Endogenous Firm Entry and the Monetary Policy ‘Room’*

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Abstract

We present a tractable New Keynesian model featuring endogenous firm participation. Entry-induced short-run shifts in aggregate supply shape the economy’s response to both supply and demand shocks. In particular, rising aggregate demand induces entry, expanding supply and reinforcing demand through investment expenditures of operating firms. Monetary policy influences both aggregate demand and the entry decisions of financially constrained firms, shaping cycle dynamics in economies with high entry potential. Equilibrium firm entry is determined by the “policy room”, a sufficient statistic for the effectiveness of monetary policy in both the model and empirical data.

Keywords: Monetary Policy, Policy Room, Endogenous Firm Entry
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1 Introduction

How does endogenous firm entry affect the transmission of monetary policy? In this paper, we provide a tractable New Keynesian framework featuring a rich interaction among monetary policy, endogenous entry decisions of firms, i.e., aggregate supply, and aggregate demand. An increase in aggregate demand induces a larger entry of firms, and entrants in turn pay in-kind fixed costs, lifting aggregate demand and thus initiating a self-reinforcing cycle between supply and demand. Monetary accommodation in this environment can be exceptionally effective in stimulating output: (i) the accommodative policy lifts aggregate demand from usual New Keynesian channels; (ii) it induces a larger proportion of firms to operate as firms can expect higher profits; (iii) firms, facing cash-in-advance constraints, pay less interest on their borrowing, based on which they finance in-kind entry fixed costs.[1] Our analytical approach summarizes the equilibrium measure of firm participation as a sole function of the *policy room*: a sufficient statistic for the short-run supply effect of monetary policy. Finally, we empirically test our theoretical predictions and find that a wider policy room amplifies the transmission of monetary policy.

To facilitate the analysis, we decouple endogenous firm entry from other elements of the standard New Keynesian model by separating the production process across downstream and upstream industries. At the downstream level, a fixed measure of identical but differentiated firms engage in the production of a continuum of consumption varieties, face nominal pricing rigidities, and rely on upstream industry inputs. Upstream firms, conversely, enjoy price flexibility and employ labor, feature heterogeneous productivity endowments, and are obligated to incur stochastic fixed costs to enter the market and remain operational in subsequent periods. To further simplify the problem and obtain intuitive analytical expressions, we follow the literature on endogenous entry and assume that productivity and entry costs are drawn from independent Pareto distributions.[2] Finally, we impose a cash-in-advance (or working capital) constraint that, coupled with entry costs, generates upstream industry's reliance on borrowing from capital markets, connecting entry decisions to monetary policy via loan rates.

While stylized, our model yields several interesting analytical outcomes, one of which is the formulation of a minimum policy rate, termed the *Satiation Bound* (SB) and defined as the threshold rate that guarantees full market participation of firms with comparable fixed

[1] This interest-rate effect additionally contributes to increases in firm entry and aggregate supply.
[2] For the use of Pareto distributions for tractablity purposes, see e.g., Melitz (2003).

costs. When the policy rate falls below the Satiation Bound, even the least productive firms find entry profitable. Consequently,[3] market participation becomes insensitive to additional monetary accommodation or other expansionary shocks beyond this bound, reducing policy effectiveness: output multipliers shrink while inflationary pressures strengthen.

This observation suggests that the gap between the current policy rate and the average Satiation Bound, which we define as the *policy room*, can serve as a *sufficient statistic* for the supply-side effect of monetary policy. In fact, our model generates significant correlation between our policy room measure and the potency of monetary policy, as well as the economy's responses to various shocks.

Finally, we construct an empirical measure of the policy room for the U.S. economy and test the model's key prediction that monetary policy becomes more potent as the policy room widens. The data support this mechanism: a two-percentage-point increase in policy room amplifies the effect of a one-standard-deviation (29-basis-point) monetary tightening shock, lowering output by up to three additional percentage points.

Related Literature Our setting with endogenous (upstream) firm participation follows the literature, including Bilbiie et al. (2007), Bergin and Corsetti (2008),[4] Stebunovs (2008), Kobayashi (2011) Bilbiie et al. (2012), Lee and Mukoyama (2015a,b, 2018),[5] Uusküla (2016), Hamano and Zanetti (2017). While some papers assume equity financing for newly entering firms, e.g., Bilbiie et al. (2007), Bergin and Corsetti (2008), Bilbiie et al. (2012),[6] firms finance their entry costs via borrowing from a loan market in our model, as in Stebunovs (2008), Kobayashi (2011), Uusküla (2016), so that firm participation is boosted under monetary accommodation, which aligns with the evidence in Colciago and Silvestrini (2022).[7]

[3]Because the least productive firms have already entered, further rate cuts do not prompt additional entry.

[4]Our assumption that fixed costs for market entry are paid in units of the final consumption goods aligns with the framework proposed by Bergin and Corsetti (2008). However, we deviate from their assumption of "pre-set" output procurement prices in favor of market prices.

[5]While Lee and Mukoyama (2015a,b, 2018) focus on issues of individual firm's entry/exit(s) along the business cycle, we focus more on monetary policy and aggregate business cycles stemming from endogenous firm entry and ensuing supply adjustments. In Section 2.2.5, we provide a way to connect our framework to the literature on firm entry more closely.

[6]An expansionary monetary shock usually increases aggregate demand, raising labor demand and wages. Under the equity financing structure for entrants, higher labor costs for potential entrants can lower their net present profits, reducing equity valuations and the entry rate of new firms, which is counterfactual. For "real wage rigidity" in resolving this problem, see e.g., Lewis and Poilly (2012).

[7]Colciago and Silvestrini (2022) find the empirical evidence that expansionary monetary policy leads to an initial decrease and then an overshooting in the average productivity of the economy, as well as an initial

We express the equilibrium firm participation as a sole function of the "policy room", a *sufficient statistic* we devise, in a New Keynesian framework. Up to our knowledge, we are one of the first papers that devise a *sufficient statistic* that accounts for the supply effect of demand shocks and empirically test the channel.[8] More recently, Bilbiie and Melitz (2022) demonstrate that nominal rigidities amplify the responsiveness of firm entry and exit—and thus welfare losses—to adverse supply shocks, a result that contrasts with standard New Keynesian predictions. In our approach, we disentangle nominal rigidities (experienced by downstream firms) from the firm participation decision (made by upstream firms) and focus on the supply-side effects of monetary policy, which we capture with our sufficient metric, the policy room.

Our characterization of the Satiation Bound hinges on the idea that (i) monetary expansion facilitates an upswing in firm entry, and (ii) upon the monetary policy rate reaching a specified lower bound, all potential firms associated with a particular fixed entry cost have ventured into the market. Beyond this juncture, the positive supply effects stemming from further monetary accommodation and subsequent firm entry begin to wane. This phenomenon resonates with the insights of Ulate (2021) and Abadi et al. (2022), who incorporate analogous concepts in the context of banking profitability and the negative interest rates.

Layout Section 2 presents our model with endogenous firm participation. Section 3 discusses our calibration, steady-state analysis, and comparative statics. The model economy's impulse response functions to various shocks are explored in Section 4. Section 5 provides our empirical analysis and confirms the model predictions in the data. Concluding remarks are presented in Section 6.

Derivations and proofs are detailed in Appendix A. Appendix B summarizes the equilibrium conditions, including the flexible-price and steady-state benchmarks. Appendix C provides estimation techniques of the unobservable policy room and the satiation measure based on available data. Appendix D presents supplementary tables and figures. Appendix E presents various robustness checks to our empirical analysis, and lastly, Appendix F provides the derivation of the model under a simplified framework with homogeneous entry

increase and then undershooting in the firm's entry rate. Our model replicates the initial response part.

[8]Bergin and Corsetti (2008) similarly find that monetary policy has a significant impact on the creation of new firms. Jordà et al. (2024) study long-run effects of monetary policy, through supply channels including capital stocks and the total factor productivity (TFP), while we focus on short-run supply effects of monetary policy.

costs. Appendix G offers a model with fixed costs denominated in labor rather than in-kind final good. Lastly, Appendix H analytically derives average productivity of operating firms, connecting our paper to the literature on firm entry more closely.

2 Model

2.1 Representative Household

The representative household maximizes lifetime utility given by

$$\max E_t \sum_{j=0}^{\infty} \beta^j \left[\phi_{c,t} \cdot \log\left(C_t\right) - \left(\frac{\eta}{\eta+1}\right) \cdot N_t^{\left(\frac{\eta+1}{\eta}\right)} \right],$$

where C_t is consumption, N_t is labor, and $\phi_{c,t} \equiv \exp\left(u_{c,t}\right)$ is an aggregate demand shock defined as $u_{c,t} = \rho_c \cdot u_{c,t-1} + \varepsilon_{c,t},\ \ \varepsilon_{c,t} \sim N\left(0, \sigma_c^2\right)$. The household's budget constraint is

$$C_t + \frac{B_t + L_t}{P_t} = \frac{R_{t-1}^J \left(B_{t-1} + L_{t-1}\right)}{P_t} + \frac{W_t N_t}{P_t} + \frac{\Upsilon_t}{P_t},$$

where B_t denotes government bonds, whose supply is determined by fiscal policy described in Section 2.3, and L_t denotes loans issued to firms. We assume both have the same gross interest rate represented by R_t^J, which is set by the central bank.[9,10] Υ_t captures lump-sum transfers to households. Such transfers may originate from various sources, including fiscal policies (such as subsidies to firms) or residual firm profits.

The first-order conditions are given by

$$\frac{1}{R_t^J} = \beta E_t \left[\frac{\phi_{c,t+1}}{\phi_{c,t}} \cdot \frac{C_t}{C_{t+1}\Pi_{t+1}} \right], \tag{1}$$

$$N_t^{\frac{1}{\eta}} = \phi_{c,t} \cdot C_t^{-1} \cdot \frac{W_t}{P_t}. \tag{2}$$

[9] We do not consider issues pertaining to the zero lower bound (ZLB), so it is possible for interest rates to be negative, $R_t^J < 1$.

[10] Thus, we assume that the pass through from monetary policy to government bond rates and private loan rates is perfect. Households are indifferent between investing in loans and government bonds. We will discuss this assumption in greater detail in Section 2.2.5.

2.2 Firms

The model stratifies firms into two discrete categories: those belonging to the downstream industry and those in the upstream industry. In both layers, firms operate in an environment of monopolistic competition. Notably, only downstream firms encounter nominal price rigidities à la Calvo (1983). Operational dynamics are structured such that upstream firms employ labor to generate intermediate input varieties, whose aggregator the downstream firms subsequently incorporate into the production of consumption good varieties. Representative households own firms across both industries, and consume the aggregated downstream goods.

One of the defining elements of our framework is the decision-making process for upstream firms. At each period, firms evaluate whether to continue/start operations in the next period. Should they decide to remain/enter the market, they must incur certain fixed costs, denominated in final goods, which are financed through loans from the banking sector.[11]

2.2.1 Downstream Industry: Aggregator

A representative firm, operating under perfect competition, aggregates the differentiated products produced by a continuum of downstream firms, denoted by u, spanning the interval $[0,\ 1]$. This can be formally expressed as:

$$Y_t = \left[\int_0^1 Y_t(u)^{\frac{\gamma-1}{\gamma}}\ \mathrm{d}u\right]^{\frac{\gamma}{\gamma-1}} \quad .$$

The demand for each distinct variety produced by downstream firms, as well as the aggregate price, are given by

$$\begin{aligned} Y_t(u) &= \left(\frac{P_t(u)}{P_t}\right)^{-\gamma} Y_t \ , \qquad (3) \\ P_t &= \left[\int_0^1 P_t(u)^{1-\gamma}\ \mathrm{d}u\right]^{\frac{1}{1-\gamma}} \quad , \end{aligned}$$

where $Y_t(u)$ and $P_t(u)$ are the output and prices of downstream varieties, respectively. Let $X_t = P_t Y_t$ represent the nominal aggregate expenditure, and $X_t(u) = P_t(u)Y_t(u)$ denote the expenditure for a specific downstream variety u. Given these definitions, the individual

[11] This dependency on external funding effectively functions as a cash-in-advance production constraint.

demands can be reformulated as:

$$X_t(u) = \Gamma_t \cdot P_t(u)^{1-\gamma} \;, \qquad \text{where: } \Gamma_t = X_t P_t^{\gamma-1}.$$

2.2.2 Downstream Industry: Monopolistic Competition with Sticky Prices

Consider a firm u within the downstream industry, belonging to the interval $[0, 1]$. This firm employs $J_t(u)$ units of the aggregate product from the upstream industry and produces $Y_t(u) = J_t(u)$, indicating a one-to-one transformation from input to output. Consequently, the aggregate sum of upstream products, denoted as J_t, satisfies: $J_t \equiv \int_0^1 J_t(u) \; \mathrm{d}u = \int_0^1 Y_t(u) \; \mathrm{d}u$.

The profit equation for a downstream firm u is given by

$$\Pi_t(u) = (1 + \zeta^T) P_t(u) Y_t(u) - P_t^J J_t(u) \;,$$

where P_t^J represents the price of the aggregate upstream product, and ζ^T stands for a production subsidy to downstream firms. Thus, the present discounted value of profits, which the downstream firm u seeks to maximize, can be expressed as:

$$\sum_{l=0}^{\infty} E_t \left\{ Q_{t,t+l} \left[(1 + \zeta^T) P_{t+l}(u) Y_{t+l}(u) - P_{t+l}^J J_{t+l}(u) \right] \right\} \;,$$

with $Q_{t,t+l}$ being the stochastic discount factor between time t and $t + l$.

Firms in the downstream industry face price stickiness à la Calvo (1983), characterized by a price-resetting probability of $1 - \theta$. A firm, when adjusting its price P_t^*, aims to:

$$\max_{P_t^*} \sum_{l=0}^{\infty} E_t \left\{ Q_{t,t+l} \theta^l \left[(1 + \zeta^T) P_t^* - P_{t+l}^J \right] \left(\frac{P_t^*}{P_{t+l}} \right)^{-\gamma} Y_{t+l} \right\} \;,$$

where all firms that adjust their prices select P_t^* as the revised price. The resulting first-order condition can be articulated as:

$$\frac{P_t^*}{P_t} = \frac{\sum_{l=0}^{\infty} E_t \left\{ Q_{t,t+l} \theta^l \left(\frac{(1 + \zeta^T)^{-1} \gamma}{\gamma - 1} \right) \left(\frac{P_{t+l}}{P_t} \right)^{\gamma+1} \left(\frac{P_{t+l}^J}{P_{t+l}} \right) Y_{t+l} \right\}}{\sum_{l=0}^{\infty} E_t \left\{ Q_{t,t+l} \theta^l \left(\frac{P_{t+l}}{P_t} \right)^{\gamma} Y_{t+l} \right\}} \;. \tag{4}$$

2.2.3 Upstream Industry: Aggregator

There exists a continuum of upstream firms spanning the interval $[0, 1]$, each producing a distinct variety. These firms exhibit heterogeneity in two principal dimensions: productivity, indexed by v, and operational fixed costs, indexed by m. The output of a firm, uniquely identified by the index pair mv, is defined as $J_{mv,t}$. A perfectly competitive firm aggregates these upstream varieties as:

$$J_t = \left[\int_0^1 \int_{v \in \Omega_{m,t}} J_{mv,t}^{\frac{\sigma-1}{\sigma}} \,\mathrm{d}v \,\mathrm{d}m \right]^{\frac{\sigma}{\sigma-1}} ,$$

where $\Omega_{m,t}$ denotes the subset of upstream firms sharing the same operational fixed cost m that decide to produce in period t. Given significant fixed costs, only the firms with the highest productivity levels may find production viable. The demand for an individual upstream variety (m, v), is:

$$J_{mv,t} = \left(\frac{P^J_{mv,t}}{P^J_t} \right)^{-\sigma} J_t \ . \tag{5}$$

Subsequently, the aggregate price index for the upstream product is:

$$P^J_t = \left[\int_0^1 \underbrace{\int^1_{v \in \Omega_{m,t}} \left(P^J_{mv,t}\right)^{1-\sigma} \,\mathrm{d}v}_{\equiv (P_{m,t})^{1-\sigma}} \,\mathrm{d}m \right]^{\frac{1}{1-\sigma}} = \left[\int_0^1 \left(P^J_{m,t}\right)^{1-\sigma} \,\mathrm{d}m \right]^{\frac{1}{1-\sigma}} , \tag{6}$$

where $P^J_{m,t}$ serves as the aggregate price of input for firms bearing the fixed costs indexed by m. We further define the nominal expenditure on a given upstream variety as $X^J_{mv,t} = P^J_{mv,t} J_{mv,t}$, and the aggregate expenditure as $X^J_t = P^J_t J_t$, so

$$X^J_{mv,t} = \Gamma^J_t \cdot P^{1-\sigma}_{mv,t} \ , \qquad \text{where: } \Gamma^J_t = X^J_t \left(P^J_t\right)^{\sigma-1} . \tag{7}$$

Using equation (3), we can express the aggregate input demand of downstream firms as:

$$J_t = \int_0^1 Y_t(u) \,\mathrm{d}u = Y_t \underbrace{\int_0^1 \left(\frac{P_t(u)}{P_t} \right)^{-\gamma} \mathrm{d}u}_{\equiv \Delta_t} = Y_t \Delta_t \ , \tag{8}$$

where

$$\Delta_t = (1-\theta)\left(\frac{P_t^*}{P_t}\right)^{-\gamma} + \theta \Pi_t^{\gamma} \Delta_{t-1} \ , \tag{9}$$

represents a measure of price dispersion. Utilizing equation (8), equation (7) can be expressed as $\Gamma_t^J = (P_t^J)^{\sigma} Y_t \Delta_t$.

2.2.4 Upstream Industry: Monopolistic Competition, Loans, and Entry Decisions

The production function for an arbitrary firm (m, v) features diminishing returns to scale and is given by

$$J_{mv,t} = \varphi_{mv,t} \cdot N_{mv,t}^{\alpha}, \qquad \text{with } 0 < \alpha \leq 1 \ ,$$

where $N_{mv,t}$ denotes the labor employed, and $\varphi_{mv,t}$ is a firm-specific productivity assumed to follow a Pareto distribution, $\varphi_{mv,t} \overset{\text{iid}}{\sim} \mathcal{P}\left(\left(\frac{\kappa-1}{\kappa}\right) A_t, \kappa\right)$, with A_t being the average aggregate productivity. A higher κ implies that the productivity distribution is more concentrated around its mean, A_t. The cumulative distribution function is given by:

$$\Psi(\varphi_{mv,t}) = 1 - \left(\frac{\left(\frac{\kappa-1}{\kappa}\right) A_t}{\varphi_{mv,t}}\right)^{\kappa} \ ,$$

with the probability distribution function defined as $\psi(\varphi_{mv,t}) \equiv \Psi'(\varphi_{mv,t})$. As our focus is on the aggregate behavior of the business cycle, we are agnostic about an individual firm (m, v)'s productivity process: the only information that is important is that firm productivity has a cross-sectional distribution $\varphi_{mv,t} \overset{\text{iid}}{\sim} \mathcal{P}\left(\left(\frac{\kappa-1}{\kappa}\right) A_t, \kappa\right)$. Section 2.2.5 discusses an interpretation of the model in which the relative ranks of firm productivities and fixed costs are preserved across time; in this case, a highly ranked firm is always more likely to operate under any realization of shocks.

Profit Function: Firms must pay a pre-determined in-kind fixed cost, $F_{m,t-1}$, in the preceding period (i.e., at $t-1$) to operate in each period t.[12] This cost, which covers expenses such as equipment acquisition, is assumed to be financed through loans financed at the prevailing gross rate, R_{t-1}^J. The profit for an upstream firm, if it chooses to operate in period

[12]Online Appendix G provides an alternative model where rather than pay a pre-determined in-kind fixed cost, firms must hire a pre-determined labor amount in the preceding period in order to operate. We show that the qualitative implication of our framework does not change: for example, a positive demand shock induces more firms to operate, which in turn demand higher labor. It pushes up labor costs as well as the purchasing power of the household, leading to higher aggregate demand and in turn inducing more firms to operate, ad infinitum.

t, is:

$$\Pi^J_{mv,t} = \underbrace{\left(1+\zeta^J\right) P^J_{mv,t} J_{mv,t}}_{\equiv r_{mv,t}} - W_t N_{mv,t} - R^J_{t-1} P_{t-1} F_{m,t-1} \ , \tag{10}$$

where ζ^J is a production subsidy to upstream firms and $r_{mv,t}$ represents their revenue. These upstream firms operate in a monopolistically competitive market and are not subject to nominal rigidities, setting prices as a constant markup over marginal costs (if they decide to produce), formally:

$$P^J_{mv,t} = \left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha}\right) W_t \varphi_{mv,t}^{-\frac{1}{\alpha}} J_{mv,t}^{\frac{1-\alpha}{\alpha}} \ . \tag{11}$$

By substituting the derived price equation into equation (10) and using the demand equations (5) and (7), we can rewrite the profit function as:

$$\Pi^J_{mv,t} = \Xi_t \cdot \varphi_{mv,t}^{\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}} - R^J_{t-1} P_{t-1} F_{m,t-1} \ , \tag{12}$$

where

$$\Xi_t \equiv \frac{\alpha+\sigma(1-\alpha)}{(\sigma-1)\alpha} \left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha}\right)^{\frac{-\sigma}{\alpha+\sigma(1-\alpha)}} W_t^{\frac{\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}} (\Gamma^J_t)^{\frac{1}{\alpha+\sigma(1-\alpha)}} \ . \tag{13}$$

Operation Decision: Firms' decision to operate is taken one-period ahead in $t-1$, and is based on their expected profits and associated costs in t. We assume that firms know at $t-1$ their forthcoming productivity for period t, $\varphi_{mv,t}$. However, they remain uninformed about other eventual shocks that could impact individual demand in t.[13] Should firm (m, v) decide to operate, it will subsequently hire labor in time t from the spot market, realizing profits as described in equation (12).

Given the cross-sectional distribution, we can pinpoint the productivity threshold, $\varphi^*_{m,t}$, below which a firm would not participate and thus expect zero profit. Firms with the same fixed cost, $F_{m,t-1}$, and their productivity below this threshold will opt out of the market for

[13]This assumption contrasts with Burnside et al. (1993), where labor decisions precede the realization of shocks. In our model, the decision to enter the market precedes the realization of other demand shocks. For simplicity, we assume that firms possess perfect foresight regarding their next period's productivity.

period t. Using equation (12), the formal representation of $\varphi^*_{m,t}$ is:

$$E_{t-1}\left[\xi_t \cdot \Xi_t\right] \cdot \left(\varphi^*_{m,t}\right)^{\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}} - R^J_{t-1} P_{t-1} F_{m,t-1} = 0 \,, \qquad \text{where: } \xi_t = \frac{Q_{t-1,t}}{E_{t-1}\left[Q_{t-1,t}\right]} \,. \tag{14}$$

It's important to note that this threshold, $\varphi^*_{m,t}$, is based on *ex-ante* expected profits. Once a firm (m, v) commits to market entry, unforeseen shocks could potentially push profits into negative figures. Considering the inherent lower limit on productivity, $\left(\frac{\kappa-1}{\kappa}\right) A_t$, the actual productivity threshold for entry becomes $\max\left\{\varphi^*_{m,t}, \left(\frac{\kappa-1}{\kappa}\right) A_t\right\}$.[14] The proportion of firms with a fixed cost $F_{m,t-1}$ that decide to operate in t is denoted as $M_{m,t}$ and is given by

$$M_{m,t} \equiv Prob\left(\varphi_{mv,t} \geq \varphi^*_{m,t}\right) = \min\left\{\left(\frac{E_{t-1}\left[\xi_t \cdot \Xi_t\right]\left[\left(\frac{\kappa-1}{\kappa}\right) A_t\right]^{\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}}}{R^J_{t-1} P_{t-1} F_{m,t-1}}\right)^{\frac{\kappa[\alpha+\sigma(1-\alpha)]}{\sigma-1}}, 1\right\} , \tag{15}$$

where we use (14) to substitute for $\varphi^*_{m,t}$ in the last expression. From this equation, we can derive the following proposition:

Proposition 1 *For upstream firms with a fixed cost of $F_{m,t-1}$, $\underline{M_{m,t} = 1}$ when the policy rate R^J_{t-1} is below a threshold $R^{J,*}_{m,t-1}$ given by*

$$R^{J,*}_{m,t-1} \equiv \frac{E_{t-1}\left[\xi_t \cdot \Xi_t\right]\left[\left(\frac{\kappa-1}{\kappa}\right) A_t\right]^{\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}}}{P_{t-1} F_{m,t-1}} \,. \tag{16}$$

We refer to this threshold, $R^{J,*}_{m,t-1}$, as the "satiation bound" (SB) for firms of fixed cost type m.

As the policy rate, R^J_{t-1}, decreases, more firms with the fixed cost $F_{m,t-1}$ opt for market operation in time t due to the reduced loan repayment costs. Upon the policy rate reaching the type-specific bound $R^{J,*}_{m,t-1}$, all firms sharing the fixed cost $F_{m,t-1}$ (or lower) decide to become operational in t, leading to stagnation in additional market entry for firms of cost type m and below. This fixed cost type-specific lower bound on the policy rate, $R^{J,*}_{m,t-1}$, is hence termed the satiation bound (SB).

In addition to the conventional intertemporal substitution effect captured by the Euler equation (1), monetary policy wields influence over the market entry decisions of upstream

[14] If $\varphi^*_{m,t}$ is below $\left(\frac{\kappa-1}{\kappa}\right) A_t$, then all firms categorized by fixed cost m will operate in t.

firms. This, in turn, impacts the input market's prices and quantities, cascading onto the aggregate economy via downstream product markets. Upon the rate hitting the SB for firms with the fixed cost $F_{m,t-1}$, no supplementary entries occur, rendering the supply-side effect of monetary policy ineffectual for such firms.

Loan Demand: From equation (15), we derive the expression for the aggregate real loan demand of firms with a fixed cost type m:

$$\frac{L_{m,t-1}}{P_{t-1}} = M_{m,t} \cdot F_{m,t-1} \ . \tag{17}$$

Firms opting to operate in period t borrow an amount $L_{m,t-1}$ to acquire final goods equivalent to $F_{m,t-1}$. This acquisition of final goods connects the operation decisions of firms to the economy's aggregate demand via the loan channel.

Fixed Cost Distribution: We assume that the fixed costs of upstream firms, $F_{m,t}$, are distributed as another Pareto distribution, $F_{m,t} \overset{\text{iid}}{\sim} \mathcal{P}\left(\left(\frac{\omega-1}{\omega}\right) F_t, \omega\right)$, where F_t represents the average fixed cost associated with running a business, and $\omega > 1$ is the parameter that determines the variance of the distribution. The associated cumulative distribution function is:

$$H(F_{m,t}) = 1 - \left(\frac{\left(\frac{\omega-1}{\omega}\right) F_t}{F_{m,t}}\right)^{\omega} \ , \tag{18}$$

and its probability distribution function is denoted by $h(F_{m,t}) \equiv H'(F_{m,t})$. From Proposition 1, we obtain the probability measure of fixed cost types $F_{m,t-1}$ that are fully satiated, that is, the share of all firms with fixed cost $F_{m,t-1}$ that have already entered the market by time t, thus resulting in $M_{m,t} = 1$. This leads us to the following proposition:

Proposition 2 *Given the distribution in equation (18), the probability that $M_{m,t} = 1$ is:*

$$Pr\left(R^J_{t-1} \leq R^{J,*}_{m,t-1}\right) = Pr\left(F_{m,t-1} \leq \underbrace{\frac{E_{t-1}\left[\xi_t \cdot \Xi_t\right]\left[\left(\frac{\kappa-1}{\kappa}\right) A_t\right]^{\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}}}{R^J_{t-1} P_{t-1}}}_{\equiv F^*_{t-1}}\right) \equiv H\left(F^*_{t-1}\right) \ ,$$

where F^*_{t-1} is the fixed cost threshold as defined above. All firms with a fixed cost $F_{m,t-1}$ less than or equal to F^*_{t-1}, irrespective of their productivity values $\varphi_{mv,t}$, opt to produce in period t. We term F^*_{t-1} the "full satiation fixed cost threshold".

Proposition 2 can be interpreted as follows: If a firm's fixed cost, $F_{m,t-1}$, is sufficiently low —below the threshold F^*_{t-1}— then even a firm with the lowest productivity would still deem operations in period t profitable. Consequently, all firms bearing the same fixed cost, regardless of their respective productivities, are active in period t.

Upstream Industry Aggregation: The price aggregator for operating upstream firms, denoted by P^J_t, can be expressed as:

$$\frac{P^J_t}{P_t} = \left(\frac{W_t}{P_t A_t}\right) \cdot \left(\frac{Y_t \Delta_t}{A_t}\right)^{\frac{1-\alpha}{\alpha}} \cdot \left[\frac{\Theta_3}{1+\Theta_4 \cdot H\left(F^*_{t-1}\right)}\right]^{\left(\frac{\alpha+\sigma(1-\alpha)}{\alpha(\sigma-1)}\right)} , \tag{19}$$

where $\Theta_3 = \frac{\kappa[\alpha+\sigma(1-\alpha)]+(\omega-1)(\sigma-1)}{\Theta_1 \omega(\sigma-1)}$ and $\Theta_4 = \frac{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}{\omega(\sigma-1)}$ are constants. The aggregate measure of firms that operate during period t, represented by M_t, is given by

$$M_t = \int_0^1 \int_{v \in \Omega_{m,t}} 1 \; \mathrm{d}v \; \mathrm{d}m = 1 - \Theta_M \cdot \left[1 - H\left(F^*_{t-1}\right)\right] , \tag{20}$$

where $\Theta_M = \frac{\kappa[\alpha+\sigma(1-\alpha)]}{\kappa[\alpha+\sigma(1-\alpha)]+\omega(\sigma-1)}$. Based on equation (20), the aggregate loan demand from operational upstream firms can be derived as:

$$\frac{L_{t-1}}{P_{t-1}} = \frac{1}{P_{t-1}} \int_0^1 L_{m,t-1} \; \mathrm{d}m = F_{t-1} \cdot \left[1 - \Theta_L \cdot \left[1 - H\left(F^*_{t-1}\right)\right]^{\left(\frac{\omega-1}{\omega}\right)}\right] , \tag{21}$$

where $\Theta_L = \frac{\kappa[\alpha+\sigma(1-\alpha)]}{\kappa[\alpha+\sigma(1-\alpha)]+(\sigma-1)(\omega-1)}$ is another constant.

In equation (20), notice that as the satiation measure $H\left(F^*_{t-1}\right)$ increases, the number of operational firms at time t also increases. From equation (21), the aggregate real loan demand of firms is proportional to the average fixed cost, F_{t-1}, and grows with the satiation rate $H\left(F^*_{t-1}\right)$. Finally, in equation (19), the relative price of inputs from upstream firms relates to the technology-adjusted real wage, $\frac{W_t}{P_t A_t}$, and the aggregate demand for inputs of downstream firms, $\frac{Y_t \Delta_t}{A_t}$. When participation from upstream firms increases, as indicated by $H(F^*_{t-1})$, this relative price decreases. This is due to more upstream varieties being available to downstream firms, leading to greater competition and lower input prices. Therefore, a higher measure of operational firms can reduce marginal costs for downstream firms and mitigate inflationary pressures for households, acting as a positive shift in aggregate supply.

Average SB: We obtain the average satiation interest rate of the economy by integrating over (16), and denote it by $R_{t-1}^{J,*}$. This rate serves as a measure of the satiation propensity of upstream firms. When the prevailing policy rate R_{t-1}^{J} exceeds this average rate, a marginal reduction in R_{t-1}^{J} can induce more upstream firms to operate in the market.

According to (19), a larger measure of operational upstream firms can lower average input prices and subsequently mitigate inflation. Also, it can boost aggregate demand and increase the final good price level, as firms that operate in the market take out loans to meet fixed costs, enabling the acquisition of fixed equipment for the production of final goods. The relative size of these two countervailing effects on the price level depends on the model calibration.

Proposition 3 *The aggregate satiation bound (SB) is expressed as:*

$$R_{t-1}^{J,*} = \int_{\left(\frac{\omega-1}{\omega}\right)F_{t-1}}^{\infty} R_{m,t-1}^{J,*}\, dH\left(F_{m,t-1}\right) = \left(\frac{\omega^2}{\omega^2-1}\right)\cdot\frac{F_{t-1}^{*}}{F_{t-1}}\cdot R_{t-1}^{J}\ , \tag{22}$$

where F_{t-1}^{*} is the full satiation fixed cost threshold defined in Proposition 2.

If the threshold fixed cost for satiation, F_{t-1}^{*}, surpasses the economy's average fixed cost F_{t-1}, it signals an elevated likelihood of satiation across diverse fixed cost categories. Consequently, it results in a high value of the average SB rate, $R_{t-1}^{J,*}$, relative to the policy rate, R_{t-1}^{J}. In such a situation, a minor ease in R_{t-1}^{J} may not substantially stimulate the additional operation of upstream firms.

2.2.5 Discussion of the Model

Limit Case, $\omega \to \infty$: Under our calibration, the fixed cost distribution $H(F_{m,t})$ collapses to its mean value, F_t, thereby becoming degenerate. This results in a uniform fixed cost across all firms. The economy's state —whether fully satiated or not— is determined by the relative sizes of the policy rate R_{t-1}^{J} and the mean satiation bound, $R_{t-1}^{J,*}$. Specifically, if $R_{t-1}^{J} < R_{t-1}^{J,*}$, all upstream firms enter the market and commence production in t. This simplified version of the model yields analytically tractable equilibrium expressions. Additional insights into the equilibrium conditions for this scenario are provided in Appendix F.

Interpretation of the Model: In our model, we mostly focus on the aggregate business cycle behavior, and therefore abstract from individual firm life cycles. In each period, firms that are relatively more efficient in terms of productivity are active (i.e., in operations) while those with lower productivity are inactive for a given fixed cost type m.

One way to connect our framework to a large literature on individual firm entry/exit is to assume that the relative productivity and fixed cost ranks of firms are preserved while the cross-sectional productivity distribution $\varphi_{mv,t} \overset{\text{iid}}{\sim} \mathcal{P}\left(\left(\frac{\kappa-1}{\kappa}\right) A_t, \kappa\right)$ and fixed cost distribution $F_{m,t} \overset{\text{iid}}{\sim} \mathcal{P}\left(\left(\frac{\omega-1}{\omega}\right) F_t, \omega\right)$ change over time, as A_t and F_t are time-varying and stochastic. In other words, we can assume that $\frac{\varphi_{mv,t}}{A_t}$ and $\frac{F_{m,t}}{F_t}$ are both constants over time for any firm (m, v). This rank preservation interpretation resembles a well-known result in the literature that an individual firm (plant) productivity is strongly persistent along the business cycle (Lee and Mukoyama, 2015b; Dong et al., 2025).[15] Then, a higher-ranked firm (i.e., higher-ranked in productivity and lower-ranked in fixed cost) is more likely to operate under any realization of shocks.

