Online Appendix for
"Workers' Job Prospects and Young Firm Dynamics"*

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A Proofs of Lemmas and Propositions

Proof of Proposition 1 . With Lemma 1, the promise-keeping constraints (2.11) and (2.12) can be rephrased in terms of the current wage w for each new hires and incumbent workers:

$$\mathbf{w} = \mathbf{x} - \beta\mathbb{E}\Bigg[\Big(\delta + \big(1-\delta\big)\big(\mathbf{d}' + (1-\mathbf{d}')\mathbf{s}'\big)\Big)\mathbf{U} \tag{A.1}$$

$$+ (1-\delta)(1-\mathbf{d}')(1-\mathbf{s}')\Big(\lambda f(\theta(\mathbf{x}^{\mathbf{E}'}))\mathbf{x}^{\mathbf{E}'} + \big(1-\lambda f(\theta(\mathbf{x}^{\mathbf{E}'}))\big)\tilde{\mathbf{W}}'\Big)\Bigg] \text{ for new hires,}$$

$$\mathbf{w} = \tilde{\mathbf{W}} - \beta\mathbb{E}\Bigg[\Big(\delta + \big(1-\delta\big)\big(\mathbf{d}' + (1-\mathbf{d}')\mathbf{s}'\big)\Big)\mathbf{U} \tag{A.2}$$

$$+ (1-\delta)(1-\mathbf{d}')(1-\mathbf{s}')\Big(\lambda f(\theta(\mathbf{x}^{\mathbf{E}'}))\mathbf{x}^{\mathbf{E}'} + \big(1-\lambda f(\theta(\mathbf{x}^{\mathbf{E}'}))\big)\tilde{\mathbf{W}}'\Big)\Bigg] \text{ for incumbents,}$$

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where the first term on the right hand side of (A.1) and (A.2) shows the promised utility level for each type of worker, which in equilibrium is determined by the worker's outside options and depends on the worker's previous employment status.[1] The term in large brackets on the right hand side refers to workers' expected future value at a given firm, which depends on their posterior beliefs about firm type. Note that workers' expected future value is identical across all workers a given firm, as they share the same information about the firm. □

Proof of Lemma 2 . The proof follows the following three steps.

1. Workers' Problem: first, solving the workers' problems, the optimal choice of markets by unemployed and employed workers is as follows:

$$x^U = \kappa - (c^\gamma(\kappa - \mathbf{U}))^{\frac{1}{1+\gamma}} \text{ for unemployed workers,} \tag{A.3}$$

$$x^E_j(a_j, \tilde{P}_{j-1}, l_{j-1}, P_j) = \kappa - (c^\gamma(\kappa - \tilde{\mathbf{W}}(a_j, \tilde{P}_{j-1}, l_{j-1}, P_j)))^{\frac{1}{1+\gamma}} \text{ for employed workers at } j. \tag{A.4}$$

This shows that the market unemployed workers search in x^U is constant with respect to firms' state variables. This is because unemployed workers have no heterogeneity (both ex-ante and ex-post) and thus all choose the same market to search.

On the other hand, employed workers direct their on-the-job search to x^E_j which depends on the promised utility $\tilde{\mathbf{W}}(a_j, \tilde{P}_{j-1}, l_{j-1}, P_j)))$ offered by the current employer. In other words, their on-the-job choice depends on their opportunity cost of moving to other firms, which is a function of the current employer's state variables. Also, x^E_j is increasing in the workers' opportunity cost $\tilde{\mathbf{W}}_\mathbf{j}$, which means that the higher utility $\tilde{\mathbf{W}}_\mathbf{j}$ workers receive from their current employer, the higher utility x^E_j another firm needs to deliver to poach them successfully. In other words, workers only climb up to a labor market that provides higher utility than what they currently have, which captures the standard job ladder property in a directed search

[1] $\mathbf{x}$ is pinned down by the equilibrium submarket choices (A.3) and (A.4) for each unemployed and poached worker, and $\tilde{\mathbf{W}}$ is the total utility level firms promise to incumbent workers in equilibrium.

framework.

Notably from (A.3) and (A.4), firms' promised utility to both unemployed and employed workers in the search market does not depend on recruiting firms' characteristics, but rather only on workers' employment status. In other words, workers are not indifferent across active submarkets, and search in a specific submarket that provides a certain promised utility (at least equal to or above their outside options) upon successful job match, while firms are indifferent across active submarkets in equilibrium.

2. Firms' Problem and Joint Surplus Maximization: solving the firms' problem, the value function (2.7) can be fully replicated by the following joint surplus maximization:

$$\mathbf{V}^{prod}(a_j, \tilde{P}_{j-1}, l_j, P_j) = \max_{d'_j, s'_j, x'_j, x^{E\prime}_j, h'_j} P_j l_j^{\alpha} - c_f + \beta \mathbb{E}_j \Bigg[(1-\delta)(1-d'_j)\Big(\mathbf{V}^{prod}(a'_j, \tilde{P}_j, l'_j, P'_j) - \Big(x'_j + \frac{c}{q(\theta(x'_j))}\Big)h'_j + (1-s'_j)\lambda f(\theta(x^{E\prime}_j))x^{E\prime}_j l_j\Big) + \Big(\delta + (1-\delta)\big(d'_j + (1-d'_j)s'_j\big)\Big)\mathbf{U}' l_j \Bigg], \tag{A.5}$$

where $\mathbf{V}_j^{prod} \equiv \mathbf{J}_j^{prod} + x_j h_j + \tilde{\mathbf{W}}_{\mathbf{j}}(1-s_j)(1-\lambda f\big(\theta(x_j^E)\big))l_{j-1}$, $\mathbf{J}_j^{prod}$ is the firm value function at the production stage after search and matching, and Ω_j^{-w} denotes the contract abstracting from the wage $\{w_j(i)\}_i$.

Given that choice variables are contingent on future productivity, it can be transformed with the following value function defined at the beginning of each period:

$$\mathbf{V}_j^{init}(a_j, \tilde{P}_{j-1}, l_{j-1}, P_j) = \max_{d_j, s_j h_j, x_j^E} \delta \mathbf{U} l_{j-1} + (1-\delta)(d_j + (1-d_j)s_j)\mathbf{U} l_{j-1} + (1-\delta)(1-d_j)\Big(P_j l_j^{\alpha} - c^f - \kappa h_j + (1-s_j)\lambda f(\theta(x_j^E))x_j^E l_{j-1} + \beta \mathbb{E}_j \mathbf{V}_j^{init}(a'_j, \tilde{P}_j, l_j, P'_j)\Big).$$

Note that the first term $\delta \mathbf{U} l_{j-1}$ is independent of the variables to maximize and $(1-\delta)$ in the remaining two terms just scales the objective function. I first solve the problem for s_j, h_j, and x_j^E, maximizing

$$\max_{s_j,h_j,x_j^E} s_j\mathbf{U}l_{j-1} + P_j l_j^\alpha - c^f - \kappa h_j + (1-s_j)\lambda f(\theta(x_j^E))x_j^E l_{j-1} + \beta\mathbb{E}_j\mathbf{V}_j^{init}(a_j', \tilde{P}_j, l_j, P_j'). \tag{A.6}$$

And then $d_j = 1$ if $\mathbf{U}l_{j-1}$ is greater than the value (A.6), and $d_j = 0$, otherwise.

In a similar fashion, the free-entry condition (2.13) can be rephrased as follows:

$$\int \max_{d_j^e, l_j^e}(1-d_j^e)\Big(P_j(l_j^e)^\alpha - c^f - \kappa l_j^e + \beta\mathbb{E}_j\mathbf{V}^{init}(1, \ln P_j, l_j^e, P_j')\Big)dF_e(P_j) - c^e = 0. \tag{A.7}$$

There are four endogenous cutoffs for the current productivity draw P among operating firms: i) the upper cutoff $\mathcal{P}^h(a, \tilde{P}_{-1}, l_{-1})$ between hiring versus inaction with no quits, ii) the middle cutoff $\mathcal{P}^q(a, \tilde{P}_{-1}, l_{-1})$ between inaction with no quits versus inaction with quits, iii) the lower cutoff $\mathcal{P}^l(a, \tilde{P}_{-1}, l_{-1})$ between quits only versus quits and layoffs, and iv) the exit cutoff $\mathcal{P}^x(a, \tilde{P}_{-1}, l_{-1})$ below which firms endogenously exit. These cutoffs are generated due to the vacancy cost and operating fixed cost and endogenously determined by the beginning-of-period state variables $(a, \tilde{P}_{-1}, l_{-1})$ before the current productivity draw P.

Note that there is no case in which firms hire and separate workers at the same time. In other words, if $s_j > 0$, then $h_j = 0$ should hold, and if $h_j > 0$, then $s_j = 0$. To prove this, suppose that firms both hire and separate workers, e.g. $h_j > 0$ and $s_j > 0$. In the maximization (A.6), the first-order conditions with respect to h_j, s_j, and x_j^E are as follows (in the same order):

$$\left[\alpha P_j l_j^{\alpha-1} + \beta\frac{\partial\mathbb{E}_j{\mathbf{V}_j}^{init\prime}}{\partial l_j}\right] = \kappa, \tag{A.8}$$

$$\mathbf{U}l_{j-1} - \lambda f(\theta(x_j^E))x_j^E l_{j-1} - (1-\lambda f(\theta(x_j^E)))l_{j-1}\Big[\alpha P_j l_j^{\alpha-1} + \beta \frac{\partial \mathbb{E}_j {\mathbf{V}_j}^{init\prime}}{\partial l_j}\Big] = 0, \tag{A.9}$$

$$\lambda f'(\theta(x_j^E))\theta'(x_j^E)x_j^E l_{j-1} + \lambda f(\theta(x_j^E))l_{j-1} - \lambda f'(\theta(x_j^E))\theta'(x_j^E)l_{j-1}\Big[\alpha P_j l_j^{\alpha-1} + \beta \frac{\partial \mathbb{E}_j {\mathbf{V}_j}^{init\prime}}{\partial l_j}\Big] = 0. \tag{A.10}$$

Using (A.8) to substitute out the term $\Big[\alpha P_j l_j^{\alpha-1} + \beta \frac{\partial \mathbb{E}_j {\mathbf{V}_j}^{init\prime}}{\partial l_j}\Big]$ in (A.10), and using (C.39), I can rewrite the left-hand side of (A.10) as follows:

$$\frac{(\kappa - x_j^E)^\gamma c^{-\gamma}\Big((\frac{\kappa - x_j^E}{c})^\gamma - 1\Big)^{\frac{1}{\gamma}}(\frac{\kappa - x_j^E}{c})^\gamma}{(\frac{\kappa - x_j^E}{c})^\gamma - 1} = \frac{\Big((\kappa - x_j^E)^\gamma c^{-\gamma}\Big)^2}{\Big((\frac{\kappa - x_j^E}{c})^\gamma - 1\Big)^{1-\frac{1}{\gamma}}} > 0.$$

This term is proved to be strictly positive given that $x_j^E < \kappa - c$ for any active markets x_j^E, implying that the marginal value of x_j^E is strictly positive. Therefore, the optimal level of x_j^E reaches the upper bound: $x_j^E = \kappa - c$. Hence, for hiring firms, it follows that $f(\theta(\kappa - c)) = 0$, which makes the marginal value of s_j from (A.9) negative as follows: $\mathbf{U} - \Big[\alpha P_j l_j^{\alpha-1} + \beta \frac{\partial \mathbb{E}_j {\mathbf{V}_j}^{init\prime}}{\partial l_j}\Big] < 0$. This is due to (A.8) and $\kappa > U$, proving that hiring firms would not separate workers.

In a similar fashion, contracting firms would never hire workers, given that their marginal value of hiring can never be positive as follows: $\Big[\alpha P_j l_j^{\alpha-1} + \beta \frac{\partial \mathbb{E}_j {\mathbf{V}_j}^{init\prime}}{\partial l_j}\Big] - \kappa < 0$, given (A.8), (A.16), and $\kappa > U$. Therefore, this completes the proof that if $h_j > 0$, $s_j = 0$ needs to hold, and vice versa. The proof enables me to split the firm's problem into the following three cases.

a) Hiring Firms ($s_j = 0$ and $h_j > 0$) solve the following maximization:

$$\max_{h_j, x_j^E} P_j l_j^\alpha - c^f - \kappa h_j + \lambda f(\theta(x_j^E))x_j^E l_{j-1} + \beta \mathbb{E}_j \mathbf{V}_j^{init}(a_j', \tilde{P}_j, l_j, P_j'), \tag{A.11}$$

subject to $l_j = h_j + (1 - \lambda f(\theta(x_j^E)))l_{j-1}$ and $\tilde{P}_j = \frac{a_j \tilde{P}_{j-1} + \ln P_j}{a_j + 1}$. As discussed before, the optimal x_j^E is pinned at the upper bound $\kappa - c$, and thus, the utility level $\tilde{\mathbf{W}}_\mathbf{j}$ that firms will offer to their incumbent workers is determined by $\tilde{\mathbf{W}}_\mathbf{j} = \kappa - c$ from (A.14).

b) Inactive Firms ($s_j = 0$ and $h_j = 0$): solve the following maximization:

$$\max_{x_j^E} P_j l_j^\alpha - c^f + \lambda f(\theta(x_j^E)) x_j^E l_{j-1} + \beta \mathbb{E}_j \mathbf{V}_j{}^{init}(a_j + 1, \tilde{P}_j, l_j, P_j'), \tag{A.12}$$

where $l_j = (1 - \lambda f(\theta(x_j^E))) l_{j-1}$, and obtain the following equation determining x_j^E:

$$x_j^E + \frac{f(\theta(x_j^E))}{f'(\theta(x_j^E))\theta'(x_j^E)} - \left[\alpha P_j \Big((1 - \lambda f(\theta(x_j^E)))l\Big)^{\alpha-1} + \beta \frac{\partial \mathbb{E}_j \mathbf{V}_j{}^{init\prime}}{\partial l_j}|_{l_j = (1-\lambda f(\theta(x_j^E)))l_{j-1}}\right] = 0. \tag{A.13}$$

Note that this case holds only when $\left[\alpha P_j l_j^{\alpha-1} + \beta \frac{\partial \mathbb{E}_j \mathbf{V}_j{}^{init\prime}}{\partial l_j}|_{l_j = l_{j-1}}\right] < \kappa$, where the marginal value of h_j is strictly less than zero and thus $h_j = 0$ is optimal.

