Optimism, Net Worth Trap, and Asset Returns*

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Abstract

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1 Introduction

There has been a surge in the literature on macroeconomic models with financial frictions post 2008 financial crisis, aiming to explain crises and high risk premium levels on financial assets. However, the majority of these models neglect cognitive beliefs about the macroeconomic state, with only a few notable exceptions (e.g., Krishnamurthy and Li (2020), Maxted (2023), Camous and Van der Ghote (2023)). Our paper aims to bridge this gap by investigating how cognitive beliefs of market participants about the long-run macroeconomic state of the economy influence financial stability and affect aggregate asset returns.

This paper contributes in two key ways. First, we introduce a tractable model with two types of agents who differ in terms of their beliefs about macroeconomic growth. One type, who is more productive in capital utilization, holds optimistic beliefs about the economy's long-run growth rate, which contrasts with the other type who is less productive but has rational beliefs.[1] The optimism of the productive agents results in expectation errors in their capital returns, leading to financial instability at the aggregate level. Moreover, beyond a certain threshold level of optimism, these forecast errors generate a 'net worth trap', a phenomenon that concentrates capital exclusively in the hands of less productive agents and perpetuates long-term inefficiencies. In addition, optimism negatively impacts the welfare of rational agents, primarily by generating a net worth trap where capital prices and the economy's growth rates are depressed for a long period. This effect on capital prices from optimism generates and matches empirically documented return predictability patters, constituting the second contribution of our paper. The interaction between optimism and the net worth of productive agents in explaining the risk premium inherently suggests a two-factor asset pricing model. To validate its pricing effectiveness, we construct an optimism factor using data from The Survey of Professional Forecasters. The inclusion of optimism in the factor pricing model effectively prices the cross-section of asset returns, showcasing improved performance compared to the traditional factors typically utilized in empirical asset pricing studies.

We purposefully build a model with a strong emphasis on financial frictions and risk premia following He and Krishnamurthy (2013), Brunnermeier and Sannikov (2014), Gertler et al. (2020), etc., to study implications on financial and macroeconomic stability as well as on asset returns. In these papers, the net worth of the leveraged agents is a natural mea-

[1] This assumption aligns with the literature that portrays entrepreneurs as skilled and optimistic. See e.g., Coval and Thakor (2005). Following the literature, we call more and less productive agents 'experts' and 'households', respectively.

sure of financial stability. The main addition to this framework is a dogmatically optimistic belief of more productive users of capital, reflecting the empirical evidence found in e.g., Puri and Robinson (2007). The first main insight that comes out of the model is that optimism exacerbates financial instability, resonating with Camous and Van der Ghote (2023).[2] The mechanisms that create the instability are that optimists expect a higher growth rate of the economy than the true growth rate, and this expectation error influences their portfolio choice over risky capital. As they operate with leverage in equilibrium, a negative shock in the economy depresses their net worth due to a loss in the capital price. Subsequent negative shocks push the economy into a crisis, where capital prices further drop due to optimists' fire-selling the risky capital to less productive agents, and create two countervailing effects: (i) a positive effect that aids the optimists in rebuilding their net worth through high risk premium, and (ii) a negative effect stemming from their expectation error that counteracts the net worth rebuilding process: optimists' perceived risk-premium during a crisis is still higher than its true level, eroding the recapitalization of their net worth. The second effect arises because the optimists take their portfolio decisions based on the perceived growth rate which is higher than the true growth rate of economy. For lower levels of optimism, the first effect dominates, moving the economy toward normalcy where all capital is allocated to the productive optimists. However, beyond a threshold level of optimism, the second effect dominates, wiping out the optimists' wealth and driving the economy to allocate all capital to less productive agents. We call this novel phenomenon a "net worth trap", in which the optimists remain unable to rebuild their wealth permanently. This trap does not arise in a rational expectations benchmark with no expectation errors. While models with financial frictions and belief distortions such as Krishnamurthy and Li (2020) and Maxted (2023) feature a fat tailed stationary distribution, we offer an analytical characterization of an extreme case of the instability where the stationary distribution endogenously becomes a Dirac measure at the most inefficient region.[3] Our net worth trap can offer a belief-based explanation to an extremely slow-moving capital crisis, in line with Duffie (2010).

Beyond increasing the financial instability, macro-level optimism generates a welfare

[2] Camous and Van der Ghote (2023) builds a model based on diagnostic expectations where risk-neutral agents face binding leverage constraints and financiers of the capital do not consume. In our model, all agents are risk-averse with preferences over consumption, and optimists face a skin-in-the game constraint.

[3] In fact, our model does not have any additional assumptions such as portfolio and collateral constraints, or a large exit that is shown to be required to generate a fat-tailed distribution. See Gopalakrishna (2022) who introduces a fluctuating productivity and state-dependent exits of the experts.

loss for the rational agents in the economy. We perform a decomposition of the welfare to understand different channels contributing to welfare loss. The decomposition leads to four channels: (i) wealth effect, (ii) investment effect, (iii) capital effect, and (iv) misallocation effect. The wealth effect captures the wealth share of rational agents, and the investment effect captures the welfare attributed to capital investment. These two positive effects raise welfare as the optimism of productive agents increase. First, a larger degree of optimism increases financial instability implying a larger (smaller) wealth share of rational (optimistic) agents. Second, as higher optimism creates a higher probability of crisis, it depresses the investment rate and mechanically increases the welfare of rational agents since they have more to consume. The third effect is due to the growth rate of aggregate capital. With higher optimism levels, the aggregate capital grows slowly due to a poor investment rate, decreasing the welfare. Finally, since more capital is allocated to less productive agents as optimism increases, the misallocation effect dampens their welfare. The last two effects turn out to dominate and increase the utility loss for the rational households.

We then relax our assumption that sentimental agents are dogmatic about their belief: now, while sentimental agents are optimistic in the normal regime, they become pessimistic when the economy transitions into a crisis. This captures an endogenous swing in their sentiment, in a similar way to diagnostic expectations (see e.g., Bordalo et al. (2018), Maxted (2023)). As true risk premium is larger than the sentimental agents' perceived risk premium during a crisis, they recapitalize at a faster rate leading to a quick economic recovery. We illustrate that this stabilizing effect of a swinging sentiment effectively prevents a net worth trap. Thus, the type of beliefs have important consequences on financial stability and the asset price in the long-run.

The elicitation of objectively accurate beliefs about the underlying technological process (i.e., rational expectations) is out of the scope of this paper. In most parts, optimists in our model have static beliefs about their expected technological growth rate, and are dogmatic in their own views. In this formulation, optimism is persistent and optimists never learn from the data. This setting is in accordance with the literature where variation about individual beliefs about expected returns are due to individual fixed effects (e.g., Giglio et al. (2019)). In contrast to the models of e.g., Harrison and Kreps (1978), Scheinkman and Xiong (2003), and Geanakoplos (2010), we consider risk-averse, not risk-neutral agents. On the methodological side, we normalize the economy by technology so that our economy becomes stationary, and rely on the Kolmogorov forward equation (KFE) in characterizing the ergodic distribution of our state variable, in order to illustrate the long term dynamics and especially, the net worth trap.

Empirical implications Our model predicts that asset returns are affected by a two-way feedback loop between optimism and the wealth share of experts, stemming from a rich interaction between financial frictions and optimism. Since fluctuations in optimism amplify asset price moves, optimism can be a priced factor. To validate this model prediction, in Appendix C, we empirically construct a factor-based asset pricing model, based on the quarter-on-quarter GDP growth rate from The Survey of Professional Forecasters (SPF) as a proxy for optimism. We show that our factor model, incorporating both optimism and the net worth factor (the latter being proxied by an intermediary capital based factor following He, Kelly and Manela (2017)), improves the pricing power of a broad spectrum of test assets above and beyond the capabilities of traditional factors. The optimism factor carries a positive price of risk, both unconditionally and conditional on the exposure to net worth factor.

In time-series, the model matches the asset return predictability patterns documented empirically. The mechanism is as follows: in comparison to a rational expectations benchmark, our model features large drops in the net worth of optimists following a sequence of adverse shocks, subsequently leading to an increase in the risk premium. For moderate levels of optimism, the optimists' expectation error is dominated by the risk premium earned on risky capital, facilitating the rebuilding of their net worth. This recapitalization induces a momentum in asset prices, with the momentum effect intensifying with higher optimism.

Related literature Our paper relates to a large literature on macro-finance models with financial frictions playing a central role in amplifying shocks (e.g., He and Krishnamurthy (2013), Gertler et al. (2020) Brunnermeier and Sannikov (2014)). Recent works in this literature have turned to quantitative models that explain macroeconomic and asset pricing moments either using a rational expectations model, or using a belief based model.[4] We follow this line of literature but focus on the general equilibrium interactions between the endogenous risk and macroeconomic variables in the presence of optimism about the long-run growth. Specifically, we build a tractable model with optimism that generates a net worth trap and conditional momentum in asset returns consistent with the empirical

[4]Maxted (2023) builds a macro-finance model with diagnostic expectations, and Krishnamurthy and Li (2020) builds a model where agents update their beliefs about tail risk rationally. In a recent work, Gopalakrishna (2022) introduces stochastic productivity and state-dependent exits of experts into a canonical rational expectations model to generate amplified (in volatility and risk-premium) but 'slow-moving' financial crises.

evidence. While we focus on optimism as a major behavioral factor, it relates to the literature that focuses on both optimism and pessimism, or belief heterogeneity in general (see, for example, Harrison and Kreps (1978), Simsek (2013), and and Caballero and Simsek (2020)).[5]

We also relate to a long literature on deviations from the rational expectations, e.g., Detemple and Murthy (1997), Basak and Croitoru (2000), Basak (2000), Basak and Croitoru (2006), Gallmeyer and Hollifield (2008), Chabakauri (2015), and Dong et al. (2022).[6] The difference is that we focus on the implications on financial stability and the interaction between macroeconomy and asset prices. In this regard, it is similar to Maxted (2023) and Camous and Van der Ghote (2023). Camous and Van der Ghote (2023) introduces diagnostic expectations into a canonical macro-finance model with binding leverage constraints and shows that diagnostic expectations exacerbate financial instability. On the contrary, the diagnostic expectation decreases financial instability compared to a rational expectation benchmark in Maxted (2023). Our paper's financial stability results are closer to the former, and in addition, characterizes an extreme case of instability featuring a net worth trap where the stationary distribution has a sharp peak around the zero net worth of experts. Our assumption of beliefs about long-term growth rate of the economy relates to Bordalo et al. (2023) who find that analysts' optimism about long-term earnings growth of S&P500 firms amplifies the boom-bust cycle and explains the excess volatility of the business cycle. This finding is in line with our result that optimism about long-term technological growth increase the aggregate macroeconomic instability.

Finally, this paper relates to the empirical literature focusing on heterogeneous beliefs across different groups of market agents. For example, Welch (2000) finds there is a large degree of heterogeneity in forecasted risk premium levels even across financial economists, while Beutel and Weber (2022) point out individuals are heterogeneous both at the information acquisition and the processing stages, thereby forming their own beliefs and choosing portfolios based on those beliefs.[7] Our results are based on the *perceived* equity premium

[5] Harrison and Kreps (1978) assume that agents agree to disagree about their beliefs, and asset prices can exceed their fundamental values in that case. Simsek (2013) studies cases in which optimists borrow from pessimists using loans collateralized by the asset that optimists purchase. As pessimists attach lower values to the collateral, this collateral arrangement puts the endogenous borrowing constraint on optimists, affecting their leverage choices and asset prices.

[6] In Dong et al. (2022), optimistic investors buy risky assets with leverage provided by pessimists, pushing up asset prices like in our model. Higher asset prices relax the financial frictions imposed on high productivity firms, mitigating degrees of misallocation and raising aggregate output.

[7] The literature also points out that different groups in the economy (e.g., households, firms, professional forecasters, etc) form different expectations not just about risk premium but also about macroeconomic vari-

levels across two groups (i.e., optimists and households), providing novel implication about the interaction between optimistic belief and the crisis dynamics in general equilibrium.

