Heterogeneous Beliefs, Risk Amplification, and Asset Returns

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Before each financial crisis:

- Asset price↑, capital investment↑, and leverage↑
- Mechanism: many investors believe asset price↑ in the future ⟶ leverage↑ ⟶ risk amounts↑ ⟶ (big enough) negative shock ⟶ crisis

And then everything crashes↓: why?

- Net worth of experts (i.e., marginal investors)↓ ⟶ less productive households take up capital ⟶ asset price↓
- Market (endogenous) volatility↑ and risk-premium↑

Then we get out of crises again:

- During crises, risk-premium↑ ⟶ experts recapitalized ⟶ exit

"Boom-bust cycle with endogenous volatility"

Big Question (Main Topic)

What if investors have heterogeneous beliefs about the economy's direction (i.e., underlying data-generating process)?

- How does the belief heterogeneity affect the endogenous market volatility's amplification during crises?
- The severity, duration (of each), and frequency of crises change. How?

Observations:

1. Markets are turbulent $\longrightarrow$ it is more likely that different market participants have different ideas about the financial market's direction
2. Before and during crises:
   - $\exists$Investors betting on the market (who think market will $\uparrow$)
   - $\exists$Investors betting against the market (who think market will $\downarrow$)
   - For example, for 08'-09' on or against the US housing market

What we do

Our Framework:

- Experts and households with single capital: experts' output production technology is superior, similar to Brunnermeier and Sannikov (2014)

Introduce (exogenous) technological growth:

- Technologies of both experts and households have the same growth rate in the true data-generating process
- However, experts believe that their technological (expected) growth is higher (lower), i.e., experts are optimistic (pessimistic)
- Households believe that their technological (expected) growth is lower (higher), i.e., households are pessimistic (optimistic)

Big Findings (Adverse 'Doom-Loop')

1. Belief heterogeneity$\uparrow$ $\longrightarrow$ more amplified (endogenous) volatility$\uparrow$
2. Endogenous volatility$\uparrow$ $\longrightarrow$ belief heterogeneity about (capital) returns$\uparrow$ $\longrightarrow$ volatility$\uparrow$ $\longrightarrow$ ad infinitum

Findings

In the presence of heterogeneous beliefs: when experts are more optimistic[1]

During normal:

1. Facilitated trade: investment↑, asset price↑, and leverage↑ than the rational expectations case

2. Risk bearing↑ $\longrightarrow$ chance of entering financial crises↑

During crisis:

1. Endogenous volatility↑ and (both true and perceived) risk-premium↑: more amplification

2. $\exists$Adverse 'doom-loop' between belief heterogeneity about asset returns and the amplification of risks

3. Each crisis' duration↓ with experts' faster recapitalization, but:

Number of 'shorter-lived and more severe' crises↑↑
$\longrightarrow$ On average more time in crises per year

[1] The case where experts are pessimistic can be characterized with the opposite results

Basic framework based on Brunnermeier and Sannikov (2014)

- Continuous-time models: Basak and Cuoco (1998), He and Krishnamurthy (2011, 2013), and Di Tella (2017)[2]
- Financial frictions, heterogeneous beliefs, and/or other deviations from the rational expectations case: Harrison and Kreps (1978), Detemple and Murthy (1997), Basak and Croitoru (2000), Basak (2000), Croitoru and Basak (2004), Gallmeyer and Hollifield (2008), and Maxted (2022)[3]
- Market selection hypothesis: Blume and Easley (2006)[4]
- Heterogeneous beliefs about risk-premium, financial markets, and the macroeconomy (e.g., inflation): Welch (2000), Coibion et al. (2020), Candia et al. (2021), and Weber et al. (2022), and Beutel and Weber (2022)[5]
- Nominal rigidity (demand-determined): Caballero and Simsek (2020)

[2] Di Tella (2017) studies uncertainty shocks driving balance sheet recessions even in cases when contracting on the macroeconomic state variable is possible

[3] Maxted (2022) incorporates diagnostic expectations into a model with intermediaries based on He and Krishnamurthy (2013)

[4] Under the market selection hypothesis, markets favor agents with more accurate beliefs: it- does not hold in our framework, as markets are incomplete

[5] Beutel and Weber (2022) point out that individuals are heterogeneous both at the information acquisition and the processing stage, thereby forming their own beliefs and choosing portfolios based on them

The Economic Environment

Single capital: owned by optimists and pessimists

Optimists: produces $\underline{y_t^O = \gamma_t^O k_t^O}$, $\forall t \in [0, \infty)$ where

$$\frac{dk_t^O}{k_t^O} = \left(\Lambda^O(\boxed{\iota_t^O}) - \delta^O\right) dt, \quad \forall t \in [0, \infty)$$

Investment ratio

Their investment$= \iota_t^O y_t^O$

with technological growth:

$$\frac{d\gamma_t^O}{\gamma_t^O} = \boxed{\alpha} dt + \sigma \underbrace{dZ_t}_{\text{Brownian motion}}, \quad \forall t \in [0, \infty)$$

True (expected) growth

Pessimists: produces $\underline{y_t^P = \gamma_t^P k_t^P}$, $\forall t \in [0, \infty)$ where

$$\frac{dk_t^P}{k_t^P} = \left(\Lambda^P(\boxed{\iota_t^P}) - \delta^P\right) dt, \quad \forall t \in [0, \infty)$$