Under the rank preservation interpretation, changes in the measure of operating firms, $M_t - M_{t-1}$, represent the net entry/exit of firms in period t, as firms with higher ranks are always more likely to operate than lower-ranked firms, while M_{t-1} can be interpreted as a measure of incumbent firms. Throughout the paper, we use the term "entry" as a shorthand for endogenous firm participation, while adopting more precise language when necessary to avoid ambiguity.

Although our theory is primarily formulated in terms of endogenous firm participation per period, the model admits a broader interpretation. For instance, firm participation may also be understood as firms adjusting their business lines—expanding or contracting—in response to changes in the economic environment and monetary policy rates.

[15] Dong et al. (2025), for example, assume that the productivity process of individual firms is given by:

$$z_{j,t+1} = \begin{cases} z_{j,t} & \text{with prob } \rho_t \\ \tilde{z}_{j,t+1} & \text{with prob } (1-\rho_t), \end{cases}$$

for any firm j, i.e., productivity in period $t+1$ could stay the same as in period t, or be re-drawn from the Pareto distribution. Our specification is consistent with this individual productivity process for different levels of ρ_t: $\rho_t = 1$, i.e., fully persistent productivity process, corresponds to our rank preservation interpretation, while $\rho_t = 0$ coincides with new random draws. Actually, their estimate of $1 - \rho_t$ from 1965 to 2015 is 0.047 for U.S. firms, consistent with our rank-preservation interpretation. Our results of aggregate business cycles variables do not depend on ρ_t levels, although different ρ_t levels can potentially have reallocation effects on the aggregate economy, with additional capital and labor adjustment costs and productivity-specific financial constraints on the firm side, which is beyond the scope of our paper.

Average Productivity: The average productivity of operating firms, $A_{p,t}$, is easily calculated as[16]

$$\begin{aligned} A_{p,t} &= \frac{1}{M_t}\int_0^1 \int_{v\in\Omega_{m,t}} \varphi_{mv,t}\,\mathrm{d}v\,\mathrm{d}m \\ &= A_t \cdot \frac{\omega}{\varrho+\omega}\cdot\frac{1}{1-\Theta_M}\cdot\frac{1+(\kappa-1)\cdot\frac{\alpha+\sigma(1-\alpha)}{\omega(\sigma-1)}H\left(F_{t-1}^*\right)}{1+\kappa\cdot\frac{\alpha+\sigma(1-\alpha)}{\omega(\sigma-1)}H\left(F_{t-1}^*\right)}, \end{aligned} \tag{23}$$

with $\varrho = (\kappa-1)\frac{\alpha+\sigma(1-\alpha)}{\sigma-1}$. We can see from (23) that the average productivity $A_{p,t}$ decreases with $H\left(F_{t-1}^*\right)$, which increases with positive demand shocks (e.g., monetary accommodation). This is well-documented in the literature, including Lee and Mukoyama (2015a) and Colciago and Silvestrini (2022).

Note that Lee and Mukoyama (2015a) uncover that the differences in productivity between booms (lower) and recessions (higher) are larger for entering plants than for exiting plants. Under our interpretation that the relative productivity and fixed cost ranks of firms are preserved when the cross-sectional distributions change over time, our model can easily capture this phenomenon, as entering firms are less productive than incumbents and their participation largely depends on the realization of demand shocks.[17]

Uniform Interest Rate Assumption In our framework, firms that operate in period t pay a pre-determined in-kind fixed cost $F_{m,t-1}$ one period in advance by receiving loans, whose rate is equal to the policy rate R_{t-1}^J. In reality, firms face different effective rates depending on levels of risk premium, credit risks, and in general, financial (credit) constraints. Even if our assumption of uniform interest rates allows our model to be tractable, there is a way to account for heterogeneous loan rates across firms, which *amplify* our main supply effect.

First, monetary accommodation reduces market risk premium (Drechsler et al., 2018; Kekre and Lenel, 2022), which in turn reduces the rate at which unproductive, riskier firms borrow from the financial market, inducing those firms to finance the fixed costs and operate in the next period. Second, it reduces credit risks and a component of risk premium related

[16] Online Appendix H derives equation (23) and offers impulse response functions of $A_{p,t}$ in response to different kinds of shocks, confirming our intuition.

[17] Colciago and Silvestrini (2022) focus on changes in average firm productivity under monetary policy shocks. Our formula (23) moves beyond monetary policy, and all the effect on average productivity is summarized by $H\left(F_{t-1}^*\right)$. Hamano and Zanetti (2022) study a similar replacement effect where contractionary monetary policy cleanses unproductive firms. However, in Hamano and Zanetti (2022), the lack of competition among incumbents leads to a lower average productivity eventually.

to credit (Palazzo and Yamarthy, 2022), allowing unproductive firms with high credit risks to operate.[18] In a situation where less productive firms, whose potential profits are smaller, face tighter credit constraints (Manova, 2013; Drechsel, 2023),[19] monetary accommodation can help those firms access the debt market and finance necessary fixed costs.

Therefore, including risk premium, credit risks, and financial constraints in our model will potentially amplify the supply channel of monetary policy and other demand shocks. We leave its exact microfoundation for future research.

2.3 Shock Processes

The average fixed cost F_t is modeled as follows:

$$F_t = \phi_f \cdot \bar{Y}_t \cdot \exp(u_{f,t}) = \phi_f \cdot \frac{Y}{A} \cdot A_t \cdot \exp(u_{f,t}) \ , \tag{24}$$

where $u_{f,t} = \rho_f \cdot u_{f,t-1} + \varepsilon_{f,t}$ and $\varepsilon_{f,t}$ is normally distributed with mean 0 and variance σ_f^2. Here, $\frac{Y}{A}$ is the steady-state output level adjusted for technology, and $\bar{Y}_t = \frac{Y}{A} \cdot A_t$ represents the balanced-growth path output.[20]

For technological progress, the model adopts:

$$GA_t \equiv \frac{A_{t+1}}{A_t} = (1+\mu) \cdot \exp\{u_{a,t}\} \ ,$$

where $u_{a,t} = \rho_a \cdot u_{a,t-1} + \varepsilon_{a,t}$, and $\varepsilon_{a,t}$ is normally distributed with mean 0 and variance σ_a^2.

Additionally, government expenditure G_t is formulated as:

$$G_t = \phi_g \cdot Y_t \cdot \exp(u_{g,t}) \ , \tag{25}$$

where $u_{g,t} = \rho_g \cdot u_{g,t-1} + \varepsilon_{g,t}$, and $\varepsilon_{g,t}$ is normally distributed with mean 0 and variance σ_g^2. It is assumed that the government maintains fiscal balance, levying a lump-sum tax, i.e., $T_{g,t} = G_t$ on the representative household each period.[21]

[18] Palazzo and Yamarthy (2022) find that a hike in interest rate raises the expected loss component of CDS spreads and a related risk premium component.

[19] Drechsel (2023) provides evidence of a connection between firms' earnings and their access to debt, and studies macroeconomic implications of those earnings-based borrowing constraints. See Lian and Ma (2021) for empirical evidence on earnings-based corporate credit.

[20] We assume that F_t scales with balanced-growth-path output $\bar{Y}_t$, not the contemporaneous output Y_t. In practice, this assumption has minimal quantitative impact. Note that F_t is procyclical, which aligns with Lee and Mukoyama (2018).

[21] Considering a zero net supply of government bonds, the government's budget constraint is upheld.

2.4 Central Bank

We assume that the central bank follows a Taylor rule for interest rate determination. The formal representation of this rule is given by:

$$R_t^J = R^J \cdot \left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\tau_\pi} \left(\frac{Y_t}{\bar{Y}_t}\right)^{\tau_y} \cdot \exp\{\varepsilon_{r,t}\} ,$$

where $\varepsilon_{r,t}$ is a normally distributed idiosyncratic monetary policy shock with mean 0 and variance σ_r^2. The variable $\bar{Y}_t$ denotes the balanced-growth path output level, and $\bar{\Pi}$ indicates the steady-state trend inflation rate. Financial markets are competitive, and note that both households and firms face R_t^J in allocating their consumption intertemporally and paying fixed costs, respectively.

2.5 Aggregation

Here, we aggregate the equations presented in Section 2.2 to obtain the economy-wide conditions. Consider first the aggregate labor demand N_t, given by

$$N_t = \Theta_N \cdot \left(\frac{Y_t \Delta_t}{A_t}\right)^{\frac{1}{\alpha}} \cdot (1 + \Theta_4 H_{t-1})^{-\frac{\alpha+\sigma(1-\alpha)}{(\sigma-1)\alpha}} , \tag{26}$$

where $H_{t-1} \equiv H(F_{t-1}^*)$ for simplicity, and

$$\begin{aligned}\Theta_N = &\left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha}\right)^{\left(\frac{-\sigma}{\alpha+\sigma(1-\alpha)}\right)} \left(\frac{\kappa-1}{\kappa}\right)^{\left(\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}\right)} \left(\frac{\kappa[\alpha+\sigma(1-\alpha)]}{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}\right) \\ &\cdot \left(\frac{\omega(\sigma-1)}{\kappa[\alpha+\sigma(1-\alpha)]+(\omega-1)(\sigma-1)}\right) \Theta_3^{\left(\frac{\sigma}{\alpha(\sigma-1))}\right)} > 0 .\end{aligned} \tag{27}$$

From equation (26), it becomes evident that aggregate labor demand, N_t, is positively correlated with the demand for upstream varieties, denoted by J_t. Conversely, the demand for labor decreases as the satiation measure, H_{t-1}, rises. An increase in H_{t-1} results in a higher aggregate measure of operating firms, M_t, as indicated in equation (20). This increase consequently stimulates employment through new entrants on the extensive margin. However, this surge in market entry also exerts downward pressure on the relative input price, $\frac{P_t^J}{P_t}$, and dampens the individual labor demand of existing firms, $N_{mv,t}$, due to intensified competition. In practice, the latter effect dominates and the reduction in labor demand at the intensive margin outweighs the increase at the extensive margin induced by new market entrants, provided that J_t is held constant.

The real wage, based on the household's intratemporal optimization condition in equation (2) and equation (26), is given by

$$\frac{W_t}{P_t A_t} = \Theta_N^{\frac{1}{\eta}} \left(\frac{C_t}{A_t}\right) \left(\frac{Y_t \Delta_t}{A_t}\right)^{\frac{1}{\eta\alpha}} (1 + \Theta_4 H_{t-1})^{-\frac{\alpha+\sigma(1-\alpha)}{\eta(\sigma-1)\alpha}} \cdot \exp\{-u_{c,t}\} \ . \tag{28}$$

Substituting equation (28) into equation (19) yields:

$$\frac{P_t^J}{P_t} = \Theta_N^{\frac{1}{\eta}} \Theta_3^{\frac{\alpha+\sigma(1-\alpha)}{(\sigma-1)\alpha}} \left(\frac{C_t}{A_t}\right) \left(\frac{Y_t \Delta_t}{A_t}\right)^{\left(\frac{(1-\alpha)\eta+1}{\eta\alpha}\right)} (1 + \Theta_4 H_{t-1})^{-\frac{(1+\eta)[\alpha+\sigma(1-\alpha)]}{\eta(\sigma-1)\alpha}} \cdot \exp\{-u_{c,t}\} \ . \tag{29}$$

Equations (26), (28), and (29) confirm that, given fixed aggregate demand measures such as C_t and J_t, an increase in H_{t-1} results in a reduction of both individual and aggregate labor demand. Consequently, this drives down the equilibrium wage. Hence, an increase in the entry of upstream firms exerts a deflationary impact on the economy, signaling a positive shift in aggregate supply.

Market Clearing: Market clearing in this economy is given by

$$C_t + \frac{L_t}{P_t} + G_t = Y_t \ , \tag{30}$$

which, in conjunction with equations (21), (24), and (25), is equivalent to:

$$\frac{C_t}{Y_t} = 1 - \phi_g \cdot \exp\{u_{g,t}\} - \phi_f \cdot \left(\frac{\tilde{Y}_t}{\tilde{Y}}\right)^{-1} \cdot \left[1 - \Theta_L \cdot [1 - H_t]^{\left(\frac{\omega-1}{\omega}\right)}\right] \cdot \exp\{u_{f,t}\} \ . \tag{31}$$

Notice that real loan demand is on the left-hand side of (30). When upstream firms opt to operate in the next period, they secure loans and pay for in-kind fixed costs in terms of the final good. This raises aggregate demand as well as inflation as shown in (28) and (29): in those equations, stronger aggregate demand translates to inflation.[22]

Consequently, the entry of upstream firms into the market has the dual effect of shifting

[22] A Keynesian cross structure becomes evident in (30) when endogenous entry of upstream firms is considered. As output Y_t expands, the measure of operating upstream firms, M_t, along with their loan demand, $\frac{L_t}{P_t}$, rises, thus generating successive increments in demand.

both the aggregate supply and demand curves.[23] Depending on the relative magnitudes of these shifts, market entry can exhibit either inflationary or deflationary tendencies. Section 4 will elaborate on the economy's short-run responses to demand and supply shocks within this framework, underscoring the inherent linkage between the two.

Average SB and Satiation: Upon substituting equation (A.24) in Appendix A into (22), we obtain an expression for the average SB rate defined in Proposition 3:

$$R_t^{J,*} = \left(\frac{\omega}{\omega+1}\right) \cdot (1-H_t)^{-\frac{1}{\omega}} \cdot R_t^J \ . \tag{32}$$

This expression allows us to interpret the "policy room", denoted as $\frac{R_t^J}{R_t^{J,*}}$, as a decreasing function of the satiation measure H_t.

Corollary 1 re-expresses the policy room $\frac{R_t^J}{R_t^{J,*}}$ as a *sufficient statistic* for the aggregate participation rate of firms, M_{t+1}. Importantly, a wider policy room level (i.e., higher $\frac{R_t^J}{R_t^{J,*}}$) amplifies the impact of monetary easing on the participation of upstream firms.[24] This rests on the following straightforward logic: a higher policy rate R_t^J compared to the average SB, $R_t^{J,*}$, enlarges the scope for additional, relatively inefficient firms to operate in the market as the policy rate declines.[25] Note from equation (32) above that

$$\frac{R_t^J}{R_t^{J,*}} \leq \frac{\omega+1}{\omega} \ . \tag{33}$$

Corollary 1 *The total measure of upstream firms opting to operate in period $t+1$ is:*

$$M_{t+1} = 1 - \Theta_M \cdot \left[\left(\frac{\omega}{\omega+1}\right) \cdot \frac{R_t^J}{R_t^{J,*}}\right]^{\omega} \ , \tag{34}$$

and a decrease in the policy room $\frac{R_t^J}{R_t^{J,*}}$ generates a larger increment in M_{t+1} when starting from a higher initial policy room level.

[23] Guerrieri et al. (2023) focus on a related economy in which a sectoral supply shock—such as the closing of high-contact sectors due to Covid-19— becomes Keynesian, triggering a more substantial shift in aggregate demand than in supply, especially in multi-sector economies with incomplete markets.

[24] This is consistent with the concave and decreasing function M_{t+1} in relation to the policy rate, R_t^J, as seen in (34).

[25] This pertains to scenarios where the fixed cost cutoff F_t^* is low, thus allowing middle-range fixed cost firms with suboptimal productivity to enter the market.

Proof. Directly from equation (34), we find:

$$\frac{dM_{t+1}}{d\left(\frac{R_t^J}{R_t^{J,*}}\right)} = -\omega\Theta_M \left[\left(\frac{\omega}{\omega+1}\right)\cdot\frac{R_t^J}{R_t^{J,*}}\right]^{\omega-1}\cdot\frac{\omega}{\omega+1} < 0\ ,$$

whose absolute magnitude is increasing in the level of $\frac{R_t^J}{R_t^{J,*}}$, given $\omega > 1$. ■

Flexible Price Model: Under flexible prices, the price of consumption varieties produced by downstream firms exhibits a constant markup over the cost of upstream inputs. Mathematically, this relationship is expressed as:

$$\frac{P_t}{P_t^J} = \frac{(1+\zeta^T)^{-1}\gamma}{\gamma-1}\ . \tag{35}$$

This establishes that the flexible price equilibrium is money-neutral, signifying that the policy rate R^J exerts no influence on the real allocation of resources. Additional equilibrium conditions are provided in Appendix A.

2.6 Summary Equilibrium Conditions

For analytical tractability, balanced growth path-adjusted variables are denoted with a tilde, for example, $\tilde{Y}_t \equiv \frac{Y_t}{A_t}$. In our simulation results, we assume the government implements optimal transfers to neutralize real distortions arising from monopolistic competition. Specifically, this involves setting $\zeta^T = \frac{1}{\gamma-1}$ and $\zeta^J = \frac{1}{\sigma-1}$. A comprehensive list of equilibrium conditions is provided in Appendix B.[26]

3 Steady State Results

3.1 Calibration and Estimation

The calibrated parameters are presented in Table 1. Our model incorporates two key factors influencing the operation of upstream firms in the market: fixed costs and productivity. These two variables follow their own independent Pareto distributions. The model is

[26] These production subsidies for upstream (i.e., ζ^J) and downstream (i.e., ζ^T) firms are widely used in the New Keynesian literature with monopolistic competition (Woodford, 2003). Our quantitative and qualitative results including monetary policy transmission do not depend on these subsidies.

designed such that the proportion of operating upstream firms is sensitive to parameters associated with these Pareto distributions. Utilizing the calibrated parameters outlined in Table 1, our model effectively replicates the moments commonly targeted in the literature. Key steady-state values are displayed in Table 2.

Fixed Cost to Balanced-Growth-Path Output Ratio, ϕ_f: In Appendix C, we estimate the parameters $(\phi_f, \rho_f, \sigma_f)$ of equation (24), based on the available data on the number of establishments in the Quarterly Census of Employment and Wage (QCEW) database as a proxy for firm participation in the model.[27]

The estimated $\phi_f = 0.5547$, obtained by matching several of the model's steady-state conditions (see Appendix C.3), yields a 91% firm participation rate at the steady state (i.e., $M = 0.91$ in Table 2). This aligns closely with the numbers based on exit and entry rates of establishments, without directly targeting any of these moments. According to the Business Dynamics Statistics (BDS), the average annual exit and entry rates from 1977 to 2016 were 10.6% and 12.3%, respectively. [28]

Shape Parameters in Pareto Distributions, κ and ω: We select $\kappa = \omega = 3.4$ based on the work of Ghironi and Melitz (2005), who choose this shape parameter for the productivity distribution to align with the standard deviation of log U.S. plant sales, estimated at 1.67 by Bernard et al. (2003). In Appendix A.2, we provide a formula for our model-implied standard deviation of log revenues of upstream firms and compare with Bernard et al. (2003).

Elasticity of Substitution, γ and σ: We select $\gamma = 4.3$ for the elasticity of substitution in the downstream market, following Ghironi and Melitz (2005) where 30% mark-up of price over cost is documented.

We choose $\sigma = 3$ based on Jones (2011), who argue that the elasticity of substitution for upstream market products tend to be lower than in downstream markets. This number is also close to the number ($\sigma = 3.79$) used by Bernard et al. (2003), who calibrate the elasticity of substitution to align with U.S. plant-level and macro trade data. There, the value

[27] Appendix C offers an alternative estimation method based on the total number of employees from CES National Databases in the Bureau of Labor Statistics (BLS) as well. See Appendix C.1.

[28] The fixed cost in our model is regarded as a composite of capital and non-capital costs. In the literature, the capital-to-output cost ratio is estimated to be around 30%. According to Table 5 in Domowitz et al. (1988), the non-capital fixed cost-to-output ratio varies between 0.05 and 0.18 across industries. Summing these two components, our estimated $\phi_f = 0.5547$ is at the upper end of this range.

of $\sigma = 3.79$ is chosen to match the productivity and size advantages of U.S. exporters.[29] Our quantitative results turn out to be robust across different levels of σ around 3.

	Parameter Description	Value	Source
β	Discount factor	0.998	Standard.
η	Frisch labor supply elasticity	1	Standard.
γ	Elasticity of substitution (of downstream market)	4.3	From Ghironi and Melitz (2005): 30% mark-up of price over cost.
σ	Elasticity of substitution (of upstream market)	3	Lower elasticity of upstream market products as argued in Jones (2011).
α	labor share in the upstream production function	0.6	Standard.
θ	Calvo (1983) price stickiness	0.75	Standard.
κ	Shape parameter: Pareto distribution of productivity	3.4	Ghironi and Melitz (2005).
ω	Shape parameter: Pareto distribution of fixed cost	3.4	Keep it the same with the productivity distribution.
ϕ_f	Fixed cost - steady state output ratio	0.5547	**Estimated** (Appendix C.3)
ϕ_g	Government spending - output ratio	18%	Smets and Wouters (2007).
τ_π	Taylor parameter (inflation)	1.5	Standard.
τ_y	Taylor parameter (output)	0.15	Standard.
μ	Long-run TFP growth rate	0.005	Match a yearly growth rate at 2%.
Π	Long-run inflation	1.02	Long-run inflation target at 2%.
ρ_a	Autoregression for TFP	0.7071	Excess TFP growth process' half-life of two quarters.
ρ_c	Autoregression for demand shock	0.98	The autocorrelation of the preference shock that affects the marginal utility of consumption estimated by Nakajima (2005).
ρ_g	Autoregression for government spending	0.87	Schmitt-Grohé and Uribe (2007).

[29] Several studies, including Ghironi and Melitz (2005), Bilbiie et al. (2012), and Fasani et al. (2023), also adopt the elasticity of substitution around this number, following Bernard et al. (2003).

ρ_f	Autoregression for fixed cost	0.9011	**Estimated** (Appendix C.3).
σ_a	SD for ϵ_a	0.0064	Schmitt-Grohé and Uribe (2007).
σ_c	SD for ϵ_c	0.017	The standard deviation of the preference shock estimated by Nakajima (2005) using U.S. data on consumption, labor, and output is 0.017.
σ_g	SD for ϵ_g	0.016	Schmitt-Grohé and Uribe (2007).
σ_f	SD for ϵ_f	0.0013	**Estimated** (Appendix C.3).
σ_r	SD for ϵ_r	0.0025	25 basis points, following Fed practices.

Table 1: Calibrated parameters.

Variable	Value	Description
H	0.82	Mass of productivity-irrelevant firms.
M	0.91	Mass of firms operating in the market.
R^J	1.012	Gross risk-free rate.
$R^{J,*}$	1.296	Gross satiation rate.
$\tilde{F}^*$	0.72	Cutoff fixed cost-to-output ratio.
Δ	1.0007	Price dispersion.
$\frac{W_t}{P_t A_t}$	0.51	Real wage.
$\frac{C_t}{Y_t}$	0.36	Consumption-to-output ratio.
$\frac{W_t N_t}{P_t Y_t}$	0.6	Labor cost-to-output ratio.
$\frac{L_t/P_t}{Y_t}$	0.46	Loan-to-output ratio.

Table 2: Steady state values.

3.2 Comparative Statics

In this section, we conduct comparative statics analyses on the steady-state equilibrium under varying parameter calibrations. This will illustrate the relationship between individual parameters and the internal mechanics of the model.

Fraction of Operating Upstream Firms: The steady-state proportion of active upstream firms, denoted as M, is described by $1-\Theta_M[1-H]$, as derived from equation (20). Figure 1 visualizes how M responds to shifts in model parameters: κ, ω, ϕ_f, β, μ, and Π. We

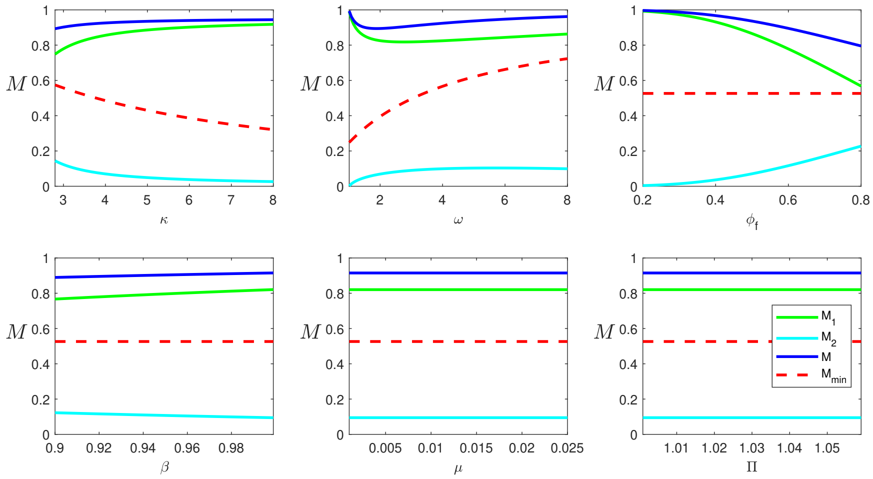


Figure 1: Comparative Statics: M.

Notes: Benchmark parameters are fixed as listed in Table 1. Ranges for κ, ω, ϕ_f, β, μ, and Π are $[2.8, 8]$, $[1.01, 8]$, $[0.2, 0.8]$, $[0.9, 0.999]$, $[0.001, 0.025]$, and $[1.001, 1.0709]$, respectively. The red dashed line marks the minimum mass of active firms, $M_{\text{min}} = 1 - \Theta_M$, attained when no firm is satiated, $H_t = 0$. We partition M into productivity-irrelevant M_1 and jointly determined M_2 components for various parameter values.

decompose M as follows:

$$
\begin{aligned}
M &= Prob(F < F^*) + \cancel{Prob(F > F^*)} \int_{F^*}^{\infty} \left(\frac{F_m}{F^*}\right)^{-\frac{\kappa[\alpha+\sigma(1-\alpha)]}{\sigma-1}} \frac{dH(F_m)}{\cancel{1 - H(F^*)}} \\
&= \underbrace{H(F^*)}_{\equiv M_1} + \underbrace{\frac{\omega(\sigma-1)}{\kappa[\alpha+\sigma(1-\alpha)] + \omega(\sigma-1)}(1 - H(F^*))}_{\equiv M_2} .
\end{aligned}
$$

Here, $M_1 = H(F^*)$ represents the mass of firms with sufficiently low fixed costs ($F_{m,t} \leq F^*$) to remain active irrespective of their productivity. M_2 comprises firms that are operational but not at the lowest fixed-cost tier; these firms do not operate if they draw a low productivity level.

The following key points can be drawn from Figure 1: (i) An increase in κ raises both M_1 and M by narrowing the productivity distribution around its mean, thereby raising the lower bound of productivity and the likelihood of satiation for any given fixed cost; (ii) An increase in ω manifests via two opposing effects on firm participation, M. On one hand, it raises the minimum fixed cost $\frac{\omega-1}{\omega}F$, thereby reducing M. On the other hand, it narrows the fixed-cost distribution around its mean F, potentially reducing the mass of high fixed-cost firms and increasing M. The total effect on M depends on the relative magnitudes of these two forces. Moreover, the satiation measure M_1 typically declines as ω increases due to an increased lower bound on fixed costs, $\frac{\omega-1}{\omega}F$, affecting firms that are typically satiated. These characteristics relating ω and M are further elaborated in Figure D.6 in Appendix D, which explores the effect of other parameters on the functional relationship between M and each parameter; (iii) An increment in ϕ_f shifts the fixed-cost distribution to the right, thereby reducing both M and M_1.

Following from equation (34), it is evident that the policy room $\frac{R^J}{R^{J,*}}$ maintains an inverse relationship with M. Variations in the parameters will produce effects on the policy room that are opposite to their impacts on M, as documented in Figure D.7 in Appendix D.

The Real Loan-to-Output Ratio: At the steady state, the following inequality is derived from equations (21) and (33):

$$\phi_f\,(1-\Theta_L) \leq \frac{L/P}{\bar{Y}} = \phi_f\left[1-\Theta_L(1-H(F^*))^{\frac{\omega-1}{\omega}}\right] = \phi_f\left[1-\Theta_L\left(\frac{\omega}{\omega+1}\frac{R^J}{R^{J,*}}\right)^{\omega-1}\right] \leq \phi_f \ ,$$

where the real loan-to-output ratio, $\frac{L/P}{\bar{Y}}$, is a decreasing function of the policy room $\frac{R^J}{R^{J,*}}$, but increasing with respect to the satiation measure $H(F^*)$, and total firm participation, M.[30]

Figure 2 describes how $\frac{L/P}{\bar{Y}}$ varies with key model parameters: κ, ω, ϕ_f, β, μ, and Π. Our observations can be summarized as follows: (i) An increase in κ raises firm participation M, as illustrated in Figure 1, and narrows the policy room $\frac{R^J}{R^{J,*}}$, as seen in equation (34) and Figure D.7, resulting on a higher aggregate loan demand; (ii) An increase in ω gives rise to conflicting outcomes: it initially depresses firm participation M when ω is below a certain threshold, which can be attributed to an increase in the minimum fixed cost of entry, $\frac{\omega-1}{\omega}F$, as seen in Figure 1. However, this negative extensive margin effect is eventually counterbalanced by a positive intensive margin effect, where each active firm incurs a greater fixed cost, hence raising the real loan-to-output ratio; (iii) An increase in ϕ_f results in a reduction of firm participation M, evident from Figure 1, thus reducing aggregate loan demand. As before, this decrease via the extensive margin is eventually neutralized by an increase via the intensive margin, where each active firm shoulders a higher fixed cost.[31] The dynamics between the policy room $\frac{R^J}{R^{J,*}}$ and the real loan-to-output ratio $\frac{L/P}{\bar{Y}}$ are cap-

[30] Note that M increases with H at the steady state as per equation (20).

[31] The functional relationship between $\frac{L/P}{\bar{Y}}$ and other parameters is further explored in Figure D.8.

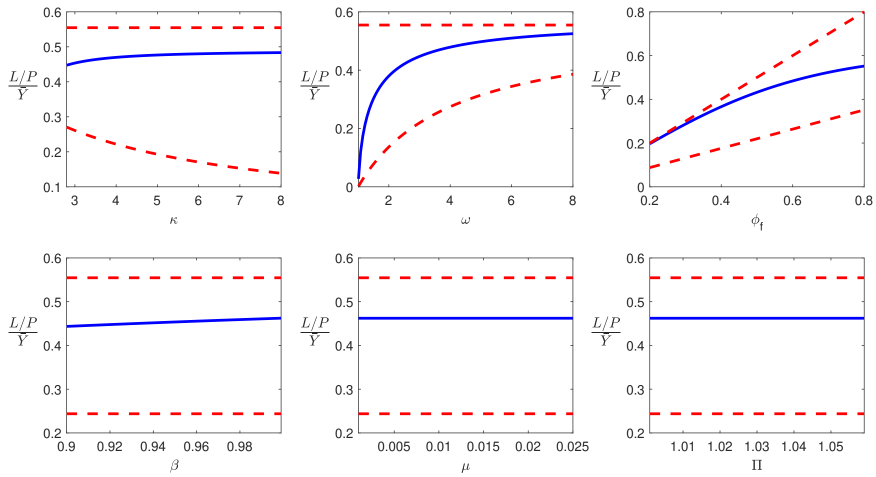


Figure 2: Comparative statistics: Output-scaled real lending.

Notes: The red-dashed lines indicate the upper and lower bounds for output-scaled lending, corresponding to ϕ_f and $\phi_f(1-\Theta_L)$, respectively.

tured in Figure 3. An increase in ϕ_f or ω lowers firm participation M and widens the policy room, $\frac{R^J}{R^{J,*}}$, with the net effect being an increase of aggregate loan issuance. In contrast, a rise in κ raises both M and $\frac{L/P}{Y}$, inducing a negative correlation with the policy room $\frac{R^J}{R^{J,*}}$.

4 Quantitative Analysis

4.1 Supply vs. Demand Shocks

Technology Shock Figure 4a shows how a positive technology shock, $u_{a,t}$, affects various variables in our model. Following the shock, a group of previously inactive firms enters the market, boosting aggregate firm participation M_t, the measure of productivity-insensitive entrants H_t, and aggregate loans $\frac{L_t}{P_t A_t}$.[32] As participating firms pay their fixed costs in units of the final consumption good, the increase in firm entry contributes to an expansion in aggregate demand, as detailed in equation (30). An uptick in market participation typically depresses the real price of inputs, $\frac{P_t^J}{P_t}$, due to heightened competition, as expressed in equation (29). Yet in this case, the rising aggregate demand effect domi-

[32]In Figures 4a and 4b, the percentage increase in the loan-to-output ratio, $\frac{L_t/P_t}{Y_t}$, equals $\frac{L_t}{P_t A_t}\frac{A}{Y}$, coming from a net rise in aggregate loan demand, $\frac{L_t}{P_t A_t}$. For small values of ϕ_f, changes in loan demand around the steady state are negligible.

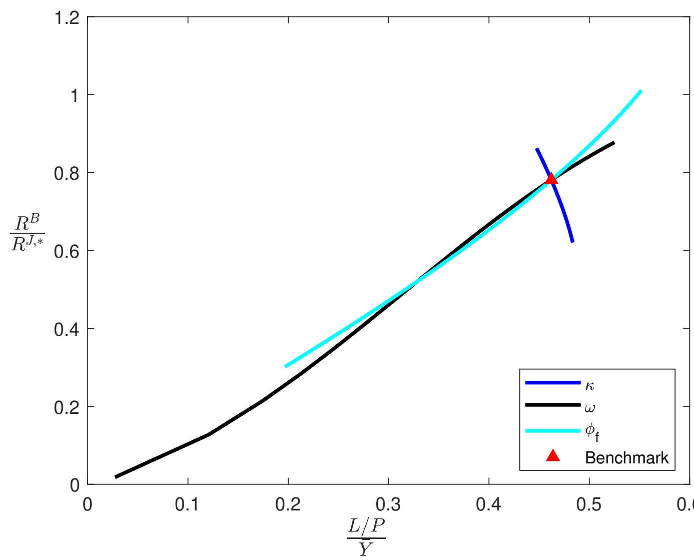


Figure 3: Policy power on output-scaled real lending.

