Do Cost-of-Living Shocks Pass Through to Wages?:
An AD-AS Framework with Quits and Wage Inflation

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Abstract

We provide a simplified version of Bloesch, Lee and Weber (2024) based on aggregate demand (AD) and aggregate supply (AS) curves.

1 Simplified Model

This section describes a two-period special case of the model in Bloesch, Lee and Weber (2024) where the economy responds to shocks in period $t = 0$ under the assumption that it returns to steady state after $t > 0$. In addition, we also assume no exogenous separations (i.e., $s = 0$); linear vacancy posting costs (i.e., $\chi = 0$); that unemployed and employed workers receive equal consumption from the household (i.e., $\xi = 1$); and that all employed workers search each period (i.e., $\lambda_{EE} = 1$) so that tightness θ_t becomes equal to V_t, the number of

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vacancies posted. Also, we assume that endowment good is zero supplied, i.e., $X_t = 0$ every period t, and that price setting of firms is fully flexible (i.e., $\psi = 0$).

Up to a first order, the following equations (derived in Appendix A) describe the model at $t = 0$ in log-deviation from the steady state:

Monetary Policy Rule (i.e., Taylor rule) $i_0 = \rho + \phi \check{\Pi}_{Y,0} + \check{\Pi}_{Y,1} + \check{\varepsilon}_0$

Euler Equation: $\check{C}_0 = \rho - i_0 + \check{\Pi}_{Y,1}$

Production Function: $\check{C}_0 = \check{A}_0 + \check{N}_0$

Law of Motion for Employment: $\check{N}_0 = \omega_V \check{V}_0$

Firm Price + Wage Optimality Conditions: $\check{\Pi}_{Y,0} = -\Omega_A \check{A}_0 + (\Omega_V + \phi_V)\, \check{V}_0$

where N is employment, V is vacancy, Π_Y is price inflation, and output Y, consumption C, and employment N are related $C_t = Y_t = A_t N_t$, so that $\check{A}_0$ is a TFP shock and $\check{\varepsilon}_0$ is a contractionary monetary policy shock. To interpret these equations, first note that all coefficients $\Omega_A, \Omega_V, \phi_V, \omega_V$ are strictly positive. Especially, in Appendix A.4, we prove that in a steady state with unemployment less than 50%, which is a reasonable assumption, $\phi_V > 0$.

The first, second and third equations above constitute our aggregate demand relationship. The first equation is a usual Taylor rule, with an adjustment that the monetary authority raises i_0 when the next period service inflation $\check{\Pi}_{Y,1}$ rises.[1] The second equation states that consumption $\check{C}_0$ is increasing in discount rate ρ, decreasing in policy rate i_0, and increasing in next period inflation $\check{\Pi}_{Y,1}$, which are all standard. Combining these equations, we obtain

AD (Monetary Policy Rule + Euler Equation): $\check{N}_0 = -\phi \check{\Pi}_{Y,0} - \check{\varepsilon}_0 - \check{A}_0,$

which states that employment N declines when inflation is high, because the monetary authority raises the real interest rate with a sensitivity governed by $\phi > 0$, and households reduce aggregate consumption demand.

The fourth equation states that when vacancies V are high, employment N is high: this is because in our simple model with a unit measure of workers where all unemployed (U) and employed (N) workers search, i.e., $\lambda_{EE} = 1$, we have tightness $\theta_t = \frac{V_t}{U_{t-1} + \lambda_{EE} N_{t-1}} = V_t$. A standard matching function states that a tighter labor market makes it easier for unemployed workers to find jobs, lowering unemployment and raising employment.

[1] As the economy returns to the steady state from period $t \geqslant 1$, at period 0, it is an economy with perfect foresight.

Finally, the last equation states that price inflation rises when marginal costs rise: a tight labor market (with higher $\check{V}_0$) means higher turnover costs (thus higher wages), leading to a higher price inflation. With high $\check{A}_0$, i.e., workers become more productive, real wage at $t = 0$ rises, so price inflation drops. Ω_A is a parameter that governs firms' market power in service pricing: when Ω_A is lower, there is less pass through from a TFP shock that affects marginal costs to the service good inflation $\check{\Pi}_{Y,0}$.

AD-AS Representation Using the law of motion for employment to substitute out for $\check{V}_0$ yields the following Aggregate Demand (AD) and Aggregate Supply (AS) curves:

$$\textbf{AD (Monetary Policy Rule + Euler Equation):}\ \check{N}_0 = -\phi\check{\Pi}_{Y,0} - \check{\varepsilon}_0 - \check{A}_0$$
$$\textbf{AS (Firm Prices + Wages + Law of Motion):}\ \check{\Pi}_{Y,0} = -\Omega_A\check{A}_0 + \left(\frac{\Omega_V + \phi_V}{\omega_V}\right)\check{N}_0$$

which allows us to analyze how shocks affect price inflation and employment (equivalently, unemployment).

