Managerial Incentives, Financial Innovation, and Risk-Management Policies*

Son Ku Kim[†] Seung Joo Lee[‡] Sheridan Titman[§]

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[†]Seoul National University (sonkukim@snu.ac.kr)

[‡]Saïd Business School, Oxford University (seung.lee@sbs.ox.ac.uk)

[§]McCombs School of Business, University of Texas at Austin (Sheridan.Titman@mccombs.utexas.edu), corresponding author

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Abstract

We study risk choices of a firm run by an effort and risk-averse manager, where the firm's initial risk exposure is only observed by the manager. By eliminating zero NPV risk, hedging allows firms to efficiently induce effort from their manager. In some settings, the manager voluntarily hedges, and asymmetric information about risk exposure has no effect on the manager's optimal compensation. However, in other cases, inducing the manager to hedge rather than speculate requires the optimal contract to directly account for hedgeable risk. When inducing hedging is sufficiently costly, the optimal contract may restrict the use of derivatives.

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1 Introduction

Corporations spend substantial amounts of resources assessing and managing their exposures to various sources of risk. In a setting with perfect information and frictionless markets, the Modigliani and Miller theorem holds, and these expenditures do not create value. However, the finance literature identifies a number of market imperfections that provide a rationale for risk management activities.[1] Most of this literature explores the role of financial constraints and implicitly assumes that the risk management choices are made by value-maximizing rather than self-interested executives.[2] In contrast, the focus of this paper is on managerial incentive issues, and in particular, risk management choices that are made by self-interested executives.

We present a model of a firm that is owned by risk neutral shareholders (i.e., the principal) and managed by a manager (i.e., the agent) who is both risk and effort averse. While the manager observes the firm's inherent (i.e., initial) risk exposure, the shareholders can only observe the distribution from which the exposure is drawn. We assume that this exposure cannot be credibly disclosed to the principal, that is, there is no communication about the firm's initial risk exposure between the principal and the agent.[3]

The shareholders in our model offer the manager a compensation contract that is designed to motivate the manager to expend effort. The contract may or may not allow the manager to take derivatives positions. If the contract allows for the use of derivatives, it is optimally designed to induce the manager to hedge rather than speculate. The contract can

[1]For previous works, see Smith and Stulz (1985), Campbell and Kracaw (1990), and DeMarzo and Duffie (1991, 1995), Froot et al. (1993), Geczy et al. (1997), Leland (1998) among others.

[2]For the role of financial (e.g., collateral) constraints in risk management activities, see e.g., Rampini and Viswanathan (2010, 2013), Rampini et al. (2014). Since both financing and risk management need collaterals, more financially constrained firms engage in less risk management, and sometimes do not hedge at all. Our framework, in contrast, abstracts from external financing constraints and focus on managerial incentive issues, noting that risk management policies of a firm are chosen by self-interested managers, not shareholders.

[3]In Appendix B, we consider a setting in which there is contractable communication between the principal and the agent, i.e., the contract can include the risk exposure disclosed by the agent, as well as the profits and the realization of the hedgeable source of uncertainty.

be contingent on the firm's observed profits (net of the profits or losses from the derivatives transactions), and the realization of hedgeable risks, both of which are commonly observable. For example, the compensation of the CEO of an oil company may be a function of the firm's profits, and the price of oil.

The use of derivatives in this paper can potentially create value through two channels. The first channel, which is our focus, is that hedging can effectively eliminate the shareholders' (i.e., the principal's) informational disadvantage about the firm's initial exposure to hedgeable risks. By doing so, hedging increases the correlation between reported earnings and managerial effort, which allows the compensation contract to more efficiently induce managerial effort. Following the existing literature on financial constraints, e.g., Smith and Stulz (1985), we also allow for a second channel that arises when negative cash flows are amplified by feedback effects, such as bankruptcy costs. As we show, even if this feedback effect is small, allowing for this second channel eliminates an equilibrium indeterminacy in the model.

Despite these two benefits of hedging, shareholders will not always allow managers to take derivative positions. The concern is that the manager, when given the opportunity to trade derivatives, may speculate rather than hedge. Given this possibility, shareholders need to offer a compensation contract that induces the manager to hedge, which is costly. If the cost of inducing the manager to take derivative positions that hedge rather than speculate is higher than the efficiency gains from hedging, derivative transactions will not be allowed.

While asymmetric information about firm's risk exposure is a key feature of our model, as we show, this asymmetry does not always create costs. The cost arising from the asymmetry depends on the curvature of the agent's indirect utility function, which is a composite of the optimal compensation contract under the counterfactual symmetric information case, i.e., when initial risk exposure is also observed by shareholders, and the agent's utility function. When this composite function is concave in output, the agent has an incentive to fully hedge. When this is the case, the inability to communicate the firm's risk exposure results in no efficiency loss. Indeed, the agent's compensation contract and the efficiency of the agency relationship is identical with and without costless communication.

The agent's indirect utility function under symmetric information, however, is not necessarily concave. For example, with power utility, the agent becomes more risk tolerant as income increases, and this implies that the slope of the optimal contract becomes steeper at higher output levels. This convex contract, combined with the agent's utility, may result in a convex indirect utility function if the agent's utility function is not too concave, i.e., the agent's relative risk aversion is not too high.[4] When this is the case, the optimal contract under symmetric information is no longer optimal when the firm's initial risk exposure cannot be credibly communicated. This is because an agent offered such a contract will take derivative positions that speculate rather than hedge.

If the manager in the above situation is allowed to take derivative positions, the optimal contract with asymmetric information differs from the benchmark symmetric information case because the agent needs to be induced to hedge rather than to speculate. As we show, this can be done by making the contract contingent on the realization of hedgeable risks as well as on the hedged profits. The optimal contract in this case penalizes the agent when both profits and hedgeable risk simultaneously have extreme realizations. More specifically, to induce hedging, the principal penalizes the agent for any realized covariance (both positive and negative) between profits and hedgeable risks. Such a contract can induce the manager to hedge rather than speculate, even if the indirect utility function under symmetric information is convex in hedged profits. However, it is not costless to alter the contract in this way, and in some situations the firm is better off not allowing the manager to take derivative positions. This depends on the level of uncertainty about the firm's initial risk

[4]Hirshleifer and Suh (1992) characterize some special cases of the agent's utility and the output distribution function that lead to the agent's indirect utility function being convex.

exposure from the perspective of shareholders, the level of the firm's initial risk exposure itself, and the magnitude of the feedback effect that can amplify negative outcomes.

While there is the large existing literature that study agency relationships and risk management, we contribute by combining insights from different strands of the literature. Intuitively, hedging creates value in the settings we examine because it allows contracts that are contingent on a measure of output to be more highly correlated with the agent's effort. In this sense, our analysis is closely related to the seminal Holmström (1979) paper, which shows that the optimal contract is a function of various state variables that can provide information about the agent's effort. Our contribution is that we extend the analysis to the case where the exposure of profits to these state variables is unknown to the principal. Specifically, we consider a setting with asymmetric information about exposures to an element of risk, which can be affected by the agent's choices (i.e., derivative positions).

It should also be noted that others have examined the optimal contract between a risk neutral principal and a risk averse agent who makes both risk and effort choices, e.g., Hirshleifer and Suh (1992), Sung (1995), Palomino and Prat (2003), DeMarzo et al. (2011), Barron et al. (2020).[5] Unlike the previous literature, we explicitly consider derivative transactions that can be distinguished from real investments. Our paper differs from the previous works in three important ways. The first is that we consider a setting where realizations of hedgeable risk, e.g., oil prices, are observable, are not affected by the agent's effort, and can thus be included in the optimal contract. The second is that in our setting, exposure to hedgeable risks is a zero net present value bet, which means that to the extent possible, the agent's exposure to this element of risk should be minimized for the purpose of maximizing the contractual efficiency. The third is that we assume that the initial exposure to hedgeable risks is observed only by the manager.

[5] Hébert (2018) assumes that the agent picks his effort and risk-shifting activities by choosing the distribution of state in a non-parametric way. Under special cost functions (e.g., Kullback-Leibler divergence), debt becomes optimal.

In summary, our model builds on the previous literature that highlights the importance of including state variables as well as output in designing optimal compensation contracts. Specifically, we provide a solution to such a problem when, in addition to effort, the agent takes a hidden action that influences the relation between the state variable, i.e., hedgeable risk, and the output.

In the risk management literature, our paper is closely related to papers by DeMarzo and Duffie (1991, 1995) and Breeden and Viswanathan (2016), who point out that hedging helps the firm's profits provide more precise information about managerial inputs. In DeMarzo and Duffie (1991, 1995) and Breeden and Viswanathan (2016), hedging allows the owners of a firm to more precisely learn about the managerial ability, which increases the value of options to either continue or abandon the firm's projects. In contrast, in our framework the owners induce the manager's effort, and with hedging, the contract can more efficiently elicit better effort. It should also be noted that the previous literature ignore the incentive issues associated with the manager's hedging choice, which is the main focus of our paper.

While we model the derivative choices of self-interested managers under moral hazard, the idea that these choices may not be made in the interests of shareholders is not new. For example, Tufano (1996) studies the gold mining industry, and finds that managerial incentives are the most important determinant of corporate derivatives choices, e.g., a firm hedges less if the compensation of management includes more options.[6] Coles et al. (2006) similarly find that a higher sensitivity of CEO wealth to stock volatility (i.e., vega) leads to the CEO to make riskier choices, including relatively more investment in R&D and higher leverage. More recently, a survey of executives by Bodnar et al. (2019) finds that the risk aversion of executives has an important effect on their risk management decisions, which is in line with our model.

[6] Knopf et al. (2002) find similar results among a large sample of firms. Also, Bakke et al. (2016), based on Financial Accounting Standard (FAS) 123R which required firms to expense options, find that an increase in the cost of using options results in a large increase in hedging activities.

Policymakers are also aware of this kind of incentive problems. For example, during the global financial crisis, Ben Bernanke stated that "compensation practices at some banking organizations have led to misaligned incentives and excessive risk-taking, contributing to bank losses and financial instability."[7] While poorly written incentive contracts are clearly inconsistent with our model, it is possible that contract changes that should have been introduced along with the introduction and growth of derivative markets, which we theoretically characterize, were in fact slow to be enacted.

The paper is organized as follows: In Section 2, we present the basic model without a derivative market. In Section 3, we formulate our model with a derivative market. Section 4 discusses model extensions, and concluding remarks are provided in Section 5. The proofs of the Lemmas and Propositions are all given in the Appendix A. We consider the case in which free communication between the principal and the agent is possible, and discuss the optimal truth-telling mechanism in Appendix B. Finally, Appendix C considers a variant of the model with discretionary real investment choices.

2 The Basic Model

We consider a two-person single-period agency model in which a risk-averse agent works for a risk-neutral principal. The principal can be thought of as the firm's shareholders, and the agent can be thought of as the firm's top manager or CEO. Alternatively, we can think of the principal as the CEO and the agent as the head of the firm's one division. Hereafter, we use the terms 'agent' and 'manager' interchangeably.

After his wage contract, which is denoted by $w(\cdot)$, is finalized, the agent chooses two actions, $a_1 \in [0, \infty)$ and $a_d \in (-\infty, +\infty)$. The first action, a_1, is a productive effort which increases expected output in a way that a high effort generates an output level that

[7] Fed press release (2009): https://www.federalreserve.gov/newsevents/pressreleases/bcreg20091022a.htm

first-order stochastically dominates the output level generated by a low effort. The agent's second action, a_d, is his derivative choice. We can think of a_d as the number of forward contracts each of which has zero upfront cost and pays η at the end of the period, where η can, for example, be the difference between the price of oil and its risk neutral expectation.

After the agent chooses a_1 and a_d, the firm's output, x, is realized and publicly observable without cost. Thus, output x can be used in the manager's wage contract. The output is determined not only by the agent's choice of (a_1, a_d) but also by the state of nature, (η, θ). For simplicity, we assume that the output function exhibits the following additively separable form:

$$x = \phi(a_1) + \sigma\theta + (R - a_d)\eta. \tag{1}$$

The first term, $\phi(a_1)$, is the firm's expected output, which is affected by a_1 but not by a_d. The firm's risk consists of two components, η and θ, where $\eta \sim N(0,1)$ represents one unit of the firm's hedgeable risks. Also, $\theta \sim N(0,1)$ represents one unit of the firm's non-hedgeable risks, where σ denotes the amount of non-hedgeable risks. We assume that η and θ are uncorrelated. As denoted by (1), the firm's total amount of non-hedgeable risk is fixed at σ. However, the firm's hedgeable risks are determined by market variables such as commodity prices, interest rates, and exchange rates, which become publicly observable after the agent chooses both a_1 and a_d.[8] Accordingly, η can also be used in the manager's wage contract if necessary. In (1), R is a random variable with $h(R)$ for its density function, denoting the firm's initial exposure to the hedgeable risks (e.g., the amount of oil underground for a drilling company). The manager can observe the true value of R after the contract is signed but before he chooses a_1 and a_d, whereas, the principal knows only its distribution. We assume that the manager's effort a_1 does not affect the firm's initial exposure to the hedgeable risks, R. However, the firm's final exposure to the hedgeable

[8] In fact, if the relevant derivative market observable is denoted as p, then $\eta = \frac{p-\overline{p}}{\sigma_p}$ where $\overline{p}$ is the expected value of p and σ_p^2 is its variance.

risks is determined by the manager's transaction a_d in the derivative market. The manager hedges, i.e., reduces the hedgeable risk, as long as $|R - a_d| < |R|$ and minimizes such risk by setting $a_d = R$. On the other hand, the manager speculates in the derivative market if $|R - a_d| > |R|$, and $a_d = 0$ implies that the manager does not trade derivatives.

It should be emphasized that we only consider "corporate level" hedging, abstracting from the possibility that managers trade derivatives on their personal account to hedge their compensation risk. In reality, financial firms either effectively ban or closely monitor derivative trades by their employees. The concern is that the ability to hedge their compensation risk distorts the incentive of managers to hedge at the firm level.[9]

In addition, we make the following assumptions:

Assumption 1. The agent's preferences on wealth and productive effort are additively separable:

$$U(w, a_1, a_d) = u(w) - v(a_1), \;\; u' > 0, u'' < 0, \; v' > 0, v'' > 0,$$

where v is the agent's disutility of exerting a productive effort.

Assumption 1 implies that the agent is risk-averse and effort-averse, and his derivatives choices have no direct effect on the agent's utility.[10]

Assumption 2. $\dfrac{\partial \phi}{\partial a_1}(a_1) \equiv \phi_1(a_1) > 0, \; \dfrac{\partial^2 \phi}{\partial a_1^2}(a_1) \equiv \phi_{11}(a_1) < 0.$

Assumption 2 indicates that the effort a_1 affects the expected output with the usual property of decreasing marginal productivity.

[9] Gao (2010) explores cases where CEO's are allowed to hedge their compensation and shows that pay-performance sensitivity (PPS) increases when executives can trade more on personal accounts. More recently, Huang et al. (2023) develop a dynamic contracting model where the agent, protected by limited liability, privately trades the market portfolio to hedge market risks contained in his compensation.

[10] For the derivative choice a_d, we assume that a direct hedging cost (e.g., option premium) is negligible compared with the nominal amount of the firm's cash flows. Therefore, we assume away costs for derivative choice a_d.

Assumption 3. The principal suffers a cost (or damage), D, when the firm is financially distressed. For analytical simplicity, we assume that the firm is financially distressed if output x is smaller than the critical level, x_b, and the firm's cost of financial stress, D, is fixed. Therefore, the principal's payoff (or utility) is $x - w(\cdot)$ if $x > x_b$ and $x - w(\cdot) - D$ if $x \leq x_b$.

Assumption 3 captures negative feedback effects that arise when firms report very low earnings. These effects include difficulties in in attracting high quality employees strategic partners, and so on. We introduce Assumption 3 in part to be consistent with the existing literature, and also, because it allows us to rule out less intuitive equilibria.[11]

2.1 The Benchmark Case

In this subsection, we consider the benchmark case in which the firm's initial exposure to the hedgeable risks, R, is known to the principal as well as the agent and there is no derivative market, i.e., $a_d = 0$.[12] Since R and η are commonly known to the principal and the agent, following the 'informativeness principle' of Holmström (1979), the optimal contract should be designed based on $y \equiv x - R\eta = \phi(a_1) + \sigma\theta$.

The optimal wage contract $w(y)$, in this case, is found by solving for the contract that maximizes the combined utilities of the principal and the agent subject to the restriction that the agent's effort a_1 is chosen to maximize the agent's utility given the contract. Thus,

[11] As long as D is a decreasing function of x, our main results in this paper will not change qualitatively because the firm's expected cost of financial stress, $Pr[x \leq x_b | a_1, a_d] \cdot D$, is a decreasing function of x.

[12] Assuming that there is no derivative market is equivalent to assuming that the agent is prohibited from trading in the derivatives market. Thus, the introduction of a derivative market later can also be understood as allowing the agent to trade in that market.

the optimization is given by

$$
\begin{aligned}
&\max_{a_1, w(\cdot)} SW \equiv \phi(a_1) - \int w(y) f(y|a_1) dy + \lambda \left(\int u(w(y)) f(y|a_1) dy - v(a_1) \right) \\
&\qquad\qquad - Pr[x \leq x_b | a_1, a_d = 0] D \\
&\qquad \text{s.t.} \quad (i) \;\; a_1 \in \arg\max_{a_1'} \int u(w(y)) f(y|a_1') dy - v(a_1'), \;\; \forall a_1', \\
&\qquad\qquad\quad (ii) \;\; w(y) \geq k, \;\; \forall y,
\end{aligned}
\tag{2}
$$

where $f(y|a_1)$ denotes a probability density function of $y \sim N(\phi(a_1), \sigma^2)$ given the agent's effort a_1, and λ is a welfare weight placed on the agent's utility in the joint benefits, whereas the last term in the joint benefits , $Pr[x \leq x_b | a_1, a_d = 0]D$, denotes the firm's expected cost of financial stress. Note that, since there is no derivative market, the probability of getting into financial distress depends only on a_1 and decreases as a_1 increases due to the first-order stochastic dominance relation of x with respect to a_1. As shown, the joint benefits are maximized subject to the agent's incentive compatibility constraint, which specifies that the agent chooses the effort that maximizes his utility, and the limited liability constraint, which specifies that the agent should receive at least k, the subsistence level of utility.[13,14]

Based on the first-order approach, instead of the optimization in (2), we solve the fol-

[13]The optimization in (2) yields a mathematically equivalent solution to the case where a principal maximizes her utility subject to an optimizing agent receiving his reservation utility level: see e.g., Holmström (1979). Our purpose here is to analyze the overall efficiency implication of financial market innovations and thus we choose to fix λ, which is usually an endogenous Lagrange multiplier in the literature.

[14]The limited liability constraint, i.e., $w(y) \geq k$, is introduced to guarantee the existence of the optimal solution for $w(y)$. This condition is needed because we assume that the signal y is normally distributed. For details about this 'unpleasantness', see Mirrlees (1974) and Jewitt et al. (2008).

lowing alternative:

$$\max_{a_1, w(\cdot)\geq k} SW \equiv \phi(a_1) - \int w(y)f(y|a_1)dy + \lambda\left(\int u(w(y))f(y|a_1)dy - v(a_1)\right)$$
$$- Pr[x \leq x_b|a_1, a_d = 0]D \tag{3}$$
$$\text{s.t. } (i) \int u(w(y))f_1(y|a_1)dy - v'(a_1) = 0,$$

where we replace the agent's incentive compatibility constraint with its first-order condition and $f_1(y|a_1) \equiv \frac{\partial f(y|a_1')}{\partial a_1'}|_{a_1'=a_1}$.[15]

To find the optimal solution $(a_1^*, w^*(y|a_1^*))$ for the optimization in (3), we first derive an optimal contract for inducing an arbitrarily given action a_1. Let $w^*(y|a_1)$ be the contract that optimally motivates the agent to choose a particular level of a_1. Then, by solving the Euler equation of the above program in (3) after fixing a_1, we derive that $w^*(y|a_1)$ must satisfy

$$\frac{1}{u'(w^*(y|a_1))} = \lambda + \mu_1^*(a_1)\frac{f_1}{f}(y|a_1), \tag{4}$$

for almost every y for which the solution in (4) satisfies $w^*(y|a_1) \geq k$, and otherwise $w^*(y|a_1) = k$. In (4), $\mu_1^*(a_1)$ denotes the optimized Lagrange multiplier for the agent's incentive compatibility constraint associated with a_1. Since $y \sim N(\phi(a_1), \sigma^2)$, (4) reduces to:

$$\frac{1}{u'(w^*(y|a_1))} = \lambda + \mu_1^*(a_1)\frac{y - \phi(a_1)}{\sigma^2}\phi_1(a_1). \tag{5}$$

[15] We assume that the first-order approach is valid. Grossman and Hart (1983) and Rogerson (1985) show that MLRP and CDFC are sufficient for the validity of the first-order approach when the signal space is of one dimension. Jewitt (1988) finds less restrictive conditions for the validity of the first-order approach, based on the agent's risk preferences as well as the distribution function of the signal. Sinclair-Desgagné (1994) shows that more general versions of MLRP and CDFC in a multi-dimensional space are sufficient for the validity of the first-order approach when the signal space is of multiple dimensions. For more recent treatments along this line, see Conlon (2009) and Jung and Kim (2015) among others. Recently, Jung et al. (2024) justifies the use of the first-order approach when the technology follows normal distributions, which corresponds to our problem in (2).

