Online Appendix for “Competition, Firm Innovation, and Growth under Imperfect Technology Spillovers”*

[NOT FOR PUBLICATION]

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A Baseline Model

A.1 Illustration of Firm Innovation Decisions

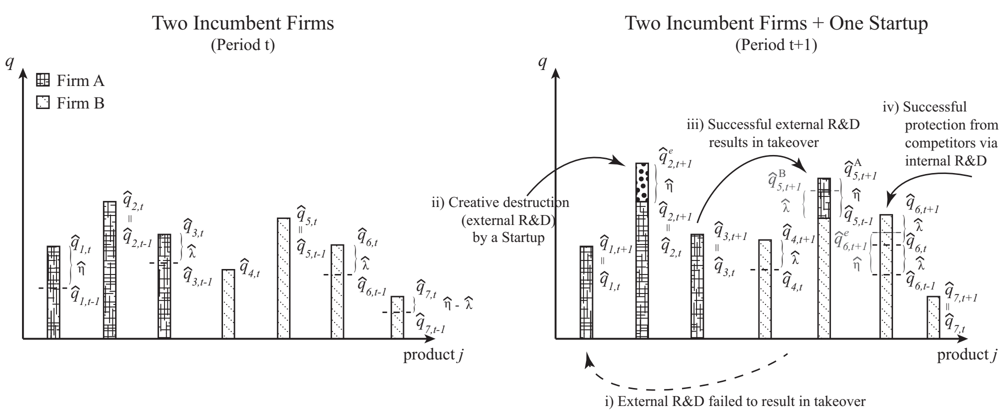

Figure 1: Firms’ Innovation and Product Quality Evolution Example

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Figure 1 illustrates the following set of examples of firm innovation decisions.[1] Suppose firm A owns products 1,2,3, and firm B owns products 4,5,6,7.

• i) Failed product takeover with coin-tossing (product 1): firm A without successful internal innovation (at t) gets $q^A_{1,t+1} = \eta q_{1,t-1}$, while firm B with successful external innovation (at t) obtains $q^B_{1,t+1} = \eta q_{1,t-1}$. A coin is tossed, and firm A keeps the product.

• ii) Successful product takeover w/o technology gap (product 2): A potential startup with successful external innovation (at t) can take over the market from firm A with no successful internal innovations (at both $t-1$ and t) as $q^e_{2,t+1} = \eta q_{2,t-1} > q^A_{2,t+1} = q_{2,t-1}$

• iii) Failed market protection w/o technological gap (product 5): firm A can take it over through successful external innovation, despite concurrently successful internal innovation by firm B as $\eta q_{5,t-1} > \lambda q_{5,t-1}$

• iv) Successful market protection with a technology gap (product 6): firm B obtains $q^B_{6,t+1} = \lambda^2 q_{6,t-1}$ with consecutively successful internal innovations from $t-1$. Rivals can only innovate up to $q^e_{6,t+1} = \eta q_{6,t-1}$, which makes firm B successfully protect the market.

A.2 Product Quality Evolution

Outsider Firms Let z^ℓ_j denote the internal innovation intensity for product line j and Δ^ℓ_j denote its technology gap. Since outside firms can only learn the lagged level of technology $q_{j,-1} = q_j/\Delta^\ell_j$, the evolution of product quality in $t+1$ occurs probabilistically as follows: for $\Delta_j = \Delta^1$, q'_j is equal to $\lambda q_{j,-1}$ with prob. $(1-\overline{x})z^1_j$, $q_{j,-1}$ with prob. $(1-\overline{x})(1-z^1_j)$, and $\eta q_{j,-1}$ with prob. $\overline{x}$; for Δ^2, q'_j is equal to $\lambda^2 q_{j,-1}$ with prob. z^2_j, $\lambda q_{j,-1}$ with prob. $(1-\overline{x})(1-z^2_j)$, and $\eta q_{j,-1}$ with prob. $\overline{x}(1-z^2_j)$; for Δ^3, q'_j is equal to $\lambda\eta q_{j,-1}$ with prob. z^3_j, $\eta q_{j,-1}$ with prob. $(1-\overline{x})(1-z^3_j) + \frac{1}{2}\overline{x}(1-z^3_j)$, and $\eta q_{j,-1}$ with prob. $\frac{1}{2}\overline{x}(1-z^3_j)$; and for Δ^4, q'_j is equal to $\lambda\frac{\eta}{\lambda}q_{j,-1}$ with prob. $(1-\overline{x})z^4_j + \frac{1}{2}\overline{x}z^4_j$, $\frac{\eta}{\lambda}q_{j,-1}$ with prob. $(1-\overline{x})(1-z^4_j)$, and $\eta q_{j,-1}$ with prob. $\overline{x}(1-z^4_j) + \frac{1}{2}\overline{x}z^4_j$.

A.3 Value Function and Optimal Innovation Decisions

The conditional expectation in the value function considers the success/failure of internal and external innovation, the arrival of the creative destruction shock, outcomes of

[1] The bar indicates log product quality $\widehat{q}_{j,t} \equiv \log(q_{j,t})$ with $\widehat{\eta} \equiv \log(\eta)$.

coin-tosses (c-t), the distribution of current period product quality q, and the distribution of the current period technology gap Δ^{ℓ}. Thus, $\mathbb{E}\big[V\big(\Phi^{f\prime}\big|\Phi^{f}\big)\big|\{z_j\}_{j\in\mathcal{J}^f},x\big] = \sum_{I_1^{\overline{x}},\ldots,I_{n_f}^{\overline{x}}=0}^{1} \sum_{I_1^{z},\ldots,I_{n_f}^{z}=0}^{1} \sum_{\text{c-t}_1,\ldots,\text{c-t}_{n_f}\ =\ \text{win}}^{\text{lose}} \sum_{I^x=0}^{1} \big[\prod_{i=1}^{n_f} \overline{x}^{I_i^{\overline{x}}}(1-\overline{x})^{1-I_i^{\overline{x}}} z_i^{I_i^z}(1-z_i)^{1-I_i^z}\big]\times\big[x^{I^x}(1-x)^{1-I^x}\big]\big(\frac{1}{2}\big)^{n_f}$ $\mathbb{E}_{q,\Delta}V\big(\big[\bigcup_{i=1}^{n_f}\big[\big\{(\Delta'_{j_i}q_{j_i},\Delta'_{j_i})\big|(q_{j_i},\Delta_{j_i}),I_i^{\overline{x}},I_i^z,\text{c-t}_i\big\}\setminus\{\mathbf{0}\}\big]\big]\bigcup\big[\big\{\big(\frac{\eta}{\Delta_{-j}}I^x q_{-j},\frac{\eta}{\Delta_{-j}}I^x\big)\big\}\setminus\{\mathbf{0}\}\big]\big)$.

Note that the first term in the value function (before $\bigcup$) is the subsets of possible realizations for $\Phi^{f\prime}$ from internal innovation, creative destruction, and coin-toss. The second term in the value function (after $\bigcup$) shows the subsets of possible realizations for $\Phi^{f\prime}$ from external innovation, where $\{q'_{j_i}\}=\{\Delta'_{j_i}q_{j_i}\}\setminus\{0\}$, and $\{q'_{-j}\}=\{\frac{\eta}{\Delta_{-j}}I^x q_{-j}\}\setminus\{0\}$. If $\Delta'_{j_i}=0$, then firm f loses product line j_i and $\{(q'_{j_i},\Delta'_{j_i})\}\setminus\{\mathbf{0}\}=\{\mathbf{0}\}\setminus\{\mathbf{0}\}=\emptyset$.

A.4 Technology Gap Portfolio Composition Distribution Transition

The range of $\widetilde{k}^1$ can be determined as follows: i) for $0\le\widetilde{k}\le\min\{n_f-k,k\}$, the two combinations preceding the term in brackets are well defined for any $\widetilde{k}^1\in[0,\widetilde{k}]\cap\mathbb{Z}$ and describe all possible cases; ii) if $n_f-k\ge k$, then $\tilde{k}>k$, $0\le\widetilde{k}-\widetilde{k}^1$, and $0\le\widetilde{k}^1\le n_f-k$ is satisfied. This gives $\widetilde{k}-k\le\widetilde{k}^1\le\widetilde{k}$; and iii) if $k\ge n_f-k$, then $\widetilde{k}>n_f-k$, $0\le\widetilde{k}-\widetilde{k}^1$, and $0\le\widetilde{k}^1\le n_f-k$ is satisfied. Thus, $\max\{0,\widetilde{k}-k\}\le\widetilde{k}^1\le n_f-k$.

By using $\widetilde{\mathbb{P}}(n_f,\widetilde{k}|n_f,k)$, the probability of $\mathcal{N}=\widetilde{\mathcal{N}}(n_f,k)$ transitioning to $\mathcal{N}'=\widetilde{\mathcal{N}}(n_f-h,\widetilde{k})$ for any $h\ge0$ without considering external innovation can be defined as follows: Take out h^1 numbers of product lines with $\Delta=\Delta^1$, and $h-h^1$ numbers of product lines with $\Delta=\Delta^2$ from $\widetilde{\mathcal{N}}(n_f,k)$, then compute the probability of $\widetilde{\mathcal{N}}(n_f-h,k-(h-h^1))$ transitioning to $\widetilde{\mathcal{N}}(n_f-h,\widetilde{k})$ with $\widetilde{\mathbb{P}}\big(n_f-h,\widetilde{k}|n_f-h,k-(h-h^1)\big)$ for all feasible h^1. Then, for $0\le h<n_f$, $n_f\ge1$, $0\le\widetilde{k}\le n_f-h$, and $0\le k\le n_f$, $\widetilde{\mathbb{P}}\big(n_f-h,\widetilde{k}|n_f,k\big)=\sum_{h^1=\max\{0,h-k\}}^{\min\{h,n_f-k\}}\Big[\begin{pmatrix} n_f-k \\ h^1\end{pmatrix}\begin{pmatrix} k \\ h-h^1\end{pmatrix}\overline{x}^h(1-z^2)^{h-h^1}\widetilde{\mathbb{P}}\big(n_f-h,\widetilde{k}|n_f-h,k-(h-h^1)\big)\Big]$; for $h=n_f\ge1$, $\widetilde{k}=0$, and $0\le k\le n_f$, $\widetilde{\mathbb{P}}\big(n_f-h,\widetilde{k}|n_f,k\big)=\overline{x}^{n_f}(1-z^2)^k$; and 0 otherwise. The range for h^1 is defined from above, ensuring $0\le h-h^1\le k$ and $0\le h^1\le n_f-k$ for any h^1.