This AD-AS representation is a useful formulation because, up to a first order, changes in employment N are a sufficient statistic for changes in labor market tightness V, wage inflation Π^w, and quits S. These always move together in the model: quits S increase with tightness θ in the model because job-to-job transitions increase when it is more likely that searching workers find a match. Wage inflation rises with tightness (and aggregate employment) because optimizing firms increase in size both by posting vacancies and by offering higher wages to improve the recruiting rate on those vacancies.

2 Impacts of Shocks

Consider the following exercises using Figure 1, which plots these AD-AS curves:

- Contractionary (positive) MP shock $\check{\varepsilon}_0$ always lowers inflation and employment (and thus vacancy, quits rate, and wage inflation), as seen in Figure 2.
- TFP shocks generate more interesting results. Positive TFP shocks $\check{A}_0 > 0$ always lower prices, but it is not clear what happens to the labor market (e.g., employment, tightness, quits rate, and wage growth) because both AD and AS curves move in response to the shocks:

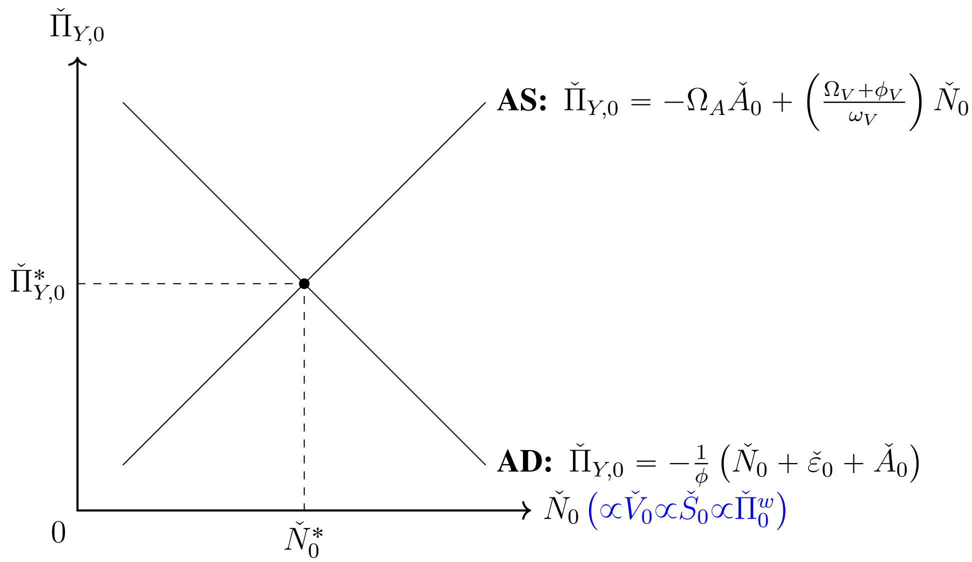
– **Very high Taylor coefficient** ϕ: If monetary policy stabilizes goods inflation $\check{\Pi}_{Y,0}$ sufficiently, then AD curve becomes very flat, i.e., service inflation $\check{\Pi}_{Y,0}$ does not vary much when employment changes. In this case, employment, tightness, quits rate, wage growth will rise (and prices will not move much). It can be understood as follows: as workers become more productive but prices are approximately fixed due to strong monetary policy responses, labor demand will increase, leading to higher real wage, vacancy, employment, tightness, and quits rate. It can be seen in Figure 3.

– **Low Taylor coefficient** ϕ **and small** Ω_A: if Ω_A is small in magnitude because, for example, firms' market power is very high and there is less room for pass through from productivity improvements (i.e., lower marginal costs) to prices,[2] then AS will not shift much. However, with low monetary responsiveness ϕ, positive TFP shocks lower marginal costs, causing deflationary pressures and lowering aggregate demand: when AD falls, prices fall but wages also fall, leading to decreases in employment, vacancies, quits rate, and wage inflation, as seen in Figure 4.

One lesson from our wage-posting model is that, at least to first order, demand (e.g., monetary policy) shocks and TFP shocks affect nominal wage growth only through their effects on labor market tightness. Here, this is summarized by vacancies $\check{V}_0$, but Bloesch, Lee and Weber (2024) show that this holds in the richer dynamic model with a more realistic calibration (wage inflation depends on both deviations in V_t (or quits) and, to a much lesser extent, U_{t-1} from steady state).

