Online Appendix for "Heterogeneous Innovations and Growth under Imperfect Technology Spillovers" (NOT FOR PUBLICATION)

Karam Jo*
Penn State

Seula Kim†
Penn State

December 12, 2025

A Proofs of Propositions

A.1 Proof of Lemma 1

Consider the following two cases: 1) no ownership change between $t-1$ and t, and 2) ownership change happens between $t-1$ and t. In scenario 1), $q_{j,t} = \Delta_{j,t} q_{j,t-1}$ with only $\Delta_{j,t} \in \{\Delta^1 = 1, \Delta^2 = \lambda\}$ as a result of own-innovation. In scenario 2), $q_{j,t} = \eta q_{j,t-2}$ holds. Let's consider all possible cases where i) $\Delta_{j,t} = 1$, ii) $\Delta_{j,t} = \lambda$, iii) $\Delta_{j,t} = \eta$, iv) $\Delta_{j,t} = \frac{\eta}{\lambda}$, v) $\Delta_{j,t} = \frac{\eta^n}{\lambda^m}$ with $n \geq m > 0$, and vi) $\Delta_{j,t} = \frac{\lambda^n}{\eta^m}$ with $n > m > 0$. These are the only possible values Δ can assume, given that product quality can only be adjusted by three step sizes (1, λ, and η) between two periods without technology regression ($q_t < q_{t-1}$).

- i) $\Delta_{j,t} = 1$: For this to be true, $q_{j,t} = q_{j,t-1}$ should hold. Since $q_{j,t} = \eta q_{j,t-2}$, we need $q_{j,t-1} = \eta q_{j,t-2}$. This is possible if there was creative destruction between $t-2$ and $t-1$, and no own-innovation between $t-3$ and $t-1$, leading to $q_{j,t-2} = q_{j,t-3}$.

*Email: karamjo@gmail.com. Address: 510 Kern Building, University Park, PA 16802.
†Email: seulakim@psu.edu. Address: 614 Kern Building, University Park, PA 16802.

- ii) $\Delta_{j,t} = \lambda$: For this to be true, $\Delta_{j,t-1} = \frac{\eta}{\lambda}$ should hold, as $\Delta_{j,t} = \frac{q_{j,t}}{q_{j,t-1}} = \frac{\eta q_{j,t-2}}{\Delta_{j,t-1} q_{j,t-2}}$. This can be possible if there were own-innovation between $t-3$ and $t-2$, and creative destruction between $t-2$ and $t-1$, but no own-innovation between $t-2$ and $t-1$. In this case, $q_{j,t-2} = \lambda q_{j,t-3}$ and $q_{j,t-1} = \eta q_{j,t-3}$ holds, and thus $\Delta_{j,t-1} = \frac{q_{j,t-1}}{q_{j,t-2}} = \frac{\eta q_{j,t-3}}{\lambda q_{j,t-3}} = \frac{\eta}{\lambda}$ follows. So we have shown that both $\Delta_{j,t} = \lambda$ and $\Delta_{j,t} = \frac{\eta}{\lambda}$ are possible, and $\Delta_{j,t} = \frac{\eta}{\lambda}$ can be realized only through creative destruction between $t-1$ and t.

- iii) $\Delta_{j,t} = \eta$: For this to be true, $q_{j,t-1} = q_{j,t-2}$ should hold. This is possible if there was neither ownership change nor own-innovation between $t-1$ and $t-2$.

- iv) $\Delta_{j,t} = \frac{\eta}{\lambda}$: This follows the illustration in case ii)

- v) $\Delta_{j,t} = \frac{\eta^n}{\lambda^m}$ with $n \geq m > 0$: Suppose this is the case. As $\Delta_{j,t} \notin \{\Delta^1 = 1, \Delta^2 = \lambda\}$, there should be an ownership change between $t-1$ and t. Thus $q_{j,t} = \eta q_{j,t-2}$ holds, implying $q_{j,t-1} = \frac{\lambda^m}{\eta^{n-1}} q_{j,t-2}$. Note that $m \leq n-1$ is not possible without technology regression. Thus, $m = n$ (as $m > n-1$ and $n \geq m > 0$). If $\frac{\lambda^m}{\eta^{m-1}} < 1$, this implies technology regression and can be ruled out. Suppose $\frac{\lambda^m}{\eta^{m-1}} > 1$. If $m = 1$, we are back to the cases ii) and iv). Suppose $m > 1$. As $\frac{\lambda^m}{\eta^{m-1}} \neq 1$ or λ, there should be an ownership change between $t-2$ and $t-1$. Thus, $q_{j,t-1} = \eta q_{j,t-3}$ holds, implying $q_{j,t-2} = \frac{\eta^m}{\lambda^m} q_{j,t-3}$. If $\Delta_{j,t} = \frac{\eta^n}{\lambda^m}$ is possible, $q_{j,t-s} = \frac{\eta^m}{\lambda^m} q_{j,t-s-1}$ holds for even numbers s, and $\frac{\lambda^m}{\eta^{m-1}} q_{j,t-s-1}$ holds for odd numbers s. Thus, in this case, either $q_{j,1} = \frac{\eta^m}{\lambda^m} q_{j,0}$ or $q_{j,1} = \frac{\lambda^m}{\eta^{m-1}} q_{j,0}$ must hold, which can be ruled out (or we assume this case does not occur). Thus, $\Delta_{j,t} = \frac{\eta^n}{\lambda^m}$ with $n \geq m > 0$ is not possible.

- vi) $\Delta_{j,t} = \frac{\lambda^n}{\eta^m}$ with $n > m > 0$: Following the same argument, this case is not possible.

Therefore $\Delta_{j,t}$ can assume only the four values of $\left\{1, \lambda, \eta, \frac{\eta}{\lambda}\right\}$.

A.2 Proof of Proposition 1

Using the conjectured value function, we can decompose the expected value into two parts with the linearity of expectation: the expected value of existing product lines $\mathbb{E}\left[\sum_{\ell=1}^{2} A_\ell \sum_{j \in \mathcal{J}^f | (\Delta'_j | \Delta_j) = \Delta^\ell} \Delta^\ell q_j\right]$ and the expected value for the new product line added through creative destruction $\mathbb{E}\left[\sum_{\ell=1}^{4} A_\ell \, I_{\left\{\eta/\Delta_j = \Delta^\ell\right\}} \, \frac{\eta}{\Delta_j} \, q_j\right]$. As the realization of own-innovation outcomes and the creative destruction are independent of the realization of creative destruction, the expected value of a new product line becomes:

$$
\begin{aligned}
&\mathbb{E}\left[\sum_{\ell=1}^{4} A_\ell \, I_{\left\{\frac{\eta}{\Delta_j}=\Delta^\ell\right\}} \, \frac{\eta}{\Delta_j}\, q_j\right] = \sum_{I^x=0}^{1} x^{I^x}(1-x)^{1-I^x}\mathbb{E}_{q_j,\Delta_j,\omega}\left[\sum_{\ell=1}^{4} A_\ell \, I_{\left\{\frac{\eta}{\Delta_j}=\Delta^\ell\right\}} \, I^x \, \frac{\eta}{\Delta_j}\, q_j\right] \\
=&x\Big[(1-\omega)ct(1-z^3)A_1\mu(\Delta^3) + (1-\omega)\left(1-(1-ct)z^4\right)A_2\lambda\mu(\Delta^4) + A_3\eta\mu(\Delta^1) \\
&+ (1-\omega)(1-z^2)A_4\frac{\eta}{\lambda}\mu(\Delta^2)\Big]\,\overline{q}\,.
\end{aligned}
$$

The terms in the bracket arise from the random property of creative destruction. The assigned product can have a technology gap of Δ^ℓ with a probability of $\mu(\Delta^\ell)$, and the probability of taking over this product line depends on its technology gap. Integrating over all possible qualities q_j over the entire set of available products gives us $\overline{q}$.[1]

The expected value of existing product lines can further be broken down into the four cases of Δ and integrated as $\sum_{\widetilde{\ell}=1}^{4}\, \mathbb{E}\left[\sum_{\ell=1}^{2} A_\ell \sum_{j\in\mathcal{J}^f|(\Delta_j'|\Delta_j=\Delta^{\widetilde{\ell}})=\Delta^\ell} \Delta^\ell q_j\right]$. To simplify the derivation, we reorder product quality q_j by its technology gap Δ_j and categorize it into the following four groups: $q_1^f = \{q_{j_1}, q_{j_2}, \ldots, q_{j_{n_f^1}}\}$; $q_2^f = \{q_{j_{n_f^1+1}}, \ldots, q_{j_{n_f^1+n_f^2}}\}$; $q_3^f = \{q_{j_{n_f^1+n_f^2+1}}, \ldots, q_{j_{n_f^1+n_f^2+n_f^3}}\}$; and $q_4^f = \{q_{j_{n_f^1+n_f^2+n_f^3+1}}, \ldots, q_{j_{n_f^1+n_f^2+n_f^3+n_f^4}}\}$, $q^f = \cup_{\widetilde{\ell}=1}^{4} q_{\widetilde{\ell}}^f$.

If $\Delta = \Delta^1$ ($\widetilde{\ell}=1$), the expected value can be rephrased as $\sum_{i=1}^{n_f^1}\left[A_1(1-\overline{x})(1-z_i^1) + \lambda A_2(1-\overline{x})z_i^1\right]q_{j_i}$; if $\Delta = \Delta^2$ ($\widetilde{\ell}=2$), it becomes $\sum_{i=n_f^1+1}^{n_f^1+n_f^2}\left[A_1(1-\overline{x})(1-z_i^2) + \lambda A_2(1-\omega\bar{x})z_i^2\right]q_{j_i}$; if $\Delta = \Delta^3$ ($\widetilde{\ell}=3$), it is $\sum_{i=n_f^1+n_f^2+1}^{n_f-n_f^4}\left[A_1\left((1-\omega)(1-ct)\overline{x}+1-\overline{x}\right)(1-z_i^3) + \lambda A_2(1-\omega\bar{x})z_i^3\right]q_{j_i}$; and if $\Delta = \Delta^4$ ($\widetilde{\ell}=4$), it is $\sum_{i=n_f-n_f^4}^{n_f}\left[A_1(1-\overline{x})(1-z_i^4) + \lambda A_2\big((1-\omega)(1-ct)\overline{x}+1-\overline{x}\big)z_i^4\right]q_{j_i}$.

The $B\overline{q}$ portion of the conjectured value function in $\mathbb{E}\left[V\left(\Phi^{f\prime} \mid \Phi^f\right)\Big|\{z_j\}_{j\in\mathcal{J}^f}, x\right]$ can be expressed as $\mathbb{E}B\overline{q}' = B(1+g)\overline{q}$, where g denotes the growth rate of product quality in a balanced growth path (BGP) equilibrium. Plugging this into the conjectured value function, we can rephrase

[1] Note that individual firms only have information about the distribution of technology gaps $\{\mu(\Delta^\ell)\}_{\ell=1}^4$ and the average quality level $\overline{q}$. That is, for an individual firm, a technology gap and product quality are independent considerations.

the original value function as:

$$
\sum_{i=1}^{n_f^1} A_1 q_{j_i} + \sum_{i=n_f^1+1}^{n_f^1+n_f^2} A_2 q_{j_i} + \sum_{i=n_f^1+n_f^2+1}^{n_f-n_f^4} A_3 q_{j_i} + \sum_{i=n_f-n_f^4+1}^{n_f} A_4 q_{j_i} + B\overline{q} =
$$

$$
\max_{\substack{x\in[0,\bar{x}],\\ \{z_i\in[0,\bar{z}]\}_{i=1}^{n_f}}} \left\{ \begin{array}{l}
\sum_{i=1}^{n_f} \left[\pi q_{j_i} - \widehat{\chi} z_i^{\widehat{\psi}} q_{j_i}\right] - \overline{q}\widetilde{\chi} x^{\widetilde{\psi}} \\
+\widetilde{\beta} \sum_{i=1}^{n_f^1} \left[A_1(1-\overline{x})(1-z_i^1) + \lambda A_2(1-\overline{x}) z_i^1\right] q_{j_i} \\
+\widetilde{\beta} \sum_{i=n_f^1+1}^{n_f^1+n_f^2} \left[A_1(1-\overline{x})(1-z_i^2) + \lambda A_2(1-\omega\bar{x}) z_i^2\right] q_{j_i} \\
+\widetilde{\beta} \sum_{i=n_f^1+n_f^2+1}^{n_f-n_f^4} \left[A_1\left((1-\omega)(1-ct)\overline{x} + 1 - \overline{x}\right)(1-z_i^3) + \lambda A_2(1-\omega\bar{x}) z_i^3\right] q_{j_i} \\
+\widetilde{\beta} \sum_{i=n_f-n_f^4}^{n_f} \left[A_1(1-\overline{x})(1-z_i^4) + \lambda A_2\Big((1-\omega)(1-ct)\overline{x} + 1 - \overline{x}\Big) z_i^4\right] q_{j_i} \\
+\widetilde{\beta} x \Big[(1-\omega)ct(1-z^3)A_1\mu(\Delta^3) + (1-\omega)\left(1-(1-ct)z^4\right) A_2\lambda\mu(\Delta^4) \\
\qquad\qquad +A_3\eta\mu(\Delta^1) + (1-\omega)(1-z^2)A_4\frac{\eta}{\lambda}\mu(\Delta^2)\Big]\, \overline{q} \\
\\
+\widetilde{\beta} B(1+g)\overline{q}
\end{array} \right\}.
$$

By taking the first-order conditions with respect to each innovation intensity, we get the optimal innovation decision rules, which depend solely on technology gaps. Substituting these optimal innovation intensities into the value function, equating the left-hand side (LHS) to the right-hand side (RHS), and collecting terms, we obtain the five coefficients of the conjectured value function.

A.3 Proof of Corollary 1

Define $\widetilde{z}^\ell = \frac{\widehat{\psi}\widehat{\chi}}{\widetilde{\beta}}\left(z^\ell\right)^{(\widehat{\psi}-1)}$. Then $z^\ell > z^{\ell'} \Leftrightarrow \widetilde{z}^\ell > \widetilde{z}^{\ell'}$ for all $\ell, \ell' \in [1,4]\cap\mathbb{Z}$ under the condition $\widehat{\psi} > 1$. Given $\widetilde{z}^2 - \widetilde{z}^3 = (1-ct)(1-\omega)\overline{x}A_1 > 0$, $\widetilde{z}^2 - \widetilde{z}^1 = (1-\omega)\overline{x}\lambda A_2 > 0$, $\widetilde{z}^2 - \widetilde{z}^4 = ct(1-\omega)\overline{x}\lambda A_2 > 0$, and $\widetilde{z}^4 - \widetilde{z}^1 = (1-ct)(1-\omega)\overline{x}\lambda A_2 > 0$, we can obtain the following relationships: $z^2 > z^3$, $z^2 > z^1$, $z^2 > z^4$, and $z^4 > z^1$. Also, given $\widetilde{z}^1 = (1-\overline{x})\left[\lambda A_2 - A_1\right] > 0$ in equilibrium, $\lambda A_2 - A_1 > 0$ holds. Thus, $\widetilde{z}^3 - \widetilde{z}^1 = (1-\omega)\overline{x}\lambda A_2 - (1-\omega)(1-ct)\bar{x}A_1 > 0$ and $z^3 > z^1$ is proved. Furthermore, $\widetilde{z}^3 - \widetilde{z}^4 = (ct\lambda A_2 - (1-ct)A_1)(1-\omega)\bar{x}$, and the relationship between z^3 and z^4 depends on the range of ct, where $z^3 > z^4$ holds if $ct > \frac{A_1}{\lambda A_2 + A_1}$, and $z^3 < z^4$

holds, otherwise. Lastly, if $ct = 1$, $z^2 = z^3 > z^4$, and if $ct = 0$, $z^2 = z^4 > z^3$. Combining all, the order of $\{z^\ell\}_{\ell=1}^4$ in equilibrium is $z^2 \geq z^3, z^4 > z^1$ for $\omega \in [0,1)$ and $ct \in [0,1]$; $z^2 \geq z^3 > z^4 > z^1$ for $ct \in (\frac{A_1}{\lambda A_2 + A_1}, 1]$; and $z^2 \geq z^4 > z^3 > z^1$ for $ct \in [0, \frac{A_1}{\lambda A_2 + A_1})$.

A.4 Proof of Corollary 2

Using the optimal decision rules in Proposition 1, the partial derivatives of $\{z^\ell\}_{\ell=1}^4$ with respect to ω are (after removing the common terms) $\frac{\partial z^1}{\partial \omega}\Big|_{A_1,A_2}$: 0; $\frac{\partial z^2}{\partial \omega}\Big|_{A_1,A_2}$: $-(z^2)^{2-\widehat{\psi}} A_2 \lambda \bar{x} < 0$; $\frac{\partial z^3}{\partial \omega}\Big|_{A_1,A_2}$: $-(z^2)^{2-\widehat{\psi}}(A_2\lambda - A_1(1-ct))\bar{x} < 0$; and $\frac{\partial z^4}{\partial \omega}\Big|_{A_1,A_2}$: $-(z^2)^{2-\widehat{\psi}} A_2 \lambda \bar{x} < 0$.

For creative destruction, $\frac{\partial x}{\partial \omega}\big|_{A_1,A_2}$: $x^{2-\tilde{\psi}}\Bigg[\big(\eta A_3 - (1-z^2)A_4\frac{\eta}{\lambda}\big)\mu(\Delta^2) + \big(\eta A_3 - ct(1-z^3)A_1\big)\mu(\Delta^3) + \big(\eta A_3 - (1-(1-ct)z^4)\big)A_2\lambda\mu(\Delta^4)\Bigg] > 0$ as $A^3 > A^2 > A^4 > A^1$, which is proved as follows. $\{A_i\}_{i=1,2,3,4}$ can be rephrased as follows using the first order conditions w.r.t. $\{z^i\}_{i=1,2,3,4}$, respectively

$$
\begin{aligned}
A_1 &= \pi + (\hat{\psi} - 1)\widehat{\chi}(z^1)^{\hat{\psi}} + \tilde{\beta} A_1(1-\bar{x}) \\
A_2 &= \pi + (\hat{\psi} - 1)\widehat{\chi}(z^2)^{\hat{\psi}} + \tilde{\beta} A_1(1-\bar{x}) \\
A_3 &= \pi + (\hat{\psi} - 1)\widehat{\chi}(z^3)^{\hat{\psi}} + \tilde{\beta} A_1(1-\bar{x} + (1-\omega)(1-ct)\bar{x}) \\
A_4 &= \pi + (\hat{\psi} - 1)\widehat{\chi}(z^4)^{\hat{\psi}} + \tilde{\beta} A_1(1-\bar{x}).
\end{aligned}
$$

We know $A_2 \geq A_4 \geq A_1$, given the relationship in Corollary 1. Furthermore, let $\tilde{A} \equiv \tilde{A}(z) = \pi - \widehat{\chi}\,(z)^{\widehat{\psi}} + \widetilde{\beta}\left[A_1\left((1-\omega)(1-ct)\overline{x} + 1 - \overline{x}\right)(1-z) + A_2\lambda(1-\omega\overline{x})z\right]$, where $\tilde{A}(z^3) = A_3$. By the definition of value function, $\tilde{A}(z_3) = A_3 \geq \tilde{A}(z)$ for any $z \neq z_3$. Using $\tilde{A}(z_3) \geq \tilde{A}(z_2)$, along with the first order condition w.r.t. z_2, we can obtain that

$$
\begin{aligned}
\tilde{A}(z_3) = A_3 &= \pi - \widehat{\chi}\left(z^3\right)^{\widehat{\psi}} + \widetilde{\beta}\left[A_1\left((1-\omega)(1-ct)\overline{x} + 1 - \overline{x}\right)(1-z^3) + A_2\lambda(1-\omega\overline{x})z^3\right] \\
&\geq \tilde{A}(z_2) \\
&= \pi - \widehat{\chi}\left(z^2\right)^{\widehat{\psi}} + \widetilde{\beta}\left[A_1\left((1-\omega)(1-ct)\overline{x} + 1 - \overline{x}\right)(1-z^2) + A_2\lambda(1-\omega\overline{x})z^2\right] \\
&= \pi + (\hat{\psi} - 1)\widehat{\chi}\left(z^2\right)^{\widehat{\psi}} + \widetilde{\beta}\left[A_1\left((1-\omega)(1-ct)\overline{x}\right)(1-z^2) + A_1(1-\bar{x})\right] \\
&\geq \pi + (\hat{\psi} - 1)\widehat{\chi}\left(z^2\right)^{\widehat{\psi}} + \widetilde{\beta} A_1(1-\bar{x}) = A_2.
\end{aligned}
$$

Similarly, using $\tilde{A}(z_3) \geq \tilde{A}(z_4)$, along with the first order condition w.r.t. z_4, it is proved that

$$
\begin{aligned}
\tilde{A}(z_3) = A_3 \geq \tilde{A}(z_4)& \\
&= \pi - \widehat{\chi}\left(z^4\right)^{\widehat{\psi}} + \widetilde{\beta}\left[A_1\left((1-\omega)(1-ct)\overline{x} + 1 - \overline{x}\right)(1-z^4) + A_2\lambda(1-\omega\overline{x})z^4\right] \\
&= \pi + (\hat{\psi}-1)\widehat{\chi}\left(z^4\right)^{\widehat{\psi}} + \widetilde{\beta}\left[A_1\left((1-\omega)(1-ct)\overline{x}\right)(1-z^4) + A_2\lambda ct(1-\omega)\bar{x}z^4 + A_1(1-\bar{x})\right] \\
&\geq \pi + (\hat{\psi}-1)\widehat{\chi}\left(z^4\right)^{\widehat{\psi}} + \widetilde{\beta}A_1(1-\bar{x}) = A_4.
\end{aligned}
$$

Combining it all, it is proved that $A_3 \geq A_2 \geq A_4 \geq A_1$, and $\frac{\partial x}{\partial \omega}\Big|_{\{A_\ell\}_{\ell=1}^4} > 0$.

A.5 Proof of Corollary 3

Based on Corollary 3, we can rephrase the relationship as follows:

$$
\begin{aligned}
&\frac{\partial z^2}{\partial \overline{x}}\Big|_{A_1,A_2} > \frac{\partial z^3}{\partial \overline{x}}\Big|_{A_1,A_2} > 0, \frac{\partial z^3}{\partial \overline{x}}\Big|_{A_1,A_2} > \frac{\partial z^4}{\partial \overline{x}}\Big|_{A_1,A_2} (\gtreqless 0) \text{ if } \omega < \frac{ctA_1}{\lambda A_2 - (1-ct)A_1}, ct > \frac{\lambda A_2 + A_1}{A_1} \\
&\frac{\partial z^2}{\partial \overline{x}}\Big|_{A_1,A_2} > \frac{\partial z^4}{\partial \overline{x}}\Big|_{A_1,A_2} > \frac{\partial z^3}{\partial \overline{x}}\Big|_{A_1,A_2} > 0 \text{ if } \omega < \frac{ctA_1}{\lambda A_2 - (1-ct)A_1}, ct < \frac{\lambda A_2 + A_1}{A_1} \\
&\frac{\partial z^2}{\partial \overline{x}}\Big|_{A_1,A_2} > 0 > \frac{\partial z^3}{\partial \overline{x}}\Big|_{A_1,A_2} > \frac{\partial z^4}{\partial \overline{x}}\Big|_{A_1,A_2} \text{ if } \omega \in \left(\frac{ctA_1}{\lambda A_2 - (1-ct)A_1}, \frac{A_1}{\lambda A_2}\right), ct > \frac{\lambda A_2 + A_1}{A_1} \\
&\frac{\partial z^2}{\partial \overline{x}}\Big|_{A_1,A_2} > 0 > \frac{\partial z^3}{\partial \overline{x}}\Big|_{A_1,A_2}, \frac{\partial z^4}{\partial \overline{x}}\Big|_{A_1,A_2} (\gtreqless 0) > \frac{\partial z^3}{\partial \overline{x}}\Big|_{A_1,A_2} \text{ if } \omega \in \left(\frac{ctA_1}{\lambda A_2 - (1-ct)A_1}, \frac{A_1}{\lambda A_2}\right), ct < \frac{\lambda A_2 + A_1}{A_1} \\
&0 > \frac{\partial z^2}{\partial \overline{x}}\Big|_{A_1,A_2} > \frac{\partial z^3}{\partial \overline{x}}\Big|_{A_1,A_2} > \frac{\partial z^4}{\partial \overline{x}}\Big|_{A_1,A_2} \text{ if } \omega > \frac{A_1}{\lambda A_2}, ct > \frac{\lambda A_2 + A_1}{A_1} \\
&0 > \frac{\partial z^2}{\partial \overline{x}}\Big|_{A_1,A_2} > \frac{\partial z^4}{\partial \overline{x}}\Big|_{A_1,A_2} > \frac{\partial z^3}{\partial \overline{x}}\Big|_{A_1,A_2} \text{ if } \omega > \frac{A_1}{\lambda A_2}, ct < \frac{\lambda A_2 + A_1}{A_1}.
\end{aligned}
$$

To prove, the partial derivatives of $\{z^\ell\}_{\ell=1}^4$ with respect to $\overline{x}$ are (after removing the common terms) $\frac{\partial z^1}{\partial \overline{x}}\Big|_{A_1,A_2}$: $(z^1)^{2-\widehat{\psi}}\left[A_1 - \lambda A_2\right] < 0$ as $\lambda A_2 - A_1 > 0$; $\frac{\partial z^2}{\partial \overline{x}}\Big|_{A_1,A_2}$: $(z^2)^{2-\widehat{\psi}}(A_1 - A_2\lambda\omega) \gtrless 0$ if $\omega \lessgtr \frac{A_1}{A_2\lambda}$; $\frac{\partial z^3}{\partial \overline{x}}\Big|_{A_1,A_2}$: $(z^3)^{2-\widehat{\psi}}(1-(1-\omega)(1-ct))A_1 - A_2\lambda\omega) \gtrless 0$ if $\omega \lessgtr \frac{ctA_1}{\lambda A_2 - (1-ct)A_1}$; and $\frac{\partial z^4}{\partial \overline{x}}\Big|_{A_1,A_2}$: $(z^4)^{2-\widehat{\psi}}\left[A_1 - (1-(1-\omega)(1-ct))A_2\lambda\right]$. Given the relationship in Corollary 1, along with $A_1 - A_2\lambda\omega > (1-(1-\omega)(1-ct))A_1 - A_2\lambda\omega > A_1 - A_2\lambda$ and $A_1 - A_2\lambda\omega > A_1 - (1 - (1-\omega)(1-ct))A_2\lambda > A_1 - A_2\lambda$, we can further derive that $\frac{\partial z^2}{\partial \overline{x}} > \frac{\partial z^3}{\partial \overline{x}} > \frac{\partial z^1}{\partial \overline{x}}$ and $\frac{\partial z^2}{\partial \overline{x}} > \frac{\partial z^4}{\partial \overline{x}} > \frac{\partial z^1}{\partial \overline{x}}$, respectively. Lastly, the proof is complete with $z^3 \gtreqless z^4$ and $(1-(1-\omega)(1-ct))A_1 - A_2 \gtreqless$

$A_1 - (1-(1-\omega)(1-ct))A_2\lambda$ if $ct \gtrless \frac{A_1}{\lambda A_2 + A_1}$, respectively.

A.6 Proof of Potential Startups' Problem

With the value function defined for incumbents, we have $\mathbb{E}V(\{(q'_j, \Delta'_j)\}) = x_e\big[ct(1-z^3)A_1\mu(\Delta^3) + (1-(1-ct)z^4)\,A_2\lambda\mu(\Delta^4) + A_3\eta\mu(\Delta^1) + (1-z^2)A_4\frac{\eta}{\lambda}\mu(\Delta^2)\big]\overline{q} + x_e\big[ct(1-z^3)\mu(\Delta^3) + \big(1-(1-ct)z^4\big)\mu(\Delta^4) + \mu(\Delta^1) + (1-z^2)\mu(\Delta^2)\big]B(1+g)\overline{q}$, from which the optimal creative destruction choice for potential startups can be derived.

