Self-fulfilling Volatility and a New Monetary Policy

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What we do

Standard non-linear New Keynesian model

$\exists$a price of risk coming from

$$\underbrace{\text{Aggregate volatility}\uparrow}_{\text{Endogenous}} \Longrightarrow \underbrace{\text{precautionary saving}}_{\text{Endogenous}}\uparrow \Longrightarrow \underbrace{\text{aggregate demand}\downarrow}_{\text{Endogenous}}$$

Takeaway (Self-fulfilling volatility)

In macroeconomic models with nominal rigidities, $\exists$global solution where:

- Taylor rules (targeting inflation and output) $\longrightarrow$ $\exists$self-fulfilling rise in aggregate volatility
- Only direct volatility (e.g., risk premium) targeting can restore determinacy

Can build a similar model with explicit stock markets (in Online Appendix)

A textbook New-Keynesian model with rigid price

The representative household's problem (given B_0):

$$\max_{\{B_t, C_t, L_t\}_{t\geq 0}} \mathbb{E}_0 \int_0^\infty e^{-\rho t} \left[\log C_t - \frac{L_t^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}} \right] dt \quad \text{s.t.} \quad \dot{B}_t = i_t B_t - \bar{p} C_t + w_t L_t + D_t$$

where

- B_t: nominal bond holding, D_t includes fiscal transfer + profits
- Rigid price: $p_t = \bar{p}$ for $\forall t$ (i.e., purely demand-determined)

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Endogenous volatility

A non-linear Euler equation (in contrast to log-linearized one)

$$\mathbb{E}_t\left(\frac{dC_t}{C_t}\right) = (i_t - \rho)dt + \underbrace{\mathrm{Var}_t\left(\frac{dC_t}{C_t}\right)}_{\text{Precautionary premium}}$$

Endogenous drift

Aggregate volatility↑ $\Longrightarrow$ precautionary saving↑ $\Longrightarrow$ recession (the drift↑)

Problem: both **variance** and **drift** are endogenous, is Taylor rule enough?

Firm i: face monopolistic competition à la Dixit-Stiglitz with $Y_t^i = A_t L_t^i$ and

$$\frac{dA_t}{A_t} = g\,dt + \underbrace{\sigma}_{\text{Fundamental risk}} dZ_t$$

- dZ_t: aggregate Brownian motion (i.e., only risk source)
- (g, σ) are exogenous

Flexible price economy as benchmark: the 'natural' output Y_t^n follows

$$\frac{dY_t^n}{Y_t^n} = \left(r^n - \rho + \sigma^2\right) dt + \sigma dZ_t$$
$$= g\,dt + \sigma dZ_t = \frac{dA_t}{A_t}$$

where $r^n = \rho + g - \sigma^2$ is the 'natural' rate of interest

Non-linear IS equation

With

$$\hat{Y}_t = \ln \frac{Y_t}{Y_t^n}, \quad \underbrace{(\sigma)^2 dt = \mathrm{Var}_t\left(\frac{dY_t^n}{Y_t^n}\right)}_{\text{Benchmark volatility}}, \quad \underbrace{\left(\sigma + \sigma_t^s\right)^2 dt = \mathrm{Var}_t\left(\frac{dY_t}{Y_t}\right)}_{\text{Actual volatility}}$$

Exogenous

Endogenous

Non-linear IS equation

With

$$\hat{Y}_t = \ln \frac{Y_t}{Y_t^n}, \quad \underbrace{(\sigma)^2 dt = \mathrm{Var}_t\left(\frac{dY_t^n}{Y_t^n}\right)}_{\text{Benchmark volatility}}, \quad \underbrace{(\sigma + \sigma_t^s)^2 dt = \mathrm{Var}_t\left(\frac{dY_t}{Y_t}\right)}_{\text{Actual volatility}}$$

Exogenous

Endogenous

A non-linear IS equation (in contrast to textbook linearized one)

$$d\hat{Y}_t = \left(i_t - \underbrace{\left(r^n \overbrace{- \frac{1}{2}(\sigma + \sigma_t^s)^2 + \frac{1}{2}\sigma^2}^{\text{New terms}} \right)}_{\equiv r_t^T} \right) dt + \sigma_t^s \, dZ_t \qquad (1)$$

