{jt+1} + (1-d_{jt+1})s_{jt+1}\big)\mathbf{U_{t+1}} l_{jt} \\
&\qquad + (1-\delta)(1-d_{jt+1})\Big(\mathbf{V}^{prod}(a_{jt+1}, \tilde{P}_{jt}, l_{jt+1}, P_{jt+1}) - \kappa h_{jt+1} + (1-s_{jt+1})\lambda f(\theta(x^E_{jt+1}))x^E_{jt+1} l_{jt}\Big)\Big].
\end{aligned}
$$

Given that choice variables are contingent on future productivity, it can be transformed with the following value function defined at the beginning of each period, $\mathbf{V}_t^{init}$:

$$
\begin{aligned}
\mathbf{V}^{init}(a_{jt}, \tilde{P}_{jt-1}, l_{jt-1}, P_{jt}) &= \max_{d_{jt}, s_{jt} h_{jt}, x^E_{jt}} \delta \mathbf{U_t} l_{jt-1} + (1-\delta)(d_{jt} + (1-d_{jt})s_{jt})\mathbf{U_t} l_{jt-1} \\
&\quad + (1-\delta)(1-d_{jt})\Big(P_{jt} l_{jt}^{\alpha} - c^f - \kappa h_{jt} + (1-s_{jt})\lambda f(\theta(x^E_{jt}))x^E_{jt} l_{jt-1} + \beta \mathbb{E}_{jt} \mathbf{V}^{init}(a_{jt+1}, \tilde{P}_{jt}, l_{jt}, P_{jt+1})\Big)
\end{aligned}
$$

subject to $l_{jt} = h_{jt} + (1-s_{jt})(1-\lambda f(\theta(x^E_{jt})))l_{jt-1}$. This can also rephrase the free-entry condition as follows:

$$
\int \max_{d^e_{jt}, l^e_{jt}} (1-d^e_{jt})\Big(P_{jt}(l^e_{jt})^{\alpha} - c^f - \kappa l^e_{jt} + \beta \mathbb{E}_{jt} \mathbf{V}^{init}(1, \ln P_{jt}, l^e_{jt}, P_{jt+1})\Big) dF_e(P_{jt}) - c^e = 0. \tag{B.48}
$$

Note that this value function has the following relationship with the firm's original value function in the main text (B.44):

$$
\begin{aligned}
\mathbf{V}^{init}(a_{jt}, \tilde{P}_{jt-1}, l_{jt-1}, P_{jt}) &= \Big(\delta + (1-\delta)\big(d_{jt} + (1-d_{jt})s_{jt}\big)\Big)\mathbf{U_t} l_{jt-1} \\
&\quad + (1-\delta)(1-d_{jt})(1-s_{jt})\Big(\lambda f(\theta(x^E_{jt}))x^E_{jt} + \tilde{\mathbf{W}}_{\mathbf{jt}}(1-s_{jt})(1-\lambda f(\theta(x^E_{jt})))\Big) l_{jt-1} \\
&\quad + (1-\delta)(1-d_{jt})\mathbf{J}(a_{jt}, \tilde{P}_{jt-1}, l_{jt-1}, P_{jt}, \Omega_{jt-1}),
\end{aligned} \tag{B.49}
$$

where the first two lines are the workers' future expected value as of the previous period, and the last line is the firm's value (2.11) in the search and matching stage. Note that d_{jt}, s_{jt}, l_{jt}, h_{jt}, x_{jt}, and $\tilde{\mathbf{W}}_{\mathbf{jt}}$ are the firm's policy functions, each of which is a function of the following set of state variables: $(a_{jt}, \tilde{P}_{jt-1}, l_{jt-1}, P_{jt})$. This relationship will be useful to draw out interpretations of equilibrium equations in the following section.

Dropping the time subscripts, it becomes:

$$
\begin{aligned}
\mathbf{V}^{init}(a, \tilde{P}, l, P) = \max_{d, s, h, x^E} & \ \delta U l + (1-\delta)(d + (1-d)s)U l + (1-\delta)(1-d)\Big(P l'^{\alpha} - c^f - \kappa h \\
&+ (1-s)\lambda f(\theta(x^E))x^E l + \beta \mathbb{E} \mathbf{V}^{init}(a+1, \tilde{P}', l', P')\Big)
\end{aligned} \tag{B.50}
$$

subject to $l' = h + (1-s)(1-\lambda f(\theta(x^E)))l$. The solution of x^E pins down $\tilde{\mathbf{W}}$ following (B.43). The expectation of P' is formed based on the posterior updated after observing P, which is $\ln P' \sim (\frac{\frac{\bar{\nu}_0}{\sigma_0^2} + \frac{a\tilde{P}+\ln P}{\sigma_\epsilon^2}}{\frac{1}{\sigma_0^2} + \frac{(a+1)}{\sigma_\epsilon^2}})$.

Note that the first term $\delta U l$ is independent of the variables to maximize and $(1-\delta)$ in the remaining two terms just scales the objective function. Thus, the maximization problem is simplified to maximize the following terms:

$$\begin{aligned}\mathbf{V}^{init}(a, \tilde{P}, l, P) = \max_{d,s,h,x^E} &(d + (1-d)s)Ul + (1-d)\Big(Pl'^{\alpha} - c^f - \kappa h + (1-s)\lambda f(\theta(x^E))x^E l \\ &+ \beta \mathbb{E}\mathbf{V}^{init}(a+1, \tilde{P}', l', P')\Big),\end{aligned}$$

subject to $l' = h + (1-s)(1-\lambda f(\theta(x^E)))l$ and $\tilde{P}' = \frac{a\tilde{P}+\ln P}{a+1}$.

I first solve the problem for s, h, x^E, and then for d, which rephrases the above maximization problem as:

$$\max\left[Ul,\ \max_{s,h,x^E} sUl + Pl'^{\alpha} - c^f - \kappa h + (1-s)\lambda f(\theta(x^E))x^E l + \beta\mathbb{E}\mathbf{V}^{init}(a+1, \tilde{P}', l', P')\right]. \tag{B.51}$$

Let's first focus on the maximization in the large bracket, which solves for optimal s, h, and x^E. Note that there is no case in which firms hire and separate workers at the same time. In other words, if $s > 0$, then $h = 0$ should hold, and if $h > 0$, then $s = 0$. This is discussed in detail in the following Section B.4.2.

B.4.1 Productivity Cutoffs

Lemma B.1. *There are four endogenous cutoffs for the current productivity draw P among operating firms: i) the upper cutoff $\mathcal{P}^h(a, \tilde{P}, l)$ between hiring versus inaction with no quits, ii) the middle cutoff $\mathcal{P}^q(a, \tilde{P}, l)$ between inaction with no quits versus inaction with quits, iii) the lower cutoff $\mathcal{P}^l(a, \tilde{P}, l)$ between quits only versus quits and layoffs, and iv) the exit cutoff $\mathcal{P}^x(a, \tilde{P}, l)$ below which firms endogenously exit. These cutoffs are endogenously determined by the beginning-of-period state variables $(a, \tilde{P}, l)$ before the current productivity draw P.*[67]

Proof. Note that all these cutoffs should depend on the other firm state variables, which are a, $\tilde{P}$, and l. Let the hiring cutoff denoted by $\mathcal{P}^h(a, \tilde{P}, l)$, the quitting cutoff denoted by $\mathcal{P}^q(a, \tilde{P}, l)$, the layoff cutoff denoted by $\mathcal{P}^l(a, \tilde{P}, l)$, and the exit cutoff denoted by $\mathcal{P}^x(a, \tilde{P}, l)$.

[67] Note that there does not exist any case in which firms find it optimal to both hire and lay off workers. More discussion can be found in Appendix B.4.2.

First, to determine the hiring cutoff, it is determined by (B.59) evaluated at $l' = l$. The reason behind this is that given $(a, \tilde{P}, l)$, if P lies in a range in which the marginal value of hiring (the right-hand side of (B.59)) becomes less than κ, then firms no longer hire any workers. The threshold of P is determined at a point where it is optimal to choose $h = 0$ from the hiring firms' problem, below which firms would never hire workers due to the reason marginal value of hiring a new worker is not high enough.

Therefore, the following equation determines the hiring productivity cutoff $\mathcal{P}^h(a, \tilde{P}, l)$:

$$\left[\alpha \mathcal{P}^h l^{\alpha-1} + \beta \frac{\partial \mathbb{E}\mathbf{V}^{init\prime}}{\partial l'}|_{\tilde{P}'=\frac{a\tilde{P}+\mathcal{P}^h}{a+1}, l'=l}\right] = \kappa, \tag{B.52}$$

where the expectation $\mathbb{E}(\cdot)$ is formed over P' based on the firm's and its workers' posteriors with the firm age $a+1$ and the average productivity $\tilde{P}' = \frac{a\tilde{P}+\mathcal{P}^h}{a+1}$ at the beginning of the next period.

Next, the quitting cutoff can be obtained as follows. Note that firms would not hire workers when

$$\left[\alpha P\Big((1-\lambda f(\theta(x^E)))l\Big)^{\alpha-1} + \beta \frac{\partial \mathbb{E}\mathbf{V}^{init\prime}}{\partial l'}|_{l'=(1-\lambda f(\theta(x^E)))l}\right] < \kappa, \tag{B.53}$$

as before. At the same time, if the marginal value of x^E is still high enough, then firms should also set x^E to the upper bound. This happens when:

$$\lambda f'(\theta(x^E))\theta'(x^E)x^E l + \lambda f(\theta(x^E))l - \lambda f'(\theta(x^E))\theta'(x^E)l\left[\alpha P l'^{\alpha-1} + \beta \frac{\partial \mathbb{E}\mathbf{V}^{init\prime}}{\partial l'}\right] > 0,$$

which can be rephrased as

$$\left[\alpha P l'^{\alpha-1} + \beta \frac{\partial \mathbb{E}\mathbf{V}^{init\prime}}{\partial l'}\right] > x^E + \frac{f(\theta(x^E))}{f'(\theta(x^E))\theta'(x^E)},$$

given $\theta'(x^E) < 0$ and $f'(\theta(x^E)) < 0$. Also, given $x^E = \kappa - c$, this can further rephrased as

$$\left[\alpha P l'^{\alpha-1} + \beta \frac{\partial \mathbb{E}\mathbf{V}^{init\prime}}{\partial l'}\right] > \kappa - c. \tag{B.54}$$

Combining (B.53) and (B.54), firms would stay inactive without allowing quits in the following range

$$\kappa - c < \left[\alpha P l^{\alpha-1} + \beta \frac{\partial \mathbb{E}\mathbf{V}^{init\prime}}{\partial l'}|_{l'=l}\right] < \kappa, \tag{B.55}$$

in other words, the quitting cutoff $\mathcal{P}^q(a, \tilde{P}, l)$ is determined by the following:

$$\left[\alpha \mathcal{P}^q l^{\alpha-1} + \beta \frac{\partial \mathbb{E}\mathbf{V}^{init\prime}}{\partial l'}|_{\tilde{P}'=\frac{a\tilde{P}+\mathcal{P}^q}{a+1}, l'=l}\right] = \kappa - c, \tag{B.56}$$

below which firms start allowing quits. Again, the expectation $\mathbb{E}(\cdot)$ is formed over P' based on the firm's and its workers' posteriors with $a+1$ and $\tilde{P}' = \frac{a\tilde{P}+\mathcal{P}^q}{a+1}$ as before.

Lastly, in regards to the layoff cutoff, it is determined by (B.71) evaluated at $l' = (1 - \lambda f(\theta(x^E)))l$ where x^E is the root of (B.72). Similar to the hiring cutoff, given $(a, \tilde{P}, l)$, if P lies in a range in which the marginal value of layoff (the left-hand side of (B.71)) becomes less than its cost (the right-hand side of (B.71)), then firms no longer lay off any workers. Therefore, the cutoff is determined at where it is optimal to choose $s = 0$ from the separating firms' problem, above which firms would never lay off workers.

Therefore, the following equation determines the layoff productivity cutoff $\mathcal{P}^l(a, \tilde{P}, l)$:

$$\begin{aligned}&\left[\alpha \mathcal{P}^l((1-\lambda f(\theta(x^E)))l)^{\alpha-1} + \beta \frac{\partial \mathbb{E}\mathbf{V}^{init\prime}}{\partial l'}|_{\tilde{P}'=\frac{a\tilde{P}+\mathcal{P}^l}{a+1}, l'=(1-\lambda f(\theta(x^E)))l}\right] \\ &= \frac{U - \lambda x^E\left(\theta(x^E)(1+\theta(x^E)^\gamma)^{-\frac{1}{\gamma}}\right)}{1-\lambda\left(\theta(x^E)(1+\theta(x^E)^\gamma)^{-\frac{1}{\gamma}}\right)},\end{aligned} \tag{B.57}$$

where $x^E = \mathbf{x^E}(a, \tilde{P}, l, \mathcal{P}^l)$ is the root of (B.72) with the set of state variables $(a, \tilde{P}, l, \mathcal{P}^l)$. Here also, the expectation $\mathbb{E}(\cdot)$ is formed over P' based on the firm's and its workers' posteriors with $a+1$ and $\tilde{P}' = \frac{a\tilde{P}+\mathcal{P}^l}{a+1}$ as before.

