

A Unified Theory of the Term-Structure and Monetary Stabilization

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2024 Annual Meeting SED

June 27, 2024

Example: IS equation with 3 maturities (short-term, 10 years, 30 years)

$$\underbrace{\hat{c}_t}_{\downarrow} = \mathbb{E}_t \left[\hat{c}_{t+1} - \left(\underbrace{\hat{r}_{t+1}^S}_{\uparrow} - \hat{\pi}_{t+1} \right) \right]$$

where

$$\hat{r}_{t+1}^S = \underbrace{i_t}_{\text{Policy rate}} + w_t^{10} \cdot (\hat{r}_{t+1}^{10} - i_t) + w_t^{30} \cdot (\hat{r}_{t+1}^{30} - i_t)$$

Up to a first-order, portfolio demand (w_t^{10}, w_t^{30}) depend on relative returns:

$$\underbrace{w_t^{10}}_{\uparrow} = w^{10} \left(\underbrace{i_t}_{\downarrow}, \underbrace{\hat{r}_{t+1}^{10}}_{\uparrow}, \underbrace{\hat{r}_{t+1}^{30}}_{\downarrow} \right)$$

- Demand elasticity with respect to returns is **finite**: market segmentation
- With $i_t \downarrow$, we have $(w_t^{10} \uparrow, w_t^{30} \uparrow)$, leading to $(\hat{r}_t^{10} \downarrow, \hat{r}_t^{30} \downarrow)$ (i.e., portfolio re-balancing), thereby $\hat{r}_{t+1}^S \downarrow$, but not one-to-one
- Then real effects: $\hat{c}_t \uparrow$

A [quantitative macroeconomic framework](#) that incorporates

- ① The general equilibrium term-structure of interest rates
- ② Multiple asset classes (government bonds and private bond)
- ③ Endogenous portfolio shares among different kinds of assets
- ④ Market segmentation across different maturities bonds (how?: [methodological contribution](#) + [estimation](#))

that makes LSAPs work in theory (a demand curve for each maturity bond slopes down)

Plus:

- ⑤ Government and central bank's explicit balance sheets
- ⑥ A micro-founded welfare criterion

which are necessary for quantitative policy experiments (e.g., [conventional](#) vs. [unconventional](#) monetary policies)

Big Findings (Conventional vs. Unconventional)

Unconventional monetary policy (e.g., yield-curve-control (YCC)) is powerful in terms of stabilization in both normal and ZLB

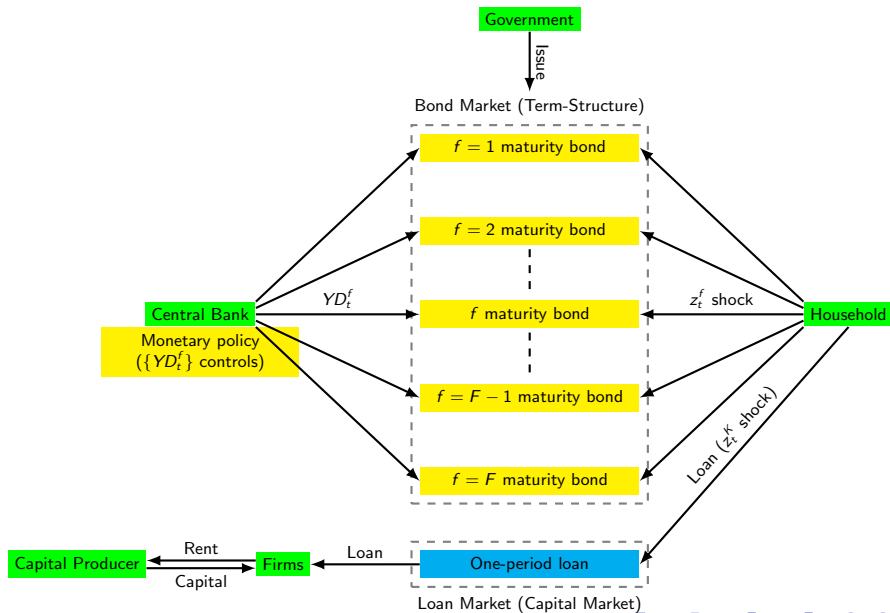
- As a drawback, the economy experiences longer ZLB regimes

