

# Higher-Order Forward Guidance

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# Motivation

## Big Question

**Forward guidance** — How does it work, exactly?

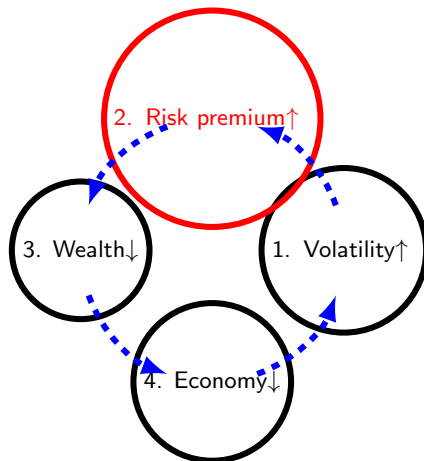
- First-order effects (level): “Interest rates will stay low” → **intertemporal substitution channel** (aggregate demand↑): e.g., Eggertsson et al. (2003), McKay et al. (2016)
- Second-order effects (volatility): reduce uncertainty, avoid worst-case scenarios, “whatever it takes” → **precautionary savings channel** (aggregate demand↑)

**This paper:** focus on central bank's strategic uncertainty management and coordination. Possible for central banks to pick an equilibrium where:

- During the ZLB (**now**): reduce aggregate volatility (and risk premium). Then aggregate demand↑
- But central banks **now** create uncertainty about where the economy ends up after the ZLB (**future**): commit less stabilization after the ZLB
- Welfare-enhancing overall

### Non-linear Two-Agent New Keynesian (TANK) model with nominal rigidities

- With an aggregate stock market + (standard) portfolio choice problem



## Output and asset price gaps

**A non-linear IS equation** (in contrast to textbook linearized one)

Fundamental volatility

$$d\hat{Q}_t = \left( i_t - \left( r^n - \frac{1}{2} \underbrace{(\sigma + \sigma_t^q)^2}_{\text{rp}_t \equiv r_t^T} + \frac{1}{2} \underbrace{\sigma^2}_{\text{rp}^n} \right) \right) dt + \sigma_t^q dZ_t$$

$$= (i_t - r_t^T) dt + \sigma_t^q dZ_t$$

$$\sigma_t^q \uparrow \longrightarrow \text{rp}_t \uparrow \longrightarrow \hat{Q}_t \downarrow \longrightarrow \hat{Y}_t \downarrow$$

What is  $r_t^T$ ?: a **risk-adjusted** natural rate of interest ( $\sigma_t^q \uparrow \longrightarrow r_t^T \downarrow$ )

$$r_t^T \equiv r^n - \frac{1}{2} \hat{r}p_t, \quad \hat{r}p_t = \underbrace{\text{rp}_t - \text{rp}_t^n}_{\text{risk-premium gap}}$$

## Monetary policy outside the ZLB

**Outside the ZLB:** can we stabilize the business cycle? Can we prevent the volatility feedback loop?

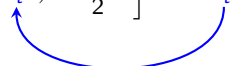
- **Yes:** Lee and Dordal i Carreras (2024, Job Market Paper)

- Under a risk-premium targeting rule:

$$i_t = r_t^T + \phi_q \hat{Q}_t$$

With  $\phi_q > 0$  (i.e., Taylor principle)  $\rightarrow \hat{Q}_t = 0$  for  $\forall t$  (unique equilibrium)

**At the ZLB,** the volatility feedback loop reappears:

$$\begin{aligned} d\hat{Q}_t &= -r_t^T dt + \sigma_t^q dZ_t \\ &= -\left[ r^n - \frac{1}{2}(\sigma + \sigma_t^q)^2 + \frac{1}{2}\sigma^2 \right] dt + \sigma_t^q dZ_t \end{aligned}$$


