Growth, Heterogeneous Beliefs, and Risk Amplification

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Before each financial crisis:

- Asset price$\uparrow$, capital investment$\uparrow$, and leverage$\uparrow$

- Mechanism: many investors believe asset price$\uparrow$ in the future $\longrightarrow$ leverage$\uparrow$ $\longrightarrow$ risk amounts$\uparrow$ $\longrightarrow$ (big enough) negative shock $\longrightarrow$ crisis

And then everything crashes$\downarrow$: why?

- Net worth of experts (i.e., marginal investors)$\downarrow$ $\longrightarrow$ less productive households take up capital $\longrightarrow$ asset price$\downarrow$

- Market (endogenous) volatility$\uparrow$ and risk-premium$\uparrow$

Then we get out of crises again:

- During crises, risk-premium$\uparrow$ $\longrightarrow$ experts recapitalized $\longrightarrow$ exit

"Boom-bust cycle with endogenous volatility"

Big Question (Main Topic)

What if investors have heterogeneous beliefs about the economy's direction (i.e., underlying data-generating process)?

- How does the belief heterogeneity affect the endogenous market volatility's amplification during crises?
- The severity, duration (of each), and frequency of crises change. How?

Observations:

1. Markets are turbulent $\longrightarrow$ it is more likely that different market participants have different ideas about the financial market's direction
2. Before and during crises:
   - $\exists$Investors betting on the market (who think market will $\uparrow$)
   - $\exists$Investors betting against the market (who think market will $\downarrow$)
   - For example, for 08'-09' on or against the US housing market

Our Framework:

- Experts and households with single capital: experts' output production technology is superior, similar to Brunnermeier and Sannikov (2014)

Introduce (exogenous) technological growth:

- Technologies of both experts and households have the same growth rate in the true data-generating process
- However, experts believe that their technological (expected) growth is higher (lower), i.e., experts are optimistic (pessimistic)
- Households believe that their technological (expected) growth is lower (higher), i.e., households are pessimistic (optimistic)

Big Findings (Adverse 'Doom-Loop')

1. Belief heterogeneity$\uparrow$ $\longrightarrow$ more amplified (endogenous) volatility$\uparrow$
2. Endogenous volatility$\uparrow$ $\longrightarrow$ belief heterogeneity about (capital) returns$\uparrow$ $\longrightarrow$ volatility$\uparrow$ $\longrightarrow$ ad infinitum

Findings

In the presence of heterogeneous beliefs: when experts are more optimistic[1]

During <u>normal</u>:

1. Facilitated trade: investment↑, asset price↑, and leverage↑ than the rational expectations case
2. Risk bearing↑ ⟶ chance of entering financial crises↑

During <u>crisis</u>:

1. Endogenous volatility↑ and (both true and perceived) risk-premium↑: more amplification
2. ∃Adverse 'doom-loop' between belief heterogeneity about asset returns and the amplification of risks
3. Each crisis' duration↓ with experts' faster recapitalization, but:

Number of 'shorter-lived and more severe' crises↑↑
⟶ On average more time in crises per year

[1] The case where experts are pessimistic can be characterized with the opposite results

Basic framework based on Brunnermeier and Sannikov (2014)

- Continuous-time models: Basak and Cuoco (1998), He and Krishnamurthy (2011, 2013), and Di Tella (2017)[2]
- Financial frictions, heterogeneous beliefs, and/or other deviations from the rational expectations case: Harrison and Kreps (1978), Detemple and Murthy (1997), Basak and Croitoru (2000), Basak (2000), Croitoru and Basak (2004), Gallmeyer and Hollifield (2008), and Maxted (2022)[3]
- Market selection hypothesis: Blume and Easley (2006)[4]
- Heterogeneous beliefs about risk-premium, financial markets, and the macroeconomy (e.g., inflation): Welch (2000), Coibion et al. (2020), Candia et al. (2021), and Weber et al. (2022), and Beutel and Weber (2022)[5]
- Nominal rigidity (demand-determined): Caballero and Simsek (2020)

