A Unified Theory of the Term-Structure and Monetary Stabilization*

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Abstract

The failure of conventional monetary policy to stabilize the economy at the zero-lower bound (ZLB) has made unconventional interventions more prevalent in recent times, which calls for a new macroeconomic framework for properly analyzing these policies. In this paper, we develop a New-Keynesian framework that incorporates the term-structure of financial markets and an active role for government and central bank's balance sheet size and composition. We show that *financial market segmentation* and the household's *endogenous portfolio reallocation* are crucial features to properly understand the effects of Large-Scale Asset Purchase (LSAP) programs. We propose a new micro-foundation based on imperfect information about expected future asset returns that easily accommodates distinct degrees of market segmentation across asset classes and maturities, while providing intuitive and tractable expressions for the household's portfolio shares. Our analysis reveals that government's issuance of risk-less bonds stimulates the economy when conventional monetary policy is constrained at the ZLB, which is consistent with the literature on the so-called "safe-asset shortage problems". We also find that central bank's bond purchases across different maturities act as a major determinant of the level and slope of the term-structure, and yield-curve-control (YCC) policies that actively manipulate long-term yields are powerful in terms of stabilization both during normal times and at the ZLB. As a drawback, YCC policies increase the likelihood of ZLB episodes and their durations, thereby locking the central bank in a position in which the short-term rate is less useful as a policy tool.

Keywords: Term Structure, Quantitative Easing, Monetary Policy

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1 Introduction

Central banks' various "unconventional" monetary policies[1] have never been more "conventional" than in recent decades, especially after the 2007-2008 Global Financial Crisis (GFC) and the subsequent Great Recession. In environments where the short-term policy rate is constrained by its lower bound (ZLB),[2] policymakers devised ways to reduce long-maturity rates, hoping that falling long-term rates would boost aggregate demand and mitigate the recessionary pressure on the economy, and to that end, central banks tremendously increased sizes of their balance sheets. Simultaneously, governments financed their spending increases by raising their debt issuance. Unfortunately, this unprecedented economic environment intensified in the wake of the Covid-19 pandemic recently and the Federal Reserve, as its policy rate hit its effective lower bound again, has undertook another round of those measures for stabilization purposes.[3]

The textbook New-Keynesian framework features a single policy rate, abstracting from the term-structure of interest rates and the presence of multiple assets. This omission is not a simplification that can be easily incorporated, as expected returns across assets and maturities are usually equalized in equilibrium in these models,[4] rendering any additional assets as a fully-dependent function of the policy rate, and therefore redundant for the study of monetary policy.

In this paper, we build a tractable New-Keynesian framework featuring the endogenous term-structure of interest rates of bond markets and private capital markets, with which we study effects of alternative monetary (conventional and unconventional) and fiscal policies. Following prior theoretical and empirical works that point out 'market segmentation' across bonds of different maturities as a critical feature in explaining the effectiveness of quantitative easing programs,[5] we provide an alternative micro-foundation that enables us to incorporate (i) bonds market segmentation, (ii) the household's endogenous portfolio choices across different asset classes and maturities, (iii) real

[1] For example, the Quantitative Easing (QE) programs, and large scale asset purchases (LSAPs) programs in general, and Operation Twist (OT) are possible forms of the unconventional monetary policy.

[2] For the long-term downward trend of neutral interest rates, see Rachel and Smith (2017). This trend amplifies the important stabilizing roles of unconventional monetary policies.

[3] The Federal Reserve lowered its short-term interest rates to a range of 0% to 0.25% in March 2020. While increasing its securities holdings and the size of its balance sheet tremendously, the Fed pledged not to raise interest rates until the eonomy reaches full employment and consistently maintain 2% inflation. The unprecedented CARES Act also provided almost $500 billion to support the Fed. See Congressional Research Service (CRS) report R46411 (2021) for the details.

[4] This result follows from the log-linearization technique and leads to the famous *expectation hypothesis*, which holds in most log-linearized New-Keynesian models. According to this hypothesis, returns on long-term bonds become just the average of expected future short-term rates.

[5] For empirical assessments of the market-segmentation hypothesis as a key determinant of the term structure, see D'Amico and King (2013) and Droste et al. (2021). For theory side, Ray (2019) adapts the preferred-habitat framework developed by Vayanos and Vila (2021) and proposes a New-Keynesian model that features bond market segmentation, revealing many interesting relationships between monetary policy and the term-structure. Gourinchas et al. (2020) and Greenwood et al. (2020) study implications of the preferred-habitat setting in joint determination of exchange rates and the term-structure of interest rates.

effects of the government and central bank's balance sheet size and composition: all building blocks necessary for understanding the transmission channel of unconventional monetary policies.[6]

Under market segmentation, our framework predicts that the total amount and maturity structure of the government's bond issuance affects the equilibrium interest rate levels and the slope of the yield curve, while the central bank's relative bond purchases across bonds of different maturities are negatively related with yields. These results are consistent with the findings of Krishnamurthy and Vissing-Jorgensen (2012) and Greenwood and Vayanos (2014),[7] who illustrate the short-run and long-run importance of both relative asset demand and supply across maturities in determining the yield curve.

We also study cyclical properties of distinct monetary interventions in the form of simple policy rules. By explicitly incorporating the government and central bank's balance sheets, our framework easily accommodates policies aimed at controlling the yields, bond supplies or a mixture of both at different maturities. We begin by focusing on the implementation of a conventional policy rule on the short-term rate and its effect of the entire yield curve and the economy. Then, we develop a more general *yield-curve-control* (YCC) policy in which the central bank directly controls the entire bond market yield curve. Our framework reveals interesting phenomena and differences across policies, which become especially relevant when the economy enters the ZLB episodes (thus, the short-term rate is constrained by the ZLB). For example, our analysis reveals that when the central bank follows a conventional monetary policy on short-term rates, a reduction in the government's risk-free bond supply is recessionary at the ZLB, as argued by the literature on safe-asset shortage (SAS) problems (see for example, Caballero and Farhi (2017) and Caballero et al. (2021)). In contrast, under the YCC policy, the central bank swiftly shifts down the entire yield curve and lowers the effective savings rate of households, boosting aggregate demand and preventing the economy's collapses.[8] We find that YCC, in general, is a more powerful policy in terms of economic stabilization which increases household's welfare compared with a conventional short-term rate policy.

However, YCC policy has interesting side-effects in forms of more frequent and prolonged ZLB episodes. Active easing of long-maturity yields imposes an additional downward pressure on the

[6]For example, (ii) endogenous portfolio decision is important: a relative fall in the short-term rate leads households to reallocate their savings into other assets and/or longer maturities bonds, thus diminishing marginal effects of further changes in the policy rate on the household's intertemporal consumption decision, and also generates spill-over effects relevant for the determination of other rates.

[7]Krishnamurthy and Vissing-Jorgensen (2012) found that a higher debt-to-GDP ratio leads to lower corporate credit spreads, and this effect becomes stronger for longer maturities. Likewise, Greenwood and Vayanos (2014) documented that the relative abundance of long term bonds supply (with respect to short term bonds) is positively correlated with the term-spread.

[8]Even under the conventional policy, a falling short-term rate reduces long-term bond yields due to the endogenous portfolio reallocation of the household, thereby reducing the effective savings rate. However, this channel is insufficient for boosting aggregate demand, especially when the economy hits the ZLB constraint and conventional policy becomes inactive.

returns of short-term bonds, which stems from the household's endogenous portfolio reallocation: falling long-term rates induce the household to pull its wealth out of long-term bonds and instead invest into (i) short-maturity bonds, imposing additional downward pressures on short-term yields, and (ii) private capital (loan) markets, reducing firms' borrowing costs, and hence the consumption prices due to lower production costs.[9] When the ZLB binds, YCC policy disproportionately controls the yields of long-term bonds, which places additional downward pressure on short-term rates and delays an exit from the ZLB. Therefore, the household's endogenous portfolio reallocation results in a feedback loop between ZLB duration and the need for YCC policies: YCC raises ZLB duration and frequency, while the economy relies more on the YCC's stabilization power during ZLB episodes. Up to our best knowledge, this result is new to the literature.[10]

We propose a new theoretical foundation for the financial market segmentation based on imperfect information about asset returns. We assume that the household is subdivided into a continuum of families and family members, each of them having distinct and imperfect information sets about those future asset returns. Then, unable to extract a common signal from the pool of heterogeneous information sets, the household evenly splits aggregate savings across its members and lets them allocate their share on the assets that they deem more profitable. This investment strategy effectively results in market segmentation, with cross-sectional dispersions in each individual's expectation of asset returns determining degrees of market segmentation associated with each class of assets. We simplify the aggregation problem of individual portfolio choices among members by modeling the differences on expected asset returns as Fréchet-distributed shocks around the respective rationally expected levels of returns.[11] This aggregation technique, which we borrow from the international trade literature (e.g., Eaton and Kortum (2002)), allows us to easily incorporate new asset varieties and distinct degrees of market segmentation across different assets and maturities, while providing analytically tractable expressions for the household's portfolio shares as a function of relative expected asset returns. Our formulation is fairly general and nests the famous *expectations hypothesis* as a one special case, allowing deviations due to imperfect information and behavioral reasons. A final benefit of this framework is that the demand elasticity of each asset class becomes a sufficient statistic for its particular degree of market segmentation, making the segmented market hypothesis easy to test and calibrate in contexts of our model.

[9] Drops in the aggregate price index further impose downward pressures on the policy rate following conventional Taylor rules, which is already constrained at the ZLB.

[10] A similar result, but obtained through the completely different channel, is presented by Karadi and Nakov (2021). The paper documents the 'QE-addiction' problems based on a model with financial frictions in which private banks get accustomed to the central bank's liquidity provisions, which reduce their incentive to recapitalize without additional QE rounds. In that environment, Karadi and Nakov (2021) derive a gradual optimal exit strategy from QE programs.

[11] For general properties of the Fréchet distribution, see Gumbel (1958).

Related Literature This paper contributes to several different strands of the literature in macroeconomics and finance. First, previous works have shown that macroeconomic factors are important in explaining behaviors of the term-structure of interest rates (e.g., Ang and Piazzesi (2003), Rudebusch and Wu (2008), and Bekaert et al. (2010)).[12] The frameworks developed in this literature are usually based on an ad-hoc affine term-structure (e.g., Duffie and Kan (1996)) without specific equilibrium micro-foundations.[13] We contribute to this literature by pinning down the term-structure of interest rates in the presence of multiple asset classes (e.g., bonds for intertemporal smoothing and private loans for productive investments) and nominal rigidities, with explicit roles of the government and central bank's balance sheets and the household's endogenous portfolio choices along the entire yield curve, which allows us to characterize how the business cycle variables and financial markets (including the term-structure) are intertwined.

Another relatively nascent literature focuses on the relationship between central bank's endogenous balance sheet composition and monetary policy (e.g. Gertler and Karadi (2011), Cúrdia and Woodford (2011), Christensen and Krogstrup (2018), Christensen and Krogstrup (2019), Karadi and Nakov (2021)). This literature provides new insights on how the central bank's large scale asset purchase programs (LSAPs) help mitigate financial market disruptions,[14] but in many cases abstracts from the study of multiple bond market maturities, and focuses instead on the aggregate expansion of central bank's balance sheets.[15] We contribute to the literature by providing a unified framework that describes how central banks can manipulate their bond portfolios in order to control targeted rates along the yield curve for stabilization purposes. Especially, our implication that an active endogenous manipulation of central bank's long-term bond holdings can be welfare-improving aligns with Sims and Wu (2021).[16]

Our analysis of the zero lower bound (ZLB) closely follows the previous literature (e.g., Swanson and Williams (2014), Caballero and Farhi (2017), and Caballero et al. (2021)) and describes additional benefits of an active manipulation of the central bank's balance sheet (size and composition along the entire yield curve) when the economy enters the ZLB. To the best of our knowledge, our paper is

[12] For example, Ang and Piazzesi (2003) found that models with business cycle factors forecast better than models with only unobservable factors by analyzing the joint dynamics of bond yields and macroeconomic variables in a VAR, with no-arbitrage as an identifying restriction.

[13] Bekaert et al. (2010) combined the no-arbitrage term-structure with a canonical New-Keynesian framework, maintaining consistency between the household's IS (intertemporal substitution) equation and the affine pricing kernel. Even though their model delivers strong contemporaneous responses of the entire term structure to various macroeconomic shocks, the household does not invest in the entire yield curve, and it does not feature a full general equilibrium.

[14] For example, Gertler and Karadi (2011) pointed out that (i) central banks are not balance sheet constrained, and (ii) as the balance sheet constraints on private intermediaries tighten during financial crises, a net benefit from the central bank's intermediation increases.

[15] Cúrdia and Woodford (2011), for example, showed that targeted asset purchases by central bank's credit expansion is effective when financial markets are highly disrupted for some *exogenous* reason.

[16] The term-structure of interest rates is abstracted away in Sims and Wu (2021). Instead, Sims and Wu (2021) assumes that a wholesale firm and fiscal authorities issue perpetuities with decaying coupon payments.

the first to characterize a general equilibrium economy featuring both the term-structure of interest rates and a possibility of the binding ZLB, together with the presence of multiple financial assets.

Layout In Section 2, we present a New-Keynesian framework incorporating capital markets and the term-structure of interest rates, and derive the main theoretical results on how imperfect information leads to market segmentation. Section 3 focuses on the steady state implications of distinct policies and the model calibration. Section 4 studies the cyclical (short-run) responses of our model to different shocks under alternative monetary policy rules and economic situations, including the ZLB. Section 5 concludes.

2 Model

2.1 Non-technical Summary

We start by providing a non-technical overview of our theoretical framework and its key components. There is a representative household and a continuum of monopolistically competitive firms producing differentiated goods and subject to price stickiness á la Calvo (1983). Firms use labor and capital in production, with the latter rented to firms by the competitive capital producer. Firms are subject to a cash-in-advance constraint when renting capital, and borrow from households through one-period loan contracts (equivalently, corporate one-period bond) in order to fulfill the capital rental payment ahead of their production. Firms also pay a wage to the household in exchange for its labor, and the household allocates its income between consumption and savings.

The household allocates its savings across a menu of different assets that includes firm loans and zero-coupon risk-free bonds. In contrast to canonical New-Keynesian models, the bond market is comprised of bonds with multiple maturities.[17] The household contains a continuum of individuals, each with a different information set regarding the assets' profitability. In order to construct its portfolio, we assume that the household evenly splits the savings across its members and lets them freely allocate their share to their preferred asset. We show that this difference in information sets across household members leads to financial market segmentation, which breaks the conventional expectations hypothesis of linearized monetary models and allows us to study the distinct impacts of different monetary policies on distinct assets and maturities.

Our framework also contains the government with an exogenous public consumption demand financed through taxation and bond issuance. By sustaining a positive steady state deficit, which is

[17] In this context, long-term bonds that exist through multiple periods are still subject to price revaluation risks each period. We regard them risk-free by the virtue of their terminal payment being known and fixed upon issuance.

a plausible assumption for most advanced economies, the government becomes the natural issuer of the entire risk-free bond market. The central bank then implements its preferred monetary policy by controlling the economy's yield curve through open market bond purchases that affect the size and composition of its balance sheet. We consider two distinct policy rules that the central bank follows. First, a *standard* policy rule that controls the short(est)-term yield through active manipulation of its short-term bond holdings, together with a passive targeting of the balance sheet volumes of longer bond maturities. A movement of the policy rate leads the household to reallocate their portfolio (i) across bonds of different maturities, and (ii) between bonds and loans. This reallocation changes the household's effective savings rate and affects its consumption through the usual intertemporal substitution channel ('demand block'). In addition, a policy movement alters the share of savings flowing into firms as private loans, which affects the effective loan rate, firms' capital demand, and the output level ('supply block').

Second, we consider a *general* monetary policy rule that targets the entire yield curve by actively trading bonds across all different maturities.[18] We show that this policy is very powerful in terms of stabilization, as it enables the central bank to lower the effective savings rate of households even at the zero-lower bound (ZLB), during which *standard* policy is ineffective. As a drawback, this policy raises the likelihood and durations of ZLB episodes, since falling long-maturity yields increases the demand for short-term bonds and imposes additional downward pressure on their yields. Albeit being optimal from a welfare perspective, the prolonged ZLB episodes that accompany such policy further amplify its usefulness and chronify its application, suggesting that more frequent ZLB spells and unconventional interventions such as Large Scale Asset Purchases (LSAPs) might become the new normal.

The key economic agents and financial markets of our model are summarized in Figure A1. Next, we formally present the main components of our model.

2.2 Representative Household

The representative household maximizes the following objective function:

$$\max_{\{C_{t+j}, N_{t+j}\}} \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \left[\log\left(C_{t+j}\right) - \left(\frac{\eta}{\eta+1}\right) \left(\frac{N_{t+j}}{\bar{N}_{t+j}}\right)^{1+\frac{1}{\eta}} \right] , \tag{1}$$

where $N_t = (\int_0^1 N_t(\nu)^{\frac{\eta+1}{\eta}} \, d\nu)^{\frac{\eta}{\eta+1}}$ is an aggregate labor index, $N(\nu)$ is labor supplied to intermediate industry ν, η is the Frisch labor supply elasticity, and $\bar{N}_t$ is the balanced growth path population, which grows at constant gross rate GN. Variable C_t is consumption of the final good.

[18]More precisely, we assume that the central bank sets a Taylor-type rule for each yield along the entire yield curve.

At each period t, the representative household can invest in f-period zero-coupon government bonds where f varies from 1 to F, and also provide loans to the firms.[19,20] Therefore, the representative household's period t budget constraint is written as

$$C_t + \frac{L_t}{P_t} + \frac{\sum_{f=1}^{F} B_t^{H,f}}{P_t} = \frac{\sum_{f=0}^{F-1} R_t^f B_{t-1}^{H,f+1}}{P_t} + \frac{R_t^K L_{t-1}}{P_t} + \int_0^1 \frac{W_t(\nu) N_t(\nu)}{P_t}\, \mathrm{d}\nu + \frac{\Lambda_t}{P_t}\,, \tag{2}$$

where L_t is the amount of one period loans to firms, with associated return R_t^K determined upon emission. $W_t(\nu)$ is the wage paid by industry ν, and Λ_t are transfers from different sources, including government's lump sum taxation and profits of the central bank and firms. $B_t^{H,f} \equiv Q_t^f \widetilde{B}_t^{H,f}$ is the nominal amount of dollars invested in the f-maturity government bond paying one dollar at the terminal period $t+f$. Q_t^f is the price of such bond, with Q_t^0 equal to one. $\widetilde{B}_t^{H,f}$ is the amount of f-maturity bonds held by the household, and we assume that the household is unable to credibly issue risk-free bonds and therefore is constrained to hold a non-negative quantity of them, $\widetilde{B}_t^{H,f} \geq 0$ for all f. Variable R_t^f is the return earned on an f-period bond, which corresponds to the rate of bond price revaluation between two adjacent quarters, $R_t^f = Q_t^f / Q_{t-1}^{f+1}$. The gross yield of a zero-coupon bond of maturity f is conventionally defined as $YD_t^f \equiv \left(Q_t^f\right)^{-\frac{1}{f}}$, and hence we can alternatively express bond return R_t^f as

$$R_t^f = \frac{\left(YD_t^f\right)^{-f}}{\left(YD_{t-1}^{f+1}\right)^{-(f+1)}}\,. \tag{3}$$

2.2.1 Individual Savings

The representative household chooses the optimal level of consumption, employment, and savings S_t, with the latter allocated either into government bonds $B_t^H = \sum_{f=1}^{F} B_t^{H,f}$ or firm loans L_t, thus satisfying $S_t = B_t^H + L_t$. To generate a downward sloping demand curve for each investment vehicle,[21] we introduce the following machinery: After deciding the savings level S_t, the household is equally split into a $[0,1]$ continuum of families that differ in their preferred savings vehicle, which can either be loans or bonds. If a family prefers to invest in the bond market instead of issuing loans, then the family is again split into a $[0,1]$ measure of family members that differ on

[19] Alternatively, we interpret it as households purchasing one-period corporate bonds

[20] Banks and financial intermediaries are abstracted away in our framework, and the representative household issues direct loans to the firms instead. Without any relevant intermediation frictions, the results of both representations are equivalent.

[21] Otherwise, linearization of the model results in the perfect equalization, in equilibrium, of all expected asset returns (including different bond maturities), which is consistent with the standard expectation hypothesis(see Froot (1989)).

the preferred bond maturity $f = 1 \sim F$. We use index m to identify a family within the continuum, and index n to refer to one of its family members. Each family m and each member n in the bond family m all have the same amount of savings S_t as the household. We solve the allocation problem recursively in the following way.

Assuming that a family m has chosen bonds as their preferred savings vehicle, its member n maximizes the expected savings return, solving the following problem.

$$\max \sum_{f=1}^{F} \mathbb{E}_{m,n,t} \left[Q_{t,t+1} R_{t+1}^{f-1} B_{m,n,t}^{H,f} \right] \text{ s.t. } B_{m,n,t}^{H} \equiv \sum_{f=1}^{F} B_{m,n,t}^{H,f} = S_t, \; B_{m,n,t}^{H,f} \geq 0, \tag{4}$$

where $\mathbb{E}_{m,n,t}$ is the expectations operator for member n in family m and $Q_{t,t+1}$ is the stochastic discount factor of the household. Due to the linear nature of the problem, we reach a corner solution in which member n allocates her entire share of savings to the bond with the highest expected discounted return.[22] Formally,

$$B_{m,n,t}^{H,i} = \begin{cases} S_t & \text{, if } i = \underset{1 \leq j \leq F}{\arg\max} \left\{ \mathbb{E}_{m,n,t} \left[Q_{t,t+1} R_{t+1}^{j-1} \right] \right\} , \\ 0 & \text{, otherwise.} \end{cases} \tag{5}$$

In the benchmark rational expectations environment, all members in the bond family m choose the same allocation, and expected discounted returns $\mathbb{E}_t \left[Q_{t,t+1} R_{t+1}^{f-1} \right]$ for any maturity f are equalized in equilibrium. This case aligns with the 'expectation hypothesis' in the log-linearized economy, where long term rates are approximated as the average of future expected short term rates.[23] Since the short term rate R_{t+1}^{0} is controlled by the central bank, longer yield maturities are fully determined by conventional monetary policy in this environment. This precludes any meaningful role for other central bank policies such as QE, despite empirical evidence on the contrary.[24]

We deviate from the expectations hypothesis and generate a downward sloping demand curve for each bond of maturity f by imposing additional structure on the portfolio allocation problem. We assume that each member n of the family has different expectations about the discounted future returns of bonds. This difference can be attributed to each agent having access to a distinct and imperfect information set (in a similar manner to Angeletos and La'O (2013)) or simply by behavioral assumptions. In addition, we assume that family m doesn't have the capacity to aggregate

[22]The exception would be if two or more bonds have exactly the same highest expected discounted return, in which case the member n would be indifferent between allocations across these bonds. This, in general, will happen with zero probability as it will become clear in the derivations below.

[23]In the linearized economy, the co-variation between $Q_{t,t+1}$ and returns R_{t+1}^{f-1} is omitted together with other higher-order terms.Thus, expected returns for each bond maturity are equalized.

[24]For example, Krishnamurthy and Vissing-Jorgensen (2011) show that LSAP interventions lower long-term interest rates

the individual information from its members and perform a centralized portfolio allocation based on signal extraction. Therefore, the family decides to equally split the savings among its members and allows them to decide on the allocation of their individual share. We assume the following functional form for member n expectations:

$$\mathbb{E}_{m,n,t}\left[Q_{t,t+1}R_{t+1}^{f-1}\right] = z_{n,t}^{f} \cdot \mathbb{E}_t\left[Q_{t,t+1}R_{t+1}^{f-1}\right], \ \forall f = 1, \ldots, F, \tag{6}$$

where the expectation operator $\mathbb{E}_{m,n,t}$ is a member-specific expectation, whereas $\mathbb{E}_t$ is the rational expectation. $z_{t,n}^{f}$ are maturity-f specific shocks to member n's expectations. Note that, *ceteris paribus*, a high realization of $z_{t,n}^{f}$ makes member n more willing to save in the f-maturity bond.

For tractability, we model $z_{t,n}^{f}$ as a Fréchet-distributed shock with location parameter zero, scale parameter z_t^f and shape parameter κ_B, and assume it to be i.i.d. across members n, maturities f and quarters t.[25] Shape parameter κ_B determines the volatility of these expectation shocks, with $\lim_{\kappa_B \to \infty} Var\left(z_{t,n}^{f}\right) = 0$. Therefore, with $z_t^f = \Gamma\left(1 - 1/\kappa_B\right)^{-1}$ [26] and $\kappa_B \to \infty$, the model collapses to the standard rational expectations case with $\mathbb{E}_{m,n,t}$ aligning with $\mathbb{E}_t$. Otherwise, individual expectations deviate from the rational expectation.[27]

We define $\lambda_t^{HB,f}$ as the probability that the f-period bond provides the highest expected discounted return to a family member n. By the properties of the Fréchet distribution, we obtain a nice analytical expression for this probability as

$$\lambda_t^{HB,f} = \left(\frac{z_t^f \mathbb{E}_t\left[Q_{t,t+1}R_{t+1}^{f-1}\right]}{\Phi_t^B}\right)^{\kappa_B}, \tag{7}$$

where $\Phi_t^B \equiv \left[\sum_{j=1}^{F}\left(z_t^j \mathbb{E}_t\left[Q_{t,t+1}R_{t+1}^{j-1}\right]\right)^{\kappa_B}\right]^{\frac{1}{\kappa_B}}$ is an aggregate index that captures the average expected discounted return across bonds of different maturities. Equation (7) implies that demand for savings in the f-maturity bond increases when its return R_{t+1}^{f-1} is relatively higher to that of the average bond return across all maturities, Φ_t^B.

Aggregating across families and family members, we obtain an expression for the household's total holdings of each f-maturity bond as

$$B_t^{H,f} = \lambda_t^{HB,f} \cdot B_t^H, \ \forall f = 1, \ldots, F, \tag{8}$$

[25] See Eaton and Kortum (2002) and Carreras et al. (2021) for applications of the Frechét-distribution and aggregation issues in international trade and macroeconomics literature.

[26] If we set the scale parameter to $z_t^f = \Gamma\left(1 - 1/\kappa_B\right)^{-1}$, then we have $\mathbb{E}(z_{n,t}^f) = 1$, and member-specific expectations fluctuate around the rational expectation.

[27] Thus, our framework nests the above benchmark case (no-arbitrage term structure) as a limiting case.

where B_t^H is the household's aggregate bond holding amounts. Using equation (8), we obtain an aggregate expression for the returns to the household's bond portfolio as

$$R_{t+1}^{HB} = \sum_{f=0}^{F-1} \lambda_t^{HB,f+1} R_{t+1}^f. \tag{9}$$

Now that we have found the allocation of savings across bond maturities, we turn our eyes into the problem of how each family m decides between depositing its savings either in bonds or loans. Family m maximizes savings returns out of the set of possible asset classes (bonds and loans in our model) by solving the following problem:

$$\max \ \mathbb{E}_{m,t}\left[Q_{t,t+1}R_{t+1}^{HB}B_{m,t}^H\right] + \mathbb{E}_{m,t}\left[Q_{t,t+1}R_{t+1}^K L_{m,t}\right] \quad \text{s.t.}$$
$$B_{m,t}^H + L_{m,t} = S_t, \ \ B_{m,t}^H \geq 0, \ \text{ and } \ L_{m,t} \geq 0.$$

Family m takes as given that if it becomes a bond family, it will follow the investment strategy outlined in equation (7) and obtain aggregate returns R_{t+1}^{HB} (equation (9)) on its bond portfolio. In the benchmark rational expectation environment, all families choose the same allocation, and in equilibrium, expected discounted returns $\mathbb{E}_{m,t}\left[Q_{t,t+1}R_{t+1}^{HB}\right]$ and $\mathbb{E}_{m,t}\left[Q_{t,t+1}R_{t+1}^{K}\right]$ are equalized, making families indifferent in their portfolio allocation.

As before, we generate a downward-sloping demands for bonds and loans in the linearized economy by assuming that each family m's expectation operator deviates from the rational expectation as follows:

$$\mathbb{E}_{m,t}\left[Q_{t,t+1}R_{t+1}^K\right] = z_{m,t}^K \cdot \mathbb{E}_t\left[Q_{t,t+1}R_{t+1}^K\right], \tag{10}$$

where $\mathbb{E}_t$ is the rational expectation whereas $\mathbb{E}_{m,t}$ is a family m-specific expectation. We model $z_{m,t}^K$ as a Fréchet-distributed shock with location parameter zero, scale parameter z_t^K and shape parameter κ_S, and assume it to be i.i.d. across families m and quarters t. As before, κ_S governs the expectation shock's volatility, satisfying $\lim_{\kappa_S\to\infty}\text{Var}\left(z_{m,t}^K\right) = 0$. Thus, when $z_t^K = \Gamma\left(1 - 1/\kappa_S\right)^{-1}$ and $\kappa_S \to \infty$, the model collapses to the standard rational expectation case with $\mathbb{E}_{m,t}$ aligning with $\mathbb{E}_t$.

Similar to what we found before, we can now aggregate decisions of each family m and find the share of aggregate savings that will be allocated to loans as

$$\lambda_t^K = \left(\frac{z_t^K \mathbb{E}_t\left[Q_{t,t+1}R_{t+1}^K\right]}{\Phi_t^S}\right)^{\kappa_S}, \tag{11}$$

where $\Phi_t^S = \left[\left(\mathbb{E}_t\left[Q_{t,t+1}R_{t+1}^{HB}\right]\right)^{\kappa_S} + \left(z_t^K\mathbb{E}_t\left[Q_{t,t+1}R_{t+1}^K\right]\right)^{\kappa_S}\right]^{\frac{1}{\kappa_S}}$ is an aggregate index that captures the average expected discounted return of bonds and loans.[28] Using equation (11), we can now express the aggregate amount of savings flowing into bonds of each maturity as

$$B_t^{H,f} = \left(1 - \lambda_t^K\right) \cdot \lambda_t^{HB,f} \cdot S_t, \;\; \forall f = 1, \ldots, F, \tag{12}$$

and the aggregate return on household savings,

$$R_t^S = \left(1 - \lambda_{t-1}^K\right) R_t^{HB} + \lambda_{t-1}^K R_t^K. \tag{13}$$

Observe that R_t^S depends on the rates of all available assets including (i) bonds of different maturities and (ii) loans (private one-period bond) with endogenous weights determined by relative returns of these assets. Finally, we can rewrite the budget constraint in equation (2) as

$$C_t + \frac{S_t}{P_t} = \frac{R_t^S S_{t-1}}{P_t} + \int_0^1 \frac{W_t(\nu) N_t(\nu)}{P_t} \, \mathrm{d}\nu + \frac{\Lambda_t}{P_t} \,. \tag{14}$$

Note that the representative household problem now resembles that of a conventional New-Keynesian model, which constitutes a remarkably tractable result given the asset variety and market segmentation that we introduce.

Remarks on aggregation: The assumption about separate information sets on asset returns (which we model as extreme type Fréchet deviations from the rational equilibrium), effectively creates market segmentations (i) between bond and loan markets, and (ii) among different maturities in the bond market, which is also empirically supported by the literature (see D'Amico and King (2013)). Shape parameters (κ_B, κ_S) control the degree of market segmentation across maturities and assets, respectively, and the conventional expectations hypothesis framework without market segmentation is nested as an special case of our model when κ_B, $\kappa_S \to \infty$. Most notably, the nested CES structure of our asset markets can be easily extended to accommodate a wide variety of assets and maturity structures. Also, the shape parameters (κ_B, κ_S) summarize the demand elasticity of financial products to movements in their expected returns (equations (7) and (11)). These elasticities can take distinct values across asset classes, and they can be easily estimated from the data in order to capture different degrees of market segmentation across assets and maturities.

[28] Observe that equation (11) implies that family preference for issuing loans increases when the return on loans R_{t+1}^K becomes relatively higher to that of the aggregate bond portfolio R_{t+1}^{HB}.

2.2.2 Optimality Conditions

The solution to the household's problem in equation (1) subject to the budget in equation (14) brings the following equilibrium conditions:

$$\left(\frac{N_t(\nu)}{\bar{N}_t}\right)^{\frac{1}{\eta}} = \left(\frac{C_t}{\bar{N}_t}\right)^{-1} \frac{W_t(\nu)}{P_t}, \tag{15}$$

$$1 = \beta \mathbb{E}_t \left[\frac{R^S_{t+1} C_t}{C_{t+1} \Pi_{t+1}} \right], \tag{16}$$

where $\Pi_{t+1} \equiv \frac{P_{t+1}}{P_t}$ is the gross inflation rate. Note that the Euler equation (16) informs us that the effective savings rate R^S_{t+1} has now become the reference rate for the household's intertemporal consumption decisions.

2.3 Capital Producer

There is a representative firm that produces capital K_t and rents it to the intermediate good producers at price P^K_t. Capital is produced by using the final good as an investment input, depreciates at rate δ and there is one-period lag for investment I_t to be deployed as new capital. The evolution of capital is thus given by

$$K_t = (1-\delta)K_{t-1} + I_{t-1}. \tag{17}$$

The profits of the capital producer are given by

$$\Lambda^K_t = P^K_t K_t - P_t I_t, \tag{18}$$

where P_t is the price index of the final good. Solving the capital producer's problem with respect to I_t, we obtain the following first order condition.

$$1 = \mathbb{E}_t \left[Q_{t,t+1} \Pi_{t+1} \left[(1-\delta) + \frac{P^K_{t+1}}{P_{t+1}} \right] \right]. \tag{19}$$

2.4 Firms

There is a continuum $\nu \in [0,1]$ of intermediate goods, each produced by a monopolist ν with the following production function employing capital and labor:

$$Y_t(\nu) = \left(\frac{K_t(\nu)}{\alpha}\right)^{\alpha} \left(\frac{A_t N_t(\nu)}{1-\alpha}\right)^{1-\alpha}, \tag{20}$$

where $A_t = \exp\left(u_t^A\right)$ is an aggregate technology, satisfying $u_t^A = \mu + u_{t-1}^A + \varepsilon_t^A,\ \varepsilon_t^A \sim N\left(0, \sigma_A^2\right)$. We define GA_t as the (gross) rate of change in A_t, thus $GA_t \equiv \frac{A_t}{A_{t-1}} = \exp(\mu + \varepsilon_t^A)$ holds.

A representative, perfectly competitive firm aggregates intermediate products into a final good according to the familiar Dixit-Stiglitz aggregator, as in

$$Y_t = \left[\int_0^1 Y_t(\nu)^{\frac{\epsilon-1}{\epsilon}}\,\mathrm{d}\nu\right]^{\frac{\epsilon}{\epsilon-1}}, \tag{21}$$

where $\epsilon > 1$ is the elasticity of substitution between varieties. Household's demand for intermediate good ν is given by

$$Y_t(\nu) = \left(\frac{P_t(\nu)}{P_t}\right)^{-\epsilon} Y_t\,, \tag{22}$$

where $P(\nu)$ is the price of intermediate ν. The aggregate price index is given by

$$P_t = \left[\int_0^1 P_t(\nu)^{1-\epsilon}\,\mathrm{d}\nu\right]^{\frac{1}{1-\epsilon}}. \tag{23}$$

Intermediate producers have sticky prices à la Calvo (1983) and reset their prices at the beginning of the quarter with probability $1-\theta$. All price-changing firms reset their prices to the same optimal price (in equilibrium) within a given period, which we denote by P_t^*. This allows us to recursively express the equation (23) as

$$P_t^{1-\epsilon} = (1-\theta)\left(P_t^*\right)^{1-\epsilon} + \theta\left(P_{t-1}\right)^{1-\epsilon}. \tag{24}$$

Intermediate producers rent capital at price P_t^K, thus paying $P_t^K K_t(\nu)$ to the capital producer at quarter t. Since firm profits are rebated to the representative household at every quarter, firms are financially constrained and need to borrow through household loans the necessary money to pay for the rented capital.[29] Formally if a firm ν borrows $L_t(\nu)$ from the household, it must satisfy

$$L_t(\nu) \geq P_t^K K_t(\nu), \tag{25}$$

where the equation holds with equality in equilibrium. If a firm ν borrows $L_t(\nu)$ amounts at period t, it pays back $R_{t+1}^K L_t(\nu)$ to the household in the period $t+1$, where R_{t+1}^K is contracted at period t.

An intermediate firm ν maximizes the discounted stream of profits, solving

$$\max \sum_{j=0}^{\infty} \mathbb{E}_t\left[\theta^j Q_{t,t+j}\cdot\left[(1+\zeta^F)P_{t+j}(\nu)Y_{t+j}(\nu) - W_{t+j}(\nu)N_{t+j}(\nu) - R_{t+j}^K P_{t+j-1}^K K_{t+j-1}(\nu)\right]\right], \tag{26}$$

[29] Inada conditions ensure that capital utilization, and therefore loan demand, will stay positive throughout the cycle.

where $Q_{t,t+j} = \beta^j \left(\frac{P_{t+j}}{P_t} \cdot \frac{C_{t+j}}{C_t} \right)^{-1}$ is the household's stochastic discount factor (SDF) between quarter t and $t+j$ and ζ^F is a production subsidy, which is employed to make sure the flexible-price steady state is efficient.[30] Note that at period $t+j$, firm ν pays back $R^K_{t+j} L_{t+j-1}(\nu)$ to the household, as it received a loan that amounts to $L_{t+j-1}(\nu)$ in the previous quarter $t+j-1$.