Mechanism: long term yields $\downarrow \Rightarrow$ portfolio shares of short term $\uparrow \Rightarrow$ short yields $\downarrow \Rightarrow$ ZLB duration $\uparrow \Rightarrow$ more reliance on LSAPs

'ZLB + LSAPs addicted economy'

» Literature

The model: environment



The representative household's problem (given B_0):

$$\begin{aligned} & \max_{\{C_{t+j}, N_{t+j}\}} \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \left[\log(C_{t+j}) - \left(\frac{\eta}{\eta+1} \right) \left(\frac{N_{t+j}}{\bar{N}_{t+j}} \right)^{1+\frac{1}{\eta}} \right] \\ & \text{subject to} \end{aligned}$$

$$C_t + \frac{L_t}{P_t} + \frac{\sum_{f=1}^F \overset{\text{Loans}}{B_t^{H,f}}}{P_t} = \frac{\sum_{f=0}^{F-1} \overset{f\text{-maturity rate}}{R_t^f} \overset{\text{Loan rate}}{B_{t-1}^{H,f+1}}}{P_t} + \frac{R_t^K L_{t-1}}{P_t} + \int_0^1 \frac{W_t(\nu) N_t(\nu)}{P_t} d\nu + \frac{\Lambda_t}{P_t}$$

Nominal bond purchase
(f -maturity)

where

- ν : intermediate firm index such that:

$$N_t = \left(\int_0^1 N_t(\nu)^{\frac{\eta+1}{\eta}} d\nu \right)^{\frac{\eta}{\eta+1}}$$

- Q_t^f is the nominal price of f -maturity bond with:

$$(\text{Return}) R_t^f = \frac{Q_t^f}{Q_{t-1}^{f+1}}, \quad (\text{Yield}) YD_t^f = \left(\frac{1}{Q_t^f} \right)^{\frac{1}{f}}$$

Total savings: $S_t = B_t^H + L_t = \sum_{f=1}^F B_t^{H,f} + L_t$

Usual bond allocation problem (Ricardian):

$$\max \sum_{f=1}^F \mathbb{E}_t \left[Q_{t,t+1} R_{t+1}^{f-1} B_t^{H,f} \right] \quad \text{s.t.} \quad \sum_{f=1}^F B_t^{H,f} = B_t^H, \quad B_t^{H,f} \geq 0$$

which gives (in equilibrium):

$$\mathbb{E}_t \left[Q_{t,t+1} R_{t+1}^{f-1} \right] = \mathbb{E}_t \left[Q_{t,t+1} R_{t+1}^0 \right], \quad \forall f \implies$$

$$\mathbb{E}_t[\hat{R}_{t+1}^{f-1}] = \hat{R}_{t+1}^0$$

↑
'Expectation hypothesis'

Our approach (Non-Ricardian):

- Split the household into a family $m \in [0, 1]$, each of which decides whether to invest in **bonds** or **loan**, subject to **expectation shock** \sim **Fréchet**
- A **bond** family m is split into members $n \in [0, 1]$, each of whom decides maturity f to invest in, subject to **expectation shock** \sim **Fréchet**

Bond portfolio (e.g., Eaton and Kortum (2002))

$$\lambda_t^{HB,f} \equiv \left(\frac{z_t^f \mathbb{E}_t [Q_{t,t+1} R_{t+1}^{f-1}]}{\Phi_t^B} \right)^{\kappa_B}$$

f -maturity share

- Deviate from expectation hypothesis $\implies \exists$ downward-sloping demand curve after log-linearization with finite demand elasticity
- Shape parameter κ_B : (inverse of) a degree of bonds market segmentation
- $z_t^f = 1$, $\kappa_B \rightarrow \infty$, then $\mathbb{E}_{m,n,t} \rightarrow \mathbb{E}_t$ (i.e., rational expectations)

