고려대학교 KOREA UNIVERSITY

DSBA Data Science & Business Analytics

Evaluation Metrics for Time Series Anomaly Detection

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[NeurIPS, 2018]

Precision Recall for Time Series

[ACM CIKM, 2019]

Time-Series Aware Precision and Recall for Anomaly Detection

Prerequisites

- Precision
  - ✓ 모델이 정답으로 예측한 것 중에서 실제 정답의 비율
  - ✓ TP / (TP+FP)

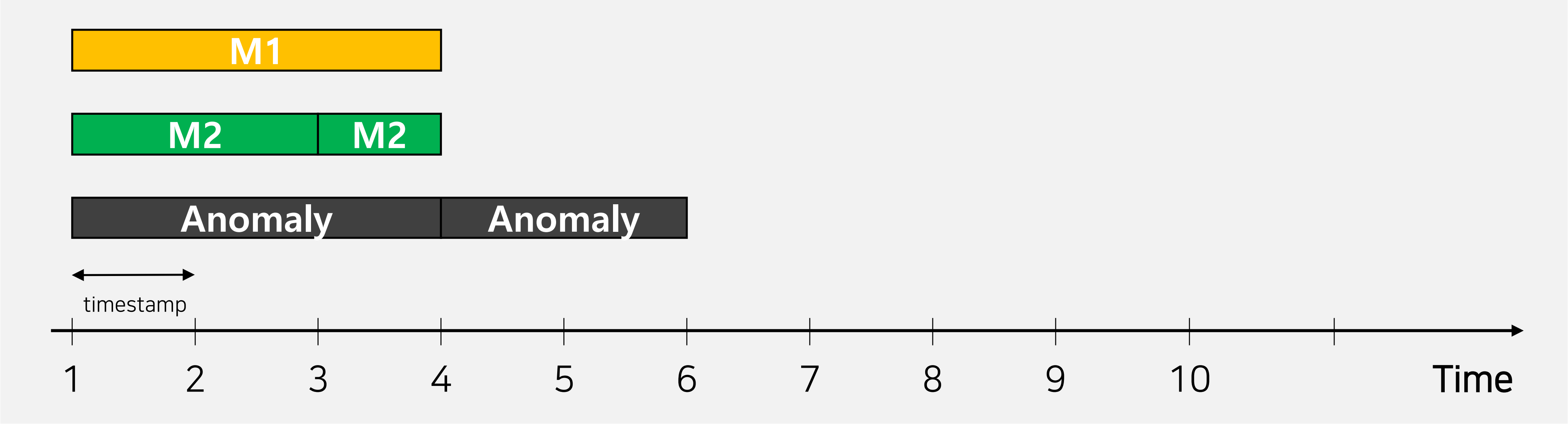
- Recall
  - ✓ 실제 정답 중에서 모델이 정답으로 예측한 비율
  - ✓ TP / (TP+FN)

		Actual class	
		Positive	Negative
Predicted class	Positive	**True Positive (TP)**	**False Positive (FP)**
	Negative	**False Negative (FN)**	**True Negative (TN)**

Motivation

- Which model is better?
  - ✓ # timestapms : 10
  - ✓ # anomaly timestamps : 5
  - ✓ 기존의 방식(Point-based)으로는 두 모델의 Precision 과 Recall 은 동일함

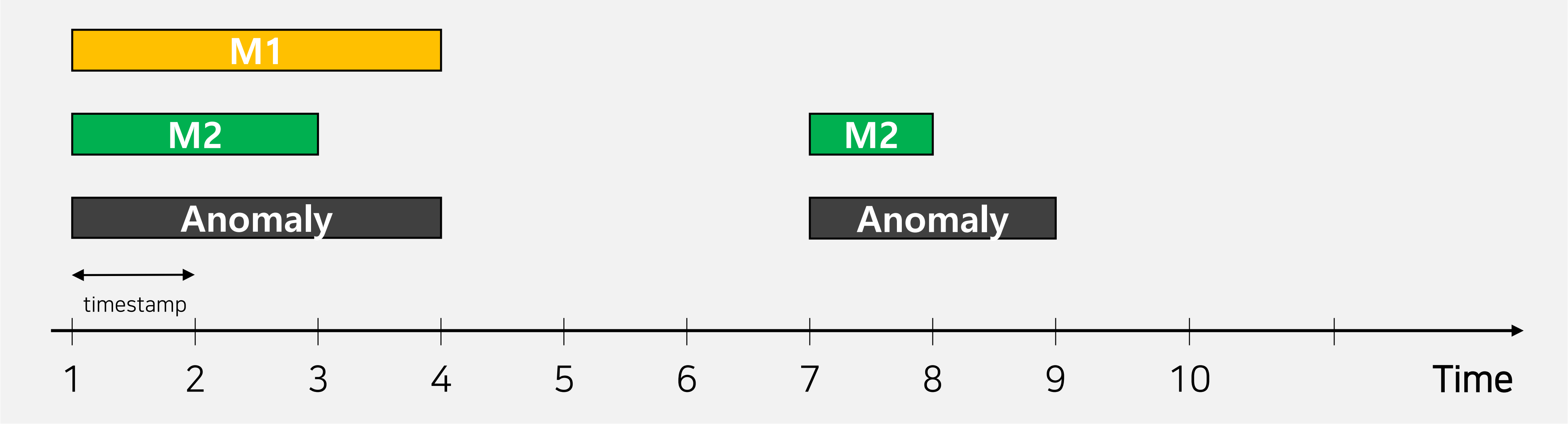
	Precision	Recall
M1	1.0 (3/3)	0.6 (3/5)
M2	1.0 (3/3)	0.6 (3/5)



Motivation

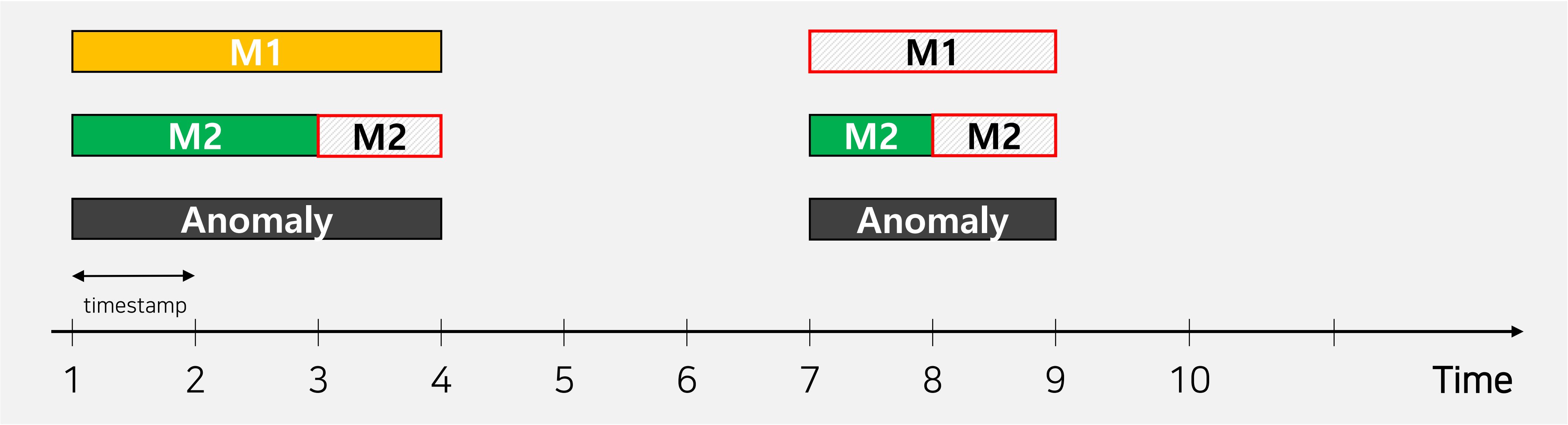
- Which model is better?
  - ✓ # timestapms : 10
  - ✓ # anomaly timestamps : 5
  - ✓ 기존의 방식(Point-based)으로는 두 모델의 Precision 과 Recall 은 동일함
  - ✓ 관점에 따라 어떤 모델을 더 높게 평가할지 결정할 수 있는 Metric 이 필요

	Precision	Recall
M1	1.0 (3/3)	0.6 (3/5)
M2	1.0 (3/3)	0.6 (3/5)



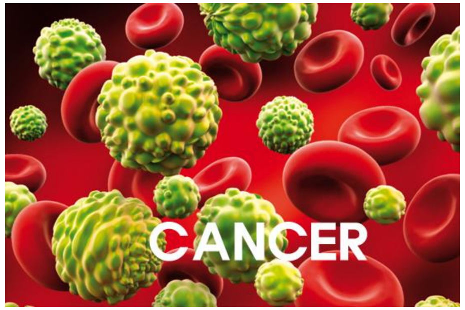
Motivation

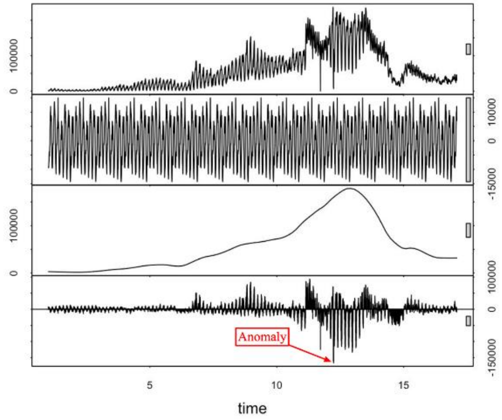
- Design Goals
  - ✓ Anomaly Range 를 부분적으로라도 탐지 한 것을 Score 에 반영 할 수 있어야 함
  - ✓ Anomaly Range 를 얼만큼 완벽하게 맞췄는지를 Score 에 반영 할 수 있어야 함
  - ✓ Missing rate / False Alarm 등에 대해 사용자가 어느 부분을 더 중요하게 생각하는지를 반영할 수 있어야 함



Motivation

- Design Goals
  - ✓ Missing rate / False Alarm 등에 대해 사용자가 어느 부분을 더 중요하게 생각하는지를 반영할 수 있어야 함
  - ➢ Cancer detection, Real-time system
    - ▪ Early response; Avoid false negatives
  - ➢ Robotic defense systems
    - ▪ Delayed response; Avoid false positives
  - ➢ Emergency breaking in self-driving cars
    - ▪ Neither too early nor too late; Avoid false negatives





[NeurIPS, 2018]