Minimizing a firm ν's production costs with respect to labor and capital, we obtain the following demand for inputs.

$$N_t(\nu) = (1-\alpha) \frac{Y_t(\nu)}{A_t} \left(\frac{\widetilde{R}^K_t \left(\frac{P^K_t}{P_t} \right)}{\frac{W_t(\nu)}{P_t A_t}} \right)^{\alpha}, \quad \frac{K_t(\nu)}{A_t} = \alpha \frac{Y_t(\nu)}{A_t} \left(\frac{\widetilde{R}^K_t \left(\frac{P^K_t}{P_t} \right)}{\frac{W_t(\nu)}{P_t A_t}} \right)^{-(1-\alpha)}, \tag{27}$$

where $\widetilde{R}^K_t = \mathbb{E}_t \left[Q_{t,t+1} R^K_{t+1} \right]$ is the expected discounted interest rate on loans to be paid back one period later.

Aggregate period t firms' profits are given as

$$\Lambda^F_t = (1+\zeta^F) P_t Y_t - \int_0^1 W_t(\nu) N_t(\nu) \, \mathrm{d}\nu - R^K_t P^K_{t-1} K_{t-1}, \tag{28}$$

which are rebated to the household in a lump-sum way.

2.5 Bond Market

The equilibrium condition in the bond market can be written as

$$B^{H,f}_t + B^{G,f}_t + B^{CB,f}_t = 0, \; \forall f = 1, \ldots, F. \tag{29}$$

where $B^{G,f}_t$ and $B^{CB,f}_t$ are nominal bonds held by government[31] and central bank, respectively. We assume government and central bank are the only agents in the economy capable of issuing riskless claims and therefore capable of holding negative bond positions. For central bank, a negative bond position can be understood as allowing for interest-bearing excess reserves, as we have witnessed after the Global Financial Crisis (GFC) and the subsequent Great Recession.[32] We let the government issue bonds, thus holding negative positions in bonds of each maturity, since risk-free assets would not exist otherwise and there will be no way for the central bank to conduct monetary policy. In particular, our specification of the technology growth GA_t and the population growth GN

[30] See Woodford (2003) and Coibion et al. (2012) for this issue in general New-Keynesian macroeconomics.

[31] Since we assume the government issues bonds across different maturities, $B^{G,f}_t \leq 0$ for all $f = 1 \sim F$.

[32] For theoretical and empirical analyses of the excess reserve's roles in conjunction with the federal fund market and interbank credit market in general, see Frost (1971), Güntner (2015), Mattingly and Abou-Zaid (2015), Primus (2017), and Ennis (2018) among others.

ensures that in steady state the government holds a non-zero (and non-explosive) amount of debt and that therefore it will always be supplying debt despite cyclical fluctuations.

When $\lambda_t^{G,f}$ and $\lambda_t^{CB,f}$ are defined as fractions of nominal f-maturity bond holdings of the government and the central bank, respectively, equation (29) becomes

$$\lambda_t^{HB,f} B_t^H + \lambda_t^{G,f} B_t^G + \lambda_t^{CB,f} B_t^{CB} = 0, \ \forall f = 1, \ldots, F. \tag{30}$$

2.6 Government

The budget constraint of the government is given by

$$G_t + \zeta^F Y_t + \frac{B_t^G}{P_t} = T_t + \frac{R_t^G B_{t-1}^G}{P_t}, \text{ with } B_t^G = \sum_{f=1}^{F} B_t^{G,f}, \ R_t^G = \sum_{f=0}^{F-1} \lambda_{t-1}^{G,f+1} R_t^f, \tag{31}$$

where B_t^G is the government's nominal bond position, G_t is real government spending, T_t are taxes, and R_t^G is the aggregate bond return that the government's portfolio $\{B_{t-1}^{G,f}\}$ yields, with $\lambda_t^{G,f}$ being a fraction of government bonds put into f-maturity bond, formally

$$B_t^{G,f} = \lambda_t^{G,f} \cdot B_t^G, \ \forall f = 1, \ldots, F, \tag{32}$$

where both $\lambda_t^{G,f}$ and B_t^G are exogenous.[33] We can rewrite the budget constraint in equation (31) as

$$\frac{B_t^G}{P_t} = \frac{R_t^G B_{t-1}^G}{P_t} - \left[\zeta_t^G + \zeta^F - \zeta_t^T\right] Y_t \,, \tag{33}$$

where $\zeta_t^G = \frac{G_t}{Y_t}$ and $\zeta_t^T = \frac{T_t}{Y_t}$ are the government spending and taxation share of GDP, respectively. Those two variables ζ_t^G and ζ_t^T are also exogenous in our framework.

2.7 Central Bank

The central bank receives the following amount of profits out of bond holding on its balance sheet:

$$\Lambda_t^{CB} = R_t^{CB} B_{t-1}^{CB} - B_t^{CB}, \text{ with } B_t^{CB} = \sum_{f=1}^{F} B_t^{CB,f}, \ R_t^{CB} = \sum_{f=0}^{F-1} \lambda_{t-1}^{CB,f+1} R_t^f, \tag{34}$$

where B_t^{CB} is central bank's total nominal bond position across different maturities and R_t^{CB} is the aggregate index of bond returns to the central bank's portfolio $\{B_{t-1}^{G,f}\}$, with $\lambda_t^{CB,f}$ being the fraction

[33] We assume away the government's optimal bond portfolio problem, and thus, assume that its gross bond positions and its portfolios across maturities are exogenously given and focus on the central bank's conduct of monetary policy.

of the central bank's bonds held at maturity F, formally

$$B_t^{CB,f} = \lambda_t^{CB,f} \cdot B_t^{CB}, \ \forall f = 1, \dots, F, \tag{35}$$

where B_t^{CB} and $\lambda_t^{CB,f}$ depend on monetary policy rules, which are to be described shortly. Central bank's profit at quarter t, Λ_t^{CB} in equation (34), is lump-sum transferred to the household as part of the total transfer Λ_t in equation (2).

2.8 Monetary Policy

Since equation (35) introduced F new equations to the model, central bank's monetary policy has F degrees of freedom to fill in for the model to have a determinate nominal equilibrium.[34] Monetary authorities might choose to implement one of the following policies:

1. For a f-maturity bond, set a rule on $B_t^{CB,f}$. Then f-maturity bond's prices (and thus its yields) adjust correspondingly.
2. For a f-maturity bond, set a rule on its yield YD_t^f (or equivalently its price Q_t^f). Then purchase amounts of f-maturity bond $B_t^{CB,f}$ adjust accordingly.
3. A combination of the previous two for different maturities.

The standard monetary policy rule in conventional New-Keynesian models can be thought of as a special case of 3. Basically, central bank controls the shortest yield YD_t^1 in those models (therefore adjusting its holding of the shortest-term bond to ensure the bond market is equilibrated) while not changing any positions on long-term bonds. The case 1 resembles textbook money supply rules, whereas not money but long-term bond supplies are controlled by the central bank in this case. A policy avenue often called 'yield-curve control (YCC)', which Japan employed in 2016, can be interpreted as the case of 2.[35] In this paper, we consider following two possible policy regimes.

Standard policy "Standard" monetary policy on the short term interest rate aligns with the case 3, with the central bank setting a rule on YD_t^1 while not manipulating longer term bonds. We assume

[34] Basically, central bank chooses its bond portfolios across different maturities $\{\lambda_t^{CB,f}\}$, as well as gross debt position B_t^{CB}, the former of which (portfolio) is usually abstracted away in standard New-Keynesian models without an explicit term-structure.

[35] For example, on September 21, the Bank of Japan combined a new long-term interest rate target with its existing short-term interest rate target to give the bank 'yield-curve control', with which the Bank of Japan set its short-term policy target—a rate paid on bank reserves—at -0.1% and capped its long-term target rate—that on 10-year government bonds—at approximately zero for the time being. For the case of the United States, see Humpage (2016) for the Fed's yield curve control policy in the WW2 era and possible benefits and costs of the policy.

that the central bank does not change its (normalized) positions for long-term bonds, as follows.

$$R^0_{t+1} \equiv YD^1_t = \max\left\{YD^{1*}_t, 1\right\}, \tag{36a}$$

$$YD^{1*}_t = \overline{YD}^1 \left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\gamma_\pi} \left(\frac{Y_t}{\bar{Y}}\right)^{\gamma_y} \cdot \exp\left(\tilde{\varepsilon}^{YD^1}_t\right), \tag{36b}$$

$$\frac{B^{CB,f}_t}{A_t \bar{N}_t P_t} = \frac{\overline{B^{CB,f}}}{A\bar{N}P} \ \forall f = 2, \ldots, F, \tag{36c}$$

where YD^{1*}_t follows a usual Taylor rule targeting inflation and output, with $\tilde{\varepsilon}^{YD^1}_t$ being a conventional monetary policy shock. When YD^{1*}_t becomes less than 1, monetary policy is constrained by zero lower bound (ZLB), thus $R^0_{t+1} \equiv YD^1_t = 1$, as implied by equation (36a).[36]

General policy A "general" monetary policy is the case in which central bank targets all the yields along the yield curve, assuming a Taylor rule for each maturity bond as

$$YD^{GP,1}_t = \max\left\{YD^{1*}_t, 1\right\}, \tag{37a}$$

$$YD^{1*}_t = \overline{YD}^f \left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\gamma^1_\pi} \left(\frac{Y_t}{\bar{Y}}\right)^{\gamma^1_y} \cdot \exp\left(\tilde{\varepsilon}^{YD^1}_t\right), \tag{37b}$$

$$YD^{GP,f}_t = \overline{YD}^{GP,f} \left(\frac{YD^{SP,f}_t}{\overline{YD}^{SP,f}}\right)^{\gamma^f_{SP}} \left[\left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\gamma^f_\pi} \left(\frac{Y_t}{\bar{Y}}\right)^{\gamma^f_y} \cdot \exp\left(\tilde{\varepsilon}^{YD^f}_t\right)\right]^{1-\gamma^f_{SP}}, \quad f \geq 2. \tag{37c}$$

In equation (37c), $YD^{SP,f}_t$ is the f-maturity yield that prevails in a counterfactual economy where the current yields are determined by the standard monetary policy, given the correct expectation that the future economy is driven by the general monetary policy rule. $\gamma^f_{SP} = 1$ corresponds to the standard monetary policy case. $\gamma^f_{SP} = 0$ case corresponds to the general monetary policy with the inflation targeting Taylor-type policy rule.[37,38]

A responsiveness to inflation γ^f_π and output γ^f_y can be different across maturities $f = 1 \sim F$, and

[36] For example, a sudden hike in the preference parameter z^1_t induces the household to raise demands for the shortest-term bond, which imposes recessionary pressures and drags the economy into ZLB episodes, as the household reduces her own consumption, and the shortest-yield does not go below 0.

[37] Standard monetary policy assumes that the central bank targets its long-term bond purchase amounts as assumed in equation (36c), while the general monetary policy controls yields of long-term bonds, targeting inflation and output components, which makes central bank's long-term bond purchase amounts endogenously determined at levels that support the controlled yields in equation (37c).

[38] A long-term yield $YD^{GP,f}_t$'s endogenous response to the business cycle fluctuation in the general policy case shares some similarities to Sims and Wu (2021), except Sims and Wu (2021) considered the endogenous response of the central bank's bond holdings (B^{CB}_t in our model) to the business cycle fluctuation (such as gross inflation Π_t) while we consider the scenario where central bank directly controls long-maturity yields to target inflation and output.

therefore, $\gamma_{SP}^{f}=0$ corresponds to the case where the maturity-f bond yield (for $\forall f \geq 2$) deviates from what is implied by the standard monetary policy, responding systematically to a fluctuation in inflation and output with the responsiveness parameters γ_{π}^{f} and γ_{y}^{f}. This formulation is convenient since the model encompasses both standard and general policy regimes, according to the value of γ_{SP}^{f}. Especially, $\gamma_{SP}^{f}=0$ corresponds to the case where the yield for maturity-f bond is given by

$$YD_{t}^{GP,f}=\overline{YD}^{GP,f}\left(\frac{\Pi_{t}}{\bar{\Pi}}\right)^{\gamma_{\pi}^{f}}\left(\frac{Y_{t}}{\bar{Y}}\right)^{\gamma_{y}^{f}}\cdot\exp\left(\tilde{\varepsilon}_{t}^{YD^{f}}\right), \tag{38}$$

as in a complete yield-curve control policy. Interim values ($0<\gamma_{SP}^{f}<1$) correspond to the cases in which monetary authority balances between targeting long-term yields directly (a complete yield-curve control) and fixing amounts of long-term bonds purchase (targeting only the short yield). $\tilde{\varepsilon}_{t}^{YD^{f}}$ is a monetary policy shock to the setting of f-maturity bond's yield.

2.9 Market Clearing

Using the bond market equilibrium (equation (29)), we can express total transfers to the household from firms, central bank, capital producer, and government as

$$\Lambda_{t}\equiv\Lambda_{t}^{F}+\Lambda_{t}^{CB}+\Lambda_{t}^{K}-P_{t}T_{t}=P_{t}Y_{t}-P_{t}G_{t}-\int_{0}^{1}W_{t}(\nu)N_{t}(\nu)\,\mathrm{d}\nu+S_{t}-R_{t}^{S}S_{t-1}-P_{t}I_{t}\,. \tag{39}$$

Combining equation (39) with the household's budget constraint (equation (2)), we obtain the following usual aggregate market clearing condition.

$$C_{t}+G_{t}+I_{t}=Y_{t}, \tag{40}$$

which can be rewritten as

$$C_{t}=(1-\zeta_{t}^{G})Y_{t}+(1-\delta)\,K_{t}-K_{t+1}. \tag{41}$$

2.10 Aggregation

Aggregating labor demand (in equation (27)) across firms results in

$$\frac{N_{t}}{\bar{N}_{t}}=(1-\alpha)^{\left(\frac{\eta}{\eta+\alpha}\right)}\left(\frac{C_{t}}{A_{t}\bar{N}_{t}}\right)^{-\alpha\left(\frac{\eta}{\eta+\alpha}\right)}\left(\frac{Y_{t}}{A_{t}\bar{N}_{t}}\right)^{\left(\frac{\eta}{\eta+\alpha}\right)}\left(\widetilde{R}_{t}^{K}\frac{P_{t}^{K}}{P_{t}}\right)^{\alpha\left(\frac{\eta}{\eta+\alpha}\right)}\Delta_{t}^{\frac{\eta}{\eta+1}}, \tag{42}$$

where Δ_t is a measure of price dispersion that can be recursively defined as

$$\Delta_t = (1-\theta)\left(\frac{P_t^*}{P_t}\right)^{-\epsilon\left(\frac{\eta+1}{\eta+\alpha}\right)} + \theta \Pi_t^{\epsilon\left(\frac{\eta+1}{\eta+\alpha}\right)} \Delta_{t-1}. \tag{43}$$

We see, from equation (42), the labor N_t that supports given (normalized) consumption and output increases with price dispersion Δ_t, which proxies the inefficiency caused by nominal rigidities. Given the (normalized) output level, a higher consumption leads to a lower N_t as the household's marginal utility of consumption falls, and firms substitute labor with capital. A rise in either $\widetilde{R}_t^K$ or $\frac{P_t^K}{P_t}$ raises the rental cost of capital, thus inducing firms to substitute capital with labor and raising N_t. This channel also can be seen in the following aggregate capital equilibrium condition.[39]

$$\frac{K_t}{A_{t-1}\bar{N}_{t-1}} = \alpha(1-\alpha)^{\frac{1-\alpha}{\eta+\alpha}} \cdot GA_t \cdot GN \cdot \left(\frac{C_t}{A_t\bar{N}_t}\right)^{\frac{\eta(1-\alpha)}{\eta+\alpha}} \left(\frac{Y_t}{A_t\bar{N}_t}\right)^{\frac{\eta+1}{\eta+\alpha}} \left(\widetilde{R}_t^K \frac{P_t^K}{P_t}\right)^{-\left(\frac{\eta(1-\alpha)}{\eta+\alpha}\right)} \Delta_t. \tag{44}$$

The aggregate (normalized) capital K_t rises when consumption, output, or price dispersion measure rise and the rental cost of capital falls. Therefore, the above two equations emphasize roles of firms' substitutions between capital and labor in the production stage.[40]

As the supply block (price-resetting decisions of firms, where price-resetting firms solve equation (26)) is similar to the one in standard New-Keynesian models,[41] we turn our eyes to the demand block. The representative household satisfies the Euler-equation (equation (16)) where R_{t+1}^S satisfies equation (13), λ_t^K satisfies equation (11), and $\lambda_t^{HB,f}$ is given by equation (7).

The equilibrium condition of the household's allocation between loans and bonds can be written as

$$\frac{B_t^H}{A_t\bar{N}_tP_t} = \frac{1}{GA_t \cdot GN}\left(\frac{1-\lambda_t^K}{\lambda_t^K}\right)\left(\frac{P_t^K}{P_t}\right)\left(\frac{K_t}{A_{t-1}\bar{N}_{t-1}}\right), \tag{45}$$

where we used $B_t^H = \sum_{f=1}^F B_t^{H,f}$ and $L_t = \int P_t^K K_t(\nu)d\nu = P_t^K K_t$. The above condition implies that households put $1-\lambda_t^K$ portion of her wealth into the bond market, while lending the remaining λ_t^K portion to firms. λ_t^K depends on endogenous rates of bonds and loans, R_t^{HB} and R_t^K, as implied by equation (11).

[39] Note that we normalize K_t by $A_{t-1}\bar{N}_{t-1}$ since K_t is determined at the previous quarter $t-1$. In contrast, aggregate labor N_t, consumption C_t, and output Y_t are all normalized by $A_t\bar{N}_t$, whereas a rental price of capital P_t^K is normalized by the nominal price index P_t.

[40] Since each firm has the usual Cobb-Douglas production function, the elasticity of substitution between capital and labor becomes 1.

[41] Price-resetting firms' optimal pricing decisions (equation (26)) and aggregation are derived in Appendix B. Derivation and Proofs.

Bond market equilibrium (equation (30)) can be rewritten as

$$\frac{B_t^H}{A_t \bar{N}_t P_t} = -\left(\frac{\lambda_t^{CB,1} - \lambda_t^{G,1}}{\lambda_t^{CB,1} - \lambda_t^{HB,1}}\right) \frac{B_t^G}{A_t \bar{N}_t P_t}. \tag{46}$$

Given $B_t^G < 0$ and $\lambda_t^{G,1} > 0$, a higher $\lambda_t^{CB,1}$ implies central bank purchases less amounts of bonds with maturity $f > 1$, thus given the total bond holding B_t^H, the household reduces $\lambda_t^{HB,1}$ and raises $\lambda_t^{HB,f}$ for $f > 1$, ensuring bond market equilibrium (equation (46) holds). Combining equation (45) and equation (46), we obtain the following equilibrium condition:

$$-\left(\frac{\lambda_t^{CB,1} - \lambda_t^{G,1}}{\lambda_t^{CB,1} - \lambda_t^{HB,1}}\right) \frac{B_t^G}{A_t \bar{N}_t P_t} = \frac{1}{GA_t \cdot GN} \left(\frac{1 - \lambda_t^K}{\lambda_t^K}\right) \left(\frac{P_t^K}{P_t}\right) \left(\frac{K_t}{A_{t-1} \bar{N}_{t-1}}\right). \tag{47}$$

2.10.1 Standard Policy

In the case of standard monetary policy (equation (36b) and equation (36c)), the monetary authority does not manipulate its (normalized) long-term bond holdings, thus normalized $B_t^{CB,f}$ is constant for $f > 1$. In this case, $\lambda_t^{HB,f}$ must satisfy

$$\lambda_t^{HB,f} = -\frac{\frac{B_t^{G,f}}{A_t \bar{N}_t P_t} + \frac{\overline{B^{CB,f}}}{A \bar{N} P}}{\frac{B_t^H}{A_t \bar{N}_t P_t}}, \ \forall f > 1. \tag{48}$$

Plugging the above equation (48) into the bond market equilibrium condition (equation (46)) yields the following equilibrium condition.

$$-\left(\frac{B_t^{G,f}}{A_t \bar{N}_t P_t} + \frac{\overline{B^{CB,f}}}{A \bar{N} P}\right) \left(\lambda_t^{HB,f}\right)^{-1} = \frac{1}{GA_t \cdot GN} \left(\frac{1 - \lambda_t^K}{\lambda_t^K}\right) \left(\frac{P_t^K}{P_t}\right) \left(\frac{K_t}{A_{t-1} \bar{N}_{t-1}}\right), \ \forall f > 1. \tag{49}$$

Other equilibrium conditions for the standard policy case are summarized in Appendix.

2.10.2 General Policy

In the case of general monetary policy in which the central bank sets rules on yields for any maturity f (equation (37c)), these yields affect the household's bond portfolio across maturities $\lambda_t^{HB,f}$ and the effective bond rate R_t^{HB} through the following relations:

$$\lambda_t^{HB,f} = \left(\frac{z_t^f \mathbb{E}_t \left[Q_{t,t+1} R_{t+1}^{f-1} \right]}{\Phi_t^B} \right)^{\kappa_B}, \quad R_{t+1}^{f-1} = \frac{\left(YD_{t+1}^{f-1} \right)^{-(f-1)}}{\left(YD_t^f \right)^{-f}}. \tag{50}$$

A change in the household's bond portfolio across maturities $\{\lambda_t^{HB,f}\}$ results in changes in effective bond rates R_t^{HB} (through equation (9)) and loan rates R_t^K (through equation (11)), effective savings rate R_t^S (through equation (13)), consumption (through Euler equation in equation (16)), and other aggregate outcomes (through equation (41), equation (42), equation (44), and equation (45)).

Other equilibrium conditions for the general policy case are summarized in Appendix.

2.11 Shock Processes

For Fréchet distributions' scale parameters z_t^f for $\forall f$ and z_t^K, we assume that both parameters follow $AR(1)$ processes, satisfying $z_t^f = \rho_z z_{t-1}^f + \varepsilon_t^{z,f}$, with $\mathrm{Var}(\varepsilon_t^{z,f}) = (\sigma_z)^2$, and $z_t^K = \rho_z^K z_{t-1}^K + \varepsilon_t^{z,K}$, with $\mathrm{Var}(\varepsilon_t^{z,K}) = (\sigma_z^K)^2$.

For government spending ratio $\zeta_t^G = \frac{G_t}{Y_t}$ and revenue ratio $\zeta_t^T = \frac{T_t}{Y_t}$, we assume the following shock processes:

$$\zeta_t^G = \frac{1}{1 + a^G \exp\left(-u_t^G\right)}, \quad \zeta_t^T = \frac{1}{1 + a^T \exp\left(-u_t^T\right)}. \tag{51}$$

where a^G and a^T are constants, and u_t^G and u_t^T follow usual AR(1) processes, given as $u_t^G = \rho_G u_{t-1}^G + \varepsilon_t^G$, $u_t^T = \rho_T u_{t-1}^T + \varepsilon_t^T$, with ε_t^G and ε_t^T being i.i.d shocks. For technological progress GA_t, we assume $GA_t = \exp(\mu + \varepsilon_t^A)$ where ε_t^A is i.i.d.

Since the government's bond shares across different maturities $\{\lambda_t^{G,f}\}$ are exogenous, we specify their processes in the following way.

$$\lambda_t^{G,1} = \frac{1}{1 + \sum_{l=2}^{F} a^{B,l} \exp\left(\tilde{u}_t^{B,l}\right)}, \quad \lambda_t^{G,f} = \frac{a^{B,f} \exp\left(\tilde{u}_t^{B,f}\right)}{1 + \sum_{l=2}^{F} a^{B,l} \exp\left(\tilde{u}_t^{B,l}\right)}, \quad \forall f > 1, \tag{52}$$

where $a^{B,f}$, $\forall f > 1$ are constants. When F is large, we reduce the number of shocks to the government's bond shares to $J \leq F$, assuming

$$\tilde{u}_t^{B,f} = \sum_{j=1}^{J} \tau_{fj}^B u_t^{B,j}, \tag{53}$$

where $u_t^{B,j}$ follows $u_t^{B,j} = \rho_B u_{t-1}^{B,j} + \varepsilon_t^{B,j}$ with $\varepsilon_t^{B,j}$ being i.i.d., $\tau^{B,j}$, $\forall j$ are constants, and $J \leq F$ holds.

In the general monetary policy case, where the central bank sets rules on yields across different maturities (as in equation (37c)), we also can reduce the state-space, by assuming monetary policy shocks to different maturities $\{\tilde{\varepsilon}_t^{YD^f}\}$ are represented as linear combinations of several factors, as

$$\tilde{\varepsilon}_t^{YD^f} = \sum_{l=1}^{L} \tau_{f,l}^{YD} \varepsilon_t^{YD^l}, \tag{54}$$

where $\tau^{YD,l}$, $\forall l$ are constants, $\varepsilon_t^{YD^l}$ are i.i.d. shocks, and $L \leq F$ holds.

All equilibrium conditions are summarized in Appendix C.1. Equilibrium Equations: Standard Policy and Appendix C.2. Equilibrium Equations: General Policy.

3 Steady-State (Long-Run) Analysis

3.1 Steady-State Relations

In the steady-state, the central bank decides the level of holdings of f-maturity bond $B^f = \lambda^{CB,f} B^{CB}$. We assume that the total bond holdings of the central bank B^{CB} are a ζ^{CB} fraction of the total government bond issuance B^G, thus $B^{CB} = \zeta^{CB} B^G$ holds.[42]

Given $\left\{\lambda^{CB,f}\right\}_{f=1}^{F}$, the bond market equilibrium (equation (30)) at the steady state can be written as

$$\lambda^{HB,f} = \frac{\lambda^{G,f} + \lambda^{CB,f}\zeta^{CB}}{1+\zeta^{CB}}. \tag{55}$$

Thus, the household's bond portfolio shares across different maturities $\{\lambda^{HB,f}\}_{f=1}^{F}$ are determined by exogenous parameters such as $\{\lambda^{G,f}, \lambda^{CB,f}\}_{f=1}^{F}$ and ζ^{CB}.[43]

The government's budget constraint (equation (33)) in the steady state can be written as

$$\frac{B^G}{A\bar{N}P} = -\left(1 - \frac{R^G}{\Pi \cdot GA \cdot GN}\right)^{-1} \left[\zeta^G + \zeta^F - \zeta^T\right] \frac{Y}{A\bar{N}}. \tag{56}$$

[42]Since $B^G < 0$ and $B^{CB} > 0$ in the steady state, we have $\zeta^{CB} < 0$.

[43]If we define R^f as steady-state value of R_{t+1}^{f-1} (f-maturity bond's one-period return), then from equation (9),

$$\sum_{f=1}^{F} \lambda^{HB,f} \left(\frac{R^f}{R^{HB}}\right) = 1$$

is satisfied. Since $\lambda^{HB,f}$, which is determined by exogenous parameters in equation (55), is determined by a relative size of R^f/R^{HB} compared with Φ^B/R^{HB} (equation (7)), we reverse-engineer the steady-state relative bond rates R^f/R^{HB} for term f through iterations. This procedure is illustrated in Appendix B.1.4. Steady-State Derivations in Section 3.1.

Given (normalized) output level $\frac{Y}{A\bar{N}}$ and primary deficit ratio $\zeta^G + \zeta^F - \zeta^T > 0$, a higher R^G raises a volume of bond issuance $|B^G|$ (since $B^G < 0$) as the government needs to pay more interests out of its own debt position.[44] A higher deficit ratio $\zeta^G + \zeta^F - \zeta^T$ leads to a larger issuance of government bonds $|B^G|$ given output, but pushes output down given the bond issuance volume.

After some manipulations of equations, we can write the equilibrium (normalized) output $\frac{Y}{A\bar{N}}$ as

$$\frac{Y}{A\bar{N}} = \xi^Y \left[(1-\zeta^G) - \xi^C \left(1 - \frac{R^G}{\Pi \cdot GA \cdot GN} \right)^{-1} \left(\frac{\lambda^K}{1-\lambda^K} \right) \right]^{-\frac{\eta}{\eta+1}} \left(R^K \right)^{-\frac{\alpha}{1-\alpha}}, \tag{57}$$

which expresses (normalized) output $\frac{Y}{A\bar{N}}$ as a function of an effective rate, R^G, the rate government pays out of its bond issuance, and a share of household's savings that flow into firms as loans, λ^K.[45] With coefficients $\xi^Y > 0$ and $\xi^C > 0$, we can execute comparative statics in equation (57). When R^K increases, two channels work in an opposite way to affect output $\frac{Y}{A\bar{N}}$. First, an increase in R^K results in a higher rental (loan) cost of capital from firms' perspectives, which reduces aggregate capital and eventually, output.[46] On the other hand, the higher R^K is, λ^K, the share of household savings flowing into firms as loans, soars up as the household shifts its fund out of the bond market and issue more loans to firms. More loans available for firms leads to the higher aggregate capital and the output level. In addition to the effect of R^K on output, note that in equation (57), an increase in R^G results in an increase in (normalized) output.[47]

Other steady-state relationships and procedures for characterizing those conditions are summarized in detail in Appendix B.1.4. Steady-State Derivations in Section 3.1.

3.2 Results

3.2.1 Calibration and Yield Curve

Using publicly available data on (i) treasury yields, (ii) federal reserve's holdings of treasury bonds, and (iii) U.S Treasury's outstanding bonds[48,49] from January 01, 1990 to January 01, 2007, we can calibrate parameters and generate the yield curve that matches with the (time) average yield curve.

[44] We assume $R^G < \Pi \cdot GA \cdot GN$, so that we can have a primary deficit $\zeta^G + \zeta^F - \zeta^T > 0$ while the government issues bonds $B^G < 0$ in the steady state. Our calibration confirms that this condition is satisfied.

[45] Coefficients ξ^Y and ξ^C in equation (57) are given in Appendix B.1.4. Steady-State Derivations in Section 3.1.

[46] Since firms' elasticity of substitution between capital and labor is 1, which is finite, replacement of capital by labor is not enough to prevent the output level from dropping.

[47] In equation (56), given the output level, a higher R^G means the government issues more bonds, thus $|B^G|$ increases. Given λ^K in equation (57), it implies that loans (and capital) rise proportionately, which would increase the output level eventually.

[48] In cases where there are missing data on yields, we relied on interpolation method in order to generate a smooth yield curve.

[49] https://fiscaldata.treasury.gov/datasets/monthly-statement-public-debt

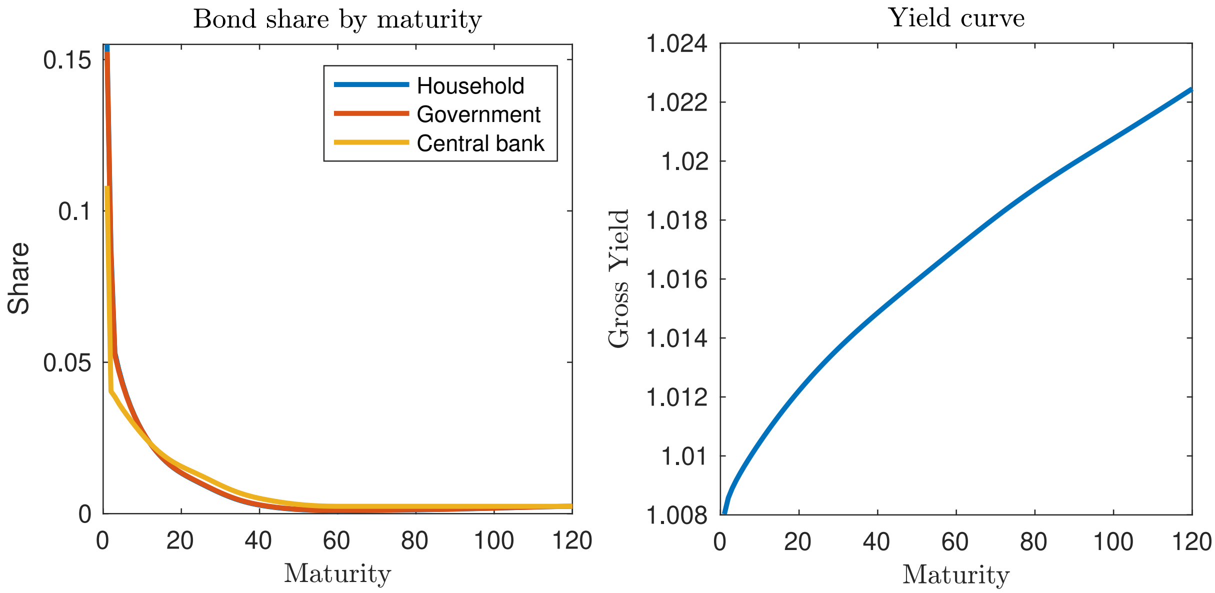

Figure 1: Steady-state bond portfolios of household, government, and central bank, and yield curve

In that purpose, we use $F = 120$ to account for maturities up to 30 years (120 quarters). Parameter values are summarized in Table A1. Most of the relevant parameters are from Coibion et al. (2012).

Remember $z^f_{t,n}$, which governs the household's portfolio demands for maturity-f bonds, follows a Fréchet distribution with z^f_t and κ_B as scale and shape parameters, respectively, and $z^K_{m,t}$, which affects the household's demands for loan issuance, follows a Fréchet distribution with z^K_t and κ_S as scale and shape parameters, respectively. Given κ_B and κ_S, scale parameters $\{z^f_t\}^F_{f=1}$ determine the household's portfolio share for f-maturity bond, and likewise, z^K affects her loan issuance demand relative to investing in the bond market.

Given shape parameters $\kappa_B = 10$ and $\kappa_S = 1$, the scale parameters $\{z^f\}^F_{f=1}$ is calibrated to match the yield curve's shape (relative yields across different maturities) while the scale parameter z^K can be calibrated to match the shortest yield YD^1 and the loan rate R^K. An exact calibration procedure for $\{z^f\}^F_{f=1}$ and z^K is described in Appendix C.3. Calibrating $\{z^f\}$ and z^K in the Steady State.

The following Figure 1 shows bond portfolio shares across maturities of each agent (household, government, and central bank) and the resultant yield curve, with calibrated z^K and $\{z^f\}^F_{f=1}$ being reported in Table A2 and Figure A2. Note that $z^1 = 1$ is especially large compared with z^f for $f \geq 2$, as the shortest-yield is historically low compared with longer-term yields, and it might account for the short-term bonds' safety and/or liquidity premium documented by many literatures, including Krishnamurthy and Vissing-Jorgensen (2012) and Caballero and Farhi (2017).

Figure A3 illustrates the yield curve's flattening behavior as we have a higher κ_B, as documented in Section 2.2.1. When $\kappa_B \to \infty$, we return to the case of expectation hypothesis, and thus obtain a flat yield curve in the steady state.

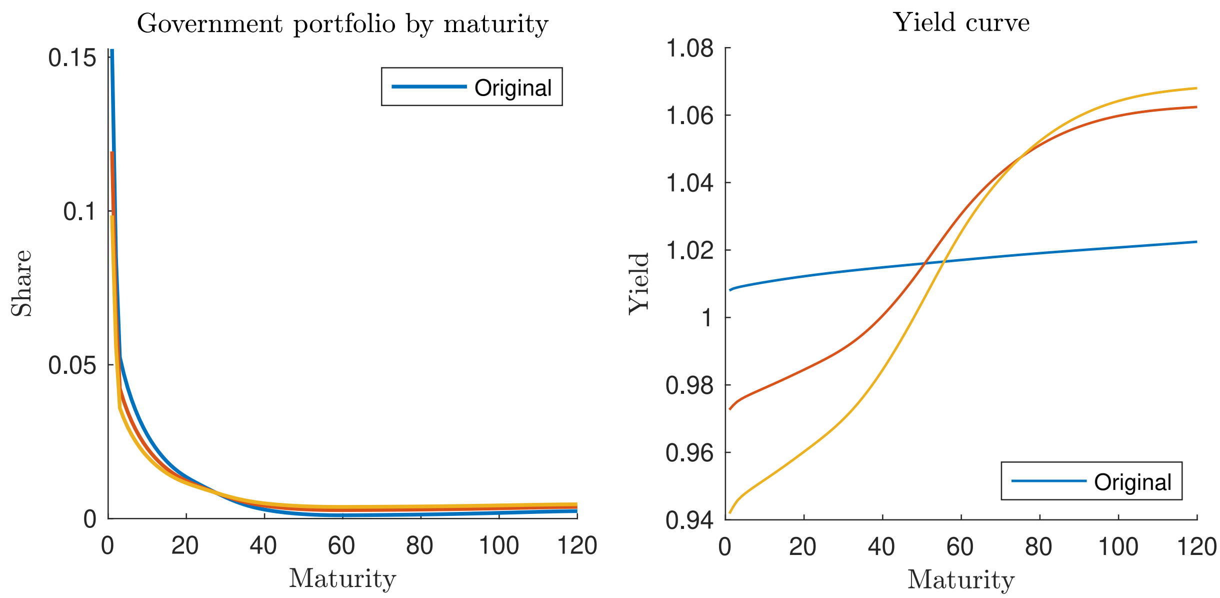

Figure 2: Government's bond issuance portfolio and yield curve

3.2.2 Government's Bond Supply and Central Bank's Bond Demand

Now, we vary bond issuance shares $\{\lambda^{G,f}\}_{f=1}^{F}$ of the government and study how the relative supply of bonds with different maturities affects the equilibrium yield curve in the steady-state. In Figure 2, the left panel describes different portfolio shares of issued government bonds, while the right panel describes resulting changes in the yield curve. It illustrates that a f-maturity bond yield is positively correlated with its relative share of supply $\lambda^{G,f}$, in line with literatures including Krishnamurthy and Vissing-Jorgensen (2012) and Greenwood and Vayanos (2014),[50] and the yield curve responds in a sensitive way to the government's bond portfolio shares. A government's larger relative issuance of a specific maturity bond pushes down its prices and raises the yields, and from the household's portfolio rebalance, it changes equilibrium returns in both bond and loan markets, affecting in turn the government's gross bond issuance, given exogenous fiscal policies.