Notes: This figure illustrates the co-movements between $\frac{R^J}{R^{J,*}}$ and $\frac{L/P}{Y}$ driven by variations in κ, ω, and ϕ_f. The solid triangular marker denotes the steady-state value under benchmark calibration.

nates, increasing real input prices along with labor demand N_t and real wages. This causes inflation Π_t and interest rates R_t^J to rise, thereby narrowing the policy room $\frac{R_t^J}{R_t^{J,*}}$.[33]

We also examine the technology shock's impact under varying levels of the fixed cost parameter, ϕ_f. Higher entry costs mean a greater steady-state prevalence of inactive firms, $1 - M$. In such conditions, a positive $u_{a,t}$ shock triggers substantial new firm entry and larger increases in M_t and H_t. The increase in aggregate demand brought by stronger entry is further amplified by the elevated fixed costs associated with a higher ϕ_f. Consequently, there's a sharper initial increase in loan demand, real input prices, wages, and labor demand, followed by a faster reversion to steady-state levels due to increased competition. In this setting, inflation Π_t shows a more moderate response due to larger shifts in firm entry.[34]

These dynamics align with the traditional AD-AS framework as follows: (i) a positive technology shock moves the supply curve rightward; (ii) it leads to an outward movement of the demand curve due to increased loan and labor demands, causing more firm entry and further shifts in the supply curve; and (iii) when entry costs are high, more inactive firms

[33] This result is consistent with the positive correlation between the policy room, $\frac{R_t^J}{R_t^{J,*}}$, and firm participation, M_t, outlined in equation (34)

[34] This observation is consistent with the findings of Cecioni (2010), who argue that greater firm entry can mitigate inflationary pressures in the U.S. economy.

enter the market after a positive supply shock. Consequently, both the aggregate supply and aggregate demand curves shift more extensively rightward, resulting in moderate inflation and increased output.

Demand Shock Figure 4b illustrates the effects of a consumption demand shock, $u_{c,t}$. The figure exhibits impulse responses that are qualitatively analogous to Figure 4a. Specifically, a positive shock to $u_{c,t}$ prompts an increase in firm entry that results in an expansion of the aggregate supply capacity of the economy.[35]

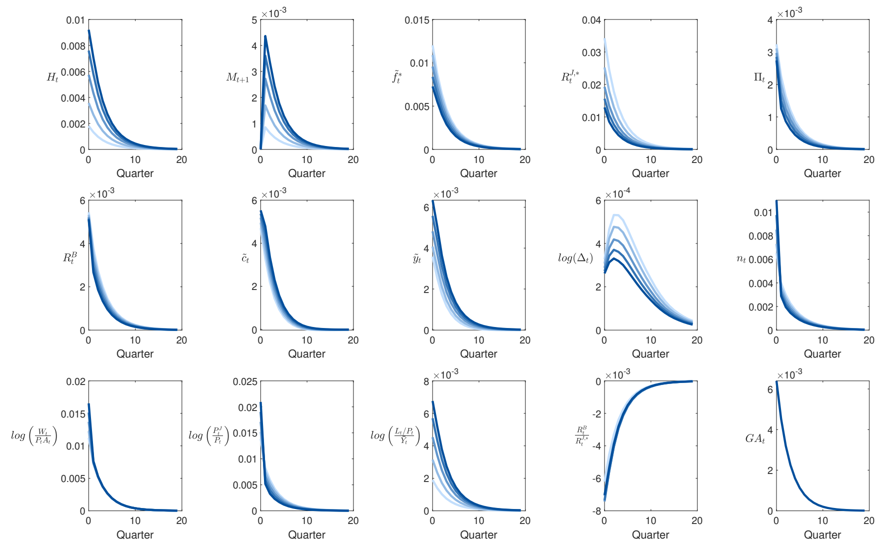
In summary, our model highlights the reciprocal relationship between supply and demand that exists as a result of endogenous firm entry. Accordingly, the initial origin of the shock —be it supply- or demand-driven— yields no qualitative distinctions in the behavior of the key variables within our model. Nonetheless, economies with a larger pool of potential new entrants generate stronger responses to shocks in the form of larger output and moderate inflation movements.

Shocks in Monetary and Fiscal Policies A monetary policy tightening shock generates an impulse response function akin to Figure 4b produced by a consumption demand shock. A rise in monetary policy rates lowers aggregate participation M_t, which in turn decrease loan demand, inflation, real wages, and production levels, as seen in Figure D.10. In Section 4.3, we study the monetary policy multiplier in greater detail.

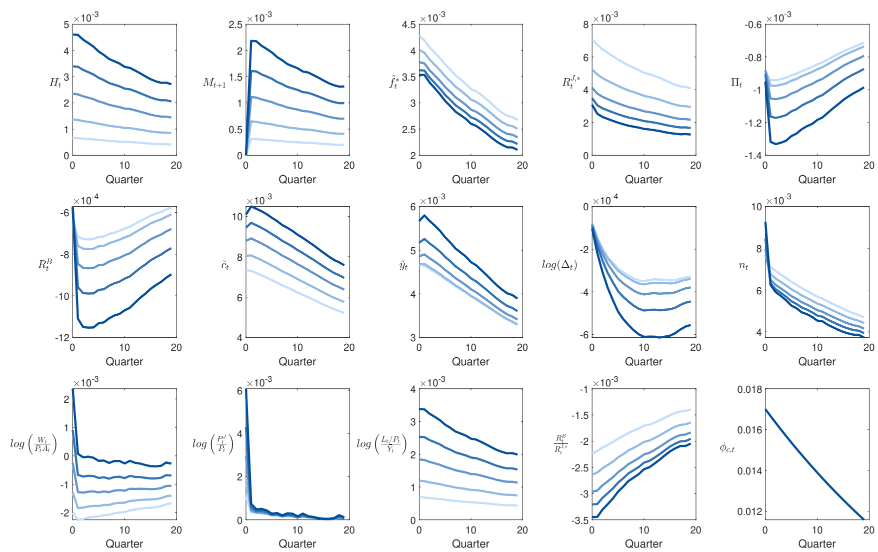
A positive government spending shock, depicted in Figure D.11, crowds out consumption via higher real interest rates while simultaneously reducing inflation through increased participation by upstream firms, as evidenced by rises in both M_t and H_t. The government spending multiplier is amplified under higher values of ϕ_f, which is attributable to stronger firm participation following the shock.

Fixed Cost Shock A positive fixed cost shock induces falls in the measure of active firms M_t and the satiation measure H_t, as depicted in Figure D.9. This decline is attributed to the elevated productivity cutoff $\varphi^*_{m,t}$, as specified in equation (14), which rises for each firm type m due to increased entry costs. This shock has dual, opposing impacts on aggregate demand: First, reduced firm participation diminishes fixed equipment demand at the extensive margin, thereby contracting aggregate demand. Second, the increased fixed costs

[35] Note one difference between Figures 4a and 4b: with our demand shocks, inflation drops due to stronger effects of additional firm entry on aggregate supply. Also, note that demand shocks are more persistent since $\rho_c >> \rho_a$ in our calibration.



(a) Impulse response functions to TFP shock.



(b) Impulse response functions to demand shock.

Notes: The figures display the impulse response functions to supply and demand shocks. Panel (a) shows the response to a one standard deviation shock (0.0064) in $u_{a,t}$, which increases the growth rate of the average productivity for upstream firms. Panel (b) shows the response to a one standard deviation shock (0.017) in $u_{c,t}$, the demand shock. The gradient blue lines denote the responses under calibrations with varying ϕ_f. From light blue to dark blue, ϕ_f values are 0.35, 0.45, 0.5547 (benchmark), 0.65, and 0.75. The following variables are plotted in deviations from their steady states: H, M, R^J, Π, and $R^{J,*}$. The remaining variables are plotted in log deviations from their steady states (in lower case letters or with a log). Δ represents the price dispersion for the downstream products.

boost demand from participating firms, thereby augmenting aggregate demand at the intensive margin. Under the model's benchmark calibration (i.e., $\phi_f = 0.5547$), the latter effect prevails, leading to a net expansion in output.[36] This subsequently results in an increase in equilibrium levels of labor demand, real wages, and inflation.

4.2 Intensive vs. Extensive Margin in Labor Adjustment

Note that changes in aggregate labor N_t as specified in equation (26) are attributable to two primary factors: (i) variations in each operating firm's labor demand, denoted $N_{mv,t}$, over time —referred to as intensive margin adjustment; and (ii) fluctuations in the number of active upstream firms M_t across business cycles —known as extensive margin adjustment. The aggregate labor N_t is formally expressed in equation (36) as:

$$N_t = \int_0^1 \int_{v \in \Omega_{m,t}} N_{mv,t} \, \mathrm{d}v \, \mathrm{d}m \; , \tag{36}$$

where the individual labor demand $N_{mv,t}$ derives from equation (A.16). We now proceed to consider an upstream firm (m, v) operating across two periods t and $t + \iota$, where $\iota \geq 1$. Utilizing equation (A.16), we define:

$$g^{\text{Density}}_{t,t+\iota} \equiv \frac{N_{mv,t+i} - N_{mv,t}}{N_{mv,t}} = \left[\frac{1 + \Theta_4 \cdot H_{t-1}}{1 + \Theta_4 \cdot H_{t+\iota-1}}\right]^{\frac{\sigma}{(\sigma-1)\alpha}} \left(\frac{\frac{Y_{t+\iota}\Delta_{t+\iota}}{A_{t+\iota}}}{\frac{Y_t\Delta_t}{A_t}}\right)^{\frac{1}{\alpha}} - 1 \; , \tag{37}$$

which represents the percentage change between periods t and $t + \iota$ in an individual firm (m, v)'s labor demand $N_{mv,t}$, contingent upon the firm's operation in both periods. Importantly, $g^{\text{Density}}_{t,t+\iota}$ is solely a function of aggregate variables, independent of the indices (m, v). We term $g^{\text{Density}}_{t,t+\iota}$ as the "intensive margin" adjustment in labor demand.

From equation (26), we can derive an expression for the percentage change in aggregate labor, N_t, denoted as $g^N_{t,t+\iota}$[37]:

$$g^N_{t,t+\iota} \equiv \frac{N_{t+\iota} - N_t}{N_t} = g^{\text{Density}}_{t,t+\iota} + (1 + g^{\text{Density}}_{t,t+\iota}) \cdot g^{\text{Entry}}_{t,t+\iota} \; , \tag{38}$$

[36] For higher levels of ϕ_f, e.g., $\phi_f = 0.75$ leads to a net decrease in output, as seen in Figure D.9.

[37] The derivation is provided in Appendix A.

where $g^{\text{Density}}_{t,t+\iota}$ is defined as in equation (37) and $g^{\text{Entry}}_{t,t+\iota}$ is given by

$$g^{\text{Entry}}_{t,t+\iota} = \frac{(H_{t+\iota-1} - H_{t-1}) + \frac{\omega(\sigma-1)}{\kappa[\alpha+\sigma(1-\alpha)]+(\omega-1)(\sigma-1)}(H_{t-1} - H_{t+\iota-1})}{H_{t-1} + \frac{\omega(\sigma-1)}{\kappa[\alpha+\sigma(1-\alpha)]+(\omega-1)(\sigma-1)}(1 - H_{t-1})} \,. \tag{39}$$

We interpret $g^{\text{Entry}}_{t,t+\iota}$ as the extensive margin adjustment in labor, triggered by changes in firm entry. According to equation (38), the total percentage change in aggregate labor comprises both intensive and extensive margin adjustments.

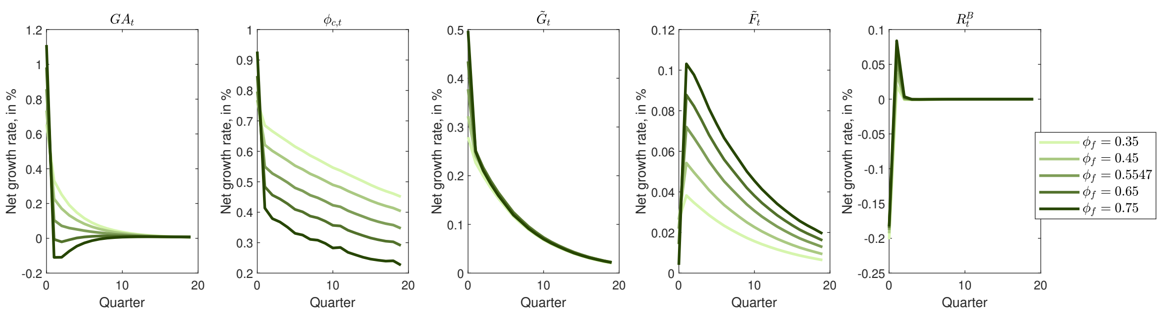


Figure 5: Decomposition of labor growth rate under different shocks: isolines on intensive margin.

Notes: Figures illustrate employment growth rate relative to pre-shock employment level. Gradient green lines indicate intensive margin responses with varying fixed cost parameter ϕ_f values. Growth rates are reported in net percentage terms.

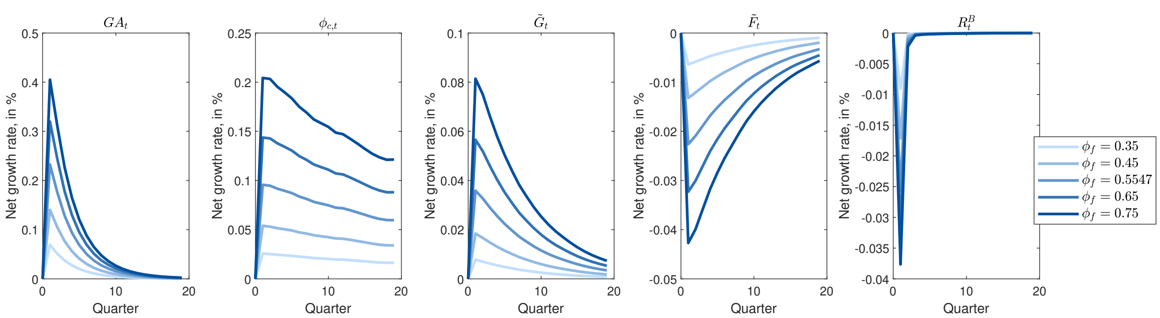


Figure 6: Decomposition of labor growth rate under different shocks: isolines on extensive margin.

Notes: Figures illustrate employment growth rate relative to pre-shock employment level. Gradient blue lines indicate extensive margin responses with varying fixed cost parameter ϕ_f values. Growth rates are reported in net percentage terms.

Figures 5 and 6 portray how intensive and extensive margins' elements respond, respectively, to different shocks. For example, for a positive fixed cost shock $u_{f,t}$, we note that (i) a negative extensive margin adjustment due to the exit of less competitive firms, and (ii) an increase in per-firm labor demand corresponding to higher aggregate output, as evidenced in Figure D.9.

In contrast, a consumption demand shock $\phi_{c,t}$ generates positive adjustments on both margins due to increased market entry and aggregate output (see Figure 4b). The extensive margin effect becomes more salient under higher ϕ_f, while the intensive margin exhibits a non-monotonic behavior. Initially, individual firms require more workers, but as market competition intensifies, labor demand flattens, as corroborated by Figure D.9.

4.3 Multipliers and the Policy Room

We now examine the influence of "initial policy room levels" on the responses of aggregate variables to shocks, commonly termed in the literature as shock multipliers. To obtain the value of multipliers outside the steady state, we simulate the model over a span of $T = 10,000$ periods, selecting 500 unique realizations denoted as $\mathbb{Y}^{\text{original}}$. For each selected realization, we extend the model dynamics up to $h = 4$ periods ahead based on two different scenarios: (i) a situation with no additional shock, which results in the original time series $\left\{\mathbb{Y}^{\text{original}}_{t+h}\right\}_{h=0}^{h=4}$; and, (ii) a case where the shock of interest is realized to be higher by a one standard deviation, giving rise to time series $\left\{\mathbb{Y}^{\text{shock}}_{t+h}\right\}_{h=0}^{h=4}$. The multiplier can subsequently be computed as $\frac{|\mathbb{Y}^{\text{shock}}_{t+h} - \mathbb{Y}^{\text{original}}_{t+h}|}{\sigma(\text{shock})}$ for horizons ranging from $h = 0$ to $h = 4$. Figure 7 plots the relationship between multipliers and initial policy room levels. The key findings are:

1. At $h = 0$, the multipliers for output and labor positively correlate with initial policy room levels. This effect is due to the higher rate of firm entry (which in turn raises equipment purchases) in response to a monetary shock when initial policy room is larger, consistent with Corollary 1. We will test this channel in the data shortly.

2. At $h = 1$, although the multipliers decline due to the shock's lack of persistence, the positive correlation with the initial policy room remains. This is explained by an increased number of firms in the market and an associated rise in supply.

3. At $h = 4$, multipliers approach zero, attributable to the lack of shock persistence.

In summary, the policy room serves as a sufficient statistic for equilibrium firm entry and is positively correlated with the multipliers for output, labor, and firm entry in response to

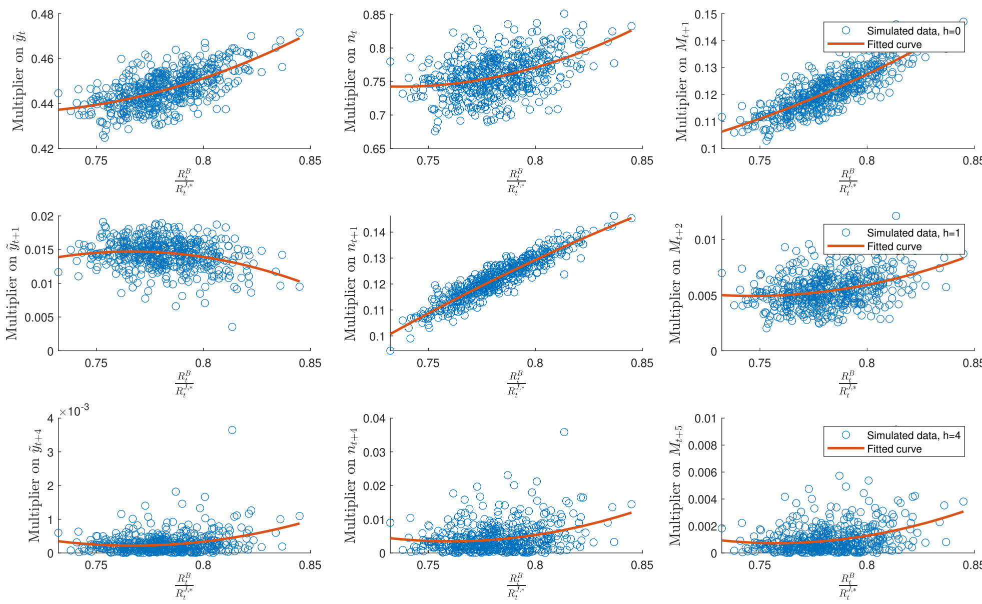


Figure 7: Scatter plot between policy room and monetary policy multipliers.

Notes: Figures plot the relationship between policy room and monetary policy multipliers on output (in logs), labor (in logs), and next period mass of operating firms (in levels). We consider the next period's mass of operating firms since the firms paying the fixed cost at t will operate on the market at $t+1$. Figures in the first to third rows display the contemporaneous multipliers ($h = 0$), multipliers after 1 quarter ($h = 1$), and multipliers after 4 quarters ($h = 4$) correspondingly. The blue circles represent the result from each simulation based on solutions from the third-order perturbation method. The red solid lines are fitted second-order polynomials.

monetary shocks. Further details can be found in Figures D.13 and D.14[38] in Appendix D, which relate closely to the discussion here.

5 Empirical Analysis

In this section, we empirically test the key implication from Corollary 1 and Section 4.3: a higher initial level of policy room enhances the efficacy of monetary policy, as it allows for a greater endogenous response in firm entry, affecting both aggregate demand and supply.

Appendix C provides two alternative methods to recover the policy room $\frac{R_t^J}{R_t^{J,*}}$ based on different model equilibrium conditions. These methods utilize data on labor markets (e.g., various employment measures) or firm participation (e.g., the number of active establishments). Here, we focus on the policy room derived from firm participation data (Version 2 in Appendix C.2).[39] Our benchmark local projection à la Jordà (2005) is specified as:

$$\begin{aligned}\tilde{y}_{t+h} = &\sum_{q=1}^{Q} \beta_{\tilde{y},q}^{(h)} \tilde{y}_{t-q} + \sum_{q=1}^{Q} \beta_{R,q}^{(h)} \left(\widehat{r_{t-q}^{J} - r_{t-q}^{J*}} \right) + \sum_{q=0}^{Q} \gamma_q^{(h)} \text{controls}_{t-q} \\ &+ \sum_{q=0}^{Q} \beta_{0,q}^{(h)} \epsilon_{t-q} + \sum_{q=0}^{Q} \beta_{0R,q}^{(h)} \epsilon_{t-q} \times \left(\widehat{r_{t-q-1}^{J} - r_{t-q-1}^{J*}} \right) + u_{t+h}^{(h)} , \end{aligned} \qquad \text{for } h = 0, \ldots, H,$$

with the following components:

1. Monetary policy shocks, ϵ_t: Acosta (2023)'s extension of Romer and Romer (2004) monetary policy shocks. In Appendix E, we present results based on Wieland and Yang (2020)'s extended series of Romer and Romer (2004).[40] Coefficient $\beta_{0,0}^{(h)}$ captures the impact of monetary policy shocks when the policy room is at its steady-state value.

2. Policy room measure, $\widehat{r_t^J - r_t^{J*}}$: Log-deviation of the policy room from the steady state, recovered using the data on the total number of establishments from the Quarterly Census of Employment and Wages (QCEW) in Appendices C.2 and C.4. We interact the monetary policy shock with the lagged policy room variable to measure

[38]Figure D.14 in Appendix D documents the relation between the policy room and the government spending multiplier, which is similar to the case of monetary policy in Figure 7.

[39]Results based on labor market variables (Version 1 in Appendix C.1) are also provided, with a comparison of the two approaches in Appendix E.

[40]These shocks are, by construction, exogenous.

the additional effectiveness of monetary policy shocks when the policy room widens, as captured by the coefficient $\beta^{(h)}_{0R,0}$.[41]

3. Controls and variables of interest: In the benchmark regression, we control for the lags of the dependent variable, the policy room, the monetary policy shock, and the interaction between the latter two. $\{\text{controls}_{t-q}\}_0^Q$ encompass any additional controls added to the regression, which, in our benchmark specification, include the current and four lags of the federal funds rates. For the dependent variable $\tilde{y}_t$, we use log consumption, log output, the unemployment rate (in percentage), and the log of the number of establishments (QCEW).

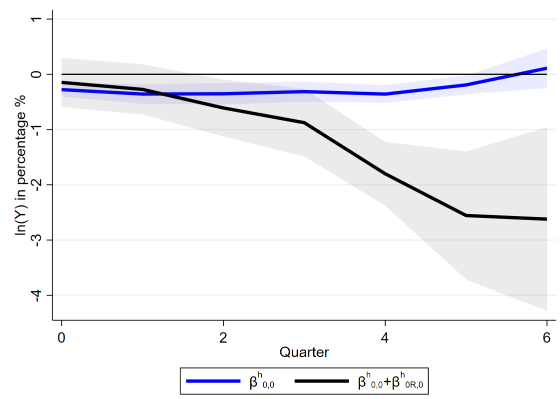
4. Number of lags Q: In our benchmark specification, we set the number of lags to four, corresponding to one year of lagged data at a quarterly frequency. In Appendix E, we discuss the robustness of our results across different lag lengths.

Figure 8 shows the impulse response functions of several variables under our benchmark regression specification. Consistent with our framework, a wider policy room significantly increases the potency of monetary policy shocks. Panel (a) demonstrates that an increase of one standard deviation (approximately 2 percentage points) in the log-policy room reduces output by up to an additional three percentage points in response to a one standard deviation (29 basis points) monetary tightening shock. Panel (d) provides further evidence supporting the model's implied interaction between supply and demand affecting the effectiveness of monetary policy shocks. When the economy suffers a monetary tightening shock under a wider policy room, firm participation tends to contract further within a period of one and a half years.

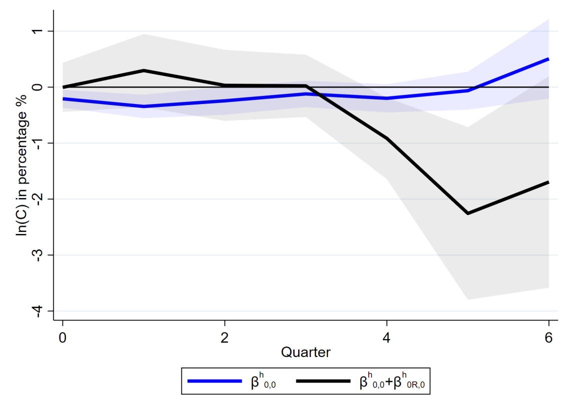
With Additional Controls We add more controls to the benchmark regression specification to test the robustness of our results. The additional controls include four lags of (i) the oil price growth rate, (ii) the long-term interest rate, (iii) the consumption growth rate, (iv) the GDP deflator, and (v) the shadow federal funds rate from Wu and Xia (2016). Figure 9 shows that the additional controls produce little to no difference in the impulse responses of the selected variables compared to the benchmark specification displayed in Figure 8.

Additional Robustness In Appendix E, we provide results based on a different method to recover the policy room $\widehat{r^J_t - r^{J*}_t}$, using employment data from the CES National Databases

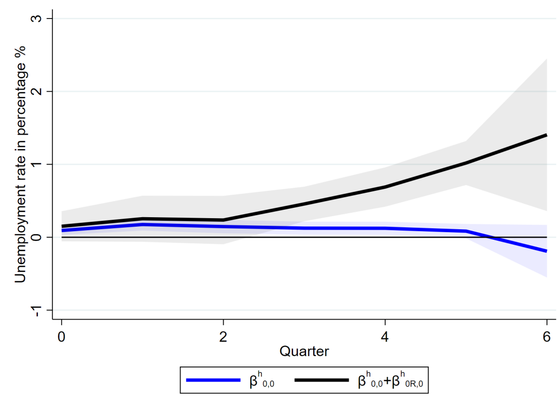
[41] We interact the monetary policy shocks with lagged policy room to avoid potential endogeneity issues.



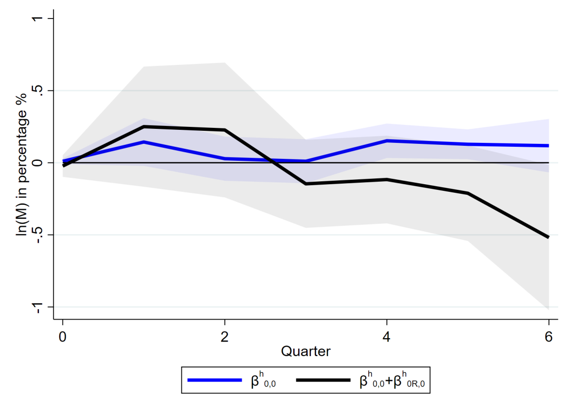
(a) log(Y), in % with Acosta (2023)



(b) log(C), in % with Acosta (2023)



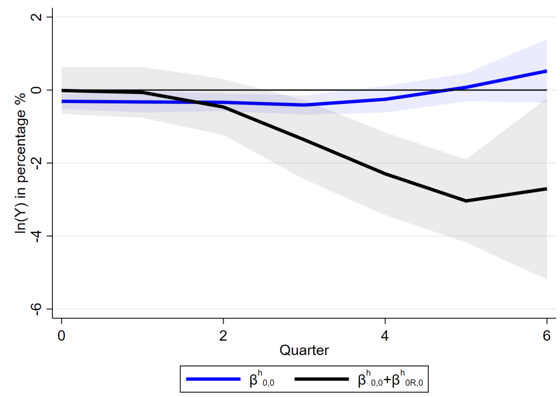
(c) Unemployment rate, in % with Acosta (2023)



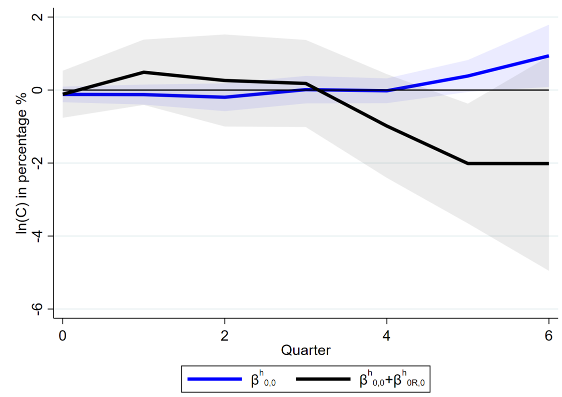
(d) log(M), in % with Acosta (2023)

Figure 8: Local projection with policy room from Version 2.

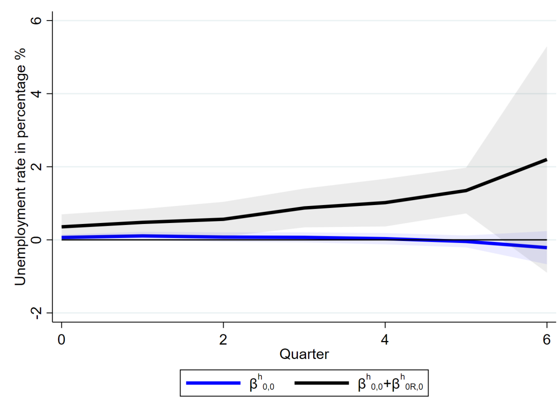
Notes: The impulse response functions are based on the benchmark regression specification, which controls for the current and four lags of the federal funds rate. The IRFs display the response to a one standard deviation (29 basis points) positive monetary policy shock and a one standard deviation (2 percentage points) increase in the log policy room, constructed based on the Version 2 measure of the variable, utilizing the number of establishments from the Quarterly Census of Employment and Wages (QCEW) dataset (see Appendix C.2). The figure reports the 95% confidence band, constructed using Newey-West standard errors under the assumption of serial correlation of the same lag length as the regression horizon, h.



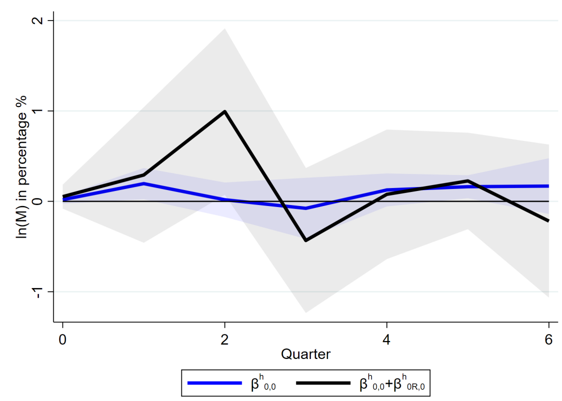
(a) log(Y), in % with Acosta (2023)



(b) log(C), in % with Acosta (2023)



(c) Unemployment rate, in % with Acosta (2023)



(d) log(M), in % with Acosta (2023)

Figure 9: Local projection with policy room from Version 2 and additional controls.

Notes: The impulse response functions extend the benchmark regression specification with the addition of the following controls: four lags of the oil price growth rate, four lags of the long-term interest rate, four lags of the consumption growth rate, four lags of the GDP deflator, and four lags of the shadow federal funds rate from Wu and Xia (2016). The figure reports the 95% confidence band, constructed using Newey-West standard errors under the assumption of serial correlation of the same lag length as the regression horizon, h.

in the Bureau of Labor Statistics (BLS) and monetary policy shocks from Wieland and Yang (2020). The results remain qualitatively similar to those in Figure 8.

6 Conclusion

In this paper, we develop a macroeconomic model with endogenous firm entry. Based on a dual-industry (i.e., upstream and downstream industries) model, we tractably embed the dynamics of endogenous firm operation within a New Keynesian model. In our model, upstream firms face stochastic fixed entry costs, denominated in the final consumption good. These firms are also constrained by cash-in-advance requirements and thus rely on capital markets for financing their fixed costs. Downstream firms, on the other hand, are subject to nominal rigidities. Our analysis reveals that demand shocks increase firm profitability and entry, thereby expanding the economy's aggregate supply. In turn, this increased participation stimulates additional demand for the final good, as firms seek to finance their entry via loans. This process initiates a self-reinforcing cycle, rendering the relationship between demand and supply non-separable under general circumstances. As a result, conventionally defined 'supply' (e.g., technology) and 'demand' (e.g., monetary policy) shocks can induce comparable patterns of business cycle co-movements. In particular, supply shifts, resulting from the entry of new firms, lead to disinflationary pressures alongside an increase in output.

Monetary policy becomes more effective in stimulating output as it affects both aggregate demand and the entry decisions of financially constrained firms, spurring additional shifts in aggregate supply. We identify a critical threshold of the policy rate for each entry fixed cost level, termed the Satiation Bound (SB). At this threshold, all firms with identical entry fixed costs fully engage in production, rendering monetary policy ineffective in further spurring economic growth through new firm participation. Based on this concept, we introduce a metric known as the "policy room", which represents the difference between the current policy rate and the average SB across firms. It turns out that there is a strong positive correlation between a room for additional firm entry, monetary policy efficacy, and the policy room: monetary policy becomes a more effective stabilizer under a higher policy room level, since there is a larger room for changes in the firm entry rate that affects aggregate supply.

While stylized and analytical in nature, our model provides a novel implication which we confirm with the data: when the policy room narrows, the extensive margin of monetary policy transmission wanes, resulting in lower output multipliers and reduced firm entry.

Competing interest statement

The authors have no competing interests to declare.

Declaration of generative AI and AI-assisted technologies in the writing process

During the preparation of this work the authors used ChatGPT in order to improve language and readability. After using this tool/service, the authors reviewed and edited the content as needed and take full responsibility for the content of the publication.