What is r_t^T?: a risk-adjusted natural rate of interest ($\sigma_t^s \uparrow \Longrightarrow r_t^T \downarrow$)

$$r_t^T \equiv r^n - \frac{1}{2} \underbrace{(\sigma + \sigma_t^s)^2}_{\text{Precautionary premium}} + \frac{1}{2}\sigma^2$$

Non-linear IS equation

Big Question

Taylor rule $i_t = r^n + \phi_y \hat{Y}_t$ for $\phi_y > 0 \implies$ perfect stabilization?

Up to a first-order (no volatility feedback): Blanchard and Kahn (1980)

- $\phi_y > 0$: Taylor principle $\implies \hat{Y}_t = 0$ with $\sigma_t^s = 0$ for $\forall t$ (unique equilibrium)

Why? (recap): without the volatility feedback:

$$d\hat{Y}_t = (i_t - r^n)\, dt + \sigma_t^s dZ_t \underbrace{=}_{\substack{\text{Under} \\ \text{Taylor rule}}} \phi_y \hat{Y}_t dt + \sigma_t^s dZ_t$$

Then,

$$\mathbb{E}_t \left(d\hat{Y}_t \right) = \phi_y \hat{Y}_t.$$

If $\hat{Y}_t \neq 0$,

$$\lim_{s \to \infty} \mathbb{E}_t \left(\hat{Y}_s \right) \to \pm\infty$$

- Foundation of modern central banking

Proposition (Fundamental Indeterminacy)

For any $\phi_y > 0$, $\exists$an equilibrium supporting a volatility $\sigma_0^s > 0$ satisfying:

1. $\mathbb{E}_t\left(d\hat{Y}_t\right) = 0$ for $\forall t$ (i.e., local martingale)
2. $\sigma_t^s \xrightarrow{a.s} \sigma_\infty^s = 0$ and $\hat{Y}_t \xrightarrow{a.s} 0$ (i.e., almost sure stabilization)
3. 0^+-possibility divergence or non-uniform integrability given by

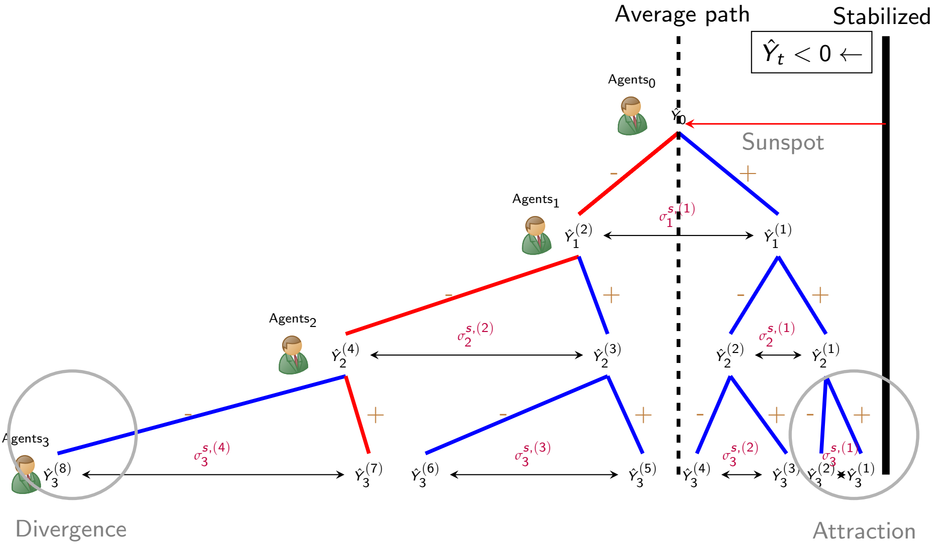
$$\mathbb{E}_0\left(\sup_{t\geq 0}\left(\sigma + \sigma_t^s\right)^2\right) = \infty$$

with

$$\lim_{K\to\infty}\sup_{t\geq 0}\left(\mathbb{E}_0\left(\sigma + \sigma_t^s\right)^2 1_{\{(\sigma+\sigma_t^s)^2\geq K\}}\right) > 0.$$