Effective bond market rates

$$R_{t+1}^{HB} = \sum_{f=0}^{F-1} \lambda_t^{HB,f+1} R_{t+1}^f$$

► Microfoundation (bond)

Loan share (e.g., Eaton and Kortum (2002))

Loan share $\lambda_t^K = \left(\frac{z_t^K \mathbb{E}_t [Q_{t,t+1} R_{t+1}^K]}{\Phi_t^S} \right)^{\kappa_S}$

- \exists downward-sloping demand curve after log-linearization (for loan and bonds)
- Shape parameter κ_S : (inverse of) a degree of market segmentation between government bonds vs loan

Effective savings rate: governs intertemporal substitution

$$\begin{aligned} R_t^S &= (1 - \lambda_{t-1}^K) R_t^{HB} + \lambda_{t-1}^K R_t^K \\ &= (1 - \lambda_{t-1}^K) \sum_{f=0}^{F-1} \lambda_{t-1}^{HB,f+1} R_t^f + \lambda_{t-1}^K R_t^K \end{aligned}$$

Enters Euler equation

► Microfoundation (loan)

Bond market equilibrium:

$$B_t^{H,f} + B_t^{G,f} + B_t^{CB,f} = 0, \quad \forall f = 1, \dots, F$$

Monetary
policy

Central bank: monetary policy through balance sheet adjustments

- **Conventional:** Taylor rules on YD_t^1 (only adjusting $B_t^{CB,1}$)
- **Yield-curve-control (YCC):** Taylor rules on $\{YD_t^f\}$ (adjusting $\{B_t^{CB,f}\}$)
- Subject to **zero lower bound (ZLB)**

» Conventional » Unconventional » Capital Producer, Firms, and Government

Steady-state U.S. calibrated yield curve (up to 30 years)

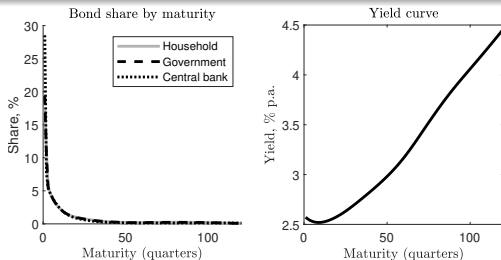


Figure: Steady-state bond portfolios of household, government, and central bank and the resultant yield curve (December 2002 - June 2007)

Estimation: $\kappa_B = 10$ from the aggregate bond portfolio data ▶▶ Estimation

Calibration: given $\kappa_B = 10$ and $\kappa_S = 6$ (from **Kekre and Lenel (2023)**)

- $\{z^f\}_{f=1}^F$ (i.e., maturity preference for a maturity- f) \Rightarrow yield curve slopes
- z^K (i.e., preference for private loan) \Rightarrow the yield curve level
- **Result:** $z^1 = 1 \gg z^f$ for $f \geq 2$ (e.g., **safety - liquidity** premium)

Short-run analysis (Impulse-responses)

A shock to the preference for the short-term bond (impulse response to z_t^1)

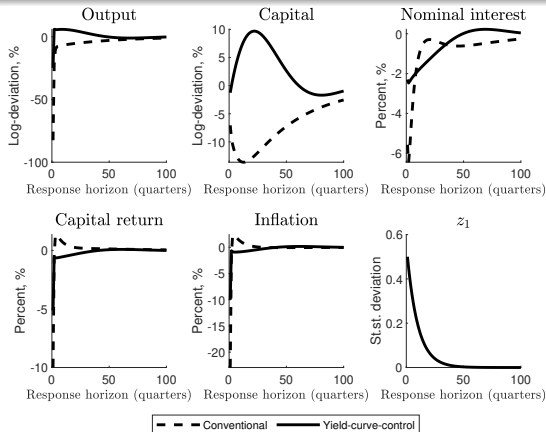


Figure: Impulse response to z_t^1 shock

With **conventional** policy

- Short yields $\downarrow \Rightarrow$ other yields, capital return, and wage $\downarrow \Rightarrow$ output \downarrow (labor supply \downarrow) and inflation \downarrow