With $\widetilde{\mathbb{P}}\big(n_f-h,\widetilde{k}|n_f,k\big)$, other possible cases can be described for each case. For example, the probability of $\mathcal{N}=(n_f,n_f-k,k,0,0)$ to $\mathcal{N}'=(n_f-h,n_f-h-\widetilde{k},\widetilde{k},0,0)$ for $h\ge-1$ is defined as $\mathbb{P}\big(n_f-h,n_f-h-\widetilde{k},\widetilde{k},0,0\,\big|n_f,n_f-k,k,0,0\big)=\widetilde{\mathbb{P}}\big(n_f-h,\widetilde{k}\big|n_f,k\big)(1-x\overline{x}_{\text{takeover}})+\widetilde{\mathbb{P}}\big(n_f-h-1,\widetilde{k}\big|n_f,k\big)\mu(\Delta^3)\frac{1}{2}x(1-z^3)+\widetilde{\mathbb{P}}\big(n_f-h-1,\widetilde{k}-1\big|n_f,k\big)\mu(\Delta^4)x\left(1-\frac{1}{2}z^4\right)$. The first term is the probability of $\mathcal{N}$ transitioning to $\mathcal{N}'$ directly via the change in the firm's existing technology gap portfolio composition with unsuccessful external innovation. The second term is the probability of $\mathcal{N}$ to $\widetilde{\mathcal{N}}(n_f - h - 1, \widetilde{k})$, where successful external innovation adds one product line with $\Delta' = \Delta^1$. Since the next period technology gap of product line j from successful external innovation is $\Delta_j' = \frac{q_j'}{q_j} = \frac{\eta q_{j,-1}}{\Delta_j q_{j,-1}} = \frac{\eta}{\Delta_j}$, firm needs to take over a product line with a technology gap of $\Delta = \Delta^3 = 1 + \eta$ to have a product line with a technology gap of Δ^1 in the next period. The third term is the probability of $\mathcal{N}$ to $\widetilde{\mathcal{N}}(n_f - h - 1, \widetilde{k} - 1)$, where successful external innovation adds one product line with $\Delta' = \Delta^2$ by taking over a product line with a technology gap of $\Delta = \Delta^4$. For $h = -1$, the first term becomes zero by the definition of $\widetilde{\mathbb{P}}(\cdot|\cdot)$. Thus this probability is well defined for any $h \geq -1$.

With the computed probabilities of transitions between technology gap portfolio compositions, we can now define the inflows and outflows of a given technology gap portfolio. Let $\mathcal{F}$ denote the total mass of firms in the economy and $\mu(\mathcal{N})$ represent the share of firms with technology gap portfolio $\mathcal{N}$. Thus, $\widetilde{\mu}(\mathcal{N}) = \mathcal{F}\mu(\mathcal{N})$. Then, for example, inflows and outflows for $\mathcal{N} = (n_f, n_f - k, k, 0, 0)$ can be described as follows: for $\mathcal{N} = (n_f, n_f - k, k, 0, 0)$ with $n_f \geq 2$, any firm whose next period technology gap portfolio is not $\mathcal{N}$ is counted as outflows, followed by $\mathit{outflow}(n_f, n_f - k, k, 0, 0) = [1 - \mathbb{P}(n_f, n_f - k, k, 0, 0 \big| n_f, n_f - k, k, 0, 0)] \times \mathcal{F}\mu(n_f, n_f - k, k, 0, 0)$. Any firm with a total number of product lines $n \geq n_f - 1$ can have a technology gap portfolio composition equal to $\mathcal{N}$ through the combinations of internal and external innovations. Thus, for the maximum number of product lines $\bar{n}_f$, $\mathit{inflow}(n_f, n_f - k, k, 0, 0) = \mathcal{F} \sum_{n=n_f-1}^{\overline{n}_f} \sum_{\widetilde{k}=0}^{n} \big[\mu(n, n - \widetilde{k}, \widetilde{k}, 0, 0)\mathbb{P}(n_f, n_f - k, k, 0, 0 \big| n, n - \widetilde{k}, \widetilde{k}, 0, 0) + \mu(n, n - 1 - \widetilde{k}I_{\{n>1\}}, \widetilde{k}I_{\{n>1\}}, 1, 0)\mathbb{P}(n_f, n_f - k, k, 0, 0 \big| n, n - 1 - \widetilde{k}I_{\{n>1\}}, \widetilde{k}I_{\{n>1\}}, 1, 0) + \mu(n, n - 1 - \widetilde{k}I_{\{n>1\}}, \widetilde{k}I_{\{n>1\}}, 0, 1)\mathbb{P}(n_f, n_f - k, k, 0, 0 \big| n, n - 1 - \widetilde{k}I_{\{n>1\}}, \widetilde{k}I_{\{n>1\}}, 0, 1)\big] - \mathcal{F}\mu(n_f, n_f - k, k, 0, 0)$ $\mathbb{P}(n_f, n_f - k, k, 0, 0 \big| n_f, n_f - k, k, 0, 0)$.[2]

B Simple Three-Period Heterogeneous Innovation Model

To analyze firms' innovation incentives and derive testable predictions, we examine a three-period economy with two product markets and three firms. In period 0, the economy starts

[2]Descriptions for other cases are available upon request.

with two product markets, market 1 and 2, with initial market-specific technologies $q_{1,0}$, and $q_{2,0}$, respectively. There are two firms in play, firm A and B. Firm A starts with product market 1 and an initial internal innovation probability $z_{1,0}$. Firm B, on the other hand, starts only with an initial external innovation probability $x_{2,0}$, and can operate and produce in period 1 (but not in period 0). If external innovation fails, firm B still keeps market 2 but produces with initial quality $q_{2,0}$. Thus, at the beginning of period 1, product qualities in the two markets are $q_{1,1} = \lambda q_{1,0}$ with probability $z_{1,0}$, $q_{1,1} = q_{1,0}$ with probability $1 - z_{1,0}$, $q_{2,1} = \eta q_{2,0}$ with probability $x_{2,0}$, and $q_{2,0}$ with probability $1 - x_{2,0}$, where $\lambda^2 > \eta > \lambda > 1$ represent innovation step sizes.

In period 1, the focal period, an outside firm engages in external innovation to potentially take over the two product markets in period 2. The success of the outside firm in external innovation is determined by the probability x_1^e for each product market. Additionally, there is a news shock in period 1 concerning the profit for period 2, possibly including an increase in foreign demand. Subsequently, the two incumbent firms utilize their given technologies to produce and invest in internal and external innovations. At the beginning of period 2, all innovation outcomes are realized, and then technological competition in each product market takes place. Only the firm with the highest technology in each product market continues producing. The economy ends after period 2.

In period 1, incumbent firm $i \in \{A, B\}$ invests $R_{j,1}^{\text{in}}$ in internal innovation for $j \in \{1, 2\}$, achieving a success probability of $z_{j,1}$. The R&D production function is $z_{j,1} = \left(R_{j,1}^{\text{in}}/\widehat{\chi} q_{j,1}\right)^{0.5}$. Successful internal innovation increases next-period product quality by $\lambda > 1$. Thus, the period 2 product quality for firm i becomes $q_{j,2}^i = \lambda q_{j,1}$ with prob. $z_{j,1}$, and $q_{j,2}^i = q_{j,1}$ with prob. $1 - z_{j,1}$. Similarly, firm i invests $R_{-j,1}^{\text{ex}}$ to learn the period 0 technology used by firm $-i \neq i$ and improve it, which determines the success probability of external innovation $x_{-j,1}$. The R&D production function is $x_{-j,1} = \left(R_{-j,1}^{\text{ex}}/\widetilde{\chi} q_{-j,0}\right)^{0.5}$, where $-j$ is owned by $-i$. Successful external innovation enhances product quality relative to the lagged-period quality by $\eta > 1$. Thus, product $-j$'s quality in period 2 for firm i is $q_{-j,2}^i = \eta q_{-j,0}$ with prob. $x_{-j,1}$, and $q_{-j,2}^i = \emptyset$ with prob. $1 - x_{-j,1}$, where the symbol $\emptyset$ means firm i failed to acquire the production technology for product $-j$.

Optimal Innovation Decisions and Theoretical Predictions Assume that in a given product market j and period t, firms receive an instantaneous profit of $\pi_{j,t}q_{j,t}$ where $q_{j,t}$ is the product quality and $\pi_{j,t}$ is a market-period-specific constant known to firms before each period begins. Because there are only two products, incumbents and the outside firm can perform external innovation on the same product. For simplicity in the model, we further assume that the outside firm can engage in external innovation only if an incumbent fails to do so, following Garcia-Macia et al. (2019). Then the profit maximization problem for firm i in product market j with quality $q_{j,1}$ in period 1 can be written as $V(q_{j,1}) = \max_{\{z_{j,1},x_{-j,1}\}} \{\pi_{j,1}q_{j,1} - \widehat{\chi}(z_{j,1})^2 q_{j,1} - \widetilde{\chi}(x_{-j,1})^2 q_{-j,0} + (1-x_{j,1})(1-x_1^e)\big[(1-z_{j,1})\pi_{j,2}q_{j,1} + z_{j,1}\pi_{j,2}\lambda q_{j,1}\big] + (x_{j,1} + (1-x_{j,1})x_1^e)\big[z_{j,1}\pi_{j,2}\lambda q_{j,1}\mathcal{I}_{\{\lambda q_{j,1} > \eta q_{j,0}\}} + \frac{1}{2}(1-z_{j,1})\pi_{j,2}q_{j,1}\mathcal{I}_{\{q_{j,1}=\eta q_{j,0}\}}\big] + x_{-j,1}\big[(1-z_{-j,1})\pi_{-j,2}\eta q_{-j,0}\mathcal{I}_{\{\eta q_{-j,0} > q_{-j,1}\}} + z_{-j,1}\pi_{-j,2}\eta q_{-j,0}\mathcal{I}_{\{\eta q_{-j,0} > \lambda q_{-j,1}\}} + \frac{1}{2}(1-z_{-j,1})\pi_{-j,2}\eta\, q_{-j,0}\mathcal{I}_{\{\eta q_{-j,0}=q_{-j,1}\}} + \frac{1}{2}z_{-j,1}\pi_{-j,2}\eta q_{-j,0}\mathcal{I}_{\{\eta q_{-j,0}=\lambda q_{-j,1}\}}\big]\}$, where $\mathcal{I}_{\{\cdot\}}$ is an indicator function that captures the possible relationships between the technologies of the three firms in period 2 within a given market. The first three terms show the period 1 profit net of total R&D cost.

The first bracket represents the incumbent's expected profit from market j if neither the incumbent nor the outside firm externally innovates the technology in market j. The second bracket represents the expected profit from market j if either the other incumbent or the outside firm succeeds in externally innovating the technology in market j. The third bracket represents the expected profit from market $-j$ if firm i succeeds in externally innovating the market $-j$ technology. The terms following $\frac{1}{2}$ account for scenarios where two firms could potentially produce the same quality product, triggering a coin-toss tiebreaker rule.