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Online Appendix: For Online Publication Only

A Derivation and Proofs

A.1 Detailed Derivation in Section 2.2

Derivation of equations (12) and (13) The price setting of a firm (m, v) is given by

$$P^J_{mv,t} = \left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha}\right) W_t \varphi_{mv,t}^{-\frac{1}{\alpha}} J_{mv,t}^{\frac{1-\alpha}{\alpha}} = \left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha}\right) W_t \varphi_{mv,t}^{-\frac{1}{\alpha}} \left[(P^J_{mv,t})^{-\sigma}\Gamma^J_t\right]^{\frac{1-\alpha}{\alpha}} ,$$

in which we can solve for $P^J_{mv,t}$ as

$$(P^J_{mv,t})^{\frac{\alpha+\sigma(1-\alpha)}{\alpha}} = \left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha}\right) W_t \varphi_{mv,t}^{-\frac{1}{\alpha}} (\Gamma^J_t)^{\frac{1-\alpha}{\alpha}} ,$$

from which we obtain

$$P^J_{mv,t} = \left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha}\right)^{\frac{\alpha}{\alpha+\sigma(1-\alpha)}} W_t^{\frac{\alpha}{\alpha+\sigma(1-\alpha)}} \varphi_{mv,t}^{-\frac{1}{\alpha+\sigma(1-\alpha)}} (\Gamma^J_t)^{\frac{(1-\alpha)}{\alpha+\sigma(1-\alpha)}} . \tag{A.1}$$

To get the revenue function $r_{mv,t}$, we start from

$$P^J_{mv,t} J_{mv,t} = \left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha}\right) W_t \varphi_{mv,t}^{-\frac{1}{\alpha}} J_{mv,t}^{\frac{1}{\alpha}} ,$$

which leads to

$$r_{mv,t} = (1+\zeta^J) P^J_{mv,t} J_{mv,t} = \left(\frac{\sigma}{(\sigma-1)\alpha}\right) W_t N_{mv,t} = (1+\zeta^J) P^J_{mv,t} \left(\frac{P^J_{mv,t}}{P^J_t}\right)^{-\sigma} J_t \tag{A.2}$$

$$= (1+\zeta^J)(P^J_{mv,t})^{1-\sigma}\Gamma^J_t = (1+\zeta^J)\left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha}\right)^{\frac{\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}} W_t^{\frac{\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}} \varphi_{mv,t}^{-\frac{(1-\sigma)}{\alpha+\sigma(1-\alpha)}} (\Gamma^J_t)^{\frac{1}{\alpha+\sigma(1-\alpha)}} .$$

Finally, we obtain the formula for the profit $\Pi^J_{mv,t}$, which is given by

$$\Pi^J_{mv,t} = r_{mv,t} - W_t N_{mv,t} - R^J_{t-1} P_{t-1} F_{m,t-1} = \frac{\alpha+\sigma(1-\alpha)}{\sigma} r_{mv,t} - R^J_{t-1} P_{t-1} F_{m,t-1} .$$

Calculating $P^J_{m,t}$ in (6): the price aggregator for firms of fixed $F_{m,t-1}$ From our notation in (6), we know that among firms with fixed cost $F_{m,t-1}$, a set of operating ones at t would be given by $\Omega_{m,t} = \left\{\varphi_{mv,t} \in \left[\max\left\{\varphi^*_{m,t}, \left(\frac{\kappa-1}{\kappa}\right) A_t\right\}, \infty\right]\right\}$. The cumulative distribution function of productivities of upstream firms that decide to produce is $\frac{\Psi(\varphi_{m,t})}{1-\Psi(\varphi^*_{m,t})}$, a truncated Pareto distribution which is itself a Pareto distribution. With the individual firm (m,v)'s pricing equation (A.1), we now can compute the aggregate price of upstream firms with fixed cost $F_{m,t-1}$ as:

$$\left(\frac{P^J_{m,t}}{P_t}\right)^{1-\sigma} = \cancel{M_{m,t}} \cdot \int_{\max\left\{\varphi^*_{m,t},\left(\frac{\kappa-1}{\kappa}\right)A_t\right\}}^{\infty} \left(\frac{P^J_{mv,t}}{P_t}\right)^{1-\sigma} \frac{\mathrm{d}\,\Psi(\varphi_{mv,t})}{\cancel{1-\Psi(\varphi^*_{m,t})}} \tag{A.3}$$

$$= \int_{\max\left\{\varphi^*_{m,t},\left(\frac{\kappa-1}{\kappa}\right)A_t\right\}}^{\infty} \left(\frac{P^J_{mv,t}}{P_t}\right)^{1-\sigma} \mathrm{d}\,\Psi(\varphi_{mv,t})$$

$$= \left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha}\right)^{\frac{\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}} \left(\frac{\kappa-1}{\kappa}\right)^{\frac{(\sigma-1)}{\alpha+\sigma(1-\alpha)}} \left(\frac{W_t}{P_tA_t}\right)^{\frac{\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}} \left[\left(\frac{\kappa-1}{\kappa}\right)A_t\right]^{\frac{(1-\sigma)}{\alpha+\sigma(1-\alpha)}}$$

$$\cdot \left(\frac{\Gamma^J_t}{(P^J_t)^\sigma A_t}\right)^{\frac{(1-\alpha)(1-\sigma)}{\alpha+\sigma(1-\alpha)}} \left(\frac{P^J_t}{P_t}\right)^{\frac{(1-\alpha)(1-\sigma)\sigma}{\alpha+\sigma(1-\alpha)}} \int_{\max\left\{\varphi^*_{m,t},\left(\frac{\kappa-1}{\kappa}\right)A_t\right\}}^{\infty} \varphi_{mv,t}^{\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}} \mathrm{d}\,\Psi(\varphi_{mv,t})$$

$$= \Theta_1 \left(\frac{W_t}{P_tA_t}\right)^{\frac{\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}} \left(\frac{Y_t\Delta_t}{A_t}\right)^{\frac{(1-\alpha)(1-\sigma)}{\alpha+\sigma(1-\alpha)}} \left(\frac{P^J_t}{P_t}\right)^{\frac{(1-\alpha)(1-\sigma)\sigma}{\alpha+\sigma(1-\alpha)}} \max\left\{\frac{\varphi^*_{m,t}}{\left(\frac{\kappa-1}{\kappa}\right)A_t},1\right\}^{-\frac{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}{\alpha+\sigma(1-\alpha)}}$$

$$= \Theta_1 \left(\frac{W_t}{P_tA_t}\right)^{\frac{\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}} \left(\frac{Y_t\Delta_t}{A_t}\right)^{\frac{(1-\alpha)(1-\sigma)}{\alpha+\sigma(1-\alpha)}} \left(\frac{P^J_t}{P_t}\right)^{\frac{(1-\alpha)(1-\sigma)\sigma}{\alpha+\sigma(1-\alpha)}} \tag{A.4}$$

$$\cdot \min\left\{\left(\frac{R^J_{t-1}P_{t-1}F_{m,t-1}}{E_{t-1}\left[\xi_t\cdot\Xi_t\right]\left[\left(\frac{\kappa-1}{\kappa}\right)A_t\right]^{\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}}}\right)^{-\frac{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}{\sigma-1}},1\right\},$$

where we define

$$\Theta_1 = \left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha}\right)^{\frac{\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}} \left(\frac{\kappa-1}{\kappa}\right)^{\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}} \left(\frac{\kappa[\alpha+\sigma(1-\alpha)]}{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}\right).$$

Reexpressing Ξ_t in equation (13) Combining equation (13) with $\Gamma^J_t = (P^J_t)^\sigma Y_t\Delta_t$, we obtain

$$\Xi_t = \frac{\alpha+\sigma(1-\alpha)}{\alpha(\sigma-1)} \left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha}\right)^{-\frac{\sigma}{\alpha+\sigma(1-\alpha)}} \left(\frac{\kappa-1}{\kappa}\right)^{\frac{\alpha(\sigma-1)}{\alpha+\sigma(1-\alpha)}} \left(\frac{P^J_t}{P_t}\right)^{\frac{\sigma}{\alpha+\sigma(1-\alpha)}}$$

$$\cdot \left(\frac{W_t}{A_tP_t}\right)^{\frac{\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}} \left[\left(\frac{\kappa-1}{\kappa}\right)A_t\right]^{\frac{\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}} P_t(Y_t\Delta_t)^{\frac{1}{\alpha+\sigma(1-\alpha)}} \tag{A.5}$$

$$= \Theta_2 \cdot \left(\frac{P^J_t}{P_t}\right)^{\frac{\sigma}{\alpha+\sigma(1-\alpha)}} \left(\frac{W_t}{A_tP_t}\right)^{\frac{\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}} \left[\left(\frac{\kappa-1}{\kappa}\right)A_t\right]^{\frac{\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}} P_t(Y_t\Delta_t)^{\frac{1}{\alpha+\sigma(1-\alpha)}},$$

where we define

$$\Theta_2 = \frac{\alpha + \sigma(1-\alpha)}{\alpha(\sigma-1)} \left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha} \right)^{-\frac{\sigma}{\alpha+\sigma(1-\alpha)}} \left(\frac{\kappa-1}{\kappa} \right)^{\frac{\alpha(\sigma-1)}{\alpha+\sigma(1-\alpha)}} .$$

Derivation of P_t^J in (19) We start from the full satiation threshold of the fixed cost F_{t-1}^* defined in Proposition 2:

$$\begin{aligned} F_{t-1}^* &= \frac{E_{t-1}\left[\xi_t \cdot \Xi_t\right] \left[\left(\frac{\kappa-1}{\kappa}\right) A_t\right]^{\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}}}{R_{t-1}^J P_{t-1}} \\ &= \Theta_2 E_{t-1} \left[\xi_t \left(\frac{P_t^J}{P_t} \right)^{\frac{\sigma}{\alpha+\sigma(1-\alpha)}} \left(\frac{W_t}{A_t P_t} \right)^{\frac{\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}} \left[\left(\frac{\kappa-1}{\kappa} \right) A_t \right]^{\frac{(\sigma-1)(1-\alpha)}{\alpha+\sigma(1-\alpha)}} \left(\frac{\Pi_t (Y_t \Delta_t)^{\frac{1}{\alpha+\sigma(1-\alpha)}}}{R_{t-1}^J} \right) \right] , \end{aligned} \tag{A.6}$$

where the second equality is from equation (A.5). From (14) and (A.6), we obtain

$$\varphi_{m,t}^* = \left(\frac{F_{m,t-1}}{F_{t-1}^*} \right)^{\frac{\alpha+\sigma(1-\alpha)}{\sigma-1}} \left(\frac{\kappa-1}{\kappa} \right) A_t , \tag{A.7}$$

and

$$R_{m,t-1}^{J,*} = \left(\frac{F_{m,t-1}}{F_{t-1}^*} \right)^{-1} R_{t-1}^J . \tag{A.8}$$

From (15), we obtain

$$M_{m,t} = \min \left\{ \left(\frac{F_{m,t-1}}{F_{t-1}^*} \right)^{-\left(\frac{\kappa[\alpha+\sigma(1-\alpha)]}{\sigma-1} \right)} , \ 1 \right\} . \tag{A.9}$$

Using equation (A.3) and (A.6), we obtain

$$\begin{aligned} \left(\frac{P_{m,t}^J}{P_t} \right)^{1-\sigma} &= \Theta_1 \cdot \left(\frac{W_t}{P_t A_t} \right)^{\frac{\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}} \left(\frac{P_t^J}{P_t} \right)^{\frac{(1-\alpha)(1-\sigma)\sigma}{\alpha+\sigma(1-\alpha)}} \left(\frac{Y_t \Delta_t}{A_t} \right)^{\frac{(1-\alpha)(1-\sigma)}{\alpha+\sigma(1-\alpha)}} \\ &\cdot \min \left\{ \left(\frac{F_{m,t-1}}{F_{t-1}^*} \right)^{-\left(\frac{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}{\sigma-1} \right)} , 1 \right\} . \end{aligned} \tag{A.10}$$

We rearrange equation (6) as:

$$
\begin{aligned}
\left(\frac{P_t^J}{P_t}\right)^{1-\sigma} &= \int_0^1 \left(\frac{P_{m,t}^J}{P_t}\right)^{1-\sigma} \mathrm{d}m \\
&= Prob\left(F_{m,t-1} \leq F_{t-1}^*\right) E_t\left[\left(\frac{P_{m,t}^J}{P_t^J}\right)^{1-\sigma} | F_{m,t-1} \leq F_{t-1}^*\right] \\
&\quad + Prob\left(F_{m,t-1} > F_{t-1}^*\right) E_t\left[\left(\frac{P_{m,t}^J}{P_t^J}\right)^{1-\sigma} | F_{m,t-1} > F_{t-1}^*\right] \qquad \text{(A.11)} \\
&= \cancel{H(F_{t-1}^*)} \int_{\left(\frac{\omega-1}{\omega}\right)F_{t-1}}^{F_{t-1}^*} \left(\frac{P_{m,t}^J}{P_t^J}\right)^{1-\sigma} \frac{\mathrm{d}H(F_{m,t-1})}{\cancel{H(F_{t-1}^*)}} + \cancel{\left[1-H(F_{t-1}^*)\right]} \int_{F_{t-1}^*}^{\infty} \left(\frac{P_{m,t}^J}{P_t^J}\right)^{1-\sigma} \frac{\mathrm{d}H(F_{m,t-1})}{\cancel{1-H(F_{t-1}^*)}} \\
&= \int_{\left(\frac{\omega-1}{\omega}\right)F_{t-1}}^{F_{t-1}^*} \left(\frac{P_{m,t}^J}{P_t^J}\right)^{1-\sigma} \mathrm{d}H(F_{m,t-1}) + \int_{F_{t-1}^*}^{\infty} \left(\frac{P_{m,t}^J}{P_t^J}\right)^{1-\sigma} \mathrm{d}H(F_{m,t-1}) \ ,
\end{aligned}
$$

where $\frac{P_{m,t}^J}{P_t^J}$ is given by (A.10). Plugging (A.10) into (A.11), we obtain

$$
\begin{aligned}
\left(\frac{P_t^J}{P_t}\right)^{1-\sigma} &= \Theta_1 \cdot \left(\frac{W_t}{P_t A_t}\right)^{\left(\frac{\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}\right)} \cdot \left(\frac{P_t^J}{P_t}\right)^{\left(\frac{(1-\alpha)(1-\sigma)\sigma}{\alpha+\sigma(1-\alpha)}\right)} \cdot \left(\frac{Y_t \Delta_t}{A_t}\right)^{\left(\frac{(1-\alpha)(1-\sigma)}{\alpha+\sigma(1-\alpha)}\right)} \\
&\quad \cdot \left[\int_{\left(\frac{\omega-1}{\omega}\right)F_{t-1}}^{F_{t-1}^*} 1 \ \mathrm{d}H\left(F_{m,t-1}\right) + \int_{F_{t-1}^*}^{\infty} \left(\frac{F_{m,t-1}}{F_{t-1}^*}\right)^{-\left(\frac{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}{\sigma-1}\right)} \mathrm{d}H\left(F_{m,t-1}\right)\right] \ , \qquad \text{(A.12)}
\end{aligned}
$$

which leads to

$$
\begin{aligned}
\left(\frac{P_t^J}{P_t}\right)^{\left(\frac{\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}\right)} &= \Theta_1 \cdot \left(\frac{W_t}{P_t A_t}\right)^{\left(\frac{\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}\right)} \cdot \left(\frac{Y_t \Delta_t}{A_t}\right)^{\left(\frac{(1-\alpha)(1-\sigma)}{\alpha+\sigma(1-\alpha)}\right)} \\
&\quad \cdot \left[H\left(F_{t-1}^*\right) + \left(\frac{\omega(\sigma-1)}{\kappa[\alpha+\sigma(1-\alpha)]+(\omega-1)(\sigma-1)}\right) \cdot \left[1 - H\left(F_{t-1}^*\right)\right]\right] \ . \qquad \text{(A.13)}
\end{aligned}
$$

Rearranging equation (A.13), we finally obtain:

$$
\frac{P_t^J}{P_t} = \left(\frac{W_t}{P_t A_t}\right) \cdot \left(\frac{Y_t \Delta_t}{A_t}\right)^{\frac{1-\alpha}{\alpha}} \cdot \left[\frac{\Theta_3}{1+\Theta_4 \cdot H\left(F_{t-1}^*\right)}\right]^{\left(\frac{\alpha+\sigma(1-\alpha)}{\alpha(\sigma-1)}\right)} \ . \qquad \text{(A.14)}
$$

where we define

$$\Theta_3 = \left(\frac{\kappa[\alpha+\sigma(1-\alpha)] + (\omega-1)(\sigma-1)}{\Theta_1\omega(\sigma-1)}\right), \quad \Theta_4 = \left(\frac{\kappa[\alpha+\sigma(1-\alpha)] - (\sigma-1)}{\omega(\sigma-1)}\right) .$$

Derivation of M_t and L_{t-1} in (20) and (21)

$$\begin{aligned}
M_t &= \int_0^1 \int_{v\in\Omega_{m,t}} 1 \, \mathrm{d}v \, \mathrm{d}m = \int_0^1 M_{m,t} \, \mathrm{d}m = \int_0^1 M_{m,t} \cdot dH(F_{m,t-1}) \qquad \text{(A.15)} \\
&= \underbrace{Prob\left(F_{t-1} \le F^*_{t-1}\right)}_{=H(F^*_{t-1})} \cdot 1 + \cancel{Prob\left(F_{t-1} > F^*_{t-1}\right)} \cdot \int_{F^*_{t-1}}^{\infty} \left(\frac{F_{m,t-1}}{F^*_{t-1}}\right)^{-\frac{\kappa[\alpha+\sigma(1-\alpha)]}{\sigma-1}} \frac{\mathrm{d}H(F_{m,t-1})}{\cancel{1-H(F^*_{t-1})}} \\
&= 1 - \Theta_M \cdot \left[1 - H\left(F^*_{t-1}\right)\right] ,
\end{aligned}$$

where

$$\Theta_M = \frac{\kappa[\alpha+\sigma(1-\alpha)]}{\kappa[\alpha+\sigma(1-\alpha)] + \omega(\sigma-1)} .$$

To derive equation (17), we start from

$$\begin{aligned}
\frac{L_{t-1}}{P_{t-1}} &= \frac{\int_0^1 L_{m,t-1} \, \mathrm{d}m}{P_{t-1}} \\
&= Prob\left(F_{m,t-1} \le F^*_{t-1}\right) \cdot \int_{\left(\frac{\omega-1}{\omega}\right)F_{t-1}}^{F^*_{t-1}} F_{m,t-1} \, \frac{\mathrm{d}H\left(F_{m,t-1}\right)}{H\left(F^*_{t-1}\right)} \\
&\quad + Prob\left(F_{m,t-1} > F^*_{t-1}\right) \cdot \int_{F^*_{t-1}}^{\infty} \left(F^*_{t-1}\right)^{\left(\frac{\kappa[\alpha+\sigma(1-\alpha)]}{\sigma-1}\right)} \cdot F_{m,t-1}^{-\left(\frac{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}{\sigma-1}\right)} \frac{\mathrm{d}H\left(F_{m,t-1}\right)}{1-H\left(F^*_{t-1}\right)} \\
&= \int_{\left(\frac{\omega-1}{\omega}\right)F_{t-1}}^{F^*_{t-1}} F_{m,t-1} \, \mathrm{d}H\left(F_{m,t-1}\right) + \int_{F^*_{t-1}}^{\infty} \left(F^*_{t-1}\right)^{\left(\frac{\kappa[\alpha+\sigma(1-\alpha)]}{\sigma-1}\right)} \cdot F_{m,t-1}^{-\left(\frac{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}{\sigma-1}\right)} \mathrm{d}H\left(F_{m,t-1}\right) ,
\end{aligned}$$

which leads to

$$\begin{aligned}
\frac{L_{t-1}}{P_{t-1}} &= F_{t-1} - \left(\frac{\omega}{\omega-1}\right)\left(\frac{\kappa[\alpha+\sigma(1-\alpha)]}{\kappa[\alpha+\sigma(1-\alpha)] + (\sigma-1)(\omega-1)}\right) \cdot F^*_{t-1} \cdot \left[1 - H\left(F^*_{t-1}\right)\right] \\
&= F_{t-1} \cdot \left[1 - \Theta_L \cdot \left[1 - H\left(F^*_{t-1}\right)\right]^{\left(\frac{\omega-1}{\omega}\right)}\right] ,
\end{aligned}$$

where

$$\Theta_L = \frac{\kappa[\alpha+\sigma(1-\alpha)]}{\kappa[\alpha+\sigma(1-\alpha)] + (\sigma-1)(\omega-1)} .$$

Derivation of N_t in equation (26) Labor $N_{mv,t}$ employed by a producing upstream firm (m,v) is given by

$$N_{mv,t} = J_{mv,t}^{\frac{1}{\alpha}} \varphi_{mv,t}^{-\frac{1}{\alpha}} = \varphi_{mv,t}^{-\frac{1}{\alpha}} \cdot \left[\left(\frac{P_{mv,t}^J}{P_t^J} \right)^{-\sigma} \cdot J_t \right]^{\frac{1}{\alpha}} \tag{A.16}$$

$$= \left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha} \right)^{\left(\frac{-\sigma}{\alpha+\sigma(1-\alpha)}\right)} \cdot \left(\frac{\kappa-1}{\kappa} \right)^{\left(\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}\right)} \cdot \left(\frac{\varphi_{mv,t}}{\left(\frac{\kappa-1}{\kappa}\right) A_t} \right)^{\left(\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}\right)}$$
$$\cdot \left(\frac{W_t}{P_t A_t} \right)^{\left(\frac{-\sigma}{\alpha+\sigma(1-\alpha)}\right)} \cdot \left(\frac{P_t^J}{P_t} \right)^{\left(\frac{\sigma}{\alpha+\sigma(1-\alpha)}\right)} \cdot \left(\frac{Y_t \Delta_t}{A_t} \right)^{\left(\frac{1}{\alpha+\sigma(1-\alpha)}\right)}$$

$$= \left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha} \right)^{\left(\frac{-\sigma}{\alpha+\sigma(1-\alpha)}\right)} \left(\frac{\kappa-1}{\kappa} \right)^{\left(\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}\right)} \left(\frac{\varphi_{mv,t}}{\left(\frac{\kappa-1}{\kappa}\right) A_t} \right)^{\left(\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}\right)}$$
$$\cdot \left[\frac{\Theta_3}{1+\Theta_4 H_{t-1}} \right]^{\left(\frac{\sigma}{(\sigma-1)\alpha}\right)} \left(\frac{Y_t \Delta_t}{A_t} \right)^{\frac{1}{\alpha}} , \tag{A.17}$$

where we use equation (5) in the second equality, equations (8) and (11) for the third equality, and equation (19) to obtain the fourth one. For convenience we define $H_{t-1} \equiv H\left(F_{t-1}^*\right)$. Now we integrate labor in (A.16) across all producing firms to obtain the aggregate labor N_t. First,

$$N_t = \int_0^1 \int_{v\in\Omega_{m,t}} N_{mv,t} \, \mathrm{d}v \, \mathrm{d}m$$
$$= \left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha} \right)^{\left(\frac{-\sigma}{\alpha+\sigma(1-\alpha)}\right)} \left(\frac{\kappa-1}{\kappa} \right)^{\left(\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}\right)} \left[\left(\frac{\kappa-1}{\kappa} \right) A_t \right]^{\left(\frac{1-\sigma}{\alpha+\sigma(1-\alpha)}\right)}$$
$$\cdot \left[\frac{\Theta_3}{1+\Theta_4 \cdot H_{t-1}} \right]^{\left(\frac{\sigma}{(\sigma-1)\alpha}\right)} \left(\frac{Y_t \Delta_t}{A_t} \right)^{\frac{1}{\alpha}} \int_0^1 \int_{v\in\Omega_{m,t}} \varphi_{mv,t}^{\left(\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}\right)} \, \mathrm{d}v \, \mathrm{d}m \tag{A.18}$$
$$= \square \int_0^1 \int_{v\in\Omega_{m,t}} \varphi_{mv,t}^{\left(\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}\right)} \, \mathrm{d}v \, \mathrm{d}m ,$$

where

$$\square = \left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha} \right)^{\left(\frac{-\sigma}{\alpha+\sigma(1-\alpha)}\right)} \left(\frac{\kappa-1}{\kappa} \right)^{\left(\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}\right)} \left[\left(\frac{\kappa-1}{\kappa} \right) A_t \right]^{\left(\frac{1-\sigma}{\alpha+\sigma(1-\alpha)}\right)}$$
$$\cdot \left[\frac{\Theta_3}{1+\Theta_4 \cdot H_{t-1}} \right]^{\left(\frac{\sigma}{(\sigma-1)\alpha}\right)} \left(\frac{Y_t \Delta_t}{A_t} \right)^{\frac{1}{\alpha}} . \tag{A.19}$$

Now, (36) leads to

$$
\begin{aligned}
N_t &= \Box \int_0^1 \int_{\max\left(\varphi^*_{m,t}, \frac{\kappa-1}{\kappa}A_t\right)} \varphi_{mv,t}^{\left(\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}\right)} \kappa \left[\left(\frac{\kappa-1}{\kappa}\right) A_t\right]^{\kappa} \varphi_{mv,t}^{-(\kappa+1)} \, \mathrm{d}\varphi_{mv,t} \, \mathrm{d}m \\
&= \Box \left[\left(\frac{\kappa-1}{\kappa}\right) A_t\right]^{\kappa} \left(\frac{\kappa[\alpha+\sigma(1-\alpha)]}{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}\right) \\
&\quad \cdot \left[\left(\frac{\kappa-1}{\kappa}\right) A_t\right]^{\left(-\frac{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}{\alpha+\sigma(1-\alpha)}\right)} \int_0^1 \max\left(\frac{\varphi^*_{m,t}}{\frac{\kappa-1}{\kappa}A_t}, 1\right)^{\left(-\frac{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}{\alpha+\sigma(1-\alpha)}\right)} \mathrm{d}m \\
&= \Box \left[\left(\frac{\kappa-1}{\kappa}\right) A_t\right]^{\left(\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}\right)} \left(\frac{\kappa[\alpha+\sigma(1-\alpha)]}{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}\right) \int_0^1 \min\left(\left(\frac{F_{m,t-1}}{F^*_{t-1}}\right)^{-\frac{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}{\sigma-1}}, 1\right) \mathrm{d}m \\
&= \Box \left[\left(\frac{\kappa-1}{\kappa}\right) A_t\right]^{\left(\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}\right)} \left(\frac{\kappa[\alpha+\sigma(1-\alpha)]}{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}\right) \\
&\quad \cdot \left[H_{t-1} + \frac{\omega(\sigma-1)}{\kappa[\alpha+\sigma(1-\alpha)]+(\omega-1)(\sigma-1)}(1-H_{t-1})\right] \\
&= \Box \left[\left(\frac{\kappa-1}{\kappa}\right) A_t\right]^{\left(\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}\right)} \left(\frac{\kappa[\alpha+\sigma(1-\alpha)]}{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}\right) \left(\frac{\omega(\sigma-1)}{\kappa[\alpha+\sigma(1-\alpha)]+(\omega-1)(\sigma-1)}\right) [1+\Theta_4 H_{t-1}] \\
&= \left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha}\right)^{\left(\frac{-\sigma}{\alpha+\sigma(1-\alpha)}\right)} \left(\frac{\kappa-1}{\kappa}\right)^{\left(\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}\right)} \left(\frac{\kappa[\alpha+\sigma(1-\alpha)]}{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}\right) \\
&\quad \cdot \left(\frac{\omega(\sigma-1)}{\kappa[\alpha+\sigma(1-\alpha)]+(\omega-1)(\sigma-1)}\right) [1+\Theta_4 H_{t-1}] \left[\frac{\Theta_3}{1+\Theta_4 \cdot H_{t-1}}\right]^{\left(\frac{\sigma}{(\sigma-1)\alpha}\right)} \left(\frac{Y_t \Delta_t}{A_t}\right)^{\frac{1}{\alpha}} \\
&= \Theta_N \cdot \left(\frac{Y_t \Delta_t}{A_t}\right)^{\frac{1}{\alpha}} \cdot (1+\Theta_4 H_{t-1})^{\frac{\alpha+\sigma(1-\alpha)}{(1-\sigma)\alpha}},
\end{aligned}
$$

where Θ_N is defined in (27).

Equilibrium conditions for downstream firms Plugging equation (29) and the expression for $Q_{t,t+l}$ into (4), we can express the resetting price in (4) in a recursive fashion as

$$
\begin{aligned}
O_t = & \left(\frac{(1+\zeta^T)^{-1}\gamma}{\gamma-1}\right) \Theta_N^{\frac{1}{\eta}} \Theta_3^{\frac{\alpha+\sigma(1-\alpha)}{(\sigma-1)\alpha}} \left(\frac{Y_t}{A_t}\right)^{\left(\frac{\eta+1}{\eta\alpha}\right)} \Delta_t^{\left(\frac{(1-\alpha)\eta+1}{\eta\alpha}\right)} (1+\Theta_4 H_{t-1})^{\frac{(1+\eta)[\alpha+\sigma(1-\alpha)]}{\eta(1-\sigma)\alpha}} \exp\{-u_{c,t}\} \\
& + \beta\theta E_t \left[\exp\{u_{c,t+1}-u_{c,t}\} \cdot \Pi_{t+1}^{\gamma} \cdot O_{t+1}\right],
\end{aligned}
\tag{A.20}
$$

and

$$
V_t = \left(\frac{C_t}{Y_t}\right)^{-1} + \beta\theta \cdot E_t \left[\exp\{u_{c,t+1}-u_{c,t}\} \cdot \Pi_{t+1}^{\gamma-1} \cdot V_{t+1}\right]. \tag{A.21}
$$

We obtain

$$\frac{P_t^*}{P_t} = \frac{O_t}{V_t} \; . \tag{A.22}$$

Due to price stickiness à la Calvo (1983), the aggregate price level can be recursively expressed as:

$$P_t^{1-\gamma} = (1-\theta)\left(P_t^*\right)^{1-\gamma} + \theta\left(P_{t-1}\right)^{1-\gamma} \; ,$$

or alternatively as:

$$\frac{P_t^*}{P_t} = \left(\frac{1-\theta}{1-\theta\cdot\Pi_t^{\gamma-1}}\right)^{\frac{1}{\gamma-1}} \; . \tag{A.23}$$

Plugging equation (A.22) into equation (9) and equation (A.23), we obtain

$$\frac{O_t}{V_t} = \left(\frac{1-\theta}{1-\theta\cdot\Pi_t^{\gamma-1}}\right)^{\frac{1}{\gamma-1}}, \;\; \Delta_t = (1-\theta)\left(\frac{O_t}{V_t}\right)^{-\gamma} + \theta\Pi_t^{\gamma}\Delta_{t-1} \; .$$

Equilibrium conditions for households We can write F_t^* as a function of H_t by using the cumulative distribution function of fixed costs in (18) and (24):

$$F_t^* = \left[1-H_t\right]^{-\frac{1}{\omega}}\left(\frac{\omega-1}{\omega}\right)\phi_f\cdot\tilde{Y}A_t\cdot\exp\left\{u_{f,t}\right\} \; . \tag{A.24}$$

Using the above (A.24), we can rearrange equation (A.6) (i.e., equation about F_t^* as:

$$\begin{aligned} R_t^J = E_t & \left[\xi_{t+1}\cdot\left(\frac{P_{t+1}^J}{P_{t+1}}\right)^{\left(\frac{\sigma}{\alpha+\sigma(1-\alpha)}\right)}\left(\frac{w_{t+1}}{P_{t+1}A_{t+1}}\right)^{\left(\frac{(1-\sigma)\alpha}{\alpha+\sigma(1-\alpha)}\right)}\frac{1}{\tilde{Y}}\Pi_{t+1}GA_{t+1}\left(\frac{Y_{t+1}\Delta_{t+1}}{A_{t+1}}\right)^{\left(\frac{1}{\alpha+\sigma(1-\alpha)}\right)}\right] \\ & \cdot\left(\frac{\Theta_2}{\left(\frac{\omega-1}{\omega}\right)\phi_f}\right)\left(\frac{\kappa-1}{\kappa}\right)^{\left(\frac{(\sigma-1)(1-\alpha)}{\alpha+\sigma(1-\alpha)}\right)}\left[1-H_t\right]^{\frac{1}{\omega}}\cdot\exp\left\{-u_{f,t}\right\} \; . \end{aligned} \tag{A.25}$$

Plugging (28) and (29) into the above (A.25), we obtain:

$$\begin{aligned} R_t^J = & \left(\frac{\Theta_2\Theta_N^{\frac{1}{\eta}}\Theta_3^{\frac{\sigma}{(\sigma-1)\alpha}}}{\left(\frac{\omega-1}{\omega}\right)\phi_f}\right)\cdot\left(\frac{\kappa-1}{\kappa}\right)^{\left(\frac{(\sigma-1)(1-\alpha)}{\alpha+\sigma(1-\alpha)}\right)}\left(1+\Theta_4H_t\right)^{\left(\frac{\alpha+\sigma(1-\alpha)+\sigma\eta}{\eta(1-\sigma)\alpha}\right)}\cdot\left(1-H_t\right)^{\frac{1}{\omega}} \\ & \cdot E_t\left[\xi_{t+1}\Pi_{t+1}\left(\frac{\frac{C_{t+1}}{A_{t+1}}}{\frac{Y_{t+1}}{A_{t+1}}}\right)\left(\frac{\frac{Y_{t+1}}{A_{t+1}}}{\tilde{Y}}\right)\left(\frac{Y_{t+1}\Delta_{t+1}}{A_{t+1}}\right)^{\left(\frac{\eta+1}{\eta\alpha}\right)}\cdot GA_{t+1}\cdot\exp\left\{-(u_{f,t}+u_{c,t+1})\right\}\right] \; . \end{aligned} \tag{A.26}$$

Finally, we can rearrange the Euler equation in (1), using (31) as follows:

$$\frac{1}{R_t^J} = \beta E_t \left[\frac{\left(\frac{C_t}{Y_t}\right)}{\left(\frac{C_{t+1}}{Y_{t+1}}\right) \widetilde{GY}_{t+1} GA_{t+1} \Pi_{t+1}} \cdot \exp\left\{u_{c,t+1} - u_{c,t}\right\} \right] , \tag{A.27}$$

where $\widetilde{GY}_{t+1} = \frac{Y_{t+1}}{Y_{t+1}} \frac{A_t}{A_{t+1}}$ and $GA_{t+1} = \frac{A_{t+1}}{A_t}$. Combining equation (A.26) and equation (A.27), we obtain

$$\exp\left\{u_{f,t} + u_{c,t}\right\} = \beta \left(\frac{\Theta_2 \cdot \Theta_N^{\frac{1}{\eta}} \cdot \Theta_3^{\frac{\sigma}{(\sigma-1)\alpha}}}{\left(\frac{\omega-1}{\omega}\right) \phi_f} \right) \left(\frac{\kappa-1}{\kappa}\right)^{\left(\frac{(\sigma-1)(1-\alpha)}{\alpha+\sigma(1-\alpha)}\right)} \cdot (1 + \Theta_4 H_t)^{\left(\frac{[\alpha+\sigma(1-\alpha)]+\sigma\eta}{\eta(1-\sigma)\alpha}\right)}$$
$$\cdot (1 - H_t)^{\frac{1}{\omega}} \cdot \left(\frac{\frac{C_t}{A_t}}{\tilde{Y}}\right) \cdot E_t \left[\left(\frac{Y_{t+1}\Delta_{t+1}}{A_{t+1}}\right)^{\left(\frac{\eta+1}{\eta\alpha}\right)} \right] . \tag{A.28}$$

Flexible price equilibrium Plugging (35) into (19), we obtain

$$\frac{W_t}{P_t A_t} = \left(\frac{(1+\zeta^T)^{-1}\gamma}{\gamma - 1}\right)^{-1} \cdot \left(\frac{Y_t \Delta_t}{A_t}\right)^{\frac{\alpha-1}{\alpha}} \cdot \left[\frac{\Theta_3}{1 + \Theta_4 \cdot H_{t-1}}\right]^{\left(\frac{\alpha+\sigma(1-\alpha)}{1-\sigma}\right)} . \tag{A.29}$$

Plugging (19) and (A.29) into (A.6) (i.e., equation about the cutoff fixed cost F_t^*), and based on the fact that there is no price dispersion under flexible prices, i.e., $\Delta_t = 1$, we obtain:

$$F_t^* = \Theta_2 \cdot \left(\frac{(1+\zeta^T)^{-1}\gamma}{\gamma-1}\right)^{-1} \cdot \left(\frac{\kappa-1}{\kappa}\right)^{\left(\frac{(\sigma-1)(1-\alpha)}{\alpha+\sigma(1-\alpha)}\right)} \cdot \left[\frac{\Theta_3}{1+\Theta_4 \cdot H_t}\right] E_t \left[\xi_{t+1} \left(\frac{\Pi_{t+1} Y_{t+1}}{R_t^J}\right)\right] . \tag{A.30}$$

By the definition of the distribution function of the fixed costs (see eq. (18)), we express:

$$[1 - H_t]^{-\frac{1}{\omega}} = \frac{F_t^*}{\left(\frac{\omega-1}{\omega}\right) F_t} = \frac{F_t^*}{\left(\frac{\omega-1}{\omega}\right) \cdot \phi_f \cdot \tilde{Y} A_t \cdot \exp\left\{u_{f,t}\right\}} . \tag{A.31}$$

Plugging equation (A.31) into equation (A.30), we obtain:

$$1 = \left(\frac{\beta\Theta_2}{\left(\frac{\omega-1}{\omega}\right) \cdot \phi_f}\right) \cdot \left(\frac{(1+\zeta^T)^{-1}\gamma}{\gamma-1}\right)^{-1} \cdot \left(\frac{\kappa-1}{\kappa}\right)^{\left(\frac{(\sigma-1)(1-\alpha)}{\alpha+\sigma(1-\alpha)}\right)} \cdot \left[\frac{\Theta_3}{1+\Theta_4 \cdot H_t}\right]$$
$$\cdot [1 - H_t]^{\frac{1}{\omega}} \cdot E_t \left[\left(\frac{\tilde{Y}_t}{\tilde{Y}}\right) \left(\frac{\frac{C_t}{Y_t}}{\frac{C_{t+1}}{Y_{t+1}}}\right) \cdot \exp\left\{u_{c,t+1} - (u_{f,t} + u_{c,t})\right\} \right] . \tag{A.32}$$

Finally, plugging (35) into (29) and based on no price dispersion under flexible prices, i.e., $\Delta_t = 1$, we obtain

$$\frac{Y_t}{A_t} = \left(\frac{(1+\zeta^T)^{-1}\gamma}{\gamma-1}\right)^{-\left(\frac{\eta\alpha}{(1-\alpha)\eta+1}\right)} \Theta_N^{-\left(\frac{\alpha}{(1-\alpha)\eta+1}\right)} \Theta_3^{-\frac{\eta[\alpha+\sigma(1-\alpha)]}{[(1-\alpha)\eta+1](\sigma-1)}} \cdot \left(\frac{C_t}{A_t}\right)^{-\left(\frac{\eta\alpha}{(1-\alpha)\eta+1}\right)}$$
$$\cdot\,(1+\Theta_4 H_{t-1})^{-\frac{(1+\eta)[\alpha+\sigma(1-\alpha)])}{(1-\sigma)[(1-\alpha)\eta+1]}} \cdot \exp\left\{\left(\frac{\eta\alpha}{(1-\alpha)\eta+1}\right)\cdot u_{c,t}\right\}. \tag{A.33}$$

From (A.32) and (A.33), we can see that the flexible price equilibrium is money-neutral.

