Heterogeneous Beliefs, Risk Amplification, and Asset Returns

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Motivation

- Budding literature on the interactions between financial frictions and investors' beliefs (Maxted, 2022; Krishnamurthy and Li, 2020; Camous and Van der Ghote, 2023; Khorrami and Mendo, 2023)
- Mostly the focus has been on diagnostic expectations or incomplete information on tail risk to explain pre-crisis frothy periods
- Empirical evidence from Bordalo et al (2023): "Overreaction of long term profit expectations emerges as a promising mechanism for reconciling Shiller's excess volatility puzzle with the business cycle"

What do we do?

- Analyze the role of investor **disagreement** in the long-term growth prospects on (i) the risk amplification and (ii) build-up to a financial crisis
- Build a tractable heterogeneous agent model with financial frictions where optimists and pessimists hold dogmatic beliefs over long-run output growth
- Tie the model predictions to the empirical predictions by building a disagreement measure from the Survey of Professional Forecasters (SPF)
- Study the cross-sectional asset pricing implications of the disagreement factor

The key model predictions are:

1. Disagreement exacerbates risk amplification and financial instability (contrary to Maxted, 2022)
2. At the stochastic steady state: **frothy periods** due to higher perceived risk premium of optimists
3. **During crisis**: higher aggregate volatility and larger risk premium compared to a benchmark rational expectations case
4. Explains the excess conditional momentum of stock returns

Cross-sectional implications:

1. A factor model with disagreement factor improves the pricing power of asset returns in the cross section, beyond conventional factors in the intermediary asset pricing literature (e.g., He, Kelly, and Manela, 2017)
2. Disagreement factor is crucial in explaining momentum and long-term reversal anomaly portfolio excess returns

Literature review

The Model

Model summary

Big Question (Main Topic)

What if investors have heterogeneous beliefs about the technological growth?

Our theory based on Brunnermeier and Sannikov (2014): when (more productive) experts are optimistic and households are pessimistic about technological growth

1. Normal $\longrightarrow$ more facilitated trade with investment$\uparrow$, asset price$\uparrow$, and leverage$\uparrow$ than the rational expectations case (i.e., **frothy periods**)
   - Experts' risk bearing during normal$\uparrow$ $\longrightarrow$ chance of entering financial crises$\uparrow$
2. Crisis $\longrightarrow$ more amplified (endogenous) volatility$\uparrow$ and (true and perceived) risk-premium$\uparrow$
   - On average, risk-premium$\uparrow$ leads faster recapitalization of experts' net worth: average duration of a crisis$\downarrow$
   - $\exists$Persistently high risk-premium unless the net worth gets recapitalized enough
3. Still, occupation time in crisis$\uparrow$

   Number of '(on average) shorter-lived and more severe' crises$\uparrow\uparrow$

   $\longrightarrow$ On average more time in crises per year

Single capital: owned by optimists and pessimists

Optimists: produces $\underline{y_t^O = \gamma_t^O k_t^O}$, $\forall t \in [0, \infty)$ where

$$\frac{dk_t^O}{k_t^O} = \left(\Lambda^O(\boxed{\iota_t^O}) - \delta^O\right) dt, \quad \forall t \in [0, \infty)$$

Investment ratio
Their investment$= \iota_t^O y_t^O$

with technological growth:

$$\frac{d\gamma_t^O}{\gamma_t^O} = \boxed{\alpha} dt + \sigma \underbrace{dZ_t}_{\text{Brownian motion}}, \quad \forall t \in [0, \infty)$$

True (expected) growth

Pessimists: produces $\underline{y_t^P = \gamma_t^P k_t^P}$, $\forall t \in [0, \infty)$ where

$$\frac{dk_t^P}{k_t^P} = \left(\Lambda^P(\underbrace{\boxed{\iota_t^P}}_{\text{Investment ratio}}) - \delta^P \right) dt, \quad \forall t \in [0, \infty)$$

Investment ratio
Their investment$= \iota_t^P y_t^P$

with the same technological growth:

$$\frac{d\gamma_t^P}{\gamma_t^P} = \boxed{\alpha} dt + \sigma \underbrace{dZ_t}_{\text{Brownian motion}}, \quad \forall t \in [0, \infty)$$

True (expected) growth

$\longrightarrow$ **Level difference**: $\gamma_t^P = l \cdot \gamma_t^O$, $\Lambda^P(\cdot) = l \cdot \Lambda^O(\cdot)$, with $l \leq 1$

- Efficiency in both production and capital formation$\downarrow$

Capital price process: (endogenous) p_t follows

$$\frac{dp_t}{p_t} = \mu_t^p dt + \boxed{\sigma_t^p} dZ_t$$

Endogenous volatility

Capital return process:

- Optimists' total return on capital:

$$dr_t^{Ok} = \underbrace{\frac{\gamma_t^O \cancel{k_t^O} - \iota_t^O \gamma_t^O \cancel{k_t^O}}{p_t \cancel{k_t^O}}}_{\text{Dividend yield}} dt + \underbrace{\left(\Lambda^O(\iota_t^O) - \delta^O + \mu_t^p\right) dt + \sigma_t^p dZ_t}_{\text{Capital gain}}$$

$$= \frac{1 - \iota_t^O}{\underbrace{q_t}_{\substack{\text{Price-earnings ratio} \\ \text{(optimists)}}}} dt + \left(\Lambda^O(\iota_t^O) - \delta^O + \mu_t^p\right) dt + \sigma_t^p dZ_t$$