The interior solutions to this problem are: for $q_{j,1} = q_{j,0}$, $z^*_{j,1} = \frac{\pi_{j,2}}{2\widehat{\chi}}(\lambda-1)(1-x^*_{j,1})(1-x^e_1)$; for $q_{j,1} = \lambda q_{j,0}$, $z^*_{j,1} = \frac{\pi_{j,2}}{2\widehat{\chi}}\big[\lambda - (1-x^*_{j,1})(1-x^e_1)\big]$; for $q_{j,1} = \eta q_{j,0}$, $z^*_{j,1} = \frac{\pi_{j,2}}{2\widehat{\chi}}\big[\lambda - \frac{1}{2} - \frac{1}{2}(1-x^*_{j,1})(1-x^e_1)\big]$; for $q_{-j,1} = q_{-j,0}$, $x^*_{-j,1} = \frac{\eta\pi_{-j,2}}{2\widetilde{\chi}}$; for $q_{-j,1} = \lambda q_{-j,0}$, $x^*_{-j,1} = \frac{\eta\pi_{-j,2}}{2\widetilde{\chi}}(1-z^*_{-j,1})$; and for $q_{-j,1} = \eta q_{-j,0}$, $x^*_{-j,1} = \frac{\eta\pi_{-j,2}}{2\widetilde{\chi}}\frac{1}{2}(1-z^*_{-j,1})$, which maximize the firm profit considering the technology gap of its own and others, as well as the potential outcomes of internal and external innovation by all firms involved.

Proposition B.1. *For each $q_{j,1}$ and for $\lambda^2 > \eta > \lambda > 1$, we can order internal innovation intensities as $z^*_{j,1}\big|_{q_{j,1}=\lambda q_{j,0}} > z^*_{1,1}\big|_{q_{j,1}=\eta q_{j,0}} > z^*_{j,1}\big|_{q_{j,1}=q_{j,0}}$. Furthermore, $\frac{\partial z^*_{j,1}}{\partial x^e_1}\big|_{q_{j,1}=\lambda q_{j,0}} > \frac{\partial z^*_{j,1}}{\partial x^e_1}\big|_{q_{j,1}=\eta q_{j,0}} > 0 > \frac{\partial z^*_{j,1}}{\partial x^e_1}\big|_{q_{j,1}=q_{j,0}}$.*

Proof. The first part is straightforward with simple algebra. The second part is proved as follows. For $q_{j,1} = q_{j,0}$, we have $\frac{\partial z_{j,1}}{\partial x_1^e} = -\frac{\pi_{j,2}}{2\widehat{\chi}}(\lambda - 1)\big[(1 - x_{j,1}) + (1 - x_1^e)\frac{\partial x_{j,1}}{\partial x_1^e}\big]$, and $\frac{\partial x_{j,1}}{\partial x_1^e} = 0$. Thus, the following is obtained: $\frac{\partial z_{j,1}}{\partial x_1^e} = -\frac{\pi_{j,2}}{2\widehat{\chi}}(\lambda-1)(1-x_{j,1}) < 0$. For $q_{j,1} = \lambda q_{j,0}$, we have $\frac{\partial z_{j,1}}{\partial x_1^e} = \frac{\pi_{j,2}}{2\widehat{\chi}}\big[1 - x_{j,1} + (1 - x_1^e)\frac{\partial x_{j,1}}{\partial x_1^e}\big]$ and $\frac{\partial x_{j,1}}{\partial x_1^e} = -\frac{\eta\pi_{j,2}}{2\widetilde{\chi}}\frac{\partial z_{j,1}}{\partial x_1^e}$. Thus, the following holds: $\frac{\partial z_{j,1}}{\partial x_1^e} = (1 - x_{j,1})\big[\frac{2\widehat{\chi}}{\pi_{j,2}} + \frac{\eta\pi_{j,2}}{2\widetilde{\chi}}(1 - x_1^e)\big]^{-1} > 0$, hence $\frac{\partial x_{j,1}}{\partial x_1^e} = -\frac{\eta\pi_{j,2}}{2\widetilde{\chi}}\frac{\partial z_{j,1}}{\partial x_1^e} < 0$. For $q_{j,1} = \eta q_{j,0}$, we have $\frac{\partial z_{j,1}}{\partial x_1^e} = \frac{\pi_{j,2}}{2\widehat{\chi}}\frac{1}{2}\big[1 - x_{j,1} + (1 - x_1^e)\frac{\partial x_{j,1}}{\partial x_1^e}\big]$, and $\frac{\partial x_{j,1}}{\partial x_1^e} = -\frac{\eta\pi_{j,2}}{2\widetilde{\chi}}\frac{1}{2}\frac{\partial z_{j,1}}{\partial x_1^e}$. This gives $\frac{\partial z_{j,1}}{\partial x_1^e} = (1-x_{j,1})\big[\frac{4\widehat{\chi}}{\pi_{j,2}} + \frac{\eta\pi_{j,2}}{4\widetilde{\chi}}(1-x_1^e)\big]^{-1} > 0$, hence $\frac{\partial x_{j,1}}{\partial x_1^e} = -\frac{1}{2}\frac{\eta\pi_{j,2}}{2\widetilde{\chi}}\frac{\partial z_{j,1}}{\partial x_1^e} < 0$ holds. With $x_{j,1}^*$ and $\frac{\eta\pi_{j,2}}{2\widetilde{\chi}} \in (0,1)$, along with the restriction $4\widehat{\chi} > \pi_{j,2}$, the following holds: $\frac{4\widehat{\chi}}{\pi_{j,2}} + \frac{\eta\pi_{j,2}}{4\widetilde{\chi}}(1 - x_1^e) > \frac{2\widehat{\chi}}{\pi_{j,2}} + \frac{\eta\pi_{j,2}}{2\widetilde{\chi}}(1 - x_1^e)$. Therefore, we get $\frac{\partial z_{j,1}^*}{\partial x_1^e}\big|_{q_{j,1}=\lambda q_{j,0}} > \frac{\partial z_{j,1}^*}{\partial x_1^e}\big|_{q_{j,1}=\eta q_{j,0}}$ □

The second part of proposition B.1 suggests that firms without a technology gap decrease their internal innovation when facing a higher probability of creative destruction in their own markets. This is because they cannot enhance their product protection through internal innovation. Conversely, firms with a significant technological advantage do not substantially increase their internal innovation in response to external innovation from outsiders, as the risk of losing their own product market is minimal. In intermediate cases, firms intensify their internal innovation response to external innovation from outsiders to reduce the probability of losing their market.

Higher innovation in period 0 increases the probability of achieving a high technology gap in period 1, thereby aiding firms in market protection. To understand how past innovation intensity influences the firm's current decision on internal innovation when facing a higher probability of encountering a competitor, x_1^e, we define the expected value of internal innovation intensity in period 1 as $\bar{z}_1^* = z_{1,1}^*\big|_{q_{1,1}=q_{1,0}}\frac{1}{2}(1 - z_{1,0}) + z_{2,1}^*\big|_{q_{2,1}=q_{2,0}}\frac{1}{2}(1 - x_{2,0}) + z_{1,1}^*\big|_{q_{1,1}=\lambda q_{1,0}}\frac{1}{2}z_{1,0} + z_{2,1}^*\big|_{q_{2,1}=\eta q_{2,0}}\frac{1}{2}x_{2,0}$, where $\frac{1}{2}$ accounts for the two products. Proposition B.1 provides the following result:

Corollary B.1 (Market-Protection Effect)**.** *The impact of period 0 innovation intensities,* $z_{1,0}$ *and* $x_{2,0}$*, on expected internal innovation in period 1 can be characterized as follows:* $\frac{\partial \bar{z}_1^*}{\partial x_1^e \partial z_{1,0}} > 0$*, and* $\frac{\partial \bar{z}_1^*}{\partial x_1^e \partial x_{2,0}} > 0$.

Proof. From $\bar{z}_1^*$, we know that $\frac{\partial \bar{z}_1^*}{\partial z_{1,0}} = \frac{1}{2}\big(z_{1,1}^*\big|_{q_{1,1}=\lambda q_{1,0}} - z_{1,1}^*\big|_{q_{1,1}=q_{1,0}}\big) > 0$ and $\frac{\partial \bar{z}_1^*}{\partial x_{2,0}} = \frac{1}{2}\big(z_{2,1}^*\big|_{q_{2,1}=\eta q_{2,0}} - z_{2,1}^*\big|_{q_{2,1}=q_{2,0}}\big) > 0$, where the signs can be derived from proposition B.1. The results follow from proposition B.1. □

Corollary B.1 suggests that intensive innovation in the previous period prompts firms to increase internal innovation in response to higher competitive pressure. As indicated by the optimal decision rule, firms' decisions regarding external innovation also depend on the past innovation decisions of other firms, which is outlined in the following proposition.

Proposition B.2. *For each $q_{j,1}$ and for $\lambda^2 > \eta > \lambda > 1$, we can order external innovation intensities as follows: $x^*_{j,1}\big|_{q_{j,1}=q_{j,0}} > x^*_{j,1}\big|_{q_{j,1}=\lambda q_{j,0}} > x^*_{j,1}\big|_{q_{j,1}=\eta q_{j,0}}$. Furthermore, $\frac{\partial x^*_{j,1}}{\partial x^e_1}\big|_{q_{j,1}=q_{j,0}} = 0$, $\frac{\partial x^*_{j,1}}{\partial x^e_1}\big|_{q_{j,1}=\lambda q_{j,0}} < 0$, and $\frac{\partial x^*_{j,1}}{\partial x^e_1}\big|_{q_{j,1}=\eta q_{j,0}} < 0$. Proof: See the proof for Proposition B.1*

Proposition B.2 implies that firms decrease external innovation if incumbents hold a higher technology advantage, as it becomes more difficult to displace them in the market through external innovation. In markets where there is a technological barrier (technology gap > 1), firms also reduce their external innovation in response to increased external innovation by outside firms. This is because incumbents in these markets respond defensively by increasing internal innovation (proposition B.1). To understand how the past innovation intensity of other firms influences a firm's current decision on external innovation, define the expected value of external innovation intensity in period 1 as $\overline{x}^*_1 = x^*_{1,1}\big|_{q_{1,1}=q_{1,0}}\frac{1}{2}(1-z_{1,0}) + x^*_{2,1}\big|_{q_{2,1}=q_{2,0}}\frac{1}{2}(1-x_{2,0}) + x^*_{1,1}\big|_{q_{1,1}=\lambda q_{1,0}}\frac{1}{2}z_{1,0} + x^*_{2,1}\big|_{q_{2,1}=\eta q_{2,0}}\frac{1}{2}x_{2,0}$. Then, the first part of proposition B.2 implies the following:

Corollary B.2 (Technological Barrier Effect)**.** *Given technology $q_{j,1}$ and period 0 innovation intensities $z_{1,0}$ and $x_{2,0}$, $\frac{\partial \overline{x}^*_1}{\partial z_{1,0}} < 0$ and $\frac{\partial \overline{x}^*_1}{\partial x_{2,0}} < 0$ hold.*

Proof. $\frac{\partial \overline{x}^*_1}{\partial z_{1,0}} = \frac{1}{2}\big(x^*_{1,1}\big|_{q_{1,1}=\lambda q_{1,0}} - x^*_{1,1}\big|_{q_{1,1}=q_{1,0}}\big) < 0$, and $\frac{\partial \overline{x}^*_1}{\partial x_{2,0}} = \frac{1}{2}\big(x^*_{2,1}\big|_{q_{2,1}=\eta q_{2,0}} - x^*_{2,1}\big|_{q_{2,1}=q_{2,0}}\big) < 0$, where the signs follow from proposition B.2 □

Corollary B.2 indicates that higher technology levels in other markets, resulting from previous innovation, act as effective technological barriers, making it challenging for outside firms to take over those product markets. This reduces firms' incentives for external innovation. Lastly, because innovation is forward-looking, changes in future profits π' are crucial factors influencing the current period's innovation. Proposition B.3 summarizes this:

Proposition B.3 (Ex-post Schumpeterian Effect)**.** *Given the expected profit $\pi_{j,2}$ in period 2, we obtain $\frac{\partial z^*_{j,1}}{\partial \pi_{j,2}} > 0 \ \forall q_{j,1}$ and $\frac{\partial x^*_{j,1}}{\partial \pi_{j,2}} > 0$ for $q_{j,1} = q_{j,0}$. The signs for $\frac{\partial x^*_{j,1}}{\partial \pi_{j,2}}$ for other technology gaps remain ambiguous.*

Proof. For $q_{j,1} = q_{j,0}$, we have $\frac{\partial z_{j,1}}{\partial \pi_{j,2}} = \frac{1}{2\widehat{\chi}}(\lambda-1)(1-x_{j,1})(1-x^e_1) - \frac{\pi_{j,2}}{2\widehat{\chi}}(\lambda-1)(1-x^e_1)\frac{\partial x_{j,1}}{\partial \pi_{j,2}}$ and $\frac{\partial x_{j,1}}{\partial \pi_{j,2}} = \frac{\eta}{2\widetilde{\chi}}$. Thus, $\frac{\partial z_{j,1}}{\partial \pi_{j,2}} = \frac{1}{2\widehat{\chi}}(\lambda-1)(1-2x_{j,1})(1-x^e_1) > 0$ iff $x_{j,1} < \frac{1}{2}$. For $q_{j,1} = \lambda q_{j,0}$, we get $\frac{\partial z_{j,1}}{\partial \pi_{j,2}} > 0$ unambiguously. For $q_{j,1} = \eta q_{j,0}$, $\frac{\partial z_{j,1}}{\partial \pi_{j,2}} = \frac{1}{2\widehat{\chi}}\left[\lambda - \frac{1}{2} - \frac{1}{2}(1-x_{j,1})(1-x^e_1)\right] + \frac{\pi_{j,2}}{2\widehat{\chi}}\frac{1}{2}(1-x^e_1)\frac{\partial x_{j,1}}{\partial \pi_{j,2}}$ and $\frac{\partial x_{j,1}}{\partial \pi_{j,2}} = \frac{\eta}{2\widetilde{\chi}}\frac{1}{2}(1-z_{j,1}) - \frac{\eta\pi_{j,2}}{2\widetilde{\chi}}\frac{1}{2}\frac{\partial z_{j,1}}{\partial \pi_{j,2}}$ are obtained, and we get $\frac{\partial z_{j,1}}{\partial \pi_{j,2}} = \left[\lambda - \frac{1}{2} - \frac{1}{2}(1-2x_{j,1})(1-x^e_1)\right]\left[2\widehat{\chi} + \frac{\eta(\pi_{j,2})^2}{2\widetilde{\chi}}\frac{1}{4}(1-x^e_1)\right]^{-1} > 0$. The sign for $\frac{\partial x_{j,1}}{\partial \pi_{j,2}}$ remains ambiguous. □

Proposition B.3 implies that any factor that affects future profits may influence firms' internal and external innovation. Specifically, an increase in expected profit from one's own market encourages firms to intensify their internal innovation efforts. However, the impact of an increase in expected profit in other markets on firms' decisions regarding external innovation is ambiguous when the local technology gap exceeds 1. This ambiguity arises because incumbents in these markets tend to increase their internal innovation efforts in response to higher expected profits, thereby allowing them to protect their markets. In cases where the local technology gap equals 1, incumbents cannot protect their markets through internal innovation alone. Consequently, an increase in expected future profit unambiguously stimulates external innovation in such scenarios. These findings highlight the diverse factors influencing internal, external, and overall innovation levels.

C Extension: Multi-External Innovation

By allowing firms to do multiple external innovations, all remain the same except for firm innovation decisions and aggregate variables.

C.1 Optimal Innovation Decision

Following Klette and Kortum (2004) and several follow-on studies, we model firms' external innovation decisions based on the number of products they produce (n_f). External innovation can be viewed as a spin-off derived from each firm's existing products. Consider product j firm f owns with quality q_j and technology gap Δ_j^ℓ. In the subsequent period, the evolution of this product can result in six cases: firm f i) loses product j and business takeover (through external innovation) fails, ii) loses product j and takeover succeeds, iii) keeps product j while both internal innovation and takeover fail, iv) keeps product j while internal innovation fails, but takeover succeeds, v) keeps product j with successful internal innovation, but takeover fails, and vi) keeps product j with successful internal innovation and takeover. Denoting the product-technology gap pair for a product that firm f acquires through successful business takeover in the next period as $\{(q', \Delta')\}$, we can write down the evolution of the product portfolio stemming from $\Phi^f = \{(q_j, \Delta_j^\ell)\}$ for $\ell \in \{1,2,3,4\}$ for each of the six cases. For example, for $\Delta_j^\ell = \Delta^2$, $\Phi_j^{f\prime} = \emptyset \cup \emptyset$ with prob. $\overline{x}(1-z^2)(1-x\overline{x}_{\text{takeover}})$, $\Phi_j^{f\prime} = \emptyset \cup \{(q', \Delta')\}$ with prob. $\overline{x}(1-z^2)(x\overline{x}_{\text{takeover}})$, $\Phi_j^{f\prime} = \{(q_j, \Delta^1)\} \cup \emptyset$ with prob. $(1-\overline{x})(1-z^2)(1-x\overline{x}_{\text{takeover}})$, $\Phi_j^{f\prime} = \{(q_j, \Delta^1)\} \cup \{(q', \Delta')\}$ with prob. $(1-\overline{x})(1-z^2)(x\overline{x}_{\text{takeover}})$, $\Phi_j^{f\prime} = \{(\Delta^2 q_j, \Delta^2)\} \cup \emptyset$ with prob. $z^2(1-x\overline{x}_{\text{takeover}})$, and $\Phi_j^{f\prime} = \{(\Delta^2 q_j, \Delta^2)\} \cup \{(q', \Delta')\}$ with prob. $z^2(x\overline{x}_{\text{takeover}})$.[3]

If the value function is additively separable with respect to each product a firm produces, we only need to solve it at the product level and aggregate it to the firm level. For product j with $\Phi_j^f = \{(q_j, \Delta^\ell)\}$, the value function is given by $V(\Phi_j^f) = \max_{z_j, x_j} \{\pi q_j - \widehat{\chi} z_j^{\widehat{\psi}} q_j - \widetilde{\chi} x_j^{\widetilde{\psi}} \overline{q} - F\overline{q} + \widetilde{\beta}\mathbb{E}[V'(\Phi_j^{f\prime})|\Phi_j^f, z_j, x_j]\}$, where $F\overline{q}$ represents fixed operating costs.[4] The value function for firm f with a portfolio of product quality and technology gap is then: $\Phi^f = \{\Phi_j^f\}_{j\in\mathcal{J}^f}$ is $V(\Phi_j^f) = \sum_{j\in\mathcal{J}^f} V(\Phi_j^f)$. The following proposition derives analytic expressions for firms' decision rules.[5]

Proposition F.1. *Given a technology gap distribution $\{\mu(\Delta^\ell)\}_{\ell=1}^4$, a fixed cost of operation equal to $F\overline{q} = \widetilde{\beta}B(1+g)\overline{q}$, and the exit value for a product given by $V(\emptyset) = \frac{B\overline{q}}{1-x\overline{x}_{\text{takeover}}}$, the value function of firm f with a product quality and technology gap portfolio of $\Phi^f \equiv \{(q_j, \Delta_j)\}_{j\in\mathcal{J}^f}$ is: $V(\Phi^f) = \sum_{\ell=1}^4 A_\ell(\sum_{j\in\mathcal{J}^f|\Delta_j=\Delta^\ell} q_j) + n_f B\overline{q}$, where $A_1 = \pi - \widehat{\chi}(z^1)^{\widehat{\psi}} + \widetilde{\beta}[A_1(1-\overline{x})(1-z^1) + \lambda A_2(1-\overline{x})z^1]$, $A_2 = \pi - \widehat{\chi}(z^2)^{\widehat{\psi}} + \widetilde{\beta}[A_1(1-\overline{x})(1-z^2) + \lambda A_2 z^2]$, $A_3 = \pi - \widehat{\chi}(z^3)^{\widehat{\psi}} + \widetilde{\beta}[A_1(1-\frac{1}{2}\overline{x})(1-z^3) + \lambda A_2 z^3]$, $A_4 = \pi - \widehat{\chi}(z^4)^{\widehat{\psi}} + \widetilde{\beta}[A_1(1-\overline{x})(1-z^4) + \lambda A_2(1-\frac{1}{2}\overline{x})z^4]$, and $B = [x\widetilde{\beta}A_{\text{takeover}} - \widetilde{\chi}x^{\widetilde{\psi}}]/[1-\widetilde{\beta}(1+g)x\overline{x}_{\text{takeover}}]$, and the optimal innovation probabilities*

[3]For simplicity, we use the unconditional probability of business takeover $x\overline{x}_{\text{takeover}}$ abusively.
[4]This is commonly assumed for tractability (Akcigit and Kerr, 2018; De Ridder, 2024; Argente et al., 2024)
[5]The analytic expression for startup decisions remains unchanged.

