A Theory of Keynesian Demand and Supply Interactions under Endogenous Firm Entry *

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Abstract

We build a macroeconomic model with endogenous firm entry where aggregate demand and supply are intertwined. Under a novel two-layers system of firms where bottom-tier firms pay random fixed costs (e.g., purchasing necessary equipment) in order to operate while a top-tier firm uses the bottom-tier produced good as a sole input and faces nominal rigidities, a bottom-tier firm relies on loans from financial markets in order to pay those fixed costs, whose rate is determined by monetary policy. In this case, a new entry by bottom-tier firms lowers the real input price for top-tier firms and shifts the aggregate supply down, while triggering an increase in aggregate demand through their equipment purchases, which spurs additional round of new entry and so on. Monetary policy can be powerful as it has impacts on aggregate demand and firms' entry decisions simultaneously. Our analytical characterization expresses the equilibrium firm entry as function of a sufficient statistic related to monetary policy.

Keywords: Monetary Policy, Satiation, Endogenous Entry

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1 Introduction

Modern macroeconomic models are usually based on the system of blocks of aggregate supply and aggregate demand, and try to disentangle shocks to those two different blocks: while positive supply shocks (e.g., positive technology or negative cost-push shocks) shifts the supply curve (e.g., New-Keynesian Phillips curve) down, positive demand shocks boosts aggregate demand and as a result, output, in the presence of nominal rigidities. However, in many cases, demand and supply shocks are convoluted (e.g., one shock leads to the other) and/or come together as in Covid-19 crisis, and econometricians usually find it hard to identify one from the other.

In this paper, we question whether supply and demand shocks in a New-Keynesian framework are separable if the firm entry is endogenous. Our motivation is as follows: for example, a negative shock to aggregate demand (e.g., monetary tightening) makes it less profitable for many potential entrants to enter the market, so the firm entry decreases, shifting aggregate supply up (or to the left). As any newly entering firm needs to build their factories and purchase necessary equipment, a drop in firm entry reduces the aggregate demand, spurring additional reduction in firm entry, and so on. During this feedback process between the two, demand shocks cause supply to shrink through firms' endogenous exits from the market, while this reduction in firm entry further lowers aggregate demand.

Figure 1, based on the classic aggregate demand (AD) and aggregate supply (AS) diagram, illustrates the opposite case: when a positive demand shock hits the economy (AD_0 to AD_1), it triggers more market entry of firms, shifting the AS curve down (AS_0 to AS_1). As new entrants purchase equipment for their operation on the market, it additionally boosts aggregate demand (AD_1 to AD_2), which spurs additional round of new firm entry (AS_1 to AS_2) and so on until the convergence happens. Note that it can explain the case where a positive supply shock, not the demand shock, kicks in first. Still the positive feedback between the two drives the business cycle. Monetary accommodation can particularly initiate this infinite feedback loop between AD and AS, but as more and more firms have already entered the market, the strength of this positive feedback will weaken as the policy rate keeps falling. Our job in this paper is to operationalize this concept in a precise manner with the help of modern macroeconomic tools.

In that purpose, we build a New-Keynesian business cycle model with two layers of firms: top-tier and bottom-tier. Top-tier firms produce intermediate goods, whose aggregator becomes final consumption good, and face nominal rigidities in price setting. Bottom-tier firms hire labor, produce inputs to the top-tier firms, and in contrast to top-tier ones, are flexible in price setting. Furthermore, bottom-tier firms, in order to operate in a given period, should pay random fixed costs in the previous period, which they draw from a given fixed cost distribution and with which they buy necessary equipment in final good terms. They also draw the next period productivity from a given productivity distribution, and enter the market only if operation can be profitable. As bottom-tier firms transfer profits to households every period, they rely on loans from financial markets to fund the fixed costs, whose rate is decided by the monetary authority.

A new market entry of bottom-tier firms raises their competition degree and thereby lowers

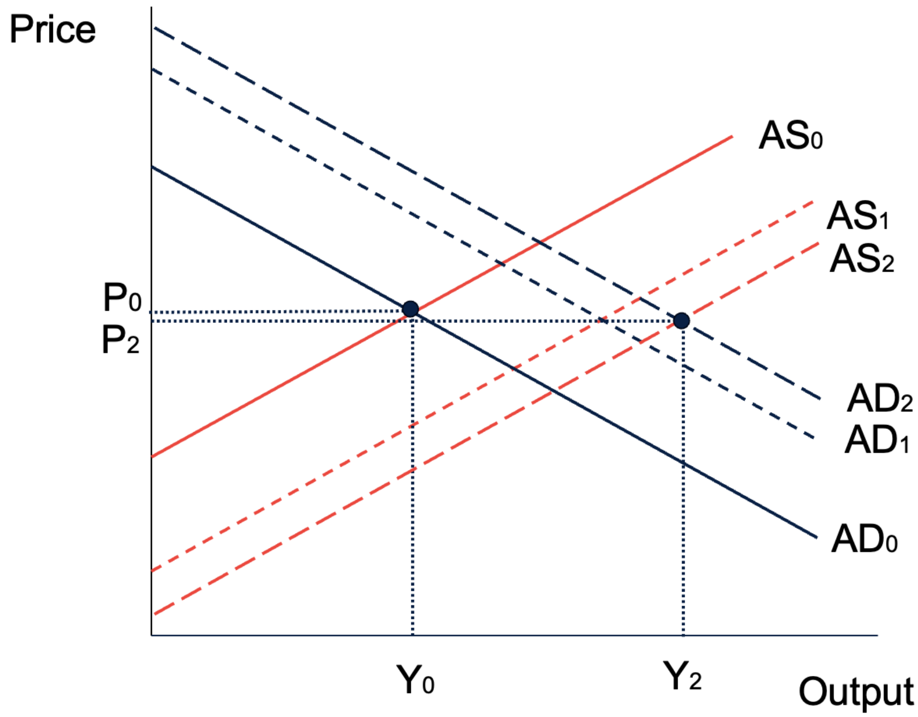

Figure 1: Convoluted aggregate demand and supply

the real (i.e., CPI-denominated) input price for top-tier firms. It lowers the real marginal cost for top-tier firms and shifts their pricing function (i.e., New-Keynesian Phillips curve or aggregate supply curve) down. In addition, it spurs more consumption demand as new entrants purchase necessary equipment in final consumption good, which invokes additional round of new entry and so on. Our steady state analysis and the calibrated impulse response exercises show that this channel is of significant power in making our results deviate from conventional macroeconomic wisdom, e.g., productivity shocks can be inflationary in our model.

Monetary accommodation can be powerful as it not only affects aggregate demand through usual channels (e.g., intertemporal substitution and wealth effects) but also it triggers firm entry by reducing the loan interest amount, adding additional upward pressures on aggregate demand. This second supply effect of monetary policy becomes weaker as the policy rate goes down, since many firms with high fixed costs or low productivity must have already entered the market under a low interest rate.

Our novel use of extreme-value distributions (e.g., Pareto distributions for both productivity and fixed cost) allows analytic expressions for many important variables. We especially define the 'satiated lower bound' (SLB) for each fixed cost type: a level of the policy rate that makes even a lowest productivity firm finds operation profitable[1] and the economy's average SLB (i.e., average across different fixed costs). It turns out that the equilibrium firm entry and other supply-related variables can be written in terms of the 'policy room', a relative ratio of the current policy rate

[1] In that case, all firms with different productivity draws that share a given fixed cost operate on the market.

to the economy's average SLB, implying that the average SLB acts as a benchmark interest rate in terms of monetary policy's supply side effects. In addition, we decompose changes in aggregate variables into the extensive margin adjustment (i.e., new entry of firms) or intensive margin adjustment (i.e., each incumbent firm changes their behaviors), which clarifies the model's interpretations.

Related literature Our business cycle setting with endogenous (bottom-tier) firm entry follows previous works in the literature, e.g., Bilbiie et al. (2007), Bergin and Corsetti (2008),[2] Stebunovs (2008), Kobayashi (2011) Bilbiie et al. (2012), Uusküla (2016), Hamano and Zanetti (2017) among others. While some papers assume equity financing for newly entering firms, e.g., Bilbiie et al. (2007), Bergin and Corsetti (2008), Bilbiie et al. (2012),[3] we assume that a bottom-tier firm that decides to operate issues a loan (i.e., borrow) from financial markets, as in e.g., Stebunovs (2008), Kobayashi (2011), Uusküla (2016), so that the firm entry is boosted under monetary accommodation, which aligns with Colciago and Silvestrini (2022).[4] In addition, we express the equilibrium firm entry as a function of the 'policy room', a sufficient statistic we devise.

Guerrieri et al. (2023) inquire about when a sectoral supply shock becomes Keynesian, i.e., a supply shock spurs a larger aggregate demand shift than its own magnitude. While their focus is on settings where (i) there are multiple sectors and markets are incomplete; (ii) the affected sector is either complementary to or uses inputs produced from unaffected sectors,[5] our two-layers structure (i.e., top-tier and bottom-tier industries) allows us to study both ways between supply and demand: in our model, supply shocks to bottom-tier firms shift aggregate demand, through their effects on labor markets and the loan demand, while demand shocks triggers shifts in the bottom-tier supply curve, which are translated into shifts in the top-tier supply through their impacts on input prices and causes another round of demand shifts, until convergence.

Our characterization of the satiation lower bound (SLB) is based on the idea that (i) monetary expansion boosts firm entry, but (ii) when the policy rate hits some lower bound we define, all the available firms of a given fixed cost have entered the market. After then, an additional monetary accommodation's positive supply effect from additional firm entry gets weaker, in a similar way

[2]Our setting that fixed costs are spent on purchasing final consumption goods are similar to Bergin and Corsetti (2008). While Bergin and Corsetti (2008) assume new entrants buy equipment from existing firms at 'pre-set' prices to enter into markets, we assume they trade at market prices.

[3]Under the equity financing for new entry, an expansionary monetary shock leads to an increase in the aggregate demand for products, raising labor demand and wage levels. Higher labor costs for potential entrants can lower their net present value (NPV), reducing the firms' entry rate, which is counterfactual. For the role of 'real wage rigidity' in resolving this problem, see Lewis and Poilly (2012).

[4]Colciago and Silvestrini (2022) find empirically that the expansionary monetary policy leads to an initial decrease and then overshooting in average productivity, while generating an initial increase and then undershooting in the firm entry rate.

[5]In Guerrieri et al. (2023), a negative supply shock to one sector have multiple countervailing effects: (i) it raises the overall price level and thereby inducing the overall consumption level to fall; (ii) it induces shifts to goods produced from unaffected sectors. This demand shifting becomes weaker when the two sectors are complementary or unaffected sectors provide inputs to the affected sector, which helps aggregate demand to fall more than the supply shock itself; (iii) the shutdown of a sector causes income losses, and with incomplete markets and borrowing constraints, causes aggregate demand to fall in general.

to Ulate (2021) and Abadi et al. (2022) who apply similar ideas to the bank profitability.

Layout In Section 2, we present our New-Keynesian framework with two layers of firms: top-tier and bottom-tier firms. Section 3 includes our calibration and steady state analysis, with some comparative statics. Section 4 studies the model economy's impulse response functions to various shocks, and Section 5 concludes. Appendix A includes additional tables and figures. Appendix B provides derivations and proofs. Appendix C summarizes the equilibrium conditions (including flexible price benchmark and steady-state ones). Finally, Appendix D provides expressions for a limit case where fixed cost distribution becomes degenerate.

2 Model

2.1 Representative Household

The representative household chooses lifetime consumption and labor $\{C_t, N_t\}$, with the preference

$$\max E_t \sum_{j=0}^{\infty} \beta^j \left[\phi_{c,t} \cdot \log\left(C_t\right) - \left(\frac{\eta}{\eta+1}\right) \cdot N_t^{\left(\frac{\eta+1}{\eta}\right)} \right],$$

where $\phi_{c,t}$ is an aggregate demand shock defined as:

$$\phi_{c,t} = \exp\left(u_{c,t}\right) \qquad \text{where: } u_{c,t} = \rho_c \cdot u_{c,t-1} + \varepsilon_{c,t}, \;\; \varepsilon_{c,t} \sim N\left(0, \sigma_c^2\right).$$

The household's budget constraint is given by

$$C_t + \frac{D_t}{P_t} + \frac{B_t}{P_t} = \frac{R_{t-1}^D D_{t-1}}{P_t} + \frac{R_{t-1}^B B_{t-1}}{P_t} + \frac{W_t N_t}{P_t} + \frac{Y_t}{P_t},$$

where D_t are bank deposits, B_t are government bonds, which are in zero net supply in equilibrium, with R_t^D and R_t^B being their respective *gross* interest rates, and Y_t are lump sum transfers to the household from e.g., fiscal policies (subsidies to firms), residual firm profits. In this paper, we do not consider issues of the zero lower bound (ZLB), so it is possible that $R_t^D < 1$.

The first-order conditions bring the following standard intertemporal and intratemporal equations:

$$\frac{1}{R_t^D} = \frac{1}{R_t^B} = \beta E_t \left[\underbrace{\frac{\phi_{c,t+1}}{\phi_{c,t}} \cdot \frac{C_t}{C_{t+1}\Pi_{t+1}}}_{\equiv Q_{t+1}} \right] \tag{1}$$

$$N_t^{\frac{1}{\eta}} = \phi_{c,t} \cdot C_t^{-1} \cdot \frac{W_t}{P_t}. \tag{2}$$

The household is indifferent between investing in bonds or deposits in equilibrium as both are risk free and note that central bank policy on R_t^B has a one-to-one pass-through on R_t^D. Note in (1) a positive shock to $\phi_{c,t}$ raises current consumption C_t.

As $R_t^D = R_t^B$ in equilibrium, $R_t^D = R_t^B = R_t^J$ holds where R_t^J is defined as the policy rate set by the central bank.

2.2 Firms

There are two layers of firms: '*top-tier industry*' firms and '*bottom-tier industry*' firms. Firms in both tiers are monopolistically competitive within each tier, but only firms in the top-tier industry face nominal price rigidities à la Calvo (1983). Bottom-tier firms hire workers and produce inputs to the top-tier firms. Top-tier firms in turn produce intermediate goods using inputs from bottom-tier firms. Both tier firms are owned by the representative households.

The key feature is that firms in the bottom-tier industry decides whether to operate or not at each period, and if they want to operate in the market, need to borrow loans from banks to cover necessary fixed-costs (e.g., buying equipment), as they do not have spare cashes.

2.2.1 Top-Tier Industry: Aggregator

A perfectly competitive firm aggregates the differentiated products made by a $u \in [0, 1]$ continuum of Top-Tier firms, formally:

$$Y_t = \left[\int_0^1 Y_t(u)^{\frac{\gamma-1}{\gamma}} \, \mathrm{d}u \right]^{\frac{\gamma}{\gamma-1}}$$

The demand for each individual Top-Tier variety and the aggregate price are given by

$$Y_t(u) = \left(\frac{P_t(u)}{P_t} \right)^{-\gamma} \cdot Y_t \tag{3}$$

and

$$P_t = \left[\int_0^1 P_t(u)^{1-\gamma} \, \mathrm{d}u \right]^{\frac{1}{1-\gamma}} \tag{4}$$

where $Y_t(u)$ and $P_t(u)$ are the output and prices of Top-Tier varieties, respectively.

We define nominal aggregate expenditures and expenditure for a top-tier variety u, as $X_t = P_t Y_t$ and $X_t(u) = P_t(u) Y_t(u)$, respectively. We can re-express the individual demand as:

$$X_t(u) = \Gamma_t \cdot P_t(u)^{1-\gamma} \qquad \text{where: } \Gamma_t = X_t P_t^{\gamma-1}.$$

2.2.2 Top-Tier Industry: Monopolistic Competition with Sticky Prices

A top-tier industry firm $u \in [0, 1]$ employs $J_t(u)$ units of aggregate product of the bottom-tier industry and produce $Y_t(u) = J_t(u)$, i.e., one-to-one production from input to output. It follows that the total aggregator of bottom-tier products, J_t, must satisfy $J_t = \int_0^1 J_t(u) \, \mathrm{d}u = \int_0^1 Y_t(u) \, \mathrm{d}u$.

The profits of a top-tier firm u is given by

$$\Pi_t(u) = (1+\zeta^T)P_t(u)Y_t(u) - P_t^J J_t(u),$$

where P_t^J is the price of the aggregate bottom-tier product, and ζ^T is the production subsidy to top-tier firms. Therefore, the present discounted value of profits that top-tier firm u maximizes is:

$$\sum_{l=0}^{\infty} E_t \left\{ Q_{t,t+l} \left[(1+\zeta^T)P_{t+l}(u)Y_{t+l}(u) - P_{t+l}^J J_{t+l}(u) \right] \right\}$$

where $Q_{t,t+l} = \beta \left[\frac{\phi_{c,t+l}}{\phi_{c,t}} \frac{C_t}{C_{t+l}\Pi_{t+1}\ldots\Pi_{t+l}} \right]$ is the stochastic discount factor between t and $t+l$. Firms in the top-tier industry face price stickiness à la Calvo (1983) with price-resetting probability $1-\theta$. Based on (3), a firm resets its price P_t^* by solving:

$$\max_{P_t^*} \sum_{l=0}^{\infty} E_t \left\{ Q_{t,t+l}\theta^l \left[(1+\zeta^T)P_t^* - P_{t+l}^J \right] \left(\frac{P_t^*}{P_{t+l}} \right)^{-\gamma} Y_{t+l} \right\}$$

where all the price-resetting firms choose P_t^* as their new price. The first-order condition yields

$$\frac{P_t^*}{P_t} = \frac{\sum_{l=0}^{\infty} E_t \left\{ Q_{t,t+l}\theta^l \left(\frac{(1+\zeta^T)^{-1}\gamma}{\gamma-1} \right) \left(\frac{P_{t+l}}{P_t} \right)^{\gamma+1} \left(\frac{P_{t+l}^J}{P_{t+l}} \right) Y_{t+l} \right\}}{\sum_{l=0}^{\infty} E_t \left\{ Q_{t,t+l}\theta^l \left(\frac{P_{t+l}}{P_t} \right)^{\gamma} Y_{t+l} \right\}} \tag{5}$$

2.2.3 Bottom-Tier Industry: Aggregator

There exists a $[0,1]$ continuum of bottom-tier firms producing differentiated products. Firms are heterogeneous in their productivities, which we index by v, and operational fixed costs, which we index by m. We define $J_{mv,t}$ as the output of a firm uniquely identified by the index pair mv. A perfectly competitive firm aggregates these bottom-tier varieties into the aggregator as:

$$J_t = \left[\int_0^1 \int_{v\in\Omega_{m,t}} J_{mv,t}^{\frac{\sigma-1}{\sigma}} \, \mathrm{d}v \, \mathrm{d}m \right]^{\frac{\sigma}{\sigma-1}} \tag{6}$$

where $\Omega_{m,t} \subseteq [0, 1]$ refers to the subset of bottom-tier firms with the same operational fixed cost, indexed by m that decide to produce during period t. Intuitively, with the large fixed costs, only the most productive firms might find it profitable to produce. The individual demand for a bottom-tier variety (m, v) is:

$$J_{mv,t} = \left(\frac{P_{mv,t}^J}{P_t^J} \right)^{-\sigma} J_t \tag{7}$$

and

$$P_t^J = \left[\int_0^1 \underbrace{\int_{v \in \Omega_{m,t}}^1 \left(P_{mv,t}^J \right)^{1-\sigma} \mathrm{d}v}_{\equiv (P_{m,t})^{1-\sigma}} \mathrm{d}m \right]^{\frac{1}{1-\sigma}} = \left[\int_0^1 \left(P_{m,t}^J \right)^{1-\sigma} \mathrm{d}m \right]^{\frac{1}{1-\sigma}}, \tag{8}$$

where P_t^J is the bottom-tier price aggregator, and $P_{m,t}^J$ the price index for the operating firms with the same index m fixed costs. As before, we define bottom-tier nominal expenditures on a given variety as $X_{mv,t}^J = P_{mv,t}^J J_{mv,t}$ and the aggregate expenditure as $X_t^J = P_t^J J_t$ as

$$X_{mv,t}^J = \Gamma_t^J \cdot P_{mv,t}^{1-\sigma}. \qquad \text{where: } \Gamma_t^J = X_t^J \left(P_t^J \right)^{\sigma-1}. \tag{9}$$

Note from (3) that we can rewrite the aggregation of upper-tier firms' input demands as

$$J_t = \int_0^1 Y_t(u)\, \mathrm{d}u = Y_t \underbrace{\int_0^1 \left(\frac{P_t(u)}{P_t} \right)^{-\gamma} \mathrm{d}u}_{\equiv \Delta_t} = Y_t \Delta_t \tag{10}$$

where

$$\Delta_t = (1-\theta) \left(\frac{P_t^*}{P_t} \right)^{-\gamma} + \theta \Pi_t^{\gamma} \Delta_{t-1} \tag{11}$$

is a price dispersion. Now we can use (10) and express (9) as $\Gamma_t^J = (P_t^J)^{\sigma} Y_t \Delta_t$.

2.2.4 Bottom Tier Industry: Monopolistic Competition, Loans, and Entry Decisions

The production of an arbitrary bottom-tier firm (m, v) features decreasing returns to scale and is given by

$$J_{mv,t} = \varphi_{mv,t} \cdot N_{mv,t}^{\alpha}, \qquad \text{with } 0 < \alpha < 1 \tag{12}$$

where $N_{mv,t}$ is labor employed, and $\varphi_{mv,t}$ is a firm-specific productivity drawn from a distribution which we specify later. The bottom-tier market is monopolistically competitive.

Profit function Firms of operational fixed cost type m that decide to operate in period t need to pay a fixed in-kind cost in terms of the top-tier aggregator, $F_{m,t-1}$, <u>in advance</u> at period $t-1$: e.g., in order to produce input products for top-tier firms, a bottom-tier firm needs to prepare necessary equipment. We assume that this fixed cost $F_{m,t-1}$ needs to be borrowed in period $t-1$ via a loan from banks at gross rate R_{t-1}^J.[6] The profit if operate at period t would be

$$\Pi_{mv,t}^J = \underbrace{\left(1+\zeta^J\right) P_{mv,t}^J J_{mv,t}}_{\equiv r_{mv,t}} - W_t N_{mv,t} - R_{t-1}^J P_{t-1} F_{m,t-1} \tag{13}$$

[6] We assume that in each period, a firm (m, v) rebates all of its profits to households. Therefore it needs to borrow $P_t F_{m,t}$ from banks for the period $t+1$ operation.

where ζ^J is the subsidy to the bottom-tier producers and $r_{mv,t}$ is their revenue. As botton-tier firms face no nominal rigidity, and they are monopolistically competitve as in (7), they charge a constant markup over marginal costs if they decide to produce, formally:

$$P^J_{mv,t} = \left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha} \right) W_t \varphi_{mv,t}^{-\frac{1}{\alpha}} J_{mv,t}^{\frac{1-\alpha}{\alpha}}. \tag{14}$$

Plugging (14) into (13), with the help of (7) and (9), we can obtain

$$\Pi^J_{mv,t} = \Xi_t \cdot \varphi_{mv,t}^{\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}} - R^J_{t-1} P_{t-1} F_{m,t-1}, \tag{15}$$

where

$$\Xi_t \equiv \frac{\alpha+\sigma(1-\alpha)}{(\sigma-1)\alpha} \left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha} \right)^{\frac{-\sigma}{\alpha+\sigma(1-\alpha)}} W_t^{\frac{\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}} (\Gamma^J_t)^{\frac{1}{\alpha+\sigma(1-\alpha)}}. \tag{16}$$

Entry decision We assume that the individual productivity draws come from a Pareto distribution, i.e., $\varphi_{mv,t} \underbrace{\sim}_{\text{i.i.d}} \mathcal{P}\left(\left(\frac{\kappa-1}{\kappa}\right) A_t, \kappa\right)$, where A_t is the average aggregate productivity,[7] with $\kappa > \sigma - 1$. The cumulative distribution function is

$$\Psi(\varphi_{mv,t}) = 1 - \left(\frac{\left(\frac{\kappa-1}{\kappa}\right) A_t}{\varphi_{mv,t}} \right)^{\kappa}$$

with the probability distribution function given by $\psi(\varphi_{mv,t}) = \Psi'(\varphi_{mv,t})$. Firms need to decide whether to produce or not in t during the previous period (i.e. at $t-1$), and if so, take the loan to pay for the fixed costs. As κ increases, the productivity distribution is more concentrated around its mean, A_t.