Investment ratio
Their investment= $\iota_t^P y_t^P$

with the same technological growth:

$$\frac{d\gamma_t^P}{\gamma_t^P} = \boxed{\alpha} dt + \sigma \underbrace{dZ_t}_{\text{Brownian motion}}, \quad \forall t \in [0, \infty)$$

True (expected) growth

$\longrightarrow$ **Level difference**: $\gamma_t^P = l \cdot \gamma_t^O$, $\Lambda^P(\cdot) = l \cdot \Lambda^O(\cdot)$, with $\underline{l \leq 1}$ (efficiency$\downarrow$)

- Efficiency in both production and capital formation$\downarrow$

Capital price process: (endogenous) p_t follows

$$\frac{dp_t}{p_t} = \mu_t^p dt + \boxed{\sigma_t^p} dZ_t$$

Endogenous volatility

Capital return process

- Optimists' total return on capital:

$$dr_t^{Ok} = \underbrace{\frac{\gamma_t^O \cancel{k_t^O} - \iota_t^O \gamma_t^O \cancel{k_t^O}}{p_t \cancel{k_t^O}}}_{\text{Dividend yield}} dt + \underbrace{\left(\Lambda^O(\iota_t^O) - \delta^O + \mu_t^p\right) dt + \sigma_t^p dZ_t}_{\text{Capital gain}}$$

$$= \frac{1 - \iota_t^O}{\underbrace{q_t}_{\substack{\text{Price-earnings ratio} \\ \text{(optimists)}}}} dt + \left(\Lambda^O(\iota_t^O) - \delta^O + \mu_t^p\right) dt + \sigma_t^p dZ_t$$

- Pessimists' total return on capital:

$$dr_t^{Pk} = \frac{\gamma_t^P \cancel{k_t^P} - \iota_t^P \gamma_t^P \cancel{k_t^P}}{p_t \cancel{k_t^P}} dt + \left(\Lambda^P(\iota_t^P) - \delta^P + \mu_t^p\right) dt + \sigma_t^p dZ_t$$

Optimists: believe γ_t^O follows

$$\frac{d\gamma_t^O}{\gamma_t^O} = \boxed{\alpha^O} dt + \sigma \underbrace{dZ_t^O}_{\text{Optimists' Brownian Motion}}, \quad \forall t \in [0, \infty)$$

Possibly different from α

even if the true process is given as

$$\frac{d\gamma_t^O}{\gamma_t^O} = \alpha dt + \sigma \underbrace{dZ_t}_{\text{True Brownian Motion}}$$

with the following consistency (see e.g., Yan (2008)):

$$\underbrace{Z_t^O}_{\text{Optimists' BM}} = \underbrace{Z_t}_{\text{True BM}} - \frac{\alpha^O - \alpha}{\sigma} t$$

Note that optimists:

- can infer a true value of σ by calculating the process' quadratic variation
- are *dogmatic*: believing the expected technological growth $\alpha^O \neq \alpha$

Pessimists: believe γ_t^P follows

$$\frac{d\gamma_t^P}{\gamma_t^P} = \boxed{\alpha^P} dt + \sigma \underbrace{dZ_t^P}_{\substack{\text{Pessimists'} \\ \text{Brownian Motion}}}, \quad \forall t \in [0, \infty)$$

Possibly different from α

even if the true process is given as

$$\frac{d\gamma_t^P}{\gamma_t^P} = \alpha dt + \sigma \underbrace{dZ_t}_{\substack{\text{True} \\ \text{Brownian Motion}}}$$

with the following consistency (see e.g., Yan (2008)):

$$\underbrace{Z_t^P}_{\text{Optimists' BM}} = \underbrace{Z_t}_{\text{True BM}} - \frac{\alpha^P - \alpha}{\sigma} t$$

Classifications:

- With $\alpha^O > \alpha > \alpha^P$: experts (households) are optimists (pessimists)
- With $\alpha^O < \alpha < \alpha^P$: experts (households) are pessimists (optimists)

Perceived capital return process

- Optimists' total return on capital:

$$dr_t^{Ok} = \underbrace{\frac{\gamma_t^O \cancel{k_t^O} - \iota_t^O \gamma_t^O \cancel{k_t^O}}{p_t \cancel{k_t^O}}}_{\text{Dividend yield}} dt + \underbrace{\left(\Lambda^O(\iota_t^O) - \delta^O + \mu_t^p\right) dt + \sigma_t^p dZ_t}_{\text{Capital gain}}$$

$$= \frac{\gamma_t^O - \iota_t^O \gamma_t^O}{p_t} dt + \left(\Lambda^O(\iota_t^O) - \delta^O + \mu_t^p + \boxed{\frac{\alpha^O - \alpha}{\sigma} \sigma_t^p}\right) dt + \sigma_t^p dZ_t^O$$

Belief (perceived) premium

- Pessimists' total return on capital:

$$dr_t^{Pk} = \frac{\gamma_t^P - \iota_t^P \gamma_t^P}{p_t} dt + \left(\Lambda^P(\iota_t^P) - \delta^P + \mu_t^p + \frac{\alpha^P - \alpha}{\sigma} \sigma_t^p\right) dt + \sigma_t^p dZ_t^P$$