- Pessimists' total return on capital:

$$dr_t^{Pk} = \frac{\gamma_t^P \cancel{k_t^P} - \iota_t^P \gamma_t^P \cancel{k_t^P}}{p_t \cancel{k_t^P}} dt + \left(\Lambda^P(\iota_t^P) - \delta^P + \mu_t^p\right) dt + \sigma_t^p dZ_t$$

Optimists: believe γ_t^O follows

$$\frac{d\gamma_t^O}{\gamma_t^O} = \boxed{\alpha^O} dt + \sigma \underbrace{dZ_t^O}_{\text{Optimists' Brownian Motion}}, \quad \forall t \in [0, \infty)$$

Possibly different from α

even if the true process is given as

$$\frac{d\gamma_t^O}{\gamma_t^O} = \alpha dt + \sigma \underbrace{dZ_t}_{\text{True Brownian Motion}}$$

with the following consistency (see e.g., Yan (2008)):

$$\underbrace{Z_t^O}_{\text{Optimists' BM}} = \underbrace{Z_t}_{\text{True BM}} - \frac{\alpha^O - \alpha}{\sigma} t$$

Note that optimists:

- can infer a true value of σ by calculating the process' quadratic variation
- are *dogmatic*: believing the expected technological growth $\alpha^O \neq \alpha$

Pessimists: believe γ_t^P follows

$$\frac{d\gamma_t^P}{\gamma_t^P} = \boxed{\alpha^P} dt + \sigma \underbrace{dZ_t^P}_{\text{Pessimists' Brownian Motion}}, \quad \forall t \in [0, \infty)$$

Possibly different from α

even if the true process is given as

$$\frac{d\gamma_t^P}{\gamma_t^P} = \alpha dt + \sigma \underbrace{dZ_t}_{\text{True Brownian Motion}}$$

with the following consistency (see e.g., Yan (2008)):

$$\underbrace{Z_t^P}_{\text{Optimists' BM}} = \underbrace{Z_t}_{\text{True BM}} - \frac{\alpha^P - \alpha}{\sigma} t$$

Classifications:

- With $\underline{\alpha^O > \alpha > \alpha^P}$: experts (households) are optimists (pessimists)
- With $\underline{\alpha^O < \alpha < \alpha^P}$: experts (households) are pessimists (optimists)

Perceived return

Financial market: capital and risk-free (zero net-supplied)

Optimists: consumption-portfolio problem (price-taker) Solution

$$\max_{\iota_t^O \geq 0, x_t \geq 0, c_{t,}^O \geq 0} \boxed{\mathbb{E}_0^O} \left[\int_0^\infty e^{-\rho^O t} \log\left(c_t^O\right) dt \right]$$

Believes dZ_t^O is the true BM

subject to

$$dw_t^O = x_t w_t^O dr_t^{Ok} + (1 - x_t) r_t w_t^O dt - c_t^O dt, \quad \text{and} \quad \underbrace{w_t^O \geq 0}_{\text{Solvency constraint}}$$

Pessimists: solve the similar problem with $\boxed{\mathbb{E}_0^P}$ $(\neq \mathbb{E}_0$ or $\mathbb{E}_0^O)$

Believes dZ_t^P is the true BM

Market clearing

Total capital $K_t = k_t^O + \underline{k}_t^P$ evolves with

$$\frac{dK_t}{dt} = \underbrace{\left(\Lambda^O\left(\iota_t^O\right) - \delta^O\right)k_t^O}_{\text{From optimists}} + \underbrace{\left(\Lambda^P\left(\underline{\iota}_t^P\right) - \delta^P\right)\underline{k}_t^P}_{\text{From pessimists}}, \quad \forall t \in [0, \infty)$$

Debt: zero net-supplied

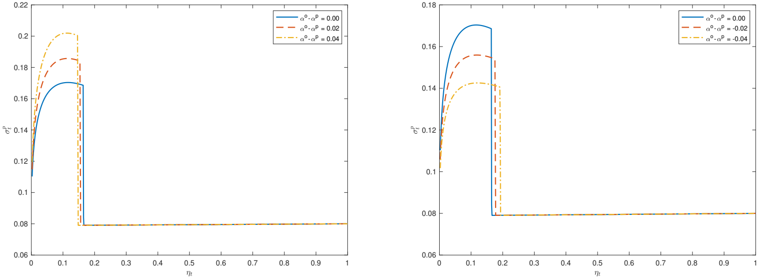
$$\underbrace{\left(w_t^O - p_t k_t^O\right)}_{\substack{\text{Optimists'} \\ \text{lending}}} + \underbrace{\left(\underline{w}_t^P - p_t \underline{k}_t^P\right)}_{\substack{\text{Pessimists'} \\ \text{lending}}} = 0$$

Good market equilibrium:

$$\underbrace{\frac{x_t^O w_t^O}{p_t}\left(\gamma_t^O - \iota_t^O \gamma_t^O\right)}_{\substack{\text{Optimists'} \\ \text{production} \\ \text{net of investment}}} + \underbrace{\frac{\underline{x}_t^P \underline{w}_t^P}{p_t}\left(\gamma_t^P - \underline{\iota}_t^P \gamma_t^P\right)}_{\substack{\text{Pessimists'} \\ \text{production} \\ \text{net of investment}}} = c_t^O + \underline{c}_t^P$$

Markov equilibrium: optimists' wealth share η_t as state variable

Details Calibration



(a) $\alpha^O \geq \alpha$ and $\alpha^P \leq \alpha$ (b) $\alpha^O \leq \alpha$ and $\alpha^P \geq \alpha$