Figure A4 illustrates opposite cases where central bank changes its bond portfolios. We observe that the central bank's relative purchase of each maturity bond is negatively related with its yield, in line with the literature documenting that the central bank's bond purchase acts as an additional demand for each maturity in segregated markets. For example, Ray (2019) and Droste et al. (2021), based on a preferred-habitat environment developed by Vayanos and Vila (2021), derived the similar implications.[51] Our specification allows us to have integrated bond markets in which the household

[50] For example, Krishnamurthy and Vissing-Jorgensen (2012) found that a higher debt-to-GDP ratio leads to lower credit spreads, and this effect gets stronger for longer maturities. Likewise, Greenwood and Vayanos (2014) documented the supply of long- relative to the short term bonds is positively correlated with the term spread, which we observe in Figure 2.

[51] Krishnamurthy and Vissing-Jorgensen (2011) empirically found that QE policies (both QE1 and QE2) worked

freely chooses her bond portfolio, subject to allocation shocks distributed Fréchet, and generates a similar implication in the steady state.

3.2.3 Other Comparative Statics

Figures A5 and A6 document comparative statics of the deficit ratio $\zeta^F + \zeta^G - \zeta^T$. A higher deficit ratio is possibly sustained only when (i) the government issues more bonds, or (ii) its effective bond rate R^G decreases, or (iii) the output falls and thus the nominal total deficit spending drops too. We first consider (i) and observe it is impossible to sustain in the long-run: if the government issues more debts in order to finance a higher deficit amount (given a fixed output level), it raises the government's effective bond return R^G (supply effect in Section 3.2.2), which results in the issuance of more bonds to finance the additional interest costs, further pushing up R^G, continuing ad infinitum. It turns out that (ii) and (iii) work together: a higher deficit ratio brings down output, consumption, and capital, which lowers the deficit size (nominal) and the government's bond issuance, pushing down its bond return R^G. A loan rate R^K remains almost unchanged and the credit spread $r^K - r^{HB}$ rises in response. Especially, our result that debt-to-GDP ratio $\frac{B^G}{Y}$ drops while the entire yield curve is shifts down in response to a deficit ratio increase is in line with prior literature including Laubach (2009).[52]

Figures A7 and A8 describe comparative statics of the scale parameter z^K that enters in the fund allocation equation between bond and loan markets (equation (11)). Given calibrated $\{z^f\}$ and for $z^K \in [0.2, 2]$, a higher z^K implies the household is more willing to issue loans to firms rather than invest in the bond market, and therefore λ^K rises in response. It brings up capital, output, and consumption in the steady-state. As household increases their loan issuance, the average marginal propensity to consume (MPC) drops, and the loan rate R^K falls, which drags the entire yield curve downward due to the household's endogenous fund reallocation, resulting ironically in a higher credit spread. As R^G falls, the government bond share with respect to GDP falls too.

Figures A9 and A10 describe comparative statics of the shape parameter κ_S that enters in the same fund allocation between bond and loan markets (equation (11)). Given calibrated $\{z^f\}$ and z^K values and for $\kappa_S \in [0.5, 3]$, a higher κ_S brings down λ^K, the household's loan share out of her total savings. It raises the loan rate R^K, and reduces capital (as firms face a higher marginal cost), output, and consumption while raising an average marginal propensity to consume (MPC). Credit spreads increase, with a higher R^K bringing up the government's effective bond return R^G and the

through several channels that affect particular assets in different manners. Especially, QE2, which had been primarily focused on treasury bonds, had a disproportionate effect on Treasuries and Agencies relative to mortgage-backed securities and corporates. Also, D'Amico and King (2013) identified QE programs' both stock and flow effects on treasury yields, and supported a view of segmented markets or imperfect substitution within the Treasury market.

[52] Laubach (2009) empirically found that a 1% point increase in the projected debt-to-GDP ratio is estimated to raise long-term interest rates by roughly 3-4 basis points.

entire yield curve. Government ends up issuing more bonds per output.

4 Short-Run Analysis

4.1 Log-linearization

In this section, we present a solution of our dynamic model under a first-order log-approximation. We use lower-case letters to denote properly normalized variables,[53] while hats correspond to their log-deviations from steady state values.[54] Since the system is quite complicated, we discuss only a few key logics and equilibrium equations here and relegate the detailed derivations and remaining equations to Appendix B. Derivation and Proofs.

Linearizing the Euler-equation (equation (16)) yields the usual dynamic IS equation with effective savings rates $\hat{r}^S_{t+1}$ of the household:

$$\hat{c}_t = \mathbb{E}_t\left[\hat{c}_{t+1} - \left(\hat{r}^S_{t+1} - \hat{\pi}_{t+1}\right)\right], \tag{59}$$

where $\hat{r}^S_t$ can be derived from equation (13) as

$$\hat{r}^S_t = \frac{\lambda^K\left(R^K - R^{HB}\right)}{R^S}\hat{\lambda}^K_{t-1} + \frac{(1-\lambda^K)R^{HB}}{R^S}\hat{r}^{HB}_t + \frac{\lambda^K R^K}{R^S}\hat{r}^K_t. \tag{60}$$

We observe $\hat{r}^S_t$ depends on the household's effective bond rates $\hat{r}^{HB}_t$ and the loan rate $\hat{r}^K_t$ in equation (60), as well as the share of savings flowing into firms as loans, $\hat{\lambda}^K_t$. Since $\hat{\lambda}^K_t$ is endogenous and determined by relative rates between bond market ($\hat{r}^{HB}_t$) and loan market ($\hat{r}^K_t$), first we characterize $\hat{r}^{HB}_t$ by linearizing equation (9) and obtain

$$\hat{r}^{HB}_t = \sum_{f=1}^{F}\frac{\lambda^{HB,f}\left(YD^{f-1}\right)^{-(f-1)}}{R^{HB}\left(YD^f\right)^{-f}}\left[\hat{\lambda}^{HB,f}_{t-1} - (f-1)\cdot\hat{yd}^{f-1}_t + f\cdot\hat{yd}^f_{t-1}\right], \tag{61}$$

where $\hat{r}^{HB}_t$ relies on yields in the previous quarter $\{\hat{yd}^f_{t-1}\}^F_{f=1}$ as well as current yields $\{\hat{yd}^{f-1}_t\}^F_{f=1}$, because a f-maturity bond's holding return is determined by its quarter-to-quarter price change and

[53] For example, we define (properly) normalized variables such as

$$k_t \equiv \frac{K_t}{A_{t-1}\bar{N}_{t-1}},\quad y_t \equiv \frac{Y_t}{A_t\bar{N}_t},\quad c_t \equiv \frac{C_t}{A_t\bar{N}_t},\quad n_t \equiv \frac{N_t}{\bar{N}_t},\quad p^K_t \equiv \frac{P^K_t}{P_t}. \tag{58}$$

where we divide K_t by $A_{t-1}\bar{N}_{t-1}$ instead of $A_t\bar{N}_t$ since K_t is determined at quarter $t-1$, not t.

[54] We have $\hat{\pi}_t = \pi_t - \bar{\pi}$, where π_t and $\bar{\pi}$ are a net inflation rate and its positive steady state level. Similarly, we have positive steady-state yields (including the short yield, which is the policy rate).

equivalently, yields. In equation (61), $\hat{r}_t^{HB}$ depends on $\hat{\lambda}_{t-1}^{HB,f}$, the share of household's bond savings put into the f-maturity bond, while $\hat{\lambda}_{t-1}^{HB,f}$ itself is endogenously determined by relative magnitudes of holding returns of bonds with different maturities. We can break this circular feedback loop by using the household's optimal bond portfolio (equation (7)) and writing bond shares $\{\hat{\lambda}_{t-1}^{HB,f}\}_{f=1}^{F}$ as functions of yields $\{\hat{yd}_{t-1}^{f}\}_{f=1}^{F}$ and $\{\hat{yd}_{t}^{f-1}\}_{f=1}^{F}$. By linearizing equation (7), we obtain

$$\hat{\lambda}_{t-1}^{HB,f} = \kappa^B \mathbb{E}_{t-1}\left[\hat{z}_{t-1}^{f} - \hat{\pi}_t + \hat{c}_{t-1} - \hat{c}_t - (f-1)\cdot \hat{yd}_t^{f-1} + f\cdot \hat{yd}_{t-1}^{f} - \hat{\phi}_{t-1}^{B}\right], \tag{62}$$

where $\hat{\phi}_t^B$ also contains $\{\hat{yd}_t^{f-1}\}_{f=1}^{F}$, $\{\hat{yd}_{t-1}^{f}\}_{f=1}^{F}$, and other aggregate variables. Plugging the above equation (62) into equation (61) allows us to express the household's effective bond rate as a function of yields along the yield curve, taking its endogenous portfolio into account.

The relation between $\hat{r}_t^K$ and λ_t^K is characterized by linearizing the household's optimal portfolio between bond and loan markets (equation (11)) and is written as

$$\hat{\lambda}_t^K = \kappa^S\left(1-\lambda^K\right)\left(\hat{z}_t^K + \mathbb{E}_t\left[\hat{r}_{t+1}^K - \hat{r}_{t+1}^{HB}\right]\right), \tag{63}$$

where hikes in preference parameter $\hat{z}_t^K$ and the relative return of loans compared to bonds $\hat{r}_{t+1}^K - \hat{r}_{t+1}^{HB}$ raises $\hat{\lambda}_t^K$. $\hat{r}_{t+1}^K$ has a direct impact on the effective savings rate $\hat{r}_{t+1}^S$ (equation (60)), and therefore, affects consumption dynamics through the intertemporal substitution channel (equation (59)). In addition, a change in $\hat{r}_{t+1}^K$ changes firms' rental cost of capital, affecting aggregate capital, output, and consumption through labor aggregation (equation (42)) and loan aggregation (equation (44)). This supply block of the economy (firms' staggered pricing and loan demands) again interacts with the demand block represented by our dynamic IS equation (equation (59)).

Standard policy: In the case of standard monetary policy, we have to consider how the change in $\hat{yd}_t^1$ (which is set by the central bank) results in different portfolio shares $\{\hat{\lambda}_t^{HB,f}\}$ across maturities. This is done by linearizing the bond market equilibrium condition[55] (equation (49)) and combining it with all other building blocks of the model. In this way, we get a joint dynamic equation for yields $\{\hat{yd}_t^f\}$, which describes a vector of yields as a function of aggregate variables, while $\hat{yd}_t^1$ follows the Taylor rule in equation (36b). The yields $\{\hat{yd}_t^f\}$ affect returns for bonds, loans, and savings through endogenous shifts in allocation of funds and feed back into the dynamic IS equation, the supply block of the model, and aggregation conditions (including labor aggregation (equation (42)) and loan aggregation (equation (44))).

[55] In the standard monetary policy case, a normalized purchase amount of $f \geq 2$-maturity bonds by central bank is fixed.

General policy: General monetary policy is similar to the standard policy in our framework, except that yields $\{\hat{yd}_t^f\}$ include Taylor rule components in addition to the counterfactual standard policy yields, as prescribed in equation (37c). All those yields jointly determine returns for bonds, loans, and savings through endogenous shifts in allocation of funds, and the effective savings rate $\hat{r}_{t+1}^S$, which governs the household's intertemporal substitution (equation (59)). In either case (standard or general), a resultant linear system becomes close to the benchmark New-Keynesian model and turns out to be tractable enough despite a complex nature of our framework.[56] Inflation targeting components on yields in general policy might help central bank achieve a better stabilization.

All other remaining linearized equilibrium conditions are summarized in Appendix C.4. Summary of Standard Policy Linearized Equations (standard policy case) and Appendix C.5. Summary of General Policy Linearized Equations (general policy case).

4.2 Welfare

To compare welfare for different policy regimes, we use a second-order approximation to the household utility function as in Woodford (2003) and Coibion et al. (2012). Because of a positive steady-state inflation ($\Pi > 1$) as in Coibion et al. (2012), our second-order approximated welfare would have a first-order constant term that represents the welfare loss caused by the steady-state's deviation from the efficient (flexible-price) steady-state.[57]

With necessary steps similar to the steps that are employed in Coibion et al. (2012), the second-order approximation to the household's welfare is summarized by the following Proposition 1.

Proposition 1 *A 2^{nd}-order approximation to the expected per-period welfare of the household would be given as*

$$\mathbb{E}U_t - \bar{U}^F = \Omega_0 + \Omega_n \text{Var}(\hat{n}_t) + \Omega_\pi \text{Var}(\hat{\pi}_t) + t.i.p + h.o.t,$$

where the coefficients Ω_0, Ω_n, and Ω_π are given in equation (245), equation (246), and equation (247). $\bar{U}^F$ is the efficient (flexible-price) steady-state utility of the household, as our approximation is around the efficient steady-state allocation.

A detailed derivation of the welfare in Proposition 1 is given in Appendix D. Welfare. Especially, $\Omega_0 < 0$ term arises with a positive steady-state inflation that causes an efficiency loss of the steady-state allocation compared to the efficient (flexible-price) steady-state. Since we fix the trend-inflation

[56]With general policy, all yields $\{\hat{yd}_t^f\}$ include counterfactual standard policy yields $\{\hat{yd}_t^{SP,f}\}$ that evolve endogenously, in addition to inflation targeting components. Therefore, our general policy solution encompasses the standard policy one as a special case, since we must obtain the endogenous dynamics of counterfactual standard policy yields even in solving the general policy equilibrium.

[57]Due to the positive trend-inflation, our steady-state features non-zero price dispersion, causing inefficiency up to a first-order. See Woodford (2003) and Coibion et al. (2012) for the non-zero price dispersion at the steady-state with a positive trend inflation.

at 2% annually, this term can actually be included in the t.i.p. The coefficients Ω_n and Ω_π that are attached to the variance terms of labor and inflation gaps, respectively, turn out to be negative, and depend on the steady-state allocation.

Our welfare characterization is useful for comparison between different monetary policy regimes: (i) standard policy, (ii) general policy, and (iii) mixed policy. Mixed policy is the policy where the central bank implements general policy only when the short rate hits the zero lower bound (ZLB). Otherwise in normal cases, mixed policy is identical to the standard policy, in which central bank controls only the shortest-maturity yield. Therefore, it is the closest to what central banks around the globe have actually implemented as their policy rules for stabilization purposes.

4.3 Results

4.3.1 Impulse-Response without the ZLB

First, we present impulse-response functions to various shocks in cases where the economy does not enter the ZLB: z^1 and z^K (scale parameters of the Fréchet distributions that govern the household's bond and loan preferences in financial markets), ε^A (technology growth shock), ε^{YD^1} (conventional monetary policy shock), and ε^T (tax, or in general, fiscal shock).

z^1 shock: First, Figure 3 presents an impulse-response to z^1 shock, which drives the household's portfolio demand for the shortest maturity bond. The blue lines depict impulse responses under the standard monetary policy framework, and red lines represent the impulse responses under the general policy setting.

With standard policy, a hike in z^1 goes hand in hand with an increase in the household's portfolio demand for the short maturity bond. It will bring the short rates down, which eventually bring other bond market returns and the capital return, as well as the wage, all down as the household re-optimizes its portfolio choices and firms substitute between capital and labor.[58] resulting in a lower inflation. Output falls as labor supply decreases with the falling wage. The initial monetary policy response is helpful for boosting aggregate demand (from consumption and investment), but is not sufficient to prevent output from falling. Therefore, under our calibration, one standard-deviation jump in z^1 reduces output by 4%.

General policy is powerful in terms of insulating the economy from the z^1 shock. The reason is simple: as z^1 shock distorts the household's bond-portfolio decisions mainly and the bond market

[58] The representative household's endogenous portfolio choice (as a function of relative rates) is crucial in generating this phenomenon. For example if the household's portfolio weight is fixed, a loan rate would rise as a positive z^1 shock reduces the household's loan issuance supply. In contrast, Ray (2019) fixes the effective weight function attached to each maturity rate, which is used in constructing the effective interest rate that governs the household's intertemporal substitution.

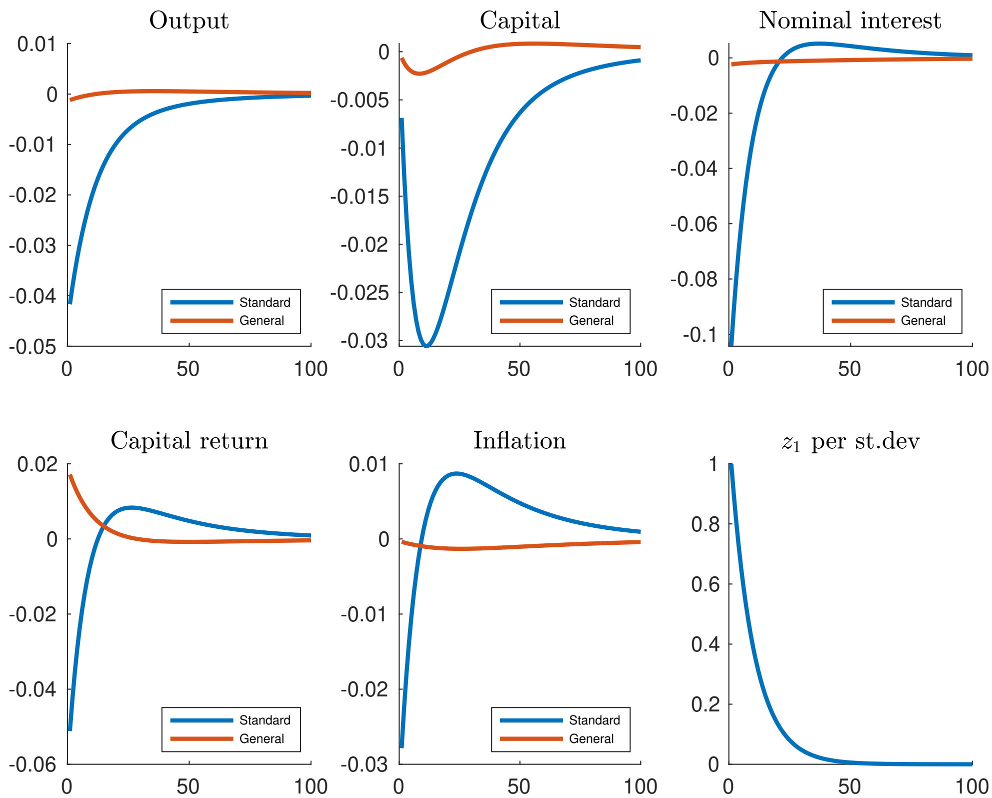

Figure 3: Impulse response to z_t^1 shock

is effectively segmented with $\kappa_B, \kappa_S < \infty$, central bank can fill the gap arising from the distortion by purchasing or selling its own bond portfolios. With a positive z^1 shock, central bank prevents the yield curve from shifting down, which can lift the effective bond market return. It raises both loan rate and wage (factor prices), and inflation does not fall. Labor supply, as well as output, remains almost unchanged.

A positive z^1 shock in this particular environment can be interpreted more broadly as a special case of bond market disruptions such as an increase in the degree of flight to safety or liquidity, as the shortest-maturity bond (federal fund market) usually features high degrees of safety and liquidity. It creates an endogenous recession with the standard policy, while general policy that allows central bank to actively manipulate the entire yield curve obtains an almost perfect stabilization.

Tax shock: Next, Figure 4 presents an impulse-response to ε^T shock that raises the government's tax revenues. In the standard policy case, an upward jump in ε^T results in a less issuance of riskless bonds by the government. It eventually pushes down bond returns, as well as the capital return and wage, from the household's endogenous portfolio reshuffling and firms' substitution between inputs as usual. Inflation jumps down in response. Monetary policy responses (policy rates falling

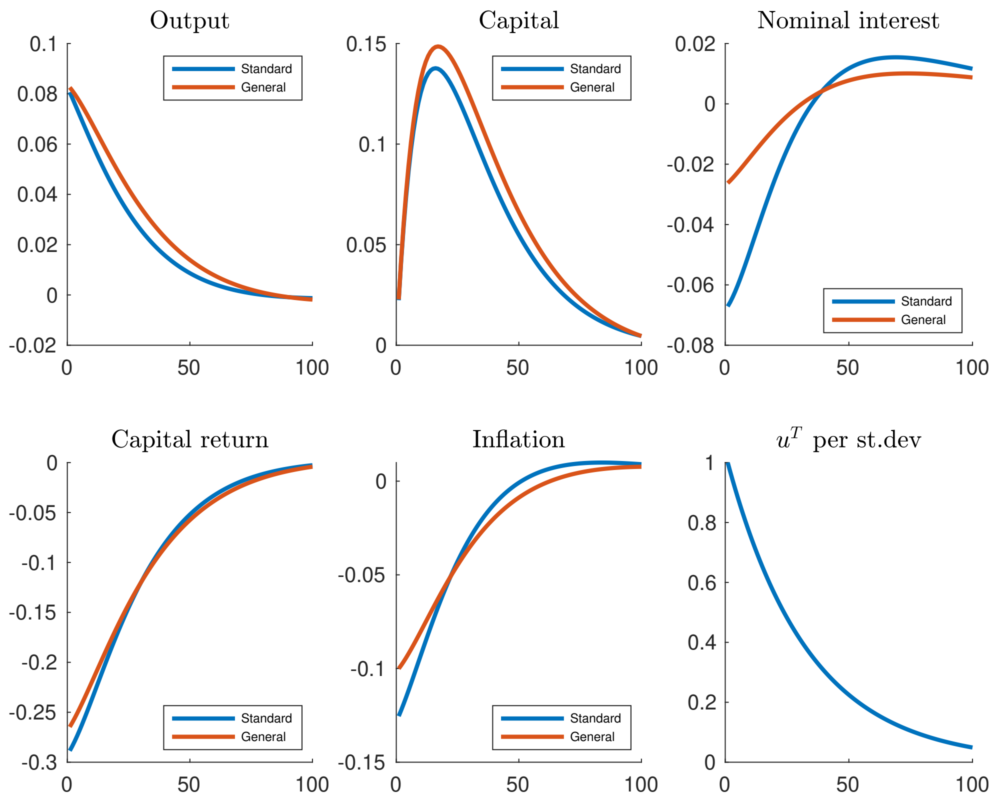

Figure 4: Impulse response to ε_t^T shock

down) boost aggregate demand, and in this case actually raises output.[59]

General policy, in this case, achieves similar stabilization effects to the standard policy, while featuring a smaller drop in the short-term yields. As the entire yield curve shifts down in response, the short-term yields need not drop as much as in the standard policy case to lower the effective interest rate that enters into the household's IS equation (equation (59)).

Other shocks: Figure A11 depicts an impulse response to z_t^K shock: a positive jump in z^K induces the household to issue more loans to the intermediate firms, raising aggregate capital and pushing down capital return. Output and inflation jump up, and thus monetary policy rate rises in response. General policy turns out to be more effective in terms of stabilizations.

Figure A12 presents the impulse response to ε_t^A shock: a positive jump in the technology growth GA_t generates similar effects to the prior literature,[60]where output rises and inflation falls down. A rising output increases firms' capital demand and brings up the capital return, while the capital level drops as firms can produce the same output levels with less inputs with a better technology. With general policy, normalized output ironically falls: as inflation falls, all yields shift downwards,

[59]Monetary policy responses are strong enough to offset negative effects a shortage of bond issuance has on output.
[60]For effects of technology shock in New-Keynesian models, see Ireland (2004).

reducing both capital return and wage. Then, the household reduces its labor supply, and normalized output falls. However, actual output (which is not normalized) increases in response to the technology shock even in the general policy case.

Figure A13 presents the impulse response to $\varepsilon_t^{YD^1}$ shock: a usual contractionary monetary policy shock brings output, inflation, and capital down, with which firms reduce their capital and labor demand. Capital return and wage fall in response, and inflation jumps down. On the other hand, general policy almost perfectly insulates the economy from the monetary policy shock. In response to the shock, central bank moves up the entire yield curve, and prevents input prices (capital return and wage) from falling. Inflation slightly increases in response. Even though a higher real effective savings rate tends to push down aggregate demand, higher factor prices raise the aggregate labor supply, thus output remains almost unchanged.

4.3.2 Impulse-Response at the ZLB

Now, we present impulse-response functions to various shocks when the monetary policy is constrained by the ZLB: z^1 and z^K (scale parameters of the Fréchet distributions that govern the household's fund allocation problems) and ε^T (tax, or in general, fiscal shock). For each structural shock to drive the economy into ZLB, the size of shocks are calibrated to be very "big", even though in reality shocks that big are extremely unlikely.

z^1 shock: Figure 5 presents an impulse-response to z^1 shock that drives the household's portfolio demand for the shortest maturity bond. The blue lines depict impulse responses under the standard monetary policy framework, and red lines represent the impulse responses under the general policy setting. Figure 5 features similar behaviors to Figure 3 (the case without ZLB), except for the short-term rate being constrained at the ZLB for a few quarters.[61]

General policy achieves almost perfect stabilization as in Figure 3. However, note that it generates a longer ZLB duration than the standard policy: when the economy enters ZLB episodes, under general policy, central bank tends to increase its purchase of long-term Treasury bonds, thus pushing down the long-term yields. This action imposes additional downward pressures on the short yields and the capital return from the household's portfolio problem,[62] which makes the ZLB constraint bind for a longer duration. With our calibration, this endogenous portfolio effect is stronger than the effect in which general policy better stabilizes the economy and make the economy get off

[61] We assume z^1 jumps up by 1000 times its standard deviation, σ_z, which occurs with very little probability. This is for the purpose of generating ZLB episodes only with z^1 shock.

[62] For example, as capital return (loan rate) falls, it tends to pull wage down from the intermediate firms' substitution between inputs, and thus inflation is likely to fall, which makes ZLB more likely to bind.

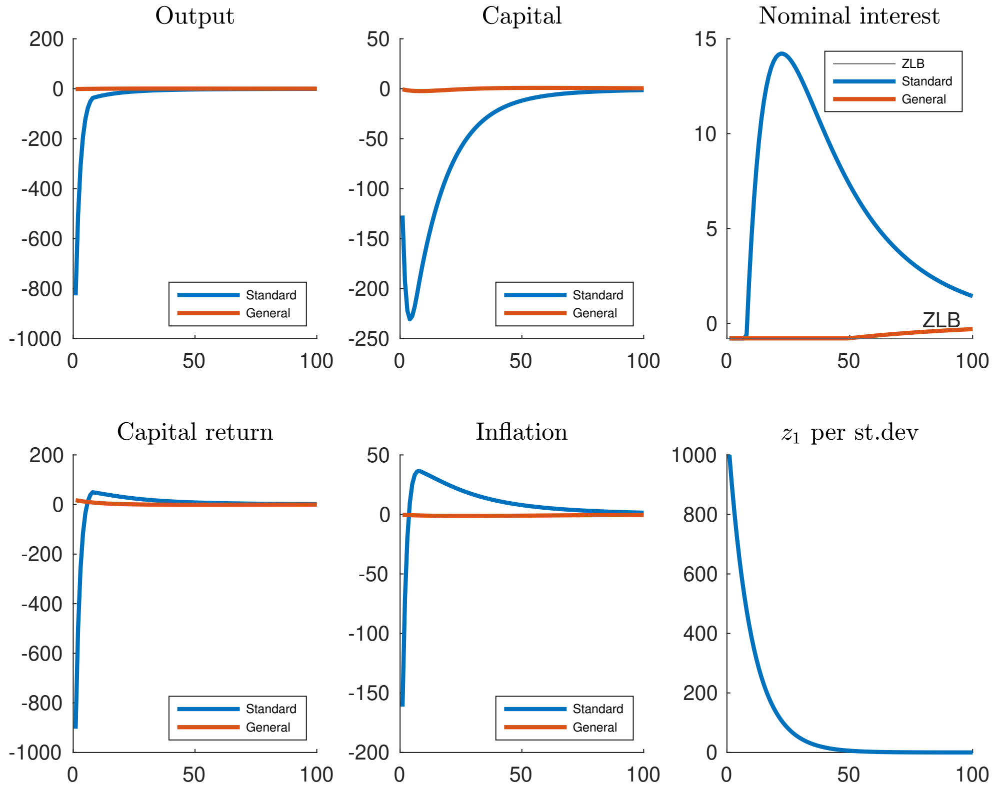

Figure 5: Impulse response to z_t^1 shock with ZLB

ZLB sooner.[63]

While general policy helps to insulate the economy from various shocks, it generates prolonged ZLB episodes when the economy becomes ZLB-constrained: with longer ZLB durations, unconventional policies' roles in terms of stabilization become more crucial, and the economy becomes more reliant on those policies itself.[64]

z^K **shock:** Figure A14 represents an impulse response to z_t^K shock at the ZLB: a big negative shock to z^K induces the household to issue less loans to intermediate firms and invest more in the bond markets. Bond rates fall and the policy rate gets constrained by the ZLB. Output, capital, inflation, and capital return jump down in response. General policy stabilizes the economy very effectively, while generating a longer ZLB episode as in Figure 5.

[63]The general policy better insulates the economy from adverse shocks, and therefore, it can lift the economy off the ZLB episodes sooner. This effect in our calibration is weaker than the endogenous portfolio effect we describe.

[64]Karadi and Nakov (2021) modeled an environment where quantitative easing (QE) policies are effective in fighting against financial disruptions to the banks, and banks get addicted to QEs. Our framework, without explicit roles of banks, emphasizes a similar phenomenon, in which the general policy generates longer ZLB episodes, which makes the economy more dependent on general policy's ammunition power in insulating the economy from the shock's adverse effects.

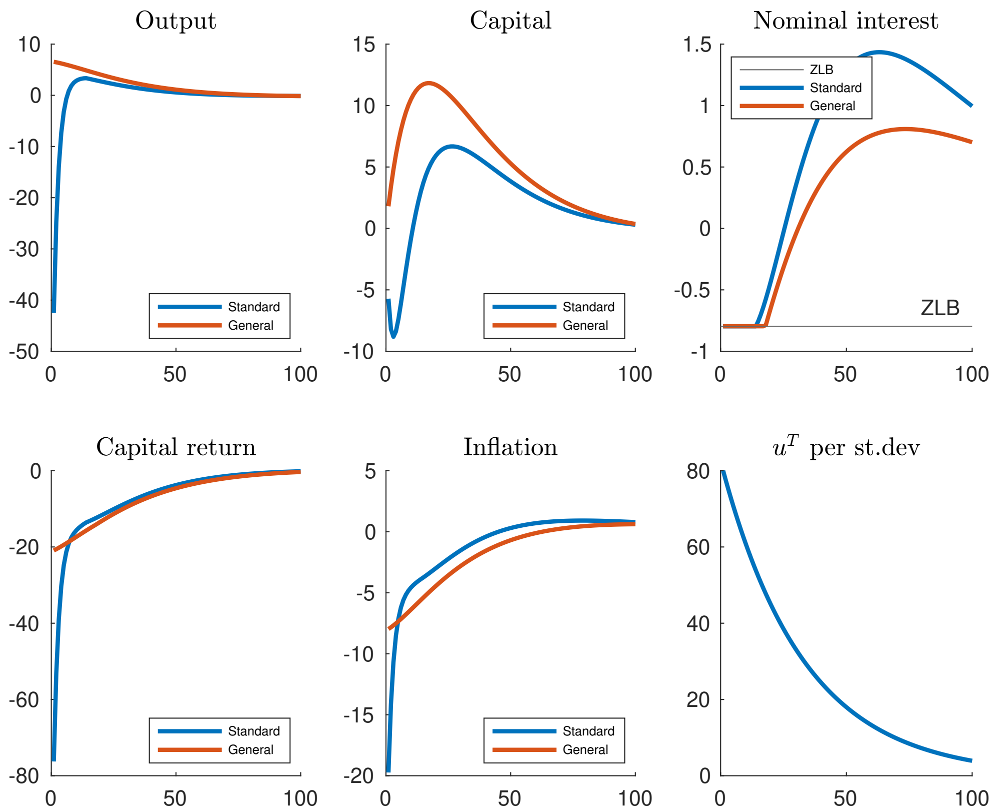

Figure 6: Impulse response to ε_t^T shock with ZLB

Tax shock: Figure 6 presents an impulse-response to a positive ε^T shock raising the government's tax revenues. In contrast to Figure 4 (non-ZLB case), the economy experiences a recession with the standard policy: with a high enough tax increase shock, the government significantly reduces its bond issuance, dragging the economy into the ZLB recession. Output, capital, inflation, and capital return all drop. This experiment emphasizes stabilizing roles of the safe bond supply at the ZLB, emphasized by many prior works including Caballero and Farhi (2017) and Caballero et al. (2021) in the global economy context. With general policy, central bank moves down the entire yield curve and lowers the household's effective savings rate. This action boosts aggregate demand, thus output and capital rise in response, and inflation and capital return fall less than in the standard policy case. Note that also in this case, our general policy generates a longer ZLB episode than the standard policy, possibly due to the same effects from changes in the household's portfolio choices.

4.3.3 Policy Comparison

Using the welfare criterion we characterize in Proposition 1, we can compare various policy regimes. As we already discussed in Section 4.2, we study (i) standard policy (equation (36a), equation (36b), equation (36c)), (ii) general policy (equation (37a), equation (37b), equation (37c)), and (iii) mixed

	Standard Policy	**General Policy**	**Mixed Policy**
Mean ZLB duration	1.6511 quarters	6.3355 quarters	7.9672 quarters
Median ZLB duration	1 quarters	1 quarters	1 quarters
ZLB frequency	8.2556%	21.8222%	21.6%
Welfare	−1.3503%	−0.90471%	−0.90302%

Table 1: Policy comparisons

policy, where central bank implements general policy only when the policy rate hits ZLB. Otherwise in normal cases, mixed policy is identical to the standard policy, in which central bank controls only the shortest-maturity yield.[65]

With these 3 monetary regimes, we calculate (i) ex-ante per-period welfare, (ii) mean and median ZLB duration, and (iii) the frequency with which the economy enters ZLB episodes.

Our findings in Table 1 can be summarized as: (i) compared with standard policy, general and mixed policy improve on welfare by around 0.4% point, (ii) general policy prolongs ZLB episodes, with higher ZLB frequency and longer ZLB duration than in the standard policy, and (iii) compared with general policy, mixed policy achieves a similar (a little bit better) welfare outcome, and while it hits the ZLB constraint less frequently, once it gets ZLB-constrained, then it generates longer ZLB durations than general policy.

In contrast to the mixed policy, general policy allows the central bank to manipulate the entire yield curve even out of ZLB periods. In cases when adverse shocks hit the economy, it lowers long rates, imposing an additional downward pressure on short-term rates, due to (i) the household's portfolio reallocation, and (ii) lower loan rates that tend to reduce the firms' costs and thus inflation. Therefore, ZLB constraints for the short-term rates are more likely to bind, and it explains a higher ZLB frequency under general policy than both standard and mixed policies. However, since mixed policy abandons the central bank's active yield curve manipulation after the economy gets off ZLB, it cannot generate a stabilizing effect as powerful as under the general policy, and features even longer ZLB duration than under the general policy.

In our framework, we did not include any welfare costs of the central bank's burgeoning balance sheets. Incorporating various economic and political costs of the central bank's direct manipulation of the yield curve have some potentials to distort our implications about various monetary policy regimes and should be an interesting topic for future research.[66]

[65]We assume: after the economy gets off the ZLB, central bank adjusts its holdings of the long-term ($f \geq 2$) maturity bonds to its steady-state holdings. It is in contrast to Karadi and Nakov (2021), who derived the optimality of gradual exit from QE policies, as banks have less power and incentive to recapitalize without additional QE policies.

[66]For example, Karadi and Nakov (2021) introduced a small quadratic efficiency cost to QE as a reduced-form proxy for un-modeled distortions and political costs of maintaining a positive central bank balance sheet.

5 Conclusion

This paper develops a New-Keynesian model that incorporates the term-structure of financial markets and an active role for government and central bank's balance sheet size and maturity structure. We show that market segmentation across assets and maturities and the household's endogenous portfolio reallocation are two necessary elements for understanding of the effects of unconventional monetary interventions. For that purpose, we show how standard techniques from the international trade literature (see Eaton and Kortum (2002)) can be employed in the macroeconomics literature to parsimoniously accommodate market segmentation arising from differences in asset return expectations. Our economy, even after log-linearization, features an equilibrium term-structure that deviates from the so-called *expectation hypothesis*, and which allows unconventional monetary policies such as LSAPs to affect the yield curve and thereby, have some stabilizing powers.

We find that government's issuance and the central bank's purchase of different bond maturities act as two major determinants of the yield curve level and slope, and government's issuance of riskless bonds stimulates the economy when conventional monetary policy is constrained by the ZLB, as documented by previous works on the so-called 'safe-asset shortage problems'. We also study different policy regimes, and reveal that yield-curve-control (YCC) interventions where the central bank actively manipulates the entire yield curve are more stabilizing than conventional policy both in normal times and during ZLB episodes. However, our YCC policy poses interesting side-effects, as it raises frequency and duration of ZLB episodes. This result comes from the portfolio balancing channel: the central bank's active easing of long-term rates imposes additional downward pressure on short-term rates by inducing households to endogenously rebalance their portfolios. Therefore, unconventional policies are *addictive*: central banks resort to them as the most powerful tools at the ZLB, but in doing so perpetuate the ZLB conditions that render conventional policy ineffective.

Now that the balance sheet of central banks dramatically expanded in most advanced economies as a result of the unconventional policies that were adopted following the 2007 Great Financial crisis, we believe that our framework will be useful to future research looking into the political economy implications and risks to the taxpayer that originate from an expanded central bank's balance sheet. In addition, we aim to extend our framework to the international macro setting and revisit global imbalance issues (e.g., Caballero et al. (2008, 2021)) and the global monetary cycles (e.g., Miranda-Agrippino and Rey (2021)) with endogenous fluctuations in the term-structure of interest rates.