Furthermore, this case holds when the optimal x_j^E is in the range of $x_j^E \leq \kappa - c$. If P_j is high enough so that the left-hand side of (A.13) becomes strictly greater than 0, then as before in the hiring case, the optimal solution is bound by the upper bound, i.e. $x_j^E = \kappa - c$. This holds when $\kappa - c < \left[\alpha P_j l_j^{\alpha-1} + \beta \frac{\partial \mathbb{E}_j \mathbf{V}_j{}^{init\prime}}{\partial l_j}|_{l_j = l_{j-1}}\right]$, so that the marginal value of x_j^E is strictly positive. In this case, firms would not just stay inactive but also not allow workers to quit, i.e. $l_j = l_{j-1}$.

The equilibrium utility level $\tilde{\mathbf{W}}_\mathbf{j}$ promised to incumbent workers is pinned down by the workers' optimal condition (A.4) that firms take into account:

$$\tilde{\mathbf{W}}_\mathbf{j} = \kappa - (\kappa - x_j^E)^{1+\gamma} c^{-\gamma}. \tag{A.14}$$

c) Separating Firms with Layoffs ($s_j > 0$ and $h_j = 0$) solve:

$$\max_{s_j, x_j^E} s_j \mathbf{U} l_{j-1} + P_j l_j^{\alpha} - c^f + (1-s_j)\lambda f(\theta(x_j^E)) x_j^E l_{j-1} + \beta \mathbb{E}_j \mathbf{V}_j{}^{init}(a_j+1, \tilde{P}_j, l_j, P_j'), \tag{A.15}$$

subject to $l_j = (1-s_j)(1-\lambda f(\theta(x_j^E))) l_{j-1}$. Using the two first-order conditions (A.9) and (A.10), the following equation holds, pinning down s_j:

$$\left[\alpha P_j l_j^{\alpha-1} + \beta \frac{\partial \mathbb{E}_j \mathbf{V}_j{}^{init\prime}}{\partial l_j}\right] = \frac{\mathbf{U} - \lambda x_j^E \Big(\theta(x_j^E)(1+\theta(x_j^E)^{\gamma})^{-\frac{1}{\gamma}}\Big)}{1 - \lambda \Big(\theta(x_j^E)(1+\theta(x_j^E)^{\gamma})^{-\frac{1}{\gamma}}\Big)}. \tag{A.16}$$

Combining this with (A.10), x_j^E is determined by the following equation:

$$\kappa - \mathbf{U} = c\Big[(1+\theta(x_j^E)^{\gamma})^{1+\frac{1}{\gamma}} - \lambda \theta(x_j^E)^{1+\gamma}\Big]. \tag{A.17}$$

The equilibrium utility level $\tilde{\mathbf{W}}_{\mathbf{j}}$ is pinned down by (A.14) as before.

3. Workers' Future Expected Value: Lastly, let's define $\hat{W}$ as incumbent workers' value at the beginning of a period after observing the firm's current productivity draw P_j (but before the firm's endogenous exit and layoffs). Then, given the firm and worker decision rules, $\hat{W}$ is determined and ranked by the following descending order:

i) Workers at hiring or inactive employers obtain the highest value, $(\kappa - c)$;

ii) Workers at quitting employers have a value lower than those at hiring or inactive firms (without quits) and higher than those at firms laying off workers, $\Big(\lambda f(\theta(\mathbf{x_j^E}))\mathbf{x_j^E} + \big(1 - \lambda f(\theta(\mathbf{x_j^E}))\big)\tilde{\mathbf{W}}_{\mathbf{j}}\Big)$;

iii) Workers at employers that lay off workers have a value lower than those at quitting or inactive or expanding firms but higher than unemployed workers, $\Big(\mathbf{s}_j \mathbf{U} + (1-\mathbf{s}_j)\big(\lambda f(\theta(\mathbf{x_j^E}))\mathbf{x_j^E} + \big(1 - \lambda f(\theta(\mathbf{x_j^E}))\big)\tilde{\mathbf{W}}_{\mathbf{j}}\big)\Big)$;

iv) Unemployed workers have the lowest value, $\mathbf{U}$.

The proof is as follows. It is already known that any active markets x_j^E need to be ranged below $\kappa - c$, and following (A.14), $\tilde{\mathbf{W}}_j$ is also bound by $\kappa - c$ (i.e. $x_j^E, \tilde{\mathbf{W}}_j \leq \kappa - c$ for any active markets x_j^E). Thus, the following is satisfied:

$$\big(\lambda f(\theta(\mathbf{x_j^E}))\mathbf{x_j^E} + \big(1 - \lambda f(\theta(\mathbf{x_j^E}))\big)\tilde{\mathbf{W}}_\mathbf{j}\big) \leq \kappa - c, \quad \forall \mathbf{x_j^E}, \tilde{\mathbf{W}}_\mathbf{j}. \tag{A.18}$$

Next, consider an inactive firm that allows worker quits. Using (A.14), the worker's value at this firm can be rephrased as follows:

$$\big(\lambda f(\theta(\mathbf{x_j^E}))\mathbf{x_j^E} + \big(1 - \lambda f(\theta(\mathbf{x_j^E}))\big)\tilde{\mathbf{W}}_\mathbf{j}\big) = \mathbf{x_j^E} - (\kappa - \mathbf{x_j^E})\theta(\mathbf{x_j^E})^\gamma + c\theta(\mathbf{x_j^E})^{1+\gamma}, \tag{A.19}$$

which is the weighted average of the promised utility in the current firm and the target utility in the worker's on-the-job search. Here, $\mathbf{x_j^E}$ is the solution of the equation (A.13). Furthermore, this firm finds $s_j = 0$ to be optimal and stays inactive with quits allowed. Therefore, the marginal value of s_j, the left-hand side of (A.9), has to be strictly negative with any $s_j > 0$ and equals to zero with $s_j = 0$.

Combining this with (A.13), the following can be obtained:

$$\mathbf{U} \leq \left(\mathbf{x^E} + \frac{(1 - \lambda f(\theta(\mathbf{x^E})))f(\theta(\mathbf{x^E}))}{f(\theta)\theta(\mathbf{x^E})}\right) = \mathbf{x^E} - \theta(\mathbf{x^E})^\gamma(\kappa - \mathbf{x^E}). \tag{A.20}$$

Combining it with (A.19) proves that the following holds:

$$\mathbf{U} \leq \big(\lambda f(\theta(\mathbf{x^E}))\mathbf{x^E} + \big(1 - \lambda f(\theta(\mathbf{x^E}))\big)\tilde{\mathbf{W}}\big) \tag{A.21}$$

for any firms staying inactive with quits and choosing $\mathbf{x_j^E}$ following (A.13).

Similarly, based on (A.19), the value of workers at a firm laying off workers is as follows:

$$\mathbf{s}_j\mathbf{U} + (1 - \mathbf{s}_j)\Big(\mathbf{x_j^E} - (\kappa - \mathbf{x_j^E})\theta(\mathbf{x_j^E})^\gamma + c\theta(\mathbf{x_j^E})^{1+\gamma}\Big), \tag{A.22}$$

where $\mathbf{x_j^E}$ satisfies (A.17). Furthermore, (A.17) implies that

$$\mathbf{U} = \mathbf{x^E} - (\kappa - \mathbf{x^E})\theta(\mathbf{x^E})^{\gamma} + \lambda c\theta(\mathbf{x^E})^{1+\gamma}, \tag{A.23}$$

and thus, (A.22) and (A.23) prove that the following holds for any firms laying off workers with s_j and $\mathbf{x_j^E}$ satisfying (A.9) and (A.13):

$$\mathbf{U} \leq \mathbf{sU} + (1-\mathbf{s})\Big(\mathbf{x^E} - (\kappa - \mathbf{x^E})\theta(\mathbf{x^E})^{\gamma} + c\theta(\mathbf{x^E})^{1+\gamma}\Big). \tag{A.24}$$

Combining (A.18), (A.21), and (A.24) proves i) and iv), meaning that workers obtain the highest value at a hiring or inactive firm and get the lowest value in the unemployment pool.

The rank order of workers' value between quitting firms and those laying off workers needs to be confirmed to verify ii) and iii). This can be established with the following two proofs. First, it can be proved that (A.19) is weakly increasing in $\mathbf{x_j^E}$, implying that workers get weakly higher values at a firm with higher $\mathbf{x_j^E}$. Second, the other proof to confirm is the equilibrium $\mathbf{x_j^E}$ is higher for quitting firms than contracting firms with layoffs. In other words, $\mathbf{x_j^E}$ satisfying (A.13) is higher than $\mathbf{x_j^E}$ satisfying (A.17). Then, the two proofs along with (A.21) can confirm that workers obtain higher values at quitting firms than those laying off workers.

Let's start with the first one by getting the derivative of (A.19) with respect to x^E:

$$\begin{aligned}\frac{\partial\big(\lambda f(\theta(x^E))x^E + \big(1-\lambda f(\theta(x^E))\big)\tilde{\mathbf{W}}\big)}{\partial x^E} &= \frac{\partial\Big(x^E - (\kappa - x^E)\theta(x^E)^{\gamma} + c\theta(x^E)^{1+\gamma}\Big)}{\partial x^E} \\ &= 1 - \frac{\theta^{\gamma}(\frac{\kappa - x^E}{c})^{-\gamma}}{1-(\frac{\kappa - x^E}{c})^{-\gamma}} = 0,\end{aligned} \tag{A.25}$$

which implies that for any non-binding optimal solutions for x_j^E in (A.13), worker values conditional on not being separated are the same.

It is already seen in the previous discussion from the equations (A.13) and (A.13) that the optimal x_j^E chosen by quitting firms is: $\mathbf{U} \leq x_j^E + \frac{(1-\lambda f(\theta(x_j^E)))f(\theta(x_j^E))}{f'(\theta)\theta'(x_j^E)}$, while the choice of firms laying off workers is pinned down by the following equation: $\mathbf{U} = x_j^E + \frac{(1-\lambda f(\theta(x_j^E)))f(\theta(x_j^E))}{f'(\theta)\theta'(x_j^E)}$. Therefore, in order to confirm the former is higher than the latter, it is sufficient to prove the following terms are increasing in x_j^E: $x_j^E + \frac{(1-\lambda f(\theta(x_j^E)))f(\theta(x_j^E))}{f'(\theta)\theta'(x_j^E)} = x_j^E - \theta^\gamma(\kappa - x_j^E) + \lambda c\theta^{\gamma+1}$.

These terms satisfy the following property:

$$\frac{\partial\Big(x_j^E - \theta^\gamma(\kappa - x_j^E) + \lambda c\theta^{\gamma+1}\Big)}{\partial x_j^E} = \frac{\partial\Big(x_j^E - \theta^\gamma(\kappa - x_j^E) + c\theta^{\gamma+1}\Big)}{\partial x_j^E} - (1-\lambda)c(\gamma+1)\theta^\gamma\frac{\partial\theta(x_j^E)}{\partial x_j^E}$$
$$> 0,$$

given that (A.25) makes the first term on the right-hand side zero and $\frac{\partial\theta(x^E)}{\partial x^E} < 0$. Therefore, it is proved that: $\frac{\partial\Big(x^E + \frac{(1-\lambda f(\theta(x^E)))f(\theta(x^E))}{f'(\theta)\theta'(x^E)}\Big)}{\partial x^E} > 0$. This suggests that the optimal x^E is higher for quitting firms than those laying off workers. Lastly, this fact along with (A.21) and (A.25) finalizes the proof for ii) and iii). □

Proof of Proposition 3 . Given the properties (2.2) and (2.3) and the assumption of log normality, there is a point of $\ln P$, $\hat{P} = \frac{\mu^{old}\sigma^{young} - \mu^{young}\sigma^{old}}{\sigma^{young} - \sigma^{old}}$, with which the following relationship can be derived between the cdf functions for young and old firms who are equally high performing (i.e., $\tilde{P} > \bar{\nu}_0$):

$$F^{old}(\ln P) \geq F^{young}(\ln P), \forall \ln P \text{ if } \ln P \geq \hat{P}$$
$$F^{old}(\ln P) \leq F^{young}(\ln P), \forall \ln P \text{ if } \ln P \leq \hat{P}$$

Similarly, the following relationship holds for the cdf functions between young and old firms who are equally low performing (i.e., $\tilde{P} < \bar{\nu}_0$):

$$F^{old}(\ln P) \leq F^{young}(\ln P), \forall \ln P \text{ if } \ln P \geq \hat{P}$$

$$F^{old}(\ln P) \geq F^{young}(\ln P), \forall \ln P \text{ if } \ln P \leq \hat{P}.$$

This indicates that if firms are equally high-performing, the posterior distribution for young firms first-order stochastically dominates that of older firms within the range of $\ln P > \hat{P}$, while the posterior distribution for older firms exhibits first-order stochastic dominance over that of younger firms within the range of $\ln P < \hat{P}$. Conversely, this relationship is reversed for the other group of firms that are equally low-performing.

As before let $\hat{W}$ denote for workers' value at the beginning of a period after observing the draw of firm's current productivity. The proof for Lemma 2 has already shown that $\hat{W}$ increases as firm has a better employment status with a higher draw of productivity. Further assume that the state-contingent utility $\tilde{W}$ is an increasing function of P (so that $\hat{W}$ is continuously increasing in P) and has a same functional form across firm age for simplicity. Then, it follows based on the first-order stochastic dominance that

$$\begin{aligned}\int_{\hat{P}} \hat{W} dF^{old}(\ln P) &\leq \int_{\hat{P}} \hat{W} dF^{young}(\ln P)\\ \int^{\hat{P}} \hat{W} dF^{old}(\ln P) &\geq \int^{\hat{P}} \hat{W} dF^{young}(\ln P)\end{aligned} \tag{A.26}$$

for high-performing firms (i.e., $\tilde{P} > \bar{\nu}_0$), and

$$\begin{aligned}\int_{\hat{P}} \hat{W} dF^{old}(\ln P) &\geq \int_{\hat{P}} \hat{W} dF^{young}(\ln P)\\ \int^{\hat{P}} \hat{W} dF^{old}(\ln P) &\leq \int^{\hat{P}} \hat{W} dF^{young}(\ln P)\end{aligned} \tag{A.27}$$

for low-performing firms (i.e., $\tilde{P} < \bar{\nu}_0$).