Outline The remainder of the paper is organised as follows: Section 2 sets out the basic framework. In particular, Section 2.4 characterizes the equilibrium in an analytic way and Section 2.5 provides simulation results and discusses our model's implications. Section 3 concludes. Appendix A provides additional figures, and Appendix B provides omitted proofs and derivations. Appendix C provides our quantitative analysis in regards to the conditional excess momentum in asset returns and cross-sectional asset pricing implications.

2 The Model

We develop a continuous-time framework with two types of agents: optimists and households, based on which we study how optimism about technological growth affect leverage choices, asset prices and the endogenous financial volatility, where the endogenous volatility itself affects the optimists' degree of optimism in asset returns. We initially assume that optimists have dogmatic optimism about technological growth, but will relax this assumption to incorporate a swinging sentiment that depends on macroeconomic environments.

Our setting is analytical yet tractable, and incorporates exogenous technological growth and heterogeneous beliefs in a general equilibrium sense. Our model is built on e.g., Basak (2000) and Brunnermeier and Sannikov (2014).

2.1 Model Setup

We begin with the complete probability space $(\Omega, \mathcal{F}, \mathcal{P})$ which is endowed with a standard Brownian motion Z_t. We assume that $Z_0 = 0$, almost surely. All economic activity will be assumed to take place in the horizon $[0, \infty)$. Let

$$\mathcal{F}^Z(t) \triangleq \sigma\{Z_s;\ 0 \leq s \leq t\}, \forall t \in [0, \infty)$$

ables: see e.g., Coibion et al. (2020), Candia et al. (2021), and Weber et al. (2022). As risk premium depends on the business cycle (e.g., Cooper and Priestley (2009)), we can expect that the forecasted risk-premium levels across groups would differ. For equity premium, Rapach et al. (2012) uncover that despite the failure of *individual* out of sample forecasts to outperform the historical average, *combinations* of individual forecasts deliver significant out-of-sample gains relative to the historical average on a consistent basis over time.

be the filtration generated by $Z(\dot{)}$ and let $\mathcal{N}$ denote the $\mathcal{P}$- null subsets of $\mathcal{F}^Z(\infty)$. We shall use the augmented filtration as follows:

$$\mathcal{F}(t) \triangleq \sigma\left\{\mathcal{F}^Z(t) \cup \mathcal{N}\right\}, \forall t \in [0, \infty).$$

One should interpret the $\sigma-$algebra $\mathcal{F}(t)$ as the information available to agents at time t in a complete information setting, in the sense that if $\omega \in \Omega$ is the true state of nature and if $A \in \mathcal{F}(t)$, then all agents will know whether $\omega \in A$.

We consider an economy with two types of agents, 'optimists' and 'households'. Both types of agents can own capital, but the former are able to use capital in a more productive way.[8]

2.1.1 Technology

The aggregate amount of capital in the economy is denoted by K_t and capital owned by an individual agent i by k_t^i , where $t \in [0, \infty)$ indicates time. The physical capital k_t^O held by optimists produces output at rate:

$$y_t^O \triangleq \gamma_t^O k_t^O, \quad \forall t \in [0, \infty) \tag{1}$$

per unit of time, where γ_t^O is an exogenous productivity parameter, which evolves according to:[9]

$$\frac{d\gamma_t^O}{\gamma_t^O} \triangleq \alpha dt + \sigma dZ_t, \quad \forall t \in [0, \infty), \tag{2}$$

where dZ_t is the aggregate standard Brownian motion defined above. The output is modeled as a numeraire, and therefore, its price is normalized to one. Capital owned by individual optimists, with state space $F^k \subseteq \mathbb{R}$, satisfies the following Ito's process:

$$\frac{dk_t^O}{k_t^O} \triangleq \left(\Lambda^O(\iota_t^O) - \delta^O\right) dt, \quad \forall t \in [0, \infty), \tag{3}$$

where ι_t^O is the portion of the generated output (i.e., $y_t^O = \gamma_t^O k_t^O$) used in creating new capital (i.e., the investment made during an infinitesimal period $(t, t + dt)$ is $\iota_t^O \gamma_t^O{}_t k_t^O dt$).

[8] Our assumption that more productive intermediaries are more optimistic than the rational households is from the literature including Coval and Thakor (2005) and Chabakauri (2015). Still our analysis can embed the opposite case in which more productive intermediaries are pessimistic about growth.

[9] Therefore, unlike Brunnermeier and Sannikov (2014), we have the stochastic growth which we model as exogenous (i.e., α and σ in (2) are exogenous). Later we normalize our economy by γ_t^O to make it stationary.

In (3), function $\Lambda^O(\cdot)$, which satisfies $\Lambda^O(0) = 0$, $\Lambda^{O\prime}(0) = 1$, $\Lambda^{O\prime}(\cdot) > 0$, and $\Lambda^{O\prime\prime}(\cdot) < 0$, represents a standard investment technology with adjustment costs. In cases where there is no investment, the capital managed by optimists depreciates at rate δ^O. The concavity of $\Lambda^O(\cdot)$ represents some investment formation friction, which is interpreted as adjustment costs of converting the output to the new capital and vice versa.[10]

Households are less productive. The capital managed by the households, with corresponding state space $F^k \subseteq \mathbb{R}$, produces the following output:

$$y_t^H \triangleq \gamma_t^H k_t^H, \quad \forall t \in [0, \infty), \tag{4}$$

with γ_t^H is given by $\gamma_t^H = l \cdot \gamma_t^O \leq \gamma^O$, where $l \leq 1$, and evolves according to:

$$\frac{d\gamma_t^H}{\gamma_t^H} = \frac{d\gamma_t^O}{\gamma_t^O}, \quad \forall t \in [0, \infty). \tag{5}$$

In other words, optimists have a proportionally higher productivity than households, where the proportionality is given by $l \leq 1$, and productivity of two groups of agents grows at the same rate at every instant. Finally, the capital owned by households, which we denote by k_t^H follows:

$$\frac{dk_t^H}{k_t^H} \triangleq \left(\Lambda^H(\iota_t^H) - \delta^H\right) dt, \quad \forall t \in [0, \infty), \tag{6}$$

where in the same way as above, ι_t^H is a capital-owning pessimist's investment rate per unit of output. The state space F^k satisfies the same conditions as for optimists and $\delta^H \geq 0$ is the depreciation rate when the capital is managed by households. We assume that $\Lambda^H(\iota) = l \cdot \Lambda^O(\iota)$ for $\forall \iota$ with $l < 1$ for simplicity.[11]

2.1.2 Preferences

Optimists and households have preferences that are generally characterized by the instantaneous utility function $u^i(c_t^i) : \mathbb{R}_+ \to \mathbb{R}$, where $i \in \{O, H\}$, and each group i has a constant

[10]In contrast to the literature (e.g., Brunnermeier and Sannikov (2014)), we abstract from the capital risk usually assumed. Its inclusion does not change our results qualitatively.

[11]Therefore, we effectively assume that both (i) households' productivity in turning capital into output; and (ii) their productivity in turning output into capital are both lower than of optimists with the same proportionality $l \leq 1$. This is for tractability and even if we assume different proportionality for the two productivity measures, most results in this paper do not change qualitatively. Thus, we interchangeably call optimists 'experts'.

discount factor ρ^i. The consumption space defined above must also be square-integrable:

$$\int_0^\infty \left|c_t^i\right|^2 dt < \infty.$$

With $c^i \equiv \{c_t^i\}_{t=0}^\infty$, each agent of type $i \in \{O, H\}$ maximizes her expected lifetime utility given by:

$$U(c^i) \triangleq \int_0^\infty e^{-\rho^i t} u^i(c_t^i)dt, \quad \forall t \in [0, \infty). \tag{7}$$

The utility function obeys the standard assumption that $u^i(c_t^i)$ is continuously differentiable, increasing, and concave for $i \in \{O, H\}$.

2.1.3 Market for Capital

Both optimists and households have the opportunity to trade the physical capital in a competitive market. We denote the equilibrium market price of capital in terms of the output by p_t with corresponding state space $F^p \subseteq \mathbb{R}$, and assume it satisfies the following endogenous Ito's process:

$$\frac{dp_t}{p_t} \triangleq \mu_t^p dt + \sigma_t^p dZ_t, \tag{8}$$

where μ_t^p and σ_t^p are drift and volatility of the capital price process (8), respectively. Based on the definition, capital k_t^O costs $p_t k_t^O$ for optimists. Note that, in equilibrium, p_t, μ_t^p and σ_t^p are determined endogenously.

2.1.4 Return to Capital

A straightforward application of Ito's lemma in (3) and (8) reveals that when an optimists hold k_t^O units of capital at price p_t, the total value of this capital (i.e., $p_t k_t^O$) evolves according to:

$$\frac{d(p_t k_t^O)}{p_t k_t^O} = \left(\Lambda^O(\iota_t) - \delta^O + \mu_t^p\right) dt + \sigma_t^p dZ_t. \tag{9}$$

Hence, the total return that experts earn from capital (per unit of wealth invested) is given by:

$$dr_t^{Ok} = \frac{\gamma_t^O - \iota_t^O \gamma_t^O}{p_t} dt + \left(\Lambda^O(\iota_t^O) - \delta^O + \mu_t^p\right) dt + \sigma_t^p dZ_t. \tag{10}$$

Similarly, a household earns the return of

$$dr_t^{Hk} = \frac{\gamma_t^H - \iota_t^H \gamma_t^H}{p_t} dt + \left(\Lambda^H(\iota_t^H) - \delta^H + \mu_t^p\right) dt + \sigma_t^p dZ_t. \tag{11}$$

2.1.5 Optimism and Consistency

Rational Households The households are rational, i.e., they know that their productivity γ_t^H follows the process in (2) and (5). They observe realized dZ_t in each period, and have a full knowledge of the above aggregate processes, i.e., equations (8), (9), (10), and (11).

Optimists Optimists, in contrast, observe their productivity γ_t^O at any instant, but have incomplete information on its exact dynamics. With (2), we know that γ_t^O actually follows

$$\frac{d\gamma_t^O}{\gamma_t^O} = \alpha dt + \sigma dZ_t, \quad \forall t \in [0, \infty). \tag{12}$$

While optimists observe the left-hand side, $\frac{d\gamma_t^O}{\gamma_t^O}$, they do not observe realized dZ_t, an actual Brownian motion, and do not know true α. Instead, optimists believe that their productivity γ_t^O follows

$$\frac{d\gamma_t^O}{\gamma_t^O} = \alpha^O dt + \sigma dZ_t^O, \quad \forall t \in [0, \infty), \tag{13}$$

where α^O is possibly different from α and Z_t^O is their own 'perceived' Brownian motion. We assume $\alpha^O > \alpha$, i.e., the optimists believe that their productivity's growth rate is higher than its true rate α in (2). Optimists are *dogmatic* and do not learn from the realized data, in a similar manner to Yan (2008) and Chabakauri (2015).[12]

As optimists do not observe dZ_t and believe that the true Brownian motion is dZ_t^O, they believe that the aggregate capital price follows:

$$\frac{dp_t}{p_t} \triangleq \mu_t^{p,O} dt + \sigma_t^p dZ_t^O. \tag{14}$$

where the relation between $\mu_t^{p,O}$ in (14) and μ_t^p in (8) is to be determined *in equilibrium* by the next consistency condition. Optimists observe $\mu_t^{p,O}$, not μ_t^p.

[12]This assumption of dogmatic optimism can be restrictive but yields meaningful intuitions of how optimistic expectation generates counterintuitive results about macroeconomic crises, especially a net-worth trap. Later in Section 2.5.3 we relax this assumption to incorporate a swinging sentiment of optimists that depends on macroeconomic environments.

Consistency From (12), and (13), we obtain the following aggregate consistency condition for optimists:

$$Z_t^O = Z_t - \frac{\alpha^O - \alpha}{\sigma} t. \tag{15}$$

In other words, optimists regard Z_t^O, not Z_t, as the Brownian motion driving the business cycle, while Z_t^O is not a Brownian motion under the rational expectations. With $\alpha^O \geq \alpha$,[13] we call equation (15) the optimism process. This condition acts as an additional equilibrium condition in our model.