Figure: Endogenous Volatility σ_t^p as a function of η_t

Efficient and crisis regions: threshold η^ψ

- With $\underline{\alpha^O > \alpha > \alpha^P}$, $\eta^\psi\downarrow$ as $\alpha^O\uparrow$: even with low wealth, optimists' demand for capital is strong (so leverage$\uparrow$)
- And then crisis (i.e., $\eta \leq \eta^\psi$) $\longrightarrow$ more risk amplification: $\sigma_t^p\uparrow$

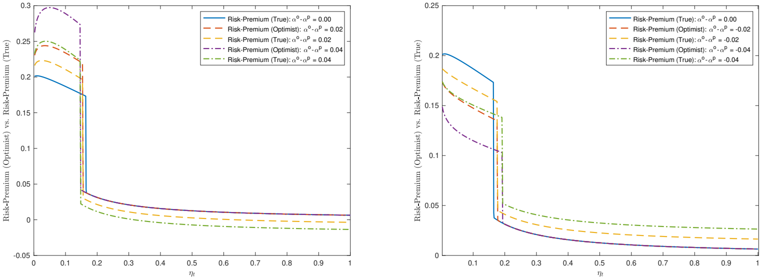
Endogenous volatility: two channels

Capital price volatility σ_t^p is given by

$$\sigma_t^p \left(1 - (x_t - 1) \frac{\frac{dq(\eta_t)}{q(\eta_t)}}{\frac{d\eta_t}{\eta_t}} \right) \equiv \sigma_t^p \left(1 - (x_t - 1)\, \varepsilon_{q,\eta}\right) = \underbrace{\sigma}_{\text{Exogenous volatility}}$$

- $\varepsilon_{q,\eta}$ is the elasticity of the price-earnings ratio (i.e., normalized capital price) with respect to the experts' wealth share η_t
- 'Market illiquidity' effect: $\alpha^O\uparrow \longrightarrow \varepsilon_{q,\eta}\uparrow \longrightarrow \sigma_t^p\uparrow$
- 'Leverage' effect: $\alpha^O\uparrow \longrightarrow x_t\uparrow \longrightarrow \sigma_t^p\uparrow$

Risk-premium (true and perceived: optimists)



(a) $\alpha^O \geq \alpha$ and $\alpha^P \leq \alpha$

(b) $\alpha^O \leq \alpha$ and $\alpha^P \geq \alpha$

Figure: Risk-Premium (Optimists' and True Value) as a Function of η_t

When experts are more optimistic (i.e., higher α^O):

- Higher perceived risk premium
- Higher true conditional risk premium: faster recapitalization of experts $\longrightarrow$ **excess time-series momentum** in capital return

Risk-free Other graphs

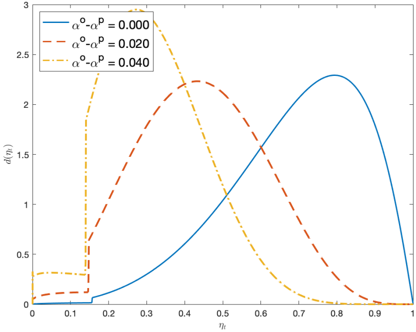


Figure: Ergodic Distribution of η_t

- In the stochastic steady state, higher optimism of experts $\longrightarrow$ more leverage: increasing the probability that a crisis occurs
- Once crisis hits, higher optimism of experts $\longrightarrow$ higher risk premium helping them to recapitalize faster
- Higher occupancy time in crisis on average

Empirical Analysis

Empirical: run the following regression with monthly S&P500 excess return:

$$r^e_{t+h} = \alpha(h) + \beta_1(h) \times r^e_t + \underbrace{\beta_2(h)}_{\text{Excess conditional momentum}} \times r^e_t \times 1_{\text{Recession}} + \epsilon_{t+h}$$

Model-implied: simulate the model for 1,000 times for 5,000 years and run the following regression:

$$r^e_{t+h} = \alpha(h) + \beta_{1,\text{model}}(h) \times r^e_t + \underbrace{\beta_{2,\text{model}}(h)}_{\text{Excess conditional momentum}} \times r^e_t \times 1_{\text{Recession}} + \epsilon_{t+h}$$

Conditional predictability

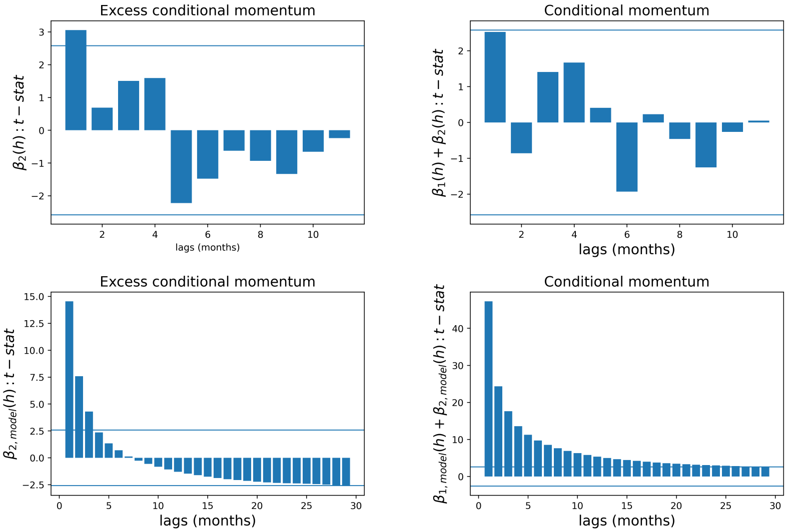


Figure: Time series return predictability: the data is at monthly frequency from 1945 till 2022. The bottom two panels present the model implied autocorrelation coefficients.