A.2 Calibration of (κ, ω) in Section 3.1

Following intutions of Bernard et al. (2003), we calculate the model-implied standard deviation of revenues and productivities of operating upstream firms.

Derivations on the cross-sectional standard deviations of sales and productivities We start from the formula for the revenue $r_{mv,t}$ generated by a firm (m, v) in (A.2):

$$r_{mv,t} = (1+\zeta^J)\left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha}\right)^{\frac{\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}} W_t^{\frac{\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}} (\Gamma_t^J)^{\frac{1}{\alpha+\sigma(1-\alpha)}} \varphi_{mv,t}^{\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}}, \tag{A.34}$$

where

$$\varphi_{m,t}^* = \left(\frac{R_{t-1}^J P_{t-1} F_{m,t-1}}{E_{t-1}\left[\xi_t \cdot \Xi_t\right]}\right)^{\frac{\alpha+\sigma(1-\alpha)}{\sigma-1}}. \tag{A.35}$$

We can calculate the cross-sectional standard deviation of an individual firm's revenue and productivity by calculating the variance:

$$\begin{aligned}
\sigma^2\left(\log r_{mv,t}\right) &= \left(\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}\right)^2 \sigma^2\left(\log \varphi_{mv,t}\right) \\
&= \left(\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}\right)^2 \sigma^2\left(\log\frac{\varphi_{mv,t}}{\varphi_{m,t}^*} + \log\varphi_{m,t}^*\right) \\
&= \left(\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}\right)^2 \left[\sigma^2\left(\log\frac{\varphi_{mv,t}}{\varphi_{m,t}^*}\right) + \sigma^2\left(\log\varphi_{m,t}^*\right)\right],
\end{aligned} \tag{A.36}$$

where for the second line we use the property that (i) $\phi_{mv,t}|\phi_{mv,t} \geq \phi_{m,t}^*$ follows a Pareto distribution; (ii) distributions of productivities and fixed costs are independent of each other.

Therefore,

$$\sigma^2\left(\log r_{mv,t}\right) = \left(\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}\right)^2\left[\sigma^2\left(\log\frac{\varphi_{mv,t}}{\varphi^*_{m,t}}\right)+\left(\frac{\alpha+\sigma(1-\alpha)}{\sigma-1}\right)^2\sigma^2\left(\log F_{m,t-1}\right)\right]$$
$$= \left(\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}\right)^2\left[\frac{1}{\kappa^2}+\left(\frac{\alpha+\sigma(1-\alpha)}{\sigma-1}\right)^2\frac{1}{\omega^2}\right],$$

which implies

$$\sigma\left(\log r_{mv,t}\right) = \frac{\sigma-1}{\alpha+\sigma(1-\alpha)}\sqrt{\frac{1}{\kappa^2}+\left(\frac{\alpha+\sigma(1-\alpha)}{\sigma-1}\right)^2\frac{1}{\omega^2}},$$

and

$$\sigma\left(\log \varphi_{mv,t}\right) = \sqrt{\frac{1}{\kappa^2}+\left(\frac{\alpha+\sigma(1-\alpha)}{\sigma-1}\right)^2\frac{1}{\omega^2}}.$$

Revenue heterogeneity in our model With $\kappa = \omega = 3.4$, our model predicts the standard deviation of upstream firms' revenues to be 0.44. The residual variability in Bernard et al. (2003) stem from some factors we do not account for, such as taste heterogeneity or different demand weights for product types. Additionally, their estimates are based on U.S. manufacturing plants, whereas our framework focuses on upstream firms.

Regarding productivity variability, the standard deviation of log productivity for operating upstream firms in our model becomes 0.4 when $\kappa = \omega = 3.4$. According to Bernard et al. (2003), their model-generated standard deviation of log value-added per worker is 0.35, while the empirical figure stands at 0.75.[1] Given the potential for measurement errors, our calibration is closely aligned with their model-generated moment and falls within a plausible range.

[1] Bernard et al. (2003) note that some degree of under-prediction could result from measurement errors in Census data.

A.3 Detailed Derivation in Section 4.2

Intensive vs. extensive margin labor adjustments: derivation of (38) From (36), (A.19), and (A.20), we know that the aggregate labor N_t can be written as

$$N_t = \left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha}\right)^{\left(\frac{-\sigma}{\alpha+\sigma(1-\alpha)}\right)} \left(\frac{\kappa-1}{\kappa}\right)^{\left(\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}\right)} \left(\frac{\kappa[\alpha+\sigma(1-\alpha)]}{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}\right)$$
$$\cdot \left[\frac{\Theta_3}{1+\Theta_4\cdot H_{t-1}}\right]^{\left(\frac{\sigma}{(\sigma-1)\alpha}\right)} \left(\frac{Y_t\Delta_t}{A_t}\right)^{\frac{1}{\alpha}} \left[H_{t-1}+\frac{\omega(\sigma-1)}{\kappa[\alpha+\sigma(1-\alpha)]+(\omega-1)(\sigma-1)}(1-H_{t-1})\right]$$
$$= \Theta_{DN}\left[\frac{\Theta_3}{1+\Theta_4\cdot H_{t-1}}\right]^{\left(\frac{\sigma}{(\sigma-1)\alpha}\right)} \left(\frac{Y_t\Delta_t}{A_t}\right)^{\frac{1}{\alpha}} \underbrace{\left[H_{t-1}+\frac{\omega(\sigma-1)}{\kappa[\alpha+\sigma(1-\alpha)]+(\omega-1)(\sigma-1)}(1-H_{t-1})\right]}_{\equiv SN_t^I},$$

where

$$\Theta_{DN} \equiv \left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha}\right)^{\left(\frac{-\sigma}{\alpha+\sigma(1-\alpha)}\right)} \left(\frac{\kappa-1}{\kappa}\right)^{\left(\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}\right)} \left(\frac{\kappa[\alpha+\sigma(1-\alpha)]}{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}\right). \tag{A.37}$$

From (A.37), we obtain for $\forall\iota$

$$\frac{N_{t+\iota}-N_t}{N_t} = \underbrace{\left[\frac{1+\Theta_4\cdot H_{t-1}}{1+\Theta_4\cdot H_{t+\iota-1}}\right]^{\left(\frac{\sigma}{(\sigma-1)\alpha}\right)} \left(\frac{Y_{t+\iota}\Delta_{t+\iota}/A_{t+\iota}}{Y_t\Delta_t/A_t}\right)^{\frac{1}{\alpha}} - 1}_{=g^{\text{Density}}_{t,t+\iota}} \tag{A.38}$$

$$+\left\{1+\underbrace{\left[\left[\frac{1+\Theta_4\cdot H_{t-1}}{1+\Theta_4\cdot H_{t+\iota-1}}\right]^{\left(\frac{\sigma}{(\sigma-1)\alpha}\right)} \left(\frac{Y_{t+\iota}\Delta_{t+\iota}/A_{t+\iota}}{Y_t\Delta_t/A_t}\right)^{\frac{1}{\alpha}} - 1\right]}_{=g^{\text{Density}}_{t,t+\iota}}\right\}\cdot \underbrace{\frac{SN^E_{t,t+\iota}}{SN^I_t}}_{\equiv g^{\text{Entry}}_{t,t+\iota}}. \tag{A.39}$$

Therefore, by (37) and the definition of the decomposition in (38), we obtain (39):

$$g^{\text{Entry}}_{t,t+\iota} \equiv \frac{SN^E_{t,t+\iota}}{SN^I_t} = \frac{(H_{t+\iota-1}-H_{t-1})+\frac{\omega(\sigma-1)}{\kappa[\alpha+\sigma(1-\alpha)]+(\omega-1)(\sigma-1)}(H_{t-1}-H_{t+\iota-1})}{H_{t-1}+\frac{\omega(\sigma-1)}{\kappa[\alpha+\sigma(1-\alpha)]+(\omega-1)(\sigma-1)}(1-H_{t-1})}. \tag{A.40}$$

B Summary of Equilibrium Conditions

B.1 Sticky Price Equilibrium (i.e., Original Model)

$$\exp\{u_{f,t}+u_{c,t}\}=\beta\left(\frac{\Theta_2\cdot\Theta_N^{\frac{1}{\eta}}\cdot\Theta_3^{\frac{\sigma}{(\sigma-1)\alpha}}}{\left(\frac{\omega-1}{\omega}\right)\phi_f}\right)\left(\frac{\kappa-1}{\kappa}\right)^{\left(\frac{(\sigma-1)(1-\alpha)}{\alpha+\sigma(1-\alpha)}\right)}\cdot(1+\Theta_4H_t)^{\left(\frac{\alpha+\sigma(1-\alpha)+\sigma\eta}{\eta(1-\sigma)\alpha}\right)}$$
$$\cdot(1-H_t)^{\frac{1}{\omega}}\cdot\left(\frac{\tilde{C}_t}{\tilde{Y}}\right)\cdot E_t\left[\left(\tilde{Y}_{t+1}\Delta_{t+1}\right)^{\left(\frac{\eta+1}{\eta\alpha}\right)}\right]$$

$$\frac{1}{R_t^J}=\beta E_t\left[\frac{\tilde{C}_t}{\tilde{C}_{t+1}GA_{t+1}\Pi_{t+1}}\cdot\exp\{u_{c,t+1}-u_{c,t}\}\right]$$

$$\frac{\tilde{C}_t}{\tilde{Y}_t}=1-\phi_g\cdot\exp\{u_{g,t}\}-\phi_f\cdot\left(\frac{\tilde{Y}_t}{\tilde{Y}}\right)^{-1}\cdot\left[1-\Theta_L\cdot[1-H_t]^{\left(\frac{\omega-1}{\omega}\right)}\right]\cdot\exp\{u_{f,t}\}$$

$$O_t=\left(\frac{(1+\zeta^T)^{-1}\gamma}{\gamma-1}\right)\Theta_N^{\frac{1}{\eta}}\Theta_3^{\frac{\alpha+\sigma(1-\alpha)}{(\sigma-1)\alpha}}\tilde{Y}_t^{\left(\frac{\eta+1}{\eta\alpha}\right)}\Delta_t^{\left(\frac{(1-\alpha)\eta+1}{\eta\alpha}\right)}(1+\Theta_4H_{t-1})^{\frac{(1+\eta)[\alpha+\sigma(1-\alpha)]}{\eta(1-\sigma)\alpha}}\exp\{-u_{c,t}\}$$
$$+\beta\theta E_t\left[\exp\{u_{c,t+1}-u_{c,t}\}\cdot\Pi_{t+1}^{\gamma}\cdot O_{t+1}\right]$$

$$V_t=\left(\frac{\tilde{C}_t}{\tilde{Y}_t}\right)^{-1}+\beta\theta\cdot E_t\left[\exp\{u_{c,t+1}-u_{c,t}\}\cdot\Pi_{t+1}^{\gamma-1}\cdot V_{t+1}\right]$$

$$\frac{O_t}{V_t}=\left(\frac{1-\theta}{1-\theta\cdot\Pi_t^{\gamma-1}}\right)^{\frac{1}{\gamma-1}}$$

$$\Delta_t=(1-\theta)\left(\frac{O_t}{V_t}\right)^{-\gamma}+\theta\Pi_t^{\gamma}\Delta_{t-1}$$

$$R_t^J=R^J\cdot\left(\frac{\Pi_t}{\Pi}\right)^{\tau_\pi}\left(\frac{\tilde{Y}_t}{\tilde{Y}}\right)^{\tau_y}\cdot\exp\{\varepsilon_{r,t}\},\ \varepsilon_{r,t}\sim N\left(0,\sigma_r^2\right)$$

$$\tilde{F}_t^*\equiv\frac{F_t^*}{A_t}=[1-H_t]^{-\frac{1}{\omega}}\left(\frac{\omega-1}{\omega}\right)\phi_f\cdot\tilde{Y}\cdot\exp\{u_{f,t}\}$$

$$R_t^{J,*}=\left(\frac{\omega}{\omega+1}\right)\cdot(1-H_t)^{-\frac{1}{\omega}}\cdot R_t^B$$

$$N_t=\Theta_N\cdot\left(\tilde{Y}_t\Delta_t\right)^{\frac{1}{\alpha}}\cdot(1+\Theta_4H_{t-1})^{\frac{\alpha+\sigma(1-\alpha)}{(1-\sigma)\alpha}}$$

$$g_{t,t+1}^{Density}=\left[\frac{1+\Theta_4\cdot H_{t-1}}{1+\Theta_4\cdot H_t}\right]^{\left(\frac{\sigma}{(\sigma-1)\alpha}\right)}\left(\frac{\tilde{Y}_{t+1}\Delta_{t+1}}{\tilde{Y}_t\Delta_t}\right)^{\frac{1}{\alpha}}-1$$

$$g_{t,t+1}^{Entry}=\frac{(H_t-H_{t-1})+\frac{\omega(\sigma-1)}{\kappa[\alpha+\sigma(1-\alpha)]+(\omega-1)(\sigma-1)}(H_{t-1}-H_t)}{H_{t-1}+\frac{\omega(\sigma-1)}{\kappa[\alpha+\sigma(1-\alpha)]+(\omega-1)(\sigma-1)}(1-H_{t-1})}$$

$$\frac{W_t}{P_tA_t}=\Theta_N^{\frac{1}{\eta}}\left(\tilde{C}_t\right)\left(\tilde{Y}_t\Delta_t\right)^{\frac{1}{\eta\alpha}}(1+\Theta_4H_{t-1})^{\frac{\alpha+\sigma(1-\alpha)}{\eta(1-\sigma)\alpha}}\cdot\exp\{-u_{c,t}\}$$

$$
\begin{aligned}
\frac{P_t^J}{P_t} &= \Theta_N^{\frac{1}{\eta}} \Theta_3^{\frac{\alpha+\sigma(1-\alpha)}{(\sigma-1)\alpha}} \left(\tilde{C}_t\right) \left(\tilde{Y}_t \Delta_t\right)^{\left(\frac{(1-\alpha)\eta+1}{\eta\alpha}\right)} \left(1+\Theta_4 H_{t-1}\right)^{\frac{(1+\eta)[\alpha+\sigma(1-\alpha)]}{\eta(1-\sigma)\alpha}} \cdot \exp\left\{-u_{c,t}\right\} \\
M_{t+1} &= 1 - \Theta_M \cdot [1 - H_t] \\
\frac{L_t/P_t}{\bar{Y}_t} &= \phi_f \cdot \left[1 - \Theta_L \cdot [1-H_t]^{\left(\frac{\omega-1}{\omega}\right)}\right] \cdot \exp\left\{u_{f,t}\right\} \\
GA_t &= (1+\mu) \cdot \exp\left\{u_{a,t}\right\}
\end{aligned}
$$

Shock processes:

$$
\begin{aligned}
u_{a,t} &= \rho_a \cdot u_{a,t-1} + \varepsilon_{a,t},\ \varepsilon_{a,t} \sim N\left(0,\ \sigma_a^2\right) \\
u_{c,t} &= \rho_c \cdot u_{c,t-1} + \varepsilon_{c,t},\ \varepsilon_{c,t} \sim N\left(0, \sigma_c^2\right) \\
u_{g,t} &= \rho_g \cdot u_{g,t-1} + \varepsilon_{g,t},\ \varepsilon_{g,t} \sim N\left(0,\ \sigma_g^2\right) \\
u_{f,t} &= \rho_f \cdot u_{f,t-1} + \varepsilon_{f,t},\ \varepsilon_{f,t} \sim N\left(0,\ \sigma_f^2\right)
\end{aligned}
$$

Parameters:

$$
\begin{aligned}
\Theta_1 &= \left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha}\right)^{\frac{\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}} \left(\frac{\kappa-1}{\kappa}\right)^{\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}} \left(\frac{\kappa[\alpha+\sigma(1-\alpha)]}{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}\right) \\
\Theta_2 &= \frac{\alpha+\sigma(1-\alpha)}{\alpha(\sigma-1)} \left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha}\right)^{-\frac{\sigma}{\alpha+\sigma(1-\alpha)}} \left(\frac{\kappa-1}{\kappa}\right)^{\frac{\alpha(\sigma-1)}{\alpha+\sigma(1-\alpha)}} \\
\Theta_3 &= \left(\frac{\kappa[\alpha+\sigma(1-\alpha)]+(\omega-1)(\sigma-1)}{\Theta_1\omega(\sigma-1)}\right) \\
\Theta_4 &= \left(\frac{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}{\omega(\sigma-1)}\right) \\
\Theta_N &= \left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha}\right)^{\left(\frac{-\sigma}{\alpha+\sigma(1-\alpha)}\right)} \left(\frac{\kappa-1}{\kappa}\right)^{\left(\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}\right)} \left(\frac{\kappa[\alpha+\sigma(1-\alpha)]}{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}\right) \\
&\quad \cdot \left(\frac{\omega(\sigma-1)}{\kappa[\alpha+\sigma(1-\alpha)]+(\omega-1)(\sigma-1)}\right) \Theta_3^{\left(\frac{\sigma}{\alpha(\sigma-1))}\right)} > 0 \\
\Theta_M &= \frac{\kappa[\alpha+\sigma(1-\alpha)]}{\kappa[\alpha+\sigma(1-\alpha)]+\omega(\sigma-1)} \\
\Theta_L &= \frac{\kappa[\alpha+\sigma(1-\alpha)]}{\kappa[\alpha+\sigma(1-\alpha)]+(\sigma-1)(\omega-1)}
\end{aligned}
$$

B.2 Flexible Price Equilibrium

$$1 = \left(\frac{\beta\Theta_2}{\left(\frac{\omega-1}{\omega}\right)\cdot\phi_f}\right)\cdot\left(\frac{(1+\zeta^T)^{-1}\gamma}{\gamma-1}\right)^{-1}\cdot\left(\frac{\kappa-1}{\kappa}\right)^{\left(\frac{(\sigma-1)(1-\alpha)}{\alpha+\sigma(1-\alpha)}\right)}\cdot\left[\frac{\Theta_3}{1+\Theta_4\cdot H_t}\right][1-H_t]^{\frac{1}{\omega}}$$
$$\cdot E_t\left[\left(\frac{\tilde{Y}_t}{\tilde{Y}}\right)\left(\frac{\tilde{C}_t/\tilde{Y}_t}{\tilde{C}_{t+1}/\tilde{Y}_{t+1}}\right)\cdot\exp\left\{u_{c,t+1}-(u_{f,t}+u_{c,t})\right\}\right]$$

$$\tilde{Y}_t = \left(\frac{(1+\zeta^T)^{-1}\gamma}{\gamma-1}\right)^{-\left(\frac{\eta\alpha}{(1-\alpha)\eta+1}\right)}\Theta_N^{-\left(\frac{\alpha}{(1-\alpha)\eta+1}\right)}\Theta_3^{-\frac{\eta[\alpha+\sigma(1-\alpha)]}{[(1-\alpha)\eta+1](\sigma-1)}}\cdot\tilde{C}_t^{-\left(\frac{\eta\alpha}{(1-\alpha)\eta+1}\right)}$$
$$\cdot(1+\Theta_4 H_{t-1})^{-\frac{(1+\eta)[\alpha+\sigma(1-\alpha)])}{(1-\sigma)[(1-\alpha)\eta+1]}}\cdot\exp\left\{\left(\frac{\eta\alpha}{(1-\alpha)\eta+1}\right)\cdot u_{c,t}\right\}$$

$$\frac{\tilde{C}_t}{\tilde{Y}_t} = 1-\phi_g\cdot\exp\{u_{g,t}\}-\phi_f\cdot\left(\frac{\tilde{Y}_t}{\tilde{Y}}\right)^{-1}\cdot\left[1-\Theta_L\cdot[1-H_t]^{\left(\frac{\omega-1}{\omega}\right)}\right]\cdot\exp\{u_{f,t}\}$$

$$\tilde{F}_t^* \equiv \frac{F_t^*}{A_t} = [1-H_t]^{-\frac{1}{\omega}}\left(\frac{\omega-1}{\omega}\right)\phi_f\cdot\tilde{Y}\cdot\exp\{u_{f,t}\}$$

$$R_t^J = R^J\cdot\left(\frac{\Pi_t}{\Pi}\right)^{\tau_\pi}\left(\frac{\tilde{Y}_t}{\tilde{Y}}\right)^{\tau_y}\cdot\exp\{\varepsilon_{r,t}\}$$

$$R_t^{J,*} = \left(\frac{\omega}{\omega+1}\right)\cdot(1-H_t)^{-\frac{1}{\omega}}\cdot R_t^B$$

Shock processes:

$$GA_t = (1+\mu)\cdot exp\{u_{a,t}\}$$
$$u_{a,t} = \rho_a\cdot u_{a,t-1}+\varepsilon_{a,t}$$
$$u_{c,t} = \rho_c\cdot u_{c,t-1}+\varepsilon_{c,t}$$
$$u_{g,t} = \rho_g\cdot u_{g,t-1}+\varepsilon_{g,t}$$
$$u_{f,t} = \rho_f\cdot u_{f,t-1}+\varepsilon_{f,t}$$
$$\varepsilon_{c,t} \sim N\left(0,\sigma_c^2\right)$$
$$\varepsilon_{a,t} \sim N\left(0,\ \sigma_a^2\right)$$
$$\varepsilon_{g,t} \sim N\left(0,\ \sigma_g^2\right)$$
$$\varepsilon_{f,t} \sim N\left(0,\ \sigma_f^2\right)$$
$$\varepsilon_{r,t} \sim N\left(0,\sigma_r^2\right)$$

B.3 Steady State Conditions

$$
\begin{aligned}
R^B &= \beta^{-1}(1+\mu)\Pi \\
\Delta &= \left(\frac{1-\theta}{1-\theta\Pi^{\gamma}}\right)\left(\frac{1-\theta\Pi^{\gamma-1}}{1-\theta}\right)^{\left(\frac{\gamma}{\gamma-1}\right)} \\
\frac{\Theta_3\cdot[1-H]^{\frac{1}{\omega}}}{1+\Theta_4\cdot H} &= \left(\frac{\kappa-1}{\kappa}\right)^{\left(\frac{(1-\sigma)(1-\alpha)}{\alpha+\sigma(1-\alpha)}\right)}\left[\frac{(1+\zeta^T)^{-1}\gamma}{\gamma-1}\right]\left[\frac{1-\theta\Pi^{\gamma}}{1-\theta\Pi^{\gamma-1}}\right]\left[\frac{1-\beta\theta\Pi^{\gamma-1}}{1-\beta\theta\Pi^{\gamma}}\right]\left(\frac{\left(\frac{\omega-1}{\omega}\right)\phi_f}{\beta\cdot\Theta_2}\right) \\
\tilde{Y} &= \frac{\left(\frac{\beta\Theta_2\Theta_N^{\frac{1}{\eta}}\Theta_3^{\frac{\sigma}{(\sigma-1)\alpha}}}{\left(\frac{\omega-1}{\omega}\right)\phi_f}\right)^{-\left(\frac{\eta\alpha}{\eta+1}\right)}\left(\frac{\kappa-1}{\kappa}\right)^{\left(\frac{-\eta\alpha(\sigma-1)(1-\alpha)}{[\alpha+\sigma(1-\alpha)](\eta+1)}\right)}(1+\Theta_4 H)^{-\frac{[\alpha+\sigma(1-\alpha)]+\sigma\eta}{(\eta+1)(1-\sigma)}}(1-H)^{-\frac{\eta\alpha}{\omega(\eta+1)}}}{\Delta\cdot\left[1-\phi_g-\phi_f\cdot\left[1-\Theta_L\cdot[1-H]^{\left(\frac{\omega-1}{\omega}\right)}\right]\right]^{\left(\frac{\eta\alpha}{\eta+1}\right)}} \\
\tilde{C} &= \left[1-\phi_g-\phi_f\cdot\left[1-\Theta_L\cdot[1-H]^{\left(\frac{\omega-1}{\omega}\right)}\right]\right]\cdot\tilde{Y} \\
M &= 1-\Theta_M\cdot[1-H] \\
\tilde{F}^* &= [1-H]^{-\frac{1}{\omega}}\left(\frac{\omega-1}{\omega}\right)\phi_f\cdot\tilde{Y} \\
R^{J,*} &= \left(\frac{\omega}{\omega+1}\right)\cdot(1-H)^{-\frac{1}{\omega}}\cdot\beta^{-1}(1+\mu)\Pi \\
\frac{R^{J,*}}{R^B} &= \left(\frac{\omega}{\omega+1}\right)\cdot(1-H)^{-\frac{1}{\omega}} \\
N &= \Theta_N\cdot\tilde{Y}^{\frac{1}{\alpha}}\cdot\Delta^{\frac{1}{\alpha}}\cdot(1+\Theta_4 H)^{\frac{\alpha+\sigma(1-\alpha)}{(1-\sigma)\alpha}} \\
\frac{W}{PA} &= \Theta_N^{\frac{1}{\eta}}\tilde{C}\tilde{Y}^{\frac{1}{\eta\alpha}}\Delta^{\frac{1}{\eta\alpha}}(1+\Theta_4 H)^{\frac{\alpha+\sigma(1-\alpha)}{\eta(1-\sigma)\alpha}} \\
\frac{P_t^J}{P_t} &= \Theta_N^{\frac{1}{\eta}}\Theta_3^{\frac{\alpha+\sigma(1-\alpha)}{(\sigma-1)\alpha}}\tilde{C}(\tilde{Y}\Delta)^{\left(\frac{(1-\alpha)\eta+1}{\eta\alpha}\right)}(1+\Theta_4 H)^{\frac{(1+\eta)[\alpha+\sigma(1-\alpha)]}{\eta(1-\sigma)\alpha}} \\
O &= \frac{(1+\zeta^T)^{-1}\gamma}{\gamma-1}\cdot\frac{\Theta_N^{\frac{1}{\eta}}\Theta_3^{\frac{\alpha+\sigma(1-\alpha)}{(\sigma-1)\alpha}}\tilde{Y}^{\left(\frac{\eta+1}{\eta\alpha}\right)}\Delta^{\frac{(1-\alpha)\eta+1}{\eta\alpha}}(1+\Theta_4 H)^{\frac{(1+\eta)[\alpha+\sigma(1-\alpha)]}{\eta(1-\sigma)\alpha}}}{1-\beta\theta\Pi^{\gamma}} \\
V &= \frac{\left(\frac{\tilde{C}}{\tilde{Y}}\right)^{-1}}{1-\beta\theta\Pi^{\gamma-1}} \\
\frac{L/P}{\tilde{Y}} &= \phi_f\left[1-\Theta_L(1-H)^{\frac{\omega-1}{\omega}}\right]
\end{aligned}
$$

C Estimation of Satiation Measure H_t and the Policy Room

Since the satiation measure H_t and the policy room $\frac{R_t^B}{R_t^{J,*}}$ are not directly observable, we propose two methods to infer these unobservable measures from the data. By estimating the H_t series from observable data, the policy room measure can be straightforwardly derived, given the close relationship between the two.

C.1 Version 1: Estimation of Satiation Measure H_t

First, we can divide equation (26) by its steady-state expression to obtain:

$$\frac{N_t}{N} = \left(\frac{\tilde{Y}_t}{\tilde{Y}} \cdot \frac{\Delta_t}{\Delta}\right)^{\frac{1}{\alpha}} \cdot \left(\frac{1+\Theta_4 H_{t-1}}{1+\Theta_4 H}\right)^{\frac{\alpha+\sigma(1-\alpha)}{(1-\sigma)\alpha}} .$$

Taking logs, we obtain:

$$\hat{n}_t = \frac{1}{\alpha}\left(\hat{\tilde{y}}_t + \widehat{\log(\Delta_t)}\right) + \left(\frac{\alpha+\sigma(1-\alpha)}{(1-\sigma)\alpha}\right)\widehat{\log(1+\Theta_4 H_{t-1})}.$$

We now proceed by replacing $\hat{n}_t$, $\hat{\tilde{y}}_t$ and $\widehat{\log(\Delta_t)}$ with the HP-filtered empirical estimates based on data on employment, real GDP, and price dispersion, respectively.[2] Once we have these empirical estimates, we plug them into the previous equation, through which we obtain an estimate of $\widehat{\log(1+\Theta_4 H_{t-1})}$ as:

$$\text{Estimate}\left(\widehat{\log(1+\Theta_4 H_{t-1})}\right) = \left(\frac{(1-\sigma)\alpha}{\alpha+\sigma(1-\alpha)}\right)\left[\hat{n}_t - \frac{1}{\alpha}\left(\hat{\tilde{y}}_t + \widehat{\log(\Delta_t)}\right)\right],$$

which leads to

$$\text{Estimate}(H_{t-1}) = \frac{1}{\Theta_4}\cdot\left[\exp\left\{\text{Estimate}\left(\widehat{\log(1+\Theta_4 H_{t-1})}\right) + \log(1+\Theta_4 H)\right\} - 1\right].$$

For the data on N_t, we use (i) the number of employees, (ii) average weekly hours, (iii) index of average weekly hours, all from CES National Databases in the Bureau of Labor Statistics (BLS). Figure C.1 depicts H_t series recovered with this method. We can observe that H_t is strongly procyclical.

[2] Notice that we HP-filter the logs of each variable, not their levels.

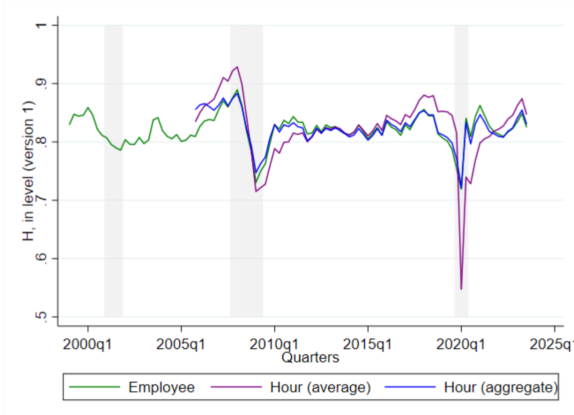


Figure C.1: Satiation measure H_t, Version 1

Notes: The three series are estimated based on three sources of information on the employment N. Employee: number of employees. Hour (average): average weekly hour. Hour (aggregate): aggregate weekly hour, in thousands. All from BLS CES National Databases.

C.2 Version 2: Estimation of Satiation Measure H_t

Instead of relying on employment data, we directly use the number of establishments from the Quarterly Census of Employment and Wages (QCEW) as a proxy for fluctuations in firm participation, denoted as M_{t+1}, at business cycle frequencies.[3] Dividing equation (15) by its steady state equivalent expression, we obtain:

$$\frac{M_{t+1}}{M} = \frac{1 - \Theta_M \cdot [1 - H_t]}{1 - \Theta_M \cdot [1 - H]} .$$

Taking logs on both sides, with $m_t \equiv \log M_t$:

$$\hat{m}_{t+1} = \log\left(1 - \widehat{\Theta_M \cdot [1} - H_t]\right) ,$$

so we can back out H_t as:

$$H_t = -\frac{1}{\Theta_M} \cdot \left[1 - \Theta_M - \exp\left\{\hat{m}_{t+1} + \log\left(1 - \Theta_M \cdot [1 - H]\right)\right\}\right] ,$$

where $\hat{m}_{t+1}$ is the HP-filtered data of the log of firm participation. The spikiness of estimated H under Version 2 comes from the spikiness of the number of establishments data.