We assume that at $t-1$ firms know their next period productivity at t, $\varphi_{mv,t}$, but do not foresee other shocks that affect individual demand at t.[8] A firm that decides in $t-1$ to enter the market in period t hires the labor in the spot market in t and enjoys profits in (15). Therefore, according to productivity draws, we can characterize the marginal firm with productivity cutoff $\varphi^*_{m,t}$ that makes zero profit in expectation. Firms with the same fixed cost $F_{m,t-1}$, whose productivity draw $\varphi_{mv,t}$ is below the cutoff $\varphi^*_{m,t}$, do not enter the market in t. From (15), $\varphi^*_{m,t}$ is formally given by

$$E_{t-1}\left[\xi_t \cdot \Xi_t\right] \cdot \left(\varphi^*_{m,t}\right)^{\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}} - R^J_{t-1} P_{t-1} F_{m,t-1} = 0 \qquad \text{where: } \xi_t = \frac{Q_{t-1,t}}{E_{t-1}\left[Q_{t-1,t}\right]} \tag{17}$$

Note that this cutoff $\varphi^*_{m,t}$ is based on expected profits. After a firm (m, v) decides to enter, adverse shocks might reduce profits to the negative territory.

[7]Therefore, we assume that firms of different fixed cost types (i.e., different m values) feature the same average productivity.

[8]In contrast to Burnside et al. (1993) where labor is decided before the realization of shocks, we assume that entry is decided before other demand shocks are realized. For simplicity, we assume firms have a perfect foresight of their productivity realization in the next period.

Due to the productivity lower bound, $\frac{\kappa-1}{\kappa}A_t$, the actual productivity cutoff for entry would be given by $\max\left\{\varphi^*_{m,t}, \left(\frac{\kappa-1}{\kappa}\right)A_t\right\}$.[9] The measure of firms with a given fixed cost $F_{m,t-1}$ that decide to actively produce in t, which we denote by $M_{m,t}$ is therefore given by

$$\begin{aligned} M_{m,t} &= Prob\left(\varphi_{mv,t} \geq \varphi^*_{m,t}\right) \\ &= \min\left\{\left(\frac{E_{t-1}\left[\xi_t \cdot \Xi_t\right]\left[\left(\frac{\kappa-1}{\kappa}\right)A_t\right]^{\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}}}{R^J_{t-1}P_{t-1}F_{m,t-1}}\right)^{\frac{\kappa[\alpha+\sigma(1-\alpha)]}{\sigma-1}}, 1\right\}, \end{aligned} \tag{18}$$

where in the second equality, we use (17) for $\varphi^*_{m,t}$. From (18), we have the following proposition:

Proposition 1 *For bottom-tier firms with fixed cost $F_{m,t-1}$, $\underline{M_{m,t} = 1}$ when the policy rate R^J_{t-1} is below some threshold $R^{J,*}_{m,t-1}$, given by*

$$R^{J,*}_{m,t-1} \equiv \frac{E_{t-1}\left[\xi_t \cdot \Xi_t\right]\left[\left(\frac{\kappa-1}{\kappa}\right)A_t\right]^{\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}}}{P_{t-1}F_{m,t-1}}. \tag{19}$$

We call $R^{J,*}_{m,t-1}$ the 'satiated lower bound' (SLB) for the firms of fixed cost type m.

As the policy rate R^J_{t-1} decreases, a measure of firms of a given fixed cost $F_{m,t-1}$ that operates at period t increases as the loan repayment cost $R^J_{t-1}P_{t-1}F_{m,t-1}$ drops. When the policy rate R^J_{t-1} hits the type-specific lower bound $R^{J,*}_{m,t-1}$ that is decreasing in $F_{m,t-1}$, all bottom-tier firms that share the fixed cost $F_{m,t-1}$ have already entered the market for period t and there is no additional entry from firms of the same fixed cost type. We call this (fixed-cost) type-specific lower bound $R^{J,*}_{m,t-1}$ the satiated lower bound (SLB).

Note that in our model, monetary policy affects market entry decisions of bottom-tier firms. This turns out to affect the input market price and quantity, and in turn have impacts on top-tier product markets and the economy's aggregate fluctuation. Still we also have a usual intertemporal substitution (i.e., demand) effect of monetary policy, which can be seen in (1). When the rate hits the SLB for firms with fixed cost $F_{m,t-1}$, no additional entry (i.e., supply effect) occurs from those firms, and the supply-side effect of monetary policy is muted for those firms.

Interestingly, we can see as well that if

$$F_{m,t-1} = \frac{E_{t-1}\left[\xi_t \cdot \Xi_t\right]}{P_{t-1}}\left[\left(\frac{\kappa-1}{\kappa}\right)A_t\right]^{\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}}, \tag{20}$$

then $R^{J,*}_{m,t-1} = 1$ so SLB corresponds to the usual zero lower bound (ZLB).[10]

[9] If $\varphi^*_{m,t} < \frac{\kappa-1}{\kappa}A_t$, then all firms of fixed cost type m operate at t.

[10] Note that the right hand side of (20) is m-independent, so (20) holds for only one fixed cost type m, if there is a dispersion in the fixed cost $F_{m,t-1}$ across firms.

Loan demand Based on equation (18), we can obtain the expression for total (real) loan demand from firms of fixed cost type m as:

$$\frac{L_{m,t-1}}{P_{t-1}} = M_{m,t} \cdot F_{m,t-1}. \tag{21}$$

Firms that decide to operate at t borrows $L_{m,t-1}$ to purchase $\frac{L_{m,t-1}}{P_{t-1}}$ amounts of final goods, which add to the economy's aggregate demand. Thus, we observe that entry decisions of firms affect aggregate demand through the loan channel.

Fixed cost distribution We assume that the fixed costs $F_{m,t}$ of bottom-tier firms follow a Pareto distribution as well, i.e., $F_{m,t} \underbrace{\sim}_{\text{i.i.d}} \mathcal{P}\left(\left(\frac{\omega-1}{\omega}\right) F_t, \omega\right)$, where F_t is the average fixed cost of operating a business and $\omega > 1$ is the parameter controlling the variance of the distribution. The cumulative distribution function is given by

$$H(F_{m,t}) = 1 - \left(\frac{\left(\frac{\omega-1}{\omega}\right) F_t}{F_{m,t}}\right)^{\omega} \tag{22}$$

with the probability distribution function given by $h(F_{m,t}) = H'(F_{m,t})$. From Proposition 1, we can now express the probability measure of fixed cost types $F_{m,t-1}$ that are fully satiated, i.e., a probability that for a draw $F_{m,t-1} \sim \mathcal{P}\left(\left(\frac{\omega-1}{\omega}\right) F_t, \omega\right)$, all the firms with the same fixed cost $F_{m,t-1}$ have entered the market at t, i.e., $M_{m,t} = 1$. It is given by the following proposition:

Proposition 2 *Now with the distribution in (22), the probability that $\underline{M_{m,t} = 1}$ is given by:*

$$Pr\left(R^J_{t-1} \leq R^{J,*}_{m,t-1}\right) = Pr\left(F_{m,t-1} \leq \underbrace{\frac{E_{t-1}\left[\xi_t \cdot \Xi_t\right]\left[\left(\frac{\kappa-1}{\kappa}\right) A_t\right]^{\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}}}{R^J_{t-1} P_{t-1}}}_{\equiv F^*_{t-1}}\right) \equiv H\left(F^*_{t-1}\right) \tag{23}$$

where F^*_{t-1} is the fixed cost cutoff: all firms with fixed cost $F_{m,t-1} \leq F^*_{t-1}$, regardless of their productivity draws $\varphi_{mv,t}$, decide to produce at t. We call F^*_{t-1} the full satiation fixed cost threshold.

Proposition 2 can be understood as follows: when a firm's fixed cost $F_{m,t-1}$ is low enough so that it is lower than some threshold F^*_{t-1}, then a firm with its productivity draw $\varphi_{mv,t}$ at the minimum, i.e., $\frac{\kappa-1}{\kappa} A_t$, still finds operating in period t profitable. Therefore, all firms that have the same fixed cost, regardless of their productivity draws, all operate in period t.

Bottom-tier industry: aggregator again P_t^J, the price aggregator for operating bottom-tier firms is given by

$$\frac{P_t^J}{P_t} = \left(\frac{W_t}{P_t A_t}\right) \cdot \left(\frac{Y_t \Delta_t}{A_t}\right)^{\frac{1-\alpha}{\alpha}} \cdot \left[\frac{\Theta_3}{1 + \Theta_4 \cdot H\left(F_{t-1}^*\right)}\right]^{\left(\frac{\alpha+\sigma(1-\alpha)}{\alpha(\sigma-1)}\right)} \tag{24}$$

where

$$\Theta_3 = \left(\frac{\kappa[\alpha+\sigma(1-\alpha)] + (\omega-1)(\sigma-1)}{\Theta_1 \omega(\sigma-1)}\right), \quad \Theta_4 = \left(\frac{\kappa[\alpha+\sigma(1-\alpha)] - (\sigma-1)}{\omega(\sigma-1)}\right). \tag{25}$$

The total measure of firms that operate in period t, which we denote by M_t, can be written as

$$M_t = \int_0^1 \int_{v \in \Omega_{m,t}} 1 \, \mathrm{d}v \, \mathrm{d}m = 1 - \Theta_M \cdot \left[1 - H\left(F_{t-1}^*\right)\right], \tag{26}$$

where

$$\Theta_M = \frac{\kappa[\alpha+\sigma(1-\alpha)]}{\kappa[\alpha+\sigma(1-\alpha)] + \omega(\sigma-1)}.$$

Finally, we can calculate the aggregate loan demand from operating bottom-tier firms as follows:

$$\frac{L_{t-1}}{P_{t-1}} = \frac{1}{P_{t-1}} \int_0^1 L_{m,t-1} \, \mathrm{d}m = F_{t-1} \cdot \left[1 - \Theta_L \cdot \left[1 - H\left(F_{t-1}^*\right)\right]^{\left(\frac{\omega-1}{\omega}\right)}\right] \tag{27}$$

where

$$\Theta_L = \frac{\kappa[\alpha+\sigma(1-\alpha)]}{\kappa[\alpha+\sigma(1-\alpha)] + (\sigma-1)(\omega-1)}.$$

Interpretations are simple: From (26), we see that as the satiation measure $H(F_{t-1}^*)$, i.e., a measure of fixed cost types that make all the firms across their productivity draws operate in the market, increases, the total measure of firms that operate at t also increases. From (27), the firms' total loan demand (in-kind) increases with the satiation rate $H(F_{t-1}^*)$. Also, note that it is increasing in F_{t-1}, the economy's average fixed-cost.

In (24), we observe that relative price of inputs produced by bottom-tier firms is increasing in both (technology adjusted) real wage $\frac{W_t}{P_t A_t}$ and (technology adjusted) top-tier firms' total demand for inputs $\frac{Y_t \Delta_t}{A_t}$, both of which usually rise when aggregate demand becomes stronger. Interestingly, more participation from bottom-tier firms, i.e., when $H(F_{t-1}^*)$ rises, it decreases, since there are more bottom-tier varieties available to top-tier firms, and the competition pushes down the average input price. This can in turn reduce marginal costs for top-tier firms and alleviates the economy's inflation.

Average satiation lower bound (SLB) Finally, we can compute the economy-wide average satiation interest rate based on equation (B.6). This variable, which we denote by $R_{t-1}^{J,*}$, will allow us to gauge the average satiation degree of bottom-tier firms. When the current policy rate R_{t-1}^J is a way too high compared with $R_{t-1}^{J,*}$, a small reduction in the policy rate can induce a wave of new entry into the market from bottom-tier firms. This additional entry will push down the average input price and inflation as well, as seen in (24), but might increase aggregate demand and push up the price level as those new entrants issue new loans to pay fixed costs (i.e., buy necessary equipment in final goods).

Proposition 3 *The average satiation lower bound (SLB) is given by*

$$R_{t-1}^{J,*} = \int_{\left(\frac{\omega-1}{\omega}\right)F_{t-1}}^{\infty} R_{m,t-1}^{J,*}\, dH\left(F_{m,t-1}\right) = \left(\frac{\omega^2}{\omega^2-1}\right) \cdot \frac{F_{t-1}^{*}}{F_{t-1}} \cdot R_{t-1}^{J}, \tag{28}$$

which is written in terms of the cutoff fixed cost F_{t-1}^{*} relative to the economy's average fixed cost F_{t-1}.

When the full satiation fixed cost threshold F_{t-1}^{*} is too high compared with the average fixed cost F_{t-1}, it means that it is highly likely to have satiation across different fixed cost draws. It in turn implies that the average SLB, $R_{t-1}^{J,*}$ relative to the current rate R_{t-1}^{J}, is high enough. In those cases, a small reduction in R_{t-1}^{J} will not boost a big wave of new entry of bottom-tier firms.

Limit case: $\omega \to \infty$ When $\omega \to \infty$, the fixed cost distribution $H(F_{m,t})$ collapses to its mean at F_t (i.e., degenerate), and there is only one fixed cost type of firms. In this case, the economy features full satiation or not according to the relative size of the policy rate, R_{t-1}^{J}, and the average satiation lower bound (SLB), $R_{t-1}^{J,*}$. If $R_{t-1}^{J} < R_{t-1}^{J,*}$, then all bottom-tier firms have already entered the market and produced their inputs, for example. We characterize equilibrium conditions in this case in Appendix D.

Now we define shock processes, government policies, and close the model.

2.3 Shock Processes

We assume that the average fixed cost F_t follows

$$F_t = \phi_f \cdot \bar{Y}_t \cdot \exp(u_{f,t}) = \phi_f \cdot \tilde{Y} A_t \cdot \exp(u_{f,t}), \tag{29}$$

where $u_{f,t} = \rho_f \cdot u_{f,t-1} + \varepsilon_{f,t}$ with $\varepsilon_{f,t} \sim N\left(0,\ \sigma_f^2\right)$. Note that $\bar{Y}$ is the technology de-denominated output level at the steady state, and $\bar{Y}_t \equiv \bar{Y} A_t$ is the balanced-growth path level of output.[11] For the technology process, we assume

$$GA_t \equiv \frac{A_{t+1}}{A_t} = (1+\mu) \cdot \exp\left\{u_{a,t}\right\}, \tag{30}$$

where $u_{a,t} = \rho_a \cdot u_{a,t-1} + \varepsilon_{a,t}$ where $\varepsilon_{a,t} \sim N\left(0,\ \sigma_a^2\right)$. Finally, the government expenditure G_t is a stochastic fraction of output Y_t, formally:

$$G_t = \phi_g \cdot Y_t \cdot \exp(u_{g,t}), \tag{31}$$

[11] We assume that F_t is proportional to $\bar{Y}_t = \bar{Y} A_t$, rather than a current output Y_t. The reason is that we believe the technology de-trended fixed cost distribution moving one-to-one with the business cycle Y_t is not realistic. Still, this assumption turns out to be not very important quantitatively.

where $u_{g,t} = \rho_g \cdot u_{g,t-1} + \varepsilon_{g,t}$ where $\varepsilon_{g,t} \sim N\left(0,\ \sigma_g^2\right)$. We assume that the government runs a balanced budget, and in each period it collects a lump sum tax $T_{g,t} = G_t$ from the representative household.[12]

2.4 Central Bank

We assume that the central bank follows a Taylor rule on interest rates, given by:

$$R_t^B = R_t^J = R^J \cdot \left(\frac{\Pi_t}{\Pi}\right)^{\tau_\pi} \left(\frac{Y_t}{\bar{Y}_t}\right)^{\tau_y} \cdot \exp\left\{\varepsilon_{r,t}\right\} \qquad \text{where: } \varepsilon_{r,t} \sim N\left(0, \sigma_r^2\right) \tag{32}$$

where $\varepsilon_{r,t}$ is the usual monetary policy shock, and $\bar{Y}_t$ is the balanced-growth path level of output. $\bar{\Pi}$ is the trend-inflation at the steady state. We can see from (29) that $\bar{Y} = \frac{\bar{Y}_t}{A_t}$.

2.5 Aggregation

Now we aggregate conditions in Section 2.2 to obtain the economy-wide conditions. First, the aggregate labor N_t demand is given by

$$N_t = \Theta_N \cdot \left(\frac{Y_t \Delta_t}{A_t}\right)^{\frac{1}{\alpha}} \cdot \left(1 + \Theta_4 H_{t-1}\right)^{-\frac{\alpha+\sigma(1-\alpha)}{(\sigma-1)\alpha}} \qquad \text{where: } H_{t-1} \equiv H(F_{t-1}^*) \tag{33}$$

where

$$\begin{aligned}\Theta_N = &\left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha}\right)^{\left(\frac{-\sigma}{\alpha+\sigma(1-\alpha)}\right)} \left(\frac{\kappa-1}{\kappa}\right)^{\left(\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}\right)} \left(\frac{\kappa[\alpha+\sigma(1-\alpha)]}{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}\right) \\ &\cdot \left(\frac{\omega(\sigma-1)}{\kappa[\alpha+\sigma(1-\alpha)]+(\omega-1)(\sigma-1)}\right) \Theta_3^{\left(\frac{\sigma}{\alpha(\sigma-1)}\right)} > 0.\end{aligned} \tag{34}$$

From (33), we observe that the aggregate labor demand increases with the aggregate demand for bottom-tier varieties, i.e., $J_t = Y_t\Delta_t$ from (10), but decreases with the satiation measure H_{t-1}. When the satiation measure H_{t-1} rises, a total measure of operating firms, M_t in (26), increases as well. It lowers the relative input price $\frac{P_t^J}{P_t}$ and the individual bottom-tier firm's labor demand $N_{mv,t}$. This intensive margin reduction in labor demand outweighs the extensive margin increase in labor demand from new entrants, taking the aggregate demand measure $J_t = Y_t\Delta_t$ as given.

Using the household's intratemporal optimization condition in (2) and equation (33), the real wage is given by:

$$\frac{W_t}{P_t A_t} = \Theta_N^{\frac{1}{\eta}} \left(\frac{C_t}{A_t}\right) \left(\frac{Y_t \Delta_t}{A_t}\right)^{\frac{1}{\eta\alpha}} \left(1 + \Theta_4 H_{t-1}\right)^{-\frac{\alpha+\sigma(1-\alpha)}{\eta(\sigma-1)\alpha}} \cdot \exp\left\{-u_{c,t}\right\}. \tag{35}$$

[12]Note that the government bond is in zero net supply, so the government's dynamic budget constraint holds.

When plugging (35) into equation (24), we obtain

$$\frac{P_t^J}{P_t} = \Theta_N^{\frac{1}{\eta}} \Theta_3^{\frac{\alpha+\sigma(1-\alpha)}{(\sigma-1)\alpha}} \left(\frac{C_t}{A_t}\right) \left(\frac{Y_t \Delta_t}{A_t}\right)^{\left(\frac{(1-\alpha)\eta+1}{\eta\alpha}\right)} (1+\Theta_4 H_{t-1})^{-\frac{(1+\eta)[\alpha+\sigma(1-\alpha)]}{\eta(\sigma-1)\alpha}} \cdot \exp\left\{-u_{c,t}\right\}. \tag{36}$$

From (33), (35), and (36), we see that given the aggregate demand measures, e.g., C_t and $J_t = Y_t \Delta_t$, as H_{t-1} increases, the relative input price drops and it causes the individual and aggregate labor demand to drop as well. It pushes down the equilibrium wage eventually. So here, more entry of bottom-tier firms is deflationary, reflecting positive shifts in aggregate supply.

Market clearing Market clearing in this economy is represented by

$$C_t + \frac{L_t}{P_t} + G_t = Y_t, \tag{37}$$

which with (27), (29), and (31) can be written as

$$\frac{C_t}{Y_t} = 1 - \phi_g \cdot \exp\left\{u_{g,t}\right\} - \phi_f \cdot \left(\frac{\tilde{Y}_t}{\tilde{Y}}\right)^{-1} \cdot \left[1 - \Theta_L \cdot [1-H_t]^{\left(\frac{\omega-1}{\omega}\right)}\right] \cdot \exp\left\{u_{f,t}\right\}. \tag{38}$$

Note in the clearing equation in (37) that the real loan enters in the left hand side. When bottom-tier firms decide to operate in the next period, they issue loans from banks, and buy final goods (e.g., equipment) using the loan proceeds. It raises aggregate demand and becomes inflationary as seen in (35) and (36) above: higher aggregate demand is inflationary there.[13] Therefore, bottom-tier firms' entry to the market both shifts the aggregate supply and demand curves to the right, roughly speaking, and depending on the relative magnitudes of two shifts, can be both inflationary and deflationary. We will see in Section 4 that the economy's short run responses to demand shocks and supply shocks would be similar, as demand shock causes supply shifts, while supply shock causes demand shifts in our framework. Therefore, demand shock and supply shock are truly intertwined in our model.

In Guerrieri et al. (2023), a sectoral supply shock (e.g., high-contact sectors are shut down due to Covid-19) is more likely to become Keynesian, i.e., it triggers a larger aggregate demand shift than its own magnitude, when there are multiple sectors and markets are incomplete. While they focus on the economy where the affected (to the supply shock) sector is either complementary to or uses inputs produced from unaffected sectors, our two-layers structure (i.e., top-tier and bottom-tier industries) allows us to study both ways between supply and demand: in our model, supply shocks to bottom-tier firms shift aggregate demand, through their effects on labor markets and the loan demand, while demand shocks triggers shifts in the bottom-tier supply curve, which are translated into shifts in the top-tier supply through their impacts on input prices and causes

[13] We observe the typical Keynesian-cross structure in (37) when endogenous bottom-tier firm entry is considered. When Y_t increases, the measure of operating bottom-tier firms M_t and their loan demand $\frac{L_t}{P_t}$ increase in response, it generates additional rounds of boosting demand: $Y\uparrow$ leads to $\frac{L}{P}\uparrow$, which leads to $Y\uparrow$, ad infinitum.

another round of demand shifts, ad infinitum.

Top-tier firms and households Top-tier firms are standard, except that their real marginal cost is now $\frac{P_t^J}{P_t}$ given in (36). Households are standard, except that they are subject to demand shocks $\phi_{c,t}$. We summarize equilibrium conditions (e.g., pricing à la Calvo (1983)) in Appendix B.

Average satiation lower bound (SLB) and satiation Plugging equation (B.22) into equation (28) we obtain an expression for the average SLB rate

$$R_t^{J,*} = \left(\frac{\omega}{\omega+1}\right) \cdot (1-H_t)^{-\frac{1}{\omega}} \cdot R_t^J \tag{39}$$

which implies the 'policy room' compared with the satiation lower bound (SLB), $\frac{R_t^J}{R_t^{J,*}}$, is decreasing in the satiation measure, $H(F_t^*)$.