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Appendix A. Additional Figures and Tables

A.1. Section 2

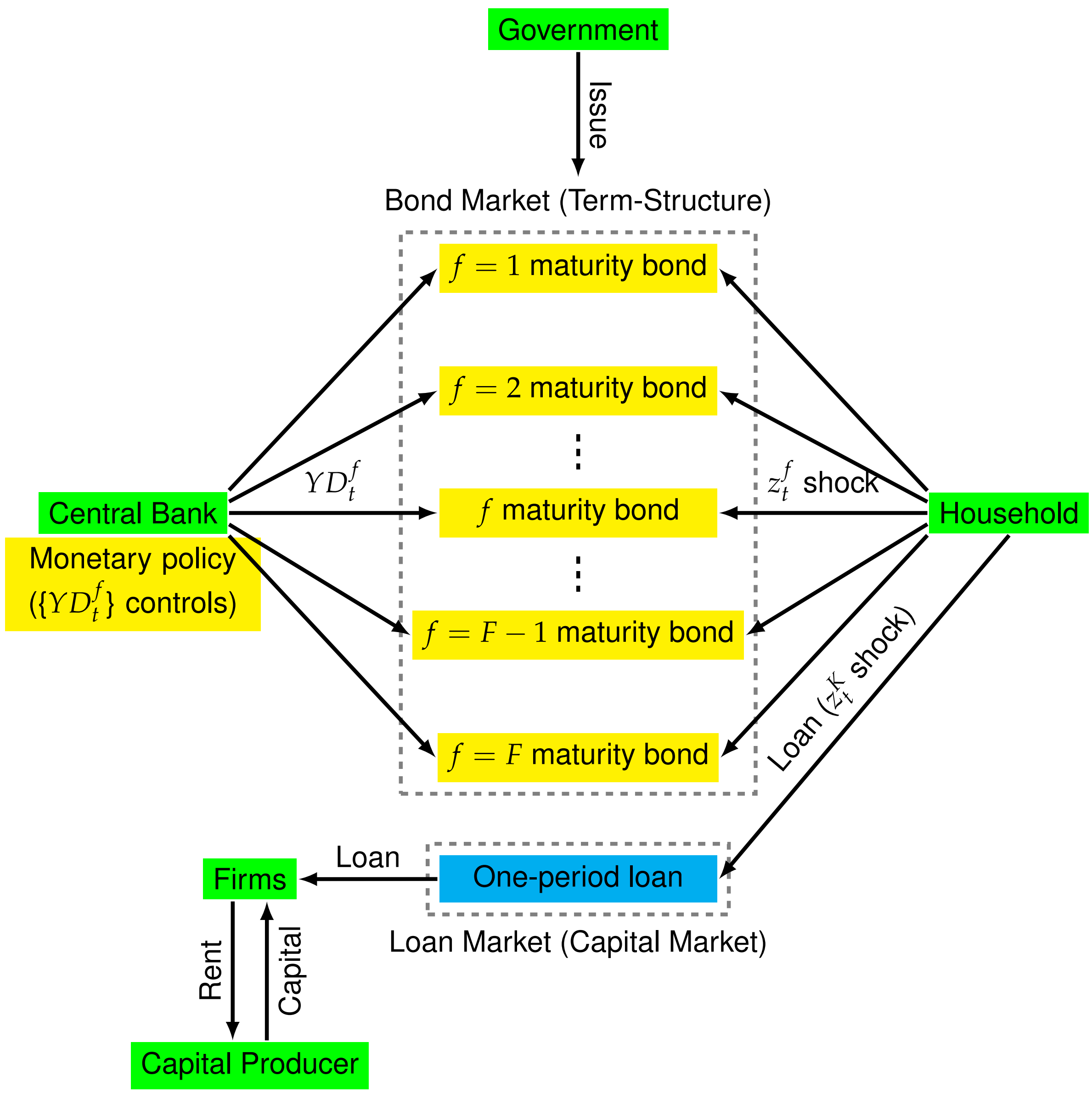

Figure A1: Markets, Agents, and Mechanisms: Household invests her wealth in the bond market or issues loans to intermediate good producers, which in turn rely on loans issued by the household to rent capital from the capital producer. There are bonds of $f = 1 \sim F$ number of maturities issued by the government. With the standard monetary policy, central bank controls the shortest maturity yield while not adjusting a purchase amount for longer-term ones, whereas with the general monetary policy, central bank controls all the yields to target business-cycle variables (in our model, inflation targeting).

A.2. Section 3

	Households	
β	0.998	Discount factor
η	1	Frisch labor elasticity
GN	1.015	Population growth rate
	Intermediate good firms	
μ	0.00375	Technology growth rate
GA	1.003757	Gross technology growth rate
α	0.25	Capital income share
ϵ	7	Elasticity of substitution between differentiated goods
θ	0.55	Calvo price stickiness parameter
σ_A	0.0090	Standard deviation of technology shock
δ	0.025	Capital depreciation rate
	Term structure	
κ_B	10	Bond maturity shape (volatility) parameter (Fréchet)
κ_S	1	Capital shape (volatility) parameter (Fréchet)
ρ_z	0.9	Autoregressive coefficient: maturity scale (mean) parameter (z_t^f)
ρ_z^K	0.9	Autoregressive coefficient: capital scale (mean) parameter (z_t^K)
σ_z	0.001	Standard deviation: maturity scale (mean) parameter (z_t^f)
σ_z^K	0.001	Standard deviation: capital scale (mean) parameter (z_t^K)
	Government	
ζ^F	0.1667	Government subsidy to firms (optimal)
ζ^G	0.1533	Government expenditure per GDP
a^G	5.5217	Government expenditure coefficient
$\zeta^F+\zeta^G-\zeta^T$	0.0119	Government deficit per GDP
ζ^T	0.3081	Government tax revenue per GDP
ρ_G	0.97	Autoregressive coefficient: government expenditure shock
ρ_T	0.97	Autoregressive coefficient: government tax revenue shock
σ_G	0.0037	Standard deviation: government expenditure shock
σ_T	0.0037	Standard deviation: government tax revenue shock
	Central bank	
ζ^{CB}	-0.050	Central bank's balance sheet per issued government bond values
$\bar{\pi}$	$\frac{0.02}{4}=0.005$	Trend inflation (steady-state inflation)
γ_π^1	1.5	Taylor rule coefficient of YD_t^1: responsiveness to inflation
$\gamma_\pi^{f\geq 2}$	1.5	Taylor rule coefficient of $YD_t^{f\geq 2}$: responsiveness to inflation
γ_y	1.5	Taylor rule coefficient: responsiveness to output
$\gamma_y^{f\geq 2}$	1.5	Taylor rule coefficient of $YD_t^{f\geq 2}$: responsiveness to output
σ^{YD^1}	0.0024	Standard deviation: monetary policy shock (for YD_t^1)
$\sigma^{YD^{f\geq 2}}$	10^{-8}	Standard deviation: monetary policy shock (for $YD_t^{f\geq 2}$)
τ^{YD}	$I_{F\times F}$	State reduction matrix (for $YD_t^{f\geq 2}$)

Table A1: Parameter values[67]

[67] In order to estimate ρ_B and $\sigma^{B,j}$ in equation (53), we implement the principal component analysis (PCA) for the time-series of government bond portfolio shares and take only 7 most salient components.

Calibrated steady-state parameters		
$\{z^f\}$	See Figure A2	Bond maturity scale (mean) parameters
z^K	0.1941	Capital scale (mean) parameter
$\frac{C}{A\bar{N}}$	2.4905	Normalized consumption
$\frac{Y}{A\bar{N}}$	4.0118	Normalized output
$\frac{K}{A\bar{N}}$	21.0701	Normalized capital
$\frac{C}{Y}$	0.6208	Consumption per GDP
$\frac{K}{Y}$	5.2521	Capital per GDP
$\frac{P^K}{P}$	0.0459	Normalized rental price of capital
$\lambda^{HB,f}$	See Figure 1	Household's bond portfolio
λ^K	0.1720	Household's loan share out of total savings
R^K	1.0852	Household's loan rates
YD^f	See Figure 1	Equilibrium yield curve

Table A2: Steady-state values with parameters in Table A1

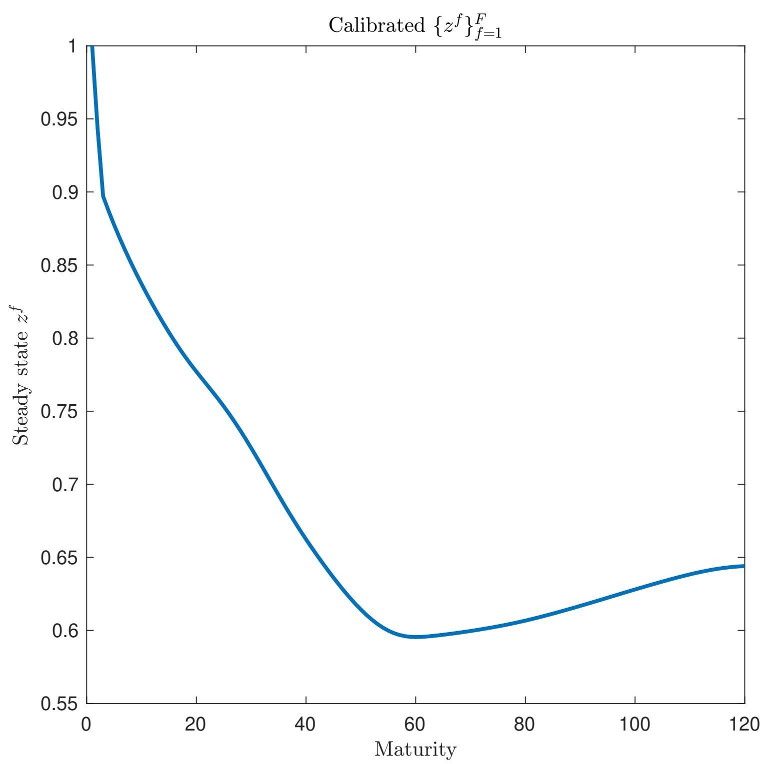

Figure A2: Calibrated scale parameters of the Fréchet distribution: z^f

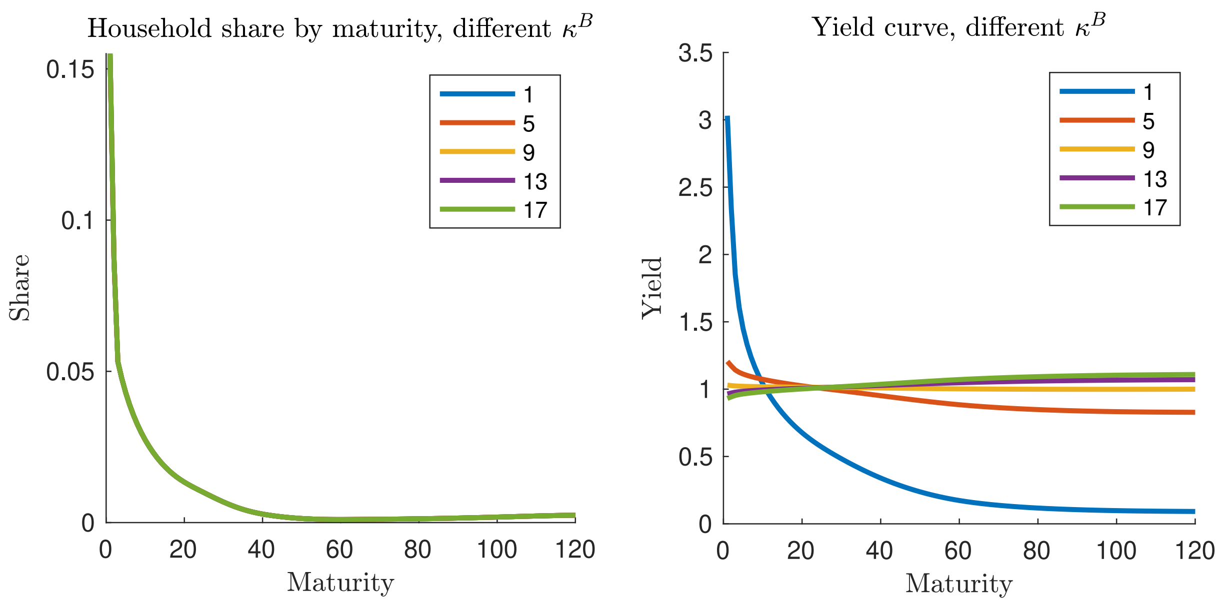

Figure A3: Variations in κ_B (scale parameter): when $\kappa_B \to \infty$, we return to the expectation hypothesis case, where all the discounted expected returns are equalized, and thus obtain a flat yield curve in the steady state.

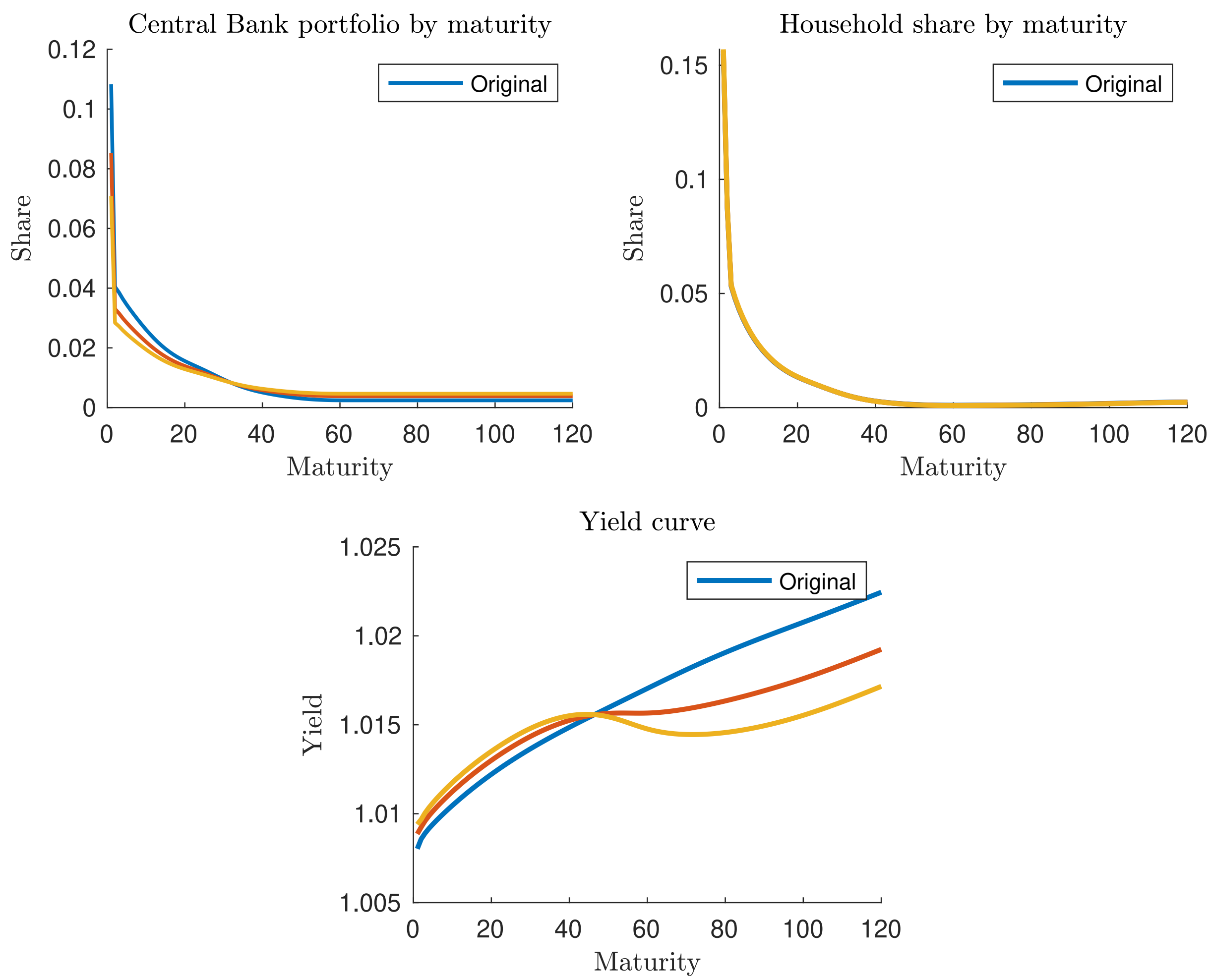

Figure A4: Variations in central bank's bond portfolio across maturities: central bank's relative purchase of bonds with different maturities is negatively related with yields, in line with literatures documenting that the central bank's bond purchase (such as QEs and LSAPs in general) reduces an yield for the bond of targeted maturiy in segregated markets.

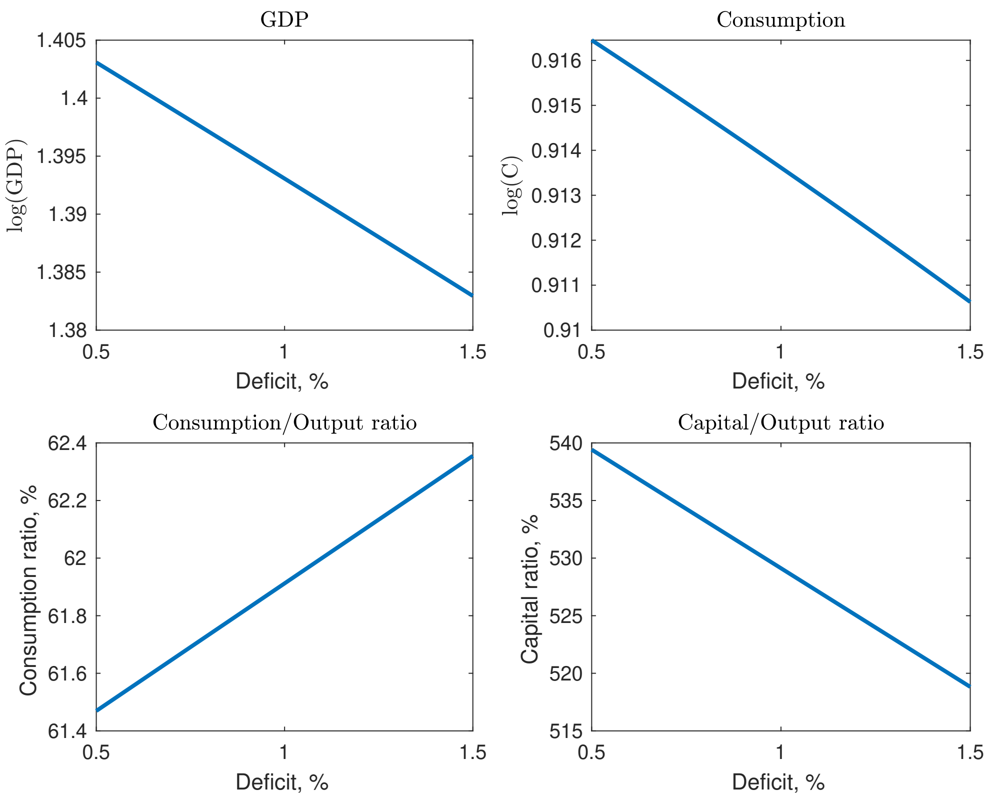

Figure A5: Variations in deficit ratio $\zeta^F + \zeta^G - \zeta^T$: a higher deficit ratio ends up hurting the economy: given that it is sustained only when the government issues more treasury bonds[68]or its effective bond rate R^G falls, a higher deficit ratio reduces output, consumption, and capital, which leads to drops in the deficit size (nominal) and the government's bond issuance, pushing down its bond return R^G. A credit spread rises in response.

[68]If government issues more treasury debts to finance a higher deficit given output, it will raise the government's effective bond return R^G, which forces government to issue more bonds and then pushes up R^G, ad infinitum, which is not sustained in the long run.

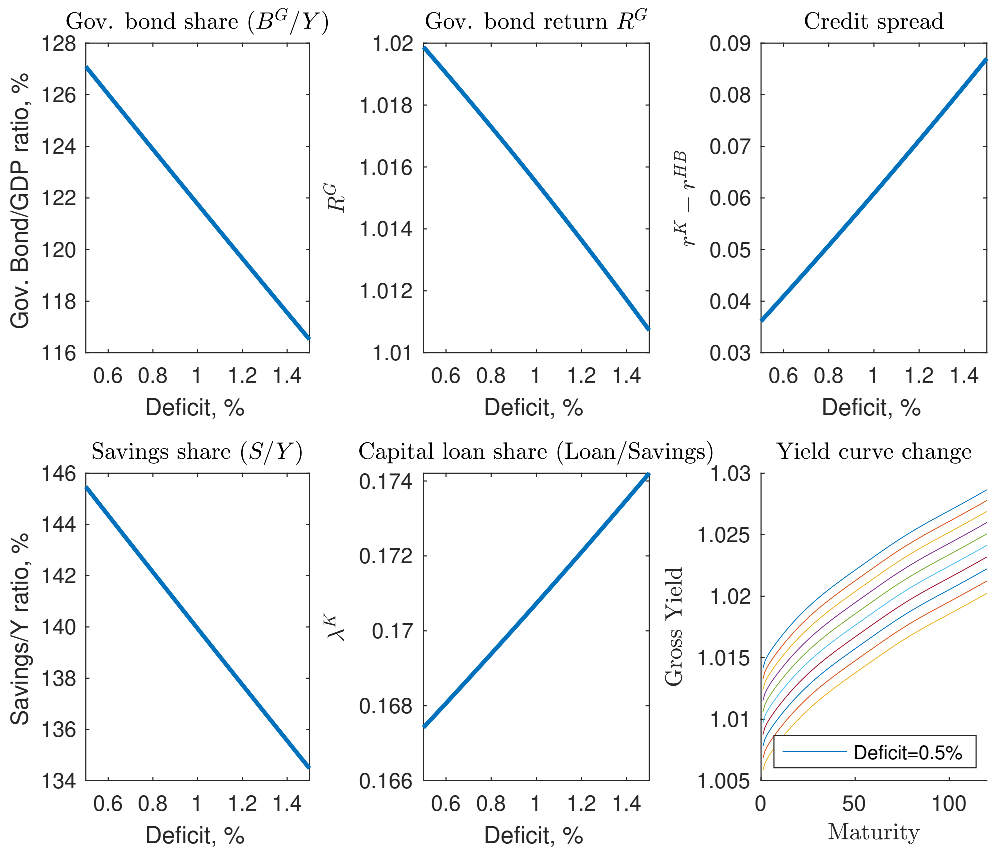

Figure A6: Variations in deficit ratio $\zeta^F + \zeta^G - \zeta^T$: a higher deficit ratio ends up hurting the economy: given that it is sustained only when the government issues more treasury bonds[69] or its effective bond rate R^G falls, a higher deficit ratio reduces output, consumption, and capital, which leads to drops in the deficit size (nominal) and the government's bond issuance, pushing down its bond return R^G. A credit spread rises in response.

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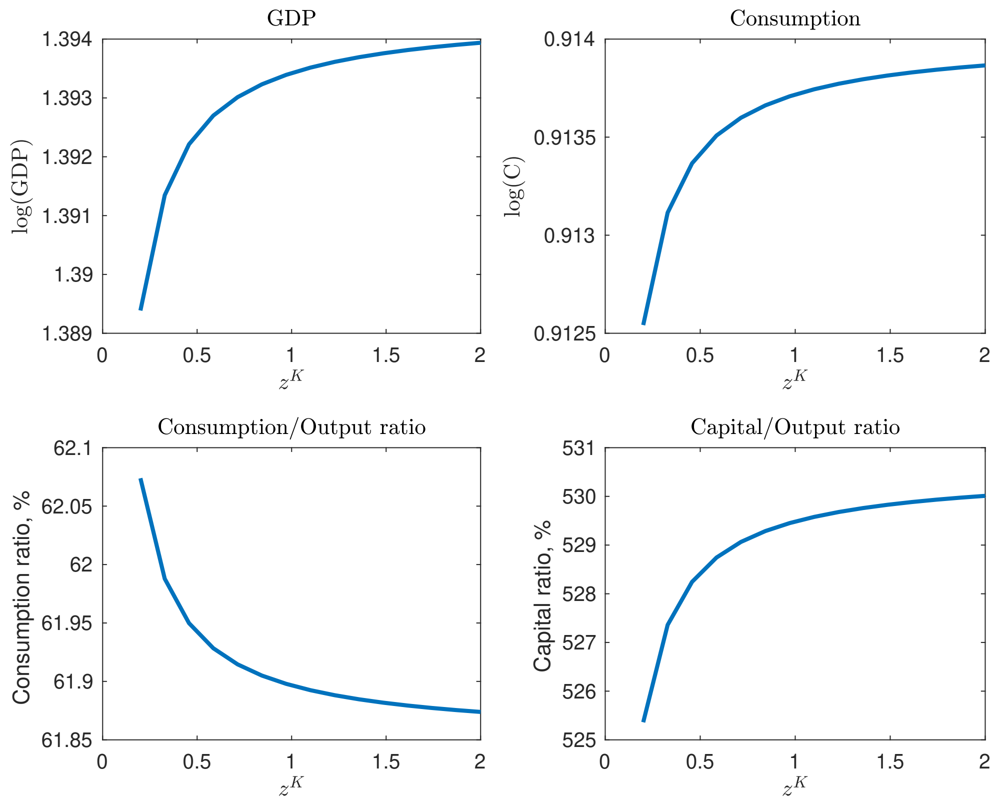

Figure A7: Variations in scale parameter z^K: given calibrated $\{z^f\}$ and for $z^K \in [0.2, 2]$, a higher z^K tends to push up λ^K, the household's capital loan share out of her total savings, thus bringing up capital, output, consumption in the steady-state. It reduces an average marginal propensity to consume (MPC). Interestingly, a positive z^K shock shifts down the entire yield curve, as well as the capital return (the loan rate R^K), from the household's endogenous fund reallocation, resulting in a higher credit spread.[70] As R^G falls, government bond share with respect to GDP also falls.

[70] Therefore, the bond market experiences larger drops in yields than the loan market experiences a falling loan rate.

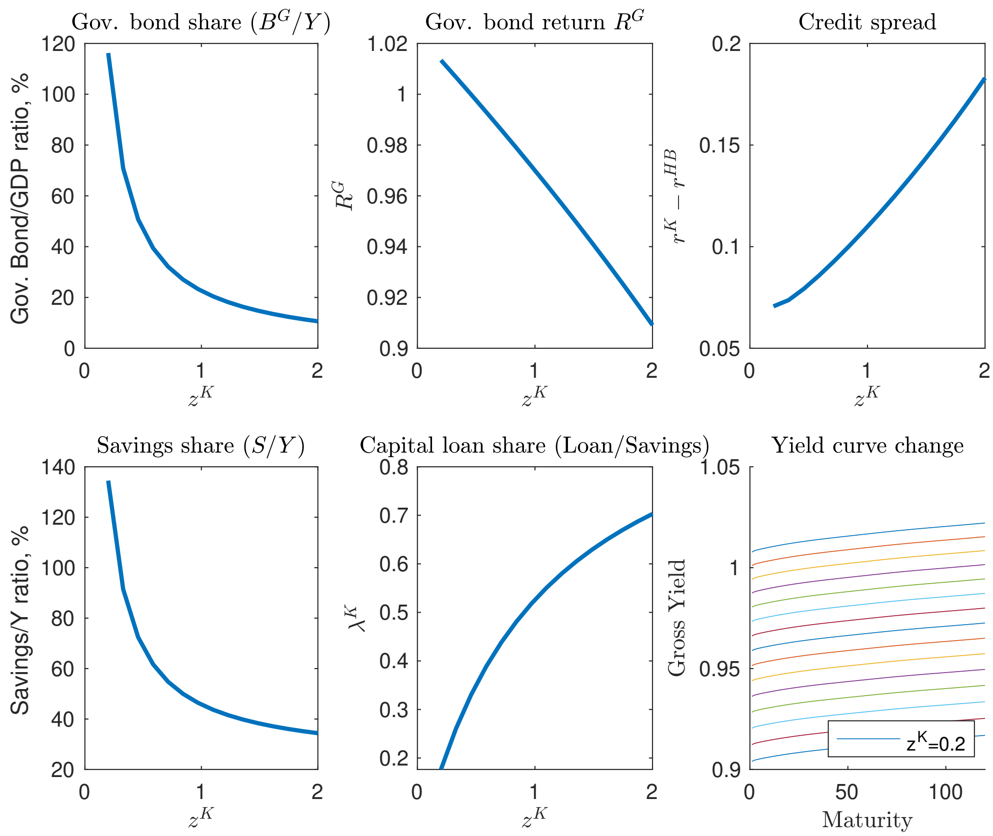

Figure A8: Variations in scale parameter z^K: given calibrated $\{z^f\}$ and for $z^K \in [0.2, 2]$, a higher z^K tends to push up λ^K, the household's capital loan share out of her total savings, thus bringing up capital, output, consumption in the steady-state. It reduces an average marginal propensity to consume (MPC). Interestingly, a positive z^K shock shifts down the entire yield curve, as well as the capital return (the loan rate R^K), from the household's endogenous fund reallocation, resulting in a higher credit spread.[71] As R^G falls, government bond share with respect to GDP also falls.

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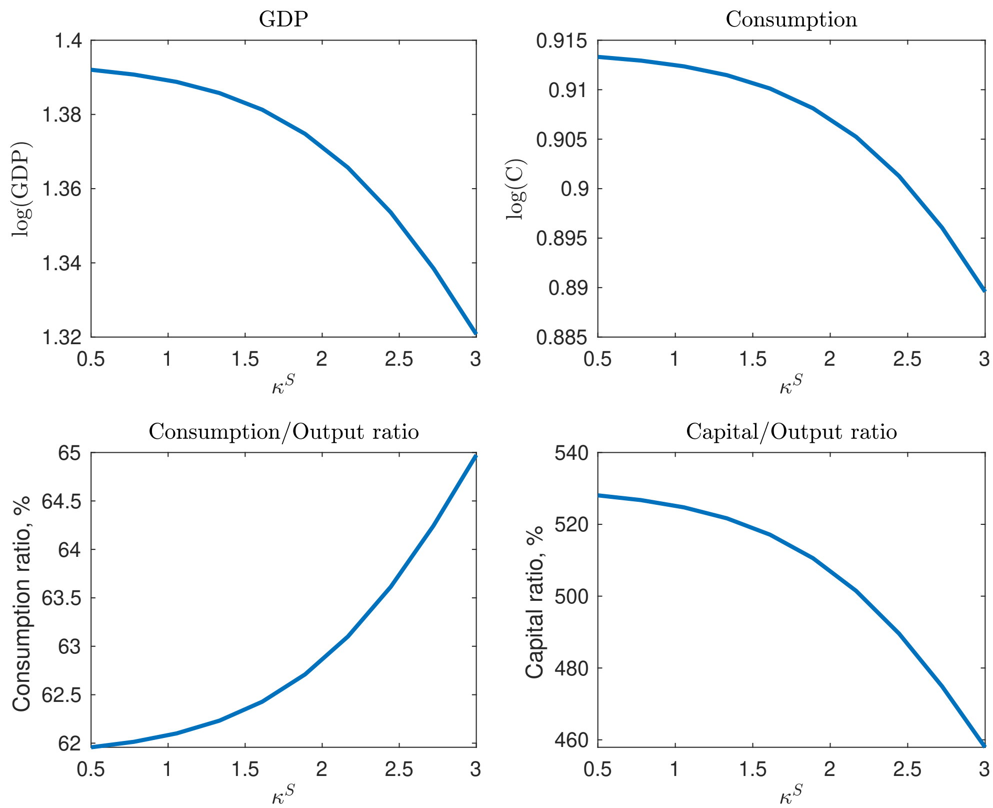

Figure A9: Variations in shape parameter κ_S: given calibrated $\{z^f\}$ and z^K values and for $\kappa_S \in [0.5, 3]$, a higher κ_S tends to reduce λ^K, the household's capital loan share out of total savings. It pushes down capital (as we have a higher R^K, rental rate of capital for firms), output, and consumption while raising an average marginal propensity to consume (MPC). Credit spreads increase while a higher R^K dragging government's bond return R^G and the entire yield curve up. Government ends up issuing more bonds per output.

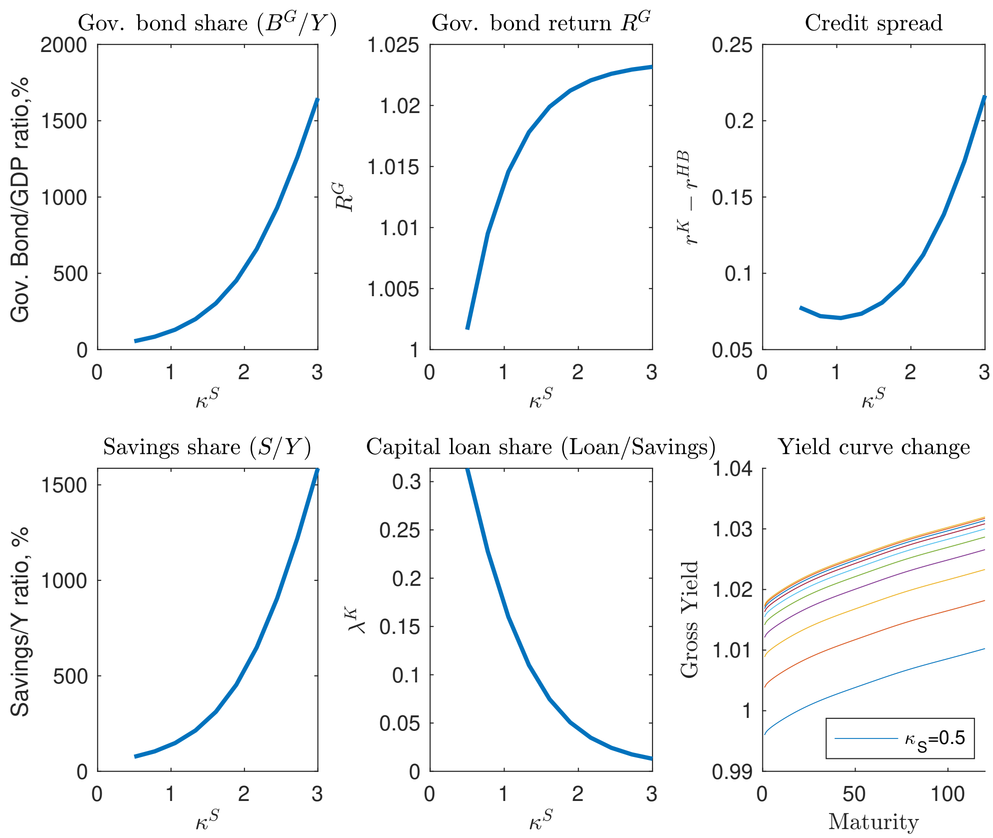

Figure A10: Variations in shape parameter κ_S: given calibrated $\{z^f\}$ and z^K values and for $\kappa_S \in [0.5, 3]$, a higher κ_S tends to reduce λ^K, the household's capital loan share out of total savings. It pushes down capital (as we have a higher R^K, rental rate of capital for firms), output, and consumption while raising an average marginal propensity to consume (MPC). Credit spreads increase while a higher R^K dragging government's bond return R^G and the entire yield curve up. Government ends up issuing more bonds per output.

A.3. Section 4.3.1

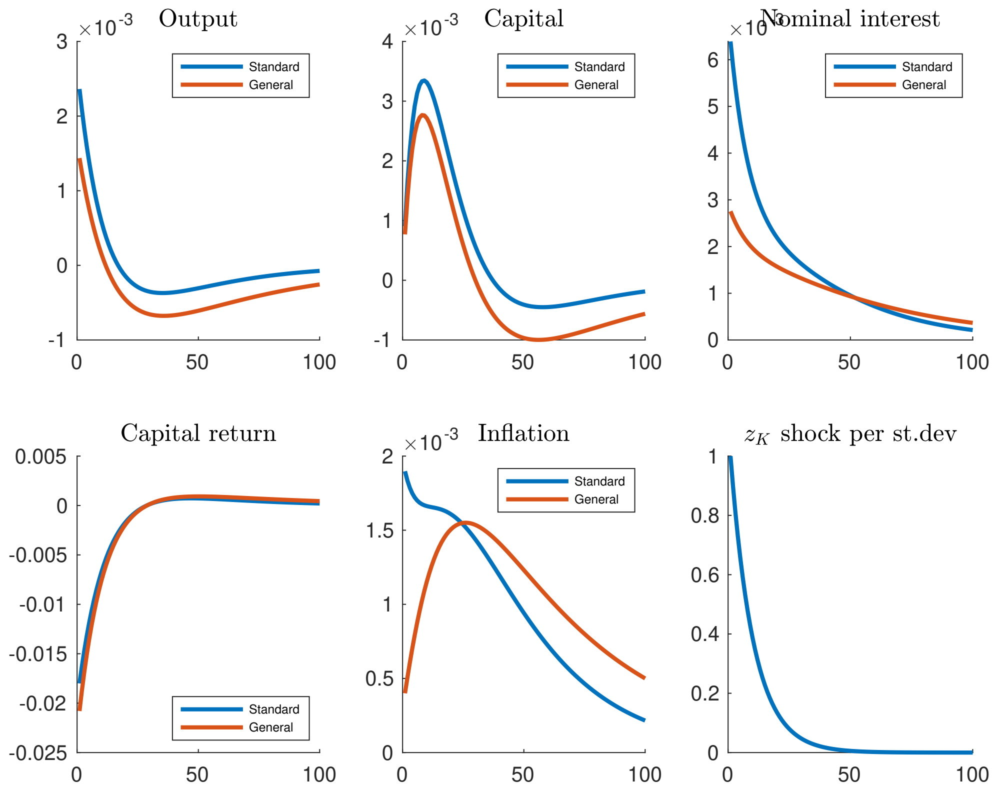

Figure A11: Impulse response to z_t^K shock: a positive z^K shock incentivizes the household to issue more loans, raising aggregate capital and pushing down the capital return. It raises output and inflation,[72]thus monetary policy rate rises in response. General policy turns out to be better-stabilizing.