[3] As shown in Figure C.2, we also derive the H_t series using various measures of firm participation, including the number of establishments from the Business Employment Dynamics (BED) of the BLS.

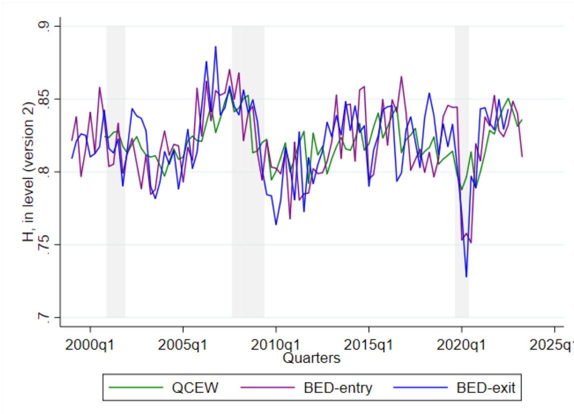


Figure C.2: Satiation measure H_t, Version 2.

Notes: The time series are estimated based on three sources of information on the operating firms M. The green line is based on the number of establishments in the QCEW database. The purple (blue) line is the number of establishments based on firm entry (exit) information in BED. The equation is:
Number of establishments $= \frac{\text{number of establishments with employment gains (loss)}}{\text{percentage of establishments with employment gains (loss)}}$.

Comparison Figure C.3 compares the H_t series recovered by Version 1 (based on the number of employees) and Version 2 (based on the number of establishments from the QCEW database). Version 1 generates a more volatile H_t series. As our model lacks physical capital, using the labor demand formula (i.e., equation (26)) to recover the satiation measure H_t might overemphasize its role in driving labor demand fluctuations, leading to more volatile H_t time series.

In our baseline empirical specification in Section 5, we use Version 2, based on the number of establishments from the QCEW database, as the benchmark. Results based on Version 1, which uses the total number of employees to measure the H_t series, are provided in Appendix E as additional robustness checks.

C.3 Estimation of the Fixed Cost Process: ϕ_f, ρ_f, and σ_f

We start from:

$$\left(\frac{L_t}{P_t Y_t}\right)\left(\frac{\tilde{Y}_t}{\tilde{Y}}\right) = \phi_f \cdot \left[1 - \Theta_L \cdot (1 - H_t)^{\frac{\omega-1}{\omega}}\right] \cdot \exp\left(u_t^f\right) ,$$

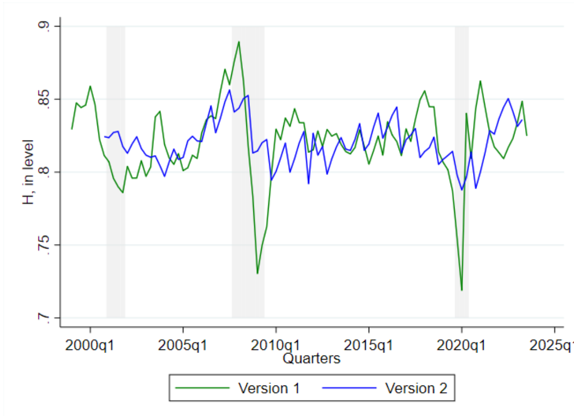


Figure C.3: Satiation Measure H_t, Version 1 vs. Version 2.

Notes: The H time series within each panel are estimated using two different methods. The green line is based on Version 1 with N measured by the total number of employees. The blue line is based on Version 2 with M measured by the number of establishments from the QCEW database.

where the left-hand side is written in the current (private) loan-to-GDP ratio and the output gap. Taking logs and rearranging, we obtain:

$$u_t^f = \log\left(\frac{L_t}{P_t Y_t}\right) + \tilde{y}_t - \log(\phi_f) - \log\left[1 - \Theta_L \cdot (1 - H_t)^{\frac{\omega-1}{\omega}}\right] \, . \tag{C.1}$$

Once we have an estimate for H_t series and ϕ_f, we back out u_t^f from (C.1) and estimate the AR(1) process parameters ρ_f and σ_f as follows:

$$u_t^f = \rho_f \cdot u_{t-1}^f + \varepsilon_t^f, \quad \varepsilon_t^f \sim N\left(0, \sigma_f^2\right) \, . \tag{C.2}$$

Appendix C.3.1 explains in detail how we obtain ϕ_f, ρ_f, and σ_f based on equation (C.1) and equation (C.2).

Figure C.4 plots the time series of nominal private debt to nominal GDP ratio measured by debt securities and loans to GDP ratio for nonfinancial corporate business, which we use as a proxy for $\frac{L_t}{P_t Y_t}$.[4]

[4]Data source: https://fred.stlouisfed.org/graph/?g=VLW.

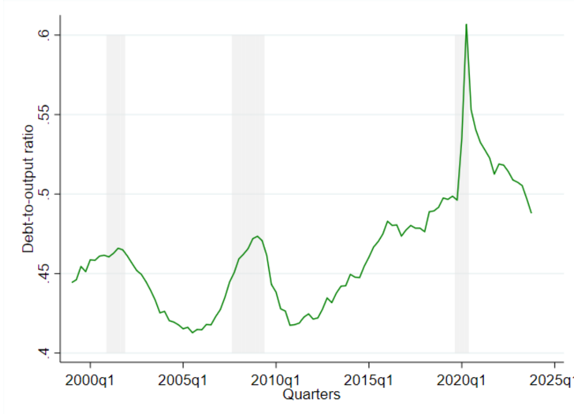


Figure C.4: Private Loan-to-output ratio.

C.3.1 Detailed Estimation Procedure

Step 1 Given the calibration in Table 1, we solve for ϕ_f and H from the following two steady state expressions:

$$\frac{\Theta_3 \cdot [1-H]^{\frac{1}{\omega}}}{1+\Theta_4 \cdot H} = \left(\frac{\kappa-1}{\kappa}\right)^{\left(\frac{(1-\sigma)(1-\alpha)}{\alpha+\sigma(1-\alpha)}\right)} \left[\frac{(1+\zeta^T)^{-1}\gamma}{\gamma-1}\right] \left[\frac{1-\theta\Pi^{\gamma}}{1-\theta\Pi^{\gamma-1}}\right] \left[\frac{1-\beta\theta\Pi^{\gamma-1}}{1-\beta\theta\Pi^{\gamma}}\right] \left(\frac{\left(\frac{\omega-1}{\omega}\right)\phi_f}{\beta\cdot\Theta_2}\right) ,$$

and

$$\frac{L}{P\bar{Y}} = \phi_f \left[1-\Theta_L(1-H)^{\frac{\omega-1}{\omega}}\right] ,$$

where we calibrate the steady state loan-to-output ratio to its sample average:

$$\frac{L}{P\bar{Y}} = \text{Average}\left\{\frac{L_t}{P_t Y_t}\right\} ,$$

and obtain a system of two equations in the unknown ϕ_f and H values, which we find numerically.

Step 2 Compute H_t (either from Version 1 or Version 2) using the above calculated ϕ_f.

Step 3 Back out u_t^f from equation (C.1) using the estimated series for H_t and the parameter estimate for ϕ_f. Then, estimate the parameters ρ_f and σ_f in equation (C.2) via OLS regression and the volatility of the regression residuals, respectively.

C.4 Estimation of the Policy Room $\frac{R_t^B}{R_t^{J,*}}$

As policy room $\frac{R_t^B}{R_t^{J,*}}$ is not observable, we use the estimated H_t series (either from Version 1 or Version 2) in recovering the $\frac{R_t^B}{R_t^{J,*}}$ series. Rearranging equation (32), we obtain:

$$\frac{R_t^B}{R_t^{J,*}} = \left(\frac{\omega+1}{\omega}\right) \cdot (1-H_t)^{\frac{1}{\omega}} \ ,$$

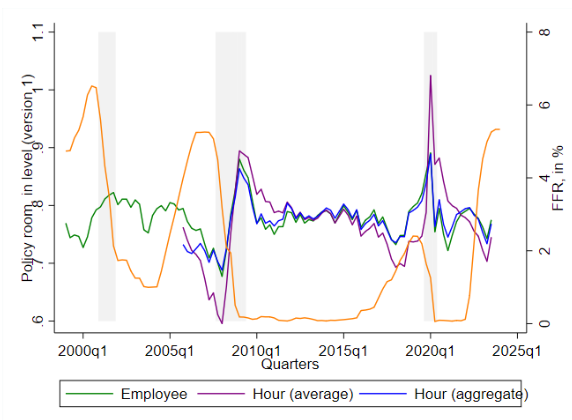
which we can estimate by plugging in the values of estimated H_t. Dividing the previous equation by its steady-state value, we obtain:

$$\left(\frac{R_t^B}{R_t^{J,*}}\right) \div \left(\frac{R^B}{R^{J,*}}\right) = \left(\frac{1-H_t}{1-H}\right)^{\frac{1}{\omega}} \ ,$$

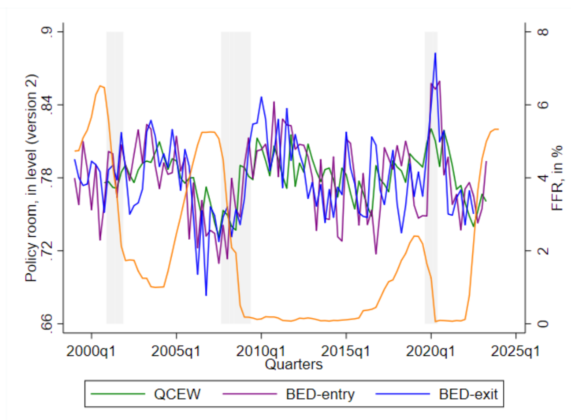
which in log-deviations becomes:

$$\widehat{r_t^B - r_t^{J,*}} = \frac{1}{\omega} \cdot \log\widehat{(1-H_t)} \ , \tag{C.3}$$

which is the expression that we use as a proxy for the policy room in our empirical analysis. Figure C.5 depicts the policy room series based on Version 1 and Version 2, respectively. We observe that the policy room tends to spike during recessions where the policy rate tends to be low and around zero (i.e., zero lower bound).



(a) Policy room: Version 1



(b) Policy room: Version 2

Figure C.5: Policy room, Version 1 and Version 2.

Notes: For Version 1, the three series are estimated based on three sources of information on the employment N. Employee: number of employees. Hour (average): average weekly hour. Hour (aggregate): aggregate weekly hour, in thousands. The yellow solid line is the federal funds rates. For Version 2, the three time series are estimated based on three sources of information on the operating firms M. The green line is based on the number of establishments in QECW database. The purple (blue) line is the number of establishments based on firm entry (exit) information in BED. The yellow line is the federal funds rates. The equation is:

Number of establishments $= \frac{\text{number of establishments with employment gains (loss)}}{\text{percentage of establishments with employment gains (loss)}}$.

D Additional Tables and Figures

D.1 Section 3.2

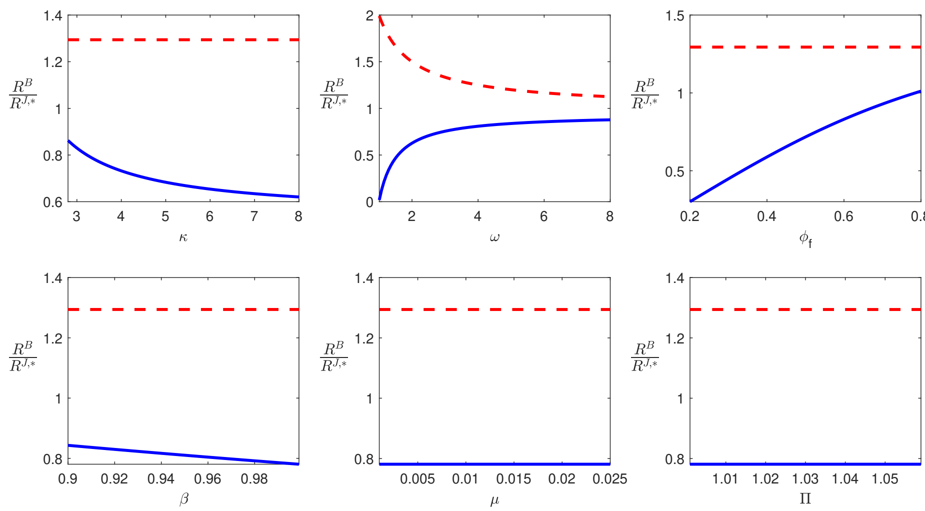


Figure D.6: Comparative Statics, M.

Notes: This figure displays how variations in other structural parameters affect the relation between M and the structural parameters.

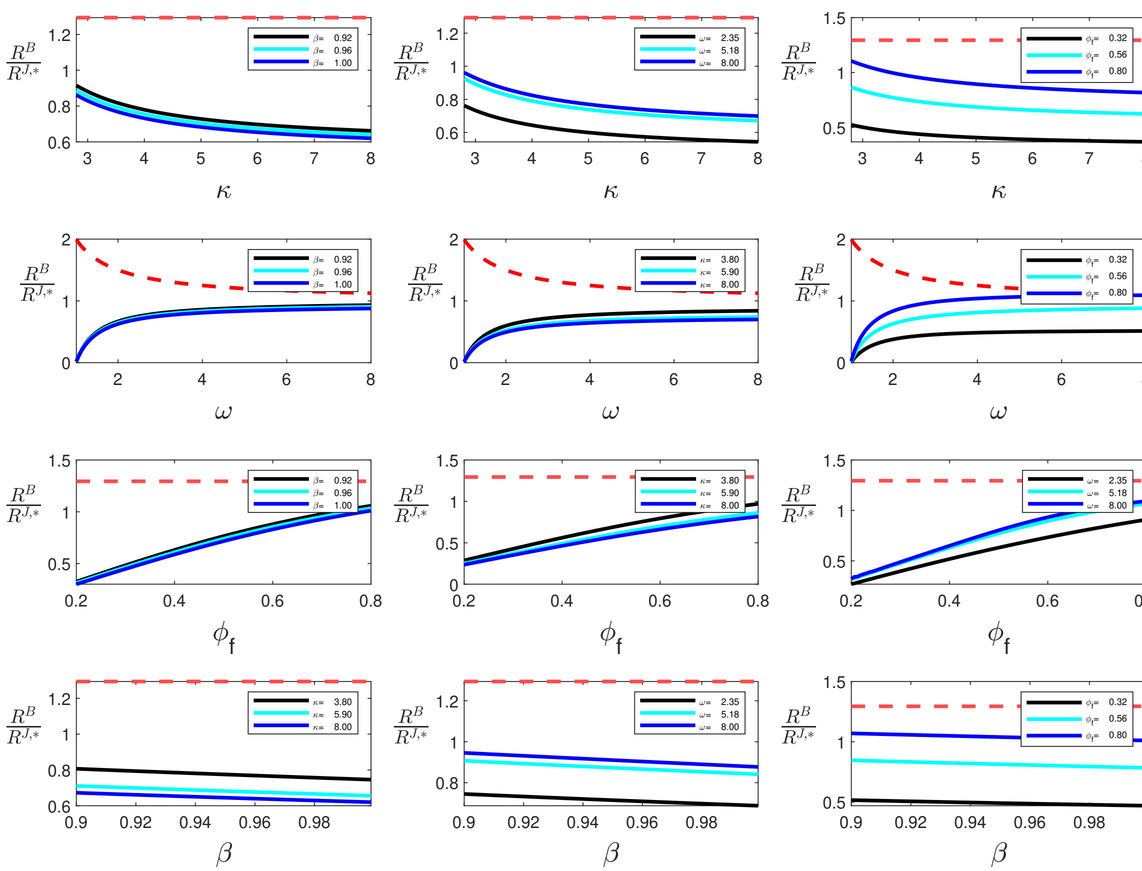


Figure D.7: Comparative Statics, Policy Room.

Notes: This figure display how κ, ω, and ϕ_f affect the relationship between the policy room and the parameters.

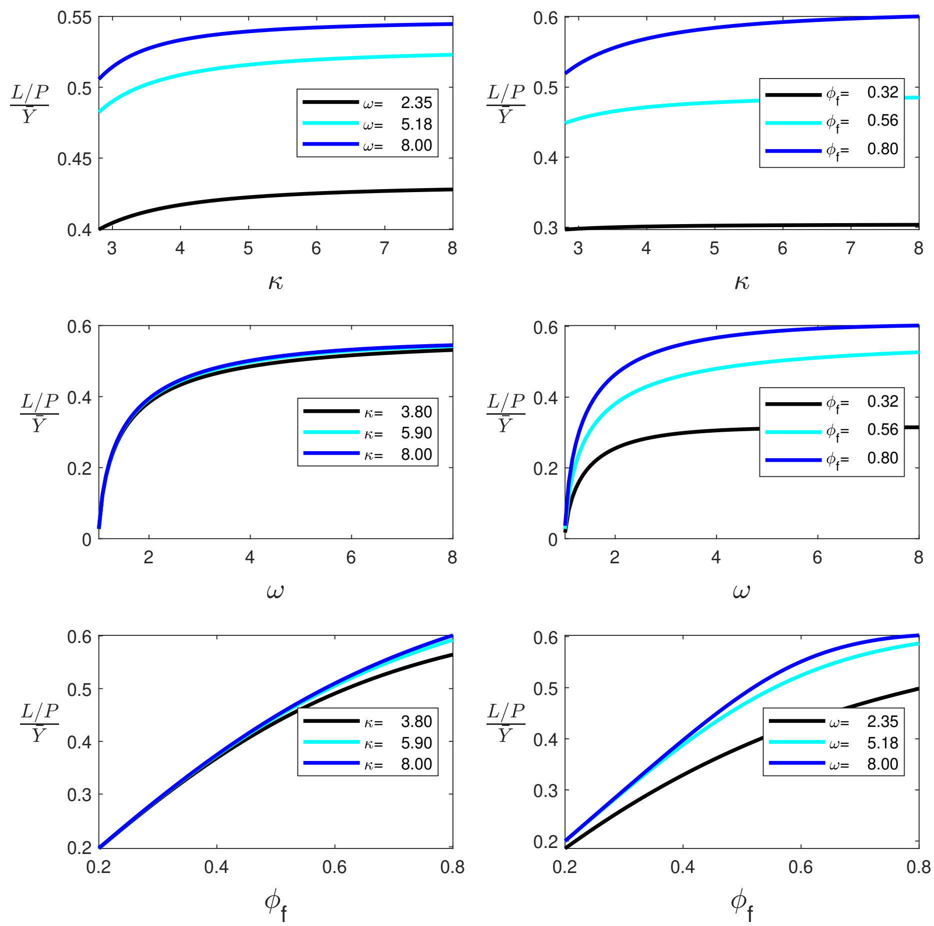


Figure D.8: Comparative Statics, Loan-to-output ratio.

Notes: This figure display how κ, ω, and ϕ_f affect the relationship between $\frac{L/P}{Y}$ and the parameters.

D.2 Section 4.1

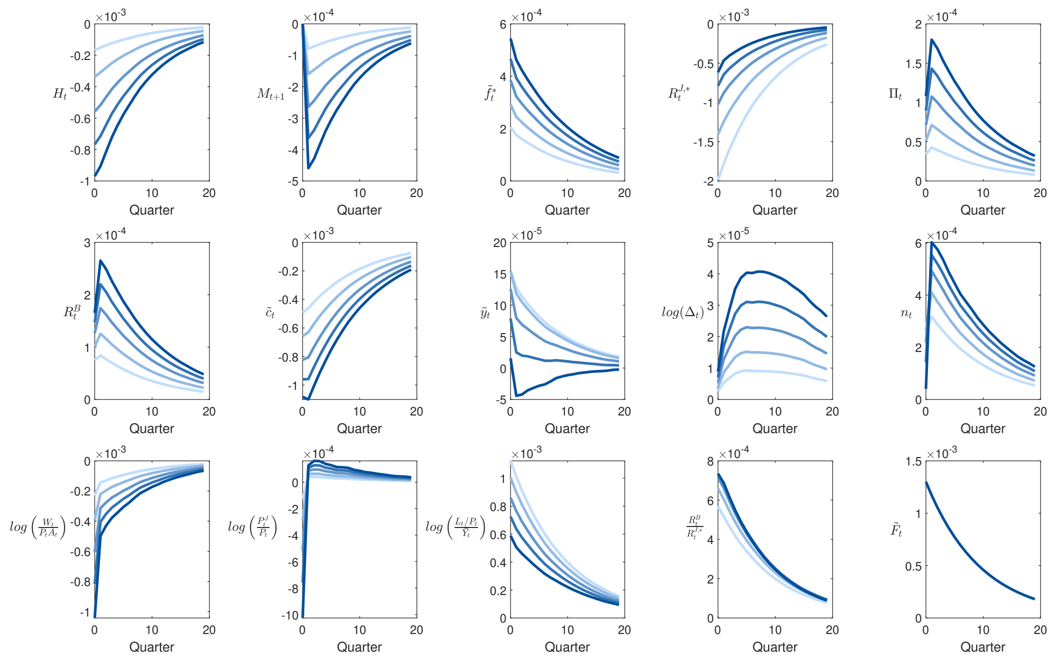


Figure D.9: Impulse response functions to fixed cost shock.

Notes: The figures display the impulse response functions to a one standard deviation shock (0.0013) in $u_{f,t}$, the fixed cost shock. The gradient blue lines denote the responses under calibrations with varying ϕ_f. From light blue to dark blue, ϕ_f values are 0.35, 0.45, 0.5547 (benchmark), 0.65, and 0.75. The following variables are plotted in deviations from their steady states: H, M, R^B, Π, and $R^{J,*}$. The remaining variables are plotted in log deviations from their steady states (in lower case letters or with a log). Δ represents the price dispersion for the downstream products. $W_t/(P_tA_t)$ is the real wage. P_t^J/P_t measures the aggregate price for the upstream products or the input price for the downstream firms.

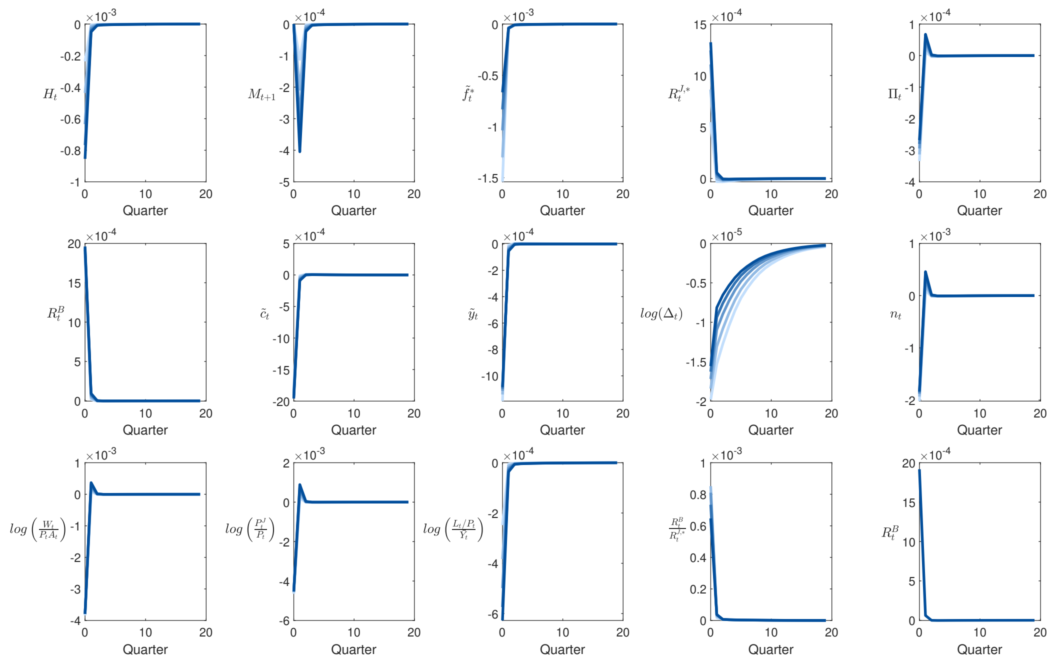


Figure D.10: Impulse response functions to monetary policy shock.

Notes: The figures display the impulse response functions to a one standard deviation shock (0.0025) in $\epsilon_{r,t}$, the monetary policy shock. The gradient blue lines denote the responses under calibrations with varying ϕ_f. From light blue to dark blue, ϕ_f values are 0.35, 0.45, 0.5547 (benchmark), 0.65, and 0.75. The following variables are plotted in deviations from their steady states: H, M, R^B, Π, and $R^{J,*}$. The remaining variables are plotted in log deviations from their steady states (in lower case letters or with a log). Δ represents the price dispersion for the downstream products. $W_t/(P_tA_t)$ is the real wage. P_t^J/P_t measures the aggregate price for the upstream products or the input price for the downstream firms.

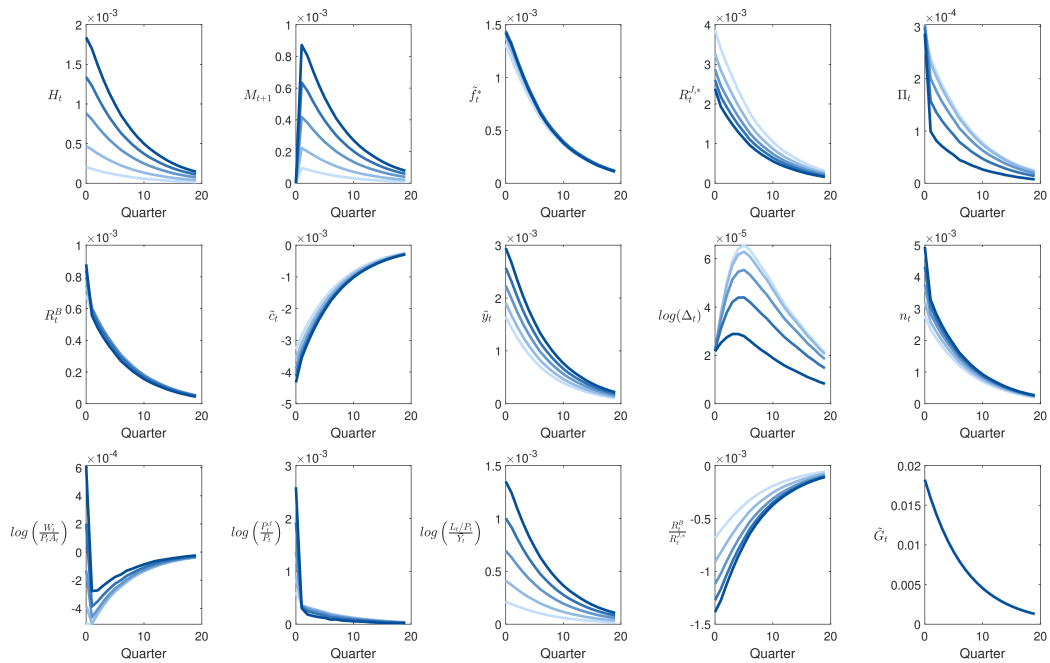


Figure D.11: Impulse response functions to government spending shock.

Notes: The figures display the impulse response functions to a one standard deviation shock (0.016) in $u_{g,t}$, which denotes the government spending shock. The gradient blue lines denote the responses under calibrations with varying ϕ_f. From light blue to dark blue, ϕ_f values are 0.35, 0.45, 0.5547 (benchmark), 0.65, and 0.75. The following variables are plotted in deviations from their steady states: H, M, R^B, Π, and $R^{J,*}$. The remaining variables are plotted in log deviations from their steady states (in lower case letters or with a log). Δ represents the price dispersion for the downstream products. $W_t/(P_t A_t)$ is the real wage. P_t^J/P_t measures the aggregate price for the upstream products or the input price for the downstream firms.

D.3 Section 4.3

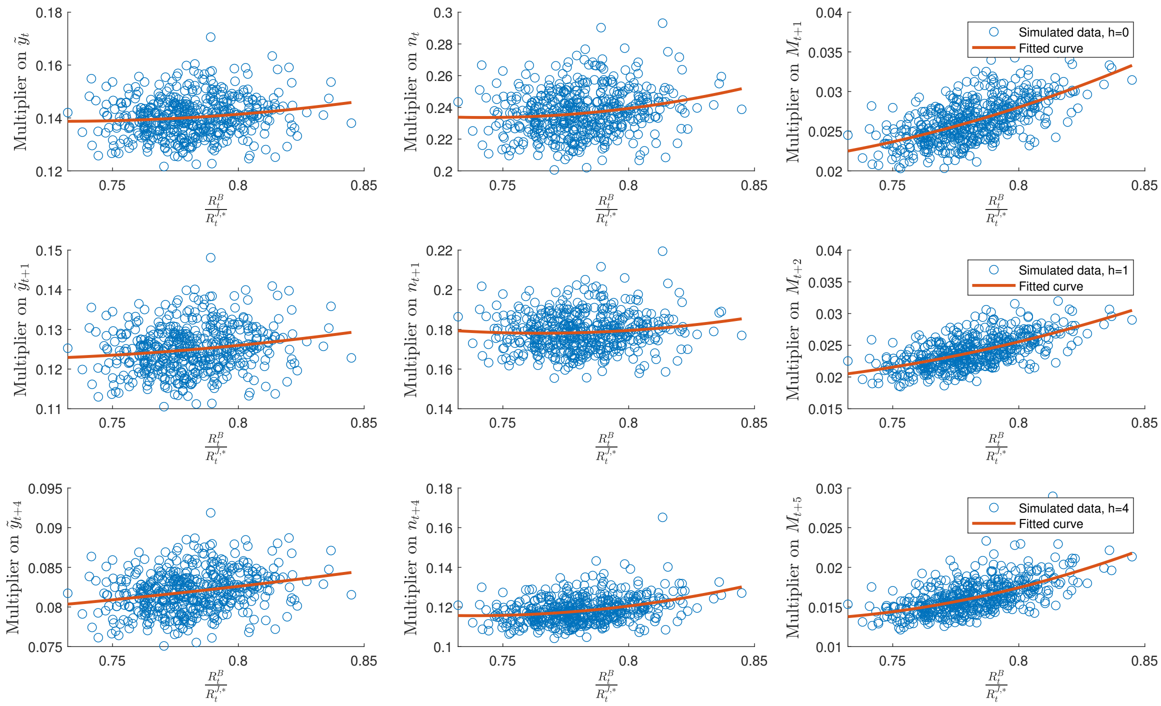


Figure D.12: Scatter plot between policy room and government spending multipliers.

Notes: Figures plot the relationship between policy room and government spending multipliers on output (in logs), labor (in logs), and next period mass of operating firms (in levels). We consider the next period's mass of operating firms since the firms paying the fixed cost at t will operate on the market at $t+1$. Figures in the first to third rows display the contemporaneous multipliers ($h = 0$), multipliers after 1 quarter ($h = 1$), and multipliers after 4 quarters ($h = 4$) correspondingly. The blue circles represent the result from each simulation based on solutions from the third-order perturbation method. The red solid lines are fitted second-order polynomials.

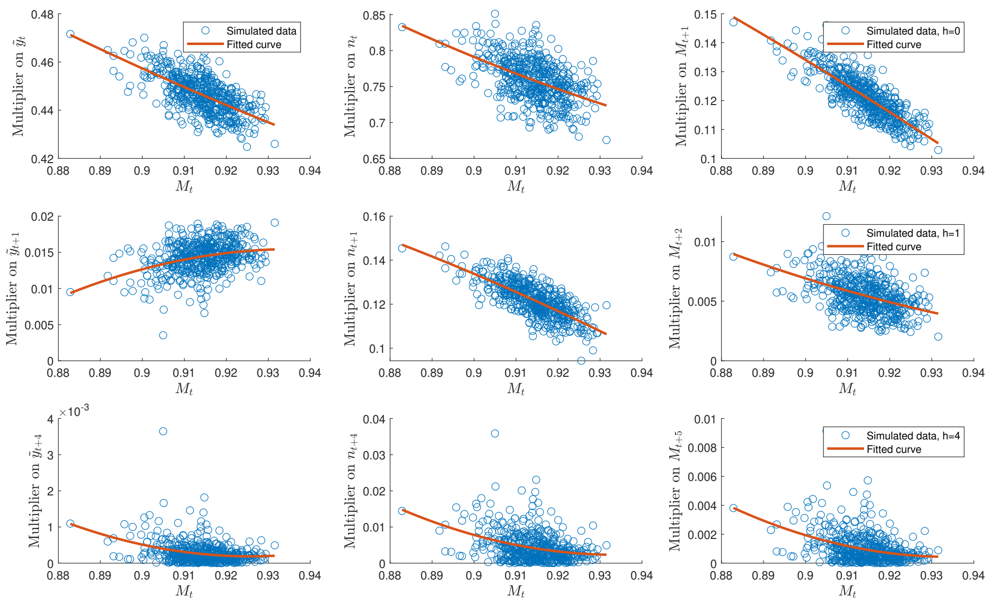


Figure D.13: Scatter plot between the mass of firms and monetary policy multipliers.

Notes: Figures plot the relationship between the current mass of operating firms and monetary policy multipliers on output (in logs), labor (in logs), and next period mass of operating firms (in levels). We consider the next period's mass of operating firms since the firms paying the fixed cost at t will operate on the market at $t + 1$. Figures in the first to third rows display the contemporaneous multipliers ($h = 0$), multipliers after 1 quarter ($h = 1$), and multipliers after 4 quarters ($h = 4$) correspondingly. The blue circles represent the result from each simulation based on solutions from the third-order perturbation method. The red solid lines are fitted second-order polynomials.