[2] Di Tella (2017) studies uncertainty shocks driving balance sheet recessions even in cases when contracting on the macroeconomic state variable is possible

[3] Maxted (2022) incorporates diagnostic expectations into a model with intermediaries based on He and Krishnamurthy (2013)

[4] Under the market selection hypothesis, markets favor agents with more accurate beliefs: it does not hold in our framework, as markets are incomplete

[5] Beutel and Weber (2022) point out that individuals are heterogeneous both at the information acquisition and the processing stage, thereby forming their own beliefs and choosing portfolios based on them

The Economic Environment

Single capital: owned by optimists and pessimists

Optimists: produces $\underline{y_t^O = \gamma_t^O k_t^O}$, $\forall t \in [0, \infty)$ where

$$\frac{dk_t^O}{k_t^O} = \left(\Lambda^O\Big(\boxed{\iota_t^O} \Big) - \delta^O \right) dt, \quad \forall t \in [0, \infty)$$

Investment ratio
Their investment$= \iota_t^O y_t^O$

with technological growth:

$$\frac{d\gamma_t^O}{\gamma_t^O} = \boxed{\alpha}\, dt + \sigma \underbrace{dZ_t}_{\text{Brownian motion}}, \quad \forall t \in [0, \infty)$$

True (expected) growth

Setting: pessimist

Pessimists: produces $\underline{y_t^P = \gamma_t^P k_t^P}$, $\forall t \in [0, \infty)$ where

$$\frac{dk_t^P}{k_t^P} = \left(\Lambda^P(\boxed{\iota_t^P}) - \delta^P\right) dt, \quad \forall t \in [0, \infty)$$

Investment ratio
Their investment$= \iota_t^P y_t^P$

with the same technological growth:

$$\frac{d\gamma_t^P}{\gamma_t^P} = \boxed{\alpha} dt + \sigma \underbrace{dZ_t}_{\text{Brownian motion}}, \quad \forall t \in [0, \infty)$$

True (expected) growth

$\longrightarrow$ **Level difference**: $\gamma_t^P = l \cdot \gamma_t^O$, $\Lambda^P(\cdot) = l \cdot \Lambda^O(\cdot)$, with $\underline{l \leq 1}$ (efficiency$\downarrow$)

Capital price process: (endogenous) p_t follows

$$\frac{dp_t}{p_t} = \mu_t^p dt + \boxed{\sigma_t^p} dZ_t$$

Endogenous volatility

Capital return process

- Optimists' total return on capital:

$$dr_t^{Ok} = \underbrace{\frac{\gamma_t^O \cancel{k_t^O} - \iota_t^O \gamma_t^O \cancel{k_t^O}}{p_t \cancel{k_t^O}}}_{\text{Dividend yield}} dt + \underbrace{\left(\Lambda^O(\iota_t^O) - \delta^O + \mu_t^p\right) dt + \sigma_t^p dZ_t}_{\text{Capital gain}}$$

$$= \frac{1 - \iota_t^O}{\underbrace{q_t}_{\substack{\text{Price-earnings ratio} \\ \text{(optimists)}}}} dt + \left(\Lambda^O(\iota_t^O) - \delta^O + \mu_t^p\right) dt + \sigma_t^p dZ_t$$

- Pessimists' total return on capital:

$$dr_t^{Pk} = \frac{\gamma_t^P \cancel{k_t^P} - \iota_t^P \gamma_t^P \cancel{k_t^P}}{p_t \cancel{k_t^P}} dt + \left(\Lambda^P(\iota_t^P) - \delta^P + \mu_t^p\right) dt + \sigma_t^p dZ_t$$

Optimists: believe γ_t^O follows

$$\frac{d\gamma_t^O}{\gamma_t^O} = \boxed{\alpha^O} dt + \sigma \underbrace{dZ_t^O}_{\substack{\text{Optimists'} \\ \text{Brownian Motion}}}, \quad \forall t \in [0, \infty)$$