- Empirical disagreement is computed as

$$D_t = \frac{f_{75} - f_{25}}{|f_{50}|}$$

where f_k: $k\%$ percentile analyst forecast of quarter-on-quarter GDP growth rate for the $T+2^{\text{th}}$ quarter ahead at date T, from the Survey of Professional Forecasters (SPF) Disagreement measure

- Model-implied disagreement is computed as the component of leverage attributable to disagreement:

$$x_t^{\text{net}} = x_t - x_t^{\text{REE}}$$

where x_t^{REE}: leverage under the rational expectations

- **Disagreement dummy**: $1_d = 1$ if $D_t \geq D_{\text{median}}$ (empirics) or $x_t^{\text{net}} \geq x_{\text{median}}^{\text{net}}$ (theory)

Run the following predictability regression:

$$r^e_{t+h} = \alpha(h) + \beta_1(h) r^e_t + \beta_2(h) \times r^e_t \times 1_{Recession} + \underbrace{\beta_3(h)}_{\text{Excess conditional momentum}} \times r^e_t \times 1_{Recession} \times 1_d + \epsilon_{t+h}$$

Role of disagreement

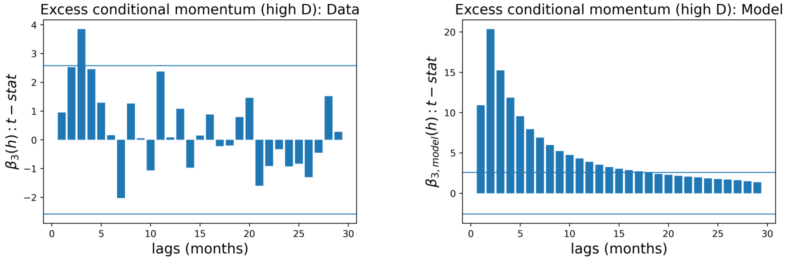


Figure: Role of disagreement: the left panel presents empirical autocorrelation coefficients from regressing the excess return on S&P500 on its lagged excess return. The data is at monthly frequency from 1945 till 2022. The left panel presents the conditional t-stats when the disagreement is high. The right panels presents the model-implied conditional t-stats when the disagreement is high.

- Define a factor $d_t = \Delta \log (1 + D_t)$

Run two-stage Fama-McBeth with $f_t \equiv [\underbrace{M_t}_{\substack{\text{Market} \\ \text{excess return}}}, \underbrace{\eta_t}_{\substack{\text{HKM equity} \\ \text{share}}}, \underbrace{d_t}_{\text{Disagreement}}]'$ with first stage:

$$R^e_{i,t} = a_i + \beta'_{i,t} f_t + v_{i,t}$$

and the second stage

$$\mathbb{E}[R^e_{i,t}] = \alpha_i + \hat{\beta}_{i,f} \lambda_f + \epsilon_i$$

- Test assets (1970Q1 - 2022Q4): 25 size and book-to-market portfolios; 24 size and momentum sorted portfolios; 10 long-term reversal portfolios; 25 profitability and investment portfolios; 10 maturity sorted US treasury bond portfolios from CRSP Fama bond dataset with maturities in 6 month intervals up to 5 years

- Other asset classes (1970Q1 - 2012Q4): 18 option portfolios; 20 CDS portfolios; 12 FX portfolios used in He, Kelly, and Manela (2017)

Risk-exposure (first-stage)

	Equities		Equities and Bonds	
	Two-factor	Three-factor	Two-factor	Three-factor
Mean excess return	2.06	2.06	1.88	1.88
Std. excess return	0.69	0.69	0.84	0.84
Mean β_M	1.0	1.0	0.9	0.9
Std β_M	0.23	0.23	0.37	0.37
Mean β_η	0.09	0.09	0.08	0.08
Std β_η	0.11	0.11	0.11	0.11
Mean β_d	-	0.004	-	0.004
Std β_d	-	0.04	-	0.04
Assets	85	85	95	95
Quarters	211	211	211	211
Controls	Yes	Yes	Yes	Yes

Table 1: Expected returns and risk exposures. Equity assets include 25 size and book-to-market portfolios, 25 size and momeutum portfolios, 10 long-term reversal portfolios, and 25 profitability and investment portfolios. Bond portfolios include 10 maturity sorted portfolios from CRSP Fama bond portfolio dataset. Mean and std. of excess return is the difference in mean return and risk free rate of the corresponding test assets. The frequency is quarterly and time period is from 1970Q1 till 2022Q4. The mean and std. of betas $(\beta_W, \beta_\eta, \beta_d)$ measure the average and standard deviation of exposure of the excess return to market factor, intermediary capital ratio, and disagreement measure respectively.