By combining equations (8), (14), and (15), in equilibrium we obtain

$$\mu_t^{p,O} = \mu_t^p + \frac{\alpha^O - \alpha}{\sigma} \sigma_t^p, \tag{16}$$

where optimists observe only the left-hand side, $\mu_t^{p,O}$, and observe neither μ_t^p nor $\frac{\alpha^O - \alpha}{\sigma} \sigma_t^p$, as they do not know α. From (10) and (15), the optimists believe that the total return that they earn from capital holding (per unit of wealth invested) would be given by

$$dr_t^{Ok} = \underbrace{\left[\frac{\gamma_t^O - \iota_t^O \gamma_t^O}{p_t} + \left(\Lambda^O(\iota_t^O) - \delta^O + \mu_t^{p,O} \right) \right]}_{\text{Drift of } dr_t^{Ok} \text{ process in optimists' mind}} dt + \sigma_t^p \underbrace{dZ_t^O}_{\substack{\text{Optimists'} \\ \text{Brownian motion}}}. \tag{17}$$

where $\mu_t^{p,O}$ is observed by the optimists, and provided by equation (16) in equilibrium.

Two points regarding equation (17) are important to note: (i) with $\alpha^O \geq \alpha$, optimists believe that the expected capital gain they earn when investing in physical capital is higher than implied under the rational expectations; (ii) the degree of optimism in terms of the 'expected' return (i.e., $\frac{\alpha^O - \alpha}{\sigma} \sigma_t^p$) becomes proportional to the endogenous capital price volatility σ_t^p: thereby, a higher endogenous risk σ_t^p mechanically raises the degree of optimism in asset returns.

2.2 Consumption-Portfolio Problems

Optimists Optimists can invest in the physical capital, or the risk free asset, which is zero net supplied. The non-monetary net worth w_t^O of each optimist, who invests fraction x_t^O of

[13] The case of $\alpha^O < \alpha$ would correspond to the case in which experts are pessimists. Our analytical results do not rely upon the relative sizes of α^O though.

her wealth in capital and consumes with the rate c_t^O, evolves according to:

$$dw_t^O = x_t^O w_t^O dr_t^{Ok} + (1 - x_t^O) r_t w_t^O dt - c_t^O dt, \tag{18}$$

where r_t is the risk-free interest rate prevailing in the economy. Note that r_t is an equilibrium object to be determined endogenously. x_t^O represents the share of wealth the optimists invest in capital. Later, it will turn out that in most cases optimists use greater-than-0 leverage (i.e., $x_t^O > 1$). In most cases, we call x_t^O 'leverage' or 'leverage multiple' of optimists.

Formally, each optimist solves

$$\max_{x_t^O \geq 0, c_{t,}^O \geq 0} \mathbb{E}_0^O \left[\int_0^\infty e^{-\rho t} u^O \left(c_t^O \right) dt \right], \tag{19}$$

subject to the solvency constraint $w_t^O \geq 0$ and the dynamic budget constraint (18). In optimization (19), the expectation operator $\mathbb{E}_0^O$ means that optimists believe dZ_t^O, not dZ_t, is the true Brownian motion. Therefore in characterizing (18), they use (17) for the capital return process dr_t^{Ok} instead of (10) with the true dZ_t. Note that we assume away short-sale for capital, thereby $x_t^O \geq 0$.

Households In a similar way to optimists' problem in (19), the non-monetary net worth w_t^H of households, who invest fraction x_t^H of their wealth in capital and consume with rate c_t^H, would follow

$$dw_t^H = x_t^H w_t^H dr_t^{Hk} + w_t^H (1 - x_t^H) r_t dt - c_t^H dt. \tag{20}$$

Formally, each household solves

$$\max_{x_t^H \geq 0, c_{t,}^H \geq 0} \mathbb{E}_0 \left[\int_0^\infty e^{-\rho t} u \left(c_t^H \right) dt \right], \tag{21}$$

subject to the solvency constraint $w_t^H \geq 0$ and the dynamic budget constraints (20). Unlike optimists, in optimization (21), the expectation operator $\mathbb{E}_0$ means that households have the rational expectations: they believe dZ_t is the true Brownian motion. Therefore in characterizing (20), they use (11) for the capital return process dr_t^{Hk} with the true dZ_t. Note that we assume away short-sale for capital, thereby $x_t^H \geq 0$.

2.3 Equilibrium and Market Clearing

Intuitively, an equilibrium with full information is characterized by a map from shock histories $\{Z_S, s \in [0,t]\}$, to the prices p_t and asset allocations such that, given prices, agents maximize their expected utilities[14]and markets clear.[15] To define our equilibrium more formally, we denote the set of optimists by an interval $I = [0,1]$ and index any optimist (i.e., expert) by $i \in I = [0,1]$. Similarly, we denote a set of the households by $J = (1,2]$ with index $j \in J = (1,2]$. Since optimists deviate from the rational expectations with their own perceived shocks Z_t^O, they have their own expectation operator $\mathbb{E}_0^O$ as shown in (19). Note that in equilibrium, every agent in the same group (i.e., optimists or households) chooses the same consumption and portfolio decisions.

We now state the market clearing conditions and formally define the equilibrium. The three markets that must clear in equilibrium at any given instant are the physical capital, consumption good, and risk-free debt markets.

Capital Market The total amount of the capital demanded by optimists and households is equal to the aggregate supply of capital in the economy: i.e.,

$$\int_0^1 k_t^O di + \int_1^2 k_t^H dj = K_t, \ \ \forall t \in [0,\infty), \tag{22}$$

where K_t is the total supply of capital. The total supply of capital in the model is not fixed as both optimists and households invest in the new capital through their investments. The following (23) describes the evolution of the total supply of capital:

$$dK_t \triangleq \left(\int_0^1 \left(\Lambda^O\left(\iota_t^O\right) - \delta^O\right) k_t^O di + \int_1^2 \left(\Lambda^H\left(\iota_t^H\right) - \delta^H\right) \underline{k}_t^H dj \right) dt, \ \ \forall t \in [0,\infty). \tag{23}$$

Good Market Whatever is produced and not invested, has to be consumed. That is,[16]

$$\int_0^1 k_t^O \left(\gamma_t^O - \iota_t^O \gamma_t^O\right) di + \int_1^2 k_t^H \left(\gamma_t^H - \iota_t^H \gamma_t^H\right) dj = \int_0^1 c_t^O di + \int_1^2 c_t^H dj, \ \ \forall t \in [0,\infty). \tag{24}$$

[14]The optimists solve optimization (19) subject to their dynamic budget constraint (18) and the solvency $w_t^O \geq 0$ while the households solve optimization (21) subject to their dynamic budget constraint (20) and the solvency $w_t^P \geq 0$.

[15]The physical capital, output, and debt markets must clear in equilibrium.

[16]In equation (24), we use the fact that the investment rate at time t is given by $i_t^i \gamma_t^i k_t^i$ for $i \in [0,2]$.

Debt Market The debt market clearing condition implies that the value of the debt that optimists receive should be equal to the value of loans that the households extend, namely,

$$\int_0^1 \left(w_t^O - p_t k_t^O\right) di + \int_1^2 \left(w_t^H - p_t k_t^H\right) dj = 0. \tag{25}$$

By defining all three markets, we are in a position to define the economy's equilibrium.

Definition 1 *The equilibrium consists of the stochastic processes of (i) the price of capital p_t, (ii) interest rate r_t, (iii) investment rate and consumption, i.e. $\left\{\left(k_t^O, \iota_t^O, c_t^O\right), t \geq 0\right\}$ for optimists, and $\left\{\left(k_t^H, \iota_t^H, c_t^H\right), t \geq 0\right\}$ for households, which should satisfy the following three conditions:*

1. Given their perceived Brownian motion Z_t^O, the optimists $O \in [0, 1]$ solve optimization (19) subject to their dynamic budget constraint (18) and the solvency $w_t^O \geq 0$. Households $H \in (1, 2]$ solve optimization (21) subject to their dynamic budget constraint (20) and the solvency $w_t^j \geq 0$, under the rational expectations.

2. The capital (i.e., (22) and (23)), consumption (i.e., (24)), and debt (i.e., (25)) markets clear.

3. The consistency condition (15) between Z_t^O and Z_t holds.

2.4 Equilibrium Characterization

In this section, we discuss how to find the equilibrium price p_t with its endogenous drift μ_t^p and volatility σ_t^p, the optimists and households' consumption and portfolio decisions, the risk-free interest rates r_t, given the history of perceived shock processes $\left\{Z_s^O, Z_s, 0 \leq s\right\}$. We first start with some definitions.

Definition 2 *The aggregate wealth of both optimists and households is given by summing up their individual wealth*[17] *respectively, that is,*

$$W_t^O = \int_0^1 w_t^O di, \;\; W_t^H = \int_1^2 w_t^H dj, \quad \forall t \in [0, \infty).$$

[17]Each optimist chooses the same consumption and portfolio decision, and thus has the same wealth w_t^O. Likewise, each household owns the same wealth level w_t^H in equilibrium.

Observe that the capital market clearing condition (22) and the debt market clearing condition (25) become[18]

$$W_t^O + W_t^H = p_t K_t \tag{26}$$

where K_t is the supply of aggregate capital that follows the process (23). Finally, the good market equilibrium condition (24) can be written as

$$\int_0^1 \frac{x_t^O w_t^O}{p_t} \left(\gamma_t^O - \iota_t^O \gamma_t^O\right) di + \int_1^2 \frac{x_t^H w_t^H}{p_t} \left(\gamma_t^H - \iota_t^H \gamma_t^H\right) dj = \int_0^1 c_t^O di + \int_1^2 c_t^H dj. \tag{27}$$

We now characterize the equilibrium based on the economy's state variable: the wealth share of optimists. By representing each variable in terms of optimists' wealth share which is bounded between 0 and 1, we characterize the equilibrium price and quantity variables in a similar manner to the literature (e.g., He and Krishnamurthy (2013), Brunnermeier and Sannikov (2014)).

The proportion of wealth that optimists possesses which we denote by η_t is given by,

$$\eta_t = \frac{W_t^O}{W_t^O + W_t^H} \underbrace{=}_{(26)} \frac{W_t^O}{p_t K_t}. \tag{28}$$

which we postulate follows the process:

$$\frac{d\eta_t}{\eta_t} = \mu_t^\eta(\eta_t)dt + \sigma_t^\eta(\eta_t)dZ_t. \tag{29}$$

The economy's dynamics is driven by η_t in our Markov equilibrium as in the literature (e.g., He and Krishnamurthy (2013), Brunnermeier and Sannikov (2014)). We have

$$x_t^O \leq \frac{1}{\eta_t}, \tag{30}$$

which translates to the fact that the maximum 'leverage' that optimists can obtain is bounded above by $\frac{1}{\eta_t}$. We identify two regions: the first one is when (30) binds and the second one is when leverage is strictly less than $\frac{1}{\eta_t}$, i.e., when (30) does not bind. We call the first region '*normal*' and the second region by '*crisis*'. In other words, in the *normal* region, all the

[18]Equation (26) is equivalent to

$$\int_0^1 x_t^O w_t^O di + \int_1^2 x_t^H w_t^H dj = p_t K_t, \quad \forall t \in [0, \infty).$$

physical capital will be owned by optimists, while in the *crisis* regime, some of the capital must be purchased by households.

Internal Investment Hereafter, we define the investment functions $\Lambda^O(\cdot)$ that optimists use as follows:

$$\Lambda^O(\iota_t^O) = \frac{1}{k}\left(\sqrt{1+2k\iota_t^O}-1\right), \ \ \forall t \in [0,\infty), \tag{31}$$

which satisfies all the standard assumptions in Section 2.1.1. We do not allow disinvestment, thus $i_t^O \geq 0$. Note that we use the mathematical form in (31) of $\Lambda^O(i_t^i)$ for simplicity and acknowledge that our results do not change qualitatively even if we use different forms for $\Lambda^O(\cdot)$ that satisfy conditions in Section 2.1.1. Similarly, we define the internal investment function $\Lambda^H(i_t^H)$ of households as

$$\Lambda^H(\iota_t^H) = l \cdot \Lambda^O(\iota_t^H), \ \ \forall \iota_t^H. \tag{32}$$

From now on, we express our equilibrium with the following normalized asset price:

Definition 3 *The normalized asset price q_t is defined as $q_t \equiv \dfrac{p_t}{\gamma_t^O}$.*

The normalized asset price q_t can be interpreted also as the price-earnings ratio of physical capital for optimists. It turns out we can characterize the 'stationary' equilibrium when we write our model in terms of q_t instead of p_t, as we have exogenous growth.