Then, the optimized joint benefits associated with a_1 in this case is given by

$$SW^*(a_1) = \phi(a_1) - C^*(a_1) - \lambda v(a_1) - Pr[x \leq x_b | a_1, a_d = 0]D, \tag{6}$$

where

$$C^*(a_1) \equiv \int \left(w^*(y|a_1) - \lambda u(w^*(y|a_1))\right) f(y|a_1) dy \tag{7}$$

represents the efficiency loss in this case compared with the full information case as shown in Kim (1995). In other words, $C^*(a_1)$ measures the agency cost arising from motivating the agent to take a particular action a_1.

The optimized joint benefits for inducing a_1 in (6) can also be regrouped into two parts such as

$$SW^*(a_1) = EAR^*(a_1) - Pr[x \leq x_b | a_1, a_d = 0]D, \tag{8}$$

where

$$EAR^*(a_1) \equiv \int (x - w^*(y|a_1)) f(y|a_1) dy + \lambda \left[\int u(w^*(y|a_1)) f(y|a_1) dy - v(a_1) \right], \tag{9}$$

represents the firm's efficiency which purely comes from the agency relation, in which the agent is to be induced to take a_1 under $w^*(y|a_1)$, whereas, as explained earlier, $Pr[x \leq x_b | a_1, a_d = 0]D$ is the firm's expected cost of financial stress given $(a_1, a_d = 0)$.

Finally, the optimal action a_1^* can be found by solving

$$a_1^* \in \arg\max_{a_1} SW^*(a_1). \tag{10}$$

To simplify notation, we use $w^*(y) \equiv w^*(y|a_1^*)$ and $SW^* \equiv SW^*(a_1^*)$.

2.2 When the Principal Does Not Know the Firm's Risk Exposure

In this subsection, we consider the case in which the firm's initial exposure to hedgeable risks, R, is observed by the agent but not by the principal. As in Section 2.1, we also assume that there is no derivative market (i.e., $a_d = 0$), and rule out communication between the principal and the agent, i.e., the agent cannot communicate observed R to the principal.[16] Thus, the compensation contract must be based on (x, η), i.e., $w = w(x, \eta)$, and the principal's optimization is:

$$\begin{aligned}
\max_{a_1(\cdot), w(\cdot) \geq k} SW^N \equiv & \int_R \left[\int_{x,\eta} (x - w(x,\eta))\, g(x,\eta|a_1(R), R) dx d\eta \right] h(R) dR \\
& + \lambda \int_R \left(\int_{x,\eta} u(w(x,\eta)) g(x,\eta|a_1(R), R) dx d\eta - v(a_1(R)) \right) h(R) dR \\
& - \int_R Pr[x \leq x_b | a_1(R), a_d = 0] D \cdot h(R) dR \\
\text{s.t. } (i) \;\; & a_1(R) \in \arg\max_{a_1} \int_{x,\eta} u(w(x,\eta)) g(x,\eta|a_1, R) dx d\eta - v(a_1), \forall R,
\end{aligned} \tag{11}$$

where

$$g(x,\eta|a_1,R) = \frac{1}{2\pi\sigma} \exp\left(-\frac{1}{2} \left(\frac{(x - \phi(a_1) - R\eta)^2}{\sigma^2} + \eta^2 \right) \right) \tag{12}$$

denotes the probability density function of (x, η) given (a_1, R) when $a_d = 0$.

Let $\left(a_1^N(R), w^N(x,\eta)\right)$ be the solution for the optimization program in (11). Then, the optimal contract, $w^N(x,\eta)$, can be written as:

$$\frac{1}{u'(w^N(x,\eta))} = \lambda + \int_R \mu_1(R) \left[\frac{g_1(x,\eta|a_1^N(R), R)}{\int_{R'} g(x,\eta|a_1^N(R'), R') h(R') dR'} \right] h(R) dR, \tag{13}$$

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[16] In general, communication between principals and agents are likely to be very costly, especially when the principal is actually composed of multiple shareholders. For a more detailed discussion about communication costs, see Laffont and Martimort (1997). In Appendix B, we study the optimal truth-telling mechanism when communication between the principal and the agent is possible.

when $w^N(x,\eta) \geq k$ in (13) and $w^N(x,\eta) = k$ otherwise. In the above equation, $\mu_1(R)$ is the optimized Lagrange multiplier attached to the incentive constraint for a_1 given R.

We define SW^N as the optimized joint benefits in this case. Thus,

$$SW^N \equiv \int_R \left[\phi(a_1^N(R)) - C^N(a_1^N(R)) - \lambda v(a_1^N(R)) - Pr[x \leq x_b | a_1^N(R), a_d = 0]D\right] h(R)dR, \tag{14}$$

where

$$C^N(a_1^N(R)) \equiv \int_{x,\eta} [w^N(x,\eta) - \lambda u(w^N(x,\eta))] g(x,\eta | a_1^N(R), R) dx d\eta \tag{15}$$

denotes the agency cost arising from inducing $a_1^N(R)$ given a realized value of R.

SW^N in this case is lower than SW^* of Section 2.1, since R, which is an informative signal about the agent's effort, can no longer be used in the compensation contract. This is summarized in the following Proposition 1.

Proposition 1. *When there is no derivative market (i.e., $a_d = 0$) and any communication between the principal and the agent is not possible, the principal's inability to observe the firm's exposure to hedgeable risks, R, lowers welfare, i.e.,*

$$SW^N < SW^*.$$

Intuitively, when the principal observes the firm's initial risk exposure, R, this information can be used for designing a compensation contract which eliminates the influence of the hedgeable risks, i.e., $w = w^*(y \equiv x - R\eta)$.[17] However, if R is not observable and cannot be communicated, this is impossible.

[17] As we explained in Section 2.1, this is related to the 'informativeness principle' in Holmström (1979), which shows a signal has a positive value (i.e., should be used in contracts) if it affects the local likelihood ratio.

3 When Managers Can Trade Derivatives

We now turn to our original model specification, where the firm's initial exposure to hedgeable risks, R, is not known and cannot be communicated to the principal. However, there is a derivative market and the agent can choose any level of a_d (i.e., a_d is not fixed at 0).

Since the firm's initial exposure to hedgeable risks, R, is assumed to be known only to the agent before he takes (a_1, a_d), the agent's choice of a_d can be thought of as his hidden choice of $b \equiv R - a_d$. Then, the pricipal's optimization program in this case reduces to

$$\begin{aligned}
\max_{a_1, b, w(\cdot) \geq k} SW^o \equiv & \int_{x,\eta} (x - w(x,\eta))\, g(x,\eta|a_1,b) dx d\eta + \lambda \left[\int_{x,\eta} u(w(x,\eta)) g(x,\eta|a_1,b) dx d\eta - v(a_1) \right] \\
& - Pr[x \leq x_b | a_1, b \equiv R - a_d] D \\
\text{s.t. } & (i) \;\; a_1 \in \arg\max_{a_1'} \int_{x,\eta} u(w(x,\eta)) g(x,\eta|a_1',b) dx d\eta - v(a_1'), \forall a_1', \\
& (ii) \;\; b \in \arg\max_{b'} \int_{x,\eta} u(w(x,\eta)) g(x,\eta|a_1,b') dx d\eta, \forall b',
\end{aligned} \tag{16}$$

where

$$g(x,\eta|a_1,b) = \frac{1}{2\pi\sigma} \exp\left(-\frac{1}{2} \left(\frac{(x - \phi(a_1) - b\eta)^2}{\sigma^2} + \eta^2 \right) \right). \tag{17}$$

Let $(a_1^o, b^o, w^o(x,\eta))$ be the optimal solution for the above program in (16). To derive the optimal solution, especially the optimal contract, $w^o(x,\eta)$, we take the following steps.

Since the principal can rationally anticipate the agent's choice of $b \equiv R - a_d$ given a wage contract, for analytical simplicity, we start by considering a wage contract based on $z(\hat{b})$ only, i.e., $w(x,\eta) = w(z(\hat{b}))$, where $z(\hat{b}) \equiv x - \hat{b}\eta$ and $\hat{b}$ is the principal's beliefs of the agent's choice of $b \equiv R - a_d$ given the contract. Then, in order for the principal's beliefs to be consistent, it must be that the agent actually chooses a_d satisfying $b \equiv R - a_d = \hat{b}$ given the contract, $w(z(\hat{b}))$.

Note that, since

$$z(\hat{b}) \equiv x - \hat{b}\eta = \phi(a_1) + \left(b - \hat{b}\right)\eta + \sigma\theta, \tag{18}$$

if the agent actually chooses a_d satisfying $b \equiv R - a_d = \hat{b}$ when $w(z(\hat{b}))$ is designed, we have

$$z(\hat{b}) = \phi(a_1) + \sigma\theta = y. \tag{19}$$

This indicates two things. First, as long as it is guaranteed that the agent will actually chooses $b = \hat{b}$ when $w(z(\hat{b}))$ is designed, $w^*(z(\hat{b})|a_1)$, the optimal contract for inducing a certain a_1 based on $z(\hat{b})$, should have a similar form as the contract in (5) of Section 2.1. That is, $w^*(z(\hat{b})|a_1)$ satisfies

$$\frac{1}{u'(w^*(z(\hat{b})|a_1))} = \lambda + \mu_1\left(a_1|\hat{b}\right)\frac{z(\hat{b}) - \phi(a_1)}{\sigma^2}\phi_1(a_1), \tag{20}$$

if $w^*(z(\hat{b})|a_1) \geq k$ in (20), and $w^*(z(\hat{b})|a_1) = k$ otherwise. In (20), $\mu_1\left(a_1|\hat{b}\right)$ is the optimized Lagrange multiplier for the incentive constraint for inducing a certain a_1 given $\hat{b}$.

Second, since $z(\hat{b})$ is independent of $\hat{b}$ if the agent actually chooses $b = \hat{b}$ under $w(z(\hat{b}))$, which $\hat{b}$ to be induced is a matter of indifference as far as maximizing the firm's efficiency from the agency relation is concerned. To see this more precisely, we decompose the joint benefits (i.e., SW^o) into two parts as shown in equations (8) and (9) such as

$$SW^o(a_1, \hat{b}) = EAR^o(a_1, \hat{b}) - Pr[x \leq x_b|a_1, \hat{b}]D, \tag{21}$$

where

$$EAR^o(a_1,\hat{b}) \equiv \int_{x,\eta}(x - w^*(z(\hat{b})|a_1))g(x,\eta|a_1,\hat{b})dxd\eta \\ + \lambda\left[\int_{x,\eta}u(w^*(z(\hat{b})|a_1))g(x,\eta|a_1,\hat{b})dxd\eta - v(a_1)\right], \tag{22}$$

represents the firm's efficiency which purely comes from the agency relation, in which the agent is induced to take a_1 given $b = \hat{b}$ under $w^*(z(\hat{b})|a_1)$. As shown in (19), since $z(\hat{b})$ is independent of $\hat{b}$, and, as can be seen from (17), since $g(x,\eta|a_1,\hat{b})$ is also independent of $\hat{b}$, as long as the agent actually takes $b = \hat{b}$ under $w^*(z(\hat{b})|a_1)$ in (20), which $\hat{b}$ to be induced is a matter of indifference in maximizing $EAR^o(a_1,\hat{b})$ for any given a_1.[18]

However, the firm's expected cost of financial stress, $Pr[x \leq x_b|a_1,\hat{b}]D$, will be minimized when $\hat{b} = 0$ (i.e., $a_d = R$, corresponding to complete hedging) for any given a_1. Thus, we obtain that the solution, $(a_1^o, b^o, w^o(x,\eta))$, for the optimization program in (16) should satisfy $b^o = 0$ (i.e., $a_d = R$) and $w^o(x,\eta) = w^*(z(0)|a_1^o)$ in (20), where a_1^o satisfies

$$a_1^o \in \arg\max_{a_1} SW^o(a_1,\hat{b} = 0),$$

as long as the agent actually takes $b = 0$ when $w^*(z(0)|a_1^o)$ is designed.

Note that, since $z(0) = x = \phi(a_1) + \sigma\theta = y$ after the agent completely hedges in the derivative market (i.e., $b = b^o = 0$), the optimal contract in this case, $w^*(z(0)|a_1^o)$, reduces to $w^*(x|a_1^o)$ which has the same contractual form as the one in (5) but depends on x instead of $y \equiv x - R\eta$. That is, the optimal contract in this case, $w^*(x|a_1^o)$, satisfies

$$\frac{1}{u'(w^*(x|a_1^o))} = \lambda + \mu_1^*(a_1^o)\frac{x - \phi(a_1^o)}{\sigma^2}\phi_1(a_1^o), \tag{23}$$

[18]This issue of the matter of indifference associated with which $\hat{b}$ to be induced will be proven in a more general setting later in Lemma 1.

if $w^*(x|a_1^o) \geq k$ in (23), and $w^*(x|a_1^o) = k$ otherwise. In (23), $\mu_1^*(a_1^o)$ is the optimized Lagrange multiplier of the incentive constraint for inducing a_1^o.[19]

The above discussion about the solution for the above program in (16), i.e., $(a_1^o, b^o = 0, w^o(x,\eta) = w^*(x|a_1^o))$, however, is valid only when the agent *voluntarily* takes $b = 0$ (i.e., $a_d = R$) when $w^*(x|a_1^o)$ in (23) is designed. Thus, an important question of whether the agent will actually choose $b = 0$ when $w^*(x|a_1^o)$ in (23) is offered still remains.

Given the optimal contract, $w^*(x|a_1^o)$, the agent's risk behaviors depend on the curvature of his indirect utility $V(\cdot)$ such as

$$V(x) \equiv u(w^*(x|a_1^o)). \tag{24}$$

If $V(x)$ is convex (concave) in x, then the agent wants to raise (reduce) the level of risk embedded in x if possible. In general, the curvature of the agent's indirect utility function $V(x)$ depends on the distribution of random state variables (θ, η) as well as the utility function $u(\cdot)$ itself. In this model, however, since all the state variables are assumed to be normal, the agent's utility function, $u(\cdot)$, mainly determines his risk behaviors. To see how different utility functions affect this curvature differently, we consider the case where the agent has constant relative risk aversion with degree $1-t$, where $t < 1$ (i.e., $u(w) = \frac{1}{t}w^t, t < 1$). Then, we obtain from (23) that

$$w^*(x|a_1^o) = \left(\lambda + \mu_1^*(a_1^o)\left(\frac{x - \phi(a_1^o)}{\sigma^2}\right)\phi_1(a_1^o)\right)^{\frac{1}{1-t}}, \tag{25}$$

and the agent's indirect utility under this wage contract is

$$V(x) \equiv u(w^*(x|a_1^o)) = \frac{1}{t}\left(\lambda + \mu_1^*(a_1^o)\left(\frac{x - \phi(a_1^o)}{\sigma^2}\right)\phi_1(a_1^o)\right)^{\frac{t}{1-t}}. \tag{26}$$

[19]Note that a_1^o here can be different from a_1^* defined in optimization (10), since the probability $Pr[x \leq x_d|a_1, b]$ is affected by a change from $b = R$ in Section 2.1 to $b = 0$ here.

The above equation shows that the agent's indirect utility $V(x)$ becomes strictly *convex* in x[20] if $t > \frac{1}{2}$, *linear* if $t = \frac{1}{2}$, and *concave* if $t < \frac{1}{2}$ for x satisfying $w^*(x) \geq k$. If we assume $w^*(x) = k$ only for sufficiently low x, as far as the agent's induced risk preferences are concerned, the agent acts as if he is risk-loving if $t > \frac{1}{2}$ (i.e., the agent's relative risk aversion is lower than $\frac{1}{2}$), as if he is risk-neutral if $t = \frac{1}{2}$, and as if he is risk averse if $t < \frac{1}{2}$.

Voluntary hedging case With a concave indirect utility function $V(x)$, the agent has an incentive to minimize the risk of output x. Thus, his optimal strategy is to eliminate the firm's risk exposure, R, by choosing $a_d = R$.[21] As a result, $w^*(x|a_1^o)$ in (23) works well as the optimal contract in this case.

In this case, the introduction of a derivative market unambiguously increases social welfare (i.e., $SW^N < SW^o$) via two channels. To see these two channels more precisely, we decompose the changes in welfare by the introduction of a derivative market as follows.

$$SW^o - SW^N = (SW^o - SW^*) + \left(SW^* - SW^N\right). \tag{27}$$

The first term on the right-hand side of equation (27), which represents the welfare change *from* the benchmark case, is always positive in this case (i.e., $SW^o > SW^*$). The introduction of a derivative market improves on the firm's efficiency by reducing the firm's chance of getting into the financially stressful situation compared with the benchmark case of Section 2.1 (i.e., $Pr[x \leq x_b|a_1, R - a_d = 0] < Pr[x \leq x_b|a_1, R], \forall a_1$) because it gives the agent the opportunity to hedge completely (i.e., $a_d = R$). Furthermore, the second term on the right-hand side of equation (27), which represents the welfare change *to* the benchmark case, is also positive in this case (i.e., $SW^* > SW^N$). The agent's voluntary hedging in

[20]It is widely known in the literature that $\mu_1^*(a_1^o) > 0$. For the proof, see Holmström (1979), Jewitt (1988), Jung and Kim (2015), Jung et al. (2024) among others.

[21]Since $x = \phi(a_1) + \sigma\theta + (R - a_d)\eta$, the agent can minimize the risk of x by choosing $a_d = R$.

the derivative market improves on the firm's efficiency from the agency relation by effectively eliminating the principal's informational disadvantage about the firm's risk exposure, R. Actually, the agent's hedging in the derivative market provides the principal with better (i.e., more precise) information about the agent's hidden choice of a_1 compared with when there is no derivative market. It becomes thereby easier for the principal to control the agent's hidden action a_1.[22]

In sum, the introduction of a derivative market provides hedging opportunities for firms, and thus improves social welfare by reducing their expected cost of financial stress (i.e., $SW^o > SW^*$), which is rather well recognized in the literature as the main benefit of having a hedging opportunity. However, as is explicitly shown in this paper, there is another benefit of hedging in the derivative market, that is, improving on the firm's efficiency from the agency relation by providing the principal with better information about the agent's hidden effort choice a_1.

This result is summarized in the following Proposition 2.

Proposition 2. *When the agent's indirect utility defined as $V(x) = u(w^*(x|a_1^o))$ in (24) is concave in output x, the agent will voluntarily choose $a_d = R$ (i.e., $b = 0$, complete hedging) in the derivative market given $w^*(x|a_1^o)$, which eliminates the welfare loss that would arise from the principal's informational disadvantage about the firm's risk-exposure R. Thus, we have that the solution for the optimization program in (16), $(a_1^o, b^o, w^o(x, \eta))$, becomes $(a_1^o, 0, w^*(x|a_1^o))$ and*

$$SW^N < SW^* < SW^o.$$

As we discussed above, for the agent with constant relative risk aversion with degree $1 - t$ (i.e., $u(w) = \frac{1}{t}w^t$, $t < 1$), for example, if the agent's preferences show a higher risk

[22]For detailed discussion on the value of information in the agency setting, see Kim (1995).

aversion than $t = \frac{1}{2}$ (i.e., $t < \frac{1}{2}$), the agent will have a concave indirect utility, $V(x)$, and thereby the above Proposition 2 holds.