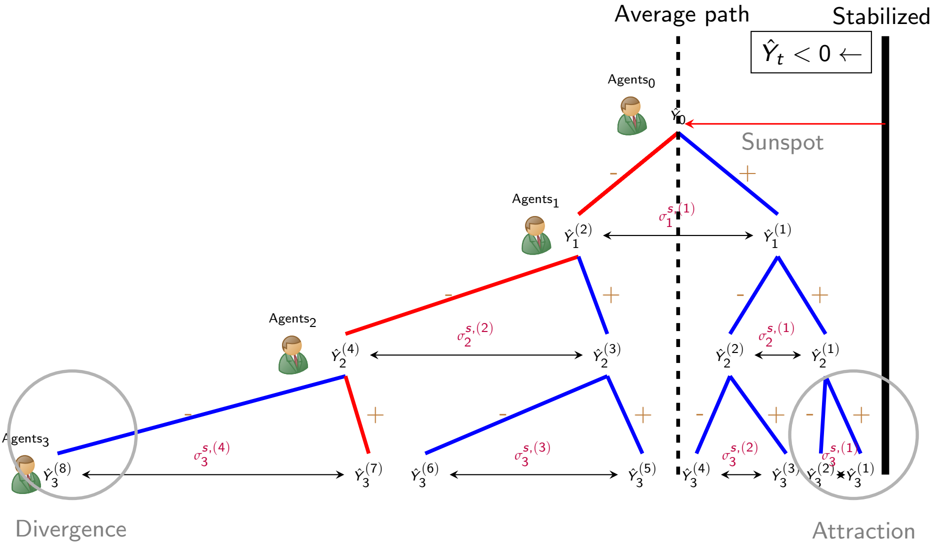
Aggregate volatility↑ possible through the intertemporal coordination of agents

- Called a "martingale equilibrium" - non-stationary equilibrium

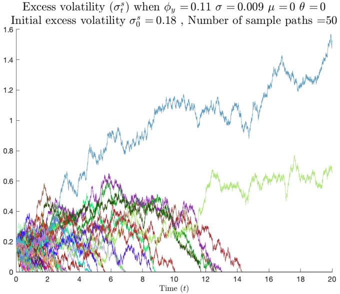


Stabilized as **attractor**: $\sigma_t^s \xrightarrow{a.s} \sigma_\infty^s = 0$ and $\hat{Y}_t \xrightarrow{a.s} 0$

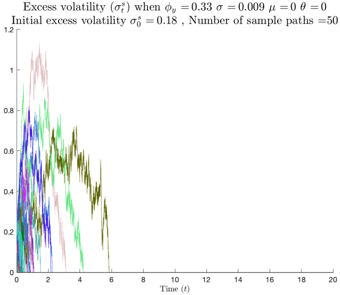
Key: a path-dependent intertemporal aggregate demand strategy



But divergence with 0^+-probability: $\mathbb{E}_0 \left(\sup_{t \geq 0} (\sigma + \sigma_t^s)^2 \right) = \infty$



(a) With Taylor coefficient $\phi_y = 0.11$



(b) With Taylor coefficient $\phi_y = 0.33$

Figure: Martingale equilibrium: with $\phi_y = 0.11$ (Figure 1a) and $\phi_y = 0.33$ (Figure 1b)

Potential stationary equilibria?

