# Ignorance is Bliss: Ex-Ante vs. Ex-Post Information Systems in an Agency Model

Jin yong Jung
Kangnam University

Son Ku Kim
Seoul National University

Seung Joo Lee
Oxford University

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# Motivation

Starting from the canonical principal-agent model à la Holmstrom (1979):

- Usually, the agent's utility $u(s)$ depends only on $s$, the monetary payment

- Some other factors, $t$, that affect the agent's utility and also are beyond the principal and the agent's controls

- Then, maybe $u(s, t)$ instead of $u(s)$

- $t$ is eventually revealed in the end, so can be used in contracts: $s(x, t)$

## Big Question (Main Topic)

In the presence of the moral-hazard problem, if the principal designs (i) the timing of when $t$ is revealed to the agent; (ii) the amount of information about $t$ revealed to the agent,

What is her optimal strategy?

# Motivation

[Timing] the principal designs the timing of the revelation of $t$

1. Before the agent signs on the contract?:

   Pre-Contract Information System

2. After the agent signs on the contract, but before takes an action (i.e., effort)

   Post-Contract Ex-Ante System

3. After takes an action

   Post-Contract Ex-Post System

[Amount] the principal designs how much of $t$ is revealed ex ante

Finer vs. Coarser Partitions

## Example 1 (Match)

A worker infers about his personal match to the new workplace through on-site visits and initial induction programs offered by the company.

- Fitness to the culture, aptitude for the work's nature, a sense of belonging to different peer groups, and a sense of accomplishment from the job, '$t$', affect his satisfaction: $u(s, t)$

The firm's hiring manager is deciding whether and when to holds the events:

- Before he signs on the contract;
- After he signs on the contract but before starts his job
- Never offers those programs for new hires ex ante and let them figure out in the end (after the new hires take an action)

Eventually, both the hiring manger and the worker will realize the degree of their match

- Final monetary payments to the new hire $s(x, t)$ can rely on the revealed information about match, '$t$'.

## Scenario 2 (Pension)

A company provides its workers with various pension plans

- Payouts from the pension plans, '$t$', are determined by a pension manager's portfolio decisions and the state of market, beyond the company CEO's control
- Pension payouts, '$t$', affect workers' job satisfaction: $u(s, t)$

The company's CEO knows the track records of the pension manager's performance during the past few years, and is considering whether and when she should reveal those data to her new employee:

- Before he signs on the contract;
- After he signs on the contract but before starts his job
- Never offers the relevant information ex ante and let the worker figure out (after he takes an action)

Eventually, the pension performance will be revealed

- Final monetary payments to the worker $s(x, t)$ might depend on the revealed pension performance, '$t$'.

# Findings

## Big Question (Main Topic)

1. Among the following options, which information system yields the best efficiency?
   - Pre-contract system
   - Post-contract ex-ante system
   - Post-contract ex-post system

2. Should the principal offer the relevant information as precise as possible ex ante?
   - An amount of information about '$t$'

## Answer

1. In terms of efficiency:

$$\text{Pre-contract} < \text{Post-contract ex-ante} < \text{Post-contract ex-post}$$

2. More precise ex-ante information$\uparrow \longrightarrow$ efficiency$\downarrow$
   - The agent will use the given ex-ante information in pursuit of his own interests, not on the principal's behalf

**Our contribution**:

- Prove the results in a standard principal-agent setting à la Holmstrom (1979)

# The Formulation

## Setting

**Single period agency setting**: the principal and the agent

**Actions**: $a$ action, and $\theta$ state of nature with $\mathbb{E}(\theta) = 0$

$$\text{Output} \quad x = \underbrace{a}_{\text{Expected output}} + \underbrace{\theta}_{\text{State of nature}} \tag{1}$$

**Preference of the agent**:

$$U(s, t, a) = u(s, t) - v(a) \tag{2}$$

where

$$v'(a) > 0, v''(a) > 0, \underbrace{v'''(a) > 0}_{\text{Convexity}\uparrow\uparrow} \tag{3}$$

**Contract**: t is revealed in the end (i.e., contractible), thus s(x,t)

# Pre-Contract Information System

The agent knows the value of $t$ even before signing on the contract

The principal designs $(a^p(t), s^p(x,t))$ to maximize:

$$\max_{a(t)\in A, s(x,t)\in S} \underbrace{\int_t \int_x (x - s(x,t))\, f(x|a(t)) h(t) dxdt}_{\text{Expected welfare of the principal}} \qquad \text{s.t:}$$

$$\begin{aligned} &(i) \quad \boxed{\int_x u(s(x,t),t) f(x|a(t)) dx - v(a(t)) \geq \overline{U}}, \ \forall t \in T, \\ &(ii) \quad \boxed{a(t) \in \arg\max_{a'} \int_x u(s(x,t),t) f(x|a') dx - v(a'), \ \forall a' \in A}, \ \forall t \in T, \\ &(iii) \quad s(x,t) \geq 0, \quad \forall (x,t) \in X \times T. \end{aligned} \tag{4}$$