[72]Note that an inflection point arises in the inflation path with the standard policy, as rising output and aggregate demand push up inflation, while a lower capital return (and wage) tends to bring it down (two countervailing forces).

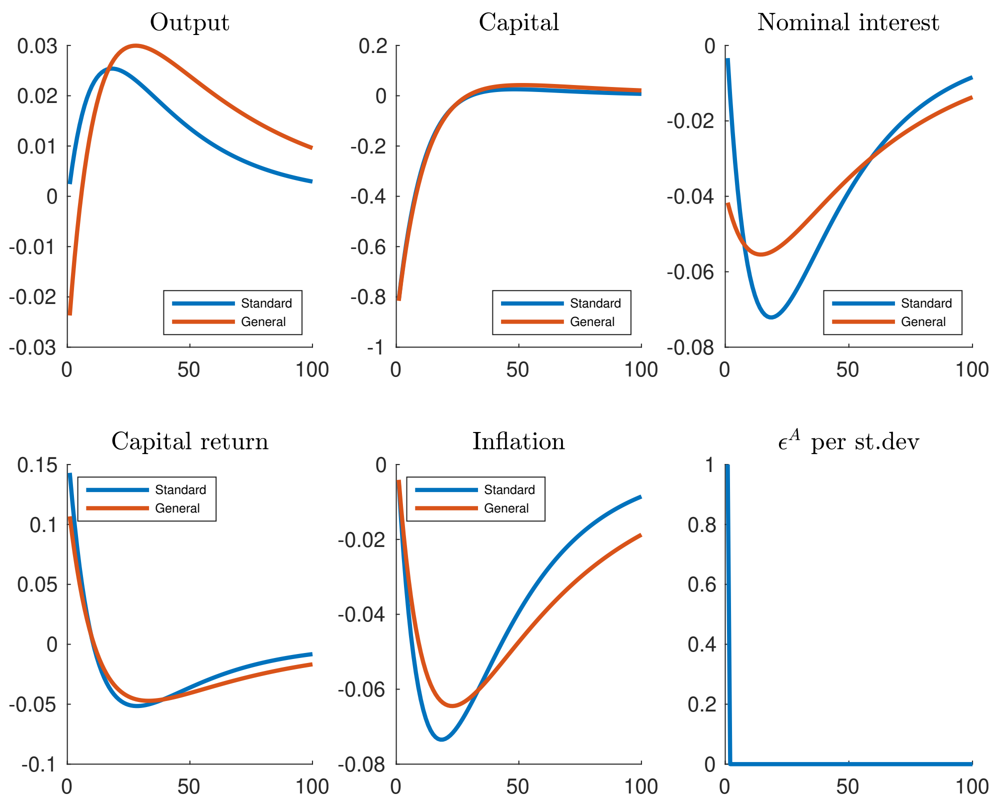

Figure A12: Impulse response to ε_t^A shock: a positive technology growth shock generates similar effects to the prior literature,[73]where output rises and inflation falls down. A rising output raises firms' capital demand and brings up the capital return, while the capital level actually drops with a better technology. With general policy, (normalized) output ironically falls: as inflation falls, all yields shift down, bringing down both capital return and wage, compared to the standard case. Then, the household reduces her labor supply, and normalized output falls. However, actual output (which is not normalized) increases in response to the technology shock even in the general policy case.

[73]For example, see Ireland (2004).

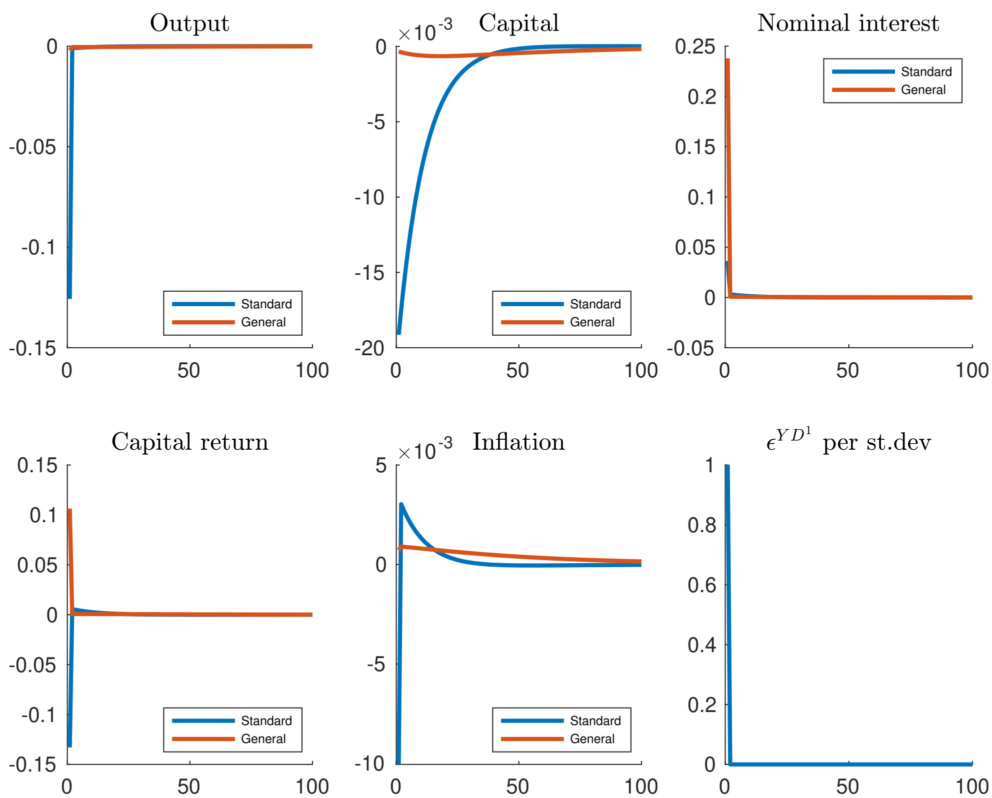

Figure A13: Impulse response to $\varepsilon_t^{YD^1}$ shock: a usual contractionary monetary policy shock pushes down output, inflation, and capital. As firms reduce their inputs demand, capital return and wage fall, which brings inflation down. On the other hand, general policy almost perfectly insulates the economy from the shock. As the policy shock hits the economy, central bank shifts up the entire yield curve, which prevents input prices (capital return and wage) from falling, and inflation slightly increases. Even though a higher real effective savings rate reduces aggregate demand, a higher wage raises the aggregate labor supply, thus output remains almost unchanged.

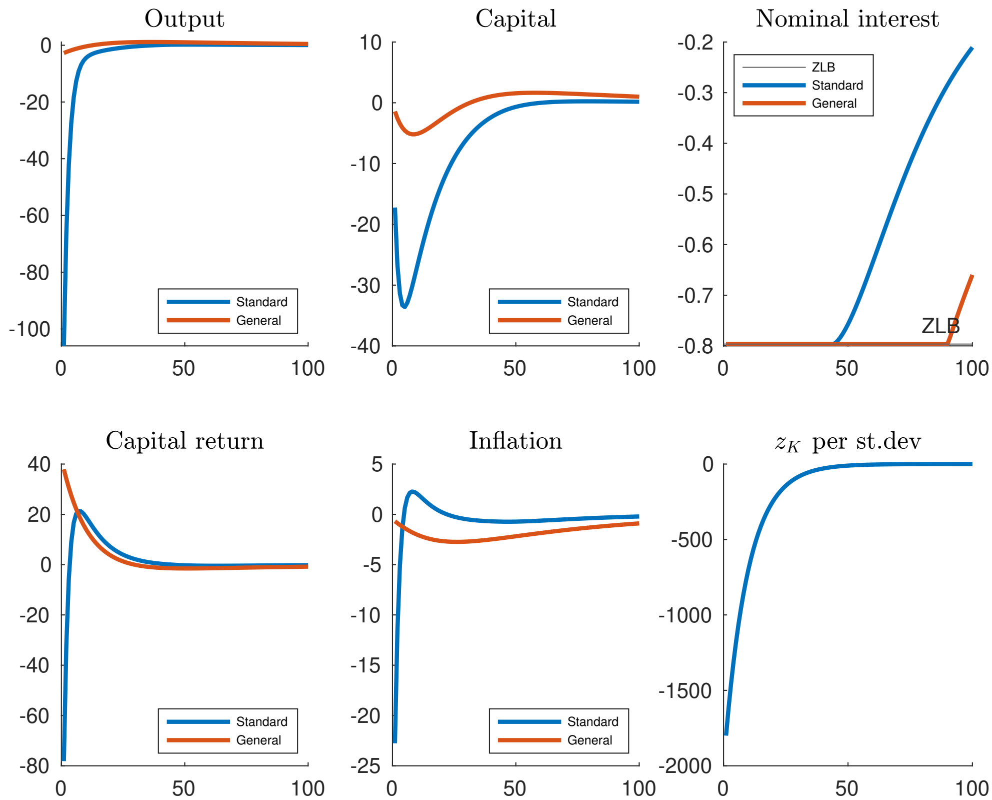

Figure A14: Impulse response to z_t^K shock with ZLB: a big negative shock to z^K induces the household to issue less loans to intermediate firms and invest more in bond markets. Bond rates fall and the policy rate gets constrained by ZLB. Output, capital, inflation, and capital return all jump down in response. General policy is effective in stabilizing the economy, while generating a longer ZLB episode as in Figure 5.

Appendix B. Derivation and Proofs

B.1. Detailed Derivations in Section 2

B.1.1. Detailed Derivations in Section 2.4

As in equation (26), an intermediate firm ν maximizes the discounted stream of profits

$$\max \sum_{j=0}^{\infty} \mathbb{E}_t \left[\theta^j Q_{t,t+j} \cdot \left[(1+\zeta^F) \cdot P_{t+j}(\nu) Y_{t+j}(\nu) - W_{t+j}(\nu) N_{t+j}(\nu) - R^K_{t+j} P^K_{t+j-1} K_{t+j-1}(\nu) \right] \right], \tag{64}$$

where $Q_{t,t+j}$ is the firm's stochastic discount factor between periods t and $t+j$ and ζ^F is a production subsidy. Solving for the optimal resetting price at period t P_t^*, we obtain

$$\frac{P_t^*}{P_t} = \frac{\mathbb{E}_t \left[\sum_{j=0}^{\infty} \theta^j Q_{t,t+j} \left(\frac{P_{t+j}}{P_t} \right)^{\epsilon+1} Y_{t+j} \left(\frac{(1+\zeta^F)^{-1}\epsilon}{\epsilon - 1} \right) \left(\frac{MC_{t+j|t}(\nu)}{P_{t+j}} \right) \right]}{\mathbb{E}_t \left[\sum_{j=0}^{\infty} \theta^j Q_{t,t+j} \left(\frac{P_{t+j}}{P_t} \right)^{\epsilon} Y_{t+j} \right]}, \tag{65}$$

where subindex $t+j|t$ represents the value of the variable conditional on the firm having reset its price last time at period t, and $MC_{t+j|t}(\nu)/P_t$ is the real marginal cost of production, defined as[74]

$$\frac{MC_{t+j|t}(\nu)}{P_{t+j}} = \left(\widetilde{R}^K_{t+j} \cdot \frac{P^K_{t+j}}{P_{t+j}} \right)^{\alpha} \left(\frac{W_{t+j|t}(\nu)}{P_{t+j} A_{t+j}} \right)^{1-\alpha}. \tag{66}$$

B.1.2. Detailed Derivation in Section 2.10

Using equations equation (15), equation (20), equation (22) and equation (66) we can express firm-specific marginal costs as a function of the aggregate variables as in

$$\frac{MC_{t+j|t}(\nu)}{P_{t+j}} = (1-\alpha)^{\frac{1-\alpha}{\eta+\alpha}} \left(\frac{C_{t+j}}{A_{t+j}\bar{N}_{t+j}} \right)^{\frac{\eta(1-\alpha)}{\eta+\alpha}} \left(\frac{Y_{t+j}}{A_{t+j}\bar{N}_{t+j}} \right)^{\frac{1-\alpha}{\eta+\alpha}} \left(\widetilde{R}^K_{t+j} \frac{P^K_{t+j}}{P_{t+j}} \right)^{\alpha\left(\frac{\eta+1}{\eta+\alpha}\right)} \left(\frac{P^*_t}{P_{t+j}} \right)^{-\left(\frac{\epsilon(1-\alpha)}{\eta+\alpha}\right)}. \tag{67}$$

Similarly, we integrate loan and labor demand across the continuum of firms and obtain the following expressions for the loan and labor aggregation conditions.

$$\frac{K_t}{A_{t-1}\bar{N}_{t-1}} = \alpha(1-\alpha)^{\frac{1-\alpha}{\eta+\alpha}} \cdot GA_t \cdot GN \cdot \left(\frac{C_t}{A_t\bar{N}_t} \right)^{\frac{\eta(1-\alpha)}{\eta+\alpha}} \left(\frac{Y_t}{A_t\bar{N}_t} \right)^{\frac{\eta+1}{\eta+\alpha}} \left(\widetilde{R}^K_t \frac{P^K_t}{P_t} \right)^{-\left(\frac{\eta(1-\alpha)}{\eta+\alpha}\right)} \Delta_t, \tag{68}$$

$$\frac{N_t}{\bar{N}_t} = (1-\alpha)^{\left(\frac{\eta}{\eta+\alpha}\right)} \left(\frac{C_t}{A_t\bar{N}_t} \right)^{-\alpha\left(\frac{\eta}{\eta+\alpha}\right)} \left(\frac{Y_t}{A_t\bar{N}_t} \right)^{\left(\frac{\eta}{\eta+\alpha}\right)} \left(\widetilde{R}^K_t \frac{P^K_t}{P_t} \right)^{\alpha\left(\frac{\eta}{\eta+\alpha}\right)} \Delta_t^{\frac{\eta}{\eta+1}}, \tag{69}$$

[74] It can be derived using the optimal demand formula for labor and capital (equation (27)).

where Δ_t is a measure of price-dispersion that can be recursively defined as

$$\Delta_t = (1-\theta)\left(\frac{P_t^*}{P_t}\right)^{-\epsilon\left(\frac{\eta+1}{\eta+\alpha}\right)} + \theta \Pi_t^{\epsilon\left(\frac{\eta+1}{\eta+\alpha}\right)} \Delta_{t-1}. \tag{70}$$

Plugging the equation (67) and the expressions for Q_{t+j} into the optimal resetting price equation (equation (65)), we obtain

$$\left(\frac{P_t^*}{P_t}\right)^{1+\epsilon\left(\frac{1-\alpha}{\eta+\alpha}\right)} = \frac{\mathbb{E}_t\left[\sum_{j=0}^{\infty}(\theta\beta)^j (1-\alpha)^{\frac{1-\alpha}{\eta+\alpha}}\left(\frac{(1+\varsigma_F)^{-1}\epsilon}{\epsilon-1}\right)\left(\frac{C_{t+j}}{A_{t+j}\bar{N}_{t+j}}\right)^{-\alpha\left(\frac{\eta+1}{\eta+\alpha}\right)}\left(\frac{Y_{t+j}}{A_{t+j}\bar{N}_{t+j}}\right)^{\frac{\eta+1}{\eta+\alpha}}\left(\frac{P_{t+j}}{P_t}\right)^{\epsilon\left(\frac{\eta+1}{\eta+\alpha}\right)}\left(\widetilde{R}^K_{t+j}\frac{P^K_{t+j}}{P_{t+j}}\right)^{\alpha\left(\frac{\eta+1}{\eta+\alpha}\right)}\right]}{\mathbb{E}_t\left[\sum_{j=0}^{\infty}(\theta\beta)^j\left(\frac{P_{t+j}}{P_t}\right)^{\epsilon-1}\left(\frac{C_{t+j}}{A_{t+j}\bar{N}_{t+j}}\right)^{-1}\left(\frac{Y_{t+j}}{A_{t+j}\bar{N}_{t+j}}\right)\right]}. \tag{71}$$

We can simplify this expression as

$$\frac{P_t^*}{P_t} = \left(\frac{F_t}{H_t}\right)^{\frac{1}{1+\epsilon\left(\frac{1-\alpha}{\eta+\alpha}\right)}}, \tag{72}$$

where F_t and H_t are recursively written as

$$\begin{aligned} F_t &= (1-\alpha)^{\frac{1-\alpha}{\eta+\alpha}}\left(\frac{(1+\varsigma_F)^{-1}\epsilon}{\epsilon-1}\right)\left(\frac{C_t}{A_t\bar{N}_t}\right)^{-\alpha\left(\frac{\eta+1}{\eta+\alpha}\right)}\left(\frac{Y_t}{A_t\bar{N}_t}\right)^{\frac{\eta+1}{\eta+\alpha}}\left(\widetilde{R}^K_t\frac{P^K_t}{P_t}\right)^{\alpha\left(\frac{\eta+1}{\eta+\alpha}\right)} + \theta\beta\mathbb{E}_t\left[\Pi_{t+1}^{\epsilon\left(\frac{\eta+1}{\eta+\alpha}\right)}F_{t+1}\right], \\ H_t &= \left(\frac{C_t}{A_t\bar{N}_t}\right)^{-1}\frac{Y_t}{A_t\bar{N}_t} + \theta\beta\mathbb{E}_t\left[\Pi_{t+1}^{\epsilon-1}H_{t+1}\right]. \end{aligned} \tag{73}$$

Using equation (24) and equation (73), we obtain the following equilibrium condition for price-resetting in our framework.

$$\frac{F_t}{H_t} = \left(\frac{1-\theta}{1-\theta\Pi_t^{\epsilon-1}}\right)^{\left(\frac{1}{\epsilon-1}\right)\left[1+\epsilon\left(\frac{1-\alpha}{\eta+\alpha}\right)\right]}. \tag{74}$$

We now rewrite equation (16) as

$$1 = \beta\cdot\mathbb{E}_t\left[\frac{R^S_{t+1}}{\Pi_{t+1}GA_{t+1}GN}\cdot\frac{\left(\frac{C_t}{A_t\bar{N}_t}\right)}{\left(\frac{C_{t+1}}{A_{t+1}\bar{N}_{t+1}}\right)}\right].$$

Since R^S_{t+1} depends on bonds return R^{HB}_{t+1} and loans return R^K_{t+1} while shares of savings that flow into bonds $(1-\lambda^K_t)$ and loans (λ^K_t) are endogenous, we start from analyzing R^{HB}_{t+1}.

We can rewrite the aggregate return indices as functions of the bond yields $\{YD_t^f\}_{f=1}^F$ as

$$R_t^j = \sum_{f=0}^{F-1} \lambda_{t-1}^{j,f+1} \frac{\left(YD_t^f\right)^{-f}}{\left(YD_{t-1}^{f+1}\right)^{-(f+1)}},\ \ j \in \{H, G, CB\},$$

and also the household's bond portfolio share as

$$\lambda_t^{HB,f} = \left(\frac{\mathbb{E}_t \left[\frac{\beta \cdot z_t^f}{\Pi_{t+1} \cdot GA_{t+1} \cdot GN} \cdot \frac{\left(\frac{C_t}{A_t \bar{N}_t}\right)}{\left(\frac{C_{t+1}}{A_{t+1}\bar{N}_{t+1}}\right)} \cdot \frac{\left(YD_{t+1}^{f-1}\right)^{-(f-1)}}{\left(YD_t^f\right)^{-f}} \right]}{\Phi_t^B} \right)^{\kappa_B},\ \ \forall f,$$

$$\Phi_t^B = \left[\sum_{j=1}^{F} \mathbb{E}_t \left[\frac{\beta \cdot z_t^j}{\Pi_{t+1} \cdot GA_{t+1} \cdot GN} \cdot \frac{\left(\frac{C_t}{A_t \bar{N}_t}\right)}{\left(\frac{C_{t+1}}{A_{t+1}\bar{N}_{t+1}}\right)} \cdot \frac{\left(YD_{t+1}^{j-1}\right)^{-(j-1)}}{\left(YD_t^j\right)^{-j}} \right]^{\kappa_B} \right]^{\frac{1}{\kappa_B}}.$$

Now we find the equilibrium condition for the bond shares of the agents. Using bond market equilibrium condition (equation (29)), we obtain

$$\lambda_t^{HB,f} = \frac{B_t^{G,f} + B_t^{CB,f}}{B_t^G + B_t^{CB}} = \frac{\lambda_t^{G,f} B_t^G + \lambda_t^{CB,f} B_t^{CB}}{B_t^G + B_t^{CB}}. \tag{75}$$

We can rearrange the previous expression as

$$\lambda_t^{CB,f} = \lambda_t^{HB,f} + \left(\lambda_t^{HB,f} - \lambda_t^{G,f}\right) \cdot \frac{B_t^G}{B_t^{CB}}. \tag{76}$$

Summing across maturities from $f = 2$ to F, and using $\sum_{f=2}^F \lambda_t^{j,f} = 1 - \lambda_t^{j,1},\ j \in \{H, G, CB\}$ we obtain

$$\sum_{f=2}^{F} \lambda_t^{CB,f} = 1 - \lambda_t^{HB,1} + \left(\lambda_t^{G,1} - \lambda_t^{HB,1}\right) \cdot \frac{B_t^G}{B_t^{CB}}. \tag{77}$$

Plugging equation (77) into equation (76) and after some rearrangements, we obtain

$$\lambda_t^{CB,f} = \frac{\lambda_t^{HB,f}\left(\lambda_t^{CB,1} - \lambda_t^{G,1}\right) - \lambda_t^{G,f}\left(\lambda_t^{CB,1} - \lambda_t^{HB,1}\right)}{\lambda_t^{HB,1} - \lambda_t^{G,1}},\ \ f > 1. \tag{78}$$

Now, we can obtain an expression for central bank's bond holdings using equation (77) as

$$B_t^{CB} = \left(\frac{\lambda_t^{HB,1} - \lambda_t^{G,1}}{\lambda_t^{CB,1} - \lambda_t^{HB,1}} \right) \cdot B_t^G. \tag{79}$$

Combining equation (29) and equation (79) we obtain

$$\frac{B_t^H}{A_t \bar{N}_t} = - \left(\frac{\lambda_t^{CB,1} - \lambda_t^{G,1}}{\lambda_t^{CB,1} - \lambda_t^{HB,1}} \right) \cdot \frac{B_t^G}{A_t \bar{N}_t}. \tag{80}$$

Combining $L_t = \lambda_t^K S_t$ and $B_t^H = (1 - \lambda_t^K) S_t$ with $L_t = P_t^K K_t$, we obtain

$$\frac{B_t^H}{A_t \bar{N}_t P_t} = \frac{1}{GA_t \cdot GN} \left(\frac{1 - \lambda_t^K}{\lambda_t^K} \right) \left(\frac{P_t^K}{P_t} \right) \left(\frac{K_t}{A_{t-1} \bar{N}_{t-1}} \right). \tag{81}$$

Using $B_t^H = -\left(B_t^G + B_t^{CB} \right)$, equation (79), and bond-market equilibrium condition (equation (29)), we get the following equation, which is equation (47).

$$- \left(\frac{\lambda_t^{CB,1} - \lambda_t^{G,1}}{\lambda_t^{CB,1} - \lambda_t^{HB,1}} \right) \cdot \frac{B_t^G}{A_t \bar{N}_t P_t} = \frac{1}{GA_t \cdot GN} \left(\frac{1 - \lambda_t^K}{\lambda_t^K} \right) \left(\frac{P_t^K}{P_t} \right) \left(\frac{K_t}{A_{t-1} \bar{N}_{t-1}} \right). \tag{82}$$

B.1.3. Standard Policy in Section 2.10.1

Using bond market equilibrium (equation (29)) with $\sum_{f=2}^{F} \lambda_t^{HB,f} = 1 - \lambda_t^{HB,1}$, we get

$$B_t^H = - \frac{\sum_{i=2}^{F} \left(B_t^{G,i} + B_t^{CB,i} \right)}{1 - \lambda_t^{HB,1}}. \tag{83}$$

Combining equation (83) with equation (49), we obtain the equilibrium set of equations,

$$\frac{\lambda_t^{HB,f}}{1 - \lambda_t^{HB,1}} = \frac{\frac{B_t^{G,f}}{A_t \bar{N}_t P_t} + \overline{\frac{B^{CB,f}}{A\bar{N}P}}}{\sum_{i=2}^{F} \left(\frac{B_t^{G,i}}{A_t \bar{N}_t P_t} + \overline{\frac{B^{CB,i}}{A\bar{N}P}} \right)}, \quad \forall f > 1. \tag{84}$$

Combining equation (81), equation (83) and equation (49) yields the following equilibrium equation

$$- \frac{\sum_{i=2}^{F} \left(\frac{B_t^{G,i}}{A_t \bar{N}_t P_t} + \overline{\frac{B^{CB,i}}{A\bar{N}P}} \right)}{1 - \lambda_t^{HB,1}} = \frac{1}{GA_t \cdot GN} \left(\frac{1 - \lambda_t^K}{\lambda_t^K} \right) \left(\frac{P_t^K}{P_t} \right) \left(\frac{K_t}{A_{t-1} \bar{N}_{t-1}} \right), \tag{85}$$

where normalized bond positions of the central bank are exogenously given.

Finally, combining equation (84) and equation (85) we finally obtain the equation (49).

$$-\left(\frac{B_t^{G,f}}{A_t\bar{N}_tP_t}+\frac{\overline{B^{CB,f}}}{A\bar{N}P}\right)\cdot\left(\lambda_t^{HB,f}\right)^{-1}=\frac{1}{GA_t\cdot GN}\left(\frac{1-\lambda_t^K}{\lambda_t^K}\right)\left(\frac{P_t^K}{P_t}\right)\left(\frac{K_t}{A_{t-1}\bar{N}_{t-1}}\right),\ \forall f>1. \tag{86}$$

B.1.4. Steady-State Derivations in Section 3.1

In the steady state, the central bank decides the level of bond holdings of each maturity $B^{CB,f}$ that it wants to hold. It can be calibrated to match the data of central bank's balance sheet. Given $\left\{\lambda^{CB,f}\right\}$ and the size of its portfolio B^{CB}, which is ζ^B fraction of total government bond issuance satisfying $B^{CB}=\zeta^{CB}\cdot B^G$, we can obtain an steady state expression for the household bond shares as

$$\lambda^{HB,f}=\frac{\lambda^{G,f}+\lambda^{CB,f}\cdot\zeta^B}{1+\zeta^B}. \tag{87}$$

From the definition of R^{HB} we have

$$\sum_{f=1}^{F}\lambda^{HB,f}\cdot\left(\frac{R^f}{R^{HB}}\right)=1.$$

together with equation (160) and equation (161) rearranged as:

$$\lambda^{HB,f}=\left(\frac{z^f\cdot\frac{R^f}{R^{HB}}}{\tilde{\Phi}^B}\right)^{\kappa},\ \forall f,\ \text{with}\ \tilde{\Phi}^B=\left[\sum_{j=1}^{F}\left[z^j\cdot\frac{R^j}{R^{HB}}\right]^{\kappa}\right]^{1/\kappa}. \tag{88}$$

The above equation (87) and equation (88) jointly determine the steady state yields and household shares. Unfortunately, there is no analytical expression for them and we have to solve for the steady state values numerically. How we proceed, relying on simple iterations:

1. Assume some initial guess for $\left\{R^{f,guess}/R^{HB}\right\}_{f=1}^{F}$
2. Construct $\tilde{\Phi}^{B,old}$ using previous guess with $\tilde{\Phi}^B$ in equation (88)
3. Update estimates on $\left\{R^f/R^{BH}\right\}_{f=1}^{F}$ with the following rules

$$\frac{R^{1,new}}{R^{HB}}=\frac{1-\sum_{f=2}^{F}\lambda^{HB,f}\left(\frac{R^f}{R^{BH}}\right)}{\lambda^{HB,1}},\quad \frac{R^{f,new}}{R^{BH}}=\left(\lambda^{HB,f}\right)^{\frac{1}{\kappa}}\left(z^f\right)^{-1}\tilde{\Phi}^{B,old},\ f>1$$

4. Construct new household shares $\lambda^{HB,f,new}$ by plugging $\left\{R^{f,new}/R^{HB}\right\}_{f=1}^{F}$ into equation (88). Compute the discrepancy between these shares and the true ones found in equation (87). If the error is big, set $R^{f,guess}/R^{HB}=R^{f,new}/R^{HB}$ and repeat from step 2. until convergence.

Using equation (185) and equation (196) we obtain

$$R^{HB} = \frac{\beta^{-1}\Pi \cdot GA \cdot GN}{1-\lambda^K} - \frac{\lambda^K}{1-\lambda^K} R^K. \tag{89}$$

We can rewrite R^G as

$$R^G = \Xi \cdot R^{HB}, \;\; \Xi = \sum_{f=1}^{F} \lambda^{G,f} \cdot \left(\frac{R^f}{R^{HB}}\right),$$

and using equation (89) it becomes

$$R^G = \Xi \cdot \left[\frac{\beta^{-1}\Pi \cdot GA \cdot GN}{1-\lambda^K} - \frac{\lambda^K}{1-\lambda^K} R^K\right]. \tag{90}$$

We obtain an expression for price dispersion as

$$\Delta = \left[\frac{1-\theta}{1-\theta\Pi^{\epsilon\left(\frac{\eta+1}{\eta+\alpha}\right)}}\right]\left(\frac{1-\theta\Pi^{\epsilon-1}}{1-\theta}\right)^{\left(\frac{\epsilon}{\epsilon-1}\right)\left(\frac{\eta+1}{\eta+\alpha}\right)}. \tag{91}$$

From the capital producer's optimization (equation equation (197)), we obtain an expression for P^K

$$\frac{P^K}{P} = \beta^{-1} \cdot GA \cdot GN - (1-\delta). \tag{92}$$

The equilibrium government bonds are obtained from its budget constraint (equation (173)) and written as

$$\frac{B^G}{P\bar{N}A} = -\left(1 - \frac{R^G}{\Pi \cdot GA \cdot GN}\right)^{-1}\left[\zeta^G + \zeta^F - \zeta^T\right]\left(\frac{Y}{A\bar{N}}\right). \tag{93}$$

The model needs government to be a borrower, so $B^G < 0$ at steady-state. Also, we would like to match the data in which the government runs primary deficit $\zeta^G + \zeta^F - \zeta^T > 0$. The only way to achieve that is by having $R^G < \Pi \cdot GA \cdot GN$. Plugging $B^{CB} = \zeta^{CB} \cdot B^G$ and equation (92) into equation (82) yields

$$\frac{K}{A\bar{N}} = -\left(1+\zeta^{CB}\right) \cdot GA \cdot GN \cdot \left(\frac{1}{\beta^{-1} \cdot GA \cdot GN - (1-\delta)}\right)\left(\frac{\lambda^K}{1-\lambda^K}\right) \cdot \left(\frac{B^G}{A\bar{N}P}\right). \tag{94}$$

By plugging equation (93) into the previous equation (94), we obtain

$$\frac{K}{A\bar{N}} = \xi^K \left(1 - \frac{R^G}{\Pi \cdot GA \cdot GN}\right)^{-1}\left(\frac{\lambda^K}{1-\lambda^K}\right)\left(\frac{Y}{A\bar{N}}\right), \tag{95}$$

$$\text{with } \xi^K = \left(1+\zeta^{CB}\right)\left[\zeta^G + \zeta^F - \zeta^T\right]\left(\frac{\beta \cdot GA \cdot GN}{GA \cdot GN - \beta(1-\delta)}\right). \tag{96}$$

By plugging the previous equation (95) into the market clearing condition (equation (184)), we obtain the following relation between consumption and output.

$$\frac{C}{A\bar{N}} = \left[(1-\zeta^G) - \xi^C\left(1 - \frac{R^G}{\Pi \cdot GA \cdot GN}\right)^{-1}\left(\frac{\lambda^K}{1-\lambda^K}\right)\right]\left(\frac{Y}{A\bar{N}}\right), \tag{97}$$
$$\text{with } \xi^C = \left[1 - \frac{1-\delta}{GA \cdot GN}\right]\xi^K.$$

The steady state representation of firms' pricing (equation (167) and equation (168)) can be written as

$$F = \xi^F\left[(1-\zeta^G) - \xi^C\left(1 - \frac{R^G}{\Pi \cdot GA \cdot GN}\right)^{-1}\left(\frac{\lambda^K}{1-\lambda^K}\right)\right]^{-\alpha\left(\frac{\eta+1}{\eta+\alpha}\right)}\left(\frac{Y}{A\bar{N}}\right)^{(1-\alpha)\left(\frac{\eta+1}{\eta+\alpha}\right)}\left(R^K\right)^{\alpha\left(\frac{\eta+1}{\eta+\alpha}\right)}, \tag{98}$$
$$\text{with } \xi^F = (1-\alpha)^{\frac{1-\alpha}{\eta+\alpha}}\left[1 - \theta\beta\Pi^{\epsilon\frac{\eta+1}{\eta+\alpha}}\right]^{-1}\left(\frac{(1+\zeta^F)^{-1}\epsilon}{\epsilon-1}\right)\left[\frac{GA \cdot GN - (1-\delta)\beta}{\Pi \cdot GA \cdot GN}\right]^{\alpha\left(\frac{\eta+1}{\eta+\alpha}\right)},$$

$$H = \left[1 - \theta\beta\Pi^{\epsilon-1}\right]^{-1}\left[(1-\zeta^G) - \xi^C\left(1 - \frac{R^G}{\Pi \cdot GA \cdot GN}\right)^{-1}\left(\frac{\lambda^K}{1-\lambda^K}\right)\right]^{-1}. \tag{99}$$

Plugging equation (98) and equation (99) into firms' optimal price-resetting equation (equation (74)) and rearranging the resulting equation, we obtain

$$\frac{Y}{A\bar{N}} = \xi^Y \cdot \left[(1-\zeta^G) - \xi^C\left(1 - \frac{R^G}{\Pi \cdot GA \cdot GN}\right)^{-1}\left(\frac{\lambda^K}{1-\lambda^K}\right)\right]^{-\left(\frac{\eta}{\eta+1}\right)}\left(R^K\right)^{-\left(\frac{\alpha}{1-\alpha}\right)}, \tag{100}$$
$$\text{with } \xi^Y = \left(\xi^F\right)^{-\left(\frac{\eta+\alpha}{(1-\alpha)(\eta+1)}\right)}\left[1 - \theta\beta\Pi^{\epsilon-1}\right]^{-\left(\frac{\eta+\alpha}{(1-\alpha)(\eta+1)}\right)}\left(\frac{1-\theta}{1-\theta \cdot \Pi^{\epsilon-1}}\right)^{\left(\frac{\eta+\alpha+\epsilon(1-\alpha)}{(\epsilon-1)(1-\alpha)(\eta+1)}\right)}. \tag{101}$$

Finally, plugging equation (95), equation (97) and equation (100) into loan aggregation equation (equation (44)) and rearranging properly, we obtain the following relation.

$$\xi^{R^K} = R^K\left(1 - \frac{R^G}{\Pi \cdot GA \cdot GN}\right)^{-1}\left(\frac{\lambda^K}{1-\lambda^K}\right) \tag{102}$$
$$= \alpha(1-\alpha)^{\frac{1-\alpha}{\eta+\alpha}} \cdot GA \cdot GN \cdot \Delta\left(\frac{GA \cdot GN - (1-\delta)\beta}{\Pi \cdot GA \cdot GN}\right)^{-\left(\frac{\eta(1-\alpha)}{\eta+\alpha}\right)}\left(\xi^K\right)^{-1}\left(\xi^Y\right)^{(1-\alpha)\left(\frac{\eta+1}{\eta+\alpha}\right)}.$$

As ξ^{R^K} is constant, after plugging equation equation (90) into equation (102) and rearranging the equation, we obtain

$$R^K = \left[1 - \frac{\Xi\xi^{R^K}}{\Pi \cdot GA \cdot GN}\right]^{-1}\left[\left(\frac{1-\lambda^K}{\lambda^K}\right) - \frac{\beta^{-1}\Xi}{\lambda^K}\right]\xi^{R^K}. \tag{103}$$

Finally, by plugging equation (103) into equation (89), we get

$$R^{HB} = \frac{\beta^{-1}\Pi \cdot GA \cdot GN}{1-\lambda^K} - \left[1 - \frac{\Xi \cdot \xi^{R^K}}{\Pi \cdot GA \cdot GN}\right]^{-1} \left[1 - \frac{\beta^{-1}\Xi}{1-\lambda^K}\right] \xi^{R^K}. \tag{104}$$

Equations equation (103) and equation (104) plugged into equation (193) form a non-linear equation of the unknown λ^K. We obtain its value by relying on computational methods, and then we can back out the rest of the steady state variables. Once we back out R^K and λ^K, we can back out R^{HB} using equation (89). After that, we can then simply back out bond returns as

$$R^f = R^{HB} \cdot \left(\frac{R^f}{R^{HB}}\right).$$

Now that we have found the bond returns, we can recursively obtain the bond yields using

$$YD^f = \left[R^f \cdot \left(YD^{f-1}\right)^{f-1}\right]^{1/f},$$

where $YD^0 = 1$, which we use to get started with the recursion from $f = 1$ to F.