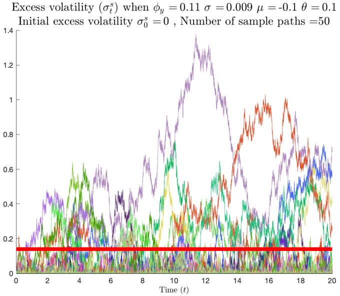
Conjecture: Ornstein-Uhlenbeck process with endogenous volatility $\{\sigma_t^s\}$

$$d\hat{Y}_t = \left(i_t - \underbrace{\left(r^n \overbrace{-\frac{1}{2}(\sigma + \sigma_t^s)^2 + \frac{1}{2}\sigma^2}^{\text{New terms}} \right)}_{\equiv r_t^T} \right) dt + \sigma_t^s \, dZ_t \tag{2}$$

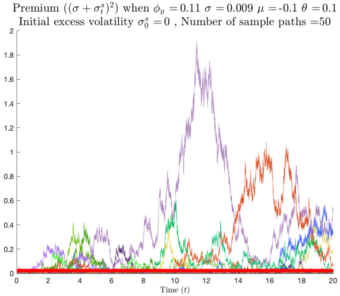
$$= \underbrace{\theta}_{>0} \cdot \left[\underbrace{\mu}_{\gtreqless 0} - \hat{Y}_t \right] dt + \sigma_t^s dZ_t$$

- μ as an *approximate* average of $\hat{Y}_t$
- θ as a speed of mean reversion
- $i_t = r^n + \phi_y \hat{Y}_t$ (i.e., Taylor rule) stays the same

With $\mu < 0$



(a) Endogenous volatility σ_t^s



(b) Precautionary premium $(\sigma + \sigma_t^s)^2$

Figure: Ornstein-Uhlenbeck equilibrium: endogenous volatility $\{\sigma_t^s\}$ (Figure 2a) and the precautionary premium$\{(\sigma + \sigma_t^s)^2\}$ (Figure 2b)

- Even with $\sigma_0^s = 0$ (no initial volatility) $\Longrightarrow$ stationary $\{\sigma_t^s\}$ process

Another case

New monetary policy:

$$i_t = r^n + \phi_y \hat{Y}_t - \boxed{\frac{1}{2}\left(\underbrace{(\sigma + \sigma^s_t)^2}_{\equiv \mathrm{pp}_t} - \underbrace{\sigma^2}_{\equiv \mathrm{pp}^n}\right)}$$

Aggregate volatility targeting?

- Restores a **determinacy** and **stabilization**, but what does it mean?

Leading to:

Ito term

Ito term

$$\boxed{i_t + pp_t - \frac{1}{2}pp_t} = \boxed{r^n + pp^n - \frac{1}{2}pp^n} + \underbrace{\phi_y \hat{Y}_t}_{\text{Business cycle targeting}}$$

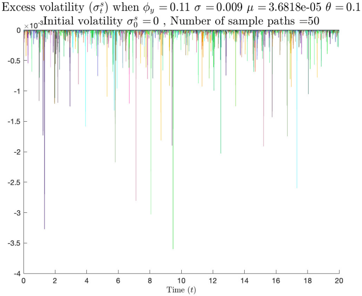
$$i_t + pp_t = \rho + \frac{\mathbb{E}_t(d\log Y_t)}{dt} \qquad r^n + pp^n = \rho + \frac{\mathbb{E}_t(d\log Y_t^n)}{dt}$$

- A % change of (i.e., return on) aggregate output (i.e., demand), not just the policy rate, follows Taylor rules

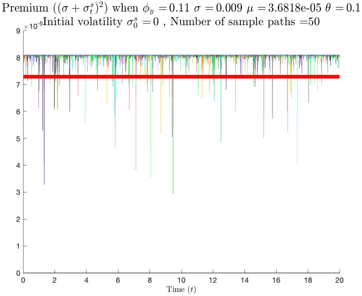
Model with stock markets

Thank you very much!
(Appendix)

With $0 < \mu < \dfrac{\sigma^2}{2\phi_y}$



(a) Endogenous volatility σ_t^s



(b) Precautionary premium $(\sigma + \sigma_t^s)^2$

Figure: Ornstein-Uhlenbeck equilibrium: endogenous volatility $\{\sigma_t^s\}$ (Figure 3a) and the precautionary premium$\{(\sigma + \sigma_t^s)^2\}$ (Figure 3b)

