Beliefs and the Net Worth Trap

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Resilience and the net worth trap: Brunnermeier (2024)

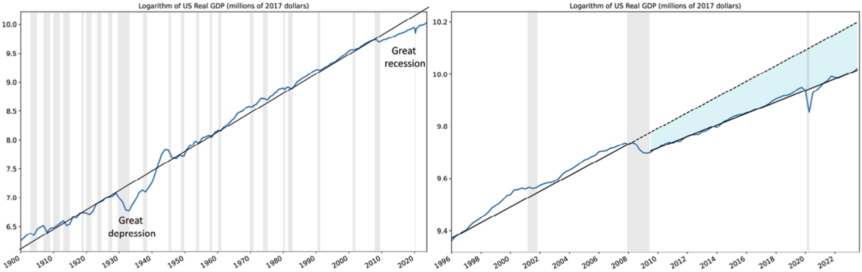


Figure 1. Panel A depicts the log level of U.S. GDP from 1900 to 2023, while Panel B zooms in level from 1996 onwards. Shaded areas show recession periods. (Color figure can be viewed at wileyonlinelibrary.com)

Motivation

- Budding literature on the interactions between financial frictions and investors' beliefs (Krishnamurthy and Li, 2020; Maxted, 2023; Camous and Van der Ghote, 2023)
- Mostly the focus has been on diagnostic expectations or incomplete information on tail risk to explain pre-crisis frothy periods

What do we do?

- Analyze the role of intermediary's (or expert's) **distorted beliefs** about the long-term growth prospects on the creation of net worth trap, i.e., perennial crisis
- Build a tractable heterogeneous agent general equilibrium model with financial frictions in which experts hold dogmatic distorted beliefs over long-run output growth
- **Net worth trap**: optimists never *recapitalize* due to their expectation error, generating extremely slow-moving capital crisis

Usually, fast recapitalization in the model due to high risk premium during crises:
– hard to generate slow-moving capital

The Model

Setting: experts

Single capital: owned by experts and (rational) households

Experts: produces $y_t^O = \gamma_t^O k_t^O, \quad \forall t \in [0, \infty)$ where

$$\frac{dk_t^O}{k_t^O} = \left(\Lambda^O(\underbrace{\iota_t^O}_{\substack{\text{Investment ratio} \\ \text{Their investment}= \iota_t^O y_t^O}}) - \delta^O\right) dt, \quad \forall t \in [0, \infty)$$

with technological growth:

$$\frac{d\gamma_t^O}{\gamma_t^O} = \underbrace{\alpha}_{\text{True (expected) growth}} dt + \sigma \underbrace{dZ_t}_{\text{Brownian motion}}, \quad \forall t \in [0, \infty)$$

Setting: rational households

Households: produces $y_t^H = \gamma_t^H k_t^H, \quad \forall t \in [0,\infty)$ where

$$\frac{dk_t^H}{k_t^H} = \left(\Lambda^H(\ \iota_t^H\ ) - \delta^H\right) dt, \quad \forall t \in [0,\infty)$$

Investment ratio
Their investment$= \iota_t^H y_t^H$

with the same technological growth:

$$\frac{d\gamma_t^H}{\gamma_t^H} = \alpha\ dt + \sigma \underbrace{dZ_t}_{\text{Brownian motion}}, \quad \forall t \in [0,\infty)$$

True (expected) growth

$\longrightarrow$ **Level difference**: $\gamma_t^H = l \cdot \gamma_t^O$, $\Lambda^H(\cdot) = l \cdot \Lambda^O(\cdot)$, with $l \leq 1$

- Efficiency in both production and capital formation$\downarrow$

Capital return

Endogenous volatility

Capital price process: (endogenous) p_t follows

$$\frac{dp_t}{p_t} = \mu_t^p dt + \sigma_t^p \, dZ_t$$

Capital return process:

- Experts' total return on capital:

$$dr_t^{Ok} = \underbrace{\frac{\gamma_t^O k_t^O - \iota_t^O \gamma_t^O k_t^O}{p_t k_t^O}}_{\text{Dividend yield}} dt + \underbrace{\left(\Lambda^O(\iota_t^O) - \delta^O + \mu_t^p\right) dt + \sigma_t^p dZ_t}_{\text{Capital gain}}$$

$$= \frac{1 - \iota_t^O}{q_t} dt + \left(\Lambda^O(\iota_t^O) - \delta^O + \mu_t^p\right) dt + \sigma_t^p dZ_t$$