A.7 Proof of Proposition 2

In this model, the output growth rate is the same as the product quality growth rate. For product j with quality q_j and a technology gap of $\Delta_j = \Delta^\ell$, we can derive the following law of motion of q_j:

$$
\begin{array}{lllll}
\Delta^1: & q'_j = \Delta^1 q_j & \text{prob.} & (1-\overline{x})(1-z^1) \\
 & q'_j = \Delta^2 q_j & \text{prob.} & (1-\overline{x})z^1 \\
 & q'_j = \Delta^3 q_j & \text{prob.} & \overline{x} \\
 & q'_j = \Delta^4 q_j & \text{prob.} & 0 \\
\Delta^3: & q'_j = \Delta^1 q_j & \text{prob.} & 1-(1-\omega\bar{x})z^3 \\
 & q'_j = \Delta^2 q_j & \text{prob.} & (1-\omega\bar{x})z^3 \\
 & q'_j = \Delta^3 q_j & \text{prob.} & 0 \\
 & q'_j = \Delta^4 q_j & \text{prob.} & 0
\end{array}
\qquad
\begin{array}{lllll}
\Delta^2: & q'_j = \Delta^1 q_j & \text{prob.} & (1-\overline{x})(1-z^2) \\
 & q'_j = \Delta^2 q_j & \text{prob.} & (1-\omega\bar{x})z^2 \\
 & q'_j = \Delta^3 q_j & \text{prob.} & 0 \\
 & q'_j = \Delta^4 q_j & \text{prob.} & \overline{x}(1-(1-\omega)z^2) \\
\Delta^4: & q'_j = \Delta^1 q_j & \text{prob.} & (1-\overline{x})(1-z^4) \\
 & q'_j = \Delta^2 q_j & \text{prob.} & z^4 + \overline{x}(1-z^4) \\
 & q'_j = \Delta^3 q_j & \text{prob.} & 0 \\
 & q'_j = \Delta^4 q_j & \text{prob.} & 0.
\end{array}
$$

Following this, we can compute the expected growth rate of q_j ($\mathbb{E}\big[q'_j \mid q_j\big]/q_j - 1$) and the aggregate growth rate in (23) by taking the expectation across all product lines.

Using the share of products owned by domestic incumbents ($s_d = \mathcal{F}_d/\mathcal{F}$), the definition of $\overline{x}$, and the evolution of product quality, the growth rate can be decomposed as follows:

$$
\begin{aligned}
g = & \underbrace{(\lambda-1)s_d\Big[(1-\overline{x})z^1\mu(\Delta^1) + (1-\omega\overline{x})z^2\mu(\Delta^2) + (1-\omega\overline{x})z^3\mu(\Delta^3) + \left[(1-\omega)(1-ct)\overline{x} + (1-\overline{x})\right]z^4\mu(\Delta^4)\Big]}_{\text{own innovation by domestic incumbents}} \\
& + (\lambda-1)(1-s_d)\Big[(1-\overline{x})z^1\mu(\Delta^1) + (1-\omega\overline{x})z^2\mu(\Delta^2) + (1-\omega\overline{x})z^3\mu(\Delta^3)
\end{aligned}
$$

$$\underbrace{+\left[(1-\omega)(1-ct)\overline{x}+(1-\overline{x})\right]z^4\mu(\Delta^4)\Big]}_{\text{own innovation by outsider incumbents}}$$

$$\underbrace{(\overline{\Delta^{ex}}-1)\mathcal{F}_d x\mu(\overline{\Delta^{ex}})}_{\text{c.d. by domestic incumbents}}+\underbrace{(\overline{\Delta^{ex}}-1)\mathcal{E}_d x_e\mu(\overline{\Delta^{ex}})}_{\text{c.d. by domestic startups}}+\underbrace{(\overline{\Delta^{ex}}-1)\overline{x}_o\mu(\overline{\Delta^{ex}})}_{\text{c.d. by outsider firms}},$$

where

$$\begin{aligned}\overline{\Delta^{ex}} =& \Big[\eta\mu(\Delta^1)+(\eta/\lambda)(1-\omega)(1-z^2)\mu(\Delta^2)+(1-\omega)ct(1-z^3)\mu(\Delta^3)\\ &+\lambda(1-\omega)(1-(1-ct)z^4)\mu(\Delta^4)+\eta\omega(1-\mu(\Delta^1))\Big]/\overline{x}_{takeover}, \text{ and}\end{aligned}$$

$$\mu(\overline{\Delta^{ex}}) = \overline{x}_{takeover}.$$

A.8 Proof of Proposition 3

$$\begin{aligned}\frac{\partial g}{\partial\omega} =& -\bar{x}\Big[\lambda z^2-\eta+(\eta/\lambda)(1-z^2)\Big]\mu(\Delta^2)-\bar{x}\Big[(1-z^3)+\lambda z^3-\eta\Big]\mu(\Delta^3)-\bar{x}\Big[\lambda-\eta\Big]\mu(\Delta^4)\\ &+\frac{\partial z^1}{\partial\omega}\Big[(\lambda-1)(1-\bar{x}))\mu(\Delta^1)\Big]+\frac{\partial z^2}{\partial\omega}\Big[(\lambda(1-\omega\bar{x})-(\eta/\lambda)(1-\omega)\bar{x})\mu(\Delta^2)\Big]\\ &+\frac{\partial z^3}{\partial\omega}\Big[(\lambda-1)(1-\omega\bar{x}))\mu(\Delta^3)\Big]+\frac{\partial z^4}{\partial\omega}\Big[(\lambda-1)(1-\bar{x}))\mu(\Delta^4)\Big]\\ &+\frac{\partial\bar{x}}{\partial\omega}\Big[(-(1-z^1)-\lambda z^1+\eta)\mu(\Delta^1)+(-(1-z^2)-\lambda\omega z^2+\eta\omega+(\eta/\lambda)(1-\omega)(1-z^2))\mu(\Delta^2)\\ &\qquad+(-\omega(1-z^3)-\lambda\omega z^3+\eta\omega)\mu(\Delta^3)+(-(1-z^4)+\lambda(1-\omega-z^4)+\eta\omega)\mu(\Delta^4)\Big]\end{aligned}$$

Note that the expression can be decomposed into the direct effect of ω on the success probabilities of own innovation and creative-destruction takeovers, holding the level of own innovation and creative destruction fixed, and the overall effect operating through shifts in the composition of innovation.

The first line captures the direct effects. Note that in the first line, the bracket in the first term is $-\eta+\eta/\lambda+(\lambda-\eta/\lambda)z^2 < -\eta+\eta/\lambda+\lambda-\eta/\lambda < 0$, and the bracket in the second term is $-(\eta-1)+(\lambda-1)z^3 < -(\eta-1)+(\lambda-1) < 0$, so that the first two terms are positive. Also, the last term is also positive given $\eta > \lambda$. Therefore, the direct effect of higher ω with reduced learning time on growth is positive, holding the level of innovations $(\{z^i\}_{i=1,2,3,4}, \bar{x}$ fixed. Although higher

ω reduces the success probability of own innovation in markets with a positive technology gap, it also increases the probability of a successful creative-destruction takeover, and the latter effect dominates.

The remaining lines show the indirect effects through the endogenous responses of own innovation and creative destruction to changes in learning time. The sign of this effect is ambiguous, depending on the composition changes in own innovation and creative destruction. The effect can further be disentangled by the parts attributed to own innovation and creative destruction changes as follows. The second and third lines show the effects through the changes in own innovation. The first term in the second line becomes zero as $\frac{\partial z^1}{\partial \omega} = 0$, which is the effect of own innovation in markets without technology gap; the second term in the second line shows the part attributed to own innovation in markets with a gap of Δ^2, of which the sign is negative as the bracket in this term is $\lambda - \eta/\lambda + \omega(\eta/\lambda - \lambda) > 0$ and $\frac{\partial z^2}{\partial \omega} < 0$; and the first and second terms in the third line show the effect attributed to own innovation changes in markets of a gap of Δ^3 and Δ^4, respectively, and the sign of both terms is negative with $\frac{\partial z^3}{\partial \omega} < 0$ and $\frac{\partial z^4}{\partial \omega} < 0$. This pattern is intuitive: own innovation does not respond to learning-time changes when there is no technology gap, but optimal innovation intensities decline with the gap if learning time decreases (with higher ω).

The last two lines capture the creative-destruction channel, of which the sign is positive. This can be proved as the sign of the brackets in the last two lines becomes $(-(1 - z^1) - \lambda z^1 + \eta) = (\eta - 1) - (\lambda - 1)z^1 > (\lambda - 1)(1 - z^1) > 0, (-(1 - z^2) - \lambda\omega z^2 + \eta\omega + (\eta/\lambda)(1 - \omega)(1 - z^2)) = \omega(\eta - \lambda z^2 - \eta/\lambda(1 - z^2)) + (\eta/\lambda - 1)(1 - z^2) = \omega(\eta - \eta/\lambda - (\lambda - \eta/\lambda)z^2 + (\eta/\lambda - 1)(1 - z^2) > 0, (-\omega(1 - z^3) - \lambda\omega z^3 + \eta\omega) = \omega(\eta - 1 - z^3(\lambda - 1)) > 0, (1 - z^4)(\lambda - 1) + (\eta - \lambda)\omega > 0$, respectively. This is consistent with the finding that creative destruction increases if learning time is reduced with higher ω.

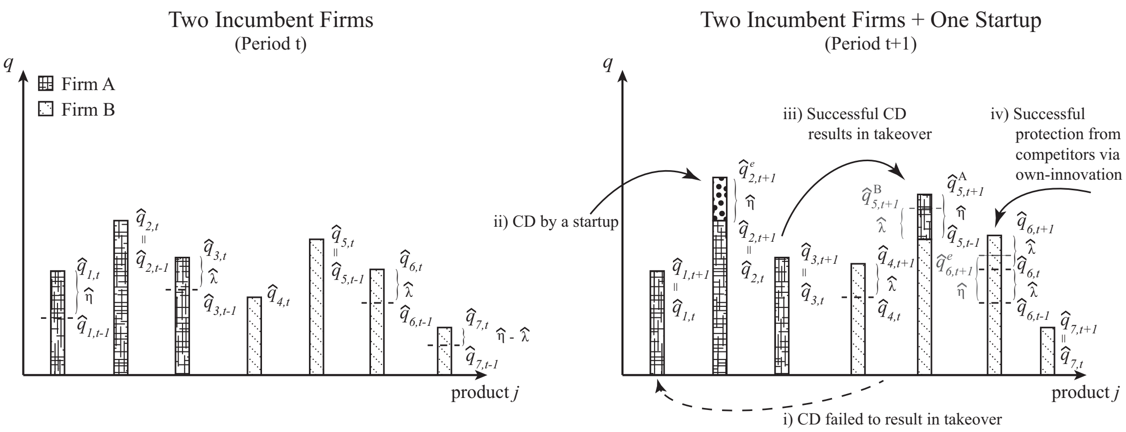

Figure B.1: Firms' Innovation and Product Quality Evolution Example

B Baseline Model

B.1 Illustration of Firm Innovation Decisions

Figure B.1 illustrates the following set of examples of firm innovation decisions.[2] Suppose firm A owns products 1,2,3, and firm B owns products 4,5,6,7. For simplicity, assume $\omega = 0$.

• i) Failed product takeover with coin-tossing (product 1): firm A without successful own-innovation (at t) gets $q^A_{1,t+1} = \eta q_{1,t-1}$, while firm B with successful creative destruction (CD) (at t) obtains $q^B_{1,t+1} = \eta q_{1,t-1}$. A coin is tossed, and firm A keeps the product.

• ii) Successful product takeover w/o technology gap (product 2): A potential startup with successful creative destruction (at t) can take over the market from firm A with no successful own-innovations (at both $t-1$ and t) as $q^e_{2,t+1} = \eta q_{2,t-1} > q^A_{2,t+1} = q_{2,t-1}$

• iii) Failed market protection w/o technological gap (product 5): firm A can take it over through successful creative destruction, despite concurrently successful own-innovation by firm B as $\eta q_{5,t-1} > \lambda q_{5,t-1}$

• iv) Successful market protection with a technology gap (product 6): firm B obtains $q^B_{6,t+1} = \lambda^2 q_{6,t-1}$ with consecutively successful own-innovations from $t-1$. Rivals can only innovate up to $q^e_{6,t+1} = \eta q_{6,t-1}$, which makes firm B successfully protect the market.

[2]The bar indicates log product quality $\widehat{q}_{j,t} \equiv \log(q_{j,t})$ with $\widehat{\eta} \equiv \log(\eta)$.

B.2 Product Quality Evolution

Outsider Firms Since outside firms can learn the frontier technology (q_j) with probability ω and the lagged technology ($q_{j,-1} = q_j/\Delta_j^\ell$) with probability $1-\omega$, the evolution of product quality for outside firms in $t+1$ occurs probabilistically as follows: for $\Delta_j = \Delta^1$, $\{q_j'\}$ equals $\emptyset$ with prob. $(1-x)$, and $\{\eta q_j\}$ with prob. x; for Δ^2, $\{q_j'\}$ equals $\emptyset$ with prob. $(1-\omega)xz_j^2 + (1-x)$, $\{\frac{\eta}{\lambda}q_j\}$ with prob. $(1-\omega)x(1-z_j^2)$, and $\{\eta q_j\}$ with prob. ωx; for Δ^3, $\{q_j'\}$ equals $\emptyset$ with prob. $(1-\omega)x[1-ct(1-z_j^3)]+(1-x)$, $\{q_j\}$ with prob. $(1-\omega)xct(1-z_j^3)$, and $\{\eta q_j\}$ with prob. ωx; and for Δ^4, $\{q_j'\}$ equals $\emptyset$ with prob. $(1-x)+(1-\omega)x(1-ct)z_j^4$, $\{\lambda q_j\}$ with prob. $(1-\omega)x[1-(1-ct)z_j^4]$, and $\{\eta q_j\}$ with prob. ωx.

B.3 Value Function and Optimal Innovation Decisions

The conditional expectation in the value function considers the success/failure of own-innovation and creative destruction, the arrival of the creative destruction shock, outcomes of coin-tosses (c-t), the distribution of current period product quality q, and the distribution of the current period technology gap Δ^ℓ. Thus, $\mathbb{E}\big[V\big(\Phi^{f\prime}\big|\Phi^f\big)\big|\{z_j\}_{j\in\mathcal{J}^f}, x\big] = \sum_{I_1^{\overline{x}},\ldots,I_{n_f}^{\overline{x}}=0}^{1} \sum_{I_1^{z},\ldots,I_{n_f}^{z}=0}^{1} \sum_{\text{c-t}_1,\ldots,\text{c-t}_{n_f}=\text{win}}^{\text{lose}} \sum_{I^x=0}^{1} \Big[\prod_{i=1}^{n_f} \overline{x}^{I_i^{\overline{x}}}(1-\overline{x})^{1-I_i^{\overline{x}}} z_i^{I_i^z}(1-z_i)^{1-I_i^z}\Big] \times \big[x^{I^x}(1-x)^{1-I^x}\big]\big(\frac{1}{2}\big)^{n_f}\mathbb{E}_{q,\Delta}V\Big(\Big[\bigcup_{i=1}^{n_f}\Big[\big\{(\Delta_{j_i}'q_{j_i}, \Delta_{j_i}')\big|(q_{j_i}, \Delta_{j_i}), I_i^{\overline{x}}, I_i^z, \text{c-t}_i\big\} \setminus \{\mathbf{0}\}\Big]\Big] \bigcup \Big[\big\{\big(\frac{\eta}{\Delta_{-j}}I^x q_{-j}, \frac{\eta}{\Delta_{-j}}I^x\big)\big\} \setminus \{\mathbf{0}\}\Big]\Big)$. Note that the first term in the value function (before $\bigcup$) is the subsets of possible realizations for $\Phi^{f\prime}$ from own-innovation, creative destruction, and coin-toss. The second term in the value function (after $\bigcup$) shows the subsets of possible realizations for $\Phi^{f\prime}$ from creative destruction, where $\{q_{j_i}'\} = \{\Delta_{j_i}'q_{j_i}\} \setminus \{0\}$, and $\{q_{-j}'\} = \{\frac{\eta}{\Delta_{-j}}I^x q_{-j}\} \setminus \{0\}$. If $\Delta_{j_i}' = 0$, then firm f loses product line j_i and $\{(q_{j_i}', \Delta_{j_i}')\} \setminus \{\mathbf{0}\} = \{\mathbf{0}\} \setminus \{\mathbf{0}\} = \emptyset$.

B.4 Technology Gap Portfolio Composition Distribution Transition

The range of $\widetilde{k}^1$ can be determined as follows: i) for $0 \le \widetilde{k} \le \min\{n_f - k, k\}$, the two combinations preceding the term in brackets are well defined for any $\widetilde{k}^1 \in [0, \widetilde{k}] \cap \mathbb{Z}$ and describe all possible cases; ii) if $n_f - k \ge k$, then $\widetilde{k} > k$, $0 \le \widetilde{k} - \widetilde{k}^1$, and $0 \le \widetilde{k}^1 \le n_f - k$ is satisfied. This gives $\widetilde{k} - k \le \widetilde{k}^1 \le \widetilde{k}$; and iii) if $k \ge n_f - k$, then $\widetilde{k} > n_f - k$, $0 \le \widetilde{k} - \widetilde{k}^1$, and $0 \le \widetilde{k}^1 \le n_f - k$ is satisfied. Thus, $\max\{0, \widetilde{k} - k\} \le \widetilde{k}^1 \le n_f - k$.

By using $\widetilde{\mathbb{P}}(n_f,\widetilde{k}|n_f,k)$, the probability of $\mathcal{N}=\widetilde{\mathcal{N}}(n_f,k)$ transitioning to $\mathcal{N}'=\widetilde{\mathcal{N}}(n_f-h,\widetilde{k})$ for any $h\geq 0$ without considering creative destruction can be defined as follows: Take out h^1 numbers of product lines with $\Delta=\Delta^1$, and $h-h^1$ numbers of product lines with $\Delta=\Delta^2$ from $\widetilde{\mathcal{N}}(n_f,k)$, then compute the probability of $\widetilde{\mathcal{N}}(n_f-h,k-(h-h^1))$ transitioning to $\widetilde{\mathcal{N}}(n_f-h,\widetilde{k})$ with $\widetilde{\mathbb{P}}\big(n_f-h,\widetilde{k}|n_f-h,k-(h-h^1)\big)$ for all feasible h^1. Then, for $0\leq h<n_f$, $n_f\geq 1$, $0\leq\widetilde{k}\leq n_f-h$, and $0\leq k\leq n_f$, $\widetilde{\mathbb{P}}\big(n_f-h,\widetilde{k}|n_f,k\big)=\sum_{h^1=\max\{0,h-k\}}^{\min\{h,n_f-k\}}\Big[\binom{n_f-k}{h^1}\binom{k}{h-h^1}\overline{x}^h(1-z^2)^{h-h^1}\widetilde{\mathbb{P}}\big(n_f-h,\widetilde{k}|n_f-h,k-(h-h^1)\big)\Big]$; for $h=n_f\geq 1$, $\widetilde{k}=0$, and $0\leq k\leq n_f$, $\widetilde{\mathbb{P}}\big(n_f-h,\widetilde{k}|n_f,k\big)=\overline{x}^{n_f}(1-z^2)^k$; and 0 otherwise. The range for h^1 is defined from above, ensuring $0\leq h-h^1\leq k$ and $0\leq h^1\leq n_f-k$ for any h^1.

With $\widetilde{\mathbb{P}}\big(n_f-h,\widetilde{k}|n_f,k\big)$, other possible cases can be described for each case. For example, the probability of $\mathcal{N}=(n_f,n_f-k,k,0,0)$ to $\mathcal{N}'=(n_f-h,n_f-h-\widetilde{k},\widetilde{k},0,0)$ for $h\geq -1$ is defined as $\mathbb{P}\big(n_f-h,n_f-h-\widetilde{k},\widetilde{k},0,0\,\big|n_f,n_f-k,k,0,0\big)=\widetilde{\mathbb{P}}\big(n_f-h,\widetilde{k}\big|n_f,k\big)(1-x\overline{x}_{\text{takeover}})+\widetilde{\mathbb{P}}\big(n_f-h-1,\widetilde{k}\big|n_f,k\big)\mu(\Delta^3)\frac{1}{2}x(1-z^3)+\widetilde{\mathbb{P}}\big(n_f-h-1,\widetilde{k}-1\big|n_f,k\big)\mu(\Delta^4)x\left(1-\frac{1}{2}z^4\right)$. The first term is the probability of $\mathcal{N}$ transitioning to $\mathcal{N}'$ directly via the change in the firm's existing technology gap portfolio composition with unsuccessful creative destruction. The second term is the probability of $\mathcal{N}$ to $\widetilde{\mathcal{N}}(n_f-h-1,\widetilde{k})$, where successful creative destruction adds one product line with $\Delta'=\Delta^1$. Since the next period technology gap of product line j from successful creative destruction is $\Delta_j'=\frac{q_j'}{q_j}=\frac{\eta q_{j,-1}}{\Delta_j q_{j,-1}}=\frac{\eta}{\Delta_j}$, firm needs to take over a product line with a technology gap of $\Delta=\Delta^3=1+\eta$ to have a product line with a technology gap of Δ^1 in the next period. The third term is the probability of $\mathcal{N}$ to $\widetilde{\mathcal{N}}(n_f-h-1,\widetilde{k}-1)$, where successful creative destruction adds one product line with $\Delta'=\Delta^2$ by taking over a product line with a technology gap of $\Delta=\Delta^4$. For $h=-1$, the first term becomes zero by the definition of $\widetilde{\mathbb{P}}(\cdot|\cdot)$. Thus this probability is well defined for any $h\geq -1$.

With the computed probabilities of transitions between technology gap portfolio compositions, we can now define the inflows and outflows of a given technology gap portfolio. Let $\mathcal{F}$ denote the total mass of firms in the economy and $\mu(\mathcal{N})$ represent the share of firms with technology gap portfolio $\mathcal{N}$. Thus, $\widetilde{\mu}(\mathcal{N})=\mathcal{F}\mu(\mathcal{N})$. Then, for example, inflows and outflows for $\mathcal{N}=(n_f,n_f-k,k,0,0)$ can be described as follows: for $\mathcal{N}=(n_f,n_f-k,k,0,0)$ with $n_f\geq 2$, any firm whose next period technology gap portfolio is not $\mathcal{N}$ is counted as outflows, followed by $\mathit{outflow}(n_f,n_f-k,k,0,0)=$

$[1-\mathbb{P}(n_f, n_f-k, k, 0, 0 \big| n_f, n_f-k, k, 0, 0)] \times \mathcal{F}\mu(n_f, n_f-k, k, 0, 0)$. Any firm with a total number of product lines $n \geq n_f - 1$ can have a technology gap portfolio composition equal to $\mathcal{N}$ through the combinations of own-innovation and creative destructions. Thus, for the maximum number of product lines $\bar{n}_f$, $\mathit{inflow}(n_f, n_f-k, k, 0, 0) = \mathcal{F}\sum_{n=n_f-1}^{\bar{n}_f}\sum_{\widetilde{k}=0}^{n}\big[\mu(n, n-\widetilde{k}, \widetilde{k}, 0, 0)\mathbb{P}(n_f, n_f - k, k, 0, 0 \big| n, n-\widetilde{k}, \widetilde{k}, 0, 0) + \mu(n, n-1-\widetilde{k}I_{\{n>1\}}, \widetilde{k}I_{\{n>1\}}, 1, 0)\mathbb{P}(n_f, n_f-k, k, 0, 0 \big| n, n-1-\widetilde{k}I_{\{n>1\}}, \widetilde{k}I_{\{n>1\}}, 1, 0) + \mu(n, n-1-\widetilde{k}I_{\{n>1\}}, \widetilde{k}I_{\{n>1\}}, 0, 1)\mathbb{P}(n_f, n_f-k, k, 0, 0 \big| n, n-1-\widetilde{k}I_{\{n>1\}}, \widetilde{k}I_{\{n>1\}}, 0, 1)\big] - \mathcal{F}\mu(n_f, n_f-k, k, 0, 0)\mathbb{P}(n_f, n_f-k, k, 0, 0 \big| n_f, n_f-k, k, 0, 0)$.[3]

C Simple Three-Period Heterogeneous Innovation Model

To analyze firms' innovation incentives and derive testable predictions, we examine a three-period economy with two product markets and three firms. For simplicity, we assume $\omega = 0$ and $ct = \frac{1}{2}$.[4] In period 0, the economy starts with two product markets, market 1 and 2, with initial market-specific technologies $q_{1,0}$, and $q_{2,0}$, respectively. There are two firms in play, firm A and B. Firm A starts with product market 1 and an initial own-innovation probability $z_{1,0}$. Firm B, on the other hand, starts only with an initial creative destruction probability $x_{2,0}$, and can operate and produce in period 1 (but not in period 0). If creative destruction fails, firm B still keeps market 2 but produces with initial quality $q_{2,0}$. Thus, at the beginning of period 1, product qualities in the two markets are $q_{1,1} = \lambda q_{1,0}$ with probability $z_{1,0}$, $q_{1,1} = q_{1,0}$ with probability $1 - z_{1,0}$, $q_{2,1} = \eta q_{2,0}$ with probability $x_{2,0}$, and $q_{2,0}$ with probability $1 - x_{2,0}$, where $\lambda^2 > \eta > \lambda > 1$ represent innovation step sizes.

In period 1, the focal period, an outside firm engages in creative destruction to potentially take over the two product markets in period 2. The success of the outside firm in creative destruction is determined by the probability x_1^e for each product market. Additionally, there is a news shock in period 1 concerning the profit for period 2, possibly including an increase in foreign demand. Subsequently, the two incumbents utilize their given technologies to produce and invest in own-innovation and creative destructions. At the beginning of period 2, all innovation outcomes are realized, and then technological competition in each product market takes place. Only the firm with the highest technology in each product market continues producing. The economy ends after period

[3]Descriptions for other cases are available upon request.

[4]This simplifying assumption is innocuous for our results as shown in Appendix Sections H.3, H.6, and H.7.

2.

In period 1, incumbent firm $i \in \{A, B\}$ invests $R^{\text{in}}_{j,1}$ in own-innovation for $j \in \{1, 2\}$, achieving a success probability of $z_{j,1}$. The R&D production function is $z_{j,1} = \left(R^{\text{in}}_{j,1}/\widehat{\chi} q_{j,1}\right)^{0.5}$. Successful own-innovation increases next-period product quality by $\lambda > 1$. Thus, the period 2 product quality for firm i becomes $q^i_{j,2} = \lambda q_{j,1}$ with prob. $z_{j,1}$, and $q^i_{j,2} = q_{j,1}$ with prob. $1 - z_{j,1}$. Similarly, firm i invests $R^{\text{ex}}_{-j,1}$ to learn the period 0 technology used by firm $-i \neq i$ and improve it, which determines the success probability of creative destruction $x_{-j,1}$. The R&D production function is $x_{-j,1} = \left(R^{\text{ex}}_{-j,1}/\widetilde{\chi} q_{-j,0}\right)^{0.5}$, where $-j$ is owned by $-i$. Successful creative destruction enhances product quality relative to the lagged-period quality by $\eta > 1$. Thus, product $-j$'s quality in period 2 for firm i is $q^i_{-j,2} = \eta q_{-j,0}$ with prob. $x_{-j,1}$, and $q^i_{-j,2} = \emptyset$ with prob. $1 - x_{-j,1}$, where the symbol $\emptyset$ means firm i failed to acquire the production technology for product $-j$.