Precision Recall for Time Series

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Proposed Metrics

- Range-based recall

Notation	Description
R, R_i	set of real anomaly ranges, the i^{th} real anomaly range
P, P_j	set of predicted anomaly ranges, the j^{th} predicted anomaly range
N, N_r, N_p	number of all points, number of real anomaly ranges, number of predicted anomaly ranges
α	relative weight of existence reward
$\gamma()$, $\omega()$, $\delta()$	overlap cardinality function, overlap size function, positional bias function

$$Recall_T(R, P) = \frac{\Sigma_{i=1}^{N_r} Recall_T(R_i, P)}{N_r}$$

$$Recall_T(R_i, P) = \alpha \times \boxed{ExistenceReward(R_i, P)} + (1 - \alpha) \times \boxed{OverlapReward(R_i, P)}$$

$$\boxed{ExistenceReward(R_i, P)} = \begin{cases} 1 & , if\ \Sigma_{j=1}^{N_p} |R_i \cap P_j| \geq 1 \\ 0 & , \text{otherwise} \end{cases}$$

$$\boxed{OverlapReward(R_i, P)} = \boxed{CardinalityFactor(R_i, P)} \times \sum_{j=1}^{N_p} \omega(R_i, R_i \cap P_j, \delta)$$

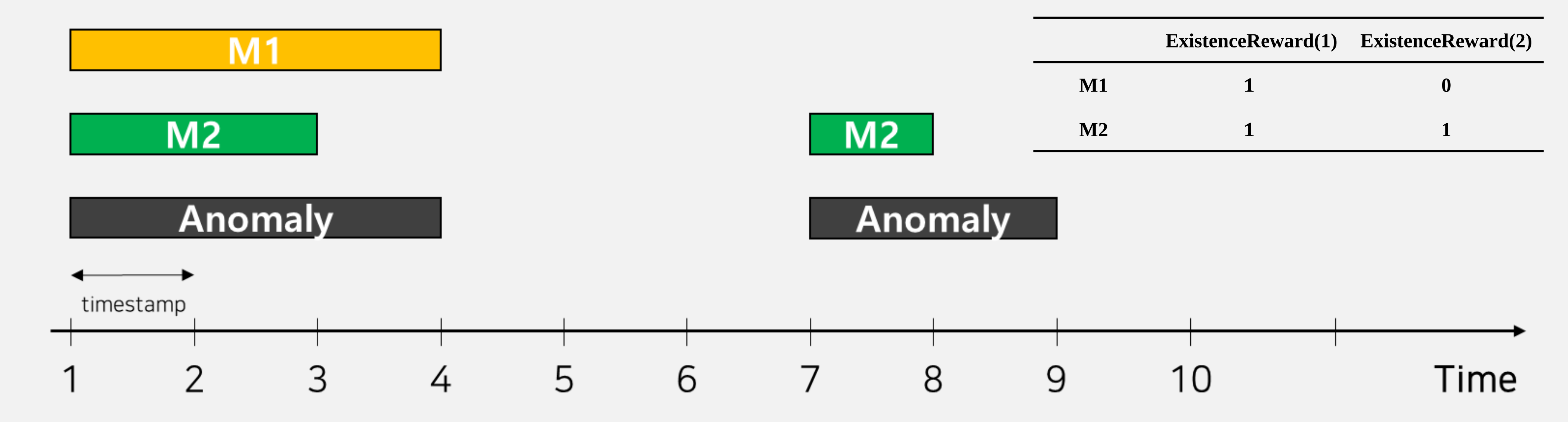
$$\boxed{CardinalityFactor(R_i, P)} = \begin{cases} 1 & , if\ R_i\ overlaps\ with\ at\ most\ one\ P_j \in P \\ \gamma(R_i, P) & , \text{otherwise} \end{cases}$$

Proposed Metrics

- Range-based recall – ExistenceReward

$$Recall_T(R_i, P) = \alpha \times \boxed{ExistenceReward(R_i, P)} + (1 - \alpha) \times \boxed{OverlapReward(R_i, P)}$$

$$\boxed{ExistenceReward(R_i, P)} = \begin{cases} 1 & , if\ \Sigma_{j=1}^{N_p} |R_i \cap P_j| \geq 1 \\ 0 & , \text{otherwise} \end{cases}$$



	ExistenceReward(1)	ExistenceReward(2)
M1	1	0
M2	1	1

Proposed Metrics

- Range-based recall – OverlapReward 中 CardinalityFactor

$$Recall_T(R_i, P) = \alpha \times \boxed{ExistenceReward(R_i, P)} + (1-\alpha) \times \boxed{OverlapReward(R_i, P)}$$

$$\boxed{OverlapReward(R_i, P)} = \boxed{CardinalityFactor(R_i, P)} \times \sum_{j=1}^{N_p} \omega(R_i, R_i \cap P_j, \delta)$$

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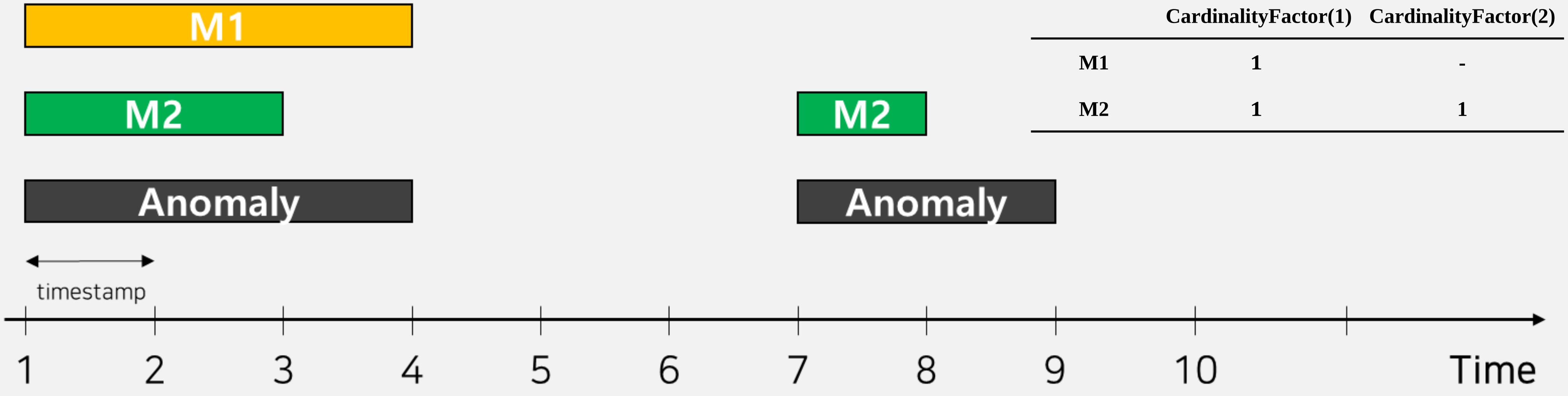

	CardinalityFactor(1)	CardinalityFactor(2)
M1	1	-
M2	1	1

Proposed Metrics

- Range-based recall – OverlapReward 中 CardinalityFactor(γ)

$$Recall_T(R_i, P) = \alpha \times \boxed{ExistenceReward(R_i, P)} + (1-\alpha) \times \boxed{OverlapReward(R_i, P)}$$

$$\boxed{OverlapReward(R_i, P)} = \boxed{CardinalityFactor(R_i, P)} \times \sum_{j=1}^{N_p} \omega(R_i, R_i \cap P_j, \delta)$$

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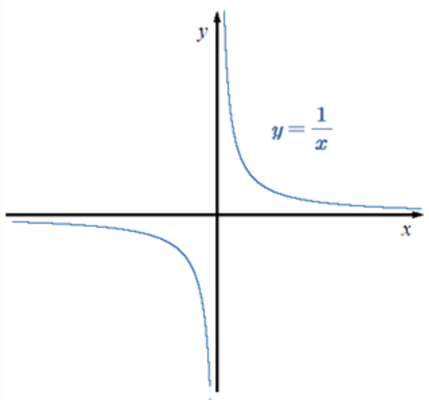

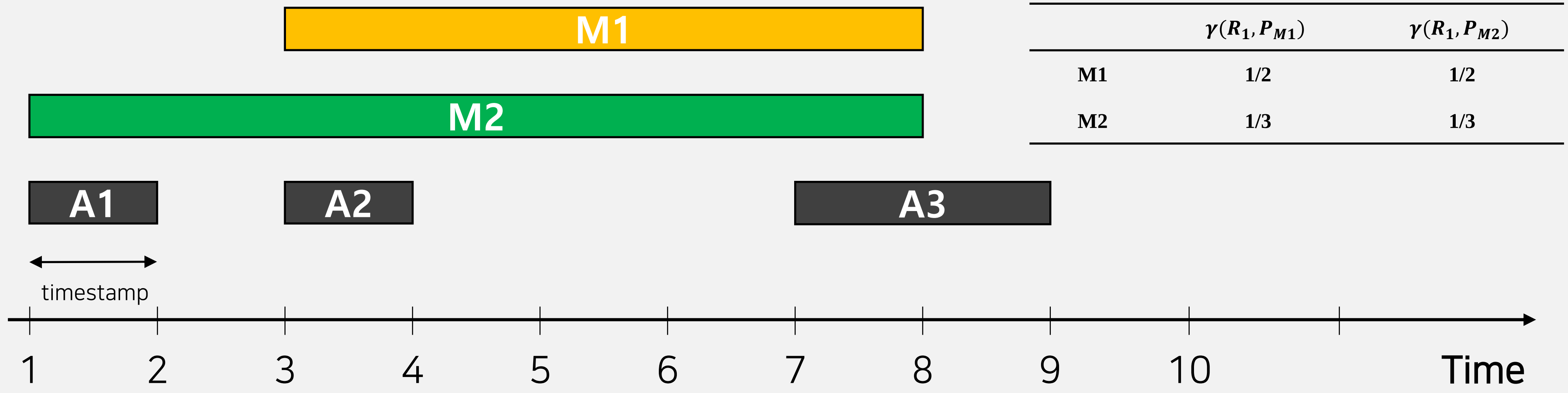

	$\gamma(R_1, P_{M1})$	$\gamma(R_1, P_{M2})$
M1	1/2	1/2
M2	1/3	1/3

Proposed Metrics

- Range-based recall – OverlapReward 中 ω, δ

$$Recall_T(R_i, P) = \alpha \times \boxed{ExistenceReward(R_i, P)} + (1-\alpha) \times \boxed{OverlapReward(R_i, P)}$$

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```
function ω(AnomalyRange, OverlapSet, δ)
    MyValue ← 0
    MaxValue ← 0
    AnomalyLength ← length(AnomalyRange)
    for i ← 1, AnomalyLength do
        Bias ← δ(i, AnomalyLength)
        MaxValue ← MaxValue + Bias
        if AnomalyRange[i] in OverlapSet then
            MyValue ← MyValue + Bias
    return MyValue/MaxValue
```