The next corollary describes M_{t+1}, the measure of operating bottom-tier firms in period $t+1$, as a function of the policy room $\frac{R_t^J}{R_t^{J,*}}$ at t, which implies that the policy room $\frac{R_t^J}{R_t^{J,*}}$ acts as a *sufficient statistic* for the aggregate firm participation rate M_{t+1}. Furthermore, when we start from a higher level of the policy room, the same decrease in the policy room results in a higher increase in the number of bottom-tier firms that enter the market.[14] The intuition is simple: when the current policy rate, R_t^J, is too high compared with the average SLB, $R_t^{J,*}$,[15] then the economy has a larger room for more new entrants to operate as the policy rate goes down. Note that from the above (39) that

$$\frac{R_t^J}{R_t^{J,*}} \leq \frac{\omega+1}{\omega}. \tag{40}$$

Corollary 1 *The total measure of bottom-tier firms that decide to operate in period $t+1$, M_{t+1}, can be written as*

$$M_{t+1} = 1 - \Theta_M \cdot \left[\left(\frac{\omega}{\omega+1}\right) \cdot \frac{R_t^J}{R_t^{J,*}}\right]^{\omega}. \tag{41}$$

A given drop in the monetary policy room $\frac{R_t^J}{R_t^{J,*}}$ yields a higher increase in M_{t+1} as we start from the higher policy room.

Proof. From (41), we obtain

$$\frac{dM_{t+1}}{d\left(\frac{R_t^J}{R_t^{J,*}}\right)} = -\Theta_M \left[\left(\frac{\omega}{\omega+1}\right) \cdot \frac{R_t^J}{R_t^{J,*}}\right]^{\omega-1} \cdot \frac{\omega}{\omega+1} < 0,$$

[14] M_{t+1} as a function of the policy rate R_t^J, seen in (41), is decreasing and concave, and this result naturally follows.

[15] It corresponds to the case where F_t^*, the fixed cost cutoff, is relatively low. In that case, a small reduction in the policy rate can induce firms of middle-ranged fixed costs that have low productivity draws to enter the market, since these middle-ranged fixed costs types (i.e., around F_t^*) are not satiated yet.

whose magnitude itself is increasing in the level of $\frac{R_t^J}{R_t^{J,*}}$ as $\omega > 1$. ■

Flexible Price Model Under flexible prices, the price of top-tier produced products is a constant markup over the price of the bottom-tier produced input, formally:

$$\frac{P_t}{P_t^J} = \frac{(1+\zeta^T)^{-1}\gamma}{\gamma - 1}. \tag{42}$$

The flexible price equilibrium is money-neutral (i.e., R^J has no effect on the real allocation). Other equilibrium conditions are provided in Appendix B.

2.6 Summary Equilibrium Conditions

We define the balanced growth path-adjusted variables with tilde-variables, for example, $\tilde{Y}_t = \frac{Y_t}{A_t}$. In our simulation results, we assume that the government sets up optimal transfers to eliminate real distortions caused by monopolistic competition, that is $\zeta^T = \frac{1}{\gamma-1}$ and $\zeta^J = \frac{1}{\sigma-1}$. The equilibrium conditions are summarized in Appendix C.

3 Steady State Results

3.1 Calibration

Calibrated parameters are provided in Table 1. Our model relies on two layers of requirements for bottom-tier firms in operating on the market: fixed cost and productivity, and and those two variables follow independent Pareto distributions. We rely on the model's feature that the ratio of operating bottom-tier firms depends on the parameters related to those two Pareto distributions. Under our calibrated parameters in Table 1, our model could match the ratios or moments targeted by other models with different specifications. Some key steady state values are provided in Table 2.

Fixed cost to balanced growth path output ratio: ϕ_f We set $\phi_f = 0.37$ based on the following two reasons: According to the Business Dynamics Statistics (BDS), average annual exit and entry rates were 10.6% and 12.3% from 1977-2016. Our pick $\phi_f = 0.37$ generates a steady-state participation rate $M = 0.9$, with the exit rate equal to 10%. Second, our fixed cost could be interpreted as a sum of capital cost and non-capital fixed cost. The capital cost to output ratio is around 30% in the standard literature. The non-capital fixed cost (e.g., plant overhead labor and central office expenditure) to output ratio is from 0.05 to 0.18 varying across industries according to Table 5 in Domowitz et al. (1988). Our steady-state fixed cost-to-output ratio is 0.37, which is within this range.

Shape parameters in Pareto distributions: κ and ω We pick $\kappa = \omega = 3.4$ based on Ghironi and Melits (2005), who choose the shape parameter of the productivity distribution to be 3.4 to match the standard deviation of log US plant sales, which equals 1.67 according to the estimation in Bernard et al. (2003). In fact, in our model, the standard deviation of log sales of operating bottom-tier firms is given by[16]

$$\sigma\left(\log r_{mv,t}\right) = \frac{\sigma - 1}{\alpha + \sigma(1-\alpha)} \sqrt{\frac{1}{\kappa^2} + \left(\frac{\alpha + \sigma(1-\alpha)}{\sigma - 1}\right)^2 \frac{1}{\omega^2}}. \tag{43}$$

With $\kappa = \omega = 3.4$, the model-generated standard deviation of bottom-tier firms' revenues is 0.51. According to Bernard et al. (2003), the unexplained variability comes from some features we do not consider (e.g., heterogeneity in tastes over varieties and demand weights on different types of products) or it arises because while varieties in our framework are based on the bottom-tier firms, Bernard et al. (2003) estimate the parameter from manufacturing plants in the US.

In terms of the variability in productivity, in our model, the standard deviation of log productivity of operating bottom-tier firms is given by

$$\sigma\left(\log \varphi_{mv,t}\right) = \sqrt{\frac{1}{\kappa^2} + \left(\frac{\alpha + \sigma(1-\alpha)}{\sigma - 1}\right)^2 \frac{1}{\omega^2}}, \tag{44}$$

which equals to 0.36 when $\kappa = \omega = 3.4$. According to the estimation of the standard deviation of the log value added per worker in Bernard et al. (2003), their model-simulated moment is 0.35, and the estimated moment from actual data is 0.75.[17] Considering possible measurement errors, our variability is close to their model estimate and is within the reasonable range.

Elasticity of substitution: γ and σ We pick $\gamma = \sigma = 3.79$ based on Bernard et al. (2003), who choose the elasticity of substitution to fit US plant and macro trade data. More specifically, they pick 3.79 to match the productivity and size advantages of exporters in the US plant-level data.[18]

The standard calibration in the literature, $\gamma = 4.3$, delivers 30% markup over marginal costs. In our model, while top-tier firms do not have fixed costs and their marginal costs equal average input costs, bottom-tier firms need to pay period-by-period fixed costs to stay on operation. Thus, their average total cost is greater than the marginal cost. Although $\gamma = 3.79$ generates a higher markup over marginal costs, we believe it delivers a reasonable markup over average costs when considering both layers of firms.[19]

[16] Equations (43) and (44) is derived in Appendix B.

[17] Bernard et al. (2003) argue that some degree of under-prediction can be due to measurement errors in the Census data.

[18] Previous works including Ghironi and Melits (2005), Bilbiie et al. (2012), and Fasani et al. (2023) use the same calibrated elasticity of substitution level as Bernard et al. (2003).

[19] Jones (2011) considers the substitutability and complementarity of intermediate goods by assuming two different levels of elasticities of substitution, 3 and 0.5, for final goods intermediate goods, respectively. We pick the same level of the elasticity of substitution in two layers, instead. Actually, the relative sizes of γ and σ rely on interpretations of the model. If we regard bottom-tier firms as producers of some fundamental products including electricity, transportation

	Parameter Description	Value	Target
β	Discount factor	0.99	Average annualized real interest rate of 3.5%
η	Fisch labor supply elasticity	1	Standard
γ	Elasticity of substitution (of top-tier market)	3.79	Calibrated by Bernard et al. (2003) to fit the US plant and macro trade data
σ	Elasticity of substitution (of bottom-tier market)	3.79	Set to be the same as top-tier products.
α	labor share in the bottom-tier production function	0.7	Standard
θ	Calvo (1983) price stickiness	0.75	Standard
κ	Shape parameter: Pareto distribution of productivity	3.4	Ghironi and Melits (2005)
ω	Shape parameter: Pareto distribution of fixed cost	3.4	Keep it the same with the productivity distribution
ϕ_f	Fixed cost - steady state output ratio	0.37	The steady state mass of firms operating in the market $M = 0.9$. The real loan to output ratio, $\frac{L}{PAY}$, equals 30%.
ϕ_g	Government spending - output ratio	18%	Smets and Wouters (2007)
τ_π	Taylor parameter (inflation)	1.5	Standard
τ_y	Taylor parameter (output)	0.15	Standard
μ	Long-run TFP growth rate	0.005	Match a yearly growth rate at 2%
Π	Long-run inflation	1.02	Long-run inflation target at 2%
ρ_a	Autoregression for TFP	0.95	Smets and Wouters (2007)
ρ_c	Autoregression for demand shock	0.6	The autocorrelation of the preference shock that affects the marginal utility of consumption estimated by Nakajima (2005)
ρ_g	Autoregression for government spending	0.97	Smets and Wouters (2007)
ρ_f	Autoregression for fixed cost	0.8	Gutiérrez et al. (2005) use data on entry, investment, and stock market valuations of the US economy to recover entry cost shocks. The estimated persistence is 0.72
σ_a	SD for ϵ_a	0.5	Within admissible intervals in Smets and Wouters (2007)
σ_c	SD for ϵ_c	0.2	The standard deviation of the preference shock estimated by Nakajima (2005) using U.S. data on consumption, labor, and output is 0.017.
σ_g	SD for ϵ_g	0.2	In Smets and Wouters (2007), the estimated admissible interval is $[0.48, 0.58]$. For our purposes, we do not need large disturbances to generate sizable responses.
σ_f	SD for ϵ_f	0.2	Gutiérrez et al. (2005) uses data on entry, investment, and stock market valuations of the US to recover entry cost shocks. The estimated standard deviation is 0.087.
σ_r	SD for ϵ_r	0.08	In Smets and Wouters (2007), the estimated admissible interval is $[0.22, 0.27]$. For our purposes, we do not need large disturbances to generate sizable responses.

Table 1: Calibrated parameters

Variable	Value	Meaning
H	0.74	Mass of productivity-irrelevant firms
M	0.9	Mass of firms operating in the market
R^B	1.02	Gross risk-free rate
$R^{J,*}$	1.17	Gross satisation rate
$\tilde{F}^*$	0.43	Cutoff fixed cost-to-output ratio
Δ	1.0006	Price dispersion
$\frac{W_t}{P_t A_t}$	0.67	Real wage
$\frac{C_t}{Y_t}$	0.52	Consumption-to-output ratio
$\frac{W_t N_t}{P_t Y_t}$	0.7	Labor cost-to-output ratio
$\frac{L_t}{P_t Y_t}$	0.3	Loan-to-output ratio

Table 2: Steady state values

3.2 Comparative statics

In this section, we do comparative statics exercises on the steady state equilibrium with varying parameters.

The fraction of operating bottom-tier firms A proportion of bottom-tier firms that operate in the market, M, is given by $1 - \Theta_M[1 - H]$ at the steady state from (26). Figure 2 shows how the steady state mass of firms active on the market, M, changes with parameters in our model: κ, ω, ϕ_f, β, μ, and Π.

We especially decompose M into two parts in the following way:

$$\begin{aligned} M &= Prob(F < F^*) + \cancel{Prob(F > F^*)} \int_{F^*}^{\infty} \left(\frac{F_m}{F^*}\right)^{-\frac{\kappa[\alpha+\sigma(1-\alpha)]}{\sigma-1}} \frac{dH(F_m)}{\cancel{1 - H(F^*)}} \\ &= \underbrace{H(F^*)}_{\equiv M_1} + \underbrace{\frac{\omega(\sigma-1)}{\kappa[\alpha+\sigma(1-\alpha)] + \omega(\sigma-1)}(1 - H(F^*))}_{\equiv M_2}. \end{aligned} \tag{45}$$

M_1 is the mass of fixed cost types that are satiated, i.e., a measure of index m such that $M_{m,t} = 1$. This term represents the mass of firms whose fixed costs are low enough, i.e., $F_{m,t} \le F^*$ that even firms with the lowest productivity still operate in the market. M_2, on the other hand, is the mass of operating firms whose fixed cost types are not satiated, i.e., there are some firms that share the same fixed cost but do not produce in the market due to bad productivity draws.

We summarize our findings from Figure 2 as follows: (i) when κ increases, the productivity distribution becomes more concentrated around its mean, and a probability of very low produc-

service, and raw materials, then those products have less substitutability, implying $\sigma < \gamma$. In contrast, if we regard them as producing different brands of the same product, then it is more likely that those products are substitutable, and we have $\sigma > \gamma$. We are agnostic about the possible interpretation and pick $\gamma = \sigma = 3.79$.

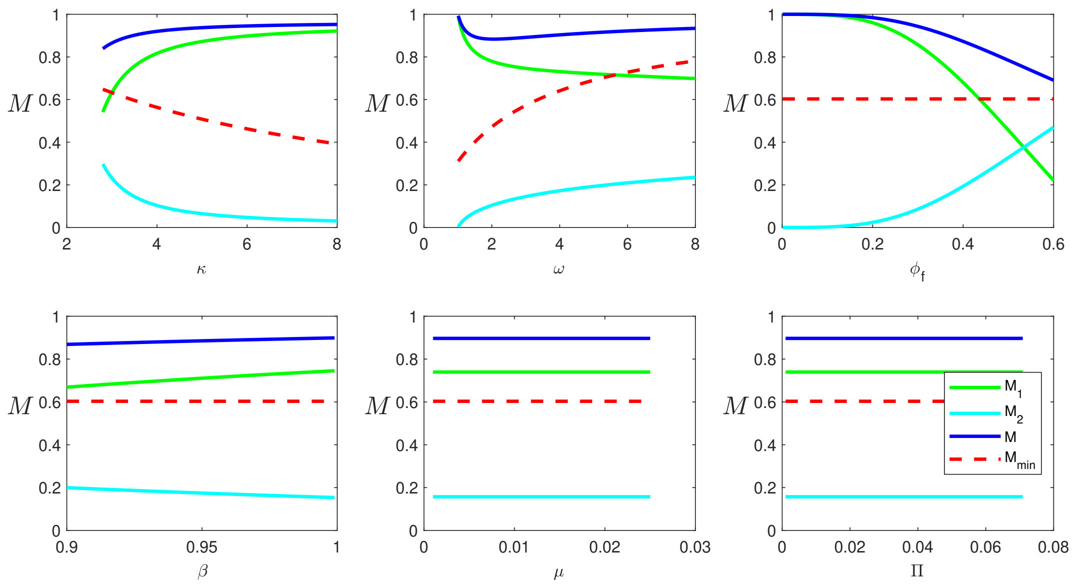

Figure 2: Comparative statistics: M

Notes: Other parameters are fixed at the benchmark level in Table 1. The range for κ, ω, ϕ_f, β, μ, and Π are $[2.8, 8]$, $[1.01, 8]$, $[0.001, 0.6]$, $[0.9, 0.999]$, $[0.001, 0.025]$, and $[1.001, 1.0709]$ respectively. The red dashed line denotes the minimum mass of firms operating in the market, $M_{min} = 1 - \Theta_M$, which is attained when no fixed cost type of firms is satiated, i.e., $H_t = 0$. We decompose M into a productivity-irrelevant part, M_1, and a jointly determined part, M_2 under different parameter values.

tivity draws shrinks as well. M_1 and M both increase consequentially, because the productivity lower bound is now higher, which makes more likely that a given fixed cost type is satiated, and also any firm operates in the market; (ii) as ω increase, it has two countervailing effects: while it raises the minimum fixed cost $\frac{\omega-1}{\omega}F$ and therefore reduces firm participation M, it makes the fixed cost distribution more concentrated around its mean F and reduce the mass of high fixed cost firms, which might raise the participation rate M. Depending on the relative sizes of these two effects, ω can both raise or lower M. The satiation measure M_1 usually decreases in ω since a rise in ω leads to an increase in the lowest fixed cost $\frac{\omega-1}{\omega}F$, and the low fixed cost types are the ones that are usually satiated. These general features about ω and M can be also seen in Figure A.1 in Appendix A where we study how changes in other parameters affect the functional relation of M with each parameter; (iii) when ϕ_f rises, it shifts the fixed cost distribution to the right, and lowers both M and M_1.

Due to (41), we know that the policy room $\frac{R^J}{R^{J,*}}$ is negatively related to M. Therefore changes in parameters would have opposite effects on the policy room from their impacts on M in Figure 2. We document this in Figure A.2 in Appendix A.

The real loan to output ratio From (27) and (40), the following holds at the steady state:

$$\phi_f\left(1-\Theta_L\right) \leq \frac{L}{P \cdot A \cdot \bar{Y}} = \phi_f\left[1-\Theta_L(1-H(F^*))^{\frac{\omega-1}{\omega}}\right] = \phi_f\left[1-\Theta_L\left(\frac{\omega}{\omega+1}\frac{R^J}{R^{J,*}}\right)^{\omega-1}\right] \leq \phi_f,$$

where the real loan to GDP ratio, $\frac{L}{PA\bar{Y}}$, is decreasing in the policy room $\frac{R^J}{R^{J,*}}$ and increasing in the satiation measure $H(F^*)$ and the total participation measure M as well.[20]

Figure 3 shows how $\frac{L}{PA\bar{Y}}$ changes with our model parameters: κ, ω, ϕ_f, β, μ, and Π. We can summarize our findings from Figure 3 as follows: (i) A rising κ raises M from Figure 2 and lowers the policy room $\frac{R^J}{R^{J,*}}$ from (41) and Figure A.2, therefore increasing the economy's aggregate loan demand as more firms enter the market; (ii) When ω increases, it invokes two opposite forces: in Figure 2, a higher ω initially lowers the participation M when ω is not too big as the minimum fixed cost $\frac{\omega-1}{\omega}F$ rises. This (negative) extensive margin effect on the loan demand is outweighed by the (positive) intensive margin effect in which each participating firm pays higher fixed cost; (iii) An increase in ϕ_f reduces firm participation M from Figure 2, thereby lowers the aggregate loan demand. This (negative) extensive margin effect on the loan demand is outweighed by the (positive) intensive margin effect in which each participating firm pays higher fixed cost, again.[21]

This can be seen in Figure 4, where we draw co-movements between the policy room $\frac{R^J}{R^{J,*}}$ and the real loan to GDP ratio $\frac{L}{PA\bar{Y}}$. An increase in ϕ_f or ω lowers M and raises $\frac{R^J}{R^{J,*}}$ from (41), which might reduce the aggregate loan amount $\frac{L}{A}$. This (negative) extensive margin adjustment in loan issuance is dominated by the positive intensive margin adjustment where each operating firm's loan issuance gets bigger under higher ϕ_f or ω, generating a positive correlation between $\frac{R^J}{R^{J,*}}$

[20] Note that in (26), M is increasing in H at the steady state.

[21] Figure A.3 studies how changes in other parameters affect the functional relation of $\frac{L}{PAY}$ with each parameter.

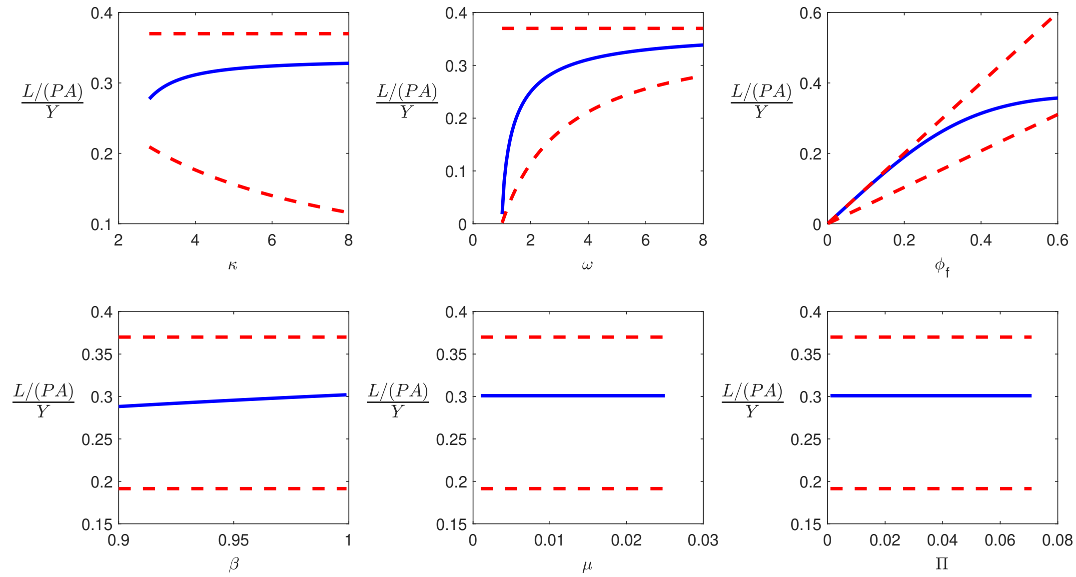

Figure 3: Comparative statistics: Output-scaled real lending

Notes: The red-dashed lines are the upper bound and lower bound for the output-scaled lending, ϕ_f and $\phi_f(1-\Theta_L)$ correspondingly

and $\frac{L}{PA\bar{Y}}$. In contrast, an increase in κ will lead to increases in both participation M and the loan issuance $\frac{L}{PA\bar{Y}}$, generating a negative correlation between the policy room $\frac{R^J}{R^{J,*}}$ and $\frac{L}{PA\bar{Y}}$.

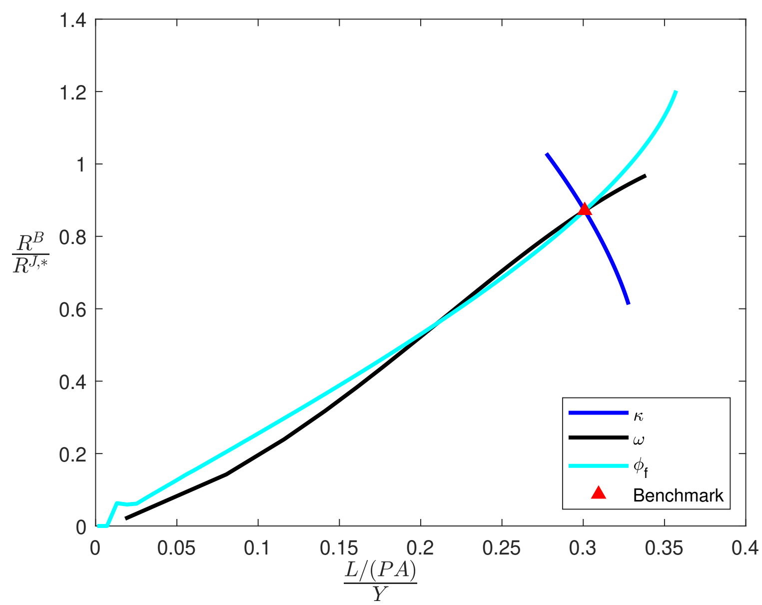

Figure 4: Policy power on output-scaled real lending

Notes: This figure display how changes in κ, ω, and ϕ_f drive comovements between $\frac{R^J}{R^{J,*}}$ and $\frac{\frac{L}{PA}}{Y}$. The solid triangular marker denotes the steady state value under the benchmark calibration.