Risk-price (second-stage)

	Equities		Equities and Bonds	
	Two-factor	Three-factor	Two-factor	Three-factor
Market	-0.01	-0.01	0.01	0.01
t-Stat Shanken	(-0.71)	(-0.5)	(1.17)	(1.05)
Intermediary	-0.01	0.0	0.02	0.02
t-Stat Shanken	(-0.29)	(0.02)	(1.08)	(1.09)
Disagreement	-	0.07	-	0.07
t-Stat Shanken	-	(2.91)	-	(2.81)
MAPE %	2.0	1.79	2.22	2.08
Adj. R2	0.00	0.18	0.23	0.35
Assets	85	85	95	95
Quarters	211	211	211	211

Table 2: Risk price estimates for equities and US government bond portfolios. Equity test assets include 25 size and book-to-market portfolios, 25 size and momentum portfolios, 10 long-term reversal portfolios, and 25 profitability and investment sorted portfolios. The 'Equity and bonds' portfolio include all of the above assets, plus 10 maturity sorted US government bond portfolios taken from the CRSP Fama bond portfolio dataset. The data is at quarterly frequency from 1970Q1 till 2022Q4. The factors are market, intermediary capital ratio, and disagreement. The disagreement factor d_t is computed as growth rate in the inter-quartile dispersion of GDP forecast scaled by median forecast by analysts. The forecast data is taken from The Survey of Professional Forecasters. The standard errors of risk price estimates are adjusted for generated regressor problem using Shanken correction. MAPE denotes the mean absolute pricing error in annualized terms.

Cross-sectional fit:two factors
Cross-sectional fit:three factors

More test assets (first-stage)

	HKM		HKM+Momentum	
	Two-factor	Three-factor	Two-factor	Three-factor
Mean excess return	0.85	0.85	1.18	1.18
Std. excess return	1.31	1.31	1.32	1.32
Mean β_M	0.46	0.46	0.61	0.61
Std. β_M	0.45	0.45	0.47	0.47
Mean β_η	0.03	0.03	0.05	0.04
Std. β_η	0.09	0.09	0.1	0.1
Mean β_d	-	0.002	-	0.002
Std. β_d	-	0.03	-	0.04
Assets	94	94	129	129
Quarters	171	171	171	171
Controls	Yes	Yes	Yes	Yes

Table 3: Expected returns and risk exposures. HKM assets include 25 size and book-to-market portfolios, 10 U.S. Govt. bond portfolios, 10 U.S. corporate bond portfolios, 18 option portfolios, 20 CDS portfolios, and 12 foreign exchange portfolios taken from HKM2017. HKM+Momentum includes 25 size and momentum portfolios and 10 long-term reversal portfolios in addition to the HKM assets. Mean and std. of excess return is the difference in mean return and risk free rate of the corresponding test assets. The frequency is quarterly and time period is from 1970Q1 till 2012Q4. The mean and std. of betas ($\beta_W, \beta_\eta, \beta_d$) measure the average and standard deviation of exposure of the excess return to market factor, intermediary capital ratio, and disagreement measure respectively.

More test assets (second-stage)

	HKM		HKM+Momentum	
	Two-factor	Three-factor	Two-factor	Three-factor
Market	0.02	0.01	0.02	0.01
t-stat Shanken	(1.46)	(0.83)	(1.59)	(0.97)
Intermediary	0.09	0.10	0.06	0.07
t-stat Shanken	(4.19)	(3.09)	(2.86)	(2.14)
Disagreement	-	0.1	-	0.12
t-stat Shanken	-	(1.93)	-	(2.93)
MAPE %	1.66	1.34	2.35	1.97
Adj. R2	0.83	0.89	0.59	0.73
Assets	94	94	129	129
Quarters	171	171	171	171

Table 4: Risk price estimates for HKM and HKM+Momentum portfolios. HKM assets include 25 size and book-to-market portfolios, 10 U.S. Govt. bond portfolios, 10 U.S. corporate bond portfolios, 18 option portfolios, 20 CDS portfolios, and 12 foreign exchange portfolios taken from HKM2017. HKM+Momentum includes 25 size and momentum portfolios and 10 long-term reversal portfolios in addition to the HKM assets. The data is at quarterly frequency from 1970Q1 till 2012Q4. The factors are market, intermediary capital ratio, and disagreement. The disagreement factor d_t is computed as growth rate in the inter-quartile dispersion of GDP forecast scaled by median forecast by analysts. The forecast data is taken from The Survey of Professional Forecasters. The standard errors of risk price estimates are adjusted for generated regressor problem using Shanken correction. MAPE denotes the mean absolute pricing error in annualized terms.

▶▶ Cross-sectional fit:two factors ▶▶ Cross-sectional fit:three factors ▶▶ Two-way portfolio sorts

Thank you very much!
(Appendix)

Basic framework based on Brunnermeier and Sannikov (2014)

- Continuous-time models: Basak and Cuoco (1998), He and Krishnamurthy (2011, 2013), Di Tella (2017)[1]
- Frictions, heterogeneous beliefs, and deviations from the rational expectations case: Harrison and Kreps (1978), Detemple and Murthy (1997), Basak and Croitoru (2000), Basak (2000), Croitoru and Basak (2004), Gallmeyer and Hollifield (2008), Maxted (2023)[2]
- Heterogeneous beliefs about risk-premium, financial markets, and the macroeconomy (e.g., inflation): Welch (2000), Coibion et al. (2020), Candia et al. (2021), Weber et al. (2022), Beutel and Weber (2022)[3]
- Nominal rigidity (demand-determined): Caballero and Simsek (2020)
- Intermediary and capital-share based empirical asset pricing: He, Kelly, and Manela (2017), Lettau, Ludvigson, and Ma (2019)
- Momentum during crises: Cujean and Hesler (2017)

[1] Di Tella (2017) studies uncertainty shocks driving balance sheet recessions even in cases when contracting on the macroeconomic state variable is possible

[2] Maxted (2023) incorporates diagnostic expectations into a model with intermediaries based on He and Krishnamurthy (2013)