are $z^1 = \big[\widetilde{\beta}\big[(1-\overline{x})\lambda A_2 - (1-\overline{x})A_1\big]/\big[\widehat{\psi}\widehat{\chi}\big]^{\frac{1}{\widehat{\psi}-1}}$, $z^2 = \big[\big[\widetilde{\beta}\big[\lambda A_2 - (1-\overline{x})A_1\big]/\big[\widehat{\psi}\widehat{\chi}\big]\big]^{\frac{1}{\widehat{\psi}-1}}$, $z^3 = \big[\big[\widetilde{\beta}\big[\lambda A_2 - \big(1-\frac{1}{2}\overline{x}\big)A_1\big]/\big[\widehat{\psi}\widehat{\chi}\big]\big]^{\frac{1}{\widehat{\psi}-1}}$, $z^4 = \big[\big[\widetilde{\beta}\big[\lambda\big(1-\frac{1}{2}\overline{x}\big)A_2 - \big(1-\overline{x}\big)A_1\big]/\big[\widehat{\psi}\widehat{\chi}\big]\big]^{\frac{1}{\widehat{\psi}-1}}$, *and* $x = \big[\big[\widetilde{\beta}\big[A_{\text{takeover}} + \overline{x}_{\text{takeover}}B(1+g)\big]/\big[\widetilde{\psi}\widetilde{\chi}\big]\big]^{\frac{1}{\widetilde{\psi}-1}}$, *where* g *is the average product quality growth rate in the economy,* A_{takeover} *is the ex-ante value of a product line obtained from successful takeover, defined as* $A_{\text{takeover}} \equiv \frac{1-z^3}{2}A_1\mu(\Delta^3) + \big(1-\frac{z^4}{2}\big)A_2\lambda\mu(\Delta^4) + A_3\eta\mu(\Delta^1) + (1-z^2)A_4\frac{\eta}{\lambda}\mu(\Delta^2)$*, and* $\overline{x}_{\text{takeover}} = \mu(\Delta^1) + (1-z^2)\mu(\Delta^2) + \frac{1}{2}(1-z^3)\mu(\Delta^3) + \big(1-\frac{1}{2}z^4\big)\mu(\Delta^4)$.

Proof. Suppose the value function is additively separable with respect to each product a firm produces. Then, we can rewrite the expected future value term for each technology gap case as follows: for Δ^1, $\mathbb{E}\big[V'\big(\Phi^{f\prime}\big)|\Phi^f, z^1, x\big] = (1-\overline{x})(1-z^1)V'\big(\big\{(q_j,\Delta^1)\big\}\big) + (1-\overline{x})z^1V'\big(\big\{(\Delta^2 q_j,\Delta^2)\big\}\big) + x\mathbb{E}V'\big(\big\{(q',\Delta')\big\}\big) + \overline{x}(1-x\overline{x}_{\text{takeover}})V'(\emptyset)$; for Δ^2, $\mathbb{E}\big[V'\big(\Phi^{f\prime}\big)|\Phi^f, z^2, x\big] = (1-\overline{x})(1-z^2)V'\big(\big\{(q_j,\Delta^1)\big\}\big) + z^2V'\big(\big\{(\Delta^2 q_j,\Delta^2)\big\}\big) + x\mathbb{E}V'\big(\big\{(q',\Delta')\big\}\big) + \overline{x}(1-z^2)(1-x\overline{x}_{\text{takeover}})V'(\emptyset)$, for Δ^3, $\mathbb{E}\big[V'\big(\Phi^{f\prime}\big)|\Phi^f, z^3, x\big] = \big(1-\frac{1}{2}\overline{x}\big)(1-z^3)V'\big(\big\{(q_j,\Delta^1)\big\}\big) + z^3V'\big(\big\{(\Delta^2 q_j,\Delta^2)\big\}\big) + x\mathbb{E}V'\big(\big\{(q',\Delta')\big\}\big) + \frac{1}{2}\overline{x}(1-z^3)(1-x\overline{x}_{\text{takeover}})V'(\emptyset)$; and for Δ^4, $\mathbb{E}\big[V'\big(\Phi^{f\prime}\big)|\Phi^f, z^4, x\big] = (1-\overline{x})(1-z^4)V'\big(\big\{(q_j,\Delta^1)\big\}\big) + \big(1-\frac{1}{2}\overline{x}\big)z^4V'\big(\big\{(\Delta^2 q_j,\Delta^2)\big\}\big) + x\mathbb{E}V'\big(\big\{(q',\Delta')\big\}\big) + \overline{x}\big(1-\frac{1}{2}z^4\big)(1-x\overline{x}_{\text{takeover}})V'(\emptyset)$.

Using the guessed value function $V(\big\{(q_j,\Delta^\ell)\big\}) = A_\ell q_j + B\overline{q}$, solving for the FONCs with respect to z^ℓ and x, and applying the suggested forms for fixed costs and the exit value, we obtain the analytic expressions for the optimal innovation decisions. For example, if $\Delta^\ell = \Delta^1$, we get $A_1 q_j + B\overline{q} = \pi q_j - \widehat{\chi}z_j^{\widehat{\psi}}q_j - \widetilde{\chi}x_j^{\widetilde{\psi}}\overline{q} + \widetilde{\beta}\Big[(1-\overline{x})(1-z^1)A_1 q_j + (1-\overline{x})z^1A_2\Delta^2 q_j + x_j\left[A_{\text{takeover}} + \overline{x}_{\text{takeover}}(1+g)B\right]\overline{q}\Big]$, as the fixed cost of operation and the exit value cancel out some terms associated with B. The FONC with respect to z_j is $\frac{\partial}{\partial z_j} = \widehat{\psi}\widehat{\chi}z_j^{\widehat{\psi}-1} = \widetilde{\beta}\Big[(1-\overline{x})A_2\Delta^2 - (1-\overline{x})A_1\Big]$. This equation provides the optimal internal innovation decision for Δ^1 case, which only depends on the technology gap. The FONC with respect to x_j is $\frac{\partial}{\partial x_j} = \widetilde{\psi}\widetilde{\chi}x_j^{\widetilde{\psi}-1} = \widetilde{\beta}\left[A_{\text{takeover}} + \overline{x}_{\text{takeover}}(1+g)B\right]$. This equation provides the optimal external innovation x, which is independent of both product quality and technology gap. Collecting terms with q_j gives us the expression for A_1, which only depends on the technology gap, and collecting terms with $\overline{q}$ gives us the expression for B, which is independent of both product quality and technology gap. The remaining three technology gap cases follow the same process. These results confirm the additive separability of the value function with respect to each product-technology gap pair. □

C.2 Technology Gap Distribution Transition

From the quality evolution for incumbents (in the main text) and outsiders (Section A.2) the inflows and outflows for technology gap distribution ($\mu(\Delta^\ell)$) are defined as follows: for Δ^1, inflow is $(1-z^2)(1-\overline{x})\mu(\Delta^2)+(1-z^3)\mu(\Delta^3)+(1-z^4)(1-\overline{x})\mu(\Delta^4)$ and outflow is $(\overline{x}+z^1(1-\overline{x}))\mu(\Delta^1)$; for Δ^2, inflow is $z^1(1-\overline{x})\mu(\Delta^1)+z^3\mu(\Delta^3)+(z^4+(1-z^4)\overline{x})\mu(\Delta^4)$ and outflow is $(1-z^2)\mu(\Delta^2)$; for Δ^3, inflow is $\overline{x}\mu(\Delta^1)$ and outflow is $\mu(\Delta^3)$; and for Δ^4, inflow is $(1-z^2)\overline{x}\mu(\Delta^2$ and outflow is $\mu(\Delta^4)$.

C.3 Aggregate Variables

Aggregate Creative Destruction Arrival Rate Firms do external innovation for each product they own simultaneously. Given the unit mass of products, there is a unit mass of external innovation trials by incumbent firms each period. Defining $s_d = \mathcal{F}_d/\mathcal{F}$ as the share (the total mass) of domestic products and $s_o = \mathcal{F}_o/\mathcal{F}$ as the outside counterpart, we can write the aggregate creative destruction arrival rate as $\overline{x} = s_d x + \mathcal{E}_d x_e + \underbrace{s_o x + \mathcal{E}_o}_{\equiv \overline{x}_o}$, where $\mathcal{E}_o$ is the total mass of potential outside entrants with successful external innovation. As we assume the symmetry between domestic and outside firms, the outsiders' external innovation intensity is also x. As $s_d + s_o = 1$, we can rewrite $\overline{x}$ as $\overline{x} = x + \mathcal{E}_d x_e + \mathcal{E}_o$.

Aggregate Productivity Growth Decomposition The total mass of domestic external innovation trials is the share of products owned by domestic firms s_d, given the unit mass assumption. Thus, we can replace the mass of domestic firms ($\mathcal{F}_d$) with s_d and obtain the following decomposition as in the single external innovation setup:

$$
\begin{aligned}
g = & \underbrace{\left(\Delta^2-1\right) s_d \left[(1-\overline{x})z^1\mu(\Delta^1)+z^2\mu(\Delta^2)+z^3\mu(\Delta^3)+\left(1-\overline{x}/2\right)z^4\mu(\Delta^4)\right]}_{\text{internal innovation by domestic incumbents}} \\
& + \underbrace{\left(\Delta^2-1\right)\left(1-s_d\right)\left[(1-\overline{x})z^1\mu(\Delta^1)+z^2\mu(\Delta^2)+z^3\mu(\Delta^3)+\left(1-\overline{x}/2\right)z^4\mu(\Delta^4)\right]}_{\text{internal innovation by foreign firms}} \\
& + \underbrace{\left(\overline{\Delta^{\text{ex}}}-1\right) s_d x \mu(\overline{\Delta^{\text{ex}}})}_{\text{external innov. by domestic incumbents}} + \underbrace{\left(\overline{\Delta^{\text{ex}}}-1\right) \mathcal{E}_d x_e \mu(\overline{\Delta^{\text{ex}}})}_{\text{external innov. by domestic startups}} + \underbrace{\left(\overline{\Delta^{\text{ex}}}-1\right) \overline{x}_o \mu(\overline{\Delta^{\text{ex}}})}_{\text{external innov. by foreign firms}} .
\end{aligned}
$$

Aggregate Domestic R&D Expenses Similarly, the aggregate domestic R&D expenses can be rephrased as $R_d = \widehat{\chi} \sum_{\ell=1}^{4} \left[\int_0^1 q_j \mathcal{I}_{\{\Delta_j = \Delta^\ell, j \in \mathcal{D}\}} dj \right] (z^\ell)^{\widehat{\psi}} + s_d \widetilde{\chi} \overline{q} x^{\widetilde{\psi}} + \mathcal{E}_d \widetilde{\chi}_e (x_e)^{\widetilde{\psi}_e} \overline{q}$.