Note that as $\tilde{P}$ increases, the posterior mean of the distribution for P increases following equation (2.2). This leads to lower productivity cutoffs determining firm status such as hiring, being inactive, laying off workers, or exiting, assuming constant vacancy cost and operating fixed cost. Additionally, $\hat{P}$ is increasing in $\tilde{P}$ as follows:

$$\frac{\partial \hat{P}}{\partial \tilde{P}} = \frac{\partial}{\partial \tilde{P}}\Big(\frac{\mu^{old}\sigma^{young} - \mu^{young}\sigma^{old}}{\sigma^{young} - \sigma^{old}}\Big) = \frac{\sigma^{young}\sigma^{old}(a^{old} - a^{young})}{\sigma^{young} - \sigma^{old}} > 0$$

Thus, as $\tilde{P}$ increases, there will be a point after which the quitting productivity cutoff meets with or goes above $\hat{P}$. Let this upper cutoff be denoted by $\bar{\tilde{P}}^{H}$. Conversely, as $\tilde{P}$ decreases, there will be a point after which the layoff productivity cutoff meets with or goes above $\hat{P}$. Let this be denoted by $\bar{\tilde{P}}^{L}$. Note that $\bar{\tilde{P}}^{H}$ and $\bar{\tilde{P}}^{L}$ can be either higher or lower than $\bar{\nu}_0$, which depends on firm state variables and can only be computed numerically. This lemma only verifies the existence of such cutoffs.

Then, we obtain the following equality for firms having $\tilde{P} \geq \bar{\tilde{P}}^{H}$,

$$\int_{\hat{P}} \hat{W} dF^{old}(\ln P) = \int_{\hat{P}} \hat{W} dF^{young}(\ln P),$$

as $\hat{W}$ is constant ($\kappa - c$) for quitting or hiring firms. Combining it with (A.26), we have the following relationship for workers' future expected value between young and old firms:

$$\mathbb{E}^{old}[W] = \int \hat{W} dF^{old}(\ln P) \geq \mathbb{E}^{young}[W] = \int \hat{W} dF^{young}(\ln P).$$

Similarly, for firms having $\tilde{P} \leq \bar{\tilde{P}}^{L}$, we get the following equality

$$\int^{\hat{P}} \hat{W} dF^{old}(\ln P) = \int^{\hat{P}} \hat{W} dF^{young}(\ln P),$$

as $\hat{W}$ is constant for firms laying off workers ($\mathbf{s}_j\mathbf{U} + (1 - \mathbf{s}_j)\big(\lambda f(\theta(\mathbf{x_j^E}))\mathbf{x_j^E} + \big(1 - \lambda f(\theta(\mathbf{x_j^E}))\big)\tilde{\mathbf{W}}_\mathbf{j}\big)$ with $\mathbf{x_j^E}$ and $\tilde{\mathbf{W}}_\mathbf{j}$ pinned down by (A.17)) or exiting (U). Thus, combined with (A.27), we get the following relationship

$$\mathbb{E}^{old}[W] = \int \hat{W} dF^{old}(\ln P) \leq \mathbb{E}^{young}[W] = \int \hat{W} dF^{young}(\ln P).$$

$\square$

B Bayesian Learning

Suppose that initial prior is $\nu_j \sim N(\bar{\nu}_0, \sigma_0^2)$, and there is an observation of $\ln P_{jt} = \nu_j + \varepsilon_{jt}$ such that $\varepsilon_{jt} \sim N(0, \sigma_\varepsilon^2)$, $\ln P_{jt}|_{\nu_j} \sim N(\bar{\nu}_0, \sigma_0^2 + \sigma_\varepsilon^2)$. Following the Bayes' rule,

$$f(\nu_j | \ln P_{jt}) \propto f(\nu_j) f(\ln P_{jt}|\nu_j),$$

we have:

$$\begin{aligned}
f(\nu_j | \ln P_{jt}) \propto f(\nu_j) f(\ln P_{jt}|\nu_j) &= \left(\frac{1}{\sqrt{2\pi\sigma_0^2}} \exp\left(-\frac{(\nu_j - \bar{\nu}_0)^2}{2\sigma_0^2}\right)\right)\left(\frac{1}{\sqrt{2\pi\sigma_\varepsilon^2}} \exp\left(-\frac{(\ln P_{jt} - \nu_j)^2}{2\sigma_\varepsilon^2}\right)\right) \\
&\propto \left(\frac{1}{\sqrt{2\pi\sigma_0^2\sigma_\varepsilon^2}} \exp\left(-\frac{\left(\nu_j - \left(\frac{\sigma_\varepsilon^2\bar{\nu}_0 + \sigma_0^2 \ln P_{jt}}{\sigma_\varepsilon^2 + \sigma_0^2}\right)\right)^2}{2\frac{\sigma_0^2\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma_0^2}}\right)\right),
\end{aligned}$$

which implies that

$$f(\nu_j | \ln P_{jt}) \sim N\left(\frac{\sigma_\varepsilon^2\bar{\nu}_0 + \sigma_0^2 \ln P_{jt}}{\sigma_\varepsilon^2 + \sigma_0^2}, \frac{\sigma_0^2\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma_0^2}\right).$$

Thus, the mean and standard deviation of the posterior distribution are

$$\bar{\nu}_{jt} = \frac{\sigma_\varepsilon^2\bar{\nu}_{jt-1} + \sigma_{jt-1}^2 \ln P_{jt}}{\sigma_{jt-1}^2 + \sigma_\varepsilon^2} = \frac{\frac{\bar{\nu}_{jt-1}}{\sigma_{jt-1}^2} + \frac{\ln P_{jt}}{\sigma_\varepsilon^2}}{\frac{1}{\sigma_{jt-1}^2} + \frac{1}{\sigma_\varepsilon^2}}, \tag{B.28}$$

$$\sigma_{jt}^2 = \frac{\sigma_{jt-1}^2\sigma_\varepsilon^2}{\sigma_{jt-1}^2 + \sigma_\varepsilon^2} = \frac{1}{\frac{1}{\sigma_{jt-1}^2} + \frac{1}{\sigma_\varepsilon^2}}. \tag{B.29}$$

By iterating (B.28) and (B.29) backward and using $a_{jt} + 1 = a_{jt+1}$, I can rewrite them as (2.2) and (2.3) in the main text.

Furthermore, the following relationships between the two sufficient statistics and the posterior mean at the beginning of each period t can be derived:

$$\frac{\partial \bar{\nu}_{jt-1}}{\partial \tilde{P}_{jt-1}} = \frac{a_{jt}\frac{1}{\sigma_\varepsilon^2}}{\frac{1}{\sigma_0^2} + a_{jt}\frac{1}{\sigma_\varepsilon^2}} > 0 \tag{B.30}$$

$$\frac{\partial \bar{\nu}_{jt-1}}{\partial a_{jt}} = \frac{(\tilde{P}_{jt-1} - \bar{\nu}_0)}{\sigma_0^2 \sigma_\varepsilon^2 \left(\frac{1}{\sigma_0^2} + a_{jt}\frac{1}{\sigma_\varepsilon^2}\right)^2} \begin{cases} \geq 0 & \text{if} \quad \tilde{P}_{jt-1} \geq \bar{\nu}_0 \\ < 0 & \text{if} \quad \tilde{P}_{jt-1} < \bar{\nu}_0 \end{cases}. \tag{B.31}$$

Equation (B.30) implies that the posterior mean increases in the average productivity level. As firms are observed to have higher average productivity, their prospects improve. Moreover, (B.31) shows that firm age affects job prospects differently depending on the firm's cumulative average productivity. Specifically, if firm j's average productivity is above the initial cross-sectional mean, a higher age implies a better inferred type, while if a firm's average productivity is below the cross-sectional mean, a higher age implies a worse inferred type.

Also, one can derive the following relationship between firm age and the posterior standard deviation:

$$\frac{\partial \sigma_{jt-1}^2}{\partial a_{jt}} = -\frac{1}{\sigma_\varepsilon^2 \left(\frac{1}{\sigma_0^2} + a_{jt}\frac{1}{\sigma_\varepsilon^2}\right)^2} < 0, \tag{B.32}$$

which implies that as a firm ages, learning gets less noisy, and the posterior converges to a degenerate distribution centered at the true type ν_j.

C Equilibrium Labor Markets

Starting with the firms' problem, only submarkets that satisfy (2.17) are searched by firms. This implies that in equilibrium, the following complementary slackness condition should hold for any active labor submarket x_t:

$$\theta(x_t)\Big(\frac{c}{q(\theta(x_t))} + x_t - \kappa\Big) = 0, \tag{C.33}$$

where κ is the minimized cost value as follows:

$$\kappa \equiv \min\Big(\frac{c}{q(\theta(x))} + x\Big). \tag{C.34}$$

I assume a CES matching function

$$M(S(x), V(x)) = (S(x)^{-\gamma} + V(x)^{-\gamma})^{-\frac{1}{\gamma}}, \tag{C.35}$$

which is common across labor submarkets x. $S(x)$ and $V(x)$ are the total number of searching workers and vacancies, respectively, in each labor submarket x.[2]

Using the matching function in (C.35), the job finding rate $f(\cdot)$ and filling rate $q(\cdot)$ for each submarket x are given by:

$$f(\theta(x)) = \theta(x)(1 + \theta(x)^{\gamma})^{-\frac{1}{\gamma}} \tag{C.36}$$

$$q(\theta(x)) = (1 + \theta(x)^{\gamma})^{-\frac{1}{\gamma}}, \tag{C.37}$$

where $\theta(x)$ is the ratio of total vacancies to searching workers, $\frac{V(x)}{S(x)}$, in each submarket x. Based on this, the firm's complementary slackness condition (C.33) can be rewritten as follows:

$$\theta(x)\left(\frac{c}{(1 + \theta(x)^{\gamma})^{-\frac{1}{\gamma}}} + x - \kappa\right) = 0. \tag{C.38}$$

Based on the conditions (C.34) and (C.38), there are different labor submarkets in the equilibrium with labor market tightness determined as follows:

$$\theta(x) = \begin{cases} \left(\left(\frac{\kappa - x}{c}\right)^{\gamma} - 1\right)^{\frac{1}{\gamma}} & \text{if} \quad x < \kappa - c \\ 0 \quad \text{if} \quad x \geq \kappa - c, \end{cases} \tag{C.39}$$

[2]Note that the job searchers $S(x)$ are workers searching either from the unemployment pool (if x is the optimal market for unemployed workers to search in) or on the job (if x is the optimal market for workers employed at j to search in).

implying that $\theta(\cdot)$ is decreasing in x, and if x is greater or equal to $\kappa - c$, there are no firms posting vacancies, so that the market becomes inactive, i.e., $\theta(x) = 0$.

D Uncertainty and Job Prospects

In this section, I discuss how the degree of uncertainty in the economy affects model outcomes. The following proposition shows how the learning process depends on the degree of productivity noise, σ_ε.[3]

Proposition A.1. *If productivity noise σ_ε increases, high performing firms have a relatively lower posterior mean, while low performing firms have a relatively higher posterior mean, for any given age and average observed performance. Furthermore, higher noise increases the posterior variance for all firms.*

Proof.

$$\frac{\partial \bar{\nu}_{jt-1}}{\partial \sigma_\varepsilon^2} = \left(\frac{a_{jt}}{\sigma_\varepsilon^2 \sigma_0^2}\right) \frac{(\bar{\nu}_0 - \tilde{P}_{jt-1})}{\left(\frac{1}{\sigma_0^2} + \frac{a_{jt}}{\sigma_\varepsilon^2}\right)^2} \begin{cases} > 0 & \text{if} \quad \tilde{P}_{jt-1} < \bar{\nu}_0 \\ < 0 & \text{if} \quad \tilde{P}_{jt-1} > \bar{\nu}_0 \end{cases}$$

$$\frac{\partial \sigma_{jt-1}}{\partial \sigma_\varepsilon^2} = \left(\frac{a_{jt}}{\sigma_\varepsilon^2}\right) \frac{1}{\left(\frac{1}{\sigma_0^2} + \frac{a_{jt}}{\sigma_\varepsilon^2}\right)^2} > 0$$

□

Proposition A.1 implies that higher noise reduces the prospects at high performing firms, while improving the prospects of low performing firms, all else equal. This is because agents are less certain about firms' actual type.

Proposition A.2. *As productivity noise σ_ε rises, firms' average observed productivity becomes less informative about firms' actual type.*

[3]Recall that the dispersion of shock σ_ε refers to the degree of noise in the economy, while the dispersion of firm types σ_0 indicates the signal level. Thus, for a given level of signal σ_0, the dispersion σ_ε measures the degree of uncertainty in the economy. In the empirical section below, I directly estimate the noise-to-signal ratio $\frac{\sigma_\varepsilon}{\sigma_0}$ to proxy the level of uncertainty in different industries over time.

Proof.

$$\frac{\partial}{\partial \sigma_\varepsilon^2}\left(\frac{\partial \bar{\nu}_{jt-1}}{\partial \tilde{P}_{jt-1}}\right) = -\left(\frac{a_{jt}}{\sigma_\varepsilon^4 \sigma_0^2}\right)\frac{1}{\left(\frac{1}{\sigma_0^2}+\frac{a_{jt}}{\sigma_\varepsilon^2}\right)^2} < 0$$

□

Proposition A.2 shows that the positive relationship in (B.30) between the average productivity level and the posterior mean is dampened as productivity noise rises in the economy. Both Propositions A.1 and A.2 imply that slow learning harms the prospects of high performing firms.

Proposition A.3. *For $\frac{\sigma_\varepsilon}{\sigma_0} < 1$, the effect of firm age on the speed of updating posteriors is more pronounced as noise increases.*[4]

Proof. With $\frac{\sigma_\varepsilon}{\sigma_0} < 1$, $\forall\, a_{jt} \geq 1$

$$\frac{\partial}{\partial \sigma_\varepsilon^2}\left(\frac{\partial \bar{\nu}_{jt-1}}{\partial a_{jt}}\right) = \frac{(\tilde{P}_{jt-1}-\bar{\nu}_0)}{\sigma_\varepsilon^4\sigma_0^2\left(\frac{1}{\sigma_0^2}+\frac{a_{jt}}{\sigma_\varepsilon^2}\right)^3}\left(\frac{a_{jt}}{\sigma_\varepsilon^2}-\frac{1}{\sigma_0^2}\right)\begin{cases} > 0 & \text{if} \quad \tilde{P}_{jt-1} > \bar{\nu}_0 \\ < 0 & \text{if} \quad \tilde{P}_{jt-1} < \bar{\nu}_0 \end{cases}$$

$$\frac{\partial}{\partial \sigma_\varepsilon^2}\left(\frac{\partial \sigma_{jt-1}^2}{\partial a_{jt}}\right) = -\frac{\left(\frac{a_{jt}}{\sigma_\varepsilon^2}-\frac{1}{\sigma_0^2}\right)}{\sigma_\varepsilon^4\left(\frac{1}{\sigma_0^2}+\frac{a_{jt}}{\sigma_\varepsilon^2}\right)^3} < 0.$$

□

Proposition A.3 shows how the degree of noise affects the learning process at different firm ages. As in (B.31), firm age affects learning about firm type in a different way depending on firms' observed performance. Specifically, firms with high average performance have better prospects due to a higher posterior mean when they are older, while firms with low average performance have better prospects due to a higher posterior mean when they are younger. Furthermore, the posterior variance decreases monotonically in firm age as seen in (B.32). Proposition A.3 shows that as the noise level rises, such age effects get more pronounced for $\frac{\sigma_\varepsilon}{\sigma_0} < 1$.