- Even with $\sigma_0^s = 0$ (no initial volatility) $\Longrightarrow$ stationary $\{\sigma_t^s\}$ process

Go back

The model with a stock market + portfolio decision

Go back

Standard demand-determined environment

$\sigma_t^s \uparrow \implies$ precautionary saving $\implies$ precautionary premium$\uparrow \implies$ output$\downarrow$

We can build a **theoretical framework with explicit stock markets** where

Financial volatility$\uparrow \implies$ risk premium$\uparrow \implies$ wealth$\downarrow \implies$ output$\downarrow$

- Wealth-dependent aggregate demand Mechanism
- Two-Agents New Keynesian (TANK) model from Dordal i Carreras and Lee (2024)
- Now, sticky price so $\underline{\pi_t \neq 0}$: Phillips curve à la Calvo (1983)

Skip the detail

Identical **capitalists** and **hand-to-mouth workers**

- **Capitalists**: consumption - portfolio decision (between stock and bond)
- **Workers**: supply labors to firms (hand-to-mouth)

1. **Technology**

Fundamental risk (Exogenous)

$$\frac{dA_t}{A_t} = \underbrace{g}_{\text{Growth}} \cdot dt + \sigma \cdot \underbrace{dZ_t}_{\text{Aggregate shock}}$$

2. **Hand-to-mouth workers**: supply labor + solves the following problem

$$\max_{C_t^w, N_t^w} \frac{\left(\frac{C_t^w}{A_t}\right)^{1-\varphi}}{1-\varphi} - \frac{\left(N_t^w\right)^{1+\chi_0}}{1+\chi_0} \quad \text{s.t.} \quad p_t C_t^w = w_t N_t^w$$

- Hand-to-mouth assumption can be relaxed, without changing implications

3. **Firms**: production using labor + pricing à la Calvo (1983)

4. **Financial market**: zero net-supplied risk-free bond + stock (index) market

Capitalists: standard portfolio and consumption decisions (very simple)

1. Total financial wealth $a_t = p_t A_t Q_t$, where (real) stock price Q_t follows:

$$\frac{dQ_t}{Q_t} = \mu_t^q \cdot dt + \sigma_t^q \cdot dZ_t$$

Financial risk (Endogenous)

- μ_t^q and σ_t^q are both endogenous (to be determined)

2. Each solves the following optimization (standard)

$$\max_{C_t, \theta_t} \mathbb{E}_0 \int_0^\infty e^{-\rho t} \log C_t dt \quad \text{s.t.}$$

$$da_t = (a_t(i_t + \theta_t(i_t^m - i_t)) - p_t C_t)dt + \theta_t a_t \left(\sigma + \sigma_t^q\right) dZ_t$$

- Aggregate consumption of capitalists ∝ aggregate financial wealth

$$C_t = \rho A_t Q_t$$

- Equilibrium **risk-premium** is determined by the total risk

$$i_t^m - i_t \equiv \text{rp}_t = \boxed{\left(\sigma + \sigma_t^q\right)^2}$$

Dividend yield: dividend yield$= \rho$, as in Caballero and Simsek (2020)

- A positive feedback loop between asset price $\Longleftrightarrow$ dividend (output)

Determination of nominal stock return $d\mathbf{l}_t^m$

$$d\mathbf{l}_t^m = \underbrace{\left[\underbrace{\rho}_{\text{Dividend yield}} + \underbrace{\underbrace{\pi_t}_{\text{Inflation}} + g + \mu_t^q + \overbrace{\sigma\sigma_t^q}^{\text{Covariance}}}_{\text{Capital gain}} \right]}_{= \underbrace{i_t^m}_{\text{Drift}} = \underbrace{i_t}_{\text{Monetary policy}} + \underbrace{\left(\sigma + \sigma_t^q\right)^2}_{\text{Risk-premium}}} dt + \underbrace{\left(\sigma + \sigma_t^q\right)}_{\text{Risk term}} dZ_t$$

- Close the model with supply-side (i.e., Phillips curve) and $\{i_t\}$ rule

Flexible price economy allocations (benchmark)