2.4.1 Solving the Consumption-Portfolio Problems

An optimist solves the optimization (19) subject to her wealth dynamics (18) and the solvency $w_t^O \geq 0$. Likewise, each household optimizes her lifetime utility (21) subject to the evolution of his wealth (20) and the solvency $w_t^H \geq 0$. We focus on the case where all the optimists and households have the same logarithmic utility function, i.e., $u^O(c_t^O) = \log c_t^O$, $u^H(c_t^H) = \log c_t^H$ for mathematical tractability.

The Proposition 1 solves the respective optimal choices of optimists and households for consumption c_t^{O*}, c_t^{H*}, leverage x_t^O, x_t^H, and investment ι_t^{O*}, ι_t^{H*}, and the equilibrium risk free interest rate r_t^*. $x_t^H \geq 0$ must hold since the short-sale of capital is not allowed.

Proposition 1 *Assume optimists and households have the logarithmic utility, i.e., $u^O(c_t^O) = \log c_t^O$, and $u^H(c_t^H) = \log c_t^H$. Then,*

1. *The optimal consumption c_t^{O*} is given by $c_t^{O*} = \rho w_t^O$. The household's consumption c_t^{H*} is given by $c_t^{H*} = \rho w_t^H$.*

2. *The equilibrium interest rate r_t^* is given by:*

$$r_t^* = \left(\frac{\gamma_t^O - \iota_t^O \gamma_t^O}{p_t} + \Lambda^O(\iota_t^O) - \delta^O + \mu_t^p + \frac{\alpha^0 - \alpha}{\sigma} \sigma_t^p \right) - x_t^O \left(\sigma_t^p\right)^2,$$

where x_t^O is each optimist's optimal portfolio choice (i.e., leverage multiple) as defined in (18). Given this r_t^*, the optimal portfolio choice of households x_t^H as defined in (20) is given by:[19]

$$x_t^H = \max \left\{ \frac{\left(\frac{\gamma_t^H - \iota_t^H \gamma_t^H}{p_t} + \Lambda^H(\iota_t^H) - \delta^H + \mu_t^p \right) - r_t^*}{\left(\sigma_t^p\right)^2}, 0 \right\}.$$

3. *The optimal investment rates ι_t^{O*} and ι_t^{H*} are given by:*

$$i_t^{O*}(q_t) = i_t^{H*}(q_t) = \frac{q_t^2 - 1}{2k}.$$

Note that given i_t^O, μ_t^p, σ_t^p, and x_t, the equilibrium interest rate r_t^* increases due to the optimism measure, i.e., $\alpha^O - \alpha$. If optimists believe that the expected capital gain they earn when investing in capital is higher, they will try to get more loans from households, raising the equilibrium interest rate r_t^*.

Finally, in order to conclude with the characterization of equilibrium, we need to derive the evolution of the state variable η_t. First, we define the fraction of physical capital held by optimists and pessimists by

$$\psi_t \equiv \frac{k_t^O}{K_t}, \;\; 1 - \psi_t = \frac{k_t^P}{K_t}, \tag{33}$$

where the aggregate capital K_t follows the process in (23). Then, the leverage multiples of optimists and households, i.e., x_t^O and x_t^H, respectively, can be characterized from (28) and

[19]Therefore, if the households' *perceived* risk-premium levels are below 0, they only invest in risk-free loans issued by optimists.

(33) as:

$$x_t = \frac{\psi_t}{\eta_t} \qquad \underline{x}_t = \frac{1-\psi_t}{1-\eta_t}. \tag{34}$$

2.4.2 The Aggregate Dynamics

Based on the above definitions, the evolution of the proportion of wealth held by optimists can be characterized in the following Proposition 2. Note that in Proposition 2, we express the aggregate dynamics in the true Brownian motion dZ_t.

Proposition 2 *The evolution of optimists' wealth relative to the entire economy is given by*

$$\frac{d\eta_t}{\eta_t} = \mu^\eta(\eta_t)dt + \sigma^\eta(\eta_t)dZ_t \tag{35}$$

where

$$\mu^\eta(\eta_t) = \left(\frac{\psi_t - \eta_t}{\eta_t}\sigma_t^p\right)^2 - \frac{\psi_t - \eta_t}{\eta_t}\frac{\alpha^O - \alpha}{\sigma}\sigma_t^p + \frac{1-\iota_t^O}{q_t} + (1-\psi_t)\left(\delta^H - \delta^O\right) + (1-l)\left(1-\psi_t\right)\Lambda^O(i_t^O) - \rho,$$

and

$$\sigma^\eta(\eta_t) = \frac{\psi_t - \eta_t}{\eta_t}\sigma_t^p.$$

Now by knowing the drift μ_t^η and the volatility σ_t^η of our state variable η, we calculate the the drift μ_t^p and the volatility σ_t^p of the price of capital p_t.

Proposition 3 *When q_t is a function of state variable η_t with $q_t = q(\eta_t)$, the drift μ_t^p of the price of capital p_t is given by*

$$\mu_t^p = \alpha + \frac{q'(\eta_t)}{q_t}\mu^\eta(\eta_t)\eta_t + \frac{1}{2}\sigma^\eta(\eta_t)\eta_t\frac{q''(\eta_t)}{q(\eta_t)} + \sigma\sigma^\eta(\eta_t)\eta_t\frac{q'(\eta_t)}{q(\eta_t)}, \tag{36}$$

where $\sigma_t^\eta(\eta_t)$ is given by Proposition 2, and the volatility of the price of capital, σ_t^p, is given by

$$\sigma_t^p = \frac{\sigma}{1 - (x_t\eta_t - \eta_t)\dfrac{q'(\eta_t)}{q(\eta_t)}}. \tag{37}$$

We can observe from (35) that optimism (i.e., $\alpha^O > \alpha$) given $\psi_t \geq \eta_t$ (which holds in the efficient region[20]) lowers the drift $\mu^\eta(\eta_t)$. It implies that the optimism hinders the process

[20]In the efficient region, all the capital is held by optimists, thereby $\psi_t = 1$.

of optimists' wealth recapitalization and tends to lower their wealth share in the long run, potentially generating a 'net worth trap' when the level of optimism, i.e., $\alpha^O - \alpha$, is high enough. This is caused by the optimists' expectation errors (i.e., wrong judgment about the capital return), and our main focus in Section 2.5.

We derive a closed formed solution for leverage multiple and a first-order differential equation for the price of capital in Appendix B.1.

2.5 Model Analysis

Now, we solve our model numerically under the following calibration. The parameters with optimism $\alpha^O = 0.13$ are chosen to target a probability of crisis of 5%. $\alpha^O = \alpha = 0.07$ case corresponds to the case where both optimists and households are rational, i.e., $\mathbb{E}_t^O = \mathbb{E}_t$.

	l	δ^O	δ^H	ρ	χ	σ	k	α	α^O set
Values	0.4	0	0	0.03	1	0.08	18	0.07	[0.07, 0.13, 0.19]

Table 1: Parameterization for $\alpha^O \geq \alpha$

2.5.1 Risk Amplification from the Optimism

Figure 1 presents the equilibrium quantities of the model for different levels of optimism.[21] In the economy's stochastic steady state, all capital is operated by the optimists who are sufficiently capitalized (i.e., η_t is high). Since the optimists have a superior ability to manage capital, this 'efficient region' of the economy features a high capital price with low endogenous volatility. The q-theory result implies that investment rate is high, and that in turn leads to higher growth rates in both capital and output. A series of negative shocks then drains the net worth of optimists since they operate with leverage, as seen in Figure 1b. Once the wealth share falls sufficiently below a threshold, say η^ψ, optimists start fire-selling capital to the inefficient agents (i.e., rational households) who demand a high risk premium to hold the risky asset since they are not as productive in operating the capital as the optimists. This inefficient "crisis" region features low capital price and investment, high endogenous volatility, and a low output. These features are common to many macro-finance models, e.g., Brunnermeier and Sannikov (2014, 2016b). The role of optimism in

[21] Figure A1 in Appendix A provides other equilibrium quantities not included in Figure 1.

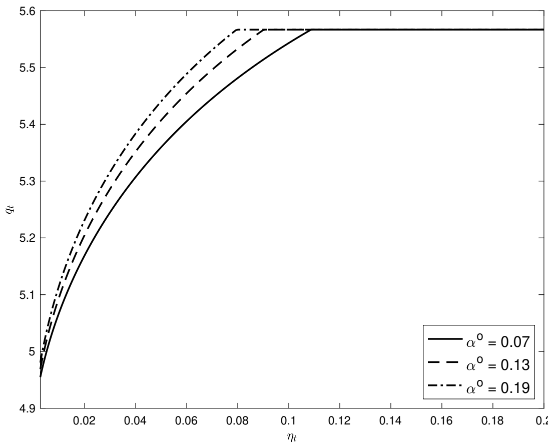
our model is to change the equilibrium allocation in two ways: by affecting the threshold at which optimists fire-sell capital, and the amplification that the fire-selling generates. The effect of optimism is particularly salient in the crisis region with little effect around the stochastic steady state. For example, optimism $\alpha^O - \alpha$ does not affect the price of capital q_t in the steady state since optimists operate all capital and the 'risk-adjusted' return that the households obtain from holding capital is less than the prevailing risk-free rate.[22] Thus they prefer to invest in risk-free bonds issued by optimists. However, in the inefficient region where less productive households hold a portion of capital, the degree of optimism $\alpha^O - \alpha$ starts to matter. As the optimism increases, the threshold for the efficient region η^ψ falls. That is, a higher optimism leads to a larger part of the state space where optimists operate all capital by taking on more leverage. In particular, the fact that they take on more leverage in the normal region precipitates to a stronger fire-selling effect as evidenced by a steeper slope of capital price as seen in Figure 1a, characterizing the boom-bust cycles documented in the literature (see Krishnamurthy and Li (2020), for example).

The role of optimism as risk amplifiers on top of the financial accelerator role can be seen by inspecting the endogenous volatility of price σ_t^p shown in Figure 1c. As optimism increases, the endogenous volatility σ_t^p is more amplified in the crisis region. Actually, the endogenous risk can be written as

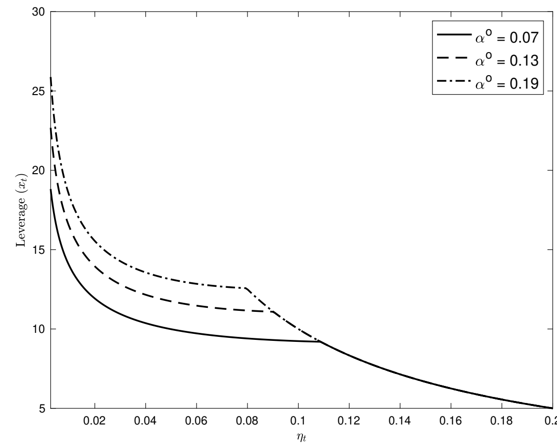
$$\sigma_t^p \left(1 - (x_t - 1) \frac{\frac{dq(\eta_t)}{q(\eta_t)}}{\frac{d\eta_t}{\eta_t}} \right) \equiv \sigma_t^p \left(1 - (x_t - 1)\, \varepsilon_{q,\eta}\right) = \sigma, \tag{38}$$

where $\varepsilon_{q,\eta}$ is defined as the elasticity of the price-earnings ratio (i.e., the normalized capital price) with respect to the optimists' wealth share η_t. We observe that optimism raises the price volatility through a higher leverage (*leverage effect*). Second, a greater optimism leads to a higher price elasticity (*elasticity effect*), captured by $\epsilon_{q,\eta}$, and magnifies the leverage effect. The elasticity effect arises due to the intense fire-selling of optimists explained earlier. In short, a higher optimism causes a boom phase of excessive leverage, which leads to a strong fire-sales of risky capital to the inefficient agents, both of which contributing to a higher price volatility σ_t^p in the economy's crisis region. One point worth noting is that we capture a belief-driven adverse loop: a higher optimism increases endogenous risk σ_t^p in

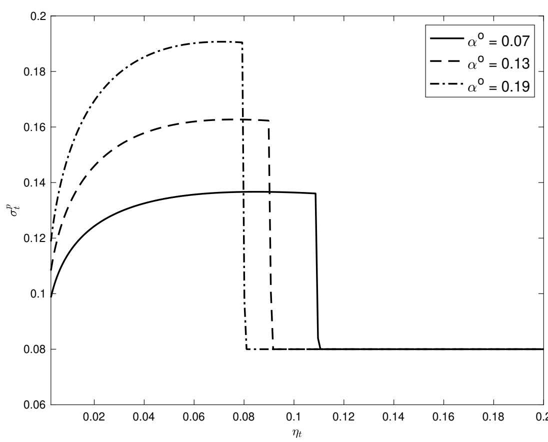
[22] We observe in Figure 1d that the *true* risk premium is negative around the stochastic steady state, which aligns with the fact that households do not hold capital at all in those regions. In contrast, the *perceived* risk premium of optimists is positive in both regions.



