Do Cost-of-Living Shocks Pass Through to Wages?

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Abstract

We develop a novel, tractable New Keynesian model where firms post wages and workers search on the job, motivated by microeconomic evidence on wage setting. Because firms set wages to avoid costly turnover, the rate that workers quit their jobs features prominently in the model's wage Phillips curve, matching U.S. evidence that wage growth tightly correlates with workers' quit rate. We then examine the response of wages to cost-of-living shocks, i.e., shocks that raise the price of household's consumption goods but do not affect the marginal product of labor. Such shocks pass through to wages only to the extent that higher cost of living improves worker's outside options, such as competing jobs or unemployment, relative to their current job. However, higher cost of living lowers real wages at all jobs evenly, and unemployment is rarely a credible outside option. Cost-of-living shocks thus have little to no effect on relative outside options and therefore wages. We conclude that wage posting and on-the-job search, which are prevalent in labor markets such as the United States, limit the scope for pass through from prices to wages and elevate voluntary quits as the primary predictor of nominal wage growth.

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1 Introduction

In the economic recovery following the COVID pandemic, economies throughout the world experienced both rapid price inflation and nominal wage growth. This experience has generated interest in the relationship between price and wage growth and raised concerns among policymakers of a wage-price spiral. However, mainstream tools for analyzing the relationship between price inflation and wage growth assume wage setting mechanisms that are counterfactual for economies such as the United States, namely that wages are set unilaterally by unions representing workers. In many advanced economies, union membership has declined dramatically, and evidence suggests that in the United States wage posting is the most common, if not dominant, method of wage determination.[1] In light of this evidence and the recent experience of inflation, we ask, *when firms set both prices and wages, through what mechanisms do workers' wages respond to shocks to cost of living, and how large is this response?*

To answer this, we extend the wage posting model in Bloesch and Larsen (2023) into a Dynamic, Stochastic General Equilibrium (DSGE) environment where workers search on the job, and firms set both prices and wages subject to nominal rigidities in the form of standard, convex adjustment costs. Since hiring is costly, firms are incentivized to pay sufficiently high wages to quickly fill vacancies and discourage workers from quitting. The threat of workers quitting into unemployment is low, so wages are primarily determined by firms competing for already-employed workers. We analytically derive the wage Phillips curve in this environment, and show how it can be written as a simple relationship between nominal wage growth and log deviations in the quit rate and the unemployment rate alone. Calibrated to match U.S. data on worker flows, our model predicts that fluctuations in quits are the most important in predicting wage growth, while unemployment, the forcing variable in standard sticky-wage models such as Galí (2011), has almost no weight. Estimating this wage Phillips curve in reduced form on US data, we find empirically that quits dominates unemployment in predicting wage growth, providing a validation of the model.

We then consider how wages respond to a cost-of-living shock: i.e., a shock that raises the cost of households' consumption bundle without affecting the marginal product of labor. To model a pure cost-of-living shock, we assume that workers consume two goods: a labor-intensive services bundle (e.g., haircuts) and an endowment good (e.g., unprepared food or energy). Negative shocks to the quantity of endowment good thus raise the price of workers' consumption basket without affecting the marginal product of labor, allowing us to study how

[1] See Hall and Krueger (2012); Lachowska et al. (2022); Di Addario et al. (2023).

a “pure” cost-of-living shock passes through to wages. This cost-of-living shock raises firms' optimal wage only if it affects turnover costs by making workers harder to recruit or more likely to quit. In our benchmark model, we show that a higher price level has no effect on these probabilities: since a higher price level changes the real wages of all jobs proportionally, cost-of-living shocks do not make workers more likely to quit or harder to recruit from other firms; and if unemployed workers' purchasing power is also equally eroded by higher prices, then a higher price level leaves the relative desirability of employment and unemployment unchanged. Since the probability that a worker quits or is recruited is unchanged, higher cost of living has no effect on wages in equilibrium.

We then consider a model extension where a higher cost of living *does* affect the probability that a worker quits or is recruited, namely that unemployment benefits are indexed to inflation, while nominal wages are not.[2]. Increases in the cost of living now make unemployment relatively more desirable, making unemployed workers more difficult to recruit and employed workers more likely to quit into unemployment. We show, however, that the presence of on-the-job search renders pass through from cost-of-living shocks to wages quantitatively small, as competition for already employed workers continues to dominate firm wage setting decisions even when the relative desirability of unemployment improves. Thus, a quantitatively realistic amount of on-the-job search severely limits the pass through of cost-of-living shocks to wages even when a higher cost of living makes employment relatively less attractive.[3]

While stylized, our model is consistent with a range of recent microeconomic evidence on how wages are determined. Our model captures the result in Jäger et al. (2020) that wages are insensitive to the flow value of unemployment benefits, even for workers who were hired directly from unemployment. This feature arises for two reasons. First, in our model calibrated to U.S. data, the value of unemployment is significantly below the value of employment, so even sizable changes in the flow value of unemployment benefits do not make unemployment a credible outside option. Second, because firms post wages rather than bargain, all workers are paid the same regardless of their previous employment status. This common wage policy is further supported by the finding in Di Addario et al. (2023) that workers' prior employer has small effects on workers' current wages in a large majority of occupations, as well as results in

[2]This is economically similar but notationally simpler than assuming that there are direct utility benefits from leisure, provided that that the elasticity of substitution between leisure and consumption is not one; see Appendix B.1

[3]We consider other channels through which higher cost of living may affect wage growth: workers search on-the-job more frequently when the cost of living rises, and that workers become more wage sensitive when cost of living is higher. We show in Appendix A and Supplementary Appendix F, respectively, that the effects on wages from these channels are also quantitatively small.

Hall and Krueger (2012) and Lachowska et al. (2022) that most workers in the United States face wage posting rather than bargaining. Lastly, our model features finite elasticities of hiring and separations rates with respect to firms wage policies (i.e., when a firm raises its wages, workers join the firm more quickly and leave the firm more slowly), as has been extensively documented in the monopsony literature such as in Bassier et al. (2022) and Datta (2023).

While prior research has modeled the relationship between job-to-job mobility and wage growth,[4] our setting provides a tractability advantage: if firms are ex-ante identical and adjust prices and wages subject to pricing frictions à la Rotemberg (1982), then our model features a symmetric equilibrium with a single wage alongside endogenous worker flows between firms (and unemployment). This outcome is compatible with workers' on-the-job search due to the presence of idiosyncratic, worker-specific preference shocks over workplaces, so that workers will sometimes choose to switch jobs even when firms offer identical wages.

Other recent studies have explored supply shocks and the response of wages in equilibrium. We differ from Lorenzoni and Werning (2023a,b) where workers set wages via unions and Gagliardone and Gertler (2023) where workers bargain and wages are rigid in real terms. Unlike these works and other papers which study oil or other shocks which affect the marginal product of labor, we study a shock which only affects workers' cost of living and focus on understanding whether the pass through of cost-of-living shocks to wages amplifies inflationary shocks in the modern U.S. economy. Given this focus, we also abstract from assuming *ad hoc* real wage rigidity, which mechanically generates pass through from cost-of-living changes to wages, noting that there is little evidence to suggest this type of indexation is widely used in the United States at present.[5] Similarly, while there is a long tradition of modelling nominal wage rigidity in New Keynesian models by assuming workers are unionized following the tractable approach in Erceg et al. (2000), we refrain from assuming that workers are unionized, noting that only 11.3% of U.S. workers were unionized as of 2022 (Shierholz et al., 2023); we also show that doing so is important for our results, as assuming unions set wages does imply pass through from prices to wages in response to a cost-of-living shock.

There is also a large macro-labor literature that embeds search frameworks and labor market frictions in macroeconomic models to study implications for the business cycle. de la Bar-

[4]For example, Moscarini and Postel-Vinay (2016a) extend the wage posting model of Burdett and Mortensen (1998) into a dynamic setting, and Birinci et al. (2022) assume a three-party bargaining protocol to determine wages. Faccini and Melosi (2023) study the relationship of job-to-job mobility and price inflation.

[5]Evidence on the use of cost-of-living adjustments (COLAs) comes from studies of large union contracts, which now cover only a small share of U.S. employment. Even within unionized workers, the share covered by contracts with COLAs has shrunk dramatically since the 1970s. See e.g., Christiano et al. (2016), footnote 4, for discussion.

rera i Bardalet (2023) develops a similar model of labor market monopsony with on-the-job search and finds that increased monopsony power flattens the wage Phillips curve. Moscarini and Postel-Vinay (2023) study a model where workers search on the job and bargain when they receive an outside offer, resulting in a wage Phillips curve in which the distribution of wages and misallocation of workers matters for wage growth. Instead, we assume wage-posting and study a simpler setting without misallocation, where workers and firms are homogeneous in their productivity, and study the implications for pass through from cost-of-living shocks to prices. A strand of recent work (Hajdini et al., 2023; Pilossoph and Ryngaert, 2023; Pilossoph et al., 2023) develops search models which capture the fact that inflation raises the rate at which workers search for job opportunities, studying the implications for wage growth. While our baseline model abstracts from this mechanism, we present an extension in the Appendix A where workers' on-the-job search intensity is increasing in the price level, suggesting the general equilibrium effects on wage growth are small in our setting—consistent with the fact that U.S. workers believe that the pass-through from aggregate inflation to their income is low (Hajdini et al., 2023).[6]

Our results suggest that in a setting such as the United States where few workers operate under collective bargaining agreements with cost-of-living adjustments, and where firms' wage setting decision reflects competition for already-employed rather than for unemployed workers, the ability for workers to reclaim real wages in response to a supply shock that raises their cost of living is limited. There is thus a little scope for supply-shock induced wage-price spirals fueled by workers' ability to command higher nominal wages in response to higher nominal prices.

Layout Section 2 presents stylized facts from U.S. data, demonstrating the tight correlation between quits and wage inflation that motivates our model's assumptions of wage posting and on-the-job search, as well as a weak relationship between price and wage inflation. Section 3 presents our benchmark dynamic New Keynesian model with on-the-job search and wage posting firms. Section 4 demonstrates that our wage-posting model with on-the-job search implies little scope for pass through from prices to wages: specifically, Section 4.1 demonstrates this result quantitatively in our model, and also shows that monetary policy shocks cause

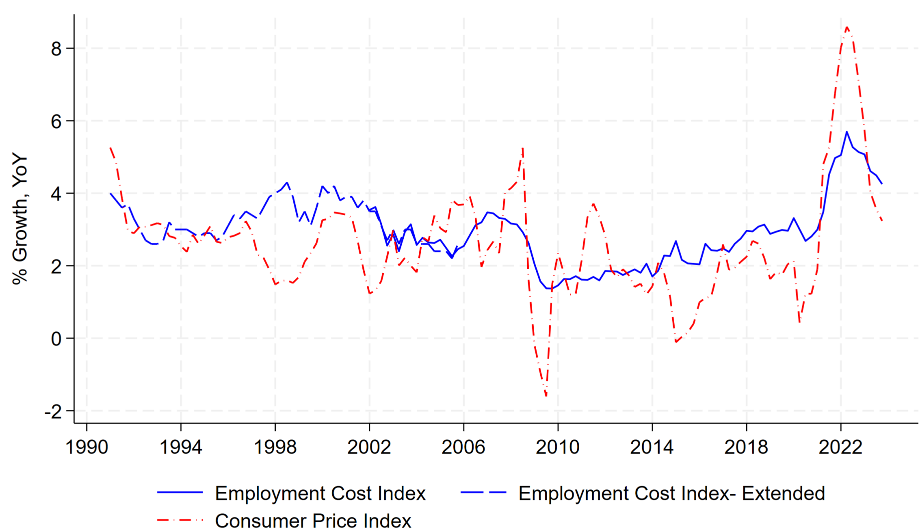
[6]Appendix A uncovers that an increase in on-the-job search caused by a cost-of-living shock has only modest effects on wage growth. While a greater threat of worker separation incentivizes firms to raise wages when prices rise, there is an important offsetting general equilibrium effect: a greater number of searchers lowers labor market tightness, making it easier for firms to replace departing workers. The net result is that wages respond minimally in general equilibrium when workers search on the job more frequently in response to higher cost of living.

wages and prices to co-move. Section 4.2 works through an extension of our baseline model, in which our cost-of-living shock makes unemployment more desirable and causes firms to raise wages, and shows that our assumption of on-the-job search renders this channel quantitatively small. Section 4.3 shows that our model's structural wage Phillips curve matches the empirical wage Phillips curve estimated from the data in Section 2, and Section 5 provides analytic comparison of our benchmark model to neoclassical labor supply models and union wage setting models commonly used in the literature and in which changes in prices do pass through to wages. Section 6 concludes.[7]

2 Stylized Facts on the Wage Phillips Curve

Before proceeding to our formal framework, we present two stylized facts about the wage Phillips curve. First, nominal wage growth, measured by the employment cost index and price inflation are weakly related at high frequencies. Second, nominal wage growth is tightly correlated with the quits rate.

Figure 1: Wage Growth and Headline Price Inflation

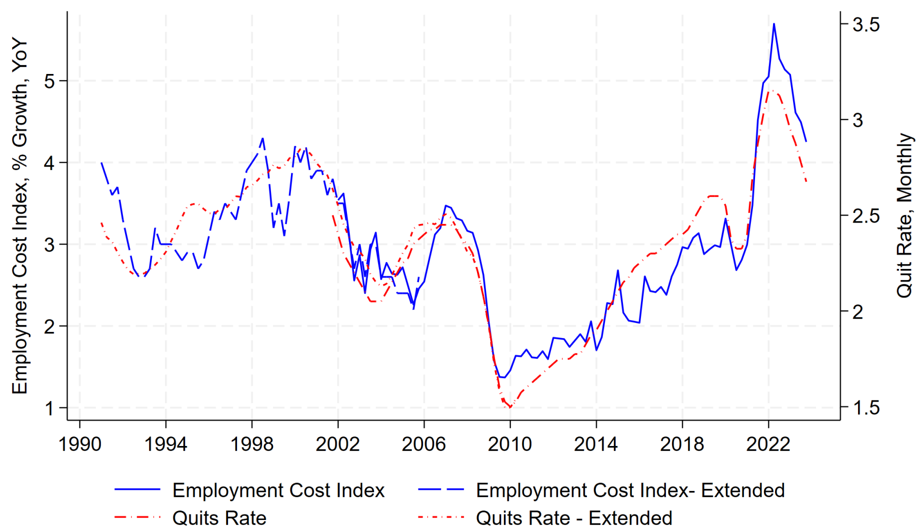


Notes: Between 1990-2019, nominal wage growth and price inflation were weakly correlated. Nominal wage growth and price inflation both surged during the COVID pandemic and recovery.

[7] Online Appendix and Supplementary Appendix provide additional related results.

Figure 1 plots the time series of four quarter growth in the employment cost index wages and salaries for private industries and four quarter growth in the consumer price index, beginning in the fourth quarter of 1990 when the employment cost index data is available. Outside of the post-COVID period, nominal wage growth and price inflation are weakly correlated. Price inflation and wage growth most visibly diverge in 2011 and 2015, when inflation rose and subsequently fell, while nominal wage growth was changing only gradually.

Figure 2: Wage Growth and Quits



Notes: There is a strong correlation between quits, measured here as the four quarter moving average of quits per hundred employees from the Job Openings and Labor Turnover Survey (JOLTS), and year-over-year wage growth.

Figure 2 plots the relationship between the four quarter moving average of the quit rate, which is measured as quits per hundred employees from the Job Openings and Labor Turnover Survey (JOLTS), and the four quarter growth in the employment cost index. This figure shows an extension of the result documented by Faberman and Justiniano (2015) that quits are highly related to growth in the employment cost index, and is related to results documented by Moscarini and Postel-Vinay (2017) that nominal wage growth is well-predicted by job-to-job transitions. [8] The fact that a large share of quits are in fact job-to-job transitions

[8]The empirical measure of quits include various labor market transitions: job-to-job transitions without a period of non-employment, job-to-job transitions with a period of non-employment, and voluntary quits into non-employment. Qiu (2022) shows finds that 3% of workers transition from employment to non-participation each month (most of which appear voluntary) and Elsby et al. (2010) find that only 16% of workers who quit enter a period of unemployment.

motivates the inclusion of on-to-job search in our model; we will show later that including a realistic quantity of on-the-job search (i.e., by calibrating our model to U.S. data) has important implications for the pass through of cost-of-living shocks to nominal wages.

To estimate the empirical wage Phillips curve more formally, we combine versions of the employment cost wage, quits, and price inflation data over time. For comparison, we also include the unemployment rate and the unemployment gap (the difference between the unemployment rate and the natural rate of unemployment), which are the traditional labor market indicators for estimating the wage Phillips curve (Galí, 2011).[9] Formally, in quarter t, letting W_t be the nominal wage, Q_t be quits, and U_t be the unemployment rate, we estimate the following regression, annualizing nominal growth for readability:

$$100 \times (\ln W_t - \ln W_{t-1}) = \beta_0 + \beta_Q \ln Q_t + \beta_U \ln U_t + \beta_\pi \times 100 \times (\underbrace{\ln P_{t-1} - \ln P_{t-2}}_{\equiv \pi_{t-1}}) + \varepsilon_t. \quad (1)$$

Table 1 reports the results: the empirical estimate $\hat{\beta}_Q$ is much larger than $\hat{\beta}_U$.[10] Indeed, $\hat{\beta}_U$ is not generally significant at conventional levels and is not of the expected sign once we include quits. These results are robust to the inclusion of the COVID pandemic and recent recovery. In the following section, we will develop a model that is capable of matching the empirical correlations in equation (1): specifically, we will write down a structural wage Phillips curve of the similar form of (1), where we will have both $\hat{\beta}_Q > \hat{\beta}_U$ and $\hat{\beta}_U$ small but positive when using both quits and unemployment as our measures of labor market tightness in the wage Phillips curve.

Table 1 also reports the coefficient of lagged price inflation on wage growth $\hat{\beta}_\pi$. The coefficient on inflation is fairly small, with elasticities all below 0.15. Comparing columns (1) and (2), we see that the coefficient on price inflation falls when adding quits, suggesting that the unemployment rate is failing to pick up fluctuations in both wage and price inflation that the quit rate absorbs. Column (3) shows that prior to COVID, there was no significant

[9]For the quits data, from 2010Q3-2023, we use the private sector quit rate JTS1000QUR from the St. Louis Federal Reserve FRED database, aggregated by averaging at the quarterly level. Prior to 2000, we use the quarterly private sector quits rate from Davis et al. (2012). Between 2001Q1 and 2010Q2, we use the average of these two series. Similarly for the employment cost index, we use the employment cost index wages and salaries series for private industry workers ECIWAG from FRED beginning in 2005. Prior to 2001, we use the SIC industry basis of the employment cost index for private industry wages and salaries, series ECS20002I, from the Bureau of Labor Statistics. From 2001-2005, we take the average of these two wage series.

[10]Column 1 should be interpreted as "for a 100% increase in the unemployment rate (i.e., double the unemployment), there is an annualized 2.2% point decrease in gross wage growth". Likewise, Column 2 implies that 100% increase in the quits rate (i.e., double the quits) would result in 4.65% point increase in wage growth.

relationship between price inflation and wage growth after conditioning on quits. Including the unemployment gap or lagging unemployment in columns (4) and (5), respectively, does not change the results.

Table 1: Time Series Regression of Wage Growth on Labor Market Variables, 1990Q4-2023

VARIABLES	(1) ECI	(2) ECI	(3) ECI	(4) ECI	(5) ECI
$\ln U_t$	-0.4850***	-0.0388	0.1375		
	(0.0727)	(0.0986)	(0.1054)		
$\ln Q_t$		0.9737***	1.1175***	0.9815***	0.9995***
		(0.1553)	(0.1875)	(0.1657)	(0.1223)
$\ln U_t - \ln U_t^*$				-0.0325	
				(0.0967)	
$\ln U_{t-1}$					-0.0226
					(0.0748)
π_{t-1}	0.1411***	0.0882***	0.0365	0.0880***	0.0877***
	(0.0480)	(0.0331)	(0.0321)	(0.0329)	(0.0328)
Observations	136	136	119	136	136

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Notes: Results from a quarterly regression of wage growth measured using the Employment Cost Index (ECI) on unemployment, the quit rate, and lagged price inflation, as specified in equation (1). While Column 1 shows that a regression of wage growth on unemployment alone yields a familiar negative sign, including quits flips the sign and reduces the significance to below conventional levels as seen in Column 2. Columns 3 and 4 demonstrates that this result is robust to dropping the COVID pandemic and recovery, and also to measuring unemployment in log deviations from it's natural rate as estimated by the CBO, $\ln(U_t) - \ln(U_t^*)$. The final column demonstrates that using lagged U_t doesn't alter the results.

In Section 4.3, we will revisit the wage Phillips curve evidence, comparing the empirical wage Phillips curve with our model-implied wage Phillips curve. Our model-implied wage Phillips curve will include a real wage "catch-up" term on the right hand side that is a function of cumulative past inflation, and so the model-implied wage Phillips curve will take a slightly different form. We will show that the elasticity of wages with respect to this model-implied catch-up term will be theoretically and empirically very small.

3 Model

This section builds a model where firms post wages and workers search on the job, and calibrates that model to U.S. Data. We will then go on to use the model to provides a structural foundation for the empirical OLS regression (1), finding that our calibration implies structural coefficients for β_Q and β_U that are consistent with the empirical coefficients $\hat{\beta}_Q$ and $\hat{\beta}_U$. Finally, we will then show that this model implies that there is little scope for pass through from prices to wages in response to a cost-of-living shock.

In laying out the model, we first describe the problem of a firm posting wages in the presence of recruiting costs and on-the-job search. When deciding whether to raise wages, the firm trades off between a higher wage bill and lower turnover costs. Lower turnover costs come from the fact that a higher wage increases the probability that the firm recruits a particular searcher, regardless of whether that searcher is already employed or unemployed (the recruiting rate), while also lowering the probability that incumbent workers leave (the separation rate). Because the firm's problem does not depend directly on the price level in partial equilibrium, increases in workers' cost of living can only affect wages through their effects on these recruiting or separation rates.

We then describe the solution to the worker's problem, which determines firms' recruiting and separation rates. Since a change in the price level affects the real wages offered by all firms proportionally, changes in the price level can relatively improve workers' outside option, and raise wages, only if changes in the price level make unemployment relatively more attractive. If this is the case, this can lead to pass through from cost of living to wages, as firms must now offer a higher wage to retain the same number of workers as before. However, these considerations are quantitatively small when (i) most workers already vastly prefer a job to unemployment and/or when (ii) most searching workers already have a job, rendering the value of unemployment irrelevant when considering whether to accept a new job offer. Moreover, this channel need not exist at all if changes in the price level do not affect the desirability of unemployment, as in our benchmark model described below.

Structure There are two goods in the economy: an endowment good X_t and services Y_t. They are combined into an aggregate consumption good, C_t, according to the CES function

$$C_t = \left(\alpha_Y^{\frac{1}{\eta}} Y_t^{\frac{\eta-1}{\eta}} + \alpha_X^{\frac{1}{\eta}} X_t^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}, \tag{2}$$

with corresponding aggregate price index[11]

$$P_t = \left(\alpha_Y P_{y,t}^{1-\eta} + \alpha_X P_{x,t}^{1-\eta}\right)^{\frac{1}{1-\eta}} . \tag{3}$$

Workers are hired by firms to produce services Y_t, so that their real wage is determined by the nominal wage offered in that sector divided by the aggregate price level P_t. The total amount of endowment good $X_t = 1$ is given.

Cost-of-Living Shock Our "pure" cost of living shock is a decline in the endowment good X_t which raises its price, $P_{x,t}$, and hence the price level P_t in (3). This is a pure cost of living shock in the sense that it raises the cost of living for workers without affecting their marginal products, unlike an oil shock, for example, which affects both. The point of considering such a shock is not to downplay the role or importance of oil shocks to many modern economies, but to highlight how these shocks propagate and question whether a "wage price spiral" amplifies their effects on the price level.

Firm's Wage-Posting Problem We now turn to the determination of the nominal wage. We assume that perfectly-competitive retailers bundle service types j according to a standard Dixit-Stiglitz production function with an associated ideal price index:

$$\begin{aligned} Y_t &= \left(\int \left(Y_t^j\right)^{\frac{\epsilon-1}{\epsilon}} dj\right)^{\frac{\epsilon}{\epsilon-1}} , \\ P_{y,t} &= \left(\int \left(P_{y,t}^j\right)^{1-\epsilon} dj\right)^{\frac{1}{1-\epsilon}} , \end{aligned}$$

yielding product demand for variety j:

$$\frac{Y_t^j}{Y_t} = \left(\frac{P_{y,t}^j}{P_{y,t}}\right)^{-\epsilon} . \tag{4}$$

The firm j produces only with labor according to $Y_t^j = N_{jt}$. Firm j sets nominal wages W_{jt} each period, which is assumed to be the same for all workers in the firm, including new hires. Workers separate from firm j with probability $S_t(W_{jt}|\{W_{kt}\}_{k\neq j})$ each period, with $S_t'(W_{jt}|\{W_{kt}\}_{k\neq j}) < 0$: firms retain a higher share of workers each period by paying a higher wage, given other firms' wages. The firm can recruit workers by posting vacancies

[11] We assume $\alpha_X + \alpha_Y = 1$ with $\alpha_X > 0$ and $\alpha_Y > 0$ as usual.

V_{jt}, and the probability that a vacancy successfully results in a hire is $R_t(W_{jt}|\{W_{kt}\}_{k\neq j})$, with $R_t'(W_{jt}|\{W_{kt}\}_{k\neq j}) > 0$.[12] The firm pays a convex, per-vacancy hiring cost, $c\left(\frac{V_{jt}}{N_{j,t-1}}\right)^{\chi} W_t$, to post V_t vacancies, where W_t is the aggregate wage, $c > 0$ and $\chi \geqslant 0$. Finally, the firm is also subject to price and wage adjustment frictions à la Rotemberg (1982).

Given this, each firm j maximizes the present discounted value of profits, solving

$$\max_{\substack{\{P^j_{y,t}\},\{Y^j_t\},\\ \{N_{jt}\},\{W_{jt}\},\{V^j_t\}}} \sum_{t=0}^{\infty}\left(\frac{1}{1+\rho}\right)^t \Bigg(P^j_{y,t}Y^j_t - W_{jt}N_{jt} - c\left(\frac{V_{jt}}{N_{j,t-1}}\right)^{\chi} V_{jt}W_t - \frac{\psi}{2}\left(\frac{P^j_{y,t}}{P^j_{y,t-1}} - 1\right)^2 Y^j_t P^j_{y,t}$$
$$- \frac{\psi^w}{2}\left(\frac{W_{jt}}{W_{j,t-1}} - 1\right)^2 W_{jt}N_{jt}\Bigg) \tag{5}$$

subject to the law of motion for employment

$$N_{jt} = (1 - S_t(W_{jt}))N_{j,t-1} + V_{jt}R_t(W_{jt}) \tag{6}$$

and the product demand equation (4). From inspecting equations (5) and (6), we can observe that the service sector firm chooses the wage (and other choice variables) taking as given the choices of other service sector firms (embodied in the price index and aggregate output of the service sector), parameters, and the separation and recruiting rates $S_t(\cdot)$ and $R_t(\cdot)$. Note that since the vacancy-posting cost is denominated in labor (i.e., priced by the aggregate wage W_t), the aggregate price level P_t does not appear directly in (5). Thus, in partial equilibrium, the only way that changes in the price level can impact the firm's wage setting decision is through changes in $S_t(\cdot)$ and $R_t(\cdot)$, which will be determined by the solution to workers' optimization problem described in Section 3.2.

3.1 Symmetric Equilibrium

To make this relationship between the separation and recruiting rates and the firm's choice of wage clearer, we derive a wage Phillips curve from the firm's first order conditions, assuming for the moment that a symmetric equilibrium, where all firms offer the same aggregate wage W_t, exists. Under this assumption, the wage Phillips curve expresses nominal wage growth as exclusively a function of aggregate, endogenous labor market variables: vacancies, employ-

[12] How retention and separation functions $R_t(W_{jt}|\{W_{kt}\}_{k\neq j})$ and $S_t(W_{jt}|\{W_{kt}\}_{k\neq j})$ depend on wages set by other service firms will be derived after we describe households' and workers' problems in Section 3.2. We write $R_t(\cdot)$ and $S_t(\cdot)$ solely as functions of W_{jt} set by firm j solely for readability.

ment, recruiting and separation rates, and recruiting and separation elasticities, again with no direct role for aggregate price index P_t.

Denote $\varepsilon_{R,W}$ and $\varepsilon_{S,W}$ as the elasticities of the recruiting function $R_t(W_{jt})$ and the separation function $S_t(W_{jt})$ with respect to the wage W_{jt}. Then in any symmetric equilibrium where $W_{jt} = W_t$, $N_{jt} = N_t$, $V_{jt} = V_t$, $P_t^j = P_t$, and $Y_t^j = Y_t$, the wage Phillips curve characterizing nominal wage growth curve is given by:

$$\begin{aligned}\psi^w \left(\Pi_t^w - 1\right) \Pi_t^w + 1 =& c(1+\chi) \left(\frac{V_t}{N_{t-1}}\right)^\chi \left[\frac{V_t}{N_t}\varepsilon_{R,W_t} + (-\varepsilon_{S,W_t}) \frac{N_{t-1}}{N_t} \frac{S_t(W_t)}{R_t(W_t)}\right] \\ &+ \frac{1}{1+\rho}\psi^w \left(\Pi_{t+1}^w - 1\right) \left(\Pi_{t+1}^w\right)^2 \frac{N_{t+1}}{N_t}.\end{aligned} \tag{7}$$

Thus, in any symmetric equilibrium where firms solve an optimization problem of the form of (5), the wage Phillips curve will be a function of the current and expected future paths of the job vacancy rate V_t, employment N_t, the recruiting and separation rates $R_t(W_t)$ and $S_t(W_t)$, and their elasticities, denoted $\varepsilon_{R,W_t} > 0$ and $\varepsilon_{S,W_t} < 0$ following conventions in the monopsony literature; see e.g. Bloesch and Larsen (2023).[13]

Interpretation Taking each term step by step, this wage Phillips curve captures how competition for workers affects firms' optimal wage growth. The first term $(V_t/N_{t-1})^\chi$ captures the convex cost of posting vacancies: since firms must post vacancies to attract workers, higher marginal vacancy posting costs raises the value of both recruiting a worker the firm has matched with as well as retaining existing workers. If getting a worker in the door is more valuable, then firms will want to pay higher wages. The next term, within brackets, includes the recruiting elasticity term ε_{R,W_t}, which captures how sensitive the probability of hiring a matched worker is to the wage. If this recruiting elasticity is elevated, the workers' acceptance probability will be more sensitive to the wage, increasing the incentive at the margin for a firm to raise its wage. This ε_{R,W_t} is multiplied by the number of vacancies V_t. Next is the separation elasticity term ε_{S,W_t}. This elasticity is negative, so the negative of the separation elasticity is positive. A more steeply negative separation elasticity means that workers' likelihood of quitting is more sensitive to wages, so the more negative this value is, the greater the incentive to raise wages at the margin. Lastly, we have the separation rate $S_t(W_t)$ and recruiting rate $R_t(W_t)$. A higher separation rate indicates that workers have more opportunities to quit,

[13] Appendix B.2 derives the firm's first-order conditions in (5), including the price Phillips curve and wage Phillips curve (7).

increasing pressure for firms to raise wages. Analogously, when the recruiting rate $R_t(W_t)$ is higher, workers are easier to hire, lowering the pressure for firms to raise wages.

The next section describes the household and workers' optimization problems, which determine the recruiting and separation functions faced by firms. Having done so, we can then log-linearize and simplify the above (7) to evaluate the model's ability to match the empirical wage Phillips curve discussed in Section 2. We do this in Section 4.3, before turning to the implications of our benchmark model for pass-through in Section 4.

3.2 Households and Workers

This section derives the household and worker block of the model. We deviate from the standard assumption in the New Keynesian literature of perfect consumption insurance within the household by assuming that households only imperfectly insure the consumption of workers who are unemployed, consistent with evidence that unemployed workers consume less than employed workers (see e.g., Chodorow-Reich and Karabarbounis (2016)). We assume that workers themselves choose whether to take a particular job offer, and make employment decisions based on relative wages and consumption levels, in addition to idiosyncratic firm-specific preference shocks. Workers' mobility decisions aggregate up into the firms' recruiting and separation functions. Households smooth aggregate consumption within the household over time, yielding a standard Euler equation, making the labor block easy to integrate into a standard New Keynesian setting.

Frictional Markets Workers and firms match according to random search in a frictional market. As mentioned above, each firm j posts V_{jt} vacancies, and aggregate vacancies are $V_t = \int V_{jt}dj$. Each period, employed workers can search on the job with some constant, exogenous probability $\lambda_{EE} \in (0,1)$, and unemployed workers can always search.[14] The unemployment rate is defined as U_t, so the total mass of searchers $\mathcal{S}_t$ is $\mathcal{S}_t = \lambda_{EE}(1-U_{t-1})+U_{t-1}$.[15]

[14]This simplifying assumption mechanically shuts down the possibility that on-the-job search intensity increases with the price level (Hajdini et al., 2023; Pilossoph and Ryngaert, 2023); In Appendix A, we relax this assumption and show that the scope for pass-through from cost-of-living shocks to wages remains small in our model even if workers search more intensely as prices rise.

[15]In terms of timing, firms post wages at the beginning of each period t (understanding that this will determine their separation and recruiting rates, and thus this period's output and labor force, through the law of motion for N_t in equation (6)). Then, all workers who were unemployed last period $t-1$ as well as some workers who were employed last period search.

Matching is random and follows a constant returns to scale matching function $M_t(V_t, \mathcal{S}_t)$,

$$M_t(V_t, \mathcal{S}_t) = \frac{\mathcal{S}_t V_t}{(\mathcal{S}_t^\nu + V_t^\nu)^{\frac{1}{\nu}}},$$

with $\nu = 2$ following the literature. Labor market tightness is $\theta_t = \frac{V_t}{\mathcal{S}_t}$. The job finding rate for workers is $f(\theta_t) = \frac{M_t}{\mathcal{S}_t}$ is increasing in tightness θ_t, and the probability that a vacancy is matched with a worker $g(\theta_t) = \frac{M_t}{V_t}$ is a decreasing function of tightness θ. The share of searchers who are employed is $\phi_{E,t} = \frac{\lambda_{EE}(1-U_{t-1})}{\mathcal{S}_t}$, and the share of searchers who are unemployed is $\phi_{U,t} = 1 - \phi_{E,t} = \frac{U_{t-1}}{\mathcal{S}_t}$. The job finding rate for workers $f(\theta_t)$ and vacancy-filling rate $g(\theta_t)$ are given by

$$f(\theta_t) = \frac{\theta_t}{(1+\theta_t^\nu)^{\frac{1}{\nu}}}, \quad g(\theta_t) = \frac{1}{(1+\theta_t^\nu)^{\frac{1}{\nu}}}.$$

Households A representative household has a unit mass $i \in [0, 1]$ of members who can work. Households seek to maximize the discounted present value of their members' utility, which is log in consumption. Without loss of generality, assume that unemployed household members must each have the same consumption level, C_t^u.[16] Then letting $C_t(i, j(i))$ denote the consumption of worker i in state $j(i)$, where $j(i)$ indicates the firm i is employed at, the household's objective function becomes

$$\sum_{t=0}^{\infty} \left(\frac{1}{1+\rho}\right)^t \left[U_t \ln(C_t^u) + \int_0^{1-U_t} \ln\left(C_t(i, j(i))\right) di \right].$$

The household is allowed to choose C_t^u (effectively, an unemployment benefit) and also a linear tax/subsidy on employed workers, who consume their income each period:

$$C_t\left((i, j(i)\right) = \tau_t \frac{W_{jt}}{P_t}$$

subject to the following budget constraint: letting D_t be nominal dividend payments from services firms (who profit from monopoly and monopsony power) and perfectly competitive goods firms (who receive the endowment X_t and sell it, rebating the proceeds to households), B_t be nominal bond holdings in zero net supply paying nominal interest rate i_t, and finally let-

[16]This is not restrictive, as given our other assumptions the household will always choose to equalize consumption across unemployed agents due to diminishing marginal utility of consumption.

ting $\bar{W}_t \equiv \frac{1}{1-U_t}\int_0^{1-U_t} W_{j(i)t}di$ be the average wage of employed workers, the budget constraint is

$$U_t C_t^u = \frac{D_t}{P_t} - \frac{B_t}{P_t} + \frac{(1+i_{t-1})B_{t-1}}{P_t} + (1-\tau_t)(1-U_t)\frac{\bar{W}_t}{P_t}. \tag{8}$$

To make further progress in delivering a tractable model with households' standard consumption Euler equation, we impose an *ad hoc* consumption sharing rule within the household requiring that unemployed workers' consumption must be a *constant* fraction of employed workers' average consumption:

$$\frac{\bar{C}_t^e}{C_t^u} = \xi, \tag{9}$$

where $\xi \geqslant 1$ and $\bar{C}_t^e \equiv \frac{1}{1-U_t}\int_0^{1-U_t} C(i,j(i))di$ is the average consumption of employed. This rule allows us to capture the fact that the ratio of unemployed and employed consumption is relatively constant over the business cycle (Chodorow-Reich and Karabarbounis, 2016). Moreover, it can be thought of as the result of a household facing an incentive-insurance trade-off: by insuring unemployed workers less and making unemployment relatively worse (lower ξ), the household encourages workers to take jobs and become employed by taking consumption away from unemployed workers with higher marginal utility of consumption. Note that in Section 4.2, we will study extension of this model in which the household implements a different unemployment insurance scheme in which the consumption ratio is not held constant.

3.2.1 Symmetric Equilibrium Features a Standard Euler Equation

In a symmetric equilibrium where all firms set the same wage, and so $W_{jt} = W_t$, the household's problem under constraints (8) and (9) simplifies to choosing aggregate consumption C_t and bond holdings B_t to maximize

$$\sum_{t=0}^{\infty}\left(\frac{1}{1+\rho}\right)^t \ln\left(\frac{C_t}{(1-U_t)\xi + U_t}\right)$$

subject to the simplified budget constraint

$$C_t = \frac{D_t}{P_t} - \frac{B_t}{P_t} + \frac{(1+i_{t-1,t})B_{t-1}}{P_t} + (1-U_t)W_t.$$

Optimization then yields the standard consumption Euler equation with log-utility given by:

$$C_t^{-1} = \frac{1}{1+\rho} \frac{1+i_{t,t+1}}{\Pi_{t+1}} C_{t+1}^{-1}. \tag{10}$$

Workers Workers get utility from consumption and an idiosyncratic preference draw ι. ι represents how much workers like their current job at firm j, which is redrawn every period and is i.i.d. Workers draw a similar preference shock each period during unemployment (note that the household does not take the idiosyncratic preference shocks into account when solving the problem described above). Workers are myopic and consider their utility only one period at a time, which for worker i in state j is given by[17]

$$\mathcal{V}_t(i,j) = \ln\left(C_t(i,j)\right) + \iota_{ijt}.$$

Workers are allowed to search on the job with probability λ^{EE}, and conditional on searching, are matched with a vacancy with probability $f(\theta)$. Workers are allowed to consider unemployment with probability λ^{EU}. Consider a worker i currently employed at firm j who successfully matches with firm k's vacancy. She will move to firm k only if $\mathcal{V}_t(i,k) \geqslant \mathcal{V}_t(i,j)$. Let us define $s_{jk}(W_{jt}, W_{kt})$ as the probability that the worker is poached from firm j to firm k.