Aggregate Consumption Households own both final goods and domestic intermediate producers. They fund the R&D expenses of domestic potential startups and pay the exit value to domestic incumbents. The households earn labor income from final goods producer (wL), operating fixed costs from intermediate producers ($s_d F \overline{q}$), as well as profits from both producers ($\Pi = 0$ and $\sum_{j \in \mathcal{D}} \pi q_j > 0$). Intermediate producers' profits include the exit value if their product is taken over and their own external innovation fails. Thus, the household budget constraint is $wL + s_d F \overline{q} + \int j \in \mathcal{D} \{\pi q_j - F \overline{q}\} + (1 - x \overline{x}_{\text{takeover}}) V(\emptyset) = C + \mathcal{E}_d \widetilde{\chi}_e (x_e)^{\widetilde{\psi}_e} \overline{q} + (1 - x \overline{x}_{\text{takeover}}) V(\emptyset)$. With the final goods producers' profit function $\Pi = Y - \int_{j \in \mathcal{D}} p_j y_j dj - \int_{j \notin \mathcal{D}} p_j y_j dj - wL$, the aggregate consumption is $C = Y - \int_{j \notin \mathcal{D}} p_j y_j dj - Y_d - R_d$.

D Solution Algorithm

In the model, $\{z^\ell\}_{\ell=1}^4$ are functions of $\overline{x}$; g is a function of $\overline{x}$, $\{z^\ell\}_{\ell=1}^4$, and $\{\mu(\Delta^\ell)\}_{\ell=1}^4$; x is a function of $\overline{x}$ and $\{\mu(\Delta^\ell)\}_{\ell=1}^4$; x_e is a function of $\overline{x}$ and $\{\mu(\Delta^\ell)\}_{\ell=1}^4$; and $\overline{x}$ is a function of x, and x_e. Therefore, we can solve for an equilibrium of the model by iterating over the value for the aggregate creative destruction arrival rate $\overline{x}$.

For the extended model with multiple external innovation: i) Guess values for $\overline{x}$, g and the technology gap distribution $\{\mu(\Delta^\ell)\}_{\ell=1}^4$; ii) Using the guess of $\overline{x}$, compute $\{A_\ell\}_{\ell=1}^4$, and $\{z^\ell\}_{\ell=1}^4$; iii) Using the guess of $\overline{x}$, g, and $\{\mu(\Delta^\ell)\}_{\ell=1}^4$, compute B, x, x_e. Next, compute the stationary $\{\mu_\infty(\Delta^\ell)\}_{\ell=1}^4$, based on the guess of $\{\mu_\infty(\Delta^\ell)\}_{\ell=1}^4$, innovation decision rules, and the following law of motion $\mu_{n+1}(\Delta^\ell) = \mu_n(\Delta^\ell) + \mathit{inflow}_n(\Delta^\ell) - \mathit{outflow}_n(\Delta^\ell)$ for each $\ell \in \{1, 2, 3, 4\}$. Lastly, compute g_∞ with $\{\mu_\infty(\Delta^\ell)\}_{\ell=1}^4$; iv) Compute $\overline{x}' = x + \mathcal{E}_d x_e + \mathcal{E}_o$; v) If $\overline{x} \neq \overline{x}'$, set $\overline{x} = \overline{x}'$, $g = g_\infty$, and $\{\mu(\Delta^\ell)\}_{\ell=1}^4 = \{\mu_\infty(\Delta^\ell)\}_{\ell=1}^4$, use them as new guess, and return to ii); vi) Repeat ii) through v) until the convergence of $\overline{x}$; and vii) Simulate the model over 10,000 products for 1,200 years and compute the moments averaged across the last 150 years.

E Other Theoretical Results

Table E1: Changes in Innovation Values

Description	Variables	Before	After	% Change
Innovation Values	A_1	0.160	0.158	-1.1%
	A_2	0.173	0.172	-1.0%
	A_3	0.182	0.180	-1.0%
	A_4	0.165	0.163	-1.1%
	B	0.011	0.011	-2.6%

Table E2: Aggregate Growth Rate Decomposition

Description	Before	After	% Change
Average productivity growth (g, %)	2.229	2.242	0.6%
Growth by outside firms (g_o, %)	0.312	0.510	63.3%
Growth by domestic firms (g_d, %)	1.888	1.680	-11.0%
Growth from domestic internal innovation (%)	1.047	0.927	-11.4%
Growth from domestic external innovation (%)	0.656	0.571	-13.0%
Growth from domestic startups (%)	0.186	0.182	-1.7%

Table E3: Aggregate Growth Rate Decomposition, Holding Mass Fixed

Description	Before	After	% Change
Average productivity growth by domestic firms (%)	1.888	1.875	-0.7%
Growth from domestic internal innovation (%)	1.047	1.048	0.1%
Growth from domestic external innovation (%)	0.656	0.645	-1.7%
Growth from domestic startups (%)	0.186	0.182	-1.7%

Table E4: Changes in Firm Innovation in High Ext. Innov. Cost Economy

Description	Variables	Before	After	% Change
Creative destruction arrival rate by outside firms	$\overline{x}_o$	1.361	2.406	76.8%
Aggregate creative destruction arrival rate	$\overline{x}$	8.966	9.636	7.5%
Prob. of internal innovation ($\Delta^1 = 1$)	z^1	20.581	20.300	-1.4%
Prob. of internal innovation ($\Delta^2 = \lambda$)	z^2	50.357	51.024	1.3%
Prob. of internal innovation ($\Delta^3 = \eta$)	z^3	36.483	36.744	0.7%
Prob. of internal innovation ($\Delta^4 = \frac{\eta}{\lambda}$)	z^4	35.469	35.662	0.5%
Prob. of external innovation, incumbents	x	0.380	0.363	-4.6%
Prob. of external innovation, potential startups	x_e	7.285	6.954	-4.6%

F Counterfactual: Increased Competitive Pressure by Domestic Startups

We increase the mass of potential domestic startups ε_d by 15.2%, which raises the creative destruction arrival rate $\overline{x}$ from 21.5% to 21.9% (1.51% increase, equivalent to the main

Table E5: Aggregate Growth Decomposition, Low Creativity Economy, Holding Mass Fixed

Description	Before	After	% Change
Average productivity growth by domestic firms (%)	1.397	1.378	-1.4%
Growth from domestic internal innovation (%)	0.991	0.994	0.3%
Growth from domestic external innovation (%)	0.017	0.016	-5.3%
Growth from domestic startups (%)	0.388	0.368	-5.3%

counterfactual exercise). Table F1 and Panel A in Table F2 present the results. The firm-level responses remain the same as before, while the total mass of domestic incumbents and startups increases. Thus, the moments related to the number of domestic firms and startups help identify the source behind the increased competitive pressure (domestic startups vs outside firms). Also, Panel B in Table F2 displays the growth decomposition, where the aggregate growth increases (unlike the main exercise), but domestic external innovation decreases as before.

Table F1: Changes in Firm Innovation: Economy with More Potential Startups

Description	Variables	Before	After	% Change
Creative destruction arrival rate by outside firms	$\overline{x}_o$	3.30	3.04	-7.94%
Aggregate creative destruction arrival rate	$\overline{x}$	21.53	21.85	1.51%
Prob. of internal innovation ($\Delta^1 = 1$)	z^1	16.87	16.80	-0.42%
Prob. of internal innovation ($\Delta^2 = \lambda$)	z^2	57.83	57.95	0.20%
Prob. of internal innovation ($\Delta^3 = \eta$)	z^3	39.66	39.72	0.14%
Prob. of internal innovation ($\Delta^4 = \frac{\eta}{\lambda}$)	z^4	37.35	37.37	0.06%
Prob. of external innovation, incumbents	x	16.76	16.54	-1.35%
Prob. of external innovation, potential startups	x_e	4.02	3.97	-1.35%

Table F2: Aggregate Moment Change: Economy with More Potential Startups

Description	Before	After	% Change
Panel A: Changes in the Aggregate Moments			
Total mass of domestic firms	0.386	0.416	7.6%
Total mass of domestic startups	0.029	0.033	13.4%
R&D to sales ratio (%)	4.579	4.512	-1.5%
Avg. number of products	2.290	2.164	-5.5%
Panel B: Changes in the Aggregate Growth and Decomposition			
Average productivity growth by domestic firms (%)	1.89	1.93	2.3%
Growth from domestic internal innovation (%)	1.05	1.07	1.8%
Growth from domestic external innovation (%)	0.66	0.65	-0.1%
Growth from domestic startups (%)	0.19	0.21	13.2%

G Data Appendix

G.1 Summary Statistics

Table G1 and G2 present summary statistics.

Table G1: The Whole Universe of Patenting Firms vs. Regression Sample in 1992

	All patenting firms	Regression sample
Average number of patents	6.15	8.86
	(19.46)	(24.10)
Average self-citation rate	0.0434	0.0540
	(0.0899)	(0.0941)
Innovation intensity	0.055	0.093
	(0.25)	(0.33)
Number of industries operating	2.34	5.43
	(3.67)	(6.94)
Employment	511.7	1988.0
	(1869.0)	(3835.0)
Patent stock	6.45	35.22
	(26.61)	(64.37)
Employment growth	0.07	0.06
	(0.60)	(0.40)
Firm age	12.33	15.65
	(6.76)	(9.42)
7yr patent growth		-0.854
		(1.312)
7yr self-citation ratio growth		0.356
		(1.322)
Number of firms	26,500	3,100

Note: Innovation intensity in 2000 is 0.183(0.58), the seven-year patent growth in 2000 is -1.07(1.207), and the seven-year self-citation ratio growth in 2000 is 0.282(1.304).

Table G2: Foreign Competition Shock Related Measures

	NTR gap	Dnstream NTR g.	Upstream NTR g.	NTR rate	Non-NTR r.
Mean	0.291	0.138	0.203	0.027	0.303
(Std. dev.)	(0.127)	(0.060)	(0.073)	(0.022)	(0.134)
cov(, NTR gap)		0.485	0.434	0.412	0.969
cov(, Up. NTR g.)		0.204			

G.2 Real Effect of the Two Types of Innovation

Table G3: Real Effect of Innovation on Employment Growth and Industry Added

	ΔEmployment	#industries added
#patents	0.036***	0.102***
	(0.010)	(0.011)
Avg. self-citation	-0.256**	-0.158**
	(0.109)	(0.079)
Log payroll	-0.027***	0.113***
	(0.008)	(0.006)
Firm age	-0.004**	0.002
	(0.002)	(0.002)
Innovation intensity	-0.004	-0.006**
	(0.003)	(0.003)
Past 5yr Δpat in own tech.	0.001	-0.000
	(0.001)	(0.001)
Observations	5,400	5,400
Fixed effects	jt	jt

Note: The baseline set of controls is included. The estimates for industry (j) and year (t) fixed effects, and the coefficient associated with the binary indicator are suppressed due to disclosure restrictions. Robust standard errors adjusted for clustering at the firm-level are displayed below each coefficient. Observations are unweighted. Observation counts are rounded due to Census Bureau disclosure avoidance procedures. * p < 0.1, ** p < 0.05, *** p < 0.01.