```
function δ(i, AnomalyLength)          ▷ Flat bias
    return 1
function δ(i, AnomalyLength)   ▷ Front-end bias
    return AnomalyLength - i + 1
function δ(i, AnomalyLength)    ▷ Back-end bias
    return i
function δ(i, AnomalyLength)       ▷ Middle bias
    if i ≤ AnomalyLength/2 then
        return i
    else
        return AnomalyLength - i + 1
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Proposed Metrics

$$Recall_T(R_i, P) = \alpha \times \boxed{ExistenceReward(R_i, P)} + (1 - \alpha) \times \boxed{OverlapReward(R_i, P)}$$

- Range-based recall – OverlapReward 中 ω, δ

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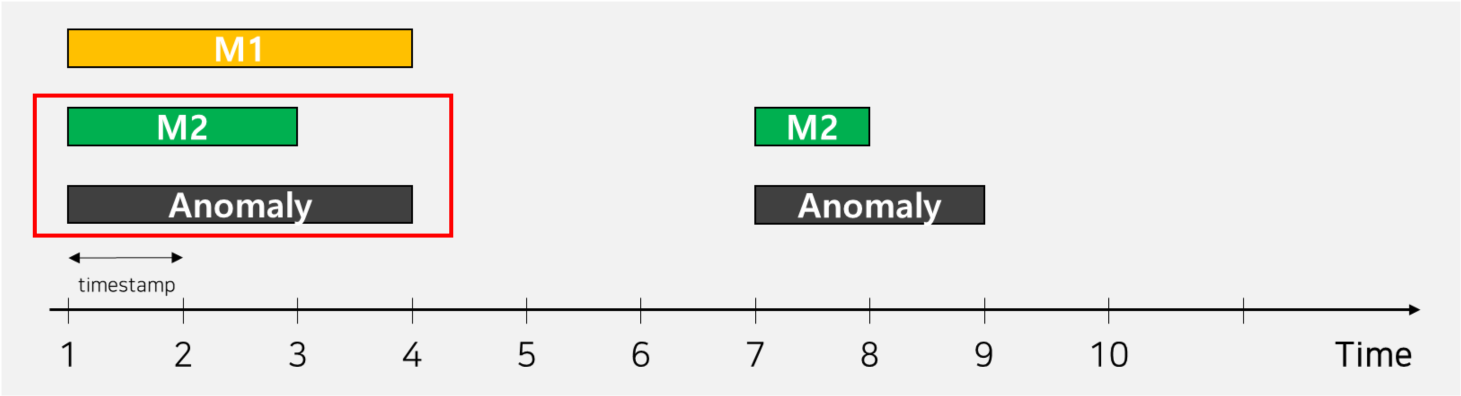


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```

Model	Range	BiasMode	AnomalyLength	i	Bias	MaxValue	MyValue	Output
M2	R1	Flat	3	0	-	0	0	-
				1	1	1	1	-
				2	1	2	2	
				3	1	3	2	2/3

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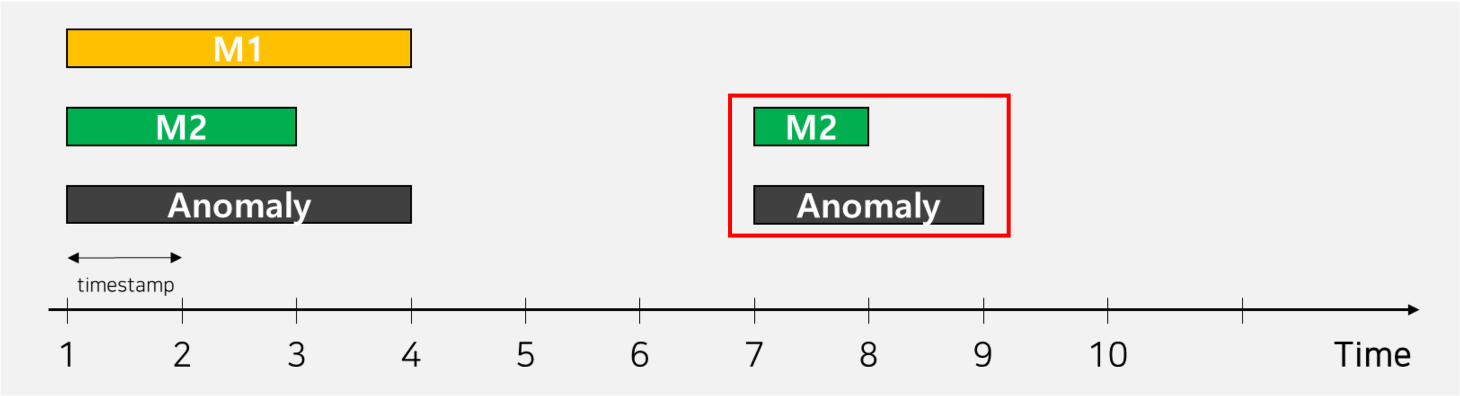

Model	Range	BiasMode	AnomalyLength	i	Bias	MaxValue	MyValue	Output
M2	R2	Flat	2	0	-	0	0	-
				1	1	1	1	-
				2	1	2	1	1/2

function ω(AnomalyRange, OverlapSet, δ)
  MyValue $\leftarrow 0$
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  AnomalyLength $\leftarrow$ length(AnomalyRange)
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    Bias $\leftarrow$ $\boxed{\delta(\text{i, AnomalyLength})}$
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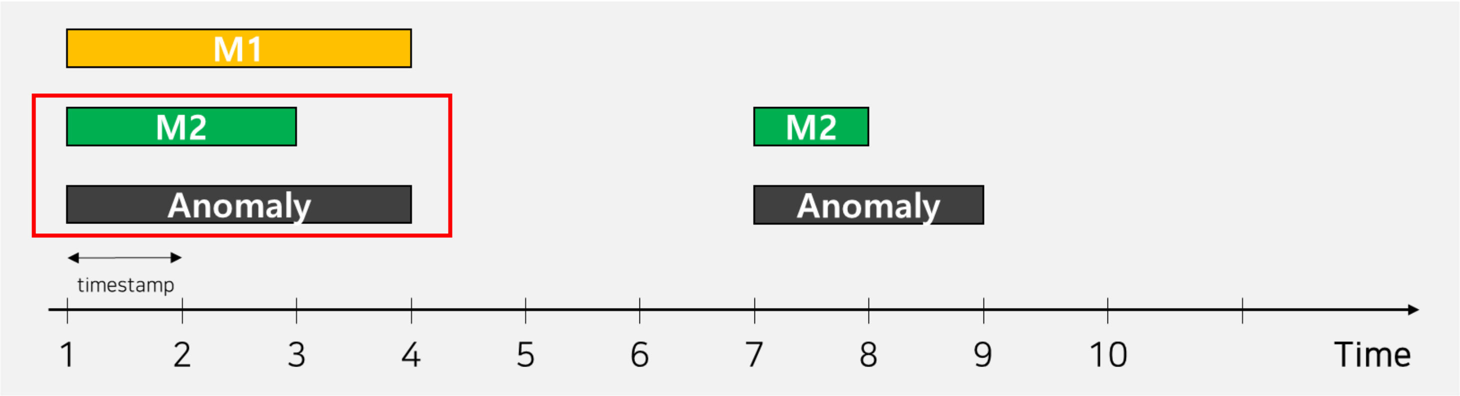


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```

Model	Range	BiasMode	AnomalyLength	i	Bias	MaxValue	MyValue	Output
M2	R1	Front	3	0	-	0	0	-
				1	3(3-1+1)	3	3	-
				2	2(3-2+1)	5	5	
				3	1(3-3+1)	6	6	5/6

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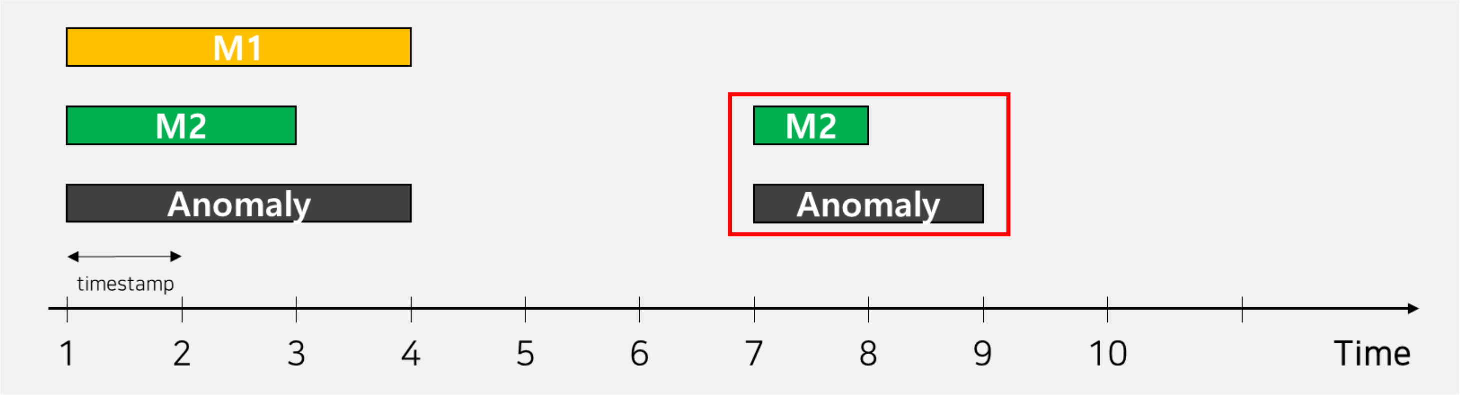

Model	Range	BiasMode	AnomalyLength	i	Bias	MaxValue	MyValue	Output
M2	R2	Front	2	0	-	0	0	-
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				2	1(2-2+1)	3	2	2/3