Price-earnings ratio
(experts)

Capital return for households

Beliefs of experts

Experts dogmatically believe γ_t^O follows

$$\frac{d\gamma_t^O}{\gamma_t^O} = \alpha^O dt + \sigma \underbrace{dZ_t^O}_{\substack{\text{Experts'} \\ \text{Brownian Motion}}}, \quad \forall t \in [0, \infty)$$

where $\alpha^O > \alpha$ corresponds to optimism and $\alpha^O < \alpha$ corresponds to pessimism

while the true process is given as

$$\frac{d\gamma_t^O}{\gamma_t^O} = \alpha dt + \sigma \underbrace{dZ_t}_{\substack{\text{True} \\ \text{Brownian Motion}}}$$

With the following consistency in equilibrium:

$$Z_t^O = Z_t - \frac{\alpha^O - \alpha}{\sigma} t$$

Biased

Perceived capital return

Optimization

Financial market: capital and risk-free (zero net-supplied)

Experts: consumption-portfolio problem (price-taker)

$$\max_{\iota_t^O, x_t^O \geq 0, c_t^O \geq 0} \mathbb{E}_0^O \left[\int_0^\infty e^{-\rho t} \log\left(c_t^O\right) dt \right]$$

Believes dZ_t^O is
the true Brownian motion

subject to

$$dw_t^O = x_t^O w_t^O dr_t^{Ok} + (1 - x_t^O) r_t w_t^O dt - c_t^O dt, \quad \text{and} \quad \underbrace{w_t^O \geq 0}_{\text{Solvency constraint}}$$

Solution

Rational households: solve the similar problem with $\mathbb{E}_0$ ($\neq \mathbb{E}_0^O$)

- Correctly understanding that dZ_t is the Brownian motion

Market clearing

Total capital $K_t = k_t^O + k_t^H$ evolves with

$$\frac{dK_t}{dt} = \underbrace{\left(\Lambda^O\left(\iota_t^O\right) - \delta^O\right) k_t^O}_{\text{From experts}} + \underbrace{\left(\Lambda^H\left(\iota_t^H\right) - \delta^H\right) k_t^H}_{\text{From households}}, \quad \forall t \in [0, \infty)$$

Debt: zero net-supplied

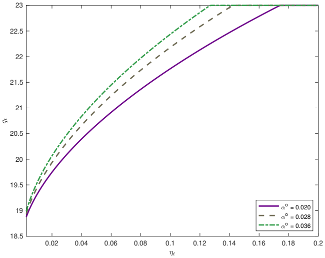
$$\underbrace{\left(w_t^O - p_t k_t^O\right)}_{\substack{\text{Experts'}\\ \text{lending}}} + \underbrace{\left(w_t^H - p_t k_t^H\right)}_{\substack{\text{Households'}\\ \text{lending}}} = 0$$

Good market equilibrium:

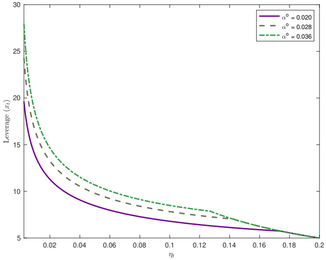
$$\underbrace{\frac{x_t^O w_t^O}{p_t}\left(\gamma_t^O - \iota_t^O \gamma_t^O\right)}_{\substack{\text{Experts'}\\ \text{production}\\ \text{net of investment}}} + \underbrace{\frac{x_t^H w_t^H}{p_t}\left(\gamma_t^H - \iota_t^H \gamma_t^H\right)}_{\substack{\text{Households'}\\ \text{production}\\ \text{net of investment}}} = c_t^O + c_t^H$$

Markov equilibrium: experts' wealth share η_t as state variable

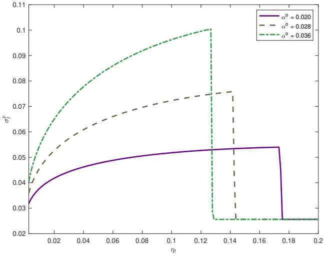
Details Calibration



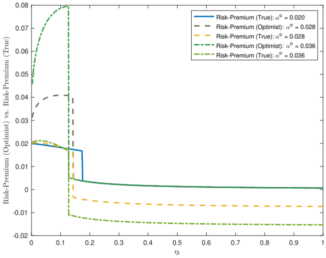
(a) Capital price q_t



(b) Leverage multiple x_t



(c) Endogenous volatility σ_t^p



(d) *Perceived-true* risk-premium

Ergodic distribution of the state variable η_t (optimism)