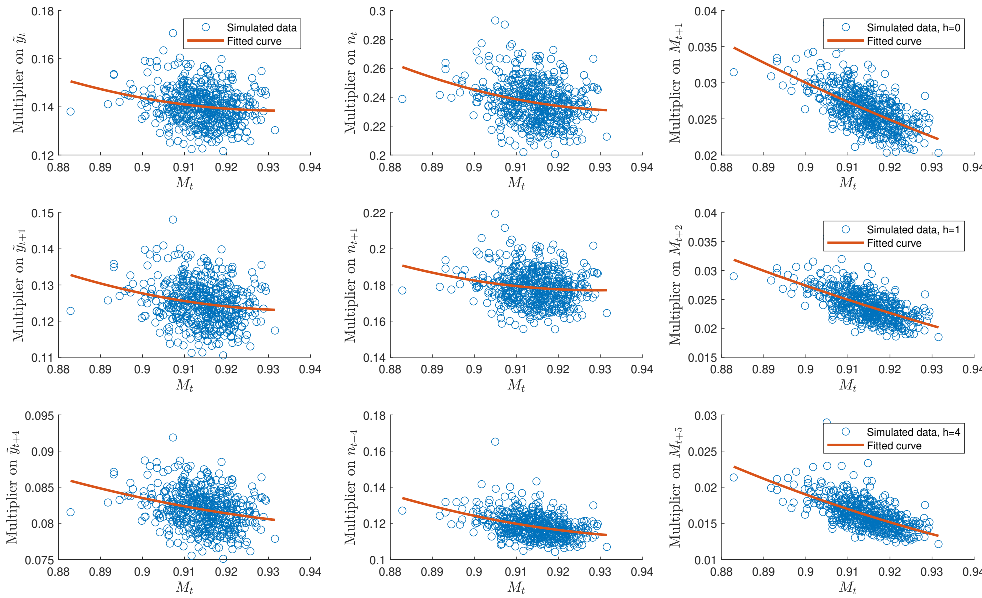


Figure D.14: Scatter plot between the mass of firms and government spending multipliers.

Notes: Figures plot the relationship between the current mass of operating firms and government spending multipliers on output (in logs), labor (in logs), and next period mass of operating firms (in levels). We consider the next period's mass of operating firms since the firms paying the fixed cost at t will operate on the market at $t+1$. Figures in the first to third rows display the contemporaneous multipliers ($h = 0$), multipliers after 1 quarter ($h = 1$), and multipliers after 4 quarters ($h = 4$) correspondingly. The blue circles represent the result from each simulation based on solutions from the third-order perturbation method. The red solid lines are fitted second-order polynomials.

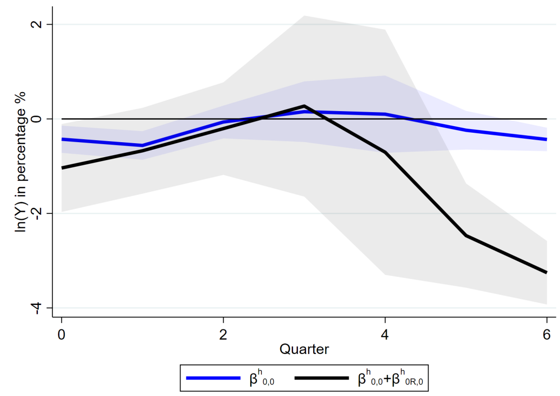
E Robustness in Section 5

E.1 Robustnes: the Policy Room Recovered by Version 1

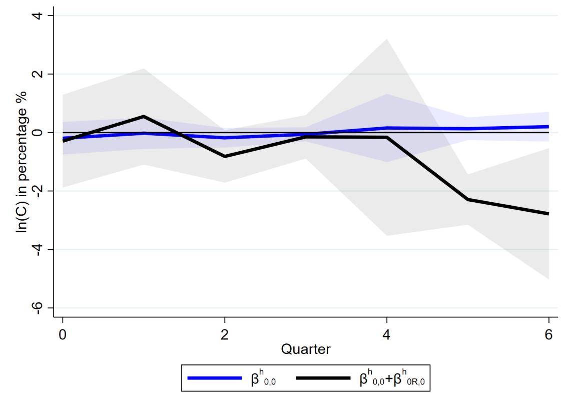
Now, we run our benchmark regression based on the policy room measure recovered by Version 1 in Appendix C.1 and Appendix C.4, based on the total number of employees from CES National Databases in the Bureau of Labor Statistics (BLS). In this case, we use monetary shock series from either Wieland and Yang (2020) or Acosta (2023), who both extended the shock series of Romer and Romer (2004).

Figures E.15 (with Wieland and Yang (2020) monetary policy shock series) and E.16 (with Acosta (2023) shock series) display the impulse response functions of output, consumption, and unemployment to monetary policy shocks and the interaction of monetary policy shocks with policy room deviation constructed from Version 1 (with the employment measured by the number of employees from BLS). Overall results are similar, even if they become less significant with Acosta (2023) and the policy room constructed with Version 1. As our model lacks physical capital, using the formula for labor demand (i.e., equation (26)) to recover the satiation measure H_t and the policy room $\frac{R_t^B}{R_t^{J,*}}$ might overemphasize the role of the policy room in driving labor demand fluctuations, lowering the significance of the results.

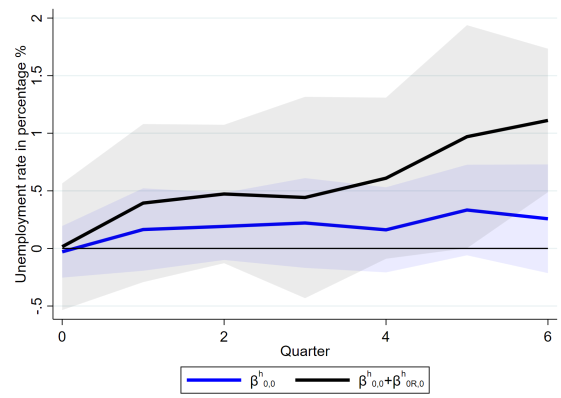
With Additional Controls We add more controls to our benchmark regression with the policy room measure recovered from Version 1 and test the robustness of our results. The controls are current and four lags of federal funds rates, four lags of oil price growth rate, four lags of long-term interest rate, four lags of consumption growth rate, four lags of GDP deflator, four lags of shadow federal funds rate from Wu and Xia (2016). Figure E.17 shows that the additional controls produce little to no difference in the impulse responses of the selected variables compared to the benchmark specification displayed in Figure E.16.



(a) log(Y) with Wieland and Yang (2020)



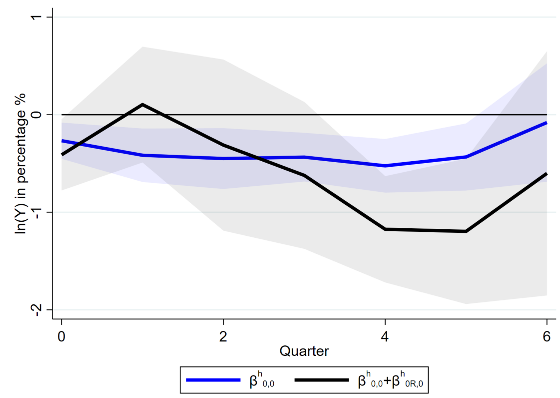
(b) log(C) with Wieland and Yang (2020)



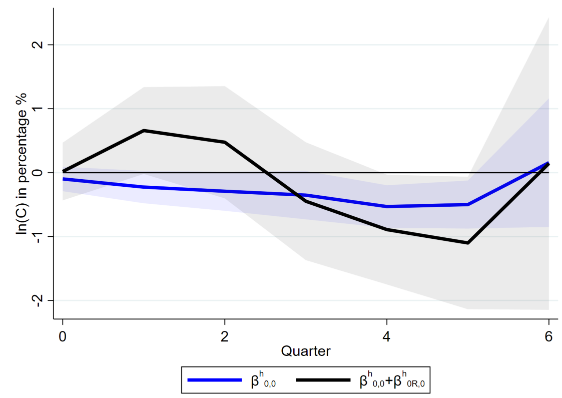
(c) Unemployment rate with Wieland and Yang (2020)

Figure E.15: Local projection with policy room from Version 1.

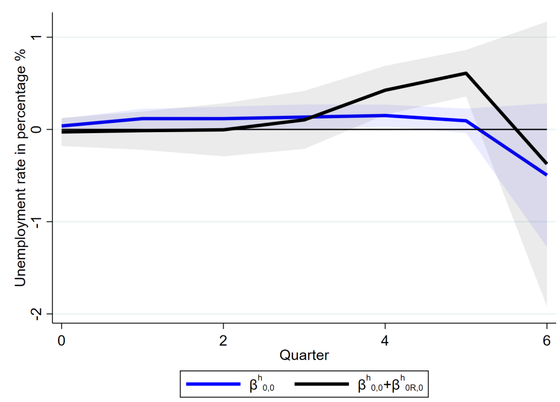
Notes: The impulse response functions are based on the benchmark regression with monetary policy shocks from Wieland and Yang (2020), which controls for current and four lags of federal funds rate. The figure reports the 95% confidence band, constructed using Newey-West standard errors under the assumption of serial correlation of the same lag length as the regression horizon, h.



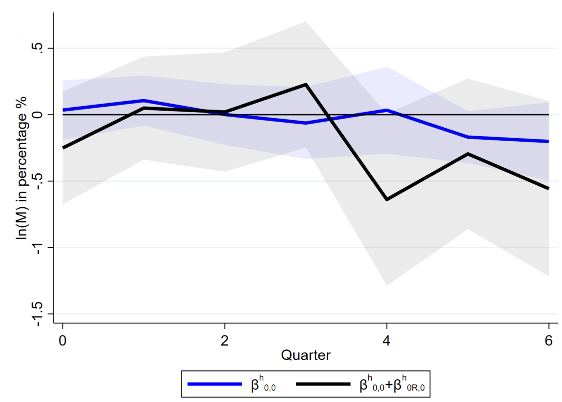
(a) log(Y), in % with Acosta (2023)



(b) log(C), in % with Acosta (2023)



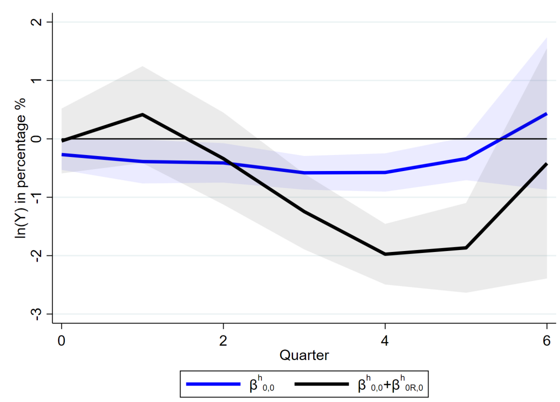
(c) Unemployment rate, in % with Acosta (2023)



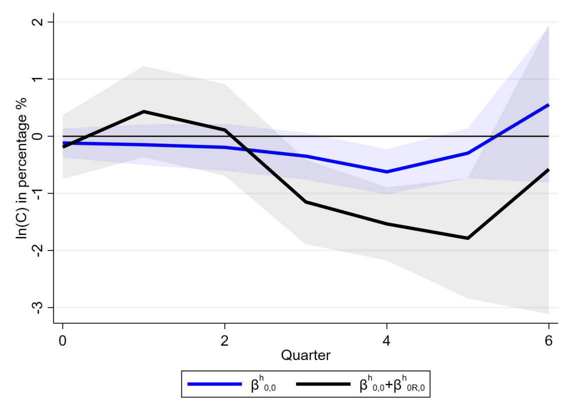
(d) log(M), in % with Acosta (2023)

Figure E.16: Local projection with policy room from Version 1.

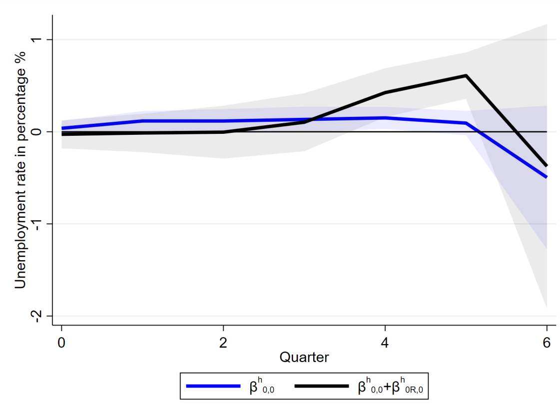
Notes: The impulse response functions are based on the benchmark regression with monetary policy shocks from Acosta (2023), which controls for current and four lags of federal funds rate. The figure reports the 95% confidence band, constructed using Newey-West standard errors under the assumption of serial correlation of the same lag length as the regression horizon, h.



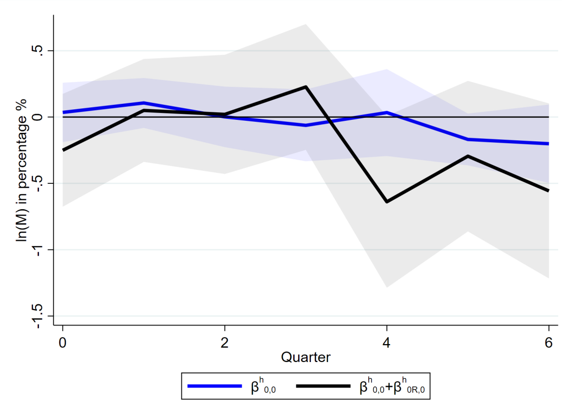
(a) log(Y), in % with Acosta (2023)



(b) log(C), in % with Acosta (2023)



(c) Unemployment rate, in % with Acosta (2023)

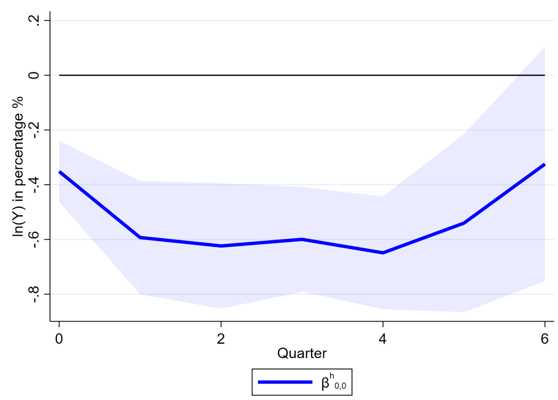


(d) log(M), in % with Acosta (2023)

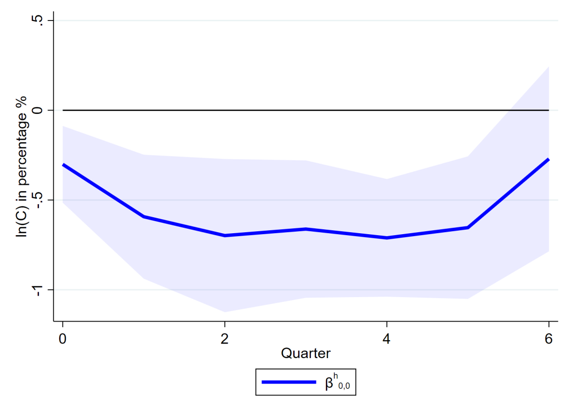
Figure E.17: Local projection with policy room from Version 1 and additional controls.

Notes: The impulse responses functions are for the local projection with the following additional controls: four lags of the oil price growth rate, four lags of the long-term interest rate, four lags of the consumption growth rate, four lags of the GDP deflator, and four lags of the shadow federal funds rate from Wu and Xia (2016). The figure reports the 95% confidence band, constructed using Newey-West standard errors under the assumption of serial correlation of the same lag length as the regression horizon, h.

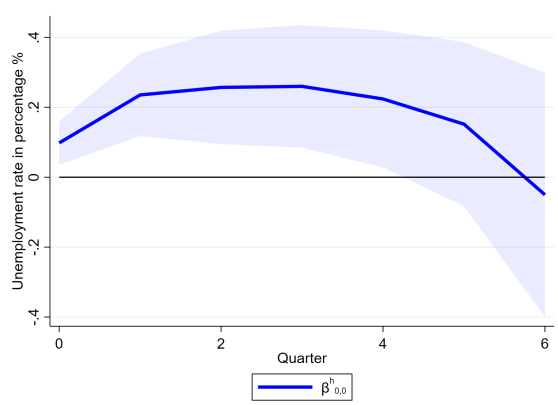
Without Interaction We test the effectiveness of monetary shocks without introducing the interaction term with the policy room, i.e., $\beta_{0R,p}^{(h)} = 0$. Figures E.19 (with Wieland and Yang (2020) shocks) and E.18 (with Acosta (2023) shocks) illustrate the impulse response functions of log output, log private consumption and the unemployment rate to a unit of contractionary monetary policy shocks following the method proposed by Romer and Romer (2004). We observe that monetary policy has mostly significant effects on output, consumption, unemployment rate, and firm entry.



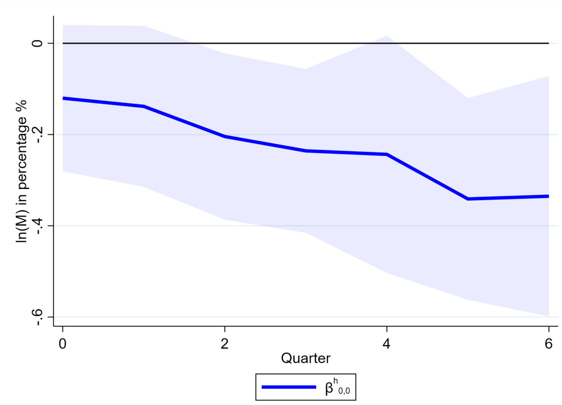
(a) log(Y) - Acosta (2023)



(b) log(C) - Acosta (2023)



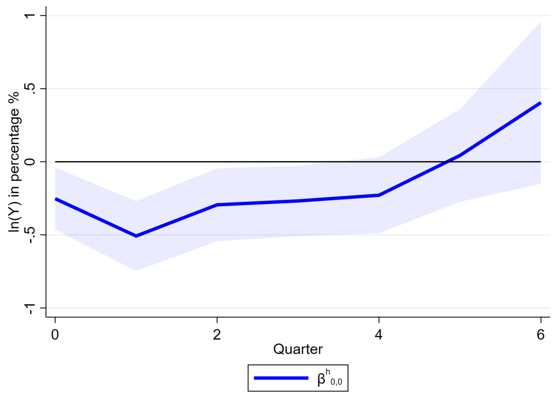
(c) Unemployment rate - Acosta (2023)



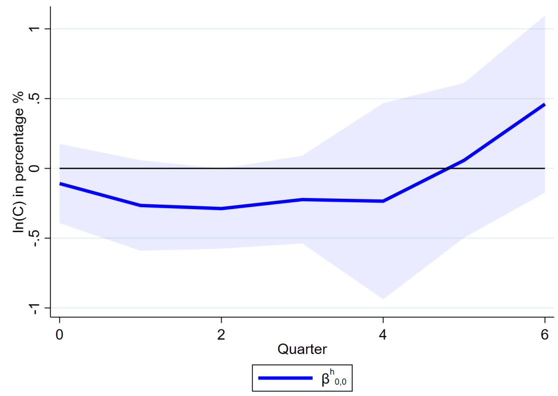
(d) log(M) - Acosta (2023)

Figure E.18: Local projection without interactions.

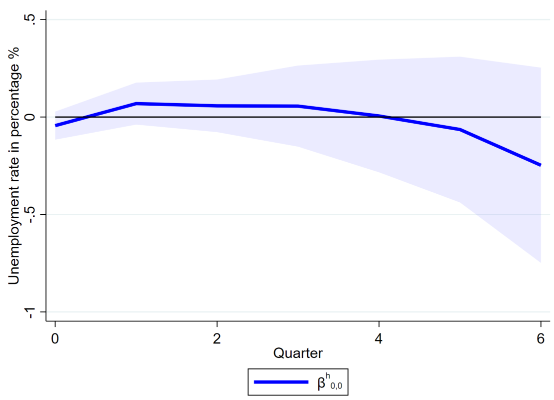
Notes: The impulse response functions are based on the benchmark regression without the interaction term, with monetary policy shocks from Acosta (2023), and controls for current and four lags of federal funds rate. The figure reports the 95% confidence band, constructed using Newey-West standard errors under the assumption of serial correlation of the same lag length as the regression horizon, h.



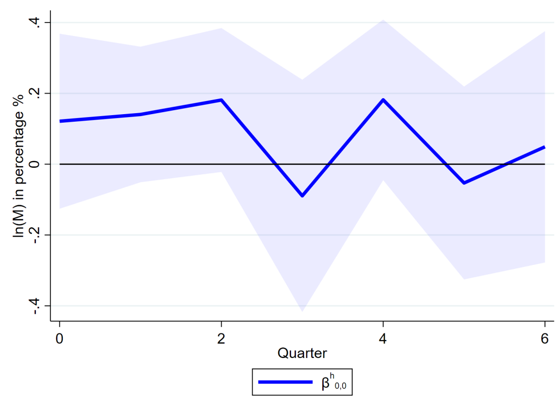
(a) log(Y) - Wieland and Yang (2020)



(b) log(C) - Wieland and Yang (2020)



(c) Unemployment rate - Wieland and Yang (2020)



(d) log(M) - Wieland and Yang (2020)

Figure E.19: Local projection without interactions.

Notes: The impulse response functions are based on the benchmark regression without the interaction term, with monetary policy shocks from Wieland and Yang (2020), and controls for current and four lags of federal funds rate. The figure reports the 95% confidence band, constructed using Newey-West standard errors under the assumption of serial correlation of the same lag length as the regression horizon, h.

E.2 Robustness: lag length

We summarize the results with the number of lags different from four, our benchmark number used in Section 5 and Appendix E.1, as follows:

Change 1 Number of lags being 2 instead of 4

(a) Impulse response functions with shocks of Wieland and Yang (2020) become smoother, with less spiky (also wider and insignificant) confidence bands.

(b) Impulse response functions with shocks of Acosta (2023) are robust when the number of lags in controls changes from 4 to 2, making the results look slightly more significant (the initial positive responses of β_{OR} become smaller and insignificant for output and consumption, thus the later negative responses stand out more significant for output and consumption. Also, the confidence bands for unemployment become narrower for Version-1-based policy room.

(c) Impulse response functions with shocks of Acosta (2023) with Version-2-based policy room (our benchmark result in Section 5) do not change much.

Change 2 Number of lags being 6 instead of 4

(a) Controlling more lags makes the confidence bands of the results with shocks of Wieland and Yang (2020) narrower.

(b) With the number of lags being 6 and the policy room recovered by Version 1, the results become worse with shocks of Acosta (2023): initial positive responses in β_{OR} become more significant and later negative responses are less significant for output and consumption, with larger confidence bands for responses in unemployment.

(c) Impulse response functions with shocks of Acosta (2023) with Version-2-based policy room (our benchmark result in Section 5) do not change much.[5]

[5]The results with too many controls display more spikiness with not-well-behaved confidence intervals though.

F Limiting Case with $\omega \to \infty$

When $\omega \to +\infty$, the Pareto distribution $H(F_{m,t})$ of the fixed costs collapse to its mean, F_t. In this scenario, it is trivial to see that $P^J_{m,t} = P^J_t$. For P^J_t, we plug equation (A.5) into equation (A.3), and obtain

$$\frac{P^J_t}{P_t} = \begin{cases} \Theta_1^{-\frac{\alpha+\sigma(1-\alpha)}{(\sigma-1)\alpha}} \left(\frac{W_t}{P_t A_t}\right) \cdot \left(\frac{Y_t \Delta_t}{A_t}\right)^{\frac{1-\alpha}{\alpha}} \\ \quad \cdot \left(\frac{R^J_{t-1} F_{t-1}}{\Theta_2 E_{t-1}\left[\xi_t \left(\frac{P^J_t}{P_t}\right)^{\frac{\sigma}{\alpha+\sigma(1-\alpha)}} \left(\frac{W_t}{P_t A_t}\right)^{-\frac{(\sigma-1)\alpha}{\alpha+\sigma(1-\alpha)}} \left(\frac{\kappa-1}{\kappa} A_t\right) \Pi_t \left(\frac{Y_t \Delta_t}{A_t}\right)^{\frac{1}{\alpha+\sigma(1-\alpha)}}\right]} \right)^{\frac{[\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)](\alpha+\sigma(1-\alpha))}{(\sigma-1)^2\alpha}} \\ \underline{\text{if } R^J_t > R^{J,*}_t}, \\ \\ \Theta_1^{-\left(\frac{\alpha+\sigma(1-\alpha)}{(\sigma-1)\alpha}\right)} \left(\frac{W_t}{P_t A_t}\right) \cdot \left(\frac{Y_t \Delta_t}{A_t}\right)^{\frac{1-\alpha}{\alpha}} \quad \underline{\text{if } R^J_t \leq R^{J,*}_t}. \end{cases} \tag{F.1}$$

Plugging (F.1) into (A.5), we can obtain

$$\Xi_t = \begin{cases} \Theta_5 \cdot \left(\frac{W_t}{P_t A_t}\right) \left[\left(\frac{\kappa-1}{\kappa}\right) A_t\right]^{\frac{1}{\alpha+\sigma(1-\alpha)}} \cdot P_t \left(\frac{Y_t \Delta_t}{A_t}\right)^{\frac{1}{\alpha}} \\ \quad \cdot \left(\frac{R^J_{t-1} F_{t-1}}{\Theta_2 E_{t-1}\left[\xi_t \left(\frac{P^J_t}{P_t}\right)^{\frac{\sigma}{\alpha+\sigma(1-\alpha)}} \left(\frac{W_t}{P_t A_t}\right)^{-\frac{(\sigma-1)\alpha}{\alpha+\sigma(1-\alpha)}} \left(\frac{\kappa-1}{\kappa} A_t\right)^{\frac{(\sigma-1)(1-\alpha)}{\alpha+\sigma(1-\alpha)}} \Pi_t (Y_t \Delta_t)^{\frac{1}{\alpha+\sigma(1-\alpha)}}\right]} \right)^{\frac{\sigma}{\sigma-1}\left(\frac{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}{(\sigma-1)\alpha}\right)} \\ \underline{\text{if } R^J_t > R^{J,*}_t}, \\ \\ \Theta_5 \cdot \left(\frac{W_t}{P_t A_t}\right) \left[\left(\frac{\kappa-1}{\kappa}\right) A_t\right]^{\frac{1}{\alpha+\sigma(1-\alpha)}} \cdot P_t \left(\frac{Y_t \Delta_t}{A_t}\right)^{\frac{1}{\alpha}} \quad \underline{\text{if } R^J_t \leq R^{J,*}_t}, \end{cases} \tag{F.2}$$

where we define

$$\Theta_5 = \Theta_1^{-\left(\frac{\sigma}{(\sigma-1)\alpha}\right)} \Theta_2 \left(\frac{\kappa-1}{\kappa}\right)^{\frac{\alpha(1-\sigma)-1}{\alpha+\sigma(1-\alpha)}}.$$

Now that $M_t = M_{m,t}$, $L_t = L_{m,t}$, $R^{J,*}_t = R^{J,*}_{m,t}$ and $\varphi^*_t = \varphi^*_{m,t}$, we can substitute (F.2) into (14), (15), (16), and (17) to obtain following analytical expressions:

$$R_t^{J,*} = \Theta_5 \cdot E_t \left[\xi_{t+1} \left(\frac{\kappa - 1}{\kappa} A_{t+1} \right)^{\frac{\sigma}{\alpha+\sigma(1-\alpha)}} \left(\frac{w_{t+1}}{P_{t+1} A_{t+1}} \right) \frac{\Pi_{t+1}}{F_t} \left(\frac{Y_{t+1} \Delta_{t+1}}{A_{t+1}} \right)^{\frac{1}{\alpha}} \right],$$

and

$$\varphi_t^* = \left(\frac{R_t^J}{R_t^{J,*}} \right)^{\left(\frac{\alpha+\sigma(1-\alpha)}{\sigma-1} \right)} \left[\left(\frac{\kappa - 1}{\kappa} \right) A_{t+1} \right], \tag{F.3}$$

$$M_{t+1} = \begin{cases} \left(\frac{R_t^J}{R_t^{J,*}} \right)^{-\left(\frac{\kappa[\alpha+\sigma(1-\alpha)]}{\sigma-1} \right)} & \underline{\text{if } R_t^J > R_t^{J,*}}, \\ 1 & \underline{\text{if } R_t^J \leq R_t^{J,*}}, \end{cases} \tag{F.4}$$

$$L_t = \begin{cases} \left(\frac{R_t^J}{R_t^{J,*}} \right)^{-\left(\frac{\kappa[\alpha+\sigma(1-\alpha)]}{\sigma-1} \right)} \cdot F_t & \underline{\text{if } R_t^J > R_t^{J,*}}, \\ F_t & \underline{\text{if } R_t^J \leq R_t^{J,*}}. \end{cases} \tag{F.5}$$

We observe: if $R_t^J \leq R_t^{J,*}$, where $R_t^{J,*}$ is defined in (22), all firms are satiated and the loan amount made to firms is equal to F_t, the fixed cost that operating firms need to pay one period in advance.

G Model with Fixed Costs Denominated in Labor

Now, we change the formulation in Section 2 and assume that instead of in-kind fixed costs denominated in final goods, firms must hire a pre-determined labor, $\frac{F_{m,t-1}}{A_{t-1}}$, in the preceding period (i.e., at $t-1$) to operate in each period t.

Now, the firm participation condition (14) becomes

$$E_{t-1}\left[\xi_t \cdot \Xi_t\right] \cdot \left(\varphi^*_{m,t}\right)^{\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}} - R^J_{t-1}\frac{W_{t-1}}{A_{t-1}}F_{m,t-1} = 0, \quad \text{where: } \xi_t = \frac{Q_{t-1,t}}{E_{t-1}\left[Q_{t-1,t}\right]}\ . \tag{G.1}$$

Thus, equation (15) becomes

$$M_{m,t} \equiv Prob\left(\varphi_{mv,t} \geq \varphi^*_{m,t}\right) = \min\left\{ \left(\frac{E_{t-1}\left[\xi_t \cdot \Xi_t\right]\left[\left(\frac{\kappa-1}{\kappa}\right)A_t\right]^{\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}}}{R^J_{t-1}\frac{W_{t-1}}{A_{t-1}}F_{m,t-1}} \right)^{\frac{\kappa[\alpha+\sigma(1-\alpha)]}{\sigma-1}}, 1 \right\},$$

and the satiation bound in (16) changes to

$$R^{J,*}_{m,t-1} \equiv \frac{E_{t-1}\left[\xi_t \cdot \Xi_t\right]\left[\left(\frac{\kappa-1}{\kappa}\right)A_t\right]^{\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}}}{\frac{W_{t-1}}{A_{t-1}}F_{m,t-1}}\ .$$

Therefore, F^*_{t-1} defined in Proposition 2 will be given by

$$\begin{aligned}
F^*_{t-1} =& \frac{E_{t-1}\left[\xi_t \cdot \Xi_t\right]\left[\left(\frac{\kappa-1}{\kappa}\right)A_t\right]^{\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}}}{R^J_{t-1}\frac{W_{t-1}}{A_{t-1}}} \\
=& \frac{E_{t-1}\left[\xi_t \cdot \Xi_t\right]\left[\left(\frac{\kappa-1}{\kappa}\right)A_t\right]^{\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}}}{R^J_{t-1}P_{t-1}} \cdot \left(\frac{W_{t-1}}{P_{t-1}A_{t-1}}\right)^{-1} \\
=& \Theta_2 E_{t-1}\left[\xi_t\left(\frac{P^J_t}{P_t}\right)^{\frac{\sigma}{\alpha+\sigma(1-\alpha)}}\left(\frac{W_t}{A_tP_t}\right)^{\frac{\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}}\left[\left(\frac{\kappa-1}{\kappa}\right)A_t\right]^{\frac{(\sigma-1)(1-\alpha)}{\alpha+\sigma(1-\alpha)}}\left(\frac{\Pi_t(Y_t\Delta_t)^{\frac{1}{\alpha+\sigma(1-\alpha)}}}{R^J_{t-1}}\right)\right] \\
& \cdot \left(\frac{W_{t-1}}{P_{t-1}A_{t-1}}\right)^{-1}.
\end{aligned} \tag{G.2}$$

Loan amounts for the fixed cost type m in (17) should be

$$L_{m,t} = M_{m,t} \cdot \frac{W_{t-1}}{A_{t-1}}F_{m,t-1},$$

which leads in aggregation to

$$\begin{aligned}\frac{L_{t-1}}{\frac{W_{t-1}}{A_{t-1}}} &= \frac{1}{\frac{W_{t-1}}{A_{t-1}}}\int_0^1 L_{m,t-1}\,\mathrm{d}m = \int_0^1 M_{m,t}\cdot F_{m,t-1}dm \\ &= F_{t-1}\cdot\left[1-\Theta_L\cdot\left[1-\underbrace{H\left(F^*_{t-1}\right)}_{=H_{t-1}}\right]^{\left(\frac{\omega-1}{\omega}\right)}\right].\end{aligned}$$

Note that the only difference from (21) is that instead of final good price aggregator P_{t-1}, we now have $\frac{W_{t-1}}{A_{t-1}}$. This result follows because we have the same formula for the measure of operating firms M_t given in (20).