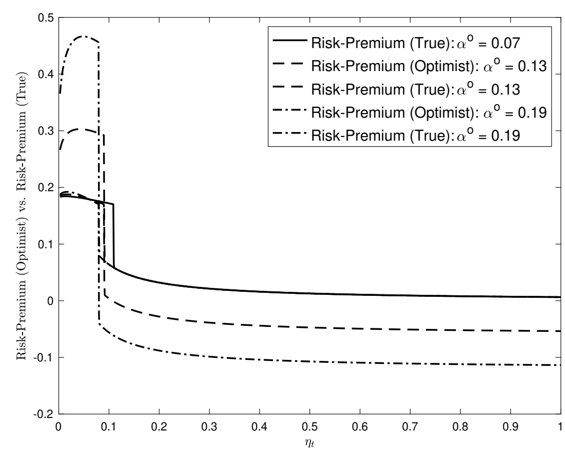
(a) Capital price q_t



(b) Leverage multiple x_t



(c) Endogenous volatility σ_t^p



(d) *Perceived* and *true* risk-premium

Figure 1: Equilibrium variables as functions of η_t

a crisis, raising the degree of optimism in the expected capital return as seen in (17), which in turn amplifies the endogenous risk even further.

Lastly, when optimists are sufficiently capitalized, a greater optimism leads to a lower interest rate, as seen in Figure A1c of Appendix C.3. In the crisis region where optimists fire-sell capital to households, a higher optimism creates a stronger precautionary saving motive due to a greater endogenous volatility, pushing down the risk free rate.[23] Most of the effects explained thus far stem from the fact that the risk premium perceived by optimists

[23]When optimists start deleveraging and fire-sell their capital assets, the interest rate r_t drops in a discontinuous manner. It is anticipated: as optimists suddenly fire-sell the capital and reduce their leverage demand, the risk-free rate should drop to clear the bond market. After the economy enters the inefficient region, i.e., $\eta_t \leq \eta^{\psi}$, the interest rate level becomes continuous again.

are different from the true risk premium of capital. This expectation error affects the net worth of the optimists strong enough to generate an amplified boom-bust cycle. Optimism makes the productive agents in the economy to take on excessive leverage precisely because they perceive the risk premium to be higher than it actually is.

The expectation error stemming from optimism can drain the net worth of optimists at the stochastic steady state, and raise the occupation time of economy in a crisis. In fact, the static comparative plot in Figure 1 reveals that there is a smaller measure of the state space where capital is misallocated to unproductive households from fire-sales, when optimism is higher. However, since the expectation errors, by which optimists' perceived risk-premium is higher than its true level, are increasing in the optimism degree, their net worth can be drained at a high rate, trapping their wealth share at low levels and causing inefficiency for a long time.

This important channel explains a novel phenomenon called as 'net-worth trap' which we explain next by characterizing the stationary distribution of the optimists' wealth share.

Net Worth Trap Figure 2 draws the stationary distribution of η_t in the presence of optimism: we observe that under the benchmark rational expectations model, i.e., $\alpha^O = \alpha$, the time the economy spends the most is around its stochastic steady state.[24] We observe that optimism generates a larger density mass in the crisis region, with sharper peaks inside the crisis region, especially around $\eta \simeq 0$. When negative shocks shift the economy toward the inefficient region, a higher optimism, on average, makes the economy more vulnerable to negative shocks through its effects on leverages, increasing its occupancy time in a crisis. In fact, even moderate levels of optimism generates this effect in which all of the capital is allocated to inefficient agents in the economy for most of the time. In a case where optimism is higher than a given threshold, in addition to this inefficient allocation, the probability of optimists to rebuild their net worth becomes zero. We call this phenomenon a "net worth trap", since the economy gets trapped in this state and remains perennially inefficient.

The origins of the net worth trap lies in the endogenous risk and expectation errors of optimists. There are two opposing forces in a crisis that affect the net worth of optimists. First, risk premium (both true and perceived) is higher in a crisis, which loads positively on the expected growth rate of optimists' net worth. This is because optimists are leveraged

[24] In generating Figure 2, we introduce the exogenous exit of optimistic experts, as otherwise we obtain the degenerate steady state distribution at $\eta_t = 1$ in the rational expectations benchmark when the discount rates for optimists and households are equal. We use the exit rate= 7.5%. Other than generating a well-behaved stationary distribution, the introduction of exits of optimists has no effect on other equilibrium properties.

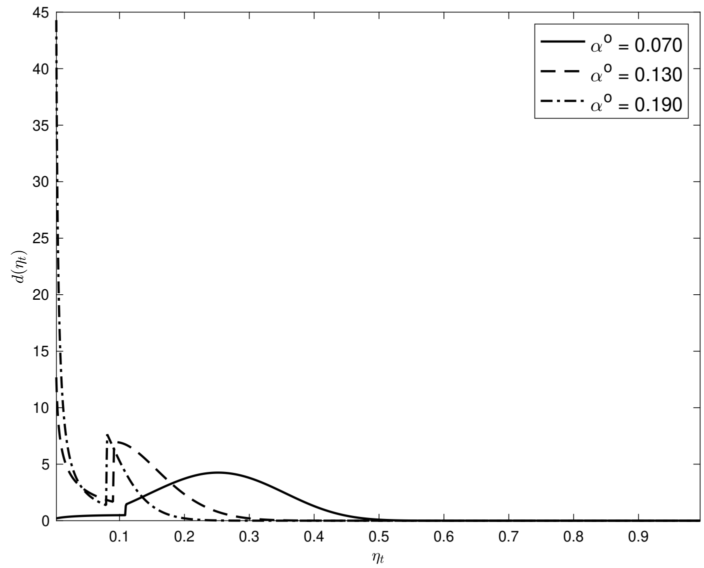
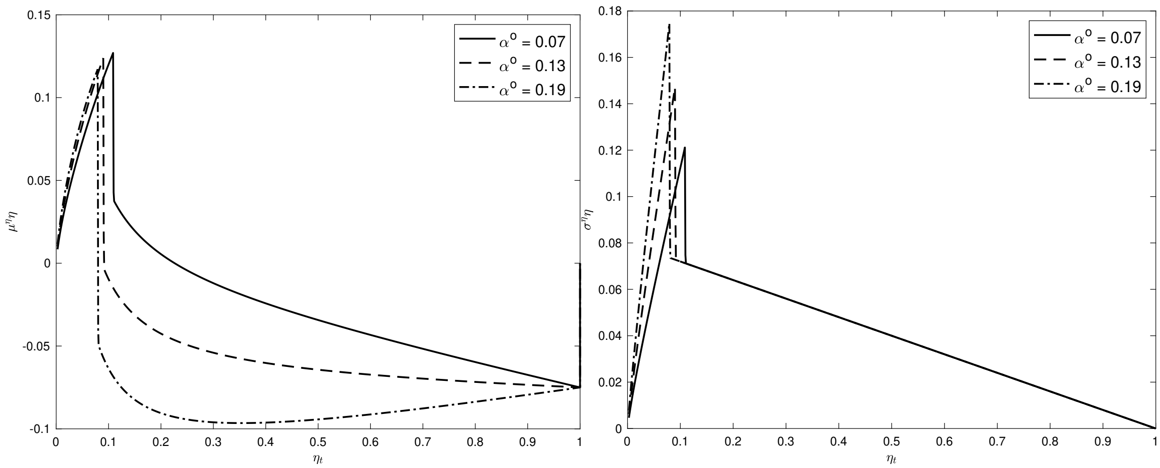


Figure 2: Stationary distribution of η_t and the net worth trap

in risky capital and earn a premium for holding it, helping them to recapitalize faster. The other force is the expectation error from optimistic beliefs about the long run growth rate of productivity. When the optimism α^O is high enough, the second expectation error channel dominates the first recapitalization channel. This effect drains the net worth of experts and (i) increases the likelihood of fire-selling capital in the steady state, and (ii) increases the occupancy time in a crisis. The effect of this expectation error on the drift of wealth share of optimists can be seen in Figures 3a. The drift $\mu^\eta(\eta)\eta$ becomes more negative in the efficient region. Moreover, its endogenous risk, captured by $\sigma^\eta(\eta)\eta$, increases in optimism during a crisis, but does not change with optimism in the efficient region, as shown in Figure 3b. The growth rate of optimists' net wealth share (i.e., drift) in the efficient region relative to its endogenous volatility is a crucial variable determining the probability of recapitalization for optimists.

We next show even for a moderate level of optimism, the growth rate of the optimists' wealth share relative to its endogenous volatility becomes low enough (due to high expectation errors) to create a two-peaked stationary distribution with an infinite mass at $\eta = 0$. For high optimism levels, the *relative* growth rate becomes low enough to force the probability of recapitalization of optimists to zero, as documented by the next Proposition 4.

Proposition 4 (Net Worth Trap) *There exists a threshold level of optimism $\bar{\alpha}^O$ above which the economy is trapped at $\eta = 0$, and the probability of recapitalization for optimists goes to zero.*



(a) $\mu_\eta(\eta_t) \cdot \eta_t$ as a function of η_t (b) $\sigma^\eta(\eta_t) \cdot \eta_t$ as a function of η_t

Figure 3: Wealth share dynamics: drift and volatility

Proof. We analyze the tail of the stationary density to pin down a threshold value of optimism that generates a net worth trap. From (B.35) in Appendix B.2, the asymptotic solution for the stationary density $d(\eta)$ when $\eta \sim 0$ is given by

$$d(\eta) \sim \left(\frac{2\mu^\eta(0)}{(\sigma^\eta)^2(0)} - 1\right)\eta^{\frac{2\mu^\eta(0)}{(\sigma^\eta)^2(0)}-2} \tag{39}$$

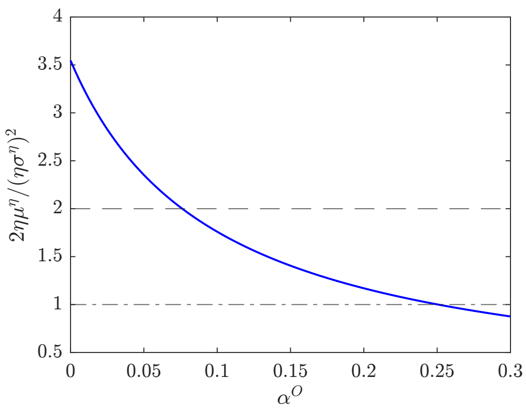
where the ratio $\tilde{D}_0 \equiv \frac{2\mu^\eta(0)}{(\sigma^\eta)^2(0)}$ determines the existence of a degenerate distribution around $\eta \sim 0$. If $\tilde{D}_0$ goes below 1, then the stationary distribution $d(\eta)$ becomes degenerate (i.e., Dirac-delta) around $\eta = 0$.[25]

Appendix B.2 offers an analytic proof, providing a threshold of $\alpha^O - \alpha$ that generates a permanent trap of net worth with zero recapitalization chance: if $\alpha^O - \alpha$ becomes above the threshold, the economy is permanently trapped at $\eta \sim 0$. Here, we provide key intuitions based on the simulation result.