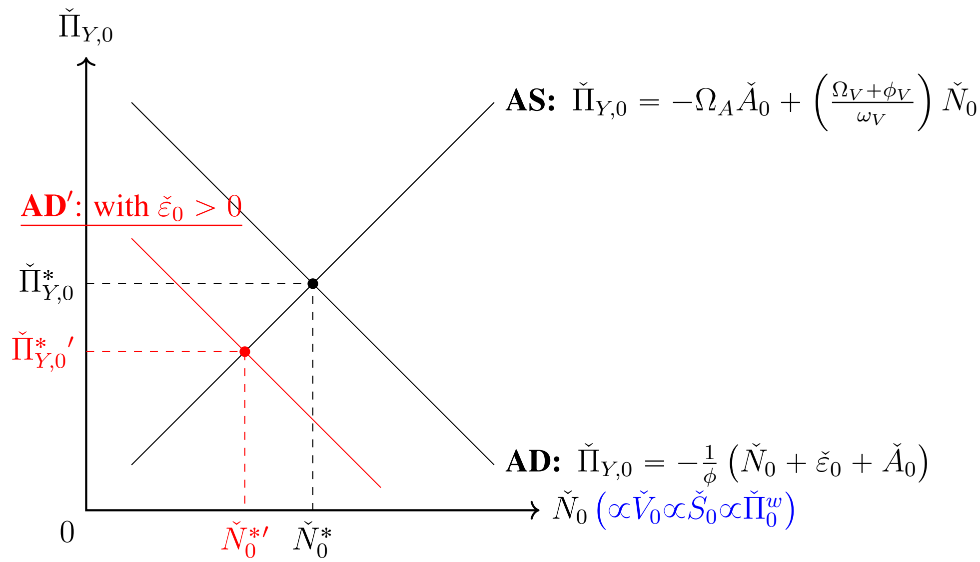
[2] For example, $\epsilon \simeq 1$ will be such a case, as shown in Appendix A.

Figure 1: Aggregate Demand-Aggregate Supply Framework with Quits, Vacancies, and Wage Inflation



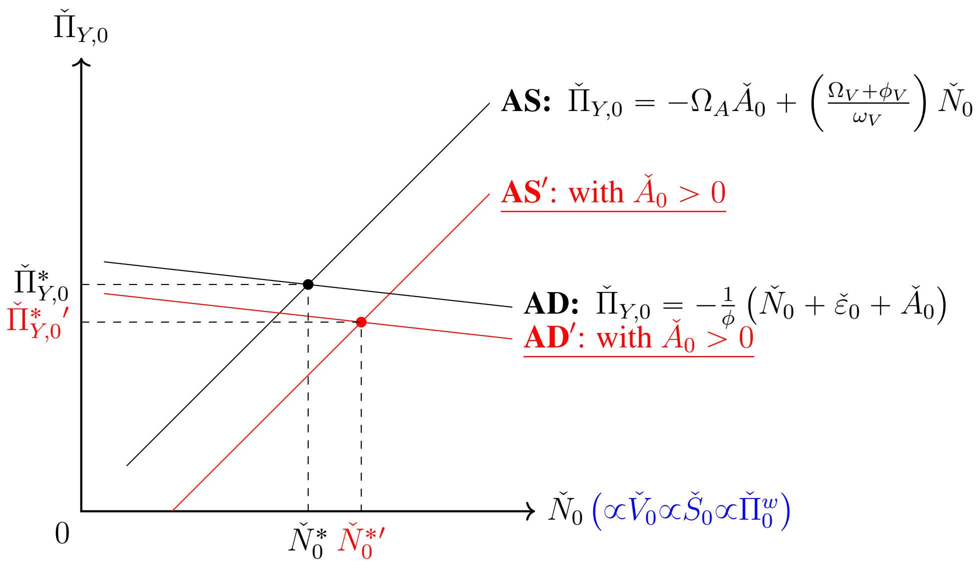
Notes: See text for derivations of the AD and AS curves based on sticky wages and flexible prices. All coefficients $\Omega_A, \Omega_V, \phi_V, \omega_v$ are strictly positive, and $\phi > 0$ governs the strength of the monetary authority's response to inflation.