4 Impulse Response Functions

4.1 Supply vs. Demand Shocks

Technology shock Figure 5 illustrates effects of the technology shock to bottom-tier firms under different ϕ_f levels. In response to a positive TFP growth shock, more bottom-tier firms decide to operate (i.e., M_t and H_t increase) and the aggregate technology-demeaned loan demand $\frac{L_t}{P_t A_t}$ increases in response,[22] raising aggregate demand from (37). While a higher M_t (and H_t) tends to reduce the real input price $\frac{P_t^J}{P_t}$ from (36) and the labor demand N_t from (33), now a higher aggregate demand is a dominating force and pushes up demands for bottom-tier produced inputs, raising real input prices and real wages in equilibrium, due to a higher labor demand N_t by those firms. As top-tier firms face higher real input prices, inflation Π_t jumps up in response, and the interest rate R_t^J spikes up as well. Finally, the policy room $\frac{R_t^J}{R_t^{J,*}}$ jumps down, in an alignment with rising M_t from (41).

Note that as ϕ_f becomes higher, the economy's extensive margin adjustment becomes stronger, leading to more drastic increases in M_t and H_t. The (additional) loan demand by those new entrants, in addition to higher fixed costs paid by incumbent firms under higher ϕ_f, leads to more amplified initial increases in the real input price $\frac{P_t^J}{P_t}$, real wage $\frac{W_t}{P_t A_t}$, labor demand N_t, and output Y_t. Later, more fierce competition from higher M_t and H_t starts to dominate this effect of higher aggregate demand, i.e., a higher supply of bottom-tier firms dominates the higher aggregate demand in (36), and $\frac{P_t^J}{P_t}$, $\frac{W_t}{P_t A_t}$, and N_t all drop, and converge to the steady state faster. Inflation Π_t, which is forward looking, jumps up less under higher ϕ_f as a result.[23]

This can be understood with a traditional aggregate demand and aggregate supply (i.e., AD-AS) model: (i) with a positive technology shock, the supply curve shifts to the right; (ii) a positive shift in supply pushes the demand curve out due to a rise in demand for loans, with which firms purchase equipment for operations (in terms of the final good) and labors, causing additional impacts on supply, ad infinitum; (iii) the supply curve shifts more to the right under higher ϕ_f, leading to less rise in inflation and higher increase in output.

Demand shock Figure 6 illustrates effects of the consumption demand shock to the reresentative household under different ϕ_f levels. Interestingly, it generates a qualitatively similar impulse response function to Figure 5: a positive shock to consumption demand induces more bottom-tier firms to operate in the market, raising aggregate demand, causing another round of positive shift in firm entry, raising aggregate demand again, ad infinitum.

In sum, we observe that supply and demand shocks are truly intertwined in our framework, and which shock is the first one to move the economy does not matter qualitatively.

[22] In Figures 5 and 6, % decreases in $L_t/(P_t A_t \tilde{Y}_t)$ are lower than increases in $\tilde{Y}_t$, and therefore $L_t/(P_t A_t)$ increases in response to the shock. When ϕ_f is small, fixed costs are small in general, and the loan demand L_t/P_t are insignificant. In those cases, $L_t/(P_t A_t \tilde{Y}_t)$ drops as $\tilde{Y}_t$ increases but $L_t/(P_t A_t)$ remains around 0.

[23] It aligns with Cecioni (2010) who argues that a rise in the number of firms significantly lowers the US inflation.

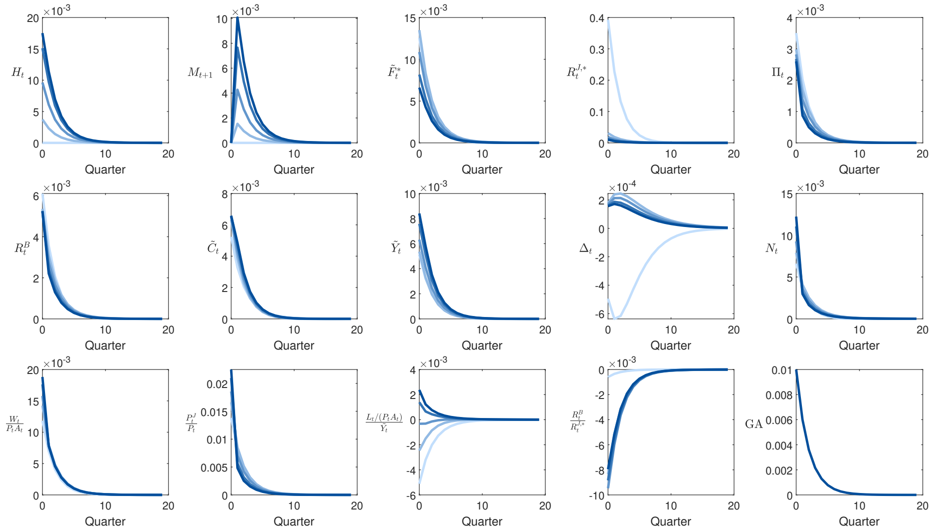

Figure 5: Impulse response functions to TFP shock

Notes: The figures display the deviation for 1 standard deviation (0.01) in $u_{a,t}$ which increases the growth rate of the average productivity for bottom-tier firms. The autoregressive coefficient is 0.6. The gradient blue lines denote the responses under calibration with varying ϕ_f. From the light blue to the dark blue, ϕ_fs are 0.02, 0.25, 0.37 (benchmark), 0.5, and 0.6, with corresponding Ms equal to 0.99, 0.96, 0.9, 0.78, and 0.69. The variables below are plotted in deviations from their steady states: H, M, R^B, Π, and $R^{j,*}$. The rest of the variables are plotted in log deviations from their steady states. Δ is the price dispersion for the top-tier products.

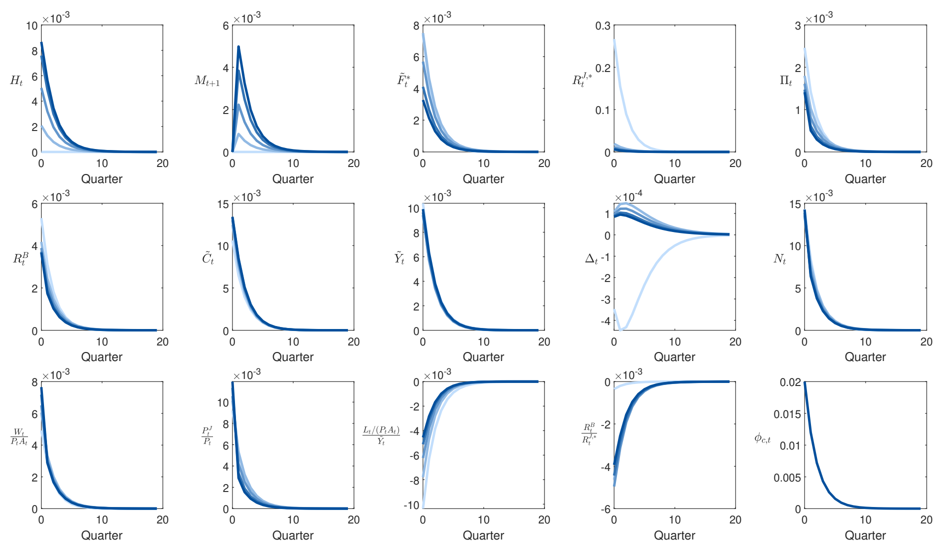

Figure 6: Impulse response functions to demand shock

Notes: The figures display the deviation for 1 standard deviation (0.08) in $u_{c,t}$, the demand shock. The autoregressive coefficient is 0.6. The gradient blue lines denote the responses under calibration with varying ϕ_f. From the light blue to the dark blue, ϕ_f are 0.02, 0.25, 0.37 (benchmark), 0.5, and 0.6, with corresponding Ms equal to 0.99, 0.96, 0.9, 0.78, and 0.69. The below variables are plotted in deviations in level from their steady states: H, M, R^B, Π, and $R^{j,*}$. The rest of the variables are plotted in deviations in logs from their steady states.

Other shocks In Appendix A, we provide impulse response functions to fixed cost shocks $u_{f,t}$ (in Figure A.4), monetary policy shocks $\varepsilon_{r,t}$ (in Figure A.5), and government spending shocks $u_{g,t}$ (in Figure A.6). In response to a positive fixed cost shock $u_{f,t}$, firm entry M_t and the satiation measure H_t drop, because the productivity cutoff $\varphi^*_{m,t}$ defined in (17) rises for every type m as each firm needs to pay higher fixed costs for their operation: thus the extensive margin effect on aggregate demand is negative. However, now that each incumbent (i.e., operating) firm needs to pay higher fixed costs, adding positive pressure on aggregate demand, this intensive margin adjustment in aggregate demand turns out to outweigh the negative extensive margin adjustment from firm exits and raises equilibrium output under our parametrization. Thus, each incumbent firm produces more,[24] demands more labor, and the equilibrium wage rises in response. It raises inflation Π_t as well. Under a higher ϕ_f, the (negative) extensive margin change becomes bigger, triggering a larger negative shift in supply, generating higher inflation and smaller increases in output.

Monetary policy tightening generates the opposite impulse response function to the case of positive demand shocks in Figure 6. It discourages firm entry, reducing M_t and H_t. The reduction in aggregate demand lowers inflation and further induces firms' exits from the market. A positive government spending shock generates an interesting dynamics: while it crowds out consumption through higher real interest rates, inflation drops due to higher participation by bottom-tier firms (i.e., M_t and H_t rise). The government spending multiplier gets bigger under higher ϕ_f, which is due to a larger degree of new entry of bottom-tier firms.

4.2 Intensive vs. Extensive Margin Adjustment in Labor

Aggregate labor N_t in (33) changes over time due to two reasons: (i) each operating firm's labor demand, $N_{mv,t}$ changes over time (i.e., intensive margin adjustment); (ii) the number of bottom-tier firms that operate, M_t, fluctuate along the business cycle (i.e., extensive margin adjustment). As the aggregate labor N_t in (33) is defined as

$$N_t = \int_0^1 \int_{v \in \Omega_{m,t}} N_{mv,t} \, \mathrm{d}v \, \mathrm{d}m, \tag{46}$$

where the individual labor demand $N_{mv,t}$ is from (B.14). Now, let us assume that a bottom-tier firm (m, v) operates in both periods t and $t+\iota$, for some $\iota \geq 1$. From (B.14), we define

$$g^{\text{Density}}_{t,t+\iota} \equiv \frac{N_{mv,t+i} - N_{mv,t}}{N_{mv,t}} = \left[\frac{1 + \Theta_4 \cdot H_{t-1}}{1 + \Theta_4 \cdot H_{t+\iota-1}}\right]^{\left(\frac{\sigma}{(\sigma-1)\alpha}\right)} \left(\frac{\frac{Y_{t+\iota}\Delta_{t+\iota}}{A_{t+\iota}}}{\frac{Y_t \Delta_t}{A_t}}\right)^{\frac{1}{\alpha}} - 1, \tag{47}$$

[24] As incumbent firms pay more fixed costs under a positive $u_{f,t}$ shock, the demand for the final consumption good rises, thereby increasing the demand for bottom-tier produced inputs as well. Therefore, under our parametrization, negative supply shocks are translated into positive demand shocks here.

which is % change between t and $t+\iota$ of an individual firm (m, v)'s labor demand, $N_{mv,t}$, if the firm operates in both periods. Note that this intensive margin change $g_{t,t+\iota}^{\text{Density}}$ does not depend on indexes (m, v), i.e., is a function of aggregate variables only. We define $g_{t,t+\iota}^{\text{Density}}$ as the 'intensive margin' adjustment in labor demand.

From (33), we can obtain a formula for the % change in the aggregate labor N_t, which is given by[25]

$$g_{t,t+\iota}^{N} \equiv \frac{N_{t+\iota} - N_t}{N_t} = g_{t,t+\iota}^{\text{Density}} + (1 + g_{t,t+\iota}^{\text{Density}}) \cdot g_{t,t+\iota}^{\text{Entry}}, \tag{48}$$

where $g_{t,t+\iota}^{\text{Density}}$ is defined in (47) and $g_{t,t+\iota}^{\text{Entry}}$ is defined as

$$g_{t,t+\iota}^{\text{Entry}} = \frac{(H_{t+\iota-1} - H_{t-1}) + \frac{\omega(\sigma-1)}{\kappa[\alpha+\sigma(1-\alpha)]+(\omega-1)(\sigma-1)}(H_{t-1} - H_{t+\iota-1})}{H_{t-1} + \frac{\omega(\sigma-1)}{\kappa[\alpha+\sigma(1-\alpha)]+(\omega-1)(\sigma-1)}(1 - H_{t-1})}. \tag{49}$$

We define $g_{t,t+\iota}^{\text{Entry}}$ as the 'extensive' margin adjustment in labor, caused by changes in firm entry. From (48), we can see that the total % change in aggregate labor can be attributed to both intensive and extensive margin adjustments.

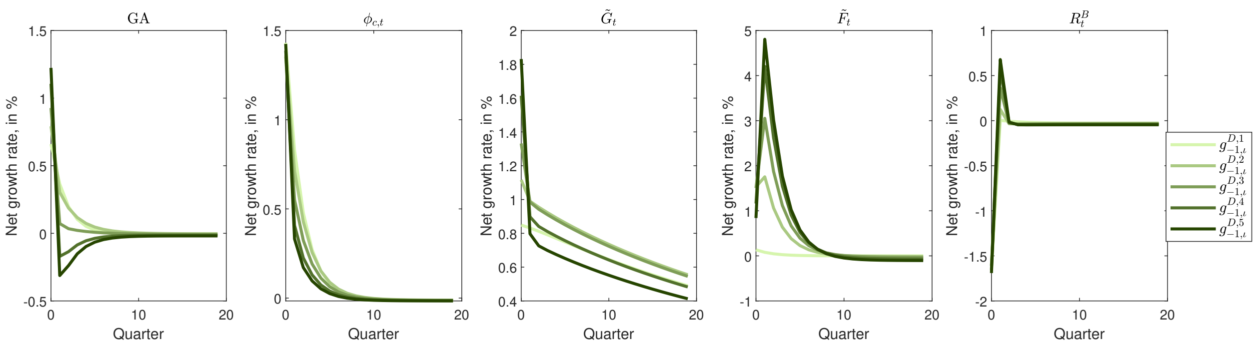

Figure 7: Decomposition of labor growth rate: isolines on intensive margin

Notes: The figures display the employment growth rate compared with the employment level before the shock ($t = -1$) after the one-time shock. The gradient green lines denote the responses in the intensive margin under calibration with varying ϕ_f. From the light green to the dark green lines, ϕ_f are 0.02, 0.25, 0.37 (benchmark), 0.5, and 0.6, with corresponding Ms equal to 0.99, 0.96, 0.9, 0.78, and 0.69. The growth rates are in net rate and in percentage.

Figure 7 and 8 depicts responses of intensive (i.e., $g_{t,t+\iota}^{\text{Density}}$) and extensive (i.e., $g_{t,t+\iota}^{\text{Entry}}$) margins to different shocks, respectively. In response to a positive fixed cost shock (i.e., $u_{f,t}$), we observe that (i) due to exits of bottom-tier firms from the market, the extensive margin adjustment in labor demand is negative; (ii) however, as operating firms pay larger fixed costs (and thus spending more on final goods using the loan proceeds) and aggregate output is higher in Figure A.4, each of them hires more workers to meet the demand for inputs as seen in Figure 7.

[25] We derive (48) in Appendix B.

For consumption demand shock $\phi_{c,t}$, we can see that both extensive and intensive margin adjustments in labor demand are positive, as more firms enter the market and aggregate output is higher as seen in Figure 6. Under higher ϕ_f, the extensive margin effect becomes more significant, while the intensive margin effect is not monotonic: initially each operating firm demands a higher number of workers, but eventually due to more fierce competition between firms caused by the higher entry (i.e., M_t and H_t rise more under higher ϕ_f as seen in Figure A.4), the labor demand's intensive margin response becomes more muted later under higher ϕ_f.

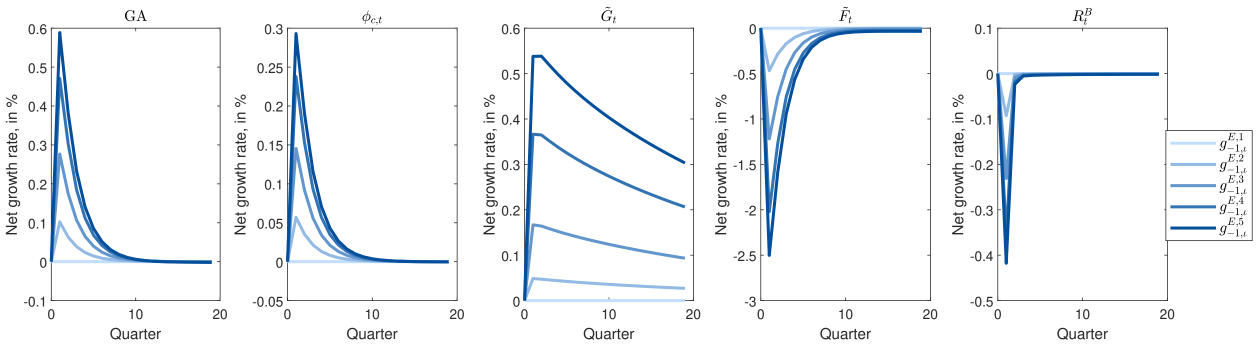

Figure 8: Decomposition of labor growth rate: isolines on extensive margin

Notes: The figures display the employment growth rate compared with the employment level before the shock ($t = -1$) after the one-time shock. The gradient blue lines denote the responses in the extensive margin under calibration with varying ϕ_f. From the light blue to the dark blue lines, ϕ_f are 0.02, 0.25, 0.37 (benchmark), 0.5, and 0.6, with corresponding Ms equal to 0.99, 0.96, 0.9, 0.78, and 0.69. The growth rates are in net rate and in percentage.

5 Conclusion

In this paper, our primary objective is to build a microfounded macroeconomic framework where aggregate demand and aggregate supply are intertwined. In that purpose, we build a novel two-layers system of firms where bottom-tier firms need to pay fixed costs (e.g., purchasing necessary equipment and building factories) in advance in order to operate in the next period while a top-tier firm uses the composite good (i.e., aggregator) produced from the bottom-tier industry as a sole input and faces nominal rigidities. As bottom-tier firms possesses no cash before deciding to enter the market, they need to rely on loans from financial markets in order to pay those fixed costs. Since fixed costs are in final consumption good terms, a new entry by a bottom-tier firm spurs a positive shock to aggregate demand. Our analytical characterization allows us to express the equilibrium firm entry as a function of the policy room, a level of current interest rate relative to the average satiation lower bound (SLB), where the average SLB depicts the rate that makes the firm entry saturated, i.e., most bottom-tier firms have already entered the market. In addition, our general equilibrium framework allows us to decompose changes in aggregate variables (e.g., labor) into extensive margin (i.e., from new firm entries) and intensive margin (i.e., incumbent firms) components, which provide additional insights on how the model works.

When a positive demand shock hits the economy, it triggers more bottom-tier firms to enter the market, pushing down marginal costs for top-tier firms and thereby the aggregate supply curve as well. Due to loan demands, this new entry adds additional upward pressures on aggregate demand, invoking a new round of firm entry, and this positive feedback between demand and supply keeps going on until the convergence happens. Therefore, supply shocks are demand shocks, and vice versa in our framework.

We believe that our framework sheds a light on how the Keynesian demand and endogenous firm entry can interact in a tractable manner in a macroeconomic model.

References

Abadi, Joseph, Markus Brunnermeier, and Yann Koby, "The Reversal Interest Rate," *working paper*, 2022.

Bergin, Paul R. and Giancarlo Corsetti, "The extensive margin and monetary policy," *Journal of Monetary Economics*, 2008, *55* (7), 1222–1237.

Bernard, Andrew B., Jonathan Eaton, J. Bradford Jensen, and Samuel Kortum, "Plants and Productivity in International Trade," *American Economic Review*, 2003, *93* (4), 1268–1290.

Bilbiie, Florin O., Fabio Ghironi, and Marc J. Melitz, "Endogenous Entry, Product Variety, and Business Cycles," *Journal of Political Economy*, 2012, *120* (2), 304–345. Publisher: The University of Chicago Press.

_ , _ , _ , Virgiliu Midrigan, and Julio J. Rotemberg, "Monetary Policy and Business Cycles with Endogenous Entry and Product Variety [with Comments and Discussion]," *NBER Macroeconomics Annual*, 2007, *22*, 299–379. Publisher: The University of Chicago Press.

Burnside, Craig, Martin Eichenbaum, and Sergio Rebelo, "Labor Hoarding and the Business Cycle," *Journal of Political Economy*, 1993, *101* (2), 245–273.

Calvo, Guillermo, "Staggered prices in a utility-maximizing framework," *Journal of Monetary Economics*, 1983, *12* (3), 383–398.

Cecioni, Martina, "Firm Entry, Competitive Pressures and the U.S. Ináation Dynamics," *Temi di discussione (Economic working papers), Bank of Italy, Economic Research Department*, 2010.

Colciago, Andrea and Riccardo Silvestrini, "Monetary policy, productivity, and market concentration," *European Economic Review*, 2022, *142*, 103999.

Domowitz, Ian, Glenn Hubbard, and Bruce C. Petersen, "Market Structure and Cyclical Fluctuations in U.S. Manufacturing," *The Review of Economics and Statistics*, 1988, *70* (1), 55–66.

Fasani, Stefano, Haroon Mumtaz, and Lorenza Rossi, "Monetary policy uncertainty and firm dynamics," *Review of Economic Dynamics*, January 2023, *47*, 278–296.

Ghironi, Fabio and Marc J. Melits, "International Trade and Macroeconomic Dynamics with Heterogeneous Firms*," *The Quarterly Journal of Economics*, 2005, *120* (3), 865–915.

Guerrieri, Veronica, Guido Lorenzoni, Ludwig Straub, and Iván Werning, "Macroeconomic Implications of COVID-19: Can Negative Supply Shocks Cause Demand Shortages?," *The American Economic Review*, 2023, *112* (5), 1437–1474.

Gutiérrez, Germán, Callum Jones, and Thomas Philippon, "Entry costs and aggregate dynamics," *The Review of Economics and Statistics*, 2005, *49* (5), 1331–1360.

Hamano, Masashige and Francesco Zanetti, "Endogenous product turnover and macroeconomic dynamics," *Review of Economic Dynamics*, 2017, *26*, 263–279.

Jones, Charles I, "Intermediate Goods and Weak Links in the Theory of Economic Development," *American Economic Journal: Macroeconomics*, April 2011, *3* (2), 1–28.

Kobayashi, Teruyoshi, "Firm entry, credit availability and monetary policy," *Journal of Economic Dynamics and Control*, 2011, *35* (8), 1245–1272.

Lewis, Vivien and Céline Poilly, "Firm entry, markups and the monetary transmission mechanism," *Journal of Monetary Economics*, 2012, *59* (7), 670–685.

Nakajima, Tomoyuki, "A business cycle model with variable capacity utilization and demand distur- bances," *The Review of Economics and Statistics*, 2005, *49* (5), 1331–1360.

Smets, Frank and Rafael Wouters, "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach," *American Economic Review*, 2007, *97* (3), 586–606.