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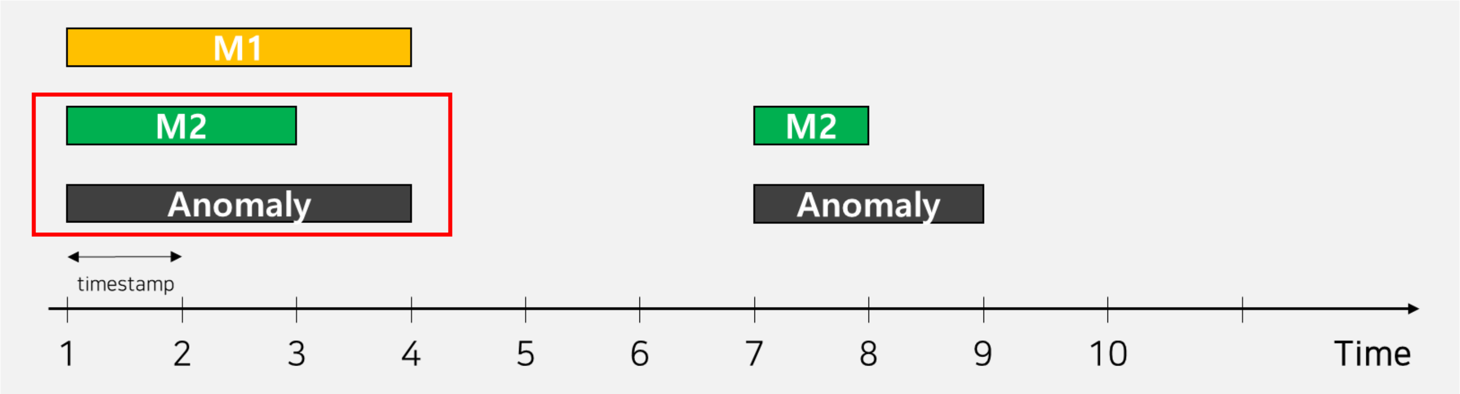


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				3	3	6	3	1/2

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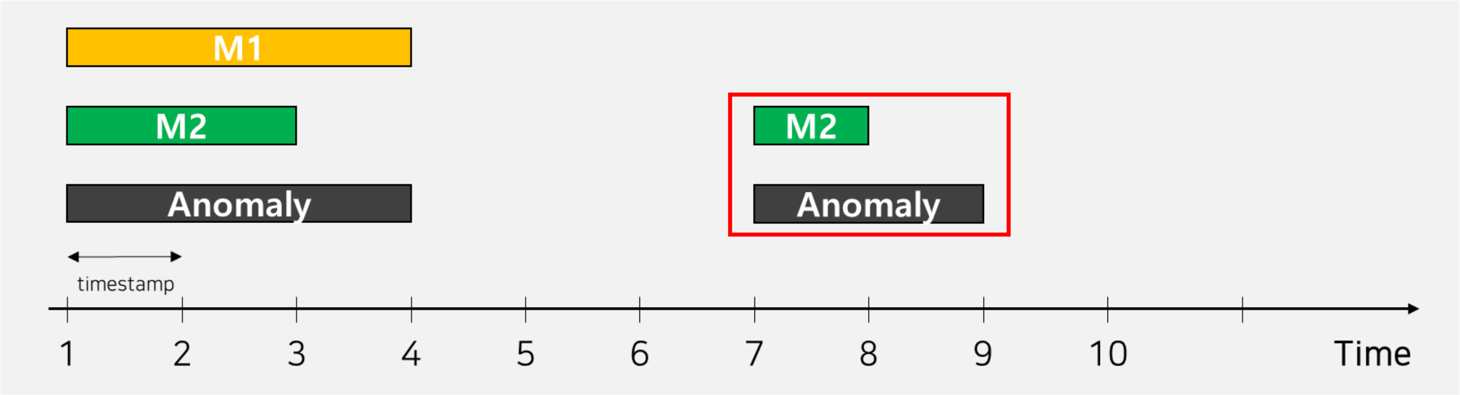
- Range-based recall – OverlapReward 中 ω, δ



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				2	2	3	1	1/3

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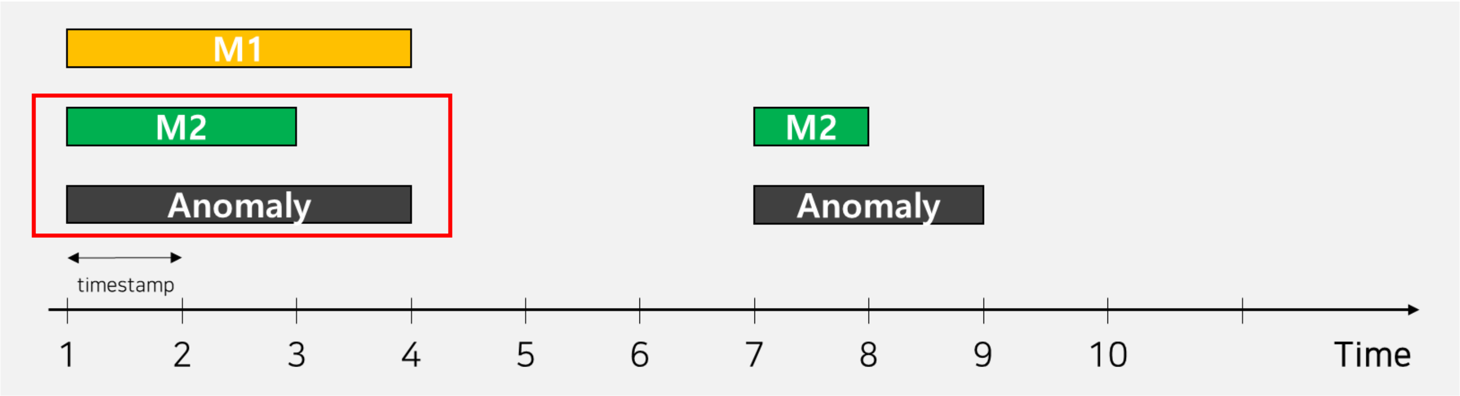

Model	Range	BiasMode	AnomalyLength	i	Bias	MaxValue	MyValue	Output
M2	R1	Middle	3	0	-	0	0	-
				1	1	1	1	-
				2	2(3-2+1)	3	3	
				3	1(3-3+1)	4	3	3/4

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function ω(AnomalyRange, OverlapSet, δ)
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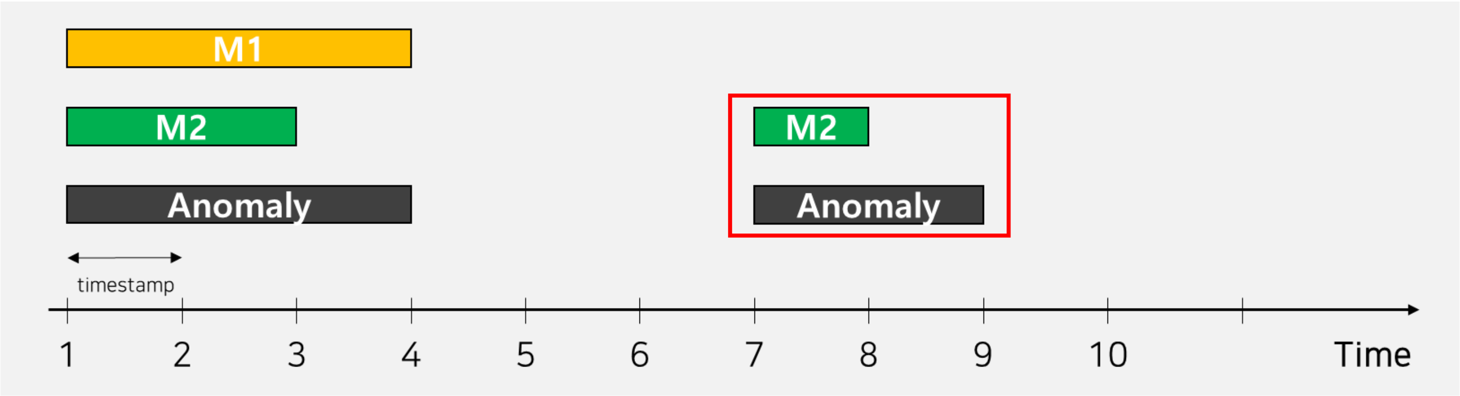
- Range-based recall – OverlapReward 中 ω, δ



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				2	1(2-2+1)	2	1	1/2

```
function ω(AnomalyRange, OverlapSet, δ)
    MyValue ← 0
    MaxValue ← 0
    AnomalyLength ← length(AnomalyRange)
    for i ← 1, AnomalyLength do
        Bias ← δ(i, AnomalyLength)
        MaxValue ← MaxValue + Bias
        if AnomalyRange[i] in OverlapSet then
            MyValue ← MyValue + Bias
    return MyValue/MaxValue
```

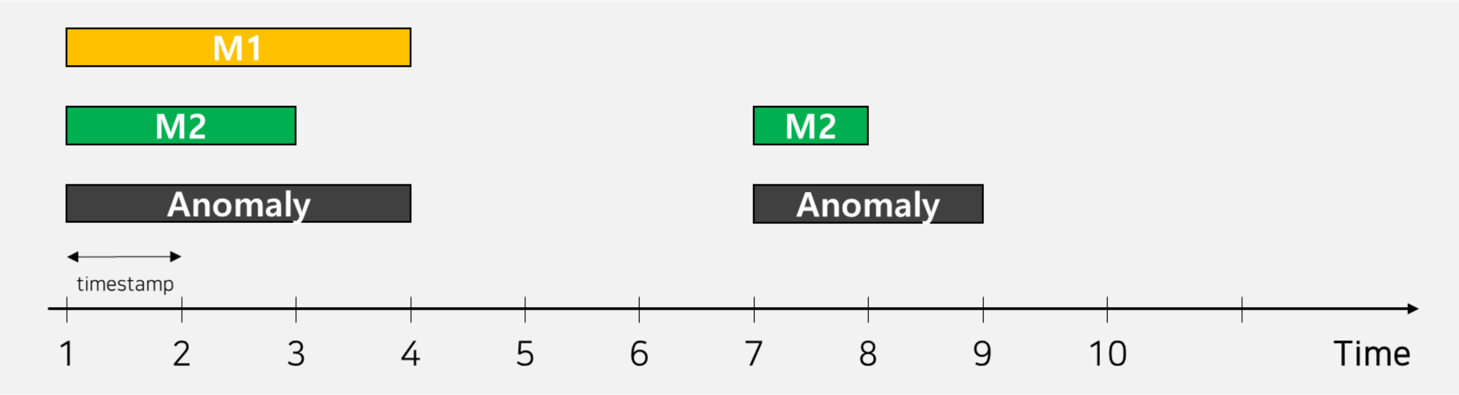
```
function δ(i, AnomalyLength)          ▷ Flat bias
    return 1
function δ(i, AnomalyLength)          ▷ Front-end bias
    return AnomalyLength - i + 1
function δ(i, AnomalyLength)          ▷ Back-end bias
    return i
function δ(i, AnomalyLength)          ▷ Middle bias
    if i ≤ AnomalyLength/2 then
        return i
    else
        return AnomalyLength - i + 1
```