Figure 2: Monetary Policy Shock $\breve{\varepsilon}_0 > 0$



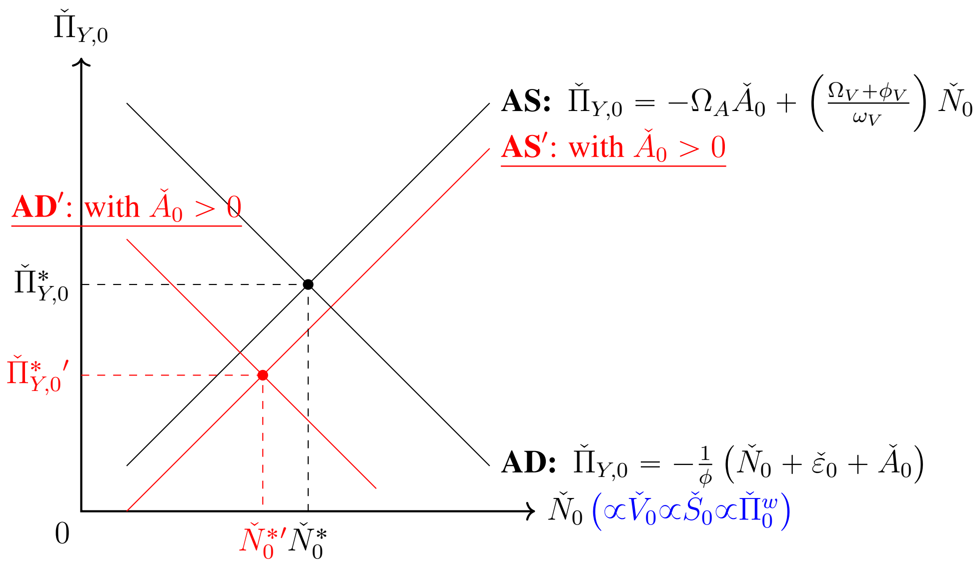
Notes: See text for derivations of the AD and AS curves based on sticky wages and flexible prices. All coefficients $\Omega_A, \Omega_V, \phi_V, \omega_v$ are strictly positive, and $\phi > 0$ governs the strength of the monetary authority's response to inflation.

Figure 3: TFP Shock $\check{A}_0 > 0$ with High $\phi >> 1$



Notes: See text for derivations of the AD and AS curves based on sticky wages and flexible prices. All coefficients $\Omega_A, \Omega_V, \phi_V, \omega_v$ are strictly positive, and $\phi > 0$ governs the strength of the monetary authority's response to inflation.

Figure 4: TFP Shock $\check{A}_0 > 0$ with Low ϕ and Small Ω_A



Notes: See text for derivations of the AD and AS curves based on sticky wages and flexible prices. All coefficients $\Omega_A, \Omega_V, \phi_V, \omega_v$ are strictly positive, and $\phi > 0$ governs the strength of the monetary authority's response to inflation.

References

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A Derivation for Section 1

Here, we describe how to derive the AD and AS curves described in Section 1 and plotted in Figure 1. We do so by analyzing a two-period special case of the dynamic model developed by Bloesch, Lee and Weber (2024). We solve for the economy's response to some shock at $t = 0$ under the assumption that we return to steady state at $t > 0$. As we explained in Section 1, we also assume no exogenous separations (i.e., $s = 0$); that unemployed and employed workers receive the same consumption (i.e., $\xi = 1$); and that all employed workers search (i.e., $\lambda_{EE} = 1$) each period so that tightness θ_t equals V_t, the number of vacancies posted; and fully flexible prices (i.e., $\psi = 0$).

A.1 Firm's Wage-Posting Problem

Perfectly-competitive retailers bundle service types j according to a standard Dixit-Stiglitz production function with an associated ideal price index:

$$\begin{aligned} Y_t &= \left(\int \left(Y_t^j \right)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}}, \\ P_{y,t} &= \left(\int \left(P_{y,t}^j \right)^{1-\epsilon} dj \right)^{\frac{1}{1-\epsilon}}, \end{aligned}$$

yielding product demand for variety j:

$$\frac{Y_t^j}{Y_t} = \left(\frac{P_{y,t}^j}{P_{y,t}} \right)^{-\epsilon}. \tag{1}$$

The firm j produces only with labor according to production function $Y_t^j = A_t^j N_{jt}$. Firm j sets its nominal wages W_{jt} each period, which is assumed to be the same for all workers in the firm, including new hires. Workers separate from firm j with probability $S(W_{jt}|\{W_{kt}\}_{k\neq j})$ each period, with $S'(W_{jt}|\{W_{kt}\}_{k\neq j}) < 0$: firms retain a higher share of workers each period by paying a higher wage, given other firms' wages. The firm recruits workers by posting vacancies V_{jt}, and the probability that a vacancy successfully results in a hire is $R(W_{jt}|\{W_{kt}\}_{k\neq j})$, with $R'(W_{jt}|\{W_{kt}\}_{k\neq j}) > 0$.[3] The firm pays a linear, per-vacancy hiring cost cW_t to post V_t vacancies, where W_t is the aggregate wage and $c > 0$. Finally, the firm is also subject to wage adjustment frictions à la Rotemberg (1982).