## ZLB from fundamental volatility shock

**Thought experiment:** fundamental volatility  $\sigma \uparrow$ :  $\bar{\sigma}$  on  $[0, T]$  (e.g., [Werning \(2012\)](#)) and comes back to  $\underline{\sigma}$  with  $\bar{\sigma} > \underline{\sigma}$

- $\bar{r} \equiv r^n(\underline{\sigma}) = \rho + g - \underline{\sigma}^2 > 0$ : no ZLB before,  $t < 0$ , or after,  $t > T$
- $\underline{r} \equiv r^n(\bar{\sigma}) = \rho + g - \bar{\sigma}^2 < 0$ : ZLB binds for  $0 \leq t \leq T$

**Assume:** perfect stabilization (i.e.,  $\hat{Q}_t = 0$ ) is achievable outside ZLB, i.e.,

$$i_t = \bar{r} - \frac{1}{2} \hat{r} p_t + \phi_q \hat{Q}_t, \quad \text{with } \phi_q > 0$$

**Result:** perfect stabilization of risk-premia gap (i.e., excess uncertainty) inside the ZLB

- Recursive argument: full stabilization at  $T$  implies  $\hat{Q}_T = 0 \rightarrow \sigma_{T-dt}^q = 0$ , and so on (so  $\hat{r} p_t = 0$  for  $\forall t$ )

## ZLB path (full stabilization after $T$ )

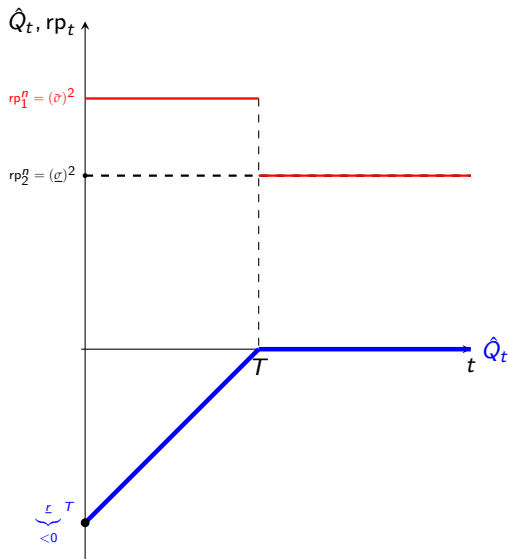


Figure: ZLB dynamics (Benchmark)

Traditional forward guidance (keep  $i_t = 0$  until  $\hat{T}^{\text{TFG}} > T$ ) [» Details](#)