- $\sigma_t^{q,n} = 0$, Q_t^n, $N_{W,t}^n$, C_t^n, r^n (natural rate), rp^n (natural risk-premium)

Gap economy (log deviation from the flexible price economy)

- With asset price gap $\hat{Q}_t \equiv \ln \frac{Q_t}{Q_t^n} = \hat{C}_t$ and π_t

Proposition (Dynamic IS)

A dynamic gap economy can be described with the following equations:

1. $\mathbb{E}_t d\pi_t = (\rho\pi_t - \kappa\hat{Q}_t)dt$ with $\kappa > 0$
2. $d\hat{Q}_t = (i_t - \pi_t - \boxed{r_t^T})dt + \sigma_t^q \, dZ_t$ where $r_t^T = r^n - \frac{1}{2}(\text{rp}_t - \text{rp}^n)$

$$\equiv r^n - \frac{1}{2}\hat{\text{rp}}_t$$

where $\text{rp}_t = (\sigma + \sigma_t^q)^2$ and $\text{rp}^n = \sigma^2 \Longrightarrow \hat{\text{rp}}_t \equiv \text{rp}_t - \text{rp}^n$

Now, with asset (stock) price gap $\hat{Q}_t$:

Real volatility

$$d\hat{Q}_t = \left(i_t - \pi_t - \underbrace{\left(r^n - \frac{1}{2}\boxed{(\sigma + \sigma_t^q)^2} + \frac{1}{2}\boxed{\sigma^2} \right)}_{\mathrm{rp}_t \equiv r_t^T \qquad \mathrm{rp}^n} \right) dt + \sigma_t^q \, dZ_t \tag{3}$$

Here

$$\sigma_t^q \uparrow \implies \mathrm{rp}_t \uparrow \implies \hat{Q}_t \downarrow \implies \hat{Y}_t \downarrow$$

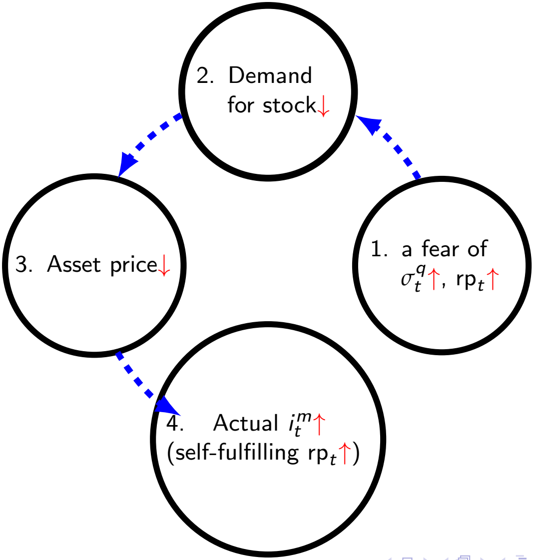
Monetary policy: Taylor rule to Bernanke and Gertler (2000) rule

$$i_t = r^n + \phi_\pi \pi_t + \phi_y \underbrace{\hat{y}_t}_{=\zeta \hat{Q}_t}$$

$$= r^n + \phi_\pi \pi_t + \phi_q \hat{Q}_t, \quad \text{where} \quad \underbrace{\phi \equiv \phi_q + \frac{\kappa(\phi_\pi - 1)}{\rho} > 0}_{\text{Taylor principle}}$$

Multiple equilibria

- **How?**: countercyclical risk-premium with conventional Taylor rules



Is a sunspot $\sigma_0^q \neq 0$ supported by a rational expectations equilibrium?