Observation (Belief heterogeneity in asset returns)

(Endogenous) volatility↑ $\longrightarrow$ belief heterogeneity in asset return↑

Financial market: capital and risk-free (zero net-supplied)

Optimists: consumption-portfolio problem (price-taker) Solution

$$\max_{\iota_t^O \geq 0, x_t \geq 0, c_{t,}^O \geq 0} \boxed{\mathbb{E}_0^O} \left[\int_0^\infty e^{-\rho^O t} \log \left(c_t^O \right) dt \right]$$

Believes dZ_t^O is the true BM

subject to

$$dw_t^O = x_t w_t^O dr_t^{Ok} + (1 - x_t) r_t w_t^O dt - c_t^O dt, \quad \text{and} \quad \underbrace{w_t^O \geq 0}_{\text{Solvency constraint}}$$

Pessimists: solve the similar problem with $\boxed{\mathbb{E}_0^P}$ ($\neq \mathbb{E}_0$ or $\mathbb{E}_0^O$)

Believes dZ_t^P is the true BM

Total capital $K_t = k_t^O + \underline{k}_t^P$ evolves with

$$\frac{dK_t}{dt} = \underbrace{\left(\Lambda^O\left(\iota_t^O\right) - \delta^O\right) k_t^O}_{\text{From optimists}} + \underbrace{\left(\Lambda^P\left(\underline{\iota}_t^P\right) - \delta^P\right) \underline{k}_t^P}_{\text{From pessimists}}, \quad \forall t \in [0, \infty)$$

Debt is zero net-supplied as

$$\underbrace{\left(w_t^O - p_t k_t^O\right)}_{\substack{\text{Optimists'} \\ \text{lending}}} + \underbrace{\left(\underline{w}_t^P - p_t \underline{k}_t^P\right)}_{\substack{\text{Pessimists'} \\ \text{lending}}} = 0$$

Good market equilibrium is represented by

$$\underbrace{\frac{x_t^O w_t^O}{p_t}\left(\gamma_t^O - \iota_t^O \gamma_t^O\right)}_{\substack{\text{Optimists'} \\ \text{production} \\ \text{net of investment}}} + \underbrace{\frac{\underline{x}_t^P \underline{w}_t^P}{p_t}\left(\gamma_t^P - \underline{\iota}_t^P \gamma_t^P\right)}_{\substack{\text{Pessimists'} \\ \text{production} \\ \text{net of investment}}} = c_t^O + \underline{c}_t^P$$

Proportion of optimists' wealth as state variable, similarly to Brunnermeier and Sannikov (2014):

$$\eta_t \equiv \frac{w_t^O}{w_t^O + \underline{w}_t^P} \underbrace{=}_{\text{Debt market equilibrium}} \frac{w_t^O}{p_t K_t}$$

which leads to:

$$x_t \leq \frac{1}{\eta_t}$$

- When it binds - 'normal' (all capital is owned by experts)
- When it does not bind - 'crisis' (less productive households must hold capital)

Under Markov equilibrium: normalized variables depend only on η_t

$$\longrightarrow q_t = q(\eta_t),\ x_t = x(\eta_t),\ \underbrace{\psi_t}_{\text{Capital share (optimists)}} = \psi(\eta_t)$$

Analysis: Markov Equilibrium

Investment function

$$\Lambda^O(\iota_t^O) = \frac{1}{k}\left(\sqrt{1+2k\iota_t^O} - 1\right), \quad \forall t \in [0,\infty)$$

with

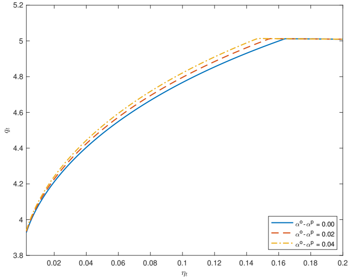
$$\Lambda^P(\iota_t) = l \cdot \Lambda^O(\iota_t), \quad \forall \iota_t \tag{1}$$

Parametrization:

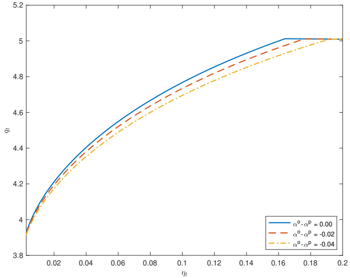
	l	δ^O	δ^P	ρ^O	ρ^P	χ	σ	k	α
Values	0.4	0	0	0.07	0.065	1	0.08	18	0.05

Table: Parameterization

- $\underline{\alpha^O > \alpha > \alpha^P}$ case (i.e., experts are optimistic):
  $$\alpha^O = \{0.05, 0.06, 0.07\}, \quad \alpha^P = \{0.05, 0.04, 0.03\}, \quad \alpha^O + \alpha^P = 0.1$$
- $\underline{\alpha^O < \alpha < \alpha^P}$ case (i.e., experts are pessimistic):
  $$\alpha^O = \{0.1, 0.09, 0.08\}, \quad \alpha^P = \{0.1, 0.11, 0.12\}, \quad \alpha^O + \alpha^P = 0.1$$