Optimal Innovation Decisions and Theoretical Predictions Assume that in a given product market j and period t, firms receive an instantaneous profit of $\pi_{j,t} q_{j,t}$ where $q_{j,t}$ is the product quality and $\pi_{j,t}$ is a market-period-specific constant known to firms before each period begins. Because there are only two products, incumbents and the outside firm can perform creative destruction on the same product. For simplicity in the model, we further assume that the outside firm can engage in creative destruction only if an incumbent fails to do so, following Garcia-Macia et al. (2019). Then the profit maximization problem for firm i in product market j with quality $q_{j,1}$ in period 1 can be written as $V(q_{j,1}) = \max_{\{z_{j,1}, x_{-j,1}\}} \Big\{ \pi_{j,1} q_{j,1} - \widehat{\chi}(z_{j,1})^2 q_{j,1} - \widetilde{\chi}(x_{-j,1})^2 q_{-j,0} + (1 - x_{j,1})(1 - x^e_1)\left[(1 - z_{j,1})\pi_{j,2} q_{j,1} + z_{j,1}\pi_{j,2}\lambda q_{j,1}\right] + (x_{j,1} + (1 - x_{j,1})x^e_1)\left[z_{j,1}\pi_{j,2}\lambda q_{j,1}\mathcal{I}_{\{\lambda q_{j,1} > \eta q_{j,0}\}} + \frac{1}{2}(1 - z_{j,1})\pi_{j,2} q_{j,1}\mathcal{I}_{\{q_{j,1} = \eta q_{j,0}\}}\right] + x_{-j,1}\Big[(1 - z_{-j,1})\pi_{-j,2}\eta q_{-j,0}\mathcal{I}_{\{\eta q_{-j,0} > q_{-j,1}\}} + z_{-j,1}\pi_{-j,2}\eta q_{-j,0}\mathcal{I}_{\{\eta q_{-j,0} > \lambda q_{-j,1}\}} + \frac{1}{2}(1 - z_{-j,1})\pi_{-j,2}\eta q_{-j,0}\mathcal{I}_{\{\eta q_{-j,0} = q_{-j,1}\}} + \frac{1}{2} z_{-j,1}\pi_{-j,2}\eta q_{-j,0}\mathcal{I}_{\{\eta q_{-j,0} = \lambda q_{-j,1}\}}\Big]\Big\}$, where $\mathcal{I}_{\{\cdot\}}$ is an indicator function that captures the possible relationships between the technologies of the three firms in period 2 within a given market. The first three terms show the period 1 profit net of total R&D cost.

The first bracket represents the incumbent's expected profit from market j if neither the incumbent nor the outside firm succeeds in creative destruction in market j. The second bracket represents the expected profit from market j if either the other incumbent or the outside firm succeeds in creative destruction in market j. The third bracket represents the expected profit from market $-j$ if firm i succeeds in creative destruction in market $-j$. The terms following $\frac{1}{2}$ account for scenarios where two firms could potentially produce the same quality product, triggering a coin-toss tiebreaker rule.

The interior solutions to this problem are: for $q_{j,1} = q_{j,0}$, $z^*_{j,1} = \frac{\pi_{j,2}}{2\widehat{\chi}}(\lambda-1)(1-x^*_{j,1})(1-x^e_1)$; for $q_{j,1} = \lambda q_{j,0}$, $z^*_{j,1} = \frac{\pi_{j,2}}{2\widehat{\chi}}\big[\lambda-(1-x^*_{j,1})(1-x^e_1)\big]$; for $q_{j,1} = \eta q_{j,0}$, $z^*_{j,1} = \frac{\pi_{j,2}}{2\widehat{\chi}}\big[\lambda-\frac{1}{2}-\frac{1}{2}(1-x^*_{j,1})(1-x^e_1)\big]$; for $q_{-j,1} = q_{-j,0}$, $x^*_{-j,1} = \frac{\eta\pi_{-j,2}}{2\widetilde{\chi}}$; for $q_{-j,1} = \lambda q_{-j,0}$, $x^*_{-j,1} = \frac{\eta\pi_{-j,2}}{2\widetilde{\chi}}(1-z^*_{-j,1})$; and for $q_{-j,1} = \eta q_{-j,0}$, $x^*_{-j,1} = \frac{\eta\pi_{-j,2}}{2\widetilde{\chi}}\frac{1}{2}(1-z^*_{-j,1})$, which maximize the firm profit considering the technology gap of its own and others, as well as the potential outcomes of own-innovation and creative destruction by all firms involved.

Proposition C.1. *For each* $q_{j,1}$ *and for* $\lambda^2 > \eta > \lambda > 1$*, we can order own-innovation intensities as* $z^*_{j,1}\big|_{q_{j,1}=\lambda q_{j,0}} > z^*_{1,1}\big|_{q_{j,1}=\eta q_{j,0}} > z^*_{j,1}\big|_{q_{j,1}=q_{j,0}}$*. Furthermore,* $\frac{\partial z^*_{j,1}}{\partial x^e_1}\big|_{q_{j,1}=\lambda q_{j,0}} > \frac{\partial z^*_{j,1}}{\partial x^e_1}\big|_{q_{j,1}=\eta q_{j,0}} > 0 > \frac{\partial z^*_{j,1}}{\partial x^e_1}\big|_{q_{j,1}=q_{j,0}}$*.*

Proof. The first part is straightforward with simple algebra. The second part is proved as follows. For $q_{j,1} = q_{j,0}$, we have $\frac{\partial z_{j,1}}{\partial x^e_1} = -\frac{\pi_{j,2}}{2\widehat{\chi}}(\lambda-1)\big[(1-x_{j,1}) + (1-x^e_1)\frac{\partial x_{j,1}}{\partial x^e_1}\big]$, and $\frac{\partial x_{j,1}}{\partial x^e_1} = 0$. Thus, the following is obtained: $\frac{\partial z_{j,1}}{\partial x^e_1} = -\frac{\pi_{j,2}}{2\widehat{\chi}}(\lambda-1)(1-x_{j,1}) < 0$. For $q_{j,1} = \lambda q_{j,0}$, we have $\frac{\partial z_{j,1}}{\partial x^e_1} = \frac{\pi_{j,2}}{2\widehat{\chi}}\big[1-x_{j,1} + (1-x^e_1)\frac{\partial x_{j,1}}{\partial x^e_1}\big]$ and $\frac{\partial x_{j,1}}{\partial x^e_1} = -\frac{\eta\pi_{j,2}}{2\widetilde{\chi}}\frac{\partial z_{j,1}}{\partial x^e_1}$. Thus, the following holds: $\frac{\partial z_{j,1}}{\partial x^e_1} = (1-x_{j,1})\big[\frac{2\widehat{\chi}}{\pi_{j,2}} + \frac{\eta\pi_{j,2}}{2\widetilde{\chi}}(1-x^e_1)\big]^{-1} > 0$, hence $\frac{\partial x_{j,1}}{\partial x^e_1} = -\frac{\eta\pi_{j,2}}{2\widetilde{\chi}}\frac{\partial z_{j,1}}{\partial x^e_1} < 0$. For $q_{j,1} = \eta q_{j,0}$, we have $\frac{\partial z_{j,1}}{\partial x^e_1} = \frac{\pi_{j,2}}{2\widehat{\chi}}\frac{1}{2}\big[1-x_{j,1} + (1-x^e_1)\frac{\partial x_{j,1}}{\partial x^e_1}\big]$, and $\frac{\partial x_{j,1}}{\partial x^e_1} = -\frac{\eta\pi_{j,2}}{2\widetilde{\chi}}\frac{1}{2}\frac{\partial z_{j,1}}{\partial x^e_1}$. This gives $\frac{\partial z_{j,1}}{\partial x^e_1} = (1-x_{j,1})\big[\frac{4\widehat{\chi}}{\pi_{j,2}} + \frac{\eta\pi_{j,2}}{4\widetilde{\chi}}(1-x^e_1)\big]^{-1} > 0$, hence $\frac{\partial x_{j,1}}{\partial x^e_1} = -\frac{1}{2}\frac{\eta\pi_{j,2}}{2\widetilde{\chi}}\frac{\partial z_{j,1}}{\partial x^e_1} < 0$ holds. With $x^*_{j,1}$ and $\frac{\eta\pi_{j,2}}{2\widetilde{\chi}} \in (0,1)$, along with the restriction $4\widehat{\chi} > \pi_{j,2}$, the following holds: $\frac{4\widehat{\chi}}{\pi_{j,2}} + \frac{\eta\pi_{j,2}}{4\widetilde{\chi}}(1-x^e_1) > \frac{2\widehat{\chi}}{\pi_{j,2}} + \frac{\eta\pi_{j,2}}{2\widetilde{\chi}}(1-x^e_1)$. Therefore, we get $\frac{\partial z^*_{j,1}}{\partial x^e_1}\big|_{q_{j,1}=\lambda q_{j,0}} > \frac{\partial z^*_{j,1}}{\partial x^e_1}\big|_{q_{j,1}=\eta q_{j,0}}$ □

The second part of proposition C.1 suggests that firms without a technology gap decrease their own-innovation when facing a higher probability of creative destruction in their own markets. This is because they cannot enhance their product protection through own-innovation. Conversely, firms with a significant technological advantage do not substantially increase their own-innovation in response to creative destruction from outsiders, as the risk of losing their own product market is minimal. In intermediate cases, firms intensify their own-innovation response to creative destruction from outsiders to reduce the probability of losing their market.

Higher innovation in period 0 increases the probability of achieving a high technology gap in period 1, thereby aiding firms in market protection. To understand how past innovation intensity influences the firm's current decision on own-innovation when facing a higher probability of encountering a competitor, x_1^e, we define the expected value of own-innovation intensity in period 1 as $\bar{z}_1^* = z_{1,1}^*\big|_{q_{1,1}=q_{1,0}}\frac{1}{2}(1-z_{1,0}) + z_{2,1}^*\big|_{q_{2,1}=q_{2,0}}\frac{1}{2}(1-x_{2,0}) + z_{1,1}^*\big|_{q_{1,1}=\lambda q_{1,0}}\frac{1}{2}z_{1,0} + z_{2,1}^*\big|_{q_{2,1}=\eta q_{2,0}}\frac{1}{2}x_{2,0}$, where $\frac{1}{2}$ accounts for the two products. Proposition C.1 provides the following result:

Corollary C.1 (Market-Protection Effect)**.** *The impact of period 0 innovation intensities, $z_{1,0}$ and $x_{2,0}$, on expected own-innovation in period 1 can be characterized as follows: $\frac{\partial \bar{z}_1^*}{\partial x_1^e \partial z_{1,0}} > 0$, and $\frac{\partial \bar{z}_1^*}{\partial x_1^e \partial x_{2,0}} > 0$.*

Proof. From $\bar{z}_1^*$, we know that $\frac{\partial \bar{z}_1^*}{\partial z_{1,0}} = \frac{1}{2}\left(z_{1,1}^*\big|_{q_{1,1}=\lambda q_{1,0}} - z_{1,1}^*\big|_{q_{1,1}=q_{1,0}}\right) > 0$ and $\frac{\partial \bar{z}_1^*}{\partial x_{2,0}} = \frac{1}{2}\left(z_{2,1}^*\big|_{q_{2,1}=\eta q_{2,0}} - z_{2,1}^*\big|_{q_{2,1}=q_{2,0}}\right) > 0$, where the signs can be derived from proposition C.1. The results follow from proposition C.1. □

Corollary C.1 suggests that intensive innovation in the previous period prompts firms to increase own-innovation in response to higher competitive pressure. As indicated by the optimal decision rule, firms' decisions regarding creative destruction also depend on the past innovation decisions of other firms, which is outlined in the following proposition.

Proposition C.2. *For each $q_{j,1}$ and for $\lambda^2 > \eta > \lambda > 1$, we can order creative destruction intensities as follows: $x_{j,1}^*\big|_{q_{j,1}=q_{j,0}} > x_{j,1}^*\big|_{q_{j,1}=\lambda q_{j,0}} > x_{j,1}^*\big|_{q_{j,1}=\eta q_{j,0}}$. Furthermore, $\frac{\partial x_{j,1}^*}{\partial x_1^e}\big|_{q_{j,1}=q_{j,0}} = 0$, $\frac{\partial x_{j,1}^*}{\partial x_1^e}\big|_{q_{j,1}=\lambda q_{j,0}} < 0$, and $\frac{\partial x_{j,1}^*}{\partial x_1^e}\big|_{q_{j,1}=\eta q_{j,0}} < 0$. Proof: See the proof for Proposition C.1*

Proposition C.2 implies that firms decrease creative destruction if incumbents hold a higher technology advantage, as it becomes more difficult to displace them in the market through creative destruction. In markets where there is a technological barrier (technology gap > 1), firms also reduce their creative destruction in response to increased creative destruction by outside firms. This is because incumbents in these markets respond defensively by increasing own-innovation (proposition C.1). To understand how the past innovation intensity of other firms influences a firm's current decision on creative destruction, define the expected value of creative destruction intensity in period 1 as $\bar{x}_1^* = x_{1,1}^*\big|_{q_{1,1}=q_{1,0}}\frac{1}{2}(1-z_{1,0}) + x_{2,1}^*\big|_{q_{2,1}=q_{2,0}}\frac{1}{2}(1-x_{2,0}) + x_{1,1}^*\big|_{q_{1,1}=\lambda q_{1,0}}\frac{1}{2}z_{1,0} + x_{2,1}^*\big|_{q_{2,1}=\eta q_{2,0}}\frac{1}{2}x_{2,0}$. Then, the first part of proposition C.2 implies the following:

Corollary C.2 (Technological Barrier Effect)**.** *Given technology $q_{j,1}$ and period 0 innovation intensities $z_{1,0}$ and $x_{2,0}$, $\frac{\partial \bar{x}_1^*}{\partial z_{1,0}} < 0$ and $\frac{\partial \bar{x}_1^*}{\partial x_{2,0}} < 0$ hold.*

Proof. $\frac{\partial \overline{x}_1^*}{\partial z_{1,0}} = \frac{1}{2}\big(x_{1,1}^*\big|_{q_{1,1}=\lambda q_{1,0}} - x_{1,1}^*\big|_{q_{1,1}=q_{1,0}}\big) < 0$, and $\frac{\partial \overline{x}_1^*}{\partial x_{2,0}} = \frac{1}{2}\big(x_{2,1}^*\big|_{q_{2,1}=\eta q_{2,0}} - x_{2,1}^*\big|_{q_{2,1}=q_{2,0}}\big) < 0$, where the signs follow from proposition C.2 □

Corollary C.2 indicates that higher technology levels in other markets, resulting from previous innovation, act as effective technological barriers, making it challenging for outside firms to take over those product markets. This reduces firms' incentives for creative destruction. Lastly, because innovation is forward-looking, changes in future profits π' are crucial factors influencing the current period's innovation. Proposition C.3 summarizes this:

Proposition C.3 (Ex-post Schumpeterian Effect)**.** *Given the expected profit $\pi_{j,2}$ in period 2, we obtain $\frac{\partial z_{j,1}^*}{\partial \pi_{j,2}} > 0\ \forall q_{j,1}$ and $\frac{\partial x_{j,1}^*}{\partial \pi_{j,2}} > 0$ for $q_{j,1} = q_{j,0}$. The signs for $\frac{\partial x_{j,1}^*}{\partial \pi_{j,2}}$ for other technology gaps remain ambiguous.*

Proof. For $q_{j,1} = q_{j,0}$, we have $\frac{\partial z_{j,1}}{\partial \pi_{j,2}} = \frac{1}{2\widehat{\chi}}(\lambda-1)(1-x_{j,1})(1-x_1^e) - \frac{\pi_{j,2}}{2\widehat{\chi}}(\lambda-1)(1-x_1^e)\frac{\partial x_{j,1}}{\partial \pi_{j,2}}$ and $\frac{\partial x_{j,1}}{\partial \pi_{j,2}} = \frac{\eta}{2\widetilde{\chi}}$. Thus, $\frac{\partial z_{j,1}}{\partial \pi_{j,2}} = \frac{1}{2\widehat{\chi}}(\lambda-1)(1-2x_{j,1})(1-x_1^e) > 0$ iff $x_{j,1} < \frac{1}{2}$. For $q_{j,1} = \lambda q_{j,0}$, we get $\frac{\partial z_{j,1}}{\partial \pi_{j,2}} > 0$ unambiguously. For $q_{j,1} = \eta q_{j,0}$, $\frac{\partial z_{j,1}}{\partial \pi_{j,2}} = \frac{1}{2\widehat{\chi}}\big[\lambda - \frac{1}{2} - \frac{1}{2}(1-x_{j,1})(1-x_1^e)\big] + \frac{\pi_{j,2}}{2\widehat{\chi}}\frac{1}{2}(1-x_1^e)\frac{\partial x_{j,1}}{\partial \pi_{j,2}}$ and $\frac{\partial x_{j,1}}{\partial \pi_{j,2}} = \frac{\eta}{2\widetilde{\chi}}\frac{1}{2}(1-z_{j,1}) - \frac{\eta\pi_{j,2}}{2\widetilde{\chi}}\frac{1}{2}\frac{\partial z_{j,1}}{\partial \pi_{j,2}}$ are obtained, and we get $\frac{\partial z_{j,1}}{\partial \pi_{j,2}} = \big[\lambda - \frac{1}{2} - \frac{1}{2}(1-2x_{j,1})(1-x_1^e)\big]\big[2\widehat{\chi} + \frac{\eta(\pi_{j,2})^2}{2\widetilde{\chi}}\frac{1}{4}(1-x_1^e)\big]^{-1} > 0$. The sign for $\frac{\partial x_{j,1}}{\partial \pi_{j,2}}$ remains ambiguous. □

Proposition C.3 implies that any factor that affects future profits may influence firms' own-innovation and creative destruction. Specifically, an increase in expected profit from one's own market encourages firms to intensify their own-innovation efforts. However, the impact of an increase in expected profit in other markets on firms' decisions regarding creative destruction is ambiguous when the local technology gap exceeds 1. This ambiguity arises because incumbents in these markets tend to increase their own-innovation efforts in response to higher expected profits, thereby allowing them to protect their markets. In cases where the local technology gap equals 1, incumbents cannot protect their markets through own-innovation alone. Consequently, an increase in expected future profit unambiguously stimulates creative destruction in such scenarios. These findings highlight the diverse factors influencing own-innovation, creative destruction, and overall innovation levels.

D Extension: Stochastic Innovation Step Size

In this section, we extend our baseline model by relaxing the constant innovation step size assumption. We demonstrate that the predictions of our baseline model remain robust without assuming that $\lambda^2 > \eta$. Thus, $\lambda^2 > \eta$ is an innocuous simplifying assumption serving only to clarify the exposition of the main mechanism and reduce computational burden. For simplicity, we assume $\omega = 0$ and $ct = \frac{1}{2}$ in this proof, which is innocuous for our results.[5]

Following Garcia-Macia et al. (2019), we let firms draw innovation step sizes from probability distributions. Successful innovation improves product quality by a step size drawn from a distribution. For own-innovation, $\lambda \sim \widehat{\mu}(\lambda)$, where $\lambda \in [\lambda_L, \lambda_U]$ with mean $\overline{\lambda}$; for creative destruction, $\eta \sim \widetilde{\mu}(\eta)$, where $\eta \in [\eta_L, \eta_U]$ with mean $\overline{\eta}$. Here, $\lambda_L \geq 1$ and $\eta_L \geq 1$ hold. To be consistent with our empirical findings in Section I.6.1, we assume $\overline{\eta} \geq \overline{\lambda}$. Under this setup, the technology gap is continuous, taking values $\Delta \in [1, \eta_U]$.

Innovation by Incumbents Consider firm A, which owns product 1 with quality q_{1t} and technology gap Δ_{1t}, where $q_{1t} = \Delta_{1t} q_{1t-1}$, and $\Delta_{1t} \in [1, \eta_U]$. For simplicity, assume firms exit the economy in $t+1$ after receiving profits from their products. If firm A retains product 1 in $t+1$, it receives a profit of πq_{1t+1} and zero otherwise. Furthermore, if firm A succeeds in taking over product 2 owned by firm B, it receives a profit of πq_{2t+1}. The value function of firm A in t is then $V(q_{1t}, \Delta_{1t}) = \max_{z_{1t}, x^A_{2t}} \big\{ \pi \Delta_{1t} q_{1t-1} - \widehat{\chi} z_{1t}^{\widehat{\psi}} \Delta_{1t} q_{1t-1} - \widetilde{\chi} (x^A_{2t})^{\widetilde{\psi}} q_{2t-1} + \widetilde{\beta} \mathbb{E}_{\{\lambda_{jt}, \eta_{jt}\}_{j=1}^2} \big[(1 - z_{1t})(1 - x^B_{1t} + x^B_{1t} Pr(\Delta_{1t} \geq \eta_{1t})) \pi \Delta_{1t} q_{1t-1} + z_{1t}(1 - x^B_{1t} + x^B_{1t} Pr(\Delta_{1t}\lambda_{1t} \geq \eta_{1t})) \pi \Delta_{1t} \lambda_{1t} q_{1t-1} + x^A_{2t}(z_{2t} Pr(\eta_{2t} > \Delta_{2t}\lambda_{2t}) + (1 - z_{2t}) Pr(\eta_{2t} > \Delta_{2t})) \pi \eta_{2t} q_{2t-1} \big] \big\}$. The first three terms represent current period profits net of R&D costs for own-innovation ($\widehat{\chi} z_{1t}^{\widehat{\psi}} \Delta_{1t} q_{1t-1}$) and creative destruction ($\widetilde{\chi}(x^A_{2t})^{\widetilde{\psi}} q_{2t-1}$). The terms inside the expectation operator correspond to the expected profits from the existing product (product 1) and that from taking over product 2 through creative destruction. Here, z_{2t}, x^B_{1t}, and Δ_{2t} represent firm B's respective counterparts for own-innovation and creative destruction intensities, and technology gap.

Taking the first-order conditions with respect to z_{1t} and x^A_{2t} yields firm A's optimal own-innovation decision $z^*_{1t} = \big(\widetilde{\beta}\pi / \widehat{\chi}\widehat{\psi} \big)^{1/(\widehat{\psi}-1)} \big[(\overline{\lambda} - 1)(1 - x^B_{1t}) + x^B_{1t} \mathbb{E}_{\lambda_{1t}, \eta_{1t}} \{ \lambda_{1t} Pr(\Delta_{1t}\lambda_{1t} \geq \eta_{1t}) \} - x^B_{1t} \mathbb{E}_{\eta_{1t}} \{ Pr(\Delta_{1t} \geq \eta_{1t}) \} \big]^{1/(\widehat{\psi}-1)}$, and optimal creative destruction decision $x^{A*}_{2t} = \big(\widetilde{\beta}\pi / \widetilde{\chi}\widetilde{\psi} \big)^{1/(\widetilde{\psi}-1)}$

[5] See discussions in Appendix Sections H.3, H.6, and H.7.

$\left[z_{2t}\mathbb{E}_{\lambda_{2t},\eta_{2t}}\{\eta_{2t}Pr(\eta_{2t} > \Delta_{2t}\lambda_{2t})\} + (1 - z_{2t})\mathbb{E}_{\eta_{2t}}\{\eta_{2t}Pr(\eta_{2t} > \Delta_{2t})\}\right]^{1/(\widetilde{\psi}-1)}$. The following proposition shows that the changes in firms' own-innovation decisions in response to increasing competition mirror those in the baseline model. To prove this analytically, we assume that the two step sizes are drawn from uniform distributions, $\mathcal{U}(\cdot,\cdot)$.

Proposition D.1 (Market-Protection Effect)**.** *Suppose $\lambda \sim \mathcal{U}(\lambda_L, \lambda_U)$ and $\eta \sim \mathcal{U}(\eta_L, \eta_U)$, with $\overline{\eta} \geq \overline{\lambda}$, $\lambda_U > \eta_L$, and equal variances. Then, $\frac{\partial z^*_{1t}(x^B_{1t},\Delta_{1t})}{\partial x^B_{1t}}$ is hump-shaped with respect to the technology gap Δ_{1t} and is positive over a region that includes $[\frac{\eta_L}{\lambda_L}, \eta_U)$ when $\frac{\lambda_U}{\lambda_L} \in (1, 4)$. Additionally, $\frac{\partial z^*_{1t}(x^B_{1t},\Delta_{1t})}{\partial x^B_{1t}}\Big|_{\Delta_{1t}=\eta_U} = 0$, while the sign of $\frac{\partial z^*_{1t}(x^B_{1t},\Delta_{1t})}{\partial x^B_{1t}}\Big|_{\Delta_{1t}=1}$ remains ambiguous.*

Proof. The sign of $\frac{\partial z^*_{1t}(x^B_{1t},\Delta_{1t})}{\partial x^B_{1t}}$ follows the sign of $** = -(\overline{\lambda} - 1) - \mathbb{E}_{\eta_{1t}}\{Pr(\Delta_{1t} \geq \eta_{1t})\} + \mathbb{E}_{\lambda_{1t},\eta_{1t}}\{\lambda_{1t}Pr(\Delta_{1t}\lambda_{1t} \geq \eta_{1t})\}$. Depending on the value of Δ_{1t}, there are three cases to consider: Case 1, where $\Delta_{1t}\lambda_L < \eta_L$; Case 2, where $\Delta_{1t}\lambda_L \in [\eta_L, \eta_U)$; and Case 3, where $\Delta_{1t}\lambda_L = \eta_U$. Without loss of generality, we normalize $\lambda_L = 1$. In Case 1, we have $\mathbb{E}_{\eta_{1t}}\{Pr(\Delta_{1t} \geq \eta_{1t})\} = 0$, and $\mathbb{E}_{\lambda_{1t},\eta_{1t}}\{\lambda_{1t}Pr(\Delta_{1t}\lambda_{1t} \geq \eta_{1t})\} = \frac{\Delta_{1t}\lambda_U - \eta_L}{6\Delta^2_{1t}(\eta_U-\eta_L)(\lambda_U-\lambda_L)}(2\Delta^2_{1t}\lambda^2_U - \Delta_{1t}\lambda_U\eta_L - \eta^2_L)$. Then, we can show that $\frac{\partial **}{\partial \Delta_{1t}} > 0$ as $\lambda_U > \eta_L > \Delta_{1t} \geq 1$, and $\frac{\partial^2 **}{\partial \Delta^2_{1t}} > 0$. Thus, $**$ is an increasing and convex function of Δ_{1t}. Furthermore, $** \,|_{\Delta_{1t}\nearrow\eta_L/\lambda_L} > 0$ when $\frac{\lambda_U}{\lambda_L} \in (1, 4)$. The sign of $\frac{\partial **}{\partial \Delta_{1t}}\Big|_{\Delta_{1t}=1}$ is ambiguous. In Case 2, we have $\mathbb{E}_{\eta_{1t}}\{Pr(\Delta_{1t} \geq \eta_{1t})\} = \frac{\Delta_{1t}-\eta_L}{\eta_U-\eta_L}$, and $\mathbb{E}_{\lambda_{1t},\eta_{1t}}\{\lambda_{1t}Pr(\Delta_{1t}\lambda_{1t} \geq \eta_{1t})\} = \frac{1}{2(\eta_U-\eta_L)(\lambda_U-\lambda_L)}\left[\lambda^2_U(\eta_U - \eta_L) + \lambda^2_L\eta_L - \frac{1}{3\Delta^2_{1t}}(2\Delta^3_{1t}\lambda^3_L + \eta^3_U)\right]$. Then, we can show that $\frac{\partial^2 **}{\partial \Delta^2_{1t}} < 0$, $\frac{\partial **}{\partial \Delta_{1t}}\Big|_{\Delta_{1t}=\eta_L/\lambda_L} > 0$, $\frac{\partial **}{\partial \Delta_{1t}}\Big|_{\Delta_{1t}\nearrow\eta_U} < 0$, and $**\,\Big|_{\Delta_{1t}\nearrow\eta_U} = 0$. Thus, $**$ is a concave function of Δ_{1t}, achieving a maximum at $\Delta^*_{1t} \in (\eta_L/\lambda_L, \eta_U)$. Since $**\,\Big|_{\Delta_{1t}\nearrow\eta_U} = 0$, it follows that $**\,\Big|_{\Delta_{1t}=\Delta^*_{1t}} > 0$. As in Case 1, $** \,|_{\Delta_{1t}=\eta_L/\lambda_L} > 0$ when $\frac{\lambda_U}{\lambda_L} \in (1, 4)$. In Case 3, we have $\mathbb{E}_{\eta_{1t}}\{Pr(\Delta_{1t} \geq \eta_{1t})\} = 1$ and $\mathbb{E}_{\lambda_{1t},\eta_{1t}}\{\lambda_{1t}Pr(\Delta_{1t}\lambda_{1t} \geq \eta_{1t})\} = \overline{\lambda}$. Therefore, $** = 0$. □

The condition $\lambda_U/\lambda_L \in (1, 4)$ implies that the average quality improvement ranges from 0% to 150%. Thus, this condition is most likely satisfied in the real application. Furthermore, $\frac{\partial z^*_{1t}(x^B_{1t},\Delta_{1t})}{\partial x^B_{1t}} > 0$ in a region near $\Delta_{1t} = \eta_U$, even without imposing this condition. Although the sign of $\frac{\partial **}{\partial \Delta_{1t}}\Big|_{\Delta_{1t}=1}$ is ambiguous, numerical analysis shows that it is negative as long as η_L and λ_L is not significantly different. For example, in our baseline model calibration, we have $\overline{\lambda} = 1.04$ and $\overline{\eta} = 1.075$, which implies $\frac{\eta_L}{\lambda_L} = 1.034$, and $\frac{\lambda_U}{\lambda_L} = 1.08$. These values satisfy $\frac{\partial **}{\partial \Delta_{1t}}\Big|_{\Delta_{1t}=1} < 0$.