Proposed Metrics

$$Recall_T(R_i, P) = \alpha \times \boxed{ExistenceReward(R_i, P)} + (1 - \alpha) \times \boxed{OverlapReward(R_i, P)}$$

- Range-based recall – OverlapReward 中 ω, δ

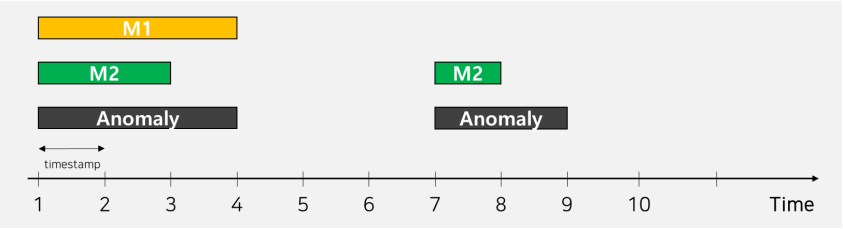
$$\boxed{OverlapReward(R_i, P)} = \boxed{CardinalityFactor(R_i, P)} \times \sum_{j=1}^{N_p} \omega(R_i, R_i \cap P_j, \delta)$$



Model	Range	Flat	Front	Back	Middle
M1	R1	1	1	1	1
	R2	0	0	0	0
M2	R1	2/3	5/6(▲)	1/2(▼)	3/4(▲)
	R2	1/2	2/3(▲)	1/3(▼)	1/2

Proposed Metrics

- Range-based recall – Summary



$$Recall_T(R, P) = \frac{\Sigma_{i=1}^{N_r} Recall_T(R_i, P)}{N_r}$$

$$Recall_T(R_i, P) = \alpha \times ExistenceReward(R_i, P) + (1 - \alpha) \times OverlapReward(R_i, P)$$

Model	Range	$Recall_T$	ExistenceReward	OverlapReward	CardinalityFactor	ω(δ=Flat)
M1	R1	1	1	1	1	1
	R2	0	0	0	-	0
M2	R1	0.83	1	2/3	1	2/3
	R2	0.75	1	1/2	1	1/2

Model	Alpha	$Recall$	$Recall_T$
M1	0.5	0.6 (3/5)	0.5 (1+0)/2
M2	0.5	0.6 (3/5)	0.79 (0.83+0.75)/2

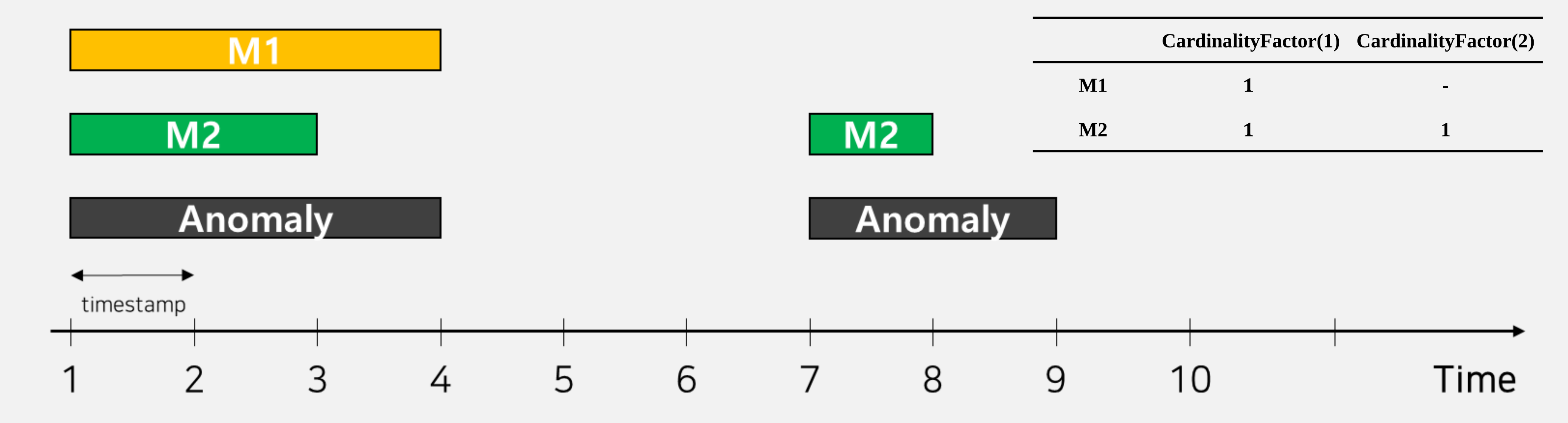
Proposed Metrics

- Range-based precision

Notation	Description
R, R_i	set of real anomaly ranges, the i^{th} real anomaly range
P, P_j	set of predicted anomaly ranges, the j^{th} predicted anomaly range
N, N_r, N_p	number of all points, number of real anomaly ranges, number of predicted anomaly ranges
α	relative weight of existence reward
$\gamma()$, $\omega()$, $\delta()$	overlap cardinality function, overlap size function, positional bias function

$$Precision_T(R, P) = \frac{\Sigma_{i=1}^{N_r} Precision_T(R, P_i)}{N_p}$$

$$Precision_T(R, P_i) = CardinalityFactor(P_i, R) \times \sum_{j=1}^{N_r} \omega(P_i, P_i \cap R_j, \delta)$$



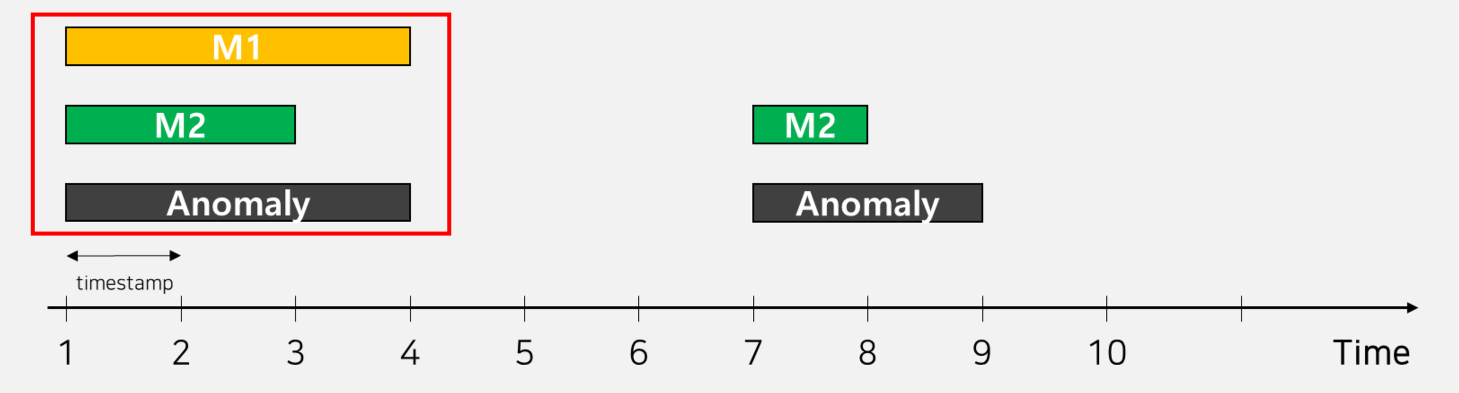
	CardinalityFactor(1)	CardinalityFactor(2)
M1	1	-
M2	1	1

Proposed Metrics

- Range-based precision – OverlapReward 中 ω, δ

$$Precision_T(R, P) = \frac{\Sigma_{i=1}^{N_r} Precision_T(R, P_i)}{N_p}$$

$$Precision_T(R, P_i) = CardinalityFactor(P_i, R) \times \sum_{j=1}^{N_r} \omega(P_i, P_i \cap R_j, \delta)$$



Model	Range	BiasMode	AnomalyLength	i	Bias	MaxValue	MyValue	Output
M1	R1	Flat	3	0	-	0	0	-
				1	1	1	1	-
				2	1	2	2	-
				3	1	3	3	3/3
M2	R1	Flat	2	0	-	0	0	-
				1	1	1	1	-
				2	1	2	2	2/2

function ω(AnomalyRange, OverlapSet, δ)
  MyValue $\leftarrow 0$
  MaxValue $\leftarrow 0$
  AnomalyLength $\leftarrow$ length(AnomalyRange)
  for i $\leftarrow 1$, AnomalyLength **do**
    Bias $\leftarrow \boxed{\delta(\text{i, AnomalyLength})}$
    MaxValue $\leftarrow$ MaxValue + Bias
    if AnomalyRange[i] in OverlapSet **then**
      MyValue $\leftarrow$ MyValue + Bias
  return MyValue/MaxValue

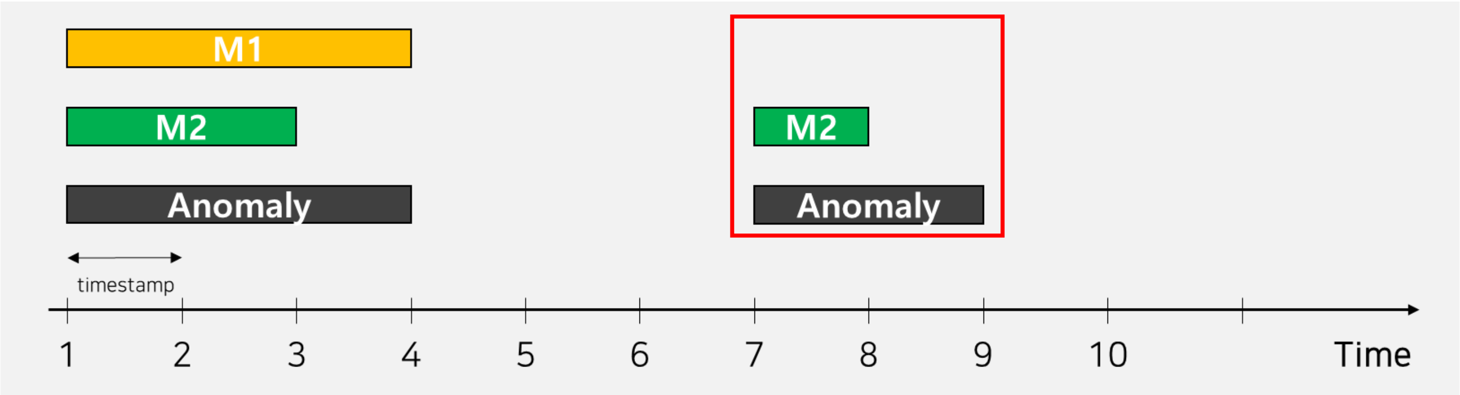
function δ(i, AnomalyLength) ▷ Flat bias
  return 1
function δ(i, AnomalyLength) ▷ Front-end bias
  return AnomalyLength - i + 1
function δ(i, AnomalyLength) ▷ Back-end bias
  return i
function δ(i, AnomalyLength) ▷ Middle bias
  if i $\leq$ AnomalyLength/2 **then**
    return i
  else
    return AnomalyLength - i + 1