Given this, each firm j maximizes the present discounted value of profits, solving

$$\max_{\substack{\{P^j_{y,t}\},\{Y^j_t\},\\ \{N_{jt}\},\{W_{jt}\},\{V^j_t\}}} \sum_{t=0}^{\infty} \left(\frac{1}{1+\rho}\right)^t \left(P^j_{y,t}Y^j_t - W_{jt}N_{jt} - cV_{jt}W_t - \frac{\psi^w}{2}\left(\frac{W_{jt}}{W_{j,t-1}} - 1\right)^2 W_{jt}N_{jt}\right) \tag{2}$$

subject to the law of motion for employment

$$N_{jt} = (1 - S(W_{jt}))N_{j,t-1} + V_{jt}R(W_{jt}) \tag{3}$$

and the product demand equation (1). From inspecting equations (2) and (3), we can observe that the service sector firm chooses the wage (and other choice variables) taking as given the choices of other service sector firms (embodied in the price index and aggregate output of the service sector), parameters, and the separation and recruiting rates $S(\cdot)$ and $R(\cdot)$ which are decreasing and increasing functions of W_{jt}, respectively.

Equilibrium: We focus on a symmetric equilibrium where $P^j_{y,t} = P_{y,t}$, $V_{j,t} = V_t$, $W_{jt} = W_t$, $A^j_t = A_t \ \forall j$. Then defining λ_t as the Lagrange multiplier on the law of motion for employment, the firm's problem yields the following first order conditions:

FOC on Wages: The marginal cost of raising the wage W_t is equal to N_t, because the wage bill is W_tN_t, plus adjustment costs terms multipled by ψ^w, which an optimizing firm equates with the marginal benefit: the marginal number of new workers $V_tR'(W_t) - N_{t-1}S'(W_t)$ times

[3]How retention and separation functions $R(W_{jt}|\{W_{kt}\}_{k\neq j})$ and $S(W_{jt}|\{W_{kt}\}_{k\neq j})$ depend on wages set by other firms and is derived from the choices of households and workers, whose optimization problem is described in Appendix A.2 and Bloesch, Lee and Weber (2024). We write $R(\cdot)$ and $S(\cdot)$ solely as functions of W_{jt} set by firm j solely for readability.

their shadow value, λ_t as follows:

$$N_t + \psi^w(\Pi_t^w - 1)\Pi_t^w N_t + \underbrace{\cancel{\frac{\psi^w}{2}(\Pi_t^w - 1)^2 N_t}}_{\simeq 0} - \frac{1}{1+\rho}\psi^w \left(\Pi_{t+1}^w - 1\right)\left(\Pi_{t+1}^w\right)^2 N_{t+1}$$
$$= \lambda_t \left(V_t R'(W_t) - N_{t-1} S'(W_t)\right)$$

where we define the aggregate wage inflation $\Pi_t^w = \frac{W_t}{W_{t-1}}$ and approximate with $(\Pi_t^w - 1)^2 \simeq 0$.

FOC on Vacancies: The marginal (shadow) value of a worker is determined by the first order condition for vacancies: an optimizing firm chooses vacancies so that the marginal value of a new worker is equal to the marginal cost,

$$\lambda_t = \frac{c}{R(W_t)} W_t.$$

where the right hand side is the effective marginal cost for firms per unit hire.

FOC on Prices: we simplify by assuming that prices are flexible.[4] An optimizing firm sets the price so that the marginal revenue of a worker equals a standard markup over marginal cost of that worker, which includes wage-adjustment costs (though these effects are second-order) and also the recruitment costs

$$P_{y,t} A_t = \frac{\epsilon}{\epsilon - 1}\left(W_t + \cancel{\frac{\psi^w}{2} \underbrace{(\Pi_t^w - 1)^2}_{\simeq 0} W_t A_t N_t} + \lambda_t - \frac{1}{1+\rho}\lambda_{t+1}(1 - S(W_{t+1})) \right).$$

A.2 Closing the Model: Households, Workers, and a Monetary Policy Rule

There are a unit mass of workers so that $N_t + U_t = 1$. The household and workers' problem is fully described in Bloesch, Lee and Weber (2024). In brief, households tax and provide unemployment benefits to workers (either employed or unemployed) while trading zero-net supplied bonds such that aggregate consumption follows a standard Euler equation. Workers choose between job offers when they arrive: both unemployed and employed workers search each period and match with a firm with probability $f(\theta_t)$, which is an increasing function of market

[4] In Bloesch, Lee and Weber (2024), we assume that prices are sticky à la Rotemberg (1982) as well.