We assume that ι follows a Type-1 extreme value distribution with variance γ^{-1} for tractability. Following the consumption sharing rule in (8) and (9), $s_{jk}(W_{jt}, W_{kt})$ is given by

$$s_{jk}\left(W_{jt}, W_{kt}\right) = \frac{\left(\tau_t \frac{W_{kt}}{P_t}\right)^\gamma}{\left(\tau_t \frac{W_{kt}}{P_t}\right)^\gamma + \left(\tau_t \frac{W_{jt}}{P_t}\right)^\gamma} = \frac{W_{kt}^\gamma}{W_{kt}^\gamma + W_{jt}^\gamma}, \tag{11}$$

which is decreasing in W_{jt}: if firm j pays a higher wage, workers are less likely to be poached. Notice also that the probability a worker switches jobs is only a function of the relative *nominal* wage. The worker takes as given the internal tax rate set by the household τ_t and the price level P_t, both of which are unchanged regardless of which job the worker chooses.

Now consider a worker who is deciding whether to quit into unemployment. Let the average wage of employed workers in worker i's household be $\bar{W}_t$, which determines consumption

[17] The absence of utility from leisure, which may be greater in unemployment, can be viewed as a simplifying assumption: we can introduce leisure without changing the results provided that the elasticity of substitution between leisure and consumption is one. See Appendix B.1 for further discussion on how assuming a different elasticity affects the results.

in unemployment through $C_t^u = \frac{\bar{C}_t^e}{\xi} = \tau_t \frac{\bar{W}_t}{\xi P_t}$. Since a worker i who is currently employed at firm j quits into unemployment only if $\mathcal{V}(i,j) \geqslant \mathcal{V}(i, \text{unemployed})$, thus the probability that a worker voluntarily quits into unemployment $s_{ju}(W_{jt})$ is given by

$$s_{ju}(W_{jt}) = \frac{\left(\frac{1}{\xi}\tau \frac{\bar{W}_t}{P_t}\right)^\gamma}{\left(\frac{1}{\xi}\tau \frac{\bar{W}_t}{P_t}\right)^\gamma + \left(\tau \frac{W_{jt}}{P_t}\right)^\gamma} = \frac{\left(\frac{\bar{W}_t}{\xi}\right)^\gamma}{\left(\frac{\bar{W}_t}{\xi}\right)^\gamma + W_{jt}^\gamma}, \tag{12}$$

which is decreasing in W_{jt} but does not depend on the price level P_t.

These individual transition probabilities aggregate up into the firm's separation rate $S(W_{jt})$: each period, a share of workers $s \in (0,1)$ exogenously separate while the remainder $(1-s)$ endogenously separate if they receive an opportunity that they prefer to their current job (either another job, or the chance to exit to unemployment). Recalling that $f(\theta_t)\lambda_{EE}$ denotes the probability that a particular employed worker is allowed to search on the job and matches to another firm, and that λ_{EU} denote the probabilities that an employed worker is allowed to consider quitting into unemployment, the separation rate is written as

$$S_t(W_{jt}) \equiv S_t(W_{jt}|\{W_{kt}\}_{k\neq j}) = s+(1-s)\left[\lambda_{EE} f(\theta_t) \int s_{jk}(W_{jt}, W_{kt}) z(W_{kt}) dk + \lambda_{EU} s_{ju}(W_{jt})\right], \tag{13}$$

where $z(W_{kt})$ is an endogenous density function of outside posted wages. Note that $S_t(\cdot)$ is a decreasing function of W_{jt}, i.e. $S_t'(W_{jt}) < 0$, since all of it's components are decreasing in W_{jt}; in other words, the firm's separation rate falls as the wage rises.

Analogously to the individual separation probabilities, there are probabilities that a matched worker is recruited into the firm conditional on whether the worker is employed or unemployed. Consider a worker employed at firm k that encounters firm j's vacancy. The probability that firm j successfully poaches the worker $r(W_{jt}, W_{kt})$ is:

$$r_{kj}(W_{kt}, W_{jt}) = \frac{\left(\tau_t \frac{W_{jt}}{P_t}\right)^\gamma}{\left(\tau_t \frac{W_{kt}}{P_t}\right)^\gamma + \left(\tau_t \frac{W_{jt}}{P_t}\right)^\gamma} = \frac{W_{jt}^\gamma}{W_{kt}^\gamma + W_{jt}^\gamma}, \tag{14}$$

which is increasing in W_{jt} and is a function of relative wages.

Now consider an unemployed worker who is matched with firm j's vacancy. The probability that the worker takes the job with firm j is defined as $r_{uj}(W_{jt})$ and is equal to

$$r_{uj}(W_{jt}) = \frac{\left(\tau_t \frac{W_{jt}}{P_t}\right)^\gamma}{\left(\frac{1}{\xi}\tau_t \frac{\bar{W}_t}{P_t}\right)^\gamma + \left(\tau_t \frac{W_{jt}}{P_t}\right)^\gamma} = \frac{W_{jt}^\gamma}{\left(\frac{\bar{W}_t}{\xi}\right)^\gamma + W_{jt}^\gamma}, \tag{15}$$

which is increasing in W_{jt}.

We can use (14) and (15) to write firm j's recruiting rate, defined as the share of vacancies that successfully result in hiring a worker is the following. Recalling that $g(\theta_t)$ denotes the probability that a vacancy is matched with a worker, and that $\phi_{E,t}$ and $\phi_{U,t}$ denote the share of searchers who are employed and unemployed, respectively, we can write the recruiting rate as:

$$R_t(W_{jt}) \equiv R_t(W_{jt}|\{W_{kt}\}_{k\neq j}) = g(\theta_t)\left[\phi_{E,t}\int_k r_{kj}(W_{kt}, W_{jt})\omega(W_{kt})dk + \phi_{U,t} r_{uj}(W_{jt})\right]. \tag{16}$$

where $\omega(W_{kt})$ is the distribution of wages that workers are currently employed at. The recruiting rate $R_t(W_{jt})$ is an increasing function because all of its components r_{kj} and r_{uj} are also increasing in W_{jt}. In other words, a higher wage improves the firms odds of recruiting workers through its vacancies.

Forward-Looking Workers Supplementary Appendix E analyzes the case where workers are forward looking, reaching the conclusion that doing so adds considerably to the complexity of the model and burdens discussion without changing the dynamics of the model's response to cost-of-living shocks. We thus proceed here to make the simplifying assumption that workers are myopic.

3.2.2 Symmetric Equilibrium Features Simple Separation and Recruiting Functions

In a symmetric equilibrium where all the firms set the same wage, i.e., $W_{jt} = W_t$ for $\forall j$, both $S(\cdot)$ and $R(\cdot)$ becomes functions of tightness θ_t and simplify from (13) and (16) to

$$S_t = s + (1-s)\left(\lambda_{EE} f(\theta_t)\frac{1}{2} + \lambda_{EU}\left(\frac{1}{1+\xi^\gamma}\right)\right) \tag{17}$$

$$R_t = g(\theta_t)\left(\phi_{E,t}\frac{1}{2} + \phi_{U,t}\left(\frac{\xi^\gamma}{1+\xi^\gamma}\right)\right). \tag{18}$$

where the aggregate wage W_t does not appear on the right hand side of (17) and (18). Therefore, in a symmetric equilibrium, both separation and recruiting rates S_t and R_t become independent of aggregate wage W_t. This is because all the competing firms set the same wage level, and the relative desirability of employment over unemployment is independent of the wage due to household's consumption sharing rule (9).

The reason for the absence of the price level P_t in the separation rate formula (17) is similar: fundamentally the price level is irrelevant to a worker considering choosing between two different nominal wage offers. Also driving this result is the fact that we have assumed the price level is irrelevant for workers considering choosing between working and *unemployment*. This is because the households' consumption sharing rule (9) fixes the relative consumption of employed and unemployed workers at ξ, which naturally appears in equations (17) and (18) above: the higher real consumption ratio ξ is on average, the more likely unemployed workers are to prefer the state of employment to that of unemployment, so S_t decreases with ξ and R_t increases with ξ all else equal. Note that relaxing our assumption that consumption ratio ξ is constant will not change the result about pass through: the issue is that the price level, P_t, does not affect both S_t and R_t in equilibrium. Fixing unemployment benefits at some nominal level, for example, would still result in the relative attractiveness of employment and unemployment being insensitive to the price level by the same logic that applies to employed workers choosing between nominal wages at two different jobs.

As we will show in Section 4.1, there is no pass-through at all in our benchmark case where the relative consumption of employed and unemployed workers is fixed at ξ. However, the assumption that the desirability unemployment (formally, the probability of preferring a job offer at aggregate wage W_t to the unemployment state) is constant and completely independent of the aggregate price level is strong; this would not be true if, for example, we had assumed that the representative household insures unemployed households by guaranteeing them some constant, real unemployment benefit b (i.e., if unemployment benefits are perfectly indexed to inflation), or if we had assumed that workers derive some utility from leisure, as well as consumption, and that leisure utility is systematically higher while unemployed. We discuss the former case in Section 4.2; Appendix B.1 discusses the worker's problem with leisure, which has similar implications but requires more burdensome notation.[18]

[18] This is because we must take a stance on the elasticity of substitution between leisure and consumption; Appendix B.1 demonstrates that if leisure and consumption have an elasticity of substitution of one, then changes in the price level have no effect on the relative desirability of employment for a given nominal wage, as in the benchmark case with fixed consumption ratios.

3.3 Equilibrium Selection

We can close the model with a simple Taylor rule, with a potentially persistent policy shock $\varepsilon_{i,t}$:

$$1 + i_t = \Pi_{Y,t}^{\phi_\Pi}(1 + \varepsilon_{i,t}) \tag{19}$$

with $\phi_\Pi = 2$, and solve for a symmetric equilibrium. Our symmetric equilibrium consists of sequences of all endogenous prices and quantities satisfying that: (i) firms choose identical sequences such that $W_{jt} = W_t$, $N_{jt} = N_t$, $V_{jt} = V_t$, $P_{y,t}^j = P_{y,t}$, (ii) workers and households maximize utility, (iii) firms maximize profits, (iv) product markets clear, and (v) labor market flows add up.

We linearize these necessary conditions in a symmetric equilibrium around a non-stochastic steady state, and solve for the unique solution. While there is a unique, symmetric equilibrium (for our given parameter values) we cannot rule out and leave unexplored the possibility of non-symmetric equilibria where *ex ante* identical firms choose different wages. The fact that we have one wage in equilibrium, while still having worker flows between unemployment and various firms due to idiosyncratic shocks, buys us a highly-tractable dynamic model with on-the-job search.

3.4 Calibration

We choose standard values for most parameters, and choose other parameters governing the labor search block of the model to match the U.S. data: Table 2 lists the model's calibrated parameters, some of which are chosen to target moments in U.S. data given in Table 3.

Specifically, we calibrate the model to match U.S. data on labor market flows during the period 2015-2019 to capture the approximately full-employment conditions that existed prior to the COVID shock. Data on the unemployment rate and separation rate come from the Bureau of Labor Statistics (BLS) and the Job Openings and Labor Turnover Survey (JOLTS). We set $\xi = 2$, which is higher than in Chodorow-Reich and Karabarbounis (2016) but closer to what maximizes steady-state utility for the household in our setting; the results are largely insensitive to changing this parameter.

Table 2: Parameters in the Monthly Benchmark New Keynesian Model

Parameter	Value	Meaning	Reason
λ_{EE}	.14	OTJ search probability	Match EE rates
λ_{EU}	.30	Opportunity to quit probability	Match voluntary EU rate, Qiu (2022)
ξ	2	Consumption ratio: C_t^e/C_t^u	See Notes below
s	.01	Exogenous separation rate	Match JOLTS monthly separation Rate
γ	6	Variance^{-1} of idiosyncratic preferences	Match $\varepsilon_{R,W} - \varepsilon_{S,W}$
ϵ	10	Elasticity of substitution of services	
ψ	100	Services price adjustment cost	
ψ^w	100	Wage adjustment cost	
η	1	Services/endowment good EOS	
α_X	.2	Endowment good's share in CES Utility	
χ	1	Convexity in vacancy posting costs	Bloesch and Larsen (2023)
c	30	Hiring cost shifter	Targeting U
ρ	.004	Discount Rate	Monthly model

Table 3: Selected Model Moments and Data in Steady State

Targeted Moment	Meaning	Model	Data	Source
U	Unemployment rate	.044	.044	BLS
S	Monthly separation rate	.036	.036	JOLTS
$\varepsilon_{R,W} - \varepsilon_{S,W}$	Recruiting minus separation elasticities	4.4	4.2	Bassier et al. (2022)

Notes: We calibrate the model to match labor market flows of the U.S. economy during 2015-2019 to capture the approximately full employment conditions that existed prior to the COVID shock. Data on the unemployment rate and separation rate come from the Bureau of Labor Statistics (BLS) and the Job Openings and Labor Turnover Survey (JOLTS). We set $\xi = 2$, higher than in Chodorow-Reich and Karabarbounis (2016) but closer to what maximizes steady-state utility for the household in our setting; the results are largely insensitive to changing this parameter.

4 Implications for Pass-Through from Prices to Wages

This section studies the effects of a cost-of-living shock as defined in Section 3, considering the following thought experiment: what happens to nominal wages when an unanticipated, temporary, negative shock to X_0 raises the price level at $t = 0$? In each case, we will consider what happens when monetary policy holds N_t (and thus Y_t) fixed, i.e., stabilizes labor markets.

Section 4.1 demonstrates quantitatively both that cost-of-living shocks do not move wages, and that monetary policy shocks cause job-to-job quits and wages to co-move, as in the data.

Section 4.2 presents quantitative results for pass through in an extension to the model in which increases in the price level make unemployment more attractive for a given nominal wage, demonstrating that quantitatively on-the-job search mutes the pass through of prices to wages in response to a cost-of-living shock. Finally, Section 5 compares analytical results for baseline wage posting model of Section 3, in which there is no increase in wages in response to the shock, to other standard models of labor supply used in the DSGE literature: specifically, a model with neoclassical labor supply and a model where unions set wages (Erceg et al., 2000).

4.1 Response of Wages to Cost-of-living and Monetary Policy Shocks: Baseline Model

This section analyzes the response of the baseline model described in Section 3 in response to both cost-of-living shocks (which do not move wages) and monetary policy shocks (which do move wages). We use the calibration presented in Table 2, except where noted, including the simplifying assumption $\eta = 1$, in light of the analytical finding that this corner case is not important for determining pass through in our model when monetary policy stabilizes total labor N_t in the service sector, as shown in Section 5 (and accompanying Appendix C.3).

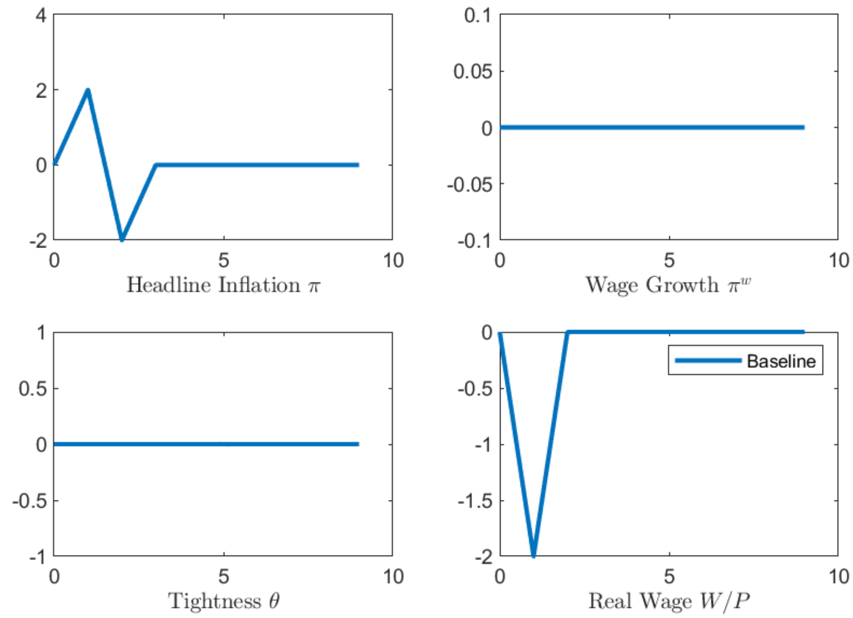
4.1.1 Cost-of-Living Shocks

We subject the economy to a 10% negative quantity shock of the endowment good X_t from $X_t = 1$. Given the assumption of a unit elasticity of substitution between services and goods in final aggregation, i.e., $\eta = 1$, this implies a 10% relative price shock to good X_t and an increase in the overall price index P_t. Additionally, we assume that the monetary authority leaves nominal interest rates pegged:[19] given the household's Euler equation (10) and the fact that $\eta = 1$ implies constant expenditure shares across services Y_t and endowments X_t, this experiment effectively holds aggregate demand for services constant, leaving the labor market unaffected; see Supplementary Appendix J for details, which works through the experiment in a 2-period version of the model and provides analytical results.[20]

[19] It turns out that monetary pegging corresponds to the Taylor rule (19), as service price inflation $\Pi_{Y,t}$ remains unchanged in response to our cost-of-living shocks.

[20] Therefore, pegged monetary policy in this environment effectively stabilizes N_t, as assumed in Section 5 unless firms charge higher service price $P_{y,t}$ in response to a cost-of-living shock, which is not the case without technology shocks. As seen in our price Phillips curve (B.11) in Appendix B that depends on wage inflation and the same set of labor market variables, our cost-of-living shocks do not affect the service price inflation as it does not affect equilibrium wage or those labor market variables. Supplementary Appendix H derives a log-linearized price Phillips curve and discuss how equilibrium service prices are determined.

Figure 3: Impulse Response to a 10% Negative Shock to Supply of Endowment Good



Notes: This figure presents the effects of a decreased supply of the endowment good X_t under a nominal interest rate peg, which we identify as a pure cost-of-living shock. Given the assumption of a unit elasticity between services and goods in final aggregation, this implies a 10% relative price shock to good X_t and an increase in the overall price index P_t. Given the household's Euler equation and constant expenditure shares, the nominal interest rate peg experiment effectively holds aggregate demand for services constant. Since the shock does not affect the relative attractiveness of unemployment and working, the recruiting and separation elasticities faced by firms are also unchanged as discussed in Section 3.2.2: the result is no change in vacancy posting, no change in tightness, and no change in the *nominal* wage, which causes *real* wages to fall as shown in the last panel.

Note that our nonlinear wage Phillips curve (7) in Section 3.1 expresses wage inflation as exclusively a function of aggregate, endogenous labor market variables: vacancies, employment, recruiting and separation rates, and recruiting and separation elasticities, with no direct role for aggregate price index P_t. This is the case for our linearized wage Phillips curve (21) as well where wage inflation is driven by fluctuations in quits rate and unemployment rate only. In this environment, unless the cost-of-living shock affects those labor market outcomes in equilibrium, there is no effect on wages.

Note that a crucial underlying mechanism is that due to household's consumption sharing scheme (9), the cost of living shock does not affect the relative attractiveness of unemployment and working, thereby not changing the recruiting and separation elasticities faced by firms as well: in general equilibrium, there is no change in vacancy posting, in tightness, in the *nominal* wage, which causes *real* wages to fall as shown in the last panel of Figure 3. In Section 4.2, we relax this assumption and assume that a higher aggregate price level changes those elasticities by changing the relative desribility of unemployment and employment. In that case, we will have the pass-through, which is not quantitatively significant.

Constant Relative Risk Aversion (CRRA) Utility Our result that a cost-of-living shock does not move wages given that monetary policy stabilizes labor markets depends crucially on the household's log preference with unit risk aversion (or elasticity of intertemporal substitution). Under higher-than-1 risk aversion, a rise in price level leads to higher recruiting and separation elasticities, thus providing an incentive for firms to raise wages to reduce turnover costs. As we document in Supplementary Appendix F however, this effect is quantitatively very small: under reasonable risk aversion levels, in response to around 2% price increase, a rise in wage growth is less than 0.1%.

4.1.2 Monetary Policy Shock

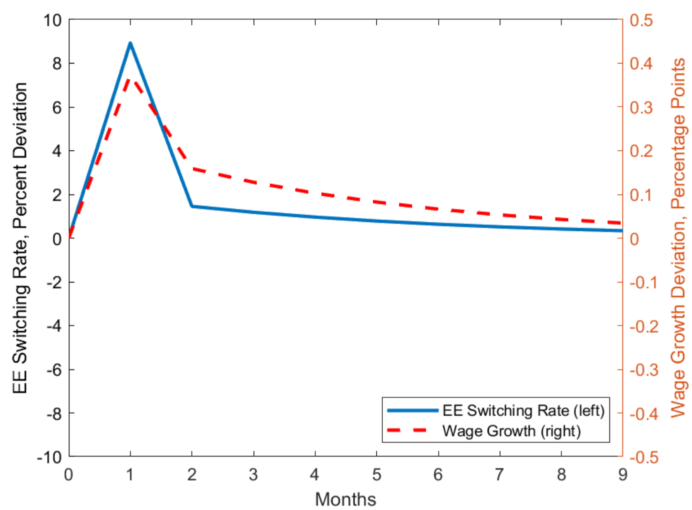
Now, we consider an expansionary monetary policy shock, subjecting the economy to a one period, 1 percentage point decrease in nominal interest rates, with a monthly persistence of 0.8. On impact, wage growth and employment-to-employment transitions rise together, as seen in Figure 4. Lower nominal interest rates raise demand for service consumption, which increases demand for labor. Firms then post more vacancies, increasing opportunities for workers to find other jobs, which raises job-to-job transitions, while also increasing competition among firms for workers, leading to higher wages. Supplementary Appendix J.1 provides analytical results about employment-to-employment transitions and wage growth based on a 2-period version of the model.

This result demonstrates that our model can rationalize comovements between quits rate and wage growth, documented in Figure 2 and the literature (Faberman and Justiniano, 2015; Moscarini and Postel-Vinay, 2017), through demand shocks like monetary policy shocks.

4.2 Extension: Cost-of-Living Shocks with Inflation-Indexed UI

This section revisits the experiment of Section 4.1.1 while relaxing the assumption that the relative desirability of unemployment and employment is held fixed by the household, allowing

Figure 4: Expansionary 1% Decrease in the Policy Rate



Notes: This figure plots the effects of a 1% decrease in nominal interest rates in our benchmark model. Both nominal wage growth and employment-to-employment transitions increase as lower nominal interest rates increase demand for consumption, which increases demand for labor. Firms post more vacancies, increasing opportunities for workers to find other jobs, which raises job-to-job transitions, while also increasing competition for workers, which raises wages. This result demonstrates that the model can rationalize comovements between quits and wage growth, documented in e.g., Faberman and Justiniano (2015); Moscarini and Postel-Vinay (2017), through demand shocks like monetary policy shocks.

the relative desirability of unemployment to rise along with the price level. We will show how on-the-job search mutes the pass through from wages to prices in this variant of the model.

4.2.1 Separation and Recruiting Rates

We now assume that households no longer fix the ratio of consumption between employed and unemployed workers, but instead guarantee unemployed workers some inflation-indexed quantity of consumption, b. For a given nominal wage, an increase in the price level now raises the relative consumption of unemployed agents, making unemployment more desirable. To see this, note that the probability that a worker separates from employment to unemployment is now

$$s_{ju}(W_{jt}) = \frac{b^\gamma}{\left(\frac{W_{jt}}{P_t}\right)^\gamma + b^\gamma}.$$

The separation rate s_{ju} from employment to unemployment now depends on the price level: at a given nominal wage, higher prices makes unemployment attractive. Similarly, the new recruiting function from unemployment is

$$r_{uj}(W_{jt}) = \frac{\left(\frac{W_{jt}}{P_t}\right)^\gamma}{\left(\frac{W_{jt}}{P_t}\right)^\gamma + b^\gamma},$$

where now we see that a higher price level makes recruiting from unemployment more difficult at a given nominal wage, by the same logic.

In a symmetric equilibrium where $W_{jt} = W_t$ for $\forall j$, the separation and recruiting rates become

$$S_t = s + (1-s)\left(\lambda_{EE} f(\theta_t)\frac{1}{2} + \lambda_{EU}\frac{b^\gamma}{\left(\frac{W_t}{P_t}\right)^\gamma + b^\gamma}\right),$$

and

$$R_t = g(\theta_t)\left(\phi_{E,t}\frac{1}{2} + \phi_{U,t}\frac{\left(\frac{W_t}{P_t}\right)^\gamma}{\left(\frac{W_t}{P_t}\right)^\gamma + b^\gamma}\right).$$

Unlike the benchmark case represented by (17) and (18), the price level P_t affects the recruiting and separation rates via the probability of quitting into unemployment and the probability of successfully recruiting unemployed workers.

All the other model equations (i.e. firms' first-order conditions and the Taylor rule) remain unchanged; Appendix B.4 shows how we can to derive an Euler equation in this setting which is identical to that used above, given appropriate assumptions on the representative household's optimization problem. We calibrate the model with a choice for unemployment benefit b instead of ξ; we set $b = 0.4$ which results in a steady-state consumption ratio for employed to unemployed agents of 2, so that this moment is the same at the steady state as in the benchmark model with $\xi = 2$.[21,22]

[21] Recall from the discussion in Table 2 that the consumption ratio $\xi = 2$ is higher than in Chodorow-Reich and Karabarbounis (2016) but closer to what maximizes the steady-state utility of households. While results in the benchmark model are insensitive to changing ξ, modifying the model to allow for pass-through from cost of living shocks to wages as we do here implies changes in b matter: lowering b might raise or reduce the pass-through of cost of living to wages depending on λ_{EE}, but does not affect the result that on-the-job search λ_{EE} mutes this pass-through. Quantitatively, changes in b do not affect the level of pass-through much.

[22] Supplementary Appendix I derives a log-linearized wage Phillips curve in this case with real b.

4.2.2 Result and the Role of On-the-Job Search λ_{EE}

First, we demonstrate that incorporating on-the-job search helps the model capture the empirical fact that changes in unemployment benefits do not seem to affect workers wages much in practice, even for new workers who are hired out of unemployment as shown by Jäger et al. (2020). Figure 5 demonstrates that in a model without on-the-job search, where $\lambda_{EE} \to 0$, changing unemployment benefit b has large effects on the equilibrium real wage, seen by examining the gaps between the blue solid line and dashed red line, for example. At our value of $\lambda = .14$ calibrated to U.S. data, we see that the same changes in b have almost no change in the equilibrium real wage offered by firms, as in the data. Thus, beyond the fact that incorporating on-the-job search is important to capture the fact that quits in Figure 2 are mostly job-to-job quits rather than quits into nonemployment, on-the-job search helps capture the near irrelevance of unemployment benefits for the wage.[23]

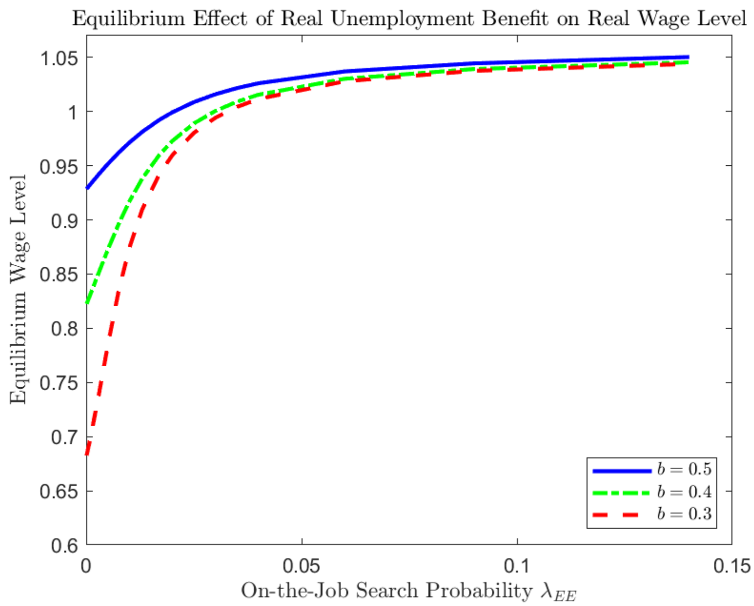
Next we consider how the response of wages to a cost-of-living shock differs under different frequencies of on-the-job search λ_{EE}. Figure 6 presents the impulse response function of wage growth to a cost-of-living shock in the log-linearized model under our standard calibration with $\lambda_{EE} = 0.14$: in the solid blue line following our benchmark calibration, we see that the effect on wage growth is quantitatively small. Intuitively, this is because the increase in the desirability of unemployment is not quantitatively relevant to firms who worry mainly about the risk of losing their workers to other firms, and recruiting workers on the job, than about quits to unemployment and/or recruiting unemployed workers, which our baseline model calibrated to U.S. data assumes is relatively uncommon.

To illustrate the importance of on-the-job search in delivering this result, we also estimate the impulse response function in the version of the model without on-the-job search, where the probability of being allowed to search on the job λ_{EE} is nearly zero, given by the red dashed line, finding that the response of wages becomes considerably larger. When λ_{EE} is low, firms' main concern when deciding wages becomes attracting unemployed workers into employment and discouraging quits to unemployment, since workers almost never have the chance to leave for to join another firm. Thus, firms raise wages more aggressively in response to a cost-of-living shock.[24]

[23]We can show qualitatively the identical result for the effect of unemployment benefits on the wage in the baseline model by varying ξ instead of b.

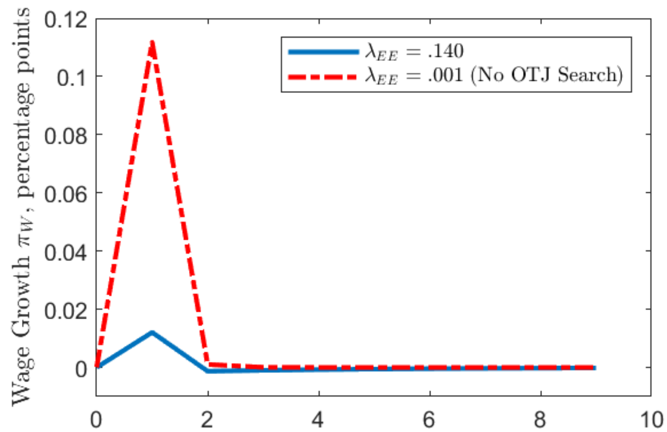
[24]Supplementary Appendix J confirms this result analytically in a 2-period version of the quantitative model.

Figure 5: On-the-Job Search Mutes the Effect of Changing Unemployment Benefits on Equilibrium Real Wages



Notes: In the model with fixed real unemployment benefits described in Section 4.2, eliminating the role of on-the-job search and sending $\lambda_{EE} \to 0$, means that changes in unemployment benefit b have large effects on the equilibrium real wage (denominated in the price of aggregate consumption P), seen by examining the gaps between the blue solid line and dashed red line, for example. This is because without on-the-job search, firms set wages mostly considering the problem of recruiting unemployed workers, which makes the level of b important in their wage-setting problem. At our value of $\lambda = .14$ calibrated to U.S. data, where firms mostly recruit from other firms, we see that the same changes in b have almost no change in the equilibrium real wage offered, as in the data: the three lines lie on top of each other at this point.

Figure 6: On-the-Job Search Mutes the Pass-Through of Cost of Living Shocks to Wages



Notes: This Figure presents the effects of a decreased supply of the endowment good X_t under a nominal interest rate peg, as in Figure 3, in a variant of the benchmark model where increased cost of living raises the desirability of unemployment as described in Section 4.2. While there is now some pass through from the cost-of-living shock to wages, on-the-job search significantly dampens this result, seen by comparing the results in calibrations where the on-the-job search probability, λ_{EE} is calibrated to match U.S. data (the solid blue line) to a calibration where workers are almost never allowed to search on the job (the dashed red line).

Potential Extension with Endogenous λ_{EE} Recent work by Pilossoph and Ryngaert (2023) find that inflation raises the rate at which workers search for job opportunities, and Pilossoph et al. (2023) study how an endogenous change in search probability affects workers wages in partial equilibrium. In Appendix A, we incorporate a simple extension where on-the-job search intensity λ_{EE} rises when real wages fall. We find that in general equilibrium, the pass-through of cost-of-living shocks to wages remains small, as greater on-the-job search intensity lowers labor market tightness, offsetting the positive wage impulse in partial equilibrium from greater on-the-job search.

4.3 Comparing Empirical and Model-Implied Wage Phillips Curves

In this section, we linearize the non-linear wage Phillips curve (7) and recast the log-linearized wage Phillips curve in terms of observable labor market variables. Because many labor market variables are linear combinations of each other in the log-linearized version of the model, there are many ways to express our wage Phillips curve in terms of observable variables. We focus on two specifications: in the first specification, wages depend on the job vacancy rate, the unemployment rate, and a real wage term. This is an interesting specification because we can test directly if the coefficients on vacancies and unemployment are symmetric (equal in magnitude but oppositely signed) as would be implied by search models without on the job search, such as Gagliardone and Gertler (2023). The second specification writes wage growth as a function of quits (instead of vacancies), the unemployment rate, and a real wage term, allowing us to evaluate the role of quits in our model and compare our model to the motivating evidence in Section 2.

Our calibrated model generates three predictions, all of which are supported in time-series estimates of the aggregate wage Phillips curve: (i) in a regression with vacancies and unemployment, the weight on vacancies is larger than the weight on unemployment; (ii) in a regression with quits and unemployment, quits dominates and the coefficient on unemployment is near zero; and (iii) the "catch up" of wages after higher cost of living pushes down the real wage is very weak. All these wage Phillips curve coefficients are untargeted moments, as the model was calibrated only to match labor market flows and the sensitivity of workers' mobility decisions to relative wages.

Linearized Wage Phillips Curve with Vacancies We begin by presenting the log-linearized wage Phillips curve, where wage inflation is a function of log deviations in the the vacancy rate $\check{V}_t$, the unemployment rate $\check{U}_t$, and a real wage term $\check{\tilde{w}}$

$$\check{\Pi}_t^w = \phi_V \check{V}_t + \phi_U \check{U}_{t-1} + \phi_{\tilde{w}} \check{\tilde{w}}_t + \frac{1}{1+\rho} \check{\Pi}_{t+1}^w, \tag{20}$$

where $\tilde{w}_t = \sum_{s=0}^{t-1} \Pi_s^w - \sum_{s=0}^{t} \Pi_s$ is a real wage term that takes into account the real wage last period and the realized price inflation in period t.[25] Therefore $\phi_{\tilde{w}}$ represents direct pass-through of prices to wages. When we estimate the OLS regression, we truncate the real wage term to include wage growth and price inflation up to 12 quarters prior, i.e. $s = t - 12$.

Panel A of Table 4 reports the model-implied and estimated coefficients in equation (20). In both the model and data, the coefficient on vacancies ϕ_V is positive, the coefficient on unemployment ϕ_U is negative, and the coefficient on real wages $\phi_{\tilde{w}}$ is negative. The magnitude of ϕ_V is larger than the magnitude of ϕ_U, suggesting that ratio of job openings to unemployed workers V/U does not summarize the effect of labor market variables on wage growth. Instead, fluctuations in the job openings rate have bigger effects on wages than similar percent fluctuations in the unemployment rate. While these coefficients are the result of the implicit

[25] Derivations cna be found in Appendix I.

weights of vacancies and unemployment that determine the recruiting rate, separation rate, and their elasticities, there is an intuitive reason why vacancies matter more than unemployment in wage growth: unemployed workers are not the only searchers in the labor market. Recall that labor market tightness is defined as $\theta = V/\mathcal{S}$, where the mass of searchers $\mathcal{S}$ is defined as $\mathcal{S} = U + \lambda_{EE}(1-U)$. As such, fluctuations in U have less of a proportionate effect on tightness θ than do fluctuations in V, and consequently less of an effect on wages.

Table 4 also reports coefficients for different values of convexity of vacancy costs χ, showing that the relative importance of V in affecting wage growth relative to U increases with the convexity of vacancy posting costs (recalling that per vacancy costs take the form of $c(V_t/N_{t-1})^{\chi}$). This is intuitive, as when χ is higher, each additional increment of vacancies raises the marginal cost of recruiting even more, incentivizing firms to raise wages more when firms are posting vacancies.

The third row of Panel A of Table 4 reports the model implied coefficients for our extended model with inflation-indexed unemployment benefits, which includes a non-zero coefficient on real wages. This coefficient is negative, since lower real wages would imply catch-up wage growth, as high cost of living improves the relative desirability of unemployment, making recruiting and retention more difficult for firms. However, the coefficient is very small, implying that a 10% drop in the real wage would increase quarterly nominal wage growth by only 0.3%.

The fourth and final row of Panel A in Table 4 reports the estimated OLS coefficients for equation (20) using US data from the fourth quarter of 1990 through 2023.[26] Consistent with the model, the coefficient on V is larger in absolute value than is the coefficient on U, though the difference is not quite as large as implied by the model. The coefficient on the catch-up term $\phi_{\tilde{w}}$ reports and elasticity of -0.019, indicating that a 10 percent decline in real wages would generate a 0.2 percent higher nominal wage growth in a quarter, which is similar to the magnitude implied by our "inflation-indexed UI" extension.

Linearized Wage Phillips Curve with Quits Returning to earlier evidence and motivation on quits and wage growth, we can rewrite the wage Phillips curve so that wage growth depends on the quits rate. To write down a wage Phillips curve of the similar form as equation (1), note that we can define quits Q_t as all separations S_t less the exogenous separations s:

$$Q_t = S_t - s,$$

[26] Prior to 2001, the job openings rate comes from the Help-Wanted Index in Barnichon (2010) and Michaillat and Saez (2022). Beginning in 2001, the jobs opening rate comes from the Job Openings and Labor Turnover Survey (JOLTS).