Speculation case On the other hand, if the agent's indirect utility $V(x) = u(w^*(x|a_1^o))$ in (24) becomes convex in output x, the agent will chooses $|a_d| = \infty$ (i.e., infinite speculation), given $w^*(x|a_1^o)$. Thus, $w^*(x|a_1^o)$ in (23) can be no longer optimal, and the principal should revise $w^*(x|a_1^o)$ to another contract to restrict the agent's such unlimited speculation.[23] To derive the new optimal contract, $w^o(x, \eta)$, which is different from $w^*(x|a_1^o)$ in this case, we start with the following lemma.

Lemma 1. *As far as maximizing only the firm's efficiency from the agency relation is concerned, if $w^o(x, \eta)$ is the optimal contract which induces (a_1^o, b^o), where $-\infty < b^o < +\infty$, then $w^o(t, \eta)$ inducing (a_1^o, b_1), where $t \equiv x + (b^o - b_1)\eta$, is also the optimal contract in the sense that*

$$EAR^o(a_1^o, b^o) = EAR^t(a_1^o, b_1),$$

where

$$\begin{aligned} EAR^o(a_1, b) \equiv & \int_{x,\eta} (x - w^o(x, \eta)) g(x, \eta | a_1, b) dx d\eta \\ & + \lambda \left[\int_{x,\eta} u(w^o(x, \eta)) g(x, \eta | a_1, b) dx d\eta - v(a_1) \right], \end{aligned} \tag{28}$$

and

$$\begin{aligned} EAR^t(a_1, b) \equiv & \int_{t,\eta} (x - w^o(t, \eta)) g(t, \eta | a_1, b) dt d\eta \\ & + \lambda \left[\int_{t,\eta} u(w^o(t, \eta)) g(t, \eta | a_1, b) dt d\eta - v(a_1) \right]. \end{aligned} \tag{29}$$

It is already explained that which b to be induced for maximizing only the firm's efficiency from the agency relation (i.e., EAR) by designing $w^*(z(\hat{b})|a_1)$ in (20) is a matter of indifference. Lemma 1 proves this in a more general way. Lemma 1 actually shows that

[23] As explained earlier, this will be the case, for example, when the agent with constant relative risk aversion with degree $1 - t$ (i.e., $u(w) = \frac{1}{t} w^t, t < 1$) has less risk aversion than $\frac{1}{2}$ (i.e., $t > \frac{1}{2}$).

even if $V(x) \equiv u(w^*(x|a_1^o))$ in (24) is convex in x, and thereby the agent prefers speculation to hedging given $w^*(x|a_1^o)$, to which level the principal should limit the agent's risk choice by revising the contract is also a matter of indifference as far as maximizing only the firm's efficiency from the agency relation is concerned.

However, if we take the firm's concern to reduce its expected cost of financial stress into consideration, we can easily see that, as in the previous case in which $V(x) \equiv u(w^*(x|a_1^o))$ is concave in x, the principal should induce the agent to completely hedge (i.e., $b^o = 0$) even in this case. Therefore, given that $V(x) \equiv u(w^*(x|a_1^o))$ in (24) is convex in x, the optimal solution for b for the program in (16) will be 0 (i.e., $b^o = 0$), and the new optimal contract, $w^o(x, \eta)$, inducing the agent to take $(a_1^o, b = 0)$ must solve the following optimization problem:

$$
\begin{aligned}
\max_{w(\cdot)\geq k} SW^o \equiv & \int_{x,\eta} (x - w(x,\eta))\, g(x,\eta|a_1^o, b=0)dxd\eta + \lambda \left[\int_{x,\eta} u(w(x,\eta))g(x,\eta|a_1^o, b=0)dxd\eta - v(a_1^o) \right] \\
& - Pr[x \leq x_b | a_1^o, b=0] \cdot D \\
\text{s.t. } (i) & \int_{x,\eta} u(w(x,\eta))g_1(x,\eta|a_1^o, b=0)dxd\eta - v'(a_1^o) = 0, \\
(ii) \ & b = 0 \in \arg\max_b \int_{x,\eta} u(w(x,\eta))g(x,\eta|a_1^o, b)dxd\eta, \forall b.
\end{aligned}
\tag{30}
$$

The optimization program in (30) takes *optimal* a_1^o as given, and relies on the first-order approach for the incentive constraint associated with a_1, as we do in the optimization program in (3).[24] Note that, however, we do not use the same first-order approach for the incentive constraint associated with the agent's risk choice b. The following Lemma 2 demonstrates the reason why we cannot use the first-order approach for the incentive constraint associated with b.

Lemma 2. *If $w^*(x|a_1^o)$ in (23) is designed, the agent will be indifferent between taking b*

[24] $g_1(x, \eta|a_1, b)$ is defined as a partial derivative of $g(x, \eta|a_1, b)$ with respect to a_1.

and taking $-b$, $\forall b$.

Lemma 2 shows that if $w^*(x|a_1^o)$ in (23) is offered, the agent's expected utility becomes symmetric around $b = 0$ (i.e., $a_d = R$) in the space of b (i.e., in the space of a_d). Since $\int u(w^*(x|a_1^o))g(x,\eta|a_1^o,b)dxd\eta$ is continuous and differentiable in b, Lemma 2 implies that

$$\int u(w^*(x|a_1^o))g_b(x,\eta|a_1^o,b=0)dzd\eta = 0.^{25} \tag{31}$$

Note that $(a_1^o, w^*(x|a_1^o))$ is the solution for the optimization program in (30), as long as the agent's taking $b = 0$ (complete hedging) is guaranteed under $w^*(x|a_1^o)$. Also, equation (31) shows that $w^*(x|a_1^o)$ always satisfies the first-order condition of constraint (ii) at $b = 0$ regardless of whether $w^*(x|a_1^o)$ actually guarantees the agent's taking $b = 0$ or not. This tells that if we replace the original 'argmax' constraint (ii) in (30) with the its first-order condition, we will always end up with $w^*(x|a_1^o)$ in (23) as the optimal contract. However, since what we consider here is the case in which $V(x) \equiv u(w^*(x|a_1^o))$ is convex in x, the agent will take $b = \pm\infty$ instead of $b = 0$ given $w^*(x|a_1^o)$. This indicates that using the first-order approach for the incentive constraint associated with b cannot be justified in this case.

Thus, by following Grossman and Hart (1983), we replace the incentive constraint for b (i.e., (ii) in (29)) with:

$$\int u(w(x,\eta))\left(g(x,\eta|a_1^o,b=0) - g(x,\eta|a_1^o,b)\right)dxd\eta \geq 0,\ \forall b, \tag{32}$$

which indicates that the manager's indirect utility is maximized when he takes $b = 0$ (i.e., $a_d = R$).

Since $(a_1^o, b^o, w^o(x,\eta))$ is already defined as the solution for the optimization program

[25] We also define $g_b(x,\eta|a_1,b)$ as a partial derivative of $g(x,\eta|a_1,b)$ with respect to b.

in (16), and since we know that the optimal level of b (i.e., b^o) should be 0 (i.e., complete hedging), the optimal contract in this case, $w^o(x,\eta)$ should solve:[26]

$$
\begin{aligned}
\max_{w(\cdot)\geq k} SW^o \equiv & \int_{x,\eta} (x - w(x,\eta))\, g(x,\eta|a_1^o, b=0)dxd\eta + \lambda\left[\int_{x,\eta} u(w(x,\eta))g(x,\eta|a_1^o,b=0)dxd\eta - v(a_1^o)\right] \\
& - Pr[x \leq x_b|a_1^o, b=0]\cdot D \\
\text{s.t. } (i) & \int_{x,\eta} u(w(x,\eta))g_1(x,\eta|a_1^o,b=0)dxd\eta - v'(a_1^o) = 0, \\
(ii) & \int_{x,\eta} u(w(x,\eta))(g(x,\eta|a_1^o,b=0) - g(x,\eta|a_1^o,b))dxd\eta \geq 0,\ \forall b.
\end{aligned} \tag{33}
$$

The Euler equation of the above program in (33) yields the optimal contract, $w^o(x,\eta)$, that satisfies

$$
\frac{1}{u'(w^o(x,\eta))} = \lambda + \mu_1^o(a_1^o)\frac{x-\phi(a_1^o)}{\sigma^2}\phi_1(a_1^o) + \underbrace{\int \mu_b^o(b)\left(1 - \frac{g(x,\eta|a_1^o,b)}{g(x,\eta|a_1^o,b=0)}\right)db}_{\text{Additional term to (23)}}, \tag{34}
$$

for (x,η) satisfying $w^o(x,\eta)\geq k$ in (34) and $w^o(x,\eta) = k$ otherwise. In (34), $\mu_1^o(a_1^o)$ and $\mu_b^o(b)$ are the optimized Lagrange multipliers associated with the first constraint (i.e., (i)) and the second constraint for a particular b (i.e., (ii)), respectively.[27]

As we formally show in the Appendix A, from equation (34), we can derive the following properties of the optimal contract $w^o(x,\eta)$:

Proposition 3. *When the agent's indirect utility $V(x)\equiv u(w^*(x|a_1^o))$ in (24) is convex in output x, the agent should be motivated to hedge completely (i.e., $b=0$ or $a_d = R$) by a new contract, $w^o(x,\eta)$ in (34), which (i) satisfies $w^o(x,\eta) = w^o(x,-\eta)$ for all x,η; (ii) penalizes the agent for having a high realization of $(x-\phi(a_1^o))^2\eta^2$, and (iii) for any given*

[26]Note that a_1^o here may be different from a_1^o defined in (22) because $w^o(x,\eta)$ here may be different from $w^*(z(0)|a_1^o)\equiv w^*(x|a_1^o)$ included in (22). Yet, we use the same notation a_1^o to avoid notational complexity.

[27]For general reference about the variational approach to the optimization program in (33), see e.g., Luenberger (1969).

output x and $(x-\phi(a_1^o))^2\eta^2$, pays more for a higher η^2.

Proposition 3 can be understood as follows. Since the agent is induced to choose $b=0$ by $w^o(x,\eta)$, $x=\phi(a_1^o)+\sigma\theta$ will be generated, which is independent of η. Thus, η is now irrelevant in inducing a_1^o (as x does not depend on η under $b=0$). Furthermore, since η is symmetrically distributed around 0, we have $w^o(x,\eta)=w^o(x,-\eta)$ for all x to minimize the amount of risk imposed on the agent.

With output $x=\phi(a_1^o)+b\eta+\sigma\theta$ given a_1^o, b can be expressed as $Cov(x,\eta)\equiv\mathbb{E}((x-\phi(a_1^o))\eta)$. Thus, the agent's derivative choice a_d, or equivalently his choice of the firm's adjusted exposure to hedgeable risks $b=R-a_d$ can be best measured by the covariance between output x and derivative market observable η. If the agent fully hedges (i.e., $b=0$), the covariance between output x and derivative market observable η becomes zero, whereas any other $b\neq 0$ generates non-zero covariance.

Thus, by penalizing any (positive or negative) covariance between x and η, the principal can effectively induce full hedging (i.e., $b=0$) from the agent. In our single period model, any positive or negative sample covariance $\widehat{Cov}=(x-\phi(a_1^o))\eta=b\eta^2+\sigma\theta\eta$, instead of the population covariance is used in a way that a higher $|\widehat{Cov}|$ pays a lower compensation in $w^o(x,\eta)$. More precisely, as shown in the proof in the Appendix A, since the optimization program in (33) is symmetric around $b=0$, the optimal contract $w^o(x,\eta)$ punishes positive and negative sample covariance $(x-\phi(a_1^o))\eta$ in a symmetric way, i.e., penalizes higher $((x-\phi(a_1^o)\eta)^2$.

On the other hand, if the sample covariance $\widehat{Cov}=(x-\phi(a_1^o))\eta=b\eta^2+\sigma\theta\eta$ is high, not because of the agent's speculation (i.e., $b\neq 0$) but from a high level of the realized market observable, $|\eta|$, the principal will take it into account and penalize less in $w^o(x,\eta)$. Actually, given the output level, x, and the sample covariance, $\widehat{Cov}$, $w^o(x,\eta)$ pays more for a higher $|\eta|$.

Welfare implication Unlike the previous case where $V(x) \equiv u(w^*(x|a_1^o))$ is concave in x, if $V(x) \equiv u(w^*(x|a_1^o))$ is convex in x, however, the social welfare can be lowered by the introduction of a derivative market.

To see how the introduction of the derivative market affects social welfare in this case more precisely, we consider equation (27) here again.

As in the previous case where $V(x) = u(w^*(x|a_1^o))$ is concave in x, the second term on the right-hand side of equation (27), $\left(SW^* - SW^N\right)$, is also positive in this case, indicating that there arises the same informational gain from the derivative market.

Such informational gain will be costlessly obtained in the previous case as explained in Proposition 2. However, in this case where $V(x) \equiv u(w^*(x|a_1^o))$ is convex in x, it will be obtained with an extra agency cost since the principal should revise the agent's contract from $w^*(x|a_1^o)$ to $w^o(x,\eta)$ to motivate the agent to choose $a_d = R$ (i.e., $b = 0$). On the other hand, the introduction of a derivative market improves on social welfare by lowering the firm's chance of getting into the financially stressful situation through $a_d = R$ (i.e., $b = 0$) as in the previous case.

Thus, the first term on the right-hand side of equation (27), $(SW^o - SW^*)$, can be positive or negative in this case. Especially, if the extra agency cost for motivating the agent to hedge (i.e., $a_d = R$) by changing the optimal contract from $w^*(x|a_1^o)$ to $w^o(x,\eta)$ dominates the gain from lowering the firm's expected cost of financial stress, $(SW^o - SW^*)$ can be negative, which does not occur in the case where $V(x) \equiv u(w^*(x|a_1^o))$ is concave in x.

Note that SW^N depends on the density function of R, $h(R)$, whereas SW^* does not. This actually indicates that the informational gain from the introduction of a derivative market, $\left(SW^* - SW^N\right)$, depends on σ_R^2 in a way that it decreases as σ_R^2 decreases. It is rather obvious that if the randomness of R (say, σ_R^2) becomes small, then the benefits from knowing R also becomes small, and thus $\left(SW^* - SW^N\right) \to 0$ as $\sigma_R^2 \to 0$.

On the other hand, the gain from reducing the firm's expected cost of financial stress (i.e., the positive effect on $(SW^o - SW^*)$) due to the introduction of a derivative market depends crucially on the size of R itself rather than the randomness of R, σ_R^2, whereas the extra agency cost (i.e., the negative effect on $(SW^o - SW^*)$) by its introduction is independent of both the size of R as well as its variability σ_R^2. This is mainly because both $w^*(x|a_1^o)$ and $w^o(x, \eta)$ are independent of both R and $h(R)$.[28] Actually, when the firm's initial exposure to hedgeable risks, R, is small, the efficiency gain from reducing the firm's expected cost of financial stress becomes small as well.

Consequently, if the firm's initial exposure to hedgeable risks, R, is small, and its randomness (i.e., σ_R^2) is small as well, then it is possible that the introduction of a derivative market lowers social welfare, i.e., $SW^o < SW^N$. In other words, sometimes, banning the agent's access to derivative markets can be welfare improving from the social welfare perspective.

This is summarized in the following proposition.

Proposition 4. *When the agent's indirect utility given $w^*(x|a_1^o)$ in equation (23), $V(x)$, is convex in output x, then social welfare can be lowered by the introduction of a derivative market, i.e., $SW^o < SW^N$. This may happen especially when the firm's initial exposure to hedgeable risks, R, is small and its randomness, σ_R^2, is also small.*

4 Extensions

In this section we briefly discuss two ways in which the model can be extended. We first consider communication between the principal and the agent and describe conditions under which the agent truthfully reveals the firm's risk exposure to the principal. We then consider the case where the agent has discretion over the firm's risky project choice as well as the derivative choice.

[28] Also, note that it is assumed that there is no hedging cost.

Communication between the principal and the agent Appendix B considers the possibility of communication between the principal and the agent about the value of the firm's initial risk exposure, R, that is observed only by the agent. We show that under some conditions, the agent, given the compensation contract that is optimal under symmetric information, will truthfully reveal the firm's risk exposure. These conditions, which are identical to the condition under which the agent fully hedges, illustrates how hedging improves the efficiency of the agency relationship by eliminating a cost associated with asymmetric information.

Discretionary project choices Appendix C considers the case where the firm's risk and expected rate of return is endogenous. Specifically, instead of (1) where the firm's amount of non-hedgeable risks, σ, is given, we assume

$$x = \phi(a_1, a_2) + a_2\theta + (R - a_3)\eta, \tag{35}$$

where the agent chooses the amount of the firm's non-hedgeable risks, a_2. We interpret the agent's second action a_2 as his (risky) project choice and assume that riskier projects have higher expected output, i.e., $\phi_2(a_1, a_2) > 0$. As in (1), a_1 and a_3 denote the agent's effort and derivative choices.

It is interesting to compare the results in Appendix C to the analysis in Section 3 that takes the real investment choice as given. Recall that in Section 2, we start from the benchmark case where R is observed by the principal, which reduces the problem to the canonical principal-agent model (e.g., Holmström (1979)). The agent's indirect utility function under the optimal contract $w^*(x|a_1^o)$ in this benchmark scenario becomes $V(x) \equiv u(w^*(x|a_1^o))$. As we show, (i) if $V(x)$ is concave (convex) in x, then the agent will choose to perfectly hedge (infinitely speculate) when there is a derivatives market and (ii) $V(x)$ is more likely to be concave (convex) when the agent's utility function exhibits higher (lower) risk-aversion. Therefore, a "less risk-averse" manager is more likely to speculate in a derivative market given the benchmark optimal contract.

In cases with a flexible project choice a_2, we obtain the *opposite* result: a "more risk-averse" manager tends to speculate infinitely when derivative transaction is allowed. When the agent's risk aversion is sufficiently high, the principal will design a contract to induce the manager to choose a higher project risk level a_2, to benefit from the positive risk-return trade-off. Such a contract will reward a higher level of risk taking, which can in turn induce the manager to speculate infinitely in the presence of a derivative market. It can be understood as a side effect of inducing "productive" project risk-taking (i.e., $\phi_2(a_1, a_2) > 0$) through an incentive contract. A contract that promotes risk-taking in the real investment choice induces the manager to speculate infinitely with the introduction of derivative markets, as he effectively acts as if he is risk-loving under the contract. Therefore, when the manager with a sufficient level of risk-aversion has discretion about project choice, the optimal contract is more likely to explicitly account for hedgeable risk or restrict the use of derivatives.

5 Conclusion

The literature on risk management is vast and growing, but to a large extent it ignores issues that are the most relevant to large public firms. In particular, most of the literature focuses on financially constrained firms, and does not account for the fact that risk management choices are made by self-interested managers rather than by value-maximizing equity holders.

While information asymmetries generally play an important role in agency relationships, as we show, the ability to hedge can sometimes nullify the negative effects of asymmetric information about a firm's risk exposure. Specifically, we show that under some conditions, a self-interested manager, compensated on the firm's hedged profits, as in Holmström (1979), makes the hedging choice that would have been chosen by fully informed shareholders. Asymmetric information about risk exposure, however, is not always costless. In other situations, the manager's compensation contract must be altered to motivate him to hedge appropriately.[29] In these situations, derivatives markets may or may not contribute to welfare. If the required alteration in the manager's compensation contract is too costly, the firm may be better off banning the use of derivatives.

While we present our model in the context of a relationship between the CEO of a corporation and its shareholders, it can also be applied to the relationship between the CEO and the heads of the firm's divisions. In such a setting, the division heads can be interpreted as the agents, each of whom is supposed to report to the firm's CEO, who may not observe each division's risk exposure. The CEO thus has to design a contract for each division head that elicits information about the division's risk exposure and simultaneously induces effort.

There are three reasons why information about each division's exposure to hedgeable risks can be useful for the CEO. The first reason, which we emphasize in this framework, is that by taking out the effect of hedgeable risks, the contract for each division head can be designed to more efficiently induce effort. The second reason, which is considered in DeMarzo and Duffie (1991, 1995) and Breeden and Viswanathan (2016), is that the better-informed CEO may be able to better allocate resources among different divisions. The third reason is that, by aggregating information from the divisions, the CEO can provide a more accurate estimate of the firm's total risk exposure to the firm's board of directors, who can then use this information to better evaluate and compensate the CEO.