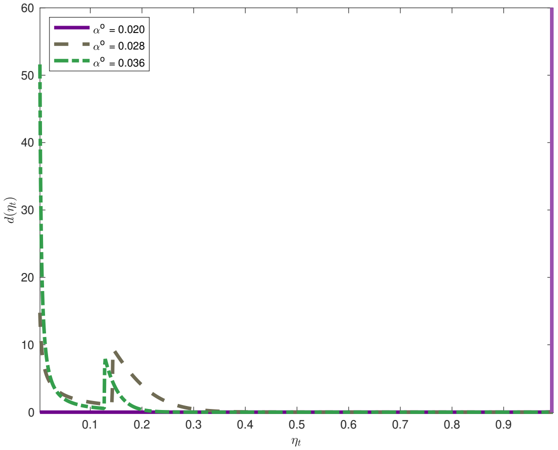


Figure: Stationary distribution of η_t and the net worth trap

Behavior of $\eta_t \sim 0$ Drift and volatility of η_t process

Net worth trap: perennial crisis

Two countervailing forces

- Once crisis hits, higher optimism of experts $\longrightarrow$ higher risk premium helping them to recapitalize faster
- Expectation error of experts preventing them from recapitalizing (stronger)

Proposition (Net Worth Trap)

There exists a threshold level of belief beyond which the economy is trapped at $\eta = 0$, and the probability of recapitalization for experts converges to zero. For the **optimistic** *case, i.e., $\alpha^O > \alpha$, the threshold is determined by*

$$\alpha^O - \alpha > \sigma\sqrt{\Gamma_0^2\sigma^2 + 2\Delta_0}, \tag{1}$$

and for the **pessimistic** *case, i.e., $\alpha^O < \alpha$, the threshold is given by*

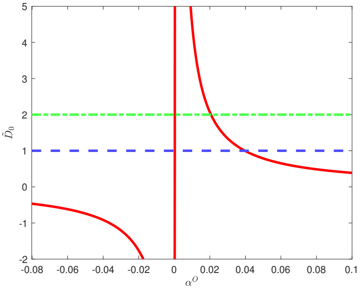
$$\alpha^O - \alpha < -\min\left\{\sigma\sqrt{\Gamma_0^2\sigma^2 + 2\Delta_0}, \max\left\{\sigma^2\left(1+\Gamma_0\right), \Delta_0 + \frac{1}{2}\sigma^2\right\}\right\}. \tag{2}$$

Without short-sale constraint and complete markets

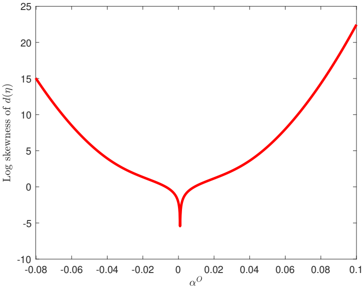
Net worth trap: perennial crisis

Around $\eta \sim 0$:

$$d(\eta) \sim \left(\underbrace{\frac{2\mu^{\eta}(0)}{(\sigma^{\eta})^2(0)}}_{\equiv \tilde{D}_0} - 1 \right) \eta^{\frac{2\mu^{\eta}(0)}{(\sigma^{\eta})^2(0)} - 2} \tag{3}$$



(a) Tail analysis of stationary distribution



(b) Skewness of the distribution around $\eta \sim 0$

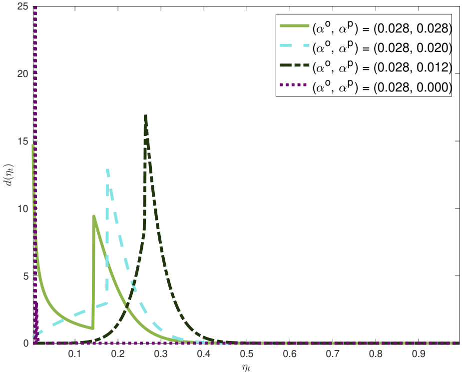
Welfare effects

From dogmatic to swinging beliefs

Now, the log-run growth rate perceived by experts

$$O_t = 1_{\psi_t < 1} \cdot \alpha^P + 1_{\psi_t = 1} \cdot \alpha^O$$

- Experts are optimistic at the stochastic steady state, but become pessimistic in crisis (similar to diagnostic expectations)