Table 4: Structural Wage Phillips Curve Coefficients vs. OLS Coefficients

Panel A: Vacancies V_t and Unemployment U_{t-1}			
Source	ϕ_V	ϕ_U	$\phi_{\tilde{w}}$
Baseline Model ($\chi = 1$)	5.49	-0.90	0
Baseline Model ($\chi = 0$)	2.85	-1.90	0
Real Unemployment Benefit Model ($\chi = 1$)	5.49	-0.90	-.030
OLS using ECI 1990-Present	0.40***	-0.22*	-.019*
	(0.12)	(0.12)	(.010)

Panel B: Quits Q_t and Unemployment U_{t-1}			
Source	β_Q	β_U	$\beta_{\tilde{w}}$
Baseline Model ($\chi = 1$)	7.38	0.27	0
Baseline Model ($\chi = 0$)	6.39	-0.33	0
Real Unemployment Benefit Model ($\chi = 1$)	7.38	0.27	.0426
OLS using ECI 1990-Present	1.11***	-0.04	-.021***
	(0.16)	(0.07)	(.007)
	(0.0016)	(0.0008)	

Standard errors in parentheses (Newey-West; 4 lags)
*** p<0.01, ** p<0.05, * p<0.1

Notes: The models' structural wage Phillips curve is broadly in line with the OLS estimates from U.S. data, putting much more weight on quits than unemployment. The table compares OLS estimates with results from two calibrations: the baseline calibration with $\chi = 1$ (convex vacancy posting costs) and $\chi = 0$ (linear vacancy posting costs). See Table 2 for other parameter choices. The models' structural wage Phillips curve is converted into quarterly frequency.

so quits in the model captures both voluntary job-to-job quits and voluntary quits from employment into unemployment. Since the quits rate itself is a function of vacancies, unemployment, and the real wage, the wage Phillips curve (20) can in turn be written in terms of quits $\check{Q}_t$, unemployment $\check{U}_{t-1}$, and real wage $\check{\tilde{w}}_t$ as[27]

$$\check{\Pi}^w_t = \beta_Q \check{Q}_t + \beta_U \check{U}_{t-1} + \beta_{\tilde{w}} \check{\tilde{w}}_t + \frac{1}{1+\rho} \check{\Pi}^w_{t+1} \tag{21}$$

for some positive $\beta_Q > 0$ and β_U of indeterminate sign which depends on the calibration (both coefficients are functions of parameters and steady state values). The exact value of β_U will depend on whether the relative weights on V and U in predicting Π^w and Q, which we derive in Appendix B.3.1.

[27]For this part of the derivation, i.e., expressing $\check{Q}_t$ in terms of $\check{V}_t$, $\check{U}_{t-1}$, and $\check{\tilde{w}}_t$, see Appendix I.

The possibility of a positive coefficient on the unemployment rate may seem surprising, since the coefficient on the unemployment rate in the wage Phillips curve traditionally is negative. However, this can arise because the quits rate already contains information on the unemployment rate: because workers' quit rate depends on their job finding probability $f(\theta_t)$, which itself depends on tightness θ_t and subsequently on the unemployment rate U_{t-1}, the extent that the unemployment rate affects wage growth may be entirely or more than entirely accounted for by its effect on wages through quits. Therefore after controlling for quits, the coefficient on the unemployment rate may be "wrong-signed". This happens to occur in the first and third rows of Panel B in Table 4 when $\chi = 1$. Regardless of the sign of β_U, for reasonable calibrations we find that β_U is almost exactly zero in the model once also conditioning on the quits rate, which is confirmed by the empirical estimates in the fourth row of 4 Panel B, as well as in Table 1.

Just as the inclusion of the quit rate can flip the sign of β_U, including the quits rate may also flip the sign of $\beta_{\tilde{w}}$, at least in theory. If unemployment benefits are inflation indexed, then an increase in the cost of living will lower real wages, make recruiting and retention more difficult, and push nominal wages up. However, this effect will be partially reflected in the quit rate: when real wages fall and unemployment becomes relatively more desirable, quits will rise in response. Therefore, as was the case with unemployment, there is information about the level of the real wage embedded in the quits rate. As a consequence, once accounting for the quits rate, the coefficient on the real wage term may become positive, as least in theory. As the fourth row of Panel B shows, the OLS regression with quits, unemployment, and the real wage term still produces a negative coefficient of -0.021, which is not predicted by this calibration. However, the coefficient is still remarkably small, consistent with very weak pass-through as shown in Figure 5.

In total, the wage Phillips curve predicted by our model matches three facts about the US wage Phillips curve since 1990: (i) in a regression with vacancies and unemployment, the weight on vacancies is larger than the weight on unemployment; (ii) in a regression with quits and unemployment, quits dominates and the coefficient on unemployment is near zero; and (iii) the catch up of wages after higher cost of living pushes down the real wage is very weak. These results, combined with the model matching microeconomic evidence on how wages are determined, lead us to conclude that our model is a good representation of wage determination and the wage Phillips curve for the United States.

Hiring Costs Throughout this paper, we assumed that all of firms' recruiting costs are in posting vacancies. In practice, a larger portion of the cost of acquiring new workers occurs after the search and matching process (Blatter et al., 2012). If firms recruiting costs were primarily in hiring or training costs, rather than vacancy costs, would it matter for our wage Phillips curves? In Supplementary Appendix G, we assume in addition to the convex vacancy costs, a firm needs to pay a direct hiring cost, which may be convex in the number of new hires each period as in e.g., Moscarini and Postel-Vinay (2016b). Even in this case, we show that the log-linearized wage Phillips curve can be written solely in vacancies (or quits rate) and unemployment, and the vacancy rate (or quits rate) still becomes a much more important driver of wage growth than unemployment. The coefficient on unemployment might be positive or negative depending on the convexity of the hiring cost function, as in Table 4. Therefore, our result that wage growth is mostly driven by vacancy creation or quits is robust across different model specifications.

5 Comparison with Other Wage Setting Models

In this section, we show how our framework with wage posting and on-the-job search differs from two common labor blocks in macroeconomic models: the standard neoclassical labor supply model and the sticky wage model with differentiated labor and union wage setting as in Erceg et al. (2000). In all three models, pass-through of cost-of-living shocks ultimately depends on workers' labor supply responses, based on how the cost-of-living shock affects the relatively desirability of work and non-work (work hours and leisure in the neoclassical and union models, employment and unemployment in our model). In practice for the neoclassical and union wage setting models, it will matter greatly how much non-labor income is generated from the cost-of-living shock: if non-labor income goes up, this generates a wealth effect that decreases household labor supply, which results in higher nominal wage growth. In our setting, because unemployment is rarely a desirable outside option, wages are primarily determined by competition for already employed workers, so changes in the relative value of employment versus unemployment has little effect on wages.

Pass-Through with Neoclassical Labor Supply and Flexible Wages Consider an alternative model where firms still have sticky prices and a similar production environment (so the consumption and price aggregators (2) and (3) are unchanged), but the labor supply block is neoclassical. Households maximize

$$\sum_{t=0}^{\infty} \beta^t \left(\ln C_t - \frac{1}{1+\frac{1}{\nu}} N_t^{1+\frac{1}{\nu}} \right)$$

given the usual budget constraint

$$C_t = \frac{D_t}{P_t} - \frac{B_t}{P_t} + \frac{(1+i_{t-1,t})B_{t-1}}{P_t} + \frac{W_t}{P_t} N_t,$$

so that labor N_t is hired in a spot market and there is no unemployment. Under the assumption that the central bank stabilizes employment $N_t = N$ in response to a negative shock to quantity of the endowment good X_t at time $t = 0$, wages rise only if $\eta < 1$, i.e., the elasticity of substitution between X_t and Y_t is relatively weak. The sign and magnitude of the response in wages depends on the strength of *income* and *substitution* effects, governed by the elasticity of intertemporal substitution of consumption utility, and *wealth* effects when workers are endowed both with leisure and the good X_t. Because utility of consumption is log, income and substitution effects on labor supply from a lower real wage cancel out. Therefore wealth effects will determine the labor supply response of households. The direction of those wealth effects are governed by η, the degree of substitution between X_t and Y_t. When $\eta < 1$, a cost of living shock as described above generates positive wealth effects, as prices for the endowment good rise more than one-for-one with the decline in quantity. This makes the household want to supply less labor, so firms must raise wages if N_t is to be stabilized at its pre-shock level.[28] If $\eta > 1$, the opposite logic will hold: workers will want to work more, and wages will actually fall in response to the shock. Thus, even in a perfectly competitive labor market, workers' wages can respond to a cost-of-living shock even when their productivity stays constant.[29]

Pass-Through with Differentiated Labor and Union Wage Setting Given the consumption and price aggregators (2) and (3), now let us assume that households now supply multiple types of labor; unions set wages for each type to maximize household utility subject to downward-sloping labor demand for each type from a "labor packer" which packages each labor type $\{N_t(i)\}$ into aggregate labor $N_t = \left(\int_0^1 N_t(i)^{\frac{1+\nu}{\nu}} di \right)^{\frac{\nu}{1+\nu}}$ which is purchased at wage W_t by services firms. Wages are sticky as in Erceg et al. (2000) and Galí et al. (2012) as unions

[28] See Appendix C.1 for a proof.

[29] Appendix C.1 also analyzes the non-log case of a general consumption utility with the elasticity of intertemporal substitution σ possibly different from 1 in this neoclassical labor supply model.

only occasionally receive the chance to reset their wage. The aggregate household maximizes

$$\sum_{t=0}^{\infty} \beta^t \left(\ln C_t - \int_0^1 \frac{1}{1+\frac{1}{\nu}} N_t(i)^{1+\frac{1}{\nu}} di \right),$$

given the same budget constraint above. We can again prove that in response to a shock to X_0 shock at $t = 0$, there is pass-through only if $\eta < 1$. We prove this result in Appendix C.2. The similar result as in neoclassical labor block arises because income, substitution, and wealth effects operate in a similar way, with the only difference being that wages are sticky and unions mark up wages above the marginal rate of substitution.

Our Setting: Wage Posting and On-the-Job Search In the two common alternative models discussed in this section (a neoclassical labor block and a union wage setting model with differentiated labor), the response of wages to a cost-of-living shock is ultimately a function of workers' labor supply response to higher cost of living. The importance of workers' labor supply response is also present in our setting of wage posting and on-the-job search: in our baseline model, by construction we eliminate the role of wealth effects by fixing ratio of consumption by employed and unemployed household members, and so workers' labor supply is unchanged. In our extension of inflation-adjusted unemployment benefits, workers are more likely to quit into unemployment and less likely to accept job offers from unemployment when the cost of living goes up: the discrete choice analogue to a decrease in labor supply. Our setting ultimately delivers *quantitatively* low pass-through because on-the-job search makes workers' extensive margin labor supply response nearly irrelevant in firms' wage setting decision, as illustrated in Section 4.2.2.

6 Conclusion

This paper develops a model of wage determination with labor market frictions where firms both set prices and post wages, subject to nominal rigidities in price and wage setting, and workers search on the job. Calibrated to match U.S. data on worker flows, the model implies a simple wage Phillips curve expressing nominal wage growth as a function of log deviations of quits and unemployment from their long-run natural (steady state) values that is quantitatively in line with results for recent U.S. aggregate data.

We then analyze the propagation of cost-of-living shocks in the model economy. Because firms set wages to avoid costly turnover, such shocks pass through to wages only to the extent that higher cost of living improves worker's outside options, such as competing jobs or unemployment, relative to their current job. As higher cost of living lowers real wages at all jobs evenly, and unemployment is rarely a credible outside option in the modern U.S. economy, we find that cost-of-living shocks have little to no effect on relative outside options and therefore wages.

While stylized, our model is consistent with a range of recent microeconomic evidence on how wages are determined, including the result in Jäger et al. (2020) that wages are insensitive to the flow value of unemployment benefits, and direct evidence on the preponderance of wage posting (Hall and Krueger, 2012; Lachowska et al., 2022; Di Addario et al., 2023). Admittedly, our simple model does abstract from the fact that there are a minority of unionized workers in the United States, and workers with automatic COLAs, for whom prices would pass through into wages. However, our results suggest that in a setting such as the United States where few workers operate under collective bargaining agreements with cost-of-living adjustments, and where firms' wage setting decision reflects competition for already-employed rather than for unemployed workers, the ability for most workers to reclaim real wages in response to a supply shock that raises their cost of living is limited. We conclude that there is little scope for supply-shock induced wage-price spirals specifically fueled by workers' ability to command higher nominal wages in response to higher nominal prices.

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A Extension with Variable On-the-Job Search Intensity

Our baseline model features an exogenous, constant on-the-job search probability of λ_{EE}, which we calibrate to match U.S. data. However, it is possible that employed workers may respond to a pure cost-of-living shock by searching more intensely. Both Pilossoph and Ryngaert (2023) and Hajdini et al. (2023) provide evidence from household surveys that this is indeed the case.

Motivated by these findings, we solve a version of the model where λ_{EE} is assumed to rise along with inflation according to a reduced form, *ad hoc* relationship calibrated to match the results in Pilossoph and Ryngaert (2023). Specifically, we assume:

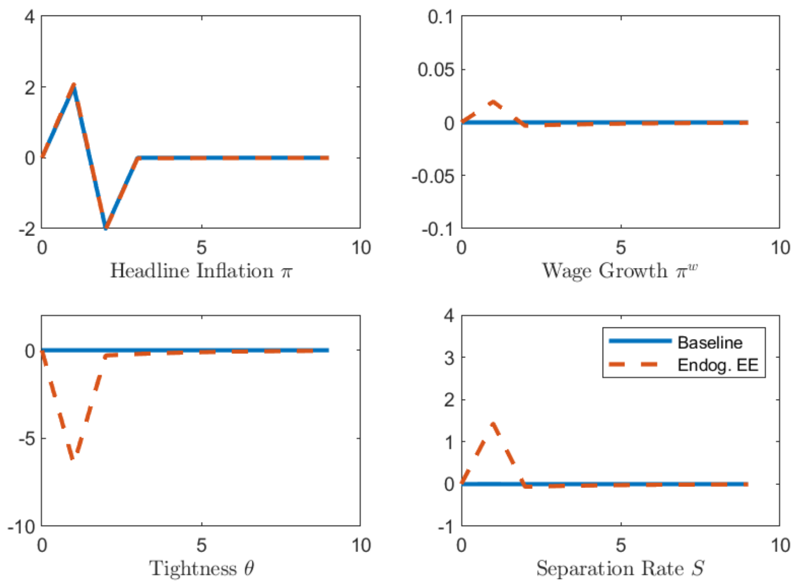
$$\lambda_{EE,t} = \lambda_{EE,0} \left(\frac{W_t}{P_t} \right)^{-m},$$

where $\lambda_{EE,0}$ is chosen to target the same steady-state value for λ_{EE} as in the benchmark model, and $m = 4$ to match the fact that Pilossoph and Ryngaert (2023) find that in response to a one percentage point increase in inflation expectations (and thus a 1% decline in expect real wages), the probability that an employed worker searches on the job rises by 0.57 percentage points.[1] With a share of 14.9% of employed workers typically searching, this represents a (0.0057/0.149) $\approx$ 4 percent increase in search probability, yielding an elasticity of search probability with respect to expected real wages of -4.

We then revisit the response in the model to a shock to the quantity of the endowment good X_t. Note that here, there are two contrasting effects of allowing for endogenous on-the-job search probability. In response to the inflationary shock, workers search more which induces firms to raise wages in order to retain workers (more searchers means more workers find jobs they prefer to their current jobs, due to the idiosyncratic preference shocks over workplaces). However, as separation rates rise, so do recruiting rates: with more searchers, tightness falls, and thereby firms can afford to lower wages and still recruit the same number of workers are before. Figure A.1 plots the impulse responses of headline inflation, wage growth, labor market tightness, and the separation rate to the shock to the quantity of endowment good X_t. We can observe the net effect of the shock is an extremely limited pass-through from cost-of-living to wages: separations and wage growth rise, pushing firms to want to raise wages, but

[1] See equation (3) and accompanying Table 3 of Pilossoph and Ryngaert (2023). Hajdini et al. (2023) estimate a much smaller effect in an information treatment RCT, finding that a one percentage point increase in inflation expectations raises the probability that a household respondent will "apply for a job(s) that pays more" by only about 0.11 percentage points.

Figure A.1: Impulse Response with Endogenous On-the-Job Search



Notes: This figure presents the effects of a decreased supply of the endowment good X_t under a nominal interest rate peg, i.e. the same experiment as in Figure 3, but comparing the benchmark case (solid blue line) of the main text to the case described in Section A where the job-to-job search probability increases along with the price level (dashed red line). In this second model, as on-the-job search rises, separations rise, inducing firms to raise wages to retain workers. But at the same time, tightness θ_t falls due to the increasing number of searchers, pushing firms to lower wages. The net effect is the modest increase in wages in the top right panel, so that overall there is very little pass-through from the aggregate price to wages even when the probability of on-the-job search rises in response to lower real wages. Note that the axes are in percent deviations, so the axis for wage growth is comparable to Figure 6.

on the other hand tightness θ_t falls due to the increasing number of searchers, pushing firms to lower wages. In sum, wages respond positively but modestly in response to the cost of living shock.

Online Appendix for
Do Cost-of-Living Shocks Pass Through to Wages?

JUSTIN BLOESCH SEUNG JOO LEE JACOB P. WEBER

B Derivations and Proofs

B.1 Worker's Problem with Utility From Leisure

This section reviews the worker's problem, deriving the probability a worker chooses a particular job j over outside offer k or unemployment. We then show that allowing for utility from leisure, as well as consumption, will not generally overturn the result that the price level does not affect the worker's optimal choice unless the elasticity of substitution between leisure and consumption is different from one.

Discrete choice with Type-1 extreme value preference draws Suppose worker i in state j (which could be working at firm j, for example), gets utility $\mathcal{U}(ijt)$ plus a draw ι_{ijt} that is distributed type-1 extreme value:

$$\mathcal{V}_t(i,j) = \mathcal{U}(ijt) + \iota_{ijt}$$

Let ι_{ijt} have variance $\frac{1}{\gamma}$. Then given options two states j and k, the probability that the worker chooses j is

$$\frac{\exp(\gamma\mathcal{U}(ijt))}{\exp\left(\gamma\mathcal{U}(ijt)\right) + \exp\left(\gamma\mathcal{U}(ikt)\right)}.$$

Suppose now that utility $\mathcal{U}$ is a function of log consumption: $\mathcal{U}(ijt) = \log(C_t(i,j))$. This is the case in the main text. Then the probability of choosing j is

$$\frac{C_t(i,j)^\gamma}{C_t(i,j)^\gamma + C_t(i,k)^\gamma}.$$

Case with a more general utility function Consider now the more general form

$$\mathcal{V}(ijt) = \log\left(U\left(C_t(i,j), \ell_t(i,j)\right)\right) + \iota_{ijt}$$

where $\ell_t(i,j)$ is the leisure i gets in state j at time t, which nests the above case. For simplicity, denote utility while unemployed by $U\left(C_t(i,u), \ell_t(i,u)\right)$, and while employed by $U\left(C_t(i,e), \ell_t(i,e)\right)$; then the probability of an unemployed worker taking a job when matched is now:

$$\frac{1}{1+\left(\frac{U(C_t(i,u),\ell_t(i,u))}{U(C_t(i,e),\ell_t(i,e))}\right)^{\gamma}} \tag{B.1}$$

Proposition 1 *In partial equilibrium (i.e. holding all other equilibrium prices and quantities fixed) the probability that an unemployed worker takes a job in our general setting, (B.1), is invariant to changes in the price level P_t if and only if $\frac{\partial}{\partial P_t}\frac{U(C_t(i,u),\ell_t(i,u))}{U(C_t(i,e),\ell_t(i,e))}=0$.*

CES preference To make progress, consider the case with CES preferences:

$$U=\left(aC^{\frac{\rho-1}{\rho}}+(1-a)\ell^{\frac{\rho-1}{\rho}}\right)^{\frac{\rho}{\rho-1}}$$

where ρ is the elasticity of substitution. We write $U(C,\ell)=U\left(\frac{I}{P},\ell\right)$, imposing $C=\frac{I}{P}$ for both types, who differ only in the nominal spending I (i.e. I_e for employed and I_u for unemployed)[2] Noting constant returns to scale (CRS) yields

$$\frac{U\left(\frac{I_t(i,u)}{P_t},\ell_t(i,u)\right)}{U\left(\frac{I_t(i,e)}{P_t},\ell_t(i,e)\right)}=\frac{U\left(I_t(i,u),P_t\ell_t(i,u)\right)}{U\left(I_t(i,e),P_t\ell_t(i,e)\right)},$$

and using the property of CES functions: $\frac{\partial}{\partial P}U(I,P\ell)=(1-a)U(\cdot)^{\frac{1}{\rho}}(P\ell)^{-\frac{1}{\rho}}l$, we can show:

$$\begin{aligned}\frac{\partial}{\partial P_t}\frac{U\left(\frac{I_t(i,u)}{P_t},\ell_t(i,u)\right)}{U\left(\frac{I_t(i,e)}{P_t},\ell_t(i,e)\right)}=&\frac{(1-a)P_t^{\frac{1}{\rho}}}{U(I_t(i,e),P_t\ell_t(i,e))}\\&\cdot\left[U(I_t(i,u),P_t\ell_t(i,u))^{\frac{1}{\rho}}\ell_t(i,u)^{1-\frac{1}{\rho}}\right.\\&\left.-U(I_t(i,e),P_t\ell_t(i,e))^{\frac{1}{\rho}}\ell_t(i,e)^{1-\frac{1}{\rho}}\frac{U(I_t(i,u),P_t\ell_t(i,u))}{U(I_t(i,e),P_t\ell_t(i,e))}\right]\end{aligned}$$

which becomes 0 when $\rho\to 1$, i.e. the Cobb-Douglas case. Therefore, under the unit elasticity of substitution between consumption and leisure, Proposition 1 still holds.

[2] Here, the result does not depend on the case where we impose a tax-and-transfer scheme to keep $I_e/I_u = C_e/C_u$ constant over the business cycle as in Section 3.2.

B.2 Firm's Problem and Derivation of the wage Phillips curve in (7)

The firm's problem is:[3]

$$\max_{\substack{\{P^j_{y,t}\},\{N_{jt}\}\\ \{W_{jt}\},\{V_{j,t}\}}} \sum_{t=0}^{\infty}\left(\frac{1}{1+\rho}\right)^t \left(P^j_{y,t}Y^j_t - W_{jt}N_{jt} - c\left(\frac{V_{j,t}}{N_{j,t-1}}\right)^{\chi} V_{j,t}W_t - \frac{\psi}{2}\left(\frac{P^j_{y,t}}{P^j_{y,t-1}}-1\right)^2 Y^j_t P^j_{y,t} \right.$$
$$\left. - \frac{\psi^w}{2}\left(\frac{W_{jt}}{W_{j,t-1}}-1\right)^2 W_{jt}N_{jt}\right) \tag{B.2}$$

subject to

$$N_{jt} = (1-S_t(W_{jt}))N_{j,t-1} + R_t(W_{jt})V_{j,t}. \tag{B.3}$$

Output is produced with labor with linear production: $Y^j_t = A^j_t N_{jt}$,[4] and Dixit-Stiglitz demand $\frac{Y^j_t}{Y_t} = \left(\frac{P^j_{y,t}}{P_{y,t}}\right)^{-\epsilon}$, hence $N_{jt} = \left(\frac{P^j_{y,t}}{P_{y,t}}\right)^{-\epsilon}\frac{Y_t}{A^j_t}$ with $\epsilon > 1$. The Lagrangian is given by:

$$\mathcal{L} = \sum_{t=0}^{\infty}\left(\frac{1}{1+\rho}\right)^t \left((P^j_{y,t})^{1-\epsilon}(P_{y,t})^{\epsilon} Y_t - W_{jt}\left(\frac{P^j_{y,t}}{P_{y,t}}\right)^{-\epsilon}\frac{Y_t}{A^j_t} - c\,(V_{j,t})^{1+\chi}\left(\frac{P^j_{y,t-1}}{P_{y,t-1}}\right)^{\epsilon\chi}\left(\frac{Y_{t-1}}{A^j_{t-1}}\right)^{-\chi} W_t \right.$$
$$- \frac{\psi}{2}\left(\frac{P^j_{y,t}}{P^j_{y,t-1}}-1\right)^2 (P^j_{y,t})^{1-\epsilon}(P_t)^{\epsilon}Y_t - \frac{\psi^w}{2}\left(\frac{W_{jt}}{W_{j,t-1}}-1\right)^2 W_{jt}\left(\frac{P^j_{y,t}}{P_{y,t}}\right)^{-\epsilon}\frac{Y_t}{A^j_t}$$
$$\left. + \lambda^j_t\left[-\left(\frac{P^j_{y,t}}{P_{y,t}}\right)^{-\epsilon}\frac{Y_t}{A^j_t} + V_{j,t}R_t(W_{jt}) + (1-S_t(W_{jt}))\left(\frac{P^j_{y,t-1}}{P_{y,t-1}}\right)^{-\epsilon}\frac{Y_{t-1}}{A^j_{t-1}}\right]\right).$$

The first order conditions are:

$$\mathcal{L}_{W_{jt}} = -\left(\frac{P^j_{y,t}}{P_{y,t}}\right)^{-\epsilon}\frac{Y_t}{A^j_t} + \lambda^j_t\left(V_{j,t}R'_t(W_{jt}) - S'_t(W_{jt})\left(\frac{P^j_{y,t-1}}{P_{y,t-1}}\right)^{-\epsilon}\frac{Y_{t-1}}{A^j_{t-1}}\right)$$
$$- \frac{\psi^w}{2}\left(\frac{W_{jt}}{W_{j,t-1}}-1\right)^2\left(\frac{P^j_{y,t}}{P_{y,t}}\right)^{-\epsilon}\frac{Y_t}{A^j_t} - \psi^w\left(\frac{W^j_t}{W_{j,t-1}}-1\right)\frac{1}{W_{j,t-1}}W_tN_{jt}$$
$$+ \frac{1}{1+\rho}\psi^w\left(\frac{W_{j,t+1}}{W_{jt}}-1\right)\frac{W^j_{t+1}}{(W_{jt})^2}W_{j,t+1}N_{j,t+1} = 0. \tag{B.4}$$

[3]Note that here we assume that vacancy costs are denominated in labor; see Bloesch and Weber (2023) for microfoundations and Appendix C.3 for additional implications. We also use the aggregate wage W_t rather than the firm-specific wage W_{jt} to simplify the firm's wage setting problem.

[4]Instead of production function $Y^j_t = N^j_t$ assumed in Section 3, we assume a linear technology $Y^j_t = A^j_t N^j_t$ in the derivation. Later we will assume a symmetric equilibrium with $A^j_t = A_t$ for $\forall j$.

and

$$\mathcal{L}_{V_{j,t}} = -c(1+\chi)(V_{j,t})^{\chi}\left(\frac{P^j_{y,t-1}}{P_{y,t-1}}\right)^{\epsilon\chi}\left(\frac{Y_{t-1}}{A^j_{t-1}}\right)^{-\chi} W_t + \lambda^j_t R_t(W_{jt}) = 0, \tag{B.5}$$

and

$$\begin{aligned}
\mathcal{L}_{P^j_{y,t}} =& (1-\epsilon)\left(\frac{P^j_{y,t}}{P_{y,t}}\right)^{-\epsilon} Y_t + \epsilon W_{jt}\left(P^j_{y,t}\right)^{-1}\left(\frac{P^j_{y,t}}{P_{y,t}}\right)^{-\epsilon}\frac{Y_t}{A^j_t} \\
&- \frac{c\epsilon}{1+\rho}\chi\left(V_{j,t+1}\right)^{1+\chi}\left(P^j_{y,t}\right)^{-1}\left(\frac{P^j_{y,t}}{P_{y,t}}\right)^{\epsilon\chi}\left(\frac{Y_t}{A^j_t}\right)^{-\chi} W_{t+1} \\
&- \psi\left(\frac{P^j_{y,t}}{P^j_{y,t-1}} - 1\right)\frac{1}{P^j_{y,t-1}}(P^j_{y,t})^{1-\epsilon}P^{\epsilon}_{y,t}Y_t - (1-\epsilon)\frac{\psi}{2}\left(\frac{P^j_{y,t}}{P^j_{y,t-1}} - 1\right)^2\left(\frac{P^j_{y,t}}{P_{y,t}}\right)^{-\epsilon} Y_t \\
&+ \frac{1}{1+\rho}\psi\left(\frac{P^j_{t+1}}{P^j_t} - 1\right)\frac{P^j_{t+1}}{\left(P^j_{y,t}\right)^2}\left(P^j_{t+1}\right)^{1-\epsilon}P^{\epsilon}_{t+1}Y_{t+1} \\
&+ \epsilon\frac{\psi^w}{2}\left(\frac{W_{jt}}{W_{j,t-1}} - 1\right)^2\frac{W_{jt}}{P_{y,t}}\left(\frac{P^j_{y,t}}{P_{y,t}}\right)^{-\epsilon-1}\frac{Y_t}{A^j_t} \\
&+ \lambda^j_t\epsilon\left(P^j_{y,t}\right)^{-1}\left(\frac{P^j_{y,t}}{P_{y,t}}\right)^{-\epsilon}\frac{Y_t}{A^j_t} - \frac{1}{1+\rho}\lambda^j_{t+1}\epsilon(1 - S_t(W_{j,t+1}))(P^j_{y,t})^{-1}\left(\frac{P^j_{y,t}}{P_{y,t}}\right)^{-\epsilon}\frac{Y_t}{A^j_t} = 0.
\end{aligned} \tag{B.6}$$

Equilibrium We focus on one particular equilibrium where $P^j_{y,t} = P_{y,t}$, $V_{j,t} = V_t$, $W_{jt} = W_t$, $A^j_t = A_t\ \forall j$. Then we can summarize the above equations as follows:

FOC on Wages in (B.4) : If we define the aggregate wage inflation $\Pi^w_t = \frac{W_t}{W_{t-1}}$ and approximate with $(\Pi^w_t - 1)^2 \simeq 0$, equation (B.4) becomes

$$\begin{aligned}
&-N_t + \frac{\lambda_t}{P_{y,t}}\left(\frac{W_t}{P_{y,t}}\right)^{-1}\left(V_t R'_t(W_t)W_t - N_{t-1}S'_t(W_t)W_t\right) - \psi^w\left(\Pi^w_t - 1\right)\Pi^w_t N_t \\
&+ \frac{1}{1+\rho}\psi^w\left(\Pi^w_{t+1} - 1\right)\left(\Pi^w_{t+1}\right)^2 N_{t+1} = 0.
\end{aligned} \tag{B.7}$$

This is important so that we have the real wage and real Lagrange multiplier, i.e., $\frac{\lambda_t}{P_{y,t}}$ in our equilibrium equations.

FOC on vacancies in (B.5) :

$$-c(1+\chi)\left(\frac{V_t}{N_{t-1}}\right)^{\chi}\frac{W_t}{P_{y,t}}+\frac{\lambda_t}{P_{y,t}}R_t(W_t)=0. \tag{B.8}$$

Plugging in (B.8) into (B.7) and rearranging gives:

$$\begin{aligned}
N_t+\psi^w\left(\Pi_t^w-1\right)\Pi_t^w N_t &= c(1+\chi)\left(\frac{V_t}{N_{t-1}}\right)^{\chi}\frac{1}{R(W_t)}\left(V_t R_t'(W_t)W_t-N_{t-1}S_t'(W_t)W_t\right)\\
&\quad+\frac{1}{1+\rho}\psi^w\left(\Pi_{t+1}^w-1\right)(\Pi_{t+1}^w)^2 N_{t+1}\\
&= c(1+\chi)\left(\frac{V_t}{N_{t-1}}\right)^{\chi}\left(V_t\underbrace{\frac{R_t'(W_t)W_t}{R_t(W_t)}}_{\equiv\varepsilon_{R,W_t}}-N_{t-1}\frac{S_t(W_t)}{R_t(W_t)}\underbrace{\frac{S_t'(W_t)W_t}{S_t(W_t)}}_{\equiv\varepsilon_{S,W_t}}\right)\\
&\quad+\frac{1}{1+\rho}\psi^w\left(\Pi_{t+1}^w-1\right)(\Pi_{t+1}^w)^2 N_{t+1}.
\end{aligned}$$

Dividing by N_t in both sides, we obtain

$$\begin{aligned}
\psi^w\left(\Pi_t^w-1\right)\Pi_t^w+1 =&c(1+\chi)\left(\frac{V_t}{N_{t-1}}\right)^{\chi}\left(\frac{V_t}{N_t}\varepsilon_{R,W_t}-\frac{N_{t-1}}{N_t}\frac{S_t(W_t)}{R_t(W_t)}\varepsilon_{S,W_t}\right)\\
&+\frac{1}{1+\rho}\psi^w\left(\Pi_{t+1}^w-1\right)(\Pi_{t+1}^w)^2\frac{N_{t+1}}{N_t},
\end{aligned} \tag{B.9}$$

which is the wage Phillips curve in our model.

FOC on pricing in (B.6) :

$$\begin{aligned}
&(1-\epsilon)+\epsilon\frac{W_t}{P_{y,t}}A_t^{-1}-\frac{1}{1+\rho}c\epsilon\chi\left(V_{t+1}\right)^{1+\chi}\frac{P_{y,t+1}}{P_{y,t}}\frac{W_{t+1}}{P_{y,t+1}}Y_t^{-1-\chi}A_t^{\chi}+\epsilon\frac{\psi^w}{2}\underbrace{\left(\Pi_t^w-1\right)^2}_{\simeq 0}\frac{W_t}{P_{y,t}}N_t\\
&\quad-\psi\left(\frac{P_{y,t}}{P_{y,t-1}}-1\right)\frac{P_{y,t}}{P_{y,t-1}}-(1-\epsilon)\frac{\psi}{2}\underbrace{\left(\frac{P_{y,t}}{P_{y,t-1}}-1\right)^2}_{\simeq 0}+\frac{1}{1+\rho}\psi\left(\frac{P_{y,t+1}}{P_{y,t}}-1\right)\left(\frac{P_{y,t+1}}{P_{y,t}}\right)^2\frac{Y_{t+1}}{Y_t}\\
&\quad+\frac{\lambda_t}{P_{y,t}}\epsilon A_t^{-1}-\frac{1}{1+\rho}\frac{\lambda_{t+1}}{P_{y,t+1}}\frac{P_{y,t+1}}{P_{y,t}}\epsilon(1-S_t(W_{t+1}))A_t^{-1}=0,
\end{aligned} \tag{B.10}$$

where we approximate $(\frac{P_{y,t+1}}{P_{y,t}}-1)^2\approx 0$ as above. If we define the service inflation $\frac{P_{y,t}}{P_{y,t-1}}=\Pi_{Y,t}$, (B.10) can be written as

$$\frac{\psi}{\epsilon}(\Pi_{Y,t}-1)\Pi_{Y,t}Y_t+\frac{\epsilon-1}{\epsilon}Y_t=\frac{W_t}{P_{y,t}}N_t+\frac{1}{1+\rho}\frac{\psi}{\epsilon}(\Pi_{Y,t+1}-1)\Pi^2_{Y,t+1}Y_{t+1}$$
$$+N_t\left(\frac{-c\chi}{1+\rho}\left(\frac{V_{t+1}}{N_t}\right)^{1+\chi}\Pi_{Y,t+1}\frac{W_{t+1}}{P_{y,t+1}}+\frac{\lambda_t}{P_{y,t}}-\frac{1}{1+\rho}\frac{\lambda_{t+1}}{P_{y,t+1}}\Pi_{Y,t+1}(1-S_t(W_{t+1}))\right) \tag{B.11}$$

which is our price Phillips curve.

B.3 Linearized Wage Phillips Curve

A Log-Linear Wage Phillips Curve We log-linearize the wage Phillips curve in (7), except we leave in the second order term, $\frac{\psi^w}{2}\left(\Pi_t^w-1\right)^2$, which we dropped when we derive (7).

$$\frac{\psi^w}{2}\left(\Pi_t^w-1\right)^2+\psi^w\left(\Pi_t^w-1\right)\Pi_t^w+1=c(1+\chi)\left(\frac{V_t}{N_{t-1}}\right)^{\chi}\left(\frac{V_t}{N_t}\varepsilon_{R,W_t}-\frac{N_{t-1}}{N_t}\frac{S_t(W_t)}{R_t(W_t)}\varepsilon_{S,W_t}\right)$$
$$+\frac{1}{1+\rho}\psi^w\left(\Pi^w_{t+1}-1\right)\left(\Pi^w_{t+1}\right)^2\frac{N_{t+1}}{N_t}. \tag{B.12}$$

It is worth pointing out that equation (B.12) would hold even if we added other factors of production (e.g., we could have Cobb-Douglass production with capital or some other inputs including oil), and is unaffected by the presence of price rigidities (e.g., if we had flexible or price rigidity à la Rotemberg (1982), (B.12) would be the same).