[4]In Section 4, I externally calibrate both σ_ε and σ_0 using estimated values from the Census data. These estimates are consistent with the assumption that $\frac{\sigma_\varepsilon}{\sigma_0} < 1$.

Corollary A.1. *For $\frac{\sigma_\varepsilon}{\sigma_0} < 1$, the difference in job prospects between otherwise similar firms of different ages increases in the degree of noise.*

Proof. Suppose there are two firms, firm 1 and firm 2, having the same average productivity $\tilde{P}$. Let a_1 and a_2 be the ages of firms 1 and 2, respectively, where $a_1 > a_2 \geq 1$. Also, let $\bar{\nu}_1$ and $\bar{\nu}_2$ be the posterior means for firms 1 and 2, respectively. From previous results, we have

$$\begin{aligned}
\bar{\nu}_1 > \bar{\nu}_2 \quad &\text{if} \quad \tilde{P} > \bar{\nu}_0 \\
\bar{\nu}_1 < \bar{\nu}_2 \quad &\text{if} \quad \tilde{P} < \bar{\nu}_0.
\end{aligned}$$

Then the following relationship holds:

$$\frac{\partial(\bar{\nu}_1 - \bar{\nu}_2)}{\partial \sigma_\varepsilon^2} = \frac{\frac{(a_1 - a_2)(\tilde{P} - \bar{\nu}_0)}{\sigma_0^2 \sigma_\varepsilon^4}\left(\frac{a_1 a_2}{\sigma_\varepsilon^4} - \frac{1}{\sigma_0^4}\right)}{\left(\frac{1}{\sigma_0^2} + \frac{a_1}{\sigma_\varepsilon^2}\right)^2 \left(\frac{1}{\sigma_0^2} + \frac{a_2}{\sigma_\varepsilon^2}\right)^2} \begin{cases} > 0 & \text{if} \quad \tilde{P} > \bar{\nu}_0 \\ < 0 & \text{if} \quad \tilde{P} < \bar{\nu}_0, \end{cases}$$

so that the gap between $\bar{\nu}_1$ and $\bar{\nu}_2$ increases in σ_ε^2. □

Overall, higher noise particularly harms the job prospects of young firms with high performance. Although higher noise generally harms firms with high performance, as shown in Propositions A.1 and A.2, the damage is more pronounced to young firms, following Proposition A.3 and Corollary A.1. This is because the speed of updating over the firm life cycle is dragged out as noise increases, widening the gap in job prospects between young and mature firms.

E Welfare Implications

Proposition A.4. *The model's decentralized block-recursive allocation given the level of uncertainty is constrained efficient. However, the decentralized allocation is distorted relative to the social optimum if the planner could eliminate uncertainty about firm type.*

Proof: First, I prove that given the level of uncertainty about firms' productivity type *(given σ_ε and σ_0), the model's block-recursive equilibrium can be replicated by a constrained social planner's problem and thus is efficient.*

Suppose that a social planner is constrained by both of the search and information frictions as in the market economy. The social planner aims to maximize the following welfare function:

$$\max_{\substack{u_t, v_t, M_t^e, \mathbf{G}(a_{t+1}, \tilde{P}_t, l_t),\\ \mathbf{d}(a_t, \tilde{P}_{t-1}, l_{t-1}, P_t),\\ \mathbf{s}(a_t, \tilde{P}_{t-1}, l_{t-1}, P_t),\\ \mathbf{h}(a_t, \tilde{P}_{t-1}, l_{t-1}, P_t),\\ \theta(a_t, \tilde{P}_{t-1}, l_{t-1}, P_t),\\ \theta^{\mathbf{E}}(a_t, \tilde{P}_{t-1}, l_{t-1}, P_t),\\ \mathbf{l}(a_t, \tilde{P}_{t-1}, l_{t-1}, P_t),\\ \theta_t^U, d_t^e(P_t), l_t^e(P_t)}} \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \Big\{ u_t b - c v_t$$
$$+ \sum_{(a_t, \tilde{P}_{t-1}, l_{t-1}, P_t), a_t \geq 1} \mathbf{G}(a_t, \tilde{P}_{t-1}, l_{t-1}) f_{(a_t, \tilde{P}_{t-1})}(P_t)$$
$$* \left(1 - \mathbf{d}(a_t, \tilde{P}_{t-1}, l_{t-1}, P_t)\right) \left(P_t l_t^\alpha - c_f\right)$$
$$+ M_t^e \left(\sum_{P_t} f^e(P_t)(1 - d_t^e(P_t)) \left(P_t (l_t^e(P_t))^\alpha - c_f\right) - c_e \right) \Big\}, \tag{E.40}$$

subject to

$$l_t = (1 - s_t)(1 - \lambda f(\theta_t^E)) l_{t-1} + h_t \tag{E.41}$$

$$v_t = \theta_t^U u_t + \sum_{(a_t, \tilde{P}_{t-1}, l_{t-1}, P_t)} \lambda \theta_t^E(a_t, \tilde{P}_{t-1}, l_{t-1}, P_t) l_{t-1} \mathbf{G}(a_t, \tilde{P}_{t-1}, l_{t-1}) f_{(a_t, \tilde{P}_{t-1})}(P_t) \tag{E.42}$$

$$u_t = (1 - f(\theta_t^U)) u_{t-1} + \sum_{(a_t, \tilde{P}_{t-1}, l_{t-1}, P_t)} (d_t + (1 - d_t) s_t) l_{t-1} \mathbf{G}(a_t, \tilde{P}_{t-1}, l_{t-1}) f_{(a_t, \tilde{P}_{t-1})}(P_t) \tag{E.43}$$

$$\mathbf{G}(a_{t+1}, \tilde{P}_t, l_t) = \sum_{\tilde{P}_{t-1}, l_{t-1}} \mathbf{G}(a_t, \tilde{P}_{t-1}, l_{t-1}) f_{(a_{t+1}, \tilde{P}_t)} \left((a_t + 1) \tilde{P}_t - a_t \tilde{P}_{t-1} \right)$$

$$* (1 - \mathbf{d}(a_t, \tilde{P}_{t-1}, l_{t-1}, P_t))\mathbb{I}_{l(a_t, \tilde{P}_{t-1}, l_{t-1}, P_t) = l_t} \textit{ for } a_t \geq 1 \tag{E.44}$$

$$\mathbf{G}(1, \tilde{P}_{t-1}, l_{t-1}) = \begin{cases} M_t^e f^e(\tilde{P}_{t-1})(1 - d^e(\tilde{P}_{t-1})), \textit{ if } l_{t-1} = l_t^e(\tilde{P}) \\ 0, \textit{ otherwise} \end{cases}$$

$$h_t(1 - d_t) = f(\theta_t^U)u_t \textit{ for firms searching in market } \theta^U \textit{ (i.e. } \theta(a_t, \tilde{P}_{t-1}, l_{t-1}, P_t) = \theta^U\textit{)} \tag{E.45}$$

$$h_t(1 - d_t) = \lambda f(\theta_t^E)(1 - s_t)l_{t-1} \textit{ for firms poaching from market } \theta^E \textit{ (i.e. } \theta(a_t, \tilde{P}_{t-1}, l_{t-1}, P_t) = \theta^E\textit{)} \tag{E.46}$$

The first line in the objective function shows the utility for unemployed workers and search cost that the social planer takes into account. The second line presents the value of operating incumbent firms, and the last line indicates the value of successful entrant firms.

Equation (E.47) can be rephrased as the following problem with an identifier j for each firm j and their birth year t_0^j:

$$\begin{aligned} \max_{\substack{u_t, v_t, M_t^e, \theta_t^U \\ \left\{d_t^j, s_t^j, h_t^j, \theta_t^j, \theta_t^{Ej}, l_t^j\right\}_{j, a_t^j \geq 1}}} \quad & \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \Big\{ \int_j \Bigg(\Big(\prod_{\tau=t_0^j}^{t} (1 - d_\tau^j) \left(P_t^j (l_t^j)^\alpha - c_f\right) \Big) \mathbb{I}_{t_0^j < t} \\ & + \left((1 - d_t^j) \left(P_t^j (l_t^j)^\alpha - c_f\right) M_t^e - M_t^e c_e\right) \mathbb{I}_{t_0^j = t} \Bigg) dj \\ & + u_t b - c v_t \Big\}, \end{aligned} \tag{E.47}$$

subject to

$$l_t^j = (1 - s_t^j)(1 - \lambda f(\theta_t^{Ej}))l_{t-1}^j + h_t^j \tag{E.48}$$

$$v_t = \theta_t^U u_t + \int_j \left(\prod_{\tau=t_0^j}^{t} (1 - d_\tau^j)(1 - s_t^j)\lambda \theta_t^{Ej} l_{t-1}^j \right) \mathbb{I}_{t_0^j < t} dj \tag{E.49}$$

$$u_t = (1 - f(\theta_t^U))u_{t-1} + \int_j \left(\prod_{\tau=t_0^j}^{t-1} (1 - d_\tau^j)(d_t^j + (1 - d_t^j)s_t^j)l_{t-1}^j \right) \mathbb{I}_{t_0^j < t} dj \tag{E.50}$$

$$h_t^j(1 - d_t^j) = f(\theta_t^U)u_t \textit{ for firm } j \textit{ searching in market } \theta^U \tag{E.51}$$

$$h_t^j(1 - d_t^j) = \lambda f(\theta_t^{Ek})(1 - s_t^k)l_{t-1}^k \textit{ for firm } j \textit{ poaching workers in market } \theta^{Ek} \tag{E.52}$$

$$M_t^e \int_j (1 - d_t^j)\mathbb{I}_{t_0^j = t} dj = \int_j \left(\prod_{\tau=t_0^j}^{t-1} (1 - d_\tau^j)d_t^j \right) \mathbb{I}_{t_0^j < t} dj \tag{E.53}$$

Combining (E.49), (E.51), and (E.52), along with the relationship $\theta_t = \frac{f(\theta_t)}{q(\theta_t)}$ gives the following equation:

$$v_t = \int_j \left(\prod_{\tau=t_0^j}^{t} (1 - d_\tau^j) \frac{h_t^j}{q(\theta_t^j)} \right) dj, \tag{E.54}$$

where θ_t^j is the market that firm j search in, i.e. $\theta_t^j \in \{\theta_t^U, \{\theta_t^{Ek}\}_k\}$.

Then, rephrasing (E.47) by replacing l_t^j with (E.48), v_t with (E.54), and using Langrangian multipliers μ_t for (E.50) and $\eta(\theta_t^j)$ for (E.51) and (E.52), the following is obtained:

$$\begin{aligned}
\max_{\substack{u_t, M_t^e, \theta_t^U \\ \left\{d_t^j, s_t^j, h_t^j, \theta_t^j, \theta_t^{Ej}\right\}_{j, a_t^j \geq 1}}} \quad & \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \Bigg\{ \int_j \Bigg(\prod_{\tau=t_0^j}^{t-1} (1 - d_\tau^j)(1 - d_t^j) \Big(P_t^j \left((1 - s_t^j)(1 - \lambda f(\theta_t^{Ej}))l_{t-1}^j + h_t^j \right)^\alpha \\
& - c_f - c\frac{h_t^j}{q(\theta_t^j)} - \eta(\theta_t^j)h_t^j + \eta(\theta_t^{Ej})\lambda f(\theta_t^{Ej})(1 - s_t^j)l_{t-1}^j \\
& + \mu_t(d_t^j + (1 - d_t^j)s_t^j)l_{t-1}^j \Bigg) \mathbb{I}_{t_0^j < t} \\
& + (1 - d_t^j) \left(P_t^j h_t^j - c_f - c\frac{h_t^j}{q(\theta_t^j)} - \eta(\theta_t^j)h_t^j - c_e \right) M_t^e \mathbb{I}_{t_0^j = t} \Bigg) dj
\end{aligned}$$

$$+ u_t b - \mu_t(u_t - u_{t-1}(1 - f(\theta_t^U))) + \eta(\theta_t^U)u_{t-1}f(\theta_t^U)\Bigg\},$$
(E.55)

Here, pick a competitive equilibrium U_t and $x(\theta_t^j)$ and replace $\mu_t = U_t$, $\eta_t(\theta_t^j) = x_{jt}$ s.t. $\theta_t^j = \theta(x_{jt})$, $\eta_t(\theta_t^{Ej}) = x_{jt}^E$ s.t. $\theta_t^{Ej} = \theta(x_{jt}^E)$, and $\eta_t(\theta_t^U) = x_t^U$ s.t. $\theta_t^U = \theta(x_t^U)$.

Rewriting (E.55)), I have:

$$\begin{aligned}
\max_{\substack{u_t, M_t^e, \theta_t^U \\ \left\{d_t^j, s_t^j, h_t^j, \theta_t^j, \theta_t^{Ej}\right\}_{j, a_t^j \geq 1}}} \quad & \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \Bigg\{ \int_j \Bigg(\prod_{\tau = t_0^j}^{t-1} (1 - d_\tau^j)(1 - d_t^j) \Big(P_t^j \left((1 - s_t^j)(1 - \lambda f(\theta(x_{jt}^E)) l_{t-1}^j + h_t^j\right)^\alpha \\
& - c_f - (\frac{c}{q(\theta(x_{jt}))} + x_{jt}) h_t^j + x_{jt}^E (\lambda f(\theta(x_{jt}^E))(1 - s_t^j) l_{t-1}^j \\
& + U_t(d_t^j + (1 - d_t^j)s_t^j) l_{t-1}^j \Big) \mathbb{I}_{t_0^j < t} \\
& + \Big((1 - d_t^j) \Big(P_t^j (h_t^j)^\alpha - c_f - (\frac{c}{q\theta(x_{jt})} + x_t^j) h_t^j - c_e \Big) M_t^e \Big) \mathbb{I}_{t_0^j = t} \Bigg) dj \\
& + u_t b - U_t(u_t - u_{t-1}(1 - f(\theta_t^U))) + \eta(\theta_t^U) u_{t-1} f(\theta_t^U) \Bigg\}.
\end{aligned} \tag{E.56}$$

Note that the first three lines are equivalent to the incumbent firms' and entrants' problems in the market equilibrium. Solving the last line with respect to u_t and θ_t^U gives the following two first-order conditions:

$$b - U_t + \beta\left(U_t(1 - f(\theta_{t+1}^U)) + f(\theta_{t+1}^U)x_{t+1}(\theta_{t+1}^U)\right) = 0 \tag{E.57}$$

$$-f'(\theta_t^U)U_t + f'(\theta_t^U)x_t(\theta_t^U) + x_t'(\theta_t^U)f(\theta_t^U) = 0, \tag{E.58}$$

where (E.57) is equivalent to the unemployed workers' value function, and (E.58) is identical to their optimal choice in the competitive equilibrium.