[3] Beutel and Weber (2022) point out that individuals are heterogeneous both at the information acquisition and processing stages, thereby forming their own beliefs and choosing portfolios based on them

Perceived capital return process

- Optimists' total return on capital:

$$dr_t^{Ok} = \underbrace{\frac{\gamma_t^O \cancel{k_t^O} - \iota_t^O \gamma_t^O \cancel{k_t^O}}{p_t \cancel{k_t^O}}}_{\text{Dividend yield}} dt + \underbrace{\left(\Lambda^O(\iota_t^O) - \delta^O + \mu_t^p\right) dt + \sigma_t^p dZ_t}_{\text{Capital gain}}$$

$$= \frac{\gamma_t^O - \iota_t^O \gamma_t^O}{p_t} dt + \left(\Lambda^O(\iota_t^O) - \delta^O + \mu_t^p + \boxed{\frac{\alpha^O - \alpha}{\sigma} \sigma_t^p}\right) dt + \sigma_t^p dZ_t^O$$

Belief (perceived) premium

- Pessimists' total return on capital:

$$dr_t^{Pk} = \frac{\gamma_t^P - \iota_t^P \gamma_t^P}{p_t} dt + \left(\Lambda^P(\iota_t^P) - \delta^P + \mu_t^p + \frac{\alpha^P - \alpha}{\sigma} \sigma_t^p\right) dt + \sigma_t^p dZ_t^P$$

Observation (Belief heterogeneity in asset returns)

(Endogenous) volatility↑ $\longrightarrow$ belief heterogeneity in asset return↑

Optimists' optimal portfolio decision (e.g., Merton (1971))

$$x_t = \frac{\left(\frac{\gamma_t^O - \iota_t^O \gamma_t^O}{p_t} + \Lambda^O(\iota_t^O) - \delta^O + \mu_t^p + \boxed{\frac{\alpha^0 - \alpha}{\sigma}\sigma_t^p}\right) - r_t^*}{(\sigma_t^p)^2}$$

New term: from optimism

For $\alpha^O > \alpha$ (experts = optimists)

- Given the risk-free r_t^* and the endogenous volatility σ_t^p, optimism (i.e., $\alpha^O\uparrow$ from α) raises the optimists' leverage↑ and capital demand↑, i.e., booms
- Optimists bear 'too much' risk on their balance sheets $\longrightarrow$ crisis when dZ_t is negative enough (entering crisis more frequently, i.e., frothy periods)

$\sigma_t^p\uparrow \longrightarrow$ has two effects on leverage x_t:

- $\sigma_t^p\uparrow$ lowers x_t as the required risk-premium level↑
- $\sigma_t^p\uparrow$ raises x_t as it raises the degree of optimism on asset returns

Proportion of optimists' wealth as state variable, similarly to Brunnermeier and Sannikov (2014):

$$\eta_t \equiv \frac{w_t^O}{w_t^O + \underline{w}_t^P} \underbrace{=}_{\text{Debt market equilibrium}} \frac{w_t^O}{p_t K_t}$$

which leads to:

$$x_t \leq \frac{1}{\eta_t}$$

- When it binds - 'normal' (all capital is owned by experts)
- When it does not bind - 'crisis' (less productive households must hold capital)

Under Markov equilibrium: normalized variables depend only on η_t

$$\longrightarrow q_t = q(\eta_t),\ x_t = x(\eta_t),\ \underbrace{\psi_t}_{\text{Capital share (optimists)}} = \psi(\eta_t)$$

Investment function

$$\Lambda^O(\iota_t^O) = \frac{1}{k}\left(\sqrt{1+2k\iota_t^O} - 1\right), \quad \forall t \in [0, \infty)$$

with

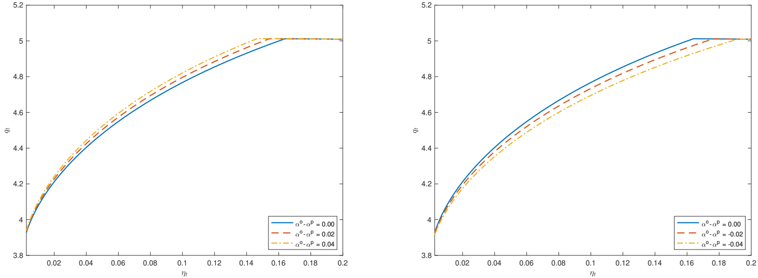
$$\Lambda^P(\iota_t) = l \cdot \Lambda^O(\iota_t), \quad \forall \iota_t \tag{1}$$

Parametrization: target 5% chance of crisis

	l	δ^O	δ^P	ρ^O	ρ^P	χ	σ	k	α
Values	0.4	0	0	0.07	0.065	1	0.08	18	0.05

Table: Parameterization

- $\underline{\alpha^O > \alpha > \alpha^P}$ case (i.e., experts are optimistic):
  $$\alpha^O = \{0.05, 0.06, 0.07\}, \quad \alpha^P = \{0.05, 0.04, 0.03\}, \ \alpha^O + \alpha^P = 0.1$$
- $\underline{\alpha^O < \alpha < \alpha^P}$ case (i.e., experts are pessimistic):
  $$\alpha^O = \{0.05, 0.04, 0.03\}, \quad \alpha^P = \{0.05, 0.06, 0.07\}, \ \alpha^O + \alpha^P = 0.1$$