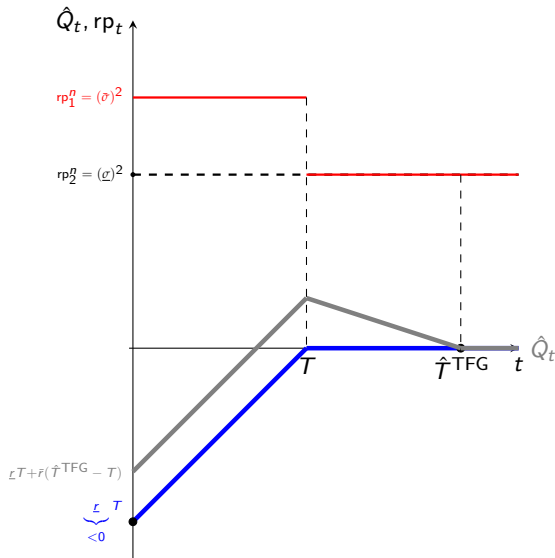


Figure: ZLB dynamics with forward guidance until  $\hat{T}^{\text{TFG}} > T$



## Alternative forward guidance policies

### Big Question

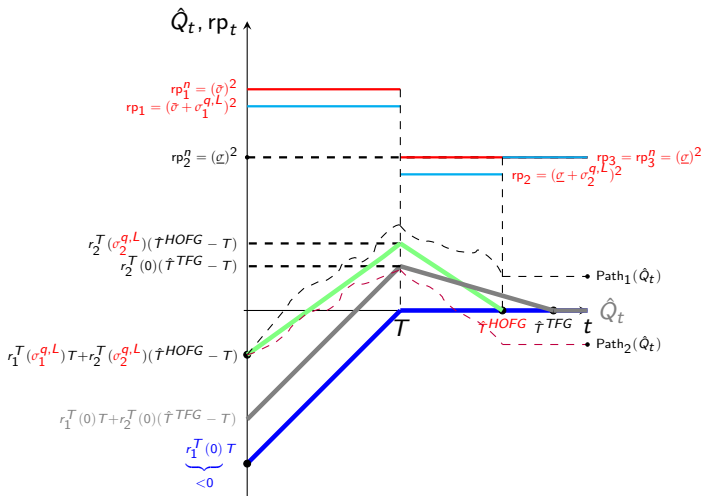
Can we do even better than the traditional forward guidance?

What if we reduce aggregate uncertainty via  $\sigma_t^q < 0$ ?

- Then  $rp_t = (\bar{\sigma} + \sigma_t^q)^2 < rp_t^n$ , raising stock prices and aggregate demand

But how?

- Nominal rigidities  $\rightarrow$  demand-determined production (and hence, wealth)
- Policy challenge: the central bank *must convince* households to “coordinate” on this particular equilibrium  $\rightarrow$  *higher-order forward guidance*
- Give up perfect stabilization in the future (no stabilization at all)
- Imagine the central bank pegs the policy rate at  $i_t = \bar{r}$  after zero rate periods

[▶▶ Details](#)

### Proposition (Optimal commitment path)

At optimum,  $\sigma_1^{q,L} < 0 = \sigma_1^{q,n}$ ,  $\sigma_2^{q,L} < 0 = \sigma_2^{q,n}$ , and  $\hat{T}^{HOFG} < \hat{T}^{TFG}$

# Optimal policy

## Proposition (Optimal forward guidance policy)

Optimal higher-order forward guidance (HOFG) always results in an equal or lower expected quadratic loss than the traditional guidance policy

## Proof

With  $(\sigma_1^{q,L}, \sigma_2^{q,L}, \hat{T}^{\text{HOFG}}) = (0, 0, \hat{T}^{\text{TFG}})$ , solutions coincide

## Remarks:

- Alternative higher-order forward guidance policy implementations **are** possible
- This paper shows **HOFG** dominates **TFG** in a simple setting

**Extension:** still higher-order forward guidance policy, now with stochastic stabilization after  $\hat{T}^{\text{HOFG}}$ . Return to stabilization with  $\nu dt$  probability after  $\hat{T}^{\text{HOFG}}$

- Central bank commits to stabilizing the economy after  $\hat{T}^{\text{HOFG}}$  with some probability. Expected stabilization after  $1/\nu$  quarters
- $\nu = 0$ : the above higher-order forward guidance
- $\nu = \infty$ : the traditional forward guidance policy

**Big discontinuity:**

$$\lim_{\nu \rightarrow +\infty^-} \mathbb{L}^{Q,*}(\{\hat{Q}_t\}_{t \geq 0}, \nu) < \underbrace{\mathbb{L}^{Q,*}(\{\hat{Q}_t\}_{t \geq 0}, \nu = \infty)}_{\text{Traditional forward guidance}}$$

- Slight probability that stabilization might not happen  $\rightarrow$  **HOFG** possible

## Policy implication

### Real World Example (Covid-19 and the Federal Reserve)

#### Flexible Average Inflation Targeting (FAIT) (2020)

- Commitment to delaying stabilization – by allowing inflation to “moderately” overshoot its target after periods of persistent undershooting at the ZLB
- “Moderate” overshooting of the business cycle now is allowed: nudging agents toward a favorable equilibrium with lower volatility

**HOFG** equilibrium → supported by fiscal policy as a unique equilibrium [» Details](#)

- Zero transfer along the equilibrium path (out-of-equilibrium threat)
- Draghi’s “whatever it takes” speech → lower periphery yields without actual expenditures, coordinating agents to an equilibrium with lower risk premium (Acharya et al., 2019)