Stebunovs, Viktors, "Finance as a Barrier to Entry: U.S. Bank Deregulation and Business Cycle," *working paper*, 2008.

Ulate, Mauricio, "Going Negative at the Zero Lower Bound: The Effects of Negative Nominal Interest Rates," *American Economic Review*, 2021, *111* (1), 1–40.

Uusküla, Lenno, "Monetary transmission mechanism with firm turnover," *Journal of Macroeconomics*, 2016, *50*, 1–18.

Appendix A Additional Tables and Figures

A.1 Section 3.2

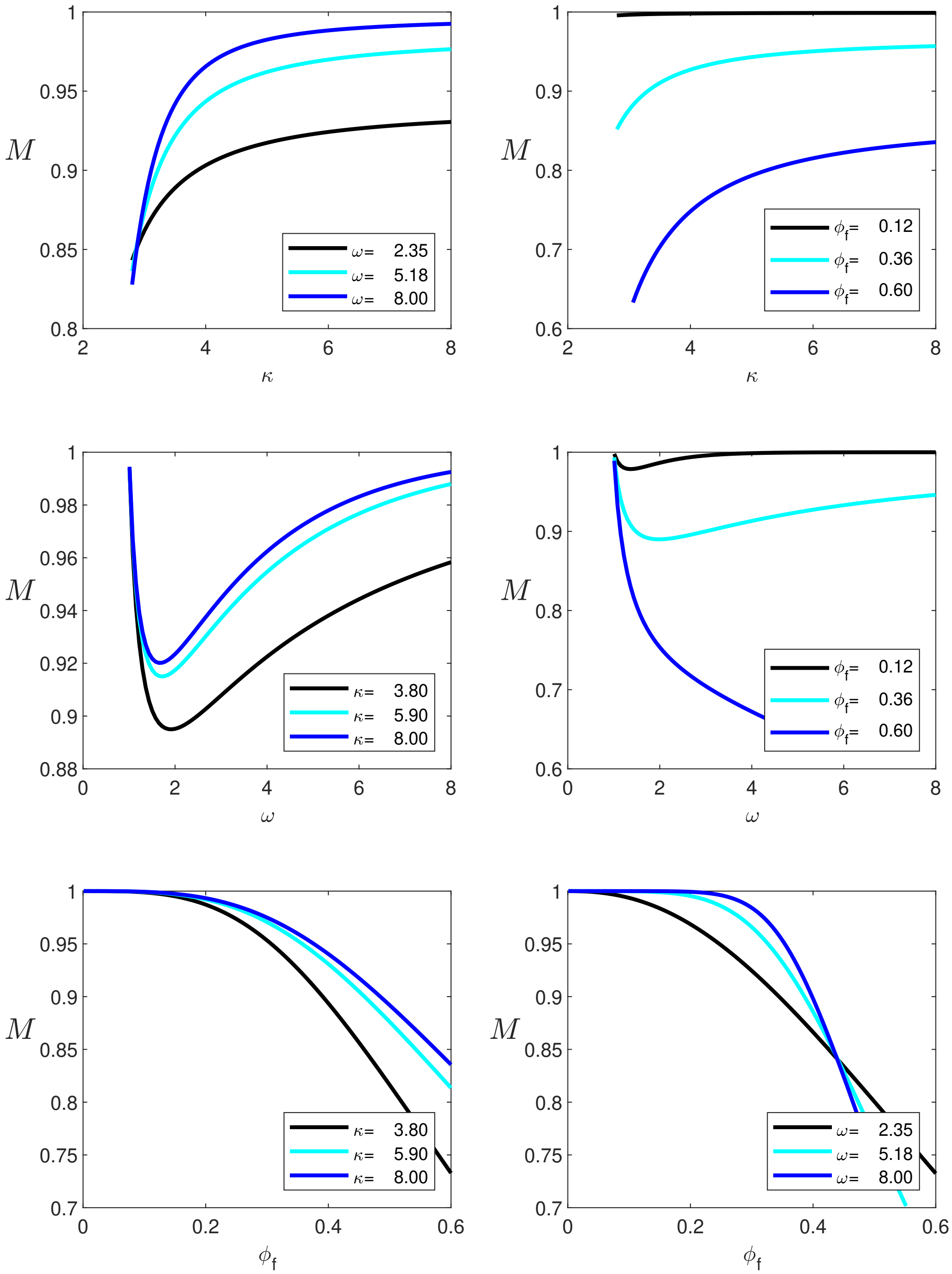

Figure A.1: Comparative Statics: M

Notes: This figure displays how variations in other structural parameters affect the relation between M and the structural parameters

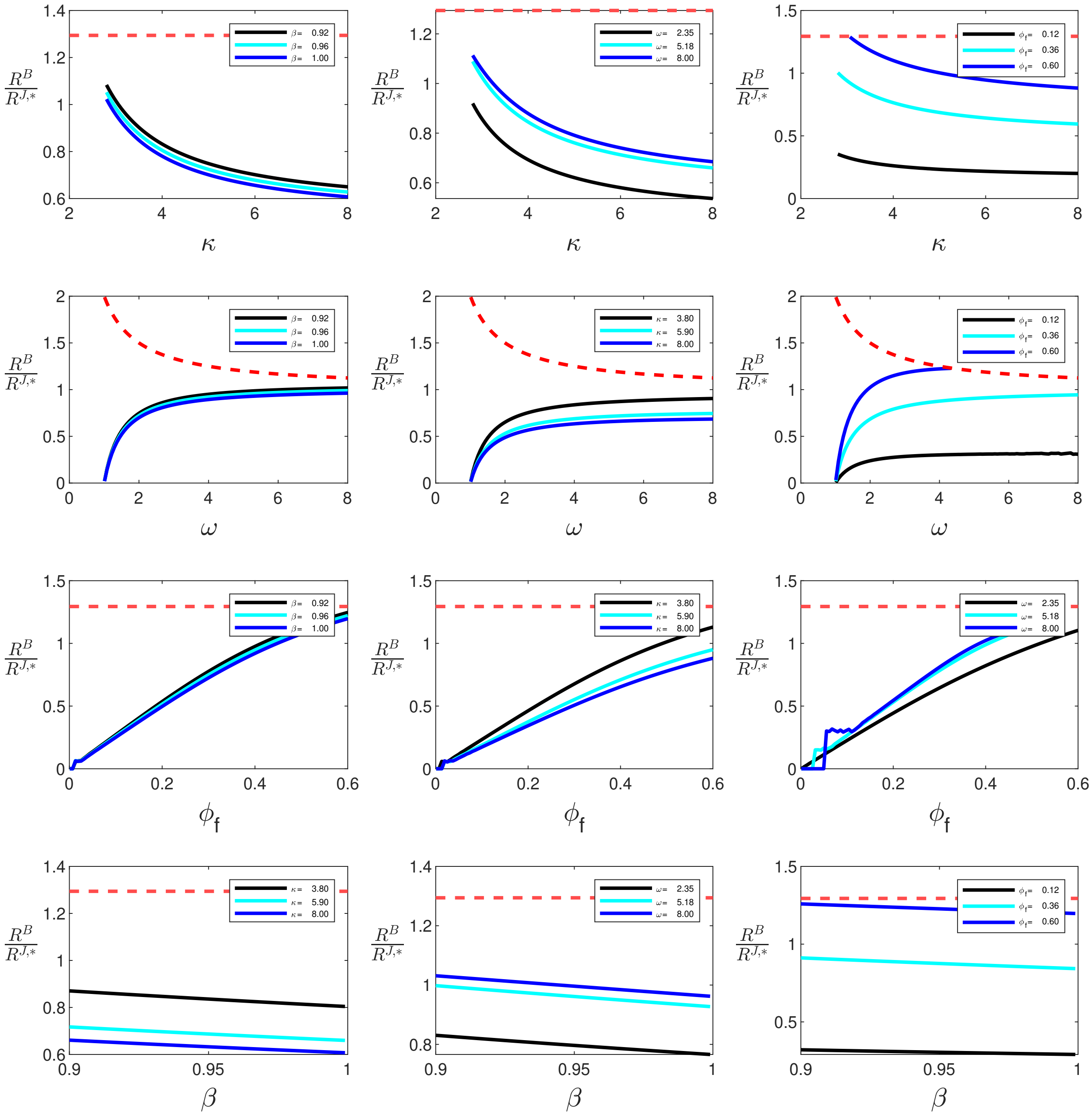

Figure A.2: Comparative Statics: Policy Room

Notes: This figure display how κ, ω, and ϕ_f affect the relationship between the policy room and the parameters

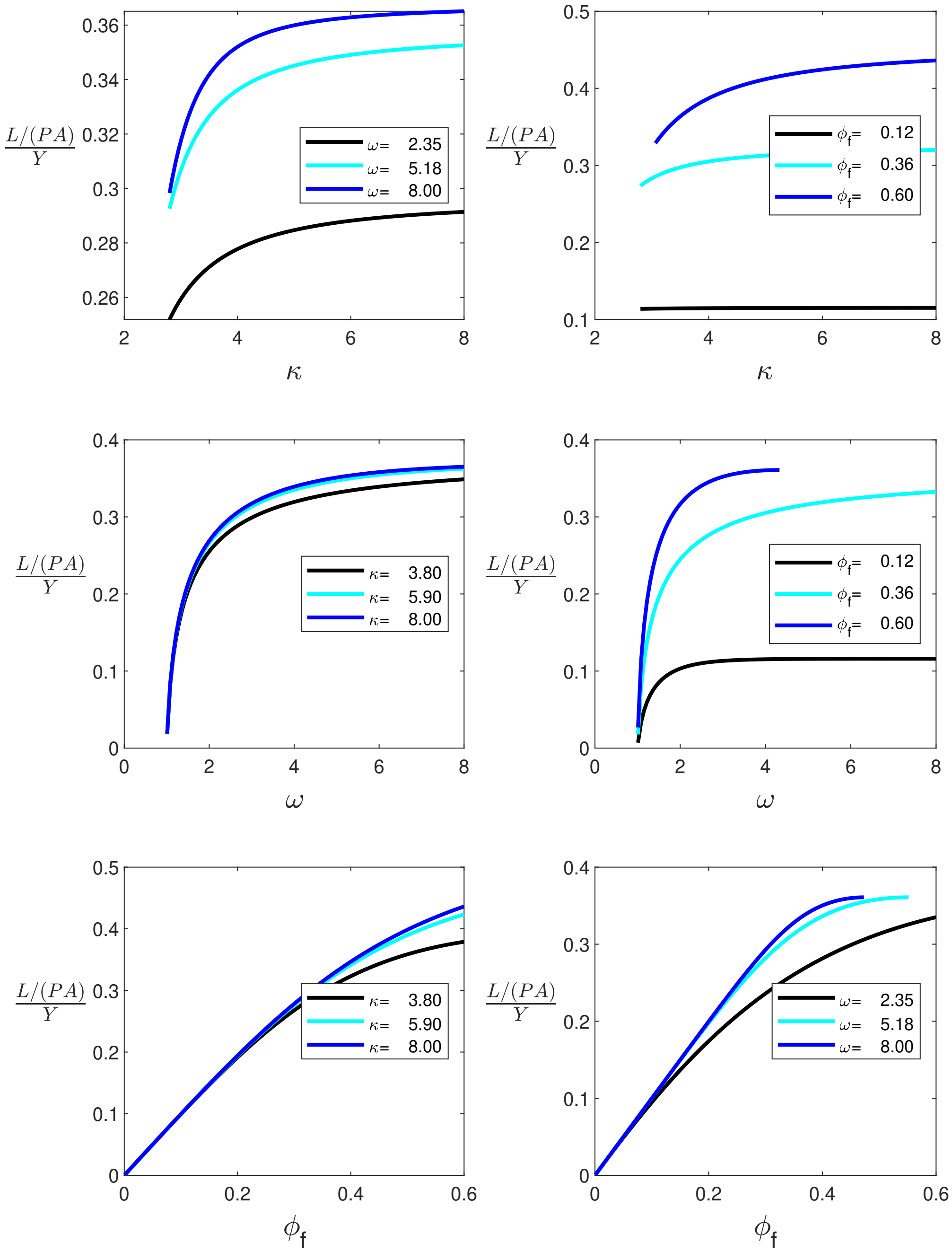

Figure A.3: Comparative Statics: Output-Scaled Loan

Notes: This figure display how κ, ω, and ϕ_f affect the relationship between $\frac{\frac{L}{PA}}{Y}$ and the parameters

A.2 Section 4.1

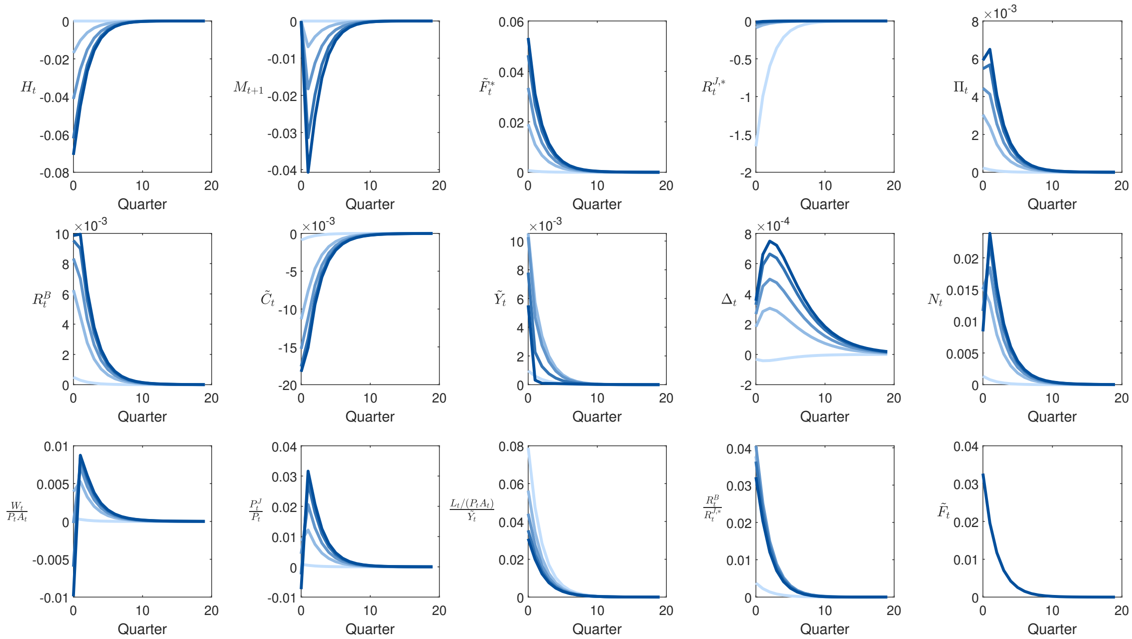

Figure A.4: Impulse response functions to fixed cost shock

Notes: The figures display the deviation for 1 positive standard deviation (0.08) in $u_{f,t}$, the fixed cost shock. The autoregressive coefficient is 0.6. The gradient blue lines denote the responses under calibrations with varying ϕ_f. From the light blue to the dark blue, ϕ_f are 0.02, 0.25, 0.37 (benchmark), 0.5, and 0.6, with corresponding Ms equal to 0.99, 0.96, 0.9, 0.78, and 0.69. The variables below are plotted in deviations from their steady states: H, M, R^B, Π, and $R^{j,*}$ (net interest rate). The rest of the variables are plotted in log deviations from their steady states. Δ is the price dispersion for the top-tier products. $W_t/(P_t A_t)$ is the real wage. P_t^J/P_t measures the aggregate price for the bottom-tier products or the input price for the top-tier firms.

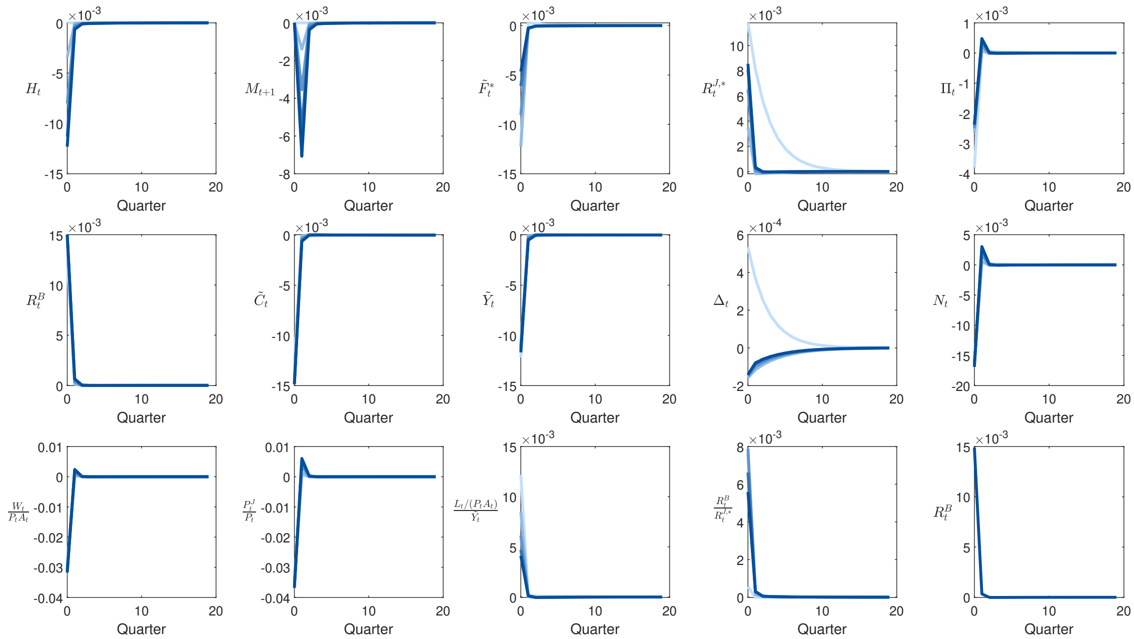

Figure A.5: Impulse response functions to monetary policy shock

Notes: The figures display the deviation for 1 positive standard deviation (0.02) in $\epsilon_{r,t}$, the monetary policy shock. The gradient blue lines denote the responses under calibrations with varying ϕ_f. From the light blue to the dark blue, ϕ_f are 0.02, 0.25, 0.37 (benchmark), 0.5, and 0.6, with corresponding Ms equal to 0.99, 0.96, 0.9, 0.78, and 0.69. The variables below are plotted in deviations from their steady states: H, M, R^B, Π, and $R^{j,*}$ (net interest rate). The remaining variables are plotted in log deviations from their steady states. Δ is the price dispersion for the top-tier products.$W_t/(P_t A_t)$ is the real wage. P_t^J/P_t measures the aggregate price for the bottom-tier products or the input price for the top-tier firms.

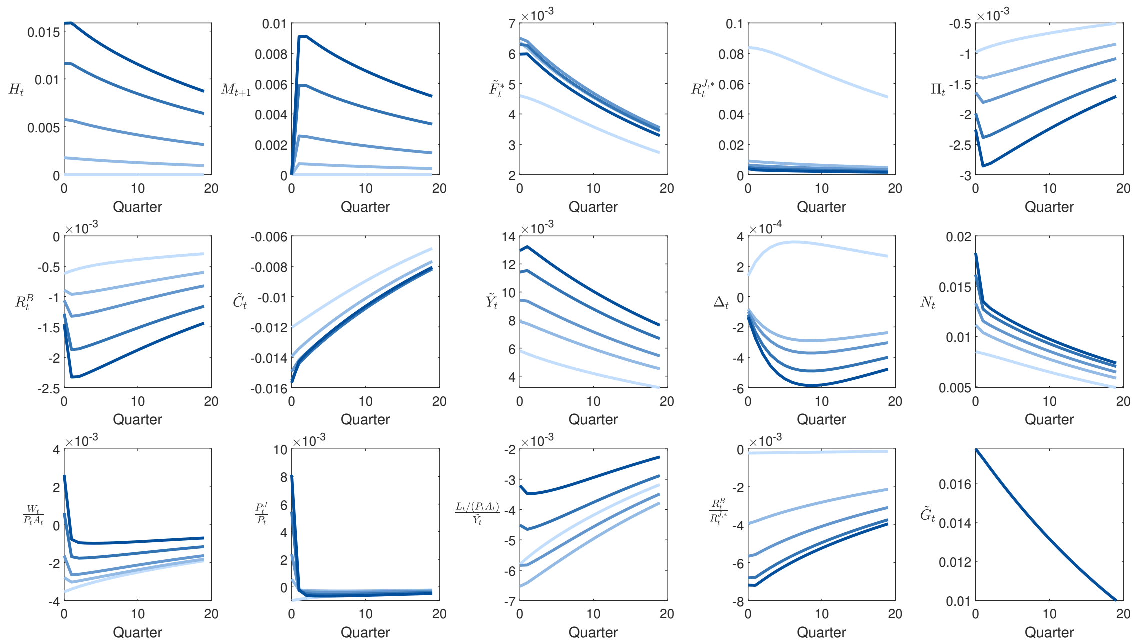

Figure A.6: Impulse response functions to government spending shock

Notes: The figures display the deviation for 1 positive standard deviation (0.08) in $u_{g,t}$ which denotes the government spending shock. The autoregressive coefficient is 0.97. The gradient blue lines denote the responses under calibration with varying ϕ_f. From the light blue to the dark blue, ϕ_f are 0.02, 0.25, 0.37 (benchmark), 0.5, and 0.6, with corresponding Ms equal to 0.99, 0.96, 0.9, 0.78, and 0.69. The variables below are plotted in level deviations from their steady states: H, M, R^B, Π, and $R^{j,*}$ (net interest rate). The remaining variables are plotted in log deviations from their steady states. Δ is the price dispersion for the top-tier products. $W_t/(P_t A_t)$ is the real wage. P_t^J/P_t measures the aggregate price for the bottom-tier products or the input price for the top-tier firms.

