A Higher-Order Forward Guidance

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Model 1 (standard New-Keynesian with rigid price): with

$$\hat{Y}_t = \ln\frac{Y_t}{Y_t^n}, \quad \underbrace{(\underset{\text{Exogenous}}{\sigma})^2 dt = \mathrm{Var}_t\left(\frac{dY_t^n}{Y_t^n}\right)}_{\text{Benchmark volatility}}, \quad \underbrace{(\sigma + \underset{\text{Endogenous}}{\sigma_t^s})^2 dt = \mathrm{Var}_t\left(\frac{dY_t}{Y_t}\right)}_{\text{Actual volatility}}$$

A non-linear IS equation (in contrast to textbook linearized one)

$$d\hat{Y}_t = \left(i_t - \underbrace{\left(r^n \overbrace{-\frac{1}{2}(\sigma + \sigma_t^s)^2 + \frac{1}{2}\sigma^2}^{\text{New terms}}\right)}_{\equiv r_t^T}\right) dt + \sigma_t^s \, dZ_t \tag{1}$$

Monetary policy: Taylor rule

$$i_t = r^n + \phi_y \hat{Y}_t \quad \text{where} \quad \phi_\pi > 0 \text{ (Taylor principle)}$$

allows self-fulfilling stock price volatility σ_0^s

Model 2 (model with stock markets and portfolio decisions) : asset (stock) price gap $\hat{Q}_t$ follows

Fundamental volatility

$$d\hat{Q}_t = \left(i_t - \pi_t - \underbrace{\left(r^n - \frac{1}{2}\underbrace{\boxed{(\sigma + \sigma_t^q)^2}}_{\mathrm{rp}_t} + \frac{1}{2}\underbrace{\boxed{\sigma^2}}_{\mathrm{rp}^n} \right)}_{\equiv r_t^T} \right) dt + \sigma_t^q \, dZ_t$$

Here

$$\sigma_t^q \uparrow \Longrightarrow \mathrm{rp}_t \uparrow \Longrightarrow \hat{Q}_t \downarrow \Longrightarrow \hat{Y}_t \downarrow$$

Monetary policy: Taylor rule to Bernanke and Gertler (2000) rule

$$i_t = r^n + \phi_\pi \pi_t + \phi_y \underbrace{\hat{y}_t}_{=\zeta \hat{Q}_t}$$

$$= r^n + \phi_\pi \pi_t + \phi_q \hat{Q}_t, \quad \text{where} \quad \underbrace{\phi \equiv \phi_q + \frac{\kappa(\phi_\pi - 1)}{\rho} > 0}_{\text{Taylor principle}}$$

allows self-fulfilling stock price volatility σ_0^q

Thought experiment: fudamental volatility $\sigma\uparrow$: $\underline{\sigma}$ to $\bar{\sigma}$ on $[0, T]$ (e.g., Werning (2012)) and comes back to $\underline{\sigma}$ with

- $r_1^n \equiv \rho + g - \underline{\sigma}^2 > 0$: no ZLB before
- $r_2^n \equiv \rho + g - \bar{\sigma}^2 < 0$: now ZLB binds (on the stabilized equilibrium path)

Assume: perfect stabilization is achievable outside ZLB

- Central bank always can use risk-premium targeting as given by

$$i_t = r_1^n + \phi_\pi \pi_t + \phi_q \hat{Q}_t - \frac{1}{2} \hat{rp}_t$$

with

$$\phi \equiv \phi_q + \frac{\kappa(\phi_\pi - 1)}{\rho} > 0$$

1. ZLB path (full stabilization after T)

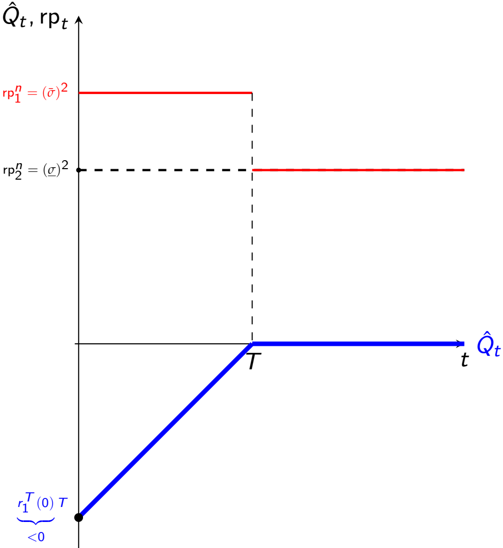


Figure: ZLB dynamics (Benchmark)