To ease interpretation, we rewrite this using $T_t\equiv\frac{V_t}{N_{t-1}}$ and $g_t\equiv\frac{N_t}{N_{t-1}}$:

$$0=\frac{\psi^w}{2}\left(\Pi_t^w-1\right)^2+\psi^w\left(\Pi_t^w-1\right)\Pi_t^w+1-c(1+\chi){T_t}^{\chi}{g_t}^{-1}\left(T_t\varepsilon_{R,W_t}-\frac{S_t(W_t)}{R_t(W_t)}\varepsilon_{S,W_t}\right)$$
$$-\frac{1}{1+\rho}\psi^w\left(\Pi^w_{t+1}-1\right)\left(\Pi^w_{t+1}\right)^2g_{t+1}. \tag{B.13}$$

We can suppress the dependence of $S_t(\cdot)$ and $R_t(\cdot)$ on W_t (since we know that in equilibrium, S_t and R_t are not functions of the aggregate wage W_t): we rewrite (B.13) as:

$$0=F\left(\ln(\Pi_t^w),\ln(\Pi^w_{t+1}),\ln(S_t),\ln(R_t),\ln(\varepsilon_{R,t}),\ln(\varepsilon_{S,t}),\ln(T_t),\ln(g_t),\ln(g_{t+1})\right),$$

and take a linear approximation around a zero wage-inflation steady state with variables $\ln(\Pi_t^w)$, $\ln(\Pi^w_{t+1})$, $\ln(S_t)$, $\ln(R_t)$, $\ln(\varepsilon_{R,t})$, $\ln(\varepsilon_{S,t})$, $\ln(T_t)$, $\ln(g_t)$, and $\ln(g_{t+1})$. We first calculate derivatives of $F(\cdot)$ with respect to each variable as follows:

$$
\begin{aligned}
F_{\ln(\Pi_t^w)} &= \psi^w \Pi_t^w \left(2(\Pi_t^w - 1) + \Pi_t^w\right) \\
F_{\ln(\Pi_{t+1}^w)} &= -\frac{\psi^w g_{t+1}}{1+\rho} \left(\Pi_{t+1}^w (\Pi_{t+1}^w)^2 + (\Pi_{t+1}^w - 1) 2 (\Pi_{t+1}^w)^2\right) \\
F_{\ln(S_t)} &= c(1+\chi) T_t^{\chi} g_t^{-1} \frac{S_t}{R_t} \varepsilon_{S,t} \\
F_{\ln(R_t)} &= -c(1+\chi) T_t^{\chi} g_t^{-1} \frac{S_t}{R_t} \varepsilon_{S,t} \\
F_{\ln(\varepsilon_{R,t})} &= -c(1+\chi) T_t^{\chi+1} g_t^{-1} \varepsilon_{R,t} \\
F_{\ln(\varepsilon_{S,t})} &= c(1+\chi) T_t^{\chi} g_t^{-1} \frac{S_t}{R_t} \varepsilon_{S,t} \\
F_{ln(T_t)} &= -c(1+\chi) g_t^{-1} \left((1+\chi) T_t^{\chi+1} \varepsilon_{R,t} - \chi T_t^{\chi} \frac{S_t}{R_t} \varepsilon_{S,t}\right) \\
F_{\ln(g_t)} &= c(1+\chi) T_t^{\chi} g_t^{-1} \left(T_t \varepsilon_{R,t} - \frac{S_t}{R_t} \varepsilon_{S,t}\right) \\
F_{\ln(g_{t+1})} &= -\frac{1}{1+\rho} \psi^w \left(\Pi_{t+1}^w - 1\right) (\Pi_{t+1}^w)^2 g_{t+1},
\end{aligned}
$$

which at the steady state with zero wage inflation can be written as

$$
\begin{aligned}
F_{\ln(\Pi_t^w)} &= \psi^w \\
F_{\ln(\Pi_{t+1}^w)} &= -\frac{\psi^w}{1+\rho} \\
F_{\ln(S_t)} &= c(1+\chi) T^{\chi} g^{-1} \frac{S}{R} \varepsilon_S = c(1+\chi) \frac{T^{\chi+1}}{g} \varepsilon_S \\
F_{\ln(R_t)} &= -c(1+\chi) T^{\chi} g^{-1} \frac{S_t}{R_t} \varepsilon_{S,t} = -c(1+\chi) \frac{T^{\chi+1}}{g} \varepsilon_S \\
F_{\ln(\varepsilon_{R,t})} &= -c(1+\chi) \frac{T^{\chi+1}}{g} \varepsilon_R \\
F_{\ln(\varepsilon_{S,t})} &= c(1+\chi) \frac{T^{\chi+1}}{g} \varepsilon_S \\
F_{ln(T_t)} &= -c(1+\chi) \frac{T^{\chi+1}}{g} \left((1+\chi)\varepsilon_R - \chi \varepsilon_S\right) \\
F_{\ln(g_t)} &= c(1+\chi) \frac{T^{\chi+1}}{g} (\varepsilon_R - \varepsilon_S) > 0 \\
F_{\ln(g_{t+1})} &= 0
\end{aligned}
$$

where we made use of the fact that $T = \frac{V}{N} = \frac{S}{R}$ in steady state, which we obtain from (G.2). We can see that the assumption above that $\frac{\psi^w}{2}\left(\Pi_t^w - 1\right)^2 \approx 0$ is correct in the sense that it drops out in our first-order approximation. We also find that in a zero inflation steady-state, there is no role for expectations of future employment growth in our first-order approximation.

Let the magenta terms above be collected as $\kappa \equiv c(1+\chi)\frac{T^{\chi+1}}{g}$. Then the first order approximation of $F(\cdot)$ around its steady state is given by[5]

$$
\begin{aligned}
0 = & \psi^w \check{\Pi}_t^w - \frac{\psi^w}{1+\rho}\check{\Pi}_{t+1}^w + \kappa\varepsilon_S\left(\check{S}_t - \check{R}_t\right) + \kappa\left(\varepsilon_S\check{\varepsilon}_{S,t} - \varepsilon_R\check{\varepsilon}_{R,t}\right) \\
& + \kappa(\chi\varepsilon_S - (1+\chi)\varepsilon_R)\check{T}_t + \kappa(\varepsilon_R - \varepsilon_S)\check{g}_t,
\end{aligned}
$$

which we can rewrite as

$$
\begin{aligned}
\check{\Pi}_t^w = & \underbrace{-\frac{\kappa\varepsilon_S}{\psi^w}\left(\check{S}_t - \check{R}_t\right) - \frac{\kappa}{\psi^w}\left(\varepsilon_S\check{\varepsilon}_{S,t} - \varepsilon_R\check{\varepsilon}_{R,t}\right) - \frac{\kappa(\chi\varepsilon_S - (1+\chi)\varepsilon_R)}{\psi^w}\check{T}_t}_{\text{Three Labor Market “Tightness” Terms}} \\
& + \underbrace{-\frac{\kappa(\varepsilon_R - \varepsilon_S)}{\psi^w}\check{g}_t}_{\text{Employment Growth}} + \underbrace{\frac{1}{1+\rho}\check{\Pi}_{t+1}^w}_{\text{Expectations}}
\end{aligned}
\tag{B.14}
$$

Note that the law of motion for employment in (G.2) that the firm faces implies:

$$
g_t = (1 - S_t) + R_t T_t \tag{B.15}
$$

Log linearizing (B.15) yields:

$$
\frac{1}{S}\check{g}_t = \check{R}_t + \check{T}_t - \check{S}_t,
$$

which leads to

$$
\check{S}_t - \check{R}_t = \check{T}_t - \frac{1}{S}\check{g}_t \tag{B.16}
$$

Plugging (B.16) into the log-linear wage Phillips curve in (B.14) and assuming $g_{=1}$, we obtain

$$
\check{\Pi}_t^w = \underbrace{\frac{\kappa\left(-\varepsilon_R + \frac{1+S}{S}\varepsilon_S\right)}{\psi^w}\check{g}_t}_{\text{Employment Growth}} + \underbrace{\frac{\kappa}{\psi^w}\left(\varepsilon_R\check{\varepsilon}_{R,t} - \varepsilon_S\check{\varepsilon}_{S,t}\right) + \frac{\kappa(1+\chi)(\varepsilon_R - \varepsilon_S)}{\psi^w}\check{T}_t}_{\text{Two Labor Market “Tightness” Terms}} + \underbrace{\frac{1}{1+\rho}\check{\Pi}_{t+1}^w}_{\text{Expectations}}. \tag{B.17}
$$

where $\kappa \equiv c(1+\chi)T^{\chi+1}$. From (B.17), we observe that stronger monopsony, i.e., a lower

[5] We let $\check{X}_t \equiv \ln X_t - \ln X$ for any X_t. If $X_t < 0$, then we let $\check{X}_t \equiv \frac{X_t - X}{X}$.

$\varepsilon_R - \varepsilon_S$, flattens the wage Phillips curve, as documented in de la Barrera i Bardalet (2023). We summarize this in the following Proposition 2.

Proposition 2 *The wage Phillips curve in (B.17) becomes flatter as the recruiting elasticity net of the separation elasticity, $\varepsilon_R - \varepsilon_S$, falls.*

Further Simplification Plugging (B.15) into (B.14) yields:

$$\begin{aligned}\check{\Pi}_t^w =& \frac{\kappa}{\psi^w}\left(-S\left(\varepsilon_R-\varepsilon_S\right)\left(\check{T}_t+\check{R}_t-\check{S}_t\right)-\varepsilon_S\left(\check{S}_t-\check{R}_t\right)+\left(\varepsilon_R\check{\varepsilon}_{R,t}-\varepsilon_S\check{\varepsilon}_{S,t}\right)+\left(\varepsilon_R+\chi\left(\varepsilon_R-\varepsilon_S\right)\right)\check{T}_t\right)\\ &+\frac{1}{1+\rho}\check{\Pi}_{t+1}^w,\end{aligned} \tag{B.18}$$

which leads to[6]

$$\begin{aligned}\check{\Pi}_t^w =& \frac{\kappa}{\psi^w}\left[\underbrace{\left(-\varepsilon_S+S(\varepsilon_R-\varepsilon_S)\right)}_{>0}\left(\check{S}_t-\check{R}_t\right)+\underbrace{\left(\varepsilon_R+(\chi-S)\left(\varepsilon_R-\varepsilon_S\right)\right)}_{>0}\check{T}_t+\left(\varepsilon_R\check{\varepsilon}_{R,t}-\varepsilon_S\check{\varepsilon}_{S,t}\right)\right]\\ &+\frac{1}{1+\rho}\check{\Pi}_{t+1}^w.\end{aligned} \tag{B.19}$$

B.3.1 Reduced Form Log-Linear Wage Phillips Curve in Only Quits and Unemployment

Estimating this regression (21) in the data via OLS shows that the regression puts more weight on quits than on unemployment, as documented in Table 1. As explained in Section 2, the regression yields a surprising empirical result for the sign of the coefficient on unemployment: replacing vacancies with quits, the sign on unemployment flips, and becomes positive. In other words, holding quits constant, a higher unemployment rate is correlated with *higher* wage growth!

Intriguingly, our model's benchmark calibration actually captures this result: when $\chi = 1$, i.e., firms' vacancy costs are convex, we find a much larger coefficient on quits than on unemployment, where the coefficient on unemployment is relatively small and positive. In

[6] As $\chi = 1$ and $S = 3.6\%$, $\chi > S$ at our steady state.

showing this, our strategy is to first express the above (I.1) into the following form:

$$\check{\Pi}_t^w = \phi_V \check{V}_t + \phi_U \check{U}_{t-1} + \frac{1}{1+\rho}\check{\Pi}_{t+1}^w \tag{B.20}$$

for some ϕ_V and ϕ_U, which are complex collections of model parameters and steady-state values. And then we use the fact that quits, which can also be decomposed into deviations of vacancy and unemployment, can be viewed as an imperfect proxy for "true" tightness since higher tightness leads to a higher rate of quits. A higher unemployment (vacancy) rate lowers (raises) tightness, and thus reduces (raises) quits. If $\check{Q}_t \equiv g_{Q,V}\check{V}_t + g_{Q,U}\check{U}_{t-1}$ where $g_{Q,U} < 0$ is of magnitude large enough, then equation (B.20) becomes

$$\Pi_t^w = \underbrace{\frac{\phi_V}{g_{Q,V}}}_{\equiv \beta_Q > 0} \check{Q}_t + \underbrace{\left(\phi_U - \phi_V \frac{g_{Q,U}}{g_{Q,V}}\right)}_{\equiv \beta_U} \check{U}_{t-1} + \frac{1}{1+\rho}\check{\Pi}_{t+1}^w, \tag{B.21}$$

possibly yielding a positive β_U.

Derivation To begin to simplify the wage Phillips curve (I.1), we decompose all of the following right-hand-side variables, $\check{Q}_t$, $\check{T}_t$, $\check{R}_t$, $\check{\varepsilon}_{R,t}$, and $\check{\varepsilon}_{S,t}$ into vacancy and unemployment deviations. The tightness term, $T_t = \frac{V_t}{N_{t-1}}$, is simple: in log deviations from steady state, it becomes

$$\check{T}_t = \check{V}_t + \frac{U}{1-U}\check{U}_{t-1}.$$

As for the rest, we will show that we can write the decompositions as follows:

1. $\check{R}_t \equiv g_{R,V}\check{V}_t + g_{R,U}\check{U}_{t-1}$

   Derivation: Recall that the recruiting function is

$$R_t = g(\theta_t)\left(\phi_{E,t}\frac{1}{2} + \phi_{U,t}\left(\frac{\xi^\gamma}{1+\xi^\gamma}\right)\right).$$

   For practical purposes we define $\mathcal{C} \equiv \frac{\xi^\gamma}{1+\xi^\gamma}$, which is increasing in the ratio of consumption for employed to unemployed workers. Then we obtain:

$$g_{R,V} = -\frac{\theta^2}{1+\theta^2},$$

and

$$g_{R,U} = \frac{\theta^2}{1+\theta^2} \cdot \frac{U(1-\lambda_{EE})}{\lambda_{EE}(1-U)+U} + \frac{0.5\phi_E}{0.5\phi_E + \mathcal{C}\phi_U} \cdot \frac{U}{1-U} \cdot \frac{\lambda_{EE}\phi_E - \lambda_{EE} - \phi_E}{\lambda_{EE}}$$
$$+ \frac{\mathcal{C}\phi_U}{0.5\phi_E + \mathcal{C}\phi_U} \cdot (1 - \phi_U(1-\lambda_{EE})).$$

2. $\check{S}_t \equiv g_{S,V}\check{V}_t + g_{S,U}\check{U}_{t-1}$ and $\check{Q}_t \equiv g_{Q,V}\check{V}_t + g_{Q,U}\check{U}_{t-1}$

   Derivation: Recall that the quit function $Q_t = S_t - s$ is given by

$$Q_t = (1-s)\left(\lambda_{EE} f(\theta_t)\frac{1}{2} + \lambda_{EU}\left(\frac{1}{1+\xi^\gamma}\right)\right)$$

   Then, we obtain:

$$g_{Q,V} = \frac{0.5\lambda_{EE} f}{0.5\lambda_{EE} f + \lambda_{EU}(1-\mathcal{C})} \cdot \frac{1}{1+\theta^2},$$

   and

$$g_{Q,U} = -\frac{0.5\lambda_{EE} f}{0.5\lambda_{EE} f + \lambda_{EU}(1-\mathcal{C})} \cdot \frac{1}{1+\theta^2} \cdot \frac{U(1-\lambda_{EE})}{\lambda_{EE}(1-U)+U}.$$

3. $\check{\varepsilon}_{R,t} = g_{\varepsilon_{R,U}}\check{U}_{t-1}$

   Derivation: Note that in equilibrium, $\varepsilon_{R,t}$ is given by

$$\varepsilon_{R,t} = \frac{\cancel{g(\theta_t)}\gamma\left(\frac{\phi_{E,t}}{4} + \phi_{U,t}\frac{\xi^{-\gamma}}{(1+\xi^{-\gamma})^2}\right)}{\cancel{g(\theta_t)}\left(0.5\phi_{E,t} + \left(\frac{\xi^\gamma}{1+\xi^\gamma}\right)\phi_{U,t}\right)} = \frac{\gamma\left(\frac{\phi_{E,t}}{4} + \phi_{U,t}\mathcal{C}(1-\mathcal{C})\right)}{0.5\phi_{E,t} + \phi_{U,t}\mathcal{C}},$$

   from which we obtain

$$g_{\varepsilon_{R,U}} = \left(\frac{0.25\phi_E}{0.25\phi_E + \mathcal{C}(1-\mathcal{C})\phi_U} - \frac{0.5\phi_E}{0.5\phi_E + \mathcal{C}\phi_U}\right)\frac{U}{1-U}\frac{\lambda_{EE}\phi_E - \lambda_{EE} - \phi_E}{\lambda_{EE}}$$
$$+ \left(\frac{\mathcal{C}(1-\mathcal{C})\phi_U}{0.25\phi_E + \mathcal{C}(1-\mathcal{C})\phi_U} - \frac{\mathcal{C}\phi_U}{0.5\phi_E + \mathcal{C}\phi_U}\right)(1 - \phi_U(1-\lambda_{EE})).$$

4. $\check{\varepsilon}_{S,t} = g_{\varepsilon_{S,V}}\check{V}_t + g_{\varepsilon_{S,U}}\check{U}_{t-1}$

Derivation: Note that in equilibrium $\varepsilon_{S,t}$ is given by

$$\varepsilon_{S,t} = \frac{-(1-s)\gamma\left(f(\theta_t)\lambda_{EE}\frac{1}{4} + \mathcal{C}(1-\mathcal{C})\lambda_{EU}\right)}{s + (1-s)\left(0.5\cdot\lambda_{EE}f(\theta_t) + (1-\mathcal{C})\,\lambda_{EU}\right)},$$

from which we obtain

$$g_{\varepsilon_{S,V}} = \left(\frac{0.25\lambda_{EE}f}{0.25\lambda_{EE}f + \mathcal{C}(1-\mathcal{C})\lambda_{EU}} - \frac{0.5(1-s)\lambda_{EE}f}{s+(1-s)(0.5\lambda_{EE}f + (1-\mathcal{C})\lambda_{EU})}\right)\frac{1}{1+\theta^2},$$

and

$$g_{\varepsilon_{S,U}} = -g_{\varepsilon_{S,V}}\cdot\frac{U(1-\lambda_{EE})}{\lambda_{EE}(1-U)+U}.$$

Decomposing Wage Growth into Vacancies and Unemployment Combining these results, we can plug in and rewrite the wage Phillips curve just in terms of vacancies and unemployment. Let $\Delta_1 \equiv -\varepsilon_S + S(\varepsilon_R - \varepsilon_S)$ and let $\Lambda_1 \equiv \varepsilon_R + (\chi - S)(\varepsilon_R - \varepsilon_S)$. Then the wage Phillips curve (I.1) can be written as:

$$\begin{aligned}\check{\Pi}^w_t =& \underbrace{\frac{\kappa}{\psi^w}\left[\Lambda_1 + \Delta_1\left(g_{S,V} - g_{R,V}\right) - \varepsilon_S g_{\varepsilon_S,V}\right]}_{\equiv\phi_V>0}\check{V}_t \\ &+ \underbrace{\frac{\kappa}{\psi^w}\left[\frac{U}{1-U}\Lambda_1 + \Delta_1\left(g_{S,U} - g_{R,U}\right) + \varepsilon_R g_{\varepsilon_R,U} - \varepsilon_S g_{\varepsilon_S,U}\right]}_{\equiv\phi_U<0}\check{U}_{t-1} + \frac{1}{1+\rho}\check{\Pi}^w_{t+1}.\end{aligned} \tag{B.22}$$

Under our calibration in Table 2, quantitatively (I.2) becomes

$$\check{\Pi}^w_t = 10^{-2}\times\left(1.83\check{V}_t - 0.3\check{U}_{t-1}\right) + \frac{1}{1+\rho}\check{\Pi}^w_{t+1} \tag{B.23}$$

Decomposing Wage Growth into Quits and Unemployment: First, note $\check{Q}_t \equiv g_{Q,V}\check{V}_t + g_{Q,U}\check{U}_{t-1}$ yields

$$\check{V}_t = \frac{1}{g_{Q,V}}\check{Q}_t - \frac{g_{Q,U}}{g_{Q,V}}\check{U}_{t-1},$$

which with (B.23) yields:

$$\Pi_t^w = \underbrace{\frac{\phi_V}{g_{Q,V}}}_{\equiv \beta_Q > 0} \check{Q}_t + \underbrace{\left(\phi_U - \phi_V \frac{g_{Q,U}}{g_{Q,V}}\right)}_{\equiv \beta_U} \check{U}_{t-1} + \frac{1}{1+\rho} \check{\Pi}_{t+1}^w$$
$$= 10^{-2} \times \left(2.46\check{Q}_t + 0.0916\check{U}_{t-1}\right) + \frac{1}{1+\rho} \check{\Pi}_{t+1}^w \tag{B.24}$$

at a monthly frequency, where β_Q dominates β_U in magnitude under our calibration, and β_U becomes positive. Thus, we prove equation (21).

A Simpler Wage Phillips Curve With No On-the-job Search Here, we argue that convex vacancy costs and on-the-job search combine to make vacancies more important in the wage Phillips curve, i.e., $|\phi_V|$ is significantly bigger than $|\phi_U|$, than when $\chi \approx 0$ and $\lambda_{EE} = 0$, the case where $\check{V}_t - \check{U}_{t-1}$, i.e., $\widecheck{\left(\frac{V_t}{U_{t-1}}\right)}$, becomes a sufficient statistic that explains wage growth.[7] In doing this, we will assume that $s \simeq 0$ and $\mathcal{C} \equiv \frac{\xi^\gamma}{1+\xi^\gamma} \simeq 1$, both of which hold approximately under our calibration.

First, we demonstrate that as we eliminate OTJ search and let $\lambda_{EE} \to 0$, the decomposition of the wage Phillips curve into $\check{V}_t$ and $\check{U}_{t-1}$ in (I.2) simplifies considerably. The first term $\check{T}_t$ remains:

$$\lim_{\lambda_{EE}\to 0} \check{T}_t = \check{V}_t + \frac{U}{1-U}\check{U}_{t-1}.$$

As for the rest, we will show that we can write the decompositions as follows:

1. $\lim_{\lambda_{EE}\to 0} \check{R}_t = -\frac{\theta^2}{1+\theta^2}\left(\check{V}_t - \check{U}_{t-1}\right)$ since $\phi_U \to 1$ (and $\phi_E \to 0$) as we shut down on-the-job search, i.e., $\lambda_{EE} \to 0$.
2. $\lim_{\lambda_{EE}\to 0} \check{S}_t = 0$
3. $\lim_{\lambda_{EE}\to 0} \check{\varepsilon}_{R,t} = 0$.
4. $\lim_{\lambda_{EE}\to 0} \check{\varepsilon}_{S,t} = 0$

Which means that the wage Phillips curve simplifies to:

$$\lim_{\lambda_{EE}\to 0} \check{\Pi}_t^w = \frac{\kappa}{\psi^w}\left[\underbrace{\left(-\varepsilon_S + S(\varepsilon_R - \varepsilon_S)\right)}_{\equiv \Delta_1} \frac{\theta^2}{1+\theta^2} \widecheck{\left(\frac{V_t}{U_{t-1}}\right)} + \underbrace{\left(\varepsilon_R + (\chi - S)\left(\varepsilon_R - \varepsilon_S\right)\right)}_{\equiv \Lambda_1} \check{T}_t\right] + \frac{1}{1+\rho}\check{\Pi}_{t+1}^w$$

[7] In other words, when $\chi \approx 0$ and $\lambda_{EE} = 0$ with very small s, $\phi_V \simeq \phi_U$.

Noting that:

$$\lim_{\lambda_{EE}\to 0} \theta_t = \frac{V_t}{U_{t-1}}$$

So we have further:

$$\lim_{\lambda_{EE}\to 0} \check{\Pi}_t^w = \frac{\kappa}{\psi^w}\left[\underbrace{\left(-\varepsilon_S + S(\varepsilon_R - \varepsilon_S)\right)}_{\equiv \Delta_1}\frac{\theta^2}{1+\theta^2}\check{\theta}_t + \underbrace{\left(\varepsilon_R + (\chi - S)\left(\varepsilon_R - \varepsilon_S\right)\right)}_{\equiv \Lambda_1}\check{T}_t\right] + \frac{1}{1+\rho}\check{\Pi}_{t+1}^w$$

As we further assume that $s \simeq 0$ and $\mathcal{C} \simeq 1$, we can find the following results for the steady-state values underlying Λ_1 and Δ_1:

$$\lim_{\lambda_{EE}\to 0} S = s + (1-s)\lambda_{EU}(1-\mathcal{C}) = 0$$
$$\lim_{\lambda_{EE}\to 0} \varepsilon_R = \gamma(1-\mathcal{C}) = 0$$
$$\lim_{\lambda_{EE}\to 0} \varepsilon_S = \frac{-(1-s)\gamma\left(\mathcal{C}(1-\mathcal{C})\lambda_{EU}\right)}{s + (1-s)\left(1-\mathcal{C}\right)\lambda_{EU}} = -\gamma$$

Which implies that $\Delta_1 = -\gamma$ and $\Lambda_1 = \gamma\chi$. So with $\chi = 0$, our wage Phillips curve in terms of $\check{V}_t$ and $\check{U}_{t-1}$ simplifies to:

$$\lim_{\lambda_{EE}\to 0} \check{\Pi}_t^w = \frac{\kappa}{\psi^w}\gamma\left(\frac{\theta^2}{1+\theta^2}\right)\check{\theta}_t + \frac{1}{1+\rho}\check{\Pi}_{t+1}^w$$

.

Therefore, the wage Phillips curve can be written entirely in terms of market tightness θ_t when there is no on-the-job search, as in Gagliardone and Gertler (2023).

In sum, as the exogenous separation rate $s \to 0$, the consumption ratio $\xi \to \infty$ or $\mathcal{C} \to 1$ (so unemployed workers always take jobs) and $\lambda_{EE} \to 0$, we have that $S \to 0$, $\varepsilon_R \to 0$, and $\varepsilon_S \to -\gamma$. Then there is complete weight on θ in the wage Phillips curve, and if $\lambda_{EE} = 0$, $\theta_t = \frac{V_t}{U_{t-1}}$. In contrast, in our setting where $\chi = 1$, which implies a convex vacancy cost, and $\lambda_{EE} > 0$, we see that $|\phi_V|$ is much higher than $|\phi_U|$ as seen in (B.23), and β_S is much higher than β_U as seen in (B.24).

B.4 Euler Equation With Fixed Real Unemployment Benefits

This section shows how the assumptions in Section 4.2.2 can be made consistent with the standard Euler equation of the household, given appropriate assumptions on how the household reallocates consumption. Recall the goal in Section 4.2.2 was to modify the model so that the desirability of unemployment varied with the price level; here, we show one way to make that model consistent with the standard Euler equation (10) used throughout the main text.

Suppose that when unemployed, household members are guaranteed some quantity b of real consumption goods and receive no other income (e.g., some nominal unemployment benefit perfectly indexed to inflation). When employed, they receive a nominal wage W_t. The household takes b, market wages W_t, and the price level P_t as given, but can smooth all members consumption by choosing a proportional "top-up" each period, multiplying each type of worker's income by $1 + \tau_t$. This yields consumption levels

$$\begin{aligned} C_t^u =& b(1+\tau_t) \\ C_t^e =& \frac{W_t}{P_t}(1+\tau_t), \end{aligned}$$

and total consumption

$$C_t = U_t b(1+\tau_t) + (1-U_t)\frac{W_t}{P_t}(1+\tau_t). \tag{B.25}$$

Making the top-up proportional and identical in both states u and e implies that as the household smooths consumption, it does not affect the relative attractiveness of unemployment and employment: the $1 + \tau_t$ terms cancel out separation and recruiting probabilities from unemployment $s_{ju}(W_{jt})$ and $r_{uj}(W_{jt})$ presented in Section 4.2.2.

Continuing on to derive the Euler equation, the household maximizes

$$\begin{aligned} &\sum_{t=0}^{\infty}\left(\frac{1}{1+\rho}\right)^t (U_t \ln(C_t^u) + (1-U_t)\ln(C_t^e)) \\ =&\sum_{t=0}^{\infty}\left(\frac{1}{1+\rho}\right)^t \left(U_t \ln(b(1+\tau_t)) + (1-U_t)\ln\left(\frac{W_t}{P_t}(1+\tau_t)\right)\right) \\ =&\sum_{t=0}^{\infty}\left(\frac{1}{1+\rho}\right)^t \left(U_t \ln(b) + (1-U_t)\ln\left(\frac{W_t}{P_t}\right) + \ln(1+\tau_t)\right). \end{aligned}$$

The household's budget constraint can be written as:

$$(1+\tau_t)\left(U_t b + (1-U_t)\frac{W_t}{P_t}\right) + \frac{B_t}{P_t} = \frac{D_t}{P_t} + (1-U_t)\frac{W_t}{P_t} + \frac{(1+i_{t-1,t})B_{t-1}}{P_t}.$$

The household's Lagrangian function is given by:

$$\mathcal{L} = \sum_{t=0}^{\infty} \left(\frac{1}{1+\rho}\right)^t \Bigg(U_t \ln(b) + (1-U_t)\ln\left(\frac{W_t}{P_t}\right) + \ln(1+\tau_t)$$
$$+ \lambda_t \left[-(1+\tau_t)\left(U_t b + (1-U_t)\frac{W_t}{P_t}\right) - \frac{B_t}{P_t} + \frac{D_t}{P_t} + \frac{(1+i_{t-1,t})B_{t-1}}{P_t} \right] \Bigg).$$

The household's only choice variables are τ_t and B_t. The first order conditions are

$$\mathcal{L}_{\tau_t} = 0 : \frac{1}{1+\tau_t} = \lambda_t \left(U_t b + (1-U_t)\frac{W_t}{P_t}\right),$$

$$\mathcal{L}_{B_t} = 0 : \frac{\lambda_t}{P_t} = \left(\frac{1}{1+\rho}\right)\lambda_{t+1}\frac{(1+i_{t,t+1})B_t}{P_{t+1}}.$$

Plugging in the expression for aggregate consumption in equation (B.25) into the first order condition on τ_t yields the standard Euler equation used in the main text:

$$C_t^{-1} = \frac{1}{1+\rho}\frac{1+i_{t,t+1}}{\pi_{t,t+1}}C_{t+1}^{-1}.$$

C Analytical Results for Pass-Through Across Different Classes of Models

This section analyzes the pass-through of prices to wages in response to a temporary decline in the endowment good X_t, assuming that monetary policy stabilizes the business cycle holding N_t fixed. We consider the following variations of the model which alter the labor block in Section 3. We work through the case of (i) a sticky-price, flexible-wage New Keynesian model where workers supply labor in a frictionless market, (ii) a flexible price, sticky-wage New Keynesian model where wages are set by unions as in Erceg et al. (2000); Galí et al. (2012); and (iii) our benchmark model in Section 3.

As in the paper, we assume throughout that consumption is a CES bundle of services Y_t, produced with labor, and goods X_t which households receive as an endowment (equivalently, perfectly competitive firms receive X_t and sell it for pure profit, rebating the proceeds to households as dividends). We have

$$C_t = \left(\alpha_Y^{\frac{1}{\eta}} Y_t^{\frac{\eta-1}{\eta}} + \alpha_X^{\frac{1}{\eta}} X_t^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}} \tag{C.1}$$

and

$$P_t = \left(\alpha_Y P_{y,t}^{1-\eta} + \alpha_X P_{x,t}^{1-\eta}\right)^{\frac{1}{1-\eta}}$$

C.1 Sticky-Price, Flexible Wage New Keynesian Model

We assume here that $P_{y,t}$ is set subject to some nominal rigidities as in the benchmark model in Section 3 (i.e. Rotemberg adjustment costs), but where firms hire labor in a standard spot market with flexible nominal wage W_t, so there is no unemployment. The household chooses paths for consumption and labor (and zero net supply nominal bonds) to maximize:

$$\sum_{t=0}^{\infty} \beta^t \left(\frac{\sigma}{\sigma-1} C_t^{\frac{\sigma-1}{\sigma}} - \frac{1}{1+\frac{1}{\nu}} N_t^{1+\frac{1}{\nu}}\right)$$

subject to the budget constraint,

$$C_t = \frac{D_t}{P_t} - \frac{B_t}{P_t} + \frac{(1+i_{t-1,t})B_{t-1}}{P_t} + \frac{W_t}{P_t} N_t.$$

This yields the following intratemporal optimality condition:

$$N_t^{\frac{1}{\nu}} = W_t \frac{C_t^{\frac{-1}{\sigma}}}{P_t}$$

So in our model with Neoclassical labor supply, the following decomposition must hold to first order:

$$\frac{1}{\nu}\check{N}_t = \check{W}_t - \frac{1}{\sigma}\underbrace{\left(\check{P}_t + \check{C}_t\right)}_{=\widecheck{P_t C_t}} + \frac{1-\sigma}{\sigma}\check{P}_t$$

So that when monetary policy fixes $\check{N}_t = 0$, we have

$$\frac{\widecheck{W_t}}{P_{y,t}} = \frac{1}{\sigma}\frac{\widecheck{P_t C_t}}{P_{y,t}} + \frac{\sigma-1}{\sigma}\frac{\widecheck{P_t}}{P_{y,t}} \tag{C.2}$$

Now we can write the two right hand side terms as functions of the shock X_t: first note that CES demand implies

$$\frac{P_t}{P_{y,t}} = \left(\frac{Y_t}{C_t}\right)^{\frac{1}{\eta}} \alpha_y^{-\frac{1}{\eta}}.$$

Under our experiment where monetary policy stabilizes N_t, and hence Y_t, from (C.1) we have to first order that $\check{C}_t = \alpha_x^{\frac{1}{\eta}} \left(\frac{X}{C}\right)^{\frac{\eta-1}{\eta}} \check{X}_t$ and

$$\widecheck{\frac{P_t}{P_{y,t}}} = -\frac{1}{\eta}\left(\alpha_x^{\frac{1}{\eta}}\left(\frac{X}{C}\right)^{\frac{\eta-1}{\eta}} \check{X}_t\right) \tag{C.3}$$

so that when X_t falls, the price of the aggregate consumption bundle in terms of the labor-intensive good, $\frac{P_t}{P_{y,t}}$ goes up (i.e. we need more units of Y-good to buy one unit of C-good). We also have for aggregate spending,

$$\widecheck{\frac{P_t C_t}{P_{y,t}}} = \frac{\eta - 1}{\eta}\left(\alpha_x^{\frac{1}{\eta}}\left(\frac{X}{C}\right)^{\frac{\eta-1}{\eta}} \check{X}_t\right)$$

So that aggregate nominal spending may either rise, or fall, depending on η. With Cobb-Douglas utility with $\eta = 1$, nominal spending is unchanged. Consider the effects of negative shock to X_t on the wage when monetary policy holds N_t fixed, and examine equation (C.2):

- We can see when $\eta = \sigma = 1$, the wage denoted in units of the service good or numeraire, i.e., $\frac{W_t}{P_{y,t}}$ remains unchanged.
- With Cobb-Douglas preferences, $\eta = 1$, we see from (C.3) that the relative price still rises, so everything depends on σ: if $\sigma > 1$, as is commonly assumed in macro applications, then there is positive pass through from prices to wages.
- If $\sigma = 1$, $\eta < 1$ then there is positive pass-through from prices to wages. When it is hard to substitute away from X_t, and total expenditure rises.

Discussion: Even in a perfectly competitive labor market, workers' wages can respond to an increased cost of living even when their productivity is unaffected by the shock. The sign and magnitude of the response depends on the strength of income and substitution effects (governed by σ) and wealth effects (governed by η) stemming from a change in $P_{x,t}X_t$, where we obtain

$$\frac{\widetilde{P_{x,t}X_t}}{P_{y,t}} = \underbrace{\frac{\eta-1}{\eta}}_{<0} \underbrace{\check{X}_t}_{<0} > 0 \tag{C.4}$$

when $\eta < 1$, so that households' non-labor income from endowment good X_t increases and so does their wealth, possibly lowering labor supply due to the wealth effect.

In specifications where $\sigma \geqslant 1$ and $\eta < 1$, the decline in X_t makes workers prefer leisure; thus if monetary policy is holding leisure (and labor) fixed, the wage must rise in equilibrium.

C.2 Flexible Price, Sticky Wage New Keynesian Model

We now consider the effect of a temporary fall in X_t when wages are sticky as in Erceg et al. (2000), again analyzing the shock under the assumption that monetary policy stabilizes aggregate labor output N_t. Specifically, we assume that households now supply multiple types of labor; unions set wages for each type to maximize household utility subject to facing CES demand for each type from a "labor packer" which packages each labor type $N_t(i)$ into aggregate labor $N_t = \left(\int_0^1 N_t(i)^{\frac{1+\nu}{\nu}} di\right)^{\frac{\nu}{1+\nu}}$ which is purchased at wage W_t by services firms—and in our setting, combined with X_t to form consumption C_t. Wages are sticky because unions only occasionally receive the chance to reset their wage.

Households now maximize the following: specializing to log utility with $\sigma = 1$,

$$\sum_{t=0}^{\infty} \beta^t \left(\ln C_t - \int_0^1 \frac{1}{1+\frac{1}{\nu}} N_t(i)^{1+\frac{1}{\nu}} di\right),$$

subject to the budget constraint, $C_t = \frac{D_t}{P_t} - \frac{B_t}{P_t} + \frac{(1+i_{t-1,t})B_{t-1}}{P_t} + W_t N_t$. Under these assumptions, we can derive the following standard wage Phillips curve (see e.g., Galí, 2011; Galí et al., 2012):

$$\check{\Pi}_t^w = \beta E\{\check{\Pi}_{t+1}^w\} + \lambda\left(-\check{W}_t + \widetilde{P_t C_t} + \frac{1}{\nu}\check{N}_t\right).$$

for some constant $\lambda > 0$. Analyzing this case is only harder than the flexible wage case of Section C.1 because of the presence of the forward-looking term Π_{t+1}^w. To make progress, rewrite this in relative price terms:

$$\check{\Pi}_t^w = \beta E\{\check{\Pi}_{t+1}^w\} + \lambda\left(-\frac{\widetilde{W_t}}{P_{y,t}} + \frac{\widetilde{P_t C_t}}{P_{y,t}} + \frac{1}{\nu}\check{N}_t\right). \tag{C.5}$$

Consider the household's budget constraint in equilibrium: using the fact that bonds are in zero net supply, and expanding the D_t term by denoting d_t the dividends potentially paid by services firms (zero, if prices are flexible and they are perfectly competitive), this is

$$P_t C_t = W_t N_t + P_{x,t} X_t + d_t.$$

How will each term in this equation respond to an X_t shock? Rewriting, we have:

$$\frac{P_t C_t}{P_{y,t}} = \frac{W_t}{P_{y,t}} N_t + \frac{P_{x,t}}{P_{y,t}} X_t + \frac{d_t}{P_{y,t}}.$$

Now recall that given fixed N_t CES demand yields:

$$\widetilde{\frac{P_t C_t}{P_{y,t}}} = \frac{\eta - 1}{\eta} \left(\alpha_x^{\frac{1}{\eta}} \left(\frac{X}{C} \right)^{\frac{\eta-1}{\eta}} \check{X}_t \right)$$

If $\eta < 1$, then $\frac{P_0 \check{C}_0}{P_{y,0}}$ rises in response to a negative X_0 shock and $\frac{P_t \check{C}_t}{P_{y,t}}$ is zero in other periods ($t > 0$) when there is no shock. From (C.4), we see the middle term, in its deviation from steady state, is zero when there is no shock. Thus, we obtain for all $t > 0$:

$$0 = \frac{WN}{PC} \widetilde{\frac{W_t}{P_{y,t}}} + \frac{d}{PC} \check{d}_t.$$

If there are no time-varying profits, e.g., if prices are flexible, then we have that $\check{d}_t = 0$ and thus $\widetilde{\frac{W_t}{P_{y,t}}} = 0$. As a result, the forward looking wage Phillips curve (C.5) implies $\check{\Pi}_t^w = 0$ for all $t > 0$, and the wage Phillips curve for the initial period greatly simplifies to

$$\pi_0^w = \lambda \left(-\check{W}_0 + P_0 \check{C}_0 \right)$$

Given that wage inflation is defined as $\check{\Pi}_t^w = \check{W}_t - \check{W}_{t-1}$ with $\check{W}_{-1} = 0$, we can write

$$\check{W}_0 = \lambda (-\check{W}_0 + P_0 \check{C}_0)$$

Divide by $P_{y,0}$ to apply our above results for $\frac{P_0 \check{C}_0}{P_{y,0}}$ and find that when $\eta < 1$, the right hand side is positive for a negative X shock, and we thus have positive pass-through to wages.

Discussion: As discussed in above Section C.1, depending on the strength of income, substitution, and wealth effects (governed by η), wages can either rise or fall in response to the shock. Here for $\sigma = 1$, we again find that $\eta < 1$ implies pass through from prices to wages in response to the X_t shock. The analysis with sticky wages is not that different from the flexible wage case.