tightness θ_t, or the number of job openings divided by the number of searchers, which simplifies here when all employed and unemployed workers search to just $\theta_t = \frac{V_t}{\lambda_{EE}N_t+U_t} = V_t$.[5] Because we assume workers have a time-varying, idiosyncratic utility associated with working at a particular firm, we have job-to-job quits and endogenous quits into unemployment even though the household transfer scheme ensures that workers never have higher consumption in unemployment: the household's tax and transfer scheme fixes consumption to be equal in both the employed and unemployed states; we also rule out exogenous separations here, so that we describe separations S_t as quits, which include job-to-job quits and quits into unemployment.[6]

With these assumptions, the law of motion for employment characterizes "labor supply" as an increasing function of tightness, which is here equivalent to vacancies posted:

$$N_t = (1 - S(V_t))N_{t-1} + V_t R(V_t)$$

where the separation and recruiting rates, $S(\cdot)$ and $R(\cdot)$, which arise from the workers' problem, are defined in Bloesch, Lee and Weber (2024), who show that in a symmetric equilibrium these two no longer depend on the wage, but are instead solely functions of labor market tightness (here, V_t): with our simplifying assumptions, these functions become

$$R(V_t) = \frac{g(V_t)}{2}, \tag{4}$$

$$S(V_t) = \frac{f(V_t) + \lambda_{EU}}{2}, \tag{5}$$

where $g(V_t)$ is a decreasing function. To see that N_t is increasing in V_t, note the our definitions for $g(V_t) = \frac{1}{(1+V_t^2)^{\frac{1}{2}}}$ and $f(V_t) = V_t g(V_t)$ imply that

$$N_t = S(V_t)\,(1 - N_{t-1}) + N_{t-1} - \frac{\lambda_{EU}}{2} \tag{6}$$

which is increasing in tightness V_t because both $f(V_t)$ and quits $S(V_t)$ are increasing in tightness V_t. This equation makes a strong, but intuitive, prediction: when the labor market is hot, vacancies, quits and employment will all be jointly high.

[5]Note that this is a special case of the model in Bloesch, Lee and Weber (2024) where all employed workers are allowed to search each period ($\lambda_{EE} = 1$) so that labor market tightness $\theta_t = \frac{V_t}{\lambda_{EE}N_{t-1}+U_{t-1}} = V_t$.

[6]Formally, this exposition sets exogenous quit rate $s = 0$ and the fixed consumption ratio for the employed over the unemployed targeted by the household to be $\xi = 1$ in Bloesch, Lee and Weber (2024), who consider the general case where $\xi, s \geqslant 0$ as well as alternative modeling assumptions where the household does not fix the consumption of employed vs. unemployed households.

Next, we characterize "labor demand" from the firm's side. Using the first order condition for vacancies to substitute out for λ_t, we can find that the real wage $\bar{W}_t = \frac{W_t}{P_{y,t}}$ is determined by the first-order condition for prices, which simplifies to:

$$\bar{W}_t = \left(1 + \underbrace{\frac{c}{R(V_t)}}_{=\frac{\bar{\lambda}_t}{W_t}}\right)^{-1} \left[\frac{\epsilon - 1}{\epsilon} A_t + \frac{(1 - S(V_{t+1}))\Pi_{Y,t+1}}{1+\rho} \bar{\lambda}_{t+1}\right]$$

where $\bar{\lambda}_{t+1} \equiv \frac{\lambda_{t+1}}{P_{y,t+1}}$. When the labor market is tight, λ_t is high which means marginal costs are high, which translates into a lower real wage. In log differences, this equation links wage inflation and price inflation to changes in marginal costs.

We close the model by assuming the monetary authority reduces aggregate demand whenever current and/or expected inflation is high: combining the Euler equation with log utility and the nominal interest rate rule $1 + i_t = (1 + \rho)\Pi^{\phi}_{Y,t}\Pi_{Y,t+1}\varepsilon_t$, we obtain

$$\frac{1}{C_t} = \frac{\Pi^{\phi}_{Y,t}\varepsilon_t}{C_{t+1}}$$

which shows that consumption $C_t = Y_t = A_t N_t$ is decreasing in current inflation.[7] Note that an increase in ε_t is a contractionary monetary policy shock.