With **yield-curve-control** (YCC): stabilizing (filling gaps in bond demand) ▶

ZLB impulse response to z_t^1

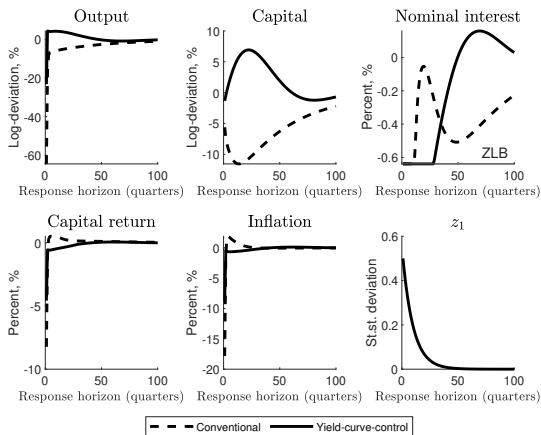


Figure: ZLB impulse response to z_t^1 shock

With **yield-curve-control** (YCC): stabilizing (filling gaps in bond demand)

- But duration of ZLB episodes \uparrow

Long-term rates $\downarrow \Rightarrow$ ZLB duration \uparrow

\gg ZLB IRF (z_t^K)

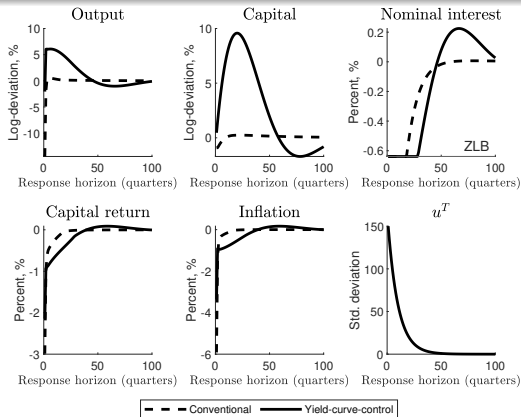


Figure: ZLB impulse response to ϵ_t^T shock

With **conventional** policy: non-Ricardian

- Tax $\uparrow \Rightarrow$ bond supply $\downarrow \Rightarrow$ ZLB \Rightarrow recessions (Caballero and Farhi, 2017)

With **yield-curve-control** (YCC): stabilizing

- But **duration** of ZLB episodes \uparrow

We also consider:

- **Mixed policy**: central bank starts controlling long-term rates only when FFR hits ZLB, thus **YCC** only at the ZLB

	Conventional	Yield-Curve-Control	Mixed Policy
Mean ZLB duration	4.5533 quarters	6.2103 quarters	5.5974 quarters
Median ZLB duration	3 quarters	3 quarters	2 quarters
ZLB frequency	15.9596%	13.4242%	17.4141%
Welfare	-1.393%	-1.2424%	-1.3662%

Table: Policy comparisons (ex-ante)

ZLB duration: Conventional < Mixed < YCC

ZLB frequency: YCC < Conventional < Mixed

Welfare: Conventional < Mixed < YCC

Thank you very much!
(Appendix)

- The term-structure and macroeconomy: [Ang and Piazzesi \(2003\)](#), [Rudebush and Wu \(2008\)](#), [Bekaert et al. \(2010\)](#)
- Central bank's endogenous balance sheet size as an another form of monetary policy: [Gertler and Karadi \(2011\)](#), [Cúrdia and Woodford \(2011\)](#), [Christensen and Krogstrup \(2018, 2019\)](#), [Karadi and Nakov \(2021\)](#), [Sims and Wu \(2021\)](#)
- Zero lower bound (ZLB) and issuance of safe bonds: [Swanson and Williams \(2014\)](#), [Caballero and Farhi \(2017\)](#), [Caballero et al. \(2021\)](#)
- Welfare criterion with a trend inflation: [Coibion et al. \(2012\)](#)
- Preferred-habitat term-structure (and limited risk-bearing): [Greenwood et al. \(2020\)](#), [Vayanos and Vila \(2021\)](#), [Gourinchas et al. \(2021\)](#), [Kekre et al. \(2023\)](#)
- Preferred-habitat term-structure and the real economy in New-Keynesian macroeconomics: [Ray \(2019\)](#), [Droste, Gorodnichenko, and Ray \(2021\)](#)