C.3 Wage Posting Model with On-the-job Search and Nominal Rigidities

This section analyzes the baseline model in Section 3 to elaborate the conditions under which there is no pass-through from prices to wages. In our benchmark model, there is no pass-through from prices to wages: in response to an X_t shock, when monetary policy perfectly stabilizes N_t, it also perfectly stabilizes wage inflation. We demonstrate both that this relies on the assumption that vacancy costs are denominated in labor. If vacancy costs are denominated in final goods, then headline inflation passes through into wages, even when monetary policy stabilizes the labor market. The title of this section reflects the fact that it does not matter for the analysis here whether prices or wages are sticky, so long as the presence of nominal rigidities allows monetary authorities to stabilize N_t.

To demonstrate the role of how adjustment costs are denominated, we generalize the firms problem slightly: let P_t^V denote the nominal price in which vacancy costs are denominated, and let P_t^ψ be the nominal price in which wage adjustment costs are denominated (which we will show will not matter). Then firm j maximizes present-discounted revenues, less costs (abstracting from price adjustment costs, which do not affect the wage Phillips curve), given by

$$\sum_{t=0}^{\infty}\left(\frac{1}{1+\rho}\right)^t\left(P_{y,t}^jY_t^j - W_t^jN_t^j - c(V_t^j)^{1+\chi}(N_{t-1}^j)^{-\chi}P_t^V - \frac{\psi^w}{2}\left(\frac{W_t^j}{W_{t-1}^j}-1\right)^2 N_tP_t^\psi\right),$$

subject to the law of motion for employment,

$$N_t^j = (1 - S_t(W_t^j))N_{t-1}^j + V_t^jR_t(W_t^j)$$

and some production and demand functions for Y_t^j. Combining the firm's first order conditions for V_t^j and W_t^j, and assuming a symmetric equilibrium, yields a nonlinear wage Phillips curve:

$$\psi^w\left(\Pi_t^w-1\right)\Pi_t^w P_t^{\psi}+W_t=P_t^V c(1+\chi)\left(\frac{V_t}{N_{t-1}}\right)^{\chi}\left(\frac{V_t}{N_t}\varepsilon_{R,W_t}-\frac{N_{t-1}}{N_t}\frac{S_t(W_t)}{R_t(W_t)}\varepsilon_{S,W_t}\right)$$
$$+\frac{\psi^w}{1+\rho}\left(\Pi_{t+1}^w-1\right)\Pi_{t+1}^w\frac{N_{t+1}}{N_t}P_{t+1}^{\psi}.$$

Gather the labor market tightness terms in $Z_t\equiv c(1+\chi)\left(\frac{V_t}{N_{t-1}}\right)^{\chi}\left(\frac{V_t}{N_t}\varepsilon_{R,W_t}-\frac{N_{t-1}}{N_t}\frac{S_t(W_t)}{R_t(W_t)}\varepsilon_{S,W_t}\right)$ and log-linearize, defining $\pi_t^V\equiv\frac{P_t^V}{P_{t-1}^V}$, let $\breve{\omega}_t\equiv\sum_{s=0}^{t}(\pi_t^w-\pi_t^V)$, obtaining

$$\breve{\pi}_t^w=\frac{ZP^V}{\psi^w P^{\psi}}\sum_{s=0}^{\infty}\left(\frac{1}{1+\rho}\right)^s\left(\breve{Z}_{t+s}-\breve{\omega}_{t+s}\right).$$

When monetary policy stabilizes employment, and $\breve{N}_t=0$, it does follow that $\breve{Z}_t=0$, as proved in Section C.3.1 so the wage Phillips curve reduces to:

$$\breve{\pi}_t^w=\frac{ZP^V}{\psi^w P^{\psi}}\sum_{s=0}^{\infty}\left(\frac{1}{1+\rho}\right)^s\left(-\breve{\omega}_{t+s}\right).$$

If the cost of posting vacancies is denominated in labor, so $P_t^V=W_t$, then $-\breve{\omega}_t=0$ and monetary policy stabilizes wage growth as well as employment.

C.3.1 Showing That $\breve{N}_t=0$ implies $\breve{Z}_t=0$

To see this result, first note that holding $N_t=N$ constant implies that the number of searchers $\mathcal{S}=\lambda_{EE}N+\lambda_{EU}(1-N)$ is constant; the shares appearing in the definitions of the separation and recruiting rates, $\phi_{E,t}$ and $\phi_{U,t}$ in equations (17) and (18), are thus also constant. This means the tightness term θ_t is constant so long as V_t is constant. If V_t and therefore θ_t are constant, then the separation rates and elasticities in Z_t are also held constant. Ergo, all we must do is show that V_t is constant.

To do so, write the law of motion for employment when $N_t=N_{t-1}=N$, plugging in for the separation and recruiting rates, to yield $V_t\cdot R_t=N\cdot S_t$ that can be written as:

$$V_t g\left(\frac{V_t}{\mathcal{S}}\right)\left(\phi_E\frac{1}{2}+\phi_U\left(\frac{\xi^{\gamma}}{1+\xi^{\gamma}}\right)\right)-Nf\left(\frac{V_t}{\mathcal{S}}\right)(1-s)\frac{\lambda_{EE}}{2}=N\left[s+(1-s)\left(\lambda_{EU}\left(\frac{1}{1+\xi^{\gamma}}\right)\right)\right].$$

Now using our definition for g, rewrite the left hand side in terms of f:

$$\mathcal{S}f\left(\frac{V_t}{\mathcal{S}}\right)\left(\phi_E\frac{1}{2}+\phi_U\left(\frac{\xi^\gamma}{1+\xi^\gamma}\right)\right)-Nf\left(\frac{V_t}{\mathcal{S}}\right)(1-s)\frac{\lambda_{EE}}{2}=N\left[s+(1-s)\left(\lambda_{EU}\left(\frac{1}{1+\xi^\gamma}\right)\right)\right]$$

leading to

$$f\left(\frac{V_t}{\mathcal{S}}\right)=\frac{N\left[s+(1-s)\left(\lambda_{EU}\left(\frac{1}{1+\xi^\gamma}\right)\right)\right]}{\mathcal{S}\left(\phi_E\frac{1}{2}+\phi_U\left(\frac{\xi^\gamma}{1+\xi^\gamma}\right)\right)-N(1-s)\frac{\lambda_{EE}}{2}}$$

Thus, there is a unique solution for $V_t = V$ for a given N (the steady state value). So we conclude that when monetary policy stabilizes N_t, V_t and θ_t are also stabilized, and Z_t is stabilized.

D Weight on Vacancies and Unemployment

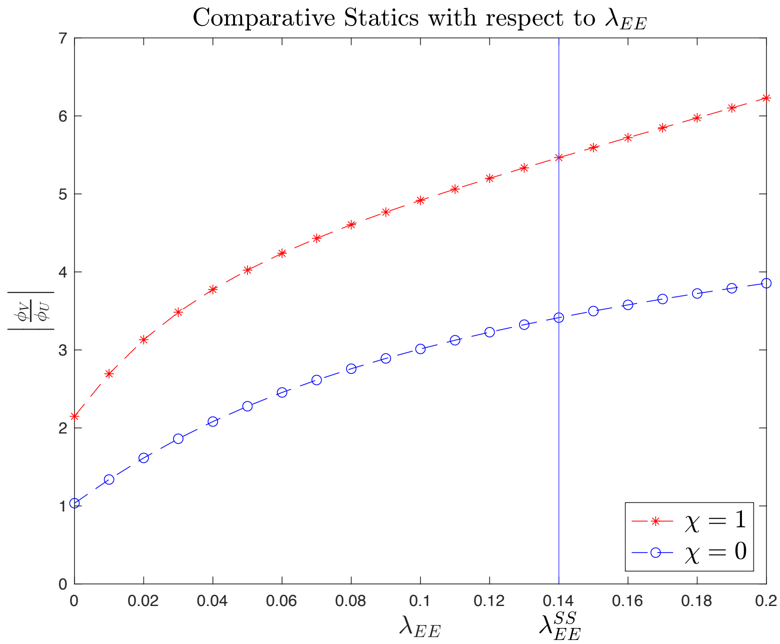
We start from the log-linearized wage Phillips curve (I.2), written in vacancy and unemployment rates :

$$\check{\Pi}^w_t=\phi_V\check{V}_t+\phi_U\check{U}_{t-1}+\frac{1}{1+\rho}\check{\Pi}^w_{t+1} \tag{D.1}$$

for some coefficients $\phi_V > 0$ and $\phi_U < 0$. In Appendix C.3, we prove that the absence of aggregate price inflation in the right hand side of the linearized wage Phillips curve (D.1) is stemming from the fact that the vacancy-creating cost is denominated in labor, not the final good. Based on our calibration in Table 2, we now illustrate how varying the probability of being allowed to search on the job, λ_{EE}, affects the predictions of the model about the relative importance of vacancies, as opposed to unemployment, in the wage Phillips curve.

Since both $\phi_V > 0$ and $\phi_U < 0$ are complex collections of model parameters and steady-state values, to consider their relative magnitudes, we proceed numerically, and specialize to particular parameter choices. As we see in (B.23), ϕ_V is much larger in magnitude than ϕ_U. This result turns out to stem both from the presence of on-the-job search ($\lambda_{EE} > 0$) and also from the convexity of vacancy costs ($\chi > 0$). Figure D.2 shows how the relative importance of vacancies in explaining wage growth, represented by the ratio of coefficients in (D.1), $\left|\frac{\phi_V}{\phi_U}\right|$, increases monotonically in on-the-job search intensity λ_{EE} under the benchmark calibration $\chi = 1$ and also when $\chi = 0$, or a linear cost of posting a vacancy which is commonly assumed in the search literature. The limit case where $\chi = 0$ and $\lambda_{EE} \to 0$ is of particular interest as a benchmark: as Appendix B.3.1 shows, at the limit where $\lambda_{EE} \to 0$ and $\chi \to 0$, $\left|\frac{\phi_V}{\phi_U}\right|$ converges to one and wage growth becomes solely a function of market tightness $\theta_t = \frac{V_t}{U_{t-1}}$[8] following the literature: see e.g., Gagliardone and Gertler (2023).

Figure D.2: In Economies with More On-the-job Search, Vacancies Matter More in the Wage Phillips Curve



Notes: The red, starred line plots the effects of a change in on-the-job search intensity λ_{EE}, holding all other model parameters constant at their values in Table 2, on the ratio of the coefficients in equation (D.1): $\check{\Pi}_t^w = \phi_V \check{V}_t + \phi_U \check{U}_{t-1} + \frac{1}{1+\rho}\check{\Pi}_{t+1}^w$. The blue dotted line repeats the exercise but with $\chi = 0$, a linear cost of vacancy posting. The vertical line marks the value for λ_{EE} used in our benchmark calibration. The relative importance of vacancies in explaining wage inflation, compared with unemployment, increases with both λ_{EE} and χ.

We acknowledge that simply pointing out the coefficient on V is larger than U does not technically imply that variations in U are less important in explaining wage growth: if U has a much higher variance than V, it can have a small coefficient while still playing a large role. To show more formally how rising $\left|\frac{\phi_V}{\phi_U}\right|$ diminishes the importance of unemployment in explaining wage growth, consider the variance decomposition of wage growth in the model under the assumption that we can ignore the inflation expectations term:[9]

[8] With $\lambda_{EE} = 0$, $\theta_t = \frac{V_t}{S_t} = \frac{V_t}{U_{t-1}}$.

[9] For example, assuming firms have constant inflation expectations, $\mathbf{E}_t \Pi_{t+1}^w = \Pi^w$, or "adaptive" expectations $\mathbf{E}_t \Pi_{t+1}^w = \Pi_t^w$. Alternatively, we might view (D.2) as an approximation when ρ is high, permitting us to ignore the many covariance cross-terms complicating the expression when solved forward.

$$\mathrm{Var}\left(\check{\Pi}_t^w\right) = \left(\frac{\phi_V}{\phi_U}\right)^2 \mathrm{Var}\left(\check{V}_t\right) + \mathrm{Var}\left(\check{U}_{t-1}\right) + \underbrace{2\frac{\phi_V}{\phi_U}}_{<0} \mathrm{Cov}\left(\check{V}_t, \check{U}_{t-1}\right) \tag{D.2}$$

Now consider the exercise in Figure D.2, which increases λ_{EE} holding other parameters constant, raising $\left|\frac{\phi_V}{\phi_U}\right|$. Given that the covariance term $\mathrm{Cov}\left(\check{V}_t, \check{U}_{t-1}\right)$ in (D.2) is strongly negative (both empirically, and also in any reasonably calibrated model), the importance of unemployment in explaining wage growth falls monotonically as we increase the amount of on-the-job search, and convexity of the vacancy costs, in the model.

Supplementary Appendix for
Do Cost-of-Living Shocks Pass Through to Wages?

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E Cost-of-Living Shocks and Forward-Looking Workers

This section relaxes the assumption that workers are completely myopic. While relaxing this assumption increases the complexity of the model, we show here that it barely matters for the dynamics: the response of wage inflation and other key labor market variables to a cost-of-living shock is nearly identical, assuming the monetary authority stabilizes output as well as inflation.

To understand why, note that the only part of the model that changes is the firm's problem, and more specifically, the first order condition for wages. Even relaxing myopia, it continues to be the case that stabilizing output Y_t (and hence N_t) stabilizes the firm's FOC condition for wages and hence wage inflation. However, this is in spite of the fact that this FOC becomes much more complicated because separation and recruiting rates at time t now depend not just on the wage at time t, but also on the wage promised for all $T > t$. The reason for this is that now when a worker considers joining a firm, they know they may remain there for more than one period. They thus care not just about the wage offered today, but also about the wage promised in the future.

Technically speaking, this makes the firm's problem time-inconsistent: when making a plan for wages, it is optimal to promise to offer a high wage in the future in order to raise the recruiting rate today "for free." But when the time arrives to pay the higher wage, the firm will be tempted to renege. This shows up in the firm's problem as a first order condition for wages which is non-stationary, as the first period is special, which introduces additional difficulty both in solving and discussing the model.

To make progress, and keep the model with forward looking workers as similar as possible to that in the main text to ease comparison, we again study the effect of an unanticipated "MIT shock" to the cost of living that becomes known at some time $t = 0$. At that date, we assume that firms are paying wages that they committed to at some point in the distant past, $t = -\infty$ so that at $t = -1$ the economy has converged to a long run steady state which we will describe below.

In letting firms respond to this MIT shock, we solve for the responses that the firms would have found optimal had they known about the shock back in the infinite past. More formally, we assume firms knew at $t = -\infty$ that there was some chance p of the shock occurring at time t, and solve for the economy's response to the shock under that optimal plan as $p \to 0$, so that the shock is entirely unanticipated.[10]

As in the main text, we will solve for a symmetric, perfect-foresight equilibrium where all firms set the same wage. Unlike in the main text, we will solve for the solution to the nonlinear models (with and without myopia) to better compare their behavior and make the point that the assumption of myopic workers is largely-innocuous simplifying assumption.

E.1 Firm's Problem With Dynamic Separation and Recruiting Rates

Recall that the only choice facing workers occurs when they are offered a chance to take a job. With probability $\lambda_{EE} f(\theta_t)$, workers meet alternative jobs, and with probability λ_{EU} they are allowed to consider quitting into unemployment. When workers meet alternate jobs, or consider the unemployment state, they draw idiosyncratic utilities ι from a type-1 extreme value distribution with scale parameter γ. They then choose the state which yields the greatest utility: for example, let $\mathcal{V}_{jt}$ be the value of being employed at firm j and $\bar{\mathcal{V}}_t$ be the value of being employed at the other matched firm. Then a worker at firm j who matches with another firm of value $\bar{\mathcal{V}}_t$ maximizes:

$$\max\{\mathcal{V}_{jt} + \iota_{jt}, \bar{\mathcal{V}}_t + \iota_{kt}\}.$$

Given this, the expected value of having a choice between two value functions next period is $\mathcal{V}^E(\mathcal{V}^a, \mathcal{V}^b) = \frac{1}{\gamma} \ln \left(\exp\left(\gamma \mathcal{V}^a\right) + \exp\left(\gamma \mathcal{V}^b\right)\right)$. So the worker's Bellman equation at firm j is thus the following: letting $\mathcal{V}_t^u$ be the value of the unemployed state at time t, and assuming all firms other than j have identical value $\bar{V}_t$ (which is without loss of generality, given that we will search for a symmetric equilibrium later),

$$\begin{aligned}
\mathcal{V}_{jt} &= \ln\left(\frac{\tau_t W_{jt}}{P_t}\right) + \beta s \mathcal{V}_{t+1}^u \\
&\quad + \beta(1-s)\Big[\lambda_{EE} f(\theta_{t+1}) \mathcal{V}^E(\mathcal{V}_{j,t+1}, \bar{\mathcal{V}}_{t+1}) + \lambda_{EU} \mathcal{V}^E(\mathcal{V}_{j,t+1}, \mathcal{V}_{t+1}^u) + (1 - \lambda_{EE} f(\theta_{t+1}) - \lambda_{EU}) \mathcal{V}_{j,t+1}\Big] \\
&\equiv \mathcal{F}_t(\mathcal{V}_{j,t+1})
\end{aligned}$$

[10] In the context of time-inconsistent optimal monetary policy, this is equivalent to the "timeless approach" to computing optimal policy under commitment.

where we write this function $\mathcal{F}_t$ as time-varying because of its dependence on the real wage w_t, tax rate $1-\tau_t$, tightness θ_{t+1}, and $\bar{\mathcal{V}}_{t+1}$. We also assume workers discount the future at rate β, which need not equal $\frac{1}{1+\rho}$. This allows us to easily nest the myopic worker case where $\frac{1}{1+\rho} > \beta = 0$. In practice, we solve a version of model where workers discount the future with $\beta = .96$ which corresponds to an annual discount rate of $\beta^{12} \approx .60$, consistent with experimental evidence; see Michaillat and Saez (2021) for a discussion and summary of this evidence. So we can plug in the following:

$$\mathcal{V}^E(\mathcal{V}_{j,t+1}, \bar{\mathcal{V}}_{t+1}) = \frac{1}{\gamma}\ln\left(\exp\left(\gamma\mathcal{V}_{j,t+1}\right) + \exp\left(\gamma\bar{\mathcal{V}}_{t+1}\right)\right)$$
$$\mathcal{V}^E(\mathcal{V}_{j,t+1}, \mathcal{V}^u_{t+1}) = \frac{1}{\gamma}\ln\left(\exp\left(\gamma\mathcal{V}_{j,t+1}\right) + \exp\left(\gamma\mathcal{V}^u_{t+1}\right)\right)$$

To see that $\mathcal{V}_{jt}$ depends on the whole path of nominal wages, $\{W_{jt}\}_{t=0}^{\infty}$, we evaluate the following derivative: for $\mathcal{S} \geqslant 1$,

$$\begin{aligned}\frac{d\mathcal{V}_{jt}}{dW_{j,t+\mathcal{S}}} &= \frac{d}{dW_{j,t+\mathcal{S}}}\mathcal{F}_t\Bigg(\mathcal{F}_{t+1}\Bigg(\ldots\mathcal{F}_{t+\mathcal{S}-1}(\mathcal{V}_{j,t+\mathcal{S}})\ldots\Bigg)\Bigg) \\ &= \mathcal{F}'_t(\mathcal{V}_{j,t+1}) \times \mathcal{F}'_{t+1}(\mathcal{V}_{j,t+2}) \times \ldots \times \mathcal{F}'_{t+\mathcal{S}-1}(\mathcal{V}_{j,t+\mathcal{S}})\frac{d\mathcal{V}_{j,t+\mathcal{S}}}{dW_{j,t+\mathcal{S}}}\end{aligned}$$

where

$$\mathcal{F}'_t(\mathcal{V}_{j,t+1}) = \beta(1-s)\left[\frac{\lambda_{EE}f(\theta_{t+1})}{1+\exp\left(\gamma(\bar{\mathcal{V}}_{t+1}-\mathcal{V}_{j,t+1})\right)} + \frac{\lambda_{EU}}{1+\exp\left(\gamma(\mathcal{V}^u_{t+1}-\mathcal{V}_{j,t+1})\right)} + 1 - \lambda_{EE}f(\theta_{t+1}) - \lambda_{EU}\right]$$

and since the final term $\frac{d\mathcal{V}_{j,t+\mathcal{S}}}{dW_{j,t+\mathcal{S}}} = \frac{1}{W_{j,t+\mathcal{S}}} > 0$, we can see that $\frac{d\mathcal{V}_{jt}}{dW_{j,t+\mathcal{S}}} > 0$. Promising a higher wage in the future makes a job offer at firm j more attractive, and helps recruit workers today.

In a symmetric equilibrium with $\bar{\mathcal{V}}_t = \mathcal{V}_{jt}$ always, and where the household fixes $\bar{\mathcal{V}}_t - \mathcal{V}^u_t$ at some value $\ln\xi$ (by varying the consumption of the unemployed),[11] this simplifies greatly to

$$\mathcal{F}'_t(\mathcal{V}_{j,t+1} = \bar{\mathcal{V}}_{t+1}) = \beta(1-s)\underbrace{\left[\frac{\lambda_{EE}f(\theta_{t+1})}{2} + \frac{\lambda_{EU}}{1+\xi^{-\gamma}} + 1 - \lambda_{EE}f(\theta_{t+1}) - \lambda_{EU}\right]}_{\equiv P(\theta_{t+1})<1}.$$

[11] With log utility and myopia ($\beta = 0$), we have $\bar{\mathcal{V}}_t - \mathcal{V}^u_t = \ln C^e_t - \ln C^u_t = \ln\xi$ implying $\frac{C^e_t}{C^u_t} = \xi$ as before.

Thus in a symmetric equilibrium, and for $\mathcal{S} \geqslant 1$ we can write down these derivatives as:

$$\frac{d\mathcal{V}_{jt}}{dW_{j,t+\mathcal{S}}} = \frac{(\beta(1-s))^{\mathcal{S}} \prod_{k=1}^{\mathcal{S}} P(\theta_{t+k})}{W_{j,t+\mathcal{S}}},$$

and for $\mathcal{S} = 0$:

$$\frac{d\mathcal{V}_{jt}}{dW_{j,t}} = \frac{1}{W_{j,t}}.$$

So we obtain

$$P(\theta_{t+1}) = -\frac{\lambda_{EE} f(\theta_{t+1})}{2} + \frac{\lambda_{EU}}{1+\xi^{-\gamma}} + 1 - \lambda_{EU}$$

which is decreasing in θ_{t+1}. It can be understood as follows: as future market tightness θ_{t+1} increases, it is easier for workers to switch firms in which they work in the future, so a future wage increase leads to less increase in the current value $\mathcal{V}_{jt}$ at period t.

Now that we know the value function $\mathcal{V}_{jt}$ depends on the whole path of wages, we rewrite our recruiting and separation rates as follows:

$$R_t(\mathcal{V}_{jt}) \equiv g(\theta_t)\left[\phi_{E,t}\left(\frac{\exp(\gamma(\mathcal{V}_{jt} - \bar{\mathcal{V}}_t))}{1+\exp(\gamma(\mathcal{V}_{jt} - \bar{\mathcal{V}}_t)}\right) + \phi_{U,t}\left(\frac{\exp(\gamma(\mathcal{V}_{jt} - \mathcal{V}^u_t))}{1+\exp(\gamma(\mathcal{V}_{jt} - \mathcal{V}^u_t)}\right)\right]$$

$$S_t(\mathcal{V}_{jt}) \equiv s + (1-s)\left[\lambda_{EE} f(\theta_t)\left(\frac{\exp(-\gamma(\mathcal{V}_{jt} - \bar{\mathcal{V}}_t))}{1+\exp(-\gamma(\mathcal{V}_{jt} - \bar{\mathcal{V}}_t)}\right) + \lambda_{EU}\left(\frac{\exp(-\gamma(\mathcal{V}_{jt} - \mathcal{V}^u_t))}{1+\exp(-\gamma(\mathcal{V}_{jt} - \mathcal{V}^u_t)}\right)\right]$$

Where we write that R_t and S_t are time varying because of changes labor market conditions (tightness θ_t and employed/unemployed searcher shares $\phi_{E,t}$ and $\phi_{U,t}$) and competition from both other firms and unemployment ($\bar{\mathcal{V}}_t$ and $\mathcal{V}^u_t$).

Note that this impacts only the FOCs for vacancies and wages, and the law of motion for employment. We can no longer derive a nice nonlinear wage Phillips Curve (though our price Phillips curve is, happily, unchanged). Putting changes in red, the firm's problem is now

$$\max_{\substack{\{P^j_{y,t}\},\{N_{jt}\}\\ \{W_{jt}\},\{V_{j,t}\}}} \sum_{t=0}^{\infty}\left(\frac{1}{1+\rho}\right)^t \Bigg(P^j_{y,t}Y^j_t - W_{jt}N_{jt} - c\left(\frac{V_{j,t}}{N_{j,t-1}}\right)^{\chi} V_{j,t}W_t - \frac{\psi}{2}\left(\frac{P^j_{y,t}}{P^j_{y,t-1}} - 1\right)^2 Y^j_t P^j_{y,t}$$

$$- \frac{\psi^w}{2}\left(\frac{W_{jt}}{W_{j,t-1}} - 1\right)^2 W_{jt}N_{jt}\Bigg)$$

subject to

$$N_{jt} = (1 - S_t({\color{red}\mathcal{V}_{jt}}))N_{j,t-1} + R_t({\color{red}\mathcal{V}_{jt}})V_{j,t}.$$

Output is produced with labor with the linear production: $Y_t^j = A_t^j N_{jt}$, and Dixit-Stiglitz demand, so $\frac{Y_t^j}{Y_t} = \left(\frac{P_{y,t}^j}{P_{y,t}}\right)^{-\epsilon}$, hence $N_{jt} = \left(\frac{P_{y,t}^j}{P_{y,t}}\right)^{-\epsilon} \frac{Y_t}{A_t^j}$ with $\epsilon > 1$.

Here we can see why only the first order condition with respect to wages will changes: the only new component is that the derivatives of the recruiting and separation rates R_t and S_t will be different. Note that their levels will be unchanged: in a symmetric equilibrium, R_t and S_t will have the same functional forms as before, in the main text: formally in a symmetric equilibrium with $\mathcal{V}_{jt} = \bar{\mathcal{V}}_t$, and assuming households set $\bar{\mathcal{V}}_t - \mathcal{V}_t^u = \ln \xi$ by adjusting the consumption of unemployed households, i.e., adjusting τ_t, we have:

$$
\begin{aligned}
R_t &\equiv g(\theta_t)\left[\phi_{E,t}\left(\frac{1}{2}\right) + \phi_{U,t}\left(\frac{\xi^\gamma}{1+\xi^\gamma}\right)\right] \\
S_t &\equiv s + (1-s)\left[\lambda_{EE} f(\theta_t)\left(\frac{1}{2}\right) + \lambda_{EU}\left(\frac{\xi^{-\gamma}}{1+\xi^{-\gamma}}\right)\right].
\end{aligned}
$$

Note that we continue to assume that the representative household imposes taxes and transfers to satisfy a standard Euler equation: while this may no longer necessarily be optimal, we make this assumption to facilitate direct comparison with the model in the main text.

Equilibrium and Steady State We focus on one particular equilibrium where $P_{y,t}^j = P_{y,t}$, $V_{j,t} = V_t$, $W_{jt} = W_t$, $A_t^j = A_t \; \forall j$. All equations in our model in the main body, and the firms problem, are unchanged except for the first order condition for the wage, W_t: defining for convenience

$$
\begin{aligned}
\Delta_t &\equiv N_t + \psi^w(\Pi_t^w - 1)\Pi_t^w N_t - \frac{1}{1+\rho}\psi^w\left(\Pi_{t+1}^w - 1\right)\left(\Pi_{t+1}^w\right)^2 N_{t+1}, \\
\Lambda_t &\equiv \frac{\lambda_t}{P_{y,t}}\left(V_t R_t' - S_t' N_{t-1}\right)
\end{aligned}
$$

where λ_t is the co-state variable associated with the law of motion for N_t, we can write the FOC for wage W_t as

$$
\begin{aligned}
\text{for } t = 0: &\quad \frac{W_0}{P_{y,0}}\Delta_0 = \Lambda_0 \\
\text{for } t \geqslant 1: &\quad \frac{W_t}{P_{y,t}}\Delta_t = \Lambda_t + \beta(1-s)\frac{P(\theta_t)}{\Pi_{y,t}}\left(\frac{W_{t-1}}{P_{y,t-1}^y}\Delta_{t-1}\right).
\end{aligned}
$$

In the long run, assuming that firms committed to a wage plan arbitrarily far in the past ($t = -\infty$), we should converge to a steady state satisfying the FOC for W_t with $t \geqslant 1$, i.e. not the equation in the initial period. Thus we look for a solution which solves the following: in a zero-inflation steady state, and noting that $P(\theta_t) < 1$, the wage satisfies:

$$\frac{W}{P^y}\Delta = \frac{\Lambda}{1 - \beta(1-s)P(\theta)}.$$

Note that if $\beta = 0$, and workers are myopic, we recover the expression for the real wage in steady state for the myopic worker model in the main body. Given this equation, and all the other equations in the model, we can again solve numerically for a zero-inflation steady state of the model (assuming the monetary authority targets $\Pi_t = 1$).

This makes it easy to see the time inconsistency in the firm's optimal wage plan: if we plug in the long run steady state here for the $t = 0$ constraint, and consider what firms choose for the real wage given Λ, Δ, θ, we see that the long run steady state does not satisfy the FOC at $t = 0$: the wage is too high by a factor of $1 - \beta(1-s)P(\theta)$. In short, given the chance to re-optimize, firms choose a lower wage than the one committed to in the infinite past, because that future commitment once helped with contemporary recruitment.

Convergence to the Long Run Steady State Assume an economy as described above without aggregate risk and where firms made their wage plans an infinitely long time ago, so that the economy is now in the long run steady state described above. When firms first set their wage plan, they initially choose to promise a wage below the long run value, and later would want to renege, but this is not permitted. Figure E.3 plots the transition to this long run steady state if we allow firms to choose new wage plans. All impulse responses are shown as percent deviations from the long-run steady state, so that we can confirming the intuition described above for the wage. Re-optimization acts like an expansionary shock: firms immediately lower nominal wages, but increase in size by recruiting through promising higher nominal wages in the future and by posting more vacancies. This causes market tightness to rise in the aggregate, which puts upward pressure on firms' marginal costs. The net effect is a modest amount of inflation as these costs are passed on to consumers, which the monetary authority responds to by raising real interest rates.

Response to an MIT Cost-of-Living Shock X_t When firms commit to their wage plan at some time $t = -\infty$, we assume that firms know that an X_t MIT shock might hit the economy

Nonlinear Dynamic Model Response to a One-Time Firm Reoptimization Allowed at $t = 0$

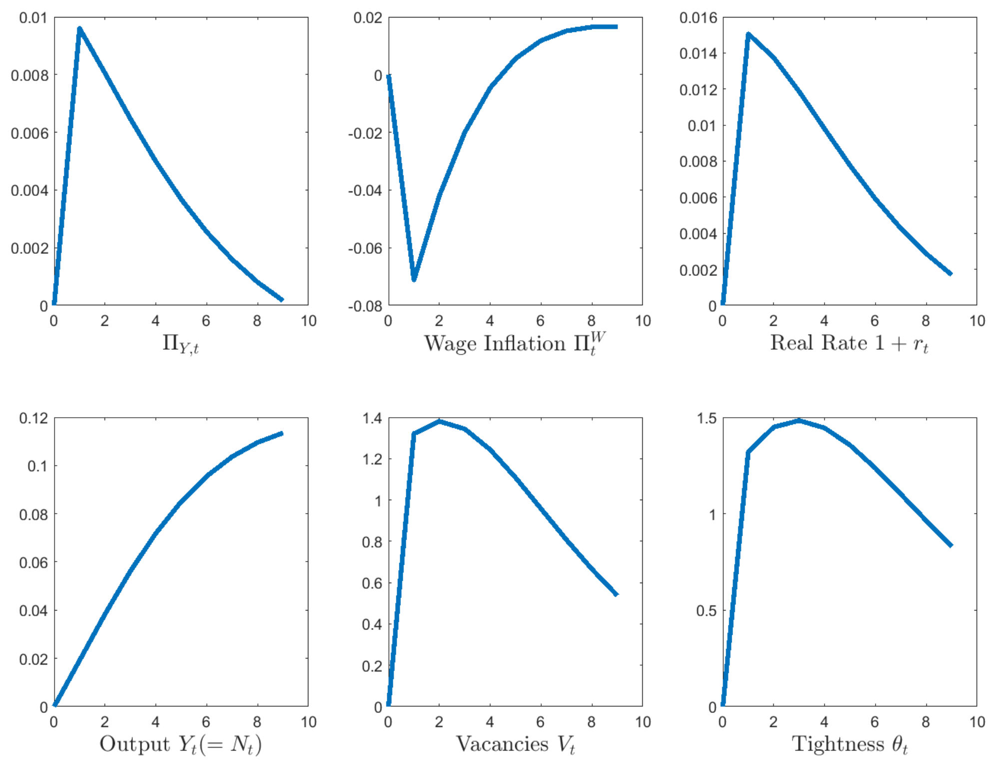


Figure E.3: The effects of allowing firms to reoptimize and choose new paths for wages and all other choice variables, which they then commit to following forever. All impulse responses are shown as percent deviations from the long run steady state.

at date $t = 0$, but that the probability of that MIT shock is effectively zero (in the limit). However, we can characterize what their wage plan looks like in the case that the shock hits: this is plotted in Figure E.4. That wage plan respects the FONC for wages for $t \geqslant 1$, not $t = 0$, written above.

Replicating the experiment in the main text, where the central bank perfectly stabilizes $N_t = N$ in response to the shock, yields identical impulse response functions for both models. Because of this, we instead plot the slightly more interesting case where the monetary authority follows an active Taylor rule that imperfectly stabilizes output, and hence domestic employment: $1 + i_t = (1 + \rho) \left(\frac{\Pi_t}{\Pi}\right)^{\phi_\Pi} \left(\frac{Y_t}{Y}\right)^{\phi_Y}$, where here $\phi_Y = \phi_\Pi = 2$. The responses in both models remain very similar, though they are no longer identical: as the monetary authority responds to the inflationary shock by raising real interest rates, the price of domestic output falls and domestic consumption Y_t remains basically flat. Wage inflation remains extremely modest, but positive—and is slightly more positive on impact for the "myopic" model.

We conclude by noting that the assumption of myopic workers is a largely-innocuous simplifying assumption which is not critical to obtaining the results in the main text.

Nonlinear Model Response to an MIT X_t Shock: Myopic vs. Forward-Looking Workers

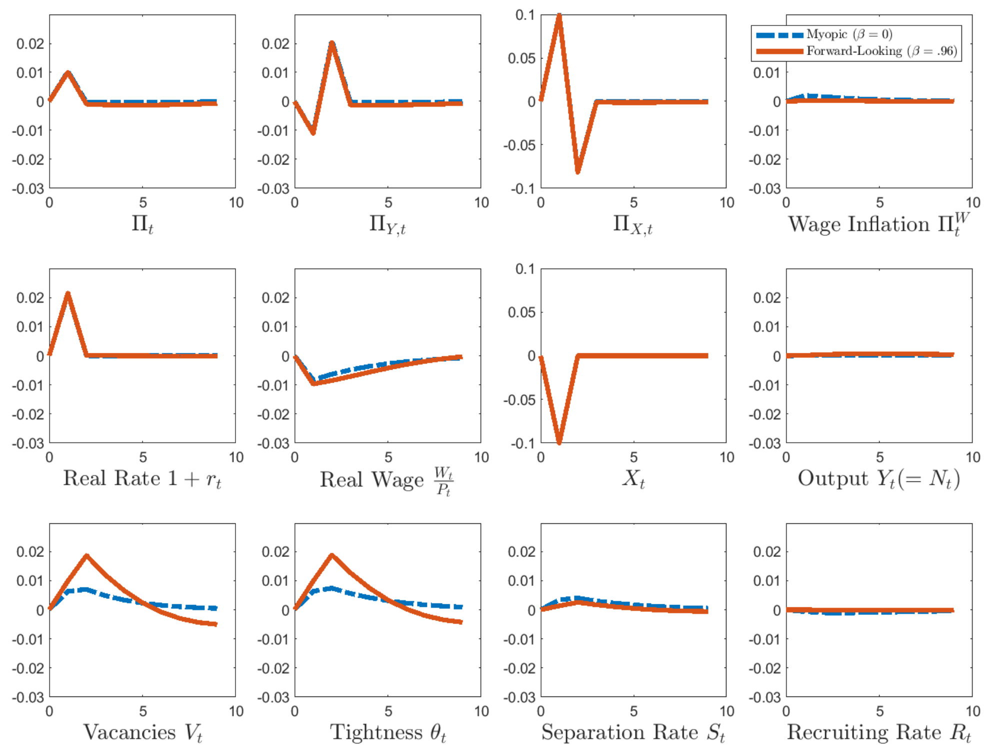


Figure E.4: The effects of a negative shock to X_t assuming the central bank follows an active Taylor rule: $1 + i_t = (1 + \rho) \left(\frac{\Pi_t}{\Pi}\right)^{\phi_\Pi} \left(\frac{Y_t}{Y}\right)^{\phi_Y}$, where here $\phi_Y = \phi_\Pi = 2$. The responses in both models are very similar: the monetary authority responds to the inflationary shock by raising real interest rates. The price of domestic output falls and domestic consumption Y_t remains basically flat. Wage inflation remains extremely modest, but positive—and is slightly more positive on impact for the "myopic" model. All impulse responses are shown as percent deviations from the long run steady state.

F Constant Relative Risk Aversion (CRRA) Utility

In this section, we deviate from our log-preference assumption and assume instead that the per-period utility function is given by $\frac{C_t^{1-\sigma}}{1-\sigma}$ featuring σ as relative risk aversion and the inverse of elasticity of intertemporal substitution. A worker working at firm j, receiving wage W_{jt} and facing τ_t as tax rate, will consume $C_t^e = \tau_t \frac{W_{jt}}{P_t}$. Under the same consumption sharing rule $\frac{C_t^e}{C_t^u} = \xi$ within a household, a unemployed person will consume $C_t^u = \frac{\tau_t}{\xi}\frac{\bar{W}_t}{P_t}$, where $\bar{W}_t$ is the average wage of employed workers as defined in Section 3.2.