Appendix B Derivation and Proofs

B.1 Detailed Derivation in Section 2.2

Derivation of equations (15) and (16) We start from the price setting of a firm (m, v), given by

$$P^J_{mv,t} = \left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha}\right) W_t \varphi_{mv,t}^{-\frac{1}{\alpha}} J_{mv,t}^{\frac{1-\alpha}{\alpha}} = \left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha}\right) W_t \varphi_{mv,t}^{-\frac{1}{\alpha}} \left[(P^J_{mv,t})^{-\sigma}\Gamma^J_t\right]^{\frac{1-\alpha}{\alpha}},$$

in which we can solve for $P^J_{mv,t}$ as

$$(P^J_{mv,t})^{\frac{\alpha+\sigma(1-\alpha)}{\alpha}} = \left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha}\right) W_t \varphi_{mv,t}^{-\frac{1}{\alpha}} (\Gamma^J_t)^{\frac{1-\alpha}{\alpha}}$$

from which we obtain

$$P^J_{mv,t} = \left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha}\right)^{\frac{\alpha}{\alpha+\sigma(1-\alpha)}} W_t^{\frac{\alpha}{\alpha+\sigma(1-\alpha)}} \varphi_{mv,t}^{-\frac{1}{\alpha+\sigma(1-\alpha)}} (\Gamma^J_t)^{\frac{(1-\alpha)}{\alpha+\sigma(1-\alpha)}}. \tag{B.1}$$

To get the revenue function $r_{mv,t}$, we start from

$$P^J_{mv,t} J_{mv,t} = \left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha}\right) W_t \varphi_{mv,t}^{-\frac{1}{\alpha}} J_{mv,t}^{\frac{1}{\alpha}},$$

which leads to

$$\begin{aligned} r_{mv,t} &= (1+\zeta^J) P^J_{mv,t} J_{mv,t} = \left(\frac{\sigma}{(\sigma-1)\alpha}\right) W_t N_{mv,t} = (1+\zeta^J) P^J_{mv,t} \left(\frac{P^J_{mv,t}}{P^J_t}\right)^{-\sigma} J_t \\ &= (1+\zeta^J)(P^J_{mv,t})^{1-\sigma}\Gamma^J_t = (1+\zeta^J)\left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha}\right)^{\frac{\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}} W_t^{\frac{\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}} \varphi_{mv,t}^{-\frac{(1-\sigma)}{\alpha+\sigma(1-\alpha)}} (\Gamma^J_t)^{\frac{1}{\alpha+\sigma(1-\alpha)}}. \end{aligned} \tag{B.2}$$

Finally, we obtain the formula for the profit $\Pi^J_{mv,t}$, which is given by

$$\Pi^J_{mv,t} = r_{mv,t} - W_t N_{mv,t} - R^J_{t-1} P_{t-1} F_{m,t-1} = \frac{\alpha+\sigma(1-\alpha)}{\sigma} r_{mv,t} - R^J_{t-1} P_{t-1} F_{m,t-1}.$$

Calculating $P^J_{m,t}$ in (8): the price aggregator for firms of fixed $F_{m,t-1}$ From our notation in (8), we know that among firms with fixed cost $F_{m,t-1}$, a set of operating ones at t would be given by $\Omega_{m,t} = \left\{\varphi_{mv,t} \in \left[\max\left\{\varphi^*_{m,t}, \left(\frac{\kappa-1}{\kappa}\right) A_t\right\}, \infty\right]\right\}$. The cumulative distribution function of productivities of bottom-tier firms that decide to produce is $\frac{\Psi(\varphi_{m,t})}{1-\Psi(\varphi^*_{m,t})}$, a truncated Pareto distribution which is itself a Pareto distribution. With the individual firm (m, v)'s pricing equation (B.1), we now can compute the aggregate price of bottom-tier firms with fixed cost $F_{m,t-1}$ as:

$$\left(\frac{P^J_{m,t}}{P_t}\right)^{1-\sigma} = \cancel{M_{m,t}} \cdot \int_{\max\left\{\varphi^*_{m,t},\left(\frac{\kappa-1}{\kappa}\right)A_t\right\}}^{\infty} \left(\frac{P^J_{mv,t}}{P_t}\right)^{1-\sigma} \frac{\mathrm{d}\,\Psi(\varphi_{mv,t})}{\cancel{1-\Psi(\varphi^*_{m,t})}} \tag{B.3}$$

$$= \int_{\max\left\{\varphi^*_{m,t},\left(\frac{\kappa-1}{\kappa}\right)A_t\right\}}^{\infty} \left(\frac{P^J_{mv,t}}{P_t}\right)^{1-\sigma} \mathrm{d}\,\Psi(\varphi_{mv,t})$$

$$= \left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha}\right)^{\frac{\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}} \left(\frac{\kappa-1}{\kappa}\right)^{\frac{(\sigma-1)}{\alpha+\sigma(1-\alpha)}} \left(\frac{W_t}{P_t A_t}\right)^{\frac{\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}} \left[\left(\frac{\kappa-1}{\kappa}\right)A_t\right]^{\frac{(1-\sigma)}{\alpha+\sigma(1-\alpha)}}$$

$$\cdot \left(\frac{\Gamma^J_t}{(P^J_t)^\sigma A_t}\right)^{\frac{(1-\alpha)(1-\sigma)}{\alpha+\sigma(1-\alpha)}} \left(\frac{P^J_t}{P_t}\right)^{\frac{(1-\alpha)(1-\sigma)\sigma}{\alpha+\sigma(1-\alpha)}} \int_{\max\left\{\varphi^*_{m,t},\left(\frac{\kappa-1}{\kappa}\right)A_t\right\}}^{\infty} \varphi_{mv,t}^{\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}} \mathrm{d}\,\Psi(\varphi_{mv,t})$$

$$= \Theta_1 \left(\frac{W_t}{P_t A_t}\right)^{\frac{\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}} \left(\frac{Y_t\Delta_t}{A_t}\right)^{\frac{(1-\alpha)(1-\sigma)}{\alpha+\sigma(1-\alpha)}} \left(\frac{P^J_t}{P_t}\right)^{\frac{(1-\alpha)(1-\sigma)\sigma}{\alpha+\sigma(1-\alpha)}} \max\left\{\frac{\varphi^*_{m,t}}{\left(\frac{\kappa-1}{\kappa}\right)A_t}, 1\right\}^{-\frac{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}{\alpha+\sigma(1-\alpha)}}$$

$$= \Theta_1 \left(\frac{W_t}{P_t A_t}\right)^{\frac{\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}} \left(\frac{Y_t\Delta_t}{A_t}\right)^{\frac{(1-\alpha)(1-\sigma)}{\alpha+\sigma(1-\alpha)}} \left(\frac{P^J_t}{P_t}\right)^{\frac{(1-\alpha)(1-\sigma)\sigma}{\alpha+\sigma(1-\alpha)}} \min\left\{\left(\frac{R^J_{t-1}P_{t-1}F_{m,t-1}}{E_{t-1}\left[\xi_t\cdot\Xi_t\right]\left[\left(\frac{\kappa-1}{\kappa}\right)A_t\right]^{\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}}}\right)^{-\frac{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}{\sigma-1}}, 1\right\}$$

where we define

$$\Theta_1 = \left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha}\right)^{\frac{\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}} \left(\frac{\kappa-1}{\kappa}\right)^{\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}} \left(\frac{\kappa[\alpha+\sigma(1-\alpha)]}{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}\right).$$

Reexpressing Ξ_t in equation (16) Combining equation (16) with $\Gamma^J_t = (P^J_t)^\sigma Y_t\Delta_t$, we obtain

$$\Xi_t = \frac{\alpha+\sigma(1-\alpha)}{\alpha(\sigma-1)} \left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha}\right)^{-\frac{\sigma}{\alpha+\sigma(1-\alpha)}} \left(\frac{\kappa-1}{\kappa}\right)^{\frac{\alpha(\sigma-1)}{\alpha+\sigma(1-\alpha)}} \left(\frac{P^J_t}{P_t}\right)^{\frac{\sigma}{\alpha+\sigma(1-\alpha)}}$$

$$\cdot \left(\frac{W_t}{A_t P_t}\right)^{\frac{\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}} \left[\left(\frac{\kappa-1}{\kappa}\right)A_t\right]^{\frac{\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}} P_t(Y_t\Delta_t)^{\frac{1}{\alpha+\sigma(1-\alpha)}} \tag{B.4}$$

$$= \Theta_2 \cdot \left(\frac{P^J_t}{P_t}\right)^{\frac{\sigma}{\alpha+\sigma(1-\alpha)}} \left(\frac{W_t}{A_t P_t}\right)^{\frac{\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}} \left[\left(\frac{\kappa-1}{\kappa}\right)A_t\right]^{\frac{\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}} P_t(Y_t\Delta_t)^{\frac{1}{\alpha+\sigma(1-\alpha)}}$$

where we define

$$\Theta_2 = \frac{\alpha+\sigma(1-\alpha)}{\alpha(\sigma-1)} \left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha}\right)^{-\frac{\sigma}{\alpha+\sigma(1-\alpha)}} \left(\frac{\kappa-1}{\kappa}\right)^{\frac{\alpha(\sigma-1)}{\alpha+\sigma(1-\alpha)}}.$$

Derivation of P_t^J in (24) We start from the full satiation threshold of the fixed cost F_{t-1}^* defined in Proposition 2:

$$
\begin{aligned}
F_{t-1}^* &= \frac{E_{t-1}\left[\xi_t \cdot \Xi_t\right]\left[\left(\frac{\kappa-1}{\kappa}\right)A_t\right]^{\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}}}{R_{t-1}^J P_{t-1}} \\
&= \Theta_2 E_{t-1}\left[\xi_t\left(\frac{P_t^J}{P_t}\right)^{\frac{\sigma}{\alpha+\sigma(1-\alpha)}}\left(\frac{W_t}{A_t P_t}\right)^{\frac{\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}}\left[\left(\frac{\kappa-1}{\kappa}\right)A_t\right]^{\frac{(\sigma-1)(1-\alpha)}{\alpha+\sigma(1-\alpha)}}\left(\frac{\Pi_t(Y_t\Delta_t)^{\frac{1}{\alpha+\sigma(1-\alpha)}}}{R_{t-1}^J}\right)\right]
\end{aligned} \tag{B.5}
$$

where the second equality is from equation (B.4). From (17) and (B.5), we obtain

$$
\varphi_{m,t}^* = \left(\frac{F_{m,t-1}}{F_{t-1}^*}\right)^{\frac{\alpha+\sigma(1-\alpha)}{\sigma-1}}\left(\frac{\kappa-1}{\kappa}\right)A_t, \quad R_{m,t-1}^{J,*} = \left(\frac{F_{m,t-1}}{F_{t-1}^*}\right)^{-1} R_{t-1}^J. \tag{B.6}
$$

From (18), we obtain

$$
M_{m,t} = \min\left\{\left(\frac{F_{m,t-1}}{F_{t-1}^*}\right)^{-\left(\frac{\kappa[\alpha+\sigma(1-\alpha)]}{\sigma-1}\right)}, 1\right\}. \tag{B.7}
$$

Using equation (B.3) and (B.5), we obtain

$$
\left(\frac{P_{m,t}^J}{P_t}\right)^{1-\sigma} = \Theta_1 \cdot \left(\frac{W_t}{P_t A_t}\right)^{\frac{\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}}\left(\frac{P_t^J}{P_t}\right)^{\frac{(1-\alpha)(1-\sigma)\sigma}{\alpha+\sigma(1-\alpha)}}\left(\frac{Y_t\Delta_t}{A_t}\right)^{\frac{(1-\alpha)(1-\sigma)}{\alpha+\sigma(1-\alpha)}}\min\left\{\left(\frac{F_{m,t-1}}{F_{t-1}^*}\right)^{-\left(\frac{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}{\sigma-1}\right)}, 1\right\} \tag{B.8}
$$

We rearrange equation (8) as:

$$
\begin{aligned}
\left(\frac{P_t^J}{P_t}\right)^{1-\sigma} &= \int_0^1\left(\frac{P_{m,t}^J}{P_t}\right)^{1-\sigma} \mathrm{d}m \\
&= Prob\left(F_{m,t-1} \le F_{t-1}^*\right) E_t\left[\left(\frac{P_{m,t}^J}{P_t^J}\right)^{1-\sigma} | F_{m,t-1} \le F_{t-1}^*\right] \\
&\quad + Prob\left(F_{m,t-1} > F_{t-1}^*\right) E_t\left[\left(\frac{P_{m,t}^J}{P_t^J}\right)^{1-\sigma} | F_{m,t-1} > F_{t-1}^*\right] \\
&= \cancel{H(F_{t-1}^*)}\int_{\left(\frac{\omega-1}{\omega}\right)F_{t-1}}^{F_{t-1}^*}\left(\frac{P_{m,t}^J}{P_t^J}\right)^{1-\sigma}\frac{\mathrm{d}H(F_{m,t-1})}{\cancel{H(F_{t-1}^*)}} + \cancel{[1-H(F_{t-1}^*)]}\int_{F_{t-1}^*}^{\infty}\left(\frac{P_{m,t}^J}{P_t^J}\right)^{1-\sigma}\frac{\mathrm{d}H(F_{m,t-1})}{\cancel{1-H(F_{t-1}^*)}} \\
&= \int_{\left(\frac{\omega-1}{\omega}\right)F_{t-1}}^{F_{t-1}^*}\left(\frac{P_{m,t}^J}{P_t^J}\right)^{1-\sigma}\mathrm{d}H(F_{m,t-1}) + \int_{F_{t-1}^*}^{\infty}\left(\frac{P_{m,t}^J}{P_t^J}\right)^{1-\sigma}\mathrm{d}H(F_{m,t-1})
\end{aligned} \tag{B.9}
$$

where $\frac{P^J_{m,t}}{P^J_t}$ is given by (B.8). Plugging (B.8) into (B.9), we obtain

$$\begin{aligned}\left(\frac{P^J_t}{P_t}\right)^{1-\sigma} &= \Theta_1 \cdot \left(\frac{W_t}{P_t A_t}\right)^{\left(\frac{\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}\right)} \cdot \left(\frac{P^J_t}{P_t}\right)^{\left(\frac{(1-\alpha)(1-\sigma)\sigma}{\alpha+\sigma(1-\alpha)}\right)} \cdot \left(\frac{Y_t \Delta_t}{A_t}\right)^{\left(\frac{(1-\alpha)(1-\sigma)}{\alpha+\sigma(1-\alpha)}\right)} \\ &\cdot \left[\int_{\left(\frac{\omega-1}{\omega}\right)F_{t-1}}^{F^*_{t-1}} 1 \, \mathrm{d}H\left(F_{m,t-1}\right) + \int_{F^*_{t-1}}^{\infty} \left(\frac{F_{m,t-1}}{F^*_{t-1}}\right)^{-\left(\frac{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}{\sigma-1}\right)} \mathrm{d}H\left(F_{m,t-1}\right)\right],\end{aligned} \tag{B.10}$$

which leads to

$$\begin{aligned}\left(\frac{P^J_t}{P_t}\right)^{\left(\frac{\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}\right)} &= \Theta_1 \cdot \left(\frac{W_t}{P_t A_t}\right)^{\left(\frac{\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}\right)} \cdot \left(\frac{Y_t \Delta_t}{A_t}\right)^{\left(\frac{(1-\alpha)(1-\sigma)}{\alpha+\sigma(1-\alpha)}\right)} \\ &\cdot \left[H\left(F^*_{t-1}\right) + \left(\frac{\omega(\sigma-1)}{\kappa[\alpha+\sigma(1-\alpha)]+(\omega-1)(\sigma-1)}\right) \cdot \left[1 - H\left(F^*_{t-1}\right)\right]\right]\end{aligned} \tag{B.11}$$

Rearranging equation (B.11), we finally obtain:

$$\frac{P^J_t}{P_t} = \left(\frac{W_t}{P_t A_t}\right) \cdot \left(\frac{Y_t \Delta_t}{A_t}\right)^{\frac{1-\alpha}{\alpha}} \cdot \left[\frac{\Theta_3}{1 + \Theta_4 \cdot H\left(F^*_{t-1}\right)}\right]^{\left(\frac{\alpha+\sigma(1-\alpha)}{\alpha(\sigma-1)}\right)} \tag{B.12}$$

where we define

$$\Theta_3 = \left(\frac{\kappa[\alpha+\sigma(1-\alpha)]+(\omega-1)(\sigma-1)}{\Theta_1 \omega(\sigma-1)}\right), \quad \Theta_4 = \left(\frac{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}{\omega(\sigma-1)}\right).$$

Derivation of M_t and L_{t-1} in (26) and (27)

$$\begin{aligned}M_t &= \int_0^1 \int_{v\in\Omega_{m,t}} 1 \, \mathrm{d}v \, \mathrm{d}m = \int_0^1 M_{m,t} \, \mathrm{d}m = \int_0^1 M_{m,t} \cdot dH(F_{m,t-1}) \\ &= \underbrace{Prob\left(F_{t-1} \leq F^*_{t-1}\right)}_{=H(F^*_{t-1})} \cdot 1 + \cancel{Prob\left(F_{t-1} > F^*_{t-1}\right)} \cdot \int_{F^*_{t-1}}^{\infty} \left(\frac{F_{m,t-1}}{F^*_{t-1}}\right)^{-\frac{\kappa[\alpha+\sigma(1-\alpha)]}{\sigma-1}} \frac{\mathrm{d}H(F_{m,t-1})}{\cancel{1 - H\left(F^*_{t-1}\right)}} \\ &= 1 - \Theta_M \cdot \left[1 - H\left(F^*_{t-1}\right)\right]\end{aligned} \tag{B.13}$$

where

$$\Theta_M = \frac{\kappa[\alpha+\sigma(1-\alpha)]}{\kappa[\alpha+\sigma(1-\alpha)]+\omega(\sigma-1)}.$$

To derive equation (21), we start from

$$
\begin{aligned}
\frac{L_{t-1}}{P_{t-1}} &= \frac{\int_0^1 L_{m,t-1}\,\mathrm{d}m}{P_{t-1}} \\
&= Prob\left(F_{m,t-1} \le F^*_{t-1}\right) \cdot \int_{\left(\frac{\omega-1}{\omega}\right)F_{t-1}}^{F^*_{t-1}} F_{m,t-1}\,\frac{\mathrm{d}H\left(F_{m,t-1}\right)}{H\left(F^*_{t-1}\right)} \\
&\quad + Prob\left(F_{m,t-1} > F^*_{t-1}\right) \cdot \int_{F^*_{t-1}}^{\infty} \left(F^*_{t-1}\right)^{\left(\frac{\kappa[\alpha+\sigma(1-\alpha)]}{\sigma-1}\right)} \cdot F_{m,t-1}^{-\left(\frac{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}{\sigma-1}\right)}\,\frac{\mathrm{d}H\left(F_{m,t-1}\right)}{1-H\left(F^*_{t-1}\right)} \\
&= \int_{\left(\frac{\omega-1}{\omega}\right)F_{t-1}}^{F^*_{t-1}} F_{m,t-1}\,\mathrm{d}H\left(F_{m,t-1}\right) + \int_{F^*_{t-1}}^{\infty} \left(F^*_{t-1}\right)^{\left(\frac{\kappa[\alpha+\sigma(1-\alpha)]}{\sigma-1}\right)} \cdot F_{m,t-1}^{-\left(\frac{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}{\sigma-1}\right)}\,\mathrm{d}H\left(F_{m,t-1}\right),
\end{aligned}
$$

which leads to

$$
\begin{aligned}
\frac{L_{t-1}}{P_{t-1}} &= F_{t-1} - \left(\frac{\omega}{\omega-1}\right)\left(\frac{\kappa[\alpha+\sigma(1-\alpha)]}{\kappa[\alpha+\sigma(1-\alpha)]+(\sigma-1)(\omega-1)}\right) \cdot F^*_{t-1} \cdot \left[1-H\left(F^*_{t-1}\right)\right] \\
&= F_{t-1} \cdot \left[1 - \Theta_L \cdot \left[1-H\left(F^*_{t-1}\right)\right]^{\left(\frac{\omega-1}{\omega}\right)}\right]
\end{aligned}
$$

where

$$
\Theta_L = \frac{\kappa[\alpha+\sigma(1-\alpha)]}{\kappa[\alpha+\sigma(1-\alpha)]+(\sigma-1)(\omega-1)}.
$$

Derivation of N_t in equation (33) Labor $N_{mv,t}$ employed by a producing bottom-tier firm (m, v) is given by

$$
\begin{aligned}
N_{mv,t} &= J_{mv,t}^{\frac{1}{\alpha}} \varphi_{mv,t}^{-\frac{1}{\alpha}} = \varphi_{mv,t}^{-\frac{1}{\alpha}} \cdot \left[\left(\frac{P^J_{mv,t}}{P^J_t}\right)^{-\sigma} \cdot J_t\right]^{\frac{1}{\alpha}} \qquad \text{(B.14)} \\
&= \left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha}\right)^{\left(\frac{-\sigma}{\alpha+\sigma(1-\alpha)}\right)} \cdot \left(\frac{\kappa-1}{\kappa}\right)^{\left(\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}\right)} \cdot \left(\frac{\varphi_{mv,t}}{\left(\frac{\kappa-1}{\kappa}\right)A_t}\right)^{\left(\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}\right)} \\
&\quad \cdot \left(\frac{W_t}{P_t A_t}\right)^{\left(\frac{-\sigma}{\alpha+\sigma(1-\alpha)}\right)} \cdot \left(\frac{P^J_t}{P_t}\right)^{\left(\frac{\sigma}{\alpha+\sigma(1-\alpha)}\right)} \cdot \left(\frac{Y_t \Delta_t}{A_t}\right)^{\left(\frac{1}{\alpha+\sigma(1-\alpha)}\right)} \\
&= \left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha}\right)^{\left(\frac{-\sigma}{\alpha+\sigma(1-\alpha)}\right)} \left(\frac{\kappa-1}{\kappa}\right)^{\left(\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}\right)} \left(\frac{\varphi_{mv,t}}{\left(\frac{\kappa-1}{\kappa}\right)A_t}\right)^{\left(\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}\right)} \left[\frac{\Theta_3}{1+\Theta_4 H_{t-1}}\right]^{\left(\frac{\sigma}{(\sigma-1)\alpha}\right)} \left(\frac{Y_t \Delta_t}{A_t}\right)^{\frac{1}{\alpha}},
\end{aligned}
$$

where we use (7) in the second equality, (10) and (14) for the third equality, and (24) to obtain the fourth one. For convenience we define $H_{t-1} \equiv H\left(F^*_{t-1}\right)$. Now we integrate labor in (B.14) across all producing firms to obtain the aggregate labor N_t. First,

$$\begin{aligned} N_t &= \int_0^1 \int_{v\in\Omega_{m,t}} N_{mv,t}\, \mathrm{d}v\, \mathrm{d}m \\ &= \left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha}\right)^{\left(\frac{-\sigma}{\alpha+\sigma(1-\alpha)}\right)} \left(\frac{\kappa-1}{\kappa}\right)^{\left(\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}\right)} \left[\left(\frac{\kappa-1}{\kappa}\right) A_t\right]^{\left(\frac{1-\sigma}{\alpha+\sigma(1-\alpha)}\right)} \\ &\cdot \left[\frac{\Theta_3}{1+\Theta_4\cdot H_{t-1}}\right]^{\left(\frac{\sigma}{(\sigma-1)\alpha}\right)} \left(\frac{Y_t\Delta_t}{A_t}\right)^{\frac{1}{\alpha}} \int_0^1 \int_{v\in\Omega_{m,t}} \varphi_{mv,t}^{\left(\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}\right)}\, \mathrm{d}v\, \mathrm{d}m \\ &= \square \int_0^1 \int_{v\in\Omega_{m,t}} \varphi_{mv,t}^{\left(\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}\right)}\, \mathrm{d}v\, \mathrm{d}m, \end{aligned} \tag{B.15}$$

where

$$\square = \left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha}\right)^{\left(\frac{-\sigma}{\alpha+\sigma(1-\alpha)}\right)} \left(\frac{\kappa-1}{\kappa}\right)^{\left(\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}\right)} \left[\left(\frac{\kappa-1}{\kappa}\right) A_t\right]^{\left(\frac{1-\sigma}{\alpha+\sigma(1-\alpha)}\right)} \left[\frac{\Theta_3}{1+\Theta_4\cdot H_{t-1}}\right]^{\left(\frac{\sigma}{(\sigma-1)\alpha}\right)} \left(\frac{Y_t\Delta_t}{A_t}\right)^{\frac{1}{\alpha}}. \tag{B.16}$$

Now, (B.15) leads to

$$\begin{aligned} N_t &= \square \int_0^1 \int_{\max\left(\varphi^*_{m,t}, \frac{\kappa-1}{\kappa}A_t\right)} \varphi_{mv,t}^{\left(\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}\right)} \kappa \left[\left(\frac{\kappa-1}{\kappa}\right) A_t\right]^{\kappa} \varphi_{mv,t}^{-(\kappa+1)}\, \mathrm{d}\varphi_{mv,t}\, \mathrm{d}m \\ &= \square \left[\left(\frac{\kappa-1}{\kappa}\right) A_t\right]^{\kappa} \left(\frac{\kappa[\alpha+\sigma(1-\alpha)]}{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}\right) \\ &\cdot \left[\left(\frac{\kappa-1}{\kappa}\right) A_t\right]^{\left(-\frac{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}{\alpha+\sigma(1-\alpha)}\right)} \int_0^1 \max\left(\frac{\varphi^*_{m,t}}{\frac{\kappa-1}{\kappa}A_t}, 1\right)^{\left(-\frac{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}{\alpha+\sigma(1-\alpha)}\right)} \mathrm{d}m \\ &= \square \left[\left(\frac{\kappa-1}{\kappa}\right) A_t\right]^{\left(\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}\right)} \left(\frac{\kappa[\alpha+\sigma(1-\alpha)]}{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}\right) \int_0^1 \min\left(\left(\frac{F_{m,t-1}}{F^*_{t-1}}\right)^{-\frac{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}{\sigma-1}}, 1\right) \mathrm{d}m \\ &= \square \left[\left(\frac{\kappa-1}{\kappa}\right) A_t\right]^{\left(\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}\right)} \left(\frac{\kappa[\alpha+\sigma(1-\alpha)]}{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}\right) \\ &\cdot \left[H_{t-1} + \frac{\omega(\sigma-1)}{\kappa[\alpha+\sigma(1-\alpha)]+(\omega-1)(\sigma-1)}(1-H_{t-1})\right] \\ &= \square \left[\left(\frac{\kappa-1}{\kappa}\right) A_t\right]^{\left(\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}\right)} \left(\frac{\kappa[\alpha+\sigma(1-\alpha)]}{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}\right) \left(\frac{\omega(\sigma-1)}{\kappa[\alpha+\sigma(1-\alpha)]+(\omega-1)(\sigma-1)}\right) [1+\Theta_4 H_{t-1}] \\ &= \left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha}\right)^{\left(\frac{-\sigma}{\alpha+\sigma(1-\alpha)}\right)} \left(\frac{\kappa-1}{\kappa}\right)^{\left(\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}\right)} \left(\frac{\kappa[\alpha+\sigma(1-\alpha)]}{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}\right) \\ &\cdot \left(\frac{\omega(\sigma-1)}{\kappa[\alpha+\sigma(1-\alpha)]+(\omega-1)(\sigma-1)}\right) [1+\Theta_4 H_{t-1}] \left[\frac{\Theta_3}{1+\Theta_4\cdot H_{t-1}}\right]^{\left(\frac{\sigma}{(\sigma-1)\alpha}\right)} \left(\frac{Y_t\Delta_t}{A_t}\right)^{\frac{1}{\alpha}} \\ &= \Theta_N \cdot \left(\frac{Y_t\Delta_t}{A_t}\right)^{\frac{1}{\alpha}} \cdot (1+\Theta_4 H_{t-1})^{\frac{\alpha+\sigma(1-\alpha)}{(1-\sigma)\alpha}}, \end{aligned} \tag{B.17}$$

where Θ_N is defined in (34).