The next proposition shows that this extended model also has the technological barrier effect.

Proposition D.2 (Technological Barrier Effect)**.** *High own-innovation intensity by an incumbent (z_{2t}) as well as a high technological barrier in the target market (Δ_{2t}) both discourage creative destruction by rival firms. Formally, $\frac{\partial x^A_{2t}(z_{2t},\Delta_{2t})}{\partial z_{2t}} < 0$, and $\frac{\partial x^A_{2t}(z_{2t},\Delta_{2t})}{\partial \Delta_{2t}} < 0$.*

Proof. Since $\lambda_{2t} \geq 1$ and $\eta_{2t} \geq 1$, we have $\mathbb{E}_{\eta_{2t}}\{\eta_{2t}Pr(\eta_{2t} > \Delta_{2t})\} > \mathbb{E}_{\eta_{2t},\lambda_{2t}}\{\eta_{2t}Pr(\eta_{2t} > \Delta_{2t}\lambda_{2t})\}\ \forall\ \Delta_{2t} \geq 1$. Thus, $\frac{\partial x^A_{2t}(z_{2t},\Delta_{2t})}{\partial z_{2t}} < 0$. Furthermore, $\mathbb{E}_{\eta_{2t},\lambda_{2t}}\{\eta_{2t}Pr(\eta_{2t} > \Delta_{2t}\lambda_{2t})\}$ is a decreasing function of Δ_{2t}. Thus, $\frac{\partial x^A_{2t}(z_{2t},\Delta_{2t})}{\partial \Delta_{2t}} < 0$. $\square$

E Extension: Multi-Creative Destruction

We extend our baseline model by allowing firms to do multiple creative destructions. The household's problem, as well as the production decisions of the final good producer and intermediate producers, remain unchanged. We therefore focus on intermediate producers' innovation decisions and aggregate variables that are affected by the multi-creative destruction.

E.1 Optimal Innovation Decision

Following Klette and Kortum (2004) and several follow-on studies, we model firms' creative destruction decisions based on the number of products they produce (n_f). Creative destruction can be viewed as a spin-off derived from each firm's existing products. Consider product j firm f owns with quality q_j and technology gap Δ^ℓ_j. In the subsequent period, the evolution of this product can result in six cases: firm f i) loses product j and business takeover (through creative destruction) fails, ii) loses product j and takeover succeeds, iii) keeps product j while both own-innovation and takeover fail, iv) keeps product j while own-innovation fails, but takeover succeeds, v) keeps product j with successful own-innovation, but takeover fails, and vi) keeps product j with successful own-innovation and takeover. Denoting the product-technology gap pair for a product that firm f acquires through successful business takeover in the next period as $\{(q', \Delta')\}$, we can write down the evolution of the product portfolio stemming from $\Phi^f = \{(q_j, \Delta^\ell_j)\}$ for $\ell \in \{1,2,3,4\}$ for each of the six cases. For example, for $\Delta^\ell_j = \Delta^2$, $\Phi^{f\prime}_j = \emptyset \cup \emptyset$ with prob. $\overline{x}(1-(1-\omega)z^2)(1-x\overline{x}_{\text{takeover}})$, $\Phi^{f\prime}_j = \emptyset \cup \{(q', \Delta')\}$ with prob. $\overline{x}(1-(1-\omega)z^2)(x\overline{x}_{\text{takeover}})$, $\Phi^{f\prime}_j = \{(q_j, \Delta^1)\} \cup \emptyset$ with prob. $(1-\overline{x})(1-z^2)(1-x\overline{x}_{\text{takeover}})$, $\Phi^{f\prime}_j = \{(q_j, \Delta^1)\} \cup \{(q', \Delta')\}$ with prob. $(1-\overline{x})(1-z^2)(x\overline{x}_{\text{takeover}})$,

$\Phi_j^{f\prime} = \{(\Delta^2 q_j, \Delta^2)\} \cup \emptyset$ with prob. $(1-\omega\bar{x})z^2(1-x\overline{x}_{\text{takeover}})$, and $\Phi_j^{f\prime} = \{(\Delta^2 q_j, \Delta^2)\} \cup \{(q', \Delta')\}$ with prob. $(1-\omega\bar{x})z^2(x\overline{x}_{\text{takeover}})$.[6]

If the value function is additively separable with respect to each product a firm produces, we only need to solve it at the product level and aggregate it to the firm level. For product j with $\Phi_j^f = \{(q_j, \Delta^\ell)\}$, the value function is given by $V(\Phi_j^f) = \max_{z_j, x_j} \{\pi q_j - \widehat{\chi} z_j^{\widehat{\psi}} q_j - \widetilde{\chi} x_j^{\widetilde{\psi}} \overline{q} - F\overline{q} + \widetilde{\beta}\mathbb{E}\big[V'\big(\Phi_j^{f\prime}\big)|\Phi_j^f, z_j, x_j\big]\}$, where $F\overline{q}$ represents fixed operating costs.[7] The value function for firm f with a portfolio of product quality and technology gap is then: $\Phi^f = \big\{\Phi_j^f\big\}_{j\in\mathcal{J}^f}$ is $V\big(\Phi_j^f\big) = \sum_{j\in\mathcal{J}^f} V\big(\Phi_j^f\big)$. The following proposition derives analytic expressions for firms' decision rules.[8]

Proposition E.1. *Given a technology gap distribution $\big\{\mu\big(\Delta^\ell\big)\big\}_{\ell=1}^4$, a fixed cost of operation equal to $F\overline{q} = \widetilde{\beta}B(1+g)\overline{q}$, and the exit value for a product given by $V(\emptyset) = \frac{B\overline{q}}{1-x\overline{x}_{\text{takeover}}}$, the value function of firm f with a product quality and technology gap portfolio of $\Phi^f \equiv \big\{(q_j, \Delta_j)\big\}_{j\in\mathcal{J}^f}$ is: $V(\Phi^f) = \sum_{\ell=1}^4 A_\ell\big(\sum_{j\in\mathcal{J}^f|\Delta_j=\Delta^\ell} q_j\big) + n_f B\overline{q}$, where $A_1 = \pi - \widehat{\chi}(z^1)^{\widehat{\psi}} + \widetilde{\beta}\big[A_1(1-\overline{x})(1-z^1) + \lambda A_2(1-\overline{x})z^1\big]$, $A_2 = \pi - \widehat{\chi}(z^2)^{\widehat{\psi}} + \widetilde{\beta}\big[A_1(1-\overline{x})(1-z^2) + \lambda A_2(1-\omega\bar{x})z^2\big]$, $A_3 = \pi - \widehat{\chi}(z^3)^{\widehat{\psi}} + \widetilde{\beta}\big[A_1\big((1-\omega)(1-ct)\bar{x} + 1 - \bar{x}\big)(1-z^3) + \lambda A_2(1-\omega\bar{x})z^3\big]$, $A_4 = \pi - \widehat{\chi}(z^4)^{\widehat{\psi}} + \widetilde{\beta}\big[A_1(1-\overline{x})(1-z^4) + \lambda A_2\big((1-\omega)(1-ct)\bar{x} + 1 - \bar{x}\big)z^4\big]$, and $B = \big[x\widetilde{\beta}A_{\text{takeover}} - \widetilde{\chi}x^{\widetilde{\psi}}\big]/\big[1 - \widetilde{\beta}(1+g)x\overline{x}_{\text{takeover}}\big]$, and the optimal innovation probabilities are $z^1 = \big[\widetilde{\beta}\big[(1-\overline{x})\lambda A_2 - (1-\overline{x})A_1\big]/\big[\widehat{\psi}\widehat{\chi}\big]\big]^{\frac{1}{\widehat{\psi}-1}}$, $z^2 = \big[\big[\widetilde{\beta}\big[\lambda A_2(1-\omega\bar{x}) - (1-\overline{x})A_1\big]/\big[\widehat{\psi}\widehat{\chi}\big]\big]^{\frac{1}{\widehat{\psi}-1}}$, $z^3 = \big[\big[\widetilde{\beta}\big[\lambda A_2(1-\omega\bar{x}) - \big((1-\omega)(1-ct)\bar{x} + 1 - \bar{x}\big)A_1\big]/\big[\widehat{\psi}\widehat{\chi}\big]\big]^{\frac{1}{\widehat{\psi}-1}}$, $z^4 = \big[\big[\widetilde{\beta}\big[\lambda\big((1-\omega)(1-ct) + 1 - \bar{x}\big)A_2 - \big(1-\overline{x}\big)A_1\big]/\big[\widehat{\psi}\widehat{\chi}\big]\big]^{\frac{1}{\widehat{\psi}-1}}$, and $x = \big[\big[\widetilde{\beta}\big[A_{\text{takeover}} + \overline{x}_{\text{takeover}}B(1+g)\big]/\big[\widetilde{\psi}\widetilde{\chi}\big]\big]^{\frac{1}{\widetilde{\psi}-1}}$, where g is the average product quality growth rate in the economy, A_{takeover} is the ex-ante value of a product line obtained from successful takeover, defined as $A_{\text{takeover}} \equiv (1-z^3)(1-\omega)ctA_1\mu(\Delta^3) + (1-\omega)(1-(1-ct)z^4)A_2\lambda\mu(\Delta^4) + A_3\eta\mu(\Delta^1) + (1-\omega)(1-z^2)A_4\frac{\eta}{\lambda}\mu(\Delta^2) + (1-\mu(\Delta^1))A_3\eta\omega$, and $\overline{x}_{\text{takeover}} = \mu(\Delta^1) + (1-(1-\omega)z^2)\mu(\Delta^2) + (\omega + (1-\omega)ct(1-z^3))\mu(\Delta^3) + \big(1-(1-\omega)(1-ct)z^4\big)\mu(\Delta^4)$.*

Proof. Suppose the value function is additively separable with respect to each product a firm produces. Then, we can rewrite the expected future value term for each technology gap case as follows: for Δ^1, $\mathbb{E}\big[V'\big(\Phi^{f\prime}\big)|\Phi^f, z^1, x\big] = (1-\overline{x})(1-z^1)V'\big(\big\{(q_j, \Delta^1)\big\}\big) + (1-\overline{x})z^1 V'\big(\big\{(\Delta^2 q_j, \Delta^2)\big\}\big) +$

[6]For simplicity, we use the unconditional probability of business takeover $x\overline{x}_{\text{takeover}}$ abusively.
[7]This is commonly assumed for tractability (Akcigit and Kerr, 2018; De Ridder, 2024; Argente et al., 2024)
[8]The analytic expression for startup decisions remains unchanged.

$x\mathbb{E}V'\big(\{(q',\Delta')\}\big)+\overline{x}(1-x\overline{x}_{\text{takeover}})V'(\emptyset)$; for Δ^2, $\mathbb{E}\big[V'\big(\Phi^{f\prime}\big)|\Phi^f,z^2,x\big]=(1-\overline{x})(1-z^2)V'\big(\{(q_j,\Delta^1)\}\big)+(1-\omega\bar{x})z^2V'\big(\{(\Delta^2q_j,\Delta^2)\}\big)+x\mathbb{E}V'\big(\{(q',\Delta')\}\big)+\overline{x}(1-(1-\omega)z^2)(1-x\overline{x}_{\text{takeover}})V'(\emptyset)$, for Δ^3, $\mathbb{E}\big[V'\big(\Phi^{f\prime}\big)|\Phi^f,z^3,x\big]=\big((1-\omega)(1-ct)\overline{x}+1-\bar{x}\big)(1-z^3)V'\big(\{(q_j,\Delta^1)\}\big)+(1-\omega\bar{x})z^3V'\big(\{(\Delta^2q_j,\Delta^2)\}\big)+x\mathbb{E}V'\big(\{(q',\Delta')\}\big)+\overline{x}(\omega+(1-\omega)ct(1-z^3))(1-x\overline{x}_{\text{takeover}})V'(\emptyset)$; and for Δ^4, $\mathbb{E}\big[V'\big(\Phi^{f\prime}\big)|\Phi^f,z^4,x\big]=(1-\overline{x})(1-z^4)V'\big(\{(q_j,\Delta^1)\}\big)+\big((1-\omega)(1-ct)\bar{x}+1-\bar{x}\big)z^4V'\big(\{(\Delta^2q_j,\Delta^2)\}\big)+x\mathbb{E}V'\big(\{(q',\Delta')\}\big)+\overline{x}\big(1-(1-\omega)(1-ct)z^4\big)(1-x\overline{x}_{\text{takeover}})V'(\emptyset)$.

Using the guessed value function $V(\{(q_j,\Delta^\ell)\})=A_\ell q_j+B\overline{q}$, solving for the FONCs with respect to z^ℓ and x, and applying the suggested forms for fixed costs and the exit value, we obtain the analytic expressions for the optimal innovation decisions. For example, if $\Delta^\ell=\Delta^1$, we get $A_1q_j+B\overline{q}=\pi q_j-\widehat{\chi}z_j^{\widehat{\psi}}q_j-\widetilde{\chi}x_j^{\widetilde{\psi}}\overline{q}+\widetilde{\beta}\Big[(1-\overline{x})(1-z^1)A_1q_j+(1-\overline{x})z^1A_2\Delta^2q_j+x_j\left[A_{\text{takeover}}+\overline{x}_{\text{takeover}}(1+g)B\right]\overline{q}\Big]$, as the fixed cost of operation and the exit value cancel out some terms associated with B. The FONC with respect to z_j is $\frac{\partial}{\partial z_j}=\widehat{\psi}\widehat{\chi}z_j^{\widehat{\psi}-1}=\widetilde{\beta}\Big[(1-\overline{x})A_2\Delta^2-(1-\overline{x})A_1\Big]$. This equation provides the optimal own-innovation decision for Δ^1 case, which only depends on the technology gap. The FONC with respect to x_j is $\frac{\partial}{\partial x_j}=\widetilde{\psi}\widetilde{\chi}x_j^{\widetilde{\psi}-1}=\widetilde{\beta}\left[A_{\text{takeover}}+\overline{x}_{\text{takeover}}(1+g)B\right]$. This equation provides the optimal creative destruction x, which is independent of both product quality and technology gap. Collecting terms with q_j gives us the expression for A_1, which only depends on the technology gap, and collecting terms with $\overline{q}$ gives us the expression for B, which is independent of both product quality and technology gap. The remaining three technology gap cases follow the same process. These results confirm the additive separability of the value function with respect to each product-technology gap pair. □

E.2 Technology Gap Distribution Transition

From the quality evolution for incumbents (in the main text) and outsiders (Section B.2) the inflows and outflows for technology gap distribution ($\mu(\Delta^\ell)$) are defined as follows: for Δ^1, inflow is $(1-z^2)(1-\overline{x})\mu(\Delta^2)+(1-\omega\overline{x})(1-z^3)\mu(\Delta^3)+(1-z^4)(1-\overline{x})\mu(\Delta^4)$ and outflow is $(\overline{x}+z^1(1-\overline{x}))\mu(\Delta^1)$; for Δ^2, inflow is $z^1(1-\overline{x})\mu(\Delta^1)+(1-\omega\overline{x})z^3\mu(\Delta^3)+((1-\omega)\overline{x}+(1-\overline{x})z^4)\mu(\Delta^4)$ and outflow is $(1-(1-\omega\overline{x})z^2)\mu(\Delta^2)$; for Δ^3, inflow is $\overline{x}\mu(\Delta^1)+\omega\overline{x}(\mu(\Delta^2)+\mu(\Delta^4))$ and outflow is $(1-\omega\overline{x})\mu(\Delta^3)$; and for Δ^4, inflow is $(1-\omega)(1-z^2)\overline{x}\mu(\Delta^2)$ and outflow is $\mu(\Delta^4)$.

E.3 Aggregate Variables

Aggregate Creative Destruction Arrival Rate Firms do creative destruction for each product they own simultaneously. Given the unit mass of products, there is a unit mass of creative destruction trials by incumbent firms each period. Defining $s_d = \mathcal{F}_d/\mathcal{F}$ as the share (the total mass) of domestic products and $s_o = \mathcal{F}_o/\mathcal{F}$ as the outside counterpart, we can write the aggregate creative destruction arrival rate as $\overline{x} = s_d x + \mathcal{E}_d x_e + \underbrace{s_o x + \mathcal{E}_o}_{\equiv \overline{x}_o}$, where $\mathcal{E}_o$ is the total mass of potential outside entrants with successful creative destruction. As we assume the symmetry between domestic and outside firms, the outsiders' creative destruction intensity is also x. As $s_d + s_o = 1$, we can rewrite $\overline{x}$ as $\overline{x} = x + \mathcal{E}_d x_e + \mathcal{E}_o$.

Aggregate Productivity Growth Decomposition The total mass of domestic creative destruction trials is the share of products owned by domestic firms s_d, given the unit mass assumption. Thus, we can replace the mass of domestic firms ($\mathcal{F}_d$) with s_d and obtain the following decomposition as in the single creative destruction setup:

$$
\begin{aligned}
g = & \underbrace{\left(\Delta^2 - 1\right) s_d \left[(1-\overline{x})z^1\mu(\Delta^1) + (1-\omega\overline{x})(z^2\mu(\Delta^2) + z^3\mu(\Delta^3)) + ((1-\omega)(1-ct)\overline{x} + 1 - \overline{x})\, z^4\mu(\Delta^4)\right]}_{\text{own-innovation by domestic incumbents}} \\
& + \underbrace{\left(\Delta^2 - 1\right)(1-s_d)\left[\qquad\qquad \text{same expression as above} \qquad\qquad\right]}_{\text{own-innovation by foreign firms}} \\
& + \underbrace{\left(\overline{\Delta^{\text{ex}}} - 1\right) s_d x \mu(\overline{\Delta^{\text{ex}}})}_{\text{creative destr. by domestic incumbents}} + \underbrace{\left(\overline{\Delta^{\text{ex}}} - 1\right) \mathcal{E}_d x_e \mu(\overline{\Delta^{\text{ex}}})}_{\text{creative destr. by domestic startups}} + \underbrace{\left(\overline{\Delta^{\text{ex}}} - 1\right) \overline{x}_o \mu(\overline{\Delta^{\text{ex}}})}_{\text{creative destr. by foreign firms}}.
\end{aligned}
$$

Aggregate Domestic R&D Expenses Similarly, the aggregate domestic R&D expenses can be rephrased as $R_d = \widehat{\chi}\sum_{\ell=1}^{4}\left[\int_0^1 q_j \mathcal{I}_{\{\Delta_j=\Delta^\ell, j\in\mathcal{D}\}}dj\right](z^\ell)^{\widehat{\psi}} + s_d\widetilde{\chi}\overline{q}x^{\widetilde{\psi}} + \mathcal{E}_d\widetilde{\chi}_e(x_e)^{\widetilde{\psi}_e}\overline{q}$.

Aggregate Consumption Households own both final goods and domestic intermediate producers. They fund the R&D expenses of domestic potential startups and pay the exit value to domestic incumbents. The households earn labor income from final goods producer (wL), operating fixed costs from intermediate producers ($s_d F\overline{q}$), as well as profits from both producers ($\Pi = 0$ and $\sum_{j\in\mathcal{D}} \pi q_j > 0$). Intermediate producers' profits include the exit value if their product is taken over and their own creative destruction fails. Thus, the household budget constraint is $wL + s_d F\overline{q} +$

$\int j \in \mathcal{D}\{\pi q_j - F\overline{q}\} + (1 - x\overline{x}_{\text{takeover}})V(\emptyset) = C + \mathcal{E}_d \widetilde{\chi}_e (x_e)^{\widetilde{\psi}_e} \overline{q} + (1 - x\overline{x}_{\text{takeover}})V(\emptyset)$. With the final goods producers' profit function $\Pi = Y - \int_{j \in \mathcal{D}} p_j y_j dj - \int_{j \notin \mathcal{D}} p_j y_j dj - wL$, the aggregate consumption is $C = Y - \int_{j \notin \mathcal{D}} p_j y_j dj - Y_d - R_d$.

F Extension: Duopolistic Competition

This section extends the baseline model to incorporate duopolistic competition in the product market, where two firms—one frontier and one laggard—operate. As in the baseline model, firms can engage in both creative destruction and own-innovation. In the case of creative destruction, innovation is undirected, and the successful firm takes over the leadership position in the target market. In the case of two incumbent firms in a product market having the same probability to be taken over from creative destruction, either firm can be taken over with equal probability. The firm not displaced becomes the laggard, positioned one step behind the new leader. We follow the functional assumptions of Cavenaile et al. (2019) and assume $\omega = 0$ and $ct = \frac{1}{2}$ for tractability.[9]

F.1 Final Goods Producer

The final goods producer manufactures the final good using a continuum of differentiated intermediate goods indexed by $j \in [0, 1]$ as follows:

$$Y_t = \exp\left[\frac{1}{1-\theta}\int_0^1 \ln\left((y_{jt}^f)^{1-\theta} + (y_{jt}^{-f})^{1-\theta}\right)dj\right], \tag{1}$$

where y_{jt}^f and y_{jt}^{-f} are the quantity of good j, provided by firm f and the other competitor $-f$ in the market, respectively. The market is competitive, with the price normalized to one, and producers take input prices as given.

Given this, they solve the following maximization and obtain demand function:

$$\max_{y_{jt}^f, y_{jt}^{-f}} Y - \int_0^1 (p_{jt}^f y_{jt}^f + p_{jt}^{-f} y_{jt}^{-f}) dj$$

[9]This simplifying assumption does not matter for our results as shown in Appendix Sections H.3, H.6, and H.7.

subject to (1). The first order conditions give the following demand functions and relationship:

$$p_{jt}^f = Y\frac{(y_{jt}^f)^{-\theta}}{(y_{jt}^f)^{1-\theta} + (y_{jt}^{-f})^{1-\theta}} \tag{2}$$

$$\frac{p_{jt}^f}{p_{jt}^{-f}} = \Big(\frac{y_{jt}^{-f}}{y_{jt}^f}\Big)^\theta \tag{3}$$

F.2 Intermediate Goods Producers

The intermediate goods producers produce and sell differentiated intermediate goods to final good producers. There are two firms, leader F and laggard L, who exert duopolistic market power in each product market. Intermediate goods are produced using final goods as input at a constant unit input cost. Assuming CRS production function, the profit-maximization problem is given by:

$$\pi_{jt}^f = \max_{y_{jt}^f} p_{jt}^f y_{jt}^f - \frac{y_{jt}^f}{q_{jt}^f} \tag{4}$$

subject to (2). q_{jt}^f is the quality of the product produced by firm f.

The first-order condition gives

$$y_{jt}^f = q_{jt}^f \frac{(1-\theta)Y_t(y_{jt}^{-f}/y_{jt}^f)^{1-\theta}}{(1+(y_{jt}^{-f}/y_{jt}^f)^{1-\theta})^2}, \tag{5}$$

and given its symmetry, and using (3), we have the following relationship:

$$\frac{y_{jt}^f}{y_{jt}^{-f}} = \frac{q_{jt}^f}{q_{jt}^{-f}} = \Big(\frac{p_{jt}^f}{p_{jt}^{-f}}\Big)^{-1/\theta}. \tag{6}$$

Let $\lambda^{\hat{\Delta}jtf} \equiv \frac{qjtf}{q_{jt}^{-f}}$ define the quality gap of firm f relative to the other firm, where $\hat{\Delta}_{jt}^f$ represents the gap in its innovation rungs. By combining (2), (5), and (6), the profit becomes a time-invariant function of the quality gap between the two firms:

$$\pi_{jt}^f = \frac{1+\theta(q_{jt}^{-f}/q_{jt}^f)^{1-\theta}}{(1+(q_{jt}^{-f}/q_{jt}^f)^{1-\theta})^2}Y_t = \frac{1+\theta\lambda^{-\hat{\Delta}_{jt}^f(1-\theta)}}{(1+\lambda^{-\hat{\Delta}_{jt}^f(1-\theta)})^2}Y_t \equiv \pi(\hat{\Delta}_{jt}^f)Y_t. \tag{7}$$

F.3 Innovation

As in the baseline, firms can do own-innovation and creative destruction. If doing own-innovation, to produce innovation intensity z_{jt}, firms incur the following R&D cost: $R_{jt}^{own} = z_{jt}^{\hat{\psi}} \widehat{\chi} Y_t$, and if doing creative destruction, firms need to pay the following R&D cost for (per-product) innovation intensity x_t: $R_{jt}^{cd} = x_t^{\tilde{\psi}} \tilde{\chi} Y_t n_t$.