Possibly different from α

Pessimists: believe γ_t^P follows

$$\frac{d\gamma_t^P}{\gamma_t^P} = \alpha^P dt + \sigma \underbrace{dZ_t^P}_{\substack{\text{Pessimists'} \\ \text{Brownian Motion}}}, \quad \forall t \in [0, \infty)$$

with the following consistency:

$$Z_t^O = \underbrace{Z_t}_{\text{True BM}} - \frac{\alpha^O - \alpha}{\sigma} t, \quad Z_t^P = \underbrace{Z_t}_{\text{True BM}} - \frac{\alpha^P - \alpha}{\sigma} t$$

- With $\alpha^O > \alpha > \alpha^P$: experts (households) are optimists (pessimists)
- With $\alpha^O < \alpha < \alpha^P$: experts (households) are pessimists (optimists)

Perceived capital return process

- Optimists' total return on capital:

$$dr_t^{Ok} = \underbrace{\frac{\gamma_t^O \cancel{k_t^O} - \iota_t^O \gamma_t^O \cancel{k_t^O}}{p_t \cancel{k_t^O}}}_{\text{Dividend yield}} dt + \underbrace{\left(\Lambda^O(\iota_t^O) - \delta^O + \mu_t^p\right) dt + \sigma_t^p dZ_t}_{\text{Capital gain}}$$

$$= \frac{\gamma_t^O - \iota_t^O \gamma_t^O}{p_t} dt + \left(\Lambda^O(\iota_t^O) - \delta^O + \mu_t^p + \underbrace{\boxed{\frac{\alpha^O - \alpha}{\sigma}\sigma_t^p}}_{\text{Belief (perceived) premium}}\right) dt + \sigma_t^p dZ_t^O$$

- Pessimists' total return on capital:

$$dr_t^{Pk} = \frac{\gamma_t^P - \iota_t^P \gamma_t^P}{p_t} dt + \left(\Lambda^P(\iota_t^P) - \delta^P + \mu_t^p + \frac{\alpha^P - \alpha}{\sigma}\sigma_t^p\right) dt + \sigma_t^p dZ_t^P$$

Observation (Belief heterogeneity in asset returns)

(Endogenous) volatility$\uparrow$ $\longrightarrow$ belief heterogeneity in asset return$\uparrow$

Financial market: capital and risk-free (zero net-supplied)

Optimists: consumption-portfolio problem (price-taker) Solution

$$\max_{i_t \geq 0, x_t \geq 0, c_t^O \geq 0} \boxed{\mathbb{E}_0^O} \left[\int_0^\infty e^{-\rho^O t} \log\left(c_t^O\right) dt \right]$$

Believes dZ_t^O is the true BM

subject to

$$dw_t^O = x_t w_t^O dr_t^{Ok} + (1 - x_t) r_t w_t^O dt - c_t^O dt, \quad \text{and} \quad \underbrace{w_t^O \geq 0}_{\text{Solvency constraint}}$$

Pessimists: solve the similar problem with $\boxed{\mathbb{E}_0^P}$ $(\neq \mathbb{E}_0 \text{ or } \mathbb{E}_0^O)$

Believes dZ_t^P is the true BM

Total capital $K_t = k_t^O + \underline{k}_t^P$ evolves with

$$\frac{dK_t}{dt} = \underbrace{\left(\Lambda^O\left(\iota_t^i\right) - \delta^O\right) k_t^O}_{\text{From optimists}} + \underbrace{\left(\Lambda^P\left(\underline{\iota}_t^P\right) - \delta^P\right) \underline{k}_t^P}_{\text{From pessimists}}, \quad \forall t \in [0, \infty)$$

Debt is zero net-supplied as

$$\underbrace{\left(w_t^O - p_t k_t^O\right)}_{\substack{\text{Optimist}\\ \text{lending}}} + \underbrace{\left(\underline{w}_t^P - p_t \underline{k}_t^P\right)}_{\substack{\text{Pessimist}\\ \text{lending}}} = 0$$