Now, the probability that a worker chooses firm j paying W_{jt} relative to market wage $\bar{W}_t$ is given by

$$r_{mj}\left(\bar{W}_t, W_{jt}|P_t\right) = \frac{e^{\frac{\gamma}{1-\sigma}\left(\frac{\tau_t W_{jt}}{P_t}\right)^{1-\sigma}}}{e^{\frac{\gamma}{1-\sigma}\left(\frac{\tau_t W_{jt}}{P_t}\right)^{1-\sigma}} + e^{\frac{\gamma}{1-\sigma}\left(\frac{\tau_t \bar{W}_t}{P_t}\right)^{1-\sigma}}} = \frac{1}{1 + e^{\frac{\gamma}{1-\sigma}\left(\frac{\tau_t}{P_t}\right)^{(1-\sigma)}\left(\bar{W}_t^{1-\sigma} - W_{jt}^{1-\sigma}\right)}}$$

where subscript m denotes market, the recruiting rate r_{mj} depends P_t explicitly. The probability that a unemployed worker chooses firm j paying W_{jt} is

$$r_{uj}\left(\bar{W}_t, W_{jt}|P_t\right) = \frac{e^{\frac{\gamma}{1-\sigma}\left(\frac{\tau_t W_{jt}}{P_t}\right)^{1-\sigma}}}{e^{\frac{\gamma}{1-\sigma}\left(\frac{\tau_t W_{jt}}{P_t}\right)^{1-\sigma}} + e^{\frac{\gamma}{1-\sigma}\left(\frac{\tau_t \bar{W}_t}{\xi P_t}\right)^{1-\sigma}}} = \frac{1}{1 + e^{\frac{\gamma}{1-\sigma}\left(\frac{\tau_t}{P_t}\right)^{(1-\sigma)}\left(\left(\frac{\bar{W}_t}{\xi}\right)^{1-\sigma} - W_{jt}^{1-\sigma}\right)}}$$

which again depends directly on P_t. Under the symmetric equilibrium with $W_{jt} = \bar{W}_t$, a rise in P_t raises the recruiting rate r_{uj} from the unemployed, when $\sigma > 1$, giving an incentive for firms to post higher wages.

Thus, the recruiting function $R(W_{jt}|P_t)$ is then

$$R_t\left(W_{jt}|P_t\right) = g(\theta_t)\left[\phi_{E,t}\frac{1}{1 + e^{\frac{\gamma}{1-\sigma}\left(\frac{\tau_t}{P_t}\right)^{(1-\sigma)}\left(\bar{W}_t^{1-\sigma} - W_{jt}^{1-\sigma}\right)}} + \phi_{U,t}\frac{1}{1 + e^{\frac{\gamma}{1-\sigma}\left(\frac{\tau_t}{P_t}\right)^{(1-\sigma)}\left(\left(\frac{\bar{W}_t}{\xi}\right)^{1-\sigma} - W_{jt}^{1-\sigma}\right)}}\right]$$

Then we have that $R'(w_{jt})W_{jt}$ is given by

$$\begin{aligned} R_t'(W_{jt})W_{jt} =& g(\theta_t)\phi_{E,t}\left(1 + e^{\frac{\gamma}{1-\sigma}\left(\frac{\tau_t}{P_t}\right)^{(1-\sigma)}\left(\bar{W}_t^{1-\sigma} - W_{jt}^{1-\sigma}\right)}\right)^{-2} e^{\frac{\gamma}{1-\sigma}\left(\frac{\tau_t}{P_t}\right)^{(1-\sigma)}\left(\bar{W}_t^{1-\sigma} - W_{jt}^{1-\sigma}\right)}\gamma\left(\frac{\tau_t W_{jt}}{P_t}\right)^{1-\sigma} \\ &+ g(\theta_t)\phi_{U,t}\left(1 + e^{\frac{\gamma}{1-\sigma}\left(\frac{\tau_t}{P_t}\right)^{(1-\sigma)}\left(\left(\frac{\bar{W}_t}{\xi}\right)^{1-\sigma} - W_{jt}^{1-\sigma}\right)}\right)^{-2} e^{\frac{\gamma}{1-\sigma}\left(\frac{\tau_t}{P_t}\right)^{(1-\sigma)}\left(\left(\frac{\bar{W}_t}{\xi}\right)^{1-\sigma} - W_{jt}^{1-\sigma}\right)}\gamma\left(\frac{\tau_t W_{jt}}{P_t}\right)^{1-\sigma} \end{aligned}$$

which is increasing in P_t in the symmetric equilibrium when $\sigma > 1$.[12] As the recruiting (and separation) elasticities ε_{R,W_t} and ε_{S,W_t} are increasing in P_t, a cost-of-living shock can provide an incentive for firms to raise wages in response in this case.

As special case, if $s = 0$, $\lambda_{EU} = 0$, then $\phi_{E,t} = 1$ and we obtain

$$\frac{\partial \varepsilon_{R,W_t}}{\partial P_t}\frac{P_t}{\varepsilon_{R,W_t}} = \frac{\partial \varepsilon_{S,W_t}}{\partial P_t}\frac{P_t}{\varepsilon_{S,W_t}} = \sigma - 1 > 0.$$

Euler Equation In this case, the household's consumption Euler equation takes a slightly different form. First, their preference is given by

$$\sum_{t=0}^{\infty}\left(\frac{1}{1+\rho}\right)^t\left[U_t\frac{(C_t^u)^{1-\sigma}}{1-\sigma} + (1-U_t)\frac{(C_t^e)^{1-\sigma}}{1-\sigma}\right]. \tag{F.1}$$

We keep assuming that the household is constrained by fairness considerations to choose $\frac{C_t^e}{C_t^u} = \xi$.

First, from the aggregate consumption, we obtain

$$C_t = (1-U_t)C_t^e + U_tC_t^u = \left((1-U_t)\xi + U_t\right)C_t^u$$

leading to

$$C_t^u = \frac{C_t}{(1-U_t)\xi + U_t}.$$

Now the household's per-period utility in (F.1) can be written as

$$\begin{aligned} U_t\frac{(C_t^u)^{1-\sigma}}{1-\sigma} + (1-U_t)\frac{(C_t^e)^{1-\sigma}}{1-\sigma} &= \frac{(C_t^u)^{1-\sigma}}{1-\sigma}\left[U_t + (1-U_t)\left(\underbrace{\frac{C_t^e}{C_t^u}}_{=\xi}\right)^{1-\sigma}\right] \\ &= \frac{(C_t)^{1-\sigma}}{1-\sigma}\cdot\frac{U_t + (1-U_t)\xi^{1-\sigma}}{\left[U_t + (1-U_t)\xi\right]^{1-\sigma}} \end{aligned}$$

Thus the household effectively maximizes:

$$\sum_{t=0}^{\infty}\left(\frac{1}{1+\rho}\right)^t\left[\frac{(C_t)^{1-\sigma}}{1-\sigma}\cdot\frac{U_t + (1-U_t)\xi^{1-\sigma}}{\left[U_t + (1-U_t)\xi\right]^{1-\sigma}}\right]$$

[12] We can prove this property similarly for the separation function $S_t(W_{jt})$.

subject to

$$C_t = (1 - U_t)\frac{W_t}{P_t} + NWI_t - B_t + (1 + r_{t-1,t})B_{t-1}$$

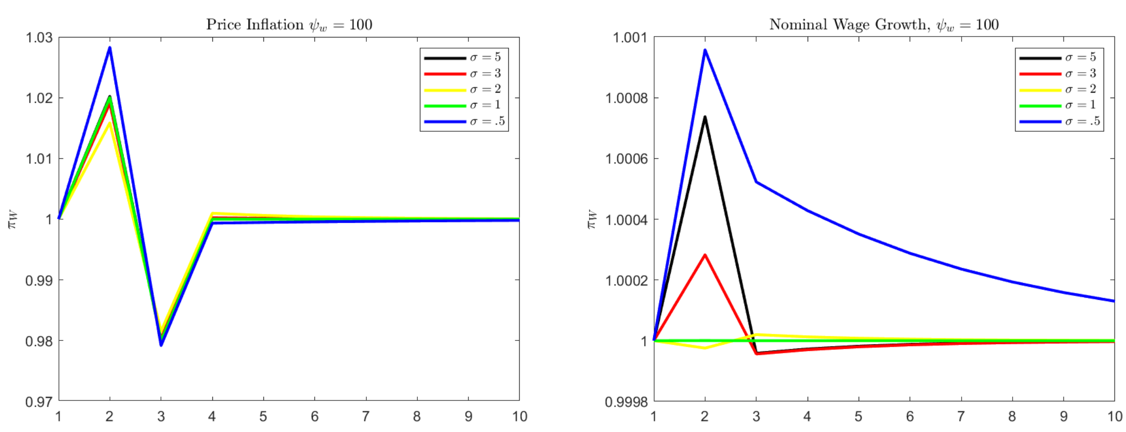
by choosing real bonds B_t and consumption C_t, where income including real non-wage income NWI_t (dividends paid out by firms) and real wage income $\left(\frac{W_t}{P_t}\right)$ is taken as given by the household, $r_{t-1,t}$ is the real rate between $t-1$ and t, and B_t are real bonds in zero net supply. Optimization requires that the household's choices obey the following consumption Euler equation:

$$\frac{C_t^{-\sigma}}{P_t} \cdot \underbrace{\frac{U_t + (1 - U_t)\xi^{1-\sigma}}{[U_t + (1 - U_t)\xi]^{1-\sigma}}}_{\equiv f(U_t)} = \frac{1}{1+\rho}(1 + i_{t,t+1})\frac{C_{t+1}^{-\sigma}}{P_{t+1}} \cdot \underbrace{\frac{U_{t+1} + (1 - U_{t+1})\xi^{1-\sigma}}{[U_{t+1} + (1 - U_{t+1})\xi]^{1-\sigma}}}_{\equiv f(U_{t+1})} \tag{F.2}$$

Note that (F.2) becomes a standard Euler equation

$$\frac{C_t^{-\sigma}}{P_t} = \frac{1}{1+\rho}(1 + i_{t,t+1})\frac{C_{t+1}^{-\sigma}}{P_{t+1}} \tag{F.3}$$

when the labor market is stabilized by monetary policy, i.e., $U_t = U_{t+1} = \bar{U}$ for $\forall t$. As assumed throughout the paper, we will assume that the monetary policy stabilizes labor market, i.e., (F.2) and (F.3) are both satisfied.



(a) Price inflation under different σ

(b) Wage inflation under different σ

Figure F.5: Impulse Response to a 10% Negative Shock to Supply of Endowment Good

Figure F.5 illustrates that (i) when $\sigma > 1$, wage inflation can rise in response to a pure cost-of-living shock, as it raises the recruiting and separation elasticities and firms are thus willing to offer higher wages in equilibrium; (ii) but still the magnitude of a rise in wage growth under our preferred calibration is small: in response to around 2% price increase, a rise in wage growth is less than 0.1%.

Monetary Policy So the Euler equation (F.3) when the labor market is stabilized can be re-written as

$$i_{t,t+1} = (1+\rho) \cdot \frac{P_{t+1}}{P_t} \cdot \left(\frac{C_{t+1}}{C_t}\right)^{\sigma} = (1+\rho) \cdot \frac{P_{t+1}C_{t+1}}{P_tC_t} \cdot \left(\frac{C_{t+1}}{C_t}\right)^{\sigma-1}. \tag{F.4}$$

In our Dixit-Stiglitz structure with $\eta = 1$, we have constant expenditure shares on endowment good X and service good Y, i.e., $\alpha_Y P_t C_t = P_{y,t} Y_t$ for $\forall t$. With $N_t = Y_t = \bar{N}$ for $\forall t$, it implies

$$\frac{P_{t+1}C_{t+1}}{P_tC_t} = \frac{P_{Y,t+1}}{P_{Y,t}}.$$

which if plugged into (F.4) leads to

$$i_{t,t+1} = (1+\rho) \cdot \underbrace{\frac{P_{Y,t+1}}{P_{Y,t}}}_{\equiv \Pi_{Y,t+1}} \cdot \left(\frac{C_{t+1}}{C_t}\right)^{\sigma-1}.$$

If period t is the timing of a cost-of-living shock, we have $\frac{C_{t+1}}{C_t} > 1$. If $\sigma > 1$ (i.e., their elasticity of substitution is less than 1), interest rate $i_{t,t+1}$ needs to rise from ρ since otherwise, households' demand for service goods at period t becomes higher than $\bar{Y} = \bar{N}$, destabilizing labor market at t. Also, since wage W_{t+1} rises due to higher price P_t raising the recruiting and separation elasticities under $\sigma > 1$, raising the marginal cost for firms, firms raise their prices, i.e., $\Pi_{Y,t+1} > 1$. Both terms raise $i_{t,t+1}$ from ρ.

Log-Utility Case In the previous log-preference case (i.e., $\sigma = 1$), we had the following Euler equation:

$$\frac{1}{P_tC_t} = \frac{1}{1+\rho}(1+i_{t,t+1})\frac{1}{P_{t+1}C_{t+1}}. \tag{F.5}$$

With interest rate pegging $i_t = \rho$ for $\forall t$, we equalize intertemporal consumption expenditure, i.e., $P_tC_t = P_{t+1}C_{t+1}$ and it equalizes intertemporal Y_t-consumption expenditure, i.e., $P_{Y,t}Y_t = P_{Y,t+1}Y_{t+1}$. Since wage stays at the steady state if labor market is stabilized under $\sigma = 1$, price $P_{Y,t}$ does not change, and labor market becomes actually stabilized.

G With Hiring Costs

Now, in addition to the direct vacancy-creating costs in the firm optimization (5), we assume that an intermediate firm pays a hiring cost which is convex in the number of new employees hired in each period. In this environment, the firm j maximizes

$$\max_{\substack{\{P^j_{y,t}\},\{N^j_t\}\\ \{W^j_t\},\{V^j_t\}}} \sum_{t=0}^{\infty}\left(\frac{1}{1+\rho}\right)^t \Bigg(P^j_{y,t}Y^j_t - W^j_t N^j_t - c_v\left(\frac{V^j_t}{N^j_{t-1}}\right)^{\chi_v} V^j_t W_t - \underbrace{c_h\left(\frac{H^j_t}{N^j_{t-1}}\right)^{\chi_h} H^j_t W_t}_{\text{Hiring cost}}$$
$$- \frac{\psi}{2}\left(\frac{P^j_{y,t}}{P^j_{y,t-1}}-1\right)^2 Y^j_t P^j_{y,t} - \frac{\psi^w}{2}\left(\frac{W^j_t}{W^j_{t-1}}-1\right)^2 W^j_t N^j_t\Bigg) \tag{G.1}$$

subject to

$$N^j_t = (1-S_t(W^j_t))N^j_{t-1} + \underbrace{R_t(W^j_t)V^j_t}_{\equiv H^j_t}. \tag{G.2}$$

We will have that physical output is produced with labor with the linear production: $Y^j_t = A^j_t N^j_t$. The Lagrangian then can be written as:

$$\mathcal{L} = \sum_{t=0}^{\infty}\left(\frac{1}{1+\rho}\right)^t \Bigg(P^j_{y,t}Y^j_t - W^j_t N^j_t - c_v\left(\frac{V^j_t}{N^j_{t-1}}\right)^{\chi_v} V^j_t W_t - \underbrace{c_h\left(\frac{V^j_t R(W^j_t)}{N^j_{t-1}}\right)^{\chi_h} V^j_t R_t(W^j_t) W_t}_{\text{Hiring cost}}$$
$$- \frac{\psi}{2}\left(\frac{P^j_{y,t}}{P^j_{y,t-1}}-1\right)^2 Y^j_t P^j_{y,t} - \frac{\psi^w}{2}\left(\frac{W^j_t}{W^j_{t-1}}-1\right)^2 W^j_t N^j_t$$
$$+ \lambda^j_t\left[-N^j_t + \underbrace{V^j_t R_t(W^j_t)}_{=H^j_t} + (1-S_t(W^j_t))N^j_{t-1}\right]\Bigg).$$

where due to the Dixit-Stiglitz structure, the labor demand N^j_t is given by

$$N^j_t = \left(\frac{P^j_{y,t}}{P_{y,t}}\right)^{-\epsilon}\frac{Y_t}{A^j_t}.$$

First order conditions We write the first order conditions under the symmetric equilibrium, where $N_t^j = N_t$, $W_t^j = W_t$, $V_t^j = V_t$, and $\lambda_t^j = \lambda_t$. The first order condition with V_t^j is given by

$$-c_v(1+\chi_V)\left(\frac{V_t}{N_{t-1}}\right)^{\chi_V} W_t - c_h(1+\chi_h)\left(\frac{V_t}{N_{t-1}}\right)^{\chi_h}(R_t(W_t))^{1+\chi_h}W_t + \lambda_t R_t(W_t) = 0, \tag{G.3}$$

which leads to

$$\lambda_t = c_v\frac{(1+\chi_V)\left(\frac{V_t}{N_{t-1}}\right)^{\chi_V}}{R_t(W_t)}W_t + c_h(1+\chi_h)\left(\frac{H_t}{N_{t-1}}\right)^{\chi_h}W_t, \tag{G.4}$$

where λ_t can be interpreted as a shadow value of a worker. It increases with c_h, a shifter in the hiring cost function.

The first order condition with W_t^j is given by

$$\begin{aligned}\frac{\psi^w}{2}\left(\frac{W_t}{W_{t-1}}-1\right)^2 + \psi^w\left(\frac{W_t}{W_{t-1}}-1\right)\frac{W_t}{W_{t-1}} + 1 =& \lambda_t\left(R_t'(W_t)\frac{V_t}{N_t} - \frac{N_{t-1}}{N_t}S_t'(W_t)\right)\\ &- c_h(1+\chi_h)R_t'(W_t)\left(\frac{H_t}{N_{t-1}}\right)^{\chi_h}\frac{V_t}{N_t}W_t\\ &+ \frac{1}{1+\rho}\psi^w\left(\frac{W_{t+1}}{W_t}-1\right)\frac{W_{t+1}}{W_t^2}W_{t+1}\frac{N_{t+1}}{N_t}.\end{aligned} \tag{G.5}$$

Combining equations (G.4) and (G.5), we obtain

$$\begin{aligned}&\frac{\psi^w}{2}\left(\frac{W_t}{W_{t-1}}-1\right)^2 + \psi^w\left(\Pi_t^w-1\right)\Pi_t^w + 1\\ =& \left(\underbrace{W_t c_v\frac{(1+\chi_V)\left(\frac{V_t}{N_{t-1}}\right)^{\chi_V}}{R_t(W_t)} + W_t c_h(1+\chi_h)\left(\frac{H_t}{N_{t-1}}\right)^{\chi_h}}_{=\lambda_t}\right)\left(R_t'(W_t)\frac{V_t}{N_t} - \frac{N_{t-1}}{N_t}S_t'(W_t)\right)\\ &- c_h(1+\chi_h)R_t'(W_t)\left(\frac{H_t}{N_{t-1}}\right)^{\chi_h}\frac{V_t}{N_t}W_t + \frac{1}{1+\rho}\psi^w\left(\Pi_{t+1}^w-1\right)\frac{W_{t+1}}{W_t^2}W_{t+1}\frac{N_{t+1}}{N_t}.\end{aligned} \tag{G.6}$$

Equation (G.6) can be rewritten as

$$\begin{aligned}\frac{\psi^w}{2}\left(\Pi_t^w-1\right)^2+\psi^w\left(\Pi_t^w-1\right)\Pi_t^w+1=&W_t c_v\frac{(1+\chi_V)\left(\frac{V_t}{N_{t-1}}\right)^{\chi_V}}{R_t(W_t)}\left(R_t'(W_t)\frac{V_t}{N_t}-\frac{N_{t-1}}{N_t}S_t'(W_t)\right)\\&+W_t c_h(1+\chi_h)\left(\frac{H_t}{N_{t-1}}\right)^{\chi_h}\left(-\frac{N_{t-1}}{N_t}S_t'(W_t)\right)\\&+\frac{1}{1+\rho}\psi^w\left(\Pi_{t+1}^w-1\right)\left(\Pi_{t+1}^w\right)^2\frac{N_{t+1}}{N_t},\end{aligned}$$

which lead to the following wage Phillips curve with hiring costs:

$$\begin{aligned}\frac{\psi^w}{2}\left(\Pi_t^w-1\right)^2+\psi^w\left(\Pi_t^w-1\right)\Pi_t^w+1=&c_v(1+\chi_V)\left(\frac{V_t}{N_{t-1}}\right)^{\chi_V}\left(\varepsilon_{R,W_t}\frac{V_t}{N_t}-\frac{N_{t-1}}{N_t}\frac{S_t(W_t)}{R_t(W_t)}\varepsilon_{S,W_t}\right)\\&+\underbrace{c_h(1+\chi_h)\left(\frac{H_t}{N_{t-1}}\right)^{\chi_h}S_t(W_t)\frac{N_{t-1}}{N_t}(-\varepsilon_{S,W_t})}_{\text{New term}}\\&+\frac{1}{1+\rho}\psi^w\left(\Pi_{t+1}^w-1\right)\left(\Pi_{t+1}^w\right)^2\frac{N_{t+1}}{N_t}.\end{aligned}\tag{G.7}$$

In the case of convex vacancy costs, i.e., $\chi_v>0$, and linear hiring costs, i.e., $\chi_h=0$, the wage Phillips curve becomes

$$\begin{aligned}\frac{\psi^w}{2}\left(\Pi_t^w-1\right)^2+\psi^w\left(\Pi_t^w-1\right)\Pi_t^w+1=&c_v(1+\chi_V)\left(\frac{V_t}{N_{t-1}}\right)^{\chi_V}\left(\varepsilon_{R,W_t}\frac{V_t}{N_t}-\frac{N_{t-1}}{N_t}\frac{S_t(W_t)}{R_t(W_t)}\varepsilon_{S,W_t}\right)\\&+c_hS_t(W_t)\frac{N_{t-1}}{N_t}(-\varepsilon_{S,W_t})+\frac{1}{1+\rho}\psi^w\left(\Pi_{t+1}^w-1\right)\left(\Pi_{t+1}^w\right)^2\frac{N_{t+1}}{N_t}.\end{aligned}$$

If instead we have linear vacancy costs, i.e., $\chi_v=0$ and convex hiring costs, i.e., $\chi_h>0$, the wage Phillips curve is given by

$$\begin{aligned}\frac{\psi^w}{2}\left(\Pi_t^w-1\right)^2+\psi^w\left(\Pi_t^w-1\right)\Pi_t^w+1=&c_v\left(\varepsilon_{R,W_t}\frac{V_t}{N_t}-\frac{N_{t-1}}{N_t}\frac{S_t(W_t)}{R_t(W_t)}\varepsilon_{S,W_t}\right)\\&+c_h(1+\chi_h)\left(\frac{H_t}{N_{t-1}}\right)^{\chi_h}S_t(W_t)g_t^{-1}(-\varepsilon_{S,W_t})\\&+\frac{1}{1+\rho}\psi^w\left(\Pi_{t+1}^w-1\right)\left(\Pi_{t+1}^w\right)^2g_{t+1},\end{aligned}$$

where we define $g_t\equiv\frac{N_t}{N_{t-1}}$ as employment growth.

No vacancy cost For simplicity, let us assume $c_v \to 0$, $c_h > 0$. With $B_t \equiv \frac{H_t}{N_{t-1}} = R_t T_t$ where $T_t \equiv \frac{V_t}{N_{t-1}}$ as defined in Appendix B.3, we can express equation (G.7) as

$$0 = F\left(\ln(\Pi_t^w), \ln(\Pi_{t+1}^w), \ln g_t, \ln g_{t+1}, \ln B_t, \ln \varepsilon_{S,W_t}, \ln S_t\right),$$

where

$$\begin{aligned}
F_{\ln(\Pi_t^w)} &= \psi^w \Pi_t^w \left(2(\Pi_t^w - 1) + \Pi_t^w\right) = \psi^w \\
F_{\ln(\Pi_{t+1}^w)} &= -\frac{\psi^w g_{t+1}}{1+\rho}\left(\Pi_{t+1}^w(\Pi_{t+1}^w)^2 + (\Pi_{t+1}^w - 1)2(\Pi_{t+1}^w)^2\right) = -\frac{\psi^w}{1+\rho} \\
F_{\ln g_t} &= -c_h(1+\chi_h)B_t^{\chi_h} \cdot S_t \cdot (-g_t^{-1})(-\varepsilon_{S,W_t}) = c_h(1+\chi_h)B^{\chi_h} \cdot S \cdot (-\varepsilon_S) \equiv \kappa_h > 0 \\
F_{\ln(g_{t+1})} &= -\frac{1}{1+\rho}\psi^w \left(\Pi_{t+1}^w - 1\right)(\Pi_{t+1}^w)^2 g_{t+1} = 0 \\
F_{ln(B_t)} &= -c_h(1+\chi_h)\chi_h \cdot B_t^{\chi_h} \cdot S_t \cdot g_t^{-1}(-\varepsilon_{S,W_t}) = -\chi_h \kappa_h \\
F_{\ln(\epsilon_{S,W_t})} &= c_h(1+\chi_h)B_t^{\chi_h} g_t^{-1}\epsilon_{S,W_t} = -\kappa_h \\
F_{\ln(S_t)} &= c_h(1+\chi_h)B_t^{\chi_h} g_t^{-1}\epsilon_{S,W_t} = -\kappa_h.
\end{aligned}$$

at the steady state. Therefore, up to a first order, we obtain

$$0 = \psi^w \check{\Pi}_t^w - \frac{\psi^w}{1+\rho}\check{\Pi}_{t+1}^w + \kappa_h\left(\check{g}_t - \chi_h \check{B}_t - \check{\varepsilon}_{S,W_t} - \check{S}_t\right),$$

leading to the following linearized wage Phillips curve:

$$\check{\Pi}_t^w = \frac{1}{1+\rho}\check{\Pi}_{t+1}^w + \underbrace{\frac{\kappa_h}{\psi^w}}_{>0}\left(\check{S}_t + \check{\varepsilon}_{S,W_t} + \chi_h \check{B}_t - \check{g}_t\right). \tag{G.8}$$

Equation (G.8) is straightforward to interpret: higher separation $\check{S}_t$ and more negative separation elasticity,[13] i.e., $\check{\varepsilon}_{S,W_t} > 0$, raise wage growth. With $\chi_h > 0$, i.e., convex hiring costs, higher $\check{B}_t$ implies higher marginal costs of new hires, thereby incentivizing a firm to raise wages so that it does not want to lose its current employees. Finally, higher $\check{g}_t$ (employment growth) means there is less incentive of firms to raise wages.

[13] In log-linearizing the non-linear wage Phillips curve (G.7), we use the following definition for negative ε_{S,W_t}:

$$\check{\varepsilon}_{S,W_t} = \frac{\varepsilon_{S,W_t} - \varepsilon_S}{\varepsilon_S},$$

which implies that $\check{\varepsilon}_{S,W_t} > 0$ when ε_{S,W_t} is more negative than its steady state level ε_S.

More Simplification To rearrange (G.8) so that it is represented in vacancy and unemployment, we use the followings we derived in Appendix B.3:

$$
\begin{aligned}
\check{S}_t &= g_{S,V}\check{V}_t + g_{S,U}\check{U}_{t-1} \\
\check{\varepsilon}_{S,W_t} &= g_{\varepsilon_S,V}\check{V}_t + g_{\varepsilon_S,U}\check{U}_{t-1},
\end{aligned}
\tag{G.9}
$$

and

$$
\check{B}_t = \check{R}_t + \check{T}_t = (g_{R,V} + 1)\,\check{V}_t + \left(g_{R,U} + \frac{U}{1-U}\right)\check{U}_{t-1}, \tag{G.10}
$$

with

$$
\begin{aligned}
\check{g}_t &= S(\check{R}_t + \check{T}_t - \check{S}_t) \\
&= S(g_{R,V} + 1 - g_{S,V})\check{V}_t + S\left(g_{R,U} + \frac{U}{1-U} - g_{S,U}\right)\check{U}_{t-1}.
\end{aligned}
\tag{G.11}
$$

Based on equations (G.9), (G.10), and (G.11), equation (G.8) can be written as

$$
\check{\Pi}^w_t = \frac{1}{1+\rho}\check{\Pi}^w_{t+1} + \phi^H_V\check{V}_t + \phi^H_U\check{U}_{t-1}, \tag{G.12}
$$

where

$$
\phi^H_V = \frac{\kappa_h}{\psi^w}\left[(g_{S,V} + g_{\varepsilon_S,V} + \chi_h(g_{R,V}+1) - S(g_{R,V} + 1 - g_{S,V})\right]
$$

and

$$
\phi^H_U = \frac{\kappa_h}{\psi^w}\left[g_{S,U} + g_{\varepsilon_S,U} + \chi_h\left(g_{R,U} + \frac{U}{1-U}\right) - S\left(g_{R,U} + \frac{U}{1-U} - g_{S,U}\right)\right].
$$

Based on $\check{Q}_t = g_{Q,V}\check{V}_t + g_{Q,U}\check{U}_{t-1}$, equation (G.12) can be again re-written as

$$
\check{\Pi}^w_t = \frac{1}{1+\rho}\check{\Pi}^w_{t+1} + \underbrace{\frac{\phi^H_V}{g_{Q,V}}}_{\equiv\beta^H_Q}\check{V}_t + \underbrace{\left(\phi^H_U - \phi^H_V\frac{g_{Q,U}}{g_{Q,V}}\right)}_{\equiv\beta^H_U}\check{U}_{t-1} \tag{G.13}
$$

which is a function of quits $\check{Q}_t$ and unemployment $\check{U}_{t-1}$. Under the current steady state levels unchanged, with parameters $c_h = 10$ and $\chi_h = 1$, we obtain

$$
\check{\Pi}^w_t = \frac{1}{1+\rho}\check{\Pi}^w_{t+1} + 10^{-3}\left(\check{V}_t + 6.8\times 10^{-2}\check{U}_{t-1}\right),
$$

where we see $\phi_V^H = 6.8 \times 10^{-5} > 0$ and

$$\frac{\phi_V}{\phi_U} = 15.151.$$

The reason we have a positive coefficient $\phi_U > 0$ on unemployment is easy to understand: higher $\check{U}_{t-1}$ (equivalently lower $\check{N}_{t-1}$) raises a marginal cost of new hire, inducing firms to raise wages to reduce turnover. This channel is absent under $\chi_h = 0$, in which case the wage Phillips curve becomes

$$\check{\Pi}_t^w = \frac{1}{1+\rho}\check{\Pi}_{t+1}^w + 10^{-3}\left(5.8\check{V}_t - 1.4 \times 10^{-2}\check{U}_{t-1}\right),$$

which depends negatively on $\check{U}_{t-1}$. Still $\frac{|\phi_V|}{|\phi_U|} = 4.14$.

Quits and Unemployment In terms of quits and unemployment, with $\chi_h = 1$, at a monthly frequency we obtain

$$\check{\Pi}_t^w = \frac{1}{1+\rho}\check{\Pi}_{t+1}^w + 10^{-3}\left(1.9 \times \check{V}_t + 0.29\check{U}_{t-1}\right),$$

where both $\beta_Q^H > 0$ and $\beta_U^H > 0$, and $\frac{\beta_Q^H}{\beta_U^H} = 6.57$.

Special case: no on-the-job search When $\lambda_{EE} \to 0$, we already know that $\check{S}_t \to 0$, $\check{\varepsilon}_{S,W_t} \to 0$, and

$$\check{R}_t \to -\frac{\theta^2}{1+\theta^2}\check{\theta}_t = -\frac{\theta^2}{1+\theta^2}(\check{V}_t - \check{U}_{t-1}). \tag{G.14}$$

We also know that

$$\check{B}_t = \check{R}_t + \check{T}_t = \underbrace{\check{R}_t + \check{V}_t}_{=\check{H}_t} + \frac{U}{1-U}\check{U}_{t-1} \tag{G.15}$$

where

$$\check{H}_t = \check{R}_t + \check{V}_t = \frac{1}{1+\theta^2}\check{V}_t + \frac{\theta^2}{1+\theta^2}\check{U}_{t-1}. \tag{G.16}$$

Finally

$$\check{g}_t = S\left(\check{R}_t + \check{T}_t - \check{S}_t\right) \to S\check{B}_t = S\left(\check{H}_t + \frac{U}{1-U}\check{U}_{t-1}\right). \tag{G.17}$$

With equations (G.14), (G.15), (G.16), and (G.17), our linearized wage Phillips curve

(G.8) can be written as

$$\begin{aligned}\check{\Pi}_t^w &= \frac{1}{1+\rho}\check{\Pi}_{t+1}^w + \underbrace{\frac{\kappa_h}{\psi^w}}_{>0}\left(\underbrace{\check{S}_t}_{\to 0} + \underbrace{\check{\varepsilon}_{S,W_t}}_{\to 0} + \chi_h \check{B}_t - \check{g}_t\right) \\ &= \frac{1}{1+\rho}\check{\Pi}_{t+1}^w + \frac{\kappa_h}{\psi^w}(\chi_h - S)\check{B}_t,\end{aligned} \tag{G.18}$$

where the right hand side is written in $\check{B}_t$. When $\chi_h = 1$, since $\chi_h > S$ at the steady state, higher $\check{B}_t$ raises wage growth since it increases the marginal cost of hiring a new worker due to the assumed convexity. In contrast, with $\chi_h = 0$, higher $\check{B}_t$ actually reduces wage growth: under linear hiring costs, marginal costs of new hires become constant, and higher $\check{B}_t$ means too many new hires, thereby inducing firms to reduce wages.

H Log-Linearized Price Phillips Curve

In this section, we log-linearize our price Phillips curve, i.e., equation (B.11). We start from our non-linear price Phillips curve equation (B.11):

$$\frac{\psi}{\epsilon}(\Pi_{Y,t}-1)\Pi_{Y,t}Y_t+\frac{\epsilon-1}{\epsilon}Y_t=\frac{W_t}{P_{y,t}}N_t+\frac{1}{1+\rho}\frac{\psi}{\epsilon}(\Pi_{Y,t+1}-1)\Pi^2_{Y,t+1}Y_{t+1}$$
$$+N_t\left(\frac{-c\chi}{1+\rho}\left(\frac{V_{t+1}}{N_t}\right)^{1+\chi}\Pi_{Y,t+1}\frac{W_{t+1}}{P_{y,t+1}}+\frac{\lambda_t}{P_{y,t}}-\frac{1}{1+\rho}\frac{\lambda_{t+1}}{P_{y,t+1}}\Pi_{Y,t+1}(1-S_{t+1}(W_{t+1}))\right) \tag{H.1}$$

We log-linearize equation (H.1). Dividing by $Y_t=A_tN_t$, the above (H.1) becomes

$$\frac{\psi}{\epsilon}(\Pi_{Y,t}-1)\Pi_{Y,t}+\frac{\epsilon-1}{\epsilon}=\underbrace{\frac{W_t}{P_{y,t}}\frac{1}{A_t}}_{\equiv mc_t}+\frac{1}{1+\rho}\frac{\psi}{\epsilon}(\Pi_{Y,t+1}-1)\Pi^2_{Y,t+1}\underbrace{\frac{N_{t+1}}{N_t}}_{\equiv g_{t+1}}\underbrace{\frac{A_{t+1}}{A_t}}_{\equiv g^A_{t+1}}$$
$$+\frac{A_{t+1}}{A_t}\left(\frac{-c\chi}{1+\rho}\left(\frac{V_{t+1}}{N_t}\right)^{1+\chi}\Pi_{Y,t+1}\underbrace{\frac{W_{t+1}}{P_{y,t+1}}\frac{1}{A_{t+1}}}_{\equiv mc_{t+1}}+\frac{\lambda_t}{P_{y,t}}\frac{1}{A_t}\underbrace{\left(\frac{A_{t+1}}{A_t}\right)^{-1}}_{\equiv(g^A_{t+1})^{-1}}\right. \tag{H.2}$$
$$\left.-\frac{1}{1+\rho}\frac{\lambda_{t+1}}{P_{y,t+1}}\frac{1}{A_{t+1}}\Pi_{Y,t+1}(1-S_{t+1}(W_{t+1}))\right)$$

where with $T_t\equiv\frac{V_t}{N_{t-1}}$, we use the shadow cost of vacancy creation (i.e., the first order condition for vacancy) given by:

$$\frac{\lambda_t}{P_{y,t}}\frac{1}{A_t}=\frac{W_t}{P_{y,t}}\frac{1}{A_t}\cdot R(W_t)^{-1}c(1+\chi)T_t^{\chi}=mc_t\cdot R_t^{-1}\cdot c(1+\chi)T_t^{\chi}. \tag{H.3}$$

Then the above equation (H.2) can be written as

$$0=\frac{\psi}{\epsilon}(\Pi_{Y,t}-1)\Pi_{Y,t}+\frac{\epsilon-1}{\epsilon}-mc_t-\frac{1}{1+\rho}\frac{\psi}{\epsilon}(\Pi_{Y,t+1}-1)\Pi^2_{Y,t+1}\cdot g_{t+1}\cdot g^A_{t+1}$$
$$-g^A_{t+1}\left(\frac{-c\chi}{1+\rho}(T_{t+1})^{1+\chi}\Pi_{Y,t+1}mc_{t+1}+mc_t\cdot R_t^{-1}\cdot c(1+\chi)T_t^{\chi}\left(g^A_{t+1}\right)^{-1}\right.$$
$$\left.-\frac{1}{1+\rho}mc_{t+1}\cdot R_{t+1}^{-1}\cdot c(1+\chi)T_{t+1}^{\chi}\Pi_{Y,t+1}(1-S_{t+1}(W_{t+1}))\right)$$
$$\equiv F\left(\ln(\Pi_{Y,t}),\ln(\Pi_{Y,t+1}),\ln(mc_t),\ln(mc_{t+1}),\ln g_{t+1},\ln g^A_{t+1},\ln T_t,\ln T_{t+1},\ln R_t,\ln R_{t+1},\ln S_{t+1}\right)$$

Steady State Calculation First, from the above equation, we know at the steady state with $\Pi_Y = 1$,

$$\frac{\epsilon-1}{\epsilon} - mc - \left(-\frac{c\chi}{1+\rho}T^{1+\chi}mc + mc\cdot c(1+\chi)T^{\chi}R^{-1} - \frac{1}{1+\rho}\cdot mc\cdot R^{-1}c(1+\chi)T^{\chi}(1-S)\right) = 0$$

which leads to

$$mc = \frac{\dfrac{\epsilon-1}{\epsilon}}{1 - \dfrac{c\chi}{1+\rho}T^{1+\chi} + \dfrac{\rho+S}{1+\rho}\cdot c(1+\chi)T^{\chi}R^{-1}}$$

at the steady state, with $T = \frac{V}{N} = \frac{S}{R}$. We first calculate derivatives of $F(\cdot)$ with respect to each variable at the steady state as follows: first, $F_{\ln(g_{t+1})} = 0$. And

$$F_{\ln(\Pi_{Y,t})} = \frac{\psi}{\epsilon}$$

$$F_{\ln(mc_t)} = -mc[1 + R^{-1}\cdot c(1+\chi)T^{\chi}]$$

$$F_{\ln(g^A_{t+1})} = \frac{mc\cdot cT^{\chi}}{1+\rho}[\chi T + (1+\chi)R^{-1}\cdot(1-S)]$$

$$F_{\ln(T_{t+1})} = \frac{mc\cdot cT^{\chi}}{1+\rho}\chi(1+\chi)[T + R^{-1}(1-S)]$$

$$F_{\ln(R_{t+1})} = -mc\cdot R^{-1}\cdot c(1+\chi)T^{\chi}\cdot\frac{1-S}{1+\rho}$$

$$F_{\ln(\Pi_{Y,t+1})} = -\frac{1}{1+\rho}\frac{\psi}{\epsilon} + \frac{mc\cdot cT^{\chi}}{1+\rho}[\chi T + (1+\chi)R^{-1}(1-S)]$$

$$F_{\ln(mc_{t+1})} = \frac{mc\cdot cT^{\chi}}{1+\rho}[\chi T + (1+\chi)R^{-1}\cdot(1-S)]$$

$$F_{\ln(T_t)} = -\chi\cdot mc\cdot R^{-1}\cdot c(1+\chi)T^{\chi}$$

$$F_{\ln(R_t)} = mc\cdot R^{-1}\cdot c(1+\chi)T^{\chi}$$

$$F_{\ln(S_{t+1})} = -mc\cdot R^{-1}\cdot c(1+\chi)T^{\chi}\cdot\frac{S}{1+\rho}.$$

Therefore, equation (H.2) can be approximated up to a first order as

$$\begin{aligned}
&\frac{\psi}{\epsilon}\check{\Pi}_{Y,t} + \left(-\frac{1}{1+\rho}\frac{\psi}{\epsilon} + \frac{mc\cdot cT^{\chi}}{1+\rho}[\chi T + (1+\chi)R^{-1}(1-S)]\right)\check{\Pi}_{Y,t+1} \\
&- mc[1 + R^{-1}\cdot c(1+\chi)T^{\chi}]\check{mc}_t + \frac{mc\cdot cT^{\chi}}{1+\rho}[\chi T + (1+\chi)R^{-1}\cdot(1-S)]\check{mc}_{t+1} \\
&+ \frac{mc\cdot cT^{\chi}}{1+\rho}[\chi T + (1+\chi)R^{-1}\cdot(1-S)]\check{g}^A_{t+1} \\
&- \chi\cdot mc\cdot R^{-1}\cdot c(1+\chi)T^{\chi}\check{T}_t + \frac{mc\cdot cT^{\chi}}{1+\rho}\chi(1+\chi)[T + R^{-1}(1-S)]\check{T}_{t+1} \\
&+ mc\cdot R^{-1}\cdot c(1+\chi)T^{\chi}\check{R}_t - mc\cdot R^{-1}\cdot c(1+\chi)T^{\chi}\cdot\frac{1-S}{1+\rho}\check{R}_{t+1} \\
&- mc\cdot R^{-1}\cdot c(1+\chi)T^{\chi}\cdot\frac{S}{1+\rho}\check{S}_{t+1} = 0
\end{aligned}$$

which leads to our linear price Phillips curve with the following form:

$$
\begin{aligned}
\check{\Pi}_{Y,t} =& b_{\pi}\check{\Pi}_{Y,t+1} + b_{mc,0}\check{mc}_t - b_{mc,1}\left(\check{mc}_{t+1} + g^A_{t+1}\right) \\
&+ b_{T,0}\check{T}_t - b_{T,1}\check{T}_{t+1} - b_{R,0}\check{R}_t + b_{R,1}\check{R}_{t+1} + b_{S,1}\check{S}_{t+1}
\end{aligned} \tag{H.4}
$$

where all the coefficients b_{π}, $b_{mc,0}$, $b_{mc,1}$, $b_{T,0}$, $b_{T,1}$, $b_{R,0}$, $b_{R,1}$, $b_{S,1}$ are positive. For example,

$$
b_{mc,1} = \frac{\epsilon}{\psi} \cdot \frac{mc \cdot cT^{\chi}}{1+\rho}[\chi T + (1+\chi)R^{-1} \cdot (1-S)] > 0. \tag{H.5}
$$

Intuition Our linear price Phillips curve (H.4) looks different from the standard Phillips curve in a canonical New Keynesian model. What is going on?