Proposed Metrics

- Range-based recall – OverlapReward 中 ω, δ

$$Precision_T(R, P) = \frac{\Sigma_{i=1}^{N_r} Precision_T(R, P_i)}{N_p}$$

$$Precision_T(R, P_i) = CardinalityFactor(P_i, R) \times \sum_{j=1}^{N_r} \omega(P_i, P_i \cap R_j, \delta)$$



Model	Range	BiasMode	AnomalyLength	i	Bias	MaxValue	MyValue	Output
M1	-	-	-	-	-	-	-	-
M2	R2	Flat	1	0	-	0	0	-
				1	1	1	1	1/1

function ω(AnomalyRange, OverlapSet, δ)
    MyValue $\leftarrow 0$
    MaxValue $\leftarrow 0$
    AnomalyLength $\leftarrow$ length(AnomalyRange)
    for i $\leftarrow 1$, AnomalyLength **do**
        Bias $\leftarrow$ $\boxed{\delta(\text{i, AnomalyLength})}$
        MaxValue $\leftarrow$ MaxValue + Bias
        if AnomalyRange[i] in OverlapSet **then**
            MyValue $\leftarrow$ MyValue + Bias
    return MyValue/MaxValue

function δ(i, AnomalyLength) ▷ Flat bias
    return 1

function δ(i, AnomalyLength) ▷ Front-end bias
    return AnomalyLength - i + 1

function δ(i, AnomalyLength) ▷ Back-end bias
    return i

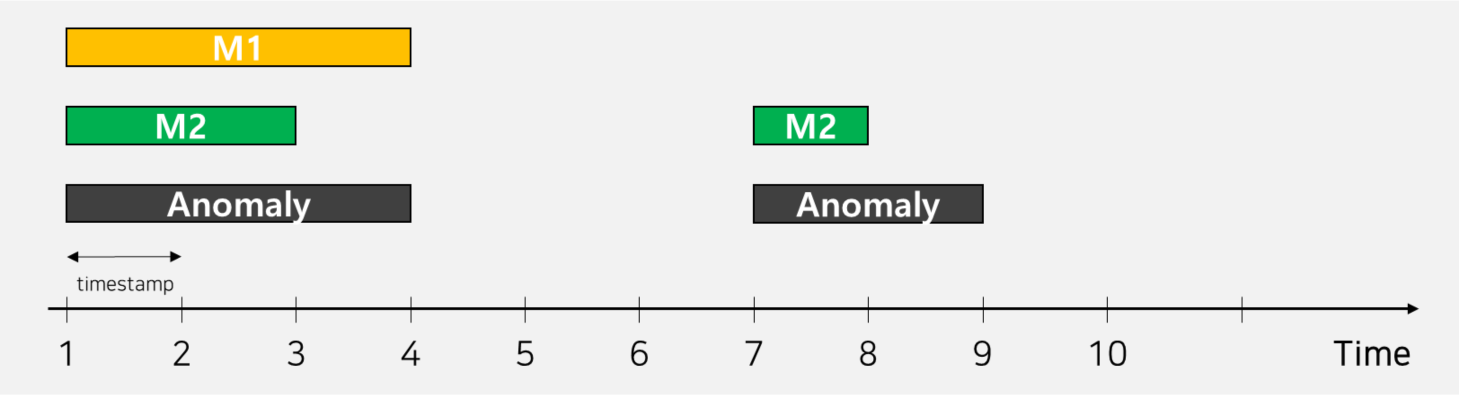
function δ(i, AnomalyLength) ▷ Middle bias
    if i $\leq$ AnomalyLength/2 **then**
        return i
    else
        return AnomalyLength - i + 1

Proposed Metrics

- Range-based precision – OverlapReward 中 ω, δ

$$Precision_T(R, P) = \frac{\Sigma_{i=1}^{N_r} Precision_T(R, P_i)}{N_p}$$

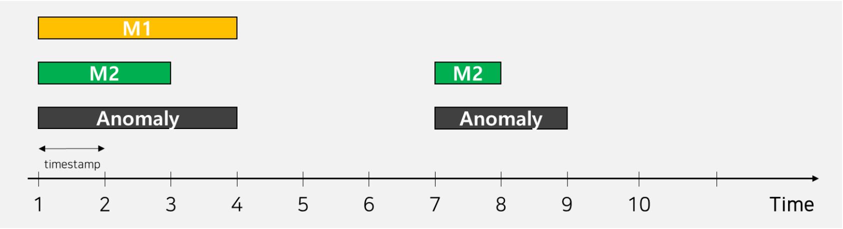
$$Precision_T(R, P_i) = CardinalityFactor(P_i, R) \times \sum_{j=1}^{N_r} \omega(P_i, P_i \cap R_j, \delta)$$



Model	Range	Flat	Front	Back	Middle
M1	R1	1	1	1	1
	R2	0	0	0	0
M2	R1	1	1	1	1
	R2	1	1	1	1

Proposed Metrics

- Range-based precision – Summary



$$Precision_T(R,P) = \frac{\Sigma_{i=1}^{N_r} Precision_T(R,P_i)}{N_p}$$

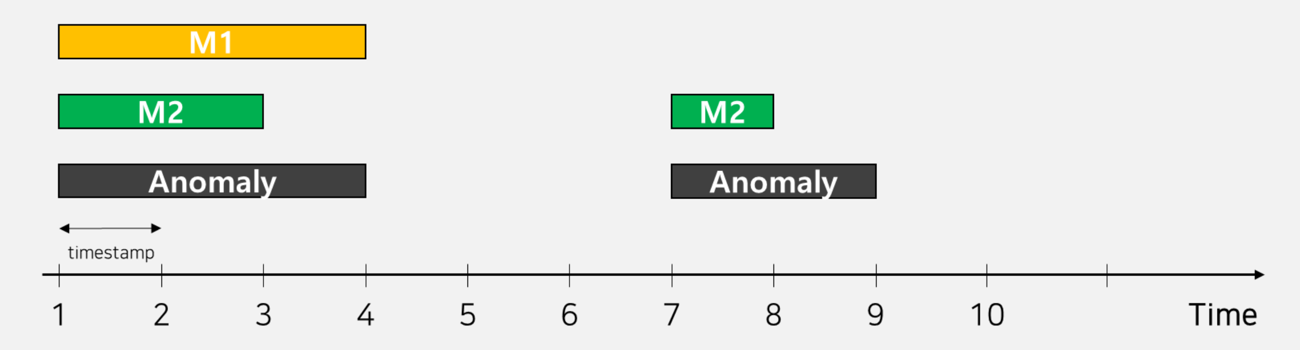
$$Precision_T(R,P_i) = CardinalityFactor(P_i,R) \times \sum_{j=1}^{N_r} \omega(P_i, P_i \cap R_j, \delta)$$

Model	Range	$Precision_T$	CardinalityFactor	ω
M1	R1	1	1	1
	R2	0	-	0
M2	R1	1	1	1
	R2	1	1	1

Model	*Precision*	$Precision_T$
M1	1.0	0.5
M2	1.0	1.0

Proposed Metrics

- Range-based precision & recall – Summary
  - ✓ M2 is better than M1



	$Precision$	$Precision_T$	$Recall$	$Recall_T$
M1	**1.0**	**0.5**	**0.6**	**0.5**
M2	**1.0**	**1.0**	**0.6**	**0.79**

[ACM CIKM, 2019]

Time-Series Aware Precision and Recall for Anomaly Detection

Considering Variety of Detection Result and Addressing Ambiguous Labeling

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Reason for Selection

학습 및 검증

1) (모델 학습) 학습 데이터셋만을 사용하여 모델을 학습합니다.

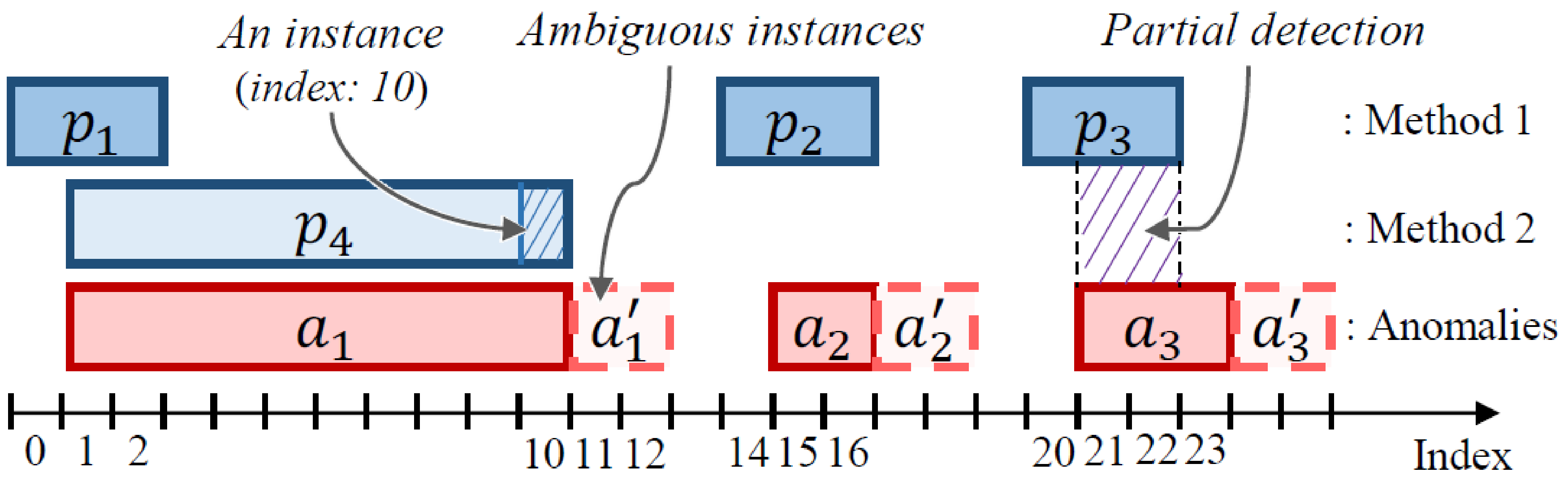
2) (모델 검증) 평가도구(TaPR)와 검증 데이터셋으로 모델을 검증합니다.