Participation for $\forall t$

Incentive Compatibility for $\forall t$

# Pre-Contract Information System

**Optimal contract**:

$$\frac{1}{u_s(s^p(x,t),t)} = \lambda^p(t) + \mu^p(t)\frac{f_a}{f}(x|a^p(t)), \tag{5}$$

where (5) has a solution $s^p(x,t) \geq 0$ and otherwise $s^p(x,t) = 0$

- Note that $(\lambda^p(t), \mu^p(t))$ are endogenous

**Welfare of the principal**:

$$PW^p \equiv \int_t \int_x (x - s^p(x,t))\, f(x|a^p(t)) h(t) dxdt. \tag{6}$$

# Post-Contract Ex-Ante System

The agent knows $t$ after signing on the contract but before choosing his effort level

The principal designs $(a^*(t), s^*(x,t))$ to maximize:

$$\max_{a(t)\in A, s(x,t)\in S} \underbrace{\int_t \int_x (x - s(x,t))\, f(x|a(t)) h(t) dx dt}_{\text{Expected welfare of the principal}} \quad \text{s.t:}$$

$$\begin{aligned}
&(i) \quad \boxed{\int_t \left( \int_x u(s(x,t),t) f(x|a(t)) dx - v(a(t)) \right) h(t) dt \geq \overline{U}}, \ \forall t \in T, \\
&(ii) \quad \boxed{a(t) \in \arg\max_{a'} \int_x u(s(x,t),t) f(x|a') dx - v(a'), \ \forall a' \in A}, \ \forall t \in T, \\
&(iii) \quad s(x,t) \geq 0, \quad \forall (x,t) \in X \times T.
\end{aligned} \tag{7}$$

Participation on average $t$

Incentive Compatibility for $\forall t$

# Post-Contract Ex-Ante System

**Optimal contract**:

$$\frac{1}{u_s(s^*(x,t),t)} = \lambda^* + \mu^*(t)\frac{f_a}{f}(x|a^*(t)) \tag{8}$$

where (8) has a solution $s^*(x,t) \geq 0$ and otherwise $s^*(x,t) = 0$

- Note that $(\lambda^*, \mu^*(t))$ are endogenous

**Welfare of the principal**:

$$PW^* \equiv \int_t \int_x (x - s^*(x,t))\, f(x|a^*(t))h(t)dxdt \tag{9}$$

# Post-Contract Ex-Post System

The agent knows $t$ after signing on the contract and after choosing his effort level

The principal designs $(a^o, s^o(x,t))$ to maximize:

$$\max_{a(t)\in A, s(x,t)\in S} \underbrace{\int_t \int_x (x - s(x,t))\, f(x|a(t)) h(t)\, dx dt}_{\text{Expected welfare of the principal}} \quad \text{s.t:}$$

$$\begin{aligned} &(i) \quad \boxed{\int_t \left( \int_x u(s(x,t),t) f(x|a(t)) dx - v(a(t)) \right) h(t) dt \geq \overline{U}}, \\ &(ii) \quad \boxed{a \in \arg\max_{a'} \int_t \int_x u(s(x,t),t) f(x|a') h(t) dx dt - v(a'), \ \forall a' \in A}, \\ &(iii) \quad s(x,t) \geq 0, \quad \forall (x,t) \in X \times T. \end{aligned} \tag{10}$$

Participation on average $t$

Incentive Compatibility on average $t$

# Post-Contract Ex-Post System

**Optimal contract**:

$$\frac{1}{u_s(s^o(x,t),t)} = \lambda^o + \mu^o \frac{f_a}{f}(x|a^o) \tag{11}$$

where (11) has a solution $s^o(x,t) \geq 0$ and otherwise $s^o(x,t) = 0$

- Note that $(\lambda^o, \mu^o)$ are endogenous

**Welfare of the principal**:

$$PW^o \equiv \int_t \int_x (x - s^o(x,t)) f(x|a^o) h(t) dxdt. \tag{12}$$

# Comparison

## Comparison 1

**Lemma (Pre-Contract vs. Post-Contract Ex-Ante)**

$$PW^p \leq PW^*$$

1. Compared with the post-contract ex-ante system:
    - Under the pre-contract system, the agent's (PC) holds for each possible $t$
    - Under the post-contract ex-ante system, the agent's (PC) holds only on average across $t$, which is less costly to the principal
    - Similar to Sobel (1993)
2. Therefore, in terms of efficiency:

$$\text{Pre-Contract} \leq \text{Post-Contract Ex-Ante}$$

## Comparison 2

**Lemma (Post-Contract Ex-Ante vs. Post-Contract Ex-Post)**

$$PW^* \leq PW^o$$

Under the post-contract ex-ante system:

- The agent's induced effort level $a(t)$ depends on $t$, so effort is a random variable
- Under the post-contract ex-post system, $a(t) = a^o$ is uniform across $t$

$v''(a) > 0$

- Harder to satisfy the agent's (PC) on average (across different $t$-realizations)
- Why? The average $\mathbb{E}\left(v(a(t))\right) \uparrow$

$v'''(a) > 0$

- Harder to satisfy the agent's (IC) for each $t$
- Why? The average $\mathbb{E}\left(v'(a(t))\right) \uparrow$

Proof is based on Kim (1995) and Jewitt et al. (2008)

# Comparison 3

## Lemma (Special Case 1)

If the agent's utility function satisfies $\frac{u_{ss}}{u_{sss}} = \frac{u_{st}}{u_{sst}}$, and the agent's limited liability constraint is not binding for any $(x, t)$, then

$$\frac{\partial^2}{\partial x \partial t} s^o(x, t) = 0. \tag{13}$$

In this case, the optimal contract $s^o(x, t)$ under the post-contract ex-post system features the same incentive (sensitivity to $x$) for $\forall t$

- In this case, post-contract ex-ante $\simeq$ post-contract ex-post

## Proposition (Equivalence between ex-post and ex-ante)

If $\frac{u_{ss}}{u_{sss}} = \frac{u_{st}}{u_{sst}}$, and the limited liability constraint is not binding for any $(x, t)$, the principal is indifferent between the post-contract ex-ante information system and the post-contract ex-post information system, i.e.,

$$PW^p \leq PW^* = PW^o. \tag{14}$$

Furthermore, in this case,

$$a^*(t) = a^o, \quad \forall t \in T. \tag{15}$$

# Comparison 4

## Proposition (Special Case 2)

If $\frac{u_s}{u_{ss}} = \frac{u_t}{u_{st}}$ and the limited liability constraint is not binding for any $(x, t)$, the principal is indifferent among the three information systems, i.e.,

$$PW^p = PW^* = PW^o. \tag{16}$$

**Special Case 1**:

$$u(s, t) = u(s + k(t)) + l(t) \tag{17}$$

**Special Case 2**:

$$u(s, t) = u(s + k(t)) \tag{18}$$

# Information Partition

# Two information partitions

Still $t$ is revealed in the end: so contractible s(x,t)

**Information system** $N$: partitions on $T = [\underline{t}, \overline{t}]$, $\{T_1, T_2, ....., T_j, ...., T_N\}$

- The agent ex ante knows which partition $i = 1, 2, ..., N$ the true $t$ belongs

**Information system** $N^+$: partitions $\{T_1, T_2, ....., T_j^-, T_j^+, ...., T_N\}$

- Now $T_j$ is decomposed into $T_j^-$ and $T_j^+$

- Finer (more precise) information system

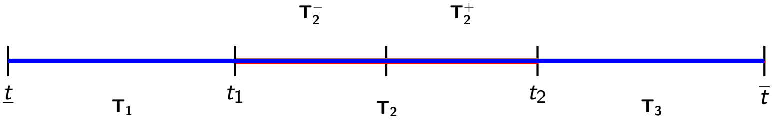


Figure: An Example of Information Systems $N$ and $N^+$

## Information system $N$

The agent's action $a(T_i)$ depends on a partition $T_i$

The principal designs $\left(a^N(T_i), s^N(x,t)\right)$ to maximize:

$$\max_{\substack{\{a(T_i)\in A\}_{1\leq i\leq N} \\ s(x,t)\in S}} \sum_{i=1}^{N} p_i \left(\int_{t\in T_i}\int_x (x - s(x,t))\, f\,(x|a(T_i))\, h(t|T_i)dxdt\right) \qquad \text{s.t.}$$

$$(i) \quad \boxed{\sum_{i=1}^{N} p_i \left(\int_{t\in T_i}\int_x u(s(x,t),t) f\,(x|a(T_i))\, h(t|T_i)dxdt - v(a(T_i))\right) \geq \overline{U}},$$

$$(ii) \quad \boxed{\int_{t\in T_i}\int_x u(s(x,t),t) f_a\,(x|a(T_i))\, h(t|T_i)dxdt = v'(a(T_i)),}\ , \forall T_i \in T$$

$$(iii) \quad s(x,t) \geq 0, \quad \forall (x,t) \in X \times T. \qquad (19)$$

Participation on average $T_i$

Information system $N^+$ entails a similar optimization

Incentive Compatibility for given $T_i$

## Comparison $N$ vs. $N^+$

**The principal's welfare under $N$:**

$$PW^N \equiv \sum_{i=1}^{N} p_i \left( \int_{t \in T_i} \int_x \left( x - s^N(x,t) \right) f\left( x | a^N(T_i) \right) h(t|T_i) dxdt \right) \tag{20}$$