Therefore, this shows that we can find a solution for the constrained social planner's *problem to be competitive equilibrium. In other words, under both search and information frictions, the competitive equilibrium is the first best allocation. This is consistent with standard directed search literature.*

Next, I prove the social planner's problem under no uncertainty and confirm the inefficiency of the decentralized allocation. In other words, if there is no uncertainty about the firm's productivity type ($\sigma_\varepsilon = 0$ and given σ_0), the model's decentralized block-recursive equilibrium can be replicated by a social planner's problem with a search friction only, and thus is efficient.

Now we assume that the social planner can see exact firm type. Thus, the information friction is no longer existent. In that case, the social planner's problem can be written as:

$$\max_{\substack{u_t, v_t, M_t^e, g(l_t),\\ \mathbf{d}(\nu, l_{t-1}),\\ \mathbf{s}(\nu, l_{t-1}),\\ \mathbf{h}(\nu, l_{t-1}),\\ \theta(\nu, l_{t-1}),\\ \theta^{\mathbf{E}}(\nu, l_{t-1}),\\ \mathbf{l}(\nu, l_{t-1}),\\ \theta_t^U, d_t^e(\nu), l_t^e(\nu)}} \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \Bigg\{ u_t b - c v_t + \sum_{(\nu, l_{t-1})} g(l_{t-1}) f(\nu)(1 - \mathbf{d}(\nu, l_{t-1}))\left(e^\nu l_t^\alpha - c_f\right)$$

$$+ M_t^e \left(\sum_{\nu} f(\nu)(1 - d_t^e(\nu))\left(e^\nu l_t^e(\nu)^\alpha - c_f\right) - c_e \right) \Bigg\}, \quad \text{(E.59)}$$

subject to

$$l_t = (1 - s_t)(1 - \lambda f(\theta_t^E)) l_{t-1} + h_t \quad \text{(E.60)}$$

$$v_t = \theta_t^U u_t + \sum_{(\nu, l_{t-1})} \lambda \theta_t^E(\nu, l_{t-1}) l_{t-1} g(l_{t-1}) f(\nu) \quad \text{(E.61)}$$

$$u_t = (1 - f(\theta_t^U)) u_{t-1} + \sum_{\nu, l_{t-1}} (d_t + (1 - d_t) s_t) l_{t-1} g(l_{t-1}) f(\nu) \quad \text{(E.62)}$$

$$g(l_t) = \sum_{\nu, l_{t-1}} f(\nu) g(l_{t-1})(1 - \mathbf{d}(\nu, l_{t-1}) \mathbb{I}_{l_{(\nu, l_{t-1})} = l_t} \quad \text{(E.63)}$$

$$+\sum_{\nu} M_t^e f(\nu)(1-d_t^e(\nu))\mathbb{I}_{l_t^e(\nu)=l_t} \tag{E.64}$$

$$h_t(1-d_t) = f(\theta_t^U)u_t \textit{ for firms searching in market } \theta^U \textit{ (i.e. } \theta(\nu, l_{t-1}) = \theta^U\textit{)} \tag{E.65}$$

$$h_t(1-d_t) = \lambda f(\theta_t^E)(1-s_t)l_{t-1} \textit{ for firms poaching from market } \theta^E \textit{ (i.e. } \theta(\nu, l_{t-1}) = \theta^E\textit{)} \tag{E.66}$$

Following the same trick, it is obvious to prove that the competitive equilibrium under the full information is also socially optimal as it can be replicated by the social planner's problem (E.59).

These results verify that the model's decentralized block-recursive allocation given the level of uncertainty is socially optimal. If the planner could resolve uncertainty, the decentralized allocation would be distorted due to the uncertainty.

F Computation Algorithm

F.1 Guess $\mathbf{V}^{init}$

We start with our guess $\mathbf{V}^{init0}(a, \tilde{P}, l, P)$ for $\mathbf{V}^{init}(a, \tilde{P}, l, P)$.[5]

F.2 Use Free-entry Condition

1. Get $\mathbb{E}_{P'}\mathbf{V}^{init}(1, \ln P, l^e, P')$

   For each possible grid points for P, use $\ln P' \sim N(\frac{\frac{\tilde{\nu}_0}{\sigma_0^2}+\frac{\ln P}{\sigma_\epsilon^2}}{\frac{1}{\sigma_0^2}+\frac{1}{\sigma_\epsilon}}, \frac{1}{\frac{1}{\sigma_0^2}+\frac{1}{\sigma_\epsilon}} + \sigma_\epsilon^2)$.

2. Guess κ

3. Find l^e and d^e that solves:

$$\max_{d^e, l^e}\Big[(1-d^e)\Big(P(l^e)^\alpha - c^f - \kappa l^e + \beta\mathbb{E}_{P'}\mathbf{V}^{init0}(1, \ln P, l^e, P')\Big)\Big], \tag{F.67}$$

[5]Here, for notational convenience, I will use $\tilde{P}$ and l to refer to the average log productivity and employment size in the previous period, respectively. Note that P is the current period productivity. Variables with $'$ refer to their value in the next period, i.e. $\tilde{P}'$ is the average log productivity up to the current period, l' is the current period employment size after all decisions made (for hiring, retention, and layoffs, etc.), and P' is the next period productivity.

for each possible P, and adjust κ with a bisection method until it satisfies

$$\int \max_{d^e, l^e}\Big[(1-d^e)\Big(P(l^e)^\alpha - c^f - \kappa l^e + \beta \mathbb{E}_{P'}\mathbf{V}^{init0}(1, \ln P, l^e, P')\Big)\Big] dF_e(P) = c^e,$$

where $\ln P \sim N(\bar{\nu}_0, \sigma_0^2 + \sigma_\epsilon^2)$.

F.3 Unemployed Workers' Problem

Use the solution for x^U,

$$x^U = \kappa - (c^\gamma(\kappa - \mathbf{U}))^{\frac{1}{1+\gamma}} \tag{F.68}$$

and solve a fixed-point problem for $\mathbf{U}$ from the following:

$$\mathbf{U} = b + \beta\Big((1 - f(\theta(x^U))\mathbf{U} + f(\theta(x^U))x^U\Big), \tag{F.69}$$

with (C.39).

F.4 Value Function Iteration

1. Generate $\mathbb{E}\mathbf{V}^{init0}(a+1, \tilde{P}', l', P') = \mathbb{E}\mathbf{V}^{init0}(a+1, \frac{a\tilde{P}+\ln P}{(a+1)}, l', P')$.

   Given state variables $(a, \tilde{P}, l, P)$ and $\ln P' \sim (\frac{\frac{\bar{\nu}_0}{\sigma_0^2} + \frac{a\tilde{P}+\ln P}{\sigma_\epsilon^2}}{\frac{1}{\sigma_0^2} + \frac{a+1}{\sigma_\epsilon^2}}, \frac{1}{\frac{1}{\sigma_0^2} + \frac{a+1}{\sigma_\epsilon^2}} + \sigma_\epsilon^2)$, I use the interpolation of $\mathbf{V}^{init0}$ evaluated at each $(a+1, \frac{a\tilde{P}+\ln P}{a+1}, l', P')$ and take expectation across $\ln P'$.

2. Use grid search to max $\mathcal{V}$ and obtain the argmax gridpoint l'.

   For each possible combination of l and l', given $(a, \tilde{P}, l, P)$:

   (a) Step 1: for hiring/inaction case $(l' \geq l)$

$$x^E = \kappa - c \tag{F.70}$$

$$s = 0 \tag{F.71}$$

$$h = l' - (1 - \lambda f(\theta(x^E)))l = l' - l \tag{F.72}$$

(b) Step 2: for separation case ($l' < l$)

$$x^E = \max\left(x_1^E, x_2^E\right) \tag{F.73}$$
$$s = 1 - \frac{l'}{(1-\lambda f(\theta(x^E)))l} \tag{F.74}$$
$$h = 0 \tag{F.75}$$

where x_1^E refers to the promised utility level to incumbent workers in a firm facing both layoffs and quits, and is pinned down by the root of the following:

$$\kappa - \mathbf{U} = c\Big((1+\theta(x^E)^\gamma)^{1+\frac{1}{\gamma}} - \lambda\theta(x^E)^{1+\gamma}\Big), \tag{F.76}$$

and x_2^E refers to that in a firm having quits only, and is the root of the following:

$$\frac{l-l'}{\lambda l} = f(\theta(x^E)) = \Big(1-(\frac{\kappa - x^E}{c})^{-\gamma}\Big)^{\frac{1}{\gamma}} \tag{F.77}$$
$$x^E = \kappa - c\Big(1-(\frac{l-l'}{\lambda l})^\gamma\Big)^{-\frac{1}{\gamma}}$$

Thus, from the above steps, we have

$$\mathbf{x^E}(a,\tilde{P},l,P,l'), \mathbf{s}(a,\tilde{P},l,P,l'), \mathbf{h}(a,\tilde{P},l,P,l') \tag{F.78}$$

and

$$\tilde{\mathbf{W}}(a,\tilde{P},l,P,l') = \kappa - (\kappa - x^E(a,\tilde{P},l,P,l'))^{1+\gamma}c^{-\gamma} \tag{F.79}$$

for each possible set of (l,l') and the state variables.

Using it, we find a gridpoint l' that solves the following maximization:

$$\begin{aligned}\mathcal{V}(a,\tilde{P},l,P) \equiv & \max_{l'} \mathbf{s}(a,\tilde{P},l,P,l')\mathbf{U}l + Pl'^\alpha - c^f - \kappa\mathbf{h}(a,\tilde{P},l,P,l') \\ & + (1-\mathbf{s}(a,\tilde{P},l,P,l'))\lambda f(\theta(\mathbf{x^E}(a,\tilde{P},l,P,l')))x^E(a,\tilde{P},l,P,l')l + \beta\mathbb{E}\mathbf{V}^{init0}(a+1,\frac{a\tilde{P}+\ln P}{a+1},l',P').\end{aligned} \tag{F.80}$$

3. Spline approximation for l'

Let I be the optimal index for l' that maximizes $\mathcal{V}$, given $(a, \tilde{P}, l, P)$. Now, we would like to spline approximate $\mathcal{V}$ across the points l_{I-1}, l_I, and l_{I+1} to get a proper policy function.

(a) Step 1: use the spline approximated form of $\mathcal{V}$

$$\mathcal{V} \;= \mathcal{V}_i(l) \quad \text{if} \quad l_i \leq l \leq l_{i+1}$$

where

$$\begin{aligned}
\mathcal{V}_i(l) &= a_i(l-l_i)^3 + b_i(l-l_i)^2 + c_i(l-l_i) + \mathcal{V}_i(l_i) \\
\mathcal{V}_i'(l) &= 3a_i(l-l_i)^2 + 2b_i(l-l_i) + c_i \\
\mathcal{V}_i''(l) &= 6a_i(l-l_i) + 2b_i.
\end{aligned}$$

(b) Conditions to use

$$\begin{aligned}
\mathcal{V}_i(l_i) &= \mathcal{V}_{i-1}(l_i) \\
\mathcal{V}_i'(l_i) &= \mathcal{V}_{i-1}'(l_i) \\
\mathcal{V}_i''(l_i) &= \mathcal{V}_{i-1}''(l_i)
\end{aligned}$$

$\rightarrow$ Using the functional form for $\mathcal{V}_i$ above, these conditions are rephrased as follows:

$$\Delta\mathcal{V}_i(l_i) = a_{i-1}(l_i-l_{i-1})^3 + b_{i-1}(l_i-l_{i-1})^2 + c_{i-1}(l_i-l_{i-1}) \tag{F.81}$$

$$c_i = 3a_{i-1}(l_i-l_{i-1})^2 + 2b_{i-1}(l_i-l_{i-1})^2 + c_{i-1} \tag{F.82}$$

$$2b_i = 6a_{i-1}(l_i-l_{i-1}) + 2b_{i-1} \tag{F.83}$$

(c) Generate coefficient matrix

We can convert (F.81), (F.82), and (F.83), for $i = 2, 3, ..., N$ (N is the number of l grid points), into a matrix form. Let

$$Coeff = \begin{pmatrix} a_1 & b_1 & c_1 & \dots & a_{N-1} & b_{N-1} & c_{N-1} \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \end{pmatrix}. \tag{F.84}$$