Once the three cutoffs are determined, I refer to a firm value in each case – hiring, inaction, quitting, and layoffs – as $\mathbf{V}^{init,h}$, $\mathbf{V}^{init,i}$, $\mathbf{V}^{init,q}$, and $\mathbf{V}^{init,l}$, respectively. Using these terms, the value function (B.51) can be rewritten as follows:

$$\mathbf{V}^{init}(a, \tilde{P}, l, P) = \delta U l + (1-\delta)\left(dUl + (1-d)\max\left[\mathbf{V}^{init,h}, \mathbf{V}^{init,i}, \mathbf{V}^{init,q}, \mathbf{V}^{init,l}\right]\right).$$

$\square$

B.4.2 Nonexistence of the case $h > 0$ and $s > 0$

Proposition B.1. *Hiring firms with productivity P drawn above the hiring cutoff $\mathcal{P}^h(a, \tilde{P}, l)$ do not layoff workers. Similarly, firms that layoff workers with productivity P drawn between the middle and lower productivity cutoffs $\mathcal{P}^q(a, \tilde{P}, l)$ and $\mathcal{P}^l(a, \tilde{P}, l)$ do not hire new workers.*

Proof. Suppose that firms both hire and separate workers, e.g. $h > 0$ and $s > 0$, so that they solve the following maximization problem:

$$\max_{h,s,x^E} sUl + Pl'^{\alpha} - c^f - \kappa h + (1-s)\lambda f(\theta(x^E))x^E l + \beta \mathbb{E}\mathbf{V}^{init}(a+1, \tilde{P}', l', P') \tag{B.58}$$

The first-order conditions with respect to h, s, and x^E are as follows (in the same order):

$$\Big[\alpha P l'^{\alpha-1} + \beta \frac{\partial \mathbb{E}\mathbf{V}^{init\prime}}{\partial l'}\Big] = \kappa, \tag{B.59}$$

$$Ul - \lambda f(\theta(x^E))x^E l - (1-\lambda f(\theta(x^E)))l\Big[\alpha P l'^{\alpha-1} + \beta \frac{\partial \mathbb{E}\mathbf{V}^{init\prime}}{\partial l'}\Big] = 0, \tag{B.60}$$

$$\lambda f'(\theta(x^E))\theta'(x^E)x^E l + \lambda f(\theta(x^E))l - \lambda f'(\theta(x^E))\theta'(x^E)l\Big[\alpha P l'^{\alpha-1} + \beta \frac{\partial \mathbb{E}\mathbf{V}^{init\prime}}{\partial l'}\Big] = 0. \tag{B.61}$$

Using (B.59) to substitute out the term $\Big[\alpha P l'^{\alpha-1} + \beta \frac{\partial \mathbb{E}\mathbf{V}^{init\prime}}{\partial l'}\Big]$ in (B.61), and using (2.25), I can rewrite the left-hand side of (B.61) as follows:

$$\frac{(\kappa - x^E)^{\gamma} c^{-\gamma}\Big((\frac{\kappa - x^E}{c})^{\gamma} - 1\Big)^{\frac{1}{\gamma}}(\frac{\kappa - x^E}{c})^{\gamma}}{(\frac{\kappa - x^E}{c})^{\gamma} - 1} = \frac{\Big((\kappa - x^E)^{\gamma} c^{-\gamma}\Big)^2}{\Big((\frac{\kappa - x^E}{c})^{\gamma} - 1\Big)^{1-\frac{1}{\gamma}}} > 0.$$

This term can be proved to be strictly positive given that $x^E < \kappa - c$ for any active markets x^E. This means that the marginal value of x^E is strictly positive, and thus optimal x^E reaches the upper bound:

$$x^E = \kappa - c. \tag{B.62}$$

Thus, for hiring firms it follows that $f(\theta(\kappa - c)) = 0$, which makes the marginal value of s from (B.60) negative as follows:

$$U - \Big[\alpha P l'^{\alpha-1} + \beta \frac{\partial \mathbb{E}\mathbf{V}^{init\prime}}{\partial l'}\Big] < 0. \tag{B.63}$$

This is due to (B.59) and $\kappa > U$, and shows that hiring firms can never have any marginal value of separating workers and would never separate workers.

In a similar fashion, contracting firms would never hire workers, given that their marginal value of a new hire from (B.59) is always negative:

$$\Big[\alpha P l'^{\alpha-1} + \beta \frac{\partial \mathbb{E}\mathbf{V}^{init\prime}}{\partial l'}\Big] - \kappa < 0, \tag{B.64}$$

given (B.71) and $\kappa > U$. Therefore, this completes the proof that if $h > 0$, $s = 0$ needs to hold, and vice versa.

The proof enables me to split the firm's problem into the following three cases from B.4.3 through B.4.5. $\square$

B.4.3 Hiring Firms: $s = 0$ and $h > 0$

$$\max_{h, x^E} Pl'^{\alpha} - c^f - \kappa h + \lambda f(\theta(x^E)) x^E l + \beta \mathbb{E}\mathbf{V}^{init}(a+1, \tilde{P}', l', P') \tag{B.65}$$

subject to $l' = h + (1 - \lambda f(\theta(x^E)))l$ and $\tilde{P}' = \frac{a\tilde{P} + \ln P}{a+1}$.

As before, the first-order conditions with respect to h and x^E are (B.59) and (B.61), respectively. We know the optimal x^E is pinned at the upper bound as in (B.62).

Lastly, using (B.43), the utility level $\tilde{\mathbf{W}}$ that firms will offer to their incumbent workers under this case is determined by:

$$\tilde{\mathbf{W}} = \kappa - c. \tag{B.66}$$

B.4.4 Inactive Firms: $s = 0$ and $h = 0$

Note that this case holds only when

$$\left[\alpha P l^{\alpha-1} + \beta \frac{\partial \mathbb{E}\mathbf{V}^{init\prime}}{\partial l'}|_{l'=l}\right] < \kappa,$$

where the marginal value of h is strictly less than zero and $h = 0$ is optimal. Under this case, firms need to solve the following problem:

$$\max_{x^E} Pl'^{\alpha} - c^f + \lambda f(\theta(x^E)) x^E l + \beta \mathbb{E}\mathbf{V}^{init}(a+1, \tilde{P}', l', P') \tag{B.67}$$

subject to $l' = (1 - \lambda f(\theta(x^E)))l$.

Using the first-order condition with respect to x^E in (B.61), and evaluating l' at $(1-\lambda f(\theta(x^E)))l$, we have the following equation to determine x^E:

$$x^E + \frac{f(\theta(x^E))}{f'(\theta(x^E))\theta'(x^E)} - \left[\alpha P \Big((1 - \lambda f(\theta(x^E)))l\Big)^{\alpha-1} + \beta \frac{\partial \mathbb{E}\mathbf{V}^{init\prime}}{\partial l'}|_{l'=(1-\lambda f(\theta(x^E)))l}\right] = 0. \tag{B.68}$$

Using (B.43), the equilibrium utility level $\tilde{\mathbf{W}}$ firms offer to their incumbent workers is pinned down by:

$$\tilde{\mathbf{W}} = \kappa - (\kappa - x^E)^{1+\gamma} c^{-\gamma}. \tag{B.69}$$

Note that this only holds when the optimal x^E is in the range of $\kappa \leq c$. If P is high enough so that the left-hand side of (B.68) becomes strictly greater than 0, then as before for the hiring firms, the optimal solution is bound by the upper bound, i.e. $x^E = \kappa - c$. This holds when

$$\kappa - c < \left[\alpha P l^{\alpha-1} + \beta \frac{\partial \mathbb{E}\mathbf{V}^{init\prime}}{\partial l'}|_{l'=l}\right],$$

so that the marginal value of x^E is strictly positive, and hence, the optimal x^E is bound by the upper bound $\kappa - c$. In this case, firms would not just stay inactive but also not allow workers quitting. In other words, they stay inactive not allowing quitting, i.e. $l' = l$. More details about the productivity cutoff will be supplemented in B.4.1.

B.4.5 Separating Firms with Layoffs: $s > 0$ and $h = 0$

$$\max_{s,x^E} sUl + Pl'^{\alpha} - c^f + (1-s)\lambda f(\theta(x^E))x^E l + \beta \mathbb{E}\mathbf{V}^{init}(a+1, \tilde{P}', l', P') \tag{B.70}$$

subject to $l' = (1-s)(1-\lambda f(\theta(x^E)))l$.

Note that the first-order conditions with respect to s and x^E hold the same as in (B.60) and (B.61), respectively. Rewriting (B.60) by canceling out l and using (B.40) as before, the following is obtained:

$$\left[\alpha P l'^{\alpha-1} + \beta \frac{\partial \mathbb{E}\mathbf{V}^{init\prime}}{\partial l'}\right] = \frac{U - \lambda x^E \Big(\theta(x^E)(1+\theta(x^E)^{\gamma})^{-\frac{1}{\gamma}}\Big)}{1 - \lambda\Big(\theta(x^E)(1+\theta(x^E)^{\gamma})^{-\frac{1}{\gamma}}\Big)}. \tag{B.71}$$

Substituting out the term $\left[\alpha P l'^{\alpha-1} + \beta \frac{\partial \mathbb{E}\mathbf{V}^{init\prime}}{\partial l'}\right]$ in (B.61) using (B.71), x^E is determined by the following equation:

$$\kappa - U = c\Big[(1+\theta(x^E)^{\gamma})^{1+\frac{1}{\gamma}} - \lambda\theta(x^E)^{1+\gamma}\Big]. \tag{B.72}$$

Again, the equilibrium utility level $\tilde{\mathbf{W}}$ is determined as (B.69).

B.4.6 Exiting Firms

Lastly, firms' optimal exit decision is chosen by:

$$\mathbf{d}(a, \tilde{P}, l, P) = \begin{cases} 1 & \text{if} \quad \mathbf{U}l > \max\left[\mathbf{V}^{init,h}, \mathbf{V}^{init,i}, \mathbf{V}^{init,q}, \mathbf{V}^{init,l}\right] \\ 0 & \text{otherwise.} \end{cases}$$

Letting the productivity cutoff denoted by $\mathcal{P}^x(a,\tilde{P},l)$, it is determined by the following equation:

$$\mathbf{U}l = \max\Big[\mathbf{V}^{init,h}(a,\tilde{P},l,\mathcal{P}^x(a,\tilde{P},l)), \mathbf{V}^{init,q}(a,\tilde{P},l,\mathcal{P}^x(a,\tilde{P},l)), \mathbf{V}^{init,l}(a,\tilde{P},l,\mathcal{P}^x(a,\tilde{P},l))\Big]. \tag{B.73}$$

Appendix C Workers' Future Expected Value (Job Prospects)

Recall that the employed worker's value in (2.10). Incorporating the decision rules of firms obtained in the previous section, the worker's value function can be rephrased as the following:

$$\begin{aligned}\mathbf{W}(a,\tilde{P},l,P) =& \mathbf{w} + \beta\mathbb{E}\Bigg[\Big(\delta + (1-\delta)\big(\mathbf{d}' + (1-\mathbf{d}')\mathbf{s}'\big)\Big)\mathbf{U} \\ &+ (1-\delta)(1-\mathbf{d}')(1-\mathbf{s}')\Big(\lambda f(\theta(\mathbf{x}^{\mathbf{E}'}))\mathbf{x}^{\mathbf{E}'} + \big(1-\lambda f(\theta(\mathbf{x}^{\mathbf{E}'}))\big)\tilde{\mathbf{W}}'\Big)\Bigg],\end{aligned} \tag{C.74}$$

where $\mathbf{w} = \mathbf{w}(a,\tilde{P},l,P)$ is the equilibrium wage offered by the firm, $\mathbf{d}' = \mathbf{d}(a+1,\tilde{P}',l',P')$, $\mathbf{s}' = \mathbf{s}(a+1,\tilde{P}',l',P')$, $\mathbf{x}^{\mathbf{E}'} = \mathbf{x}^{\mathbf{E}}(a+1,\tilde{P}',l',P')$, and $\tilde{\mathbf{W}}' = \tilde{\mathbf{W}}(a+1,\tilde{P}',l',P')$ are the firm's exit, layoff, retention decision rules in the next period, contingent on the realization of P'. Note that $l' = \mathbf{h}(a,\tilde{P},l,P) + (1-\lambda f(\mathbf{x}^{\mathbf{E}}(a,\tilde{P},l,P)))l$ is the next period initial employment size of the firm as a result of its hiring and retention activity in the current period. Hence, the worker value function ends up being a function of the employer's current state variable, $(a,\tilde{P},l,P)$.

As seen in Lemma 1, the promise keeping constraints (2.15) and (2.16) hold with equality at equilibrium for new hires and incumbent workers, respectively. Thus, following Proposition 1, given workers' outside option, equilibrium wages depend on the workers' expected value based on their beliefs about firms.

Now, I would like to delve into the large bracket in (C.74), which is associated with workers' expected future value. Incorporating the firm's decision rules and the productivity cutoffs, these terms be rephrased as the following:

$$\begin{aligned}&\delta\mathbf{U} + (1-\delta)\Bigg(\int_{\mathcal{P}^q}^{\infty}(\kappa - c)dF(P') + \int_{\mathcal{P}^l}^{\mathcal{P}^q}\Big(\lambda f(\theta(\mathbf{x}^{\mathbf{E}'}))\mathbf{x}^{\mathbf{E}'} + \big(1-\lambda f(\theta(\mathbf{x}^{\mathbf{E}'}))\big)\tilde{\mathbf{W}}'\Big)dF(P') \\ &+ \int_{\mathcal{P}^x}^{\mathcal{P}^l}\Big(\mathbf{s}'\mathbf{U} + (1-\mathbf{s}')\big(\lambda f(\theta(\mathbf{x}^{\mathbf{E}'}))\mathbf{x}^{\mathbf{E}'} + \big(1-\lambda f(\theta(\mathbf{x}^{\mathbf{E}'}))\big)\tilde{\mathbf{W}}'\big)\Big)dF(P') + \int_{\infty}^{\mathcal{P}^x}\mathbf{U}dF(P')\Bigg),\end{aligned} \tag{C.75}$$

where $F(\cdot)$ is the log-normal cumulative density function of productivity P', based on the worker's posterior about the firm with the corresponding mean $\bar{\nu} = \frac{\frac{\bar{\nu}_0}{\sigma_0^2} + (a+1)\frac{\tilde{P}'}{\sigma_\varepsilon^2}}{\frac{1}{\sigma_0^2} + (a+1)\frac{1}{\sigma_\varepsilon^2}}$ and variance

$\sigma^2 + \sigma_\varepsilon^2$ where $\sigma^2 = \frac{1}{\frac{1}{\sigma_0^2}+(a+1)\frac{1}{\sigma_\varepsilon^2}}$. Also, the productivity cutoffs $\mathcal{P}^q$, $\mathcal{P}^l$, $\mathcal{P}^x$ are from (B.56), (B.57), and (B.73), respectively, which are a function of the firm's state variables $(a+1, \tilde{P}', l')$ at the beginning of the next period.

The first term is the worker's value when the employer is hit by the exogenous death shock. And conditional on surviving from the shock, workers further consider the following cases expressed in the large bracket following the first term.