Initially stabilizing (e.g., Maxted (2023)), but stronger pessimism in a crisis becomes destabilizing (e.g., Camous and Van der Ghote (2023))

Thank you very much!
(Appendix)

Capital return

Capital return process:

- Households' total return on capital:

$$dr_t^{Hk} = \underbrace{\frac{\gamma_t^H k_t^H - \iota_t^H \gamma_t^H k_t^H}{p_t k_t^H}}_{\text{Dividend yield}} dt + \underbrace{\left(\Lambda^H(\iota_t^H) - \delta^H + \mu_t^p\right) dt + \sigma_t^p dZ_t}_{\text{Capital gain}}$$

$$= \underset{\substack{\uparrow \\ \text{Inefficiency } (l<1)}}{l} \times \frac{1 - \iota_t^H}{\underset{\substack{\uparrow \\ \text{Price-earnings ratio (experts)}}}{q_t}} dt + \left(\Lambda^H(\iota_t^H) - \delta^H + \mu_t^p\right) dt + \sigma_t^p dZ_t$$

Go back

Perceived capital return

Experts' total return on capital:

$$dr_t^{Ok} = \underbrace{\frac{\gamma_t^O k_t^O - \iota_t^O \gamma_t^O k_t^O}{p_t k_t^O}}_{\text{Dividend yield}} dt + \underbrace{\left(\Lambda^O(\iota_t^O) - \delta^O + \mu_t^p\right) dt + \sigma_t^p dZ_t}_{\text{Capital gain}}$$

$$= \frac{\gamma_t^O - \iota_t^O \gamma_t^O}{p_t} dt + \left(\Lambda^O(\iota_t^O) - \delta^O + \mu_t^p + \frac{\alpha^O - \alpha}{\sigma}\sigma_t^p\right) dt + \sigma_t^p \, dZ_t^O$$

Perceived Brownian motion

Belief premium

Go back

Portfolio decisions under belief distortions

Experts' optimal portfolio decision (e.g., Merton (1971))

$$x_t^O = \frac{\left(\dfrac{\gamma_t^O - \iota_t^O \gamma_t^O}{p_t} + \Lambda^O(\iota_t^O) - \delta^O + \mu_t^p + \dfrac{\alpha^0 - \alpha}{\sigma}\sigma_t^p\right) - r_t^*}{\left(\sigma_t^p\right)^2}$$

Additional term

If $\alpha^O > \alpha$ (optimism)

- Given the risk-free r_t^* and the endogenous volatility σ_t^p, optimism raises the leverage↑ and capital demand↑

σ_t^p affects leverage x_t^O in two different ways:

- σ_t^p↑ lowers x_t^O as the required risk-premium level↑
- σ_t^p↑ raises x_t^O as it raises the degree of belief premium on capital returns

Go back

Markov equilibrium

Wealth share of experts as state variable, as in Brunnermeier and Sannikov (2014):

$$\eta_t \equiv \frac{W_t^O}{W_t^O + W_t^H} = \frac{W_t^O}{p_t K_t}$$

which leads to:

$$x_t^O \leq \frac{1}{\eta_t}$$

- When it binds: "normal" (i.e., all capital is owned by experts)
- When it does not bind: "crisis" (i.e., less productive households hold some capital)

Under Markov equilibrium: normalized variables depend only on η_t

$$q_t = q(\eta_t),\ x_t^O = x(\eta_t), \quad \underbrace{\psi_t}_{\substack{\text{Capital share}\\\text{(experts)}}} = \psi(\eta_t)$$

Go back

Specification and calibration

Investment function

$$\Lambda^O(\iota_t^O) = \frac{1}{k}\left(\sqrt{1 + 2k\iota_t^O} - 1\right), \quad \forall t \in [0, \infty)$$

with

$$\Lambda^P(\iota_t) = l \cdot \Lambda^O(\iota_t), \quad \forall \iota_t \tag{4}$$