Equilibrium conditions for top-tier firms Plugging equation (36) and the expression for $Q_{t,t+l}$ into (5), we can express the resetting price in (5) in a recursive fashion as: with

$$O_t = \left(\frac{(1+\zeta^T)^{-1}\gamma}{\gamma-1}\right)\Theta_N^{\frac{1}{\eta}}\Theta_3^{\frac{\alpha+\sigma(1-\alpha)}{(\sigma-1)\alpha}}\left(\frac{Y_t}{A_t}\right)^{\left(\frac{\eta+1}{\eta\alpha}\right)}\Delta_t^{\left(\frac{(1-\alpha)\eta+1}{\eta\alpha}\right)}(1+\Theta_4 H_{t-1})^{\frac{(1+\eta)[\alpha+\sigma(1-\alpha)]}{\eta(1-\sigma)\alpha}}\exp\{-u_{c,t}\}$$
$$+\beta\theta E_t\left[\exp\{u_{c,t+1}-u_{c,t}\}\cdot\Pi_{t+1}^{\gamma}\cdot O_{t+1}\right] \tag{B.18}$$

and

$$V_t = \left(\frac{C_t}{Y_t}\right)^{-1} + \beta\theta\cdot E_t\left[\exp\{u_{c,t+1}-u_{c,t}\}\cdot\Pi_{t+1}^{\gamma-1}\cdot V_{t+1}\right], \tag{B.19}$$

we obtain

$$\frac{P_t^*}{P_t} = \frac{O_t}{V_t}. \tag{B.20}$$

Due to price stickiness à la Calvo (1983), the aggregate price level can be recursively expressed as:

$$P_t^{1-\gamma} = (1-\theta)\left(P_t^*\right)^{1-\gamma} + \theta\left(P_{t-1}\right)^{1-\gamma}$$

which can be re-expressed as:

$$\frac{P_t^*}{P_t} = \left(\frac{1-\theta}{1-\theta\cdot\Pi_t^{\gamma-1}}\right)^{\frac{1}{\gamma-1}} \tag{B.21}$$

Plugging equation (B.20) into equation (11) and equation (B.21), we obtain

$$\frac{O_t}{V_t} = \left(\frac{1-\theta}{1-\theta\cdot\Pi_t^{\gamma-1}}\right)^{\frac{1}{\gamma-1}}, \quad \Delta_t = (1-\theta)\left(\frac{O_t}{V_t}\right)^{-\gamma} + \theta\Pi_t^{\gamma}\Delta_{t-1}$$

Equilibrium conditions for households We can write F_t^* as a function of H_t by using the cumulative distribution function of fixed costs in (22) and (29):

$$F_t^* = [1-H_t]^{-1/\omega}\left(\frac{\omega-1}{\omega}\right)\phi_f\cdot\tilde{Y}A_t\cdot\exp\{u_{f,t}\}. \tag{B.22}$$

Using the above (B.22), we can rearrange equation (B.5) (i.e., equation about F_t^* as:

$$R_t^J = E_t\left[\xi_{t+1}\cdot\left(\frac{P_{t+1}^J}{P_{t+1}}\right)^{\left(\frac{\sigma}{\alpha+\sigma(1-\alpha)}\right)}\left(\frac{w_{t+1}}{P_{t+1}A_{t+1}}\right)^{\left(\frac{(1-\sigma)\alpha}{\alpha+\sigma(1-\alpha)}\right)}\frac{1}{\tilde{Y}}\Pi_{t+1}GA_{t+1}\left(\frac{Y_{t+1}\Delta_{t+1}}{A_{t+1}}\right)^{\left(\frac{1}{\alpha+\sigma(1-\alpha)}\right)}\right]$$
$$\cdot\left(\frac{\Theta_2}{\left(\frac{\omega-1}{\omega}\right)\phi_f}\right)\left(\frac{\kappa-1}{\kappa}\right)^{\left(\frac{(\sigma-1)(1-\alpha)}{\alpha+\sigma(1-\alpha)}\right)}[1-H_t]^{\frac{1}{\omega}}\cdot\exp\{-u_{f,t}\} \tag{B.23}$$

Plugging (35) and (36) into the above (B.23), we obtain:

$$R_t^J = \left(\frac{\Theta_2 \Theta_N^{\frac{1}{\eta}} \Theta_3^{\frac{\sigma}{(\sigma-1)\alpha}}}{\left(\frac{\omega-1}{\omega}\right) \phi_f} \right) \cdot \left(\frac{\kappa - 1}{\kappa} \right)^{\left(\frac{(\sigma-1)(1-\alpha)}{\alpha+\sigma(1-\alpha)} \right)} (1 + \Theta_4 H_t)^{\left(\frac{\alpha+\sigma(1-\alpha)+\sigma\eta}{\eta(1-\sigma)\alpha} \right)} \cdot (1 - H_t)^{\frac{1}{\omega}}$$
$$\cdot E_t \left[\xi_{t+1} \Pi_{t+1} \left(\frac{\frac{C_{t+1}}{A_{t+1}}}{\frac{Y_{t+1}}{A_{t+1}}} \right) \left(\frac{\frac{Y_{t+1}}{A_{t+1}}}{\bar{Y}} \right) \left(\frac{Y_{t+1} \Delta_{t+1}}{A_{t+1}} \right)^{\left(\frac{\eta+1}{\eta\alpha} \right)} \cdot GA_{t+1} \cdot \exp\left\{ -(u_{f,t} + u_{c,t+1}) \right\} \right]. \tag{B.24}$$

Finally, we can rearrange the Euler equation in (1), using (38) as follows:

$$\frac{1}{R_t^J} = \beta E_t \left[\frac{\left(\frac{C_t}{Y_t} \right)}{\left(\frac{C_{t+1}}{Y_{t+1}} \right) \widetilde{GY}_{t+1} GA_{t+1} \Pi_{t+1}} \cdot \exp\left\{ u_{c,t+1} - u_{c,t} \right\} \right] \tag{B.25}$$

where $\widetilde{GY}_{t+1} = \frac{Y_{t+1}}{Y_{t+1}} \frac{A_t}{A_{t+1}}$ and $GA_{t+1} = \frac{A_{t+1}}{A_t}$. Combining equation (B.24) and equation (B.25), we obtain

$$\exp\left\{ u_{f,t} + u_{c,t} \right\} = \beta \left(\frac{\Theta_2 \cdot \Theta_N^{\frac{1}{\eta}} \cdot \Theta_3^{\frac{\sigma}{(\sigma-1)\alpha}}}{\left(\frac{\omega-1}{\omega}\right) \phi_f} \right) \left(\frac{\kappa - 1}{\kappa} \right)^{\left(\frac{(\sigma-1)(1-\alpha)}{\alpha+\sigma(1-\alpha)} \right)} \cdot (1 + \Theta_4 H_t)^{\left(\frac{[\alpha+\sigma(1-\alpha)]+\sigma\eta}{\eta(1-\sigma)\alpha} \right)}$$
$$\cdot (1 - H_t)^{\frac{1}{\omega}} \cdot \left(\frac{\frac{C_t}{A_t}}{\bar{Y}} \right) \cdot E_t \left[\left(\frac{Y_{t+1} \Delta_{t+1}}{A_{t+1}} \right)^{\left(\frac{\eta+1}{\eta\alpha} \right)} \right] \tag{B.26}$$

Flexible price equilibrium Plugging (42) into (24), we obtain

$$\frac{W_t}{P_t A_t} = \left(\frac{(1 + \zeta^T)^{-1} \gamma}{\gamma - 1} \right)^{-1} \cdot \left(\frac{Y_t \Delta_t}{A_t} \right)^{\frac{\alpha-1}{\alpha}} \cdot \left[\frac{\Theta_3}{1 + \Theta_4 \cdot H_{t-1}} \right]^{\left(\frac{\alpha+\sigma(1-\alpha)}{1-\sigma} \right)} \tag{B.27}$$

Plugging (24) and (B.27) into (B.5) (i.e., equation about the cutoff fixed cost F_t^*), and based on the fact that there is no price dispersion under flexible prices, i.e., $\Delta_t = 1$, we obtain:

$$F_t^* = \Theta_2 \cdot \left(\frac{(1 + \zeta^T)^{-1} \gamma}{\gamma - 1} \right)^{-1} \cdot \left(\frac{\kappa - 1}{\kappa} \right)^{\left(\frac{(\sigma-1)(1-\alpha)}{\alpha+\sigma(1-\alpha)} \right)} \cdot \left[\frac{\Theta_3}{1 + \Theta_4 \cdot H_t} \right] E_t \left[\xi_{t+1} \left(\frac{\Pi_{t+1} Y_{t+1}}{R_t^J} \right) \right]. \tag{B.28}$$

By the definition of the distribution function of the fixed costs (see eq. equation (22)), we can express:

$$[1 - H_t]^{-\frac{1}{\omega}} = \frac{F_t^*}{\left(\frac{\omega-1}{\omega}\right) F_t} = \frac{F_t^*}{\left(\frac{\omega-1}{\omega}\right) \cdot \phi_f \cdot \bar{Y} A_t \cdot \exp\left\{ u_{f,t} \right\}}. \tag{B.29}$$

Plugging equation (B.29) into equation (B.28), we obtain:

$$1 = \left(\frac{\beta \Theta_2}{\left(\frac{\omega-1}{\omega}\right) \cdot \phi_f} \right) \cdot \left(\frac{(1+\zeta^T)^{-1}\gamma}{\gamma - 1} \right)^{-1} \cdot \left(\frac{\kappa - 1}{\kappa} \right)^{\left(\frac{(\sigma-1)(1-\alpha)}{\alpha+\sigma(1-\alpha)}\right)} \cdot \left[\frac{\Theta_3}{1 + \Theta_4 \cdot H_t} \right]$$
$$\cdot \left[1 - H_t\right]^{\frac{1}{\omega}} \cdot E_t \left[\left(\frac{\tilde{Y}_t}{\bar{Y}} \right) \left(\frac{\frac{C_t}{Y_t}}{\frac{C_{t+1}}{Y_{t+1}}} \right) \cdot \exp\left\{ u_{c,t+1} - \left(u_{f,t} + u_{c,t} \right) \right\} \right] \tag{B.30}$$

Finally, plugging (42) into (36) and based on no price dispersion under flexible prices, i.e., $\Delta_t = 1$, we obtain

$$\frac{Y_t}{A_t} = \left(\frac{(1+\zeta^T)^{-1}\gamma}{\gamma - 1} \right)^{-\left(\frac{\eta\alpha}{(1-\alpha)\eta+1}\right)} \Theta_N^{-\left(\frac{\alpha}{(1-\alpha)\eta+1}\right)} \Theta_3^{-\frac{\eta[\alpha+\sigma(1-\alpha)]}{[(1-\alpha)\eta+1](\sigma-1)}} \cdot \left(\frac{C_t}{A_t} \right)^{-\left(\frac{\eta\alpha}{(1-\alpha)\eta+1}\right)}$$
$$\cdot \left(1 + \Theta_4 H_{t-1}\right)^{-\frac{(1+\eta)[\alpha+\sigma(1-\alpha)])}{(1-\sigma)[(1-\alpha)\eta+1]}} \cdot \exp\left\{ \left(\frac{\eta\alpha}{(1-\alpha)\eta + 1} \right) \cdot u_{c,t} \right\} \tag{B.31}$$

From (B.30) and (B.31), we can see that the flexible price equilibrium is money-neutral.

B.2 Detailed Derivations in Section 3.1

Derivations on the cross-sectional standard deviations of sales and productivities in (43) and (44) We start from the formula for the revenue $r_{mv,t}$ generated by a firm (m, v) in (B.2):

$$r_{mv,t} = (1+\zeta^J) \left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha} \right)^{\frac{\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}} W_t^{\frac{\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}} (\Gamma_t^J)^{\frac{1}{\alpha+\sigma(1-\alpha)}} \varphi_{mv,t}^{\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}}, \tag{B.32}$$

where

$$\varphi_{m,t}^* = \left(\frac{R_{t-1}^J P_{t-1} F_{m,t-1}}{E_{t-1}\left[\xi_t \cdot \Xi_t\right]} \right)^{\frac{\alpha+\sigma(1-\alpha)}{\sigma-1}}. \tag{B.33}$$

We can calculate the cross-sectional standard deviation of an individual firm's revenue and productivity by calculating the variance:

$$\sigma^2\left(\log r_{mv,t}\right) = \left(\frac{\sigma-1}{\alpha+\sigma(1-\alpha)} \right)^2 \sigma^2\left(\log \varphi_{mv,t}\right) = \left(\frac{\sigma-1}{\alpha+\sigma(1-\alpha)} \right)^2 \sigma^2 \left(\log \frac{\varphi_{mv,t}}{\varphi_{m,t}^*} + \log \varphi_{m,t}^* \right)$$
$$= \left(\frac{\sigma-1}{\alpha+\sigma(1-\alpha)} \right)^2 \left[\sigma^2 \left(\log \frac{\varphi_{mv,t}}{\varphi_{m,t}^*} \right) + \sigma^2\left(\log \varphi_{m,t}^*\right) \right], \tag{B.34}$$

where for the second line we use the property that (i) $\phi_{mv,t}|\phi_{mv,t} \geq \phi^*_{m,t}$ follows a Pareto distribution; (ii) distributions of productivities and fixed costs are independent of each other. Therefore,

$$\begin{aligned}
\sigma^2\left(\log r_{mv,t}\right) &= \left(\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}\right)^2\left[\sigma^2\left(\log\frac{\varphi_{mv,t}}{\varphi^*_{m,t}}\right)+\left(\frac{\alpha+\sigma(1-\alpha)}{\sigma-1}\right)^2\sigma^2\left(\log F_{m,t-1}\right)\right] \\
&= \left(\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}\right)^2\left[\frac{1}{\kappa^2}+\left(\frac{\alpha+\sigma(1-\alpha)}{\sigma-1}\right)^2\frac{1}{\omega^2}\right],
\end{aligned}$$

which implies

$$\sigma\left(\log r_{mv,t}\right) = \frac{\sigma-1}{\alpha+\sigma(1-\alpha)}\sqrt{\frac{1}{\kappa^2}+\left(\frac{\alpha+\sigma(1-\alpha)}{\sigma-1}\right)^2\frac{1}{\omega^2}},$$

and

$$\sigma\left(\log \varphi_{mv,t}\right) = \sqrt{\frac{1}{\kappa^2}+\left(\frac{\alpha+\sigma(1-\alpha)}{\sigma-1}\right)^2\frac{1}{\omega^2}}.$$

B.3 Detailed Derivation in Section 4.2

Intensive vs. extensive margin labor adjustments: derivation of (48) From (B.15), (B.16), and (B.17), we know that the aggregate labor N_t can be written as

$$\begin{aligned}
N_t &= \left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha}\right)^{\left(\frac{-\sigma}{\alpha+\sigma(1-\alpha)}\right)}\left(\frac{\kappa-1}{\kappa}\right)^{\left(\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}\right)}\left(\frac{\kappa[\alpha+\sigma(1-\alpha)]}{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}\right) \qquad \text{(B.35)}\\
&\quad\cdot\left[\frac{\Theta_3}{1+\Theta_4\cdot H_{t-1}}\right]^{\left(\frac{\sigma}{(\sigma-1)\alpha}\right)}\left(\frac{Y_t\Delta_t}{A_t}\right)^{\frac{1}{\alpha}}\left[H_{t-1}+\frac{\omega(\sigma-1)}{\kappa[\alpha+\sigma(1-\alpha)]+(\omega-1)(\sigma-1)}(1-H_{t-1})\right] \\
&= \Theta_{DN}\left[\frac{\Theta_3}{1+\Theta_4\cdot H_{t-1}}\right]^{\left(\frac{\sigma}{(\sigma-1)\alpha}\right)}\left(\frac{Y_t\Delta_t}{A_t}\right)^{\frac{1}{\alpha}}\underbrace{\left[H_{t-1}+\frac{\omega(\sigma-1)}{\kappa[\alpha+\sigma(1-\alpha)]+(\omega-1)(\sigma-1)}(1-H_{t-1})\right]}_{\equiv SN^I_t} \\
&= \Theta_{DN}\left[\frac{\Theta_3}{1+\Theta_4\cdot H_{t-1}}\right]^{\left(\frac{\sigma}{(\sigma-1)\alpha}\right)}\left(\frac{Y_t\Delta_t}{A_t}\right)^{\frac{1}{\alpha}}\cdot SN^I_t \qquad \text{(B.36)}
\end{aligned}$$

Where

$$\Theta_{DN} \equiv \left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha}\right)^{\left(\frac{-\sigma}{\alpha+\sigma(1-\alpha)}\right)}\left(\frac{\kappa-1}{\kappa}\right)^{\left(\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}\right)}\left(\frac{\kappa[\alpha+\sigma(1-\alpha)]}{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}\right). \qquad \text{(B.37)}$$

From (B.35), we obtain for $\forall\iota$

$$\frac{N_{t+\iota}-N_t}{N_t}=\underbrace{\left[\frac{1+\Theta_4\cdot H_{t-1}}{1+\Theta_4\cdot H_{t+\iota-1}}\right]^{\left(\frac{\sigma}{(\sigma-1)\alpha}\right)}\left(\frac{Y_{t+\iota}\Delta_{t+\iota}/A_{t+\iota}}{Y_t\Delta_t/A_t}\right)^{\frac{1}{\alpha}}-1}_{=g^{\text{Density}}_{t,t+\iota}} \tag{B.38}$$

$$+\left\{1+\underbrace{\left[\left[\frac{1+\Theta_4\cdot H_{t-1}}{1+\Theta_4\cdot H_{t+\iota-1}}\right]^{\left(\frac{\sigma}{(\sigma-1)\alpha}\right)}\left(\frac{Y_{t+\iota}\Delta_{t+\iota}/A_{t+\iota}}{Y_t\Delta_t/A_t}\right)^{\frac{1}{\alpha}}-1\right]}_{=g^{\text{Density}}_{t,t+\iota}}\right\}\cdot\underbrace{\frac{SN^E_{t,t+\iota}}{SN^I_t}}_{\equiv g^{\text{Entry}}_{t,t+\iota}} \tag{B.39}$$

Therefore, by (47) and the definition of the decomposition in (48), we obtain

$$g^{\text{Entry}}_{t,t+\iota}\equiv\frac{SN^E_{t,t+\iota}}{SN^I_t}=\frac{(H_{t+\iota-1}-H_{t-1})+\dfrac{\omega(\sigma-1)}{\kappa[\alpha+\sigma(1-\alpha)]+(\omega-1)(\sigma-1)}(H_{t-1}-H_{t+\iota-1})}{H_{t-1}+\dfrac{\omega(\sigma-1)}{\kappa[\alpha+\sigma(1-\alpha)]+(\omega-1)(\sigma-1)}(1-H_{t-1})}, \tag{B.40}$$

which proves (49).