Proposition (Fundamental Indeterminacy)

For any $\phi > 0$, $\exists$an equilibrium supporting a volatility $\sigma_0^q > 0$ satisfying:

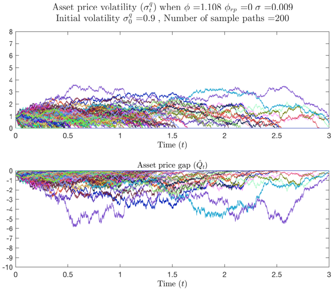
1. $\mathbb{E}_t\left(d\hat{Q}_t\right) = 0$ for $\forall t$ (i.e., local martingale)
2. $\sigma_t^q \xrightarrow{a.s} \sigma_\infty^s = 0$ and $\hat{Q}_t \xrightarrow{a.s} 0$ (i.e., almost sure stabilization)
3. 0^+-possibility divergence or non-uniform integrability given by

$$\mathbb{E}_0\left(\sup_{t\geq 0}\left(\sigma + \sigma_t^q\right)^2\right) = \infty$$

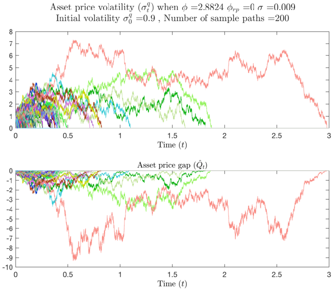
with

$$\lim_{K\to\infty}\sup_{t\geq 0}\left(\mathbb{E}_0\left(\sigma + \sigma_t^q\right)^2 1_{\left\{(\sigma+\sigma_t^q)^2 \geq K\right\}}\right) > 0.$$

1. (Almost surely) stabilized in the long run after sunspot $\sigma_0^q > 0$
   Meantime: **crisis** with volatility (risk-premium)↑, asset price↓, and business cycle↓
2. $\mathbb{E}_0\left(\sup_{t\geq 0}\left(\sigma + \sigma_t^q\right)^2\right) = \infty$: an $\epsilon \to 0$ possibility of ∞-severity crisis



(a) With $\phi_\pi = 1.5$



(b) With $\phi_\pi = 2.5$.

Figure: $\{\sigma_t^q, \hat{Q}_t\}$ dynamics when $\sigma^{q,n} = 0$ and $\sigma_0^q = 0.9$, with reasonable calibration

As monetary policy responsiveness $\phi\uparrow$

Stabilization speed$\uparrow$, $\exists$more severe crisis sample path

- $\sigma_t^q\uparrow$ by $\sigma \implies 2 - 10\%\downarrow$ in Q_t, depending on monetary responsiveness ϕ

New monetary policy:

$$i_t = r^n + \phi_\pi \pi_t + \phi_q \hat{Q}_t - \boxed{\overbrace{\underbrace{\frac{1}{2}}_{\text{Sharp}}\ \hat{rp}_t}^{\text{New targeting}}}, \quad \text{where} \quad \underbrace{\phi \equiv \phi_q + \frac{\kappa(\phi_\pi - 1)}{\rho} > 0}_{\text{Taylor principle}}$$

restores a determinacy with:

Takeaway (Ultra-divine coincidence)

One monetary tool (i_t) $\Longrightarrow$ (i) inflation, (ii) output, and (iii) risk-premium

Sharpness

Leading to:

Ito term

Ito term

$$\boxed{\underbrace{i_t + \mathrm{rp}_t}_{=i_t^m} - \frac{1}{2}\mathrm{rp}_t} = \boxed{\underbrace{r^n + \mathrm{rp}^n}_{=i_t^{m,n}} - \frac{1}{2}\mathrm{rp}^n} + \underbrace{\phi_\pi \pi_t + \phi_q \hat{Q}_t}_{\text{Business cycle targeting}}$$

$$\rho + \frac{\mathbb{E}_t(d\log a_t)}{dt} \qquad\qquad \rho + \frac{\mathbb{E}_t(d\log a_t^n)}{dt}$$

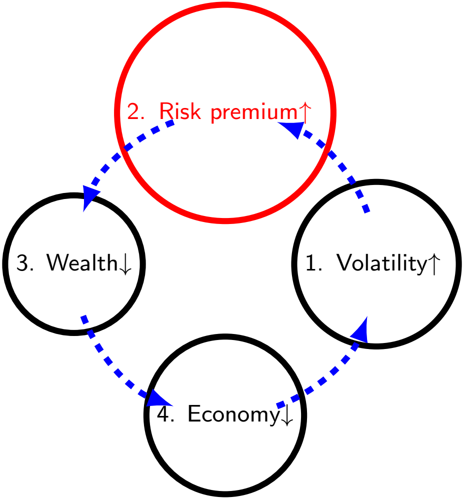
- i_t^m, not i_t, follows a Taylor rule?