F.4 Value Function

The state variables for the firm are the number of products (n_t^f), the quality gap ($\hat{\Delta}_{jt}^f$), and the quality gap from the previous period ($\Delta_{jt}^f = \frac{q_{jt}^f}{q_{jt-1}^f}$). Given that, the firm's value function can be characterized as follows:

$$\begin{aligned} V(n_t^f, \{\hat{\Delta}_{jt}^f, \Delta_{jt}^f\}_{j \in \mathcal{J}_t^f}) = \max_{\{z_{jt}^f\}_{j \in \mathcal{J}_t^f}, x_t} \sum_{j \in \mathcal{J}_t^f} \Big(\pi(\hat{\Delta}_{jt}^f) - (z_{jt}^f)^{\hat{\psi}} \widehat{\chi}\Big) Y_t - x_t^{\tilde{\psi}} \tilde{\chi} Y_t n_t \\ + \beta \mathbb{E} V(n_{t+1}^f, \{\hat{\Delta}_{jt+1}^f, \Delta_{jt+1}^f\}_{j \in \mathcal{J}_{t+1}^f}). \end{aligned} \tag{8}$$

As before, we can guess the value function by decomposing it into components associated with own-innovation and those associated with creative destruction as follows:

$$V(n_t^f, \{\hat{\Delta}_{jt}^f, \Delta_{jt}^f\}_{j \in \mathcal{J}_t^f}) = \sum_{l=1}^{4} \sum_{j \in \mathcal{J}_t^f | \Delta_{jt}^f = \Delta^l} A_l(\hat{\Delta}_{jt}^f) Y_t + B n_t Y_t, \tag{9}$$

where $\Delta^1 = 1$, $\Delta^2 = \lambda$, $\Delta^3 = \eta$, and $\Delta^4 = \frac{\eta}{\lambda}$. Rephrasing (8) with (9), we have:

$$\begin{aligned} \sum_{l=1}^{4} \sum_{j \in \mathcal{J}_t^f | \Delta_{jt}^f = \Delta^l} A_l(\hat{\Delta}_{jt}^f) Y_t + B n_t Y_t = \max_{\{z_{jt}^f\}_{j \in \mathcal{J}_t^f}, x_t} \sum_{j \in \mathcal{J}_t^f} \Big(\pi(\hat{\Delta}_{jt}^f) - (z_{jt}^f)^{\hat{\psi}} \widehat{\chi}\Big) Y_t - x_t^{\tilde{\psi}} \tilde{\chi} Y_t n_t \\ + \beta \sum_{l=1}^{4} \sum_{k=1}^{4} \int_{\hat{\Delta}_{jt+1}^f} \Big(P(\hat{\Delta}_{jt+1}^f, \Delta^k | \hat{\Delta}_{jt}^f, \Delta^l) \sum_{\substack{j \in \mathcal{J}_{t+1}^f | \Delta_{jt}^f = \Delta^l, \\ \Delta_{jt+1}^f = \Delta^k}} A_k(\hat{\Delta}_{jt+1}^f) Y_{t+1} + B n_{t+1} Y_{t+1}\Big) d\hat{\Delta}_{jt+1}^f, \end{aligned}$$

where $P(\hat{\Delta}_{jt+1}^f, \Delta^k | \hat{\Delta}_{jt}^f, \Delta^l)$ is the probability of switching from one state $(\hat{\Delta}_{jt}^f, \Delta_{jt}^f = \Delta^l)$ to another $(\hat{\Delta}^f_{jt+1}, \Delta_{jt+1} = \Delta^k)$. Note that this can be rewritten with the product-level value function $v(\hat{\Delta}^f_{jt}, \Delta^l)$ as follows:

$$V(n^f_t, \{\hat{\Delta}^f_{jt}, \Delta^f_{jt}\}_{j\in\mathcal{J}^f_t}) = \sum_{l=1}^{4} \sum_{j\in\mathcal{J}^f_t|\Delta^f_{jt}=\Delta^l} \Big(A_l(\hat{\Delta}^f_{jt})Y_t + BY_t\Big) = \sum_{l=1}^{4} \sum_{j\in\mathcal{J}^f_t|\Delta^f_{jt}=\Delta^l} v(\hat{\Delta}^f_{jt}, \Delta^l),$$

$$\text{where } v(\hat{\Delta}^f_{jt}, \Delta^l) = \max_{z^l(\hat{\Delta}^f_{jt}),x_t} \Big(\pi(\hat{\Delta}^f_{jt}) - (z^l(\hat{\Delta}^f_{jt}))^{\hat{\psi}}\hat{\chi}\Big)Y_t - x_t^{\tilde{\psi}}\tilde{\chi}Y_t$$

$$+ \beta \sum_{k=1}^{4} \int_{\hat{\Delta}^f_{jt+1}} \Big(P(\hat{\Delta}^f_{jt+1}, \Delta^k|\hat{\Delta}^f_{jt}, \Delta^l) \sum_{\substack{j\in\mathcal{J}^f_{t+1}|\Delta^f_{jt}=\Delta^l,\\ \Delta^f_{jt+1}=\Delta^k}} A_k(\hat{\Delta}^f_{jt+1})Y_{t+1} + BY_{t+1}\Big) d\hat{\Delta}^f_{jt+1}.$$

F.5 Optimal Innovation Decision

We can solve for the optimal innovation by considering the cases where firm f is the frontier, leveled (neck-and-neck), or the laggard. For the sake of brevity, we present the frontier's decision rules in this manuscript, with the decision rules for the neck-and-neck and laggard cases available upon request.

Suppose that in product market j, firm f is the frontier with the gaps $\hat{\Delta}^f_{jt}$ and Δ^ℓ_j. As before, the evolution of this product in the subsequent period depends on several contingencies, depending on the firm's success of own-innovation and potential takeovers as well as the laggard's success of own-innovation. Note that with the duopoly setup, the laggard's innovation now also affects the firm's expected value of innovation, as it influences the quality gap and profit (market share) in the market.

Let z^F and z^L be the innovation intensity of the frontier and the laggard in a product market. Denoting the product-technology gap pair for a product that firm f acquires through successful business takeover in the next period as $\{(1, \Delta')\}$, we can write down the evolution of the product portfolio stemming from $\Phi^f = \{(\hat{\Delta}_j, \Delta^\ell_j)\}$ for $\ell \in \{1, 2, 3, 4\}$ for each possible case.[10] For example, for $\Delta^\ell_j = \Delta^2$, $\Phi^{f\prime}_j = \emptyset \cup \emptyset$ with prob. $\overline{x}(1 - z^{F2})(1 - x\overline{x}_{\text{takeover}})$, $\Phi^{f\prime}_j = \emptyset \cup \{(1, \Delta')\}$ with prob. $\overline{x}(1 - z^{F2})(x\overline{x}_{\text{takeover}})$, $\Phi^{f\prime}_j = \{(\hat{\Delta}, \Delta^1)\} \cup \emptyset$ with prob. $(1 - \overline{x})(1 - z^{F2})(1 - z^L)(1 - x\overline{x}_{\text{takeover}})$, $\Phi^{f\prime}_j = \{(\hat{\Delta}, \Delta^1)\}\cup\{(1, \Delta')\}$ with prob. $(1-\overline{x})(1-z^{F2})(1-z^L)(x\overline{x}_{\text{takeover}})$, $\Phi^{f\prime}_j = \{(\hat{\Delta}-1, \Delta^1)\}\cup\emptyset$

[10] Note that successful creative destruction will result in a quality gap corresponding to a step size from the laggard in the market.

with prob. $(1-\bar{x})(1-z^{F2})z^L(1-x\overline{x}_{\text{takeover}})$, $\Phi_j^{f\prime}=\{(\hat{\Delta}-1,\Delta^1)\}\cup\{(1,\Delta')\}$ with prob. $(1-\bar{x})(1-z^{F2})z^L(x\overline{x}_{\text{takeover}})$, $\Phi_j^{f\prime}=\{(\hat{\Delta}+1,\Delta^2)\}\cup\emptyset$ with prob. $(z^{F2})(1-z^L)(1-x\overline{x}_{\text{takeover}})$, $\Phi_j^{f\prime}=\{(\hat{\Delta}+1,\Delta^2)\}\cup\{(1,\Delta')\}$ with prob. $(z^{F2})(1-z^L)(x\overline{x}_{\text{takeover}})$, $\Phi_j^{f\prime}=\{(\hat{\Delta},\Delta^2)\}\cup\emptyset$ with prob. $(z^{F2})(z^L)(1-x\overline{x}_{\text{takeover}})$, and $\Phi_j^{f\prime}=\{(\hat{\Delta},\Delta^2)\}\cup\{(1,\Delta')\}$ with prob. $(z^{F2})(z^L)(x\overline{x}_{\text{takeover}})$.

Following this step for other cases of $\Delta_j^l=\Delta^1,\Delta^3,\Delta^4$, the following analytic expressions for for the frontier's decision rules.

Proposition F.1. *Given a technology gap distribution $\{\mu(\Delta^\ell)\}_{\ell=1}^4$, the laggard innovation intensity z^L, and the exit value for exiting product given by $V^{exit}=\beta B(1+g)Y$, the value function of firm f with a technology gap portfolio of $\Phi^f\equiv\{(\hat{\Delta}_j,\Delta_j)\}_{j\in\mathcal{J}^f}$ is: $V(\Phi^f)=\sum_{\ell=1}^4\sum_{j\in\mathcal{J}^f|\Delta_j=\Delta^\ell}A_\ell(\hat{\Delta}_j)Y+n_fBY$, where $A_1(\hat{\Delta}_j)=\pi(\hat{\Delta}_j)-\widehat{\chi}(z^{F1})^{\widehat{\psi}}+\beta(1+g)\Big[(1-\bar{x})\Big((1-z^{F1})(1-z^L)A_1(\hat{\Delta}_j)+(1-z^{F1})z^LA_1(\hat{\Delta}_j-1)+z^{F1}(1-z^L)A_2(\hat{\Delta}_j+1)+z^{F1}z^LA_2(\hat{\Delta}_j)\Big)\Big]$, $A_2(\hat{\Delta}_j)=\pi(\hat{\Delta}_j)-\widehat{\chi}(z^{F2})^{\widehat{\psi}}+\beta(1+g)\Big[(1-\bar{x})\Big((1-z^{F2})(1-z^L)A_1(\hat{\Delta}_j)+(1-z^{F2})z^LA_1(\hat{\Delta}_j-1)\Big)+z^{F2}(1-z^L)A_2(\hat{\Delta}_j+1)+z^{F2}z^LA_2(\hat{\Delta}_j)\Big]$, $A_3(\hat{\Delta}_j)=\pi(\hat{\Delta}_j)-\widehat{\chi}(z^{F3})^{\widehat{\psi}}+\beta(1+g)\Big[(1-\frac{1}{2}\bar{x})\Big((1-z^{F3})(1-z^L)A_1(\hat{\Delta}_j)+(1-z^{F3})z^LA_1(\hat{\Delta}_j-1)\Big)+z^{F3}(1-z^L)A_2(\hat{\Delta}_j+1)+z^{F3}z^LA_2(\hat{\Delta}_j)\Big]$, $A_4(\hat{\Delta}_j)=\pi(\hat{\Delta}_j)-\widehat{\chi}(z^{F4})^{\widehat{\psi}}+\beta(1+g)\big[(1-\bar{x})((1-z^{F4})(1-z^L)A_1(\hat{\Delta}_j)+(1-z^{F4})z^LA_1(\hat{\Delta}_j-1)\Big)+(1-\frac{1}{2}\bar{x})\Big(z^{F4}(1-z^L)A_2(\hat{\Delta}_j+1)+z^{F4}z^LA_2(\hat{\Delta}_j)\Big)\big]$, and $B=\big[x\beta(1+g)A_{\text{takeover}}-\widetilde{\chi}x^{\widetilde{\psi}}\big]/\big[1-\widetilde{\beta}(1+g)(1+x\overline{x}_{\text{takeover}})\big]$, and the optimal innovation probabilities are $z^{F1}(\hat{\Delta}_j)=\big[\beta(1+g)(1-\overline{x})\big[(1-z^L)(A_2(\hat{\Delta}_j+1)-A_1(\hat{\Delta}_j))+z^L(A_2(\hat{\Delta}_j)-A_1(\hat{\Delta}_j-1))\big]/\big[\widehat{\psi}\widehat{\chi}\big]\big]^{\frac{1}{\widehat{\psi}-1}}$, $z^{F2}(\hat{\Delta}_j)=\big[\beta(1+g)\big((1-z^L)(A_2(\hat{\Delta}_j+1)-(1-\bar{x})A_1(\hat{\Delta}_j))+z^L(A_2(\hat{\Delta}_j)-(1-\bar{x})A_1(\hat{\Delta}_j-1))\big)\big]/\big[\widehat{\psi}\widehat{\chi}\big]\big]^{\frac{1}{\widehat{\psi}-1}}$, $z^{F3}(\hat{\Delta}_j)=\big[\beta(1+g)\big((1-z^L)(A_2(\hat{\Delta}_j+1)-(1-\frac{1}{2}\bar{x})A_1(\hat{\Delta}_j))+z^L(A_2(\hat{\Delta}_j)-(1-\frac{1}{2}\bar{x})A_1(\hat{\Delta}_j-1))\big)\big]/\big[\widehat{\psi}\widehat{\chi}\big]\big]^{\frac{1}{\widehat{\psi}-1}}$, $z^{F4}(\hat{\Delta}_j)=\big[\beta(1+g)\big((1-z^L)((1-\frac{1}{2}\bar{x})A_2(\hat{\Delta}_j+1)-(1-\bar{x})A_1(\hat{\Delta}_j))+z^L((1-\frac{1}{2}\bar{x})A_2(\hat{\Delta}_j)-(1-\bar{x})A_1(\hat{\Delta}_j-1))\big)\big]/\big[\widehat{\psi}\widehat{\chi}\big]\big]^{\frac{1}{\widehat{\psi}-1}}$, and $x=\big[(1+g)\beta\big[A_{\text{takeover}}+\overline{x}_{\text{takeover}}B\big]/\big[\widetilde{\psi}\widetilde{\chi}\big]\big]^{\frac{1}{\widetilde{\psi}-1}}$, where g is the average product quality growth rate in the economy, A_{takeover} is the ex-ante value of a product line obtained from successful takeover, defined as $A_{\text{takeover}}\equiv\frac{1-z^3}{2}A_1\mu(\Delta^3)+\big(1-\frac{z^4}{2}\big)A_2\lambda\mu(\Delta^4)+A_3\eta\mu(\Delta^1)+(1-z^2)A_4\frac{\eta}{\lambda}\mu(\Delta^2)$, and $\overline{x}_{\text{takeover}}=\mu(\Delta^1)+(1-z^2)\mu(\Delta^2)+\frac{1}{2}(1-z^3)\mu(\Delta^3)+\big(1-\frac{1}{2}z^4\big)\mu(\Delta^4)$.*

Proof. Using the guessed value function and solving for the FONCs with respect to z^ℓ and x, and applying the the exit value, we obtain the analytic expressions for the optimal innovation decisions. For example, if $\Delta^\ell=\Delta^1$, we get $A_1(\hat{\Delta}_j)Y+BY=\pi(\hat{\Delta}_j)Y-\widehat{\chi}z_j^{\widehat{\psi}}Y-\widetilde{\chi}x_j^{\widetilde{\psi}}Y+\beta(1+g)\Big[\Big\{(1-$

$\bar{x})\big((1-z_j)(1-z^L)A_1(\hat{\Delta}_j)+(1-z_j)z^L A_1(\hat{\Delta}_j-1)+z_j(1-z^L)A_2(\hat{\Delta}_j+1)+z_j z^L A_2(\hat{\Delta}_j)\big)\Big\}+$ $x_j A^{takeover}+(1+x_j\bar{x}^{takeover})B\Big]Y$, as the exit value cancel out some terms associated with B. The FONC with respect to z_j is $\frac{\partial}{\partial z_j}=\widehat{\psi}\widehat{\chi}z_j^{\widehat{\psi}-1}=\beta(1+g)(1-\overline{x})\big[(1-z^L)(A_2(\hat{\Delta}_j+1)-A_1(\hat{\Delta}_j))+$ $z^L(A_2(\hat{\Delta}_j)-A_1(\hat{\Delta}_j-1))\big]$. This equation provides the optimal own-innovation decision for Δ^1 case, which depends on the technology gap $(\hat{\Delta}_j,\Delta_j)$ and the laggard's innovation z^L. The FONC with respect to x_j is $\frac{\partial}{\partial x_j}=\widetilde{\psi}\widetilde{\chi}x_j^{\widetilde{\psi}-1}=\beta(1+g)\left[A_{\text{takeover}}+\overline{x}_{\text{takeover}}B\right]$. This equation provides the optimal creative destruction x, which is independent of the technology gaps and the laggard innovation in the market. Collecting terms associated with own-innovation gives us the expression for $A_1(\hat{\Delta}_j)$, which depends on technology gap and the laggard innovation, and collecting terms associated with creative destruction gives us the expression for B, which is independent of both technology gap and the laggard innovation. The remaining three technology gap cases for Δ_j follow the same process. These results confirm the additive separability of the value function with respect to each product-technology gap pair. □

Given the proposition, we can replicate the main results in the baseline model. First, the following corollary shows that own-innovation increases with the technology gap, but beyond a certain point, a wider technology gap can discourage further investment in own-innovation. This replicates Corollary 1 in the main text.

Corollary F.1. *In an equilibrium where $\{z^{F\ell}\}_{\ell=1}^4$ are well defined, the probabilities of own-innovation for the frontier firm satisfy $z^{F2}>z^{F3}>z^{F4}>z^{F1}$ given any levels of $\hat{\Delta}_j$ and z^L.*

Proof. Given $\hat{\Delta}_j$ and z^L, comparing z^{F1} and z^{F2} gives $(z^{F1})^{\hat{\psi}-1}-(z^{F2})^{\hat{\psi}-1}=\frac{\beta(1+g)}{\hat{\psi}\widehat{\chi}}\bar{x}((1-z^L)A_2(\hat{\Delta}_j+1)+z^L A_1(\hat{\Delta}_j))>0$. Similarly, comparing z^{F2} and z^{F3}, we have $(z^{F2})^{\hat{\psi}-1}-(z^{F3})^{\hat{\psi}-1}=\frac{\beta(1+g)}{\hat{\psi}\widehat{\chi}}\frac{1}{2}\bar{x}((1-z^L)A_1(\hat{\Delta}_j)+z^L A_1(\hat{\Delta}_j-1))>0$. Comparing z^{F1} and z^{F4} we obtain $(z^{F4})^{\hat{\psi}-1}-(z^{F1})^{\hat{\psi}-1}=\frac{\beta(1+g)}{\hat{\psi}\widehat{\chi}}\frac{1}{2}\bar{x}((1-z^L)A_2(\hat{\Delta}_j+1)+z^L A_2(\hat{\Delta}_j))>0$. Furthermore, comparing z^{F3} and z^{F4}, we get $(z^{F3})^{\hat{\psi}-1}-(z^{F4})^{\hat{\psi}-1}=\frac{\beta(1+g)}{\hat{\psi}\widehat{\chi}}\frac{1}{2}\bar{x}\Big((1-z^L)(A_2(\hat{\Delta}_j+1)-A_1(\hat{\Delta}_j))+$ $z^L(A_2(\hat{\Delta}_j)-A_1(\hat{\Delta}_j-1))\Big)>0$. This follows $z^{F1}>0$, where $(1-z^L)(A_2(\hat{\Delta}_j+1)-A_1(\hat{\Delta}_j))+$ $z^L(A_2(\hat{\Delta}_j)-A_1(\hat{\Delta}_j-1))>0$ needs to hold. Combining them all with $\hat{\psi}>1$ completes the proof. □

Furthermore, we can derive market-protection effect in Corollary 3 as before.

Corollary F.2 (Market-Protection Effect of Frontier). *With $\widetilde{\psi} \in (1, 2]$, the market-protection effect has the following sign, given any levels of $\hat{\Delta}_j$ and z^L:*

$$\left.\frac{\partial z^{F2}}{\partial \bar{x}}\right|_{A_1,A_2} > \left.\frac{\partial z^{F3}}{\partial \bar{x}}\right|_{A_1,A_2} > 0, \quad \left.\frac{\partial z^{F3}}{\partial \bar{x}}\right|_{A_1,A_2} > \left.\frac{\partial z^{F4}}{\partial \bar{x}}\right|_{A_1,A_2} \lesseqgtr 0, \quad \textit{and} \quad 0 > \left.\frac{\partial z^{F1}}{\partial \bar{x}}\right|_{A_1,A_2}.$$

Proof. Getting the derivatives of z^{Fl} with respect to $\bar{x}$ for each $l = 1, 2, 3, 4$, given $\hat{\Delta}_j$ and z^L, we get the following signs: $\frac{\partial z^{F1}}{\partial \bar{x}} = -\left(\frac{1}{\hat{\psi}\widehat{\chi}}\right)^{\frac{1}{\hat{\psi}-1}} \frac{1}{\hat{\psi}-1}(z^{F1})^{2-\hat{\psi}}\beta(1+g)((1-z^L)(A_2(\hat{\Delta}_j+1) - A_1(\hat{\Delta}_j)) + z^L(A_2(\hat{\Delta}_j) - A_1(\hat{\Delta}_j - 1))) < 0$, $\frac{\partial z^{F2}}{\partial \bar{x}} = \left(\frac{1}{\hat{\psi}\widehat{\chi}}\right)^{\frac{1}{\hat{\psi}-1}} \frac{1}{\hat{\psi}-1}(z^{F2})^{2-\hat{\psi}}\beta(1+g)((1-z^L)A_1(\hat{\Delta}_j) + z^L A_1(\hat{\Delta}_j - 1)) > 0$, $\frac{\partial z^{F3}}{\partial \bar{x}} = \left(\frac{1}{\hat{\psi}\widehat{\chi}}\right)^{\frac{1}{\hat{\psi}-1}} \frac{1}{\hat{\psi}-1}(z^{F3})^{2-\hat{\psi}}\beta(1+g)\frac{1}{2}((1-z^L)A_1(\hat{\Delta}_j) + z^L A_1(\hat{\Delta}_j - 1)) > 0$, $\frac{\partial z^{F4}}{\partial \bar{x}} = \left(\frac{1}{\hat{\psi}\widehat{\chi}}\right)^{\frac{1}{\hat{\psi}-1}} \frac{1}{\hat{\psi}-1}(z^{F4})^{2-\hat{\psi}}\beta(1+g)((1-z^L)(A_1(\hat{\Delta}_j) - \frac{1}{2}A_2(\hat{\Delta}_j+1)) + z^L(A_1(\hat{\Delta}_j - 1) - \frac{1}{2}A_2(\hat{\Delta}_j)))$ is ambiguous. Furthermore, we can derive $\frac{\partial z^{F2}}{\partial \bar{x}} > \frac{\partial z^{F3}}{\partial \bar{x}}$ using that $z^{F2} > z^{F3}$ and $z^{F1} > 0$. This completes the proof. □

In addition, the following corollary shows how the frontier's innovation and market-protection effect depends on the quality gap from the laggard, $\hat{\Delta}_j$.

Corollary F.3. *In an equilibrium where $\{z^{F\ell}\}_{\ell=1}^4$ are well defined, the effect of quality gap from the laggard (or market share) on the probabilities of own-innovation for the frontier firm is ambiguous given Δ_j.*

Proof. Taking the derivatives of own-innovation with respect to the quality gap from the laggard, holding z^L fixed for simplicity, we obtain the following expressions: $\frac{\partial z^{F1}}{\partial \hat{\Delta}_j} = \frac{1}{(\hat{\psi}-1)\hat{\psi}\widehat{\chi}}(z^{F1})^{2-\hat{\psi}}\beta(1+g)(1-\bar{x})\big((1-z^L)(A_2'(\hat{\Delta}_j+1) - A_1'(\hat{\Delta}_j)) + z^L(A_2'(\hat{\Delta}_j) - A_1'(\hat{\Delta}_j - 1))\big)$, $\frac{\partial z^{F2}}{\partial \hat{\Delta}_j} = \frac{1}{(\hat{\psi}-1)\hat{\psi}\widehat{\chi}}(z^{F2})^{2-\hat{\psi}}\beta(1+g)\big((1-z^L)(A_2'(\hat{\Delta}_j+1) - (1-\bar{x})A_1'(\hat{\Delta}_j)) + z^L(A_2'(\hat{\Delta}_j) - (1-\bar{x})A_1'(\hat{\Delta}_j - 1))\big)$, $\frac{\partial z^{F3}}{\partial \hat{\Delta}_j} = \frac{1}{(\hat{\psi}-1)\hat{\psi}\widehat{\chi}}(z^{F3})^{2-\hat{\psi}}\big[\beta(1+g)\big((1-z^L)(A_2'(\hat{\Delta}_j+1) - (1-\frac{1}{2}\bar{x})A_1'(\hat{\Delta}_j)) + z^L(A_2'(\hat{\Delta}_j) - (1-\frac{1}{2}\bar{x})A_1'(\hat{\Delta}_j - 1))\big)\big]$, $\frac{\partial z^{F4}}{\partial \hat{\Delta}_j} = \frac{1}{(\hat{\psi}-1)\hat{\psi}\widehat{\chi}}(z^{F4})^{2-\hat{\psi}}\big[\beta(1+g)\big((1-z^L)((1-\frac{1}{2}\bar{x})A_2'(\hat{\Delta}_j+1) - (1-\bar{x})A_1'(\hat{\Delta}_j)) + z^L((1-\frac{1}{2}\bar{x})A_2'(\hat{\Delta}_j) - (1-\bar{x})A_1'(\hat{\Delta}_j - 1))\big)\big]$. The signs of these terms are ambiguous, depending on multiple terms associated with $A_1'(\hat{\Delta}_j)$, $A_2'(\hat{\Delta}_j)$, and $\bar{x}$. □

The ambiguous effect arises from the decreasing returns to scale in the profit function and the structure of R&D costs. As $\hat{\Delta}_j$ increases, marginal returns to profit diminish, which reduces firms' incentives to further enhance own-innovation beyond a certain point, combined with rising R&D costs. This leads to the observed ambiguity in the results.

However, the firm's market share has an unambiguous impact on its market protection motive, as demonstrated in the following corollary.

Corollary F.4. *If $A_1(\Delta_j)$ is an increasing function of Δ_j^F, the market protection effect for the frontier (with technology gap Δ^2 or Δ^3) becomes more pronounced as the gap Δ_j^F increases.*

Proof. Getting the derivative of $\frac{\partial z^{F2}}{\partial \bar{x}}$ and $\frac{\partial z^{F2}}{\partial \bar{x}}$ with respect to $\hat{\Delta}_j$, we can obtain that $\frac{\partial^2 z_2^F}{\partial \Delta_{jt}^F \partial \bar{x}} > 0$ and $\frac{\partial^2 z_3^F}{\partial \Delta_{jt}^F \partial \bar{x}} > 0$. This implies the amplification of the market-protection effect given $\frac{\partial z^{F2}}{\partial \bar{x}} > \frac{\partial z^{F3}}{\partial \bar{x}} > 0$. $\square$

This result is intuitive because a leader with a higher market share will have greater incentives to foster innovation in order to protect its position in response to increased competitive pressure in the market.

G Solution Algorithm

In the model, $\{z^\ell\}_{\ell=1}^4$ are functions of $\overline{x}$; g is a function of $\overline{x}$, $\{z^\ell\}_{\ell=1}^4$, and $\{\mu(\Delta^\ell)\}_{\ell=1}^4$; x is a function of $\overline{x}$ and $\{\mu(\Delta^\ell)\}_{\ell=1}^4$; x_e is a function of $\overline{x}$ and $\{\mu(\Delta^\ell)\}_{\ell=1}^4$; and $\overline{x}$ is a function of x, and x_e. Therefore, we can solve for an equilibrium of the model by iterating over the value for the aggregate creative destruction arrival rate $\overline{x}$.

For the extended model with multiple creative destruction: i) Guess values for $\overline{x}$, g and the technology gap distribution $\{\mu(\Delta^\ell)\}_{\ell=1}^4$; ii) Using the guess of $\overline{x}$, compute $\{A_\ell\}_{\ell=1}^4$, and $\{z^\ell\}_{\ell=1}^4$; iii) Using the guess of $\overline{x}$, g, and $\{\mu(\Delta^\ell)\}_{\ell=1}^4$, compute B, x, x_e. Next, compute the stationary $\{\mu_\infty(\Delta^\ell)\}_{\ell=1}^4$, based on the guess of $\{\mu_\infty(\Delta^\ell)\}_{\ell=1}^4$, innovation decision rules, and the following law of motion $\mu_{n+1}(\Delta^\ell) = \mu_n(\Delta^\ell) + \mathit{inflow}_n(\Delta^\ell) - \mathit{outflow}_n(\Delta^\ell)$ for each $\ell \in \{1,2,3,4\}$. Lastly, compute g_∞ with $\{\mu_\infty(\Delta^\ell)\}_{\ell=1}^4$; iv) Compute $\overline{x}' = x + \mathcal{E}_d x_e + \mathcal{E}_o$; v) If $\overline{x} \neq \overline{x}'$, set $\overline{x} = \overline{x}'$, $g = g_\infty$, and $\{\mu(\Delta^\ell)\}_{\ell=1}^4 = \{\mu_\infty(\Delta^\ell)\}_{\ell=1}^4$, use them as new guess, and return to ii); vi) Repeat ii) through v) until the convergence of $\overline{x}$; and vii) Simulate the model over 10,000 products for 1,200 years and compute the moments averaged across the last 150 years.