Our paper: general equilibrium term-structure (without relying on factor models)
+ balance sheet quantities of government and central bank + yield-curve-control
+ novel way to generate and estimate market segmentation

Bond family m : a member n has the following expectation shock:

$$\mathbb{E}_{m,n,t} \left[Q_{t,t+1} R_{t+1}^{f-1} \right] = \mathbf{z}_{n,t}^f \cdot \mathbb{E}_t \left[Q_{t,t+1} R_{t+1}^{f-1} \right], \quad \forall f = 1, \dots, F$$

with $\mathbf{z}_{n,t}^f$ follows a **Fréchet** distribution with location parameter **0**, scale parameter \mathbf{z}_t^f , and shape parameter κ_B

Aggregation (**Eaton and Kortum (2002)**)

$$\begin{aligned} \lambda_t^{HB,f} &\equiv \mathbb{P} \left(\mathbb{E}_{m,n,t} \left[Q_{t,t+1} R_{t+1}^{f-1} \right] = \max_j \left\{ \mathbb{E}_{m,n,t} \left[Q_{t,t+1} R_{t+1}^{j-1} \right] \right\} \right) \\ &= \left(\frac{\mathbf{z}_t^f \mathbb{E}_t \left[Q_{t,t+1} R_{t+1}^{f-1} \right]}{\Phi_t^B} \right)^{\kappa_B} \end{aligned}$$

f -maturity share

- Deviate from expectation hypothesis $\implies \exists$ **downward-sloping demand curve** after log-linearization with finite demand elasticity
- Shape parameter κ_B : (inverse of) a degree of bonds market segmentation

Effective bond market rates

$$R_{t+1}^{HB} = \sum_{f=0}^{F-1} \lambda_t^{HB,f+1} R_{t+1}^f$$

Loan vs. bond decision: a family m solves the following problem

$$\max \mathbb{E}_t \left[Q_{t,t+1} R_{t+1}^{HB} B_{m,t}^H \right] + z_{m,t}^K \cdot \mathbb{E}_t \left[Q_{t,t+1} R_{t+1}^K L_{m,t} \right] \quad \text{s.t.} \\ B_{m,t}^H + L_{m,t} = S_t, \quad B_{m,t}^H \geq 0, \quad \text{and} \quad L_{m,t} \geq 0$$

with $z_{m,t}^K$ follows a Fréchet distribution with location parameter 0, scale parameter z_t^K , and shape parameter κ_S

Aggregation (Eaton and Kortum (2002))

Loan share $\lambda_t^K = \left(\frac{z_t^K \mathbb{E}_t [Q_{t,t+1} R_{t+1}^K]}{\Phi_t^S} \right)^{\kappa_S}$

- \exists downward-sloping demand curve after log-linearization (for loan and bonds)
- Shape parameter κ_S : (inverse of) a degree of market segmentation between government bonds vs loan

Effective savings rate: governs intertemporal substitution

$$R_t^S = \left(1 - \lambda_{t-1}^K \right) R_t^{HB} + \lambda_{t-1}^K R_t^K \\ = \left(1 - \lambda_{t-1}^K \right) \sum_{f=0}^{F-1} \lambda_{t-1}^{HB,f+1} R_t^f + \lambda_{t-1}^K R_t^K$$

Conventional monetary policy

Under the **conventional** monetary policy, central banks set Taylor rules on YD_t^1 (i.e., the shortest yield) while not manipulating longer term bonds holdings