First, the first term in the bracket in (C.75) shows that workers will get $\kappa - c$ conditional on the case in which their employer hires or stay inactive without losing any workers in the next period, i.e. P' is drawn above $\mathcal{P}^q(a+1, \tilde{P}', l')$. As seen in the previous sections B.4.3 and B.4.4, this firm would not allow any quits by setting the promised utility to incumbent workers to the maximum value, i.e. $\kappa - c$. Thus, in either case, workers end up obtaining the value $\kappa - c$ and staying at the firm.

Next, the second term in the bracket presents the worker's expected value when the firm stays inactive but allows quits, i.e. P' is realized in between $\mathcal{P}^l(a+1, \tilde{P}', l')$ and $\mathcal{P}^q(a+1, \tilde{P}', l')$. In this case, with probability $\lambda f(\theta(\mathbf{x}^{\mathbf{E}'}))$, the worker can make his on-the-job search successful and gain $\mathbf{x}^{\mathbf{E}'}$. Otherwise, the worker stays at the current employer and obtains $\tilde{\mathbf{W}}'$. Note that $\mathbf{x}^{\mathbf{E}} = \mathbf{x}^{\mathbf{E}}(a+1, \tilde{P}', l', P')$ and $\tilde{\mathbf{W}} = \tilde{\mathbf{W}}(a+1, \tilde{P}', l', P')$ are the employer's equilibrium retention choice (taking into account the worker's choice for x^E) following (B.43), (B.68), and (B.69).

The third term in the bracket is the worker's expected value when the firm has a possibility to lay off workers in the next period, i.e. (with P' realized between $\mathcal{P}^x$ and $\mathcal{P}^l$). Then, in the case of firm layoffs, the worker goes to the unemployment pool and consumes the value $\mathbf{U}$, which is the first term of the integral in this bracket. Otherwise, the worker needs to consider the possibility of being poached (with the probability $\lambda f(\theta(\mathbf{x}^{\mathbf{E}'}))$) or staying at the current firm (with the probability $1 - \lambda f(\theta(\mathbf{x}^{\mathbf{E}'}))$) as before, which is expressed by the remaining terms in the integral. Here, $\mathbf{x}^{\mathbf{E}} = \mathbf{x}^{\mathbf{E}}(a+1, \tilde{P}', l', P')$ is the employer's layoff decision rule following (B.71), and $\mathbf{x}^{\mathbf{E}} = \mathbf{x}^{\mathbf{E}}(a+1, \tilde{P}', l', P')$, $\tilde{\mathbf{W}} = \tilde{\mathbf{W}}(a+1, \tilde{P}', l', P')$ are the employer's retention decision rules as before following (B.43), (B.69), and (B.72).

Lastly, conditional on the firm observing P' below the exit cutoff $\mathcal{P}^x(a+1, \tilde{P}', l')$, the firm will endogenously stop operating and exit, and the worker becomes unemployed. This is reflected on the last term.

C.1 The Ranking of Workers' Value (Proof of Proposition 3)

Define $\hat{W}$ as incumbent workers' value at the beginning of a period after observing the firm's current productivity draw P (but before the firm's endogenous exit and layoffs). Then, $\hat{W}$ is ranked by the following descending order:

i) Workers at hiring or inactive employers (with $P \geq \mathcal{P}^q$) obtain the highest value, $(\kappa - c)$;

ii) Workers at quitting employers (with $P \in [\mathcal{P}^l, \mathcal{P}^q]$) have a value lower than those at hiring or inactive firms (without quits) and higher than those at firms laying off workers, $\Big(\lambda f(\theta(\mathbf{x^E}))\mathbf{x^E} + \big(1 - \lambda f(\theta(\mathbf{x^E}))\big)\tilde{\mathbf{W}}\Big)$;

iii) Workers at employers that lay off workers (with $P \leq \mathcal{P}^l$) have a value lower than those at quitting or inactive or expanding firms but higher than unemployed workers, $\Big(\mathbf{sU} + (1-\mathbf{s})\big(\lambda f(\theta(\mathbf{x^E}))\mathbf{x^E} + \big(1 - \lambda f(\theta(\mathbf{x^E}))\big)\tilde{\mathbf{W}}\big)\Big)$;

iv) Unemployed workers have the lowest value, U.

The proof is as follows. First, it is already known that any inactive markets x^E need to be ranged below $\kappa - c$. And following (B.69), $\tilde{\mathbf{W}}$ has to be bound by $\kappa - c$. In other words,

$$x^E \leq \kappa - c \quad \text{and} \quad \tilde{\mathbf{W}} \leq \kappa - c \quad \text{for any active } x^E,$$

which confirms that

$$\big(\lambda f(\theta(\mathbf{x^E}))\mathbf{x^E} + \big(1 - \lambda f(\theta(\mathbf{x^E}))\big)\tilde{\mathbf{W}}\big) \leq \kappa - c, \quad \forall \mathbf{x^E}, \tilde{\mathbf{W}}. \tag{C.76}$$

Next, consider a firm in the inaction region, $P \in [\mathcal{P}^l, \mathcal{P}^q]$, but allowing quits. Using (B.69), the worker's value at this firm can be rephrased as follows:

$$\big(\lambda f(\theta(\mathbf{x^E}))\mathbf{x^E} + \big(1 - \lambda f(\theta(\mathbf{x^E}))\big)\tilde{\mathbf{W}}\big) = \mathbf{x^E} - (\kappa - \mathbf{x^E})\theta(\mathbf{x^E})^\gamma + c\theta(\mathbf{x^E})^{1+\gamma}, \tag{C.77}$$

which is the weighted average of the promised utility in the current firm and the target utility in the worker's on-the-job search. Here, $\mathbf{x^E}$ is the solution of the equation (B.68). Furthermore, this firm finds $s = 0$ to be optimal and stays inactive with quits allowed. Therefore, the marginal value of s, the left-hand side of (B.60), has to be strictly negative with any $s > 0$ and equals to zero with $s = 0$.

Combining this with (B.68), it can be proved that

$$\mathbf{U} \leq \Big(\mathbf{x^E} + \frac{(1 - \lambda f(\theta(\mathbf{x^E})))f(\theta(\mathbf{x^E}))}{f(\theta)\theta(\mathbf{x^E})}\Big),$$

which can further be rewritten with (B.40) and (2.25) as follows:

$$\mathbf{U} \leq \mathbf{x^E} - \theta(\mathbf{x^E})^{\gamma}(\kappa - \mathbf{x^E}). \tag{C.78}$$

With (C.77) and (C.78), it is proved that

$$\mathbf{U} \leq \big(\lambda f(\theta(\mathbf{x^E}))\mathbf{x^E} + \big(1 - \lambda f(\theta(\mathbf{x^E}))\big)\tilde{\mathbf{W}}\big), \tag{C.79}$$

for any firms staying inactive with quits and choosing $\mathbf{x^E}$ following (B.68).

Similarly, let's consider a firm laying off workers after observing $P \in [\mathcal{P}^x, \mathcal{P}^l]$ in a given period. Based on (C.77), the worker's value at this firm is

$$\mathbf{sU} + (1 - \mathbf{s})\Big(\mathbf{x^E} - (\kappa - \mathbf{x^E})\theta(\mathbf{x^E})^{\gamma} + c\theta(\mathbf{x^E})^{1+\gamma}\Big), \tag{C.80}$$

where $\mathbf{x^E}$ is the solution of the equation (B.72). Furthermore, (B.72) implies that

$$\mathbf{U} = \mathbf{x^E} - (\kappa - \mathbf{x^E})\theta(\mathbf{x^E})^{\gamma} + \lambda c\theta(\mathbf{x^E})^{1+\gamma}. \tag{C.81}$$

Hence, (C.80) and (C.81) confirm that

$$\mathbf{U} \leq \mathbf{sU} + (1 - \mathbf{s})\Big(\mathbf{x^E} - (\kappa - \mathbf{x^E})\theta(\mathbf{x^E})^{\gamma} + c\theta(\mathbf{x^E})^{1+\gamma}\Big). \tag{C.82}$$

for any firms laying off workers with s and $\mathbf{x^E}$ following (B.60) and (B.68).

Combining (C.76), (C.79), and (C.82) proves i) and iv), meaning that workers obtain the highest value at a hiring or inactive firm and get the lowest value in the unemployment pool.

Lastly, the rank order of workers' value between quitting firms and those laying off workers needs to be confirmed to verify ii) and iii). This can be established with the following two proofs. First, it can be proved that (C.77) is weakly increasing in $\mathbf{x^E}$, implying that workers get weakly higher values at a firm with higher $\mathbf{x^E}$. Second, the other proof to be confirmed is the equilibrium $\mathbf{x^E}$ is higher for quitting firms than contracting firms with layoffs. In other words, $\mathbf{x^E}$ satisfying (B.68) is higher than $\mathbf{x^E}$ satisfying (B.72). Then, the two proofs along with (C.79) can confirm that workers obtain higher values at quitting firms than those laying off workers.

Let's start with the first one by getting the derivative of (C.77) with respect to x^E.

$$\frac{\partial\big(\lambda f(\theta(x^E))x^E + \big(1 - \lambda f(\theta(x^E))\big)\tilde{\mathbf{W}}\big)}{\partial x^E} = \frac{\partial\Big(x^E - (\kappa - x^E)\theta(x^E)^{\gamma} + c\theta(x^E)^{1+\gamma}\Big)}{\partial x^E}$$

$$= 1 - \frac{\theta^{\gamma}(\frac{\kappa - x^E}{c})^{-\gamma}}{1 - (\frac{\kappa - x^E}{c})^{-\gamma}} = 0. \tag{C.83}$$

This implies that for any non-binding optimal solutions for x^E in (B.68), worker values conditional on not being separated are the same.

Second, it is already seen in the previous discussion from the equations (B.68) and (B.68) that the optimal choice x^E of quitting firms follows:

$$\mathbf{U} \leq x^E + \frac{(1 - \lambda f(\theta(x^E)))f(\theta(x^E))}{f'(\theta)\theta'(x^E)},$$

while the choice of firms laying off workers is pinned down by the following:

$$\mathbf{U} = x^E + \frac{(1 - \lambda f(\theta(x^E)))f(\theta(x^E))}{f'(\theta)\theta'(x^E)}.$$

Thus, in order to confirm the former is higher than the latter, it is sufficient to prove the following terms are increasing in x^E:

$$x^E + \frac{(1 - \lambda f(\theta(x^E)))f(\theta(x^E))}{f'(\theta)\theta'(x^E)}.$$

Using (B.40), the above terms can be rephrased by

$$x^E + \frac{(1 - \lambda f(\theta(x^E)))f(\theta(x^E))}{f'(\theta)\theta'(x^E)} = x^E - \theta^{\gamma}(\kappa - x^E) + \lambda c\theta^{\gamma+1}.$$

These terms satisfy the following property:

$$\frac{\partial\Big(x^E - \theta^{\gamma}(\kappa - x^E) + \lambda c\theta^{\gamma+1}\Big)}{\partial x^E} = \frac{\partial\Big(x^E - \theta^{\gamma}(\kappa - x^E) + c\theta^{\gamma+1}\Big)}{\partial x^E} - (1-\lambda)c(\gamma+1)\theta^{\gamma}\frac{\partial\theta(x^E)}{\partial x^E} > 0,$$

given that (C.83) makes the first term on the right-hand side being zero and $\frac{\partial\theta(x^E)}{\partial x^E} < 0$. Thus, the following is proved:

$$\frac{\partial\Big(x^E + \frac{(1-\lambda f(\theta(x^E)))f(\theta(x^E))}{f'(\theta)\theta'(x^E)}\Big)}{\partial x^E} > 0,$$

implying that the optimal x^E is higher for quitting firms than those laying off workers. Lastly, this fact along with (C.79) and (C.83) finalizes the proof for ii) and iii).

Linking the findings i)-iv) to the equation (C.75), it can be shown that workers would expect higher future values at a hiring or inactive firm than a contracting firms with poaching or layoffs.

Appendix D Implications of Workers' Job Prospects

D.1 The Ranking of $\frac{\partial \mathbb{E}\mathbf{V}^{init}(a+1,\tilde{P}',l',P')}{\partial l'}$

In this section, I analyze how workers' job prospects matter for firms' decision making at the hiring or retention margin. Recalling (B.50), it can be rephrased as follows:

$$\mathbf{V}^{init}(a,\tilde{P},l,P) = [*]l + (1-\delta)(1-d)\left(J(a,\tilde{P},l',P,x,\tilde{\mathbf{W}}) - \frac{c}{q(\theta(x))}h\right), \tag{D.84}$$

where

$$[*] \equiv \delta\mathbf{U} + (1-\delta)(d+(1-d)s)\mathbf{U} + (1-\delta)(1-d)(1-s)(\lambda f(\theta(x^E))x^E + (1-\lambda f(\theta(x^E)))\tilde{\mathbf{W}}).$$

Then, iterating it one period forward and taking expectation, the following holds:

$$\frac{\partial \mathbb{E}\mathbf{V}^{init\prime}}{\partial l'} = \mathbb{E}[*'] + \frac{\partial \mathbb{E}[*']}{\partial l'}l' + (1-\delta)\frac{\partial}{\partial l'}\mathbb{E}\Big[(1-d')(\mathbf{J}' - \frac{c}{q(\theta(x'))}h')\Big], \tag{D.85}$$

which shows the expected future marginal value of a labor input. Note that this is the sum of the following three components associated with workers' job prospects and firms' own prospects about their type: i) the first term on the right-hand side is workers' future expected value, ii) the second term is the indirect effect of firm size on workers' future expected value, and iii) the last term is the firms' expected future value. These terms play a key role in firms' decision making.

Now, I prove that $\frac{\partial \mathbb{E}\mathbf{V}^{init}(a+1,\tilde{P}',l',P')}{\partial l'}$ varies across firms depending on their employment status, and the ranking holds the same as the workers' future expected value as seen in the previous section.