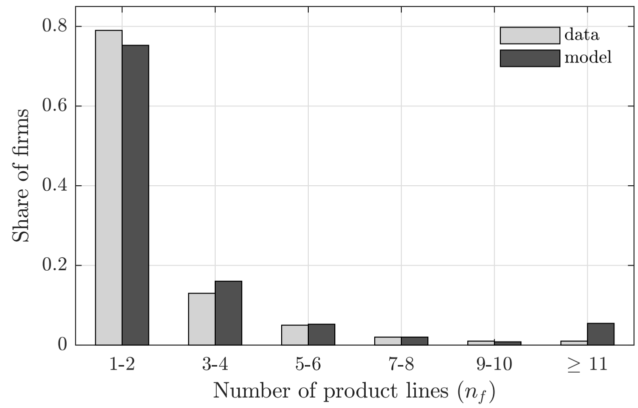

Figure H.1: Firm Size Distribution: Theory and Data

H Other Theoretical Results

H.1 Untargeted Size Distribution

Figure H.1 compares the firm size distribution (in terms of the number of products) between the model and the data, which is untargeted. While the model exhibits a thicker right tail, indicating more firms with 11 or more products, it aligns closely with the data.[11]

H.2 Counterfactual: Increasing Learning Probability

Table H.1 shows further details behind the counterfactual exercise of increasing learning probability in the U.S. Panel A shows the changes in expected value of each type of innovation. As learning probability increases, the average own-innovation values ($\{A_\ell\}_{\ell=1}^4$) decreases by 8.8%, with the most pronounced effect comes from those with a gap of Δ^2. The expected value of creative destruction (B) increases by 27.2%. This leads to the increase in creative destruction and the decline in own-innovation. As a result, Panel B displays the changes in technology gap distribution, where the mass of firms with a gap of Δ^2 declines a lot, while the counterparts for Δ^3 and Δ^4 increase due to the rise of creative destruction.

Furthermore, Table H.2 shows the growth decomposition by holding the mass of firms fixed. The growth attributed to creative destruction increases, while the growth driven by own-innovation decreases even with the fixed mass of firms for each innovation.

[11] Approximately 60% of firms are single-product firms, but they account for less than 13% of total output (Bernard et al., 2010; Kim and Jo, 2024).

Table H.1: Counterfactual: Increasing Learning Probability in the U.S.

Description	Variables	Before	After	Δ (%)
	Panel A: Changes in Innovation Values			
Average of own-innovation values	$\overline{A}$	0.059	0.054	-8.8%
Own-innovation value (for each gap)	A_1	0.050	0.047	-6.7%
	A_2	0.060	0.054	-10.1%
	A_3	0.063	0.058	-8.6%
	A_4	0.053	0.049	-8.1%
Creative destruction value	B	0.009	0.012	27.2%
	Panel B: Changes in Technology Gap Distribution			
Technology gap distribution (shares)	$\Delta^1 = 1$	0.181	0.213	17.3%
	$\Delta^2 = \lambda$	0.446	0.302	-32.3%
	$\Delta^3 = \eta$	0.325	0.434	33.2%
	$\Delta^4 = \frac{\eta}{\lambda}$	0.047	0.052	9.3%

Table H.2: Aggregate Growth Rate Decomposition, Holding Mass Fixed

Description	Before	After	% Change
Average productivity growth by domestic firms (%)	1.942	2.072	6.7%
Growth from domestic own-innovation (%)	0.741	0.485	-34.6%
Growth from domestic creative destruction (%)	0.982	1.297	32.1%
Growth from domestic startups (%)	0.219	0.290	32.4%

H.3 Counterfactual: Increasing Learning Probability with $ct = 0$

Table H.3: Counterfactual: Increasing Learning Probability with $ct = 0$

Description	Variables	Before	After	% Change
Creative destruction arrival rate by outside firms	$\overline{x}_o$	11.723	11.880	1.3%
Aggregate creative destruction arrival rate	$\overline{x}$	76.755	85.978	12.0%
Prob. of own-innovation ($\Delta^1 = 1$)	z^1	8.102	3.880	-52.1%
Prob. of own-innovation ($\Delta^2 = \lambda$)	z^2	80.437	68.531	-14.8%
Prob. of own-innovation ($\Delta^3 = \eta$)	z^3	53.721	43.736	-18.6%
Prob. of own-innovation ($\Delta^4 = \frac{\eta}{\lambda}$)	z^4	44.270	36.206	-18.2%
Prob. of creative destruction, incumbents	x	62.591	70.577	12.8%
Prob. of creative destruction, potential startups	x_e	11.842	13.352	12.8%

H.4 Counterfactual: Changes in Learning Time in the U.S. 1992 vs. 2007

Table H.4: Parameter Estimates and Target Moments

Panel A: Parameter estimates					
External calibration			Internal calibration		
Param.	Description	Value	Param.	Description	Value
β	Time discount rate	0.721	$\widehat{\chi}$	Scale, own-innov.	0.017
$\widehat{\psi}$	Curvature, own-innov.	2.000	$\widetilde{\chi}$	Scale, creative dest.	0.093
$\widetilde{\psi}$	Curvature, creative dest.	2.000	$\widetilde{\chi}^e$	Scale, startup R&D	0.301
$\widetilde{\psi}^e$	Curvature, startup R&D	2.000	λ	Step size, own-innov.	1.098
θ	Qual. share, final goods	0.109	η	Step size, creative dest.	1.176
ct	Coin toss winning prob.	0.500	ω	Learning prob.	0.329
$\mathcal{E}_d$	Mass of dome. startups	1.000	$\mathcal{E}_o$	Mass of out. entrants	0.028

Panel B: Target moments					
Moment	Data	Model	Moment	Data	Model
# of products	1.8	1.8	Productivity growth (%)	0.9	0.9
# of products added	0.2	0.1	High-growth firm growth (%)	17.0	17.1
Firm entry rate (%)	5.1	5.1	Average learning time	2.6	2.6
			Import penetration rate (%)	25.1	25.1

Table H.5: Counterfactual: Setting ω to Its 1992 Value in the 2007 U.S. Economy

Description	Variables	1992	2007	w/ ω_{1992}	Δ (%)
		Panel A: Changes in Firm Innovation			
creative destr. arrival rate by outside firms (%)	$\overline{x}_o$	11.68	9.75	9.73	-0.2%
aggregate creative destr. arrival rate (%)	$\overline{x}$	76.8	38.7	37.8	-2.3%
prob. of own-innovation ($\Delta^1 = 1$, %)	z^1	8.1	16.5	17.7	6.9%
prob. of own-innovation ($\Delta^2 = \lambda$, %)	z^2	80.4	61.1	64.2	5.1%
prob. of own-innovation ($\Delta^3 = \eta$, %)	z^3	53.7	42.3	44.7	5.7%
prob. of own-innovation (%) ($\Delta^4 = \frac{\eta}{\lambda}$, %)	z^4	44.3	38.8	40.9	5.5%
prob. of creative destr., incumbents (%)	x	62.6	27.4	26.6	-2.8%
prob. of creative destr., potential startups (%)	x_e	11.8	8.5	8.2	-2.8%
		Panel B: Changes in Innovation Values			
Average of own-innovation values	$\overline{A}$	0.059	0.077	0.078	1.6%
Creative destruction value	B	0.009	0.007	0.007	-5.6%
		Panel C: Changes in Technology Gap Distribution			
Technology gap distribution (shares)	$\Delta^1 = 1$	0.181	0.421	0.403	-4.2%
	$\Delta^2 = \lambda$	0.446	0.312	0.345	10.8%
	$\Delta^3 = \eta$	0.325	0.236	0.219	-7.3%
	$\Delta^4 = \frac{\eta}{\lambda}$	0.047	0.031	0.033	4.5%
		Panel D: Changes in the Aggregate Moments			
R&D to sales ratio (%)		4.09	2.60	2.64	1.7%
Creative destruction R&D intensity (%)		58.7	69.1	64.6	-6.5%
Average number of products		2.12	1.77	1.78	0.5%
Total mass of domestic firms		0.401	0.423	0.417	-1.4%
		Panel E: Changes in the Aggregate Growth and Decomposition			
Average productivity growth by domestic firms (%)		1.94	0.93	0.92	-0.6%
Growth from domestic own-innovation (%)		0.74	0.37	0.41	9.9%
Growth from domestic creative destruction (%)		0.98	0.39	0.36	-7.8%
Growth from domestic startups (%)		0.22	0.16	0.15	-7.0%

Table H.6: Counterfactual: Increasing Competitive Pressure in the U.S.

Description	Variables	Before	After	Δ (%)
Panel A: Changes in Innovation Values				
Average of own-innovation values	$\overline{A}$	0.059	0.059	-0.5%
Own-innovation value (for each gap)	A_1	0.050	0.049	-0.8%
	A_2	0.060	0.060	-0.6%
	A_3	0.063	0.063	-0.5%
	A_4	0.053	0.053	-0.7%
Creative destruction value	B	0.009	0.009	-2.6%
Panel B: Changes in Technology Gap Distribution				
Technology gap distribution (shares)	$\Delta^1 = 1$	0.181	0.178	-1.8%
	$\Delta^2 = \lambda$	0.446	0.446	0.1%
	$\Delta^3 = \eta$	0.325	0.328	0.8%
	$\Delta^4 = \frac{\eta}{\lambda}$	0.047	0.047	0.4%

H.5 Counterfactual: Increasing Competitive Pressure from Outside Firms

Table H.6 shows further details behind the counterfactual exercise of increasing competitive pressure by outside firms. Note that an exogenous increase in outside firm entry leads to a rise in the aggregate creative destruction arrival rate $\overline{x}$ and decreases the expected profits of both types of innovations ($\{A_\ell\}_{\ell=1}^4$ and B) in Panel A. This is known as the Schumpeterian effect. Also, as incumbents intensify own-innovation to protect their existing product lines, it shifts the distribution of technology gaps, especially for $\Delta^2, \Delta^3 > 1$. As in Panel B, it raises the average technology gap, making it harder for firms to take over product markets via creative destruction, labeled as the technological barrier effect. This leads to the decline of $\overline{x}_{\text{takeover}}$ from 56.4% to 56.2%. Here, also note that the increase in densities $\mu(\Delta^3)$ and $\mu(\Delta^4)$ is solely attributed to increased creative destruction by outside firms. The higher density of Δ^2 reflects both increased own-innovation driven by the market-protection effect and creative destruction by outside firms. Consequently, firm incentives for creative destruction and domestic firm entry get reduced, which contributes to the decline observed in x and x_e. Furthermore, keeping the mass of domestic incumbents constant, 7.7% of decline in growth can be attributed to changes in firm-level creative destruction, as shown in Table H.7.

Table H.7: Aggregate Growth Rate Decomposition, Holding Mass Fixed

Description	Before	After	% Change
Average productivity growth by domestic firms (%)	1.942	1.920	-1.11%
Growth from domestic own-innovation (%)	0.741	0.740	-0.12%
Growth from domestic creative destruction (%)	0.982	0.965	-1.73%
Growth from domestic startups (%)	0.219	0.215	-1.73%

H.6 Counterfactual: Increasing Competitive Pressure from Outside Firms with $ct = 0$

Table H.8: Counterfactual: Increasing Competitive Pressure with $ct = 0$

Description	Variables	Before	After	% Change
Creative destruction arrival rate by outside firms	$\overline{x}_o$	11.840	19.789	67.1%
Aggregate creative destruction arrival rate	$\overline{x}$	81.823	82.784	1.2%
Prob. of own-innovation ($\Delta^1 = 1$)	z^1	6.450	6.096	-5.5%
Prob. of own-innovation ($\Delta^2 = \lambda$)	z^2	81.975	82.056	0.1%
Prob. of own-innovation ($\Delta^3 = \eta$)	z^3	26.960	26.823	-0.5%
Prob. of own-innovation ($\Delta^4 = \frac{\eta}{\lambda}$)	z^4	81.975	82.056	0.1%
Prob. of creative destruction, incumbents	x	66.606	65.979	-0.9%
Prob. of creative destruction, potential startups	x_e	12.601	12.483	-0.9%

Table H.9: Changes in Aggregate Moments

Description	Before	After	% Change
Panel A: Changes in the Aggregate Moments			
R&D to sales ratio (%)	3.8	3.7	-1.5%
Creative destruction R&D intensity (%)	72.1	71.7	-0.5%
Average number of products	2.1	1.9	-5.1%
Total mass of domestic firms	0.42	0.39	-6.3%
Consumption equivalence			-3.9%
Panel B: Changes in the Aggregate Growth and Decomposition			
Average productivity growth by domestic firms (%)	1.9	1.7	-10.6%
Growth from domestic own-innovation (%)	0.5	0.4	-11.8%
Growth from domestic creative destruction (%)	1.2	1.0	-12.2%
Growth from domestic startups (%)	0.3	0.3	-1.2%

H.7 Counterfactual: Increasing Competitive Pressure from Outside Firms with A Single-Period Lag Assumption ($\omega = 0$)

We simplify the model with the exact single-period lag learning (with zero possibility of learning the frontier technology, $\omega = 0$) and verify this assumption does not affect the main results.[12] To see this, we re-calibrate the model to the U.S. manufacturing sector in 1992 on an annual basis with $\omega = 0$.[13] In this set-up, we increase increase the mass of potential outside entrants $\mathcal{E}_o$ by 83%, corresponding to the rise in import penetration ratio in the U.S. manufacturing sector from 1992 to 2007 as before.

Table H.10 shows qualitatively similar results as in the baseline exercise. An exogenous rise in the aggregate creative destruction arrival rate $\overline{x}$ drives lower expected profits for both innovations (Panel B); increases own-innovation for firms with a technology gap of $\Delta^{\ell} > 1$ (market-protection effect), which raises the average technology gap (Panel C); and thus reduces creative destruction by

[12]By assuming a fixed annual learning duration, the calibration adjusts the R&D cost parameters to align the data with the annual frequency of the model.

[13]We use the same set of annual target moments and parameters, excluding ω and the average learning time.

Table H.10: Counterfactual: Increasing Competitive Pressure in the U.S., $\omega = 0$

Description	Variables	Before	After	Δ (%)
Panel A: Changes in Firm Innovation				
creative destr. arrival rate by outside firms (%)	$\overline{x}_o$	3.3	5.5	66.4%
aggregate creative destr. arrival rate (%)	$\overline{x}$	21.5	21.9	1.51%
prob. of own-innovation ($\Delta^1 = 1$, %)	z^1	16.9	16.8	-0.43%
prob. of own-innovation ($\Delta^2 = \lambda$, %)	z^2	57.8	57.9	0.19%
prob. of own-innovation ($\Delta^3 = \eta$, %)	z^3	39.7	39.7	0.13%
prob. of own-innovation (%) ($\Delta^4 = \frac{\eta}{\lambda}$, %)	z^4	37.3	37.4	0.05%
prob. of creative destr., incumbents (%)	x	16.8	16.5	-1.33%
prob. of creative destr., potential startups (%)	x_e	4.02	3.97	-1.33%
Panel B: Changes in Innovation Values				
Average of own-innovation values	$\overline{A}$	0.167	0.165	-1.04%
Creative destruction value	B	0.011	0.011	-2.6%
Panel C: Changes in Technology Gap Distribution				
Technology gap distribution (shares)	$\Delta^1 = 1$	0.541	0.539	-0.4%
	$\Delta^2 = \lambda$	0.314	0.314	0.2%
	$\Delta^3 = \eta$	0.116	0.118	1.1%
	$\Delta^4 = \frac{\eta}{\lambda}$	0.028	0.029	1.4%
Panel D: Changes in the Aggregate Moments				
R&D to sales ratio (%)		4.6	4.5	-1.6%
Creative destruction R&D intensity (%)		63.9	63.1	-1.2%
Average number of products		2.3	2.2	-5.5%
Total mass of domestic firms		0.39	0.36	-6.4%
Panel E: Changes in the Aggregate Growth and Decomposition				
Average productivity growth by domestic firms (%)		1.9	1.7	-11.0%
Growth from domestic own-innovation (%)		1.0	0.9	-11.4%
Growth from domestic creative destruction (%)		0.7	0.6	-13.0%
Growth from domestic startups (%)		0.2	0.2	-1.7%

decreasing $\overline{x}_{\text{takeover}}$ from 73.2% to 73.0%. See more details for the changes in expected value in Table H.11.

Furthermore, Panel D summarizes how aggregate variables change in response to the increased competitive pressure from outside firms. The aggregate R&D to sales ratio of domestic incumbents drops, indicating that the decrease in creative destruction outweighs the increase in own-innovation as before. Finally, Panel E shows that the average productivity growth of domestic firms (g_d) declines. This decrease is attributed to shifts in firm-level innovation intensities and the mass of firms. As before, keeping the mass of domestic incumbents constant, 12.7% of this decline in growth can be attributed to changes in firm-level creative destruction. For more detailed breakdowns, refer to Table H.12.

Table H.11: Changes in Innovation Values, $\omega = 0$

Description	Variables	Before	After	% Change
Innovation Values	A_1	0.160	0.158	-1.1%
	A_2	0.173	0.172	-1.0%
	A_3	0.182	0.180	-1.0%
	A_4	0.165	0.163	-1.1%
	B	0.011	0.011	-2.6%

Table H.12: Aggregate Growth Rate Decomposition, Holding Mass Fixed, $\omega = 0$

Description	Before	After	% Change
Average productivity growth by domestic firms (%)	1.888	1.875	-0.7%
Growth from domestic own-innovation (%)	1.047	1.048	0.1%
Growth from domestic creative destruction (%)	0.656	0.645	-1.7%
Growth from domestic startups (%)	0.186	0.182	-1.7%

H.8 Counterfactual: Increasing Competitive Pressure from Outside Firms with No Learning Friction ($\omega = 1$)

Table H.13: Counterfactual: Increasing Competitive Pressure in the U.S., $\omega = 1$

Description	Variables	Before	After	Δ (%)
	Panel A: Changes in Firm Innovation			
creative destr. arrival rate by outside firms (%)	$\overline{x}_o$	2.0	3.3	62.8%
aggregate creative destr. arrival rate (%)	$\overline{x}$	16.1	16.5	2.0%
prob. of own-innovation ($\Delta^1 = 1$, %)	z^1	22.5	22.3	-1.0%
prob. of own-innovation ($\Delta^2 = \lambda$, %)	z^2	22.5	22.3	-1.0%
prob. of own-innovation ($\Delta^3 = \eta$, %)	z^3	22.5	22.3	-1.0%
prob. of own-innovation (%) ($\Delta^4 = \frac{\eta}{\lambda}$, %)	z^4	22.5	22.3	-1.0%
prob. of creative destr., incumbents (%)	x	12.84	12.76	-0.6%
prob. of creative destr., potential startups (%)	x_e	2.80	2.78	-0.6%

Note that when $\omega = 1$, incumbents can no longer protect their markets from competitors through own-innovation. Consequently, an increase in foreign competition reduces both own-innovation and creative destruction through the standard Schumpeterian effect. Therefore, under the immediate-learning assumption ($\omega = 1$), the model cannot generate the compositional changes in innovation observed in the data. Table H.13 illustrates this pattern.

H.9 Counterfactual: Economy with High Creative Destruction Costs

Figure H.2 shows the results across different initial levels of $\overline{x}$ (reflecting different degrees of initial competitive pressure) corresponding to varied values of $\widetilde{\chi}$ (that negatively affects $\overline{x}$) in continuum. The U.S. economy represents the highest $\overline{x}$ level in the figures. The left panel shows the initial R&D to sales ratios and their changes following a competitive pressure shock, and the right panel breaks down the latter into the changes in own-innovation and creative destruction (CD).

Across all initial values of $\widetilde{\chi}$, own-innovation R&D expenses rise as competitive pressure intensifies, while creative destruction R&D expenses decline. However, the decline in creative

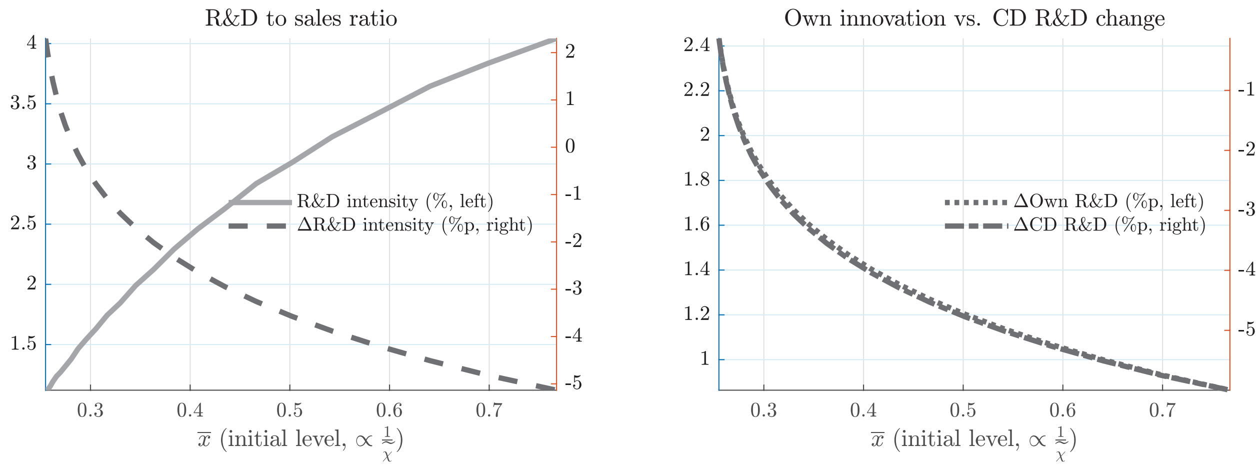

Figure H.2: Decomposition of Innovation Change Across different $\widetilde{\chi}$

destruction R&D is more pronounced when its cost $\widetilde{\chi}$ is low (high initial $\overline{x}$). While both types of innovation respond similarly across different economies, own-innovation increases more than the decline in creative destruction when creative destruction is more costly (lower initial $\overline{x}$), whereas the opposite holds when it is less costly (higher initial $\overline{x}$). Thus, in economies with high creative destruction costs, aggregate R&D rises in response to competitive pressure, in contrast to the U.S. where it declines.

Furthermore, Table H.14 and H.15 describe further details and breakdowns for the case when $\widetilde{\chi}$ is 80 times higher than the U.S.

Table H.14: Changes in Firm Innovation in High Creative Destruction Cost Economy

Description	Variables	Before	After	% Change
Creative destruction arrival rate by outside firms	$\overline{x}_o$	4.044	7.305	80.7%
Aggregate creative destruction arrival rate	$\overline{x}$	26.632	28.777	8.1%
Prob. of own-innovation ($\Delta^1 = 1$)	z^1	25.435	24.670	-3.0%
Prob. of own-innovation ($\Delta^2 = \lambda$)	z^2	64.690	65.906	1.9%
Prob. of own-innovation ($\Delta^3 = \eta$)	z^3	48.309	48.794	1.01%
Prob. of own-innovation ($\Delta^4 = \frac{\eta}{\lambda}$)	z^4	45.062	45.288	0.5%
Prob. of creative destruction, incumbents	x	1.413	1.353	-4.2%
Prob. of creative destruction, potential startups	x_e	21.380	20.478	-4.2%

Table H.15: Aggregate Growth Decomposition, Low Creativity Economy, Holding Mass Fixed

Description	Before	After	% Change
Average productivity growth by domestic firms (%)	1.264	1.235	-2.3%
Growth from domestic own-innovation (%)	0.688	0.692	0.7%
Growth from domestic creative destruction (%)	0.031	0.029	-5.9%
Growth from domestic startups (%)	0.546	0.514	-5.9%

H.10 Counterfactual: Competitive Pressure by Domestic Startups

We increase the mass of potential domestic startups ε_d by 15.2%, which raises the creative destruction arrival rate $\overline{x}$ from 76.8% to 77.9% (1.51% increase, equivalent to the main counterfactual exercise). Table H.16 and Panel A in Table H.17 present the results. The firm-level responses remain the same as before, while the total mass of domestic incumbents and startups increases. Thus, the moments related to the number of domestic firms and startups help identify the source behind the increased competitive pressure (domestic startups vs outside firms). Also, Panel B in Table H.17 displays the growth decomposition, where the aggregate growth increases (unlike the main exercise). As a result, welfare increases by 0.5% in consumption-equivalence terms.

Table H.16: Changes in Firm Innovation: Economy with More Potential Startups

Description	Variables	Before	After	% Change
Creative destruction arrival rate by outside firms	$\overline{x}_o$	11.676	10.628	-9.0%
Aggregate creative destruction arrival rate	$\overline{x}$	76.755	77.918	1.5%
Prob. of own-innovation ($\Delta^1 = 1$)	z^1	8.102	7.742	-4.5%
Prob. of own-innovation ($\Delta^2 = \lambda$)	z^2	80.437	80.650	0.3%
Prob. of own-innovation ($\Delta^3 = \eta$)	z^3	53.721	53.788	0.1%
Prob. of own-innovation ($\Delta^4 = \frac{\eta}{\lambda}$)	z^4	44.270	44.196	-0.2%
Prob. of creative destruction, incumbents	x	62.591	61.733	-1.4%
Prob. of creative destruction, potential startups	x_e	11.842	11.679	-1.4%

Table H.17: Aggregate Moment Change: Economy with More Potential Startups

Description	Before	After	% Change
Panel A: Changes in the Aggregate Moments			
Total mass of domestic firms	0.401	0.430	7.2%
Total mass of domestic startups	0.067	0.076	13.1%
R&D to sales ratio (%)	4.094	4.039	-1.3%
Avg. number of products	2.115	2.005	-5.2%
Consumption equivalence			0.5%
Panel B: Changes in the Aggregate Growth and Decomposition			
Average productivity growth by domestic firms (%)	1.942	1.982	2.1%
Growth from domestic own-innovation (%)	0.741	0.753	1.6%
Growth from domestic creative destruction (%)	0.982	0.981	-0.0%
Growth from domestic startups (%)	0.219	0.248	13.2%

Table H.18: Changes in Aggregate Moments

Description	Before	After	% Change
Panel A: Changes in the Aggregate Moments			
R&D to sales ratio (%)	4.09	4.06	-0.9%
Creative destruction R&D intensity (%)	58.6	75.3	28.5%
Average number of products	2.1	1.9	-11.2%
Total mass of domestic firms	0.40	0.46	14.6%
Consumption equivalence			1.0%
Panel B: Changes in the Aggregate Growth and Decomposition			
Average productivity growth by domestic firms (%)	1.9	2.1	8.4%
Growth from domestic own-innovation (%)	0.7	0.5	-33.3%
Growth from domestic creative destruction (%)	1.0	1.3	34.4%
Growth from domestic startups (%)	0.2	0.3	32.0%

I Data Appendix

I.1 Data Construction

To compile comprehensive data on firm innovation and foreign competition shock, we combine the USPTO PatentsView database, the Longitudinal Business Database (LBD), the Longitudinal Firm Trade Transactions Database (LFTTD), the Census of Manufactures (CMF), the Compustat Fundamental Annual database, the NBER-CES database, and the tariff data in Feenstra et al. (2002).

The LBD tracks the universe of establishments and firms in the U.S. non-farm private sector with at least one paid employee annually from 1976 onward.[14] We aggregate establishment-level data into firm-level using firm identifiers.[15] Firm size is measured by total employment or payroll, and firm age by the age of the oldest establishment of the firm when the firm is first observed in the data. The firm's main industry of operation is based on the six-digit North American Industry Classification System (NAICS) code of the establishment with the highest employment.[16]

The LFTTD tracks all U.S. international trade transactions at the firm level from 1992 onward. It provides information such as the U.S. dollar value of shipments, the origin and destination countries, and a related-party flag indicating whether the U.S. importer and the foreign exporter are related by ownership of at least 6 percent.[17]

The USPTO PatentsView database records all patents ultimately granted by the USPTO from 1976 onward.[18] This database provides comprehensive details for patents, including application and grant dates, technology class, citation, and the name and address of patent assignees. In our analyses, we rely on the citation-adjusted number of utility patents as the main measure of firm innovation.[19] Using the patent-level information, we distinguish domestic innovation from foreign innovation, and assess the extent to which each patent represents own-innovation. The patent application year is used for the innovation year.

[14] Details for the LBD and its construction can be found in Jarmin and Miranda (2002).