We replicate the findings in Akcigit and Kerr (2018) that internal innovation contributes less to firm employment growth with the following regression:

$$\Delta Y_{ijt+5} = \beta_1 Pat_{ijt} + \beta_2 Internal_{ijt} + \mathbf{X}_{ijt}\gamma_1 + \delta_{jt+5} + \varepsilon_{ijt+5}.$$

ΔY_{ijt+5} is the DHS growth of firm employment or the number of industries (six-digit NAICS) added between t and $t+5$, Pat_{ijt} is the citation adjusted number of patents (in log) at t, and $Internal_{ijt}$ is the citation-adjusted average of self-citation ratio at t for firm i in industry j. Firm and industry controls include firm age, log payroll, the past five-year U.S. patent growth in firm technology fields, innovation intensity, and public firm status. The regression is unweighted and standard errors are clustered on firm. The mean (and standard deviation) of the citation-adjusted logged number of patents is 1.284 (1.125), and the counterpart for the citation-adjusted average self-citation ratio is 0.050 (0.101).

Column 1 in Table G3 shows that for average firms, creating one more patent is associated with a 1.32 pp (3.6/2.718) increase in their employment growth as exp(1) $\approx$ 2.718. Also,

Table G4: Real Effect of Innovation on Productivity Growth, Product Added, and Product Concentration

	ΔTFPR	#products added	ΔHHI
#patents	0.118**	0.358**	-0.012
	(0.055)	(0.085)	(0.023)
Avg. self-citation	-0.027	-0.274***	0.154**
	(0.053)	(0.102)	(0.069)
Log employment	0.016	0.023	-0.050***
	(0.015)	(0.022)	(0.016)
Log payroll	-0.018	0.011	-0.004
	(0.013)	(0.021)	(0.016)
#industries	0.033**	0.308***	-0.214***
	(0.013)	(0.028)	(0.018)
#products	-0.027**	0.205***	0.435***
	(0.012)	(0.022)	(0.016)
Firm age	-0.001	-0.008***	-0.001
	(0.001)	(0.002)	(0.001)
Innovation intensity	-0.145	0.641***	-0.337**
	(0.128)	(0.16)	(0.131)
Past 5yr Δpat in own tech.	-0.000	0.000	0.001*
	(0.000)	(0.000)	(0.000)
Observations	5,700	5,700	5,700
Fixed effects	jt	jt	jt

Note: The baseline set of controls along with firm payroll, the number of operating industries and products are included. The estimates for industry (j) and the year (t) fixed effects, and the coefficient associated with the binary indicator are suppressed due to disclosure restrictions. Robust standard errors adjusted for clustering at the firm-level are displayed below each coefficient. Observations are unweighted. Observation counts are rounded due to Census Bureau disclosure avoidance procedures. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

since average firms have the average self-citation ratio of 0.05, an 1% increase in the self-citation ratio is associated with a 0.0128 pp ($-0.256\times0.05\times0.01\times100$) decrease in their employment growth.

We also replace the dependent variable with the growth of TFPR, the number of new products added (seven-digit NAICS), and the growth of within-firm product concentration. We find similar results in Table G4. In addition, we replicate the findings using an alternative set of measures for external and internal innovation. Specifically, external innovation is explicitly defined by the count of patents with a zero self-citation ratio, while internal innovation is measured by patents with a self-citation above 0% or 10%. This more direct measure of external and internal innovation exhibits consistent and even more pronounced effects, as presented in Table G5.

Table G5: Real Effect of Innovation on Productivity Growth, Product Added, and Product Concentration (Alternative Innovation Measures)

	ΔTFPR	#prod. added	ΔHHI	ΔTFPR	#prod. added	ΔHHI
#patents (self-cite=0)	0.118**	0.358**	-0.124**	0.129**	0.354***	-0.120**
	(0.055)	(0.085)	(0.055)	(0.052)	(0.081)	(0.052)
#patents (self-cite>0,10)	-0.027	-0.274***	0.134**	-0.055	-0.317***	0.152**
	(0.053)	(0.102)	(0.063)	(0.056)	(0.118)	(0.067)
Log employment	0.016	0.023	-0.050***	0.016	0.023	-0.051***
	(0.015)	(0.022)	(0.016)	(0.015)	(0.022)	(0.016)
Log payroll	-0.018	0.011	-0.002	-0.018	0.011	-0.002
	(0.013)	(0.021)	(0.016)	(0.013)	(0.021)	(0.016)
#industries	0.033**	0.308***	-0.216***	0.033**	0.307***	-0.212***
	(0.013)	(0.028)	(0.018)	(0.013)	(0.028)	(0.018)
#products	-0.027**	0.205***	0.433***	-0.027**	0.205***	0.434***
	(0.012)	(0.022)	(0.016)	(0.012)	(0.022)	(0.016)
Firm age	-0.001	-0.008***	-0.001	-0.000	-0.008***	-0.001
	(0.001)	(0.002)	(0.001)	(0.001)	(0.002)	(0.001)
Innovation intensity	-0.145	0.641***	-0.317**	-0.144	0.635***	-0.313**
	(0.128)	(0.164)	(0.130)	(0.128)	(0.164)	(0.130)
Past 5yr Δpat in own tech.	-0.000	0.000	0.001*	-0.000	0.000	0.001*
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Observations	5,700	5,700	5,700	5,700	5,700	5,700
Fixed effects	jt	jt	jt	jt	jt	jt
Internal innov cutoffs	0%	0%	0%	10%	10%	10%

Note: External innovation is defined by the number of patents with a zero self-citation ratio, and internal innovation is defined by the number of patents with a self-citation above a certain cutoff. In the first three columns, the cutoff is set at zero, whereas in the last three columns, it is set at 10%. The baseline set of controls along with firm payroll, the number of operating industries and products are included. The estimates for industry (j) and the year (t) fixed effects, and the coefficient associated with the binary indicator are suppressed due to disclosure restrictions. Robust standard errors adjusted for clustering at the firm-level are displayed below each coefficient. Observations are unweighted. Observation counts are rounded due to Census Bureau disclosure avoidance procedures. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

G.3 Parallel Pre-trend Assumption

Table G6 tests the parallel pre-trend assumption. The coefficient estimates are smaller and statistically insignificant.

Table G6: Parallel Pre-trend Test

	ΔPatents	ΔPatents	ΔSelf-cite	ΔSelf-cite
NTR gap	-0.397	-0.380	-0.554	-0.546
	(0.487)	(0.488)	(0.403)	(0.402)
$\times$ Innovation intensity		-0.195		-0.058
		(0.162)		(0.395)
NTR gap $\times$ $\mathcal{I}_{\{1992\}}$	0.523	0.500	0.252	0.259
	(0.355)	(0.362)	(0.294)	(0.290)
$\times$ Innovation intensity		0.092		-0.113
		(0.243)		(0.491)
Post $\times$ Innovation intensity		0.009		0.027
		(0.064)		(0.115)
Innovation intensity		0.036		0.022
		(0.033)		(0.082)
Past 5yr Δpat in own tech.	0.149	0.151	0.105	0.104
	(0.096)	(0.096)	(0.097)	(0.098)
Log employment	0.157***	0.156***	-0.047***	-0.047**
	(0.013)	(0.013)	(0.014)	(0.014)
Firm age	0.000	0.000	-0.009***	-0.009***
	(0.003)	(0.003)	(0.003)	(0.003)
Observations	5,000	5,000	5,000	5,000
Fixed effects	j, p	j, p	j, p	j, p
Controls	baseline	baseline	baseline	baseline

Note: The baseline set of controls is included. The estimates for industry (j) and the period (p) fixed effects, and the coefficient associated with the binary indicator are suppressed due to disclosure restrictions, and the constant is dropped as well. Robust standard errors adjusted for clustering at the level of the firms' major industries are displayed below each coefficient. Observations are unweighted. Observation counts are rounded due to Census Bureau disclosure avoidance procedures. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

G.4 Main Results (Full Tables)

Table G7: Overall Effect

	ΔPatents	ΔPatents	ΔSelf-cite	ΔSelf-cite
NTR gap × Post	0.226	0.067	0.025	0.045
	(0.230)	(0.275)	(0.260)	(0.291)
NTR gap	-2.222***	0.392	1.104***	-0.058
	(0.372)	(0.409)	(0.317)	(0.390)
Past 5yr Δpat in own tech.		0.165**		0.265***
		(0.084)		(0.083)
Log employment		0.160***		-0.026**
		(0.011)		(0.012)
Firm age		-0.006**		-0.011***
		(0.002)		(0.002)
NTR rate		-2.456		1.248
		(1.672)		(2.220)
Observations	6,500	6,500	6,500	6,500
Fixed effects	j, p	j, p	j, p	j, p
Controls	no	baseline	no	baseline

Note: The baseline controls include the past five-year U.S. patent growth in firms' own technology fields, log employment, firm age, NTR rate, and a dummy for publicly traded firms. The estimates for industry (j) and the period (p) fixed effects, and the coefficient associated with the binary indicator are suppressed due to disclosure restrictions, and the constant is dropped as well. Robust standard errors adjusted for clustering at the level of the firms' major industries are displayed below each coefficient. Observations are unweighted. Observation counts are rounded due to Census Bureau disclosure avoidance procedures. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table G8: Market-Protection Effect

	ΔPatents	ΔPatents	ΔSelf-cite	ΔSelf-cite
NTR gap $\times$ Post	0.238	0.071	-0.075	-0.062
	(0.237)	(0.283)	(0.257)	(0.291)
$\times$ Innovation intensity	0.077	-0.054	0.732**	0.795***
	(0.231)	(0.242)	(0.299)	(0.277)
NTR gap	-2.206***	0.418	1.101***	-0.005
	(0.375)	(0.412)	(0.315)	(0.394)
$\times$ Innovation intensity	-0.226	-0.161	-0.198	-0.390
	(0.158)	(0.184)	(0.231)	(0.236)
Post $\times$ Innovation intensity	-0.053	0.040	-0.179*	-0.202**
	(0.070)	(0.079)	(0.095)	(0.087)
Innovation intensity	0.080*	0.029	0.059	0.088
	(0.048)	(0.051)	(0.070)	(0.068)
Past 5yr Δpat in own tech.		0.164*		0.265***
		(0.084)		(0.083)
Log employment		0.161***		-0.025**
		(0.011)		(0.012)
Firm age		-0.005**		-0.011***
		(0.002)		(0.002)
NTR rate		-2.619		1.024
		(1.683)		(2.224)
$\times$ Innovation intensity		0.690		0.625
		(0.531)		(0.501)
Observations	6,500	6,500	6,500	6,500
Fixed effects	j, p	j, p	j, p	j, p
Controls	no	baseline	no	baseline

Note: The same set of controls, fixed effects, and disclosure policy applied as in the baseline (Table G7). * p < 0.1, ** p < 0.05, *** p < 0.01.