(a) $\alpha^O \geq \alpha$ and $\alpha^P \leq \alpha$

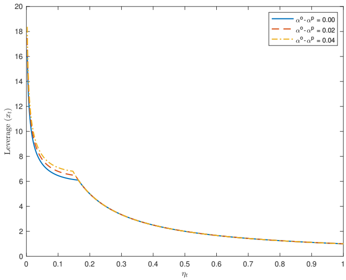


(b) $\alpha^O \leq \alpha$ and $\alpha^P \geq \alpha$

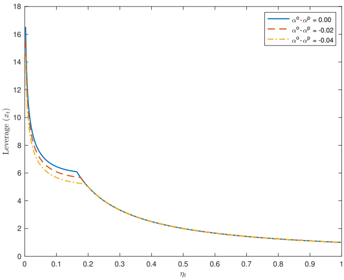
Figure: Price-earnings ratio q_t as a function of η_t

Efficient and crisis regions: threshold η^ψ

- With $\underline{\alpha^O > \alpha > \alpha^P}$, $\eta^\psi \downarrow$ as $\alpha^O \uparrow$: even with low wealth, optimists' demand for capital is strong (so leverage$\uparrow$)
- And then crisis (i.e., $\eta \leq \eta^\psi$) $\longrightarrow$ steeper decline in q_t (i.e., more elastic)



(a) $\alpha^O \geq \alpha$ and $\alpha^P \leq \alpha$



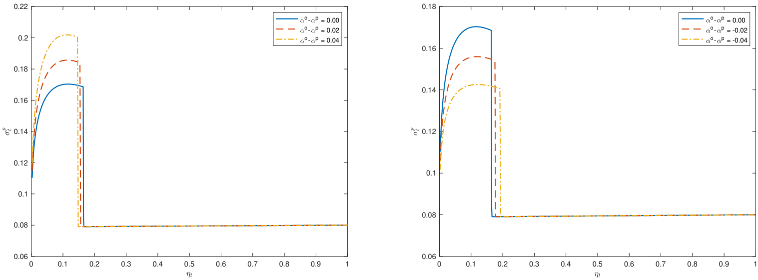
(b) $\alpha^O \leq \alpha$ and $\alpha^P \geq \alpha$

Figure: Leverage x_t as a function of η_t

Efficient and crisis regions: threshold η^ψ

- With $\underline{\alpha^O > \alpha > \alpha^P}$, $\eta^\psi \downarrow$ as $\alpha^O \uparrow$: even with low wealth, optimists' demand for capital is strong (so leverage$\uparrow$)
- And then crisis (i.e., $\eta \leq \eta^\psi$) $\longrightarrow$ higher leverage (a perceived risk-premium is high)

Endogenous volatility



(a) $\alpha^O \geq \alpha$ and $\alpha^P \leq \alpha$ (b) $\alpha^O \leq \alpha$ and $\alpha^P \geq \alpha$

Figure: Endogenous Volatility σ_t^p as a function of η_t

Efficient and crisis regions: threshold η^ψ

- With $\underline{\alpha^O > \alpha > \alpha^P}$, $\eta^\psi\downarrow$ as $\alpha^O\uparrow$: even with low wealth, optimists' demand for capital is strong (so leverage$\uparrow$)
- And then crisis (i.e., $\eta \leq \eta^\psi$) $\longrightarrow$ more risk amplification ($\sigma_t^p\uparrow$) $\longrightarrow$ belief disagreement on asset return$\uparrow$ $\longrightarrow$ amplification $\sigma_t^p\uparrow$ $\longrightarrow$ ad infinitum

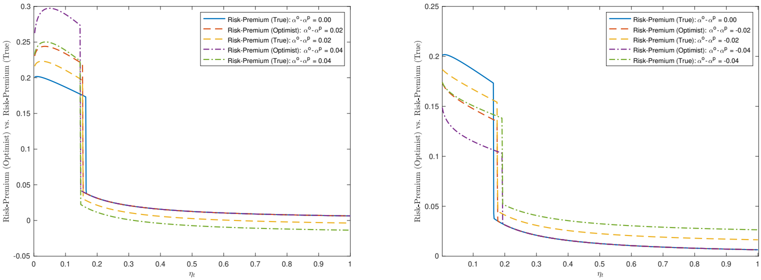
Equilibrium endogenous volatility σ_t^p is written as

$$\sigma_t^p \left(1 - (x_t - 1) \frac{\frac{dq(\eta_t)}{q(\eta_t)}}{\frac{d\eta_t}{\eta_t}} \right) \equiv \sigma_t^p \left(1 - (x_t - 1)\, \varepsilon_{q,\eta}\right) = \underbrace{\sigma}_{\substack{\text{Exogenous} \\ \text{volatility}}}$$

- $\varepsilon_{q,\eta}$ is the elasticity of the price-earnings ratio (i.e., normalized capital price) with respect to the experts' wealth share η_t

- '*Market illiquidity*' *effect*: as $\alpha^O \uparrow$, % increase in $\eta_t \longrightarrow$ higher % increases in the price of capital in the inefficient region $\longrightarrow \sigma_t^p \uparrow$

- '*Leverage*' *effect*: as $\alpha^O \uparrow$, experts take more leverage (i.e., $x_t \uparrow$) $\longrightarrow$ more fire-sale during crises $\longrightarrow \sigma_t^p \uparrow$