[29] When the indirect utility function is not concave, the optimal contract deviates from the contract specified by Holmström (1979), and in addition to being a function of hedged projects, it includes the realization of the hedgeable risk.

This description of incentives and risk choices of multi-divisional firms is especially relevant in the financial sector, where each business unit is likely to be exposed to unique risks that may be difficult to communicate. As illustrated in this paper, designing compensation contracts that optimally elicit both efforts and hedging choices can be quite complicated, and in some situations, the firm is better off banning the use of derivatives. Indeed, one of the lessons of the 2008 global financial crisis was that inappropriately compensated executives can potentially be induced to take risks that destroy considerable value.

Although our model is already quite complex, there are several extensions that may be considered in future research. The first is to consider this problem in a dynamic setting. We have shown that the optimal compensation contract sometimes penalizes the agent for realizing unusually high or low output when the payoff from the derivative trading is also unusually high or low, respectively. We interpret this as penalizing covariance between hedgeable risks and the output. In a dynamic extension of the model, where output and hedgeable risk is observed each period, the compensation contract can explicitly penalize an estimate of covariance. We conjecture that the efficiency of the compensation contract will improve as this covariance estimate becomes more accurate, increasing the gains from hedging.

A second potential extension has to do with uncertainty about the risk aversion of the agent. In our model, the risk aversion plays an important role, because it affects the curvature of the agent's indirect utility function, and thereby his risk choices. Uncertainty about the agent's risk aversion, however, makes it difficult to design a contract that induces the agent to take appropriate derivative positions, which may increase the incentive of firms to restrict the use of derivatives.

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Appendix A Proofs

Proof of Proposition 1: Consider the principal's following *alternative* maximization program:

$$\begin{aligned}\max_{a_1(\cdot),w(\cdot)\geq k} & \int_R \int_{x,\eta} \left(x - w(x,R,\eta)\right) g(x,\eta|a_1(R),R)h(R)dxd\eta dR \\ & + \lambda \int_R \left(\int_{x,\eta} u(w(x,R,\eta))g(x,\eta|a_1(R),R)dxd\eta - v(a_1(R)) \right) h(R)dR \\ & - \int_R Pr\left[x \leq x_b | a_1(R), a_d = 0\right] D \cdot h(R)dR \\ \text{s.t. } (i) & \int_{x,\eta} u(w(x,R,\eta))g_1(x,\eta|a_1(R),R)dxd\eta - v'(a_1(R)) = 0, \forall R.\end{aligned} \tag{A1}$$

Note that the above optimization program is different from the original program in (11) in that the contract here can be written based on the realized value of R, implying that the principal also observes R. If we let the Lagrange multipliers to the incentive constraint be $\mu_1(R)h(R)$, we get the following optimal contractual form:

$$\frac{1}{u'(w(x,R,\eta))} = \lambda + \mu_1(R)\frac{\underbrace{x - R\eta}_{\equiv y} - \phi(a_1(R))}{\sigma^2}\phi_1(a_1(R)), \tag{A2}$$

when $w(x,R,\eta) \geq k$. Equation (A2) implies that the optimal contract depends only on $y \equiv x - R\eta$, and it is obvious that the solution in (A1) is $(a_1^*, w^*(y) \equiv w^*(x - R\eta))$. By comparing (A1) with the program in (11) where the principal does not observe R, one can easily see that the set of wage contracts, $\{w(x,R,\eta)\}$, satisfying the incentive constraint for a given action $a_1(R)$ in the above program (A1) always contains the set of wage contracts, $\{w(x,\eta)\}$, satisfying the incentive constraint for the same action in (11). Therefore, we have

$$SW^N \leq SW^*. \tag{A3}$$

However, one can easily see that $w^*(y) = w^*(x - R\eta)$, which is a unique solution for the above program (A1), is not in the set of $\{w(x, \eta)\}$. As a result, we finally derive

$$SW^N < SW^*. \tag{A4}$$

■

Proof of Lemma 1: Since $x = \phi(a_1) + b\eta + \sigma\theta$ for any given (a_1, b), and $t \equiv x + (b^o - b_1)\eta = \phi(a_1) + (b^o - b_1)\eta + b'\eta + \sigma\theta$ for any given (a_1, b'). We have

$$x \text{ given } (a_1, b) = t \text{ given } (a_1, b'), \text{ whenever } b' = b - (b^o - b_1),$$

which implies that, if $b' = b - (b^o - b_1)$, then density function

$$g(x, \eta|a_1, b) = \frac{1}{2\pi\sigma} \exp\left(-\frac{1}{2}\left(\frac{(x - \phi(a_1) - b\eta)^2}{\sigma^2} + \eta^2\right)\right)$$

is the same as density function

$$\begin{aligned} g(t, \eta|a_1, b') &= \frac{1}{2\pi\sigma} \exp\left(-\frac{1}{2}\left(\frac{(t - \phi(a_1) - (b^o - b_1 + b')\eta)^2}{\sigma^2} + \eta^2\right)\right) \\ &= \frac{1}{2\pi\sigma} \exp\left(-\frac{1}{2}\left(\frac{(t - \phi(a_1) - b\eta)^2}{\sigma^2} + \eta^2\right)\right) \end{aligned}$$

where $t = x + (b^o - b_1)\eta$.

Thus, we have

$$\int_{x,\eta} u(w^o(x, \eta))g(x, \eta|a_1, b)dxd\eta = \int_{t,\eta} u(w^o(t, \eta))g(t, \eta|a_1, b' = b - (b^o - b_1))dtd\eta,$$

for any given (a_1, b), indicating that if the agent is induced to take (a_1^o, b^o) under $w^o(x, \eta)$, then he will also be induced to take (a_1^o, b_1) under $w^o(t, \eta)$ where $t = x + (b^o - b_1)\eta$.

Furthermore, since

$$\int_{x,\eta} w^o(x,\eta)g(x,\eta|a_1,b)dxd\eta = \int_{t,\eta} w^o(t,\eta)g(t,\eta|a_1,b' = b-(b^o-b_1))dtd\eta,$$

if $w^o(x,\eta)$ is the optimal contract which maximizes $EAR^o(a_1,b)$ in equation (28) by inducing (a_1^o,b^o), then $w^o(t,\eta)$, where $t = x+(b^o-b_1)\eta$, equally maximizes $EAR^t(a_1,b)$ in (29) by inducing (a_1^o,b_1), i.e.,

$$EAR^o(a_1^o,b^o) = EAR^t(a_1^o,b_1).$$

■

<u>Proof of Lemma 2</u>: Given that $w^*(x|a_1^o)$ described in (23) is designed,[1] if the agent takes (a_1^o,b), then his expected utility becomes:

$$\int u(w^*(x|a_1^o))g(x,\eta|a_1^o,b)dxd\eta - v(a_1^o) = \int u(w^*(x|a_1^o))q(x|a_1^o,b,\eta)l(\eta)dxd\eta - v(a_1^o), \tag{A5}$$

where $q(\cdot)$ denotes the conditional density function of x given (a_1^o,b,η) and $l(\cdot)$ denotes the density function of $\eta \sim N(0,1)$. Now, suppose the agent takes $(a_1^o,-b)$ under $w^*(x|a_1^o)$. Then,

$$\int u(w^*(x|a_1^o))g(x,\eta|a_1^o,-b)dzd\eta - v(a_1^o) = \int u(w^*(x|a_1^o))q(x|a_1^o,-b,\eta)l(\eta)dzd\eta - v(a_1^o). \tag{A6}$$

Since

$$q(x|a_1^o,b,\eta) = \frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{(x-\phi(a_1^o)-b\eta)^2}{2\sigma^2}\right), \tag{A7}$$

[1]The output x is given by $x = \phi(a_1^o)+\sigma\theta+b\eta$ given a_1^o and $b = R-a_d$.

we have

$$q(x|a_1^o, b, \eta) = q(x|a_1, -b, -\eta). \tag{A8}$$

Since $\eta \sim N(0,1)$ is symmetrically distributed around 0 and $l(\eta) = l(-\eta),\ \forall \eta$, and since $w^*(x|a_1^o)$ in (23) is independent of b, we finally have

$$\int u(w^*(x|a_1^o))g(x,\eta|a_1^o,b)dxd\eta - v(a_1^o) = \int u(w^*(x|a_1^o))g(x,\eta|a_1^o,-b)dxd\eta - v(a_1^o), \tag{A9}$$

implying that, given $w^*(x|a_1^o)$ in (23), the agent is indifferent between taking b and taking $-b$, $\forall b$.

■

Proof of Proposition 3: To prove this proposition, we start with the following Lemma 3.

Lemma 3. *If the agent's indirect utility $V(x) = u(w^*(x|a_1^o))$ in (24) is convex in output x, then the optimal contract $w^o(x,\eta)$ guaranteeing that the agent takes $a_1^o, a_d^o = R$ (i.e., $b^o = 0$), i.e., $w^o(x,\eta)$ in equation (34), must satisfy*

Property (1) *$\mu_b^o(b) \neq 0$ (> 0) for a positive Borel-measure of b.*[2]

Property (2) *$w^o(x,\eta) = w^o(x,-\eta)$ for all x,η and $\mu_b^o(b) = \mu_b^o(-b)$ for all b.*

Proof of Lemma 3.

Property (1): $\mu_b^o(b) \neq 0$ for a positive Borel-measure of b.

Assume $\mu_b^o(b) = 0$, $a.s.$ Then the optimal contract $w^o(x,\eta)$ in (34) can be written as

$$\frac{1}{u'(w^o(x,\eta))} = \lambda + \mu_1^o \frac{x - \phi(a_1^o)}{\sigma^2}\phi_1(a_1^o), \tag{A10}$$

[2] We already know $\mu_4^o(b) \geq 0$ for every b (almost surely), since it is derived from the inequality constraint at each b.

for (x,η) satisfying $w^o(x,\eta) \geq k$ in (A10) and $w^o(x,\eta) = k$, otherwise.

This indicates that $(w^o(x,\eta), \mu_1^o, a_1^o)$ becomes $(w^*(x|a_1^o), \mu_1^*(a_1^o), a_1^o)$ in this case. However, since $V(x) \equiv u(w^*(x|a_1^o))$ is convex in x by assumption, the agent will take $b = \pm\infty$ instead of $b = 0$, which contradicts with the constraint (ii) in (30).

Property (2): $w^o(x,\eta) = w^o(x,-\eta)$ for all x, η and $\mu_4^o(b) = \mu_4^o(-b)$ for all b.

We first see that given a_1^o:[3]

$$g(x,\eta|b) = \frac{1}{2\pi\sigma}\exp\left(-\frac{1}{2}\frac{(x-\phi(a_1^o)-b\eta)^2}{\sigma^2} - \frac{1}{2}\eta^2\right), \tag{A11}$$

where

$$\frac{g(x,\eta|b)}{g(x,\eta|b=0)} = \exp\left(\frac{b\eta(x-\phi(a_1^o))}{\sigma^2}\right)\exp\left(-\frac{b^2\eta^2}{2\sigma^2}\right). \tag{A12}$$

From (A11), we obtain that (i) $g(x,-\eta|b=0) = g(x,\eta|b=0)$, and (ii) $g_1(x,-\eta|b=0) = g_1(x,\eta|b=0)$. Also, from (A11), we see that

$$g(x,\eta|b) = g(x,-\eta|-b), \ \ \forall(x,\eta,b). \tag{A13}$$

Our strategy is to prove that: (i) if $w^o(x,\eta)$ is an optimal contract, then $w^o(x,-\eta)$ satisfies all the constraints in (30), (ii) based on (i), if $w^o(x,\eta)$ is an optimal contract, then $w^o(x,-\eta)$ also becomes an optimal contract, and (iii) $\mu_b^o(-b) = \mu_b^o(b)$ for $\forall b$ at the optimum.

Claim 1. If $w^o(x,\eta)$ is an optimal contract, then $w^o(x,-\eta)$ satisfies all the constraints in

[3] We suppress a_1^o in $g(x,\eta|a_1^o,b)$ in (31). Note that $g(x,\eta|a_1,b)$ yields the following likelihood ratios:

$$\frac{g_1}{g}(x,\eta|a_1,b) = \frac{x-b\eta-\phi(a_1)}{\sigma^2}\phi_1(a_1), \ \ \frac{g_b}{g}(x,\eta|a_1,b) = \frac{(x-b\eta-\phi(a_1))\eta}{\sigma^2}.$$

(30).

As $w^o(x,\eta)$ is optimal, it satisfies the constraints in (30). We start from the incentive constraint for a_1. Since $g_1(x,\eta|b=0) = g_1(x,-\eta|b=0)$, we have

$$\begin{aligned}\int u(w^o(x,-\eta))g_1(x,\eta|b=0)dxd\eta - v'(a_1^o) &= \int u(w^o(x,-\eta))g_1(x,-\eta|b=0)dxd\eta - v'(a_1^o)\\ &= \int u(w^o(x,\eta))g_1(x,\eta|b=0)dxd\eta - v'(a_1^o) = 0,\end{aligned}$$

where we use the change of variable (i.e., $-\eta$ to η) in the second equality.

Also, as $w^o(x,\eta)$ is optimal,

$$\int u(w^o(x,\eta))\left(g(x,\eta|b=0) - g(x,\eta|b)\right)dxd\eta \geq 0. \tag{A14}$$

Thus, we obtain for any given b

$$\begin{aligned}&\int u(w^o(x,-\eta))\left(g(x,\eta|b=0) - g(x,\eta|b)\right)dxd\eta\\ &\quad= \int u(w^o(x,-\eta))\left(g(x,-\eta|b=0) - g(x,-\eta|-b)\right)dxd\eta\\ &\quad= \int u(w^o(x,\eta))\left(g(x,\eta|b=0) - g(x,\eta|-b)\right)dxd\eta \geq 0,\end{aligned}$$

where the first equality is from $g(x,\eta|b=0) = g(x,-\eta|b=0)$ and (A13) and the second equality is from the change of variable (i.e., $-\eta$ to η). Thus, we proved that if $w^o(x,\eta)$ is an optimal contract, then $w^o(x,-\eta)$ satisfies all the constraints in (30).

Claim 2: If $w^o(x,\eta)$ is an optimal contract, then $w^o(x,-\eta)$ also is an optimal contract.

From **Claim 1**, $w^o(x,-\eta)$ satisfies all the constraints in (30) at $(a_1^o, b=0)$. Thus, it is sufficient to show that $w^o(x,-\eta)$ achieves the same efficiency as $w^o(x,\eta)$. This follows from:

$$\begin{aligned}
&\int (x - w^o(x,-\eta))g(x,\eta|b=0)dxd\eta + \lambda\left(\int u(w^o(x,-\eta))g(x,\eta|b=0)dxd\eta - v(a_1^o)\right) \\
&\quad = \int (x - w^o(x,-\eta))g(x,-\eta|b=0)dxd\eta + \lambda\left(\int u(w^o(x,-\eta))g(x,-\eta|b=0)dxd\eta - v(a_1^o)\right) \\
&\quad = \int (x - w^o(x,\eta))g(x,\eta|b=0)dxd\eta + \lambda\left(\int u(w^o(x,\eta))g(x,\eta|b=0)dxd\eta - v(a_1^o)\right),
\end{aligned}$$

where the first equality is from that $g(x,\eta|b=0) = (x,-\eta|b=0)$, and the second equality is from the change of variable (i.e., $-\eta$ to η). Also, note that the firm's expected bankruptcy cost, $Pr[x \le x_b|a_1^o, b=0]\cdot D$ does not change because both $w^o(x,\eta)$ and $w^o(x,-\eta)$ induce the agent to take the same $(a_1^o, b=0)$. Therefore, if $w^o(x,\eta)$ is an optimal contract, then $w^o(x,-\eta)$ becomes an optimal contract and we obtain $w^o(x,-\eta) = w^o(x,\eta)$.[4]

Claim 3. $\mu_b^o(-b) = \mu_b^o(b)$, $\forall b$.

Note from the Lagrange duality theorem (see e.g., Luenberger (1969)) that the optimal solution $(\mu_1^o, \{\mu_b^o(b)\}, w^o(\cdot))$ is the one that solves $\min_{\mu_1,\{\mu_b(\cdot)\}} \max_{w(\cdot)} \mathcal{L}$ where $\mathcal{L}$ is given by

$$\begin{aligned}
\mathcal{L} \equiv &\int (x - w(x,\eta))g(x,\eta|b=0)dxd\eta - Pr[x \le x_b|a_1^o, b=0]\cdot D \\
&+ \lambda\left(\int u(w(x,\eta))g(x,\eta|b=0)dxd\eta - v(a_1^o)\right) \\
&+ \mu_1\left(\int u(w(x,\eta))g_1(x,\eta|b=0)dxd\eta - v'(a_1^o)\right) \\
&+ \int_b \mu_b(b)\left(\int u(w(x,\eta))\left(g(x,\eta|b=0) - g(x,\eta|b)\right)dxd\eta\right)db,
\end{aligned}$$

[4]We implicitly assume that the optimal contract is unique in this environment, following the literature (e.g., Jewitt et al. (2008)).

while satisfying $\mu_b^o(b) \geq 0, \forall b$, and the following complementary slackness at the optimum:

$$\mu_b^o(b) \left(\int u(w^o(x,\eta)) \left(g(x,\eta|b=0) - g(x,\eta|b) \right) dx d\eta \right) = 0, \quad \forall b. \tag{A15}$$

The last term in the above Lagrangian $\mathcal{L}$ given the optimal contract, $w^o(x,\eta)$, can be written as

$$\begin{aligned} &\int_b \mu_4(b) \left(\int u(w^o(x,\eta)) \left(g(x,\eta|b=0) - g(x,\eta|b) \right) dx d\eta \right) db \\ &\quad = \int_b \mu_4(-b) \left(\int u(w^o(x,-\eta)) \left(g(x,\eta|b=0) - g(x,\eta|-b) \right) dx d\eta \right) db, \end{aligned} \tag{A16}$$

where we use a change of variable (i.e., b to $-b$) and $w^o(x,-\eta) = w^o(x,\eta)$. Now with (A13) and that $g(x,\eta|b=0)$ is symmetric in η around $\eta = 0$, we know:

$$\begin{aligned} &\int u(w^o(x,-\eta)) \left(g(x,\eta|b=0) - g(x,\eta|-b) \right) dx d\eta \\ &\quad = \int u(w^o(x,-\eta)) \left(g(x,-\eta|b=0) - g(x,-\eta|b) \right) dx d\eta \\ &\quad = \int u(w^o(x,\eta)) \left(g(x,\eta|b=0) - g(x,\eta|b) \right) dx d\eta, \end{aligned} \tag{A17}$$

where we use the change of variable (i.e., $-\eta$ to η) for the second equality. With (A16) and (A17), the last term in Lagrangian $\mathcal{L}$ can be therefore written as

$$\begin{aligned} &\int_b \mu_4(b) \left(\int u(w^o(x,\eta)) \left(g(x,\eta|b=0) - g(x,\eta|b) \right) dx d\eta \right) db \\ &\quad = \int_b \mu_4(-b) \left(\int u(w^o(x,\eta)) \left(g(x,\eta|b=0) - g(x,\eta|b) \right) dx d\eta \right) db. \end{aligned} \tag{A18}$$

Plugging in (A18) into the original Lagrangian $\mathcal{L}$ yields $\mu_4^o(-b) = \mu_4^o(b), \forall b$.

Claim 4. In addition, we have:

$$\int u(w^o(x,\eta))g(x,\eta|b)dxd\eta = \int u(w^o(x,\eta))g(x,\eta|-b)dxd\eta, \tag{A19}$$

which implies that the agent's indirect utility given $w^o(x,\eta)$ is symmetric in b around $b=0$. Equation (A19) follows from:

$$\begin{aligned}\int u(w^o(x,\eta))g(x,\eta|-b)dxd\eta &= \int u(w^o(x,\eta))g(x,-\eta|b)dxd\eta \\ &= \int u(w^o(x,-\eta))g(x,-\eta|b)dxd\eta \\ &= \int u(w^o(x,\eta))g(x,\eta|b)dxd\eta,\end{aligned}$$

where we use (A13) in the first equality, $w^o(x,-\eta)=w^o(x,\eta)$ in the second, and and the change of variable (i.e., $-\eta$ to η) in the third equality.