Good market equilibrium is represented by

$$\underbrace{\frac{x_t^O w_t^O}{p_t}\left(\gamma_t^O - \iota_t^O \gamma_t^O\right)}_{\substack{\text{Optimist}\\ \text{production}\\ \text{net of investment}}} + \underbrace{\frac{\underline{x}_t^P \underline{w}_t^P}{p_t}\left(\gamma_t^P - \underline{\iota}_t^P \gamma_t^P\right)}_{\substack{\text{Pessimist}\\ \text{production}\\ \text{net of investment}}} = c_t^O + \underline{c}_t^P$$

Proportion of optimists' wealth as state variable, similarly to Brunnermeier and Sannikov (2014):

$$\eta_t \equiv \frac{w_t^O}{w_t^O + \underline{w}_t^P} \underbrace{=}_{\substack{\text{Debt market}\\ \text{equilibrium}}} \frac{w_t^O}{p_t K_t}$$

which leads to:

$$x_t \leq \frac{1}{\eta_t}$$

- When it binds - 'normal' (all capital is owned by experts)
- When it does not bind - 'crisis' (less productive households must hold capital)

Under Markov equilibrium: normalized variables depend only on η_t

$$\longrightarrow q_t = q(\eta_t),\ x_t = x(\eta_t),\ \underbrace{\psi_t}_{\substack{\text{Capital share}\\ \text{(optimists)}}} = \psi(\eta_t)$$

Analysis: Markov Equilibrium

Investment function

$$\Lambda^O(i_t^O) = \frac{1}{k}\left(\sqrt{1+2ki} - 1\right), \quad \forall t \in [0, \infty), \quad i \in \{O, P\}$$

with

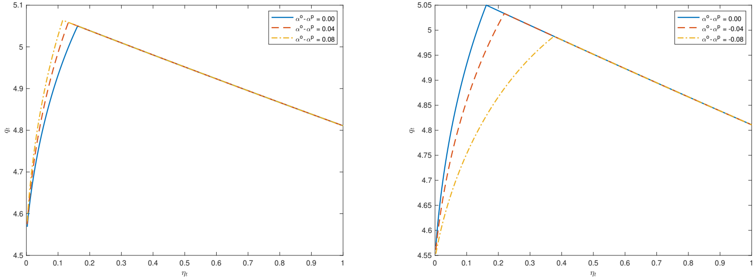
$$\Lambda^P(i_t^O) = l \cdot \Lambda^O(i_t^O) \tag{1}$$

Parametrization:

	l	δ^O	δ^P	ρ^O	ρ^P	χ	σ	k	α
Values	0.6	0	0	0.09	0.05	1	0.1	18	0.05

Table: Parameterization

- $\underline{\alpha^O > \alpha > \alpha^P}$ case (i.e., experts are optimistic):
  $\alpha^O = \{0.05, 0.07, 0.09\}, \quad \alpha^P = \{0.05, 0.03, 0.01\}, \quad \alpha^O + \alpha^P = 0.1$
- $\underline{\alpha^O < \alpha < \alpha^P}$ case (i.e., experts are pessimistic):
  $\alpha^O = \{0.05, 0.03, 0.01\}, \quad \alpha^P = \{0.05, 0.07, 0.09\}, \quad \alpha^O + \alpha^P = 0.1$



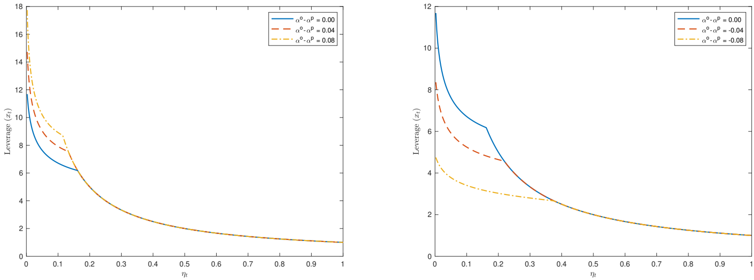
(a) $\alpha^O \geq \alpha$ and $\alpha^P \leq \alpha$