- Long-term yields fluctuate endogenously (in response to shocks + changes in short-term rate)

$$R_{t+1}^0 \equiv YD_t^1 = \max \left\{ YD_t^{1*}, \underset{\text{ZLB}}{1} \right\}$$

$$YD_t^{1*} = \overline{YD}^1 \left(\frac{YD_{t-1}^{1*}}{\overline{YD}^1} \right)^{\rho_1} \left(\frac{YD_{t-2}^{1*}}{\overline{YD}^1} \right)^{\rho_2} \underbrace{\left(\left(\frac{\Pi_t}{\bar{\Pi}} \right)^{\gamma_\pi^1} \left(\frac{Y_t}{\bar{Y}} \right)^{\gamma_y^1} \right)}_{\text{Targeting}} \cdot \exp \left(\underset{\text{MP shock } (f=1)}{\tilde{\epsilon}_t^{YD^1}} \right)^{1-(\rho_1+\rho_2)}$$

$$\underbrace{\frac{B_t^{CB,f}}{A_t \bar{N}_t P_t}}_{\text{Normalized holding of } f > 1 \text{ fixed}} = \frac{\overline{B^{CB,f}}}{\overline{A \bar{N} P}} \quad \forall f = 2, \dots, F$$

Normalized holding of $f > 1$ fixed

Unconventional monetary policy: yield-curve-control (YCC)

In the **unconventional** monetary policy case, central bank targets all yields along the yield curve, assuming the Taylor-type rule for each maturity yield

- Back out the needed purchases of each maturity $\forall f$, which are endogenous

$$R_{t+1}^0 \equiv YD_t^1 = \max \left\{ YD_t^{1*}, \underset{\substack{\uparrow \\ \text{ZLB}}}{1} \right\}$$

$$YD_t^{1*} = \overline{YD}^1 \left(\frac{YD_{t-1}^{1*}}{\overline{YD}^1} \right)^{\rho_1} \left(\frac{YD_{t-2}^{1*}}{\overline{YD}^1} \right)^{\rho_2} \underbrace{\left(\left(\frac{\Pi_t}{\overline{\Pi}} \right)^{\gamma_\pi^1} \left(\frac{Y_t}{\overline{Y}} \right)^{\gamma_y^1} \right)}_{\text{Targeting}} \cdot \exp \left(\tilde{\varepsilon}_t^{YD^1} \right)^{1-(\rho_1+\rho_2)}$$

MP shock ($f = 1$)

$$YD_t^{f*} = \overline{YD}^f \left(\frac{YD_{t-1}^{f*}}{\overline{YD}^f} \right)^{\rho_1} \left(\frac{YD_{t-2}^{f*}}{\overline{YD}^f} \right)^{\rho_2} \underbrace{\left(\left(\frac{\Pi_t}{\overline{\Pi}} \right)^{\gamma_\pi^f} \left(\frac{Y_t}{\overline{Y}} \right)^{\gamma_y^f} \right)}_{\text{Targeting}} \cdot \exp \left(\tilde{\varepsilon}_t^{YD^f} \right)^{1-(\rho_1+\rho_2)}$$

MP shock ($\forall f \geq 2$)

Firms: standard with Cobb-Douglas production (pricing à la Calvo (1983))

- $$\underbrace{L_t(\nu)}_{\text{Loan of firm } \nu} \geq \gamma(1 + \zeta^F)P_t(\nu)Y_t(\nu), \forall \nu$$

$$\frac{B_t^G}{P_t} = \frac{R_t^G B_{t-1}^G}{P_t} - \left[\underbrace{\zeta_t^G}_{\substack{\uparrow \\ \frac{G_t}{Y_t} \text{ (Exogenous)}}} + \underbrace{\zeta^F}_{\text{Production subsidy}} - \underbrace{\zeta_t^T}_{\substack{\uparrow \\ \frac{T_t}{Y_t} \text{ (Exogenous)}}} \right] Y_t, \quad R_t^G = \sum_{f=0}^{F-1} \underbrace{\lambda_{t-1}^{G,f+1}}_{\substack{\uparrow \\ \text{(Exogenous)}}} R_t^f$$

- A set of small navigation icons typically found in Beamer presentations, including symbols for back, forward, search, and other slide controls.