## Welfare comparisons

$T = 20$  quarters ZLB spell

Loss function  $\mathbb{L}$  as the (conditional) quadratic output loss per quarter:

$$\mathbb{L}_{\text{Per-period}}^Y \equiv \rho \int_0^\infty e^{-\rho t} \mathbb{E}_0 \left( \hat{Y}_t^2 \right) \approx \zeta^2 \cdot \rho \int_0^\infty e^{-\rho t} \frac{1}{s} \sum_{i=1}^s \left( \hat{Q}_t^{(i)} \right)^2 dt$$

| Policy                             | No guidance | Traditional | Higher-Order<br>(no stochastic stabilization) | Higher-Order<br>(with stoch. stab., $\nu = 1$ ) |
|------------------------------------|-------------|-------------|---|---|
| $\sigma_1^{q,L}$                   | 0           | 0           | -1.27%  | -4.13%  |
| $\sigma_2^{q,L}$                   | 0           | 0           | -0.24%  | -3.79%  |
| $\hat{T}$                          | 20          | 25.27       | 25.09   | 24.68   |
| $\mathbb{L}_{\text{Per-period}}^Y$ | 7%          | 1.93%       | 1.81%   | 1.69%   |

- Still, traditional forward guidance too strong: e.g., [McKay et al. \(2016\)](#)
- **HOFG** with  $\nu \rightarrow \infty$  but  $\nu \neq \infty$  most effective

Thank you very much!  
(Appendix)

Identical **capitalists** and **hand-to-mouth workers** (two types of agents)

- **Capitalists:** consumption - portfolio decision (between stock and bond)
- **Workers:** supply labor to firms (hand-to-mouth)

## 1. Technology

$$\frac{dA_t}{A_t} = \underbrace{g}_{\text{Growth}} \cdot dt + \underbrace{\sigma}_{\text{Aggregate shock}} \cdot \underbrace{dZ_t}_{\text{Fundamental risk (Exogenous)}}$$

## 2. Hand-to-mouth workers: solves the following problem:

$$\max_{C_t^w, N_t^w} \frac{\left(\frac{C_t^w}{A_t}\right)^{1-\varphi}}{1-\varphi} - \frac{(N_t^w)^{1+\chi_0}}{1+\chi_0} \quad \text{s.t.} \quad \bar{p}C_t^w = w_t N_t^w$$

- Hand-to-mouth assumption can be relaxed, without changing implications

## 3. Firms: Dixit-Stiglitz production using labor + perfectly rigid prices ( $\pi_t = 0$ )

## 4. Financial market: zero net-supplied risk-free bond + stock (index) market




**Capitalists:** standard portfolio and consumption decisions (very simple)

1. Stock market valuation =  $\bar{p}A_tQ_t$ , where (real) stock price  $Q_t$  follows:

$$\frac{dQ_t}{Q_t} = \mu_t^q \cdot dt + \sigma_t^q \cdot dZ_t$$

Financial risk  
(Endogenous)



- $\mu_t^q$  and  $\sigma_t^q$  are both endogenous (to be determined)

2. Each solves the following optimization (standard)

$$\max_{C_t, \theta_t} \mathbb{E}_0 \int_0^\infty e^{-\rho t} \log C_t dt \quad \text{s.t.}$$

$$da_t = (a_t(i_t + \theta_t(i_t^m - i_t)) - \bar{p}C_t)dt + \theta_t a_t(\sigma + \sigma_t^q)dZ_t$$