	Parameter Description	Value	Source (target)
ρ	Discount rate	0.03	Standard: e.g., Brunnermeier and Sannikov (2014).
α	Productivity growth	0.02	2% growth in the long run.
σ	Exogenous TFP volatility	0.0256	Schimitt-Grohé and Uribe (2007)
δ	Depreciation rate (δ^H, δ^O)	0	2% capital growth in the long run (2.5% in the stochastic steady state)
k	Investment function	851.6	Consumption-to-output ratio at 69%
l	Productivity gap	0.7	Most severe recessions: the average output drop from the trend in the Great Depression was $\sim$ 30% according to Romer (1993)

Endogenous volatility: two channels

Capital price volatility σ_t^p is given by

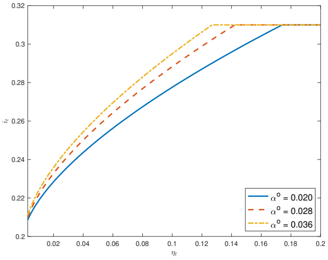
$$\sigma_t^p \left(1 - \left(x_t^O - 1 \right) \frac{\frac{dq(\eta_t)}{q(\eta_t)}}{\frac{d\eta_t}{\eta_t}} \right) \equiv \sigma_t^p \left(1 - \left(x_t^O - 1 \right) \varepsilon_{q,\eta} \right) = \underbrace{\sigma}_{\substack{\text{Exogenous} \\ \text{volatility}}}$$

- $\varepsilon_{q,\eta}$ is the elasticity of the price-earnings ratio (i.e., normalized capital price) with respect to the experts' wealth share η_t

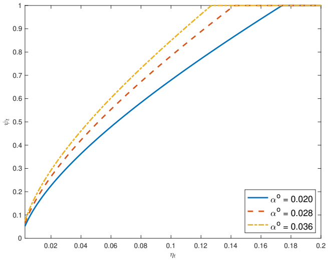
With optimism, volatility σ_t^p is amplified in a crisis through:

- "Elasticity" effect: optimism $\alpha^O \uparrow \longrightarrow \varepsilon_{q,\eta} \uparrow \longrightarrow \sigma_t^p \uparrow$
- "Leverage" effect: $\alpha^O \uparrow \longrightarrow x_t^O \uparrow \longrightarrow \sigma_t^p \uparrow$

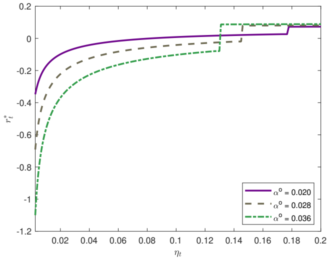
Go back



(a) Investment rate ι_t



(b) Capital share ψ_t



(c) Equilibrium interest rate r_t^*

Go back

Behavior of wealth share $\eta_t \sim 0$

Lemma

In the limit $\eta \to 0^+$, the drift $\mu^\eta(0^+)$ and diffusion $\sigma^\eta(0^+)$ of the wealth share of experts is given by

$$\mu^\eta(0^+) \equiv \lim_{\eta \to 0} \mu^\eta(\eta) = \Gamma_0(\alpha^O - \alpha) + \Gamma_0^2\sigma^2 + \Delta_0$$

$$\sigma^\eta(0^+) \equiv \lim_{\eta \to 0} \sigma^\eta(\eta) = \frac{\alpha^O - \alpha}{\sigma} + \Gamma_0\sigma.$$

where

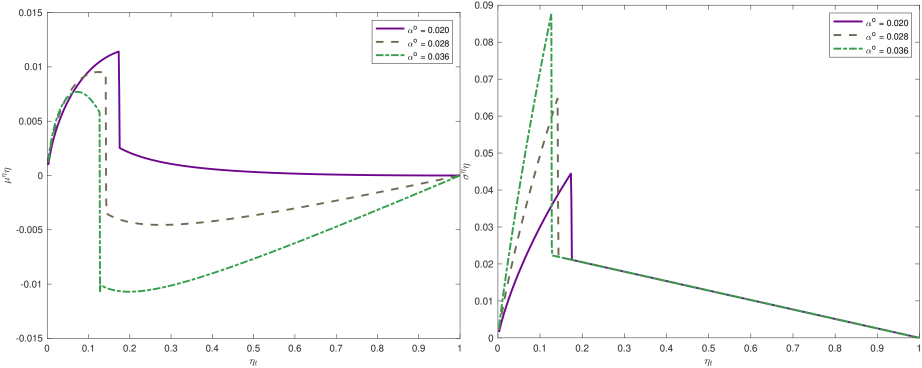
$$\Gamma_0 = \frac{1}{\sigma^2}\left[(1-l)\frac{1-\iota_0}{q_0} + (\delta^H - \delta^O) + (1-l)\Lambda^O(\iota_0)\right]$$