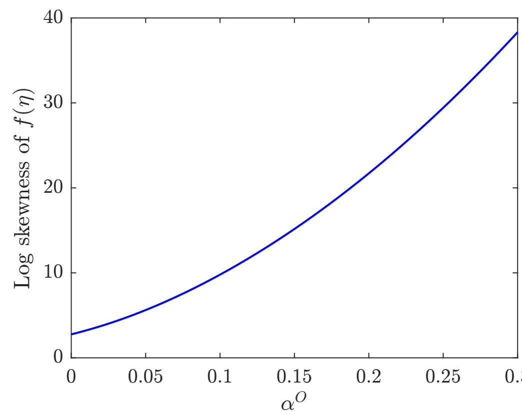
Figure (4a) plots $\tilde{D}_0$ against the optimists' perceived long-run growth rate α^O when the true growth rate is $\alpha = 0.07$. With α^O close enough to α, $\tilde{D}_0$ is far above 2, indicating a non-degenerate stationary distribution with finite mass at the point $\eta \sim 0$. In this case, even when the economy enters a crisis, the relative growth rate of optimists' net worth compared with its volatility is high (i.e., expectation errors are small) enough to enable optimists to recapitalize. Under higher optimism, the quantity $\tilde{D}_0$ becomes smaller than the threshold

[25] $\mu^\eta(0) \equiv \lim_{\eta\to 0} \mu^\eta(\eta)$ and $\sigma^\eta(0) \equiv \lim_{\eta\to 0} \sigma^\eta(\eta)$ in (39) are derived analytically in Appendix B.2.

value of 2, indicating that the density features infinite mass at the point $\eta \sim 0$. Once the optimism crosses a threshold value $\bar{\alpha}^O$, $\tilde{D}_0$ falls below 1, where the stationary distribution becomes a Dirac-delta measure at $\eta = 0$. In this case, the relative growth rate of optimists' wealth share compared with its volatility is negative (i.e., expectation errors are large) such that once the economy enters a crisis, the negative growth rate drains the wealth share until it reaches the point $\eta = 0$. At this point, the optimists can no longer recapitalize, and the economy gets trapped in this state inefficiently forever. ■



(a) Tail analysis of stationary distribution



(b) Skewness of the distribution around $\eta \sim 0$

Figure 4: The dashed line in Figure 4a indicates the threshold value of 2 below which the economy generates a net worth trap.

Corollary 1 *The asymptotic skewness of the stationary distribution $d(\eta)$ increases in optimism α^O.*

We can analyze the third moment of the distribution $d(\eta)$ when η approaches 0. Figure 4b draws $\log$-skewness of the distribution around $\eta \sim 0$. We observe that the skewnewss rises with the experts' optimism measure α^O, which is confirmed by Corollary 1.

Pessimism Interestingly, our proof of Proposition 4 in Appendix B.2 reveals that if experts are pessimistic enough, i.e., $\alpha^O << \alpha$, a net worth trap arises. The reason is different from the case of optimistic experts, however: pessimism of more productive users of capital (i.e., intermediaries) make them less willing to catch profitable (long) trades, even when the market risk premium is high enough. It can eventually drain their net worth in the long run. However, the pessimism degree must be relatively higher (than the necessary degree of optimism) to lead to a net worth trap, as shown in Appendix B.2.

General models with or without market imcompleteness In Appendix D, we illustrate that a net worth trap can arise under a different model specification with behavioral biases, emphasizing that it is not unique to our specific model specification. We uncover that regardless of whether the market is complete or incomplete, a strong degree of both optimism and pessimism generates a net worth trap.

Next, we study the welfare implications of optimism on the rational households in the economy.

2.5.2 The Household's Welfare Decomposition

We compute the welfare change of the rational households due to optimism of experts in the economy as follows:

$$\text{Welfare Change} = \mathbb{E}_0\left[\int_0^\infty e^{-\rho t}\log c_t^H dt\right] - \mathbb{E}_0\left[\int_0^\infty e^{-\rho t}\log c_t^{H,REE} dt\right] \tag{40}$$

where $c_t^{H,REE}$ is the household's consumption in the rational expectation benchmark, i.e., when $\alpha^O = \alpha$. The welfare of the households can be decomposed as

$$\mathbb{E}_0\left[\int_0^\infty e^{-\rho t}\log c_t^H dt\right] = \underbrace{\mathbb{E}_0\left[\int_0^\infty e^{-\rho t}\log(1-\eta_t) dt\right]}_{\text{Wealth effect}_+} + \underbrace{\mathbb{E}_0\left[\int_0^\infty e^{-\rho t}\log(1-\iota_t) dt\right]}_{\text{Investment effect}_+}$$
$$+ \underbrace{\mathbb{E}_0\left[\int_0^\infty e^{-\rho t}\log K_t dt\right]}_{\text{Capital effect}_-} + \underbrace{\mathbb{E}_0\left[\int_0^\infty e^{-\rho t}\log A(\psi) dt\right]}_{\text{Misallocation effect}_-} + \underbrace{\text{t.i.e.}}_{\text{Terms independent of equilibria}} \tag{41}$$

where $A(\psi) = \psi_t + l(1-\psi_t)$ is the aggregate (i.e., weighted average of capital share based on productivities) capital share in the economy, and $(1-\eta_t)$ captures the wealth share of the households. The wealth effect is an obvious direct effect that arises from a higher wealth share of households in the presence of optimism. Since the expectation errors particularly affect the optimists by draining their net worth, the share of wealth held by households is higher. A higher occupancy time at the crisis due to optimism means that the investment rate is lower, leaving the households to consume more. However, there are other general equilibrium effects that arise out of a higher occupancy time in a crisis due to optimism. First, the aggregate capital stock will be lower in the future as well due to a lower growth rate of capital. Similarly, the share of capital held by households, who are less productive than optimists, gets bigger, captured by the misallocation effect. The last two components have a negative effect on the welfare. We perform numerical simulations and show that the negative general equilibrium effects dominate the direct wealth and investment effect, leading to an overall decline in welfare. In fact, all these forces become stronger as optimism increases, and thus the total welfare of rational agents decrease in optimism as displayed in Figure 5.[26]

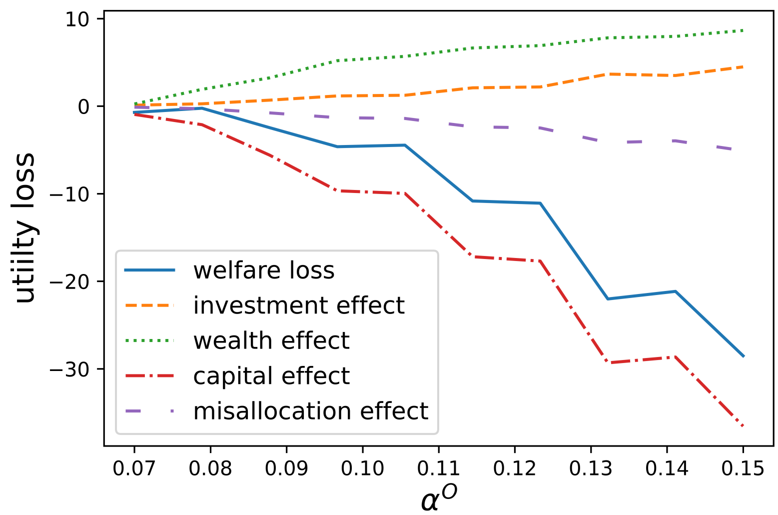


Figure 5: Decomposition of the rational agent's welfare loss

2.5.3 Swinging Sentiment: Connection to Diagnostic Expectations

Now, we relax the assumption that optimists are dogmatic about their beliefs of technological growth. We instead assume that optimists turn pessimistic when the economy transitions into a crisis. The optimism has the following functional form

$$O_t = 1_{\psi_t<1} \cdot \alpha^P + 1_{\psi_t=1} \cdot \alpha^O \tag{42}$$

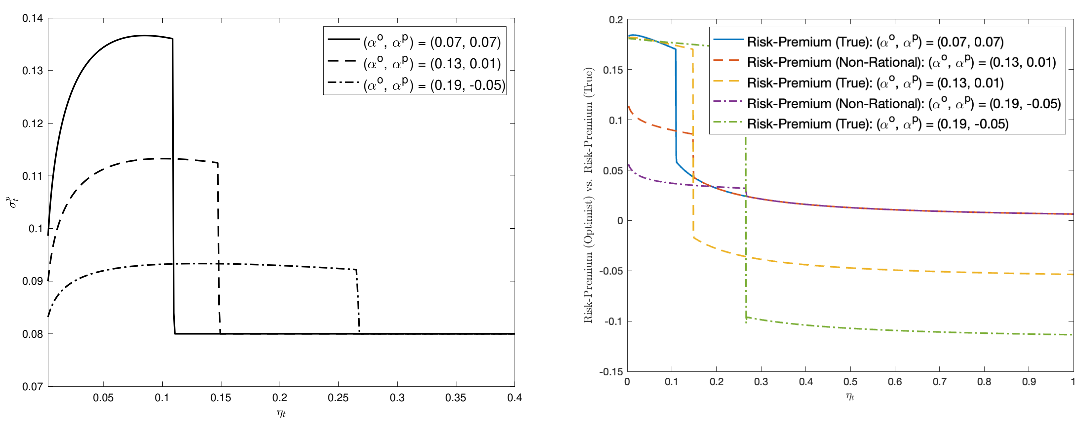
where, $\alpha^P < \alpha < \alpha^O$. Note that O_t is higher than the true α when the economy is at the stochastic steady state, but becomes lower than α when the economy enters a crisis. Since the inefficient regime itself is determined endogenously, O_t captures a swinging sentiment in an endogenous manner, resonating with the diagnostic expectations literature (e.g., Bordalo et al. (2018) and Maxted (2023)): optimists, who are more productive users of capi-

[26]Details of calculations of each effect are provided in Appendix B.3.

tal than households, are optimistic about growth in production efficiency at the stochastic steady state, thereby raising leverages and pushing up capital prices. However when the economy turns recessionary, their net worth drops, and their expectation of capital holding returns turns pessimistic. O_t following (42) captures the effects of diagnostic expectations in a simple manner.

We provide two novel insights through this variant of the model: (i) we argue that this swinging sentiment between optimism at the stochastic steady state and pessimism in a crisis will have a stabilizing effect: basically, this will eliminate a net worth trap shown in Figure 2 and lower the probability of a crisis. This result is in line with Maxted (2023); (ii) as larger reduction in optimism at a crisis generates deeper drops in asset prices, implying a meaningful interaction between financial frictions and changing sentiments, and a swing in optimism generates a bigger fluctuation in asset prices, optimism has a potential to be a priced factor in the data, leading to our empirical analysis in Section C.