To see what happens to wage inflation, we turn to the first order condition for wages—the wage Phillips Curve—-which writes wage inflation as a function of labor market tightness: again substituting for λ_t, and dropping terms approximately zero, we can write

$$1 + \psi^w(\Pi^w_t - 1)\Pi^w_t = \frac{\lambda_t}{N_t}\left(V_t R'(W_t) - N_{t-1}S'(W_t)\right) + \frac{1}{1+\rho}\psi^w\left(\Pi^w_{t+1} - 1\right)\left(\Pi^w_{t+1}\right)^2 \frac{N_{t+1}}{N_t}$$

The right hand side of this equation is generally increasing in tightness V_t: up to a first order, Bloesch, Lee and Weber (2024) show that generically in this model, we can write

$$\check{\Pi}^w_t = \phi_V \check{V}_t + \phi_U \check{U}_{t-1} + \frac{1}{1+\rho}\check{\Pi}^w_{t+1} \qquad (7)$$

[7] This is a one-sector version of Bloesch, Lee and Weber (2024) which sets $C_t = Y_t$, with no endowment good.

where the "check" variables denote log deviation from steady state, and where $\phi_V > 0$.[8]

A.3 Monetary Policy and TFP Shocks at $t = 0$

To first order, the following equations describe the model (technically we need to substitute for $\bar{\lambda}_{t+1}$ using the first order condition for vacancies but we leave it in for now):

Wage Phillips Curve: $\check{\Pi}_t^w = \phi_V \check{V}_t + \phi_U \check{U}_{t-1} + \frac{1}{1+\rho}\check{\Pi}_{t+1}^w$

Law of Motion for Employment: $\check{N}_t = \frac{S(1-N)}{N}\check{S}_t + (1-S)\check{N}_{t-1}$

Monetary Policy Rule + Euler Equation: $\check{N}_t + \check{A}_t = -\phi\check{\Pi}_{Y,t} + \check{\varepsilon}_t$

Pricing Equation: $\check{\bar{W}}_t = \Omega_A \check{A}_t - \Omega_V \check{V}_t - \Omega_S \check{S}_{t+1} + \Omega_\lambda \check{\bar{\lambda}}_{t+1} + \Omega_\Pi \check{\Pi}_{Y,t+1}$

where the last equation comes from the log-linear form of

$$\bar{W}_t = \left(1 + \frac{c}{R(V_t)}\right)^{-1}\left[\frac{\epsilon - 1}{\epsilon}A_t + \frac{(1 - S(V_{t+1}))\Pi_{Y,t+1}}{1+\rho}\bar{\lambda}_{t+1}\right]$$

and note from equation (5) we also have

$$\check{S}_t = \underbrace{\frac{f'(V)V}{f(V) + \lambda_{EU}}}_{>0}\check{V}_t.$$

Now, assume that the economy is in steady state when an unanticipated shock hits at $t = 0$. If we assume a return to steady state in $t = 1$ then letting

$$\omega_V \equiv \frac{S(1-N)}{N}\left(\frac{f'(V)V}{f(V) + \lambda_{EU}}\right) > 0,$$

[8] With $\chi = 0$ (i.e., linear vacancy costs) instead of $\chi = 1$, it turns out to be difficult to show this generically, but it is certainly true in a model with any realistic steady state: we can show that it is sufficient (not necessary) for $\phi_V > 0$ that steady state unemployment is $< 50\%$. See Appendix A.4.

the following must hold:

Wage Phillips Curve: $\check{\Pi}_0^w = \phi_V \check{V}_0$

Law of Motion for Employment: $\check{N}_0 = \omega_V \check{V}_0$

Monetary Policy Rule + Euler Equation: $\check{N}_0 = -\phi\check{\Pi}_{Y,0} - \check{\varepsilon}_0 - \check{A}_0$

Pricing Equation: $\check{\check{W}}_0 = \Omega_A \check{A}_0 - \Omega_V \check{V}_0$

Combining the wage Phillips curve and pricing equation yields the following: note the first equation has $\check{\Pi}_0^w = \ln W_0 - \ln W = \check{W}_0 = \phi_V \check{V}_0$, so that we can write:

$$\check{\Pi}_0^W - \check{\Pi}_{Y,0} = \Omega_A \check{A}_0 - \Omega_V \check{V}_0$$

and

Law of Motion for Employment: $\check{N}_0 = \omega_V \check{V}_0$

Monetary Policy Rule + Euler Equation: $\check{N}_0 = -\phi\check{\Pi}_{Y,0} - \check{\varepsilon}_0 - \check{A}_0$

Pricing + Wage Phillips Curve: $\check{\Pi}_{Y,0} = -\Omega_A \check{A}_0 + (\Omega_V + \phi_V)\check{V}_0$

Note that this last equation remains intuitive: prices rise when marginal costs rise: a tight labor market means higher turnover costs (and higher wages) which is why this is increasing in $\check{V}_0$ and decreasing in $\check{A}_0$. Then using the law of motion for employment to substitute out of for $\check{V}_0$, we derive the AD and AS curves in Figure 1:

AD (Monetary Policy Rule + Euler Equation): $\check{N}_0 = -\phi\check{\Pi}_{Y,0} - \check{\varepsilon}_0 - \check{A}_0$