From portfolio equations:

$\lambda_t^{HB,f}$ $\equiv \mathbb{P} \left(\mathbb{E}_{m,n,t} [Q_{t,t+1} R_{t+1}^{f-1}] = \max_j \left\{ \mathbb{E}_{m,n,t} [Q_{t,t+1} R_{t+1}^{j-1}] \right\} \right)$

$= \left(\frac{z_t^f \mathbb{E}_t [Q_{t,t+1} R_{t+1}^{f-1}]}{\Phi_t^B} \right)^{\kappa_B}$

f -maturity share leading to:

$$\log \left(\lambda_t^{H,f} \right) - \log \left(\lambda_t^{H,l} \right) = \alpha^f + \kappa_B \cdot E_t \left[r_{t+1}^{f-1} - r_{t+1}^{l-1} \right] + \varepsilon_t^f \quad (1)$$

Jordà local projection:

$$\log \left(\lambda_{t+h}^{H,f} \right) - \log \left(\lambda_{t+h}^{H,l} \right) = \alpha_h^f + \kappa_{B,h} \cdot \left[yd_t^f - yd_t^l \right] + \mathbf{x}_t' \beta_h^f + \varepsilon_{t+h}^f, \quad h \geq 0,$$

- Long maturity: $f = 5 \sim 10$ years and short: $l = 15 \sim 90$ days (bunching) for portfolio shares and use $f = 7$ years and $l = 1$ month for yields
- Instrument $yd_t^f - yd_t^l$ with $yd_{t-1}^f - yd_{t-1}^l$ (\perp demand shocks, e.g., z_t^f , z_t^l)
- Control variables (e.g., lagged $\log \left(\lambda_{t-1}^{H,f} \right) - \log \left(\lambda_{t-1}^{H,l} \right)$ for serial correlation)

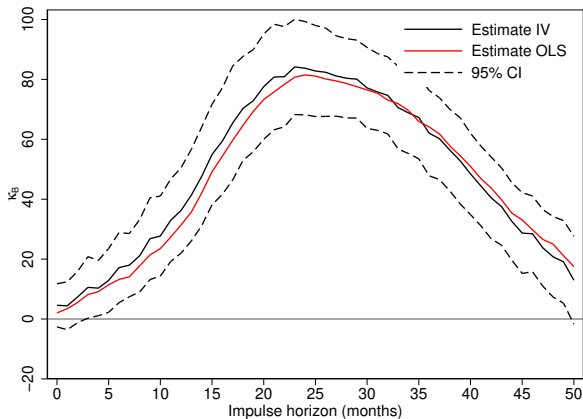


Figure: Impulse-Response to a shock in the yield spread, $yd_t^f - yd_t^l$. The figure presents the coefficient estimates for the bond portfolio elasticity, κ_B . The solid black line illustrates the estimate from the instrumental variables (IV) regression, with dashed lines indicating the 95% robust confidence intervals. The red line exhibits alternative OLS estimates. The sample period is from 2003m3 to 2019m3.

►► Go back

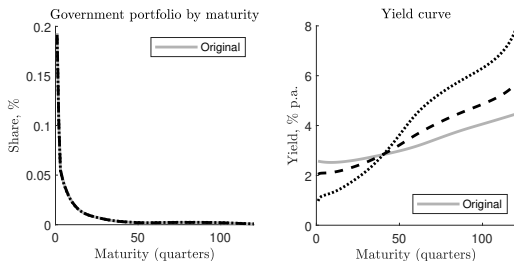


Figure: Government's bond issuance portfolio and yield curve

- Government's supply of f -maturity bond $\uparrow \Rightarrow$ its yield \uparrow (i.e., price effect)
- Similar to Krishnamurthy and Vissing-Jorgensen (2012) and Greenwood and Vayanos (2014) in the long run