Stabilizing Effect We assume that in (42) $\alpha^O - \alpha = \alpha - \alpha^P$. This assumption captures that the more optimistic intermediaries are at the stochastic steady state, the bigger a drop in their net worth becomes due to their higher leverage when negative shocks are realized, and the revision in optimists' expected growth rate of production efficiency once the economy transitions into a crisis can be bigger, following the literature on diagnostic expectations.[27]



(a) Endogenous volatility σ_t^p

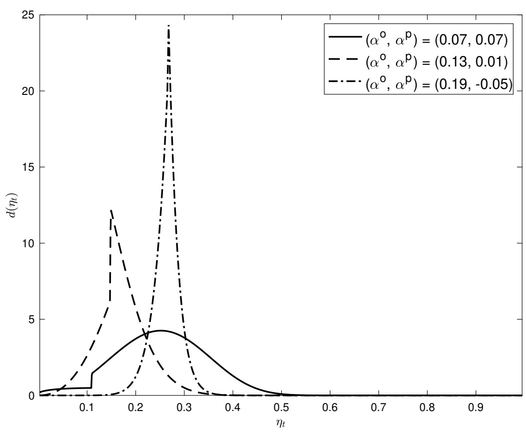
(b) *Perceived* and *true* risk-premium

Figure 6: Equilibrium variables as functions of η_t

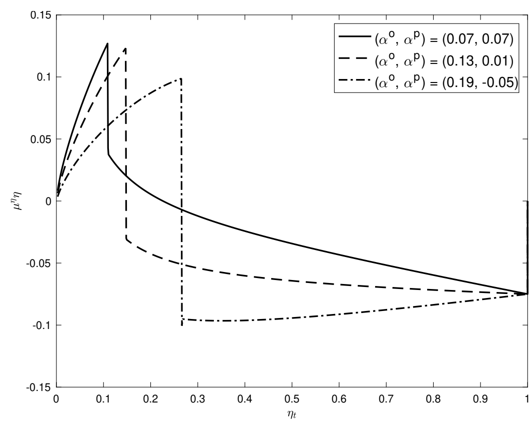
Figure 6 illustrates the model's equilibrium behavior under this new assumption: Figure

[27]This is of course a simplifying assumption, and the stabilizing effect of swinging sentiments that effectively prevent a creation of net worth trap is robust to different values of (α^O, α^P).

6a shows that in contrast to the above Figure 1c where the volatility σ_t^p is more amplified at a crisis due to optimism, now the amplification of volatility σ_t^p becomes muted: as optimists become pessimistic[28] once the economy is dragged into a crisis, the economy exits the stochastic steady state and actually enters a crisis even under a relatively high wealth share of optimists, lowering the intensity of fire sale and mitigating the amplification of risk. Figure 6b shows that optimists' perceived risk premium is higher than the true risk premium at the stochastic steady state, but falls below the true level in the crisis regime. Optimists' higher-than-true perceived risk premium in normalcy raises their risk exposure, draining the net worth so that the economy moves toward a crisis. Once the economy is in the crisis regime, their lower-than-true perceived risk-premium helps optimists to recapitalize, helping the economy exit the crisis regime and potentially lowering an ex-ante probability of a severe crisis.[29] This process similarly happens in models based on diagnostic expectations as well, as documented by Maxted (2023).



(a) Stationary distribution of η_t: no net worth trap



(b) Drift of η_t process $\mu^\eta(\eta_t) \cdot \eta_t$

Figure 7: Stabilizing effect of swinging sentiment: no net worth trap

Net Worth Trap: Revisited The above discussion can be connected to a concept of net worth trap that we present in Proposition 4. Figure 7b shows the draft of η_t process, $\mu_t^\eta \eta_t$. It confirms our previous intuitions: optimism drains optimists' wealth shares in the normal region due to their higher-than-true perceived risk-premium, but once the economy enters

[28] As optimists turn pessimistic with reduced demands for capital once the economy transitions out of the stochastic steady state, the economy enters a crisis even at a relatively high net worth share of optimists. Note that this threshold η^ψ of their net worth share is determined endogenously.

[29] For example, in Figure 7a, for an arbitrary η^* close enough to zero, the probability that the wealth share η_t drops below η^* decreases with a higher degree of swinging sentiment, i.e., higher α^O and lower α^P.

a crisis, since they turn pessimistic, their lower-than-true perceived risk-premium at a crisis lowers current asset prices[30] and facilitates the recapitalization process, pushing the wealth share toward the stochastic steady state again. This stabilizing effect of pessimism during a crisis can prevent the appearance of a net worth trap.

As seen in the stationary distribution of η_t in Figure 7a, it effectively eliminates a net worth trap illustrated in Figure 2 and makes the distribution more stable around the crisis threshold η^{ψ}. We conclude (i) a net worth trap arises when the optimism of intermediaries is dogmatic, as their higher-than-true perceived risk-premium during a crisis (i.e., expectation errors) disallows their net worth building; (ii) a swinging sentiment resonating diagnostic expectations eliminate the possibility of a perpetual crisis. All other relevant figures are provided in Figure A2 of Appendix A.

3 Conclusion

In this paper, we build a simple continuous-time economy with optimistic experts and rational households. Experts who are optimistic about the economy's long-term growth raise the probability that the economy enters a crisis, amplify the endogenous risk during crises, and potentially generate a net worth trap: a phenomenon where their wealth share is trapped around low levels for an arbitrarily long duration. Beyond a threshold level of optimism, the probability that they rebuild net worth converges to zero and the economy suffers from perennial inefficiency. It can be explained by the fact that optimistic experts take excessive leverage, based on an incorrect belief about the economy's technological growth rate. This expectation error, which makes their perceived risk-premium higher than its true value, prevents their net worth from getting recapitalized, increases the financial instability, and generates a protracted crisis. A swing in optimism, as in the case of diagnostic expectations, eliminates the net worth trap. How agents form beleifs thus has important consequences on financial stability and asset price behaviors in the long-run.

In Appendix C, we show that a factor pricing model with the intermediary factor and our new macro-optimism factor based on The Survey of Professional Forecasters (SPF) significantly improves the pricing power in the cross-section of asset returns. Lastly, our model with macro optimism quantitatively explains the empirical conditional momentum, which is concentrated during crises and crashes after market rebounds. Overall, meaningful

[30]We see from Figure A2a of Appendix A that given η_t, capital price q_t becomes lower under a lower α^P in the crisis region, thereby allowing intermediaries to relatively easily recapitalize in a crisis.

interactions between optimism and financial frictions generate interesting macroeconomic dynamics as well as behaviors of asset prices.

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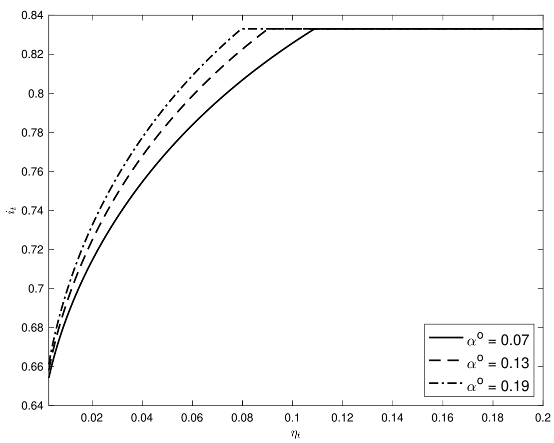
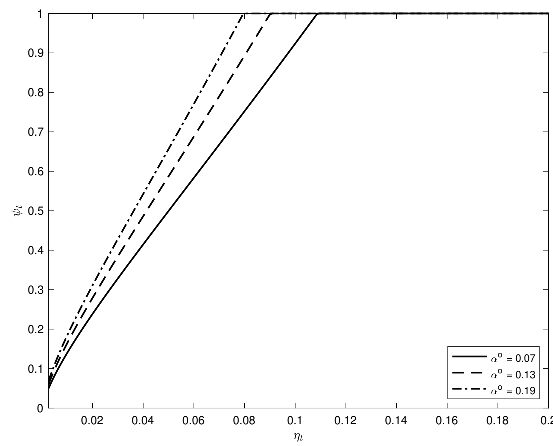
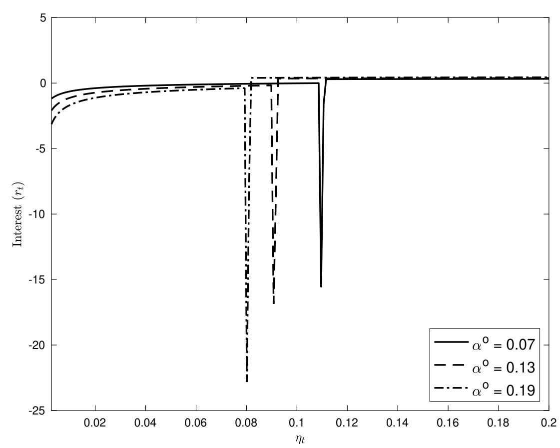
Appendix A Additional Figures



(a) Investment rate ι_t as a function of η_t

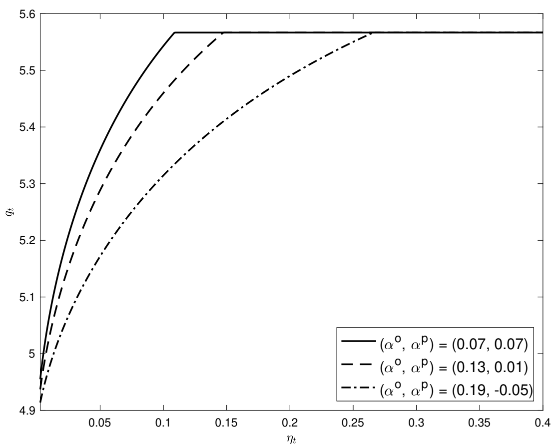


(b) Capital share ψ_t

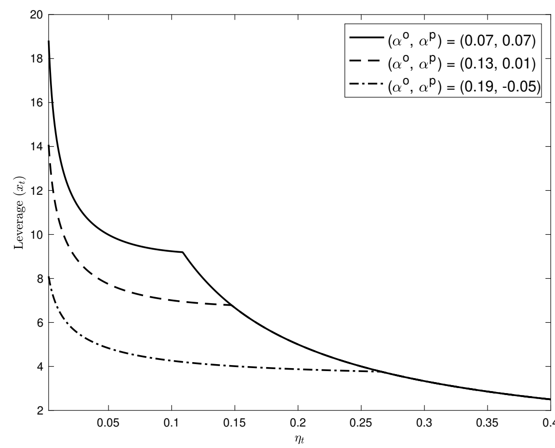


(c) Equilibrium interest rate r_t^*

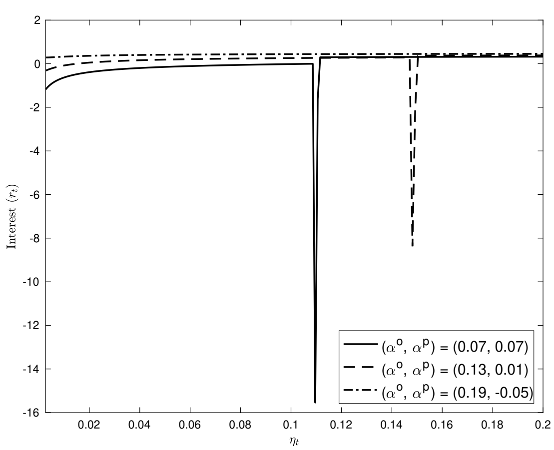
Figure A1: Other equilibrium variables as functions of η_t



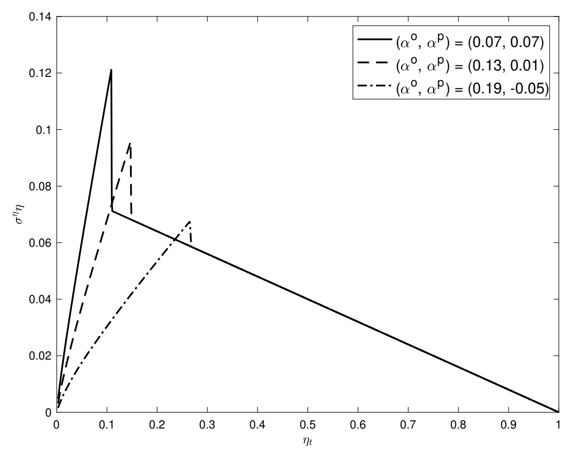
(a) Capital price q_t



(b) Leverage multiple x_t



(c) Equilibrium interest rate r_t^*



(d) $\sigma^{\eta}(\eta_t) \cdot \eta_t$ as a function of η_t

Figure A2: Equilibrium variables as functions of η_t with swinging sentiment in Section 2.5.3

Appendix B Proofs and Derivations

Proof of Proposition 1. We separately solve problems of optimists and households.