D.1.1 Hiring Firms: $s=0$ and $h>0$

For hiring firms, their value function becomes:

$$\mathbf{V}^{init}(a,\tilde{P},l,P) = \delta\mathbf{U}l + (1-\delta)\Big[Pl'^{\alpha} - c^f - \kappa\mathbf{h} + \beta\mathbb{E}\mathbf{V}^{init}(a+1,\tilde{P}',l',P')\Big],$$

where $\mathbf{h} \equiv \mathbf{h}(a,\tilde{P},l,P)$ is the firm's hiring decision rule and $l' \equiv \mathbf{h}(a,\tilde{P},l,P) + l$. Then, we have the following derivative with respect to l:

$$\frac{\partial \mathbf{V}^{init}(a,\tilde{P},l,P)}{\partial l} = \delta\mathbf{U} + (1-\delta)\Big[\alpha Pl'^{\alpha-1} + \beta\frac{\partial \mathbb{E}\mathbf{V}^{init}(a+1,\tilde{P}',l',P')}{\partial l'}\Big]$$

$$+(1-\delta)\frac{\partial \mathbf{h}}{\partial l}\Big[\alpha P{l'}^{\alpha-1}+\beta\frac{\partial \mathbb{E}\mathbf{V}^{init}(a+1,\tilde{P}',l',P')}{\partial l'}-\kappa\Big],$$

where the first line is a direct effect of l, and the second line is an indirect effect of l through its optimal hiring on the value function. With (B.59), the indirect effect becomes zero, which is consistent with the Envelope theorem. Therefore, it gets simplified as follows:

$$\frac{\partial \mathbf{V}^{init}(a,\tilde{P},l,P)}{\partial l}=\delta\mathbf{U}+(1-\delta)\kappa. \tag{D.86}$$

D.1.2 Inactive Firms: $s=0$ and $h=0$

Next, consider inactive firms who do not allow quits. Their hiring, layoff, and retention decisions are $\mathbf{h}=0$, $\mathbf{s}=0$, and $\mathbf{x^E}=0$, all of which are the function of $(a,\tilde{P},l,P)$, and this makes $l'=l$. Thus, their value function is

$$\mathbf{V}^{init}(a,\tilde{P},l,P)=\delta\mathbf{U}l+(1-\delta)\Big[Pl^{\alpha}-c^f+\beta\mathbb{E}\mathbf{V}^{init}(a+1,\tilde{P}',l,P')\Big],$$

and the first derivative of it with respect to l is

$$\frac{\partial \mathbf{V}^{init}(a,\tilde{P},l,P)}{\partial l}=\delta\mathbf{U}+(1-\delta)\Big[\alpha Pl^{\alpha-1}+\beta\frac{\partial \mathbb{E}\mathbf{V}^{init}(a+1,\tilde{P}',l,P')}{\partial l}\Big].$$

Note that this case can only happen with the range (B.55), and thus this term should be in between $[\kappa-c,\kappa]$. In other words, the following holds for this type of firms:

$$\delta\mathbf{U}+(1-\delta)(\kappa-c)\leq\frac{\partial \mathbf{V}^{init}(a,\tilde{P},l,P)}{\partial l}\leq\delta\mathbf{U}+(1-\delta)\kappa. \tag{D.87}$$

Now, consider the other case of inactive firms who allow quits. Their value function is as follows:

$$\mathbf{V}^{init}(a,\tilde{P},l,P)=\delta\mathbf{U}l+(1-\delta)\Big[P{l'}^{\alpha}-c^f+\lambda f(\theta(\mathbf{x^E}))\mathbf{x^E}l+\beta\mathbb{E}\mathbf{V}^{init}(a+1,\tilde{P}',l',P')\Big],$$

where $\mathbf{x^E}\equiv\mathbf{x^E}(a,\tilde{P},l,P)$ is their optimal retention choice, which is a root of (B.68), and $l'\equiv(1-\lambda f(\theta(\mathbf{x^E}(a,\tilde{P},l,P))))l$.

Getting the derivative as before, the following can be obtained:

$$\frac{\partial \mathbf{V}^{init}(a,\tilde{P},l,P)}{\partial l}=\delta\mathbf{U}+(1-\delta)\Big[(1-\lambda f(\theta(\mathbf{x^E})))\Big(\alpha P{l'}^{\alpha-1}+\beta\frac{\partial \mathbb{E}\mathbf{V}^{init}(a+1,\tilde{P}',l',P')}{\partial l'}\Big)+\lambda f(\theta(\mathbf{x^E}))\mathbf{x^E}\Big]$$
$$+(1-\delta)\frac{\partial \mathbf{x^E}}{\partial l}\Big[-\lambda f'(\theta)\theta'(\mathbf{x^E})l\Big(\alpha P{l'}^{\alpha-1}+\beta\frac{\partial \mathbb{E}\mathbf{V}^{init}(a+1,\tilde{P}',l',P')}{\partial l'}\Big)+\lambda f(\theta(\mathbf{x^E}))l+\lambda f'(\theta)\theta'(\mathbf{x^E})\mathbf{x^E}l\Big],$$

where the first line is a direct effect of l, and the second line is an indirect effect of l through its optimal retention on the value function. As before, using (B.68), the indirect effect becomes zero. Thus, the terms can be rephrased as follows:

$$\frac{\partial \mathbf{V}^{init}(a,\tilde{P},l,P)}{\partial l} = \delta \mathbf{U} + (1-\delta)\Big[\mathbf{x^E} + \frac{(1-\lambda f(\theta(\mathbf{x^E})))f(\theta(\mathbf{x^E}))}{f'(\theta)\theta'(\mathbf{x^E})}\Big].$$

Note that this term has to be in the following range:

$$\mathbf{U} \leq \frac{\partial \mathbf{V}^{init}(a,\tilde{P},l,P)}{\partial l} \leq \delta \mathbf{U} + (1-\delta)(\kappa - c). \tag{D.88}$$

The upper bound comes from $f'(\theta)\theta'(\mathbf{x^E}) < 0$ and $\mathbf{x^E} \leq \kappa - c$. The lower bound is from the fact that this firm never finds $s > 0$ to be optimal, which is consistent to say the left-hand side of (B.60) is strictly negative with any $s > 0$ or zero with $s = 0$. Combining this with (B.68), it can be proved that

$$\mathbf{U} \leq \Big[\mathbf{x^E} + \frac{(1-\lambda f(\theta(\mathbf{x^E})))f(\theta(\mathbf{x^E}))}{f'(\theta)\theta'(\mathbf{x^E})}\Big]$$

which gives the lower bound of (D.88).

D.1.3 Separating Firms with Layoffs: $s > 0$ and $h = 0$

For firms separating workers with explicit layoffs, their value function is:

$$\mathbf{V}^{init}(a,\tilde{P},l,P) = \delta \mathbf{U}l + (1-\delta)\Big[\mathbf{s}\mathbf{U}l + Pl'^{\alpha} - c^f + (1-\mathbf{s})\lambda f(\theta(\mathbf{x^E}))\mathbf{x^E}l + \beta \mathbb{E}\mathbf{V}^{init}(a+1,\tilde{P}',l',P')\Big],$$

where $\mathbf{s} \equiv \mathbf{s}(a,\tilde{P},l,P)$ is their layoff decision, $\mathbf{x^E} \equiv \mathbf{x^E}(a,\tilde{P},l,P)$ is their retention decision, and $l' \equiv (1-\mathbf{s}(a,\tilde{P},l,P))(1-\lambda f(\theta(\mathbf{x^E}(a,\tilde{P},l,P))))l$.

Making the first derivative of it with respect to l, it can be obtained that

$$\begin{aligned}
&\frac{\partial \mathbf{V}^{init}(a,\tilde{P},l,P)}{\partial l} \\
&= \delta \mathbf{U} + (1-\delta)\Big[\mathbf{s}\mathbf{U} + (1-\mathbf{s})(1-\lambda f(\theta(\mathbf{x^E})))\Big(\alpha P l'^{\alpha-1} + \beta \frac{\partial \mathbb{E}\mathbf{V}^{init}(a+1,\tilde{P}',l',P')}{\partial l'}\Big)(1-\mathbf{s})\lambda f(\theta(\mathbf{x^E}))\mathbf{x^E}\Big] \\
&+ (1-\delta)\frac{\partial \mathbf{s}}{\partial l}\Big[\mathbf{U}l - (1-\lambda f(\theta(\mathbf{x^E})))l\Big(\alpha P l'^{\alpha-1} + \beta \frac{\partial \mathbb{E}\mathbf{V}^{init}(a+1,\tilde{P}',l',P')}{\partial l'}\Big) - \lambda f(\theta(\mathbf{x^E}))\mathbf{x^E}l\Big] \\
&+ (1-\delta)(1-\mathbf{s})\frac{\partial \mathbf{x^E}}{\partial l}\Big[-\lambda f'(\theta)\theta'(\mathbf{x^E})l\Big(\alpha P l'^{\alpha-1} + \beta \frac{\partial \mathbb{E}\mathbf{V}^{init}(a+1,\tilde{P}',l',P')}{\partial l'}\Big) + \lambda f(\theta(\mathbf{x^E}))l \\
&+ \lambda f'(\theta)\theta'(\mathbf{x^E})\mathbf{x^E}l\Big],
\end{aligned}$$

where the first line is a direct effect of l, the second line is an indirect effect of l through its optimal layoffs, the last two lines are an indirect effect of l through its optimal retention on the value function. Note that, consistent with the Envelope theorem again, (B.71) and (B.61) make the indirect effects zero. Also, using (B.71), the first line gets even more simplified. Ultimately, the derivative becomes:

$$\frac{\partial \mathbf{V}^{init}(a, \tilde{P}, l, P)}{\partial l} = \mathbf{U}. \tag{D.89}$$

D.1.4 Exiting firms: $d = 1$

Lastly, for exiting firms, their value function is:

$$\mathbf{V}^{init}(a, \tilde{P}, l, P) = \mathbf{U}l,$$

and the derivative with respect to l is

$$\frac{\partial \mathbf{V}^{init}(a, \tilde{P}, l, P)}{\partial l} = \mathbf{U}. \tag{D.90}$$

Combining (D.86), (D.87), (D.88), (D.89), and (D.90), it can be proved that for $\frac{\partial \mathbf{V}^{init}(a,\tilde{P},l,P)}{\partial l}$, hiring firms have the highest value, inactive firms without quits have the second highest value, quitting firms have the third highest value, and firms laying off workers or exiting have the lowest value. Therefore, this implies that firms that are more expected to draw higher P' and expand in the next period will obtain a higher expected future marginal value of a labor input, $\frac{\partial \mathbb{E}\mathbf{V}^{init}(a+1,\tilde{P}',l',P')}{\partial l'}$.

This indicates that the expected future marginal value of a labor input goes in the same direction as the workers' expected value in the previous section. In other words, even after considering the indirect effect of firm size on the workers' expected value as well as the firms' own prospects, the direct effect through the workers' job prospects remains dominant to the expected future marginal value of a labor input. In the following sections, I discuss how workers' job prospects can matter for firms' choice for hiring and retention by showing that the expected future marginal value of a labor input directly affects their decision.

D.2 Implications on Productivity Cutoffs

Note from the previous section B.4 that the term $\frac{\partial \mathbb{E}\mathbf{V}^{init\prime}}{\partial l'}$ matters to determine variations of the productivity cutoffs across firms with different job prospects. Recall that there are four endogenous productivity cutoffs among operating firms, $\mathcal{P}^h$, $\mathcal{P}^q$, $\mathcal{P}^l$, and $\mathcal{P}^x$, which are determined by (B.52), (B.56), (B.57), and (B.73), respectively.

In order to see how the productivity cutoffs vary across firms with different posteriors, let's consider the following case. Suppose a firm with $(a, \tilde{P}, l)$ has the equilibrium productivity cutoffs denoted by $\mathcal{P}^h(a, \tilde{P}, l)$, $\mathcal{P}^q(a, \tilde{P}, l)$, and $\mathcal{P}^l(a, \tilde{P}, l)$, following the equations (B.52), (B.56), and (B.57), respectively. Let's consider another firm having the same age a and size l, but higher average productivity $\tilde{P}$ than the focal firm. Thus, this firm has a better posterior mean, with the same posterior variance.

Now suppose that the three productivity cutoffs remain the same for this firm at the equilibrium. Since this firm has a better posterior, it is more likely to expand and less likely to contract in the next period, and thus has a higher level of $\frac{\partial \mathbb{E}\mathbf{V}^{init}(a+1, \tilde{P}', l', P')}{\partial l'}$ following the previous discussion. However, this contradicts to the equilibrium conditions for the productivity cutoffs, as the left-hand sides of (B.52), (B.56), and (B.57) become greater than the right-hand sides of the equations that remain constant. This confirms that firms having different posteriors cannot have the same productivity cutoffs.

Note that the exact level of the productivity cutoffs can only be solved numerically. However, it can still be inferred that the productivity cutoffs would be lower for the firm having a better posterior from the following. The firm with a better posterior is expected to draw higher productivity in the next period, and this increases the expected future marginal value of a labor input following the discussion in the previous section. Thus, from the equations (B.52), (B.56), and (B.57), the expected marginal future values on the left-hand side are higher for this firm, and this will require the productivity cutoffs to go down to equate with the right-hand side by lowering the spontaneous marginal product of the firm.

D.3 Implications on Hiring and Retention Margins (Result 3)

In this section, I show how workers' job prospects affect firms' decision to hire or retain workers.

D.3.1 Hiring Margin

The equation (B.59) associated with firms' optimal hiring choice can be rephrased as follows by using (D.84) and the firms' indifference curve (2.23):

$$\alpha P l'^{\alpha-1} + \beta(1-\delta)\frac{\partial}{\partial l'}\mathbb{E}\Big[(1-d')(\mathbf{J}' - \frac{c}{q(\theta(x'))}h')\Big] = x - \beta\mathbb{E}[*'] + \frac{c}{q(\theta(x))} - \beta\frac{\partial \mathbb{E}[*']}{\partial l'}l'. \tag{D.91}$$

The left-hand side of the equation shows the marginal benefit of hiring and the right-hand side of the equation is the marginal cost of hiring.

Here, workers' job prospects are associated with two terms as follows: i) workers' expected future value ($\mathbb{E}[*']$) and ii) the derivative of workers' expected value with respect to firm size ($\frac{\partial \mathbb{E}[*']}{\partial l'} l'$). First, workers' job prospects affect firms' hiring costs. Different job prospects from the perspective of workers create wage differentials conditional on the promised utility x, which are reflected in the first two terms in the second bracket. Second, workers' expected future value is further indirectly affected by new hires, which also affect hiring costs and the net marginal value of hiring. This is the last term in the second bracket.

Furthermore, firms' own uncertain prospects about themselves affect their expected future value and hiring decision under the symmetric information structure. This is reflected in the second term in the first bracket: iii) $\frac{\partial}{\partial l'} \mathbb{E}\Big[(1-d')(\mathbf{J}' - \frac{c}{q(\theta(x'))} h')\Big]$.