B.1.5. Log-linearization

We start by log-linearizing the equations that are common to the standard policy model and the QE one, then derive the ones that are different.
Log-linearize equations equation (179), equation (180) and equation (181) to obtain

$$\hat{ga}_t = \hat{\varepsilon}^A_t, \;\; \hat{\zeta}^G_t = \frac{a^G}{1+a^G} \cdot \hat{u}^G_t, \;\; \hat{\zeta}^T_t = \frac{a^T}{1+a^T} \cdot \hat{u}^T_t. \tag{105}$$

Equations (equation (154) and equation (155)) with the help of equation (105) can be linearized as

$$\hat{c}_t = \left[\left(1-\zeta^G\right) \cdot \frac{Y}{C}\right] \left[\hat{y}_t - \frac{1}{1+a^G} \cdot \hat{u}^G_t\right] + \left[\frac{1-\delta}{GA \cdot GN} \frac{K}{C}\right] (\hat{k}_t - \hat{\varepsilon}^A_t) - \frac{K}{C}\hat{k}_{t+1}, \tag{106}$$

$$\hat{c}_t = \mathbb{E}_t \left[\hat{c}_{t+1} - \left(\hat{r}^S_{t+1} - \hat{\pi}_{t+1}\right)\right], \tag{107}$$

where I used equation (105) to solve for $\hat{\zeta}^G_t$ and $\hat{ga}_t$.
Plugging equation (106) into equation (107) and using equation (182), we obtain the following dynamic IS equation for output $\hat{y}_t$.

$$\begin{aligned}\hat{y}_t = \mathbb{E}_t \Bigg[\hat{y}_{t+1} &- \left[\frac{(1-\zeta^G)^{-1}(1-\delta)}{GA \cdot GN} \cdot \frac{K}{Y}\right] (\hat{k}_t - \hat{\varepsilon}^A_t) + (1-\zeta^G)^{-1} \left[1 + \frac{1-\delta}{GA \cdot GN}\right] \frac{K}{Y}\hat{k}_{t+1} \\ &- (1-\zeta^G)^{-1}\frac{K}{Y}\hat{k}_{t+2} - (1-\zeta^G)^{-1}\frac{C}{Y}\left(\hat{r}^S_{t+1} - \hat{\pi}_{t+1}\right) + \frac{1-\rho_G}{1+a^G} \cdot \hat{u}^G_t\Bigg].\end{aligned} \tag{108}$$

Linearizing the household's bond portfolio conditions (equation (160) and equation (161)) yields

$$\hat{\lambda}_t^{HB,f} = \kappa^B \mathbb{E}_t \left[\hat{z}_t^f - \hat{\pi}_{t+1} - \hat{ga}_{t+1} + \hat{c}_t - \hat{c}_{t+1} - (f-1)\hat{yd}_{t+1}^{f-1} + f\hat{yd}_t^f - \hat{\phi}_t^B \right], \tag{109}$$

where

$$\hat{\phi}_t^B = \mathbb{E}_t \left(-\hat{\pi}_{t+1} - \hat{ga}_{t+1} + \hat{c}_t - \hat{c}_{t+1} \right) + \sum_{j=1}^{F} \left[\frac{\beta z^j \left(YD^{j-1}\right)^{-(j-1)}}{\Pi \cdot GA \cdot GN \cdot \Phi^B \left(YD^j\right)^{-j}} \right]^{\kappa^B} \hat{z}_t^j$$
$$+ \sum_{j=1}^{F} j \left[\frac{\beta z^j \left(YD^{j-1}\right)^{-(j-1)}}{\Pi \cdot GA \cdot GN \cdot \Phi^B \left(YD^j\right)^{-j}} \right]^{\kappa^B} \hat{yd}_t^j - \sum_{j=0}^{F-1} j \left[\frac{\beta z^{j+1} \left(YD^j\right)^{-j}}{\Pi \cdot GA \cdot GN \cdot \Phi^B \left(YD^{j+1}\right)^{-(j+1)}} \right]^{\kappa^B} \mathbb{E}_t(\hat{yd}_{t+1}^j). \tag{110}$$

Combining equation (109) and equation (110), we obtain the following expression for $\hat{\lambda}_t^{HB,f}$:

$$\hat{\lambda}_t^{HB,f} = \sum_{j=1}^{F} \Psi_1^{fj} \hat{z}_t^j + \sum_{j=1}^{F} \Psi_2^{fj} \hat{yd}_t^j + \sum_{j=1}^{F} \Psi_3^{fj} \mathbb{E}_t \left[\hat{yd}_{t+1}^j \right], \tag{111}$$

where

$$\Psi_1^{fj} = \begin{cases} \left[1 - \left[\dfrac{\beta \cdot z^j \left(YD^{j-1}\right)^{-(j-1)}}{\Pi \cdot GA \cdot GN \cdot \Phi^B \left(YD^j\right)^{-j}} \right]^{\kappa^B} \right] \cdot \kappa^B & \text{, if } f = j, \\ - \left[\dfrac{\beta \cdot z^j \left(YD^{j-1}\right)^{-(j-1)}}{\Pi \cdot GA \cdot GN \cdot \Phi^B \left(YD^j\right)^{-j}} \right]^{\kappa^B} \cdot \kappa^B & \text{, if } f \neq j, \end{cases}$$

$$\Psi_2^{fj} = j \cdot \Psi_1^{fj},$$

$$\Psi_3^{fj} = \begin{cases} -j \cdot \left[1 - \left[\dfrac{\beta \cdot z^{j+1} \left(YD^j\right)^{-j}}{\Pi \cdot GA \cdot GN \cdot \Phi^B \left(YD^{j+1}\right)^{-(j+1)}} \right]^{\kappa^B} \right] \cdot \kappa^B & \text{, if } j = f-1, \\ j \cdot \left[\dfrac{\beta \cdot z^{j+1} \left(YD^j\right)^{-j}}{\Pi \cdot GA \cdot GN \cdot \Phi^B \left(YD^{j+1}\right)^{-(j+1)}} \right]^{\kappa^B} \cdot \kappa^B & \text{, if } j \neq f-1, \\ 0 & \text{, if } j = F. \end{cases}$$

We can put the system of F equation in matrix format as

$$\overrightarrow{\hat{\lambda}_t^{HB}} = \Psi_1 \cdot \overrightarrow{\hat{z}}_t + \Psi_2 \cdot \overrightarrow{\hat{yd}}_t + \Psi_3 \cdot \mathbb{E}_t \left[\overrightarrow{\hat{yd}}_{t+1} \right], \tag{112}$$

where $\{\Psi_1, \Psi_2, \Psi_3\}$ are matrices containing elements of $\left\{\Psi_1^{fj}, \Psi_2^{fj}, \Psi_3^{fj}\right\}$, with f representing rows and j

columns.
Linearizing equations equation (174) and equation (182) yields the following expression:

$$\overrightarrow{\hat{\lambda}_t^G} = \widetilde{\Xi} \cdot \overrightarrow{\hat{\tilde{u}}_t^B}, \quad \overrightarrow{\hat{\tilde{u}}_t^B} = \mathcal{T}^B \cdot \overrightarrow{\hat{u}_t^B}.$$

where $\widetilde{\Xi}$ is a matrix whose elements $\widetilde{\Xi}_{fj}$ (f-rows, j-columns) are

$$\widetilde{\Xi}_{fj} = \begin{cases} 0 & \text{, if } f = 1 \;\&\; j = f, \\ 1 - \lambda^{G,f} & \text{, if } f \geq 2 \;\&\; j = f, \\ -\lambda^{G,j} & \text{, if } j \neq f, \end{cases}$$

and similarly $\mathcal{T}^B$ is a matrix containing elements τ_{fj}^B from equation (176). By defining $\Xi = \widetilde{\Xi} \cdot \mathcal{T}^B$, we can combine the previous two equations to obtain

$$\overrightarrow{\hat{\lambda}_t^G} = \Xi \cdot \overrightarrow{\hat{u}_t^B}. \tag{113}$$

Linearizing equation (178) with the help ofequation (113) yields

$$\overrightarrow{\hat{b}_t^G} = \Xi \cdot \overrightarrow{\hat{u}_t^B} + \overrightarrow{1}_{Fx1} \cdot \hat{b}_t^G, \tag{114}$$

where $\overrightarrow{1}_{Fx1}$ is a unit vector of size F.
Log-linearizing the household's stochastic discount factor yields the following formula.

$$\hat{q}_{t,t+1} = \hat{c}_t - \hat{c}_{t+1} - \hat{\pi}_{t+1} - \hat{ga}_{t+1}. \tag{115}$$

Log-linearizing Φ_t^S in the household's portfolio between loans and bonds (equation (194)), we obtain

$$\hat{\phi}_t^S = \mathbb{E}_t\left[q_{t,t+1}\right] + \frac{\left(z^B R^{HB}\right)^{\kappa^S}}{\left(z^B R^{HB}\right)^{\kappa^S} + \left(z^K R^K\right)^{\kappa^S}} \mathbb{E}_t\left[\hat{r}_{t+1}^{HB}\right] + \frac{\left(z^K R^K\right)^{\kappa^S}}{\left(z^B R^{HB}\right)^{\kappa^S} + \left(z^K R^K\right)^{\kappa^S}} \left(\hat{z}_t^K + \mathbb{E}_t\left[\hat{r}_{t+1}^K\right]\right). \tag{116}$$

Log-linearizing the household's portfolio decision between loans and bonds (equation (193)) and making use of the previous expression (equation (116)), we obtain

$$\begin{aligned} \hat{\lambda}_t^K &= \kappa^S \cdot \left[\frac{\left(z^B R^{HB}\right)^{\kappa^S}}{\left(z^B R^{HB}\right)^{\kappa^S} + \left(z^K R^K\right)^{\kappa^S}}\right] \left(\hat{z}_t^K + \mathbb{E}_t\left[\hat{r}_{t+1}^K - \hat{r}_{t+1}^{HB}\right]\right) \\ &= \kappa^S \left(1 - \lambda^K\right) \left(\hat{z}_t^K + \mathbb{E}_t\left[\hat{r}_{t+1}^K - \hat{r}_{t+1}^{HB}\right]\right). \end{aligned} \tag{117}$$

By linearizing the formula for the effective savings rate of the household (equation (165)), we obtain

$$\hat{r}_t^S = \frac{\lambda^K\left(R^K - R^{HB}\right)}{R^S} \hat{\lambda}_{t-1}^K + \frac{(1-\lambda^K) R^{HB}}{R^S} \hat{r}_t^{HB} + \frac{\lambda^K R^K}{R^S} \hat{r}_t^K. \tag{118}$$

Log-linearizing the effective bond rates (equation (164)) of household, government, and the central bank yields

$$\hat{r}_t^j = \sum_{f=1}^{F} \frac{\lambda^{j,f}\left(YD^{f-1}\right)^{-(f-1)}}{R^j\left(YD^f\right)^{-f}} \cdot \left[\hat{\lambda}_{t-1}^{j,f} - (f-1)\hat{yd}_t^{f-1} + f\hat{yd}_{t-1}^{f}\right],\ j \in \{HB, G, CB\}, \tag{119}$$

with which we can express these equations on matrix format as

$$\hat{r}_t^j = \Psi^{j,4} \cdot \overrightarrow{\hat{\lambda}_{t-1}^{j}} - \Psi^{j,5} \cdot \overrightarrow{\hat{yd}_t} + \Psi^{j,6} \cdot \overrightarrow{\hat{yd}_{t-1}}, \qquad j \in \{HB, G, CB\}, \tag{120}$$

where $\left\{\Psi^{j,4}, \Psi^{j,5}, \Psi^{j,6}\right\}$ are $1xF$-sized matrices whose elements are defined as follows.

$$\begin{aligned}
\Psi_{1f}^{j,4} &= \frac{\lambda^{j,f}\left(YD^{f-1}\right)^{-(f-1)}}{R^j\left(YD^f\right)^{-f}}, \\
\Psi_{1f}^{j,5} &= \begin{cases} \dfrac{\lambda^{j,f+1}\left(YD^{f}\right)^{-f}}{R^j\left(YD^{f+1}\right)^{-(f+1)}} f & \text{, if } f < F,\ j \in \{HB, G, CB\}, \\ 0 & \text{, if } f = F, \end{cases} \\
\Psi_{1f}^{j,6} &= \frac{\lambda^{j,f}\left(YD^{f-1}\right)^{-(f-1)}}{R^j\left(YD^f\right)^{-f}} f.
\end{aligned}$$

By plugging equation (113) into $\hat{r}^G$ in equation (120), we obtain

$$\hat{r}_t^G = \Psi^{G,4} \cdot \Xi \cdot \overrightarrow{\hat{u}_{t-1}^{B}} - \Psi^{G,5} \cdot \overrightarrow{\hat{yd}_t} + \Psi^{G,6} \cdot \overrightarrow{\hat{yd}_{t-1}}. \tag{121}$$

By plugging equation (112) into $\hat{r}^{HB}$ in equation (120), we obtain

$$\hat{r}_t^{HB} = \Psi^{HB,4}\Psi^1 \cdot \overrightarrow{\hat{z}_{t-1}} + \left[\Psi^{HB,4}\Psi^2 + \Psi^{HB,6}\right] \cdot \overrightarrow{\hat{yd}_{t-1}} + \Psi^{HB,4}\Psi^3 \cdot \mathbb{E}_{t-1}\left[\overrightarrow{\hat{yd}_t}\right] - \Psi^{HB,5} \cdot \overrightarrow{\hat{yd}_t}. \tag{122}$$

Taking the expectation operator $\mathbb{E}_t$ on the previous equation (122), we obtain

$$\mathbb{E}_t\left[\hat{r}_{t+1}^{HB}\right] = \Psi^{HB,4}\Psi^1 \cdot \overrightarrow{\hat{z}_t} + \left[\Psi^{HB,4}\Psi^2 + \Psi^{HB,6}\right] \cdot \overrightarrow{\hat{yd}_t} + \left[\Psi^{HB,4}\Psi^3 - \Psi^{HB,5}\right] \mathbb{E}_t\left[\overrightarrow{\hat{yd}_{t+1}}\right]. \tag{123}$$

By plugging equation (117) and equation (123) into equation (118), we now obtain the expected effective savings rate as follows.[75]

$$\mathbb{E}_t\left[\hat{r}_{t+1}^{S}\right] = \Psi^7 \overrightarrow{\hat{z}}_t + \Psi^8 \overrightarrow{\hat{yd}_t} + \Psi^9 \mathbb{E}_t\left[\overrightarrow{\hat{yd}_{t+1}}\right] + \Psi^{10}\hat{r}_{t+1}^{K} + \Psi^{11}\hat{z}_t^{K}, \tag{124}$$

[75]Since $\hat{r}_{t+1}^K$ is determined at quarter t, thus $\mathbb{E}_t(\hat{r}_{t+1}^K) = \hat{r}_{t+1}^K$ holds.

where

$$\Psi^7 = \Psi^{HB,4}\Psi^1 \left[\frac{(1+\kappa^S\lambda^K)(1-\lambda^K)R^{HB} - \kappa^S(1-\lambda^K)\lambda^K R^K}{R^S}\right],$$
$$\Psi^8 = \left[\Psi^{HB,4}\Psi^2 + \Psi^{HB,6}\right] \left[\frac{(1+\kappa^S\lambda^K)(1-\lambda^K)R^{HB} - \kappa^S(1-\lambda^K)\lambda^K R^K}{R^S}\right],$$
$$\Psi^9 = \left[\Psi^{HB,4}\Psi^3 - \Psi^{HB,5}\right] \left[\frac{(1+\kappa^S\lambda^K)(1-\lambda^K)R^{HB} - \kappa^S(1-\lambda^K)\lambda^K R^K}{R^S}\right],$$
$$\Psi^{10} = \frac{\left[1+\kappa^S(1-\lambda^K)\right]\lambda^K R^K - \kappa^S(1-\lambda^K)\lambda^K R^{HB}}{R^S},$$
$$\Psi^{11} = \frac{\kappa^S\lambda^K(1-\lambda^K)\left(R^K - R^{HB}\right)}{R^S}.$$

Plugging back the expression of the household's expected bonds rate (equation (123)) into her portfolio decision between loans and bonds (equation (117)), we obtain

$$\hat{\lambda}_t^K = \kappa^S\left(1-\lambda^K\right)\left(\hat{z}_t^K + \hat{r}_{t+1}^K\right) - \Psi^{12}\cdot\overrightarrow{\hat{z}_t} - \Psi^{13}\cdot\overrightarrow{\hat{yd}_t} - \Psi^{14}\cdot\mathbb{E}_t\left[\overrightarrow{\hat{yd}_{t+1}}\right], \tag{125}$$

where

$$\Psi^{12} = \kappa^S(1-\lambda^K)\Psi^{HB,4}\Psi^1,$$
$$\Psi^{13} = \kappa^S(1-\lambda^K)\left[\Psi^{HB,4}\Psi^2 + \Psi^{HB,6}\right],$$
$$\Psi^{14} = \kappa^S(1-\lambda^K)\left[\Psi^{HB,4}\Psi^3 - \Psi^{HB,5}\right].$$

If we linearize loan aggregation equation(equation (172)), we obtain[76]

$$\hat{k}_t = \hat{ga}_t + \left(\frac{\eta+1}{\eta+\alpha}\right)\hat{y}_t - \left(\frac{\eta(1-\alpha)}{\eta+\alpha}\right)\mathbb{E}_t\left[\hat{q}_{t,t+1} + \hat{r}_{t+1}^K + \hat{p}_t^K - \hat{c}_t\right]. \tag{126}$$

By plugging equation (115) into equation (126) and using equation (105) with rearranging, we obtain

$$p_t^K = \left(\frac{\eta+1}{\eta(1-\alpha)}\right)\hat{y}_t - \left(\frac{\eta+\alpha}{\eta(1-\alpha)}\right)\left[\hat{k}_t - \hat{\varepsilon}_t^A\right] + \mathbb{E}_t\left[\hat{c}_{t+1} + \hat{\pi}_{t+1}\right] - \hat{r}_{t+1}^K. \tag{127}$$

Combining equation (182), equation (106), and equation (127) we obtain

$$\begin{aligned} p_t^K = {} & \left(\frac{\eta+1}{\eta(1-\alpha)}\right)\hat{y}_t - \left(\frac{\eta+\alpha}{\eta(1-\alpha)}\right)\left[\hat{k}_t - \hat{\varepsilon}_t^A\right] + \left[\frac{a^G}{1+a^G}\frac{Y}{C}\right]\left[\mathbb{E}_t\left[\hat{y}_{t+1}\right] - \frac{\rho_G}{1+a^G}\hat{u}_t^G\right] \\ & + \left[\frac{1-\delta}{GA\cdot GN}\frac{K}{C}\right]\hat{k}_{t+1} - \frac{K}{C}\mathbb{E}_t\left[\hat{k}_{t+2}\right] + \mathbb{E}_t\left[\hat{\pi}_{t+1}\right] - \hat{r}_{t+1}^K. \end{aligned} \tag{128}$$

[76]We have the first-order price dispersion $\hat{\Delta}_t$, generated from the positive trend inflation. We ignore its roles in most cases other than the welfare derivation. For this issue, see Woodford (2003), Coibion et al. (2012), and Carreras et al. (2016).

If we linearize the supply block (equation (167), equation (168), and equation (169)), we obtain

$$\begin{aligned}
\hat{f}_t =& \left[1-\theta\beta\Pi^{\epsilon\left(\frac{\eta+1}{\eta+\alpha}\right)}\right]\left(\frac{\eta+1}{\eta+\alpha}\right)\left[\hat{y}_t+\alpha\mathbb{E}_t\left[\hat{q}_{t,t+1}+\hat{r}^K_{t+1}+\hat{p}^K_t-\hat{c}_t\right]\right] \\
&+\theta\beta\Pi^{\epsilon\left(\frac{\eta+1}{\eta+\alpha}\right)}\mathbb{E}_t\left[\epsilon\left(\frac{\eta+1}{\eta+\alpha}\right)\hat{\pi}_{t+1}+\hat{f}_{t+1}\right],
\end{aligned} \tag{129}$$

$$\hat{h}_t = \left[1-\theta\beta\Pi^{\epsilon-1}\right][\hat{y}_t-\hat{c}_t]+\theta\beta\Pi^{\epsilon-1}\mathbb{E}_t\left[(\epsilon-1)\hat{\pi}_{t+1}+\hat{h}_{t+1}\right], \tag{130}$$

$$\hat{f}_t-\hat{h}_t = \left[1+\epsilon\left(\frac{1-\alpha}{\eta+\alpha}\right)\right]\left(\frac{\theta\Pi^{\epsilon-1}}{1-\theta\Pi^{\epsilon-1}}\right)\hat{\pi}_t. \tag{131}$$

Plugging equation (126) into equation (132) and using equation (105), we obtain

$$\hat{f}_t = \left[1-\theta\beta\Pi^{\epsilon\left(\frac{\eta+1}{\eta+\alpha}\right)}\right]\left(\frac{\eta+1}{\eta(1-\alpha)}\right)\left[\hat{y}_t-\alpha\cdot\left(\hat{k}_t-\hat{\varepsilon}^A_t\right)\right]+\theta\beta\Pi^{\epsilon\left(\frac{\eta+1}{\eta+\alpha}\right)}\mathbb{E}_t\left[\epsilon\left(\frac{\eta+1}{\eta+\alpha}\right)\hat{\pi}_{t+1}+\hat{f}_{t+1}\right]. \tag{132}$$

Plugging equation (106) into equation (130)

$$\begin{aligned}
\hat{h}_t =& \left[1-\theta\beta\Pi^{\epsilon-1}\right]\Bigg[\left[1-(1-\zeta^G)\frac{Y}{C}\right]\hat{y}_t+\left[\left(1-\zeta^G\right)\frac{Y}{C}\right]\frac{1}{1+a^G}\hat{u}^G_t-\left[\frac{1-\delta}{GA\cdot GN}\frac{K}{C}\right](\hat{k}_t-\hat{\varepsilon}^A_t) \\
&+\frac{K}{C}\hat{k}_{t+1}\Bigg]+\theta\beta\Pi^{\epsilon-1}\mathbb{E}_t\left[(\epsilon-1)\hat{\pi}_{t+1}+\hat{h}_{t+1}\right].
\end{aligned} \tag{133}$$

Linearizing the government's budget constraint (equation (173)) yields

$$\hat{b}^G_t = \frac{R^G}{\Pi\cdot GA\cdot GN}\left[\hat{r}^G_t-\hat{\pi}_t-\hat{ga}_t+\hat{b}^G_{t-1}\right] \tag{134}$$

$$-\left[\zeta^G+\zeta^F-\zeta^T\right]\left(\frac{Y}{B/P}\right)\left[\hat{y}_t+\left(\frac{\zeta^G}{\zeta^G+\zeta^F-\zeta^T}\right)\left(\frac{a^G}{1+a^G}\right)\hat{u}^G_t-\left(\frac{\zeta^T}{\zeta^G+\zeta^F-\zeta^T}\right)\left(\frac{a^T}{1+a^T}\right)\hat{u}^T_t\right]. \tag{135}$$

Using the steady state equilibrium condition (equation (93)) with equation (105) and equation (121), we can express the previous equation (135) as

$$\begin{aligned}
\hat{b}^G_t =& \frac{R^G}{\Pi\cdot GA\cdot GN}\left[\Psi^{G,4}\cdot\Xi\cdot\overrightarrow{\hat{u}^B_{t-1}}-\Psi^{G,5}\cdot\overrightarrow{\hat{yd}_t}+\Psi^{G,6}\cdot\overrightarrow{\hat{yd}_{t-1}}-\hat{\pi}_t-\hat{\varepsilon}^A_t+\hat{b}^G_{t-1}\right] \\
&+\left(1-\frac{R^G}{\Pi\cdot GA\cdot GN}\right)\left[\hat{y}_t+\left(\frac{\zeta^G}{\zeta^G+\zeta^F-\zeta^T}\right)\left(\frac{a^G}{1+a^G}\right)\hat{u}^G_t-\left(\frac{\zeta^T}{\zeta^G+\zeta^F-\zeta^T}\right)\left(\frac{a^T}{1+a^T}\right)\hat{u}^T_t\right].
\end{aligned} \tag{136}$$

Linearizing the labor aggregation condition (equation (171)) yields

$$\hat{n}_t = -\alpha\left(\frac{\eta}{\eta+\alpha}\right)\hat{c}_t+\left(\frac{\eta}{\eta+\alpha}\right)\hat{y}_t+\alpha\left(\frac{\eta}{\eta+\alpha}\right)\left[\mathbb{E}_t(\hat{q}_{t,t+1})+\hat{r}^K_{t+1}+\hat{p}^K_t\right]. \tag{137}$$

Linearizing the capital producer's optimization condition (equation (166)) yields

$$0 = \mathbb{E}_t \left[\hat{q}_{t,t+1} + \hat{\pi}_{t+1} + \left(\frac{P^K/P}{1-\delta+P^K/P} \right) \hat{p}^K_{t+1} \right]. \tag{138}$$

By plugging equation (107) and equation (115) into the previous equation (138) and rearranging, we get

$$\mathbb{E}_t \left[\hat{r}^S_{t+1} - \hat{\pi}_{t+1} \right] = \left(\frac{P^K/P}{1-\delta+P^K/P} \right) \mathbb{E}_t \left[\hat{p}^K_{t+1} \right]. \tag{139}$$

Plugging expressions on the effective savings rate (equation (124)) and the rental price of capital (equation (128)) into equation (139) we obtain

$$\begin{aligned} \hat{r}^K_{t+1} =& - \Psi^{15} \cdot \overrightarrow{\hat{z}_t} - \Psi^{16} \hat{z}^K_t - \Psi^{17} \overrightarrow{\hat{yd}_t} - \Psi^{18} \mathbb{E}_t \left[\overrightarrow{\hat{yd}_{t+1}} \right] + \Psi^{19} \mathbb{E}_t \left[\hat{\pi}_{t+1} \right] + \Psi^{20} \mathbb{E}_t \left[\hat{\pi}_{t+2} \right] + \Psi^{21} \mathbb{E}_t \left[\hat{y}_{t+1} \right] \\ &+ \Psi^{22} \mathbb{E}_t \left[\hat{y}_{t+2} \right] - \Psi^{23} \hat{k}_{t+1} + \Psi^{24} \mathbb{E}_t \left[\hat{k}_{t+2} \right] - \Psi^{25} \mathbb{E}_t \left[\hat{k}_{t+3} \right] - \Psi^{26} \hat{u}^G_t - \Psi^{20} \mathbb{E}_t \left[\hat{r}^K_{t+2} \right], \end{aligned} \tag{140}$$

where we defined

$$\begin{aligned} \Psi^{15} &= \left(\Psi^{10} \right)^{-1} \Psi^7, \\ \Psi^{16} &= \left(\Psi^{10} \right)^{-1} \Psi^{11}, \\ \Psi^{17} &= \left(\Psi^{10} \right)^{-1} \Psi^8, \\ \Psi^{18} &= \left(\Psi^{10} \right)^{-1} \Psi^9, \\ \Psi^{19} &= \left(\Psi^{10} \right)^{-1}, \\ \Psi^{20} &= \left(\Psi^{10} \right)^{-1} \left(\frac{P^K/P}{1-\delta+P^K/P} \right), \\ \Psi^{21} &= \Psi^{20} \left(\frac{\eta+1}{\eta(1-\alpha)} \right), \\ \Psi^{22} &= \Psi^{20} \left[\frac{a^G}{1+a^G} \cdot \frac{Y}{C} \right], \\ \Psi^{23} &= \Psi^{20} \left(\frac{\eta+\alpha}{\eta(1-\alpha)} \right), \\ \Psi^{24} &= \Psi^{20} \left[\frac{1-\delta}{GA \cdot GN} \cdot \frac{K}{C} \right], \\ \Psi^{25} &= \Psi^{20} \frac{K}{C}, \\ \Psi^{26} &= \Psi^{22} \frac{(\rho_G)^2}{1+a^G}. \end{aligned}$$

Finally, plugging the effective savings rate (equation (124)) into the Euler equation (equation (108)), we get

$$\hat{y}_t = \mathbb{E}_t\Big[\hat{y}_{t+1} + \Psi^{27}\hat{\pi}_{t+1} - \Psi^{28}\overrightarrow{\hat{z}}_t - \Psi^{29}\hat{z}^K_t - \Psi^{30}\overrightarrow{\hat{yd}}_t - \Psi^{31}\mathbb{E}_t\left[\overrightarrow{\hat{yd}}_{t+1}\right]$$
$$- \Psi^{32}\hat{r}^K_{t+1} - \Psi^{33}(\hat{k}_t - \hat{\varepsilon}^A_t) + \Psi^{34}\hat{k}_{t+1} - \Psi^{35}\hat{k}_{t+2} + \Psi^{36}\hat{u}^G_t\Big], \tag{141}$$

where we defined

$$\begin{aligned}
\Psi^{27} &= (1-\zeta^G)^{-1}\frac{C}{Y},\\
\Psi^{28} &= \Psi^{27}\Psi^{7},\\
\Psi^{29} &= \Psi^{27}\Psi^{11},\\
\Psi^{30} &= \Psi^{27}\Psi^{8},\\
\Psi^{31} &= \Psi^{27}\Psi^{9},\\
\Psi^{32} &= \Psi^{27}\Psi^{10},\\
\Psi^{33} &= \frac{(1-\zeta^G)^{-1}(1-\delta)}{GA\cdot GN}\frac{K}{Y},\\
\Psi^{34} &= (1-\zeta^G)^{-1}\left[1+\frac{1-\delta}{GA\cdot GN}\right]\frac{K}{Y},\\
\Psi^{35} &= (1-\zeta^G)^{-1}\frac{K}{Y},\\
\Psi^{36} &= \frac{1-\rho_G}{1+a^G}.
\end{aligned}$$

B.1.5.1. Log-linearization: Standard Policy Specific Derivations

Linearizing bond market equilibrium condition (equation (86)) using equation (105), we obtain

$$\hat{\lambda}^{HB,f}_t = \left(\frac{B^{G,f}}{B^{G,f}+B^{CB,f}}\right)\hat{b}^{g,f}_t + \hat{\varepsilon}^A_t + \frac{1}{1-\lambda^K}\hat{\lambda}^K_t - \hat{p}^K_t - \hat{k}_t,\ \ f\geq 2. \tag{142}$$

From $\lambda^{HB,1}_t = 1-\sum_{f=2}^{F}\lambda^{HB,f}_t$ we obtain

$$\hat{\lambda}^{HB,1}_t = -\sum_{f=2}^{F}\frac{\lambda^{HB,f}}{\lambda^{HB,1}}\hat{\lambda}^{HB,f}_t. \tag{143}$$

We can rearrange the previous expressions (equation (142) and equation (143)) in the matrix form as

$$\Theta^1\cdot\overrightarrow{\hat{\lambda}^{HB}_t} = \Theta^2\cdot\overrightarrow{\hat{b}^{g}_t} + \Theta^3\cdot\hat{\varepsilon}^A_t - \Theta^3\cdot\hat{p}^K_t - \Theta^3\cdot\hat{k}_t + \Theta^4\cdot\hat{\lambda}^K_t, \tag{144}$$

where $\{\Theta^1, \Theta^2\}$ are FxF-sized matrices with elements Θ^1_{jf} (row j, column f) and $\{\Theta^3, \Theta^4\}$ are $Fx1$ vectors with $j-$element Θ^3_{j1}. We define their elements as

$$\Theta^1_{jf} = \begin{cases} 1 & \text{, if } j = f, \\ \dfrac{\lambda^{HB,f}}{\lambda^{HB,1}} & \text{, if } j = 1 \& f > 1, \end{cases}$$

$$\Theta^2_{jf} = \begin{cases} \dfrac{B^{G,f}}{B^{G,f} + B^{CB,f}} & \text{, if } j > 1 \& j = f, \\ 0 & \text{, otherwise,} \end{cases}$$

$$\Theta^3_{j1} = \begin{cases} 0 & \text{, if } j = 1, \\ 1 & \text{, otherwise,} \end{cases}, \quad \Theta^4 = \frac{1}{1-\lambda^K} \cdot \Theta^3.$$

By inverting Θ^1 in equation (144), we can rewrite equation (144) as

$$\overrightarrow{\hat{\lambda}^{HB}_t} = \Theta^5 \cdot \overrightarrow{\hat{b}^{g,f}_t} + \Theta^6 \cdot \hat{\varepsilon}^A_t - \Theta^6 \cdot \hat{p}^K_t - \Theta^6 \cdot \hat{k}_t + \Theta^7 \cdot \hat{\lambda}^K_t, \tag{145}$$

where

$$\Theta^5 = \left(\Theta^1\right)^{-1} \Theta^2, \; \Theta^6 = \left(\Theta^1\right)^{-1} \Theta^3, \; \Theta^7 = \left(\Theta^1\right)^{-1} \Theta^4.$$

Plugging the government's bond portfolio (equation (114)), the household's loan share (equation (125)), and the rental price of capital (equation (128)) into equation (145), we obtain

$$\begin{aligned} \overrightarrow{\hat{\lambda}^{HB}_t} =& \Theta^8 \hat{b}^G_t - \Theta^9 \hat{y}_t - \Theta^{10} \mathbb{E}_t \left[\hat{y}_{t+1}\right] - \Theta^6 \mathbb{E}_t \left[\hat{\pi}_{t+1}\right] + \Theta^{11} \hat{k}_t - \Theta^{12} \hat{k}_{t+1} + \Theta^{13} \mathbb{E}_t \left[\hat{k}_{t+2}\right] + \Theta^{14} \hat{r}^K_{t+1} \\ & - \Theta^{15} \overrightarrow{\hat{yd}_t} - \Theta^{16} \mathbb{E}_t \left[\overrightarrow{\hat{yd}_{t+1}}\right] - \Theta^{17} \overrightarrow{\hat{z}_t} + \Theta^{18} \hat{z}^K_t - \Theta^{11} \hat{\varepsilon}^A_t + \Theta^{19} \hat{u}^G_t + \Theta^{20} \overrightarrow{\hat{u}^B_t}, \end{aligned} \tag{146}$$

where we defined

$$\Theta^8 = \Theta^5 \cdot \overrightarrow{1_{Fx1}}, \; \Theta^9 = \Theta^6 \left(\frac{\eta+1}{\eta(1-\alpha)}\right), \; \Theta^{10} = \Theta^6 \left[\frac{a^G}{1+a^G} \cdot \frac{Y}{C}\right], \; \Theta^{11} = \alpha \Theta^9,$$
$$\Theta^{12} = \Theta^6 \left[\frac{1-\delta}{GA \cdot GN} \cdot \frac{K}{C}\right], \; \Theta^{13} = \Theta^6 \frac{K}{C}, \; \Theta^{14} = \Theta^6 + \kappa^S \cdot \left(1-\lambda^K\right) \cdot \Theta^7, \; \Theta^{15} = \Theta^7 \Psi^{13},$$
$$\Theta^{16} = \Theta^7 \Psi^{14}, \; \Theta^{17} = \Theta^7 \Psi^{12}, \; \Theta^{18} = \kappa^S \left(1-\lambda^K\right) \Theta^7, \; \Theta^{19} = \Theta^{10} \frac{\rho_G}{1+a^G}, \; \Theta^{20} = \Theta^5 \Xi.$$

By plugging the household's optimal portfolio across maturities (equation (112)) into equation (146), we obtain

$$\begin{aligned} \overrightarrow{\hat{yd}_t} =& \Theta^{21} \hat{b}^G_t - \Theta^{22} \hat{y}_t - \Theta^{23} \mathbb{E}_t \left[\hat{y}_{t+1}\right] - \Theta^{24} \mathbb{E}_t \left[\hat{\pi}_{t+1}\right] + \Theta^{25} \hat{k}_t - \Theta^{26} \hat{k}_{t+1} + \Theta^{27} \mathbb{E}_t \left[\hat{k}_{t+2}\right] + \Theta^{28} \hat{r}^K_{t+1} \\ & - \Theta^{29} \mathbb{E}_t \left[\overrightarrow{\hat{yd}_{t+1}}\right] - \Theta^{30} \overrightarrow{\hat{z}_t} + \Theta^{31} \hat{z}^K_t - \Theta^{32} \hat{\varepsilon}^A_t + \Theta^{33} \hat{u}^G_t + \Theta^{34} \overrightarrow{\hat{u}^B_t}, \end{aligned} \tag{147}$$

where we defined

$$\Theta^{21} = \left[\Theta^{15} + \Psi^2\right]^{-1} \Theta^8,$$
$$\Theta^{22} = \left[\Theta^{15} + \Psi^2\right]^{-1} \Theta^9,$$
$$\Theta^{23} = \left[\Theta^{15} + \Psi^2\right]^{-1} \Theta^{10},$$
$$\Theta^{24} = \left[\Theta^{15} + \Psi^2\right]^{-1} \Theta^6,$$
$$\Theta^{25} = \left[\Theta^{15} + \Psi^2\right]^{-1} \Theta^{11},$$
$$\Theta^{26} = \left[\Theta^{15} + \Psi^2\right]^{-1} \Theta^{12},$$
$$\Theta^{27} = \left[\Theta^{15} + \Psi^2\right]^{-1} \Theta^{13},$$
$$\Theta^{28} = \left[\Theta^{15} + \Psi^2\right]^{-1} \Theta^{14},$$
$$\Theta^{29} = \left[\Theta^{15} + \Psi^2\right]^{-1} \left(\Theta^{16} + \Psi^3\right),$$
$$\Theta^{30} = \left[\Theta^{15} + \Psi^2\right]^{-1} \left(\Theta^{17} + \Psi^1\right),$$
$$\Theta^{31} = \left[\Theta^{15} + \Psi^2\right]^{-1} \Theta^{18},$$
$$\Theta^{32} = \left[\Theta^{15} + \Psi^2\right]^{-1} \Theta^{11},$$
$$\Theta^{33} = \left[\Theta^{15} + \Psi^2\right]^{-1} \Theta^{19},$$
$$\Theta^{34} = \left[\Theta^{15} + \Psi^2\right]^{-1} \Theta^{20}.$$

B.1.5.2. Log-linearization: General Policy Specific Derivations

Linearizing the Taylor rule for f-maturity bond (equation (37c)) yields

$$\hat{yd}_t^{GP,f} = \gamma_{SP}^f \hat{yd}_t^{SP,f} + \left(1 - \gamma_{SP}^f\right) \left[\gamma_\pi^f \hat{\pi}_t + \tilde{\varepsilon}_t^{YD^f}\right], \quad f \geq 2. \tag{148}$$

We define a $(F-1) \times (F-1)$ matrix Γ^{SP} with $\Gamma_{ff}^{SP} = \gamma_{SP}^{f+1}$ for $f = 1 \sim F-1$ and $\Gamma_{ij}^{SP} = 0$ for $i \neq j$. Also from the equation (54), we define $\mathcal{T}_{(f\geq 2)}^{YD}$, a $(F-1) \times L$ matrix with $\mathcal{T}_{(f\geq 2),f,l}^{YD} = \tau_{f+1,l}^{YD}$ (row f, column l)[77] and a vector of Taylor coefficients $\overrightarrow{\gamma_{\pi}}_{(f\geq 2)} = \left[\gamma_\pi^2, \ldots, \gamma_\pi^F\right]'$. If we construct such vectors as

$$\overrightarrow{\hat{yd}_t^{GP}}_{(f\geq 2)} = \left[\hat{yd}_t^{GP,2}, \ldots, \hat{yd}_t^{GP,F}\right]', \quad \overrightarrow{\hat{yd}_t^{SP}}_{(f\geq 2)} = \left[\hat{yd}_t^{SP,2}, \ldots, \hat{yd}_t^{SP,F}\right]', \tag{149}$$

[77] For large F, we reduce the state-space using a fewer number of monetary policy shocks than F.

then above equation (148) can be written in vector form as

$$\overrightarrow{\hat{yd}_t^{GP}}_{(f\geq 2)} = \Gamma^{SP}\overrightarrow{\hat{yd}_t^{SP}}_{(f\geq 2)} + (I-\Gamma^{SP})\cdot\left[\overrightarrow{\gamma_\pi}_{(f\geq 2)}\cdot\hat{\pi}_t + \mathcal{T}^{YD}_{(f\geq 2)}\cdot\overrightarrow{\varepsilon_t^{YD}}\right], \tag{150}$$

where I is the identity matrix of size $F-1$. Since $\overrightarrow{\hat{yd}_t^{SP}}$ is the yield vector that prevails in the counterfactual scenario where the current yield is determined by the standard monetary policy, it follows equation (147), expecting that the economy is driven by the general monetary policy. Thus, we can represent its dynamics as

$$\begin{aligned}\overrightarrow{\hat{yd}_t^{SP}} =& \Theta^{21}\hat{b}_t^G - \Theta^{22}\hat{y}_t - \Theta^{23}\mathbb{E}_t\left[\hat{y}_{t+1}\right] - \Theta^{24}\mathbb{E}_t\left[\hat{\pi}_{t+1}\right] + \Theta^{25}\hat{k}_t - \Theta^{26}\hat{k}_{t+1} + \Theta^{27}\mathbb{E}_t\left[\hat{k}_{t+2}\right] + \Theta^{28}\hat{r}^K_{t+1} \\ &- \Theta^{29}\mathbb{E}_t\left[\overrightarrow{\hat{yd}_{t+1}^{SP}}\right] - \Theta^{30}\overrightarrow{\hat{z}_t} + \Theta^{31}\hat{z}_t^K - \Theta^{32}\hat{\varepsilon}_t^A + \Theta^{33}\hat{u}_t^G + \Theta^{34}\overrightarrow{\hat{u}_t^B},\end{aligned} \tag{151}$$

where coefficients Θ^i for $i = 21 \sim 34$ are the same as the standard policy case, and $\overrightarrow{\hat{yd}_t^{SP}}$ and $\overrightarrow{\hat{yd}_t^{GP}}$ are defined as[78]

$$\overrightarrow{\hat{yd}_t^{GP}} = [\hat{yd}_t^{GP,1}, \overrightarrow{\hat{yd}_t^{GP}}'_{(f\geq 2)}]',\ \ \overrightarrow{\hat{yd}_t^{SP}} = [\hat{yd}_t^{GP,1}, \overrightarrow{\hat{yd}_t^{SP}}'_{(f\geq 2)}]'. \tag{152}$$

where $\hat{yd}_t^{GP,1}$ follows the Taylor rule rule in equation (37a) and equation (37b). Now that $\overrightarrow{\hat{yd}_t^{GP}}$, not $\overrightarrow{\hat{yd}_t^{SP}}$ governs agents' intertemporal decisions, equation (141) becomes

$$\begin{aligned}\hat{y}_t = \mathbb{E}_t\Big[&\hat{y}_{t+1} + \Psi^{27}\hat{\pi}_{t+1} - \Psi^{28}\overrightarrow{\hat{z}}_t - \Psi^{29}\hat{z}_t^K - \Psi^{30}\overrightarrow{\hat{yd}}_t - \Psi^{31}\mathbb{E}_t\left[\overrightarrow{\hat{yd}_{t+1}^{GP}}\right] \\ &- \Psi^{32}\hat{r}^K_{t+1} - \Psi^{33}(\hat{k}_t - \hat{\varepsilon}_t^A) + \Psi^{34}\hat{k}_{t+1} - \Psi^{35}\hat{k}_{t+2} + \Psi^{36}\hat{u}_t^G\Big].\end{aligned} \tag{153}$$

[78]There is no $\hat{yd}_t^{SP,1}$ in our formulation in equation (37c). Therefore, we use $\hat{yd}_t^{GP,1}$ instead of $\hat{yd}_t^{GP,1}$ in constructing $\overrightarrow{\hat{yd}_t^{SP}}$.