Then, we could get this by

$$Coeff = DV * inv(H), \tag{F.85}$$

where

$$H = \begin{pmatrix}
(l_2-l_1)^3 & 0 & 0 & \dots & 0 & 3(l_2-l_1)^2 & 0 & \dots & 0 & 6(l_2-l_1) & 0 & \dots & 0 & 0 & 0 \\
(l_2-l_1)^2 & 0 & 0 & \dots & 0 & 2(l_2-l_1) & 0 & \dots & 0 & 2 & 0 & \dots & 0 & 0 & 0 \\
(l_2-l_1) & 0 & 0 & \dots & 0 & 1 & 0 & \dots & 0 & 0 & 0 & \dots & 0 & 1 & 0 \\
0 & (l_3-l_2)^3 & 0 & \dots & 0 & 0 & 3(l_3-l_2)^2 & \dots & 0 & 0 & 6(l_3-l_2) & \dots & 0 & 0 & 0 \\
0 & (l_3-l_2)^2 & 0 & \dots & 0 & 0 & 2(l_3-l_2) & \dots & 0 & -2 & 2 & \dots & 0 & 0 & 0 \\
0 & (l_3-l_2) & 0 & \dots & 0 & -1 & 1 & \dots & 0 & 0 & 0 & \dots & 0 & 0 & 0 \\
\dots & \dots & \dots & \dots & \dots & 0 & 0 & \dots & 0 & 0 & 0 & \dots & 0 & 0 & 0 \\
\dots & \dots & \dots & \dots & \dots & 0 & 0 & \dots & 0 & 0 & -2 & \dots & 0 & 0 & 0 \\
\dots & \dots & \dots & \dots & \dots & 0 & -1 & \dots & 0 & 0 & 0 & \dots & 0 & 0 & 0 \\
\dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & 0 & 0 \\
\dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & 1 & 0 & 0 & \dots & 0 & 0 & 0 \\
0 & 0 & 0 & \dots & (l_N-l_{N-1})^3 & 0 & \dots & \dots & 0 & 0 & 0 & \dots & 0 & 0 & 3(l_N-l_{N-1})^2 \\
0 & 0 & 0 & \dots & (l_N-l_{N-1})^2 & 0 & \dots & \dots & 0 & 0 & 0 & \dots & -2 & 0 & 2(l_N-l_{N-1}) \\
0 & 0 & 0 & \dots & (l_N-l_{N-1}) & 0 & \dots & \dots & -1 & 0 & 0 & \dots & 0 & 0 & 1
\end{pmatrix}$$

and

$$DV = \begin{pmatrix}
\Delta\mathcal{V}(l_2) & \Delta\mathcal{V}(l_3) & 0 & \dots & \Delta\mathcal{V}(l_N) & 0 & 0 & \dots & 0 & 0 & \frac{\Delta\mathcal{V}(l_2)}{l_2-l_1} & \frac{\Delta\mathcal{V}(l_N)}{l_N-l_{N-1}} \\
\Delta\mathcal{V}(l_2) & \Delta\mathcal{V}(l_3) & 0 & \dots & \Delta\mathcal{V}(l_N) & 0 & 0 & \dots & 0 & 0 & \frac{\Delta\mathcal{V}(l_2)}{l_2-l_1} & \frac{\Delta\mathcal{V}(l_N)}{l_N-l_{N-1}} \\
\Delta\mathcal{V}(l_2) & \Delta\mathcal{V}(l_3) & 0 & \dots & \Delta\mathcal{V}(l_N) & 0 & 0 & \dots & 0 & 0 & \frac{\Delta\mathcal{V}(l_2)}{l_2-l_1} & \frac{\Delta\mathcal{V}(l_N)}{l_N-l_{N-1}} \\
\dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\
\Delta\mathcal{V}(l_2) & \Delta\mathcal{V}(l_3) & 0 & \dots & \Delta\mathcal{V}(l_N) & 0 & 0 & \dots & 0 & 0 & \frac{\Delta\mathcal{V}(l_2)}{l_2-l_1} & \frac{\Delta\mathcal{V}(l_N)}{l_N-l_{N-1}}
\end{pmatrix}$$

where the number of each matrix is the same as $3 * (N-1)$, and the number of rows in $Coeff$ and DV is $(na * n\tilde{P} * N * nP)$, and each row is for each pair of state variables $(a, \tilde{P}, l, P')$.

(d) Get the root of l'

Once we have $Coeff$, we derive the root of l' from each $\mathcal{V}_{I-1}$ and $\mathcal{V}_I$. This means to find l', such that

$$\mathcal{V}'_{I-1}(l) = a_{I-1}(l - l_{I-1})^2 + b_{I-1}(l - l_{I-1}) + c_{I-1} = 0$$

and

$$\mathcal{V}'_I(l) = a_I(l - l_I)^2 + b_I(l - l_I) + c_I = 0$$

Thus, we have four possible roots of l' from the spline approximation:

$$l' = \Big[\frac{-B_{I-1} \pm \sqrt{B_{I-1}^2 - 4A_{I-1}C_{I-1}}}{2A_{I-1}}, \frac{-B_I \pm \sqrt{B_I^2 - 4A_IC_I}}{2A_I}\Big] \tag{F.86}$$

where

$$\begin{aligned} A_i &= 3a_i \\ B_i &= 2b_i - 6a_il_i \\ C_i &= 3a_il_i^2 + 2b_il_i + c_i, \quad for \quad i \in \{I-1, I\} \end{aligned}$$

(e) Evaluate $\mathcal{V}$ and the corresponding policy function l'

We evaluate

$$\max[\mathcal{V}(l_1'), \mathcal{V}(l_2'), \mathcal{V}(l_3'), \mathcal{V}(l_4'), \mathcal{V}],$$

and obtain

$$l'(a, \tilde{P}, l, P) = argmax[\mathcal{V}(l_1'), \mathcal{V}(l_2'), \mathcal{V}(l_3'), \mathcal{V}(l_4'), \mathcal{V}]. \tag{F.87}$$

Note that $l_1' \sim l_4'$ are the roots based on (F.86), and the first $\mathcal{V}(l_1') \sim \mathcal{V}(l_4')$ are spline approximated $\mathcal{V}$ evaluated at each root, and the last $\mathcal{V}$ is the maximized value from the grid search.

(f) Managing inaction ranges

For the inaction range, such that $l_I(a, \tilde{P}, l, P) = l$, we don't use spline approximation for $\mathcal{V}(a, \tilde{P}, l, P)$.

4. Policy functions

We use (F.78) and (F.87) to back out policy functions for

$$\begin{aligned} \mathbf{x}^{\mathbf{E}}(a, \tilde{P}, l, P) &\equiv \mathbf{x}^{\mathbf{E}}(a, \tilde{P}, l, P, l') \\ \mathbf{s}(a, \tilde{P}, l, P) &\equiv \mathbf{s}(a, \tilde{P}, l, P, l') \\ \mathbf{h}(a, \tilde{P}, l, P) &\equiv \mathbf{h}(a, \tilde{P}, l, P, l'), \end{aligned}$$

and

$$\mathbf{d}(a,\tilde{P},l,P) = \begin{cases} 1 & \text{if} \quad \mathbf{U}l > \mathcal{V}(a,\tilde{P},l,P) \\ 0 & \text{otherwise.} \end{cases} \tag{F.88}$$

5. Update the Guess

$$\mathbf{V}^{init1}(a,\tilde{P},l,P) = \Big(\delta + (1-\delta)\mathbf{d}(a,\tilde{P},l,P)\Big)\mathbf{U}l + (1-\delta)(1-\mathbf{d}(a,\tilde{P},l,P))\mathcal{V}(a,\tilde{P},l,P) \tag{F.89}$$

If $|\mathbf{V}^{init0} - \mathbf{V}^{init1}| < \epsilon$, with sufficiently small ϵ, then it's done! Otherwise, replace $\mathbf{V}^{init0}$ with a new guess $\mathbf{V}^{init1}$ and reiterate from the part B.2.

G Figures for Low-performing Firms

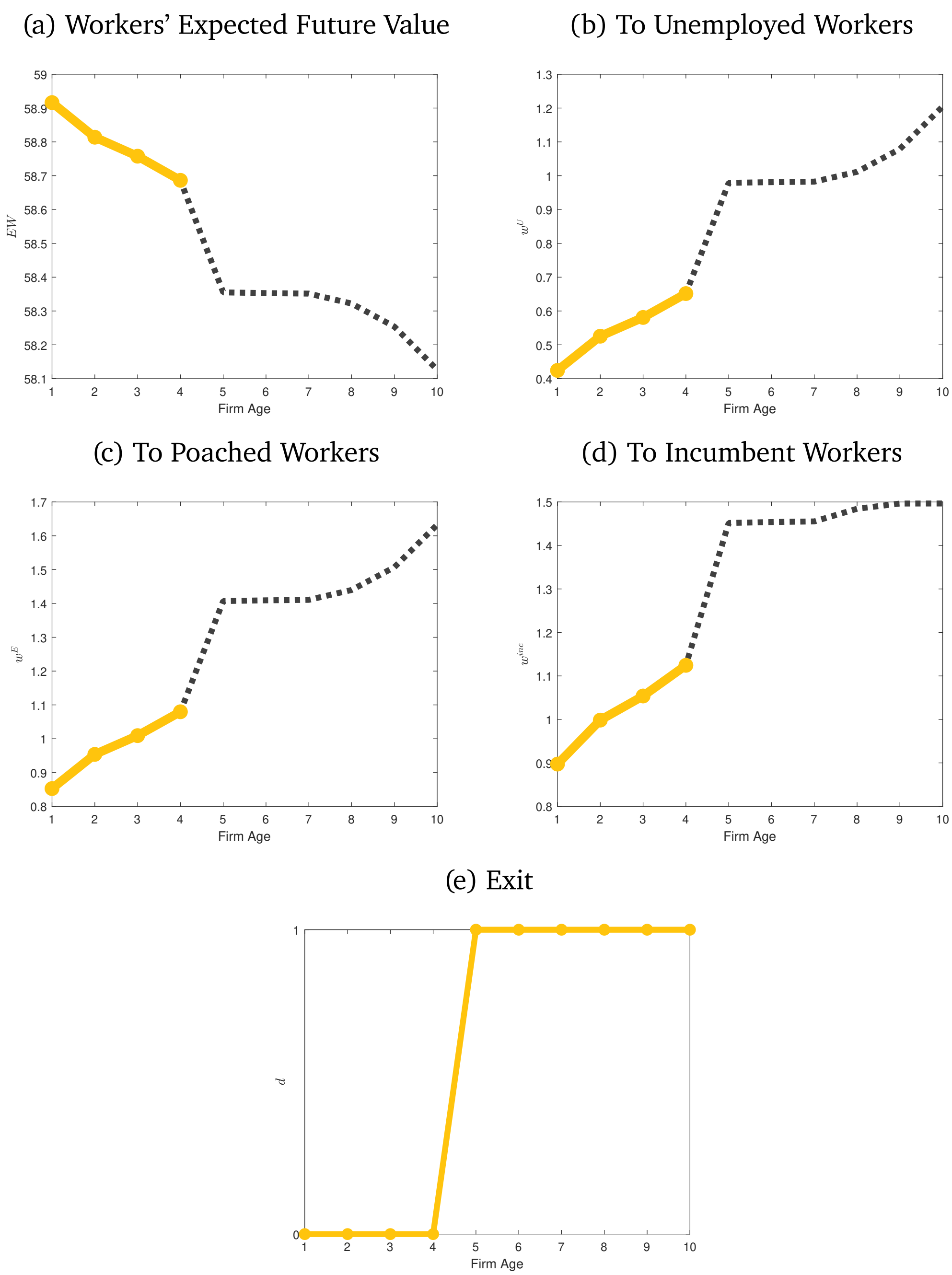

Figure G.1: Low Performing Firms (average size)

[5]The dotted grey lines indicate counterfactual series if firms continued operating.

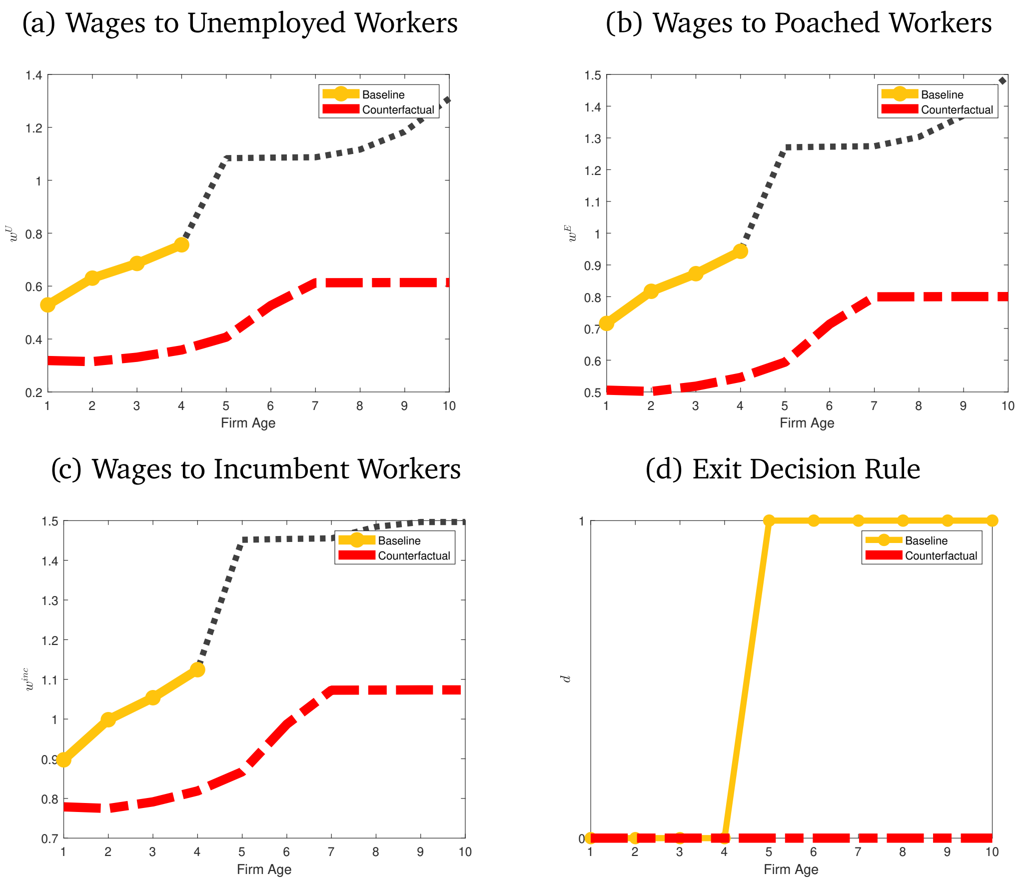

Figure G.2: Low-performing Firms: Baseline vs. Counterfactual (higher uncertainty)

[5]The dotted grey lines indicate counterfactual series if firms continued operating.

H Data Appendix

H.1 Longitudinal Business Database (LBD)

The LBD tracks the universe of U.S. business establishments and firms that have at least one paid employee, annually from 1976 onward. Establishments that are owned by a parent firm are grouped under a common firm identifier, which allows me to aggregate establishment-level activities to the firm level. The LBD contains basic information such as employment, payroll, revenue, NAICS codes, employer identification numbers, business name, and location, which enables me to measure firm size, age, entry, exit, productivity, and employment growth.[6]

H.1.1 Longitudinal Firm Identifiers

One limitation of the LBD is the lack of longitudinally consistent firm identifiers.[7] However, longitudinal consistency of firm identifiers is necessary for my analysis to track firms' history of performance as well as to estimate noise components in firm type learning process. Therefore, I construct and use longitudinal firm identifiers following Dent et al. (2018). Henceforth, I will use the term "firm identifier" to refer to the longitudinal firm identifiers constructed using this method.

