

# A Higher-Order Forward Guidance

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**Model 1** (standard New-Keynesian with rigid price): with

$$\hat{Y}_t = \ln \frac{Y_t}{Y_t^n}, \quad \underbrace{(\sigma)^2 dt = \text{Var}_t \left( \frac{dY_t^n}{Y_t^n} \right)}_{\substack{\text{Benchmark volatility} \\ \text{Exogenous}}}, \quad \underbrace{(\sigma + \sigma_t^s)^2 dt = \text{Var}_t \left( \frac{dY_t}{Y_t} \right)}_{\substack{\text{Actual volatility} \\ \text{Endogenous}}}$$

**A non-linear IS equation** (in contrast to textbook linearized one)

$$d\hat{Y}_t = \left( i_t - \underbrace{\left( r^n - \frac{1}{2}(\sigma + \sigma_t^s)^2 + \frac{1}{2}\sigma^2 \right)}_{\equiv r_t^T} \right) dt + \sigma_t^s dZ_t \quad (1)$$

New terms

**Monetary policy:** Taylor rule

$$i_t = r^n + \phi_y \hat{Y}_t \quad \text{where } \phi_\pi > 0 \text{ (Taylor principle)}$$

allows **self-fulfilling** aggregate volatility  $\sigma_0^s$

**Model 2** (model with stock markets and portfolio decisions) : asset (stock) price gap  $\hat{Q}_t$  follows

$$d\hat{Q}_t = \left( i_t - \pi_t - \underbrace{\left( r^n - \frac{1}{2} \underbrace{(\sigma + \sigma_t^q)^2}_{rp_t \equiv r_t^r} + \frac{1}{2} \underbrace{\sigma^2}_{rp^n} \right)}_{\text{Fundamental volatility}} \right) dt + \sigma_t^q dZ_t$$

Here

$$\sigma_t^q \uparrow \Rightarrow rp_t \uparrow \Rightarrow \hat{Q}_t \downarrow \Rightarrow \hat{Y}_t \downarrow$$

**Monetary policy:** Taylor rule to Bernanke and Gertler (2000) rule

$$\begin{aligned} i_t &= r^n + \phi_\pi \pi_t + \phi_y \underbrace{\hat{y}_t}_{=\zeta \hat{Q}_t} \\ &= r^n + \phi_\pi \pi_t + \phi_q \hat{Q}_t, \quad \text{where} \quad \underbrace{\phi \equiv \phi_q + \frac{\kappa(\phi_\pi - 1)}{\rho}}_{\text{Taylor principle}} > 0 \end{aligned}$$

allows self-fulfilling stock price volatility  $\sigma_t^q$

**Thought experiment** (rigid price economy with  $\pi_t = 0$ ): fundamental volatility  $\sigma \uparrow$ :  $\underline{\sigma}$  to  $\bar{\sigma}$  on  $[0, T]$  (e.g., **Werning (2012)**) and comes back to  $\underline{\sigma}$  with

- $r_1^n \equiv \rho + g - \underline{\sigma}^2 > 0$ : no ZLB before
- $r_2^n \equiv \rho + g - \bar{\sigma}^2 < 0$ : now ZLB binds (on the stabilized equilibrium path)

**Assume:** perfect stabilization is achievable outside ZLB

- Central bank always can use risk-premium targeting as given by

$$i_t = r_1^n + \phi_\pi \pi_t + \phi_q \hat{Q}_t - \frac{1}{2} \hat{p}_t$$

with

$$\phi \equiv \phi_q + \frac{\kappa(\phi_\pi - 1)}{\rho} > 0$$

# 1. ZLB path (full stabilization after $T$ )

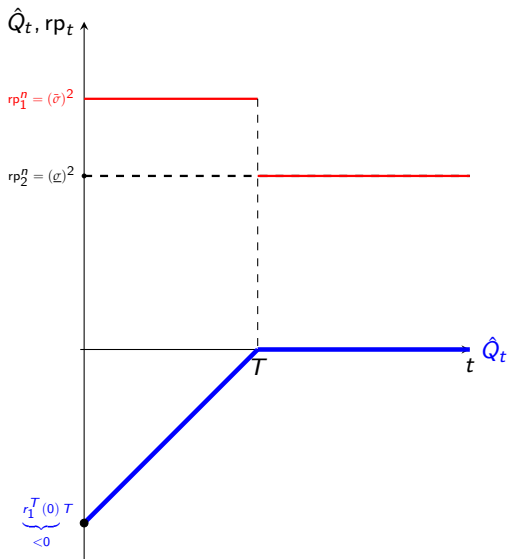


Figure: ZLB dynamics (Benchmark)