(b) $\alpha^O \leq \alpha$ and $\alpha^P \geq \alpha$

Figure: Price-earnings ratio q_t as a function of η_t

Efficient and crisis regions: threshold η^ψ

- With $\underline{\alpha^O > \alpha > \alpha^P}$, $\eta^\psi\downarrow$ as $\alpha^O\uparrow$: even with low wealth, optimists' demand for capital is strong (so leverage$\uparrow$)
- And then crisis (i.e., $\eta \leq \eta^\psi$) $\longrightarrow$ steeper decline in q_t (i.e., more elastic)

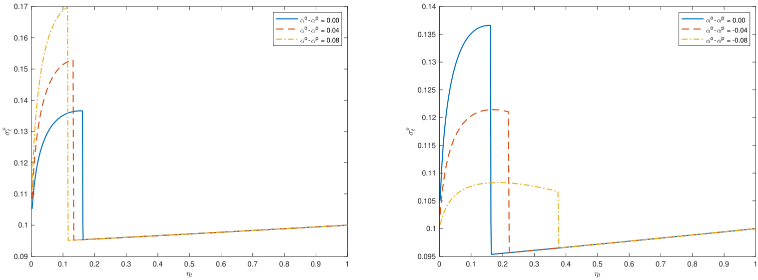


(a) $\alpha^O \geq \alpha$ and $\alpha^P \leq \alpha$ (b) $\alpha^O \leq \alpha$ and $\alpha^P \geq \alpha$

Figure: Leverage x_t as a function of η_t

Efficient and crisis regions: threshold η^ψ

- With $\underline{\alpha^O > \alpha > \alpha^P}$, $\eta^\psi \downarrow$ as $\alpha^O \uparrow$: even with low wealth, optimists' demand for capital is strong (so leverage$\uparrow$)
- And then crisis (i.e., $\eta \leq \eta^\psi$) $\longrightarrow$ higher leverage (a perceived risk-premium is high)



(a) $\alpha^O \geq \alpha$ and $\alpha^P \leq \alpha$

(b) $\alpha^O \leq \alpha$ and $\alpha^P \geq \alpha$

Figure: Endogenous Volatility σ_t^p as a function of η_t

Efficient and crisis regions: threshold η^ψ

- With $\underline{\alpha^O > \alpha > \alpha^P}$, $\eta^\psi\downarrow$ as $\alpha^O\uparrow$: even with low wealth, optimists' demand for capital is strong (so leverage$\uparrow$)
- And then crisis (i.e., $\eta \leq \eta^\psi$) $\longrightarrow$ more risk amplification ($\sigma_t^p\uparrow$) $\longrightarrow$ belief disagreement on asset return$\uparrow$ $\longrightarrow$ amplification $\sigma_t^p\uparrow$ $\longrightarrow$ ad infinitum

Equilibrium endogenous volatility σ_t^p is written as

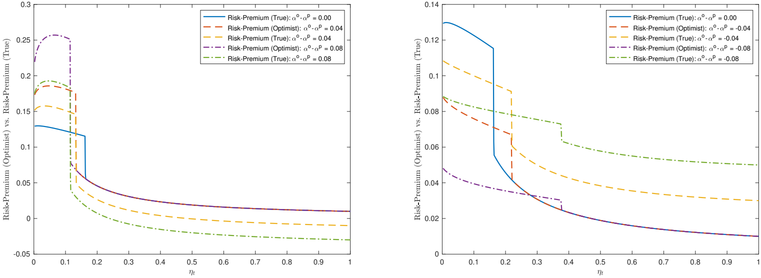
$$\sigma_t^p \left(1 - (x_t - 1) \frac{\frac{dq(\eta_t)}{q(\eta_t)}}{\frac{d\eta_t}{\eta_t}} \right) \equiv \sigma_t^p \left(1 - (x_t - 1)\,\varepsilon_{q,\eta}\right) = \underbrace{\sigma}_{\substack{\text{Exogenous}\\ \text{volatility}}}$$

- $\varepsilon_{q,\eta}$ is the elasticity of the price-earnings ratio (i.e., normalized capital price) with respect to the experts' wealth share η_t