H.2 Longitudinal Employer Household Dynamics (LEHD)

The LEHD is constructed from quarterly Unemployment Insurance (UI) system wage reports of states participating in the program, which collect quarterly earnings and employment information, along with demographic information.[8] The data cover

[6] Jarmin and Miranda (2002), Haltiwanger et al. (2016), and Chow et al. (2021) contain more detailed information about the LBD. Fort and Klimek (2018) construct time-consistent NAICS codes for LBD establishments after the implementation of a change from the SIC to NAICS in 1997.

[7] Although the redesigned LBD has a new firm identifier that links firms across time by correcting previous firm identifiers that are recycled in the old LBD, it is still not yet a true longitudinal identifier and has not yet resolved firm reorganization issues. See more discussion in Chow et al. (2021).

[8] The earnings data in the LEHD are reported on a quarterly basis, which include all forms of compensation that are taxable.

over 95 percent of private sector workers, and the length of time series varies across states covered by the LEHD. I have access to 29 states covering over 60 percent of U.S. private sector employment.[9] The data enable me to identify worker heterogeneity, employment history, and job mobility. Linking the LEHD to the LBD with a crosswalk between employer identification numbers (EINs) and state-level employer identification numbers (SEINs), I track employer information for each job. The UI data, the main source of the LEHD, assign firms a state-level employer identification number (SEIN) that captures the activity of a firm within a state.

H.2.1 Main Jobs

The LEHD defines a job as the presence of an individual-employer match, with earnings defined as the amount earned from that job during the quarter. However, it does not record the start and end dates of a job, which makes the total number of weeks during that quarter unknown. To avoid potential bias from this, I follow the literature and restrict my analysis to full-quarter main jobs that give the highest earnings in a given quarter and are present for the quarter prior to and the quarter after the focal quarter. For any worker-quarter pairs that are associated with multiple jobs paying the same earnings, I pick the job that shows up the most frequently in the worker's job history. This leaves one main job observation for each worker-quarter pair.

H.2.2 Previous Employment Status

Following Haltiwanger et al. (2018), I can identify workers' previous job using a within/adjacent quarter approach, which allows for a brief nonemployment period between workers' last day on the previous job and their first day on the contemporaneous job. Therefore, workers are identified as previously employed if they had

[9] The 29 states are AL, AZ, CA, CO, CT, DE, ID, IN, KS, MD, ME, ND, NE, NJ, NM, NV, NY, OH, OK, OR, PA, SD, TN, TX, UT, VA, WA, WI, and WY.

at least one full-quarter job within the most recent three quarters before t, and as non-employed if they had no full-quarter jobs within those three quarters.

Note that restricting the sample to full-quarter main jobs makes use of the three-quarter duration to define previous jobs. For notational convenience, let $(t - q1)$ denote the quarter prior to t, and $(t - q2)$ denote two quarters prior to t, and so on. If a worker had any full-quarter jobs at either $(t-q1)$ or $(t-q2)$, this implies that the worker must have moved to the contemporaneous job within quarter $(t - q1)$. The latter could happen if the worker had some overlapping period between $(t-q1)$ and t in job transition. If a worker had any full-quarter jobs at $(t-q3)$, this means that the worker must have left the job at $(t-q2)$, had a brief nonemployment period between $(t - q2)$ and $(t - q1)$, and joined the contemporaneous job at $(t - q1)$. Alternatively, the within quarter approach identifies workers as previously employed if they had at least one full-quarter job within the latest two quarters before t, where the previous job is defined by the most recent main full-quarter job within the most recent two quarters before t.

In the LEHD, I identify workers who did not have employment in any states during the previous period, i.e., those who had no earnings from any states in any of the three most recent quarters before time t, as unemployed. For this group, I set their previous employer fixed effect to zero and introduce a dummy variable indicating their non-employment status. Additionally, I set the previous employed fixed effect to zero and include a dummy variable for those employed in states beyond the scope of my data in the previous period, where I lack information about their previous employer and earnings.

H.3 Summary Statistics

Table H1: Summary Statistics

A. Worker-year Sample	Mean (sd)	B. Firm-year Sample	Mean (sd)
Worker Age	40.05	Firm Size	10.42
	(14.67)		(50.2)
Earnings (2009$)	9,670	Firm Age	5.492
	(27,830)		(3.347)
Earnings (log, 2009$)	8.697	Revenue (thousands, 2009$)	1,633
	(1.027)		(7,736)
Job Tenure (years)	3.66	Revenue Productivity (log, 2009$)	4.764
	(2.6)		(1.041)
Highest Education	2.68	Employment Growth (DHS)	0.0174
	(1.025)		(0.382)
Observations	50,170,000	Observations	6,959,000

Notes: The table presents summary statistics for the main regression samples. Panel A displays statistics for the worker-year level sample, while Panel B presents statistics for the firm-year level sample. The first row of each variable indicates the mean, and the second row (in brackets) displays the standard deviation. Jobs are defined by the full-quarter main job in the first quarter of each year. Highest education categorizes workers based on their highest level of education attainment (1 - Less than high school, 2 - High school, 3 - Some college, 4 - Bachelor's degree or higher). All nominal variables are adjusted to 2009 dollars. Observation counts are rounded to the nearest 10,000 to avoid potential disclosure risks.

I Full Tables

Table I2: Wage Differentials for Young Firms

	(1) Earnings Residuals	(2) Earnings Residuals
Young firm	-0.002***	-0.003***
	(0.001)	(0.001)
Young firm × High performing firm	0.015***	0.016***
	(0.001)	(0.001)
High performing firm	0.002	0.002
	(0.001)	(0.001)
Average Firm Productivity (up to $t-1$)	0.009***	0.012***
	(0.001)	(0.001)
Current Productivity (at t)	0.020***	0.015***
	(0.001)	(0.001)
Firm Size (at t)	0.017***	
	(0.001)	
Firm Size (at $t-1$)		0.013***
		(0.001)
Previous Employer (AKM)	0.267***	0.270***
	(0.001)	(0.001)
Observations	50,170,000	50,170,000
Fixed effects	Industry, State	Industry, State

Notes: The table reports the full results for the main earnings regression. Firm controls include cumulative average productivity, current productivity, and log employment size. Controls associated with worker's previous employment status are the AKM firm fixed effect associated with the previous employer and a dummy for non-employed workers in the previous period. Observation counts are rounded to the nearest 10,000 to avoid potential disclosure risks. Estimates for constant, industry, state fixed effects, the coefficient of the indicator for worker's previous non-employment status are suppressed. Observations are unweighted. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table I3: The Effect of Wage Differentials on Firm Outcomes

A. Raw Productivity	(1) Hire (firm level)	(2) Hire (SEIN level)	(3) Employment Growth (log difference)	(4) Employment Growth (DHS)
Average Earnings Residuals	-0.520***	-0.387***	-0.015***	-0.018***
	(0.020)	(0.024)	(0.000)	(0.000)
Firm Productivity	0.588***	0.302***	0.092***	0.102***
	(0.033)	(0.035)	(0.000)	(0.000)
Firm Size	7.964***	6.230***	-0.040***	-0.048***
	(0.133)	(0.068)	(0.000)	(0.000)
Firm Age	0.039***	0.007	-0.001***	-0.001***
	(0.008)	(0.008)	(0.000)	(0.000)
Observations	6,959,000	6,959,000	6,959,000	6,959,000
Fixed effects	Industry, State	Industry, State	Industry, State	Industry, State

B. Estimated Productivity	(1) Hire (firm level)	(2) Hire (SEIN level)	(3) Employment Growth (log difference)	(4) Employment Growth (DHS)
Average Earnings Residuals	-0.498***	-0.369***	-0.012***	-0.015***
	(0.0195)	(0.0244)	(0.0003)	(0.0003)
Average Productivity up to (t-1)	-0.904***	-0.845***	-0.095***	-0.108**
	(0.035)	(0.050)	(0.000)	(0.001)
Current Productivity at t	1.31***	0.924***	0.176***	0.197***
	(0.039)	(0.044)	(0.000)	(0.001)
Firm Size	7.998***	6.259***	-0.035***	-0.043***
	(0.134)	(0.068)	(0.000)	(0.000)
Firm Age	0.042***	0.009	-0.001***	-0.001***
	(0.008)	(0.008)	(0.000)	(0.000)
Observations	6,959,000	6,959,000	6,959,000	6,959,000
Fixed effects	Industry, State	Industry, State	Industry, State	Industry, State

Notes: The table reports the full results for the effect of earnings residuals on firm-level outcomes. Firm controls include firm productivity, log employment size, and age. Note that Panel A uses the raw value of firm productivity, while Panel B adopts the cross-time average value as well as the current value of the estimated firm productivity as in the main regressions. Column (1) uses the firm-level total new hires, and column (2) uses the average of the SEIN-level new hires. Observation counts are rounded to the nearest 10,000 to avoid potential disclosure risks. Estimates for constant, and industry, state fixed effects are suppressed. Observations are unweighted. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table I4: The Effect of Uncertainty on Young Firms' Wage Differentials

	(1) Earnings Residuals	(2) Earnings Residuals	(3) Earnings Residuals	(4) Earnings Residuals
Young firm	-0.001	-0.001	-0.001	-0.002
	(0.001)	(0.001)	(0.002)	(0.002)
Young firm × High performing firm	0.012***	0.012***	0.003	0.005**
	(0.002)	(0.002)	(0.002)	(0.002)
Young firm × Uncertainty (at t)	-0.004**	-0.004***		
	(0.002)	(0.002)		
Young firm × High performing firm × Uncertainty (at t)	0.006***	0.006***		
	(0.002)	(0.002)		
Young firm × Uncertainty (at $t-1$)			-0.005**	-0.004*
			(0.002)	(0.002)
Young firm × High performing firm × Uncertainty (at $t-1$)			0.016***	0.015***
			(0.003)	(0.003)
High performing firm	-0.022***	-0.022***	0.004**	0.003*
	(0.001)	(0.001)	(0.002)	(0.002)
Uncertainty	-0.033***	-0.033***	-0.067***	-0.071***
	(0.001)	(0.001)	(0.002)	(0.002)
Uncertainty × High performing firm	0.028***	0.028***	-0.004**	-0.002
	(0.001)	(0.001)	(0.002)	(0.002)
Average Firm Productivity (up to $t-1$)	0.009***	0.011***	0.009***	0.011***
	(0.000)	(0.000)	(0.000)	(0.000)
Current Productivity (at t)	0.020***	0.016***	0.020***	0.016***
	(0.000)	(0.000)	(0.000)	(0.000)
Firm Size (at t)	0.012***		0.012***	
	(0.000)		(0.000)	
Firm Size (at $t-1$)		0.010***		0.010***
		(0.000)		(0.000)
Previous Employer (AKM)	0.269***	0.271***	0.269***	0.2716***
	(0.000)	(0.000)	(0.000)	(0.000)
Observations	50,170,000	50,170,000	50,170,000	50,170,000
Fixed effects	State, Sector	State, Sector	State, Sector	State, Sector

Notes: The table reports the full results for the earnings regression interacted with industry-level uncertainty. The set of controls for firm characteristics and worker previous employment status remain the same as in the baseline regression. Columns (1) and (3) incorporate the current value of firm size, while columns (2) and (4) use the lagged value of firm size. In addition, columns (1) and (2) are based on the current level of uncertainty, whereas columns (3) and (4) utilize the lagged uncertainty value. Observation counts are rounded to the nearest 10,000 to avoid potential disclosure risks. Estimates for constant, industry, state fixed effects, the coefficient of the indicator for worker's previous non-employment status are suppressed. Observations are unweighted. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

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Table J5: Wage Differentials for Young Firms (excluding firm size)

	(1) Earnings Residuals	(2) Earnings Residuals
Young firm	-0.006***	-0.007***
	(0.001)	(0.001)
Young firm × High performing firm	0.013***	0.015***
	(0.001)	(0.001)
High performing firm	0.005***	0.004***
	(0.001)	(0.001)
Average Firm Productivity (up to $t-1$)	0.016***	0.006***
	(0.001)	(0.001)
Current Productivity (at t)		0.015***
		(0.001)
Previous Employer (AKM)	0.283***	0.281***
	(0.001)	(0.001)
Observations	50,170,000	50,170,000
Fixed effects	Industry, State	Industry, State

Notes: The table reports the earnings regression results. Firm controls include cumulative average productivity and current productivity (but not log employment size). Controls associated with worker's previous employment status are AKM firm fixed effect associated with the previous employer and a dummy for non-employed workers in the previous period. Observation counts are rounded to the nearest 10,000 to avoid potential disclosure risks. Estimates for constant, industry, state fixed effects, the coefficient of the indicator for worker's previous non-employment status are suppressed. Observations are unweighted. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table J6: Wage Differentials for Young Firms (propensity score weighted)

	(1) Earnings Residuals	(2) Earnings Residuals	(3) Earnings Residuals	(4) Earnings Residuals
Young firm	-0.007***	-0.008***	-0.003***	-0.003***
	(0.001)	(0.001)	(0.001)	(0.001)
Young firm × High performing firm	0.015***	0.018***	0.019***	0.019***
	(0.001)	(0.001)	(0.001)	(0.001)
High performing firm	0.004***	0.002*	-0.000	0.000
	(0.001)	(0.001)	(0.001)	(0.001)
Average Firm Productivity	0.017***	0.003***	0.005***	0.009***
(up to $t-1$)	(0.001)	(0.001)	(0.001)	(0.001)
Current Productivity (at t)		0.021***	0.027***	0.021***
		(0.001)	(0.001)	(0.001)
Firm Size			0.020***	
			(0.000)	
Firm Size (at $t-1$)				0.015***
				(0.000)
Previous Employer (AKM)	0.281***	0.278***	0.266***	0.269***
	(0.001)	(0.001)	(0.001)	(0.001)
Observations	50,170,000	50,170,000	50,170,000	50,170,000
Fixed effects	Industry, State	Industry, State	Industry, State	Industry, State

Notes: The table reports results for regression of earning residuals on young firm and high performing firm indicators. Firm controls include cross-time average productivity level, current productivity level, and log employment size. Controls associated with worker's previous employment status are AKM firm fixed effect associated with the previous employer and a dummy for non-employed workers in the previous period. Observation counts are rounded to the nearest 10,000 to avoid potential disclosure risks. Estimates for constant, industry, state fixed effects, the coefficient of the indicator for worker's previous non-employment status are suppressed. Observations are weighted with inverse propensity score weights of author's own construction. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table J7: Wage Differentials for Young Firms (bootstrapped standard errors)