As seen in the previous section, workers' expected future value is higher (lower) for high (low) performing young firms with better (worse) posterior beliefs. Thus, high performing young firms end up facing higher wage differentials and hiring costs compared to otherwise similar mature firms, while low performing young firms can pay a discount. This causes high performing young firms to have a lower net marginal value of hiring, while low performing young firms have a higher marginal value of hiring, compared to otherwise similar mature firms. This effect is through the term i) due to workers' uncertain job prospects at young firms.

Furthermore, as seen in the previous section D.1, the sum of the above three terms i)-iii) is higher (lower) for firms having better (worse) posterior beliefs and thus increases (decreases) their marginal value of hiring.[68] In other words, even after considering the indirect effect of new hires in ii) and the firms' own uncertain prospects in iii), the direct effect through workers' uncertain job prospects and the consequent wage differentials remains influential. Thus, workers' uncertain job prospects can create a hiring friction for young firms with high potential, leading to employment gaps between young and mature firms.

D.3.2 Retention Margin

The retention margin is another channel through which young firms' uncertain job prospects affect firm size and growth. It is characterized by (B.61). Using the relationship (D.84) again and putting the equation differently, the following can be obtained:

$$\begin{aligned} &\alpha P l'^{\alpha-1} + \beta(1-\delta)\frac{\partial}{\partial l'}\mathbb{E}\Big[(1-d')(\mathbf{J}' - \frac{c}{q(\theta(x'))}h')\Big] \\ &\quad = \tilde{\mathbf{W}} - \mathbb{E}[*'] - \beta\frac{\partial \mathbb{E}[*']}{\partial l'}l' + \frac{f(\theta(x^E)) + f'(\theta)\theta'(x^E)(x^E - \tilde{\mathbf{W}})}{f'(\theta)\theta'(x^E)}. \end{aligned} \tag{D.92}$$

[68] The separate signs of ii) and iii) cannot be analytically derived, but only the total sign can be shown.

As before, the left-hand side of the equation indicates the marginal benefit and the right-hand side shows the marginal cost of retaining workers (by increasing a unit of promised utility to incumbent workers). Note that unlike (D.91), the marginal cost here does not include vacancy cost, but does include the marginal effect of adjusting contract offers on the previously employed workers by affecting their wages in the previous period (the last term on the right-hand side). The marginal benefit and cost are equalized at the equilibrium.

Here also, the aforementioned three terms are related to beliefs about firm type: i) workers' expected future value ($\mathbb{E}[*']$), ii) the derivative of workers' expected value with respect to firm size ($\frac{\partial \mathbb{E}[*']}{\partial l'}$), and iii) the firm's expected future marginal value $\frac{\partial}{\partial l'}\mathbb{E}\Big[(1-d')(\mathbf{J}' - \frac{c}{q(\theta(x'))}h')\Big]$, where the first two terms are related to workers' uncertain job prospects, and the last term is associated with firms own prospects. As discussed before, given the utility level $\tilde{\mathbf{W}}$ promised to incumbent workers, the wage differentials through the the term i) reduce (increase) the marginal value of high (low) performing young firms relative to heir mature counterparts. And this effect is dominant to the other effects through the terms ii) and iii) as shown in Appendix D.1. This indicates that firms' retention decision is also importantly affected by workers' uncertain job prospects and the consequent wage differentials.

Related to the retention margin, the workers' job prospects also affect the layoff decision of firms. Firms' layoff decision rule is determined by (B.60), which can be rephrased as follows:

$$\begin{aligned} &-(1-\lambda f(\theta(x^E)))l\Bigg(\alpha P l'^{\alpha-1} + \beta(1-\delta)\frac{\partial}{\partial l'}\mathbb{E}\Big[(1-d')(\mathbf{J}' - \frac{c}{q(\theta(x'))}h')\Big]\Bigg) \\ &= -(1-\lambda f(\theta(x^E)))l\Big(\tilde{\mathbf{W}} - \mathbb{E}[*'] - \beta\frac{\partial \mathbb{E}[*']}{\partial l'}l'\Big) - \mathbf{U}l + \lambda f(\theta(x^E))lx^E + (1-\lambda f(\theta(x^E)))l\tilde{\mathbf{W}}, \end{aligned} \tag{D.93}$$

where the left-hand side is the marginal benefit and the right-hand side is the marginal cost of laying off a worker. To be specific, the first term on the left-hand side shows the marginal decrease of firm size with respect to an increase in separation probability, i.e. $-(1-\lambda f(\theta(x^E))) < 0$. Thus, the left-hand side characterizes a marginal effect of laying off a worker on the firm's marginal product and expected future value through employment size change.

Similarly, the first three terms on the right-hand side present the reduction of wages for dismissed workers. The remaining terms on the right-hand side show the marginal effect of higher layoff possibility on the wages paid to workers in the previous period. This is another source of the marginal cost associated with firm layoff.

As before, the aforementioned three terms are engaged in the same direction, and the firms' layoff decision is mainly affected by workers' job prospects. More importantly, firms with better (worse) workers' job prospects now have a higher (lower) marginal cost of laying off workers and are less (more) likely lay off workers due to the negative sign of $-(1-\lambda f(\theta(x^E)))$.

D.4 Uncertainty and Job Prospects

D.4.1 Proof of Proposition 4

Proof.

$$\frac{\partial \bar{\nu}_{jt-1}}{\partial \sigma_\varepsilon^2} = \left(\frac{a_{jt}}{\sigma_\varepsilon^2 \sigma_0^2}\right) \frac{(\bar{\nu}_0 - \tilde{P}_{jt-1})}{\left(\frac{1}{\sigma_0^2} + \frac{a_{jt}}{\sigma_\varepsilon^2}\right)^2} \begin{cases} > 0 & \text{if} \quad \tilde{P}_{jt-1} < \bar{\nu}_0 \\ < 0 & \text{if} \quad \tilde{P}_{jt-1} > \bar{\nu}_0 \end{cases}$$

$$\frac{\partial \sigma_{jt-1}}{\partial \sigma_\varepsilon^2} = \left(\frac{a_{jt}}{\sigma_\varepsilon^2}\right) \frac{1}{\left(\frac{1}{\sigma_0^2} + \frac{a_{jt}}{\sigma_\varepsilon^2}\right)^2} > 0$$

$\square$

D.4.2 Proof of Proposition 5

Proof.

$$\frac{\partial}{\partial \sigma_\varepsilon^2}\left(\frac{\partial \bar{\nu}_{jt-1}}{\partial \tilde{P}_{jt-1}}\right) = -\left(\frac{a_{jt}}{\sigma_\varepsilon^4 \sigma_0^2}\right) \frac{1}{\left(\frac{1}{\sigma_0^2} + \frac{a_{jt}}{\sigma_\varepsilon^2}\right)^2} < 0$$

$\square$

D.4.3 Proof of Proposition 6

Proof. With $\frac{\sigma_\varepsilon}{\sigma_0} < 1, \forall\, a_{jt} \geq 1$

$$\frac{\partial}{\partial \sigma_\varepsilon^2}\left(\frac{\partial \bar{\nu}_{jt-1}}{\partial a_{jt}}\right) = \frac{(\tilde{P}_{jt-1} - \bar{\nu}_0)}{\sigma_\varepsilon^4 \sigma_0^2 \left(\frac{1}{\sigma_0^2} + \frac{a_{jt}}{\sigma_\varepsilon^2}\right)^3} \left(\frac{a_{jt}}{\sigma_\varepsilon^2} - \frac{1}{\sigma_0^2}\right) \begin{cases} > 0 & \text{if} \quad \tilde{P}_{jt-1} > \bar{\nu}_0 \\ < 0 & \text{if} \quad \tilde{P}_{jt-1} < \bar{\nu}_0 \end{cases}$$

$$\frac{\partial}{\partial \sigma_\varepsilon^2}\left(\frac{\partial \sigma_{jt-1}^2}{\partial a_{jt}}\right) = -\frac{\left(\frac{a_{jt}}{\sigma_\varepsilon^2} - \frac{1}{\sigma_0^2}\right)}{\sigma_\varepsilon^4 \left(\frac{1}{\sigma_0^2} + \frac{a_{jt}}{\sigma_\varepsilon^2}\right)^3} < 0.$$

$\square$

D.4.4 Proof of Corollary 1

Proof. Suppose there are two firms, firm 1 and firm 2, having the same average productivity $\tilde{P}$. Let a_1 and a_2 be the ages of firms 1 and 2, respectively, where $a_1 > a_2 \geq 1$. Also, let $\bar{\nu}_1$ and $\bar{\nu}_2$

be the posterior means for firms 1 and 2, respectively. From previous results, we have

$$\bar{\nu}_1 > \bar{\nu}_2 \quad \text{if} \quad \tilde{P} > \bar{\nu}_0$$
$$\bar{\nu}_1 < \bar{\nu}_2 \quad \text{if} \quad \tilde{P} < \bar{\nu}_0.$$

Then the following relationship holds:

$$\frac{\partial(\bar{\nu}_1 - \bar{\nu}_2)}{\partial \sigma_\varepsilon^2} = \frac{\frac{(a_1-a_2)(\tilde{P}-\bar{\nu}_0)}{\sigma_0^2\sigma_\varepsilon^4}\left(\frac{a_1a_2}{\sigma_\varepsilon^4} - \frac{1}{\sigma_0^4}\right)}{\left(\frac{1}{\sigma_0^2} + \frac{a_1}{\sigma_\varepsilon^2}\right)^2\left(\frac{1}{\sigma_0^2} + \frac{a_2}{\sigma_\varepsilon^2}\right)^2} \begin{cases} > 0 & \text{if} \quad \tilde{P} > \bar{\nu}_0 \\ < 0 & \text{if} \quad \tilde{P} < \bar{\nu}_0, \end{cases}$$

so that the gap between $\bar{\nu}_1$ and $\bar{\nu}_2$ increases in σ_ε^2. □

Appendix E Welfare Implications

In this subsection, I derive welfare implications of the model as follows.

Proposition E.1. *Given the level of uncertainty about firms' productivity type (given σ_ε and σ_0), the model's block-recursive equilibrium can be replicated by a constrained social planner's problem and thus is efficient.*

Proof. Suppose that a social planner is constrained by both of the search and information frictions as in the market economy. The social planner aims to maximize the following welfare function:

$$\begin{aligned} \max_{\substack{u_t, v_t, M_t^e, \mathbf{G}(a_{t+1}, \tilde{P}_t, l_t), \\ \mathbf{d}(a_t, \tilde{P}_{t-1}, l_{t-1}, P_t), \\ \mathbf{s}(a_t, \tilde{P}_{t-1}, l_{t-1}, P_t), \\ \mathbf{h}(a_t, \tilde{P}_{t-1}, l_{t-1}, P_t), \\ \theta(a_t, \tilde{P}_{t-1}, l_{t-1}, P_t), \\ \theta^{\mathbf{E}}(a_t, \tilde{P}_{t-1}, l_{t-1}, P_t), \\ \mathbf{l}(a_t, \tilde{P}_{t-1}, l_{t-1}, P_t), \\ \theta_t^U, d_t^e(P_t), l_t^e(P_t)}} \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \Big\{ & u_t b - c v_t \\ & + \sum_{(a_t, \tilde{P}_{t-1}, l_{t-1}, P_t), a_t \geq 1} \mathbf{G}(a_t, \tilde{P}_{t-1}, l_{t-1}) f_{(a_t, \tilde{P}_{t-1})}(P_t) \\ & \qquad * \left(1 - \mathbf{d}(a_t, \tilde{P}_{t-1}, l_{t-1}, P_t)\right) \left(P_t l_t^\alpha - c_f\right) \\ & + M_t^e \left(\sum_{P_t} f^e(P_t)(1 - d_t^e(P_t)) \left(P_t (l_t^e(P_t))^\alpha - c_f\right) - c_e \right) \Big\}, \end{aligned} \tag{E.94}$$

subject to

$$l_t = (1 - s_t)(1 - \lambda f(\theta_t^E))l_{t-1} + h_t \tag{E.95}$$

$$v_t = \theta_t^U u_t + \sum_{(a_t, \tilde{P}_{t-1}, l_{t-1}, P_t)} \lambda \theta_t^E(a_t, \tilde{P}_{t-1}, l_{t-1}, P_t) l_{t-1} \mathbf{G}(a_t, \tilde{P}_{t-1}, l_{t-1}) f_{(a_t, \tilde{P}_{t-1})}(P_t) \tag{E.96}$$

$$\begin{aligned} u_t &= (1 - f(\theta_t^U))u_{t-1} \\ &\quad + \sum_{(a_t, \tilde{P}_{t-1}, l_{t-1}, P_t)} (d_t + (1 - d_t)s_t) l_{t-1} \mathbf{G}(a_t, \tilde{P}_{t-1}, l_{t-1}) f_{(a_t, \tilde{P}_{t-1})}(P_t) \end{aligned} \tag{E.97}$$

$$\begin{aligned} \mathbf{G}(a_{t+1}, \tilde{P}_t, l_t) &= \sum_{\tilde{P}_{t-1}, l_{t-1}} \mathbf{G}(a_t, \tilde{P}_{t-1}, l_{t-1}) f_{(a_{t+1}, \tilde{P}_t)} \left((a_t + 1)\tilde{P}_t - a_t \tilde{P}_{t-1} \right) \\ &\qquad * (1 - \mathbf{d}(a_t, \tilde{P}_{t-1}, l_{t-1}, P_t)) \mathbb{I}_{l(a_t, \tilde{P}_{t-1}, l_{t-1}, P_t) = l_t} \text{ for } a_t \geq 1 \end{aligned} \tag{E.98}$$

$$\mathbf{G}(1, \tilde{P}_{t-1}, l_{t-1}) = \begin{cases} M_t^e f^e(\tilde{P}_{t-1})(1 - d^e(\tilde{P}_{t-1})), \text{ if } l_{t-1} = l_t^e(\tilde{P}) \\ 0, \text{ otherwise} \end{cases}$$

$$h_t(1 - d_t) = f(\theta_t^U)u_t \text{ for firms searching in market } \theta^U \text{ (i.e. } \theta(a_t, \tilde{P}_{t-1}, l_{t-1}, P_t) = \theta^U) \tag{E.99}$$

$$h_t(1 - d_t) = \lambda f(\theta_t^E)(1 - s_t)l_{t-1} \text{ for firms poaching from market } \theta^E \text{ (i.e. } \theta(a_t, \tilde{P}_{t-1}, l_{t-1}, P_t) = \theta^E) \tag{E.100}$$

The first line in the objective function shows the utility for unemployed workers and search cost that the social planer takes into account. The second line presents the value of operating incumbent firms, and the last line indicates the value of successful entrant firms.