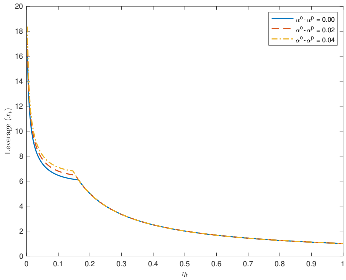


(a) $\alpha^O \geq \alpha$ and $\alpha^P \leq \alpha$ (b) $\alpha^O \leq \alpha$ and $\alpha^P \geq \alpha$

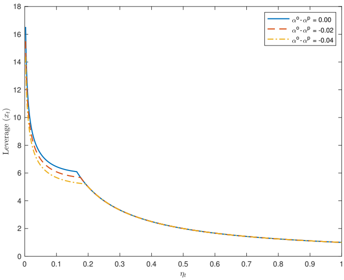
Figure: Price-earnings ratio q_t as a function of η_t

Efficient and crisis regions: threshold η^ψ

- With $\underline{\alpha^O > \alpha > \alpha^P}$, $\eta^\psi \downarrow$ as $\alpha^O \uparrow$: even with low wealth, optimists' demand for capital is strong (so leverage$\uparrow$)
- And then crisis (i.e., $\eta \leq \eta^\psi$) $\longrightarrow$ steeper decline in q_t (i.e., more elastic)



(a) $\alpha^O \geq \alpha$ and $\alpha^P \leq \alpha$



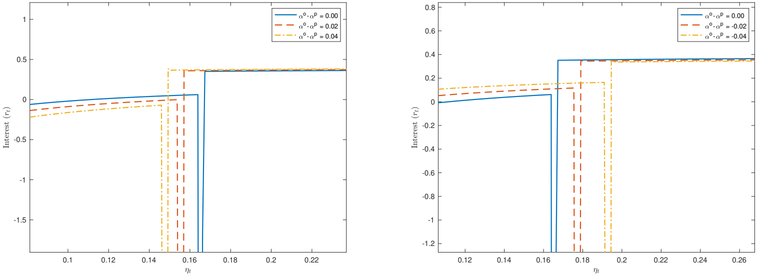
(b) $\alpha^O \leq \alpha$ and $\alpha^P \geq \alpha$

Figure: Leverage x_t as a function of η_t

Efficient and crisis regions: threshold η^ψ

- With $\underline{\alpha^O > \alpha > \alpha^P}$, $\eta^\psi \downarrow$ as $\alpha^O \uparrow$: even with low wealth, optimists' demand for capital is strong (so leverage↑)
- And then crisis (i.e., $\eta \leq \eta^\psi$) $\longrightarrow$ higher leverage (a perceived risk-premium is high)

Go back



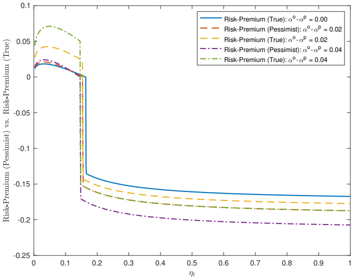
(a) $\alpha^O \geq \alpha$ and $\alpha^P \leq \alpha$

(b) $\alpha^O \geq \alpha$ and $\alpha^P \leq \alpha$

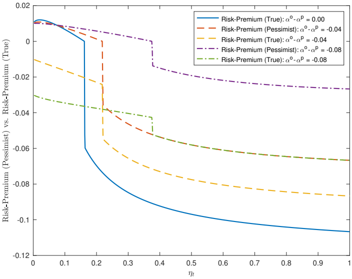
Figure: Interest Rate r_t as a function of η_t: $\alpha^O \leq \alpha$ and $\alpha^P \geq \alpha$

Efficient and crisis regions: threshold η^ψ

- Downward spike in r_t at η^ψ: the moment experts start a fire-sale of capital
- With $\underline{\alpha^O > \alpha > \alpha^P}$, a higher leverage $x_t \longrightarrow r_t\uparrow$ in 'normal'
- During crises (i.e., $\eta_t \leq \eta^\psi$), $\alpha^O\uparrow \longrightarrow r_t\downarrow$: higher demand for safety with precautionary motive



(a) $\alpha^O \geq \alpha$ and $\alpha^P \leq \alpha$



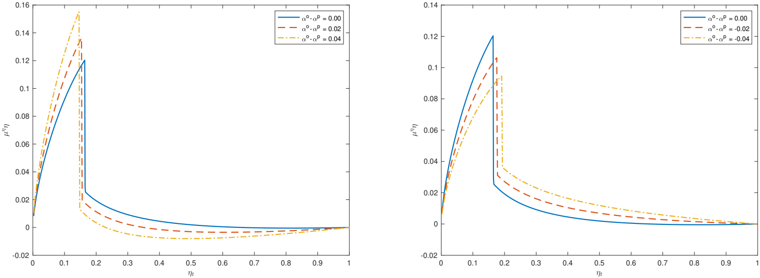
(b) $\alpha^O \leq \alpha$ and $\alpha^P \geq \alpha$

Figure: Risk-Premium (Pessimists' and True Value) as a Function of η_t

- Pessimists perceive to risk-premium to be positive only when $\eta_t \leq \eta^{\psi}$