	(1) Earnings Residuals	(2) Earnings Residuals	(3) Earnings Residuals
Young firm	-0.006***	-0.007***	-0.002***
	(0.001)	(0.001)	(0.001)
Young firm $\times$ High performing firm	0.013***	0.015***	0.015***
	(0.002)	(0.002)	(0.002)
High performing firm	0.005***	0.004*	0.002
	(0.002)	(0.002)	(0.002)
Average Firm Productivity (up to $t-1$)	0.016***	0.006***	0.009***
	(0.000)	(0.001)	(0.001)
Current Productivity (at t)		0.015***	0.020***
		(0.001)	(0.001)
Firm Size			0.017***
			(0.000)
Previous Employer (AKM)	0.283***	0.281***	0.267***
	(0.001)	(0.001)	(0.001)
Observations	50,170,000	50,170,000	50,170,000
Fixed effects	Industry, State	Industry, State	Industry, State

Notes: The table reports the earnings regression results. Firm controls include cumulative average productivity, current productivity, and log employment size. Controls associated with worker's previous employment status are the AKM firm fixed effect associated with the previous employer and a dummy for non-employed workers in the previous period. Note that the only difference from the main table is the standard errors. Observation counts are rounded to the nearest 10,000 to avoid potential disclosure risks. Estimates for constant, industry, state fixed effects, the coefficient of the indicator for worker's previous non-employment status are suppressed. Observations are unweighted. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table J8: Wage Differentials for Young Firms (with previous earnings)

	(1) Earnings Residuals	(2) Earnings Residuals	(3) Earnings Residuals	(4) Earnings Residuals	(5) Earnings Residuals	(6)	(7)
Young firm	-0.003***	-0.003***	-0.004***	-0.005***	-0.005***	-0.000	-0.001
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
Young firm × High performing firm	0.014***	0.014***	0.016***	0.018***	0.018***	0.019***	0.019***
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
High performing firm	0.001***	0.001***	-0.004	-0.010***	-0.010***	-0.012***	-0.012***
	(0.002)	(0.002)	(0.001)	(0.002)	(0.002)	(0.001)	(0.001)
Average Firm Productivity (up to $t-1$)	0.006***	0.003***	0.006***	-0.009***	-0.007***	-0.005***	-0.002***
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
Current Productivity (at t)		0.005***	0.012***		-0.003***	0.003***	0.000
		(0.001)	(0.001)		(0.001)	(0.001)	(0.001)
Firm Size (at t)			0.028***			0.018***	
			(0.001)			(0.000)	
Firm Size (at $t-1$)							0.014***
							(0.000)
Previous Employer (AKM)				0.155***	0.155***	0.141***	0.160***
				(0.001)	(0.001)	(0.001)	(0.001)
Previous Earnings	0.194***	0.194***	0.190***	0.167***	0.167***	0.167***	0.165***
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
Observations	50,170,000	50,170,000	50,170,000	50,170,000	50,170,000	50,170,000	50,170,000
Fixed effects	Industry, State	Industry, State	Industry, State	Industry, State	Industry, State	Industry, State	Industry, State

Notes: The table reports the earnings regression results. Firm controls include cumulative average productivity, current productivity, and log employment size. Controls associated with worker's previous employment status are previous earning level (in all columns) along with AKM firm fixed effect associated with the previous employer and a dummy for non-employed workers in the previous period (in the last three columns). Observation counts are rounded to the nearest 10,000 to avoid potential disclosure risks. Estimates for constant, industry, state fixed effects, the coefficient of the indicator for worker's previous non-employment status are suppressed. Observations are weighted with inverse propensity score weights of author's own construction. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table J9: Wage Differentials for Young Firms (worker skill controlled)

	(1) Earnings Residuals	(2) Earnings Residuals	(3) Earnings Residuals
Young firm	-0.008***	-0.009***	-0.004***
	(0.001)	(0.001)	(0.001)
Young firm $\times$ High performing firm	0.016***	0.017***	0.017***
	(0.001)	(0.001)	(0.001)
High performing firm	0.004***	0.002*	0.002
	(0.001)	(0.001)	(0.001)
Average Firm Productivity (up to $t-1$)	0.015***	0.005***	0.008***
	(0.001)	(0.001)	(0.001)
Current Productivity (at t)		0.014***	0.020***
		(0.001)	(0.001)
Firm Size			0.017***
			(0.000)
Previous Employer (AKM)	0.281***	0.279***	0.265***
	(0.001)	(0.001)	(0.001)
Observations	50,170,000	50,170,000	50,170,000
Fixed effects	Industry, State	Industry, State	Industry, State

Notes: The table reports the earnings regression results. Firm controls include cumulative average productivity, current productivity, and log employment size. Controls associated with worker's previous employment status are AKM firm fixed effect associated with the previous employer and a dummy for non-employed workers in the previous period. Note that the only difference from the main table is the earnings residuals, which are computed after additionally controlling for worker skills in the first stage. Observation counts are rounded to the nearest 10,000 to avoid potential disclosure risks. Estimates for constant, industry, state fixed effects, the coefficient of the indicator for worker's previous non-employment status are suppressed. Observations are unweighted. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table J10: Wage Differentials for Young Firms (with young firm risks)

	(1) Earnings Residuals	(2) Earnings Residuals	(3) Earnings Residuals
Young firm	-0.006***	-0.006***	-0.003***
	(0.001)	(0.001)	(0.001)
Young firm $\times$ High performing firm	0.013***	0.015***	0.015***
	(0.001)	(0.001)	(0.001)
High performing firm	0.005***	0.004***	0.002
	(0.001)	(0.001)	(0.001)
Average Firm Productivity (up to $t-1$)	0.016***	0.006***	0.009***
	(0.001)	(0.001)	(0.001)
Firm Productivity (at t)		0.015***	0.020***
		(0.001)	(0.001)
Firm Size			0.017***
			(0.000)
Previous Employer (AKM)	0.283***	0.281***	0.267***
	(0.001)	(0.001)	(0.001)
Young Firm Risks	-0.009***	-0.005***	0.005***
	(0.002)	(0.002)	(0.002)
Observations	50,170,000	50,170,000	50,170,000
Fixed effects	Industry, State	Industry, State	Industry, State

Notes: The table reports the earnings regression results. Firm controls include cumulative average productivity, current productivity, and log employment size. Controls associated with worker's previous employment status are AKM firm fixed effect associated with the previous employer and a dummy for non-employed workers in the previous period. In addition, the dispersion of productivity shocks for young firms is included to control for the level of unobserved risks associated with them. Observation counts are rounded to the nearest 10,000 to avoid potential disclosure risks. Estimates for constant, industry, state fixed effects, the coefficient of the indicator for worker's previous non-employment status are suppressed. Observations are unweighted. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table J11: Wage Differentials for Young Firms (firm-level previous employment)

	(1) Earnings Residuals	(2) Earnings Residuals	(3) Earnings Residuals
Young firm	-0.004***	-0.005***	-0.000
	(0.001)	(0.001)	(0.001)
Young firm $\times$ High performing firm	0.013***	0.014***	0.015***
	(0.001)	(0.001)	(0.001)
High performing firm	0.007***	0.006***	0.003**
	(0.001)	(0.001)	(0.001)
Average Firm Productivity (up to $t-1$)	0.022***	0.010***	0.013***
	(0.001)	(0.001)	(0.001)
Firm Productivity (at t)		0.017***	0.023***
		(0.001)	(0.001)
Firm Size			0.020***
			(0.000)
Previous Employer (AKM)	0.281***	0.279***	0.264***
	(0.001)	(0.001)	(0.001)
Observations	50,170,000	50,170,000	50,170,000
Fixed effects	Industry, State	Industry, State	Industry, State

Notes: The table reports the earnings regression results. Firm controls include cumulative average productivity, current productivity, and log employment size. Controls associated with worker's previous employment status are AKM firm fixed effect associated with the previous employer (estimated at the firm level, rather than the SEIN level) and a dummy for non-employed workers in the previous period. Observation counts are rounded to the nearest 10,000 to avoid potential disclosure risks. Estimates for constant, industry, state fixed effects, the coefficient of the indicator for worker's previous non-employment status are suppressed. Observations are unweighted. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table J12: Wage Differentials for Young Firms (firm-level regression)

	(1) Earnings Residuals (firm-level avg.)	(2) Earnings Residuals (firm-level avg.)	(3) Earnings Residuals (firm-level avg.)	(4) Earnings Residuals (firm-level avg.)
Young firm	-0.010***	-0.010***	-0.010***	-0.010***
	(0.001)	(0.001)	(0.001)	(0.001)
Young firm $\times$ High performing firm	0.016***	0.017***	0.019***	0.020***
	(0.001)	(0.001)	(0.001)	(0.001)
High performing firm	0.018***	0.018***	0.016***	0.017***
	(0.001)	(0.001)	(0.001)	(0.001)
Average Firm Productivity (up to $t-1$)	0.033***	0.049***	0.029***	0.043***
	(0.001)	(0.001)	(0.001)	(0.001)
Current Productivity (at t)	0.072***	0.055***	0.0746***	0.0586***
	(0.001)	(0.001)	(0.001)	(0.001)
Firm Size (at t)	0.067***		0.067***	
	(0.000)		(0.001)	
Firm Size (at $t-1$)		0.0576***		0.0562***
		(0.000)		(0.001)
Observations	6,959,000	6,959,000	6,959,000	6,959,000
Fixed effects	Industry, State	Industry, State	Industry, State	Industry, State
Weighted	No	No	Yes	Yes

Notes: The table reports the firm-level earnings regression results. The dependent variable is the average earnings residuals across workers within each firm. As before, firm-level characteristics are controlled, including cumulative average productivity, current productivity, and log employment size. Observation counts are rounded to the nearest 10,000 to avoid potential disclosure risks. Estimates for constant, industry and state fixed effects. Observations are unweighted in columns (1) and (2) and are weighted by inverse propensity score weights in columns (3) and (4). * p $<$ 0.1, ** p $<$ 0.05, *** p $<$ 0.01.

Table J13: The Effect of Wage Differentials on Firm Outcomes (propensity score weighted)

A. Raw Productivity	(1) Hire (firm level)	(2) Hire (SEIN level)	(3) Employment Growth (log diff)	(4) Employment Growth (DHS)
Average Earnings Residuals	-0.285***	-0.275***	-0.016***	-0.019***
	(0.010)	(0.041)	(0.000)	(0.000)
Firm Productivity	0.370***	0.254***	0.086***	0.095***
	(0.014)	(0.030)	(0.000)	(0.000)
Firm Size	5.426***	4.839***	-0.055***	-0.064***
	(0.071)	(0.058)	(0.000)	(0.000)
Firm Age	0.009**	-0.014*	-0.001***	-0.001***
	(0.004)	(0.007)	(0.000)	(0.000)
Observations	6,959,000	6,959,000	6,959,000	6,959,000
Fixed effects	Industry, State	Industry, State	Industry, State	Industry, State

B. Estimated Productivity	(1) Hire (firm level)	(2) Hire (SEIN level)	(3) Employment Growth (log diff)	(4) Employment Growth (DHS)
Average Earnings Residuals	-0.274***	-0.266***	-0.014***	-0.016***
	(0.010)	(0.042)	(0.000)	(0.000)
Average Productivity up to (t-1)	-0.515***	-0.504***	-0.092***	-0.103***
	(0.022)	(0.052)	(0.001)	(0.001)
Current Productivity at t	0.793***	0.646***	0.168***	0.187***
	(0.021)	(0.043)	(0.001)	(0.001)
Firm Size	5.452***	4.864***	-0.049***	-0.058***
	(0.071)	(0.059)	(0.000)	(0.000)
Firm Age	0.009**	-0.014*	-0.001***	-0.001***
	(0.004)	(0.007)	(0.000)	(0.000)
Observations	6,959,000	6,959,000	6,959,000	6,959,000
Fixed effects	Industry, State	Industry, State	Industry, State	Industry, State

Notes: The table reports the effect of earnings residuals on firm-level outcomes.Firm controls include firm productivity, log employment size, and age. Note that Panel A uses the raw value of firm productivity, while Panel B adopts the cross-time average value as well as the current value of the estimated firm productivity as in the main regressions. Column (1) uses the firm-level total new hires, and column (2) uses the average of the SEIN-level new hires. Observation counts are rounded to the nearest 10,000 to avoid potential disclosure risks. Estimates for constant, and industry, state fixed effects are suppressed. Observations are weighted with inverse propensity score weights of author's own construction. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table J14: Aggregate Implications of Uncertainty (lagged uncertainty)

	(1) Entry rate	(2) Young firm share	(3) HG young firm share	(4) HG young firm growth	(5) Productivity
Uncertainty	-0.016***	-0.050***	-0.030***	-0.041***	-0.020
(at $t-1$)	(0.004)	(0.007)	(0.005)	(0.008)	(0.018)
Observations	4,300	4,300	4,300	4,300	4,300
Fixed effects	Industry, Year	Industry, Year	Industry, Year	Industry, Year	Industry, Year

Notes: The table reports results for regression of firm entry, the share and growth of young firms, and aggregate productivity in each column on the lagged value of the uncertainty at the industry level. Observation counts are rounded to the nearest 100 to avoid potential disclosure risks. Estimates for constant, industry and year fixed effects are suppressed. Observations are unweighted. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table J15: Aggregate Implications of Uncertainty (long run, NAICS6)

A. Industry FE	(1) Entry rate	(2) Young firm share	(3) HG young firm share	(4) HG young firm growth	(5) Productivity
Uncertainty	-0.034***	-0.107***	-0.044***	-0.071***	-0.357***
	(0.007)	(0.023)	(0.009)	(0.015)	(0.087)
Observations	900	900	900	900	900

B. Long-run Avg.	(1) Entry rate	(2) Young firm share	(3) HG young firm share	(4) HG young firm growth	(5) Productivity
Uncertainty	-0.034***	-0.107***	-0.044***	-0.071***	-0.357***
	(0.007)	(0.023)	(0.009)	(0.015)	(0.087)
Observations	900	900	900	900	900

Notes: The table reports results for regression of the long-run value of firm entry, the share and growth of young firms, and aggregate productivity in each column on the counterpart for uncertainty at the industry level. Industries are defined at the NAICS6 level. Panel A is based on the industry fixed effects, and Panel B uses the long-run average value of each measure. Observation counts are rounded to the nearest 50 to avoid potential disclosure risks. Estimates for constant are suppressed. Observations are unweighted. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

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