1. From (H.4), a higher $\check{mc}_{t+1} + g^A_{t+1}$ is deflationary: higher $mc_{t+1} + g^A_{t+1}$ means higher W_{t+1} and makes hiring more costly next period, lowering tomorrow's vacancy creation amount. Thus, firms have higher incentive for firm today to raise N_{jt} to reduce tomorrow's vacancy cost: thereby lowering $P_{Y,t}$.

   - It is represented by
   $$
   \frac{\epsilon}{\psi} \cdot \frac{mc \cdot cT^{\chi}}{1+\rho} \cdot \chi T
   $$
   term in (H.5).

2. Another effect of a higher $\check{mc}_{t+1} + g^A_{t+1}$ is stemming from the labor flow equation $N_{j,t+1} = (1 - S(W_{j,t+1})N_{jt} + R(W_{j,t+1})V_{j,t+1}$: a higher W_{t+1} implies lower S_{t+1} and higher R_{t+1} rates in the next period. So a value of labor hiring now is higher, and firms have incentive to lower price to increase output demand that leads to a higher labor demand.

   - It is represented by
   $$
   \frac{\epsilon}{\psi} \cdot \frac{mc \cdot cT^{\chi}}{1+\rho} \cdot (1+\chi)R^{-1}(1-S)
   $$
   term in (H.5).

3. Current marginal costs $\check{mc}_t$ affect firms' pricing decision in two different channels: it affects direct labor cost, thereby raising $P_{Y,t}$. Also, it means a higher vacancy cost at the margin, thereby raising $P_{Y,t}$.

4. $\check{T}_t$ is inflationary: higher $\check{T}_t$ means roughly a tighter labor market: vacancy cost is higher at the margin, thus pushing overall marginal cost of hiring and price $P_{Y,t}$ up.

5. $\check{T}_{t+1}$ is deflationary: the intuition is similar to why $\check{mc}_{t+1} + g_{t+1}^A$ is deflationary: (i) firms have an incentive today to raise N_{jt} to reduce tomorrow's vacancy cost: thereby lowering $P_{Y,t}$; (ii) since hiring tomorrow is costly, it is better to hire now. Thus firms lower $P_{Y,t}$ to increase labor demand.

6. $\check{R}_t$ deflationary - vacancy today gets filled more easily so price $P_{Y,t}$ decreases.

7. $\check{R}_{t+1}$ inflationary: since hiring tomorrow becomes easier, why hire today? Let us just raise price $P_{Y,t}$ with lower labor demand N_{jt}.

8. $\check{S}_{t+1}$ inflationary: tomorrow separation is more prevalent, so firms have less incentive for hiring today: they can raise $P_{Y,t}$ and reduce labor demand.

Monetary Stabilization and Wage-Price Spiral We already know that if monetary policy stabilizes N_t so that $N_t = N$, it stabilizes all the labor market variables including T_t, R_t, and S_t, and wage as well. From equation (H.4), if $\check{mc}_t$ and $\check{mc}_{t+1}$ stay at 0 given that other labor market variables (their deviation from steady state) stay at 0 due to monetary policy, then we know $\Pi_{Y,t} = 0$, i.e., joint stabilization of price and wage can be achieved.

- If there is a supply shock, i.e., a negative shock in A_t that raises $\check{mc}_t$, then even with zero wage response, we have price inflation, as we see in (H.4) - still there is no spillover from price to wage.

Numerical Coefficients Under our calibration at monthly frequency, we obtain $b_\pi = 0.974, b_{mc,0} = 0.112, b_{mc,1} = 0.022, b_{T,0} = 0.0225, b_{T,1} = 0.0224, b_{R,0} = 0.0225, b_{R,1} = 0.0216, b_{S,1} = 8.1215 \times 10^{-4}$.

Thus,

$$\begin{aligned}\check{\Pi}_{Y,t} =& b_\pi \check{\Pi}_{Y,t+1} + b_{mc,0}\check{mc}_t - b_{mc,1}\left(\check{mc}_{t+1} + g_{t+1}^A\right) \\ &+ b_{T,0}\check{T}_t - b_{T,1}\check{T}_{t+1} - b_{R,0}\check{R}_t + b_{R,1}\check{R}_{t+1} + b_{S,1}\check{S}_{t+1}\end{aligned} \tag{H.6}$$

becomes

$$\begin{aligned}\check{\Pi}_{Y,t} =& 0.974\check{\Pi}_{Y,t+1} + 0.112\check{mc}_t - 0.022\left(\check{mc}_{t+1} + g_{t+1}^A\right) \\ &+ 0.0225\check{T}_t - 0.0224\check{T}_{t+1} - 0.0225\check{R}_t + 0.0216\check{R}_{t+1} + 0.0008\check{S}_{t+1}.\end{aligned} \tag{H.7}$$

I Log-Linearized Wage Phillips Curve with Fixed b

In this section, we derive log-linearized wage Phillips curve in our model extension with fixed b in Section 4.2. We start from equation (I.1):

$$\check{\Pi}_t^w = \frac{\kappa}{\psi^w}\left[\underbrace{\left(-\varepsilon_S + S(\varepsilon_R - \varepsilon_S)\right)}_{>0}\left(\check{S}_t - \check{R}_t\right) + \underbrace{\left(\varepsilon_R + (\chi - S)\left(\varepsilon_R - \varepsilon_S\right)\right)}_{>0}\check{T}_t + \left(\varepsilon_R \check{\varepsilon}_{R,t} - \varepsilon_S \check{\varepsilon}_{S,t}\right)\right] + \frac{1}{1+\rho}\check{\Pi}_{t+1}^w. \tag{I.1}$$

Functional Forms with Real b Now we have

$$R_t = g(\theta_t)\left(\phi_{E,t}\frac{1}{2} + \phi_{U,t}\left(\frac{\tilde{w}_t^\gamma}{\tilde{w}_t^\gamma + b^\gamma}\right)\right).$$

where $\tilde{w}_t \equiv \frac{W_t}{P_t}$ is real wage, i.e., wage denominated in price aggregator P_t. At the steady state, we assume

$$\frac{\tilde{w}^\gamma}{\tilde{w}^\gamma + b^\gamma} = \frac{\xi^\gamma}{1+\xi^\gamma} \equiv \mathcal{C}.$$

Likewise, we obtain

$$S_t = s + (1-s)\left(\lambda_{EE} f(\theta_t)\frac{1}{2} + \lambda_{EU}\left(\frac{b^\gamma}{\tilde{w}_t^\gamma + b^\gamma}\right)\right)$$

$$\varepsilon_{R,t} = \frac{\gamma\left(\frac{\phi_{E,t}}{4} + \phi_{U,t}\cdot\frac{b^\gamma \tilde{w}_t^\gamma}{\left[\tilde{w}_t^\gamma + b^\gamma\right]^2}\right)}{0.5\phi_{E,t} + \left(\frac{\tilde{w}_t^\gamma}{\tilde{w}_t^\gamma + b^\gamma}\right)\phi_{U,t}}$$

$$\varepsilon_{S,t} = \frac{-(1-s)\gamma\left(f(\theta_t)\lambda_{EE}\frac{1}{4} + \lambda_{EU}\cdot\frac{b^\gamma \tilde{w}_t^\gamma}{\left[\tilde{w}_t^\gamma + b^\gamma\right]^2}\right)}{s + (1-s)\left(0.5\cdot\lambda_{EE} f(\theta_t) + \lambda_{EU}\cdot\frac{b^\gamma}{\tilde{w}_t^\gamma + b^\gamma}\right)}$$

Derivation To begin to simplify the wage Phillips curve with fixed b (I.1), we decompose all of the following right-hand-side variables, $\check{S}_t$, $\check{T}_t$, $\check{R}_t$, $\check{\varepsilon}_{R,t}$, and $\check{\varepsilon}_{S,t}$ into vacancy, unemployment, and the real wage $\tilde{w}_t$ deviations. The tightness term, $T_t = \frac{V_t}{N_{t-1}}$, is the same: in log deviations from steady state, it becomes

$$\check{T}_t = \check{V}_t + \frac{U}{1-U}\check{U}_{t-1}.$$

For the rest, we can write the decomposition as follows:

1. $\check{R}_t \equiv g_{R,V}\check{V}_t + g_{R,U}\check{U}_{t-1} + g_{R,w}\check{\tilde{w}}_t$

   Derivation: Recall that the recruiting function is

$$R_t = g(\theta_t)\left(\phi_{E,t}\frac{1}{2} + \phi_{U,t}\left(\frac{\tilde{w}_t^\gamma}{\tilde{w}_t^\gamma + b^\gamma}\right)\right).$$

   For practical purposes we define $\mathcal{C} \equiv \frac{\xi^\gamma}{1+\xi^\gamma}$, which is increasing in the ratio of consumption for employed to unemployed workers. Then we obtain:

$$g_{R,V} = -\frac{\theta^2}{1+\theta^2},$$

   and

$$\begin{aligned} g_{R,U} =& \frac{\theta^2}{1+\theta^2}\cdot\frac{U(1-\lambda_{EE})}{\lambda_{EE}(1-U)+U} + \frac{0.5\phi_E}{0.5\phi_E + \mathcal{C}\phi_U}\cdot\frac{U}{1-U}\cdot\frac{\lambda_{EE}\phi_E - \lambda_{EE} - \phi_E}{\lambda_{EE}} \\ &+ \frac{\mathcal{C}\phi_U}{0.5\phi_E + \mathcal{C}\phi_U}\cdot(1-\phi_U(1-\lambda_{EE})), \end{aligned}$$

   and

$$g_{R,w} = \frac{\mathcal{C}\phi_U}{0.5\phi_E + \mathcal{C}\phi_U}\cdot\gamma(1-\mathcal{C}) > 0.$$

   Note that $|g_{R,w}|$ is decreasing in λ_{EE}.

2. $\check{S}_t \equiv g_{S,V}\check{V}_t + g_{S,U}\check{U}_{t-1} + g_{S,w}\check{\tilde{w}}_t$

   Derivation: Recall that the separation function S_t is given by

$$S_t = s + (1-s)\left(\lambda_{EE}f(\theta_t)\frac{1}{2} + \lambda_{EU}\left(\frac{b^\gamma}{\tilde{w}_t^\gamma + b^\gamma}\right)\right)$$

   Then, we obtain:

$$g_{S,V} = \frac{(1-s)0.5\lambda_{EE}f}{s + (1-s)\left(0.5\lambda_{EE}f + \lambda_{EU}(1-\mathcal{C})\right)}\cdot\frac{1}{1+\theta^2},$$

   and

$$g_{S,U} = -\frac{(1-s)0.5\lambda_{EE}f}{s + (1-s)\left(0.5\lambda_{EE}f + \lambda_{EU}(1-\mathcal{C})\right)}\cdot\frac{1}{1+\theta^2}\cdot\frac{U(1-\lambda_{EE})}{\lambda_{EE}(1-U)+U},$$

and

$$g_{S,w} = \frac{-\gamma(1-s)\lambda_{EU}\mathcal{C}(1-\mathcal{C})}{s+(1-s)\left(0.5\lambda_{EE}f+\lambda_{EU}(1-\mathcal{C})\right)} < 0.$$

Note that $|g_{S,w}|$ is decreasing in λ_{EE}, given θ.

3. $\check{\varepsilon}_{R,t} = g_{\varepsilon_{R,U}}\check{U}_{t-1} + g_{\varepsilon_{R,w}}\check{\tilde{w}}_t$

   Derivation: Note that in equilibrium, $\varepsilon_{R,t}$ is given by

$$\varepsilon_{R,t} = \frac{\gamma\left(\frac{\phi_{E,t}}{4} + \phi_{U,t}\cdot\frac{b^\gamma\tilde{w}_t^\gamma}{\left[\tilde{w}_t^\gamma+b^\gamma\right]^2}\right)}{\left(0.5\phi_{E,t}+\left(\frac{\tilde{w}_t^\gamma}{\tilde{w}_t^\gamma+b^\gamma}\right)\phi_{U,t}\right)}$$

   from which we obtain

$$\begin{aligned} g_{\varepsilon_{R,U}} = &\left(\frac{0.25\phi_E}{0.25\phi_E+\mathcal{C}(1-\mathcal{C})\phi_U} - \frac{0.5\phi_E}{0.5\phi_E+\mathcal{C}\phi_U}\right)\frac{U}{1-U}\frac{\lambda_{EE}\phi_E-\lambda_{EE}-\phi_E}{\lambda_{EE}} \\ &+\left(\frac{\mathcal{C}(1-\mathcal{C})\phi_U}{0.25\phi_E+\mathcal{C}(1-\mathcal{C})\phi_U} - \frac{\mathcal{C}\phi_U}{0.5\phi_E+\mathcal{C}\phi_U}\right)(1-\phi_U(1-\lambda_{EE})). \end{aligned}$$

   and

$$g_{\varepsilon_{R,w}} = -\left[\frac{\phi_U\mathcal{C}(1-\mathcal{C})}{0.25\phi_E+\phi_U\mathcal{C}(1-\mathcal{C})}\gamma(2\mathcal{C}-1) + \frac{\mathcal{C}\phi_U}{0.5\phi_E+\mathcal{C}\phi_U}\gamma(1-\mathcal{C})\right] < 0$$

   Note that $|g_{\varepsilon_{R,w}}|$ is decreasing in λ_{EE}.

4. $\check{\varepsilon}_{S,t} = g_{\varepsilon_{S,V}}\check{V}_t + g_{\varepsilon_{S,U}}\check{U}_{t-1} + g_{\varepsilon_{S,w}}\check{\tilde{w}}_t$

   Derivation: Note that in equilibrium $\varepsilon_{S,t}$ is given by

$$\varepsilon_{S,t} = \frac{-(1-s)\gamma\left(f(\theta_t)\lambda_{EE}\frac{1}{4} + \lambda_{EU}\cdot\frac{b^\gamma\tilde{w}_t^\gamma}{\left[\tilde{w}_t^\gamma+b^\gamma\right]^2}\right)}{s+(1-s)\left(0.5\cdot\lambda_{EE}f(\theta_t)+\lambda_{EU}\cdot\frac{b^\gamma}{\tilde{w}_t^\gamma+b^\gamma}\right)}$$

   from which we obtain

$$g_{\varepsilon_{S,V}} = \left(\frac{0.25\lambda_{EE}f}{0.25\lambda_{EE}f+\mathcal{C}(1-\mathcal{C})\lambda_{EU}} - \frac{0.5(1-s)\lambda_{EE}f}{s+(1-s)(0.5\lambda_{EE}f+(1-\mathcal{C})\lambda_{EU})}\right)\frac{1}{1+\theta^2},$$

and

$$g_{\varepsilon_{S,U}} = -g_{\varepsilon_{S,V}} \cdot \frac{U(1-\lambda_{EE})}{\lambda_{EE}(1-U)+U}.$$

and

$$g_{\varepsilon_s,w} = \gamma\left[\mathcal{C}\cdot\frac{(1-s)\lambda_{EU}(1-\mathcal{C})}{s+(1-s)(0.5\lambda_{EE}f+\lambda_{EU}(1-\mathcal{C}))} - (2\mathcal{C}-1)\frac{\lambda_{EU}\mathcal{C}(1-\mathcal{C})}{0.25\lambda_{EE}f+\lambda_{EU}\mathcal{C}(1-\mathcal{C})}\right]$$

Note that $g_{\varepsilon_S,w}$ is a complex function of λ_{EE}. Since $\mathcal{C} \simeq 1$ and $s \simeq 0$ under our calibration, we have $2\mathcal{C}-1 \simeq 1$ and:

$$\begin{aligned}
g_{\varepsilon_s,w} &\simeq \gamma\left[\cdot\frac{\lambda_{EU}(1-\mathcal{C})}{0.5\lambda_{EE}f+\lambda_{EU}(1-\mathcal{C})} - \frac{\lambda_{EU}(1-\mathcal{C})}{0.25\lambda_{EE}f+\lambda_{EU}(1-\mathcal{C})}\right] \\
&= -\gamma\left(\frac{0.25\lambda_{EE}f}{0.25\lambda_{EE}f+\lambda_{EU}(1-\mathcal{C})}\right)\cdot\left(\frac{\lambda_{EU}(1-\mathcal{C})}{0.5\lambda_{EE}f+\lambda_{EU}(1-\mathcal{C})}\right) \\
&\simeq -\gamma\lambda_{EU}(1-\mathcal{C})\frac{0.25\lambda_{EE}f}{0.125\lambda_{EE}^2f^2+\lambda_{EU}(1-\mathcal{C})\cdot 0.75\lambda_{EE}f+\underbrace{\lambda_{EU}^2(1-\mathcal{C})^2}_{\simeq 0}} \\
&\simeq -\gamma\lambda_{EU}(1-\mathcal{C})\frac{0.25\cdot f}{0.125\lambda_{EE}f^2+\lambda_{EU}(1-\mathcal{C})\cdot 0.75f}
\end{aligned}$$

Therefore, $|g_{\varepsilon_S,w}|$ is (approximately) decreasing in λ_{EE}.

Decomposing Wage Growth into Vacancies and Unemployment Combining these results, we can rewrite the wage Phillips curve just in terms of gaps in vacancies, unemployment, and real wage. Let $\Delta_1 \equiv -\varepsilon_S + S(\varepsilon_R - \varepsilon_S)$ and let $\Lambda_1 \equiv \varepsilon_R + (\chi - S)(\varepsilon_R - \varepsilon_S)$. Then the wage Phillips curve (I.1) can be written as:

$$\begin{aligned}
\check{\Pi}_t^w = &\underbrace{\frac{\kappa}{\psi^w}\left[\Lambda_1 + \Delta_1\left(g_{S,V} - g_{R,V}\right) - \varepsilon_S g_{\varepsilon_S,V}\right]}_{\equiv\phi_V>0}\check{V}_t \\
&+ \underbrace{\frac{\kappa}{\psi^w}\left[\frac{U}{1-U}\Lambda_1 + \Delta_1\left(g_{S,U} - g_{R,U}\right) + \varepsilon_R g_{\varepsilon_R,U} - \varepsilon_S g_{\varepsilon_S,U}\right]}_{\equiv\phi_U<0}\check{U}_{t-1} \\
&+ \underbrace{\frac{\kappa}{\psi^w}\left[\Delta_1\left(\underbrace{g_{S,w}}_{<0} - \underbrace{g_{R,w}}_{>0}\right) + \underbrace{\varepsilon_R}_{>0}\underbrace{g_{\varepsilon_R,w}}_{<0} - \underbrace{\varepsilon_S}_{<0}\underbrace{g_{\varepsilon_S,w}}_{<0}\right]}_{\equiv\phi_w<0}\check{\check{w}}_t + \frac{1}{1+\rho}\check{\Pi}_{t+1}^w.
\end{aligned} \tag{I.2}$$

and $|\phi_w|$ is decreasing in on-the-job search probability λ_{EE}, given steady-state θ. Interpretation is very simple: given $\check{V}_t$ and $\check{U}_{t-1}$, a cost of living shock lowers real wage $\check{\tilde{w}}_t$, and with $\phi_w < 0$ raises wage inflation $\check{\Pi}^w_t$. Therefore, there is pass through from price to wage as we explain in Section 4.2.

I.1 Wage Phillips Curve in Quits, Unemployment, and Real Wage

First, we log-linearize the quit function Q_t as $\check{Q}_t = g_{Q,V}\check{V}_t + g_{Q,U}\check{U}_{t-1} + g_{Q,w}\check{\tilde{w}}_t$. Recall that the quit function $Q_t = S_t - s$ is given by

$$Q_t = (1-s)\left(\lambda_{EE} f(\theta_t)\frac{1}{2} + \lambda_{EU}\left(\frac{b^\gamma}{\tilde{w}_t^\gamma + b^\gamma}\right)\right)$$

Then, we obtain:

$$g_{Q,V} = \frac{0.5\lambda_{EE} f}{0.5\lambda_{EE} f + \lambda_{EU}(1-\mathcal{C})} \cdot \frac{1}{1+\theta^2} > 0,$$

and

$$g_{Q,U} = -\frac{0.5\lambda_{EE} f}{0.5\lambda_{EE} f + \lambda_{EU}(1-\mathcal{C})} \cdot \frac{1}{1+\theta^2} \cdot \frac{U(1-\lambda_{EE})}{\lambda_{EE}(1-U)+U} < 0.$$

and

$$g_{Q,w} = \frac{-\gamma\lambda_{EU}\mathcal{C}(1-\mathcal{C})}{0.5\lambda_{EE} f + \lambda_{EU}(1-\mathcal{C})} < 0$$

Rearranging in terms of

$$\check{V}_t = \frac{\check{Q}_t - g_{Q,U}\check{U}_{t-1} - g_{Q,w}\check{\tilde{w}}_t}{g_{Q,V}},$$

the above equation (I.2) becomes

$$\begin{aligned}
\check{\Pi}^w_t =& \phi_V \check{V}_t + \phi_U \check{U}_{t-1} + \phi_w \check{\tilde{w}}_t + \frac{1}{1+\rho}\check{\Pi}^w_{t+1} \\
=& \phi_V\left(\frac{\check{Q}_t - g_{Q,U}\check{U}_{t-1} - g_{Q,w}\check{\tilde{w}}_t}{g_{Q,V}}\right) + \phi_U\check{U}_{t-1} + \phi_w\check{\tilde{w}}_t + \frac{1}{1+\rho}\check{\Pi}^w_{t+1} \\
=& \underbrace{\frac{\phi_V}{g_{Q,V}}}_{>0}\check{Q}_t + \left(\underbrace{\frac{-g_{Q,U}}{g_{Q,V}}\phi_V}_{>0} + \underbrace{\phi_U}_{<0}\right)\check{U}_{t-1} + \left(\underbrace{\frac{-g_{Q,w}}{g_{Q,V}}\phi_V}_{>0} + \underbrace{\phi_w}_{<0}\right)\check{\tilde{w}}_t + \frac{1}{1+\rho}\check{\Pi}^w_{t+1}.
\end{aligned} \tag{I.3}$$

We should then expect that the "catch-up" (see e.g., Bernanke and Blanchard (2024)) term

$\check{\check{w}}_t$ should be mitigated for the quits equation (I.3) than for the equation with vacancies (I.2). The reason is that some effect of this catch-up is absorbed by an endogenous response in quits, i.e., quits $\check{Q}_t$ will rise with a cost of living shock, given $\check{V}_t$, $\check{U}_{t-1}$, and $\check{\check{w}}_t$. If those effects are strong enough, the coefficient on $\check{\check{w}}_t$ in equation (I.3) can be positive.

It turns out that the coefficient on $\check{\check{w}}_t$ in equation (I.3) becomes positive under our calibration. At monthly frequency, we obtain

$$\check{\Pi}_t^w = 0.0183\check{V}_t - 0.003\check{U}_{t-1} \underbrace{-0.011}_{<0} \check{\check{w}}_t + \frac{1}{1+\rho}\check{\Pi}_{t+1}^w,$$

and

$$\check{\Pi}_t^w = 0.0246\check{Q}_t - 0.0009\check{U}_{t-1} \underbrace{+0.0142}_{>0} \check{\check{w}}_t + \frac{1}{1+\rho}\check{\Pi}_{t+1}^w,$$

where still quits remains as the strongest driver of wage growth.

J Two-Period Model

In this section, we build a simple two-period general equilibrium model that illustrates the following two features in a sharper way:

1. When the employed and unemployed share consumption risks according to $\frac{C_t^e}{C_t^u} = \xi$, i.e., the unemployed receives the consumption expenditure that is ξ^{-1} times that of employed workers, the cost of living shock does not affect wage and labor market outcomes in general.

2. When the unemployed benefit b_t is in real terms, which workers compare with real wage $\frac{W_t}{P_t}$ in deciding whether to join the workforce or not, a cost of living shock generates a positive wage response. This wage response becomes more muted as λ_{EE}, the on-the-job search probability, increases.

We consider 3 different points in time: $t = 0, 1, 2$. At $t = 0$, the economy is at its steady-state: the number of employed is $\bar{N}$, that of unemployed is $\bar{U} = 1 - \bar{N}$. At $t = 2$, the economy gets back to the steady state, regardless of what happens at the interim period, $t = 1$.

Demand block The policy rate is given by $i_t = \rho$ for $t = 0, 1$ (i.e., pegged) so the households' Euler equation under log-preference implies the intertemporal equalization of consumption expenditures, given by

$$P_0 C_0 = P_1 C_1 = P_2 C_2, \tag{J.1}$$

where P_t is the price aggregator (of endowment good X_t and service good Y_t which is produced by firms) at time t, and C_t is the corresponding consumption aggregator. Under the unit elasticity of substitution between goods X_t and Y_t, i.e., $\eta = 1$ in our dynamic general equilibrium model, the households' expenditures on X_t and Y_t goods become proportional, implying

$$\frac{P_{X,t} X_t}{\alpha_X} = \frac{P_{Y,t} Y_t}{\alpha_Y} = P_t C_t \tag{J.2}$$

for all $t = 0, 1, 2$. From (J.1) and (J.2), we obtain:

$$P_{Y,0} Y_0 = P_{Y,1} Y_1 = P_{Y,2} Y_2 \tag{J.3}$$

in equilibrium. We assume the perfect price rigidity for the service good sector for tractability purposes: so $P_{Y,0} = P_{Y,1} = P_{Y,2} = \bar{P}_Y$,[14] which implies $Y_0 = Y_1 = Y_2 = \bar{Y}$ where $\bar{Y}$ is the

[14]We will characterize the flexible price case later as a separate case.

steady-state level of service output. Therefore, the service output Y_1 at the interim period $t = 1$ is always at the steady state level $\bar{Y}$, regardless of shocks realized at $t = 1$. It is because the economy is demand-determined, and the household always insures their perfect consumption smoothing under pegged monetary policy.

Firm's problem Firm i, with its production function $Y_t^i = N_t^i$,[15]. solves the following optimization at $t = 1$, with its number of workers $N_0 = \bar{N}$ inherited from the previous period:

$$J(\bar{N}) = \max_{V_1^i.W_1^i} \bar{P}_Y N_1^i - W_1^i N_1^i - \kappa(W_1) \cdot V_1^i + \frac{1}{1+\rho} J(N_1^i) \tag{J.4}$$

subject to

$$N_1^i = \bar{N} = (1 - S(W_1^i|W_1))\bar{N} + R(W_1^i|W_1)V_1^i, \tag{J.5}$$

where $\kappa(W_1)V_1^i$ is a vacancy-creation cost, where $\kappa(W_1)$ is a function of aggregate wage W_1. We will later consider two cases: $\kappa(W_1) = \kappa$ (i.e., constant) and $\kappa(W_1) = \kappa W_1$ (i.e., linear function). $S(W_1^i|W_1)$ and $R(W_1^i|W_1)$ are separation and retaining probabilities, respectively, that depend on the firm's individual wage W_1^i and the aggregate wage W_1. We will use the same functional form as in our dynamic general equilibrium model. Note that in (J.4), we do not incorporate nominal wage rigidities for now. Note that due to demand-determined nature, $N_1 = \bar{N}$ is taken as given by each firm.

Solving (J.4) and (J.5) with μ_1^i as the Lagrange multiplier to (J.5) yields the followings:

- For vacancy V_1^i:

$$\mu_1^i = \frac{\kappa(W_1)}{R(W_1^i|W_1)} \tag{J.6}$$

  which implies: the value of each worker is equal to the expected cost of hiring the worker. The creation of one vacancy costs $\kappa(W_1)$ but each vacancy is filled with probability $R(W_1^i|W_1)$. This interpretation is provided in de la Barrera i Bardalet (2023) as well.

[15]With production function $Y_t^i = N_t^i$, from (J.3), we obtain that $N_0 = N_1 = N_2 = \bar{N}$.

- Wage W_1^i:

$$N_1^i = \frac{\kappa(W_1)}{R(W_1^i|W_1)}\left[R'(W_1^i|W_1)V_1^i - S'(W_1^i|W_1)\bar{N}\right]$$
$$= \frac{\kappa(W_1)}{R(W_1^i|W_1)}\left[\frac{R(W_1^i|W_1)}{W_1^i}\underbrace{\frac{R'(W_1^i|W_1)W_1^i}{R(W_1^i|W_1)}}_{=\varepsilon_{R,1}}V_1^i - \underbrace{\frac{S'(W_1^i|W_1)W_1^i}{S(W_1^i|W_1)}}_{=\varepsilon_{S,1}}\cdot\frac{S(W_1^i|W_1)}{W_1^i}\bar{N}\right] \tag{J.7}$$

which becomes

$$N_1^i = \frac{\kappa(W_1)}{W_1^i}\left[\varepsilon_{R,1}\cdot V_1^i - \varepsilon_{S,1}\cdot\frac{S(W_1^i|W_1)}{R(W_1^i|W_1)}\bar{N}\right]. \tag{J.8}$$

Envelope condition:

$$J'(\bar{N}) = (1 - S(W_1^i|W_1))\mu_1^i = (1 - S(W_1^i|W_1))\frac{\kappa(W_1)}{R(W_1^i|W_1)}. \tag{J.9}$$

Later, we will impose the equilibrium condition: $W_1^i = W_1$ and $N_1^i = N_1 = \bar{N}$.

Search and matching process For now, we use the same functional forms for $R(W_1^i|W_1)$ and $S(W_1^i|W_1)$ as in our dynamic general equilibrium model in Section 3.2. As we stated, we assume employed and unemployed share consumption risks according to $\frac{C_t^e}{C_t^u} = \xi$. Therefore, under the equilibrium condition with equal decisions across firms, i.e., $W_1^i = W_1$, $N_1^i = N_1$, $V_1^i = V_1$, the following definitions can be introduced:

- Labor market tightness θ_1:

$$\theta_1 = \frac{V_1}{\lambda_{EE}\bar{N} + 1 - \bar{N}} \tag{J.10}$$

  where λ_{EE} is the on-the-job search intensity, and we use $N_0 = \bar{N}$.

- Retaining probability $R(W_1^i = W_1|W_1)$:

$$R(W_1|W_1) = g(\theta_1)\left(\frac{1}{2}\phi_{E,1} + \frac{\xi^\gamma}{1+\xi^\gamma}\phi_{U,1}\right) \tag{J.11}$$

  where $\phi_{E,1}$ and $\phi_{U,1} \equiv 1 - \phi_{E,1}$ are fractions of employed (i.e., on-the-job searchers) and unemployed among job seekers, given by

$$\phi_{E,1} = \frac{\lambda_{EE}\bar{N}}{\lambda_{EE}\bar{N} + 1 - \bar{N}}. \tag{J.12}$$

- Separation probability $S(W_1^i = W_1 | W_1)$:

$$S(W_1|W_1) = \frac{1}{2}\lambda_{EE} f(\theta_1) + \frac{1}{1+\xi^\gamma}\lambda_{EU} \tag{J.13}$$

  where we assume zero automatic separation (i.e., $s = 0$ in our dynamic general equilibrium model), and λ_{EU} is the exogenous job-quitting probability.

- Elasticity $\varepsilon_{R,1}$ and $\varepsilon_{S,1}$: from (J.11) and (J.13), we obtain

$$\varepsilon_{R,1} = \gamma \cdot \left(\frac{\frac{1}{4}\phi_{E,1} + \phi_{U,1}\left(\frac{\xi^\gamma}{(1+\xi^\gamma)^2}\right)}{\frac{1}{2}\phi_{E,1} + \left(\frac{\xi^\gamma}{1+\xi^\gamma}\right)\phi_{U,1}} \right) \simeq \gamma \cdot \left(\frac{\frac{1}{4}\phi_{E,1}}{\frac{1}{2}\phi_{E,1} + \left(\frac{\xi^\gamma}{1+\xi^\gamma}\right)\phi_{U,1}} \right), \tag{J.14}$$

  and

$$\varepsilon_{S,1} = -\gamma \cdot \left(\frac{f(\theta_1)\lambda_{EE}\frac{1}{4} + \lambda_{EU}\frac{\xi^r}{(1+\xi^r)^2}}{0.5\lambda_{EE} f(\theta_1) + \left(\frac{1}{1+\xi^\gamma}\right)\lambda_{EU}} \right) \simeq -\frac{\gamma}{2}. \tag{J.15}$$

  where we approximate $\frac{\lambda_{EU}}{1+\xi^\gamma} \simeq 0$ and $\frac{\phi_{U,1}\xi^\gamma}{(1+\xi^\gamma)^2} \simeq 0$, which hold well under our calibration. In (J.14), our approximation is based on that the effect of higher wages in making currently unemployed people choose to work at a firm is small compared with the effect on attracting on-the-job searchers from other firms.

Equilibrium characterization Since every firm i chooses the same decisions in equilibrium, i.e., $W_1^i = W_1$, $V_1^i = V_1$, and $N_1^i = N_1 = \bar{N}$, from (J.11) and (J.13), we obtain

$$\begin{aligned} \frac{S(W_1|W_1)\bar{N}}{R(W_1|W_1)} &= \frac{\frac{1}{2}\lambda_{EE}\underbrace{f(\theta_1)}_{=\theta_1 g(\theta_1)}\bar{N} + \frac{1}{1+\xi^\gamma}\lambda_{EU}\bar{N}}{g(\theta_1)\left(\frac{1}{2}\phi_{E,1} + \frac{\xi^\gamma}{1+\xi^\gamma}\phi_{U,1}\right)} \\ &= \frac{\frac{1}{2}\phi_{E,1} g(\theta_1) V_1 + \frac{1}{1+\xi^\gamma}\lambda_{EU}\bar{N}}{g(\theta_1)\left(\frac{1}{2}\phi_{E,1} + \frac{\xi^\gamma}{1+\xi^\gamma}\phi_{U,1}\right)}. \end{aligned} \tag{J.16}$$

We then plug in (J.14), (J.15), and (J.16) to (J.8) to obtain

$$\bar{N} = N_1 = \frac{\kappa(W_1)}{W_1} \left\{ \underbrace{V_1 \left[\gamma \left(\frac{\frac{1}{2}\phi_{E,1}}{\frac{1}{2}\phi_{E,1} + \frac{\xi^\gamma}{1+\xi^\gamma}\phi_{U,1}} \right) \right]}_{\equiv \varepsilon_{11}} + \underbrace{\frac{\frac{\gamma}{2}\frac{1}{1+\xi^\gamma}\lambda_{EU}\bar{N}}{\left(\frac{1}{2}\phi_{E,1} + \frac{\xi^\gamma}{1+\xi^\gamma}\phi_{U,1}\right) g(\theta_1)}}_{\equiv \varepsilon_{21}} \right\}, \tag{J.17}$$

where $\varepsilon_{11} + \varepsilon_{21}$ in (J.17) becomes the 'effective' labor supply elasticity each firm faces. ε_{11} is about the elasticity due to those who are on-the-job search: an increase in wage attracts more on-the-job searchers from other firms and reduce the endogenous separation of current workers, and given other variables, this effect becomes more pronounced with higher measure of on-the-job searchers among job seekers, i.e., higher $\phi_{E,1}$ (thereby decrease in $\phi_{U,1}$). Eventually in equilibrium, every firm sets the same wage: $W_1^i = W_1$ for $\forall i$.

ε_{21} is the elasticity attributed to those who quit their jobs to be unemployed: a higher wage deters workers from going to be unemployed. The proportion of those who exit the labor market becomes smaller under a bigger and more competitive job market with higher λ_{EE}, i.e., higher λ_{EE} lowers ε_{21} and raises ε_{11}.