* 평가도구 'etapr.evalute()' 함수만을 사용하여 평가(코드공유 탭의 baseline 코드 참조)

* 평가도구에 대한 상세 설명은 토론 탭에 있는 "TaPR설명서" 게시물을 참조

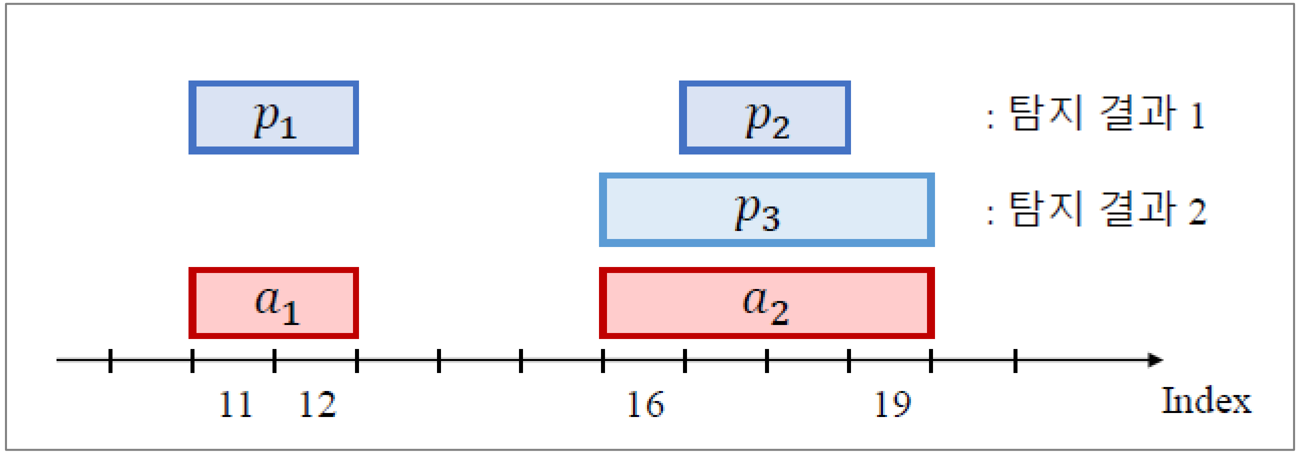
Motivation

- Which model is better?
  - ✓ RR_PR 에서 설명한 것과 같은 문제점에 대한 고민에서 출발 (giving a high score to the method that only detects a long anomaly)
  - ✓ Detection scoring(≒ExistenceReward)
  - ✓ Portion scoring(≒OverlapReward)
  - ✓ Subsequent scoring(consider ambiguous instances) 개념을 추가하여, 이에 대한 고려방안 제시



Motivation

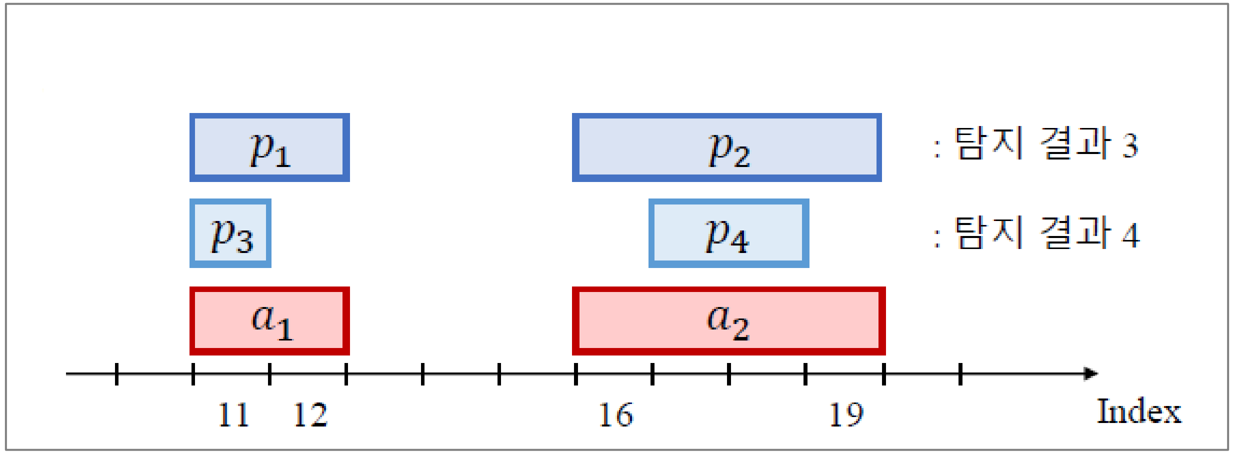
- Design Goals - ① 탐지된 공격의 다양성 평가
  - ✓ 아래 그림의 예시에서, 두 결과 모두 4초의 데이터를 탐지하였으나 결과 1은 두가지 공격의 영향을 모두 탐지 했으므로 결과 1을 더 높게 평가



[출처]
https://www.slideshare.net/daconist/etapr-237428659?ref=https://dacon.io/competitions/official/235624/talkboard/401260?page=1&dtype=recent&ptype=pub

Motivation

- Design Goals - ② 탐지의 정확성
  - ✓ 최대한 정확하게 탐지할수록 높은 점수 부여
  - ✓ 아래의 그림에서는 탐지 결과 3이 4보다 더 높게 평가됨



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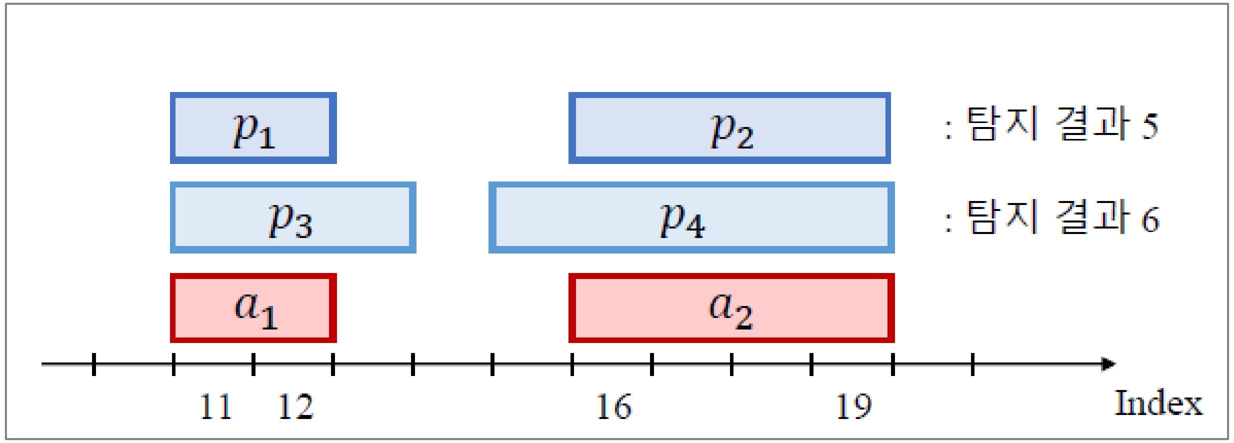
https://www.slideshare.net/daconist/etapr-237428659?ref=https://dacon.io/competitions/official/235624/talkboard/401260?page=1&dtype=recent&ptype=pub

Motivation

- Design Goals - ③ 낮은 오탐
  - ✓ 정답 외의 시각을 탐지하지 않을수록 높은 점수 부여
  - ✓ 아래의 그림에서는 탐지 결과 5가 6보다 더 높게 평가됨



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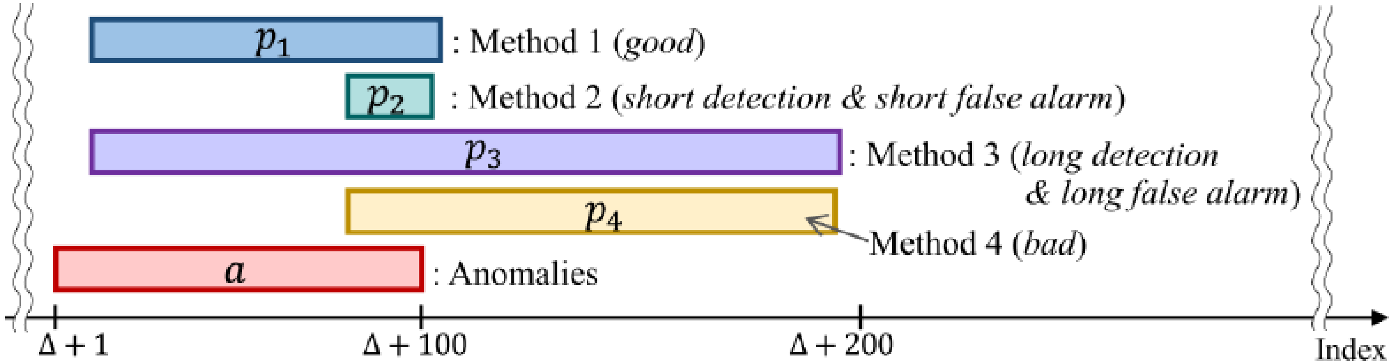
Motivation

- TaPR 은 상기 목표들을 만족하는 평가 방법을 제공
  - ✓ TaP : 예측 결과가 오탐 없이 이상 징후를 찾아내는가?(0~1)
  - ✓ TaR : 얼마나 다양한 이상 범위를 찾아내는가?(0~1)



예측 결과	TaP	TaR
p_1	High	High
p_2	High	Low
p_3	Low	High
p_4	Low	Low

[출처]
https://www.slideshare.net/daconist/etapr-237428659?ref=https://dacon.io/competitions/official/235624/talkboard/401260?page=1&dtype=recent&ptype=pub

Proposed Metrics

- Terminology

a	$instances\ in\ each\ range$	$a = \{t, t+1, \ldots, t+l-1\}$
l	$the\ number\ of\ instances\ in\ each\ range$	$\|a\|$
A	$A\ set\ of\ anomaly\ ranges$	$A = \{a_1, a_2, \ldots, a_n\}$
p	$predictions\ in\ each\ range$	$p = \{t', t'+1, \ldots, t'+l'-1\}$
P	$A\ set\ of\ prediction\ ranges$	$P = \{p_1, p_2, \ldots, p_m\}$
a'	$instances\ in\ each\ ambiguous\ range$	$a' = \{t+l, t+l+1, \ldots, t+l+\delta-1\}$
A'	$A\ set\ of\ ambiguous\ ranges$	$A' = \{a'_1, a'_2, \ldots, a'_n\}$