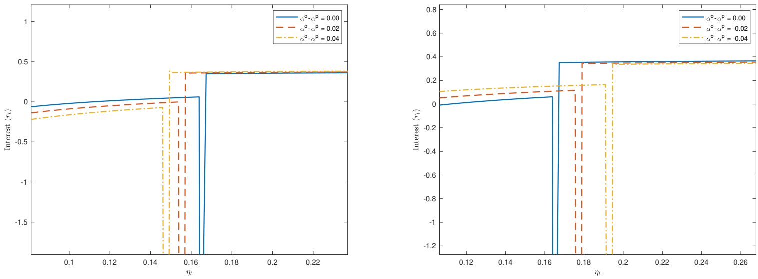


(a) $\alpha^O \geq \alpha$ and $\alpha^P \leq \alpha$ (b) $\alpha^O \leq \alpha$ and $\alpha^P \geq \alpha$

Figure: Risk-Premium (Optimists' and True Value) as a Function of η_t

Efficient and crisis regions: threshold η^ψ

- With $\underline{\alpha^O > \alpha > \alpha^P}$, $\alpha^O\uparrow \longrightarrow$ both true and optimists' perceived risk-premium$\uparrow$
- It helps optimists get recapitalized $\longrightarrow$ the economy gets out of crisis faster
- Each crisis lasts for shorter duration (i.e., shorter-lived)



(a) $\alpha^O \geq \alpha$ and $\alpha^P \leq \alpha$

(b) $\alpha^O \geq \alpha$ and $\alpha^P \leq \alpha$

Figure: Interest Rate r_t as a function of η_t: $\alpha^O \leq \alpha$ and $\alpha^P \geq \alpha$

Efficient and crisis regions: threshold η^ψ

- Downward spike in r_t at η^ψ: the moment experts start a fire-sale of capital
- With $\underline{\alpha^O > \alpha > \alpha^P}$, a higher leverage $x_t \longrightarrow r_t\uparrow$ in 'normal'
- During crises (i.e., $\eta_t \leq \eta^\psi$), $\alpha^O\uparrow \longrightarrow r_t\downarrow$: higher demand for safety with precautionary motive

Other graphs

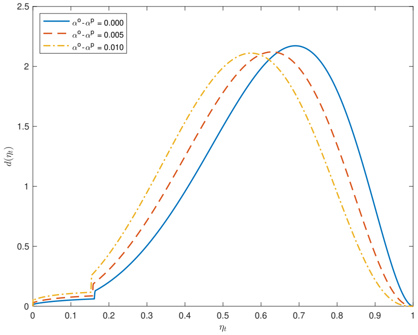


Figure: Ergodic Distribution of η_t

Efficient and crisis regions: threshold η^{ψ}

- With $\underline{\alpha^O > \alpha > \alpha^P}$, $\alpha^O\uparrow \longrightarrow$ the economy spends more time in crises per year, even if each crisis on average lasts for shorter duration
- Number of 'shorter-lived and more severe' crises$\uparrow\uparrow$: optimistic experts bear too much risk during 'normal'

Observation through model:

- Disagreement in technological growth $\longrightarrow$ more amplified risk-premium as well as risk itself

- Disagreement in prospects of the economic (technological) growth can be a separate factor in addition to a wealth share of the intermediary (i.e., experts)

Augment He, Kelly, and Manela (2017) with the disagreement factor for:

1. Cross-sectional asset pricing

2. Conditional return-predictability (e.g., during crises)
   - During crisis, more likely to have persistent return due to the behaviroal factor (i.e., momentum)
   - But the economy enters the normal, featuring reversal (i.e., reversal)
   - Can explain Cujean and Hasler (2017) that the conditional predictability is concentrated in bad times

$$D_t = \frac{f_{75} - f_{25}}{|f_{50}|}$$

- f_k: $k\%$ percentile analyst forecast of quarter-on-quarter GDP growth rate for the $T+2^{\text{th}}$ quarter ahead at date T, from Survey of Professional Forecasters (SPF)
- Then define a factor d_t as a change in $\log(1 + D_t)$

Run two-stage Fama-McBeth with $f_t \equiv [\underbrace{M_t}_{\text{Market excess return}}, \underbrace{\eta_t}_{\text{HKM equity share}}, \underbrace{d_t}_{\text{Disagreement}}]'$ with first stage:

$$R^e_{i,t} = a_i + \beta'_{i,t} f_t + v_{i,t}$$

and the second stage

$$\mathbb{E}[R^e_{i,t}] = \alpha_i + \hat{\beta}_{i,f} \lambda_f + \epsilon_i$$

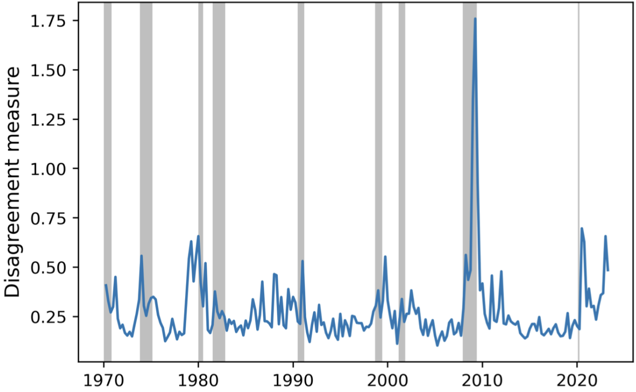


Figure 1: Disagreement is computed as the interquartile dispersion of 2nd quarter ahead GDP QoQ projection scaled by median growth projection. The data is taken from Survey of Professional Forecasters. The shaded areas represent NBER recessionary periods.