Drift of the wealth share

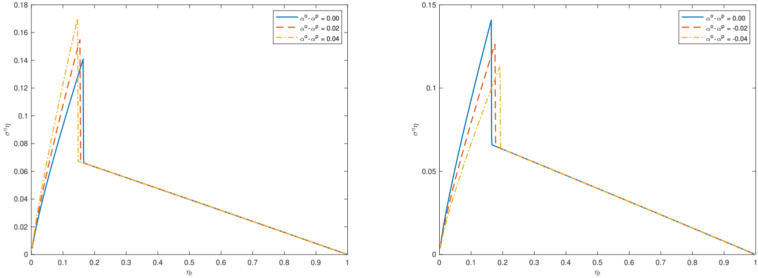
Go back



(a) $\alpha^O \geq \alpha$ and $\alpha^P \leq \alpha$ (b) $\alpha^O \leq \alpha$ and $\alpha^P \geq \alpha$

Figure: Wealth Share Drift $\mu_\eta(\eta_t) \cdot \eta_t$ as a Function of η_t

- With $\underline{\alpha^O > \alpha > \alpha^P}$, $\alpha^O \uparrow \longrightarrow$ Wealth share drift $\mu_\eta(\eta_t) \cdot \eta_t \uparrow$: recapitalized faster on average



(a) $\alpha^O \geq \alpha$ and $\alpha^P \leq \alpha$

(b) $\alpha^O \leq \alpha$ and $\alpha^P \geq \alpha$

Figure: Wealth Share Volatility $\sigma^\eta(\eta_t) \cdot \eta_t$ as a Function of η_t

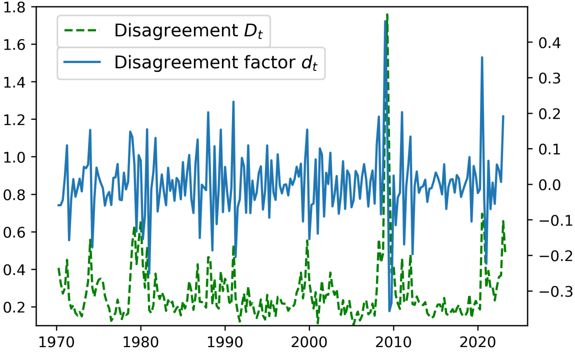


Figure: Disagreement D_t is computed as the interquartile dispersion of the 2nd quarter ahead GDP Quarter-on-Quarter projection scaled by median growth projection. It corresponds to the left axis. The data is taken from The Survey of Professional Forecasters. The disagreement factor d_t, corresponding to the right axis, is computed as the change in $\log(1 + D_t)$. The shaded areas represent NBER recessionary periods.

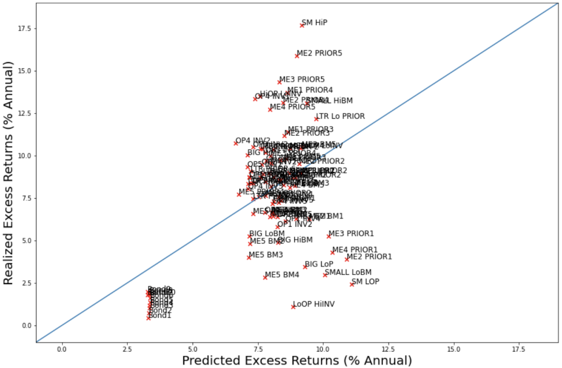


Figure 4: Pricing errors on equity and bond portfolios. Realized excess returns versus predicted excess returns using the two-factor model with market and intermediary factors. The data is at quarterly frequency and from 1970Q1 till 2022Q4.

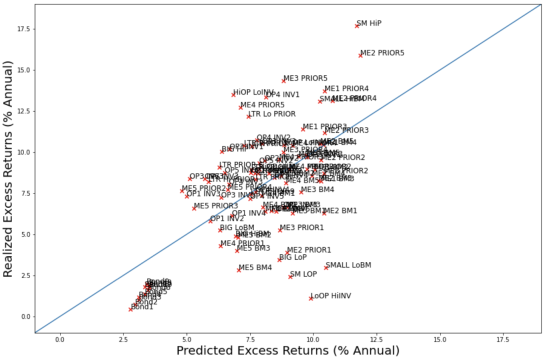


Figure 5: Pricing errors on equity and bond portfolios. Realized excess returns versus predicted excess returns using the three-factor model with market, intermediary, and disagreement factors. The data is at quarterly frequency and from 1970Q1 till 2022Q4.

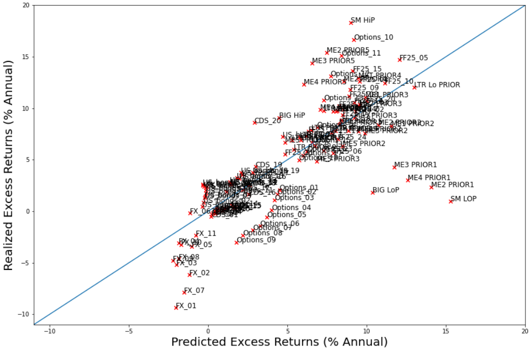


Figure: Pricing error in two-factor model.

Figure: Pricing error in three-factor model.

		Disagreement			
		(1)	(2)	(3)	(3)-(1)
Intermediary	(1)	6.47	4.71	6.83	0.36
	(2)	7.37	7.67	9.69	2.32
	(3)	7.26	9.09	9.26	2.00
	(3)-(1)	0.79	4.38	2.43	-

Table: The table reports the annualized mean excess return on equity and bond portfolios double-sorted on their exposures to the intermediary factor and the disagreement factor using the three-factor model. The data is at quarterly frequency from 1970Q1 till 2022Q4. The intermediary factor is from HKM2017, and disagreement factor is computed from the growth rate of inter-quartile dispersion in GDP projection scaled by the median projection.