Appendix C. Summary of Equilibrium Equations

C.1. Equilibrium Equations: Standard Policy

Summary of relevant equilibrium conditions:

$$(i).\ \frac{C_t}{A_t\bar{N}_t} = \left(1-\zeta_t^G\right)\left(\frac{Y_t}{A_t\bar{N}_t}\right) + \left(\frac{1-\delta}{GA_t\cdot GN}\right)\left(\frac{K_t}{A_{t-1}\bar{N}_{t-1}}\right) - \left(\frac{K_{t+1}}{A_t\bar{N}_t}\right) \tag{154}$$

$$(ii).\ 1 = \beta\cdot\mathbb{E}_t\left[\frac{R_{t+1}^S}{\Pi_{t+1}\cdot GA_{t+1}\cdot GN}\frac{\left(\frac{C_t}{A_t\bar{N}_t}\right)}{\left(\frac{C_{t+1}}{A_{t+1}\bar{N}_{t+1}}\right)}\right] \tag{155}$$

$$(iii).\ \lambda_t^{HB,1} = 1-\sum_{f=2}^{F}\lambda_t^{HB,f} \tag{156}$$

$$(iv).\ -\left(\frac{B_t^{G,f}}{A_t\bar{N}_tP_t}+\frac{\overline{B^{CB,f}}}{A\bar{N}P}\right)\left(\lambda_t^{HB,f}\right)^{-1} = \frac{1}{GA_t\cdot GN}\left(\frac{1-\lambda_t^K}{\lambda_t^K}\right)\left(\frac{P_t^K}{P_t}\right)\left(\frac{K_t}{A_{t-1}\bar{N}_{t-1}}\right),\qquad \forall f>1 \tag{157}$$

$$(v).\ YD_t^1 = \max\left\{YD_t^{1*},1\right\} \tag{158}$$

$$(vi).\ YD_t^{1*} = \overline{YD}^1\cdot\left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\gamma_\pi}\left(\frac{Y_t}{\bar{Y}}\right)^{\gamma_y}\cdot\exp\left(\tilde{\varepsilon}_t^{YD^1}\right) \tag{159}$$

$$(vii).\ \lambda_t^{HB,f} = \left(\frac{\mathbb{E}_t\left[\frac{\beta z_t^f}{\Pi_{t+1}\cdot GA_{t+1}\cdot GN}\cdot\frac{\left(\frac{C_t}{A_t\bar{N}_t}\right)}{\left(\frac{C_{t+1}}{A_{t+1}\bar{N}_{t+1}}\right)}\frac{\left(YD_{t+1}^{f-1}\right)^{-(f-1)}}{\left(YD_t^f\right)^{-f}}\right]}{\Phi_t^B}\right)^{\kappa_B},\qquad \forall f \tag{160}$$

$$(viii).\ \Phi_t^B = \left[\sum_{j=1}^{F}\mathbb{E}_t\left[\frac{\beta z_t^j}{\Pi_{t+1}\cdot GA_{t+1}\cdot GN}\cdot\frac{\left(\frac{C_t}{A_t\bar{N}_t}\right)}{\left(\frac{C_{t+1}}{A_{t+1}\bar{N}_{t+1}}\right)}\frac{\left(YD_{t+1}^{j-1}\right)^{-(j-1)}}{\left(YD_t^j\right)^{-j}}\right]^{\kappa_B}\right]^{\frac{1}{\kappa_B}} \tag{161}$$

$$(ix).\ \lambda_t^K = \left(\frac{z_t^K\mathbb{E}_t\left[Q_{t,t+1}R_{t+1}^K\right]}{\Phi_t^S}\right)^{\kappa_S} \tag{162}$$

$$(x).\ \Phi_t^S = \left[\left(\mathbb{E}_t\left[Q_{t,t+1}R_{t+1}^{HB}\right]\right)^{\kappa_S}+\left(z_t^K\mathbb{E}_t\left[Q_{t,t+1}R_{t+1}^K\right]\right)^{\kappa_S}\right]^{\frac{1}{\kappa_S}} \tag{163}$$

$$(xi).\ R_t^j = \sum_{f=0}^{F-1}\lambda_{t-1}^{j,f+1}\frac{\left(YD_t^f\right)^{-f}}{\left(YD_{t-1}^{f+1}\right)^{-(f+1)}},\ \ j\in\{HB,G,CB\} \tag{164}$$

$$(xii).\ R_t^S = \left(1-\lambda_{t-1}^K\right)R_t^{HB}+\lambda_{t-1}^KR_t^K \tag{165}$$

$$(xiii).\ 1 = \mathbb{E}_t\left[Q_{t,t+1}\Pi_{t+1}\left[(1-\delta)+\frac{P_{t+1}^K}{P_{t+1}}\right]\right] \tag{166}$$

$$(xiv).\ \ F_t = (1-\alpha)^{\frac{1-\alpha}{\eta+\alpha}} \left(\frac{(1+\varsigma_F)^{-1}\epsilon}{\epsilon-1}\right) \left(\frac{C_t}{A_t\bar{N}_t}\right)^{-\alpha\left(\frac{\eta+1}{\eta+\alpha}\right)} \left(\frac{Y_t}{A_t\bar{N}_t}\right)^{\frac{\eta+1}{\eta+\alpha}} \left(\widetilde{R}_t^K \frac{P_t^K}{P_t}\right)^{\alpha\left(\frac{\eta+1}{\eta+\alpha}\right)} + \theta\beta\mathbb{E}_t\left[\Pi_{t+1}^{\epsilon\left(\frac{\eta+1}{\eta+\alpha}\right)} F_{t+1}\right] \tag{167}$$

$$(xv).\ \ H_t = \left(\frac{C_t}{A_t\bar{N}_t}\right)^{-1} \frac{Y_t}{A_t\bar{N}_t} + \theta\beta\mathbb{E}_t\left[\Pi_{t+1}^{\epsilon-1} H_{t+1}\right] \tag{168}$$

$$(xvi).\ \ \frac{F_t}{H_t} = \left(\frac{1-\theta}{1-\theta\Pi_t^{\epsilon-1}}\right)^{\left(\frac{1}{\epsilon-1}\right)\left[1+\epsilon\left(\frac{1-\alpha}{\eta+\alpha}\right)\right]} \tag{169}$$

$$(xvii).\ \ \Delta_t = (1-\theta)\left(\frac{1-\theta\Pi_t^{\epsilon-1}}{1-\theta}\right)^{\left(\frac{\epsilon}{\epsilon-1}\right)\left(\frac{\eta+1}{\eta+\alpha}\right)} + \theta\Pi_t^{\epsilon\left(\frac{\eta+1}{\eta+\alpha}\right)}\Delta_{t-1} \tag{170}$$

$$(xviii).\ \ \frac{N_t}{\bar{N}_t} = (1-\alpha)^{\left(\frac{\eta}{\eta+\alpha}\right)} \left(\frac{C_t}{A_t\bar{N}_t}\right)^{-\alpha\left(\frac{\eta}{\eta+\alpha}\right)} \left(\frac{Y_t}{A_t\bar{N}_t}\right)^{\left(\frac{\eta}{\eta+\alpha}\right)} \left(\widetilde{R}_t^K \frac{P_t^K}{P_t}\right)^{\alpha\left(\frac{\eta}{\eta+\alpha}\right)} \Delta_t^{\frac{\eta}{\eta+1}} \tag{171}$$

$$(xix).\ \ \frac{K_t}{A_{t-1}\bar{N}_{t-1}} = \alpha(1-\alpha)^{\frac{1-\alpha}{\eta+\alpha}} \cdot GA_t \cdot GN \cdot \left(\frac{C_t}{A_t\bar{N}_t}\right)^{\frac{\eta(1-\alpha)}{\eta+\alpha}} \left(\frac{Y_t}{A_t\bar{N}_t}\right)^{\frac{\eta+1}{\eta+\alpha}} \left(\widetilde{R}_t^K \frac{P_t^K}{P_t}\right)^{-\left(\frac{\eta(1-\alpha)}{\eta+\alpha}\right)} \Delta_t \tag{172}$$

$$(xx).\ \ \frac{B_t^G}{P_tA_t\bar{N}_t} = \frac{R_t^G}{\Pi_t \cdot GA_t \cdot GN} \cdot \frac{B_{t-1}^G}{P_{t-1}A_{t-1}\bar{N}_{t-1}} - \left[\zeta_t^G + \zeta^F - \zeta_t^T\right]\left(\frac{Y_t}{A_t\bar{N}_t}\right) \tag{173}$$

$$(xxi).\ \ \lambda_t^{G,1} = \frac{1}{1+\sum_{l=2}^{F} a^{B,l}\exp\left(\tilde{u}_t^{B,l}\right)} \tag{174}$$

$$(xxii).\ \ \lambda_t^{G,f} = \frac{a^{B,f}\exp\left(\tilde{u}_t^{B,f}\right)}{1+\sum_{l=2}^{F} a^{B,l}\exp\left(\tilde{u}_t^{B,l}\right)},\ \ \forall f>1 \tag{175}$$

$$(xxiii).\ \ \tilde{u}_t^{B,f} = \sum_{j=1}^{J} \tau_{fj}^B u_t^{B,j} \tag{176}$$

$$(xxiv).\ \ u_t^{B,j} = \rho_B u_{t-1}^{B,j} + \varepsilon_t^{B,j} \tag{177}$$

$$(xxv).\ \ B_t^{G,f} = \lambda_t^{G,f} B_t^G, \qquad \forall f = 1,\ldots,F \tag{178}$$

$$(xxvi).\ \ GA_t = \exp(\mu + \varepsilon_t^A) \tag{179}$$

$$(xxvii).\ \ \zeta_t^G = \frac{1}{1+a^G\exp\left(-u_t^G\right)} \tag{180}$$

$$(xxviii).\ \ \zeta_t^T = \frac{1}{1+a^T\exp\left(-u_t^T\right)} \tag{181}$$

$$(xxix).\ \ u_t^G = \rho_G \cdot u_{t-1}^G + \varepsilon_t^G \tag{182}$$

$$(xxx).\ \ u_t^T = \rho_T \cdot u_{t-1}^T + \varepsilon_t^T \tag{183}$$

C.2. Equilibrium Equations: General Policy

Summary of relevant equilibrium conditions:

$$(i).\ \frac{C_t}{A_t\bar{N}_t} = \left(1-\zeta_t^G\right)\left(\frac{Y_t}{A_t\bar{N}_t}\right) + \left(\frac{1-\delta}{GA_t\cdot GN}\right)\left(\frac{K_t}{A_{t-1}\bar{N}_{t-1}}\right) - \left(\frac{K_{t+1}}{A_t\bar{N}_t}\right) \tag{184}$$

$$(ii).\ 1 = \beta\mathbb{E}_t\left[\frac{R_{t+1}^S}{\Pi_{t+1}\cdot GA_{t+1}\cdot GN}\frac{\left(\frac{C_t}{A_t\bar{N}_t}\right)}{\left(\frac{C_{t+1}}{A_{t+1}\bar{N}_{t+1}}\right)}\right] \tag{185}$$

$$(iii).\ -\left(\frac{\lambda_t^{CB,1}-\lambda_t^{G,1}}{\lambda_t^{CB,1}-\lambda_t^{HB,1}}\right)\frac{B_t^G}{A_t\bar{N}_tP_t} = \frac{1}{GA_t\cdot GN}\left(\frac{1-\lambda_t^K}{\lambda_t^K}\right)\left(\frac{P_t^K}{P_t}\right)\left(\frac{K_t}{A_{t-1}\bar{N}_{t-1}}\right) \tag{186}$$

$$(iv).\ \lambda_t^{CB,f} = \frac{\lambda_t^{HB,f}\cdot\left(1-\sum_{i\neq\{f,1\}}\lambda_t^{CB,i}-\lambda_t^{G,1}\right)-\lambda_t^{G,f}\cdot\left(1-\sum_{i\neq\{f,1\}}\lambda_t^{CB,i}-\lambda_t^{HB,1}\right)}{\left(\lambda_t^{HB,1}+\lambda_t^{HB,f}\right)-\left(\lambda_t^{G,1}+\lambda_t^{G,f}\right)},\qquad f=2,\ldots,F \tag{187}$$

$$(v).\ YD_t^1 = \max\left\{YD_t^{1*},1\right\} \tag{188}$$

$$(vi).\ YD_t^{1*} = \overline{YD}^1\cdot\left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\gamma_\pi^1}\left(\frac{Y_t}{\bar{Y}}\right)^{\gamma_y^1}\cdot\exp\left(\tilde{\varepsilon}_t^{YD^1}\right) \tag{189}$$

$$(vii).\ YD_t^{GP,f} = \overline{YD}^{GP,f}\cdot\left(\frac{YD_t^{SP,f}}{\overline{YD}^{SP,f}}\right)^{\gamma_{SP}^f}\left[\left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\gamma_\pi^f}\left(\frac{Y_t}{\bar{Y}}\right)^{\gamma_y^f}\cdot\exp\left(\tilde{\varepsilon}_t^{YD^f}\right)\right]^{1-\gamma_{SP}^f},\ f\geq 2 \tag{190}$$

$$(viii).\ \lambda_t^{HB,f} = \left(\frac{\mathbb{E}_t\left[\frac{\beta z_t^f}{\Pi_{t+1}\cdot GA_{t+1}\cdot GN}\frac{\left(\frac{C_t}{A_t\bar{N}_t}\right)}{\left(\frac{C_{t+1}}{A_{t+1}\bar{N}_{t+1}}\right)}\frac{\left(YD_{t+1}^{f-1}\right)^{-(f-1)}}{\left(YD_t^f\right)^{-f}}\right]}{\Phi_t^B}\right)^{\kappa_B},\ \forall f \tag{191}$$

$$(ix).\ \Phi_t^B = \left[\sum_{j=1}^F\mathbb{E}_t\left[\frac{\beta z_t^j}{\Pi_{t+1}\cdot GA_{t+1}\cdot GN}\frac{\left(\frac{C_t}{A_t\bar{N}_t}\right)}{\left(\frac{C_{t+1}}{A_{t+1}\bar{N}_{t+1}}\right)}\frac{\left(YD_{t+1}^{j-1}\right)^{-(j-1)}}{\left(YD_t^j\right)^{-j}}\right]^{\kappa_B}\right]^{\frac{1}{\kappa_B}} \tag{192}$$

$$(x).\ \lambda_t^K = \left(\frac{z_t^K\cdot\mathbb{E}_t\left[Q_{t,t+1}R_{t+1}^K\right]}{\Phi_t^S}\right)^{\kappa_S} \tag{193}$$

$$(xi).\ \Phi_t^S = \left[\left(\mathbb{E}_t\left[Q_{t,t+1}R_{t+1}^{HB}\right]\right)^{\kappa_S}+\left(z_t^K\mathbb{E}_t\left[Q_{t,t+1}R_{t+1}^K\right]\right)^{\kappa_S}\right]^{\frac{1}{\kappa_S}} \tag{194}$$

$$(xii).\ R_t^j = \sum_{f=0}^{F-1}\lambda_{t-1}^{j,f+1}\frac{\left(YD_t^f\right)^{-f}}{\left(YD_{t-1}^{f+1}\right)^{-(f+1)}}\qquad j\in\{HB,G,CB\} \tag{195}$$

$$(xiii).\ R_t^S = \left(1-\lambda_{t-1}^K\right)R_t^{HB}+\lambda_{t-1}^KR_t^K \tag{196}$$

$$(xiv).\ 1 = \mathbb{E}_t\left[Q_{t,t+1}\Pi_{t+1}\left[(1-\delta)+\frac{P_{t+1}^K}{P_{t+1}}\right]\right] \tag{197}$$

$$(xv).\;\; F_t = (1-\alpha)^{\frac{1-\alpha}{\eta+\alpha}} \left(\frac{(1+\varsigma_F)^{-1}\epsilon}{\epsilon-1}\right) \left(\frac{C_t}{A_t\bar{N}_t}\right)^{-\alpha\left(\frac{\eta+1}{\eta+\alpha}\right)} \left(\frac{Y_t}{A_t\bar{N}_t}\right)^{\frac{\eta+1}{\eta+\alpha}} \left(\mathbb{E}_t\left[Q_{t,t+1}R^K_{t+1}\right]\frac{P^K_t}{P_t}\right)^{\alpha\left(\frac{\eta+1}{\eta+\alpha}\right)} + \theta\beta\mathbb{E}_t\left[\Pi_{t+1}^{\epsilon\left(\frac{\eta+1}{\eta+\alpha}\right)}F_{t+1}\right] \tag{198}$$

$$(xvi).\;\; H_t = \left(\frac{C_t}{A_t\bar{N}_t}\right)^{-1}\frac{Y_t}{A_t\bar{N}_t} + \theta\beta\mathbb{E}_t\left[\Pi_{t+1}^{\epsilon-1}H_{t+1}\right] \tag{199}$$

$$(xvii).\;\; \frac{F_t}{H_t} = \left(\frac{1-\theta}{1-\theta\Pi_t^{\epsilon-1}}\right)^{\left(\frac{1}{\epsilon-1}\right)\left[1+\epsilon\left(\frac{1-\alpha}{\eta+\alpha}\right)\right]} \tag{200}$$

$$(xviii).\;\; \Delta_t = (1-\theta)\left(\frac{1-\theta\Pi_t^{\epsilon-1}}{1-\theta}\right)^{\left(\frac{\epsilon}{\epsilon-1}\right)\left(\frac{\eta+1}{\eta+\alpha}\right)} + \theta\Pi_t^{\epsilon\left(\frac{\eta+1}{\eta+\alpha}\right)}\Delta_{t-1} \tag{201}$$

$$(xix).\;\; \frac{N_t}{\bar{N}_t} = (1-\alpha)^{\left(\frac{\eta}{\eta+\alpha}\right)}\left(\frac{C_t}{A_t\bar{N}_t}\right)^{-\alpha\left(\frac{\eta}{\eta+\alpha}\right)}\left(\frac{Y_t}{A_t\bar{N}_t}\right)^{\left(\frac{\eta}{\eta+\alpha}\right)}\left(\mathbb{E}_t\left[Q_{t,t+1}R^K_{t+1}\right]\frac{P^K_t}{P_t}\right)^{\alpha\left(\frac{\eta}{\eta+\alpha}\right)}\Delta_t^{\frac{\eta}{\eta+1}} \tag{202}$$

$$(xx).\;\; \frac{K_t}{A_{t-1}\bar{N}_{t-1}} = \alpha(1-\alpha)^{\frac{1-\alpha}{\eta+\alpha}}\cdot GA_t\cdot GN\cdot\left(\frac{C_t}{A_t\bar{N}_t}\right)^{\frac{\eta(1-\alpha)}{\eta+\alpha}}\left(\frac{Y_t}{A_t\bar{N}_t}\right)^{\frac{\eta+1}{\eta+\alpha}}\left(\mathbb{E}_t\left[Q_{t,t+1}R^K_{t+1}\right]\frac{P^K_t}{P_t}\right)^{-\left(\frac{\eta(1-\alpha)}{\eta+\alpha}\right)}\Delta_t \tag{203}$$

$$(xxi).\;\; \frac{B^G_t}{P_tA_t\bar{N}_t} = \frac{R^G_t}{\Pi_t\cdot GA_t\cdot GN}\frac{B^G_{t-1}}{P_{t-1}A_{t-1}\bar{N}_{t-1}} - \left[\zeta^G_t + \zeta^F - \zeta^T_t\right]\left(\frac{Y_t}{A_t\bar{N}_t}\right) \tag{204}$$

$$(xxii).\;\; \tilde{\varepsilon}_t^{YD,f} = \sum_{l=1}^{L}\tau^{YD}_{fl}\varepsilon_t^{YD,l} \tag{205}$$

C.3. Calibrating $\{z^f\}$ and z^K in the Steady State

Calibration of $\{z^f\}$ Here we explain how to calibrate $\{z^f\}$ to match the yield curve. Based on data on yields of bonds with different maturities, we calculate each f-maturity bond's average holding returns $\{R^f\}$, which we would use as our calibration target.

1. Compute the return ratio $\{R^F/R^1\}$
2. Back out steady state bond shares $\{\lambda^{HB,f}\}$ using equation (87)
3. Normalize $z^1 = 1$ and obtain initial guess for $\{z^{j,guess}\}$. Set $z^{j,old} = z^{j,guess}$ in the iteration below
4. Construct $\tilde{\Phi}^{old}$ using the following formula, where the return ratios $\{R^f/R^1\}$ across maturities are obtained from the data

$$\tilde{\Phi}^{old} = \left[1 + \sum_{f=2}^{F}\left[z^j\left(\frac{R^f}{R^1}\right)\right]^{\kappa_B}\right]^{\frac{1}{\kappa_B}}$$

5. Back out new $z^{f,new}$, $f = 2, \ldots, F$ estimates using:

$$z^{f,new} = \left(\lambda^{HB,f}\right)^{\frac{1}{\kappa_B}}\left(\frac{R^f}{R^1}\right)^{-1}\tilde{\Phi}^{old}$$

6. If difference with $\tilde{\Phi}^{old}$ is large, set $z^{f,old} = z^{f,new}$ and start again from the step 4

Calibration of z^K For the calibration of z^K, first, we need to obtain data on $\{R^{HB}, R^K\}$.

1. Guess $z^{K,guess}$ and set $z^{K,old} = z^{K,guess}$
2. Solve for the steady state values of the model using $z^{K,old}$. The reason is that we do not get the data on λ^K, thus we use the model-dependent value of it
3. Construct $\tilde{\Phi}^{old}$ using the following formula, where the ratio is the one obtained from the data

$$\tilde{\Phi}^{old} = \left[1 + \left[z^K\left(\frac{R^K}{R^{HB}}\right)\right]^{\kappa_S}\right]^{\frac{1}{\kappa_S}}$$

4. Back out new $z^{K,new}$ estimates as

$$z^{K,new} = \left(\lambda^K\right)^{\frac{1}{\kappa_S}}\left(\frac{R^K}{R^{HB}}\right)^{-1}\tilde{\Phi}^{old}$$

   where λ^K comes from the model solution of the step 2
5. If difference with $z^{K,old}$ is large, set $z^{K,old} = z^{K,new}$ and start again from step 4.

C.4. Summary of Standard Policy Linearized Equations

Those are the essential equation to solve the model, other variables can be found on equations above.

$$
\begin{aligned}
&(i).\ \hat{y}_t = \mathbb{E}_t\Big[\hat{y}_{t+1} + \Psi^{27}\hat{\pi}_{t+1} - \Psi^{28}\overrightarrow{\hat{z}}_t - \Psi^{29}\hat{z}^K_t - \Psi^{30}\overrightarrow{\hat{yd}}_t - \Psi^{31}\mathbb{E}_t\left[\overrightarrow{\hat{yd}_{t+1}}\right] \\
&\qquad - \Psi^{32}\hat{r}^K_{t+1} - \Psi^{33}(\hat{k}_t - \hat{\varepsilon}^A_t) + \Psi^{34}\hat{k}_{t+1} - \Psi^{35}\hat{k}_{t+2} + \Psi^{36}\hat{u}^G_t\Big] \\
&(ii).\ \overrightarrow{\hat{yd}_t} = \Theta^{21}\hat{b}^G_t - \Theta^{22}\hat{y}_t - \Theta^{23}\mathbb{E}_t\left[\hat{y}_{t+1}\right] - \Theta^{24}\mathbb{E}_t\left[\hat{\pi}_{t+1}\right] + \Theta^{25}\hat{k}_t - \Theta^{26}\hat{k}_{t+1} + \Theta^{27}\mathbb{E}_t\left[\hat{k}_{t+2}\right] + \Theta^{28}\hat{r}^K_{t+1} \\
&\qquad - \Theta^{29}\mathbb{E}_t\left[\overrightarrow{\hat{yd}_{t+1}}\right] - \Theta^{30}\overrightarrow{\hat{z}_t} + \Theta^{31}\hat{z}^K_t - \Theta^{32}\hat{\varepsilon}^A_t + \Theta^{33}\hat{u}^G_t + \Theta^{34}\overrightarrow{\hat{u}^B_t} \\
&(iii).\ \hat{yd}^1_t = \max\left\{\hat{yd}^{1*}_t, 0\right\} \\
&(iv).\ \hat{yd}^{1*}_t = \gamma_\pi\hat{\pi}_t + \gamma_y\hat{y}_t + \tilde{\varepsilon}^{YD^1}_t,\ \ \tilde{\varepsilon}^{YD^f}_t = \sum_{l=1}^{L}\tau^{YD}_{f,l}\varepsilon^{YD^l}_t \\
&(v).\ \hat{r}^K_{t+1} = -\Psi^{15}\cdot\overrightarrow{\hat{z}_t} - \Psi^{16}\hat{z}^K_t - \Psi^{17}\overrightarrow{\hat{yd}_t} - \Psi^{18}\mathbb{E}_t\left[\overrightarrow{\hat{yd}_{t+1}}\right] + \Psi^{19}\mathbb{E}_t\left[\hat{\pi}_{t+1}\right] + \Psi^{20}\mathbb{E}_t\left[\hat{\pi}_{t+2}\right] + \Psi^{21}\mathbb{E}_t\left[\hat{y}_{t+1}\right] \\
&\qquad + \Psi^{22}\mathbb{E}_t\left[\hat{y}_{t+2}\right] - \Psi^{23}\hat{k}_{t+1} + \Psi^{24}\mathbb{E}_t\left[\hat{k}_{t+2}\right] - \Psi^{25}\mathbb{E}_t\left[\hat{k}_{t+3}\right] - \Psi^{26}\hat{u}^G_t - \Psi^{20}\mathbb{E}_t\left[\hat{r}^K_{t+2}\right] \\
&(vi).\ \hat{b}^G_t = \frac{R^G}{\Pi\cdot GA\cdot GN}\cdot\left[\Psi^{G,4}\Xi\overrightarrow{\hat{u}^B_{t-1}} - \Psi^{G,5}\overrightarrow{\hat{yd}_t} + \Psi^{G,6}\overrightarrow{\hat{yd}_{t-1}} - \hat{\pi}_t - \hat{\varepsilon}^A_t + \hat{b}^G_{t-1}\right] \\
&\qquad + \left(1 - \frac{R^G}{\Pi\cdot GA\cdot GN}\right)\left[\hat{y}_t + \left(\frac{\zeta^G}{\zeta^G+\zeta^F-\zeta^T}\right)\left(\frac{a^G}{1+a^G}\right)\hat{u}^G_t - \left(\frac{\zeta^T}{\zeta^G+\zeta^F-\zeta^T}\right)\left(\frac{a^T}{1+a^T}\right)\hat{u}^T_t\right] \\
&(vii).\ \hat{f}_t = \left[1 - \theta\beta\Pi^{\epsilon\left(\frac{\eta+1}{\eta+\alpha}\right)}\right]\left(\frac{\eta+1}{\eta(1-\alpha)}\right)\left[\hat{y}_t - \alpha\left(\hat{k}_t - \hat{\varepsilon}^A_t\right)\right] + \theta\beta\Pi^{\epsilon\left(\frac{\eta+1}{\eta+\alpha}\right)}\mathbb{E}_t\left[\epsilon\left(\frac{\eta+1}{\eta+\alpha}\right)\hat{\pi}_{t+1} + \hat{f}_{t+1}\right] \\
&(viii).\ \hat{h}_t = \left[1 - \theta\beta\Pi^{\epsilon-1}\right]\Bigg[\left[1 - (1-\zeta^G)\frac{Y}{C}\right]\hat{y}_t + \left[\left(1-\zeta^G\right)\frac{Y}{C}\right]\frac{1}{1+a^G}\hat{u}^G_t - \left[\frac{1-\delta}{GA\cdot GN}\frac{K}{C}\right](\hat{k}_t - \hat{\varepsilon}^A_t) \\
&\qquad + \frac{K}{C}\hat{k}_{t+1}\Bigg] + \theta\beta\Pi^{\epsilon-1}\mathbb{E}_t\left[(\epsilon-1)\hat{\pi}_{t+1} + \hat{h}_{t+1}\right] \\
&(ix).\ \hat{f}_t - \hat{h}_t = \left[1 + \epsilon\left(\frac{1-\alpha}{\eta+\alpha}\right)\right]\left(\frac{\theta\Pi^{\epsilon-1}}{1-\theta\Pi^{\epsilon-1}}\right)\hat{\pi}_t \\
&(x).\ u^{B,j}_t = \rho_B\cdot u^{B,j}_{t-1} + \varepsilon^{B,j}_t, \qquad \forall\ j = 1,\ldots,J \\
&(xi).\ u^G_t = \rho_G\cdot u^G_{t-1} + \varepsilon^G_t \\
&(xii).\ u^T_t = \rho_T\cdot u^G_{t-1} + \varepsilon^T_t
\end{aligned}
$$

C.5. Summary of General Policy Linearized Equations

Those are the essential equation to solve the model, other variables can be found on equations above.

$$(i).\ \hat{y}_t = \mathbb{E}_t\Big[\hat{y}_{t+1} + \Psi^{27}\hat{\pi}_{t+1} - \Psi^{28}\overrightarrow{\hat{z}}_t - \Psi^{29}\hat{z}^K_t - \Psi^{30}\overrightarrow{\hat{yd}}_t - \Psi^{31}\mathbb{E}_t\left[\overrightarrow{\hat{yd}^{GP}_{t+1}}\right]$$

$$- \Psi^{32}\hat{r}^K_{t+1} - \Psi^{33}(\hat{k}_t - \hat{\varepsilon}^A_t) + \Psi^{34}\hat{k}_{t+1} - \Psi^{35}\hat{k}_{t+2} + \Psi^{36}\hat{u}^G_t\Big]$$

$$(ii).\ \overrightarrow{\hat{yd}^{SP}_t} = \Theta^{21}\hat{b}^G_t - \Theta^{22}\hat{y}_t - \Theta^{23}\mathbb{E}_t[\hat{y}_{t+1}] - \Theta^{24}\mathbb{E}_t[\hat{\pi}_{t+1}] + \Theta^{25}\hat{k}_t - \Theta^{26}\hat{k}_{t+1} + \Theta^{27}\mathbb{E}_t\left[\hat{k}_{t+2}\right] + \Theta^{28}\hat{r}^K_{t+1}$$

$$- \Theta^{29}\mathbb{E}_t\left[\overrightarrow{\hat{yd}^{SP}_{t+1}}\right] - \Theta^{30}\overrightarrow{\hat{z}_t} + \Theta^{31}\hat{z}^K_t - \Theta^{32}\hat{\varepsilon}^A_t + \Theta^{33}\hat{u}^G_t + \Theta^{34}\overrightarrow{\hat{u}^B_t}$$

$$(iii).\ \hat{yd}^{GP,1}_t = \max\left\{\hat{yd}^{1*}_t, 0\right\} = \hat{yd}^{SP,1}_t$$

$$(iv).\ \hat{yd}^{1*}_t = \gamma_\pi\hat{\pi}_t + \gamma_y\hat{y}_t + \tilde{\varepsilon}^{YD^1}_t,\ \ \tilde{\varepsilon}^{YD^1}_t = \sum_{l=1}^{L}\tau^{YD}_{1,l}\varepsilon^{YD^l}_t,$$

$$(v).\ \hat{yd}^{GP,f}_t = \gamma^f_{SP}\hat{yd}^{SP,f}_t + \left(1-\gamma^f_{SP}\right)\left[\gamma^f_\pi\hat{\pi}_t + \gamma^f_y\hat{y}_t + \tilde{\varepsilon}^{YD^f}_t\right],\ \ \tilde{\varepsilon}^{YD^f}_t = \sum_{l=1}^{L}\tau^{YD}_{f,l}\varepsilon^{YD^l}_t, \quad f \geq 2$$

$$(vi).\ \hat{r}^K_{t+1} = -\Psi^{15}\cdot\overrightarrow{\hat{z}_t} - \Psi^{16}\hat{z}^K_t - \Psi^{17}\overrightarrow{\hat{yd}^{GP}_t} - \Psi^{18}\mathbb{E}_t\left[\overrightarrow{\hat{yd}^{GP}_{t+1}}\right] + \Psi^{19}\mathbb{E}_t[\hat{\pi}_{t+1}] + \Psi^{20}\mathbb{E}_t[\hat{\pi}_{t+2}] + \Psi^{21}\mathbb{E}_t[\hat{y}_{t+1}]$$

$$+ \Psi^{22}\mathbb{E}_t[\hat{y}_{t+2}] - \Psi^{23}\hat{k}_{t+1} + \Psi^{24}\mathbb{E}_t\left[\hat{k}_{t+2}\right] - \Psi^{25}\mathbb{E}_t\left[\hat{k}_{t+3}\right] - \Psi^{26}\hat{u}^G_t - \Psi^{20}\mathbb{E}_t\left[\hat{r}^K_{t+2}\right]$$

$$(vii).\ \hat{b}^G_t = \frac{R^G}{\Pi\cdot GA\cdot GN}\cdot\left[\Psi^{G,4}\Xi\overrightarrow{\hat{u}^B_{t-1}} - \Psi^{G,5}\overrightarrow{\hat{yd}^{GP}_t} + \Psi^{G,6}\overrightarrow{\hat{yd}^{GP}_{t-1}} - \hat{\pi}_t - \hat{\varepsilon}^A_t + \hat{b}^G_{t-1}\right]$$

$$+ \left(1 - \frac{R^G}{\Pi\cdot GA\cdot GN}\right)\left[\hat{y}_t + \left(\frac{\zeta^G}{\zeta^G+\zeta^F-\zeta^T}\right)\left(\frac{a^G}{1+a^G}\right)\hat{u}^G_t - \left(\frac{\zeta^T}{\zeta^G+\zeta^F-\zeta^T}\right)\left(\frac{a^T}{1+a^T}\right)\hat{u}^T_t\right]$$

$$(viii).\ \hat{f}_t = \left[1 - \theta\beta\Pi^{\epsilon\left(\frac{\eta+1}{\eta+\alpha}\right)}\right]\left(\frac{\eta+1}{\eta(1-\alpha)}\right)\left[\hat{y}_t - \alpha\left(\hat{k}_t - \hat{\varepsilon}^A_t\right)\right] + \theta\beta\Pi^{\epsilon\left(\frac{\eta+1}{\eta+\alpha}\right)}\mathbb{E}_t\left[\epsilon\left(\frac{\eta+1}{\eta+\alpha}\right)\hat{\pi}_{t+1} + \hat{f}_{t+1}\right]$$

$$(ix).\ \hat{h}_t = \left[1 - \theta\beta\Pi^{\epsilon-1}\right]\Big[\left[1 - (1-\zeta^G)\frac{Y}{C}\right]\hat{y}_t + \left[\left(1-\zeta^G\right)\frac{Y}{C}\right]\frac{1}{1+a^G}\hat{u}^G_t - \left[\frac{1-\delta}{GA\cdot GN}\frac{K}{C}\right](\hat{k}_t - \hat{\varepsilon}^A_t)$$

$$+ \frac{K}{C}\hat{k}_{t+1}\Big] + \theta\beta\Pi^{\epsilon-1}\mathbb{E}_t\left[(\epsilon-1)\hat{\pi}_{t+1} + \hat{h}_{t+1}\right]$$

$$(x).\ \hat{f}_t - \hat{h}_t = \left[1 + \epsilon\left(\frac{1-\alpha}{\eta+\alpha}\right)\right]\left(\frac{\theta\Pi^{\epsilon-1}}{1-\theta\Pi^{\epsilon-1}}\right)\hat{\pi}_t$$

$$(xi).\ u^{B,j}_t = \rho_B\cdot u^{B,j}_{t-1} + \varepsilon^{B,j}_t, \qquad \forall\, j = 1,\ldots,J$$

$$(xii).\ u^G_t = \rho_G\cdot u^G_{t-1} + \varepsilon^G_t$$

$$(xiii).\ u^T_t = \rho_T\cdot u^G_{t-1} + \varepsilon^T_t$$

Appendix D. Welfare

D.1. Deriving a second-order welfare

In order to approximate welfare up to a second-order, we cannot discard $\hat{\Delta}_t$, which is price dispersion's log-deviation from its steady-state value. Since we have a positive steady-state inflation, price dispersion becomes of the first-order importance, in contrast to the standard New-Keynesian models in which zero trend inflation is assumed usually and price dispersion is the second-order term. In handling this issue when calculating the welfare cost of inflation, we closely follow the pioneering work of Woodford (2003) and Coibion et al. (2012). First, we express the necessary building blocks with the price dispersion gap term, $\hat{\Delta}_t$.