G.5 Other Robustness Test

Table G9-G19 display other robustness test of the baseline regression.

Table G9: Industry-level Tariff Measures

	ΔPatents	ΔPatents	ΔSelf-cite	ΔSelf-cite
NTR gap × Post	0.016	0.011	0.005	-0.001
	(0.249)	(0.249)	(0.261)	(0.261)
× Innovation intensity		-0.032		0.760***
		(0.229)		(0.272)
Observations	6,500	6,500	6,500	6,500
Fixed effects	j, p	j, p	j, p	j, p
Controls	baseline	baseline	baseline	baseline
Weights for tariffs	major industry	major industry	major industry	major industry

Note: The industry-level tariff measures are used. All else remains the same as in the baseline regression. * p < 0.1, ** p < 0.05, *** p < 0.01.

Table G10: Foreign Competition Shock through I-O Linkages

	ΔPatents	ΔPatents	ΔSelf-cite	ΔSelf-cite
NTR gap × Post	-0.111	-0.111	-0.296	-0.424
	(0.331)	(0.342)	(0.356)	(0.355)
× Innovation intensity		-0.001		0.824***
		(0.337)		(0.288)
Observations	6,500	6,500	6,500	6,500
Fixed effects	j, p	j, p	j, p	j, p
Controls	baseline+IO	baseline	baseline	baseline

Note: The baseline set of controls is included along with the diff-in-diff terms for upstream and downstream sectors, respectively. All else remains the same as in the baseline regression. * p < 0.1, ** p < 0.05, *** p < 0.01.

Table G11: Weighted by Inverse Propensity Score

	ΔPatents	ΔPatents	ΔSelf-cite	ΔSelf-cite
NTR gap × Post	0.003	0.039	-0.394	-0.603
	(0.475)	(0.484)	(0.509)	(0.512)
× Innovation intensity		-0.045		0.893***
		(0.282)		(0.294)
Observations	6,500	6,500	6,500	6,500
Fixed effects	j, p	j, p	j, p	j, p
Controls	baseline	baseline	baseline	baseline
Regression weights	inv. propens.	inv. propens.	inv. propens.	inv. propens.

Note: Regression is weighted by the inverse of the propensity scores from logit model (y = indicator for the regression sample). All else remains the same as in the baseline regression. * p < 0.1, ** p < 0.05, *** p < 0.01.

Table G12: Standard Error Clustering on Firms

	ΔPatents	ΔPatents	ΔSelf-cite	ΔSelf-cite
NTR gap × Post	0.067	0.071	0.045	-0.062
	(0.287)	(0.290)	(0.308)	(0.312)
× Innovation intensity		-0.054		0.795***
		(0.245)		(0.277)
Observations	6,500	6,500	6,500	6,500
Fixed effects	j, p	j, p	j, p	j, p
Controls	baseline	baseline	baseline	baseline
se. cluster	firmid	firmid	firmid	firmid

Note: The standard errors are adjusted for clustering at the firm-level. All else remains the same as in the baseline regression. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table G13: Robustness Check for Innovation Intensity Measure (Firm Age, Size Effects)

	ΔPatents	ΔPatents	ΔSelf-cite	ΔSelf-cite
NTR gap × Post	-0.447	-0.342	0.805	0.292
	(0.645)	(0.691)	(0.668)	(0.641)
× Innovation intensity		-0.026		0.826***
		(0.239)		(0.284)
Observations	6,500	6,500	6,500	6,500
Fixed effects	j, p	j, p	j, p	j, p
Controls	baseline+	baseline+	baseline+	baseline+

Notes: The baseline set of controls is included along with additional controls for the set of interaction terms between innovation intensity, firm age, and size, to check robustness for potential correlations between innovation intensity, firm age, and size. All else remains the same as in the baseline regression. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table G14: Alternative Technology Barrier Measure

	ΔPatents	ΔPatents	ΔSelf-cite	ΔSelf-cite
NTR gap × Post	0.067	0.131	0.045	0.029
	(0.287)	(0.291)	(0.308)	(0.313)
× Innovation intensity		-0.058		0.066*
		(0.440)		(0.040)
Observations	6,500	6,500	6,500	6,500
Fixed effects	j, p	j, p	j, p	j, p
Controls	baseline	baseline	baseline	baseline

Note: The baseline set of controls is included, with the innovation intensity measure replaced by the past 5-year average of the inverse of the within-industry innovation intensity gap from the frontier firm as a proxy for the accumulated level of technology barriers. All else remains the same as in the baseline regression. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table G15: Alternative External Innovation Measure

	#products added	#products added	#products added
NTR gap × Post	-0.239***	-0.231***	-0.218***
	(0.068)	(0.067)	(0.063)
Observations	497,000	497,000	497,000
Fixed effects	j, p	j, p	j, p
Controls	baseline	baseline	baseline
	(innovation intensity)	(labor productivity)	(TFPR)

Note: External innovation is directly measured by the number of products added and taken as the main dependent variable. The baseline set of controls (with a different measure for technological barriers) is included. Innovation intensity is the baseline measure as before in the first column. In the second and third columns, it is replaced by the inverse gap of the firm's labor productivity or TFPR from the frontier in its operating industry as an alternative way to measure the degree of technological barriers. All else remains the same as in the baseline regression. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table G16: Alternative Internal Innovation Measure

	ΔPatents (self-cite>0)	ΔPatents (self-cite>0)	ΔPatents (self-cite>10)	ΔPatents (self-cite>10)
NTR gap × Post	0.007	0.001	0.005	-0.008
	(0.004)	(0.004)	(0.004)	(0.005)
× Innovation intensity		0.100***		0.206***
		(0.033)		(0.077)
Observations	497,000	497,000	497,000	497,000
Fixed effects	j, p	j, p	j, p	j, p
Controls	baseline	baseline	baseline	baseline

Note: Internal innovation is directly measured and taken as the main dependent variable. The first two columns measure it by the number of patents with a self-citation ratio above 0%, and the last two columns use those above 10%. All else remains the same as in the baseline regression. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table G17: The Effect on Product Concentration

	Δproduct HHI	Δproduct HHI
NTR gap × Post	-0.002	-0.019
	(0.042)	(0.012)
× Innovation intensity		0.262**
		(0.116)
Observations	497,000	497,000
Fixed effects	j, p	j, p
Controls	baseline	baseline

Note: The main dependent variable is within-firm product sales concentration. The baseline set of controls is included. All else remains the same as in the baseline regression. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table G18: Robustness Test for the Overall Response

	ΔPat. (1)	ΔPat. (2)	ΔPat. (3)	ΔPat. (4)	ΔPat. (5)	ΔPat. (6)	ΔSelf-c. (7)	ΔSelf-c. (8)	ΔSelf-c. (9)	ΔSelf-c. (10)	ΔSelf-c. (11)	ΔSelf-c. (12)
NTR gap × Post	0.074	0.059	0.023	0.118	0.075	0.067	0.030	0.048	0.081	0.114	0.149	0.166
	(0.276)	(0.276)	(0.276)	(0.271)	(0.272)	(0.272)	(0.291)	(0.292)	(0.290)	(0.290)	(0.288)	(0.287)
NTR gap	0.396	0.417	0.566	0.390	0.563	0.567	-0.067	-0.065	-0.200	-0.067	-0.206	-0.249
	(0.408)	(0.407)	(0.406)	(0.412)	(0.409)	(0.408)	(0.388)	(0.390)	(0.385)	(0.379)	(0.374)	(0.376)
Observations	6,500	6,500	6,500	6,500	6,500	6,500	6,500	6,500	6,500	6,500	6,500	6,500
Fixed effects	j, p	j, p	j, p	j, p	j, p	j, p	j, p	j, p	j, p	j, p	j, p	j, p
Controls	base+	base+	base+	base+	base+	base+	base+	base+	base+	base+	base+	base+

Note: All columns augment the baseline regression set of controls with additional variables. Specifically, columns (1),(7) include the cumulative number of patents, column (2),(8) include firm payroll, column (3),(9) include the number of industries in which firms operate, column (4),(10) include the industry-level skill, capital intensities, column (5),(11) include the number of industries and the industry-level skill, capital intensities, column (6),(12) include the number of industries, the industry-level skill, capital intensities, a dummy for firms with total imports > 0, and a dummy for firms with total exports > 0. All else remains the same as in the baseline regression. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table G19: Robustness Test for the Market-Protection Effect

	ΔPat. (1)	ΔPat. (2)	ΔPat. (3)	ΔPat. (4)	ΔPat. (5)	ΔPat. (6)	ΔSelf-c. (7)	ΔSelf-c. (8)	ΔSelf-c. (9)	ΔSelf-c. (10)	ΔSelf-c. (11)	ΔSelf-c. (12)
NTR gap × Post	0.076	0.062	0.028	0.112	0.081	0.074	-0.078	-0.059	-0.026	0.007	0.042	0.063
	(0.283)	(0.284)	(0.284)	(0.278)	(0.279)	(0.280)	(0.290)	(0.291)	(0.289)	(0.287)	(0.285)	(0.285)
× Innovation intensity	-0.055	-0.037	-0.051	0.058	-0.055	-0.029	0.798***	0.789***	0.792***	0.789***	0.787***	0.777***
	(0.242)	(0.242)	(0.239)	(0.243)	(0.240)	(0.231)	(0.278)	(0.278)	(0.280)	(0.277)	(0.279)	(0.268)
Observations	6,500	6,500	6,500	6,500	6,500	6,500	6,500	6,500	6,500	6,500	6,500	6,500
Fixed effects	j, p	j, p	j, p	j, p	j, p	j, p	j, p	j, p	j, p	j, p	j, p	j, p
Controls	base+	base+	base+	base+	base+	base+	base+	base+	base+	base+	base+	base+

Note: All columns augment the baseline set of controls with additional variables. Specifically, columns (1),(7) include the cumulative number of patents, column (2),(8) include firm payroll, column (3),(9) include the number of industries in which firms operate, column (4),(10) include the industry-level skill, capital intensities, column (5),(11) include the number of industries and the industry-level skill, capital intensities, column (6),(12) include the number of industries, the industry-level skill, capital intensities, a dummy for firms with total imports > 0, and a dummy for firms with total exports > 0. All else remains the same as in the baseline regression. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

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