Proof of Proposition 2: Given the optimal action a_1^o, we define $\widehat{Cov} \equiv (x-\phi(a_1^o))\eta$.[5] Since

$$\exp\left(\frac{b\eta(x-\phi(a_1^o))}{\sigma^2}\right) = \exp\left(\frac{b}{\sigma^2}\widehat{Cov}\right) = \sum_{k=0}^{\infty}\frac{1}{k!}\frac{b^k}{\sigma^{2k}}\widehat{Cov}^k, \tag{A20}$$

we obtain from equation (A12)

$$\frac{g(x,\eta|b)}{g(x,\eta|b=0)} = \left(\sum_{k=0}^{\infty}\frac{1}{k!}\frac{b^k}{\sigma^{2k}}\widehat{Cov}^k\right)\exp\left(-\frac{b^2\eta^2}{2\sigma^2}\right). \tag{A21}$$

[5]This is a sample covariance between x and η, as our framework is a single-period setting.

Therefore, we have

$$\int \mu_4^o(b)\left(1-\frac{g(x,\eta|b)}{g(x,\eta|b=0)}\right)db = \int \mu_4^o(b)db - \int \mu_4^o(b)\left(\sum_{k=0}^{\infty}\frac{1}{k!}\frac{b^k}{\sigma^{2k}}\widehat{Cov}^k\right)\exp\left(-\frac{b^2\eta^2}{2\sigma^2}\right)db$$
$$= \int \mu_4^o(b)db - \sum_{k=0}^{\infty}\left(\frac{1}{k!}\frac{1}{\sigma^{2k}}\underbrace{\left(\int \mu_4^o(b)b^k\exp\left(-\frac{b^2\eta^2}{2\sigma^2}\right)db\right)}_{\equiv C_k(\eta)}\right)\widehat{Cov}^k. \tag{A22}$$

When k is odd, the coefficient, $C_k(\eta)$, becomes 0 for $\forall\eta$, since the fact that $\mu_4^o(b)=\mu_4^o(-b)$ for all b from Lemma 3 implies

$$C_{k:odd}(\eta) = \int \mu_4^o(b)b^k\exp\left(-\frac{b^2\eta^2}{2\sigma^2}\right)db = \int_{b\geq 0}\left(\underbrace{\mu_4^o(b)-\mu_4^o(-b)}_{=0}\right)b^k\exp\left(-\frac{b^2\eta^2}{2\sigma^2}\right)db = 0. \tag{A23}$$

When k is even, however, the coefficient, $C_k(\eta)$, becomes strictly positive for $\forall\eta$, since the fact that $\mu_4^o(b)\neq 0$ for the non-zero measure of b from Lemma 3 implies

$$\begin{aligned}C_{k:even}(\eta) &= \int \mu_4^o(b)b^k\exp\left(-\frac{b^2\eta^2}{2\sigma^2}\right)db\\ &= \int_{b\geq 0}(\mu_4^o(b)+\mu_4^o(-b))b^k\exp\left(-\frac{b^2\eta^2}{2\sigma^2}\right)db\\ &= 2\int_{b\geq 0}\mu_4^o(b)b^k\exp\left(-\frac{b^2\eta^2}{2\sigma^2}\right)db > 0.\end{aligned} \tag{A24}$$

Therefore, (A22) can be written as

$$\begin{aligned}&\int \mu_4^o(b)\left(1-\frac{g(x,\eta|b)}{g(x,\eta|b=0)}\right)db\\ &= \int \mu_4^o(b)db - 2\sum_{k:even}^{\infty}\left(\frac{1}{k!}\frac{1}{\sigma^{2k}}\left(\int_{b\geq 0}\mu_4^o(b)b^k\exp\left(-\frac{b^2\eta^2}{2\sigma^2}\right)db\right)\right)\widehat{Cov}^k.\end{aligned} \tag{A25}$$

Consequently, by plugging (A25) into the optimal contact, $w^o(x,\eta)$, in (34) when $w^o(x,\eta)\geq$

k, we obtain

$$\frac{1}{u'(w^o(x,\eta))} = \lambda + \mu_1^o \frac{x-\phi(a_1^o)}{\sigma^2}\phi_1(a_1^o) + \underbrace{\int \mu_4^o(b)db}_{>0}$$
$$- 2\sum_{k:\text{even}}^{\infty} \underbrace{\frac{1}{k!}\frac{1}{\sigma^{2k}} \underbrace{\left(\int_{b\geq 0} \mu_4^o(b) b^k \exp\left(-\frac{b^2\eta^2}{2\sigma^2}\right) db\right)}_{\equiv C_{k:\text{even}}(\eta)>0}}_{\equiv D_{k:\text{even}}(\eta)>0} \widehat{Cov}^k. \tag{A26}$$

Since $D_{k:\text{even}}(\eta) > 0$ for all even numbers of k, given (x,η) a higher $|\widehat{Cov}|$ results in a lower compensation $w^o(x,\eta)$. Also, as $D_{k:\text{even}}(\eta) > 0$ decreases in η^2, given $(x, \widehat{Cov})$, a higher η^2 results in a higher $w^o(x,\eta)$. In sum, the principal punishes a sample covariance $|\widehat{Cov}|$ but becomes lenient when a high $|\widehat{Cov}|$ comes from the high $|\eta|$ realization, not from the agent's speculation activity ($b \neq 0$).

■

Appendix B The Truth-Telling Mechanism

We have previously assumed that there is no communication between the principal and the agent about the firm's initial exposure to hedgeable risks, R, after the contract is written. We now relax this assumption and consider the case where the agent can costlessly report the firm's risk exposure R to the principal, and receive a payoff that is contingent on the communicated risk exposure as well as on the output and hedgeable risks.

As we will show below, when $V(x) \equiv u(w^*(x|a_1^*))$ for $w^*(x|a_1^*)$ in (23) is concave in x,[1] a contract that is similar to $w^*(x|a_1^*)$ can be designed to induce the agent to truthfully reveal the firm's risk initial exposure, R. Therefore, in this case, there is no loss associated with the firm's initial risk exposure being unobservable to the principal and thus the same informational gain can be obtained by designing a truth-telling mechanism as the one from the introduction of a derivative market. The intuition is also the same as the one for the case where the manager would voluntarily hedge under $w^*(x|a_1^*)$ in the derivative market when $V(x) = u(w^*(x|a_1^*))$ is concave in x. Essentially, the truth-telling contract will allow the agent to make a *side bet* with the principal. If the agent hedges with the contract $w^*(x|a_1^*)$ after the derivative market is introduced, he would truthfully reveal what he observes (i.e., true R) to minimize the additional risk associated with this side bet even without the derivative market.

However, when $V(x) = u(w^*(x|a_1^*))$ is convex in x, any contract similar to $w^*(x|a_1^*)$ does not induce truth-telling since the agent wants to add more risks, as he would do by engaging in speculation with the derivative market. Again, a new contract must be designed to induce him to reveal the truth.

[1] Note that, in this case, optimal a_1 is a_1^* which is defined in (10), but not a_1^o defined in (23) because still there is no derivative market.

Equivalence between derivative market games and communication games Suppose the principal does not observe the firm's initial risk exposure, R, and there is no derivative market (i.e., a_d is again fixed at 0 as in Section 2). Since the agent observes R before he chooses a_1 and the communication regarding R is freely allowed, the principal can design a truth-telling mechanism, $w(x, r, \eta)$, where r represents the value of R reported by the agent. Let $a_1^T(R)$ be the agent's optimal action after observing R and $w^T(x, r, \eta)$ be the wage contract that optimally induces $a_1^T(R)$ with the agent telling the truth. Knowing that $r = R, \forall R$, under $w^T(x, r, \eta)$, we denote optimized joint benefits in this case as

$$SW^T \equiv \int \left(\phi(a_1^T(R)) - C^T(a_1^T(R)) - \lambda v(a_1^T(R)) - Pr[x \leq x_b | a_1^T(R), a_d = 0]D\right) h(R)dR, \tag{B1}$$

where

$$C^T(a_1^T(R)) \equiv \int \left(w^T(x, R, \eta) - \lambda u(w^T(x, R, \eta))\right) g(x, \eta | a_1^T(R), a_d = 0)dxd\eta \tag{B2}$$

represents the agency cost arising from inducing $a_1^T(R)$ through $w^T(x, r, \eta)$ when R is realized. In the above equation, $g(x, \eta | a_1^T(R), a_d = 0)$ denotes the joint density function of (x, η) given that $a_1^T(R)$ is chosen by the agent when a_d is fixed at 0.

As in Section 3 we first consider the case in which principal designs a wage contract, $w^*(y_r|a_1^*)$, that is the same as $w^*(x|a_1^*)$ in (23) except that it is based on $y_r \equiv x - r\eta$ instead of x. That is, $w^*(y_r|a_1^*)$ satisfies

$$\frac{1}{u'(w^*(y_r|a_1^*))} = \lambda + \mu_1^*(a_1^*)\frac{y_r - \phi(a_1^*)}{\sigma^2}\phi_1(a_1^*), \tag{B3}$$

for y_r such that $w^*(y_r|a_1^*) \geq k$ and $w^*(y_r|a_1^*) = k$ otherwise.

Note that since, without a derivative market,

$$x = \phi(a_1) + R\eta + \sigma\theta, \tag{B4}$$

we obtain

$$y_r \equiv x - r\eta = \phi(a_1) + (R - r)\eta + \sigma\theta. \tag{B5}$$

Thus, the principal's problem of designing a truth-telling mechanism in this case is similar to her problem of designing an incentive scheme based on x to induce $b = 0$ (i.e., $a_d = R$) when derivative transactions are allowed. Therefore, the optimal truth-telling contract should be based on $y_r \equiv x - r\eta$ and have the same contractual form as $w^*(x|a_1^*)$ in (23) as long as the agent's truth-telling can be guaranteed by that contract. As a result, as is the case for $w^*(x|a_1^*)$ in Section 3, we directly obtain following results for $w^*(y_r|a_1^*)$.

Lemma 4. [Hedging and Speculation with $w^*(y_r|a_1^*)$]
(1) *If $V(y_r) \equiv u(w^*(y_r|a_1^*))$ for the wage contract, $w^*(y_r|a_1^*)$, in (B3) is concave in y_r, then the manager will always report truthfully, i.e., $r = R, \forall R$, when $w^*(y_r|a_1^*)$ is offered.*
(2) *If $V(y_r) \equiv u(w^*(y_r|a_1^*))$ for the wage contract, $w^*(y_r|a_1^*)$, in (B3) is convex in y_r, then the manager will report r such that $|R - r| = \infty$ when $w^*(y_r|a_1^*)$ is offered.*

From Lemma 4, we obtain the following propositions.

Proposition 5. *When there is no derivative market and communication between the principal and the agent is costless, then $w^*(y_r|a_1^*)$ described in (B3) is the optimal truth-telling contract for the firm's hidden risk exposure, R, if $V(y_r) \equiv u(w^*(y_r|a_1^*))$ is concave in y_r. In this case,*
(1) the principal's inability to observe R does not reduce the firm's welfare (i.e., no adverse selection). That is, $SW^T = SW^*$,
(2) and the introduction of a derivative market, although it cannot provide the informational

gain for the agency relation, still improves welfare by reducing the firm's its expected cost of financial stress through hedging. That is, $SW^T < SW^o$.

Proposition 5 reaffirms that one important benefit from the derivative market is actually the principal's informational gain in the agency relation, as it allows the agent to engage in complete hedging in the derivative market. If the principal and the agent can only communicate with each other by paying huge communication costs, this benefit of having the derivative market is actually associated with saving such communication costs. In reality, the costs associated with communicating this information and updating the compensation contract based on the reavealed R may be greater than the hedging cost. As shown in (B5), in this case, allowing the manager to choose a_d in derivative transactions is observationally equivalent to allowing him to freely report the firm's realized risk exposure R. Thus, one important benefit from the derivative market in this case is obtaining the same result as the one with a truth-telling contract in a cheaper way.

On the other hand, if $V(y_r) \equiv u(w^*(y_r|a_1^*))$ is convex in y_r, the manager will not report the true R under $w^*(y_r|a_1^*)$, and shareholders have to redesign a truth-telling mechanism, $w^T(x, r, \eta)$ different from $w^*(y_r|a_1^*)$.

Proposition 6. *When there is no derivative market and communication between the principal and the agent is costless, if* $V(y_r) \equiv u(w^*(y_r|a_1^*))$ *is convex in* y_r*, for* $w^*(y_r|a_1^*)$ *described in (B3), a new contract,* $w^T(x, r, \eta)$*, should be designed to optimally induce the agent to reveal* R *truthfully. In this case,*
(1) the principal's inability to observe R *reduces the firm's welfare, that is,* $SW^T < SW^*$*,*
(2) and the introduction of a derivative market may not improve on the firm's welfare in this case. Especially, if both R *itself and* σ_R^2 *are small enough, the firm's welfare will be lowered by the introduction of a derivative market.*

As shown in Proposition 4, given that free communication between the principal and the agent about R is not available, if $V(x) \equiv u(w^*(x|a_1^o))$ in (24) is convex in x, the introduction of a derivative market may lower the firm's welfare, especially when both R and σ_R^2 are very small. A similar result is obtained in the truth-telling environment as shown in Proposition 6.

Proposition 6 illustrates that, when free communication between the principal and the agent about R is available, if $V(y_r) \equiv u(w^*(y_r|a_1^*))$ for $w^*(y_r|a_1^*)$ in (B3), is convex in y_r, the introduction of a derivative market may lower the firm's welfare especially when both R and σ_R^2 are very small. This is mainly because the obtained contract in this case, $w^T(x, r, \eta)$, is designed to induce the agent's truth-telling about R without taking the agent's possible transactions in the derivative market into consideration. But, with the derivative market being introduced, the principal should worry about inducing not only the agent's truth-telling but also his hedging when designing a compensation contract, and the optimal contract satisfying both requirements is $w^o(x, \eta)$ in (34).

In sum, in the truth-telling environment in which free communication between the principal and the agent is available, the agent's access to derivative market transactions always improves on the firm's welfare if $V(y_r) \equiv u(w^*(y_r|a_1^*))$ is concave in y_r. But, it may lower the firm's welfare if $V(y_r) \equiv u(w^*(y_r|a_1^*))$ is convex in y_r, and the principal's imposing restriction on the agent's derivative trading can be desirable.

In summary, when the communication between shareholders and the manager becomes free, the manager's access to derivative market transactions does not change the firm's welfare if $V(y_r) \equiv u(w^*(y_r|a_1^*))$ is concave in y_r, and might lower it if $V(y_r) \equiv u(w^*(y_r|a_1^*))$ is convex in y_r and no restriction on the derivative trading can be imposed by the principal.

B.1 Proofs of Appendix B

Proof of Proposition 5: From Lemma 4, we see that $w^*(y_r|a_1^*)$ is a truth-telling mechanism for the initial risk exposure, R, if $V(y_r) \equiv u(w^*(y_r|a_1^*))$ is concave in y_r because the agent's truth-telling is automatically guaranteed under $w^*(y_r|a_1^*)$. Since $r = R, \forall R$, under $w^*(y_r|a_1^*)$, we have

$$y \equiv x - R\eta = \phi(a_1) + \sigma\theta = y_r, \tag{B6}$$

and the optimal truth-telling contract $w^*(y_r|a_1^*)$ in equation (B3), which has the same contractual form as $w^*(x|a_1^*)$ in (23) except that the optimal action to be chosen by the agent is a_1^*, i.e., $a_1^T(R) = a_1^*, \forall R$ which is defined in (10) but not a_1^o defined in (23) because still there is no derivative market. Therefore, we derive

$$SW^T = SW^*, \tag{B7}$$

and we also derive that SW^T is lower than the joint benefits SW^o that will be obtained under $w^*(x|a_1^o)$ when there is a derivative market.

■

Proof of Proposition 6:

(1) As shown in Lemma 4, if $V(y_r) \equiv u(w^*(y_r|a_1^*))$ is convex in y_r, $w^*(y_r|a_1^*)$ cannot be the optimal truth-telling contract, and a new optimal truth-telling contract $w^T(x, r, \eta)$, which is different from $w^*(y_r|a_1^*)$, should be designed. Thus, it is obvious that $\text{SW}^T < SW^*$.

(2) We decompose the changes in welfare in the truth-telling environment due to the introduction of a derivative market such as:

$$SW^o - SW^T = (SW^o - SW^*) + \left(SW^* - SW^T\right). \tag{B8}$$

The second term on the right-hand side of (B8), which is always positive according to (B.1), represents saving the extra agency cost which incurs when revising the optimal contract from $w^*(y_r|a_1^*)$ to $w^T(x, r, \eta)$ is needed to induce the agent's truth-telling about R in this case. Note that this extra agency cost gets small as σ_R^2 becomes small. In a limit case where $\sigma_R^2 \to 0$, we obtain $SW^* - SW^T \to 0$.

As already explained in Proposition 4, however, the positive effect in the first term on the right-hand side of (B8) (i.e., reducing the firm's expected cost of financial stress by hedging) also becomes small as R is small, whereas its negative effect (i.e., the extra agency cost which incurs when revising the optimal contract from $w^*(y_r|a_1^*)$ to $w^o(x, \eta)$ in (34) is needed to induce the agent's complete hedging in this case[2] remains unchanged by the changes in R and/or σ_R^2.

Therefore, the introduction of a derivative market to the truth-telling environment may lower the firm's welfare if both R and σ_R^2 are small enough (i.e., $SW^o < SW^T$).

■

[2]Note that $w^o(x, \eta)$ is also a truth-telling contract.

Appendix C A Model with Discretionary Project Choice

This section extends the model in Section 2 to include the agent‘s real investment choices. Specifically, after his wage contract $w(\cdot)$ is finalized, the agent takes three kinds of actions, $a_1 \in [0, \infty)$, $a_2 \in [\underline{a}_2, \overline{a}_2]$, and $a_3 \in (-\infty, +\infty)$. The first action, a_1 is the productive effort choice, which increases expected output as before, that is, high effort generates an output level that first-order stochastically dominates the output level generated by low effort. The agent's second action a_2 is his (real) project choice. We assume there exists projects with different risks with more risky projects having higher expected output. The agent's preference is still the same as in Assumption 1. The third action a_3 is his choice in the derivatives market.[1] Although the set of projects available to the agent is bounded, i.e., $a_2 \in [\underline{a}_2, \overline{a}_2]$, the agent can choose any position in the derivatives market, i.e., $a_3 \in (-\infty, +\infty)$ as in the main text. In contrast to Section 2, we assume $D = 0$ in Assumption 3 for simplicity, i.e., we ignore the negative feedback effect that amplifies negative cash flows.

After the agent chooses a_1, a_2, and a_3, the firm's output, x, is realized and publicly observable without cost. Thereby, an output x can be used in the manager's wage contract that is denoted by w. The output is determined not only by the agent's choice of (a_1, a_2, a_3) but also by the state of nature, (η, θ). For simplicity, we assume that the output function exhibits the following additively separable form:

$$x = \phi(a_1, a_2) + a_2\theta + (R - a_3)\eta. \tag{C1}$$

Equation (C1) looks like equation (1), except that (i) the agent's project choice a_2 affects the expected output level $\phi(a_1, a_2)$; and (ii) the firm's level of non-hedgeable risk is not fixed *a priori*, but determined by the agent's project choice a_2. Now, an expected output, $\phi(a_1, a_2)$, is a function of both a_1 and a_2, whereas the agent's derivatives choice, a_3, does

[1] We use the notation a_3 instead of a_d of Section 2 for notational convenience.

not directly affect it. As in (1), we assume that (i) $\eta \sim N(0,1)$ and $\theta \sim N(0,1)$ are uncorrelated; and (ii) η is observable at the end of the contracting period, and thereby can be used in the manager's wage contract if necessary. As in the main text, the manager observes R after the contract is signed but before choosing a_1, a_2, and a_3. Again, shareholders do not observe R, but know its distribution $R \sim h(R)$. Management effort a_1 and project choice a_2 do not affect R, the firm's innate exposure to the hedgeable risks.[2] However, the firm's final risk exposure is determined by the manager's transaction a_3 in the derivative market. If $a_3 = 0$, the manager does not trade derivatives. He hedges, i.e., reduces risk, as long as $|R - a_3| < |R|$ and minimizes hedgeable risks by setting $a_3 = R$. If $|R - a_3| > |R|$, the manager speculates in the derivative market.