Period 1970Q1 till 2022Q4

- 25 size and book-to-market portfolios; 24 size and momentum sorted portfolios; 10 long-term reversal portfolios; 25 profitability and investment portfolios; 10 maturity sorted US treasury bond portfolios from CRSP Fama bond dataset with maturities in 6 month intervals up to 5 years

In addition, other asset classes: period 1970Q1 till 2012Q4

- 18 option portfolios; 20 CDS portfolios; 12 FX portfolios used in He, Kelly, and Manela (2017)

	Equities		Equities and Bonds	
	Two-factor	Three-factor	Two-factor	Three-factor
Mean excess return	2.06	2.06	1.88	1.88
Std. excess return	0.69	0.69	0.84	0.84
Mean β_M	1.0	1.0	0.9	0.9
Std β_M	0.23	0.23	0.37	0.37
Mean β_η	0.09	0.09	0.08	0.08
Std β_η	0.11	0.11	0.11	0.11
Mean β_d	-	0.004	-	0.004
Std β_d	-	0.04	-	0.04
Assets	85	85	95	95
Quarters	211	211	211	211
Controls	Yes	Yes	Yes	Yes

Table 1: Expected returns and risk exposures. Equity assets include 25 size and book-to-market portfolios, 25 size and momeutum portfolios, 10 long-term reversal portfolios, and 25 profitability and investment portfolios. Bond portfolios include 10 maturity sorted portfolios from CRSP Fama bond portfolio dataset. Mean and std. of excess return is the difference in mean return and risk free rate of the corresponding test assets. The frequency is quarterly and time period is from 1970Q1 till 2022Q4. The mean and std. of betas ($\beta_W, \beta_\eta, \beta_d$) measure the average and standard deviation of exposure of the excess return to market factor, intermediary capital ratio, and disagreement measure respectively.

Risk-price (second-stage)

	Equities		Equities and Bonds	
	Two-factor	Three-factor	Two-factor	Three-factor
Market	-0.01	-0.01	0.01	0.01
t-Stat Shanken	(-0.71)	(-0.5)	(1.17)	(1.05)
Intermediary	-0.01	0.0	0.02	0.02
t-Stat Shanken	(-0.29)	(0.02)	(1.08)	(1.09)
Disagreement	-	0.07	-	0.07
t-Stat Shanken	-	(2.91)	-	(2.81)
MAPE %	2.0	1.79	2.22	2.08
Adj. R2	0.00	0.18	0.23	0.35
Assets	85	85	95	95
Quarters	211	211	211	211

Table 2: Risk price estimates for equities and US government bond portfolios. Equity test assets include 25 size and book-to-market portfolios, 25 size and momentum portfolios, 10 long-term reversal portfolios, and 25 profitability and investment sorted portfolios. The 'Equity and bonds' portfolio include all of the above assets, plus 10 maturity sorted US government bond portfolios taken from the CRSP Fama bond portfolio dataset. The data is at quarterly frequency from 1970Q1 till 2022Q4. The factors are market, intermediary capital ratio, and disagreement. The disagreement factor d_t is computed as growth rate in the inter-quartile dispersion of GDP forecast scaled by median forecast by analysts. The forecast data is taken from The Survey of Professional Forecasters. The standard errors of risk price estimates are adjusted for generated regressor problem using Shanken correction. MAPE denotes the mean absolute pricing error in annualized terms.

Risk-price (second-stage)

	HKM		HKM+Momentum	
	Two-factor	Three-factor	Two-factor	Three-factor
Mean excess return	0.85	0.85	1.18	1.18
Std. excess return	1.31	1.31	1.32	1.32
Mean β_M	0.46	0.46	0.61	0.61
Std. β_M	0.45	0.45	0.47	0.47
Mean β_η	0.03	0.03	0.05	0.04
Std. β_η	0.09	0.09	0.1	0.1
Mean β_d	-	0.002	-	0.002
Std. β_d	-	0.03	-	0.04
Assets	94	94	129	129
Quarters	171	171	171	171
Controls	Yes	Yes	Yes	Yes

Table 3: Expected returns and risk exposures. HKM assets include 25 size and book-to-market portfolios, 10 U.S. Govt. bond portfolios, 10 U.S. corporate bond portfolios, 18 option portfolios, 20 CDS portfolios, and 12 foreign exchange portfolios taken from HKM2017. HKM+Momentum includes 25 size and momentum portfolios and 10 long-term reversal portfolios in addition to the HKM assets. Mean and std. of excess return is the difference in mean return and risk free rate of the corresponding test assets. The frequency is quarterly and time period is from 1970Q1 till 2012Q4. The mean and std. of betas ($\beta_W, \beta_\eta, \beta_d$) measure the average and standard deviation of exposure of the excess return to market factor, intermediary capital ratio, and disagreement measure respectively.