From (J.5), (J.11), and (J.13), we obtain the labor dynamics as follows:

$$\begin{aligned}
\bar{N} = N_1 &= \left[1 - \left(\frac{1}{2}\lambda_{EE} \underbrace{f(\theta_1)}_{=\theta_1 g(\theta_1)} + \frac{1}{1+\xi^\gamma}\lambda_{EU} \right) \right] \bar{N} + g(\theta_1) \left(\frac{1}{2}\phi_{E,1} + \frac{\xi^\gamma}{1+\xi^\gamma}\phi_{U,1} \right) V_1 \\
&= \bar{N} - \bar{N}\frac{1}{1+\xi^\gamma}\lambda_{EU} + g(\theta_1)V_1 \left[\left\{ \cancel{\frac{1}{2}\phi_{E,1}} + \frac{\xi^\gamma}{1+\xi^\gamma}\phi_{U,1} \right\} - \left\{ \cancel{\frac{1}{2}\phi_{E,1}} \right\} \right] \\
&= \bar{N} - \bar{N}\frac{1}{1+\xi^\gamma}\lambda_{EU} + g(\theta_1)V_1 \frac{\xi^\gamma}{1+\xi^\gamma}\phi_{U,1},
\end{aligned} \tag{J.18}$$

which implies

$$\frac{\bar{N}\frac{1}{1+\xi^\gamma}\lambda_{EU}}{\lambda_{EE}\bar{N} + 1 - \bar{N}} = f(\theta_1)\frac{\xi^\gamma}{1+\xi^\gamma}\phi_{U,1}. \tag{J.19}$$

Equations (J.17) and (J.19) constitute our equilibrium, with the condition $N_1 = Y_1 = \bar{N}$. We can theoretically elicit equilibrium W_1 and V_1 from those two equations.

Cost-of-living shock We assume that the endowment good X_t drops from its steady state level $\bar{X}$ to $X_1 < \bar{X}$ at $t = 1$ in an unanticipated manner, and see how the business cycle variables adjust at $t = 1$. From (J.17) and (J.19), a sudden drop in X_1 from $\bar{X}$ does not affect the equilibrium levels of V_1 and W_1, and from the household's Euler equation (J.3), $N_1 = \bar{N}$ remains the same. From (J.2), the only change is the price of endowment good X_t, and $P_{X,1}$ rises satisfying $P_{X,1}X_1 = \bar{P_X}\bar{X}$. The following Proposition 3 summarizes this finding.

Proposition 3 *A cost-of-living shock, i.e., a sudden drop in X_1 from $\bar{X}$, does not affect equilibrium labor market outcomes: $N_1 = \bar{N}$, W_1, and V_1. The price $P_{X,1}$ of endowment good X_1 rises so that the expenditure stays the same, i.e., $P_{X,1}X_1 = \bar{P_X}\bar{X}$.*

Flexible price case The result in Proposition 3 holds even if firms set their prices fully flexibly. As in our dynamig general equilibrium model, we assume firms are in monopolistic competition, represented by Dixit-Stiglitz aggregator with elasticity of substitution ϵ. Then

$$Y_1^i = Y_1 \left(\frac{P_{Y,1}^i}{P_{Y,1}} \right)^{-\epsilon}. \tag{J.20}$$

Each firm i solves instead the following problem:

$$J(\bar{N}) = \max_{P_{Y,1}^i, N_1^i, V_1^i, W_1^i} P_{Y,1}^i N_1^i - W_1^i N_1^i - \kappa(W_1) \cdot V_1^i + \frac{1}{1+\rho} J(N_1^i) \tag{J.21}$$

subject to (J.20) and

$$N_1^i = (1 - S(W_1^i | W_1))\bar{N} + R(W_1^i | W_1) V_1^i. \tag{J.22}$$

The solution to (J.21), with $W_1^i = W_1$, will be given by

$$\begin{aligned} P_{Y,1}^i = P_{Y,1} &= \frac{\epsilon}{\epsilon - 1} \left(W_1 + \frac{\kappa(W_1)}{R(W_1|W_1)} - \frac{1}{1+\rho} J'(N_1^i) \right) \\ &= \frac{\epsilon}{\epsilon - 1} \left(W_1 + \frac{\kappa(W_1)}{R(W_1|W_1)} - \frac{1}{1+\rho} (1 - S(W_2|W_2)) \frac{\kappa(W_2)}{R(W_2|W_2)} \right) \end{aligned} \tag{J.23}$$

where $W_2 = \bar{W}$ as the economy gets back to its steady state at $t = 2$. The term $\frac{\kappa(W_1)}{R(W_1|W_1)}$ is a cost of hiring through additional vacancy. If a firm hires at $t = 1$, it can reduce hiring at $t = 2$ by one. The last term $\frac{1}{1+\rho}(1 - S(W_2|W_2))\frac{\kappa(W_2)}{R(W_2|W_2)}$ represents this reduction in future hiring costs.[16]

[16]The decomposition of marginal costs in equation (J.23) is similarly given in de la Barrera i Bardalet (2023).

From (J.3), (J.17), and (J.23), we obtain

$$\underbrace{P_{Y,0}}_{=\bar{P}_Y}\bar{Y} = P_{Y,1}Y_1 = \frac{\epsilon}{\epsilon-1}\left[W_1 + \frac{\kappa(W_1)}{R(W_1|W_1)} - \frac{1}{1+\rho}(1-S(W_2|W_2))\frac{\kappa(W_2)}{R(W_2|W_2)}\right]$$
$$\cdot\frac{\kappa(W_1)}{W_1}\left\{\underbrace{V_1\left[\gamma\left(\frac{\frac{1}{2}\phi_{E,1}}{\frac{1}{2}\phi_{E,1}+\frac{\xi\gamma}{1+\xi\gamma}\phi_{U,1}}\right)\right]}_{\equiv\varepsilon_{11}} + \underbrace{\frac{\frac{\gamma}{2}\frac{1}{1+\xi\gamma}\lambda_{EU}\bar{N}}{\left(\frac{1}{2}\phi_{E,1}+\frac{\xi\gamma}{1+\xi\gamma}\phi_{U,1}\right)g(\theta_1)}}_{\equiv\varepsilon_{21}}\right\}, \tag{J.24}$$

which, with (J.19), constitute the flexible price equilibrium. Since (J.19) and (J.24) do not depend on X_1 or $P_{X,1}$, a cost-of-living shock, i.e., reduction in X_1 from $\bar{X}$, does not affect the labor market equilibrium outcome as in the rigid price case.

Corollary 1 *Even if the price-setting of firms is fully flexible, a cost-of-living shock, i.e., a sudden drop in X_1 from $\bar{X}$, does not affect the equilibrium labor market outcomes: $N_1 = \bar{N}$, W_1, and V_1. The price $P_{X,1}$ of endowment good X_1 rises so that the expenditure stays the same, i.e., $P_{X,1}X_1 = \bar{P_X}\bar{X}$.*

J.1 Quits rate and wage growth under demand shocks

In this section, we show analytically that a positive demand shock generates positive responses in both on-the-job switching rate $\frac{1}{2}\lambda_{EE}f(\theta_1)$[17] and wage growth. As $f(\cdot)$ is increasing, it is equivalent to a positive correlation between market tightness θ_1 and wage growth under a demand shock.

We define a positive demand shock that raises N_1 from $\bar{N}$, e.g., a reduction in the policy rate at $t = 1$ will result in a consumption boom, thereby leading to firms' higher labor demand level at $t = 1$. We start from our equilibrium conditions: instead of $\bar{N}$, we use $N_1 > \bar{N}$ there:

$$\bar{N} < N_1 = \frac{\kappa(W_1)}{W_1}\left\{\underbrace{V_1\left[\gamma\left(\frac{\frac{1}{2}\phi_{E,1}}{\frac{1}{2}\phi_{E,1}+\frac{\xi\gamma}{1+\xi\gamma}\phi_{U,1}}\right)\right]}_{\equiv\varepsilon_{11}} + \underbrace{\frac{\frac{\gamma}{2}\frac{1}{1+\xi\gamma}\lambda_{EU}\bar{N}}{\left(\frac{1}{2}\phi_{E,1}+\frac{\xi\gamma}{1+\xi\gamma}\phi_{U,1}\right)g(\theta_1)}}_{\equiv\varepsilon_{21}}\right\}, \tag{J.25}$$

[17]Quits rate includes those who voluntarily quit to unemployed as well, which is a small margin compared to the on-the-job switching part.

and

$$\bar{N} < N_1 = \bar{N} - \bar{N}\frac{1}{1+\xi^\gamma}\lambda_{EU} + g(\theta_1)V_1\frac{\xi^\gamma}{1+\xi^\gamma}\phi_{U,1}. \tag{J.26}$$

We divide into two cases according to different functional forms of $\kappa(W_1)$: (i) $\kappa(W_1) = \kappa$ (i.e., constant), and (ii) $\kappa(W_1) = \kappa W_1$ (i.e., linear) with nominal wage rigidity.

Case 1: $\kappa(W_1) = \kappa$ In this case, (J.25) becomes:

$$\bar{N} < N_1 = \frac{\kappa}{W_1}\left\{\underbrace{V_1\left[\gamma\left(\frac{\frac{1}{2}\phi_{E,1}}{\frac{1}{2}\phi_{E,1}+\frac{\xi^\gamma}{1+\xi^\gamma}\phi_{U,1}}\right)\right]}_{\equiv\varepsilon_{11}} + \underbrace{\frac{\frac{\gamma}{2}\frac{1}{1+\xi^\gamma}\lambda_{EU}\bar{N}}{\left(\frac{1}{2}\phi_{E,1}+\frac{\xi^\gamma}{1+\xi^\gamma}\phi_{U,1}\right)g(\theta_1)}}_{\equiv\varepsilon_{21}}\right\}. \tag{J.27}$$

In order to get a sharper results, we log-linearize (J.26) and obtain[18]

$$0 < \check{N}_1 = \frac{1}{1+\xi^\gamma}\lambda_{EU}\left(\underbrace{\frac{g'(\bar{\theta}_1)\bar{\theta}_1}{g(\bar{\theta}_1)}}_{\equiv-\varepsilon_{g,\theta}}\check{\theta}_1 + \check{\theta}_1\right) = \frac{1}{1+\xi^\gamma}\lambda_{EU}\left(1-\underbrace{\varepsilon_{g,\theta}}_{<1}\right)\check{\theta}_1, \tag{J.28}$$

where we assume the firm's matching elasticity $\varepsilon_{g,\theta} \geqslant 0$ of $g(\theta_1)$ is less than 1, which holds under various specification.[19] Therefore, from (J.28), $\check{\theta}_1 > 0$ when $\check{N}_1 > 0$, i.e., labor market gets tighter at $t = 1$. We then log-linearize (J.27) and use (J.28) to obtain

$$\underbrace{\frac{1}{1+\xi^\gamma}\lambda_{EU}\left(1-\underbrace{\varepsilon_{g,\theta}}_{<1}\right)\check{\theta}_1}_{=\check{N}_1} + \check{W}_1 = \left[\frac{\bar{\varepsilon}_{11}}{\bar{\varepsilon}_{11}+\bar{\varepsilon}_{21}} + \frac{\bar{\varepsilon}_{11}}{\bar{\varepsilon}_{11}+\bar{\varepsilon}_{21}}\varepsilon_{g,\theta}\right]\check{\theta}_1. \tag{J.29}$$

Since $\frac{1}{1+\xi^\gamma}\lambda_{EU}$ is small under our calibration, $\check{\theta}_1 > 0$ from (J.28) implies $\check{W}_1 > 0$ in (J.29). Thus, we generate a positive correlation between movements in wage and market tightness (on-the-job switching rate), which is summarized in the following Proposition 4.

[18] We use $\check{\theta}_1 = \check{V}_1$ as θ_1 and V_1 are proportional and $\lambda_{EE}\bar{N} + 1 - \bar{N}$ is constant.

[19] Since $f(\theta_1) = \theta_1 g(\theta_1)$, $\varepsilon_{f,\theta} \equiv \frac{g'(\bar{\theta}_1)\bar{\theta}_1}{g(\bar{\theta}_1)} = 1 - \varepsilon_{g,\theta} > 0$ under our specification, as $f(\theta_1)$ is increasing in θ_1.

Proposition 4 *When $\kappa(W_1) = \kappa$, i.e., $\kappa(W_1)$ is a constant function, both market tightness θ_1 (on-the-job switching rate $0.5\lambda_{EE} f(\theta_1)$) and wage W_1 rises in response to a positive demand shock.*

Case 2: $\kappa(W_1) = \kappa W_1$ with nominal wage stickiness Now we assume $\kappa(W_1) = \kappa W_1$ (i.e., linear function) but incorporate nominal wage rigidity à la Rotemberg (1982). Firm i solves:

$$J(\bar{N}) = \max_{V_1^i . W_1^i} \bar{P_Y} N_1^i - W_1^i N_1^i - \underbrace{\kappa(W_1)}_{\equiv \kappa W_1} \cdot V_1^i - \underbrace{\frac{\psi^W}{2}\left(\frac{W_1^i}{\bar{W}} - 1\right)^2 \bar{W} N_1^i}_{\text{Wage changing cost}} + \frac{1}{1+\rho} J(N_1^i) \quad \text{(J.30)}$$

subject to

$$N_1^i = (1 - S(W_1^i | W_1))\bar{N} + R(W_1^i | W_1) V_1^i. \quad \text{(J.31)}$$

Solving (J.30) subject to (J.31) with $W_1^i = W_1$ and $N_1^i = N_1$ yields

$$N_1 \left(1 + \psi^W \frac{W_1 - \bar{W}}{\bar{W}}\right) = \frac{\kappa \cancel{W_1}}{\cancel{W_1}} \left\{ \underbrace{V_1 \left[\gamma \left(\frac{\frac{1}{2}\phi_{E,1}}{\frac{1}{2}\phi_{E,1} + \frac{\xi^\gamma}{1+\xi^\gamma}\phi_{U,1}}\right)\right]}_{\equiv \varepsilon_{11}} + \underbrace{\frac{\frac{\gamma}{2}\left(\frac{1}{1+\xi^\gamma}\right)\lambda_{EU}\bar{N}}{\left(\frac{1}{2}\phi_{E,1} + \frac{\xi^\gamma}{1+\xi^\gamma}\phi_{U,1}\right) g(\theta_1)}}_{\equiv \varepsilon_{21}} \right\}, \quad \text{(J.32)}$$

We log-linearize (J.32) and use (J.28) to obtain

$$\frac{1}{1+\xi^\gamma}\lambda_{EU}\left(1 - \underbrace{\varepsilon_{g,\theta}}_{<1}\right)\check{\theta}_1 + \psi^W \check{W}_1 = \left[\frac{\bar{\varepsilon}_{11}}{\bar{\varepsilon}_{11} + \bar{\varepsilon}_{21}} + \frac{\bar{\varepsilon}_{11}}{\bar{\varepsilon}_{11} + \bar{\varepsilon}_{21}}\varepsilon_{g,\theta}\right]\check{\theta}_1. \quad \text{(J.33)}$$

Since $\frac{1}{1+\xi^\gamma}\lambda_{EU}$ is small under our calibration, $\check{\theta}_1 > 0$ from (J.28) implies $\check{W}_1 > 0$ in (J.33) as well. Thus, we generate a positive correlation between movements in wage and market tightness (on-the-job switching rate), which is summarized in the following Proposition 5. Finally, note that **Case 2** (which is the case in our dynamic stochastic general equilibrium model in Section 3, with $\chi = 0$) generate similar results to **Case 1**, where $\kappa(\cdot)$ is a constant function.

Proposition 5 *When $\kappa(W_1) = \kappa W_1$ and firms face nomial wage rigidities à la Rotemberg (1982), both market tightness θ_1 (on-the-job switching rate $0.5\lambda_{EE} f(\theta_1)$) and wage W_1 rises in response to a positive demand shock.*

Therefore, our simple model generates the results in Section 4.1.2.

J.2 With real benefits of unemployment

In this section, we assume that unemployed workers some inflation-indexed quantity of consumption b_1 at $t = 1$ as we do in Section 4.2.2. In those cases, all the equilibrium conditions above, i.e., (J.10), (J.11), (J.12), (J.13), (J.14), (J.15), (J.17), (J.19), hold, with

$$c(P_1, W_1) \equiv \frac{\left(\frac{W_1}{P_1}\right)^\gamma}{b_1^\gamma + \left(\frac{W_1}{P_1}\right)^\gamma}.$$

in the position of $\frac{\xi^\gamma}{1+\xi^\gamma}$. Here b_1 is the consumption-equivalent during unemployment, which an unemployed person compares with real wage $\frac{W_1}{P_1}$ in deciding whether to be back at work.

Note that $c(P_1, W_1)$ is increasing in W_1 and decreasing in P_1, where P_1 is total price aggregator of endowment good X_1 and service good Y_1. Under the rigid service prices, i.e., $P_{Y,1} = \bar{P}$, a cost-of-living shock as described above increases P_1 and lower $c(P_1, W_1)$. We ask how the economy's responses to a cost-of-living shock under this specification would differ from the above case where $c(P_1, W_1) \equiv \frac{\xi^\gamma}{1+\xi^\gamma}$. Intuitively, a rise in cost-of-living reduces the relative attractiveness of working compared with being unemployed, resulting in a lower $c(P_1, W_1)$. The equilibrium will be represented by

$$\frac{\bar{N}\left(1 - c(P_1, W_1)\right)\lambda_{EU}}{\lambda_{EE}\bar{N} + 1 - \bar{N}} = f(\theta_1)c(P_1, W_1)\phi_{U,1}. \tag{J.34}$$

and

$$\bar{N} = N_1 = \frac{\kappa(W_1)}{W_1}\left\{\underbrace{V_1\left[\gamma\left(\frac{\frac{1}{2}\phi_{E,1}}{\frac{1}{2}\phi_{E,1} + c(P_1, W_1)\phi_{U,1}}\right)\right]}_{\equiv\varepsilon_{11}} + \underbrace{\frac{\frac{\gamma}{2}\left(1 - c(P_1, W_1)\right)\lambda_{EU}\bar{N}}{\left(\frac{1}{2}\phi_{E,1} + c(P_1, W_1)\phi_{U,1}\right)g(\theta_1)}}_{\equiv\varepsilon_{21}}\right\}, \tag{J.35}$$

where we use the fact that output (and labor) remains at the steady state level due to households' perfect consumption smoothing. We assume that at the steady state, $c(\bar{P}_1, \bar{W}_1) = \bar{c} = \frac{\xi^\gamma}{1+\xi^\gamma}$.

We divide into three cases according to different functional forms of $\kappa(W_1)$: (i) $\kappa(W_1) = \kappa \cdot W_1$ (i.e., linear); (ii) $\kappa(W_1) = \kappa$ (i.e., constant), and (iii) whether we introduce nominal wage rigidity.

Case 1: $\kappa(W_1) = \kappa \cdot W_1$ In this case, (J.35) becomes:

$$\bar{N} = \kappa \left\{ \underbrace{V_1 \left[\gamma \left(\frac{\frac{1}{2}\phi_{E,1}}{\frac{1}{2}\phi_{E,1} + c(P_1, W_1)\phi_{U,1}} \right) \right]}_{\equiv \varepsilon_{11}} + \underbrace{\frac{\frac{\gamma}{2}\left(1 - c(P_1, W_1)\right) \lambda_{EU} \bar{N}}{\left(\frac{1}{2}\phi_{E,1} + c(P_1, W_1)\phi_{U,1}\right) g(\theta_1)}}_{\equiv \varepsilon_{21}} \right\}. \tag{J.36}$$

Since (J.34) and (J.36) constitute the equilibrium, an increase in P_1 will lead to an increase in W_1 so that $c(P_1, W_1) = \bar{c}$. Then other labor market variables, e.g., V_1, remain the same. Therefore, in this case, wage rises to compensate higher costs of living so that real wage stays constant, and real wage rigidity naturally arises as optimal decisions of firms.

Proposition 6 **(**$\kappa(W_1) = \kappa \cdot W_1$**)** *A rise in cost-of-living is exactly compensated by the same rate of increase in wage in equilibrium, and labor market equilibrium outcomes remain the same. The result does not depend on λ_{EE}, the on-the-job search intensity. Therefore, real wage rigidity naturally arises as optimal decisions of firms.*

Case 2: $\kappa(W_1) = \kappa$ In this case, (J.35) becomes

$$\bar{N} = \frac{\kappa}{W_1} \left\{ \underbrace{V_1 \left[\gamma \left(\frac{\frac{1}{2}\phi_{E,1}}{\frac{1}{2}\phi_{E,1} + c(P_1, W_1)\phi_{U,1}} \right) \right]}_{\equiv \varepsilon_{11}} + \underbrace{\frac{\frac{\gamma}{2}\left(1 - c(P_1, W_1)\right) \lambda_{EU} \bar{N}}{\left(\frac{1}{2}\phi_{E,1} + c(P_1, W_1)\phi_{U,1}\right) g(\theta_1)}}_{\equiv \varepsilon_{21}} \right\}. \tag{J.37}$$

If, as in the above case, W_1 rises at the same rate as P_1 so that $c(P_1, W_1)$ does not change, then (J.37) is not satisfied as its left hand side becomes smaller than $\bar{N}$. Thus, we can infer that in this case, the wage response would be generically smaller than the price increase. In order to obtain sharper results, we log-linearize (J.34) and obtain

$$-\frac{\bar{c}}{1 - \bar{c}} \check{c} = \underbrace{\frac{f'(\bar{\theta}_1)\bar{\theta}_1}{f(\bar{\theta}_1)}}_{\equiv \varepsilon_{f,\theta}} \check{\theta}_1 + \check{c} \tag{J.38}$$

with

$$\check{c} = \frac{\bar{c}_P \bar{P}_1}{\bar{c}} \check{P}_1 + \frac{\bar{c}_W \bar{W}_1}{\bar{c}} \check{W}_1. \tag{J.39}$$

Equations (J.38) and (J.39) yield

$$\check{\theta}_1 = -\frac{1}{(1-\bar{c})\varepsilon_{f,\theta}}\left(\frac{\bar{c}_P\bar{P}_1}{\bar{c}}\check{P}_1 + \frac{\bar{c}_W\bar{W}_1}{\bar{c}}\check{W}_1\right). \tag{J.40}$$

We also log-linearize (J.37) and obtain[20]

$$0 = -\check{W}_1 + \frac{\bar{\varepsilon}_{11}}{\bar{\varepsilon}_{11}+\bar{\varepsilon}_{21}}\left[\check{\theta}_1 - \frac{\bar{c}\phi_{U,1}}{\frac{1}{2}\phi_{E,1}+\bar{c}\phi_{U,1}}\check{c}\right] + \frac{\bar{\varepsilon}_{21}}{\bar{\varepsilon}_{11}+\bar{\varepsilon}_{21}}\left[-\frac{\bar{c}}{1-\bar{c}}\check{c} - \frac{\bar{c}\phi_{U,1}}{\frac{1}{2}\phi_{E,1}+\bar{c}\phi_{U,1}}\check{c} - \underbrace{\frac{g'(\bar{\theta}_1)\bar{\theta}_1}{g(\bar{\theta}_1)}}_{\equiv\varepsilon_{g,\theta}>0}\check{\theta}_1\right]. \tag{J.41}$$

If we define

$$\begin{aligned} d_W &\equiv \frac{\bar{\varepsilon}_{11}}{\bar{\varepsilon}_{11}+\bar{\varepsilon}_{21}}\left(\frac{\bar{c}\phi_{U,1}}{\frac{1}{2}\phi_{E,1}+\bar{c}\phi_{U,1}} + \frac{1}{(1-\bar{c})\varepsilon_{f,\theta}}\right) + \frac{\bar{\varepsilon}_{21}}{\bar{\varepsilon}_{11}+\bar{\varepsilon}_{21}}\left(\frac{\bar{c}}{1-\bar{c}} + \frac{\bar{c}\phi_{U,1}}{\frac{1}{2}\phi_{E,1}+\bar{c}\phi_{U,1}} + \frac{\varepsilon_{g,\theta}}{(1-\bar{c})\varepsilon_{f,\theta}}\right) \\ &= \underbrace{\frac{\bar{c}\phi_{U,1}}{\frac{1}{2}\phi_{E,1}+\bar{c}\phi_{U,1}}}_{\equiv d_{W,1}} + \underbrace{\frac{\bar{\varepsilon}_{11}}{\bar{\varepsilon}_{11}+\bar{\varepsilon}_{21}}\frac{1}{(1-\bar{c})\varepsilon_{f,\theta}} + \frac{\bar{\varepsilon}_{21}}{\bar{\varepsilon}_{11}+\bar{\varepsilon}_{21}}\left(\frac{\bar{c}}{1-\bar{c}} + \frac{\varepsilon_{g,\theta}}{(1-\bar{c})\varepsilon_{f,\theta}}\right)}_{\equiv d_{W,2}} > 0 \end{aligned} \tag{J.42}$$

then because at the steady state we have[21]

$$\frac{\bar{c}_W\bar{W}_1}{\bar{c}} = -\frac{\bar{c}_P\bar{P}_1}{\bar{c}} = \frac{\gamma}{1+\xi^\gamma} = \gamma(1-\bar{c}),$$

the wage response $\check{W}_1$ is given by

$$\check{W}_1 = \frac{d_W}{\frac{1}{\gamma(1-\bar{c})}+d_W}\check{P}_1 < \check{P}_1, \tag{J.43}$$

which is increasing in d_W. From (J.39) and (J.43), $\check{\theta}_1 > 0$ follows, i.e., labor market becomes tighter. This result is summarized in the following Proposition 7.

Proposition 7 *When $\kappa(W_1) = \kappa$, i.e., $\kappa(W_1)$ is a constant function, wage rises in response to a cost-of-living shock, but the rate of wage increase is lower than that of price aggregator, i.e., $\check{W}_1 < \check{P}_1$. As a result, labor market becomes tighter, i.e., $\check{\theta}_1 > 0$.*

[20] Again, we use $\check{\theta}_1 = \check{V}_1$ as θ_1 and V_1 are proportional and $\lambda_{EE}\bar{N} + 1 - \bar{N}$ is constant.
[21] We assume that at the steady state, $c(\bar{P}_1, \bar{W}_1) = \bar{c} = \frac{\xi^\gamma}{1+\xi^\gamma}$.

Role of on-the-job search intensity λ_{EE} At the steady state, $\frac{1}{1+\xi^{\gamma}}\lambda_{EU} \simeq 0$ under our calibration, and $\frac{\bar{\varepsilon}_{21}}{\bar{\varepsilon}_{11}+\bar{\varepsilon}_{21}} \simeq 0$ with $\frac{\bar{\varepsilon}_{11}}{\bar{\varepsilon}_{11}+\bar{\varepsilon}_{21}} \simeq 1$. Then from (J.42),

$$d_W \simeq \underbrace{\frac{\bar{c}\phi_{U,1}}{\frac{1}{2}\phi_{E,1} + \bar{c}\phi_{U,1}}}_{\equiv d_{W,1}} + \underbrace{\frac{1}{(1-\bar{c})\varepsilon_{f,\theta}}}_{\equiv d_{W,2}},$$

which is decreasing in λ_{EE} as $\phi_{E,1}$ falls and $\phi_{U,1}$ increases. Therefore, we can see from (J.43) that wage rises less under higher λ_{EE}. This result is summarized by the next Proposition 8.

Proposition 8 *Equilibrium wage rises less in response to a cost-of-living shock, under higher on-the-job search intensity λ_{EE}.*

Case 3: $\kappa(W_1) = \kappa W_1$ with nominal wage stickiness Now we go back to the first **Case 1** where $\kappa(W_1)$ is linear in W_1, but incorporate nominal wage rigidity à la Rotemberg (1982). Firm i solves:

$$J(\bar{N}) = \max_{V_1^i . W_1^i} \bar{P}_Y N_1^i - W_1^i N_1^i - \underbrace{\kappa(W_1)}_{\equiv \kappa W_1} \cdot V_1^i - \underbrace{\frac{\psi^W}{2}\left(\frac{W_1^i}{\bar{W}} - 1\right)^2 \bar{W} N_1^i}_{\text{Wage changing cost}} + \frac{1}{1+\rho} J(N_1^i) \quad \text{(J.44)}$$

subject to

$$N_1^i = (1 - S(W_1^i|W_1))\bar{N} + R(W_1^i|W_1)V_1^i. \quad \text{(J.45)}$$

Solving (J.44) subject to (J.45) with $W_1^i = W_1$ and $N_1^i = \bar{N}$ yields

$$\bar{N}\left(1 + \psi^W \frac{W_1 - \bar{W}}{\bar{W}}\right) = \frac{\kappa \cancel{W_1}}{\cancel{W_1}} \left\{ \underbrace{V_1 \left[\gamma \left(\frac{\frac{1}{2}\phi_{E,1}}{\frac{1}{2}\phi_{E,1} + c(P_1, W_1)\phi_{U,1}}\right)\right]}_{\equiv \varepsilon_{11}} + \underbrace{\frac{\frac{\gamma}{2}\left(1 - c(P_1, W_1)\right)\lambda_{EU}\bar{N}}{\left(\frac{1}{2}\phi_{E,1} + c(P_1, W_1)\phi_{U,1}\right) g(\theta_1)}}_{\equiv \varepsilon_{21}} \right\}, \quad \text{(J.46)}$$

which in log-linear form becomes

$$\psi^W \check{W}_1 = \frac{\bar{\varepsilon}_{11}}{\bar{\varepsilon}_{11} + \bar{\varepsilon}_{21}}\left[\check{\theta}_1 - \frac{\bar{c}\phi_{U,1}}{\frac{1}{2}\phi_{E,1} + \bar{c}\phi_{U,1}}\check{c}\right] + \frac{\bar{\varepsilon}_{21}}{\bar{\varepsilon}_{11} + \bar{\varepsilon}_{21}}\left[-\frac{\bar{c}}{1-\bar{c}}\check{c} - \frac{\bar{c}\phi_{U,1}}{\frac{1}{2}\phi_{E,1} + \bar{c}\phi_{U,1}}\check{c} - \underbrace{\frac{g'(\bar{\theta}_1)\bar{\theta}_1}{g(\bar{\theta}_1)}}_{\equiv \varepsilon_{g,\theta}}\check{\theta}_1\right]. \quad \text{(J.47)}$$

With (J.40) and (J.47), in equilibrium, the equilibrium wage response $\check{W}_1$ to cost-of-living shock $\check{P}_1$ is given by

$$\check{W}_1 = \frac{d_W}{\psi^W \frac{1}{\gamma(1-\bar{c})} + d_W} \check{P}_1 < \check{P}_1, \tag{J.48}$$

and Propositions 7 and 8 holds as well in this case. Again, note that **Case 3** (which is the case in our dynamic stochastic general equilibrium model in Section 3, with $\chi = 0$) generate similar results to **Case 2**, where $\kappa(\cdot)$ is a constant function.

J.3 Variable On-the-Job Search Intensity

Following Appendix A, we now assume that on-the-job probability λ_{EE} at $t = 1$ is following

$$\lambda_{EE}(P_1, W_1) \equiv \bar{\lambda}_{EE} \left(\frac{\bar{W}_1}{\bar{P}_1}\right)^m \left(\frac{W_1}{P_1}\right)^{-m} \tag{J.49}$$

with $m = 4$. A cost-of-living shock raises $\lambda_{EE,1}$. Now from

$$\phi_{E,1} = \frac{\lambda_{EE}\bar{N}}{\lambda_{EE}\bar{N} + 1 - \bar{N}}, \quad \phi_{U,1} = \frac{1 - \bar{N}}{\lambda_{EE}\bar{N} + 1 - \bar{N}}, \quad \theta_1 = \frac{V_1}{\lambda_{EE}\bar{N} + 1 - \bar{N}}, \tag{J.50}$$

we see higher $\lambda_{EE,1}$ raises $\phi_{E,1}$ and lowers $\phi_{U,1}$, i.e., more of job seekers are on-the-job searchers. We start from the equilibrium conditions with $\kappa(W_1) = \kappa$:[22]

$$N_1 = \frac{\kappa}{W_1} \left\{ \underbrace{\underbrace{\left(\lambda_{EE}\bar{N} + 1 - \bar{N}\right) \theta_1}_{=V_1} \left[\gamma \left(\frac{\frac{1}{2}\phi_{E,1}}{\frac{1}{2}\phi_{E,1} + \frac{\xi^\gamma}{1+\xi^\gamma}\phi_{U,1}}\right)\right]}_{\equiv\varepsilon_{11}} + \underbrace{\frac{\frac{\gamma}{2}\frac{1}{1+\xi^\gamma}\lambda_{EU}\bar{N}}{\left(\frac{1}{2}\phi_{E,1} + \frac{\xi^\gamma}{1+\xi^\gamma}\phi_{U,1}\right) g(\theta_1)}}_{\equiv\varepsilon_{21}} \right\}, \tag{J.51}$$

and

$$\begin{aligned} N_1 &= \bar{N} - \bar{N}\frac{1}{1+\xi^\gamma}\lambda_{EU} + g(\theta_1)V_1\frac{\xi^\gamma}{1+\xi^\gamma}\phi_{U,1} \\ &= \bar{N} - \bar{N}\frac{1}{1+\xi^\gamma}\lambda_{EU} + f(\theta_1)\frac{\xi^\gamma}{1+\xi^\gamma}\left(1 - \bar{N}\right). \end{aligned} \tag{J.52}$$

Price stickiness In contrast to Appendices J.1 and J.2 where we assume fully rigid prices, we assume a flexible form of price stickiness: in contrast to increase in W_1, service price $P_{Y,1}$

[22] From Appendices J.1 and J.2, we know that $\kappa(W_1) = \kappa$ (i.e., constant) generates similar results to our specification in Section 3 of $\kappa(W_1) = \kappa W_1$ (i.e., linear) with nominal wage stickiness.

increases to some degree. More specifically, we assume $\check{P}_{Y,1} = d_P \check{W}_1$, with $d_P > 0$, where $\check{P}_{Y,1}$ and $\check{W}_1$ are log-deviations from the steady state levels. $d_P = 0$ corresponds to the case of rigid prices.

Since $P_{Y,1} N_1 = \bar{P}_Y \bar{Y}$ holds due to the household's equal expenditure under pegged monetary policy, we know

$$\check{N}_1 = -\check{P}_{Y,1} = -d_P \check{W}_1 = \frac{1}{1+\xi^{\gamma}} \lambda_{EU} \underbrace{\varepsilon_{f,\theta}}_{>0} \check{\theta}_1 \tag{J.53}$$

where the last equality is derived from (J.52). From (J.53), we can see that if we have $\check{W}_1 > 0$ in equilibrium in response to a cost-of-living shock, i.e., $\check{P}_1 > 0$, then we need to have $\check{\theta}_1 < 0$, i.e., labor market becomes less tight. With lower θ_1, wage $\check{W}_1$ rises less in response to $\check{P}_1 > 0$ in (J.51), as θ_1 appears in ε_{11} and $g(\theta_1)$ is decreasing in θ_1: less tight labor market means that firms need not raise wage as much to attract job seekers and potential leavers.

By log-linearizing (J.50), we obtain

$$\check{\phi}_{E,1} = \bar{\phi}_{U,1} \check{\lambda}_{EE}, \;\; \check{\phi}_{U,1} = -\bar{\phi}_{E,1} \check{\lambda}_{EE} \tag{J.54}$$

with $\check{\lambda}_{EE} = -m\left(\check{W}_1 - \check{P}_1\right)$. Linearizing (J.51) yields:

$$\check{N}_1 = -\check{W}_1 + \frac{\bar{\varepsilon}_{11}}{\bar{\varepsilon}_{11} + \bar{\varepsilon}_{21}} \left[\bar{\phi}_{E,1} \check{\lambda}_{EE} + \check{\theta}_1 + (1-\chi)\check{\lambda}_{EE}\right] - \frac{\bar{\varepsilon}_{21}}{\bar{\varepsilon}_{11} + \bar{\varepsilon}_{21}} \left[\chi \check{\phi}_{E,1} + (1-\chi)\check{\phi}_{U,1} - \varepsilon_{g,\theta} \check{\theta}_1\right], \tag{J.55}$$

where

$$\chi \equiv \frac{\frac{1}{2}\bar{\phi}_{E,1}}{\frac{1}{2}\bar{\phi}_{E,1} + \frac{\xi^{\gamma}}{1+\xi^{\gamma}} \bar{\phi}_{U,1}}.$$

Combining equations (J.53), (J.54), and (J.55) with $\check{\lambda}_{EE} = -m(\check{W}_1 - \check{P}_1)$ and approximating $\frac{\bar{\varepsilon}_{21}}{\bar{\varepsilon}_{11}+\bar{\varepsilon}_{21}} \simeq 0$ with $\frac{\bar{\varepsilon}_{11}}{\bar{\varepsilon}_{11}+\bar{\varepsilon}_{21}} \simeq 1$ as before, we obtain

$$\check{W}_1 = \frac{m\left(\bar{\phi}_{EE} + 1 - \chi\right)}{1 - d_P + m\left(\bar{\phi}_{EE} + 1 - \chi\right) + \dfrac{d_P}{\dfrac{\lambda_{EU}}{1+\xi^{\gamma}} \varepsilon_{f,\theta}}} \check{P}_1 > 0. \tag{J.56}$$

Interpretation Under fully rigid prices, i.e., $d_P = 0$, then we would have

$$\check{W}_1 = \frac{m\left(\bar{\phi}_{EE} + 1 - \chi\right)}{1 + m\left(\bar{\phi}_{EE} + 1 - \chi\right)} \check{P}_1 > 0.$$

with $\check{\theta}_1 = 0$: no change in tightness. When employees engage in intensified on-the-job searches, firms offer more vacancies so that labor market tightness θ_1 remains the same: it is because under fully rigid prices, labor demand remains unchanged in response to a cost-of-living shock.

Under sticky prices following (J.53), $\check{\theta}_1 < 0$ and $\check{W}_1 > 0$ hold from equation (J.56). In equilibrium, firms raise service price in response to a cost-of-living shock, leading to lower service and labor demand. Since workers show a higher probability of on-the-job search, it reduces the market tightness θ_1. It in turn lowers the incentive of firms to raise wage to attract job seekers, resulting in muted wage responses: this effect is represented by $\frac{d_P}{\frac{\lambda_{EU}}{1+\xi^\gamma}\varepsilon_{f,\theta}}$.

On the other hand, a lower level of labor demand by service firms implies the marginal cost of wage increase in terms of wage bills (e.g., \$ increase in wage implies all workers, new hires and incumbents, benefit from it) is lower from each firm's perspective, and raises firms' incentive to raise wage: this effect is represented by d_P term in (J.56). In effect, the first effect dominates the second effect,[23] and we have muted wage increase under endogenous on-the-job search intensity following (J.49).

[23]Remember $\frac{\lambda_{EU}}{1+\xi^\gamma}$ is small.