In addition to the assumptions in Section 2, we make the following additional assumptions:

Assumption 4. $\dfrac{\partial \phi}{\partial a_1}(a_1, a_2) \equiv \phi_1(a_1, a_2) > 0,\ \dfrac{\partial^2 \phi}{\partial a_1^2}(a_1, a_2) \equiv \phi_{11}(a_1, a_2) < 0.$

Assumption 5. $\dfrac{\partial \phi}{\partial a_2}(a_1, a_2) \equiv \phi_2(a_1, a_2) > 0,\ \phi_{22}(a_1, a_2) < 0,\ \phi_2(a_1, \underline{a}_2) = \infty,$ *and* $\phi_2(a_1, \overline{a}_2) = 0.$

Assumption 6. $0 < \underline{a}_2 < \overline{a}_2 < \infty.$

Assumption 7. $\phi_{12}(a_1, a_2) \cdot a_2 < \phi_1(a_1, a_2),\ \ \forall (a_1, a_2).$

Assumptions 4 and 5 specify that a_1 affects expected output with the usual property of decreasing marginal product of effort, while a higher a_2 increases expected output as well as output variability, i.e., there is a trade-off between return and risk.[3] Assumption 6 states that there is neither a completely safe project nor a project with unbounded risk.

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[2] In general, a firm's risk exposure might depend on the real investment undertaken. Even if we allow the firm's risk exposure to be affected by the project choice a_2, most results do not change qualitatively.

[3] As noted from equation (C1), reducing the firm's non-hedgeable risks requires the firm to sacrifice a part of an expected output. This trade-off guarantees the existence of an optimal project choice a_2 in our setting.

If $\phi_{12}(a_1, a_2)$ is positive and decreasing in a_2, and $\phi_1(a_1, \underline{a}_2) \simeq 0$, $\underline{a}_2$ is close to 0, then Assumption 7 holds as we can see in Figure C.1. For example, with positive $\phi_{12}(a_1, a_2)$,[4] a_1 and a_2 are complementary in generating output. We assume this complementarity between a_1 and a_2 become weaker as the project becomes riskier, i.e., a_2 increases.

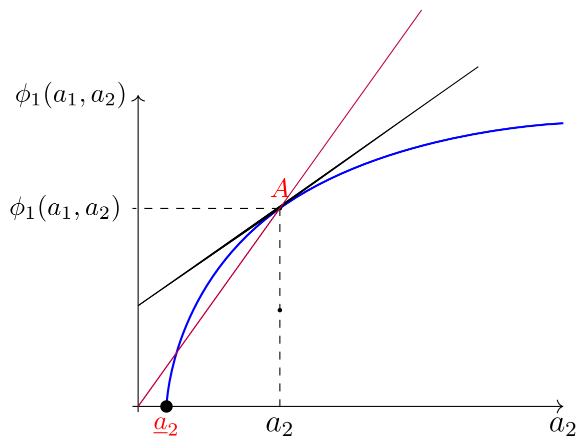

Figure C.1: Illustration of the Assumption 7

C.1 When There Is No Derivative Market

C.1.1 The Principal Knows the Firm's Exposure to the Hedgeable Risks

In this section, we consider a benchmark case where there is no derivatives market and the principal knows the firm's innate risk exposure, R. We thus specify $a_3 = 0$ so that the production function in equation (C1) reduces to

$$x = \phi(a_1, a_2) + R\eta + a_2\theta. \tag{C2}$$

Since there is no derivative market, the manager's incentive problem arises only in inducing (a_1, a_2). As R and η are observable and thus contractable, $y \equiv x - R\eta$ is a sufficient

[4]For example, if we regard the action a_1 as managing the project on a day-to-day basis, it is natural to assume that when the manager takes additional project risk a_2, the role of effort a_1 in generating output becomes more important, i.e., $\phi_1(a_1, a_2)$ rises in a_2.

statistic for (x, η) in assessing (a_1, a_2). Therefore, the principal uses y as a contractual variable to induce (a_1, a_2), and the above equation can be expressed as

$$y = \phi(a_1, a_2) + a_2\theta. \tag{C3}$$

Benchmark: without incentive problem in a_2 In general, designing a contract to optimally induce project choice a_2 as well as effort choice a_1 is quite different than designing a contract that only induces the agent's effort choice a_1 in Section 2. To illustrate this distinction, we first consider the case in which the agent's project choice, a_2, is observable, or equivalently, selected by the principal. The optimal compensation contract $w(\cdot)$, in this case, maximizes the combined utilities of the principal and agent subject to the restriction that the agent's effort a_1 is chosen to maximize his utility given the contract.

$$\begin{aligned} \max_{a_1, a_2, w(\cdot)} \phi(a_1, a_2) - \int w(y) f(y|a_1, a_2) dy + \lambda \left(\int u(w(y)) f(y|a_1, a_2) dy - v(a_1) \right) \quad \text{s.t.} \\ (i) \;\; a_1 \in \arg\max_{a_1'} \int u(w(y)) f(y|a_1', a_2) dy - v(a_1'), \;\; \forall a_1', \\ (ii) \;\; w(y) \geq k, \;\; \forall y, \end{aligned} \tag{C4}$$

where $f(y|a_1, a_2)$ denotes a probability density function of y given the agent's three actions, and λ denotes the weight placed on the agent's utility in the joint optimization. As shown, the combined utilities of the principal and the agent are maximized subject to the agent's incentive compatibility constraint, which specifies that the agent optimally chooses his effort, and his limited liability constraint, which specifies that the agent receives at least k, the subsistence level of utility.

Based on the first-order approach as in Section 2, the above maximization problem (C4) reduces to:

$$\max_{\substack{a_1,a_2\\ w(\cdot)\geq k}} \phi(a_1,a_2) - \int w(y)f(y|a_1,a_2)dy + \lambda\left(\int u(w(y))f(y|a_1,a_2)dy - v(a_1)\right) \quad \text{s.t.}$$

$$(i) \quad \int u(w(y))f_1(y|a_1,a_2)dy - v'(a_1) = 0, \tag{C5}$$

where f_1 denotes the first derivative of f taken with respect to a_1.

To find the solution $(a_1^P, a_2^P, w^P(y|a_1^P, a_2^P))$ for the above program, we first derive an optimal contract for an arbitrarily given (a_1, a_2). Let $w^P(y|a_1, a_2)$ be a contract which optimally motivates the agent to take a particular level of a_1 when an arbitrary level of a_2 is chosen by the principal. By solving the Euler equation of the above program after fixing (a_1, a_2), we derive that $w^P(y|a_1, a_2)$ must satisfy

$$\frac{1}{u'(w^P(y|a_1,a_2))} = \lambda + \mu_1(a_1,a_2)\frac{f_1}{f}(y|a_1,a_2), \tag{C6}$$

for almost every y for which (C6) has a solution $w^P(y|a_1, a_2) \geq k$, and otherwise $w^P(y|a_1, a_2) = k$. In (C6), $\mu_1(a_1, a_2)$ denotes the optimized Lagrange multiplier for the agent's incentive constraint associated with effort a_1 when the project choice is pinned down at a_2. Since $f(y|a_1, a_2)$ is a normal density function with mean $\phi(a_1, a_2)$ and variance a_2^2, (C6) is reduced to:

$$\frac{1}{u'(w^P(y|a_1,a_2))} = \lambda + \mu_1(a_1,a_2)\frac{y-\phi(a_1,a_2)}{a_2^2}\phi_1(a_1,a_2). \tag{C7}$$

Before analyzing the optimal contract, we first define given (a_1, a_2):

$$SW^P(a_1,a_2) \equiv \phi(a_1,a_2) - C^P(a_1,a_2) - \lambda v(a_1), \tag{C8}$$

which denotes the joint benefits when $w^P(y|a_1, a_2)$ is designed and a_2 is instructed by the principal where

$$C^P(a_1, a_2) \equiv \int \left(w^P(y|a_1, a_2) - \lambda u(w^P(y|a_1, a_2))\right) f(y|a_1, a_2)dy \tag{C9}$$

represents the efficiency loss of this case compared with the full information case. In other words, $C(a_1, a_2)$ measures the agency cost arising from inducing the agent to take that particular a_1 when a_2 is chosen by the principal.

We start our analysis with the following Lemma 5, which is analogous to Kim (1995).

Lemma 5. $C^P(a_1, a_2^0) < C^P(a_1, a_2^1)$ *for any given* a_1 *if* $a_2^0 < a_2^1$.

Since the principal dictates the agent's project choice a_2 here, an agency problem arises only in inducing a_1. Lemma 5 implies that under Assumption 7, when the project choice a_2 is selected by the principal, the agency cost associated with motivating the agent to take any given effort a_1, i.e., $C^P(a_1, a_2)$, is reduced if the principal chooses a less risky project. A lowered risk a_2 improves the efficiency of the agency relationship by providing a more precise signal y about the agent's effort, a_1, which enables the principal to design a contract inducing a particular a_1 in a less costly way. If $\phi_{12}(a_1, a_2)$ is large enough to break Assumption 7, then lower a_2 might lower $\phi_1(a_1, a_2)$ a lot, which in turn makes harder for the principal to give the proper incentive for the effort a_1 and raises the incentive cost $C^P(a_1, a_2)$. Assumption 7 guarantees that this incentive drawback is lower than the informational rent from lower a_2, so that a lower level of a_2 reduces the agency cost $C^P(a_1, a_2)$.

Value of hedging Lemma 5 indicates that firms should eliminate all the zero net present value risks, if possible. For example, when the agent can be induced to hedge in the derivative market, the principal can more efficiently induce the agent to expend efforts and choose project choices, given any initial risk exposure level R.

Risk-return trade-off in project choice However, given the trade-off between return and risk, i.e., $\phi_2 > 0$, the exact level of a_2 that the principal prefers will be determined by the loss in expected return as well as the benefit from achieving a more precise signal of effort. Let a_2^P be the project that is most preferred by the principal, and a_1^P the agent's optimal effort choice for the above program when a_2^P is chosen by the principal. Then, from the above optimization we obtain that $(a_1^P, a_2^P, w^P(\cdot))$ should satisfy

$$\int \left(y - w^P(y) + \lambda u(w^P(y))\right) f_2(y|a_1^P, a_2^P)dy + \mu_1(a_1^P, a_2^P) \int u(w^P(y)) f_{12}(y|a_1^P, a_2^P)dy = 0, \tag{C10}$$

where $w^P(\cdot) = w^P(\cdot|a_1^P, a_2^P)$, f_2 denotes the first derivative of f with respect to a_2 and f_{12} is the second derivative with respect to a_1 and a_2. The optimal contract $w^P(y|a_1^P, a_2^P)$ satisfies,

$$\frac{1}{u'(w^P(y|a_1^P, a_2^P))} = \lambda + \mu_1(a_1^P, a_2^P)\frac{y - \phi(a_1^P, a_2^P)}{(a_2^P)^2}\phi_1(a_1^P, a_2^P), \tag{C11}$$

for y satisfying $w^P(y|a_1^P, a_2^P) \geq k$ and $w^P(y|a_1^P, a_2^P) = k$ otherwise.

The manager's incentive to select a_2 under contract $w^P(\cdot)$ The above analysis assumes that shareholders essentially select the projects. Now, we ask whether the manager will voluntarily choose the project that would be chosen by informed shareholders, i.e., a_2^P. If the answer to this question is no, then the moral-hazard problem arises not only in motivating a_1 but also in incentivizing a_2, which implies that the optimal wage contract must be modified from the contract, $w^P(y|a_1^P, a_2^P)$, in (C11).

To formally analyze this issue, we denote $a_2^A(a_2^P)$ as a solution to

$$a_2^A(a_2^P) \in \arg\max_{a_2} \int u(w^P(y|a_1^P, a_2^P)) f(y|a_1^P, a_2)dy. \tag{C12}$$

Thus, $a_2^A(a_2^P)$ represents the project choice that the agent would take under $w^P(y|a_1^P, a_2^P)$ described in (C11) when a_2 is not enforceable. Thus, our previous question, "Will the agent voluntarily choose a_2^P when $w^P(y|a_1^P, a_2^P)$ is designed?", is equivalent to the question, "Will $a_2^A(a_2^P)$ be equal to a_2^P?"

As previously shown, the principal balances two considerations when he directs the agent to take a certain project: the informational benefits from risk reduction and the lower mean return associated with lower risk. However, the risk level chosen by the agent depends on his indirect risk preferences induced by contract $w^P(y|a_1^P, a_2^P)$, i.e., the curvature of $u(w^P(y|a_1^P, a_2^P))$ with respect to y, and the effect that a trade-off between return and risk would have on his utility *via* $w^P(y|a_1^P, a_2^P)$.

In general, the curvature of the agent's indirect utility function depends on the distribution of the random state variable and his utility function. To see how different utility functions affect this curvature differently, we again consider the case where the agent has constant relative risk aversion with degree $1 - t$ as we did in Section 2.1, where $t < 1$ (i.e., $u(w) = \frac{1}{t}w^t, t < 1$). We obtain from equation (C11) that

$$w^P(y|a_1^P, a_2^P) = \left(\lambda + \mu_1(a_1^P, a_2^P)\left(\frac{y - \phi(a_1^P, a_2^P)}{(a_2^P)^2}\right)\phi_1(a_1^P, a_2^P)\right)^{\frac{1}{1-t}}, \tag{C13}$$

and the agent's indirect utility under this wage contract is

$$u(w^P(y|a_1^P, a_2^P)) = \frac{1}{t}\left(\lambda + \mu_1(a_1^P, a_2^P)\left(\frac{y - \phi(a_1^P, a_2^P)}{(a_2^P)^2}\right)\phi_1(a_1^P, a_2^P)\right)^{\frac{t}{1-t}}. \tag{C14}$$

The above equation shows that the agent's indirect utility becomes strictly concave in y if $t < \frac{1}{2}$, linear if $t = \frac{1}{2}$, and convex if $t > \frac{1}{2}$ for y satisfying $w^P(y|a_1^P, a_2^P) \geq k$. If we assume $w^P(y|a_1^P, a_2^P) = k$ for sufficiently low y, as far as the agent's induced risk preferences are concerned, $u(w^P(y|a_1^P, a_2^P))$ makes the agent risk-loving if $t \geq \frac{1}{2}$.

Furthermore, since the compensation contract $w^P(y|a_1^P, a_2^P)$ is positively related to the absolute output level, i.e., $\mu_1(a_1^P, a_2^P) > 0$,[5] if $t \geq \frac{1}{2}$, the agent is induced to take the most risky project, i.e., $a_2^A(a_2^P) = \overline{a}_2$ when $w^P(y|a_1^P, a_2^P)$ is designed even if $\phi_2(a_1, \overline{a}_2) = 0$ by Assumption 5. However, in this case, principal prefers to have a firm's risk level a_2 lower than $\overline{a}_2$. This is because, from his standpoint, the informational benefits from risk reduction are still substantial, while the costs of risk reduction are zero at $\overline{a}_2$ (i.e., $\phi_2(\overline{a}_2) = 0$). Thus, $a_2^P < a_2^A(a_2^P)$ in this case. In other words, the principal prefers less risk than the agent under $w^P(y|a_1^P, a_2^P)$.

On the other hand, if t is close to $-\infty$ (i.e., the agent is extremely risk-averse), the agent's indirect utility function induces him to choose a lower level of risk than what the principal prefers (i.e., $a_2^A(a_2^P) < a_2^P$) even if a lower a_2 yields on average lower output.

Incentive problems associated with project choice a_2, in general, exist in all cases except for those where both of the following conditions are satisfied: (i) the agent's indirect utility is sufficiently concave and (ii) there is no trade-off between return and risk, i.e., $\phi_2 = 0, \ \forall a_2$. Under these conditions, both the principal and the agent agree that the firm should choose the least risky project, i.e., $a_2 = \underline{a}_2$, and there is no efficiency loss due to the existence of the manager's unobservable project choice. However, when either the agent's induced risk preferences are convex, or the trade-off between return and risk exists as assumed in Assumption 5, the principal and the agent will not generally agree on the firm's optimal project choice, and the compensation contract, $w^P(y|a_1^P, a_2^P)$, described in equation (C11) will no longer be optimal.

[5] For the proof of $\mu_1(a_1^P, a_2^P) > 0$, see e.g., Holmström (1979), Jewitt (1988), Jung and Kim (2015).

Optimal contracts with moral hazard in a_2 In this situation, the principal must determine the optimal compensation contract by solving the following optimization problem:

$$\max_{\substack{a_1,a_2 \\ w(\cdot)\geq k}} \phi(a_1,a_2) - \int w(y)f(y|a_1,a_2)dy + \lambda\left(\int u(w(y))f(y|a_1,a_2)dy - v(a_1)\right) \quad \text{s.t.}$$
$$(i) \;\; (a_1,a_2) \in \arg\max_{a_1',a_2'} \int u(w(y))f(y|a_1',a_2')dy - v(a_1'), \;\; \forall a_1',a_2'. \tag{C15}$$

The optimization problem (C15) accounts for the fact that the agent selects a_2 to maximize his own expected utility. If an interior solution for (a_1, a_2) exists and the first-order approach is valid, the above maximization problem can be expressed as:

$$\max_{\substack{a_1,a_2 \\ w(\cdot)\geq k}} \phi(a_1,a_2) - \int w(y)f(y|a_1,a_2)dy + \lambda\left(\int u(w(y))f(y|a_1,a_2)dy - v(a_1)\right) \quad \text{s.t.}$$
$$(i) \;\; \int u(w(y))f_1(y|a_1,a_2)dy - v'(a_1) = 0,$$
$$(ii) \;\; \int u(w(y))f_2(y|a_1,a_2)dy = 0, \tag{C16}$$

Let (a_1^*, a_2^*) be the optimal action combination for the above program. Then, by solving the Euler equation, we obtain that the optimal wage contract, $w^*(y)$, which satisfies,

$$\frac{1}{u'(w^*(y))} = \lambda + \mu_1^* \frac{f_1}{f}(y|a_1^*,a_2^*) + \mu_2^* \frac{f_2}{f}(y|a_1^*,a_2^*), \tag{C17}$$

for almost every y for which equation (C17) has a solution $w^*(y) \geq k$, and otherwise $w^*(y) = k$. μ_1^* and μ_2^* are the optimized Lagrange multipliers for both incentive constraints, respectively.

Since $f(y|a_1^*, a_2^*)$ is a normal distribution with mean $\phi(a_1^*, a_2^*)$ and variance $(a_2^*)^2$, from

(C17), we have

$$\frac{1}{u'(w^*(y))} = \lambda + \mu_1^* \underbrace{\frac{y-\phi(a_1^*,a_2^*)}{(a_2^*)^2}\phi_1^*}_{\equiv SS_1} + \mu_2^* \left[\underbrace{\frac{y-\phi(a_1^*,a_2^*)}{(a_2^*)^2}\phi_2^*}_{\equiv SS_2^1} + \underbrace{\frac{1}{a_2^*}\left(\frac{(y-\phi(a_1^*,a_2^*))^2}{(a_2^*)^2} - 1\right)}_{\equiv SS_2^2} \right], \tag{C18}$$

where we define SS_1, $SS_2 \equiv SS_2^1 + SS_2^2$ as sufficient statistics for unobservbable action a_1 and project choice a_2, respectively. Compared with (C11), (C18) shows that when both a_1 and a_2 are not observable, the optimal wage contract is based not only on the absolute output y, but also on its (standardized) deviation from the expected level, $\frac{\left(y-\phi(a_1^*,a_2^*)\right)^2}{(a_2^*)^2}$. Since $\left(y-\phi(a_1^*,a_2^*)\right)^2$ is a sample (i.e., realized) variance of a single observation with mean zero and variance $(a_2^*)^2$, the term $\frac{\left(y-\phi(a_1^*,a_2^*)\right)^2}{(a_2^*)^2}$ in (C18) can be regarded a standardized output deviation. Note that SS_2, the sufficient statistic for the project choice a_2, can be now decomposed into two parts: SS_2^1 and SS_2^2. SS_2^1 takes account of the effects that an increase in a_2 has on the mean cash flow $\phi(a_1, a_2)$,[6] while SS_2^2 is about how an increase in a_2 affects the signal y's volatility. By including the sample variance as a contractual parameter, the principal effectively motivates the agent to take the appropriate level of a_2, i.e., a_2^*. (C18) can be written in a simpler way as

$$\frac{1}{u'(w^*(y))} = \lambda + (\mu_1^*\phi_1^* + \mu_2^*\phi_2^*)\frac{y-\phi(a_1^*,a_2^*)}{(a_2^*)^2} + \mu_2^*\frac{1}{a_2^*}\left(\frac{(y-\phi(a_1^*,a_2^*))^2}{(a_2^*)^2} - 1\right), \tag{C19}$$

for y satisfying $w^*(y) \geq k$ and $w^*(y) = k$ otherwise. Here, $\phi_i^* \equiv \phi_i(a_1^*, a_2^*), i = 1, 2$. We call $w^*(y)$ as an *optimal dual-agency contract* à la Hirshleifer and Suh (1992).