Equation (E.101) can be rephrased as the following problem with an identifier j for each firm j and their birth year t_0^j:

$$\begin{aligned} \max_{\substack{u_t, v_t, M_t^e, \theta_t^U \\ \left\{ d_t^j, s_t^j, h_t^j, \theta_t^j, \theta_t^{Ej}, l_t^j \right\}_{j, a_t^j \geq 1}}} \quad & \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \Big\{ \int_j \Bigg(\Big(\prod_{\tau = t_0^j}^{t} (1 - d_\tau^j) \left(P_t^j (l_t^j)^\alpha - c_f \right) \Big) \mathbb{I}_{t_0^j < t} \\ & + \left((1 - d_t^j) \left(P_t^j (l_t^j)^\alpha - c_f \right) M_t^e - M_t^e c_e \right) \mathbb{I}_{t_0^j = t} \Bigg) dj \\ & + u_t b - c v_t \Big\}, \end{aligned} \tag{E.101}$$

subject to

$$l_t^j = (1 - s_t^j)(1 - \lambda f(\theta_t^{Ej}))l_{t-1}^j + h_t^j \tag{E.102}$$

$$v_t = \theta_t^U u_t + \int_j \left(\prod_{\tau=t_0^j}^{t} (1 - d_\tau^j)(1 - s_t^j)\lambda \theta_t^{Ej} l_{t-1}^j \right) \mathbb{I}_{t_0^j < t} dj \tag{E.103}$$

$$u_t = (1 - f(\theta_t^U))u_{t-1} + \int_j \left(\prod_{\tau=t_0^j}^{t-1} (1 - d_\tau^j)(d_t^j + (1 - d_t^j)s_t^j)l_{t-1}^j \right) \mathbb{I}_{t_0^j < t} dj \tag{E.104}$$

$$h_t^j(1 - d_t^j) = f(\theta_t^U)u_t \text{ for firm } j \text{ searching in market } \theta^U \tag{E.105}$$

$$h_t^j(1 - d_t^j) = \lambda f(\theta_t^{Ek})(1 - s_t^k)l_{t-1}^k \text{ for firm } j \text{ poaching workers in market } \theta^{Ek} \tag{E.106}$$

$$M_t^e \int_j (1 - d_t^j)\mathbb{I}_{t_0^j = t} dj = \int_j \left(\prod_{\tau=t_0^j}^{t-1} (1 - d_\tau^j) d_t^j \right) \mathbb{I}_{t_0^j < t} dj \tag{E.107}$$

Combining (E.103), (E.105), and (E.106), along with the relationship $\theta_t = \frac{f(\theta_t)}{q(\theta_t)}$ gives the following equation:

$$v_t = \int_j \left(\prod_{\tau=t_0^j}^{t} (1 - d_\tau^j) \frac{h_t^j}{q(\theta_t^j)} \right) dj, \tag{E.108}$$

where θ_t^j is the market that firm j search in, i.e. $\theta_t^j \in \{\theta_t^U, \{\theta_t^{Ek}\}_k\}$.

Then, rephrasing (E.101) by replacing l_t^j with (E.102), v_t with (E.108), and using Langrangian multipliers μ_t for (E.104) and $\eta(\theta_t^j)$ for (E.105) and (E.106), the following is obtained:

$$\begin{aligned}
\max_{\substack{u_t, M_t^e, \theta_t^U \\ \left\{d_t^j, s_t^j, h_t^j, \theta_t^j, \theta_t^{Ej}\right\}_{j, a_t^j \geq 1}}} \quad & \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \Bigg\{ \int_j \Bigg(\prod_{\tau=t_0^j}^{t-1} (1 - d_\tau^j)(1 - d_t^j) \Bigg(P_t^j \left((1 - s_t^j)(1 - \lambda f(\theta_t^{Ej}))l_{t-1}^j + h_t^j \right)^\alpha \\
& - c_f - c\frac{h_t^j}{q(\theta_t^j)} - \eta(\theta_t^j)h_t^j + \eta(\theta_t^{Ej})\lambda f(\theta_t^{Ej})(1 - s_t^j)l_{t-1}^j \\
& + \mu_t(d_t^j + (1 - d_t^j)s_t^j)l_{t-1}^j \Bigg) \mathbb{I}_{t_0^j < t} \\
& + (1 - d_t^j)\left(P_t^j h_t^j - c_f - c\frac{h_t^j}{q(\theta_t^j)} - \eta(\theta_t^j)h_t^j - c_e \right) M_t^e \mathbb{I}_{t_0^j = t} \Bigg) dj
\end{aligned}$$

$$+ u_t b - \mu_t(u_t - u_{t-1}(1 - f(\theta_t^U))) + \eta(\theta_t^U)u_{t-1}f(\theta_t^U)\Bigg\}, \quad \text{(E.109)}$$

Here, pick a competitive equilibrium U_t and $x(\theta_t^j)$ and replace $\mu_t = U_t$, $\eta_t(\theta_t^j) = x_{jt}$ s.t. $\theta_t^j = \theta(x_{jt})$, $\eta_t(\theta_t^{Ej}) = x_{jt}^E$ s.t. $\theta_t^{Ej} = \theta(x_{jt}^E)$, and $\eta_t(\theta_t^U) = x_t^U$ s.t. $\theta_t^U = \theta(x_t^U)$.

Rewriting (E.109), I have:

$$\begin{aligned}
\max_{\substack{u_t, M_t^e, \theta_t^U \\ \left\{d_t^j, s_t^j, h_t^j, \theta_t^j, \theta_t^{Ej}\right\}_{j, a_t^j \geq 1}}} \quad & \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \Bigg\{ \int_j \Bigg(\prod_{\tau=t_0^j}^{t-1} (1 - d_\tau^j)(1 - d_t^j)\Bigg(P_t^j \left((1 - s_t^j)(1 - \lambda f(\theta(x_{jt}^E))l_{t-1}^j + h_t^j\right)^\alpha \\
& - c_f - (\frac{c}{q(\theta(x_{jt}))} + x_{jt})h_t^j + x_{jt}^E(\lambda f(\theta(x_{jt}^E))(1 - s_t^j)l_{t-1}^j \\
& + U_t(d_t^j + (1 - d_t^j)s_t^j)l_{t-1}^j \Bigg)\mathbb{I}_{t_0^j < t} \\
& + \Bigg((1 - d_t^j)\Bigg(P_t^j(h_t^j)^\alpha - c_f - (\frac{c}{q\theta(x_{jt})} + x_t^j)h_t^j - c_e\Bigg)M_t^e\Bigg)\mathbb{I}_{t_0^j = t}\Bigg)dj \\
& + u_t b - U_t(u_t - u_{t-1}(1 - f(\theta_t^U))) + \eta(\theta_t^U)u_{t-1}f(\theta_t^U)\Bigg\}. \quad \text{(E.110)}
\end{aligned}$$

Note that the first three lines are equivalent to the incumbent firms' and entrants' problems in the market equilibrium. Solving the last line with respect to u_t and θ_t^U gives the following two first-order conditions:

$$b - U_t + \beta\left(U_t(1 - f(\theta_{t+1}^U)) + f(\theta_{t+1}^U)x_{t+1}(\theta_{t+1}^U)\right) = 0 \quad \text{(E.111)}$$

$$-f'(\theta_t^U)U_t + f'(\theta_t^U)x_t(\theta_t^U) + x_t'(\theta_t^U)f(\theta_t^U) = 0, \quad \text{(E.112)}$$

where (E.111) is equivalent to the unemployed workers' value function, and (E.112) is identical to their optimal choice in the competitive equilibrium. □

Therefore, this shows that we can find a solution for the constrained social planner's problem to be competitive equilibrium. In other words, under both search and information frictions, the competitive equilibrium is the first best allocation. This is consistent with standard directed search literature.

The following corollary holds under no uncertainty.

Corollary E.1. *If there is no uncertainty about the firm's productivity type ($\sigma_\varepsilon = 0$ and given σ_0), the model's decentralized block-recursive equilibrium can be replicated by a social planner's* *problem with a search friction only, and thus is efficient.*

Proof. Now we assume that the social planner can see exact firm type. Thus, the information friction is no longer existent. In that case, the social planner's problem can be written as:

$$\max_{\substack{u_t, v_t, M_t^e, g(l_t),\\ \mathbf{d}(\nu, l_{t-1}),\\ \mathbf{s}(\nu, l_{t-1}),\\ \mathbf{h}(\nu, l_{t-1}),\\ \theta(\nu, l_{t-1}),\\ \theta^{\mathbf{E}}(\nu, l_{t-1}),\\ \mathbf{l}(\nu, l_{t-1}),\\ \theta_t^U, d_t^e(\nu), l_t^e(\nu)}} \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \Bigg\{ u_t b - c v_t + \sum_{(\nu, l_{t-1})} g(l_{t-1}) f(\nu)(1 - \mathbf{d}(\nu, l_{t-1})) \left(e^{\nu} l_t^{\alpha} - c_f\right)$$

$$+ M_t^e \left(\sum_{\nu} f(\nu)(1 - d_t^e(\nu)) \left(e^{\nu} l_t^e(\nu)^{\alpha} - c_f\right) - c_e \right) \Bigg\}, \tag{E.113}$$

subject to

$$l_t = (1 - s_t)(1 - \lambda f(\theta_t^E)) l_{t-1} + h_t \tag{E.114}$$

$$v_t = \theta_t^U u_t + \sum_{(\nu, l_{t-1})} \lambda \theta_t^E(\nu, l_{t-1}) l_{t-1} g(l_{t-1}) f(\nu) \tag{E.115}$$

$$u_t = (1 - f(\theta_t^U)) u_{t-1} + \sum_{\nu, l_{t-1}} (d_t + (1 - d_t) s_t) l_{t-1} g(l_{t-1}) f(\nu) \tag{E.116}$$

$$g(l_t) = \sum_{\nu, l_{t-1}} f(\nu) g(l_{t-1})(1 - \mathbf{d}(\nu, l_{t-1}) \mathbb{I}_{l_{(\nu, l_{t-1})} = l_t} \tag{E.117}$$

$$+ \sum_{\nu} M_t^e f(\nu)(1 - d_t^e(\nu)) \mathbb{I}_{l_t^e(\nu) = l_t} \tag{E.118}$$

$$h_t(1 - d_t) = f(\theta_t^U) u_t \text{ for firms searching in market } \theta^U \text{ (i.e. } \theta(\nu, l_{t-1}) = \theta^U) \tag{E.119}$$

$$h_t(1 - d_t) = \lambda f(\theta_t^E)(1 - s_t) l_{t-1} \text{ for firms poaching from market } \theta^E \text{ (i.e. } \theta(\nu, l_{t-1}) = \theta^E) \tag{E.120}$$

Following the same trick, it is obvious to prove that the competitive equilibrium under the full information is also socially optimal as it can be replicated by the social planner's problem (E.113). □

These results verify that the model's decentralized block-recursive allocation given the level of uncertainty is socially optimal. If the planner could resolve uncertainty, the decentralized allocation would be distorted due to the uncertainty.

Appendix F Computation Algorithm

F.1 Guess $\mathbf{V}^{init}$

We start with our guess $\mathbf{V}^{init0}(a, \tilde{P}, l, P)$ for $\mathbf{V}^{init}(a, \tilde{P}, l, P)$.[69]

F.2 Use Free-entry Condition

1. Get $\mathbb{E}_{P'}\mathbf{V}^{init}(1, \ln P, l^e, P')$

   For each possible grid points for P, use $\ln P' \sim N(\frac{\frac{\bar{\nu}_0}{\sigma_0^2} + \frac{\ln P}{\sigma_\epsilon^2}}{\frac{1}{\sigma_0^2} + \frac{1}{\sigma_\epsilon}}, \frac{1}{\frac{1}{\sigma_0^2} + \frac{1}{\sigma_\epsilon}} + \sigma_\epsilon^2)$.

2. Guess κ

3. Find l^e and d^e that solves:

$$\max_{d^e, l^e}\Big[(1 - d^e)\Big(P(l^e)^\alpha - c^f - \kappa l^e + \beta\mathbb{E}_{P'}\mathbf{V}^{init0}(1, \ln P, l^e, P')\Big)\Big], \tag{F.121}$$

   for each possible P, and adjust κ with a bisection method until it satisfies

$$\int \max_{d^e, l^e}\Big[(1 - d^e)\Big(P(l^e)^\alpha - c^f - \kappa l^e + \beta\mathbb{E}_{P'}\mathbf{V}^{init0}(1, \ln P, l^e, P')\Big)\Big]dF_e(P) = c^e,$$

   where $\ln P \sim N(\bar{\nu}_0, \sigma_0^2 + \sigma_\epsilon^2)$.

F.3 Unemployed Workers' Problem

Use the solution for x^U,

$$x^U = \kappa - (c^\gamma(\kappa - \mathbf{U}))^{\frac{1}{1+\gamma}} \tag{F.122}$$

and solve a fixed-point problem for $\mathbf{U}$ from the following:

$$\mathbf{U} = b + \beta\Big((1 - f(\theta(x^U))\mathbf{U} + f(\theta(x^U))x^U\Big), \tag{F.123}$$

using (2.25).

[69] Here, for notational convenience, I will use $\tilde{P}$ and l to refer to the average log productivity and employment size in the previous period, respectively. Note that P is the current period productivity. Variables with $'$ refer to their value in the next period, i.e. $\tilde{P}'$ is the average log productivity up to the current period, l' is the current period employment size after all decisions made (for hiring, retention, and layoffs, etc.), and P' is the next period productivity.

F.4 Value Function Iteration

1. Generate $\mathbb{E}\mathbf{V}^{init0}(a+1, \tilde{P}', l', P') = \mathbb{E}\mathbf{V}^{init0}(a+1, \frac{a\tilde{P}+\ln P}{(a+1)}, l', P')$.