- '*Market illiquidity*' *effect*: as $\alpha^O \uparrow$, % increase in $\eta_t \longrightarrow$ higher % increases in the price of capital in the inefficient region $\longrightarrow \sigma_t^p \uparrow$

- '*Leverage*' *effect*: as $\alpha^O \uparrow$, experts take more leverage (i.e., $x_t \uparrow$) $\longrightarrow$ more fire-sale during crises $\longrightarrow \sigma_t^p \uparrow$

Risk-premium (true and perceived by optimists)

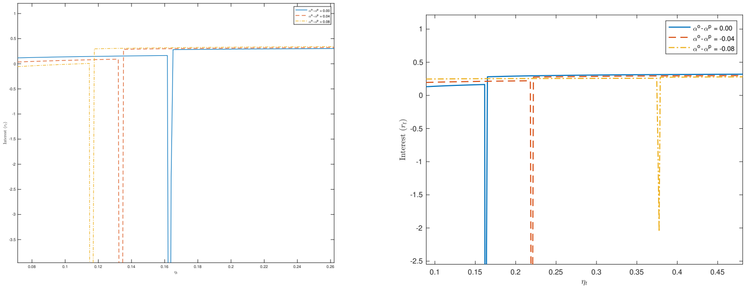


(a) $\alpha^O \geq \alpha$ and $\alpha^P \leq \alpha$ (b) $\alpha^O \leq \alpha$ and $\alpha^P \geq \alpha$

Figure: Risk-Premium (Optimists' and True Value) as a Function of η_t

Efficient and crisis regions: threshold η^ψ

- With $\underline{\alpha^O > \alpha > \alpha^P}$, $\alpha^O\uparrow \longrightarrow$ both true and optimists' perceived risk-premium$\uparrow$
- It helps optimists get recapitalized $\longrightarrow$ the economy gets out of crisis faster
- Each crisis lasts for shorter duration (i.e., shorter-lived)



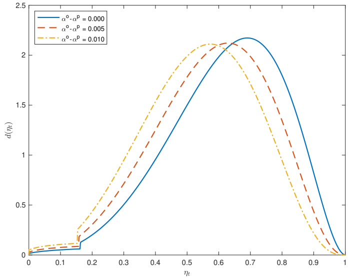
(a) $\alpha^O \geq \alpha$ and $\alpha^P \leq \alpha$ (b) $\alpha^O \geq \alpha$ and $\alpha^P \leq \alpha$

Figure: Interest Rate r_t as a function of η_t: $\alpha^O \leq \alpha$ and $\alpha^P \geq \alpha$

Efficient and crisis regions: threshold η^ψ

- Downward spike in r_t at η^ψ: the moment experts start a fire-sale of capital
- With $\underline{\alpha^O > \alpha > \alpha^P}$, a higher leverage $x_t \longrightarrow r_t\uparrow$ in 'normal'
- During crises (i.e., $\eta_t \leq \eta^\psi$), $\alpha^O\uparrow \longrightarrow r_t\downarrow$: higher demand for safety with precautionary motive

Other graphs



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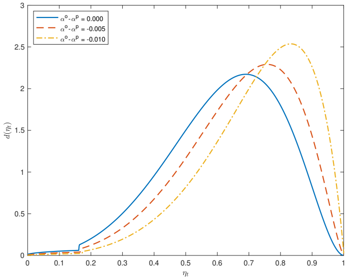


(b) $\alpha^O \leq \alpha$ and $\alpha^P \geq \alpha$

Figure: Ergodic Distribution of η_t

Efficient and crisis regions: threshold η^ψ

- With $\underline{\alpha^O > \alpha > \alpha^P}$, $\alpha^O\uparrow \longrightarrow$ the economy spends more time in crises per year, even if each crisis on average lasts for shorter duration
- Number of 'shorter-lived and more severe' crises$\uparrow\uparrow$: optimistic experts bear too much risk during 'normal'

Thank you very much!