$$\Delta_0 = \frac{1-\iota_0}{q_0} + (\delta^H - \delta^O) + (1-l)\Lambda^O(\iota_0) - \rho$$

and the quantities $\iota_0 = \lim_{\eta \to 0} \iota(\eta)$ and $q_0 = \lim_{\eta \to 0} q(\eta)$ are given in Appendix B.2.

Go back

Drift and volatility of the wealth share



(a) $\mu_\eta(\eta_t) \cdot \eta_t$ as a function of η_t (b) $\sigma^\eta(\eta_t) \cdot \eta_t$ as a function of η_t

Figure: Wealth share dynamics: drift and volatility

- With higher $\alpha^O\uparrow$, the wealth share drift $\mu_\eta(\eta_t)\eta_t\downarrow$ in stochastic steady states: more likely to enter crises

Go back

Other cases

Corollary (Without short-sale constraint)

The threshold level of belief that determines the net worth trap in an economy without a short-selling constraint is given by

$$\left|\alpha^O - \alpha\right| > \sigma\sqrt{\Gamma_0^2\sigma^2 + 2\Delta_0}, \tag{5}$$

Proposition (Complete markets)

Under complete markets with $I = 1$ *and* $\delta^H = \delta^O$, *if* $\alpha^O \neq \alpha$, *experts lose the entire wealth in the long run and the economy features a net worth trap.*

- Similar to "market selection hypothesis" à la Blume and Easley (2006) and Borovička (2020)

Go back

Does optimism hurt the household's welfare?

$$\text{Welfare Loss} = \mathbb{E}_0\left[\int_0^\infty e^{-\rho t}\log c_t^H dt\right] - \mathbb{E}_0\left[\int_0^\infty e^{-\rho t}\log c_t^{H,REE} dt\right]$$

- $c_t^{H,REE}$: household's consumption in the rational expectations benchmark

Decomposition:

$$\mathbb{E}_0\left[\int_0^\infty e^{-\rho t}\log c_t^H dt\right] = \underbrace{\mathbb{E}_0\left[\int_0^\infty e^{-\rho t}\log(1-\eta_t) dt\right]}_{\text{Wealth effect}_+} + \underbrace{\mathbb{E}_0\left[\int_0^\infty e^{-\rho t}\log(1-\iota_t) dt\right]}_{\text{Investment effect}_+}$$

$$+ \underbrace{\mathbb{E}_0\left[\int_0^\infty e^{-\rho t}\log K_t dt\right]}_{\text{Capital effect}_-} + \underbrace{\mathbb{E}_0\left[\int_0^\infty e^{-\rho t}\log A(\psi) dt\right]}_{\text{Misallocation effect}_-}$$

$$+ \underbrace{\text{t.i.e.}}_{\text{Terms independent of equilibria}}$$

- $A(\psi) = \psi_t + l(1-\psi_t)$: productivity-adjusted aggregate capital share

Go back

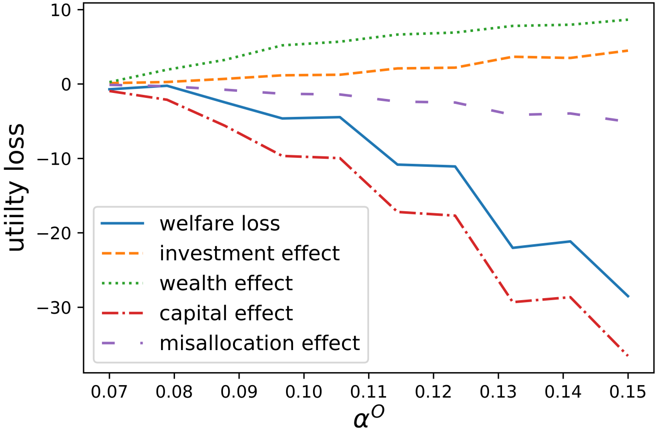


Figure: Decomposition of the rational household's welfare loss

- Overall, optimism reduces welfare of households

Go back