   Given state variables $(a, \tilde{P}, l, P)$ and $\ln P' \sim (\frac{\frac{\bar{\nu}_0}{\sigma_0^2}+\frac{a\tilde{P}+\ln P}{\sigma_\epsilon^2}}{\frac{1}{\sigma_0^2}+\frac{a+1}{\sigma_\epsilon^2}}, \frac{1}{\frac{1}{\sigma_0^2}+\frac{a+1}{\sigma_\epsilon^2}} + \sigma_\epsilon^2)$, I use the interpolation of $\mathbf{V}^{init0}$ evaluated at each $(a+1, \frac{a\tilde{P}+\ln P}{a+1}, l', P')$ and take expectation across $\ln P'$.

2. Use grid search to max $\mathcal{V}$ and obtain the argmax gridpoint l'.
   For each possible combination of l and l', given $(a, \tilde{P}, l, P)$:

   (a) Step 1: for hiring/inaction case $(l' \geq l)$

$$x^E = \kappa - c \tag{F.124}$$

$$s = 0 \tag{F.125}$$

$$h = l' - (1 - \lambda f(\theta(x^E)))l = l' - l \tag{F.126}$$

   (b) Step 2: for separation case $(l' < l)$

$$x^E = \max\Big(x_1^E, x_2^E\Big) \tag{F.127}$$

$$s = 1 - \frac{l'}{(1 - \lambda f(\theta(x^E)))l} \tag{F.128}$$

$$h = 0 \tag{F.129}$$

   where x_1^E refers to the promised utility level to incumbent workers in a firm facing both layoffs and quits, and is pinned down by the root of the following:

$$\kappa - \mathbf{U} = c\Big((1 + \theta(x^E)^\gamma)^{1+\frac{1}{\gamma}} - \lambda\theta(x^E)^{1+\gamma}\Big), \tag{F.130}$$

   and x_2^E refers to that in a firm having quits only, and is the root of the following:

$$\frac{l - l'}{\lambda l} = f(\theta(x^E)) = \Big(1 - (\frac{\kappa - x^E}{c})^{-\gamma}\Big)^{\frac{1}{\gamma}} \tag{F.131}$$

$$x^E = \kappa - c\Big(1 - (\frac{l - l'}{\lambda l})^\gamma\Big)^{-\frac{1}{\gamma}}$$

   Thus, from the above steps, we have

$$\mathbf{x^E}(a, \tilde{P}, l, P, l'), \mathbf{s}(a, \tilde{P}, l, P, l'), \mathbf{h}(a, \tilde{P}, l, P, l') \tag{F.132}$$

and

$$\tilde{\mathbf{W}}(a,\tilde{P},l,P,l') = \kappa - (\kappa - x^E(a,\tilde{P},l,P,l'))^{1+\gamma}c^{-\gamma} \tag{F.133}$$

for each possible set of (l,l') and the state variables.

Using it, we find a gridpoint l' that solves the following maximization:

$$\mathcal{V}(a,\tilde{P},l,P) \equiv \max_{l'} \mathbf{s}(a,\tilde{P},l,P,l')\mathbf{U}l + Pl'^{\alpha} - c^f - \kappa\mathbf{h}(a,\tilde{P},l,P,l')$$
$$+ (1-\mathbf{s}(a,\tilde{P},l,P,l'))\lambda f(\theta(\mathbf{x}^{\mathbf{E}}(a,\tilde{P},l,P,l')))x^E(a,\tilde{P},l,P,l')l + \beta\mathbb{E}\mathbf{V}^{init0}(a+1,\frac{a\tilde{P}+\ln P}{a+1},l',P'). \tag{F.134}$$

3. Spline approximation for l'

   Let I be the optimal index for l' that maximizes $\mathcal{V}$, given $(a,\tilde{P},l,P)$. Now, we would like to spline approximate $\mathcal{V}$ across the points l_{I-1}, l_I, and l_{I+1} to get a proper policy function.

   (a) Step 1: use the spline approximated form of $\mathcal{V}$

   $$\mathcal{V} \ = \mathcal{V}_i(l) \quad \text{if} \quad l_i \leq l \leq l_{i+1}$$

   where

   $$\begin{aligned}\mathcal{V}_i(l) &= a_i(l-l_i)^3 + b_i(l-l_i)^2 + c_i(l-l_i) + \mathcal{V}_i(l_i)\\ \mathcal{V}_i'(l) &= 3a_i(l-l_i)^2 + 2b_i(l-l_i) + c_i\\ \mathcal{V}_i''(l) &= 6a_i(l-l_i) + 2b_i.\end{aligned}$$

   (b) Conditions to use

   $$\begin{aligned}\mathcal{V}_i(l_i) &= \mathcal{V}_{i-1}(l_i)\\ \mathcal{V}_i'(l_i) &= \mathcal{V}_{i-1}'(l_i)\\ \mathcal{V}_i''(l_i) &= \mathcal{V}_{i-1}''(l_i)\end{aligned}$$

   $\rightarrow$ Using the functional form for $\mathcal{V}_i$ above, these conditions are rephrased as follows:

   $$\Delta\mathcal{V}_i(l_i) = a_{i-1}(l_i-l_{i-1})^3 + b_{i-1}(l_i-l_{i-1})^2 + c_{i-1}(l_i-l_{i-1}) \tag{F.135}$$
   $$c_i = 3a_{i-1}(l_i-l_{i-1})^2 + 2b_{i-1}(l_i-l_{i-1})^2 + c_{i-1} \tag{F.136}$$
   $$2b_i = 6a_{i-1}(l_i-l_{i-1}) + 2b_{i-1} \tag{F.137}$$

(c) Generate coefficient matrix

We can convert (F.135), (F.136), and (F.137), for $i = 2, 3, ..., N$ (N is the number of l grid points), into a matrix form. Let

$$Coeff = \begin{pmatrix} a_1 & b_1 & c_1 & ... & a_{N-1} & b_{N-1} & c_{N-1} \\ ... & ... & ... & ... & ... & ... & ... \\ ... & ... & ... & ... & ... & ... & ... \end{pmatrix}. \tag{F.138}$$

Then, we could get this by

$$Coeff = DV * inv(H), \tag{F.139}$$

where

$$H = \begin{pmatrix}
(l_2-l_1)^3 & 0 & 0 & ... & 0 & 3(l_2-l_1)^2 & 0 & ... & 0 & 6(l_2-l_1) & 0 & ... & 0 & 0 & 0 \\
(l_2-l_1)^2 & 0 & 0 & ... & 0 & 2(l_2-l_1) & 0 & ... & 0 & 2 & 0 & ... & 0 & 0 & 0 \\
(l_2-l_1) & 0 & 0 & ... & 0 & 1 & 0 & ... & 0 & 0 & 0 & ... & 0 & 1 & 0 \\
0 & (l_3-l_2)^3 & 0 & ... & 0 & 0 & 3(l_3-l_2)^2 & ... & 0 & 0 & 6(l_3-l_2) & ... & 0 & 0 & 0 \\
0 & (l_3-l_2)^2 & 0 & ... & 0 & 0 & 2(l_3-l_2) & ... & 0 & -2 & 2 & ... & 0 & 0 & 0 \\
0 & (l_3-l_2) & 0 & ... & 0 & -1 & 1 & ... & 0 & 0 & 0 & ... & 0 & 0 & 0 \\
... & ... & ... & ... & ... & 0 & 0 & ... & 0 & 0 & 0 & ... & 0 & 0 & 0 \\
... & ... & ... & ... & ... & 0 & 0 & ... & 0 & 0 & -2 & ... & 0 & 0 & 0 \\
... & ... & ... & ... & ... & 0 & -1 & ... & 0 & 0 & 0 & ... & 0 & 0 & 0 \\
... & ... & ... & ... & ... & ... & ... & ... & ... & ... & ... & ... & ... & 0 & 0 \\
... & ... & ... & ... & ... & ... & ... & ... & 1 & 0 & 0 & ... & 0 & 0 & 0 \\
0 & 0 & 0 & ... & (l_N-l_{N-1})^3 & 0 & ... & ... & 0 & 0 & 0 & ... & 0 & 0 & 3(l_N-l_{N-1})^2 \\
0 & 0 & 0 & ... & (l_N-l_{N-1})^2 & 0 & ... & ... & 0 & 0 & 0 & ... & -2 & 0 & 2(l_N-l_{N-1}) \\
0 & 0 & 0 & ... & (l_N-l_{N-1}) & 0 & ... & ... & -1 & 0 & 0 & ... & 0 & 0 & 1
\end{pmatrix}$$

and

$$DV = \begin{pmatrix}
\Delta\mathcal{V}(l_2) & \Delta\mathcal{V}(l_3) & 0 & ... & \Delta\mathcal{V}(l_N) & 0 & 0 & ... & 0 & 0 & \frac{\Delta\mathcal{V}(l_2)}{l_2-l_1} & \frac{\Delta\mathcal{V}(l_N)}{l_N-l_{N-1}} \\
\Delta\mathcal{V}(l_2) & \Delta\mathcal{V}(l_3) & 0 & ... & \Delta\mathcal{V}(l_N) & 0 & 0 & ... & 0 & 0 & \frac{\Delta\mathcal{V}(l_2)}{l_2-l_1} & \frac{\Delta\mathcal{V}(l_N)}{l_N-l_{N-1}} \\
\Delta\mathcal{V}(l_2) & \Delta\mathcal{V}(l_3) & 0 & ... & \Delta\mathcal{V}(l_N) & 0 & 0 & ... & 0 & 0 & \frac{\Delta\mathcal{V}(l_2)}{l_2-l_1} & \frac{\Delta\mathcal{V}(l_N)}{l_N-l_{N-1}} \\
... & ... & ... & ... & ... & ... & ... & ... & ... & ... & ... & ... \\
\Delta\mathcal{V}(l_2) & \Delta\mathcal{V}(l_3) & 0 & ... & \Delta\mathcal{V}(l_N) & 0 & 0 & ... & 0 & 0 & \frac{\Delta\mathcal{V}(l_2)}{l_2-l_1} & \frac{\Delta\mathcal{V}(l_N)}{l_N-l_{N-1}}
\end{pmatrix}$$

where the number of each matrix is the same as $3 * (N - 1)$, and the number of rows in $Coeff$ and DV is $(na * n\tilde{P} * N * nP)$, and each row is for each pair of state variables $(a, \tilde{P}, l, P')$.

(d) Get the root of l'

Once we have $Coeff$, we derive the root of l' from each $\mathcal{V}_{I-1}$ and $\mathcal{V}_I$. This means to find l', such that

$$\mathcal{V}'_{I-1}(l) = a_{I-1}(l - l_{I-1})^2 + b_{I-1}(l - l_{I-1}) + c_{I-1} = 0$$

and

$$\mathcal{V}_I'(l) = a_I(l - l_I)^2 + b_I(l - l_I) + c_I = 0$$

Thus, we have four possible roots of l' from the spline approximation:

$$l' = \Big[\frac{-B_{I-1} \pm \sqrt{B_{I-1}^2 - 4A_{I-1}C_{I-1}}}{2A_{I-1}}, \frac{-B_I \pm \sqrt{B_I^2 - 4A_IC_I}}{2A_I}\Big] \tag{F.140}$$

where

$$\begin{aligned} A_i &= 3a_i \\ B_i &= 2b_i - 6a_il_i \\ C_i &= 3a_il_i^2 + 2b_il_i + c_i, \quad for \quad i \in \{I-1, I\} \end{aligned}$$

(e) Evaluate $\mathcal{V}$ and the corresponding policy function l'

We evaluate

$$\max[\mathcal{V}(l_1'), \mathcal{V}(l_2'), \mathcal{V}(l_3'), \mathcal{V}(l_4'), \mathcal{V}],$$

and obtain

$$l'(a, \tilde{P}, l, P) = argmax[\mathcal{V}(l_1'), \mathcal{V}(l_2'), \mathcal{V}(l_3'), \mathcal{V}(l_4'), \mathcal{V}]. \tag{F.141}$$

Note that $l_1' \sim l_4'$ are the roots based on (F.140), and the first $\mathcal{V}(l_1') \sim \mathcal{V}(l_4')$ are spline approximated $\mathcal{V}$ evaluated at each root, and the last $\mathcal{V}$ is the maximized value from the grid search.

(f) Managing inaction ranges

For the inaction range, such that $l_I(a, \tilde{P}, l, P) = l$, we don't use spline approximation for $\mathcal{V}(a, \tilde{P}, l, P)$.

4. Policy functions

We use (F.132) and (F.141) to back out policy functions for

$$\begin{aligned} \mathbf{x^E}(a, \tilde{P}, l, P) &\equiv \mathbf{x^E}(a, \tilde{P}, l, P, l') \\ \mathbf{s}(a, \tilde{P}, l, P) &\equiv \mathbf{s}(a, \tilde{P}, l, P, l') \\ \mathbf{h}(a, \tilde{P}, l, P) &\equiv \mathbf{h}(a, \tilde{P}, l, P, l'), \end{aligned}$$

and

$$\mathbf{d}(a, \tilde{P}, l, P) = \begin{cases} 1 & \text{if} \quad \mathbf{U}l > \mathcal{V}(a, \tilde{P}, l, P) \\ 0 & \text{otherwise.} \end{cases} \tag{F.142}$$

5. Update the Guess

$$\mathbf{V}^{init1}(a, \tilde{P}, l, P) = \Big(\delta + (1-\delta)\mathbf{d}(a, \tilde{P}, l, P)\Big)\mathbf{U}l + (1-\delta)(1-\mathbf{d}(a, \tilde{P}, l, P))\mathcal{V}(a, \tilde{P}, l, P) \tag{F.143}$$

If $|\mathbf{V}^{init0} - \mathbf{V}^{init1}| < \epsilon$, with sufficiently small ϵ, then it's done! Otherwise, replace $\mathbf{V}^{init0}$ with a new guess $\mathbf{V}^{init1}$ and reiterate from the part B.2.