- A % change of (i.e., return on) aggregate wealth, not just the policy rate, follows Taylor rules

- Why? Because i_t^m, not i_t truly governs intertemporal substitution

Additional slides

Go back



- **1** → **2** comes from "non-linearity (not linearizing)"
- **2** → **3** comes from "portfolio decision" of each investor and externality
- **3** → **4** comes from the fact wealth drives aggregate demand
- **4** → **1** where business cycle has its own volatility (self-sustaining)

Go back

What if central bank uses the following alternative rule, where $\phi_{\text{rp}} \neq \frac{1}{2}$?

$$i_t = r_t^n + \phi_\pi \pi_t + \phi_q \hat{Q}_t - \boxed{\phi_{\text{rp}}} \hat{rp}_t, \quad \text{where} \quad \phi \equiv \phi_q + \frac{\kappa(\phi_\pi - 1)}{\rho} > 0$$

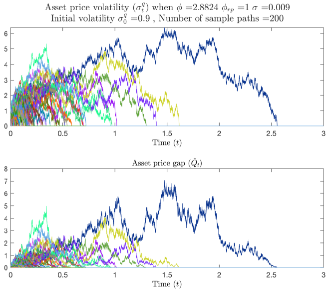
- Then still $\exists$martingale equilibrium supporting sunspot $\sigma_0^q \neq 0$
- As $|\phi_{\text{rp}} - \frac{1}{2}| \uparrow \implies$ (on average) longer time for σ_t^q to vanish
- Especially, $\phi_{\text{rp}} < 0$ (i.e., **Real Bills Doctrine**) is a bad idea. Why?

Go back

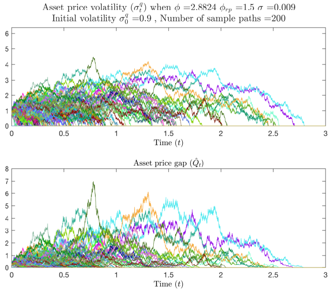
$\phi_{\mathbf{rp}} < 0$ **(Real Bills Doctrine)**	$0 < \phi_{\mathbf{rp}} < \frac{1}{2}$
(i) With $\phi_{\text{rp}}\downarrow$, convergence speed$\downarrow$ and less amplified paths (ii) $\sigma_t^q > \sigma_t^{q,n} = 0$ means a crisis ($\hat{Q}_t < 0$ and $\pi_t < 0$)	(i) With $\phi_{\text{rp}}\uparrow$, convergence speed$\uparrow$ and more amplified paths (ii) $\sigma_t^q > \sigma_t^{q,n} = 0$ means a crisis ($\hat{Q}_t < 0$ and $\pi_t < 0$)
$\phi_{\mathbf{rp}} = \frac{1}{2}$	$\phi_{\mathbf{rp}} > \frac{1}{2}$
No sunspot (Ultra-divine coincidence)	(i) With $\phi_{\text{rp}}\uparrow$, convergence speed$\downarrow$ and less amplified paths (ii) $\sigma_t^q > \sigma_t^{q,n} = 0$ means a boom ($\hat{Q}_t > 0$ and $\pi_t > 0$)
As $\phi\uparrow$, convergence speed$\uparrow$ and $\exists$ more amplified paths	

When ϕ_{rp} deviates from $\frac{1}{2}$

Go back



(a) With $\phi_{\text{rp}} = 1$



(b) With $\phi_{\text{rp}} = 1.5$.

Figure: $\{\sigma_t^q, \hat{Q}_t\}$ dynamics when $\sigma^{q,n} = 0$ and $\sigma_0^q = 0.9$, with varying $\phi_{\text{rp}} > \frac{1}{2}$