2. Traditional forward guidance (keep $\underline{i_t = 0}$ until $\hat{T} > T$)

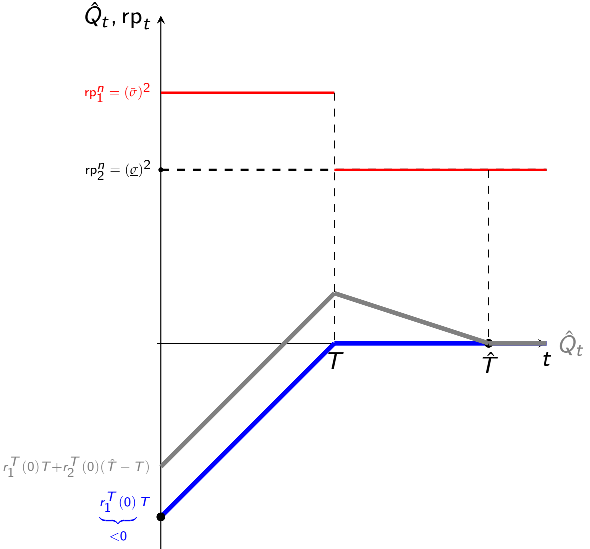


Figure: ZLB dynamics with forward guidance until $\hat{T} > T$

Higher-order intertemporal stabilization trade-off with commitment

Recall an economic mechanism in the ZLB and forward guidance

$\boxed{\text{1. Central bank achieves perfect stabilization: } \hat{Q}_t = \hat{rp}_t = 0, \forall t \geq \hat{T}}$

$\downarrow$

$\boxed{\text{2. } \hat{Q}_{\hat{T}} = 0 \text{ guarantees } \sigma_t^q = \sigma^{q,n} = 0,\ \mathrm{rp}_t = \mathrm{rp}^n \text{ for } t \leq \hat{T}}$

Still if rp^n is too high, might want to push $\{\sigma_t^q, \mathrm{rp}_t\}$ down for $\hat{Q}_t \uparrow$?

- Thus achieve $\sigma_t^q < \sigma^{q,n} = 0$, $\mathrm{rp}_t < \mathrm{rp}^n \implies \hat{Q}_t \uparrow$ at the ZLB

Take contrapositive to the above:

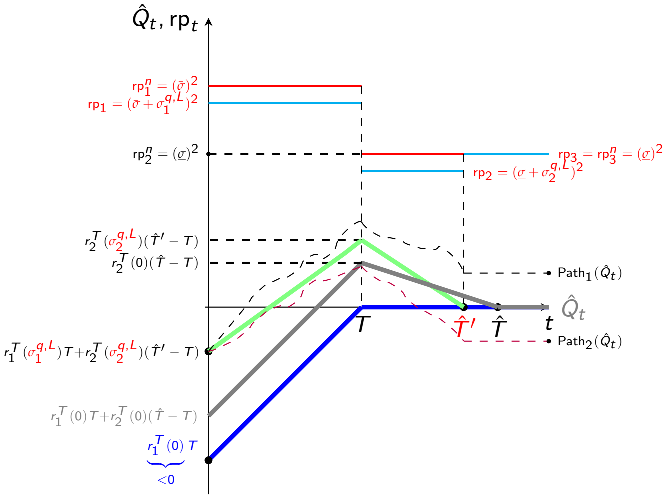
$\boxed{\neg\text{2. } \sigma_t^q < \sigma^{q,n} = 0,\ \mathrm{rp}_t < \mathrm{rp}^n \text{ for } t \leq \hat{T}}$

$\downarrow$

$\boxed{\neg\text{1. } \hat{Q}_{\hat{T}} \neq 0. \text{ Central bank commits not to perfectly stabilize the economy after } \hat{T}}$

- Giving up future financial stability $\implies \mathrm{rp}_t \downarrow$ and $\hat{Q}_t \uparrow$ now (at the ZLB)

3. Central bank picks $\{\sigma_t^q\}$ and $\{\mathrm{rp}_t\}$



Proposition (Optimal commitment path)

At optimum, $\sigma_1^{q,L} < \sigma_1^{q,n}$, $\sigma_2^{q,L} < \sigma_2^{q,n}$, and $\hat{T}' < \hat{T}$

Thank you very much!