Step 1: For any variable X, we define $\bar{X}$ as its steady-state value (with the positive trend inflation $\Pi > 1$) and $\bar{X}^F$ as its flexible price steady-state value. Also define (small) letter $\tilde{x}$ as log-deviation of X around $\bar{X}^F$, and $\hat{x}$ as log-deviation of X around $\bar{X}$.

Efficient (flexible-price) steady state With optimal production subsidy $\zeta^F = (\epsilon - 1)^{-1}$ that eliminates the monopolistic competition distortion, there is no distortion in the flexible-price steady state economy anymore.[79] In particular, individual firm's optimal price resetting condition (equation (65)) becomes

$$1 = \frac{P_t^*}{P_t} = \underbrace{\frac{(1+\zeta^F)^{-1}\epsilon}{\epsilon - 1}}_{=1} \cdot \frac{MC_t}{P_t} = \frac{MC_t}{P_t}, \tag{206}$$

where we use the fact that all firms become identical, and thus $MC_t(\nu) = MC_t$ for all $\nu \in [0,1]$. Therefore, the real marginal cost becomes 1 for all firms. Plugging the unit real marginal cost (equation (66)) into the individual firm's labor demand (equation (27)) with $W_t(\nu) = W_t$ for $\forall \nu$, we obtain

$$n_t = (1-\alpha) y_t \left(\frac{\widetilde{R}_t^K P_t^K}{P_t} \right)^{\alpha} \left(\frac{W_t}{P_t A_t} \right)^{-\alpha} = (1-\alpha) y_t \left(\frac{W_t}{P_t A_t} \right)^{-1}, \tag{207}$$

which, with the household's intra-temporal consumption-labor decision (equation (15)), becomes:

$$\frac{n_t^{\eta^{-1}}}{c_t^{-1}} = (1-\alpha) \frac{y_t}{n_t}, \tag{208}$$

which is exactly the social efficiency condition that ensures the household's marginal rate of substitution matches with the marginal rate of technical substitution. Therefore, at the flexible-price steady state, the new constant Φ, which will turn out to enter in the per-period welfare later, can be calculated as

$$\Phi \equiv (\bar{n}^F)^{1+\frac{1}{\eta}} = (1-\alpha) \frac{\bar{y}^F}{\bar{c}^F} = (1-\alpha) \frac{\bar{Y}^F}{\bar{C}^F}, \tag{209}$$

[79] A capital producing firm is perfectly competitive and therefore, our economy features no friction if it were not nominal rigidity nor trend inflation, and satisfies the first welfare theorem in the flexible price steady state.

where $\bar{n}^F$, $\bar{y}^F$, and $\bar{c}^F$ are values of normalized labor, output, and consumption, respectively.

Step 2: With equation (202) and equation (203), we obtain

$$\left(\frac{N_t}{\bar{N}_t}\right)^{1-\alpha}\left(\frac{K_t}{A_{t-1}\bar{N}_{t-1}}\right)^{\alpha} = \alpha^{\alpha}(1-\alpha)^{1-\alpha}(GA_t \cdot GN)^{\alpha}\left(\frac{Y_t}{A_t\bar{N}_t}\right)\Delta_t^{(1-\alpha)\left[\frac{\eta}{\eta+1}+\frac{\alpha}{1-\alpha}\right]}, \tag{210}$$

which is the aggregate production function with price dispersion Δ_t. Plugging steady-state (with trend-inflation) capital (equation (95)) and output (equation (100)) equations into equation (210) yields the formula for the steady-state labor, which is given as

$$\frac{N}{\bar{N}} = \xi^N\xi^Y\left[\left(1-\frac{R^G}{\Pi\cdot GA\cdot GN}\right)^{-1}\frac{\lambda^K}{1-\lambda^K}\right]^{\frac{-\alpha}{1-\alpha}}\left[(1-\zeta^G)-\xi^C\left(1-\frac{R^G}{\Pi\cdot GA\cdot GN}\right)^{-1}\frac{\lambda^K}{1-\lambda^K}\right]^{\frac{-\eta}{\eta+1}}\left(R^K\right)^{\frac{-\alpha}{1-\alpha}},$$
$$\text{with } \xi^N = (1-\alpha)\left[\frac{1-\theta}{1-\theta\Pi^{\epsilon\frac{\eta+1}{\eta+\alpha}}}\right]^{\frac{\eta+\alpha}{(\eta+1)(1-\alpha)}}\left(\frac{1-\theta\Pi^{\epsilon-1}}{1-\theta}\right)^{\frac{\epsilon}{(\epsilon-1)(1-\alpha)}}\left(\frac{\xi^K}{\alpha\cdot GA\cdot GN}\right)^{\frac{-\alpha}{1-\alpha}}. \tag{211}$$

From equation (102), equation (103), and equation (104), we observe that equilibrium steady state values of R^K, λ^K, and R^G do not depend on θ, a degree of price-stickiness. However,

$$\xi^N\xi^Y = (1-\alpha)^{\frac{\eta}{1+\eta}}\left[\frac{GA\cdot GN-\beta(1-\delta)}{\Pi\cdot GA\cdot GN}\right]^{\frac{-\alpha}{1-\alpha}}\left(\frac{\xi^K}{\alpha\cdot GA\cdot GN}\right)^{\frac{-\alpha}{1-\alpha}}\left(\frac{1-\theta\Pi^{\epsilon-1}}{1-\theta\beta\Pi^{\epsilon-1}}\cdot\frac{1-\theta\beta\Pi^{\epsilon\frac{1+\eta}{\eta+\alpha}}}{1-\theta\Pi^{\epsilon\frac{1+\eta}{\eta+\alpha}}}\right)^{\frac{\eta+\alpha}{(\eta+1)(1-\alpha)}} \tag{212}$$

is dependent on θ, we see that $\bar{n} \neq \bar{n}^F$ and define $\log X_n \equiv \log\bar{n} - \log\bar{n}^F$, which will turn out to be useful later when we calculate the household's first-order labor cost.

Step 3: Price dispersion with a positive trend inflation[80]
Delta method Before we start, we would use this approximation throughout this section. For a random variable X with $E(X) = \mu_X$, we have

$$\text{Var}\left(f(X)\right) = f'(\mu_X)^2\cdot\text{Var}(X) + \text{h.o.t.} \tag{213}$$

Price dispersion We use lower-case p_t and $p_t(\nu)$ as logarithms of P_t and $P_t(\nu)$. By applying delta method to $P_t^{1-\epsilon} = \mathbb{E}_\nu(P_t(\nu)^{1-\epsilon})$, we obtain

$$p_t = \underbrace{\int_0^1 p_t(\nu)d\nu}_{\equiv\bar{p}_t} + \frac{1}{2}\left(\frac{1}{1-\epsilon}\right)\frac{\text{Var}_\nu\left(P_t(\nu)^{1-\epsilon}\right)}{\mathbb{E}_\nu\left(P_t(\nu)^{1-\epsilon}\right)^2} + \text{h.o.t.} \tag{214}$$

where we define $\bar{p}_t \equiv \mathbb{E}_\nu(p_t(\nu))$. Applying delta method to $\text{Var}_\nu(P_t(\nu)^{1-\epsilon})$ term, we have

[80] Due to the positive trend inflation $\Pi > 1$, we have non-zero price dispersion at the steady state.

$$\mathrm{Var}_\nu\left(P_t(\nu)^{1-\epsilon}\right) = (1-\epsilon)^2 \cdot [\exp((1-\epsilon)\bar{p}_t)]^2 \cdot \mathrm{Var}_\nu(p_t(\nu)), \tag{215}$$

where we define $D_t \equiv \mathrm{Var}_\nu(p_t(\nu))$. Applying delta method to $\mathbb{E}_\nu(P_t(\nu)^{1-\epsilon})$, we obtain

$$\mathbb{E}_\nu\left(P_t(\nu)^{1-\epsilon}\right) = \exp((1-\epsilon)\bar{p}_t)\left[1+\frac{(1-\epsilon)^2}{2}D_t\right]. \tag{216}$$

Plugging equation (215) and equation (216) into equation (214), we obtain

$$p_t = \bar{p}_t + \frac{1-\epsilon}{2}\cdot\frac{D_t}{\left[1+\frac{(1-\epsilon)^2}{2}D_t\right]^2}, \tag{217}$$

which we linear-approximate around $D_t = \bar{D}$ and get[81,82]

$$\begin{aligned} p_t - \bar{p}_t &= \underbrace{\frac{1-\epsilon}{2}\cdot\frac{\bar{D}}{\left[1+\frac{(1-\epsilon)^2}{2}\bar{D}\right]^2}}_{\equiv\Theta_1^p} + \underbrace{\frac{1-\epsilon}{2}\cdot\frac{1-\frac{(1-\epsilon)^2}{2}\bar{D}}{\left[1+\frac{(1-\epsilon)^2}{2}\bar{D}\right]^3}}_{\equiv\Theta_2^p}\cdot(D_t-\bar{D}) \\ &= \Theta_1^p + \Theta_2^p(D_t-\bar{D}). \end{aligned} \tag{218}$$

Now from our original definition of price dispersion Δ_t (equation (43)), we take logarithm on both sides, linear-approximate around $\bar{D}$, and plug equation (218) into it to attain

$$\begin{aligned} \ln\Delta_t &= \ln\int_0^1\left(\frac{P_t(\nu)}{P_t}\right)^{\frac{-\epsilon(\eta+1)}{\eta+\alpha}}d\nu \\ &= \frac{\epsilon(\eta+1)}{\eta+\alpha}(p_t-\bar{p}_t) + \ln\left(1+\frac{1}{2}\left(\frac{\epsilon(\eta+1)}{\eta+\alpha}\right)^2\bar{D}\right) + \frac{\frac{1}{2}\left(\frac{\epsilon(\eta+1)}{\eta+\alpha}\right)^2}{1+\frac{1}{2}\left(\frac{\epsilon(\eta+1)}{\eta+\alpha}\right)^2\bar{D}}(D_t-\bar{D}) \\ &= \Theta_1^\Delta + \Theta_2^\Delta\cdot(D_t-\bar{D}) + \text{h.o.t}, \end{aligned} \tag{219}$$

where

$$\Theta_1^\Delta \equiv \frac{\epsilon(\eta+1)}{\eta+\alpha}\cdot\frac{1-\epsilon}{2}\cdot\frac{\bar{D}}{\left[1+\frac{(1-\epsilon)^2}{2}\bar{D}\right]^2} + \ln\left(1+\frac{1}{2}\left(\frac{\epsilon(\eta+1)}{\eta+\alpha}\right)^2\bar{D}\right), \tag{220}$$

$$\Theta_2^\Delta \equiv \frac{\epsilon(\eta+1)}{\eta+\alpha}\cdot\frac{1-\epsilon}{2}\cdot\frac{1-\frac{(1-\epsilon)^2}{2}\bar{D}}{\left[1+\frac{(1-\epsilon)^2}{2}\bar{D}\right]^3} + \frac{\frac{1}{2}\left(\frac{\epsilon(\eta+1)}{\eta+\alpha}\right)^2}{1+\frac{1}{2}\left(\frac{\epsilon(\eta+1)}{\eta+\alpha}\right)^2\bar{D}}. \tag{221}$$

[81] In textbook New-Keynesian models, we assume zero inflation at the steady state, which yields $\bar{D} = 0$.

[82] Θ_1^p and Θ_2^p are defined as in equation (218).

If we define b_t as the logarithm of the newly price-resetting firm's relative price P_t^*/P_t and $\bar{b}$ as its steady state value, we might have $\bar{b} \neq 0$ due to the trend inflation. Combining equation (72) and equation (74) and linearizing, we obtain

$$b_t \equiv p_t^* - p_t = \bar{b} + \underbrace{\frac{\theta\Pi^{\epsilon-1}}{1-\theta\Pi^{\epsilon-1}}}_{\equiv M}\hat{\pi}_t = \bar{b} + M \cdot \hat{\pi}_t, \text{ with } \bar{b} = \frac{1}{\epsilon-1}\ln\left(\frac{1-\theta}{1-\theta\Pi^{\epsilon-1}}\right). \tag{222}$$

With $D_t = \mathrm{Var}_\nu(p_t(\nu)) = \mathbb{E}_\nu((p_t(\nu) - p_t + p_t - \bar{p}_t)^2)$, we can write it as

$$\begin{aligned} D_t &= \int_0^{1-\theta}(p_t^* - p_t)^2 d\nu + 2\left(\int_0^{1-\theta}(p_t^* - p_t)d\nu\right)(p_t - \bar{p}_t) + (1-\theta)(p_t - \bar{p}_t)^2 + \int_{1-\theta}^1 (p_{t-1}(\nu) - \bar{p}_t)^2 d\nu \\ &= (1-\theta)(p_t^* - p_t)^2 + 2(1-\theta)(p_t^* - p_t)(p_t - \bar{p}_t) + (1-\theta)(p_t - \bar{p}_t)^2 + \theta D_{t-1} + \theta(\bar{p}_t - \bar{p}_{t-1})^2, \end{aligned} \tag{223}$$

where we use

$$\int_{1-\theta}^1 (p_{t-1}(\nu) - \bar{p}_t)^2 d\nu = \theta D_{t-1} + \theta(\bar{p}_{t-1} - \bar{p}_t)^2. \tag{224}$$

Conjecture Following Coibion et al. (2012), we conjecture the dynamics of D_t up to a second-order as[83]

$$D_t - \bar{D} = \kappa_D\hat{\pi}_t + Z_D(\hat{\pi}_t)^2 + F_D(D_{t-1} - \bar{D}) + G_D(D_{t-1} - \bar{D})\hat{\pi}_t + H_D(D_{t-1} - \bar{D})^2. \tag{225}$$

With no trend inflation, we would have $\pi = 0$ and $\bar{D} = 0$, thus D_t becomes the second-order variable around 0 and we would have $\kappa_D = 0$. However with steady-state inflation $\pi > 0$ and the price dispersion measure $\bar{D} > 0$, as we see in equation (225), D_t includes $\hat{\pi}_t$ term as one of its components, even though κ_D is of the first-order. Our objective here is to derive equation (225) from firms' optimal pricing behaviors and the price dispersion's effects on the aggregate price itself. Plugging equation (218) and equation (222) into equation (223) and replace $(D_t - \bar{D})$ with the conjectured form in equation (225) up to a second-order,[84]and comparing coefficients, we obtain the following set of coefficients:

$$\begin{aligned} \bar{D} &= (\bar{b} + \Theta_1^p)^2 + \frac{\theta}{1-\theta}(\bar{\pi})^2 \text{ (Steady-state value of } D_t), \\ \kappa_D &= \left[1 - 2(1-\theta)\Theta_2^p(\bar{b} + \Theta_1^p) + 2\theta\Theta_2^p\bar{\pi}\right]^{-1}\left[2(1-\theta)M(\bar{b} + \Theta_1^p) + 2\theta\bar{\pi}\right], \\ Z_D &= \left[1 - 2(1-\theta)\Theta_2^p(\bar{b} + \Theta_1^p) + 2\theta\Theta_2^p\bar{\pi}\right]^{-1}\left[(1-\theta)M^2 + 2(1-\theta)M\Theta_2^p\kappa_D + (\Theta_2^p)^2(\kappa_D)^2 + \theta - 2\theta\Theta_2^p\kappa_D\right], \\ F_D &= \left[1 - 2(1-\theta)\Theta_2^p(\bar{b} + \Theta_1^p) + 2\theta\Theta_2^p\bar{\pi}\right]^{-1}\left[\theta + 2\theta\Theta_2^p\bar{\pi}\right], \\ G_D &= \left[1 - 2(1-\theta)\Theta_2^p(\bar{b} + \Theta_1^p) + 2\theta\Theta_2^p\bar{\pi}\right]^{-1}\left[2(1-\theta)M\Theta_2^pF_D + 2(\Theta_2^p)^2\kappa_DF_D - 2\theta\Theta_2^pF_D \right. \\ &\qquad \left. + 2\theta\Theta_2^p - 2\theta(\Theta_2^p)^2\kappa_D\right], \\ H_D &= \left[1 - 2(1-\theta)\Theta_2^p(\bar{b} + \Theta_1^p) + 2\theta\Theta_2^p\bar{\pi}\right]^{-1}\left[(\Theta_2^p)^2(F_D)^2 + \theta(\Theta_2^p)^2 - 2\theta(\Theta_2^p)^2F_D\right]. \end{aligned} \tag{226}$$

[83]Following Coibion et al. (2012), we assume κ_D is of the same order as the shock processes, so that the first term becomes of a second-order. Then our log-linearized model derivation without price dispersion term is valid.

[84]In the right-hand side of the expression, $(p_t - \bar{p}_t)^2$ appears and has a second-order term $(D_t - \bar{D})^2$ from equation (218), and we use equation (225) to replace this term with terms related to $(\hat{\pi}_t)^2$, $(D_{t-1} - \bar{D})^2$, and $\hat{\pi}_t(D_{t-1} - \bar{D})$.

Consumption utility We can second-order approximate the utility of consumption as

$$u(c_t) = \log c_t = u(\bar{c}^F) + u'_{\bar{c}^F} \cdot \bar{c}^F \cdot \underbrace{\left(\frac{c_t - \bar{c}^F}{\bar{c}^F}\right)}_{=\tilde{c}_t + \cancel{\frac{1}{2}(\tilde{c}_t)^2}} + \frac{1}{2} u''_{\bar{c}^F} \cdot (\bar{c}^F)^2 \cdot \underbrace{\left(\frac{c_t - \bar{c}^F}{\bar{c}^F}\right)^2}_{=\cancel{(\tilde{c}_t)^2}} + \text{h.o.t} = u(\bar{c}^F) + \tilde{c}_t + \text{h.o.t.} \tag{227}$$

Step 4: Labor aggregation and cost
By applying Delta method (equation (213)) to the labor aggregator, which is

$$\left(\frac{N_t}{\bar{N}_t}\right)^{\frac{\eta+1}{\eta}} = \int_0^1 \left(\frac{N_t(\nu)}{\bar{N}_t}\right)^{\frac{\eta+1}{\eta}} d\nu, \tag{228}$$

we can obtain[85]

$$\tilde{n}_t - \mathbb{E}_\nu(\tilde{n}_t(\nu)) = \underbrace{\frac{\frac{1}{2}\left(\frac{\eta+1}{\eta}\right)\overline{\nabla}}{1 + \frac{1}{2}\left(\frac{\eta+1}{\eta}\right)^2 \overline{\nabla}}}_{\equiv \Theta_1^n} + \underbrace{\frac{1}{2}\left(\frac{\eta+1}{\eta}\right) \frac{1 - \frac{1}{2}\left(\frac{\eta+1}{\eta}\right)^2 \overline{\nabla}}{\left[1 + \frac{1}{2}\left(\frac{\eta+1}{\eta}\right)^2 \overline{\nabla}\right]^3}}_{\equiv \Theta_2^n} \cdot (\nabla_t - \overline{\nabla}) \tag{229}$$

where $\nabla_t \equiv \text{Var}_\nu(\log n_t(\nu))$. A second-order approximation to the firm ν-specific labor cost around the flexible-price steady state yields

$$\frac{\eta}{\eta+1}\left(\frac{N_t}{\bar{N}_t}\right)^{\frac{\eta+1}{\eta}} = \frac{\eta}{\eta+1}(\bar{n}^F)^{\frac{\eta+1}{\eta}} + \Phi\left[\tilde{n}_t(\nu) + \frac{1}{2}\left(\frac{\eta+1}{\eta}\right)\tilde{n}_t(\nu)^2\right] + h.o.t \tag{230}$$

where a constant Φ is from equation (209). Aggregating equation (230) over firms $\nu \in [0, 1]$ and plugging equation (224) results in

$$\begin{aligned}
\frac{\eta}{\eta+1}\int_0^1 \left(\frac{N_t}{\bar{N}_t}\right)^{\frac{\eta+1}{\eta}} d\nu - \frac{\eta}{\eta+1}(\bar{n}^F)^{\frac{\eta+1}{\eta}} &= \Phi\left[\mathbb{E}_\nu(\tilde{n}_t(\nu)) + \frac{1}{2}\left(\frac{\eta+1}{\eta}\right)\int_0^1 \tilde{n}_t(\nu)^2 d\nu\right] \\
&= -\Phi\left(\Theta_1^n - \frac{1}{2}\left(\frac{\eta+1}{\eta}\right)(\Theta_1^n)^2\right) + \Phi\Bigg[\left(1 - \left(\frac{\eta+1}{\eta}\right)\Theta_1^n\right)\tilde{n}_t + \frac{1}{2}\left(\frac{\eta+1}{\eta}\right)\tilde{n}_t^2 \\
&\quad + \frac{1}{2}\left(\frac{\eta+1}{\eta}\right)(\Theta_2^n)^2\left(\text{Var}_\nu(\tilde{n}_t(\nu)) - \overline{\nabla}\right)^2 - \frac{\eta+1}{\eta}\Theta_2^n \tilde{n}_t\left(\text{Var}_\nu(\tilde{n}_t(\nu)) - \overline{\nabla}\right) \\
&\quad + \left(\frac{1}{2}\left(\frac{\eta+1}{\eta}\right)(1 + 2\Theta_1^n\Theta_2^n) - \Theta_2^n\right)\left(\text{Var}_\nu(\tilde{n}_t(\nu)) - \overline{\nabla}\right) + \frac{1}{2}\left(\frac{\eta+1}{\eta}\right)\overline{\nabla}\Bigg].
\end{aligned} \tag{231}$$

Labor dispersion From individual firm's labor and capital demand (equation (27)) and the household's

[85] In the flexible-price steady-state, there is no heterogeneity among firms and thus, $\bar{n}^F(\nu) = \bar{n}^F$ for $\forall \nu \in [0, 1]$.

intra-marginal condition (equation (15)), we obtain

$$\tilde{k}_t(\nu) = \left(1 + \frac{1}{\eta}\right)\tilde{n}_t(\nu) + \text{aggregate}, \tag{232}$$

where 'aggregate' stands for aggregate variables. Therefore, we obtain

$$\tilde{y}_t(\nu) = \left(1 + \frac{\alpha}{\eta}\right)\tilde{n}_t(\nu) + \text{aggregate}, \tag{233}$$

by plugging equation (232) into an individual firm's production function $\tilde{y}_t(\nu) = \alpha\tilde{k}_t(\nu) + (1-\alpha)\tilde{n}_t(\nu)$. From the Dixit-Stiglitz good demand (equation (22)) and with equation (233), we can get

$$\mathrm{Var}_\nu(\tilde{n}_t(\nu)) = \left(\frac{\epsilon}{1+\frac{\alpha}{\eta}}\right)^2 \mathrm{Var}_\nu(p_t(\nu)), \text{ with } \overline{\nabla} = \left(\frac{\epsilon}{1+\frac{\alpha}{\eta}}\right)^2 \bar{D}. \tag{234}$$

Step 5: Constructing a welfare function: Combining the consumption utility (equation (227)) and the labor disutility (equation (231)), we can construct welfare as

$$\begin{aligned}\mathbb{E}U_t - \bar{U}^F = \mathbb{E}\Bigg[&\tilde{c}_t + \Phi\left(\Theta_1^n - \frac{1}{2}\left(\frac{\eta+1}{\eta}\right)(\Theta_1^n)^2\right) - \Phi\Bigg\{\left(1 - \left(\frac{\eta+1}{\eta}\right)\Theta_1^n\right)\tilde{n}_t + \frac{1}{2}\left(\frac{\eta+1}{\eta}\right)\tilde{n}_t^2 \\ &+ \frac{1}{2}\left(\frac{\eta+1}{\eta}\right)(\Theta_2^n)^2\left(\mathrm{Var}_\nu(\tilde{n}_t(\nu)) - \overline{\nabla}\right)^2 - \frac{\eta+1}{\eta}\Theta_2^n\tilde{n}_t\left(\mathrm{Var}_\nu(\tilde{n}_t(\nu)) - \overline{\nabla}\right) \\ &+ \left(\frac{1}{2}\left(\frac{\eta+1}{\eta}\right)(1 + 2\Theta_1^n\Theta_2^n) - \Theta_2^n\right)\left(\mathrm{Var}_\nu(\tilde{n}_t(\nu)) - \overline{\nabla}\right) + \frac{1}{2}\left(\frac{\eta+1}{\eta}\right)\overline{\nabla}\Bigg\}\Bigg].\end{aligned} \tag{235}$$

with the flexible-price steady state utility given as

$$\begin{aligned}\bar{U}^F &= \log \bar{c}^F - \frac{\eta}{\eta+1}(\bar{n}^F)^{\frac{\eta+1}{\eta}} \\ &= \frac{1}{\eta+1}\log\left[(1-\zeta^G) - \xi^C\left(1 - \frac{R^G}{\Pi\cdot GA\cdot GN}\right)^{-1}\frac{\lambda^K}{1-\lambda^K}\right] - \frac{\alpha}{1-\alpha}\log(R^K) + \log(\xi^{Y,F}) \\ &- \frac{\eta}{\eta+1}(\xi^{Y,F}\xi^{N,F})^{\frac{\eta+1}{\eta}}\left[\left(1 - \frac{R^G}{\Pi\cdot GA\cdot GN}\right)^{-1}\frac{\lambda^K R^K}{1-\lambda^K}\right]^{\frac{-\alpha(\eta+1)}{(1-\alpha)\eta}}\left[(1-\zeta^G) - \xi^C\left(1 - \frac{R^G}{\Pi\cdot GA\cdot GN}\right)^{-1}\frac{\lambda^K}{1-\lambda^K}\right]^{-1}\end{aligned} \tag{236}$$

where $\xi^{Y,F}$ and $\xi^{N,F}$ are values of ξ^Y (equation (101)) and ξ^N (equation (211)), when $\theta = 0$, satisfying

$$\left(\xi^{Y,F}\xi^{N,F}\right)^{\frac{\eta+1}{\eta}} = (1-\alpha)\left(\frac{\xi^K}{\alpha\cdot GA\cdot GN}\cdot\frac{GA\cdot GN - \beta(1-\delta)}{\Pi\cdot GA\cdot GN}\right)^{\frac{-\alpha(\eta+1)}{(1-\alpha)\eta}} \tag{237}$$

$$\xi^{Y,F} = (1-\alpha)^{\frac{-1}{\eta+1}}\left(\frac{GA\cdot GN - \beta(1-\delta)}{\Pi\cdot GA\cdot GN}\right)^{\frac{-\alpha}{1-\alpha}} \tag{238}$$

If we define $\log X_c$[86] as the log-difference in consumption between our steady state (with trend-inflation) and flexible-price steady state, we first check if $\log X_c$ is determined by exogenous parameters and trend-inflation Π. From equation (97), we see that $\log X_c = \log X_y$, where $\log X_y$ is the log-difference in output between our steady state (with trend-inflation) and the flexible-price steady state.

From equation (100), with the fact that R^G, R^K, and λ^K are all independent of price stickiness θ, $\log X_y$ can be expressed as (by comparing our steady state value of Y and the corresponding value when there is no price stickiness ($\theta = 0$))

$$\log X_y = -\frac{(\eta+\alpha)}{(1-\alpha)(\eta+1)} \log\left(\frac{1-\theta\beta\Pi^{\epsilon-1}}{1-\theta\beta\Pi^{\epsilon\frac{1+\eta}{\eta+\alpha}}}\right) + \frac{\eta+\alpha+\epsilon(1-\alpha)}{(\epsilon-1)(1-\alpha)(\eta+1)} \log\left(\frac{1-\theta}{1-\theta\Pi^{\epsilon-1}}\right) \tag{239}$$

From equation (211) and equation (212), we also can calculate $\log X_n = \log \bar{n} - \log \bar{n}^F$ as

$$\log X_n = \frac{\eta+\alpha}{(\eta+1)(1-\alpha)}\left[\log\left(\frac{1-\theta\Pi^{\epsilon-1}}{1-\theta\beta\Pi^{\epsilon-1}}\right) + \log\left(\frac{1-\theta\beta\Pi^{\epsilon\frac{1+\eta}{\eta+\alpha}}}{1-\theta\Pi^{\epsilon\frac{1+\eta}{\eta+\alpha}}}\right)\right] \tag{240}$$

With $\tilde{c}_t = \hat{c}_t + \log X_y$, $\tilde{n}_t = \hat{n}_t + \log X_n$, and the stationarity assumption (following Coibion et al. (2012)), we can get

$$\mathbb{E}\left[\tilde{c}_t - \Phi\left(1-\left(\frac{\eta+1}{\eta}\right)\Theta_1^n\right)\tilde{n}_t\right] = \log X_y - \Phi\left(1-\left(\frac{\eta+1}{\eta}\right)\Theta_1^n\right)\log X_n. \tag{241}$$

Second order terms: With $\tilde{n}_t = \hat{n}_t + \log X_n$, second-order terms can be collected as

$$\begin{aligned} -\Phi\Bigg[&\frac{1}{2}\left(\frac{\eta+1}{\eta}\right)\mathbb{E}\left(\hat{n}_t^2\right) + \frac{1}{2}\left(\frac{\eta+1}{\eta}\right)(\Theta_2^n)^2\mathbb{E}\left(\left(\mathrm{Var}_\nu(\tilde{n}_t(\nu)) - \overline{\nabla}\right)^2\right) - \frac{\eta+1}{\eta}\Theta_2^n\mathbb{E}\left(\hat{n}_t\left(\mathrm{Var}_\nu(\hat{n}_t(\nu)) - \overline{\nabla}\right)\right) \\ &+ \left(\frac{1}{2}\left(\frac{\eta+1}{\eta}\right)(1+2\Theta_1^n\Theta_2^n) - \Theta_2^n - \frac{\eta+1}{\eta}\Theta_2^n\log X_n\right)\mathbb{E}\left(\mathrm{Var}_\nu(\hat{n}_t(\nu)) - \overline{\nabla}\right)\Bigg]\Bigg], \end{aligned} \tag{242}$$

which, after we can plug equation (234) into, becomes

$$\begin{aligned} -\Phi\Bigg[&\frac{1}{2}\left(\frac{\eta+1}{\eta}\right)\mathrm{Var}\left(\hat{n}_t\right) + \frac{1}{2}\left(\frac{\eta+1}{\eta}\right)(\Theta_2^n)^2\left(\frac{\epsilon}{1+\frac{\alpha}{\eta}}\right)^4\mathbb{E}(D_t-\bar{D})^2 \\ &- \frac{\eta+1}{\eta}\Theta_2^n\left(\frac{\epsilon}{1+\frac{\alpha}{\eta}}\right)^2\mathrm{Cov}(\hat{n}_t, D_t) \\ &+ \left(\frac{1}{2}\left(\frac{\eta+1}{\eta}\right)(1+2\Theta_1^n\Theta_2^n) - \Theta_2^n\left(1+\frac{\eta+1}{\eta}\log X_n\right)\right)\left(\frac{\epsilon}{1+\frac{\alpha}{\eta}}\right)^2\mathbb{E}(D_t-\bar{D})\Bigg]\Bigg]. \end{aligned} \tag{243}$$

Finally, by plugging equation (225) into equation (243), we get the following proposition. Sine κ_D is of

[86]Then, $\hat{c}_t = \tilde{c}_t + \log X_c$ holds.

the same order as shock processes, up to a second-order, we can ignore covariance terms and the square term of D_t. Therefore, a 2^{nd}-order approximation to the expected per-period welfare would be given as

$$\mathbb{E}U_t - \bar{U}^F = \Omega_0 + \Omega_n \text{Var}(\hat{n}_t) + \Omega_\pi \text{Var}(\hat{\pi}_t), \tag{244}$$

with

$$\begin{aligned}
\Omega_0 &= \log X_y - \Phi\left(1 - \left(\frac{\eta+1}{\eta}\right)\Theta_1^n\right)\log X_n + \Phi\left(\Theta_1^n - \frac{1}{2}\left(\frac{\eta+1}{\eta}\right)(\Theta_1^n)^2\right) - \Phi\frac{1}{2}\frac{\eta+1}{\eta}(\log X_n)^2 \\
&\quad - \Phi\frac{1}{2}\left(\frac{\eta+1}{\eta}\right)\left(\frac{\epsilon}{1+\frac{\alpha}{\eta}}\right)^2\bar{D}, \\
\Omega_n &= -\Phi\frac{1}{2}\left(\frac{\eta+1}{\eta}\right), \\
\Omega_\pi &= -\Phi\left[\left(\frac{1}{2}\left(\frac{\eta+1}{\eta}\right)(1+2\Theta_1^n\Theta_2^n) - \Theta_2^n\left(1+\frac{\eta+1}{\eta}\log X_n\right)\right)\left(\frac{\epsilon}{1+\frac{\alpha}{\eta}}\right)^2\frac{Z_D}{1-F_D}\right],
\end{aligned} \tag{245}$$

where

$$\log X_y = -\frac{(\eta+\alpha)}{(1-\alpha)(\eta+1)}\log\left(\frac{1-\theta\beta\Pi^{\epsilon-1}}{1-\theta\beta\Pi^{\epsilon\frac{1+\eta}{\eta+\alpha}}}\right) + \frac{\eta+\alpha+\epsilon(1-\alpha)}{(\epsilon-1)(1-\alpha)(\eta+1)}\log\left(\frac{1-\theta}{1-\theta\Pi^{\epsilon-1}}\right), \tag{246}$$

$$\log X_n = \frac{\eta+\alpha}{(\eta+1)(1-\alpha)}\left[\log\left(\frac{1-\theta\Pi^{\epsilon-1}}{1-\theta\beta\Pi^{\epsilon-1}}\right) + \log\left(\frac{1-\theta\beta\Pi^{\epsilon\frac{1+\eta}{\eta+\alpha}}}{1-\theta\Pi^{\epsilon\frac{1+\eta}{\eta+\alpha}}}\right)\right]. \tag{247}$$

Coefficients Θ_1^n, Θ_2^n are given in equation (229) and $\bar{D}$ is given by jointly solving equation (218) and equation (226). $\kappa_D, Z_D, F_D, G_D, H_D$ are given in equation (226).