[15] An establishment corresponds to the physical location where business activity occurs. Establishments that are operated by the same entity, identified through the Economic Census and the Company Organization Survey, are grouped under a common firm identifier.

[16] Time-consistent NAICS codes for LBD establishments are constructed by Fort and Klimek (2018), and the 2012 NAICS codes are used throughout the entire analysis.

[17] Bernard et al. (2009) describe the LFTTD in greater detail.

[18] See https://patentsview.org/download/data-download-tables.

[19] See Cohen (2010) for a comprehensive review of the literature on the determination of firm/industry innovative activity and related patent measures.

We link the USPTO patent database to the LBD to track firm patenting over time. Failure to match a patent assignee with its LBD firm counterpart can mismeasure firm innovation changes.[20] Since the USPTO patent data lacks a longitudinally consistent firm identifier, we build our own crosswalk between the two datasets by adopting the internet search-aided algorithm as in Autor et al. (2020).[21,22] We pool all patents granted up to December 26, 2017, and use patent applications up to 2007 in our main analyses to avoid a right censoring issue arising from patents applied for but not yet granted.

The quinquennial CMF provides detailed information about the U.S. manufacturing establishments and products they produce. It contains product-level details such as product codes and the value of shipment. We use five-digit SIC codes (for the pre-2002 years) or seven-digit NAICS codes (for 2002 onward) to define a product. We obtain the U.S. tariff schedules from Feenstra et al. (2002) to measure the industry-level Trade Policy Uncertainty (TPU) as a proxy for foreign competitive pressure. Lastly, all nominal values are converted to 1997 U.S. dollars, using the industry-level deflator from the NBER CES Manufacturing Industry Database for manufacturing industries and the Consumer Price Index from the BEA for other industries.[23]

The UN Comtrade Database offers information on global trade flows at the six-digit HS product-level, which can be concorded to the six-digit 2012 NAICS codes using the crosswalks provided by Pierce and Schott (2009, 2012).[24] Using this data, we construct industry-level imports and exports.

For our main analyses, we use LBD and Compustat firms matched to USPTO patents, CMF firms, and industry-level trade data spanning from 1982 to 2007.

I.2 Summary Statistics

Table I.1, I.2, and I.3 present summary statistics of the main regression sample, the learning-time measures, and the competition shock (NTR gap), respectively.

[20]The USPTO assigns patent applications to self-reported firm names, which are frequently misspelled.

[21]This algorithm utilizes the machine-learning capacities of internet search engines. The entire matching methodology is outlined in our accompanying paper Ding et al. (2022). We also apply it when linking Compustat to USPTO.

[22]Our procedure links patents to the firms initially reported by the USPTO as owners and does not track ownership changes resulting from, for example, M&A activities. We expect our analysis not to be contaminated by firms substituting innovation with acquisitions of other firms, particularly given that U.S. M&A activities began declining around 2000 and did not fully recover by 2007, as shown in Phillips and Zhdanov (2023).

[23]The NBER CES data are complied by Becker et al. (2013) (http://www.nber.org/nberces/http://www.nber.org/nberces/).

[24]https://comtrade.un.org/db/default.aspx.

Table I.1: The Whole Universe of Patenting Firms vs. Regression Sample in 1992

	All patenting firms	Regression sample
Average number of patents	6.15	8.86
	(19.46)	(24.10)
Average self-citation rate	0.0434	0.0540
	(0.0899)	(0.0941)
Innovation intensity	0.055	0.093
	(0.25)	(0.33)
Number of industries operating	2.34	5.43
	(3.67)	(6.94)
Employment	511.7	1988.0
	(1869.0)	(3835.0)
Patent stock	6.45	35.22
	(26.61)	(64.37)
Employment growth	0.07	0.06
	(0.60)	(0.40)
Firm age	12.33	15.65
	(6.76)	(9.42)
7yr patent growth		-0.854
		(1.312)
7yr self-citation ratio growth		0.356
		(1.322)
Number of firms	26,500	3,100

Note: This table reports summary statistics for the main USPTO-LBD sample used in the difference-in-differences analysis. Innovation intensity in 2000 is 0.183(0.58), the seven-year patent growth in 2000 is -1.07(1.207), and the seven-year self-citation ratio growth in 2000 is 0.282(1.304).

Table I.2: Backward Citation Gaps in 1982-1999

	Learning Time (year)	
	Minimum	Average
Mean	2.4679	5.5516
Std.	2.9066	3.1477
10%	0.1667	1.7917
25%	0.5000	3.2292
50%	1.4167	5.1667
75%	3.2500	7.4292
90%	6.1667	9.7333
Observations	761,945	761,945

Note: This table reports summary statistics for the backward citation gap measures in USPTO data for 1982–1999. The first column uses the minimum gap (the baseline measure) across citations, and the second column uses the average gap across citations.

Table I.3: Foreign Competition Shock Related Measures

	NTR gap	Dnstream NTR g.	Upstream NTR g.	NTR rate	Non-NTR r.
Mean	0.291	0.138	0.203	0.027	0.303
(Std. dev.)	(0.127)	(0.060)	(0.073)	(0.022)	(0.134)
cov(, NTR gap)		0.485	0.434	0.412	0.969
cov(, Up. NTR g.)		0.204			

Note: This table reports summary statistics for the NTR-gap measures. The first column presents the baseline NTR gap, the second and third columns report the NTR gap for downstream and upstream industries, respectively, and the last two columns show the NTR and non-NTR rates.

I.3 Learning Time Patterns

In this section, we present the time-series patterns of backward citation gaps since 1980 with different types of measures. Figure I.1 presents the minimum learning gaps (left panel) and the average learning gaps (right panel) across versions that either include or exclude expired patents and self-citations. We find that learning time rises modestly in the early 1980s, remains relatively stable through the early 1990s, declines during the mid-1990s, and then shifts upward in the late 1990s, with a sharp acceleration in the mid-2000s.

To account for potential effects arising from shifts in CPC technology composition or broader trends in patenting activity, we also plot the series by CPC section in Figure I.2. In addition, we construct residualized measures that partial out CPC-group fixed effects, assignee fixed effects, and controls for the annual number of patents, assignees, and backward citations. These residualized series, shown in Figure I.3, confirm that the overall pattern is robust across technology categories and after removing composition changes in CPC groups, assignees, and the patenting environment.

While these adjustments mitigate mechanical composition effects, the remaining trend may still reflect a mixture of deeper forces, including fundamental technological shifts in the economic environment with heightened competition, rising technological complexity and scope, and strategic behavior by inventors and assignees, as well as policy changes. On the one hand, technological forces may have shifted firm innovation incentives and contributed to changes in overall learning time. For example, intensified competition can push firms to develop more integrated and interdependent technologies, increasing the sophistication of the underlying knowledge and making diffusion or replication by outsiders substantially more difficult.[25] As technologies advance, generating new ideas may become increasingly difficult, consistent with the evidence in Bloom et al. (2020).

On the other hand, policy changes such as the American Inventors Protection Act (AIPA, henceforth)—which shortens publication delays after 2000 as in Figure I.4—can also contribute to the pattern observed in the 2000s by increasing the possibility of learning the frontier technology. The effect on overall learning time can still be mixed. Hegde et al. (2023) show that AIPA reduces the forward-citation gap for the first citation and lowers duplicative patenting, pointing to a channel

[25] We have supportive data evidence on this hypothesis, presented in Appendix Section I.9, related to the "technological barrier" effects driven by incumbents' strategic innovation.

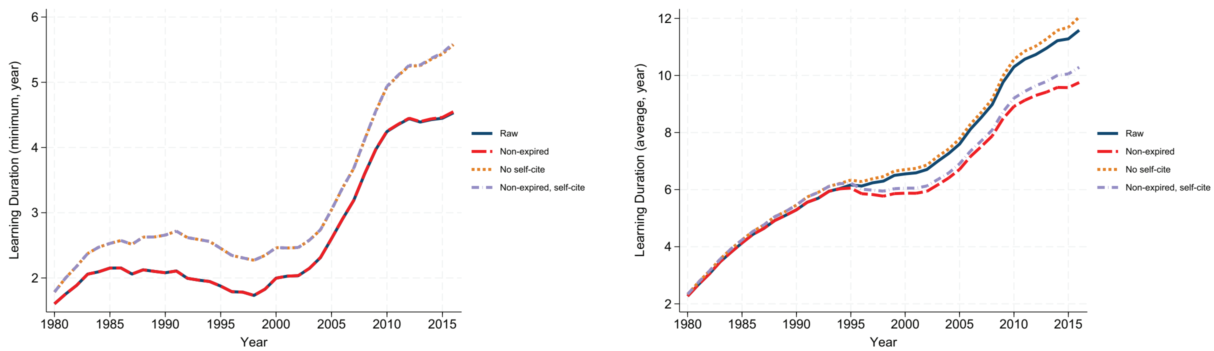

Figure I.1: Learning Gap Measures (minimum and average)

Notes: This figure plots the average learning gap based on the minimum (left panel) and the average (right panel). The navy line shows the raw series, the red line excludes expired patents, the dashed orange line excludes self-citations, and the dashed lavender line excludes both (our baseline measure).

through which earlier disclosure facilitates recognition among information-intensive firms.[26] At the same time, increased visibility does not guarantee immediate technological absorption; the effect depends on the type of information disclosed, the absorptive capacity of downstream innovators, and their innovation and IP strategies (Baruffaldi and Simeth, 2020).

Our measure, which aggregates minimum backward citation gaps across a broad set of assignees and technologies, can capture the combined influence of these forces. Our counterfactual analysis provides suggestive evidence on the role of learning probability in accounting for the change in overall learning time between 1992 and 2007. Disentangling how this channel interacts with other forces and quantifying their respective contributions remains a promising direction for future research.

[26]We present complementary evidence in Section 3.2, where CPC technologies more exposed to the AIPA exhibit a pronounced post-2000 decline in learning time.

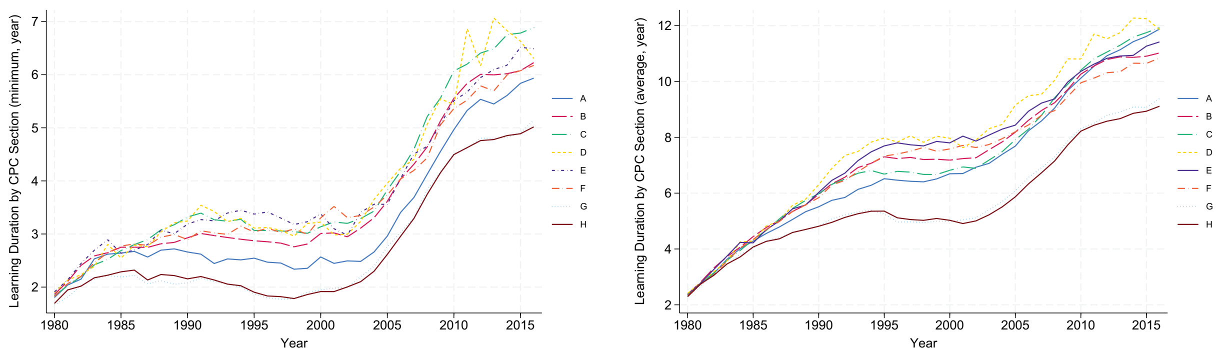

Figure I.2: Learning Gap Measures by CPC Section (minimum and average)

Notes: This figure plots the average learning gap based on the minimum (left panel) and the average (right panel) by each CPC section.

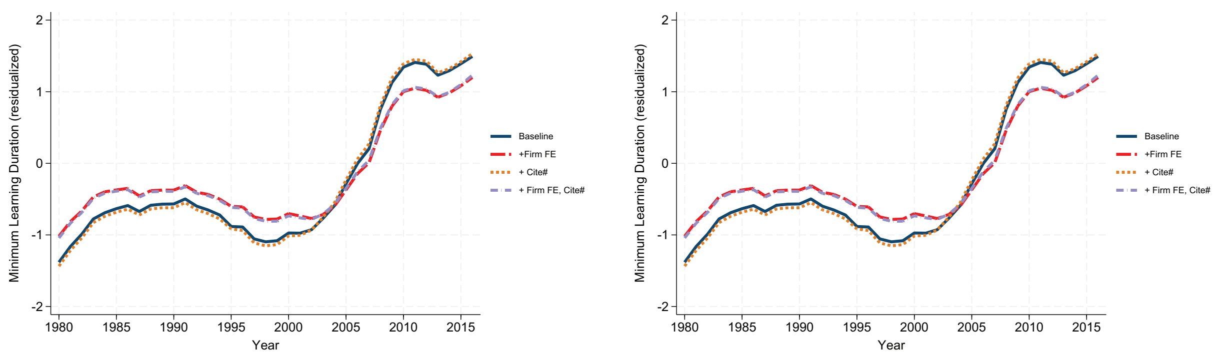

Figure I.3: Residualized Learning Gap Measures (minimum and average)

Notes: This figure plots the average residualized learning gap based on the minimum (left panel) and the average (right panel). Residualization is implemented under several specifications: the navy line controls for CPC-group fixed effects and year-level counts of patents and assignees; the red line adds assignee fixed effects to this baseline; the dashed orange line augments the baseline with controls for the number of cited patents; and the dashed lavender line includes both controls in the baseline specification.

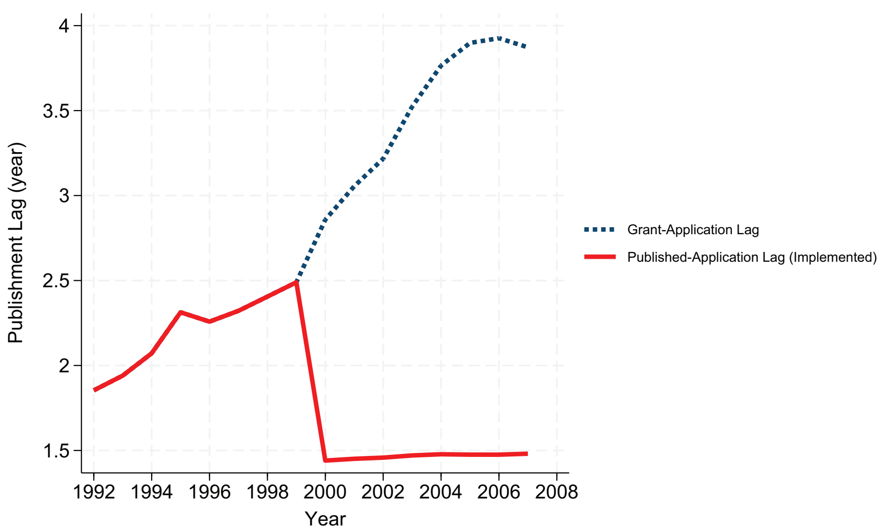

Figure I.4: Publication Lags of Patents

Notes: This figure plots the average publication lag of patents—measured as the gap between the publication date and the application year—over time. Prior to the AIPA (before 2000), publication occurred at grant, so publication lag corresponds to grant lag. After the AIPA, publication is defined as the earlier of the grant year or 1.5 years after the application year. Accordingly, the dashed line depicts the grant–application lag, while the red line depicts the publication lag; the two lines coincide prior to 2000.

I.4 Backgrounds on Trade Policy Uncertainty

Following Pierce and Schott (2016) and Handley and Limão (2017), we use the removal of trade policy uncertainty (TPU) as a measure of an exogenous competitive pressure shock. Specifically, we use the following industry-level tariff rate gaps between WTO members and non-market economies in the year 1999 as a proxy for the industry-level competitive pressure shock from China occurring in 2001.[27]

$$NTRGap_j = Non\,NTR\,Rate_j \;-\; NTR\,Rate_j \text{ (for industry } j\text{)}.$$

For multi-industry firms, we use the employment-weighted average of $NTRGap_j$.

The removal of TPU encouraged Chinese firms to enter the U.S. markets and export their products (Pierce and Schott, 2016), which captures an exogenous increase in competitive pressure by foreign firms and directly maps into an increase in $\overline{x}_o$ in our model.

I.5 Robustness Test for Learning and Innovation Heterogeneity

This section presents several robustness test conducted for Table 1. First, we rerun the regression using the average citation gaps. The results remain robust, as shown in Table I.4. Second, we consider an alternative explanation related to the technological diversity of innovations. If an innovation spans a broader range of technologies, it may naturally cite a larger number of older patents. To address this, we control for the number of backward-cited patents or/and the number of CPC classes associated with the backward-cited patents. Specifically, we include and exclude the set of technology classes linked to the focal patent itself. The results remain robust in both specifications, as shown in Table I.5, I.6 and I.7. Also, producing a novel patent may require citing a diverse set of prior patents, which could increase the likelihood of referencing very old ones. To

[27] Nonmarket economies such as China are by default subject to relatively high tariff rates, known as non-Normal Trade Relations (non-NTR) or column 2 tariffs, when they export to the U.S. On the other hand, the U.S. offers WTO member countries NTR or column 1 tariffs, which are substantially lower than non-NTR tariffs. Although the U.S. granted temporary NTR status to China from 1980, the U.S. Congress voted on a bill to revoke China's temporary NTR status every year from 1990 to 2001 after the Tiananmen Square protests in 1989. This caused uncertainty about whether the low tariffs would revert to non-NTR rates. Following an agreement on China's entry into the WTO, the U.S. Congress passed a bill granting China permanent NTR (PNTR), and PNTR was implemented on January 1, 2002. The PNTR has reduced trade policy uncertainty, more for industries with a large prior gap between NTR and non-NTR tariff rates. See Pierce and Schott (2016) for details.

Table I.4: Backward Citation Gap and Self-Citation Ratio (Average Gap)

	Citation gaps	Citation gaps	Citation gaps	Citation gaps
Self-citation ratio	-1.497***	-1.771***	-1.824***	-1.867***
	(0.018)	(0.018)	(0.020)	(0.020)
Observations	740,908	740,908	740,908	740,908
Fixed effects	none	ct	i, t	i, ct

Notes: The average backward citation gaps are used for the main dependent variable. Constant terms are omitted for brevity. Robust standard errors are displayed below each coefficient. Observations are unweighted. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

rule this out, we control for the standard deviation of citation gaps to ensure that the average is not driven by a small number of outliers with exceptionally old application dates. The results, shown in Table I.8, confirm the robustness of our findings. Lastly, the trends of patent filings in a certain technology group may also affect the learning time in general. We control for the total number of patents filings or assignees in a CPC technology group, and Table I.9 shows the robustness of the results.

Table I.5: Backward Citation Gap and Self-Citation Ratio (Number of Cited Patents)

	Citation gaps	Citation gaps	Citation gaps	Citation gaps
Self-citation ratio	-2.443***	-2.705***	-2.805***	-2.845***
	(0.014)	(0.015)	(0.017)	(0.017)
Number of cited patent	-1.122***	-1.196***	-1.247***	-1.243***
	(0.004)	(0.004)	(0.005)	(0.005)
Observations	740,908	740,908	740,908	740,908
Fixed effects	none	ct	i, t	i, ct

Note: The log number of cited patents is controlled. Constant terms are omitted for brevity. Robust standard errors are displayed below each coefficient. Observations are unweighted. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table I.6: Backward Citation Gap and Self-Citation Ratio (CPCs of Cited Patents)

	Citation gaps	Citation gaps	Citation gaps	Citation gaps
Self-citation ratio	-1.352***	-1.442***	-1.560***	-1.586***
	(0.016)	(0.016)	(0.019)	(0.019)
CPC class number	-0.209***	-0.227***	-0.236***	-0.240***
	(0.001)	(0.002)	(0.002)	(0.002)
Observations	536,029	536,029	536,029	536,029
Fixed effects	none	ct	i, t	i, ct

Note: The number of CPC classes associated with cited patents is controlled. Constant terms are omitted for brevity. Robust standard errors are displayed below each coefficient. Observations are unweighted. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table I.7: Backward Citation Gap and Self-Citation Ratio (CPCs of Cited Patents exc. own)

	Citation gaps	Citation gaps	Citation gaps	Citation gaps
Self-citation ratio	-1.365***	-1.454***	-1.577***	-1.606***
	(0.016)	(0.016)	(0.019)	(0.019)
CPC class number	-0.175***	-0.193***	-0.196***	-0.202***
	(0.001)	(0.001)	(0.002)	(0.002)
Observations	536,029	536,029	536,029	536,029
Fixed effects	none	ct	i, t	i, ct

Note: The number of CPC classes associated with cited patents, excluding the focal CPC class, is controlled. Constant terms are omitted for brevity. Robust standard errors are displayed below each coefficient. Observations are unweighted. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table I.8: Backward Citation Gap and Self-Citation Ratio (Std. of Cited Patents)

	Citation gaps	Citation gaps	Citation gaps	Citation gaps
Self-citation ratio	-1.093***	-1.237***	-1.337***	-1.382***
	(0.012)	(0.013)	(0.015)	(0.015)
Std of citation gaps	-0.120***	-0.192***	-0.197***	-0.211***
	(0.001)	(0.002)	(0.002)	(0.002)
Observations	679,347	679,347	679,347	679,347
Fixed effects	none	ct	i, t	i, ct

Note: The standard deviation of citation gaps is controlled. Constant terms are omitted for brevity. Robust standard errors are displayed below each coefficient. Observations are unweighted. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table I.9: Backward Citation Gap and Self-Citation Ratio (Patent Filing Trends)

	Citation gaps	Citation gaps	Citation gaps	Citation gaps
Self-citation ratio	-1.485***	-1.644***	-1.779***	-1.808***
	(0.014)	(0.015)	(0.017)	(0.017)
Patent numbers	-0.176***	-0.126***	-0.041***	-0.095***
	(0.006)	(0.010)	(0.007)	(0.011)
Assignee numbers	0.038***	0.042***	-0.045***	0.034**
	(0.007)	(0.012)	(0.009)	(0.014)
Observations	740,908	740,908	740,908	740,908
Fixed effects	none	ct	i, t	i, ct

Note: The log numbers of patent filings and assignees in a CPC technology group are controlled. Constant terms are omitted for brevity. Robust standard errors are displayed below each coefficient. Observations are unweighted. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

I.6 Other Dimensions of Innovation Heterogeneity

I.6.1 Quality Improvement

We also provide empirical evidence on the differentiated quality improvement between own-innovation and creative destruction ($\eta > \lambda$). Innovation quality is measured in two ways: the number of forward citations as a proxy for scientific value and the stock market response to patent news as a measure of market value (Kogan et al., 2017).[28] Since the market value of innovation is available only for publicly traded firms, we limit this analysis to patenting firms in Compustat from 1982 to 1999. We estimate the following regression:

$$Quality_{ipct} = \alpha + \beta_1 SelfCite_{ipct} + \beta_2 X_{it-1} + \delta_{ct} + \varepsilon_{ipct},$$

where $Quality_{ipct}$ represents either the log of market value (M-value) or the log of one plus the number of forward citations (S-value) for patent p by firm i in year t, within CPC subsection c; $SelfCite_{ipct}$ is the self-citation ratio for patent p; X_{it-1} is firm i's market capitalization in period $t-1$ as the baseline measure for firm size; and δ_{ct} is a CPC technology-year fixed effect.[29] δ_{ct} allows for patent quality comparisons within each market, while accounting for varying forward citation trends across technologies and years.[30]

Table I.10 displays the results. Note that patent market value is mechanically correlated with market capitalization (0.67 in our sample), as it is the product of its estimated stock return and the firm's market capitalization (Kogan et al., 2017). Also, larger firms tend to produce patents with higher self-citation ratios (correlation = 0.16). After controlling for these factors, we find a negative relationship between market value and closeness to own-innovation, which remains significant with firm fixed effects (column 2-3). Similar results hold for scientific value (columns 4-6). These findings support the view that creative destruction is of higher quality than own-innovation.[31]

Note that the market value of creative destruction reflects both the addition of a new product

[28]The data is sourced from the paper's website (github.com/KPSS2017/Technological-Innovation-Resource-Allocation-and-Growth-Extended-Data), updated through 2023.

[29]Market capitalization is calculated as the product of the closing market price (PRCC_F) and the number of common shares outstanding (CSHO).

[30]Due to the lack of product market information for patents, we use the primary CPC subsection as a proxy for product markets.

[31]The results remain robust across different firm size measures and with firm-year fixed effects. The results are available upon request.

Table I.10: Patent Quality and Self-Citation Ratio

	M-value	M-value	M-value	S-value	S-value	S-value
Self-citation	0.192***	-0.289***	-0.027***	-0.110***	-0.082***	-0.047***
	(0.008)	(0.006)	(0.005)	(0.008)	(0.008)	(0.008)
Market cap$_{-1}$		0.431***	0.289***		-0.025***	-0.043***
		(0.001)	(0.003)		(0.001)	(0.005)
Observations	360,750	360,750	360,750	360,750	360,750	360,750
Fixed effects	ct	ct	i, ct	ct	ct	i, ct

Notes: The estimates for firm (i), CPC technology-year (ct) fixed effects, and the constant are suppressed. Robust standard errors are displayed below each coefficient. Observations are unweighted. The mean (standard deviation) of the market value, scientific value, and self-citation ratio are 2.07 (1.32), 2.88 (1.22), and 0.17 (0.24), respectively. * p $<$ 0.1, ** p $<$ 0.05, *** p $<$ 0.01.

and the associated quality improvement, whereas the market value of own-innovation captures only the quality improvement. As a result, the probability that the market value of creative destruction exceeds that of own-innovation should be high. While we cannot separately identify the quality component within market value, the consistency of our findings when using scientific value—an indicator of quality improvement—supports the robustness of our conclusion.

I.6.2 Economic Outcomes

We also compare the impacts of the two innovation types on firm performance using either patenting LBD firms or CMF firms. We estimate the following regression using census years from 1982 to 1997:

$$\Delta Y_{ijt+5} = \beta_1 Pat_{ijt} + \beta_2 Self_{ijt} + \mathbf{X}_{ijt}\gamma_1 + \delta_{jt+5} + \varepsilon_{ijt+5}.$$

ΔY_{ijt+5} represents the DHS growth of firm employment, the number of industries (six-digit NAICS) added, revenue productivity growth, the number of products added, or the growth in within-firm product market concentration between t and $t+5$; Pat_{ijt} is the citation-adjusted number of patents (in log) at t; and $Self_{ijt}$ is the citation-adjusted average self-citation ratio at t for firm i in industry j. Firm and industry controls include firm age, log payroll, the past five years of U.S. patent growth in firm technology fields, innovation intensity, and public firm status.

Table I.11 shows that firm patenting is positively associated with the growth of firm-level employment and productivity, as well as the number of industries or products added. However,

Table I.11: Real Effect of Innovation on Employment Growth and Industry Added

	ΔEmployment	#Industries added	ΔTFPR	#Products added	ΔHHI
#patents	0.036***	0.102***	0.118**	0.358**	-0.012
	(0.010)	(0.011)	(0.055)	(0.085)	(0.023)
Avg. self-citation	-0.256**	-0.158**	-0.027	-0.274***	0.154**
	(0.109)	(0.079)	(0.053)	(0.102)	(0.069)
Observations	5,400	5,400	5,700	5,700	5,700
Fixed effects	jt	jt	jt	jt	jt

Notes: The baseline set of controls is included. The first two columns are based on the LBD-USPTO firms, and the last three columns are based on the CMF firms. Robust standard errors adjusted for clustering at the firm-level are displayed below each coefficient. Observations are unweighted. Observation counts are rounded due to Census Bureau disclosure avoidance procedures. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

this association weakens if the new patent has a higher self-citation ratio (or is closer to own-innovation).[32,33]

We also check the robustness using alternative innovation measures for creative destruction and own-innovation. Specifically, creative destruction is explicitly defined by the count of patents with a zero self-citation ratio, while own-innovation is measured by patents with a self-citation above 0% or 10%. This more direct measure of creative destruction and own-innovation exhibits consistent and even more pronounced effects, as presented in Table I.12.

I.7 Parallel Pre-trend Assumption

We test the parallel pre-trends assumption, a key identifying assumption for the Diff-in-Diff model. We estimate (10) for the two seven-year periods preceding the policy change, 1984-1991 and 1992-1999. Table I.13 supports the validity of the assumption, where the coefficient estimates are smaller and statistically insignificant.