**The principal's welfare under $N^+$:**

$$PW^{N^+} \equiv \sum_{i=1}^{N^+} p_i \left( \int_{t \in T_i} \int_x \left( x - s^{N^+}(x,t) \right) f\left( x | a^{N^+}(T_i) \right) h(t|T_i) dxdt \right) \tag{21}$$

**Proposition ($N$ vs. $N^+$)**

$$PW^{N^+} \leq PW^N$$

- More ex-ante information always hurts the principal's welfare (i.e., efficiency)

# Illustrative Examples

# Special utility

Now assume $u(s,t) = 2t\sqrt{s}$

Under the post-contract ex-ante system:

- Given the optimal induced $\{a^*(t)\}$, the optimal $s^*(x,t)$ minimizes the agency cost

**Agency cost (ex-ante)**:

$$AC_M^*(\{a^*(t)\}) \equiv \min_{s(x,t)\in S} \int_t \int_x s(x,t) f(x|a^*(t)) h(t) dx dt \quad \text{s.t.}$$

$$\begin{aligned} &(i) \quad \int_t \left( \int_x 2t\sqrt{s(x,t)} f(x|a^*(t)) dx - v(a^*(t)) \right) h(t) dt \geq \overline{U}, \\ &(ii) \quad \int_x 2t\sqrt{s(x,t)} f_a(x|a^*(t)) dx - v'(a^*(t)) = 0, \ \ \forall t \in T, \\ &(iii) \quad s(x,t) \geq 0, \ \ \forall (x,t) \in X \times T. \end{aligned} \tag{22}$$

We can express:

$$AC_M^*(\{a^*(t)\}) = \underbrace{AC_{M,\text{IR}}^*(\{a^*(t)\})}_{\text{Agency cost for insuring (IR) on average } t} + \underbrace{AC_{M,\text{IC}}^*(\{a^*(t)\})}_{\text{Agency cost for insuring (IC) on } \forall t} \tag{23}$$

## Special utility

Under the post-contract ex-post system:

- Fix the induced effort $a^m = \mathbb{E}\left(a^*(t)\right)$ and think of $s^m(x, t)$ that minimizes the agency cost

**Agency cost (ex-post)**:

$$\begin{aligned} AC^o_M(a^m) \equiv \min_{s(x,t)\in S} & \int_t \int_x s(x,t) f(x|a^m) h(t) dxdt \quad \text{s.t.} \\ (i) \quad & \int_t \int_x 2t\sqrt{s(x,t)} f(x|a^m) h(t) dxdt - v(a^m) \geq \overline{U}, \\ (ii) \quad & \int_t \int_x 2t\sqrt{s(x,t)} f_a(x|a^m) h(t) dxdt - v'(a^o) = 0, \\ (iii) \quad & s(x,t) \geq 0, \quad \forall (x,t) \in X \times T. \end{aligned} \tag{24}$$

Turns out that we can express:

$$AC^o_M(\{a^m\}) = \underbrace{AC^o_{M,\text{IR}}(\{a^m\})}_{\text{Agency cost for insuring (IR) on average } t} + \underbrace{AC^o_{M,\text{IC}}(\{a^m\})}_{\text{Agency cost for insuring (IC) on average } t} \tag{25}$$

Note that $(a^m, s^m(x,t))$ might not be optimal: $(a^m, s^m(x,t)) \neq (a^o, s^o_\cdot(x,t))$

# Comparison (agency cost)

**Lemma (Agency cost comparison)**

$$AC^{o}_{M,\mathrm{IR}}(a^m) < AC^{*}_{M,\mathrm{IR}}(\{a^*(t)\}) \text{ and } AC^{o}_{M,\mathrm{IC}}(a^m) < AC^{*}_{M,\mathrm{IC}}(\{a^*(t)\})$$

$v''(a) > 0$

- Harder to satisfy the agent's (PC) on average (across different $t$-realizations)
- Why? The average $\mathbb{E}\left(v(a(t))\right) \uparrow$

$v'''(a) > 0$

- Harder to satisfy the agent's (IC) for each $t$
- Why? The average $\mathbb{E}\left(v'(a(t))\right) \uparrow$