AS (Firm Prices + Wages + Law of Motion): $\check{\Pi}_{Y,0} = -\Omega_A \check{A}_0 + \left(\frac{\Omega_V + \phi_V}{\omega_V}\right)\check{N}_0$

A.4 Sufficient Condition for $\phi_V > 0$ in Equation (7)

In the text, we claimed that in a steady state with unemployment $U < 50\%$, $\phi_V > 0$. We justify this claim here. Consider the following definition for ϕ_V from the Appendix of Bloesch, Lee and Weber (2024): the coefficient on vacancies in the wage Phillips curve is defined as:

$$\phi_V \equiv \frac{\kappa}{\psi_w}\left(\Lambda_1 + \Delta_1(g_{S,V} - g_{R,V}) - \epsilon_S g_{\epsilon_S,V}\right)$$

where we have, in our special case with $s = 0, \lambda_{EE} = 1, \xi = 1, \chi = 0$ ($\mathcal{C} = 0.5$ in Appendix of Bloesch, Lee and Weber (2024)):

$$\Lambda_1 = \epsilon_R - S(\epsilon_R - \epsilon_S) = \gamma\left(\frac{1}{2} - S\right)$$
$$\Delta_1 = -\epsilon_S + S(\epsilon_R - \epsilon_S) = \gamma\left(\frac{1}{2} + S\right)$$

with $\epsilon_S g_{\epsilon_S,V} = 0$. We also have that:

$$g_{S,V} = \frac{f}{f + \lambda_{EU}} \times \frac{1}{1+\theta^2}$$
$$g_{R,V} = -\frac{\theta^2}{1+\theta^2}$$

The issue is that before, with $\chi = 1 > S$ as we assume in Bloesch, Lee and Weber (2024), we could always sign $\Lambda_1 > 0$. Now with $\chi = 0$, we cannot. To make progress, combine these results to write everything in terms of S and θ, noting that $f = \frac{\theta}{(1+\theta^2)^{\frac{1}{2}}}$:

$$\phi_v = \frac{\kappa}{\psi_w}\left(\underbrace{\gamma\left(\frac{1}{2} - S\right)}_{\Lambda_1} + \underbrace{\gamma\left(\frac{1}{2} + S\right)}_{\Delta_1}\left(\underbrace{\frac{f}{f+\lambda_{EU}} \times \frac{1}{1+\theta^2} + \frac{\theta^2}{1+\theta^2}}_{g_{S,V} - g_{R,V}}\right) - \underbrace{0}_{\epsilon_S g_{\epsilon_S,V}}\right)$$

which simplifies to

$$\phi_V = \frac{\kappa}{\psi_w} \cdot \gamma\left(\frac{1}{2} - S + \left(\frac{1}{2} + S\right)\frac{\frac{f}{f+\lambda_{EU}} + \theta^2}{1+\theta^2}\right). \tag{8}$$

Looking at equation (8), we can see if the steady state labor market is sufficiently tight, i.e. θ is high so that f is high, then $\frac{\frac{f}{f+\lambda_{EU}} + \theta^2}{1+\theta^2} \simeq 1$ and $\phi_v \simeq \frac{\kappa}{\psi_w}\gamma > 0$.

Sufficiency Now, we note that in any steady state where $U < 50\%$, ϕ_v in equation (8) is positive.

To see this, begin by noting that $N = \frac{f}{f+\lambda_{EU}}$ at the steady state. This follows from the law of motion for employment (6): $N = (1 - S)N + R(V)V$ and plugging in for $V \cdot R(V) = \frac{f(V)}{2} = S - \frac{\lambda_{EU}}{2}$, which is from equations (4) and (5). Obtaining $N = S(1 - N) + N - \frac{\lambda_{EU}}{2}$, we solve for $N = 1 - \frac{\lambda_{EU}}{2S}$ and plug in for $S = \frac{1}{2}(f + \lambda_{EU})$, leading to $N = \frac{f}{f+\lambda_{EU}}$.

Then it follows that if $N = \frac{f}{f+\lambda_{EU}} > 0.5$, i.e., $U < 0.5$, we have

$$\frac{\frac{f}{f+\lambda_{EU}} + \theta^2}{1+\theta^2} > 0.5,$$

which leads to

$$\phi_V > \frac{\kappa}{\psi_w}\gamma\left(\frac{1}{2} - S + \left(\frac{1}{2} + S\right)\frac{1}{2}\right) = \frac{\kappa}{\psi_w}\gamma\frac{1}{2}\left(1 - \underbrace{S}_{<1} + \frac{1}{2}\right) > 0$$

Finally, note that we can always find such a steady state by choosing vacancy costs c low so that θ and f are high in steady state, so that steady state employment N (unemployment U) is high enough (low enough) to ensure $\phi_V > 0$.