[32] The mean (and standard deviation) of the citation-adjusted logged number of patents is 1.284 (1.125), and the counterpart for the citation-adjusted average self-citation ratio is 0.050 (0.101). The result, along with this, implies that for average firms, creating one more patent is associated with a 1.32 pp (3.6/2.718) increase in their employment growth as $\exp(1) \approx 2.718$. Also, since average firms have the average self-citation ratio of 0.05, an 1% increase in self-citation ratio is associated with a 0.0128 pp ($-0.256\times0.05\times0.01\times100$) decrease in their employment growth.

[33] Note Akcigit and Kerr (2018) also show that own-innovation contributes less to firm employment growth, which is consistent with our result in the first column of Table I.11.

Table I.12: Real Effect of Innovation on Productivity Growth, Product Added, and Product Concentration (Alternative Innovation Measures)

	ΔTFPR	#prod. add	ΔHHI	ΔTFPR	#prod. add	ΔHHI
#patents (self-cite=0)	0.118**	0.358**	-0.124**	0.129**	0.354***	-0.120**
	(0.055)	(0.085)	(0.055)	(0.052)	(0.081)	(0.052)
#patents (self-cite>0.10)	-0.027	-0.274***	0.134**	-0.055	-0.317***	0.152**
	(0.053)	(0.102)	(0.063)	(0.056)	(0.118)	(0.067)
Observations	5,700	5,700	5,700	5,700	5,700	5,700
Fixed effects	jt	jt	jt	jt	jt	jt
Own-innov. cutoffs	0%	0%	0%	10%	10%	10%

Notes: Creative destruction is defined by the number of patents with a zero self-citation ratio, and own-innovation is defined by the number of patents with a self-citation above a certain cutoff. In the first three columns, the cutoff is set at zero, whereas in the last three columns, it is set at 10%. The baseline set of controls along with firm payroll, the number of operating industries and products are included. The estimates for industry (j) and the year (t) fixed effects, and the coefficient associated with the binary indicator are suppressed due to disclosure restrictions. Robust standard errors adjusted for clustering at the firm-level are displayed below each coefficient. Observations are unweighted. Observation counts are rounded due to Census Bureau disclosure avoidance procedures. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table I.13: Parallel Pre-trend Test

	ΔPatents	ΔPatents	ΔSelf-cite	ΔSelf-cite
NTR gap	-0.397	-0.380	-0.554	-0.546
	(0.487)	(0.488)	(0.403)	(0.402)
× Innovation intensity		-0.195		-0.058
		(0.162)		(0.395)
NTR gap × $\mathcal{I}_{\{1992\}}$	0.523	0.500	0.252	0.259
	(0.355)	(0.362)	(0.294)	(0.290)
× Innovation intensity		0.092		-0.113
		(0.243)		(0.491)
Observations	5,000	5,000	5,000	5,000
Fixed effects	j, p	j, p	j, p	j, p
Controls	baseline	baseline	baseline	baseline

Notes: The baseline set of controls is included. The estimates for industry (j) and the period (p) fixed effects, and the coefficient associated with the binary indicator are suppressed due to disclosure restrictions, and the constant is dropped as well. Robust standard errors adjusted for clustering at the level of the firms' major industries are displayed below each coefficient. Observations are unweighted. Observation counts are rounded due to Census Bureau disclosure avoidance procedures. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

I.8 Robustness Test for the Diff-in-Diff Identification

Furthermore, we perform several robustness checks as follows. First, we replace the baseline firm-level NTR gaps with the industry-level NTR gaps based on the primary industry (with the largest employment size) in which firms operate.[34] See Table I.14. Second, we include upstream and downstream competitive pressure shocks as covariates to control the effect of trade shocks through firms' I-O networks.[35] See Table I.15. The third test addresses a potential sampling bias using the inverse propensity score weights.[36,37] See Table I.16. The fourth test adjusts the level of standard error clustering to the firm level.[38] See Table I.17. The fifth test considers the potential correlation between the innovation intensity measure and firm size or age (e.g., Acemoglu et al., 2018), which may blur the effect of technological barriers. To address this concern, we control additional terms that interact innovation intensity with firm age and size. Moreover, we use an alternative measure based on the inverse of the innovation intensity gap relative to the industry frontier, averaged over the past five years, as the level of technological advantage. See Tables I.18 and I.19. The sixth test confirms the robustness of alternative measures for creative destruction and own-innovations. Creative destruction is directly measured by the number of new product added, and own-innovation is directly measured by the number of patents with a self-citation ratio above 0% or 10%. Also, we examine the impact on within-firm product market concentration. See Tables I.20, I.21, and I.22. Lastly, we include additional controls (such as the cumulative number of patents, firm payroll, the number of industries or products, industry-level skill and capital intensities, as well as dummies for importers and exporters) beyond the baseline set to eliminate potential alternative interpretations. See Tables I.23 and I.24.

[34]The baseline measure uses the employment-share weighted average of the industry-level NTR gaps, where the employment share is measured at the start year of each period and averaged across the firm's operating industries.

[35]The upstream (downstream) measure captures the effect of trade shocks propagating upstream (downstream) from an industry's buyers (suppliers). Using the 1992 BEA input-output table, we construct upstream and downstream competitive pressure shocks as the weighted averages of industry-level trade shocks. Following the approach in Pierce and Schott (2016), we assign I-O weights to zero for both upstream and downstream industries within the same three-digit NAICS broad industries for each six-digit NAICS industry.

[36]This issue can potentially arise from the selection of samples with a positive number of patents granted in the start year and in any of the last four years of each period in the regression analysis, which is inevitable to compute the self-citation ratio over two years for each period.

[37]To formulate the weights, we employ a logit regression on the entire universe of the LBD. The dependent variable is set to one if the firm belongs to the regression sample and zero otherwise. The independent variables include firm size, age, employment growth rate, industry, and a multi-unit status indicator.

[38]In our baseline analysis, we cluster the standard errors at the six-digit NAICS level as most variations in the firm-level NTR gap occur at the industry level.

Table I.14: Industry-level Tariff Measures

	ΔPatents	ΔPatents	ΔSelf-cite	ΔSelf-cite
NTR gap × Post	0.016	0.011	0.005	-0.001
	(0.249)	(0.249)	(0.261)	(0.261)
× Innovation intensity		-0.032		0.760***
		(0.229)		(0.272)
Observations	6,500	6,500	6,500	6,500
Fixed effects	j, p	j, p	j, p	j, p
Controls	baseline	baseline	baseline	baseline
Weights for tariffs	major industry	major industry	major industry	major industry

Notes: Table reports results of OLS generalized difference-in-differences regressions in which industry-level tariff measures are used. The baseline set of controls is included. The estimates for industry (j) and the period (p) fixed effects, and the coefficient associated with the binary indicator are suppressed due to disclosure restrictions, and the constant is dropped as well. Robust standard errors adjusted for clustering at the level of the firms' major industries are displayed below each coefficient. Observations are unweighted. Observation counts are rounded due to Census Bureau disclosure avoidance procedures. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table I.15: Foreign Competition Shock through I-O Linkages

	ΔPatents	ΔPatents	ΔSelf-cite	ΔSelf-cite
NTR gap × Post	-0.111	-0.111	-0.296	-0.424
	(0.331)	(0.342)	(0.356)	(0.355)
× Innovation intensity		-0.001		0.824***
		(0.337)		(0.288)
Observations	6,500	6,500	6,500	6,500
Fixed effects	j, p	j, p	j, p	j, p
Controls	baseline+IO	baseline	baseline	baseline

Notes: The baseline set of controls is included along with the diff-in-diff terms for upstream and downstream sectors, respectively. The estimates for industry (j) and the period (p) fixed effects, and the coefficient associated with the binary indicator are suppressed due to disclosure restrictions, and the constant is dropped as well. Robust standard errors adjusted for clustering at the level of the firms' major industries are displayed below each coefficient. Observations are unweighted. Observation counts are rounded due to Census Bureau disclosure avoidance procedures. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table I.16: Weighted by Inverse Propensity Score

	ΔPatents	ΔPatents	ΔSelf-cite	ΔSelf-cite
NTR gap $\times$ Post	0.003	0.039	-0.394	-0.603
	(0.475)	(0.484)	(0.509)	(0.512)
$\times$ Innovation intensity		-0.045		0.893***
		(0.282)		(0.294)
Observations	6,500	6,500	6,500	6,500
Fixed effects	j, p	j, p	j, p	j, p
Controls	baseline	baseline	baseline	baseline
Regression weights	inv. propens.	inv. propens.	inv. propens.	inv. propens.

Notes: Table reports results of OLS generalized difference-in-differences regressions in which observations are weighted by the inverse of the propensity scores from logit model (y = indicator for analysis sample). The baseline set of controls is included. The estimates for industry (j) and the period (p) fixed effects, and the coefficient associated with the binary indicator are suppressed due to disclosure restrictions. Robust standard errors adjusted for clustering at the level of the firms' major industries are displayed below each coefficient. Observation counts are rounded due to Census Bureau disclosure avoidance procedures. For the sake of space, only the main coefficients are presented. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table I.17: Standard Error Clustering on Firms

	ΔPatents	ΔPatents	ΔSelf-cite	ΔSelf-cite
NTR gap $\times$ Post	0.067	0.071	0.045	-0.062
	(0.287)	(0.290)	(0.308)	(0.312)
$\times$ Innovation intensity		-0.054		0.795***
		(0.245)		(0.277)
Observations	6,500	6,500	6,500	6,500
Fixed effects	j, p	j, p	j, p	j, p
Controls	baseline	baseline	baseline	baseline
se. cluster	firmid	firmid	firmid	firmid

Notes: Table reports results of OLS generalized difference-in-differences regressions in which robust standard errors are adjusted for clustering at the firm-level. The baseline set of controls is included. The estimates for industry (j) and the period (p) fixed effects, and the coefficient associated with the binary indicator are suppressed due to disclosure restrictions. Observations are unweighted. Observation counts are rounded due to Census Bureau disclosure avoidance procedures. For the sake of space, only the main coefficients are presented. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table I.18: Robustness Check for Innovation Intensity Measure (Firm Age, Size Effects)

	ΔPatents	ΔPatents	ΔSelf-cite	ΔSelf-cite
NTR gap $\times$ Post	-0.447	-0.342	0.805	0.292
	(0.645)	(0.691)	(0.668)	(0.641)
$\times$ Innovation intensity		-0.026		0.826***
		(0.239)		(0.284)
Observations	6,500	6,500	6,500	6,500
Fixed effects	j, p	j, p	j, p	j, p
Controls	baseline+	baseline+	baseline+	baseline+

Notes: The baseline set of controls is included along with additional controls for the set of interaction terms between innovation intensity and firm age, as well as innovation intensity and firm size, to check robustness for potential correlations between innovation intensity, firm age, and firm size. The estimates for industry (j) and the period (p) fixed effects, and the coefficient associated with the binary indicator are suppressed due to disclosure restrictions, and the constant is dropped as well. Observations are unweighted. Observation counts are rounded due to Census Bureau disclosure avoidance procedures. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table I.19: Alternative Technology Barrier Measure

	ΔPatents	ΔPatents	ΔSelf-cite	ΔSelf-cite
NTR gap $\times$ Post	0.067	0.131	0.045	0.029
	(0.287)	(0.291)	(0.308)	(0.313)
$\times$ Innovation intensity		-0.058		0.066*
		(0.440)		(0.040)
Observations	6,500	6,500	6,500	6,500
Fixed effects	j, p	j, p	j, p	j, p
Controls	baseline	baseline	baseline	baseline

Notes: The baseline set of controls is included, with the innovation intensity measure replaced by the past 5-year average of the inverse of the within-industry innovation intensity gap from the frontier firm as a proxy for the accumulated level of technology barriers. The estimates for industry (j) and the period (p) fixed effects, and the coefficient associated with the binary indicator are suppressed due to disclosure restrictions, and the constant is dropped as well. Observations are unweighted. Observation counts are rounded due to Census Bureau disclosure avoidance procedures. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table I.20: Alternative Creative Destruction Measure

	#products added	#products added	#products added
NTR gap × Post	-0.239***	-0.231***	-0.218***
	(0.068)	(0.067)	(0.063)
Observations	497,000	497,000	497,000
Fixed effects	j, p	j, p	j, p
Controls	baseline	baseline	baseline
Creative destruction measure	(innovation intensity)	(labor productivity)	(TFPR)

Notes: Creative destruction is directly measured by the number of products added and taken as the main dependent variable. The baseline set of controls (with a different measure for technological barriers) is included. Innovation intensity is the baseline measure as before in the first column. In the second and third columns, it is replaced by the inverse gap of the firm's labor productivity or TFPR from the frontier in its operating industry as an alternative way to measure the degree of technological barriers. Estimates for industry-period (jp) fixed effects as well as the constant are suppressed. Robust standard errors adjusted for clustering at the level of the firms' major industries are displayed below each coefficient. Observations are unweighted. Observation counts are rounded due to Census Bureau disclosure avoidance procedures. * p $<$ 0.1, ** p $<$ 0.05, *** p $<$ 0.01.

Table I.21: Alternative Own-Innovation Measure

	ΔPatents (self-cite>0)	ΔPatents (self-cite>0)	ΔPatents (self-cite>10)	ΔPatents (self-cite>10)
NTR gap × Post	0.007	0.001	0.005	-0.008
	(0.004)	(0.004)	(0.004)	(0.005)
× Innovation intensity		0.100***		0.206***
		(0.033)		(0.077)
Observations	497,000	497,000	497,000	497,000
Fixed effects	j, p	j, p	j, p	j, p
Controls	baseline	baseline	baseline	baseline

Notes: Own-innovation is directly measured and taken as the main dependent variable. The first two columns measure it by the number of patents with a positive self-citation ratio (self-cite $>$ 0), and the last two columns measure it by those with at least a 10% self-citation ratio (self-cite $>$ 10). The baseline set of controls is included. Estimates for industry-period (jp) fixed effects as well as the constant are suppressed. Robust standard errors adjusted for clustering at the level of the firms' major industries are displayed below each coefficient. Observations are unweighted. Observation counts are rounded due to Census Bureau disclosure avoidance procedures. * p $<$ 0.1, ** p $<$ 0.05, *** p $<$ 0.01.

Table I.22: The Effect on Product Concentration

	Δproduct HHI	Δproduct HHI
NTR gap $\times$ Post	-0.002	-0.019
	(0.042)	(0.012)
$\times$ Innovation intensity		0.262**
		(0.116)
Observations	497,000	497,000
Fixed effects	j, p	j, p
Controls	baseline	baseline

Notes: The main dependent variable is the product sales concentration within each firm. The baseline set of controls is included. Estimates for industry-period (jp) fixed effects as well as the constant are suppressed. Robust standard errors adjusted for clustering at the level of the firms' major industries are displayed below each coefficient. Observations are unweighted. Observation counts are rounded due to Census Bureau disclosure avoidance procedures. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table I.23: Robustness Test for the Market-Protection Effect (Overall Innovation)

	ΔPatents	ΔPatents	ΔPatents	ΔPatents	ΔPatents	ΔPatents
NTR gap $\times$ Post	0.076	0.062	0.028	0.112	0.081	0.074
	(0.283)	(0.284)	(0.284)	(0.278)	(0.279)	(0.280)
$\times$ Innov. intensity	-0.055	-0.037	-0.051	0.058	-0.055	-0.029
	(0.242)	(0.242)	(0.239)	(0.243)	(0.240)	(0.231)
Observations	6,500	6,500	6,500	6,500	6,500	6,500
Fixed effects	j, p	j, p	j, p	j, p	j, p	j, p
Controls	base+	base+	base+	base+	base+	base+

Notes: All columns augment the baseline set of controls with additional variables. Specifically, column (1) includes the cumulative number of patents, column (2) includes firm payroll, column (3) includes the number of industries in which firms operate, column (4) includes the industry-level skill, capital intensities, column (5) includes the number of industries and the industry-level skill, capital intensities, and column (6) includes the number of industries, the industry-level skill, capital intensities, a dummy for firms with total imports > 0, and a dummy for firms with total exports > 0. The estimates for industry (j) and the period (p) fixed effects, and the coefficients associated with the binary indicators are suppressed due to disclosure restrictions, and the constant is dropped as well. Robust standard errors adjusted for clustering at the level of the firms' major industries are displayed below each coefficient. Observations are unweighted. Observation counts are rounded due to Census Bureau disclosure avoidance procedures. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table I.24: Robustness Test for the Market-Protection Effect (Own-Innovation)

	ΔSelf-c.	ΔSelf-c.	ΔSelf-c.	ΔSelf-c.	ΔSelf-c.	ΔSelf-c.
NTR gap $\times$ Post	-0.078	-0.059	-0.026	0.007	0.042	0.063
	(0.290)	(0.291)	(0.289)	(0.287)	(0.285)	(0.285)
$\times$ Innov. intensity	0.798***	0.789***	0.792***	0.789***	0.787***	0.777***
	(0.278)	(0.278)	(0.280)	(0.277)	(0.279)	(0.268)
Observations	6,500	6,500	6,500	6,500	6,500	6,500
Fixed effects	j, p	j, p	j, p	j, p	j, p	j, p
Controls	base+	base+	base+	base+	base+	base+

Notes: All columns augment the baseline set of controls with additional variables. Specifically, column (1) includes the cumulative number of patents, column (2) includes firm payroll, column (3) includes the number of industries in which firms operate, column (4) includes the industry-level skill, capital intensities, column (5) includes the number of industries and the industry-level skill, capital intensities, and column (6) includes the number of industries, the industry-level skill, capital intensities, a dummy for firms with total imports > 0, and a dummy for firms with total exports > 0. The estimates for industry (j) and the period (p) fixed effects, and the coefficients associated with the binary indicators are suppressed due to disclosure restrictions, and the constant is dropped as well. Robust standard errors adjusted for clustering at the level of the firms' major industries are displayed below each coefficient. Observations are unweighted. Observation counts are rounded due to Census Bureau disclosure avoidance procedures. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

I.9 Technological Barrier Effect

Due to market-protection motives, we expect slower diffusion of knowledge and hence longer learning times for new innovations. To test this prediction, we estimate the following CPC group–level regression using the same generalized difference-in-differences framework.

$$\begin{aligned}\Delta y_{jp} =& \beta_1 Post_p \times NTRGap_{jp0} \times InnovIntens_{jp0} + \beta_2 Post_p \times NTRGap_{jp0} \\ &+ \beta_3 Post_p \times InnovIntens_{jp0} + \beta_4 NTRGap_{jp0} \times InnovIntens_{jp0} \\ &+ \beta_5 NTRGap_{jp0} + \beta_6 InnovIntens_{jp0} + \mathbf{X}_{jp0}\gamma + \delta_s + \delta_p + \alpha + \varepsilon_{jp}, \end{aligned} \tag{10}$$

where $p \in \{1992-1999,\ 2000-2007\}$ as before, y_{jp} is the average learning time for CPC group j, which is constructed as the forward citation-weighted mean of the minimum backward citation gaps based on citations from patents applied in CPC group j to prior patents in that CPC group, and Δy_{jp} is the DHS growth rate of y between the start-year and end-year for each period p. $NTRGap_{jp0}$ is the CPC-group–level NTR gap based on the crosswalk between six-digit NAICS and CPC groups, provided by Lybbert and Zolas (2014). $InnovIntens_{jp0}$ is the lagged five-year average of the ratio of patent applications to total employment for CPC group j, serving as a proxy for the CPC group's technological advantage. We include the numbers of patent filings and assignees in CPC group j as controls in $\mathbf{X}_{jp0}$. δ_s and δ_p is a fixed effect for the CPC section and year, respectively.

As before, we consider CPC groups that have at least one patent in the start-year and at least one patent in or before the end-year for each period and compute the DHS growth rates for the longest span in each period. We also require CPC groups to have at least one patent before the start-year of each period.

Table I.25 presents the results, where we find $\beta_1 > 0$. This indicates that learning time gets extended in technology groups with greater accumulated technological advantage. The finding is consistent with our model, in which firms' market-protection motives strategically strengthen own innovation, raising technological barriers for outsiders. The results remain robust when using average citation gaps and when measuring innovation intensity based on either real GDP or the value of shipments.

Furthermore, we see the impact on firm entry by running the following industry-level regression

Table I.25: The Effects of Market-Protection on Learning

	ΔCitation gaps	ΔCitation gaps
NTR gap $\times$ Post	-0.220	-0.229
	(0.450)	(0.448)
$\times$ Innovation intensity	0.037**	0.042**
	(0.016)	(0.017)
Observations	1,104	1,104
Fixed effects	s,p	s,p
Controls	no	baseline

Notes: Robust standard errors, adjusted for clustering at the level of the CPC technology group, are displayed below each coefficient. Observations are unweighted. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table I.26: The Effects of Market-Protection on Learning (Average Gaps)

	ΔCitation gaps	ΔCitation gaps
NTR gap $\times$ Post	-0.154	-0.167
	(0.439)	(0.434)
$\times$ Innovation intensity	0.029*	0.034**
	(0.015)	(0.016)
Observations	1,104	1,104
Fixed effects	s,p	s,p
Controls	no	baseline

Notes: The main dependent variable based on the forward citation-weighted mean of the average backward citation gaps based on citations from patents applied in CPC group to prior patents in the same CPC. Robust standard errors, adjusted for clustering at the level of the CPC technology group, are displayed below each coefficient. Observations are unweighted. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table I.27: The Effects of Market-Protection on Learning (Innovation Intensity)

	ΔCitation gaps	ΔCitation gaps	ΔCitation gaps	ΔCitation gaps
NTR gap $\times$ Post	-0.212	-0.215	-0.225	-0.226
	(0.456)	(0.455)	(0.452)	(0.450)
$\times$ Innovation intensity	3.094**	3.177**	6.442**	6.575**
	(1.336)	(1.302)	(2.701)	(2.621)
Observations	1,104	1,104	1,104	1,104
Fixed effects	s,p	s,p	s,p	s,p
Controls	no	baseline	no	baseline

Notes: We use different measures for the CPC-level innovation intensity. The first two columns are based on patent application divided by real GDP and the last two columns are based on patent application divided by real value of shipment at the CPC group level. Robust standard errors, adjusted for clustering at the level of the CPC technology group, are displayed below each coefficient. Observations are unweighted. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

for the four census years in the pre-shock period (1982-1999):

$$FirmEntry_{jt} = \beta TechBarrier_{jt} + \delta_j + \delta_t + \alpha + \varepsilon_{jt}. \tag{11}$$

Table I.28: Technological Barrier Effect

	Firm entry	Firm entry
Technological barriers	-0.012**	-0.016**
	(0.006)	(0.007)
Observation	1,300	1,300
Fixed effects	j, t	j, t
Tech. barrier thresholds	Top 5%	Top 10%

Notes: Column 1 uses the top 5th percentile, and Column 2 uses the top 10th percentile of technological barriers. Observations are unweighted. Observation counts are rounded due to Census Bureau disclosure avoidance procedures. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

$FirmEntry_{jt}$ is the firm entry rate, and $TechBarrier_{jt}$ is the technological barrier in industry j at year t. We measure the industry-level technological barrier using the skewness of the firm-level TFPR distribution (normalized by the industry frontier level).[39] Specifically, we use the top 5th or 10th percentile of this distribution to capture how far the technology level of top-performing firms is from that of the average firm within an industry. This measure reflects the intensity of innovation within the industry.

Table I.28 indicates that firm entry is lower in industries with higher technological barriers, consistent with the technological barrier effect in the model.

[39]This is the inverse of the TFPR gap in Aghion et al. (2005) (i.e., 1-TFPR gap).

References

Acemoglu, D., Akcigit, U., Alp, H., Bloom, N., Kerr, W., 2018. Innovation, reallocation, and growth. American Economic Review 108, 3450–91.

Aghion, P., Bloom, N., Blundell, R., Griffith, R., Howitt, P., 2005. Competition and innovation: An inverted-u relationship. The Quarterly Journal of Economics , 701–728.

Akcigit, U., Kerr, W.R., 2018. Growth through heterogeneous innovations. Journal of Political Economy 126, 1374–1443.

Argente, D., Lee, M., Moreira, S., 2024. The life cycle of products: Evidence and implications. Journal of Political Economy 132, 337–390.

Autor, D., Dorn, D., Hanson, G.H., Pisano, G., Shu, P., 2020. Foreign competition and domestic innovation: Evidence from us patents. American Economic Review: Insights 2, 357–374.

Baruffaldi, S.H., Simeth, M., 2020. Patents and knowledge diffusion: The effect of early disclosure. Research Policy 49, 103927.

Becker, R., Gray, W., Marvakov, J., 2013. Nber-ces manufacturing industry database: Technical notes. NBER Working Paper 5809.

Bernard, A.B., Jensen, J.B., Schott, P.K., 2009. Importers, exporters and multinationals: a portrait of firms in the us that trade goods, in: Producer dynamics: New evidence from micro data. University of Chicago Press, pp. 513–552.

Bernard, A.B., Redding, S.J., Schott, P.K., 2010. Multiple-product firms and product switching. American Economic Review 100, 70–97.

Bloom, N., Jones, C.I., Van Reenen, J., Webb, M., 2020. Are ideas getting harder to find? American Economic Review 110, 1104–1144.

Cavenaile, L., Celik, M.A., Tian, X., 2019. Are markups too high? competition, strategic innovation, and industry dynamics. Competition, Strategic Innovation, and Industry Dynamics (August 1, 2019) .

Cohen, W.M., 2010. Fifty years of empirical studies of innovative activity and performance, in: Handbook of the Economics of Innovation. Elsevier. volume 1, pp. 129–213.

De Ridder, M., 2024. Market power and innovation in the intangible economy. American Economic Review 114, 199–251.

Ding, Y., Jo, K., Kim, S., 2022. Improving Patent Assignee-Firm Bridge with Web Search Results. US Census Bureau, Center for Economic Studies.

Feenstra, R.C., Romalis, J., Schott, P.K., 2002. US imports, exports, and tariff data, 1989-2001. Technical Report. National Bureau of Economic Research.

Fort, T., Klimek, S., 2018. The Effects of Industry Classification Changes on US Employment Composition. Technical Report. US Census Bureau, Center for Economic Studies.

Garcia-Macia, D., Hsieh, C.T., Klenow, P.J., 2019. How destructive is innovation? Econometrica 87, 1507–1541.

Handley, K., Limão, N., 2017. Policy uncertainty, trade, and welfare: Theory and evidence for china and the united states. American Economic Review 107, 2731–83.

Hegde, D., Herkenhoff, K., Zhu, C., 2023. Patent publication and innovation. Journal of Political Economy 131, 1845–1903.

Jarmin, R., Miranda, J., 2002. The Longitudinal Business Database. Technical Report. US Census Bureau, Center for Economic Studies.

Kim, S., Jo, K., 2024. Product switching and young firm dynamics .

Klette, T.J., Kortum, S., 2004. Innovating firms and aggregate innovation. Journal of political economy 112, 986–1018.

Kogan, L., Papanikolaou, D., Seru, A., Stoffman, N., 2017. Technological innovation, resource allocation, and growth. The quarterly journal of economics 132, 665–712.

Lybbert, T.J., Zolas, N.J., 2014. Getting patents and economic data to speak to each other: An 'algorithmic links with probabilities' approach for joint analyses of patenting and economic activity. Research Policy 43, 530–542.

Phillips, G.M., Zhdanov, A., 2023. Venture capital investments, merger activity, and competition laws around the world. The Review of Corporate Finance Studies .

Pierce, J.R., Schott, P.K., 2009. Concording us harmonized system codes over time. Journal of Official Statistics .

Pierce, J.R., Schott, P.K., 2012. A concordance between ten-digit us harmonized system codes and sic/naics product classes and industries. Journal of Economic and Social Measurement 37, 61–96.

Pierce, J.R., Schott, P.K., 2016. The surprisingly swift decline of us manufacturing employment. American Economic Review 106, 1632–62.