- Aggregate consumption of capitalists  $\propto$  aggregate financial wealth

$$C_t = \rho A_t Q_t$$

- Equilibrium risk-premium is determined by the total risk

$$i_t^m - i_t \equiv \text{rp}_t = (\sigma + \sigma_t^q)^2$$

**Flexible price economy** as benchmark: 'natural' consumption of capitalists  $C_t^n = \rho A_t Q_t^n$  follows

$$\begin{aligned}\frac{dC_t^n}{C_t^n} &\equiv \frac{d(A_t Q_t^n)}{A_t Q_t^n} = (r^n - \rho + \sigma^2) dt + \sigma dZ_t \\ &= g dt + \sigma dZ_t = \frac{dA_t}{A_t}\end{aligned}$$

where  $r^n = \rho + g - \sigma^2$  is the 'natural' rate of interest

Define **asset price gap**

$$\hat{Q}_t = \ln \frac{Q_t}{Q_t^n}, \quad 0 = \underbrace{\text{Var}_t \left( \frac{dQ_t^n}{Q_t^n} \right)}_{\text{Benchmark volatility}}, \quad \underbrace{\left( \overset{\text{Endogenous}}{\sigma_t^q} \right)^2 dt}_{\text{Actual volatility}} = \text{Var}_t \left( \frac{dQ_t}{Q_t} \right)$$

which is proportional to **output gap**

$$\hat{Y}_t = \ln \left( \frac{Y_t}{Y_t^n} \right) \longrightarrow \hat{Y}_t = \underbrace{\zeta}_{>0} \cdot \hat{Q}_t$$

**Dividend yield:** dividend yield =  $\rho$ , as in Caballero and Simsek (2020)

- A positive feedback loop between asset price  $\Longleftrightarrow$  dividend (output)

**Determination of nominal stock return  $dI_t^m$**

$$\begin{aligned}
 dI_t^m &= \left[ \underbrace{\rho}_{\text{Dividend yield}} + \underbrace{g + \mu_t^q + \overbrace{\sigma\sigma_t^q}^{\text{Covariance}}}_{\text{Capital gain}} \right] dt + \underbrace{(\sigma + \sigma_t^q)}_{\text{Risk term}} dZ_t \\
 &= \underbrace{i_t^m}_{\text{Drift}} = \underbrace{i_t}_{\text{Monetary policy}} + \underbrace{(\sigma + \sigma_t^q)^2}_{\text{Risk-premium}}
 \end{aligned}$$

## Assume:

- Central bank commits to keep  $i_t = 0$  until  $\hat{T}^{\text{TFG}} \geq T$  (i.e., Odyssean guidance)
- Perfect stabilization (i.e.,  $\hat{Q}_t = 0$ ) afterwards, i.e., for  $t > \hat{T}^{\text{TFG}}$
- From the same arguments, risk-premium gap stabilization beforehand,  $t \leq \hat{T}^{\text{TFG}}$  (no excess volatility while  $i_t = 0$ )

**Problem:** minimize smooth quadratic welfare loss

$$\begin{aligned} \min_{\hat{T}^{\text{TFG}}} \mathbb{L}^Q(\{\hat{Q}\}_{t \geq 0}) &\equiv \mathbb{E}_0 \int_0^\infty e^{-\rho t} (\hat{Q}_t)^2 dt \\ \text{s.t. } \hat{Q}_0 &= \underbrace{\underline{r}}_{<0} T + \underbrace{\bar{r}}_{>0} (\hat{T}^{\text{TFG}} - T) \end{aligned}$$

- Smoothing the ZLB costs over time (i.e., welfare enhancing)

## Assume:

- Central bank can commit to keep  $i_t = 0$  until  $\hat{T}^{HOFG} \geq T$
- No stabilization (i.e.,  $\hat{Q}_t = \hat{Q}_{\hat{T}^{HOFG}}$ ) guaranteed afterwards,  $t \geq \hat{T}^{HOFG}$
- Pick  $\{\sigma_t^q\}$  for  $t < \hat{T}^{HOFG}$