	HKM		HKM+Momentum	
	Two-factor	Three-factor	Two-factor	Three-factor
Market	0.02	0.01	0.02	0.01
t-stat Shanken	(1.46)	(0.83)	(1.59)	(0.97)
Intermediary	0.09	0.10	0.06	0.07
t-stat Shanken	(4.19)	(3.09)	(2.86)	(2.14)
Disagreement	-	0.1	-	0.12
t-stat Shanken	-	(1.93)	-	(2.93)
MAPE %	1.66	1.34	2.35	1.97
Adj. R2	0.83	0.89	0.59	0.73
Assets	94	94	129	129
Quarters	171	171	171	171

Table 4: Risk price estimates for HKM and HKM+Momentum portfolios. HKM assets include 25 size and book-to-market portfolios, 10 U.S. Govt. bond portfolios, 10 U.S. corporate bond portfolios, 18 option portfolios, 20 CDS portfolios, and 12 foreign exchange portfolios taken from HKM2017. HKM+Momentum includes 25 size and momentum portfolios and 10 long-term reversal portfolios in addition to the HKM assets. The data is at quarterly frequency from 1970Q1 till 2012Q4. The factors are market, intermediary capital ratio, and disagreement. The disagreement factor d_t is computed as growth rate in the inter-quartile dispersion of GDP forecast scaled by median forecast by analysts. The forecast data is taken from The Survey of Professional Forecasters. The standard errors of risk price estimates are adjusted for generated regressor problem using Shanken correction. MAPE denotes the mean absolute pricing error in annualized terms.

Better fit: two-factors

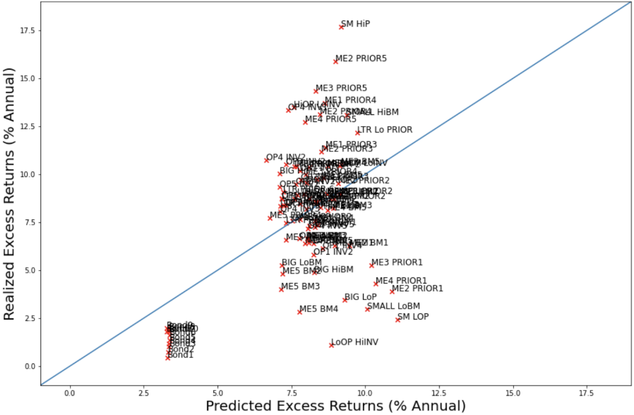


Figure 4: Pricing errors on equity and bond portfolios. Realized excess returns versus predicted excess returns using the two-factor model with market and intermediary factors. The data is at quarterly frequency and from 1970Q1 till 2022Q4.

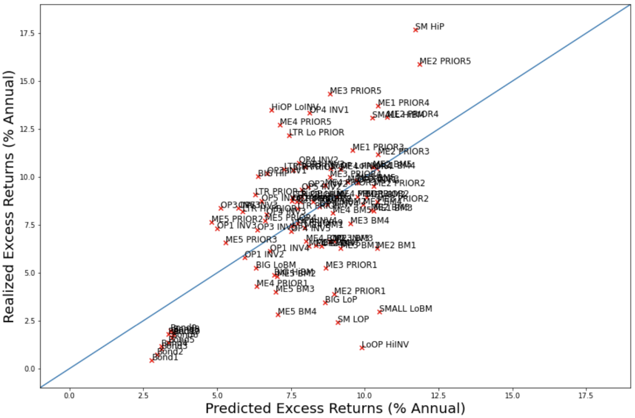


Figure 5: Pricing errors on equity and bond portfolios. Realized excess returns versus predicted excess returns using the three-factor model with market, intermediary, and disagreement factors. The data is at quarterly frequency and from 1970Q1 till 2022Q4.

Step 1: Run the following regression:

$$r^e_{t+h} = \alpha(h) + \underbrace{\beta_1(h)}_{\substack{\text{Unconditional}\\ \text{predictability}}} \times r^e_t + \underbrace{\beta_2(h)}_{\substack{\text{Conditional}\\ \text{predictability}}} \times r^e_t \times 1_{\text{Recession}} + \epsilon_{t+h}$$

Step 2: simulate our model (e.g., 1,000 times for 5,000 years) and run the following same regression:

$$r^e_{t+h} = \alpha(h) + \underbrace{\beta_{1,\text{model}}(h)}_{\substack{\text{Unconditional}\\ \text{predictability}}} \times r^e_t + \underbrace{\beta_{2,\text{model}}(h)}_{\substack{\text{Conditional}\\ \text{predictability}}} \times r^e_t \times 1_{\text{Recession}} + \epsilon_{t+h}$$

Conditional predictability

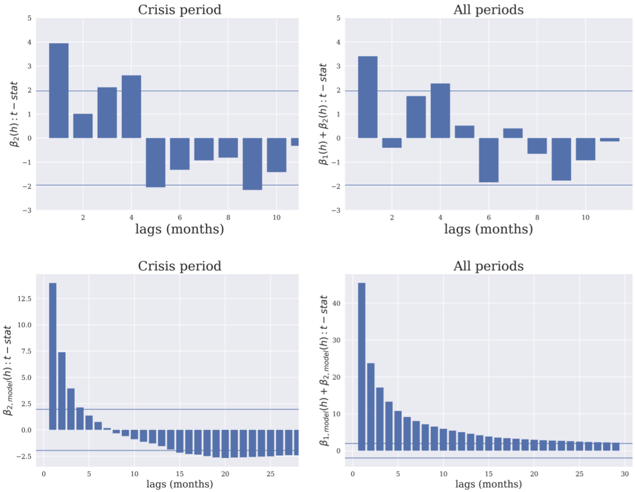


Figure 1: Time series return predictability. Top two panels represent the empirical auto-correlation from regressing excess return on S&P500 on lagged excess returns, as shown in equation (1). The data is at monthly frequency from 1945 till 2022. The right panels represent the model implied t-stats from the regression (1). The model is simulated 1000 times for 5000 years at a monthly frequency. The correlation coefficients represent average values across simulations.