We assume the same process for F_t in (24):

$$F_t = \phi_f\cdot\bar{Y}_t\cdot\exp(u_{f,t}) = \phi_f\cdot\frac{Y}{A}\cdot A_t\cdot\exp(u_{f,t}).$$

Now, since the additional labor is hired in order for firms to operate in the next period, the labor aggregation (26) becomes:

$$\begin{aligned}N_t &= \underbrace{\Theta_N\cdot\left(\frac{Y_t\Delta_t}{A_t}\right)^{\frac{1}{\alpha}}\cdot(1+\Theta_4 H_{t-1})^{-\frac{\alpha+\sigma(1-\alpha)}{(\sigma-1)\alpha}}}_{\text{Production labor}} + \underbrace{\frac{F_t}{A_t}\cdot\left[1-\Theta_L\cdot[1-H_t]^{\left(\frac{\omega-1}{\omega}\right)}\right]}_{\text{Fixed costs paid for operation in period } t+1} \\ &= \Theta_N\cdot\left(\frac{Y_t\Delta_t}{A_t}\right)^{\frac{1}{\alpha}}\cdot(1+\Theta_4 H_{t-1})^{-\frac{\alpha+\sigma(1-\alpha)}{(\sigma-1)\alpha}} + \phi_f\tilde{Y}\exp(u_{f,t})\left[1-\Theta_L\cdot[1-H_t]^{\left(\frac{\omega-1}{\omega}\right)}\right].\end{aligned}$$

From the household's intratemporal optimality condition, we again derive the equilibrium (technology-adjusted) real wage as

$$\begin{aligned}\frac{W_t}{P_t A_t} &= \frac{C_t}{A_t}\cdot\exp(-u_{c,t})\cdot N_t^{\frac{1}{\eta}} \\ &= \frac{C_t}{A_t}\cdot\exp(-u_{c,t}) \\ &\quad\cdot\left[\Theta_N\cdot\left(\frac{Y_t\Delta_t}{A_t}\right)^{\frac{1}{\alpha}}\cdot(1+\Theta_4 H_{t-1})^{-\frac{\alpha+\sigma(1-\alpha)}{(\sigma-1)\alpha}} + \phi_f\tilde{Y}\exp(u_{f,t})\left[1-\Theta_L\cdot[1-H_t]^{\left(\frac{\omega-1}{\omega}\right)}\right]\right]^{\frac{1}{\eta}}.\end{aligned}$$

The price aggregator for operating upstream firms given in (29) changes to

$$\begin{aligned}\frac{P_t^J}{P_t} &= \left(\frac{W_t}{P_t A_t}\right) \cdot \left(\frac{Y_t \Delta_t}{A_t}\right)^{\frac{1-\alpha}{\alpha}} \cdot \Theta_3^{\left(\frac{\alpha+\sigma(1-\alpha)}{\alpha(\sigma-1)}\right)} \left[1 + \Theta_4 \cdot H_{t-1}\right]^{-\left(\frac{\alpha+\sigma(1-\alpha)}{\alpha(\sigma-1)}\right)} \\ &= \frac{C_t}{A_t} \cdot \exp(-u_{c,t}) \cdot \left(\frac{Y_t \Delta_t}{A_t}\right)^{\frac{1-\alpha}{\alpha}} \cdot \Theta_3^{\left(\frac{\alpha+\sigma(1-\alpha)}{\alpha(\sigma-1)}\right)} \left[1 + \Theta_4 \cdot H_{t-1}\right]^{-\left(\frac{\alpha+\sigma(1-\alpha)}{\alpha(\sigma-1)}\right)} \\ &\quad \cdot \left[\Theta_N \cdot \left(\frac{Y_t \Delta_t}{A_t}\right)^{\frac{1}{\alpha}} \cdot (1 + \Theta_4 H_{t-1})^{-\frac{\alpha+\sigma(1-\alpha)}{(\sigma-1)\alpha}} + \phi_f \tilde{Y} \exp(u_{f,t}) \left[1 - \Theta_L \cdot [1 - H_t]^{\left(\frac{\omega-1}{\omega}\right)}\right]\right]^{\frac{1}{\eta}}.\end{aligned}$$

The market clearing equation (30) simply becomes

$$C_t + G_t = Y_t,$$

which yields

$$\frac{C_t}{Y_t} = 1 - \phi_g \cdot \exp(u_{g,t}),$$

which is simpler than (31). Intuitively, total labor demand is boosted due to those who are additionally needed to be hired for building equipment so that firms can operate in the next period. This raises real wage, which leads to a higher price aggregator for operating upstream firms, as they need to pay higher wages.

Pricing equation for downstream firms Based on

$$Q_{t,t+l} = \beta^l \frac{C_t}{C_{t+l}} \cdot \frac{\phi_{c,t+l}}{\phi_{c,t}}, \tag{G.3}$$

we recalculate O_t and V_t that determine the optimally reset price P_t^* for downstream firms. O_t in (A.20) becomes

$$\begin{aligned}O_t &= \left(\frac{(1+\zeta^T)^{-1}\gamma}{\gamma - 1}\right) \frac{P_t^J}{P_t} \cdot \frac{Y_t}{C_t} + \beta\theta E_t \left[\exp\left\{u_{c,t+1} - u_{c,t}\right\} \cdot \Pi_{t+1}^{\gamma} \cdot O_{t+1}\right] \\ &= \left(\frac{(1+\zeta^T)^{-1}\gamma}{\gamma - 1}\right) \frac{Y_t}{A_t} \cdot \exp(-u_{c,t}) \cdot \left(\frac{Y_t \Delta_t}{A_t}\right)^{\frac{1-\alpha}{\alpha}} \cdot \Theta_3^{\left(\frac{\alpha+\sigma(1-\alpha)}{\alpha(\sigma-1)}\right)} \left[1 + \Theta_4 \cdot H_{t-1}\right]^{-\left(\frac{\alpha+\sigma(1-\alpha)}{\alpha(\sigma-1)}\right)} \\ &\quad \cdot \left[\Theta_N \cdot \left(\frac{Y_t \Delta_t}{A_t}\right)^{\frac{1}{\alpha}} \cdot (1 + \Theta_4 H_{t-1})^{-\frac{\alpha+\sigma(1-\alpha)}{(\sigma-1)\alpha}} + \frac{F_t}{A_t} \left[1 - \Theta_L \cdot [1 - H_t]^{\left(\frac{\omega-1}{\omega}\right)}\right]\right]^{\frac{1}{\eta}} \\ &\quad + \beta\theta E_t \left[\exp\left\{u_{c,t+1} - u_{c,t}\right\} \cdot \Pi_{t+1}^{\gamma} \cdot O_{t+1}\right].\end{aligned}$$

Likwise, V_t in (A.21) changes to

$$V_t = \left(\frac{C_t}{Y_t}\right)^{-1} + \beta\theta \cdot E_t \left[\exp\left\{u_{c,t+1} - u_{c,t}\right\} \cdot \Pi_{t+1}^{\gamma-1} \cdot V_{t+1}\right] \ . \tag{G.4}$$

With new O_t and V_t, we obtain

$$\frac{P_t^*}{P_t} = \frac{O_t}{V_t}. \tag{G.5}$$

The aggregate price level can be recursively expressed as:

$$P_t^{1-\gamma} = (1-\theta)\left(P_t^*\right)^{1-\gamma} + \theta\left(P_{t-1}\right)^{1-\gamma} \ ,$$

or alternatively as:

$$\frac{P_t^*}{P_t} = \left(\frac{1-\theta}{1-\theta\cdot\Pi_t^{\gamma-1}}\right)^{\frac{1}{\gamma-1}} . \tag{G.6}$$

Thus, we obtain

$$\frac{O_t}{V_t} = \left(\frac{1-\theta}{1-\theta\cdot\Pi_t^{\gamma-1}}\right)^{\frac{1}{\gamma-1}}, \ \ \Delta_t = (1-\theta)\left(\frac{O_t}{V_t}\right)^{-\gamma} + \theta\Pi_t^{\gamma}\Delta_{t-1} \ .$$

Equilibrium conditions for households From (G.2), we obtain

$$\begin{aligned}
F_{t-1}^* &= \Theta_2 E_{t-1}\left[\xi_t\left(\frac{P_t^J}{P_t}\right)^{\frac{\sigma}{\alpha+\sigma(1-\alpha)}}\left(\frac{W_t}{A_tP_t}\right)^{\frac{\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}}\left[\left(\frac{\kappa-1}{\kappa}\right)A_t\right]^{\frac{(\sigma-1)(1-\alpha)}{\alpha+\sigma(1-\alpha)}}\left(\frac{\Pi_t(Y_t\Delta_t)^{\frac{1}{\alpha+\sigma(1-\alpha)}}}{R_{t-1}^J}\right)\right] \\
&\quad\cdot\left(\frac{W_{t-1}}{P_{t-1}A_{t-1}}\right)^{-1} \\
&= \left[1-H_{t-1}\right]^{-\frac{1}{\omega}}\left(\frac{\omega-1}{\omega}\right)\phi_f\cdot\tilde{Y}A_{t-1}\cdot\exp\left\{u_{f,t-1}\right\},
\end{aligned}$$

which leads to

$$\begin{aligned}
R_t^J &= \frac{\Theta_2}{\left(\frac{\omega-1}{\omega}\right)\phi_f}\left(\frac{\kappa-1}{\kappa}\right)^{\frac{(\sigma-1)(1-\alpha)}{\alpha+\sigma(1-\alpha)}}\left[1-H_t\right]^{\frac{1}{\omega}}\cdot\exp(-u_{f,t})\cdot\left(\frac{W_t}{P_tA_t}\right)^{-1} \\
&\quad\cdot E_t\left[\xi_{t+1}\left(\frac{P_{t+1}^J}{P_{t+1}}\right)^{\frac{\sigma}{\alpha+\sigma(1-\alpha)}}\left(\frac{W_{t+1}}{A_{t+1}P_{t+1}}\right)^{\frac{\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}}\cdot\frac{1}{\tilde{Y}}\underbrace{\frac{A_{t+1}}{A_t}}_{=GA_{t+1}}\cdot\Pi_{t+1}\left(\frac{Y_{t+1}\Delta_{t+1}}{A_{t+1}}\right)^{\frac{1}{\alpha+\sigma(1-\alpha)}}\right],
\end{aligned} \tag{G.7}$$

where[6]

$$\left(\frac{P^J_{t+1}}{P_{t+1}}\right)^{\frac{\sigma}{\alpha+\sigma(1-\alpha)}}\left(\frac{W_{t+1}}{A_{t+1}P_{t+1}}\right)^{\frac{\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}}$$
$$=\left(\frac{W_{t+1}}{A_{t+1}P_{t+1}}\right)\cdot\left(\frac{Y_{t+1}\Delta_{t+1}}{A_{t+1}}\right)^{\frac{(1-\alpha)\sigma}{\alpha[\alpha+\sigma(1-\alpha)]}}\cdot\Theta_3^{\left(\frac{\sigma}{\alpha(\sigma-1)}\right)}\left[1+\Theta_4\cdot H_t\right]^{-\left(\frac{\sigma}{\alpha(\sigma-1)}\right)}.$$

From the definition of $Q_{t,t+1}$ in (G.3), we obtain

$$\begin{aligned}\xi_{t+1}&=\frac{Q_{t,t+1}}{E_t(Q_{t,t+1})}=R^J_t\cdot Q_{t,t+1}\\&=\beta\cdot R^J_t\cdot\frac{\frac{C_t}{A_t}}{\frac{C_{t+1}}{A_{t+1}}}\cdot\frac{1}{\Pi_{t+1}}\cdot(GA_{t+1})^{-1}\cdot\exp(u_{c,t+1}-u_{c,t}).\end{aligned}\tag{G.8}$$

Plugging (G.8) into (G.7), equation (G.7) becomes

$$\begin{aligned}1=&\frac{\beta\Theta_2\Theta_3^{\frac{\sigma}{\alpha(\sigma-1)}}}{\left(\frac{\omega-1}{\omega}\right)\phi_f}\left(\frac{\kappa-1}{\kappa}\right)^{\frac{(\sigma-1)(1-\alpha)}{\alpha+\sigma(1-\alpha)}}\left[1-H_t\right]^{\frac{1}{\omega}}\cdot\exp(-u_{f,t})\cdot\left[1+\Theta_4H_t\right]^{-\left(\frac{\sigma}{\alpha(\sigma-1)}\right)}\\&\cdot\left(\frac{W_t}{P_tA_t}\right)^{-1}E_t\left[\frac{\frac{C_t}{A_t}}{\frac{C_{t+1}}{A_{t+1}}}\cdot\frac{W_{t+1}}{P_{t+1}A_{t+1}}\cdot\frac{1}{\tilde{Y}}\exp(u_{c,t+1}-u_{c,t})\left(\frac{Y_{t+1}\Delta_{t+1}}{A_{t+1}}\right)^{\frac{1}{\alpha}}\right].\end{aligned}\tag{G.9}$$

From the intratemporal optimality condition of households, i.e.,

$$\frac{W_{t+1}}{P_{t+1}A_{t+1}}=\frac{C_{t+1}}{A_{t+1}}\cdot N_{t+1}^{\frac{1}{\eta}}\exp(-u_{c,t+1})$$

and

$$\frac{W_t}{P_tA_t}=\frac{C_t}{A_t}\cdot N_t^{\frac{1}{\eta}}\exp(-u_{c,t}),$$

equation (G.9) finally becomes

$$\begin{aligned}\exp(u_{f,t})=&\frac{\beta\Theta_2\Theta_3^{\frac{\sigma}{\alpha(\sigma-1)}}}{\left(\frac{\omega-1}{\omega}\right)\phi_f}\left(\frac{\kappa-1}{\kappa}\right)^{\frac{(\sigma-1)(1-\alpha)}{\alpha+\sigma(1-\alpha)}}\left[1-H_t\right]^{\frac{1}{\omega}}\cdot\left[1+\Theta_4H_t\right]^{-\left(\frac{\sigma}{\alpha(\sigma-1)}\right)}\\&\cdot\frac{1}{\tilde{Y}}\cdot E_t\left[\left(\frac{N_{t+1}}{N_t}\right)^{\frac{1}{\eta}}\left(\frac{Y_{t+1}\Delta_{t+1}}{A_{t+1}}\right)^{\frac{1}{\alpha}}\right].\end{aligned}$$

[6] We change t to $t+1$ for notational convenience.

G.1 Quantitative Results

The impulse response functions (IRFs) presented below are generated using the same parameters (except the fixed cost parameter ϕ_f) as in the benchmark final-good-denominated fixed cost model. For the labor-denominated fixed cost specification, we recalibrate ϕ_f to 0.82 to ensure that the steady-state loan-to-output ratio matches the observed average value of 0.46, and that the steady-state mass of operating firms remains at 0.91, consistent with our previous calibration. The new calibration is summarized in the following Table G.1.

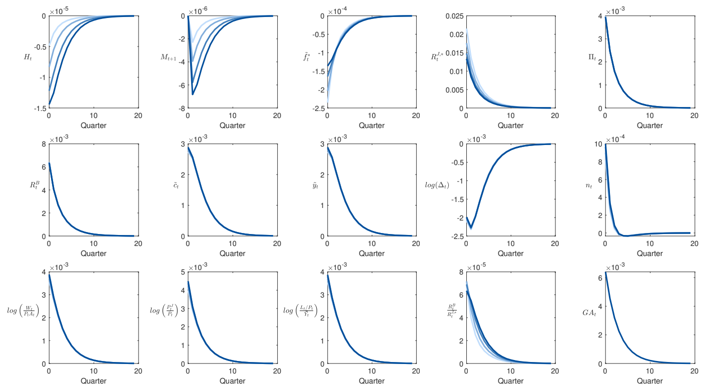
Variable	Value	Description
H	0.82 (0.82)	Mass of productivity-irrelevant firms.
M	0.91 (0.91)	Mass of firms operating in the market.
R^J	1.012	Gross risk-free rate.
$R^{J,*}$	1.295 (1.296)	Gross satiation rate.
$\tilde{F}^*$	0.70 (0.72)	Cutoff fixed cost-to-output ratio.
Δ	1.0007 (1.007)	Price dispersion.
$\frac{W_t}{P_t A_t}$	0.68 (0.51)	Real wage.
$\frac{C_t}{Y_t}$	0.82 (0.36)	Consumption-to-output ratio.
$\frac{W_t N_t}{P_t Y_t}$	1.06 (0.6)	Labor-cost-to-output ratio.
$\frac{L_t/P_t}{Y_t}$	0.46 (0.46)	Loan-to-output ratio.

Table G.1: Steady state values under labor-denominated fixed cost.

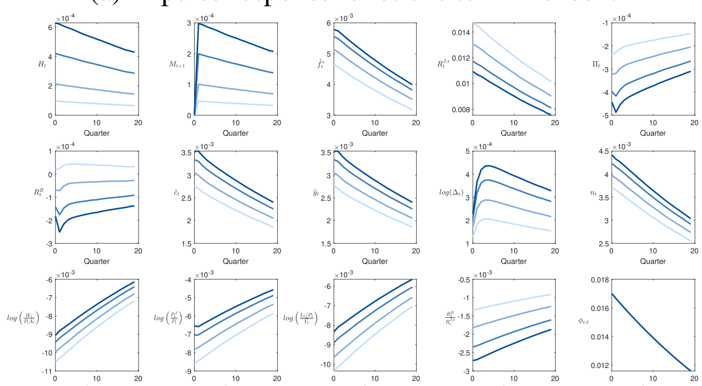
Notes: The numbers in the brackets are steady-state values in our benchmark model under the final-good-denominated fixed costs. The model with the labor-denominated fixed cost includes both production labor and the predetermined labor required for operation (corresponding to the previous final-good-denominated fixed cost component).

Overall, the model with labor-denominated fixed costs generates similar impulse response functions to our benchmark results in Section 4, except few quantitative differences: in response to positive shocks in technology growth GA_t, firm participation declines, offsetting the effect of higher productivity on output. This happens because labor becomes more efficient and expensive, as we see from increases in productivity-adjusted real wage, inducing less firms to operate in the next period since firms now need to pay labor-denominated fixed costs in order to operate in the next period. Likewise, a monetary policy tightening shock induces more participation of firms, as technology-adjusted real wages fall, and firms now pay lower wage costs for labor to operate in the next period, which dominates the opposite effect that firms need to pay higher interests on fixed costs. This phenomenon can also arise in a model with equity financing for firms entering the market (Lewis and Poilly, 2012). Nonetheless, this effect is not strong and we obtain conventional output responses from those shocks.

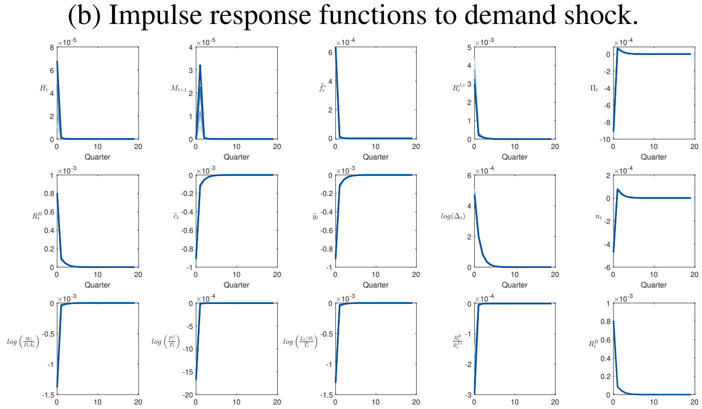
G.1.1 Impulse Response Functions



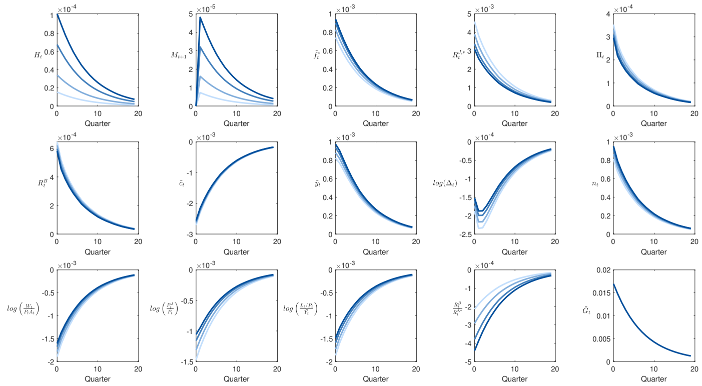
(a) Impulse response functions to TFP shock.



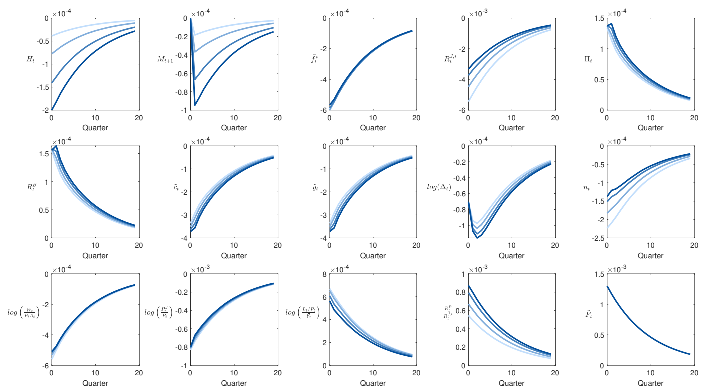
(b) Impulse response functions to demand shock.



(c) Impulse response functions to monetary policy shock.



(d) Impulse response functions to government spending shock.



(e) Impulse response functions to fixed cost shock.

Figure G.20: The figures display the impulse response functions to shocks. Panel (a) shows the response to a one standard deviation shock (0.0064) in $u_{a,t}$, which increases the growth rate of the average productivity for upstream firms. Panel (b) shows the response to a one standard deviation shock (0.017) in $u_{c,t}$, the demand shock. Panel (c) shows the impulse response functions to a one standard deviation shock (0.0025) in $\epsilon_{r,t}$, the monetary policy shock. Panel (d) shows the impulse response functions to a one standard deviation shock (0.016) in $u_{g,t}$, which denotes the government spending shock. Panel (e) shows the impulse response functions to a one standard deviation shock (0.0013) in $u_{f,t}$, the fixed cost shock. The gradient blue lines denote the responses under calibrations with varying ϕ_f. From light blue to dark blue, ϕ_f values are 0.5547 (previous benchmark), 0.65, 0.75, and 0.82 (new calibration under labor-denominated fixed costs to match loan-to-output ratio and mass of operating firms). The following variables are plotted in deviations from their steady states: H, M, R^B, Π, and $R^{J,*}$. The remaining variables are plotted in log deviations from their steady states (in lower case letters or with a log). Δ represents the price dispersion for the downstream products.

G.1.2 Labor Decomposition

As in Section 4.2, we now can decompose adjustments in labor into intensive- and extensive-margin components in our model with labor-denominated fixed costs. Since firms must hire a pre-determined labor, $\frac{F_{m,t-1}}{A_{t-1}}$, in the preceding period (i.e., at $t-1$) to operate in period t, we now differentiate production labor (i.e., labor needed for production purposes, given by (A.16)) and operation labor (i.e., labor needed as a fixed cost, given by $\frac{F_{m,t-1}}{A_{t-1}}$), and do the intensive- and extensive-margin decomposition for each type of labor.

First, note that, because the production-labor formula (A.16) remains unchanged, we can easily follow Section 4.2 to decompose production labor.

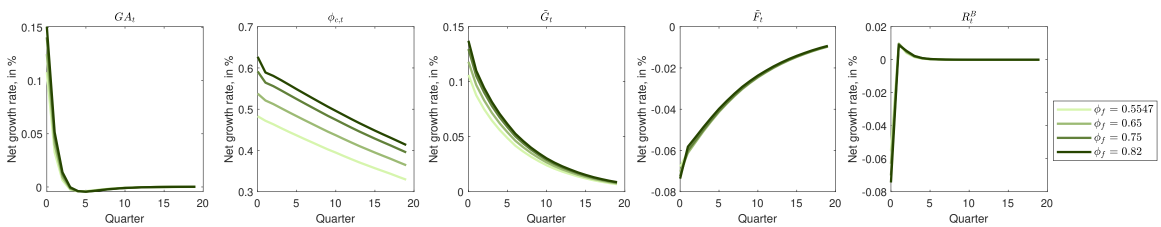


Figure G.21: Decomposition of production labor growth rate under different shocks: iso-lines on intensive margin.

Notes: Figures illustrate employment growth rate relative to pre-shock employment level. Gradient green lines indicate intensive margin responses with varying fixed cost parameter ϕ_f values. Growth rates are reported in net percentage terms.

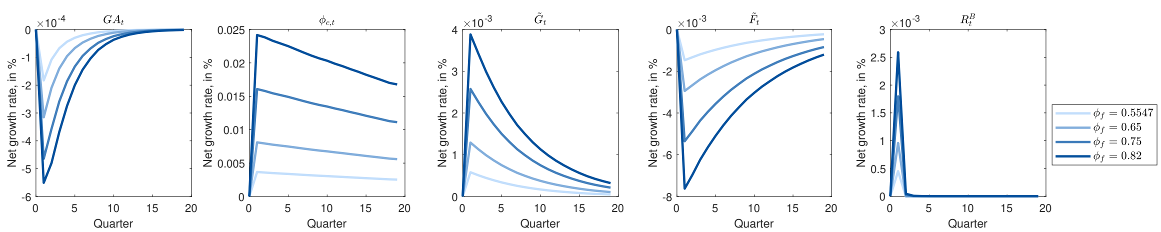


Figure G.22: Decomposition of production labor growth rate under different shocks: iso-lines on extensive margin.

Notes: Figures illustrate employment growth rate relative to pre-shock employment level. Gradient blue lines indicate extensive margin responses with varying fixed cost parameter ϕ_f values. Growth rates are reported in net percentage terms.

Overall, the new model generates similar production labor decomposition results to our benchmark model in Section 2, except few differences: for example, in response to positive fixed cost shocks, output decreases in the new model while the benchmark model of Section

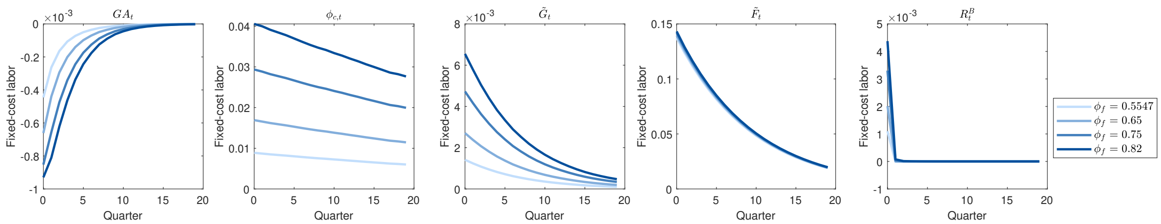


Figure G.23: Fixed-cost operation labor responses under different shocks: isolines.

Notes: Figures illustrate employment responses relative to pre-shock employment level. Gradient blue lines indicate impulse responses in fixed-cost driven operational labor demand with varying fixed cost parameter ϕ_f values.

2 yields positive responses in output (see Panel (e) of Figure G.20).[7] It generates negative intensive margin adjustment in production labor. A positive monetary policy shock pushes down wage in equilibrium, inducing more firms to operate and generating positive extensive margin adjustments in production labor.

Figure G.22 shows the impulse response functions for operation labor.[8] It is consistent with Figure G.20. For instance, in response to a fixed cost shock, there are two offsetting forces: a higher $\tilde{F}_t$ induces less participation of firms, reducing operation labor, while each firm needs to hire more operation labor for the next period operation. Overall, the net effect is positive.

[7] It is because the effect that operating firms demand final good and thus raise aggregate demand is absent in the model with labor-denominated fixed costs.

[8] All changes in operation labor are attributed to extensive-margin adjustments, as $\frac{F_{t-1}}{A_{t-1}}$ is constant on the balanced growth path.

H Average Productivity

In this section, we analytically characterize the average productivity $A_{p,t}$ of operating firms, based on the participation criteria given in equation (14), and prove that the average productivity decreases under positive demand shocks (e.g., monetary accommodation), as documented in e.g., Lee and Mukoyama (2015) and Colciago and Silvestrini (2022).[9] First, we calculate

$$
\begin{aligned}
&\int_0^1 \int_{v\in\Omega_{m,t}} \varphi_{mv,t}\ \mathrm{d}v\ \mathrm{d}m \\
&= \int_0^1 \int_{\max\left(\varphi^*_{m,t},\frac{\kappa-1}{\kappa}A_t\right)} \cancel{\varphi_{mv,t}} \cdot \kappa \left[\left(\frac{\kappa-1}{\kappa}\right)A_t\right]^{\kappa} \varphi_{mv,t}^{-(\kappa \cancel{+1})}\ \mathrm{d}\varphi_{mv,t}\ \mathrm{d}m \\
&= A_t \int_0^1 \max\left(\frac{\varphi^*_{m,t}}{\frac{\kappa-1}{\kappa}A_t},1\right)^{-(\kappa-1)} \mathrm{d}m \\
&= A_t \int_0^1 \min\left(\left(\frac{F_{m,t-1}}{F^*_{t-1}}\right)^{-(\kappa-1)\frac{\alpha+\sigma(1-\alpha)}{\sigma-1}},1\right)\ \mathrm{d}m \\
&= A_t \left[H(F^*_{t-1}) + \left(F^*_{t-1}\right)^{\varrho}\int_{F^*_{t-1}}^{\infty} F_{m,t-1}^{-\varrho}dH(F_{m,t-1})\right] \\
&= A_t \cdot \left[H_{t-1} + \frac{\omega}{\varrho+\omega}(1-H_{t-1})\right] = A_t\frac{\omega}{\varrho+\omega}\left(1+\frac{\varrho}{\omega}H_{t-1}\right),
\end{aligned}
$$

where we use

$$
\begin{aligned}
\int_{F^*_{t-1}}^{\infty} F_{m,t-1}^{-\varrho}dH(F_{m,t-1}) &= \int_{F^*_{t-1}}^{\infty} F_{m,t-1}^{-\varrho}\omega\left[\left(\frac{\omega-1}{\omega}\right)F_{t-1}\right]^{\omega} F_{m,t-1}^{-(\omega+1)}dF_{m,t-1} \\
&= \frac{\omega}{\varrho+\omega}\left[\left(\frac{\omega-1}{\omega}\right)F_{t-1}\right]^{\omega}\cdot\left(F^*_{t-1}\right)^{-(\varrho+\omega)},
\end{aligned}
$$

with

$$
\varrho = (\kappa-1)\frac{\alpha+\sigma(1-\alpha)}{\sigma-1},
$$

[9] While Colciago and Silvestrini (2022) discuss a change in average firm productivity with policy shocks as we do here, Lee and Mukoyama (2015) find that the differences in productivity between booms and recessions are larger for entering plants than for exiting plants. Under our assumption that the relative productivity ranks of firms are preserved, our model can easily capture this phenomenon, as entering firms are less productive than incumbents and their participation largely depends on the realization of demand shocks.

and
$$\frac{\varrho}{\omega} = (\kappa - 1)\frac{\alpha + \sigma(1-\alpha)}{\omega(\sigma - 1)}.$$

Since the total number of operating firms M_t is given by

$$M_t = 1 - \Theta_M \left[1 - H_{t-1}\right] = (1 - \Theta_M)\left[1 + \frac{\Theta_M}{1 - \Theta_M} H_{t-1}\right],$$

from equation (15) with
$$\frac{\Theta_M}{1 - \Theta_M} = \frac{\kappa\left[\alpha + \sigma(1-\alpha)\right]}{\omega(\sigma - 1)},$$

the average productivity $A_{p,t}$ will be given by

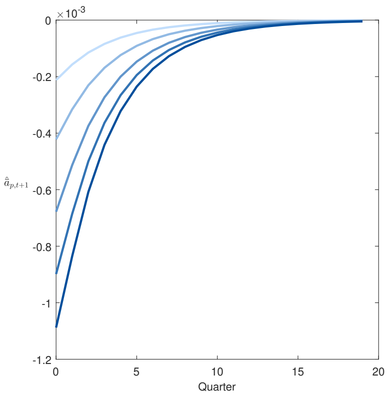
$$\begin{aligned} A_{p,t} &= \frac{1}{M_t}\int_0^1 \int_{v \in \Omega_{m,t}} \varphi_{mv,t} \, \mathrm{d}v \, \mathrm{d}m \\ &= A_t \cdot \frac{\omega}{\varrho + \omega} \cdot \frac{1}{1 - \Theta_M} \cdot \underbrace{\frac{1 + (\kappa - 1) \cdot \frac{\alpha + \sigma(1-\alpha)}{\omega(\sigma-1)} H_{t-1}}{1 + \kappa \cdot \frac{\alpha + \sigma(1-\alpha)}{\omega(\sigma-1)} H_{t-1}}}_{\text{Decreasing in } H_{t-1}}, \end{aligned} \tag{H.1}$$

which is decreasing in H_{t-1} (and thus M_t). Therefore, a positive demand shock that raises satiation H_{t-1} (and thus the measure of operating firms M_t) lowers the average productivity of operating firms $A_{p,t}$.

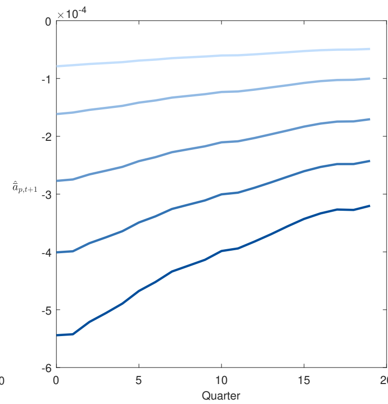
This can be seen more clearly from the log-linearized version of (H.1), which gives:

$$\hat{\hat{a}}_{p,t} = -\frac{\frac{\alpha + \sigma(1-\alpha)}{\omega(\sigma-1)} \cdot H}{1 + (\kappa - 1) \cdot \frac{\alpha + \sigma(1-\alpha)}{\omega(\sigma-1)} \cdot H} \hat{h}_{t-1},$$

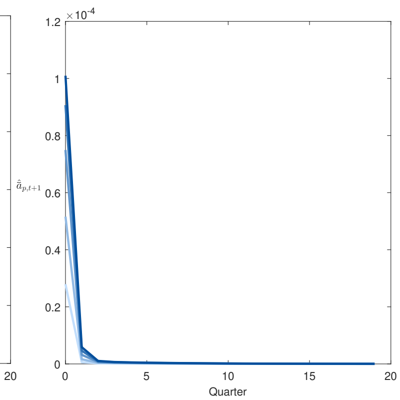
where $\hat{\hat{a}}_{p,t}$ is the log-deviation of the average productivity adjusted for technology growth, $\frac{A_{p,t}}{A_t}$. We can clearly see $\hat{\hat{a}}_{p,t}$ is decreasing in $\hat{h}_{t-1}$. Figure H.24 shows the impurse resonse functions of $A_{p,t}$ to different shocks, confirming our analytical insight.



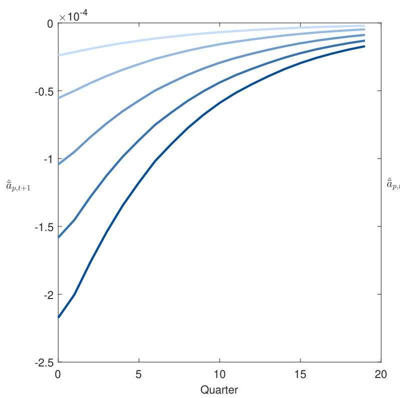
(a) Productivity shock



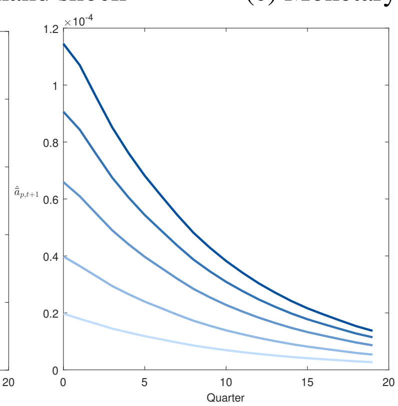
(b) Demand shock



(c) Monetary policy shock



(d) Government spending shock



(e) Fixed cost shock

Figure H.24: Impulse response function of average productivity to different shocks.

Notes: The figures display the impulse response function of the average productivity $A_{p,t}$ of operating firms, defined in (H.1), to a one standard deviation shock: (0.0064) in $u_{a,t}$ (Panel (a)), (0.017) in $u_{c,t}$ (Panel (b)), (0.0025) in $\epsilon_{r,t}$ (Panel (c)), (0.016) in $u_{g,t}$ (Panel (d)), and (0.0013) in $u_{f,t}$ (Panel (e)). The gradient blue lines denote the responses under calibrations with varying ϕ_f. From light blue to dark blue, ϕ_f values are 0.5547 (benchmark), 0.65, 0.75, and 0.82.

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