2. Traditional forward guidance (keep  $i_t = 0$  until  $\hat{t} > T$ )

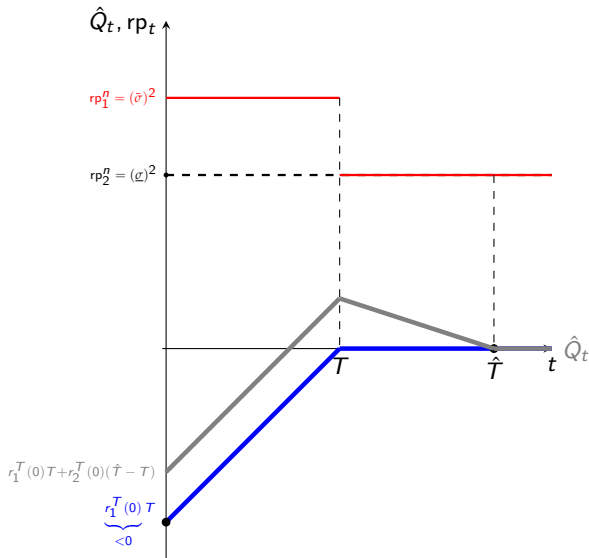


Figure: ZLB dynamics with forward guidance until  $\hat{T} > T$

**Recall** an economic mechanism in the ZLB and forward guidance

1. Central bank achieves perfect stabilization:  $\hat{Q}_t = \hat{r}p_t = 0, \forall t \geq \hat{T}$



2.  $\hat{Q}_{\hat{T}} = 0$  guarantees  $\sigma_t^q = \sigma^{q,n} = 0, rp_t = rp^n$  for  $t \leq \hat{T}$

Still if  $rp^n$  is too high, might want to push  $\{\sigma_t^q, rp_t\}$  down for  $\hat{Q}_t \uparrow$ ?

- Thus achieve  $\sigma_t^q < \sigma^{q,n} = 0, rp_t < rp^n \implies \hat{Q}_t \uparrow$  at the ZLB

Take **contrapositive** to the above:

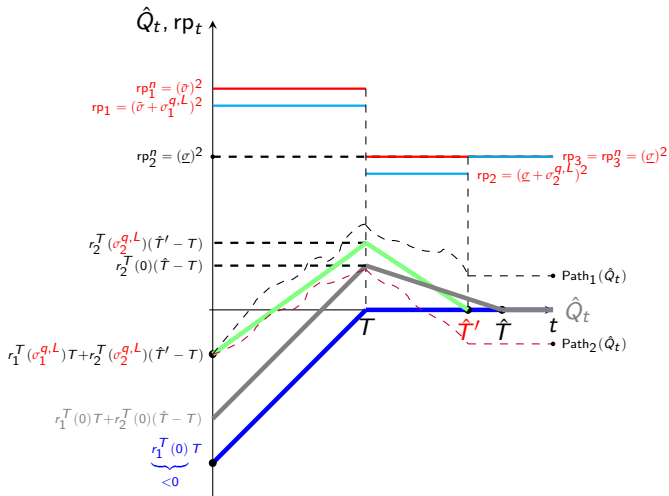
$\neg 2. \sigma_t^q < \sigma^{q,n} = 0, rp_t < rp^n$  for  $t \leq \hat{T}$



$\neg 1. \hat{Q}_{\hat{T}} \neq 0$ . Central bank commits not to perfectly stabilize the economy after  $\hat{T}$

- Giving up **future** financial stability  $\implies rp_t \downarrow$  and  $\hat{Q}_t \uparrow$  **now** (at the ZLB)

3. Central bank picks  $\{\sigma_t^q\}$  and  $\{rp_t\}$



### Proposition (Optimal commitment path)

At optimum,  $\sigma_1^{q,L} < \sigma_1^{q,n}$ ,  $\sigma_2^{q,L} < \sigma_2^{q,n}$ , and  $\hat{T}' < \hat{T}$



Thank you very much!