**Problem:** minimize smooth quadratic welfare loss

$$\min_{\sigma_1^{q,L}, \sigma_2^{q,L}, \hat{T}^{HOFG}} \mathbb{L}^Q(\{\hat{Q}\}_{t \geq 0}) \equiv \mathbb{E}_0 \int_0^\infty e^{-\rho t} (\hat{Q}_t)^2 dt,$$

$$\text{s.t. } \begin{cases} d\hat{Q}_t = -\underbrace{r_1^T (\sigma_1^{q,L})}_{<0} dt + \sigma_1^{q,L} dZ_t, & \text{for } t < T, \\ d\hat{Q}_t = -\underbrace{r_2^T (\sigma_2^{q,L})}_{>0} dt + \sigma_2^{q,L} dZ_t, & \text{for } T \leq t < \hat{T}^{HOFG}, \\ d\hat{Q}_t = 0, & \text{for } t \geq \hat{T}^{HOFG}, \end{cases}$$

with

$$\hat{Q}_0 = \underbrace{r_1^T (\sigma_1^{q,L})}_{<0} T + \underbrace{r_2^T (\sigma_2^{q,L})}_{>0} (\hat{T}^{HOFG} - T)$$

## Change:

- Central bank commits to stabilizing the economy after  $\hat{T}^{HOFG}$  with Poisson probability  $\nu$ : at each point after  $\hat{T}^{HOFG}$ ,  $\hat{Q}_t$  becomes 0 with probability  $\nu dt$

**Problem:** minimize smooth quadratic welfare loss

$$\min_{\sigma_1^{q,L}, \sigma_2^{q,L}, \hat{T}^{HOFG}} \mathbb{E}_0 \left[ \int_0^{\hat{T}^{HOFG}} e^{-\rho t} \hat{Q}_t^2 dt + \int_{\hat{T}^{HOFG}}^{\infty} e^{-\rho t} e^{-\nu(t-\hat{T}^{HOFG})} \hat{Q}_t^2 dt \right],$$

$$\text{s.t. } \begin{cases} d\hat{Q}_t = - \underbrace{r_1^T (\sigma_1^{q,L})}_{<0} dt + \sigma_1^{q,L} dZ_t, & \text{for } t < T, \\ d\hat{Q}_t = - \underbrace{r_2^T (\sigma_2^{q,L})}_{>0} dt + \sigma_2^{q,L} dZ_t, & \text{for } T \leq t < \hat{T}^{HOFG}, \\ d\hat{Q}_t = 0, & \text{for } t \geq \hat{T}^{HOFG}, \end{cases}$$

with

$$\hat{Q}_0 = \underbrace{r_1^T (\sigma_1^{q,L})}_{<0} T + \underbrace{r_2^T (\sigma_2^{q,L})}_{>0} (\hat{T}^{HOFG} - T)$$

Fiscal authority's monetary reserves  $F_t$

$$dF_t = -\theta_t a_t \tau_t dZ_t, \quad \text{with:} \quad F_0 = F_{0-} - \underbrace{\chi \theta_{0-} a_{0-}}_{\text{Instant subsidy}}, \quad (1)$$

Then capitalist's dynamic flow becomes:

$$da_t = (a_t (i_t + \theta_t (i_t^m - i_t)) - \bar{p} C_t) dt + \theta_t a_t [(\sigma_t + \sigma_t^q) + \tau_t] dZ_t, \quad (2)$$

with  $\Delta a_0 \equiv a_0 - a_{0-} = \chi \theta_{0-} a_{0-} + \bar{p} A_{0-} \underbrace{\Delta Q_0}_{\text{Asset price change}}$

## Proposition

**HOFG** equilibrium (with  $\sigma_t^{q,*}$ ) becomes a unique equilibrium under the following rule:

$$\tau_t = (\sigma_t^{q,*} - \sigma_t^q), \quad \text{and} \quad \chi = \bar{p} A_{0-} \frac{Q_0^* - Q_0}{\theta_{0-} a_{0-}}, \quad (3)$$

In this case,  $\tau_t = 0$ , and  $\chi = 0$  on the equilibrium path