(Appendix)

Optimists' optimal portfolio decision (e.g., Merton (1971))

$$x_t = \frac{\left(\frac{\gamma_t^O - \iota_t^O \gamma_t^O}{p_t} + \Lambda^O(\iota_t^O) - \delta^O + \mu_t^p + \boxed{\frac{\alpha^0 - \alpha}{\sigma}\sigma_t^p} \right) - r_t^*}{(\sigma_t^p)^2}$$

New term: from optimism

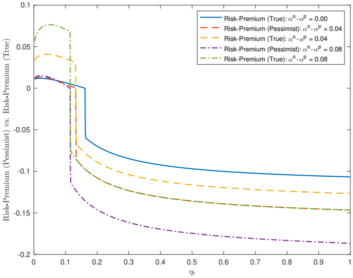
For $\alpha^O > \alpha$ (experts = optimists)

- Given the risk-free r_t^* and the endogenous volatility σ_t^p, optimism (i.e., $\alpha^O\uparrow$ from α) raises the optimists' leverage↑ and capital demand↑
- Optimists bear 'too much' risk on their balance sheets $\longrightarrow$ crisis when dZ_t is negative enough (more frequently)

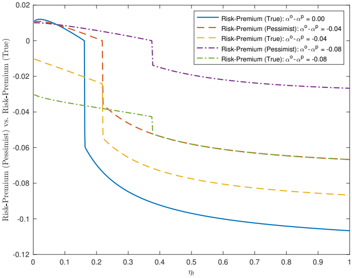
$\sigma_t^p\uparrow \longrightarrow$ has two effects on leverage x_t:

- $\sigma_t^p\uparrow$ lowers x_t as the required risk-premium level↑
- $\sigma_t^p\uparrow$ raises x_t as it raises the degree of optimism on asset returns

Go back



(a) $\alpha^O \geq \alpha$ and $\alpha^P \leq \alpha$



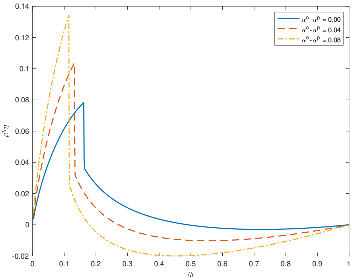
(b) $\alpha^O \leq \alpha$ and $\alpha^P \geq \alpha$

Figure: Risk-Premium (Pessimists' and True Value) as a Function of η_t

- Pessimists perceive to risk-premium to be positive only when $\eta_t \leq \eta^\psi$

Go back

Drift of the wealth share



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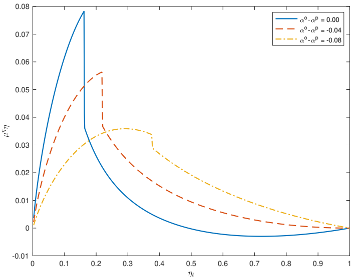
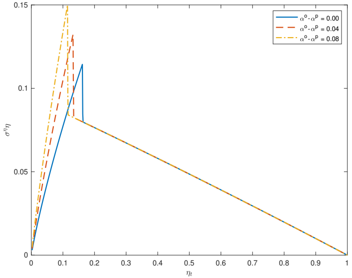


(b) $\alpha^O \leq \alpha$ and $\alpha^P \geq \alpha$

Figure: Wealth Share Drift $\mu_\eta(\eta_t) \cdot \eta_t$ as a Function of η_t

- With $\underline{\alpha^O > \alpha > \alpha^P}$, $\alpha^O\uparrow \longrightarrow$ Wealth share drift $\mu_\eta(\eta_t) \cdot \eta_t\uparrow$: recapitalized faster

Go back



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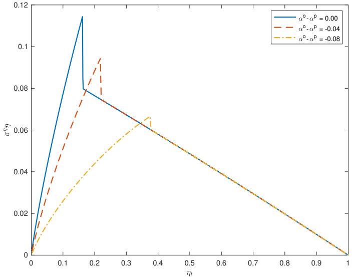

(b) $\alpha^O \leq \alpha$ and $\alpha^P \geq \alpha$

Figure: Wealth Share Volatility $\sigma^\eta(\eta_t) \cdot \eta_t$ as a Function of η_t

Go back