Proposed Metrics

- Time-Series Aware Recall

$$TaR = \alpha \times TaR^d + (1-\alpha) \times TaR^p$$

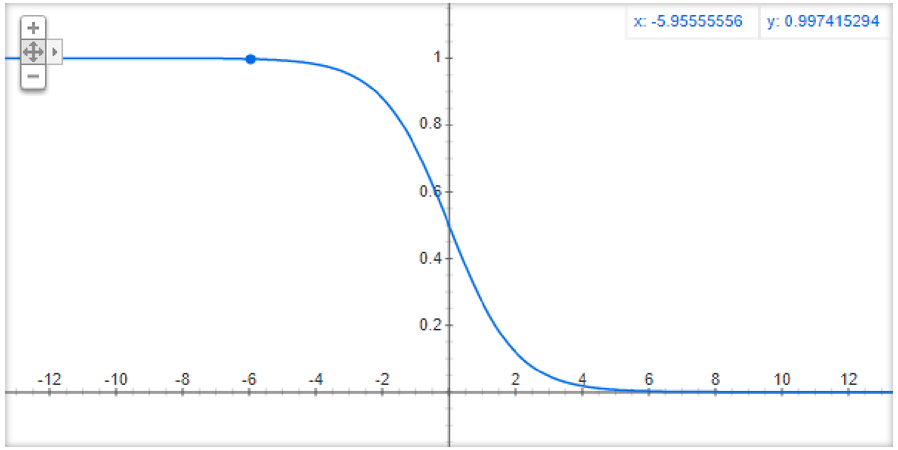
$$TaR^d = \frac{|A^d(\theta)|}{|A|},\ where\ A^d(\theta) = \{a | a \in A\ and\ \frac{\Sigma_{p \in P} O(a,p)}{|a|} \geq \theta\}$$

$$O(a,p) = |a \cap p| + S(a',p)$$

$$S(a',p) = \sum_{i \in (a' \cap p)} \frac{1}{1+e^{i'}},\ where\ i' = -6 + \frac{12(i - t_{a'} - 1)}{\delta - 1}$$

$$TaR^p = \frac{1}{|A|} \times \sum_{a \in A} min\left(1, \frac{\Sigma_{p \in P} O(a,p)}{|a|}\right)$$

1/(1+e^x) 그래프



Proposed Metrics

- Time-Series Aware Recall

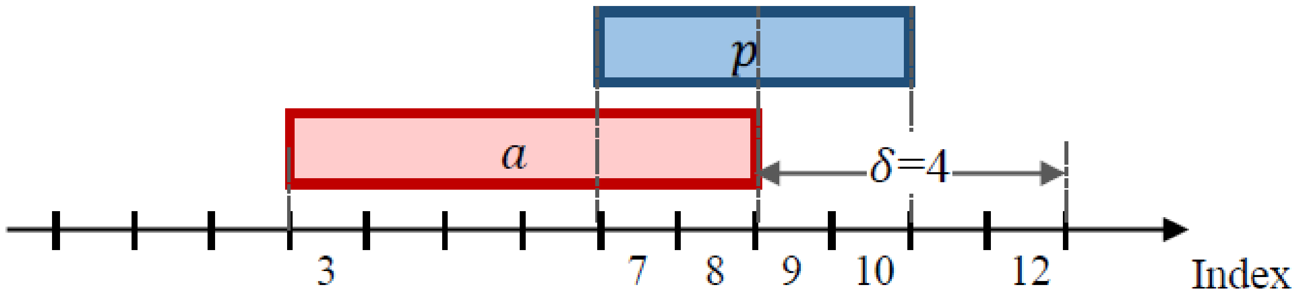
$$TaR = \alpha \times TaR^d + (1-\alpha) \times TaR^p$$

$$TaR^d = \frac{|A^d(\theta)|}{|A|}, \ where \ A^d(\theta) = \{a | a \in A \ and \ \frac{\Sigma_{p \in P} O(a,p)}{|a|} \geq \theta\}$$

$$O(a,p) = |a \cap p| + S(a',p)$$

$$S(a',p) = \sum_{i \in (a' \cap p)} \frac{1}{1+e^{i'}}, \ where \ i' = -6 + \frac{12(i - t_{a'} - 1)}{\delta - 1}$$

$$TaR^p = \frac{1}{|A|} \times \sum_{a \in A} min\left(1, \frac{\Sigma_{p \in P} O(a,p)}{|a|}\right)$$



Proposed Metrics

- Time-Series Aware Recall
  - ✓ 아래의 그림과 같이 1개의 anomaly만 있다고 가정했을 때, |A| = 1, |a| = 6
  - ✓ $3.88 / 6 = 0.6466\ (\geq \theta = 0.5)$
  - ✓ $TaR^{d} = 1,\ TaR^{p} = 0.6466,\quad \mathrm{TaR} = 0.8233(\alpha = 0.5)$

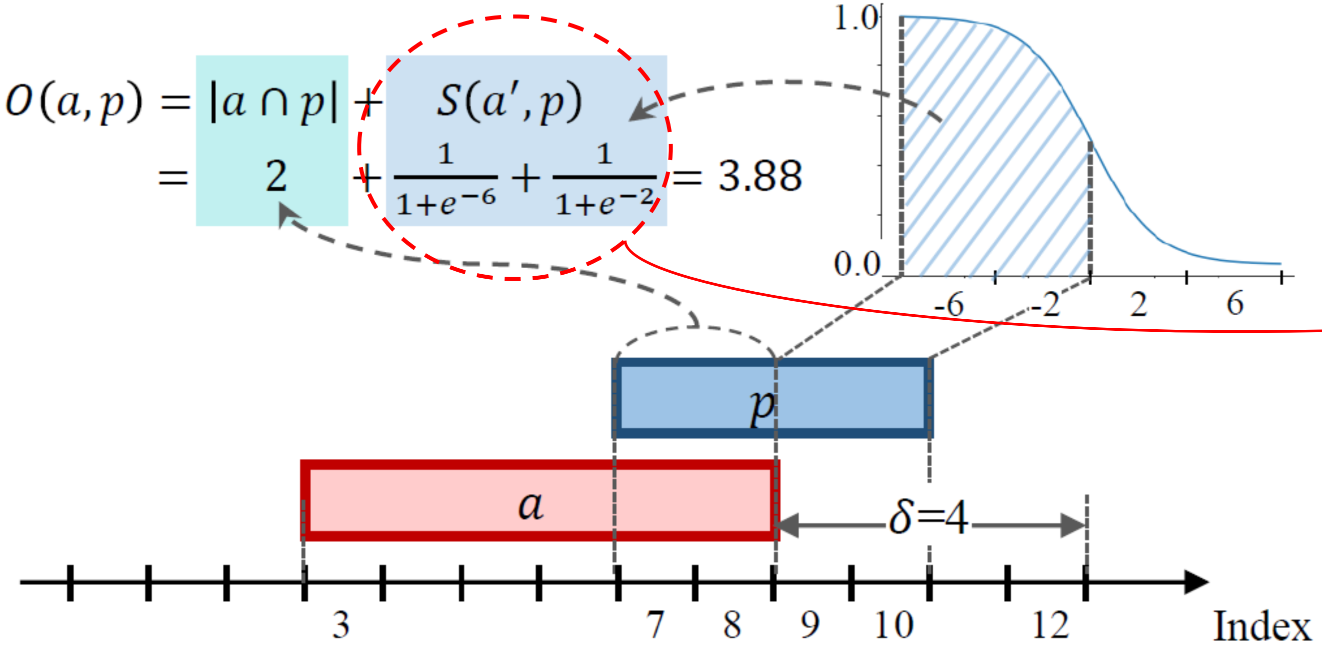
$$TaR = \alpha \times TaR^{d} + (1-\alpha) \times TaR^{p}$$

$$TaR^{d} = \frac{|A^{d}(\theta)|}{|A|},\ where\ A^{d}(\theta) = \{a | a \in A\ and\ \frac{\Sigma_{p \in P} O(a,p)}{|a|} \geq \theta\}$$

$$O(a,p) = |a \cap p| + S(a',p)$$

$$S(a',p) = \sum_{i \in (a' \cap p)} \frac{1}{1+e^{i'}},\ where\ i' = -6 + \frac{12(i - t_{a'} - 1)}{\delta - 1}$$

$$TaR^{p} = \frac{1}{|A|} \times \sum_{a \in A} min\left(1, \frac{\Sigma_{p \in P} O(a,p)}{|a|}\right)$$



-6+12(9-8-1)/-1 = -6
-6+12(10-8-1)/-1 = -2

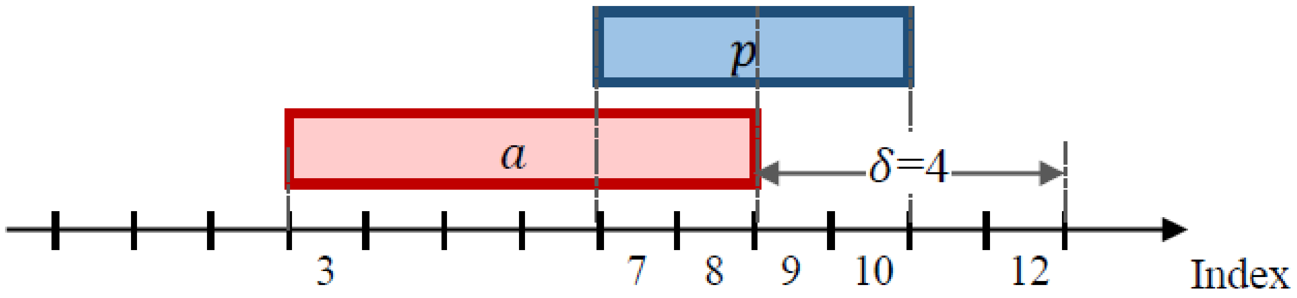
Proposed Metrics

- Time-Series Aware Precision

$$TaP = \alpha \times TaP^d + (1-\alpha) \times TaP^p$$

$$TaP^d = \frac{P^c(\theta)}{|P|},\ where\ P^c(\theta) = \{p | p \in P\ and\ \frac{\Sigma_{a \in A} O(a,p)}{|p|} \geq \theta\}$$

$$TaP^p = \frac{1}{|P|} \times \sum_{p \in P} min\left(\frac{\Sigma_{a \in A} O(a,p)}{|p|}\right)$$



$TaP^d = 1$

$TaP^p = 0.97(3.88/4)$

$TaP = 0.985(\alpha = 0.5)$

Q & A

Thank You!