The optimal dual agency contract is characterized in the following propositions.

Proposition 7. $\mu_1^*\phi_1^* + \mu_2^*\phi_2^* > 0$.

Proposition 7 implies that holding the cash flow variance constant, the manager's pay-

[6]This term is present since we assume the risk-return trade-off in a_2, i.e., $\phi(a_1, a_2)$ is increasing in a_2.

out increases when output increases, implying that the agent is rewarded for a higher effort. However, this does not necessarily mean that the contracted payout is monotonically increasing in output. For example, if $\mu_2^* < 0$ in (C19), the agent can be paid less when the output is very high.

Thus, a more interesting question has to do with the relation between the agent's rewards and the output deviation, i.e., the sign of μ_2^*.

Proposition 8. *If the principal prefers a less risky project than the agent under $w^P(y|a_1^P, a_2^P)$ in equation (C11), i.e., $a_2^P < a_2^A(a_2^P)$, then the optimal dual agency contract will penalize the agent if output differs substantially from the expected level, i.e., $\mu_2^* < 0$ for $w^*(y)$ in equation (C19). If the principal prefers a riskier project than the agent under $w^P(y|a_1^P, a_2^P)$, i.e., $a_2^P > a_2^A(a_2^P)$, then the optimal dual agency contract will reward the agent for having unusual output deviation, i.e., $\mu_2^* > 0$ for $w^*(y)$ in equation (C19).*

If the principal prefers a lower level of project risk than the agent under the contract $w^P(y|a_1^P, a_2^P)$, the contract will be revised in a way that motivates the agent to reduce risk. This can be done by setting $\underline{\mu_2^* < 0}$ in equation (C19) which penalizes the agent for the unusual output deviation and effectively makes the agent act as if he is more risk-averse. On the other hand, if the principal prefers a higher risk than the agent when $w^P(y|a_1^P, a_2^P)$ is designed, the contract is revised to motivate the agent to increase risk. This can be done by setting $\underline{\mu_2^* > 0}$ in equation (C19) which rewards the agent for unusual output deviation and effectively makes the agent act as though he is less risk-averse. As discussed earlier, the later case is more likely to occur when the manager is more risk averse and when the firm's investment opportunities offer a non-trivial trade-off between return and risk.[7]

[7] For example, in cases of constant relative risk aversion with degree $1 - t$, it is more likely that $\underline{\mu_2^* > 0}$ when $1 - t$ is higher (i.e., t is lower).

We denote the optimized joint benefits in this case as

$$SW^*(a_1^*, a_2^*) \equiv \phi(a_1^*, a_2^*) - C^*(a_1^*, a_2^*) - \lambda v(a_1^*), \tag{C20}$$

where

$$C^*(a_1^*, a_2^*) \equiv \int \left(w^*(y) - \lambda u(w^*(y))\right) f(y|a_1^*, a_2^*)dy \tag{C21}$$

denotes the agency cost arising from inducing (a_1^*, a_2^*) when a_3 is fixed at 0 and R is observable.

C.1.2 The Principal Does Not Know the Firm's Risk Exposure

Similar to Section 2.2, we consider the case of asymmetric information, where the firm's innate exposure to hedgeable risks, R, is observed only by the agent. In this case, the wage contract cannot explicitly include $y \equiv x - R\eta$ as a contractual variable. Furthermore, we rule out the possibility of any communication between principal and the agent that allows the agent to reveal R.

If principal does not observe R, the compensation contract must be based on (x, η), i.e., $w = w(x, \eta)$. The principal's maximization program in this case is thus:[8]

$$\begin{aligned}
\max_{\substack{a_1(\cdot), a_2(\cdot) \\ w(\cdot) \geq k}} & \int_R \int_{x,\eta} (x - w(x,\eta))\, g(x,\eta|a_1(R), a_2(R), R)h(R)dxd\eta dR \\
& + \lambda \int_R \left(\int_{x,\eta} u(w(x,\eta))g(x,\eta|a_1(R), a_2(R), R)dxd\eta - v(a_1(R)) \right) h(R)dR \quad \text{s.t.} \\
& (i) \;\; (a_1(R), a_2(R)) \in \arg\max_{a_1, a_2} \int_{x,\eta} u(w(x,\eta))g(x,\eta|a_1, a_2, R)dxd\eta - v(a_1), \forall R,
\end{aligned} \tag{C22}$$

[8] In this case, since the agent is the only one that observes R, his actions a_1, a_2 both depend on R, given the contract $w(x, \eta)$.

where

$$g(x,\eta|a_1,a_2,R) = \frac{1}{2\pi a_2}\exp\left(-\frac{1}{2}\left(\frac{(x-\phi(a_1,a_2)-R\eta)^2}{a_2^2}+\eta^2\right)\right) \tag{C23}$$

denotes a joint probability density function of (x,η) given (a_1,a_2,R) and $h(R)$ denotes the probability density function of R.

For each R, let $(a_1^N(R), a_2^N(R), w^N(x,\eta))$ be the solution for the optimization program (C22). If we let $\mu_1(R)$, $\mu_2(R)$ be Lagrange multipliers attached to incentive constraints in $a_1(R)$ and $a_2(R)$, respectively, the optimal contract $w^N(x,\eta)$ can be written as

$$\begin{aligned}\frac{1}{u'(w^N(x,\eta))} =&\lambda + \int_R \mu_1(R)\left[\frac{g_1(x,\eta|a_1^N(R),a_2^N(R),R)}{\int_{R'} g(x,\eta|a_1^N(R'),a_2^N(R'),R')h(R')dR'}\right]h(R)dR \\ &+ \int_R \mu_2(R)\left[\frac{g_2(x,\eta|a_1^N(R),a_2^N(R),R)}{\int_{R'} g(x,\eta|a_1^N(R'),a_2^N(R'),R')h(R')dR'}\right]h(R)dR,\end{aligned} \tag{C24}$$

when $w(x,\eta)\geq k$ and otherwise $w(x,\eta)=k$. The optimized joint benefit in this case is denoted as:

$$SW^N \equiv \int_R \left(\phi(a_1^N(R),a_2^N(R)) - C^N(a_1^N(R),a_2^N(R)) - \lambda v(a_1^N(R))\right)h(R)dR, \tag{C25}$$

where

$$C^N(a_1^N(R),a_2^N(R)) \equiv \int_{x,\eta}\left(w^N(x,\eta)-\lambda u(w^N(x,\eta))\right)g(x,\eta|a_1^N(R),a_2^N(R),R)dxd\eta \tag{C26}$$

denotes the agency cost arising from inducing $(a_1^N(R),a_2^N(R))$ given a realized value of R. As in Proposition 2, in this case, we obtain the following comparison between two welfare measures: SW^N and $SW^*(a_1^*, a_2^*)$.

Proposition 9. *When there is no derivative market and no communication is allowed between the principal and the agent, the principal's inability to observe the firm's risk exposure reduces welfare, i.e.,*

$$SW^N < SW^*(a_1^*, a_2^*).$$

Intuitively, when the principal observes the firm's risk exposure, R, this information can be used to design a wage contract that eliminates the influence of hedgeable risk, i.e., $w = w^*(y \equiv x - R\eta)$. However, if R is not observable and cannot be communicated, this is impossible.

C.2 When Managers Can Trade Derivatives

In this subsection we consider how the introduction of an opportunity to trade derivatives (i.e., when a_3 is not fixed at 0) affects the optimal contract and the firm's efficiency. Continuing from Section C.1.2, we assume that a manager's project choice, a_2, is not observable, and in addition, we assume that the derivatives choice, a_3 and the firm's risk initial exposure, R, cannot be observed by or communicated to the principal.

We closely follow the logic of Section 3: since the firm's exposure to hedgeable risks, R, is observed by the agent before he takes actions (a_1, a_2, a_3), the agent's choice of a_3 can be characterized as his choice of $b \equiv R - a_3$. Then given a compensation contract, the principal can rationally anticipate the agent's choice of $b = R - a_3$. We denote the principal's anticipation of the agent's choice of $R - a_3$ by $\hat{b}$, and define $z(\hat{b}) \equiv x - \hat{b}\eta$ as a variable that can potentially be in the wage contract, i.e., $w(z(\hat{b}))$ is a potential contract. If the principal's beliefs are to be consistent,[9] it must be the case that the agent chooses a_3

[9] As the principal predicts the agent with risk-exposure R to choose $\hat{b} = R - a_3$, a contract that relies on $\hat{b}$ induces the agent to take $b = \hat{b}$.

satisfying $b \equiv R - a_3 = \hat{b}$ given this contract.

Thus, since

$$z(\hat{b}) \equiv x - \hat{b}\eta = \phi(a_1, a_2) + (b - \hat{b})\eta + a_2\theta, \tag{C27}$$

if the principal offers the contract $w(z(\hat{b}))$ and the agent chooses a_3 satisfying $b = R - a_3 = \hat{b}$, then

$$z(\hat{b}) = \phi(a_1, a_2) + a_2\theta = y. \tag{C28}$$

Note that the maximum level of joint benefits that can be obtained in this case is $SW(a_1^*, a_2^*, a_3 = 0)$ in equation (C20).[10] Therefore, we first consider the case in which the principal designs the contract the same as $w^*(y)$ in the benchmark case (i.e., equation (C19)) but based on $z(\hat{b})$ instead of $y \equiv x - R\eta$, and examine whether the agent actually chooses $b \equiv R - a_3 = \hat{b}$ under $w^*(z(\hat{b}))$. If this is indeed the case, there is no welfare loss associated with R (and a_3) being unobservable when the agent is able to transact in the derivatives market.

The optimal contract in the benchmark case (i.e., (C19)) as a potential contract Suppose that the principal designs a contract $w^*(z(\hat{b}) \equiv x - \hat{b}\eta)$ satisfying

$$\frac{1}{u'(w^*(z(\hat{b})))} = \lambda + (\mu_1^*\phi_1^* + \mu_2^*\phi_2^*)\frac{z(\hat{b}) - \phi(a_1^*, a_2^*)}{(a_2^*)^2} + \mu_2^*\frac{1}{a_2^*}\left(\frac{\left(z(\hat{b}) - \phi(a_1^*, a_2^*)\right)^2}{(a_2^*)^2} - 1\right), \tag{C29}$$

for $z(\hat{b})$ satisfying $w^*(z(\hat{b})) \geq k$ and $w^*(z(\hat{b})) = k$ otherwise. Because $w^*(z(\hat{b}))$ in (C29) is of the same functional form as $w^*(y)$ in (C19),[11] we easily see the agent will take (a_1^*, a_2^*) under $w^*(z(\hat{b}))$ *if* he chooses a_3 satisfying $b \equiv R - a_3 = \hat{b}$. But, the real question is: "Will the agent always choose a_3 satisfying $b = \hat{b}$ when $w^*(z(\hat{b}))$ is designed and offered?".

[10]Given the contract $w(z(\hat{b}))$, if there is no incentive problem associated with $b = R - a_3$, i.e., the agent voluntarily chooses a_3 such that $R - a_3 = \hat{b}$, then we obtain the maximal joint benefit $SW(a_1^*, a_2^*, a_3 = 0)$. The issue is whether the agent would voluntarily choose a_3 such that $R - a_3 = \hat{b}$ given $w(z(\hat{b}))$.

[11]Note that $\mu_1^*, \mu_1^*, a_1^*, a_2^*$ in (C19) and (C29) are endogenous variables characterized by solving the optimization in (C15).

The following Lemma 6 provides an answer to the above question.

Lemma 6. [Speculation and Hedging with $w^*(z(\hat{b}))$]

(1) *If $\mu_2^* < 0$ for the contract, $w^*(z(\hat{b}))$, described in equation* (C29) *for any given $\hat{b}$,*[12] *then the manager will choose a_3 such that $b = \hat{b}$ when the contract $w^*(z(\hat{b}))$ is offered.*

(2) *If $\mu_2^* > 0$ for $w^*(z(\hat{b}))$ in equation* (C29) *for any given $\hat{b}$, then the manager will take a_3 such that $|R - a_3| = \infty$ when $w^*(z(\hat{b}))$ is offered.*

From Lemma 6, we directly obtain the following proposition:

Proposition 10. *If $\mu_2^* < 0$ for $w^*(z(\hat{b}))$ in (C29) for any given $\hat{b}$, then the level of $b \equiv R - a_3$ induced is a matter of indifference as long as it is bounded, i.e., $|b| < \infty$. For example, If $\mu_2^* < 0$ for $w^*(z(0))$ in (C29), the agent chooses $a_1^*, a_2^*, a_3 = R$ (i.e., $b = 0$) when $w^*(z(0))$ is offered. In this case, the optimized welfare is the same as $SW^*(a_1^*, a_2^*)$ in (C20), implying that the firm's welfare with a derivative market will be the same as it is in the case where the risk exposure is observed by the principal.*[13]

Proposition 10 is quite intuitive. If $\underline{\mu_2^* < 0}$ for $w^*(z(\hat{b}))$ in (C29), the agent is induced to engage in full hedging to minimize the variance of $z(\hat{b})$. Intuitively, the contract $w^*(z(\hat{b}))$ with $\mu_2^* < 0$ induces the agent to sacrifice expected payoffs to lower risk.[14] If the risk can be reduced through a channel that does not decrease the expected payoff (e.g., here a_3 does not have risk-return trade-off.), then agent will clearly do so. In addition, $\mu_1^*\phi_1^* + \mu_2^*\phi_2^* > 0$ means a higher $z(\hat{b})$ yields the higher compensation $w^*(z(\hat{b}))$ given its squared deviation from the average of $z(\hat{b})$.

[12] One can easily see that if $\mu_2^* < 0$ in $w^*(z(\hat{b}))$ for any given $\hat{b}$, then $\mu_2^* < 0$ in $w^*(z(\hat{b}))$ for all $\hat{b}$. This is because the principal's anticipating different $\hat{b}$ does not change the functional form of $w^*(\cdot)$.

[13] The introduction of derivative markets improves the welfare compared with the case where the principal does not observe the firm's risk-exposure R and the communication between the principal and the agent is prohibitively costly (i.e., $SW^*(a_1^*, a_2^*) > SW^N$ in Proposition 9). In practice, with $D > 0$ in Assumption 3, benefits a derivative market can provide to firms are multi-dimensional, e.g., hedging allows firms to prevent themselves from experiencing some financial distress.

[14] The optimal contract $w^*(z(\hat{b}))$ features $\mu_2^* < 0$ when $a_2^P < a_2^A(a_2^P)$, as explained in Proposition 8.

In this case, the optimal contract can be designed <u>*as if*</u> $b = R - a_3$ is observable to the principal, and it allows the principal and the agent to achieve the welfare $SW^*(a_1^*, a_2^*)$ that can be achieved when the risk exposure R is observable.

However, this is not possible if $\underline{\mu_2^* > 0}$ for $w^*(z(\hat{b}))$ in (C29), since the agent speculates infinitely, i.e., the agent chooses a_3 such that $|R - a_3| = \infty$. This is because, as shown from equation (C29), the agent will be paid an infinite amount when $z(\hat{b}) = x - \hat{b}\eta$ is either positive or negative infinity if $\mu_2^* > 0$ for $w^*(z(\hat{b}))$. Given that it is impossible to design a wage contract $w^*(z(\hat{b}))$ based on the belief $\hat{b} = \infty$, the principal has to either alter the wage contract to ensure $|R - a_3| < \infty$ or retain the optimal contract without a derivative market, $w^N(x, \eta)$ and prohibit the manager from engaging in derivative transactions.

Comparison with Section 3 It is interesting to compare the results in this section to the analysis in Section 3 that takes the real investment choice as given. Recall that in Section 2, we start from the benchmark case where R is observed by the principal, which reduces the problem to the canonical principal-agent model (e.g., Holmström (1979)). The optimal contract $w^*(x|a_1^o)$ in this benchmark scenario generates the agent's indirect utility function $V(x)$. As we show, (i) if $V(x)$ is concave (convex) in x, then the agent chooses to perfectly hedge (infinitely speculate) when there is a derivatives market and (ii) $V(x)$ is more likely to be concave (convex) when the agent's utility function features higher (lower) risk aversion. Therefore, a less risk averse manager is more likely to speculate in derivative markets given the contract $w^*(\cdot)$.

With flexible project choice a_2, we obtain the opposite result: (i) the agent with $\mu_2^* > 0$ speculates infinitely when derivative markets open; (ii) under the benchmark case without asymmetric information or a derivative market, the principal initially offers a contract with $\mu_2^* > 0$ since she prefers a higher level of project risk a_2 than the agent, implying generically that the agent's risk aversion is very high. To be specific, when the manager's risk aversion is sufficiently high, shareholders will design a contract to induce the manager to choose a higher project risk level a_2, to benefit from the positive risk-return tradeoff. Such a contract will reward a higher level of output variance (i.e., $\mu_2^* > 0$), which can in turn induce the manager to speculate infinitely, choosing $a_3 = \pm\infty$ due to the additional incentive effect from $\mu_2^* > 0$.

It can be understood as a side effect of inducing the project risk taking which is productive (i.e., $\phi_2(a_1, a_2) > 0$) through incentive contracts. A contract that induces risk taking in the real investment choice makes the manager speculate infinitely when derivative transaction is possible, as he acts as effectively risk-loving under the contract (C29) with $\mu_2^* > 0$.

Optimal contracts when $\mu_2^* > 0$ When the agent takes infinite speculation in derivative markets under the contract $w^*(z(\hat{b}))$ in (C29) with $\mu_2^* > 0$, our analysis becomes close to Section 3. First, the principal might design a new optimal contract, $w^o(x, \eta)$ to induce the agent's perfect hedging. This new optimal contract satisfies conditions in Proposition 3, and thus penalizes the agent for having both positive and negative realization of $(x - \phi(a_1^o, a_2^o))\eta$, which we regard as sample covariance between output and hedgeable risks. As the new optimal contract $w^o(x, \eta)$ imposes additional risks on the agent, it incurs efficiency costs, thereby lowering the social welfare from SW^* to SW^o, i.e., $SW^o < SW^*$.

Instead, the principal might just ban the derivative trading, in which case we go back to Section C.1.2 and achieve SW^N as welfare. When the degree of asymmetric information is small enough, i.e., the principal's prior distribution $h(R)$ on risk exposure R is tight with $\sigma_R \to 0$, then hedging benefits shrink, and therefore, the principal is better off banning the derivative trading, as summarized in Proposition 4.

C.3 Proofs of Appendix C

Proof of Lemma 5: We know from $y \sim N(\phi(a_1, a_2), a_2^2)$ that

$$\frac{y - \phi(a_1, a_2)}{a_2} \sim N(0, 1), \;\; \frac{f_1}{f}(y|a_1, a_2) = \frac{y - \phi(a_1, a_2)}{a_2^2}\phi_1(a_1, a_2) \sim N\left(0, \frac{\phi_1(a_1, a_2)^2}{a_2^2}\right). \tag{C30}$$

Therefore, we observe that if $\frac{\phi_1(a_1,a_2)}{a_2}$ is decreasing in a_2, for any pair $a_2^0 < a_2^1$, $\frac{f_1}{f}(y|a_1, a_2^0)$'s distribution is mean-preserving spread (MPS) of that of $\frac{f_1}{f}(y|a_1, a_2^1)$. Assumption 7 guarantees that this condition holds, and the following Lemma 7, a slightly changed form of Kim (1995), proves $C(a_1, a_2^0) < C(a_1, a_2^1)$ for $\forall a_1$.