Thank you very much!
(Appendix)

Optimists' optimal portfolio decision (e.g., Merton (1971))

$$x_t = \frac{\left(\frac{\gamma_t^O - \iota_t^O \gamma_t^O}{p_t} + \Lambda^O(\iota_t^O) - \delta^O + \mu_t^p + \boxed{\frac{\alpha^0 - \alpha}{\sigma}\sigma_t^p}\right) - r_t^*}{(\sigma_t^p)^2}$$

New term: from optimism

For $\alpha^O > \alpha$ (experts = optimists)

- Given the risk-free r_t^* and the endogenous volatility σ_t^p, optimism (i.e., $\alpha^O\uparrow$ from α) raises the optimists' leverage↑ and capital demand↑
- Optimists bear 'too much' risk on their balance sheets $\longrightarrow$ crisis when dZ_t is negative enough (more frequently)

$\sigma_t^p\uparrow \longrightarrow$ has two effects on leverage x_t:

- $\sigma_t^p\uparrow$ lowers x_t as the required risk-premium level↑
- $\sigma_t^p\uparrow$ raises x_t as it raises the degree of optimism on asset returns

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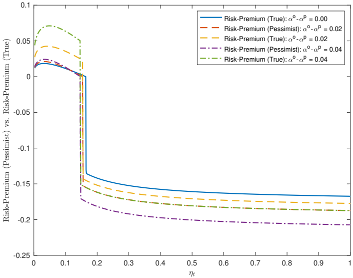
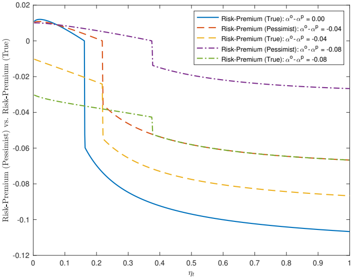
Risk-premium (true and perceived by pessimists)



(a) $\alpha^O \geq \alpha$ and $\alpha^P \leq \alpha$



(b) $\alpha^O \leq \alpha$ and $\alpha^P \geq \alpha$

Figure: Risk-Premium (Pessimists' and True Value) as a Function of η_t

- Pessimists perceive to risk-premium to be positive only when $\eta_t \leq \eta^\psi$

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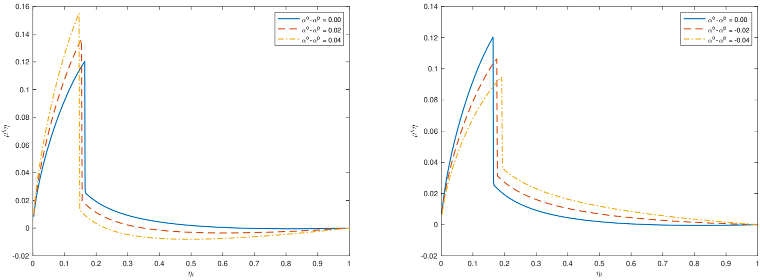
Drift of the wealth share



(a) $\alpha^O \geq \alpha$ and $\alpha^P \leq \alpha$ (b) $\alpha^O \leq \alpha$ and $\alpha^P \geq \alpha$

Figure: Wealth Share Drift $\mu_\eta(\eta_t) \cdot \eta_t$ as a Function of η_t

- With $\underline{\alpha^O > \alpha > \alpha^P}$, $\alpha^O\uparrow \longrightarrow$ Wealth share drift $\mu_\eta(\eta_t) \cdot \eta_t\uparrow$: recapitalized faster

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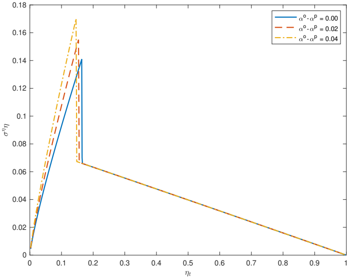
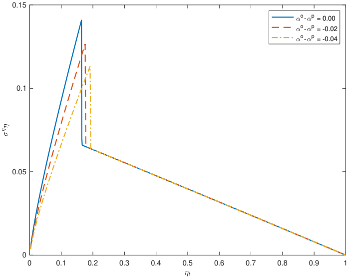
Volatility of the wealth share



(a) $\alpha^O \geq \alpha$ and $\alpha^P \leq \alpha$



(b) $\alpha^O \leq \alpha$ and $\alpha^P \geq \alpha$

Figure: Wealth Share Volatility $\sigma^\eta(\eta_t) \cdot \eta_t$ as a Function of η_t

Go back