Go back

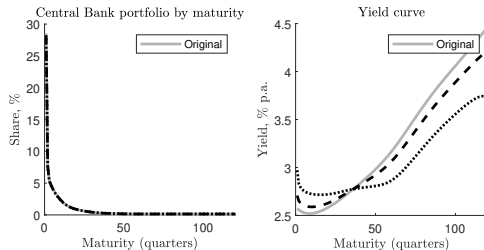


Figure: Central bank's bond demand portfolio and yield curve

- **Segmented** markets \Rightarrow QE matters in the long run

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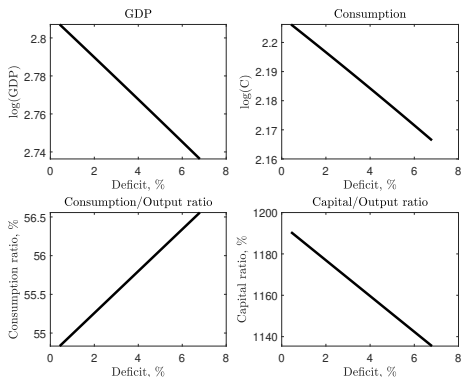


Figure: Variations in a deficit ratio $\zeta_t^G + \zeta_t^F - \zeta_t^T$

A higher deficit ratio \Rightarrow depressed economy (for $R^G \downarrow$)

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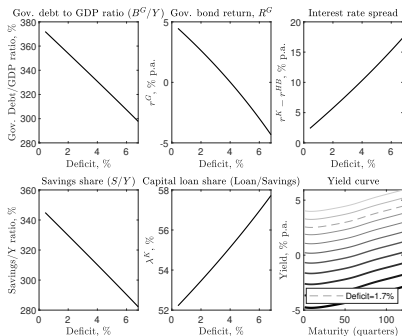


Figure: Variations in a deficit ratio $\zeta_t^G + \zeta^F - \zeta_t^T$

A higher deficit ratio \Rightarrow depressed economy (for $R^G \downarrow$)

- An entire yield curve \downarrow

ZLB impulse response to z_t^K

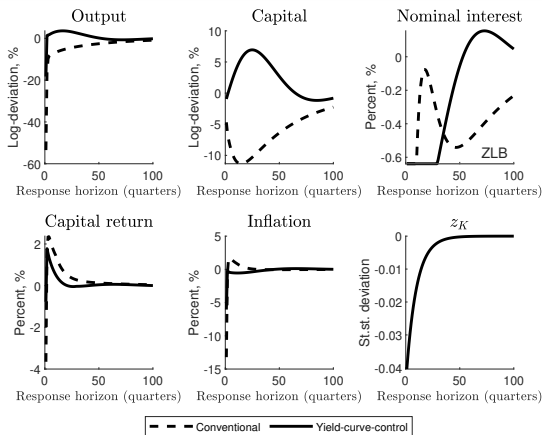


Figure: ZLB impulse response to z_t^K shock

With **yield-curve-control** (YCC): stabilizing (filling gaps in bond demand)

- But duration of ZLB episodes \uparrow

Long-term rates $\downarrow \Rightarrow$ ZLB duration \uparrow

Go back

Impulse-response to an exogenous tax hike shock

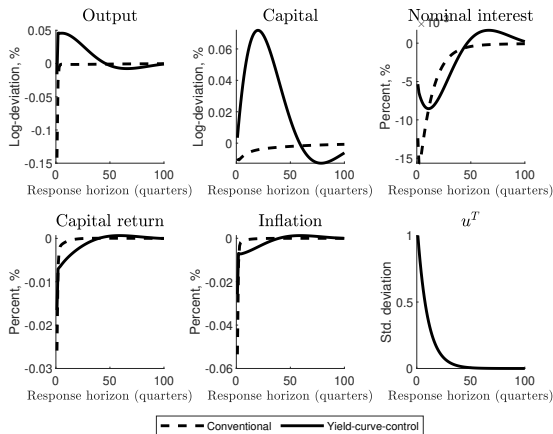


Figure: Impulse response to ϵ_t^T shock

Tax $\uparrow \Rightarrow$ bond supply $\downarrow \Rightarrow$ yields \downarrow , loan rates \downarrow , and wages \downarrow (i.e., real effects)

- The **yield-curve-control** (YCC): stabilizing