Lemma 7. *For given action a_1 and technology $h(\cdot|a_1)$, let the solution of the following optimization problem be $w_h(\cdot)$:*

$$\begin{aligned} \max_{w(\cdot)} \int (y - w(y))h(y|a_1)dy + \lambda\left(\int u(w(y))h(y|a_1)dy - v(a_1)\right) \;\; & \textit{s.t.} \\ (i) \int u(w(y))h_1(y|a_1)dy - v'(a_1) = 0, & \\ (ii)\; w(y) \geq k, \forall y. & \end{aligned} \tag{C31}$$

For two different technologies $h = f,\ g$ such that $\frac{f_1}{f}(y|a_1)$ is a mean-preserving spread of $\frac{g_1}{g}(y|a_1)$, we have:

$$C_f(a_1) \equiv \int (w_f(y) - \lambda u(w_f(y)))\, f(y|a_1)dy < \int (w_g(y) - \lambda u(w_g(y)))\, g(y|a_1)dy \equiv C_g(a_1). \tag{C32}$$

Proof. We know that the solution of (C31) would be given as

$$\frac{1}{u'(w_h(y))} = \max\left\{\lambda + \mu_h \frac{h_1}{h}(y|a_1), \frac{1}{u'(k)}\right\}, \tag{C33}$$

where μ_h is the Lagrange multiplier attached to the incentive constraint for the given a_1. If we define $q_h \equiv \lambda + \mu_h \frac{h_1}{h}(y|a_1)$, we can rewrite the optimal contract $w_h(\cdot)$ as a function of q_h so that $w_h(y) = r(q_h)$ where $r(\cdot) = (\frac{1}{u'})^{-1}(\cdot)$ is increasing and does not rely on the technology h. Therefore, (C33) can be written as

$$u'(r(q_h))q_h = 1, \tag{C34}$$

if $q_h \geq u(k)^{-1}$ and otherwise $r(q_h) = k$. Now, we obtain

$$\begin{aligned}\mathbb{E}_h\left(u(r(q_h))q_h\right) &= \int u(r(q_h)) \cdot q_h \cdot h(y|a_1)dy = \int u(r(q_h))\left[\lambda + \mu_h \frac{h_1}{h}(y|a_1)\right] h(y|a_1)dy \\ &= \lambda \underbrace{\int u(r(q_h))h(y|a_1)dy}_{\equiv B_h} + \mu_h \underbrace{\int u(r(q_h))h_1(y|a_1)dy}_{=v'(a_1)} = \lambda B_h + \mu_h v'(a_1),\end{aligned} \tag{C35}$$

where we used the fact that $r(q_h)$ satisfies the agent's incentive constraint in a_1. Following Kim (1995), we define

$$\psi(q) \equiv r(q) - u(r(q))q, \tag{C36}$$

which is <u>concave</u> in $\forall q$, since: (i) with $q \geq u(k)^{-1}$, we obtain $\psi'(q) = \cancel{r'(q)} - \cancel{u'(r(q))r'(q)q} - u(r(q)) = -u(r(q))$ as $u'(r(q))q = 1$ and $\psi''(q) = -u'(r(q))r'(q) < 0$; (ii) with $q < u(k)^{-1}$, we have $r(q) = k$ so $\psi(q)$ becomes linear.[15] Now we can introduce two different technologies $f(\cdot|a_1)$ and $g(\cdot|a_1)$ such that $\frac{f_1}{f}(y|a_1)$ is a mean-preserving spread of $\frac{g_1}{g}(y|a_1)$, and define

$$\bar{q} \equiv \lambda + \mu_f \frac{g_1}{g}(y|a_1), \tag{C37}$$

which is possibly different from q_g as μ_f is possibly different from μ_g. As $\psi(q)$ is globally

[15] We see that $\psi(q)$ is continuously differentiable at all points including $q = u(k)^{-1}$.

concave, we obtain

$$\begin{aligned}\mathbb{E}_g\left(\psi(\bar{q})\right)-\mathbb{E}_g\left(\psi(q_g)\right) &\leq \mathbb{E}_g\left(\psi'(q_g)(\bar{q}-q_g)\right)=\mathbb{E}_g\left(-u(r(q_g))(\mu_f-\mu_g)\frac{g_1}{g}\right)\\ &=(\mu_g-\mu_f)\int u(r(q_g))g_1(y|a_1)dy=(\mu_g-\mu_f)v'(a_1)\\ &=\left(\mathbb{E}_g\left(u(r(q_g))q_g\right)-\lambda B_g\right)-\left(\mathbb{E}_f\left(u(r(q_f))q_f\right)-\lambda B_f\right),\end{aligned} \tag{C38}$$

where we used (C35). Finally, it leads to the following agency cost comparison:

$$\begin{aligned}C_g(a_1)-C_f(a_1) &= \mathbb{E}_g\left(r(q_g)-\lambda B_g\right)-\mathbb{E}_f\left(r(q_f)-\lambda B_f\right)=\mathbb{E}_g(r(q_g))-\mathbb{E}_f(r(q_f))-(\lambda B_g-\lambda B_f)\\ &=\mathbb{E}_g(\psi(q_g))+\mathbb{E}_g(u(r(q_g))q_g)-\mathbb{E}_f(\psi(q_f))-\mathbb{E}_f(u(r(q_f))q_f)-(\lambda B_g-\lambda B_f)\\ &\geq \cancel{\mathbb{E}_g(\psi(q_g))}-\mathbb{E}_f(\psi(q_f))+\mathbb{E}_g(\psi(\bar{q}))-\cancel{\mathbb{E}_g(\psi(q_g))}=\mathbb{E}_g(\psi(\bar{q}))-\mathbb{E}_f(\psi(q_f))\\ &=\int\psi\left(\lambda+\mu_f\frac{g_1}{g}(y|a_1)\right)g(y|a_1)dy-\int\psi\left(\lambda+\mu_f\frac{f_1}{f}(y|a_1)\right)f(y|a_1)dy\end{aligned} \tag{C39}$$

where we used (C38) in the above (C39)'s inequality part. Finally, if $\frac{f_1}{f}(y|a_1)$ is a mean-preserving spread of $\frac{g_1}{g}(y|a_1)$, then (C39) with Rothschild and Stiglitz (1970) implies $C_g(a_1)\geq C_f(a_1)$, as $\mu_f>0$ and $\psi(\cdot)$ is globally concave.

■

Finally, with $f(y|a_1)\equiv f(y|a_1,a_2^0)$ and $g(y|a_1)\equiv f(y|a_1,a_2^1)$ in our specification, Lemma 7 proves Lemma 5.

■

Proof of Proposition 7: Assume to the contrary that $\mu_1^*\phi_1^*+\mu_2^*\phi_2^*\leq 0$. Then, pick up any two levels of y: y_1 and y_2, such that

$$y_1<y_2,\quad\text{and}\quad\frac{y_1+y_2}{2}=\phi(a_1^*,a_2^*). \tag{C40}$$

That is, y_1 and y_2 are located at the same distance from the mean value $\phi(a_1^*,a_2^*)$. If $\mu_1^*\phi_1^*+$

$\mu_2^*\phi_2^* \le 0$, we have from equation (C19) that

$$w^*(y_1) \ge w^*(y_2), \text{ and } u(w^*(y_1)) \ge u(w^*(y_2)). \tag{C41}$$

Since $f_1(y_1|a_1^*, a_2^*) = -f_1(y_2|a_1^*, a_2^*) < 0$ for any y_1 and y_2 satisfying equation (C40), we have:

$$\int u(w^*(y))f_1(y|a_1^*, a_2^*)dy \le 0, \text{ and } \int u(w^*(y))f_1(y|a_1^*, a_2^*)dy - v'(a_1^*) < 0. \tag{C42}$$

Therefore, there is a contradiction.

■

Proof of Proposition 8:

1. $\mu_2^* > 0$ if $a_2^A(a_2^P) < a_2^P$. Let us compare the following two optimizations:[16]

$$\begin{aligned} \max_{a_1,a_2,w(\cdot)\ge k} & \int (y - w(y))f(y|a_1,a_2)dy + \lambda\left(\int u(w(y))f(y|a_1,a_2)dy - v(a_1)\right) \quad \text{s.t.} \\ & (i) \quad \int u(w(y))f_1(y|a_1,a_2)dy - v'(a_1) = 0, \\ & (ii) \quad \int u(w(y))f_2(y|a_1,a_2)dy = 0, \end{aligned} \tag{C43}$$

and

$$\begin{aligned} \max_{a_1,a_2,w(\cdot)\ge k} & \int (y - w(y))f(y|a_1,a_2)dy + \lambda\left(\int u(w(y))f(y|a_1,a_2)dy - v(a_1)\right) \quad \text{s.t.} \\ & (i) \quad \int u(w(y))f_1(y|a_1,a_2)dy - v'(a_1) = 0, \\ & (ii) \quad \int u(w(y))f_2(y|a_1,a_2)dy \ge 0, \end{aligned} \tag{C44}$$

[16]Following Rogerson (1985), we replace the incentive constraint with the corresponding inequality constraint, and exploit the fact that a multiplier to the inequality constraint must be non-negative.

where the incentive constraint associated with the non-hedgeable risk choice a_2 takes the form of inequality in the latter program, instead of equality in the original optimization program.

We know that $(w^*(y), a_1^*, a_2^*, \mu_1^*, \mu_2^*)$ are the optimal solution for the first program. Let $(\hat{w}(y), \hat{a}_1, \hat{a}_2, \hat{\mu}_1, \hat{\mu}_2)$ be the optimal solution for the second program. We will show that the above two programs are equivalent in that two solutions align perfectly with each other when $a_2^A(a_2^P) < a_2^P$. Then, we can directly derive $\mu_2^* \geq 0$ when $a_2^A(a_2^P) < a_2^P$, since $\hat{\mu}_2 \geq 0$ by Kuhn-Tucker theorem.

Assume that the second constraint in the second program is not binding. Then, $\hat{\mu}_2 = 0$, and $\hat{w}(y)$ should satisfy:

$$\frac{1}{u'(\hat{w}(y))} = \lambda + \hat{\mu}_1 \frac{y - \phi(\hat{a}_1, \hat{a}_2)}{(\hat{a}_2)^2} \phi_1(\hat{a}_1, \hat{a}_2), \tag{C45}$$

for y satisfying $\hat{w}(y) \geq k$ and $\hat{w}(y) = k$ otherwise. As the second constraint is not binding, $\hat{a}_2$ becomes the best (from the principal's perspective) a_2, i.e., $\hat{a}_2 = a_2^P$. Then we must have $\hat{a}_1 = a_1^P$ and $\hat{w}(y) = w^P(y|a_1^P, a_2^P)$. Therefore, the fact that the second constraint in the second program is not binding implies

$$\int u(w^P(y|a_1^P, a_2^P)) f_2(y|a_1^P, a_2^P, a_3 = 0) dy > 0. \tag{C46}$$

However, equation (C46) implies $a_2^A(a_2^P) > a_2^P$, a contradiction.[17] Thus, the second constraint in the second program must be binding, and the above two programs are equivalent so $\mu_2^* = \hat{\mu}_2 \geq 0$. And also, $\mu_2^* \neq 0$, because $\mu_2^* = 0$ implies $a_2^A(a_2^P) = a_2^P$.

2. $\mu_2^* < 0$ if $a_2^A(a_2^P) > a_2^P$. By applying the same method as in the above case, we can

[17] We assume that $\int u(w(y|a_2^P)) f(y|a_1^P, a_2, a_3 = 0) dy$ is concave in a_2, which is based on the first-order approach associated with a_2.

easily prove it. We compare following two optimizations similar to (C43) and (C44):

$$\max_{a_1,a_2,w(\cdot)\geq k} \int (y-w(y))f(y|a_1,a_2)dy + \lambda \left(\int u(w(y))f(y|a_1,a_2)dy - v(a_1) \right) \quad \text{s.t.}$$
$$(i) \quad \int u(w(y))f_1(y|a_1,a_2)dy - v'(a_1) = 0,$$
$$(ii) \quad \int u(w(y))f_2(y|a_1,a_2)dy = 0, \tag{C47}$$

and

$$\max_{a_1,a_2,w(\cdot)\geq k} \int (y-w(y))f(y|a_1,a_2)dy + \lambda \left(\int u(w(y))f(y|a_1,a_2)dy - v(a_1) \right) \quad \text{s.t.}$$
$$(i) \quad \int u(w(y))f_1(y|a_1,a_2)dy - v'(a_1) = 0,$$
$$(ii) \quad \int u(w(y))f_2(y|a_1,a_2)dy \leq 0, \tag{C48}$$

Solutions of the above two optimization programs must be the same, and due to the property that the multiplier attached to the incentive constraint associated with a_2 in the second program must be non-positive, we conclude $\mu_2^* < 0$ when $a_2^A(a_2^P) > a_2^P$.

■

Proof of Proposition 9: Now we have the project choice $a_2(R)$ that depends on the observed R by the manager. Consider the principal's following alternative maximization program:

$$
\begin{aligned}
\max_{a_1(\cdot),a_2(\cdot),w(\cdot)} & \int_R \int_{x,\eta} (x - w(x,R,\eta))\, g(x,\eta|a_1(R),a_2(R),R)h(R)dxd\eta dR \\
& + \lambda \int_R \left(\int_{x,\eta} u(w(x,R,\eta))g(x,\eta|a_1(R),a_2(R),R)dxd\eta - v(a_1(R)) \right) h(R)dR \quad \text{s.t.} \\
& (i) \ \int_{x,\eta} u(w(x,R,\eta))g_1(x,\eta|a_1(R),a_2(R),R)dxd\eta - v'(a_1(R)) = 0, \forall R, \\
& (ii) \ \int_{x,\eta} u(w(x,R,\eta))g_2(x,\eta|a_1(R),a_2(R),R)dxd\eta = 0, \forall R, \\
& (iii) \ w(x,R,\eta) \geq k, \ \ \forall (x,\eta).
\end{aligned} \tag{C49}
$$

Note that the above program is different from the original program (C22) in that here contract can be written on the realized value of R. If we let the Lagrange multipliers to the constraints (i) and (ii) be $\mu_1(R)h(R)$ and $\mu_2(R)h(R)$ respectively, we get the following optimal contractual form:[18]

$$
\begin{aligned}
\frac{1}{u'(w(x,R,\eta))} &= \lambda + \mu_1(R)\frac{x - R\eta - \phi_R}{a_2(R)^2}\phi_{1,R} \\
&\quad + \mu_2(R)\left[-\frac{1}{a_2(R)} + \frac{x - R\eta - \phi_R}{a_2(R)^2}\phi_{2,R} + \frac{(x - R\eta - \phi_R)^2}{a_2(R)^3} \right] \\
&= \lambda + (\mu_1(R)\phi_{1,R} + \mu_2(R)\phi_{2,R})\frac{\underbrace{x - R\eta}_{\equiv y} - \phi_R}{a_2(R)^2} + \frac{\mu_2(R)}{a_2(R)}\left[\frac{(\underbrace{x - R\eta}_{\equiv y} - \phi_R)^2}{a_2(R)^2} - 1 \right],
\end{aligned} \tag{C50}
$$

when $w(x,R,\eta) \geq k$. The above equation (C50) implies that optimal contract only depends on $y \equiv x - R\eta$ and the solution $(w(x,R,\eta), a_1(R), a_2(R))$ becomes $(a_1^*, a_2^*, w^*(y) \equiv w^*(x - R\eta))$. By comparing the above (C49) with the program in (C22) where the principal does not know R, one can easily see that the set of wage contracts, $\{w(x,R,\eta)\}$, satisfying the incentive constraints for a given action combination $(a_1(R), a_2(R))$ in the above

[18] We define $\phi_R \equiv \phi(a_1(R), a_2(R))$, $\phi_{i,R} \equiv \phi_i(a_1(R), a_2(R))$ for $\forall i = 1, 2$, where $\{a_1(R), a_2(R)\}$ are optimal actions for each R.

program always contains the set of wage contracts that would be available when the principal does not know R, $\{w(x, \eta)\}$, satisfying the incentive constraints for the same action combination. Therefore, we have

$$SW^N \leq SW^*(a_1^*, a_2^*). \tag{C51}$$

However, one can easily see that $w^*(y) = w^*(x - R\eta)$ which is a unique solution for the wage contract of the above program is not in the set of $\{w(x, \eta)\}$. As a result, we finally derive

$$SW^N < SW^*(a_1^*, a_2^*). \tag{C52}$$

■

Proof of Lemma 6:

(1) Suppose $\underline{\mu_2^* < 0}$ in equation (C29) for any given $\hat{b}$. Proposition 8 implies that if the shareholders want their manager to reduce the risk through the project choice (i.e., if $a_2^P < a_2^A(a_2^P)$), the optimal contract in equation (C19) features $\mu_2^* < 0$. Note that risk reduction through the real project choice (i.e., lowering a_2) is costly to the manager in the sense that a less risky project generates the lower expected return, and thereby reduces the agent's expected payoff (i.e., $\mu_1^*\phi_1^* + \mu_2^*\phi_2^* > 0$). Thus, the fact that even costly risk reduction is encouraged by $w^*(z(\hat{b}))$ implies that any risk reduction (i.e., reducing the variance of $z(\hat{(b)}))$ in the absence of expected return reduction will be taken by the manager under $w^*(z(\hat{b}))$. Risk reduction through derivative transaction is costless to the agent because there is no risk-return trade-off for derivative transaction (i.e., manipulating a_3). Whenever taking further risk reduction is encouraged, therefore, the manager would like to do it through the derivative choices first.

Thus, the manager will choose a_3 so that $b \equiv R - a_3 = \hat{b}$ which minimizes the variance of $z(\hat{b})$, when $w^*(z(\hat{b}))$ with $\mu_2^* < 0$ is designed.

■

(2) Suppose $\underline{\mu_2^* > 0}$ for $w^*(z(\hat{b}))$ in equation (C29). Given (a_1^*, a_2^*), $z(\hat{b}) = x - \hat{b}\eta = \phi(a_1^*, a_2^*) + (b - \hat{b})\eta + a_2^*\theta$ holds. Let $w(\eta, \theta, b|w^*)$ be the wage that the manager will receive under $w^*(z(\hat{b}))$ when he takes (a_1^*, a_2^*, b) and (η, θ) are realized. Then, by substituting equation (C27) into equation (C29), we have

$$\frac{1}{u'(w(\eta,\theta,b|w^*))} = \lambda + (\mu_1^*\phi_1^* + \mu_2^*\phi_2^*)\frac{(b-\hat{b})\eta + a_2^*\theta}{(a_2^*)^2} + \mu_2^*\frac{1}{a_2^*}\left(\frac{((b-\hat{b})\eta + a_2^*\theta)^2}{(a_2^*)^2} - 1\right), \tag{C53}$$

when $w(\eta, \theta, b|w*) \geq k$ and otherwise $w(\eta, \theta, b|w*) = k$. Therefore, for two different b, say b^0 and b^1, given some realized (η, θ), we have

$$\begin{aligned}\frac{1}{u'(w(\eta,\theta,b^1|w^*))} - \frac{1}{u'(w(\eta,\theta,b^0|w^*))} =& (\mu_1^*\phi_1^* + \mu_2^*\phi_2^*)\frac{(b^1-b^0)\eta}{(a_2^*)^2} \\ &+ \mu_2^*\frac{1}{a_2^*}\left(\frac{\left((b^1-\hat{b})^2 - (b^0-\hat{b})^2\right)\eta^2 + 2a_2^*(b^1-b^0)\eta\theta]}{(a_2^*)^2}\right).\end{aligned} \tag{C54}$$

Assume that $b^1 = +\infty$ or $-\infty$, and $-\infty < b^0 < +\infty$. Since $\mu_2^* > 0$, we have

$$\frac{1}{u'(w(\eta,\theta,b^1|w^*))} - \frac{1}{u'(w(\eta,\theta,b^0|w^*))} > 0, \ \ \forall(\eta,\theta). \tag{C55}$$

Therefore, we have

$$w(\eta,\theta,b^1|w^*) > w(\eta,\theta,b^0|w^*), \ \ \forall(\eta,\theta). \tag{C56}$$

which implies that the agent takes a_3 satisfying $b = +\infty$ or $-\infty$ with $w^*(z(\hat{b}))$ with $\mu_2^* > 0$.

■