Appendix C Summary of Equilibrium Conditions

C.1 Sticky Price Equilibrium (i.e., Original Model)

$$
\begin{aligned}
\exp\left\{u_{f,t}+u_{c,t}\right\} &= \beta\left(\frac{\Theta_2\cdot\Theta_N^{\frac{1}{\eta}}\cdot\Theta_3^{\frac{\sigma}{(\sigma-1)\alpha}}}{\left(\frac{\omega-1}{\omega}\right)\phi_f}\right)\left(\frac{\kappa-1}{\kappa}\right)^{\left(\frac{(\sigma-1)(1-\alpha)}{\alpha+\sigma(1-\alpha)}\right)}\cdot\left(1+\Theta_4 H_t\right)^{\left(\frac{\alpha+\sigma(1-\alpha)+\sigma\eta}{\eta(1-\sigma)\alpha}\right)} \\
&\quad\cdot\left(1-H_t\right)^{\frac{1}{\omega}}\cdot\left(\frac{\tilde{C}_t}{\bar{Y}}\right)\cdot E_t\left[\left(\tilde{Y}_{t+1}\Delta_{t+1}\right)^{\left(\frac{\eta+1}{\eta\alpha}\right)}\right] \\
\frac{1}{R_t^J} &= \beta E_t\left[\frac{\tilde{C}_t}{\tilde{C}_{t+1}GA_{t+1}\Pi_{t+1}}\cdot\exp\left\{u_{c,t+1}-u_{c,t}\right\}\right] \\
\frac{\tilde{C}_t}{\tilde{Y}_t} &= 1-\phi_g\cdot\exp\left\{u_{g,t}\right\}-\phi_f\cdot\left(\frac{\tilde{Y}_t}{\bar{Y}}\right)^{-1}\cdot\left[1-\Theta_L\cdot\left[1-H_t\right]^{\left(\frac{\omega-1}{\omega}\right)}\right]\cdot\exp\left\{u_{f,t}\right\} \\
O_t &= \left(\frac{(1+\zeta^T)^{-1}\gamma}{\gamma-1}\right)\Theta_N^{\frac{1}{\eta}}\Theta_3^{\frac{\alpha+\sigma(1-\alpha)}{(\sigma-1)\alpha}}\tilde{Y}_t^{\left(\frac{\eta+1}{\eta\alpha}\right)}\Delta_t^{\left(\frac{(1-\alpha)\eta+1}{\eta\alpha}\right)}\left(1+\Theta_4 H_{t-1}\right)^{\frac{(1+\eta)[\alpha+\sigma(1-\alpha)]}{\eta(1-\sigma)\alpha}}\exp\left\{-u_{c,t}\right\} \\
&\quad+\beta\theta E_t\left[\exp\left\{u_{c,t+1}-u_{c,t}\right\}\cdot\Pi_{t+1}^{\gamma}\cdot O_{t+1}\right] \\
V_t &= \left(\frac{\tilde{C}_t}{\tilde{Y}_t}\right)^{-1}+\beta\theta\cdot E_t\left[\exp\left\{u_{c,t+1}-u_{c,t}\right\}\cdot\Pi_{t+1}^{\gamma-1}\cdot V_{t+1}\right] \\
\frac{O_t}{V_t} &= \left(\frac{1-\theta}{1-\theta\cdot\Pi_t^{\gamma-1}}\right)^{\frac{1}{\gamma-1}} \\
\Delta_t &= (1-\theta)\left(\frac{O_t}{V_t}\right)^{-\gamma}+\theta\Pi_t^{\gamma}\Delta_{t-1} \\
R_t^J &= R^J\cdot\left(\frac{\Pi_t}{\Pi}\right)^{\tau_\pi}\left(\frac{\tilde{Y}_t}{\bar{Y}}\right)^{\tau_y}\cdot\exp\left\{\varepsilon_{r,t}\right\},\ \varepsilon_{r,t}\sim N\left(0,\sigma_r^2\right) \\
\tilde{F}_t^{*} &= \left[1-H_t\right]^{-\frac{1}{\omega}}\left(\frac{\omega-1}{\omega}\right)\phi_f\cdot\bar{Y}\cdot\exp\left\{u_{f,t}\right\} \\
R_t^{J,*} &= \left(\frac{\omega}{\omega+1}\right)\cdot\left(1-H_t\right)^{-\frac{1}{\omega}}\cdot R_t^J \\
N_t &= \Theta_N\cdot\left(\tilde{Y}_t\Delta_t\right)^{\frac{1}{\alpha}}\cdot\left(1+\Theta_4 H_{t-1}\right)^{\frac{\alpha+\sigma(1-\alpha)}{(1-\sigma)\alpha}} \\
g_{t,t+1}^{Density} &= \left[\frac{1+\Theta_4\cdot H_{t-1}}{1+\Theta_4\cdot H_t}\right]^{\left(\frac{\sigma}{(\sigma-1)\alpha}\right)}\left(\frac{\tilde{Y}_{t+1}\Delta_{t+1}}{\tilde{Y}_t\Delta_t}\right)^{\frac{1}{\alpha}}-1 \\
g_{t,t+1}^{Entry} &= \frac{\left(H_t-H_{t-1}\right)+\frac{\omega(\sigma-1)}{\kappa[\alpha+\sigma(1-\alpha)]+(\omega-1)(\sigma-1)}\left(H_{t-1}-H_t\right)}{H_{t-1}+\frac{\omega(\sigma-1)}{\kappa[\alpha+\sigma(1-\alpha)]+(\omega-1)(\sigma-1)}\left(1-H_{t-1}\right)} \\
\frac{W_t}{P_tA_t} &= \Theta_N^{\frac{1}{\eta}}\left(\tilde{C}_t\right)\left(\tilde{Y}_t\Delta_t\right)^{\frac{1}{\eta\alpha}}\left(1+\Theta_4 H_{t-1}\right)^{\frac{\alpha+\sigma(1-\alpha)}{\eta(1-\sigma)\alpha}}\cdot\exp\left\{-u_{c,t}\right\}
\end{aligned}
$$

$$
\begin{aligned}
\frac{P_t^J}{P_t} &= \Theta_N^{\frac{1}{\eta}} \Theta_3^{\frac{\alpha+\sigma(1-\alpha)}{(\sigma-1)\alpha}} \left(\tilde{C}_t\right) \left(\tilde{Y}_t \Delta_t\right)^{\left(\frac{(1-\alpha)\eta+1}{\eta\alpha}\right)} \left(1+\Theta_4 H_{t-1}\right)^{\frac{(1+\eta)[\alpha+\sigma(1-\alpha)]}{\eta(1-\sigma)\alpha}} \cdot \exp\left\{-u_{c,t}\right\} \\
M_{t+1} &= 1 - \Theta_M \cdot [1 - H_t] \\
\frac{\frac{L_t}{A_t P_t}}{\bar{Y}} &= \phi_f \cdot \left[1 - \Theta_L \cdot [1 - H_t]^{\left(\frac{\omega-1}{\omega}\right)}\right] \\
GA_t &= (1+\mu) \cdot \exp\left\{u_{a,t}\right\}
\end{aligned}
$$

Shock processes:

$$
\begin{aligned}
u_{a,t} &= \rho_a \cdot u_{a,t-1} + \varepsilon_{a,t},\ \varepsilon_{a,t} \sim N\left(0,\ \sigma_a^2\right) \\
u_{c,t} &= \rho_c \cdot u_{c,t-1} + \varepsilon_{c,t},\ \varepsilon_{c,t} \sim N\left(0, \sigma_c^2\right) \\
u_{g,t} &= \rho_g \cdot u_{g,t-1} + \varepsilon_{g,t},\ \varepsilon_{g,t} \sim N\left(0,\ \sigma_g^2\right) \\
u_{f,t} &= \rho_f \cdot u_{f,t-1} + \varepsilon_{f,t},\ \varepsilon_{f,t} \sim N\left(0,\ \sigma_f^2\right)
\end{aligned}
$$

Parameters:

$$
\begin{aligned}
\Theta_1 &= \left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha}\right)^{\frac{\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}} \left(\frac{\kappa-1}{\kappa}\right)^{\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}} \left(\frac{\kappa[\alpha+\sigma(1-\alpha)]}{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}\right) \\
\Theta_2 &= \frac{\alpha+\sigma(1-\alpha)}{\alpha(\sigma-1)} \left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha}\right)^{-\frac{\sigma}{\alpha+\sigma(1-\alpha)}} \left(\frac{\kappa-1}{\kappa}\right)^{\frac{\alpha(\sigma-1)}{\alpha+\sigma(1-\alpha)}} \\
\Theta_3 &= \left(\frac{\kappa[\alpha+\sigma(1-\alpha)]+(\omega-1)(\sigma-1)}{\Theta_1\omega(\sigma-1)}\right) \\
\Theta_4 &= \left(\frac{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}{\omega(\sigma-1)}\right) \\
\Theta_N &= \left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha}\right)^{\left(\frac{-\sigma}{\alpha+\sigma(1-\alpha)}\right)} \left(\frac{\kappa-1}{\kappa}\right)^{\left(\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}\right)} \left(\frac{\kappa[\alpha+\sigma(1-\alpha)]}{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}\right) \\
&\quad \cdot \left(\frac{\omega(\sigma-1)}{\kappa[\alpha+\sigma(1-\alpha)]+(\omega-1)(\sigma-1)}\right) \Theta_3^{\left(\frac{\sigma}{\alpha(\sigma-1))}\right)} > 0 \\
\Theta_M &= \frac{\kappa[\alpha+\sigma(1-\alpha)]}{\kappa[\alpha+\sigma(1-\alpha)]+\omega(\sigma-1)} \\
\Theta_L &= \frac{\kappa[\alpha+\sigma(1-\alpha)]}{\kappa[\alpha+\sigma(1-\alpha)]+(\sigma-1)(\omega-1)}
\end{aligned}
$$

C.2 Flexible Price Equilibrium

$$
\begin{aligned}
1 &= \left(\frac{\beta\Theta_2}{\left(\frac{\omega-1}{\omega}\right)\cdot\phi_f}\right)\cdot\left(\frac{(1+\zeta^T)^{-1}\gamma}{\gamma-1}\right)^{-1}\cdot\left(\frac{\kappa-1}{\kappa}\right)^{\left(\frac{(\sigma-1)(1-\alpha)}{\alpha+\sigma(1-\alpha)}\right)}\cdot\left[\frac{\Theta_3}{1+\Theta_4\cdot H_t}\right][1-H_t]^{\frac{1}{\omega}} \\
&\quad\cdot E_t\left[\left(\frac{\tilde{Y}_t}{\bar{Y}}\right)\left(\frac{\tilde{C}_t/\tilde{Y}_t}{\tilde{C}_{t+1}/\tilde{Y}_{t+1}}\right)\cdot\exp\left\{u_{c,t+1}-\left(u_{f,t}+u_{c,t}\right)\right\}\right] \\
\tilde{Y}_t &= \left(\frac{(1+\zeta^T)^{-1}\gamma}{\gamma-1}\right)^{-\left(\frac{\eta\alpha}{(1-\alpha)\eta+1}\right)}\Theta_N^{-\left(\frac{\alpha}{(1-\alpha)\eta+1}\right)}\Theta_3^{-\frac{\eta[\alpha+\sigma(1-\alpha)]}{[(1-\alpha)\eta+1](\sigma-1)}}\cdot\tilde{C}_t^{-\left(\frac{\eta\alpha}{(1-\alpha)\eta+1}\right)} \\
&\quad\cdot(1+\Theta_4 H_{t-1})^{-\frac{(1+\eta)[\alpha+\sigma(1-\alpha)])}{(1-\sigma)[(1-\alpha)\eta+1]}}\cdot\exp\left\{\left(\frac{\eta\alpha}{(1-\alpha)\eta+1}\right)\cdot u_{c,t}\right\} \\
\frac{\tilde{C}_t}{\tilde{Y}_t} &= 1-\phi_g\cdot\exp\left\{u_{g,t}\right\}-\phi_f\cdot\left(\frac{\tilde{Y}_t}{\bar{Y}}\right)^{-1}\cdot\left[1-\Theta_L\cdot[1-H_t]^{\left(\frac{\omega-1}{\omega}\right)}\right]\cdot\exp\left\{u_{f,t}\right\} \\
\tilde{F}_t^* &= [1-H_t]^{-\frac{1}{\omega}}\left(\frac{\omega-1}{\omega}\right)\phi_f\cdot\bar{Y}\cdot\exp\left\{u_{f,t}\right\} \\
R_t^J &= R^J\cdot\left(\frac{\Pi_t}{\Pi}\right)^{\tau_\pi}\left(\frac{\tilde{Y}_t}{\bar{Y}}\right)^{\tau_y}\cdot\exp\left\{\varepsilon_{r,t}\right\} \\
R_t^{J,*} &= \left(\frac{\omega}{\omega+1}\right)\cdot(1-H_t)^{-\frac{1}{\omega}}\cdot R_t^J
\end{aligned}
$$

Shock processes:

$$
\begin{aligned}
GA_t &= (1+\mu)\cdot exp\left\{u_{a,t}\right\} \\
u_{a,t} &= \rho_a\cdot u_{a,t-1}+\varepsilon_{a,t} \\
u_{c,t} &= \rho_c\cdot u_{c,t-1}+\varepsilon_{c,t} \\
u_{g,t} &= \rho_g\cdot u_{g,t-1}+\varepsilon_{g,t} \\
u_{f,t} &= \rho_f\cdot u_{f,t-1}+\varepsilon_{f,t} \\
\varepsilon_{c,t} &\sim N\left(0,\sigma_c^2\right) \\
\varepsilon_{a,t} &\sim N\left(0,\ \sigma_a^2\right) \\
\varepsilon_{g,t} &\sim N\left(0,\ \sigma_g^2\right) \\
\varepsilon_{f,t} &\sim N\left(0,\ \sigma_f^2\right) \\
\varepsilon_{r,t} &\sim N\left(0,\sigma_r^2\right)
\end{aligned}
$$

C.3 Steady State Conditions

$$
\begin{aligned}
R^B &= \beta^{-1}(1+\mu)\Pi \\
\Delta &= \left(\frac{1-\theta}{1-\theta\Pi^{\gamma}}\right)\left(\frac{1-\theta\Pi^{\gamma-1}}{1-\theta}\right)^{\left(\frac{\gamma}{\gamma-1}\right)} \\
\frac{\Theta_3 \cdot [1-H]^{\frac{1}{\omega}}}{1+\Theta_4 \cdot H} &= \left(\frac{\kappa-1}{\kappa}\right)^{\left(\frac{(1-\sigma)(1-\alpha)}{\alpha+\sigma(1-\alpha)}\right)} \left[\frac{(1+\zeta^T)^{-1}\gamma}{\gamma-1}\right]\left[\frac{1-\theta\Pi^{\gamma}}{1-\theta\Pi^{\gamma-1}}\right]\left[\frac{1-\beta\theta\Pi^{\gamma-1}}{1-\beta\theta\Pi^{\gamma}}\right]\left(\frac{\left(\frac{\omega-1}{\omega}\right)\phi_f}{\beta\cdot\Theta_2}\right) \\
\bar{Y} &= \frac{\left(\frac{\beta\Theta_2\Theta_N^{\frac{1}{\eta}}\Theta_3^{\frac{\sigma}{(\sigma-1)\alpha}}}{\left(\frac{\omega-1}{\omega}\right)\phi_f}\right)^{-\left(\frac{\eta\alpha}{\eta+1}\right)} \left(\frac{\kappa-1}{\kappa}\right)^{\left(\frac{-\eta\alpha(\sigma-1)(1-\alpha)}{[\alpha+\sigma(1-\alpha)](\eta+1)}\right)} (1+\Theta_4 H)^{-\frac{[\alpha+\sigma(1-\alpha)]+\sigma\eta}{(\eta+1)(1-\sigma)}} (1-H)^{-\frac{\eta\alpha}{\omega(\eta+1)}}}{\Delta\cdot\left[1-\phi_g-\phi_f\cdot\left[1-\Theta_L\cdot[1-H]^{\left(\frac{\omega-1}{\omega}\right)}\right]\right]^{\left(\frac{\eta\alpha}{\eta+1}\right)}} \\
\tilde{C} &= \left[1-\phi_g-\phi_f\cdot\left[1-\Theta_L\cdot[1-H]^{\left(\frac{\omega-1}{\omega}\right)}\right]\right]\cdot\bar{Y} \\
M &= 1-\Theta_M\cdot[1-H] \\
\tilde{F}^* &= [1-H]^{-\frac{1}{\omega}}\left(\frac{\omega-1}{\omega}\right)\phi_f\cdot\bar{Y} \\
R^{J,*} &= \left(\frac{\omega}{\omega+1}\right)\cdot(1-H)^{-\frac{1}{\omega}}\cdot\beta^{-1}(1+\mu)\Pi \\
\frac{R^{J,*}}{R^B} &= \left(\frac{\omega}{\omega+1}\right)\cdot(1-H)^{-\frac{1}{\omega}} \\
N &= \Theta_N\cdot\bar{Y}^{\frac{1}{\alpha}}\cdot\Delta^{\frac{1}{\alpha}}\cdot(1+\Theta_4 H)^{\frac{\alpha+\sigma(1-\alpha)}{(1-\sigma)\alpha}} \\
\frac{W}{PA} &= \Theta_N^{\frac{1}{\eta}}\tilde{C}\bar{Y}^{\frac{1}{\eta\alpha}}\Delta^{\frac{1}{\eta\alpha}}(1+\Theta_4 H)^{\frac{\alpha+\sigma(1-\alpha)}{\eta(1-\sigma)\alpha}} \\
\frac{P_t^J}{P_t} &= \Theta_N^{\frac{1}{\eta}}\Theta_3^{\frac{\alpha+\sigma(1-\alpha)}{(\sigma-1)\alpha}}\tilde{C}(\bar{Y}\Delta)^{\left(\frac{(1-\alpha)\eta+1}{\eta\alpha}\right)}(1+\Theta_4 H)^{\frac{(1+\eta)[\alpha+\sigma(1-\alpha)]}{\eta(1-\sigma)\alpha}} \\
O &= \frac{(1+\zeta^T)^{-1}\gamma}{\gamma-1}\cdot\frac{\Theta_N^{\frac{1}{\eta}}\Theta_3^{\frac{\alpha+\sigma(1-\alpha)}{(\sigma-1)\alpha}}\bar{Y}^{\left(\frac{\eta+1}{\eta\alpha}\right)}\Delta^{\frac{(1-\alpha)\eta+1}{\eta\alpha}}(1+\Theta_4 H)^{\frac{(1+\eta)[\alpha+\sigma(1-\alpha)]}{\eta(1-\sigma)\alpha}}}{1-\beta\theta\Pi^{\gamma}} \\
V &= \frac{\left(\frac{\tilde{C}}{\bar{Y}}\right)^{-1}}{1-\beta\theta\Pi^{\gamma-1}} \\
\frac{\frac{L}{PA}}{\bar{Y}} &= \phi_f\left[1-\Theta_L(1-H)^{\frac{\omega-1}{\omega}}\right]
\end{aligned}
$$

Appendix D Limiting Case with $\omega \to \infty$

When $\omega \to +\infty$,the Pareto distribution $H(F_{m,t})$ of the fixed costs collapse to its mean, F_t. In this scenario, it is trivial to see that $P^J_{m,t} = P^J_t$. For P^J_t, we plug equation (B.4) into equation (B.3), and obtain

$$\frac{P^J_t}{P_t} = \begin{cases} \Theta_1^{-\frac{\alpha+\sigma(1-\alpha)}{(\sigma-1)\alpha}} \left(\frac{W_t}{P_t A_t}\right) \cdot \left(\frac{Y_t \Delta_t}{A_t}\right)^{\frac{1-\alpha}{\alpha}} \\ \quad \cdot \left(\frac{R^J_{t-1} F_{t-1}}{\Theta_2 E_{t-1} \left[\xi_t \left(\frac{P^J_t}{P_t}\right)^{\frac{\sigma}{\alpha+\sigma(1-\alpha)}} \left(\frac{W_t}{P_t A_t}\right)^{-\frac{(\sigma-1)\alpha}{\alpha+\sigma(1-\alpha)}} \left(\frac{\kappa-1}{\kappa} A_t\right) \Pi_t \left(\frac{Y_t\Delta_t}{A_t}\right)^{\frac{1}{\alpha+\sigma(1-\alpha)}} \right]} \right)^{\frac{[\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)](\alpha+\sigma(1-\alpha))}{(\sigma-1)^2\alpha}} \\ \underline{\text{if } R^J_t > R^{J,*}_t,} \\ \Theta_1^{-\left(\frac{\alpha+\sigma(1-\alpha)}{(\sigma-1)\alpha}\right)} \left(\frac{W_t}{P_t A_t}\right) \cdot \left(\frac{Y_t \Delta_t}{A_t}\right)^{\frac{1-\alpha}{\alpha}} \quad \underline{\text{if } R^J_t \le R^{J,*}_t}. \end{cases} \tag{D.1}$$

Plugging (D.1) into (B.4), we can obtain

$$\Xi_t = \begin{cases} \Theta_5 \cdot \left(\frac{W_t}{P_t A_t}\right) \left[\left(\frac{\kappa-1}{\kappa}\right) A_t \right]^{\frac{1}{\alpha+\sigma(1-\alpha)}} \cdot P_t \left(\frac{Y_t\Delta_t}{A_t}\right)^{\frac{1}{\alpha}} \\ \quad \cdot \left(\frac{R^J_{t-1} F_{t-1}}{\Theta_2 E_{t-1} \left[\xi_t \left(\frac{P^J_t}{P_t}\right)^{\frac{\sigma}{\alpha+\sigma(1-\alpha)}} \left(\frac{W_t}{P_t A_t}\right)^{-\frac{(\sigma-1)\alpha}{\alpha+\sigma(1-\alpha)}} \left(\frac{\kappa-1}{\kappa} A_t\right)^{\frac{(\sigma-1)(1-\alpha)}{\alpha+\sigma(1-\alpha)}} \Pi_t (Y_t\Delta_t)^{\frac{1}{\alpha+\sigma(1-\alpha)}} \right]} \right)^{\frac{\sigma}{\sigma-1}\left(\frac{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}{(\sigma-1)\alpha}\right)} \\ \underline{\text{if } R^J_t > R^{J,*}_t,} \\ \Theta_5 \cdot \left(\frac{W_t}{P_t A_t}\right) \left[\left(\frac{\kappa-1}{\kappa}\right) A_t \right]^{\frac{1}{\alpha+\sigma(1-\alpha)}} \cdot P_t \left(\frac{Y_t\Delta_t}{A_t}\right)^{\frac{1}{\alpha}} \quad \underline{\text{if } R^J_t \le R^{J,*}_t,} \end{cases} \tag{D.2}$$

where we define

$$\Theta_5 = \Theta_1^{-\left(\frac{\sigma}{(\sigma-1)\alpha}\right)} \Theta_2 \left(\frac{\kappa-1}{\kappa}\right)^{\frac{\alpha(1-\sigma)-1}{\alpha+\sigma(1-\alpha)}}.$$

Now that $M_t = M_{m,t}$, $L_t = L_{m,t}$, $R^{J,*}_t = R^{J,*}_{m,t}$ and $\varphi^*_t = \varphi^*_{m,t}$, we can substitute (D.2) into (17), (18), (19), and (21) to obtain following analytical expressions:

$$R_t^{J,*} = \Theta_5 \cdot E_t \left[\xi_{t+1} \left(\frac{\kappa - 1}{\kappa} A_{t+1} \right)^{\frac{\sigma}{\alpha + \sigma(1-\alpha)}} \left(\frac{w_{t+1}}{P_{t+1} A_{t+1}} \right) \frac{\Pi_{t+1}}{F_t} \left(\frac{Y_{t+1} \Delta_{t+1}}{A_{t+1}} \right)^{\frac{1}{\alpha}} \right], \tag{D.3}$$

$$\varphi_t^* = \left(\frac{R_t^J}{R_t^{J,*}} \right)^{\left(\frac{\alpha + \sigma(1-\alpha)}{\sigma - 1} \right)} \left[\left(\frac{\kappa - 1}{\kappa} \right) A_{t+1} \right], \tag{D.4}$$

$$M_{t+1} = \begin{cases} \left(\frac{R_t^J}{R_t^{J,*}} \right)^{-\left(\frac{\kappa[\alpha + \sigma(1-\alpha)]}{\sigma - 1} \right)} & \underline{\text{if } R_t^J > R_t^{J,*},} \\ 1 & \underline{\text{if } R_t^J \leq R_t^{J,*},} \end{cases} \tag{D.5}$$

$$L_t = \begin{cases} \left(\frac{R_t^J}{R_t^{J,*}} \right)^{-\left(\frac{\kappa[\alpha + \sigma(1-\alpha)]}{\sigma - 1} \right)} \cdot F_t & \underline{\text{if } R_t^J > R_t^{J,*},} \\ F_t & \underline{\text{if } R_t^J \leq R_t^{J,*}.} \end{cases} \tag{D.6}$$

We observe: if $R_t^J \leq R_t^{J,*}$, where $R_t^{J,*}$ is defined in (28), all firms are satiated and the loan amount made to firms is equal to F_t, the fixed cost that operating firms need to pay one period in advance.