Appendix G Figures for Low performing Firms

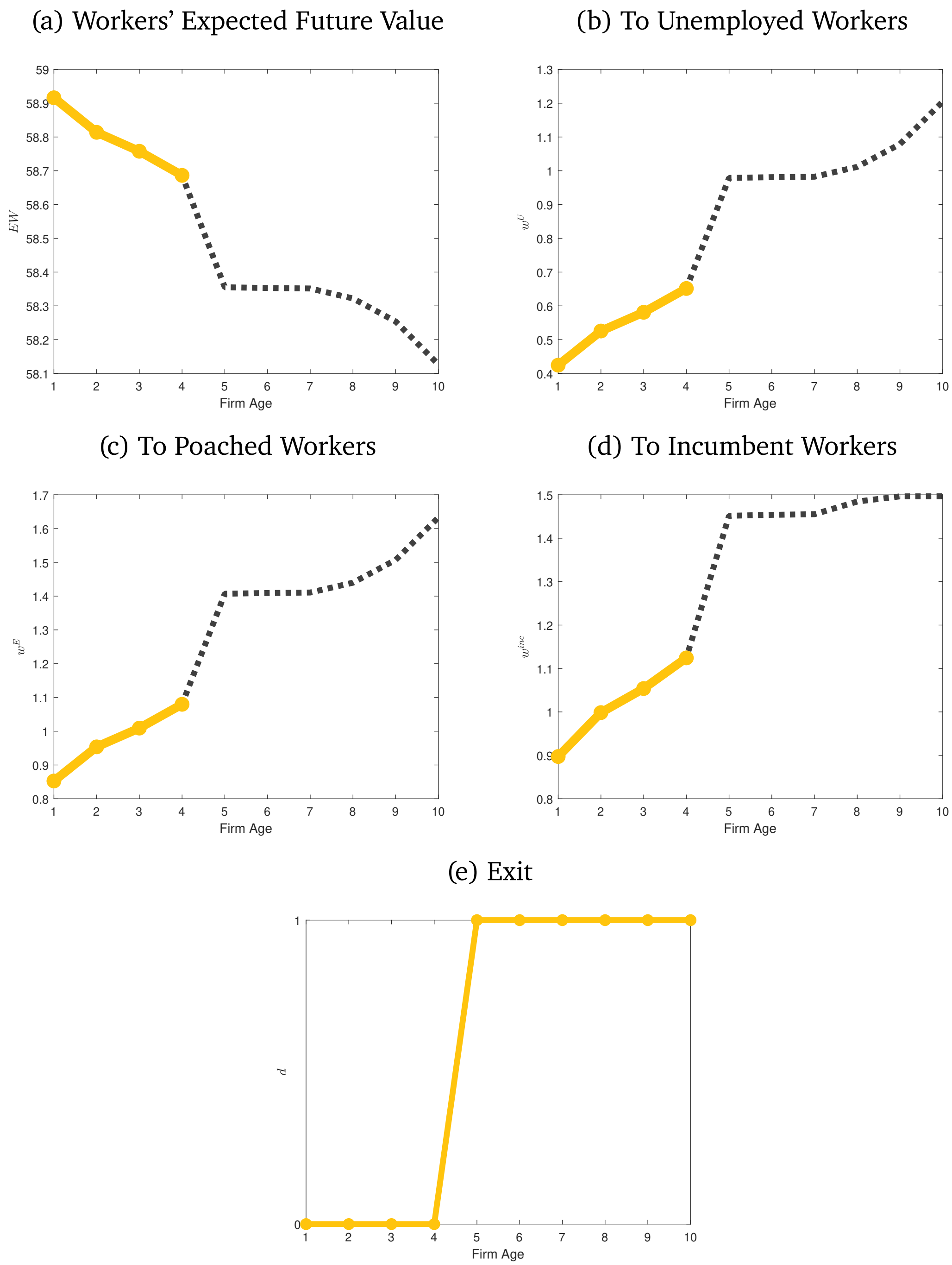

Figure G.1: Low Performing Firms (average size)[70]

[70]The dotted grey lines indicate counterfactual series if firms continued operating.

Figure G.2: Low Performing Firms: Baseline vs. Counterfactual (higher uncertainty)[71]

Appendix H Full Tables

Table H7: Wage Differentials for Young Firms

	(1) Earnings Residuals	(2) Earnings Residuals
Young firm	-0.002***	-0.003***
	(0.001)	(0.001)
Young firm × High performing firm	0.015***	0.016***
	(0.001)	(0.001)
High performing firm	0.002	0.002
	(0.001)	(0.001)
Average Firm Productivity (up to $t-1$)	0.009***	0.012***
	(0.001)	(0.001)
Current Productivity (at t)	0.020***	0.015***
	(0.001)	(0.001)
Firm Size (at t)	0.017***	
	(0.001)	
Firm Size (at $t-1$)		0.013***
		(0.001)
Previous Employer (AKM)	0.267***	0.270***
	(0.001)	(0.001)
Observations	50,170,000	50,170,000
Fixed effects	Industry, State	Industry, State

Notes: The table reports results for regression of earning residuals on young firm and high performing firm indicators. Firm controls include cross-time average productivity level, current productivity level, and log employment size. Controls associated with worker's previous employment status are AKM firm fixed effect associated with the previous employer and a dummy for non-employed workers in the previous period. Observation counts are rounded to the nearest 10,000 to avoid potential disclosure risks. Estimates for constant, industry, state fixed effects, the coefficient of the indicator for worker's previous non-employment status are suppressed. Observations are unweighted. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

[71] The dotted grey lines indicate counterfactual series if firms continued operating.

Table H8: The Effect of Uncertainty on Young Firms' Wage Differentials

	(1) Earnings Residuals	(2) Earnings Residuals
Young firm	-0.001	-0.002
	(0.002)	(0.002)
Young firm × High performing firm	0.003	0.005**
	(0.002)	(0.002)
Young firm × Uncertainty	-0.005**	-0.004*
	(0.002)	(0.002)
Young firm × High performing firm × Uncertainty (at $t-1$)	0.016***	0.015***
	(0.003)	(0.003)
Uncertainty (at $t-1$)	-0.067***	-0.071***
	(0.002)	(0.002)
High performing firm	0.004**	0.003*
	(0.002)	(0.002)
Average Firm Productivity (up to $t-1$)	0.009***	0.011***
	(0.000)	(0.000)
Current Productivity (at t)	0.020***	0.016***
	(0.000)	(0.000)
Firm Size (at t)	0.012***	
	(0.000)	
Firm Size (at $t-1$)		0.010***
		(0.000)
Previous Employer (AKM)	0.269***	0.272***
	(0.000)	(0.000)
Observations	50,170,000	50,170,000
Fixed effects	Sector, State	Sector, State

Notes: The table reports results for regression of earning residuals on young firm, high performing firm indicators, and the uncertainty measure. Firm controls include cross-time average productivity level, current productivity level, and log employment size. Controls associated with worker's previous employment status are AKM firm fixed effect and a dummy for non-employed workers in the previous period, associated with the previous employer to capture time-varying components. Observation counts are rounded to the nearest 10,000 to avoid potential disclosure risks. Estimates for constant, sector and state fixed effects, and the coefficient of the indicator for worker's previous non-employment status are suppressed. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Appendix I Robustness Checks for Regressions

Table I9: Wage Differentials for Young Firms (without firm size)

	(1) Earnings Residuals	(2) Earnings Residuals
Young firm	-0.006***	-0.007***
	(0.001)	(0.001)
Young firm $\times$ High performing firm	0.013***	0.015***
	(0.001)	(0.001)
High performing firm	0.005***	0.004***
	(0.001)	(0.001)
Average Firm Productivity (up to $t-1$)	0.016***	0.006***
	(0.001)	(0.001)
Current Productivity (at t)		0.015***
		(0.001)
Previous Employer (AKM)	0.283***	0.281***
	(0.001)	(0.001)
Observations	50,170,000	50,170,000
Fixed effects	Industry, State	Industry, State

Notes: The table reports results for regression of earning residuals on young firm and high performing firm indicators. Firm controls include cross-time average productivity level and current productivity level. Controls associated with worker's previous employment status are AKM firm fixed effect associated with the previous employer and a dummy for non-employed workers in the previous period. Observation counts are rounded to the nearest 10,000 to avoid potential disclosure risks. Estimates for constant, industry, state fixed effects, the coefficient of the indicator for worker's previous non-employment status are suppressed. Observations are unweighted. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table I10: Compensating Differentials for Young Firms (propensity score weighted)

	(1) Earnings Residuals	(2) Earnings Residuals	(3) Earnings Residuals	(4) Earnings Residuals
Young firm	-0.007***	-0.008***	-0.003***	-0.003***
	(0.001)	(0.001)	(0.001)	(0.001)
Young firm × High performing firm	0.015***	0.018***	0.019***	0.019***
	(0.001)	(0.001)	(0.001)	(0.001)
High performing firm	0.004***	0.002*	-0.000	0.000
	(0.001)	(0.001)	(0.001)	(0.001)
Average Firm Productivity	0.017***	0.003***	0.005***	0.009***
(up to $t-1$)	(0.001)	(0.001)	(0.001)	(0.001)
Current Productivity (at t)		0.021***	0.027***	0.021***
		(0.001)	(0.001)	(0.001)
Firm Size			0.020***	
			(0.000)	
Firm Size (at $t-1$)				0.015***
				(0.000)
Previous Employer (AKM)	0.281***	0.278***	0.266***	0.269***
	(0.001)	(0.001)	(0.001)	(0.001)
Observations	50,170,000	50,170,000	50,170,000	50,170,000
Fixed effects	Industry, State	Industry, State	Industry, State	Industry, State

Notes: The table reports results for regression of earning residuals on young firm and high performing firm indicators. Firm controls include cross-time average productivity level, current productivity level, and log employment size. Controls associated with worker's previous employment status are AKM firm fixed effect associated with the previous employer and a dummy for non-employed workers in the previous period. Observation counts are rounded to the nearest 10,000 to avoid potential disclosure risks. Estimates for constant, industry, state fixed effects, the coefficient of the indicator for worker's previous non-employment status are suppressed. Observations are weighted with inverse propensity score weights of author's own construction. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table I11: Wage Differentials for Young Firms (bootstrapped standard errors)

	(1) Earnings Residuals	(2) Earnings Residuals	(3) Earnings Residuals
Young firm	-0.006***	-0.007***	-0.002***
	(0.001)	(0.001)	(0.001)
Young firm $\times$ High performing firm	0.013***	0.015***	0.015***
	(0.002)	(0.002)	(0.002)
High performing firm	0.005***	0.004*	0.002
	(0.002)	(0.002)	(0.002)
Average Firm Productivity (up to $t-1$)	0.016***	0.006***	0.009***
	(0.000)	(0.001)	(0.001)
Current Productivity (at t)		0.015***	0.020***
		(0.001)	(0.001)
Firm Size			0.017***
			(0.000)
Previous Employer (AKM)	0.283***	0.281***	0.267***
	(0.001)	(0.001)	(0.001)
Observations	50,170,000	50,170,000	50,170,000
Fixed effects	Industry, State	Industry, State	Industry, State

Notes: The table reports results for regression of earning residuals on young firm and high performing firm indicators by using bootstrapped standard errors. Firm controls include cross-time average productivity level, current productivity level, and log employment size. Controls associated with worker's previous employment status are AKM firm fixed effect associated with the previous employer and a dummy for non-employed workers in the previous period. Note that the only difference from the main table is the standard errors. Observation counts are rounded to the nearest 10,000 to avoid potential disclosure risks. Estimates for constant, industry, state fixed effects, the coefficient of the indicator for worker's previous non-employment status are suppressed. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table I12: Wage Differentials for Young Firms (with previous earnings)

	(1) Earnings Residuals	(2) Earnings Residuals	(3) Earnings Residuals	(4) Earnings Residuals	(5) Earnings Residuals	(6)	(7)
Young firm	-0.003***	-0.003***	-0.004***	-0.005***	-0.005***	-0.000	-0.001
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
Young firm × High performing firm	0.014***	0.014***	0.016***	0.018***	0.018***	0.019***	0.019***
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
High performing firm	0.001***	0.001***	-0.004	-0.010***	-0.010***	-0.012***	-0.012***
	(0.002)	(0.002)	(0.001)	(0.002)	(0.002)	(0.001)	(0.001)
Average Firm Productivity (up to $t-1$)	0.006***	0.003***	0.006***	-0.009***	-0.007***	-0.005***	-0.002***
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
Current Productivity (at t)		0.005***	0.012***		-0.003***	0.003***	0.000
		(0.001)	(0.001)		(0.001)	(0.001)	(0.001)
Firm Size (at t)			0.028***			0.018***	
			(0.001)			(0.000)	
Firm Size (at $t-1$)							0.014***
							(0.000)
Previous Employer (AKM)				0.155***	0.155***	0.141***	0.160***
				(0.001)	(0.001)	(0.001)	(0.001)
Previous Earnings	0.194***	0.194***	0.190***	0.167***	0.167***	0.167***	0.165***
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
Observations	50,170,000	50,170,000	50,170,000	50,170,000	50,170,000	50,170,000	50,170,000
Fixed effects	Industry, State	Industry, State	Industry, State	Industry, State	Industry, State	Industry, State	Industry, State

Notes: The table reports results for regression of earning residuals on young firm and high performing firm indicators. Firm controls include cross-time average productivity level, current productivity level, and log employment size. Controls associated with worker's previous employment status are previous earning level (in all columns) along with AKM firm fixed effect associated with the previous employer and a dummy for non-employed workers in the previous period (in the last three columns). Observation counts are rounded to the nearest 10,000 to avoid potential disclosure risks. Estimates for constant, industry, state fixed effects, the coefficient of the indicator for worker's previous non-employment status are suppressed. Observations are weighted with inverse propensity score weights of author's own construction. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table I13: Aggregate Implications of Uncertainty (current value)

	(1) Entry rate	(2) Young firm share	(3) Young firm emp. share
Uncertainty (at t)	-0.009***	-0.013***	-0.013***
	(0.002)	(0.005)	(0.005)
Observations	4,300	4,300	4,300
Fixed effects	Industry, Year	Industry, Year	Industry, Year
	(4) HG young firm share	(5) HG young firm emp.share	(6) HG young firm avg. emp. growth
Uncertainty (at t)	-0.010***	-0.008***	-0.020***
	(0.003)	(0.002)	(0.005)
Observations	4,300	4,300	4,300
Fixed effects	Industry, Year	Industry, Year	Industry, Year

Notes: The table reports results for regression of young firm activities in each column on the contemporary value of the uncertainty at the industry level. Observation counts are rounded to the nearest 100 to avoid potential disclosure risks. Estimates for constant, industry and year fixed effects are suppressed. Observations are unweighted. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.