Optimists Since optimists believe that dZ_t^O, not dZ_t, is the true Brownian motion driving the business cycle, they believe the wealth share η_t follows:

$$\frac{d\eta_t}{\eta_t} = \left(\mu_t^\eta(\eta_t) + \frac{\alpha^O - \alpha}{\sigma}\sigma_t^\eta(\eta_t)\right) dt + \sigma_t^\eta(\eta_t) dZ_t^O, \tag{B.1}$$

which is consistent with the true η_t process in (29). Based on Merton (1971), we conjecture her value function $V(\cdot)$ will depend on her own wealth w_t^O and the aggregate wealth share of optimists η_t with the following form:

$$V\left(w_t^O, \eta_t\right) = \frac{\log w_t^O}{\rho} + f\left(\eta_t\right). \tag{B.2}$$

Based on (18) and (B.1), the Hamiltonian-Jacobi-Bellman (HJB) equation for an individual optimist's problem can be written as[1]

$$\begin{aligned}\rho V(\cdot) = \max_{x_t^O, c_t^O, i_t^O} \log c_t^O + &\left[w_t^O\left(x_t^O\left(\frac{\mathbb{E}_t^O\left(dr^{Ok}\right)}{dt}\right) + (1 - x_t^O)r_t\right) - c_t^O\right]\frac{dV_t}{dw_t^O} \\ &+ \frac{\left(x_t^O w_t^O \sigma_t^p\right)^2}{2}\frac{d^2V_t}{d\left(w_t^O\right)^2} + \left(\eta_t\left(\mu_t^\eta(\eta_t) + \frac{\alpha^O - \alpha}{\sigma}\sigma_t^\eta(\eta_t)\right)\right)\frac{dV_t}{d\eta_t} + \frac{\left(\eta_t\sigma_t^\eta(\eta_t)\right)^2}{2}\frac{d^2V_t}{d\eta_t^2}.\end{aligned}$$

The first order condition[2] with respect to c_t^O are given by:

$$\frac{1}{c_t^{O*}} = \frac{dV_t}{dw_t^O}\left(w_t^O, \eta_t\right) = \frac{1}{\rho w_t^O}, \tag{B.3}$$

which gives $c_t^{O*} = \rho w_t^O$ at optimum. As every aggregate variable including q_t, i_t^O, μ_t^p, σ_t^p, x_t^O, and r_t will be expressed as functions of the aggregate wealth share η_t, Merton (1971)

[1] Due to the assumed value function in (B.2), we ignore the cross-derivative term of $V(\cdot)$ with respect to w_t^O and η_t.

[2] We use the following relation from equation (17):

$$\mathbb{E}_t^O\left(dr^{Ok}\right) = \left(\frac{\gamma_t^O - \iota_t^O\gamma_t^O}{p_t} + \Lambda^O(\iota_t^O) - \delta^O + \mu_t^p + \frac{\alpha^0 - \alpha}{\sigma}\sigma_t^p\right)dt = \left(\frac{1-\iota_t^O}{q_t} + \Lambda^O(\iota_t^O) - \delta^O + \mu_t^p + \frac{\alpha^0 - \alpha}{\sigma}\sigma_t^p\right)dt.$$

justifies our choice of value function in (B.2). The first-order condition with respect to x_t^O gives the optimal portfolio choice given in Proposition 1.

To derive i_t^{O*}, we observe that the investment appears only in a intra-temporal manner. Thus the corresponding optimization problem is static and satisfies the first-order condition: $\Lambda^{O\prime}(i_t) = q_t^{-1}$ (i.e., marginal Tobin's q). With $\Lambda^O(i_t)$ from (31), we finally obtain:

$$i_t^{O*}(q_t) = \frac{q_t^2 - 1}{2k}. \tag{B.4}$$

Rational households Since households have the rational expectations, i.e., they believe that dZ_t, not dZ_t^O, is the true Brownian motion driving the business cycle, they believe the wealth share η_t follows:

$$\frac{d\eta_t}{\eta_t} = \mu_t^\eta(\eta_t)dt + \sigma_t^\eta(\eta_t)dZ_t, \tag{B.5}$$

which is consistent with the true η_t process in (29). Based on Merton (1971), we conjecture her value function $\underline{V}(\cdot)$ will depend on her own wealth $w_t^P H$ and the aggregate wealth share of optimists η_t as the state variable:

$$V^H\left(w_t^H, \eta_t\right) = \frac{\log w_t^H}{\rho} + f^H\left(\eta_t\right). \tag{B.6}$$

Based on (20) and (B.5), the Hamiltonian-Jacobi-Bellman (HJB) equation for an individual optimist's problem can be written as[3]

$$\begin{aligned}\rho V^H(\cdot) = \max_{x_t^H \geq 0, c_t^H, i_t^H} \log c_t^H + \left[w_t^H\left(x_t^H\left(\frac{\mathbb{E}_t^H\left(dr^{Hk}\right)}{dt}\right) + (1-x_t^H)r_t\right) - c_t^H\right]\frac{dV_t^H}{dw_t^H} \\ + \frac{\left(x_t^H w_t^H \sigma_t^p\right)^2}{2}\frac{d^2V_t^H}{d\left(w_t^H\right)^2} + \eta_t\left(\mu_t^\eta(\eta_t)\right)\frac{dV_t^H}{d\eta_t} + \frac{\left(\eta_t\sigma_t^\eta(\eta_t)\right)^2}{2}\frac{d^2V_t^H}{d\eta_t^2}.\end{aligned}$$

The first order condition with respect to c_t^G are given by:

$$\frac{1}{c_t^{H*}} = \frac{dV_t^H}{dw_t^H}\left(w_t^H, \eta_t\right) = \frac{1}{\rho w_t^H}, \tag{B.7}$$

[3]We use the following relation:

$$\mathbb{E}_t^H\left(dr^{Hk}\right) = \mathbb{E}_t\left(dr^{Hk}\right) = \left(\frac{\gamma_t^H - \iota_t^H\gamma_t^H}{p_t} + \Lambda^H(\iota_t^H) - \delta^H + \mu_t^p\right)dt.$$

which gives $c_t^{H*} = \rho w_t^H$ at optimum. As aggregate variables including q_t, i_t^H, μ_t^p, σ_t^p, x_t^H, and r_t are expressed as functions of the aggregate wealth share η_t, Merton (1971) justifies our choice of value function in (B.6). The first-order condition with respect to x_t^H gives the optimal portfolio choice given in Proposition 1.

To derive ι_t^H, we observe that the investment appears only in a intra-temporal manner. Thus the corresponding optimization problem is static and satisfies the first-order condition: $\Lambda^{H\prime}(i_t) = l \cdot q_t^{-1}$. With equation (32) that $\Lambda^H(\cdot) = l \cdot \Lambda^H(\cdot)$, we finally obtain:

$$i_t^{H*}(q_t) = \frac{q_t^2 - 1}{2k}, \tag{B.8}$$

which is the same as $i_t^{O*}(q_t)$ in (B.4).

■

Proof of Proposition 2. The aggregate wealth of optimists, W_t^O evolves with the process

$$dW_t^O = r_t W_t^O dt + \psi_t p_t K_t \left(dr_t^{Ok} - r_t dt\right) - c_t^O dt, \tag{B.9}$$

where $c_t^O = \rho W_t^O$ holds at optimum. By applying Ito's quotient rule on (28),[4] we have

$$\frac{d\eta_t}{\eta_t} = \frac{dW_t^O}{W_t^O} - \frac{d\left(p_t K_t\right)}{p_t K_t} + \left(\frac{d\left(p_t K_t\right)}{p_t K_t}\right)^2 - \frac{dW_t^O}{W_t^O}\frac{d\left(p_t K_t\right)}{p_t K_t}. \tag{B.10}$$

In addition, from the process (23) of the aggregate capital K_t, we obtain

$$\begin{aligned}\frac{1}{K_t}\frac{dK_t}{dt} &= \left(\Lambda^O\left(i_t^O\right) - \delta^O\right)\psi_t + \left(\Lambda^H\left(i_t^H\right) - \delta^H\right)(1-\psi_t) \\ &= \left(\Lambda^O\left(i_t^O\right) - \delta^O\right) - \left(\delta^H - \delta^O\right)(1-\psi_t) - (1-\psi_t)(1-l)\Lambda^O\left(i_t^O\right),\end{aligned} \tag{B.11}$$

where we used the property that $i_t^O = i_t^H$ in equilibrium as seen in (B.4) and (B.8). Applying Ito's product rule to the price process (8) and the capital process (B.11), and comparing

[4] Ito's quotient rule states that:

$$\frac{d\left(XY^{-1}\right)}{XY^{-1}} = \frac{dX}{X} - \frac{dY}{Y} + \left(\frac{dY}{Y}\right)^2 - \frac{dX}{X}\frac{dY}{Y}.$$

with (17), we obtain

$$\frac{d\left(p_t K_t\right)}{p_t K_t} = dr_t^{Ok} - \frac{1-\iota_t^O}{q_t} dt - \left(\delta^H - \delta^O\right)\left(1-\psi_t\right) dt - (1-\psi_t)(1-l)\Lambda^O\left(i_t^O\right) dt. \quad \text{(B.12)}$$

Since $\mathbb{E}_t^O\left(dr_t^{Ok}\right) - r_t = x_t^H\left(\sigma_t^p\right)^2$ from the optimists' optimal portfolio decision à la Merton (1971), and from the fact that

$$\mathbb{E}_t^O\left(dr_t^{Ok}\right) = \mathbb{E}_t\left(dr_t^{Ok}\right) + \frac{\alpha^O - \alpha}{\sigma}\sigma_t^p dt, \quad \text{(B.13)}$$

where $\mathbb{E}_t$ is the operator corresponding to the rational expectations, we plug (B.9), (B.12), and (B.13) into (B.10) and obtain

$$\begin{aligned}\frac{d\eta_t}{\eta_t} &= \left(\frac{\psi_t - \eta_t}{\eta_t}\sigma_t^p\right)^2 dt - \frac{\psi_t - \eta_t}{\eta_t}\frac{\alpha^O - \alpha}{\sigma}\sigma_t^p dt + \frac{1-\iota_t^O}{q_t} dt + \left(1-\psi_t\right)\left(\delta^H - \delta^O\right) dt \\ &\quad + \left(1-l\right)\left(1-\psi_t\right)\Lambda^O\left(i_t^O\right) dt - \rho dt + \frac{\psi_t - \eta_t}{\eta_t}\sigma_t^p dZ_t,\end{aligned} \quad \text{(B.14)}$$

where we used $c_t^O = \rho W_t^O$ in the equilibrium. From (B.14), we obtain

$$\mu^\eta(\eta_t) = \left(\frac{\psi_t - \eta_t}{\eta_t}\sigma_t^p\right)^2 - \frac{\psi_t - \eta_t}{\eta_t}\frac{\alpha^O - \alpha}{\sigma}\sigma_t^p + \frac{1-\iota_t^O}{q_t} + (1-\psi_t)\left(\delta^H - \delta^O\right) + (1-l)\left(1-\psi_t\right)\Lambda^O(i_t^O) - \rho, \quad \text{(B.15)}$$

and

$$\sigma^\eta(\eta_t) = \frac{\psi_t - \eta_t}{\eta_t}\sigma_t^p. \quad \text{(B.16)}$$

■

Proof of Proposition 3. Applying Ito's lemma to the Markov relationship $q_t = q\left(\eta_t\right)$, we obtain

$$dq_t = q'\left(\eta_t\right) d\eta_t + \frac{1}{2} q''\left(\eta_t\right)\left(d\eta_t\right)^2. \quad \text{(B.17)}$$

Plugging in equation (35) to (B.17) and using $\left(d\eta_t\right)^2 = \eta_t^2 \sigma^\eta(\eta_t)^2 dt$, we have

$$\frac{dq_t}{q_t} = \left(\eta_t \mu^\eta\left(\eta_t\right)\frac{q'\left(\eta_t\right)}{q(\eta_t)} + \frac{\eta_t^2 \sigma^\eta\left(\eta_t\right)^2}{2}\frac{q''\left(\eta_t\right)}{q(\eta_t)}\right) dt + \eta_t \sigma^\eta\left(\eta_t\right)\frac{q'\left(\eta_t\right)}{q(\eta_t)} dZ_t. \quad \text{(B.18)}$$

With the definition of normalized asset price (i.e., price-earnings ratio) $q_t = \frac{p_t}{\gamma_t^O}$, we have:[5]

$$\frac{dq_t}{q_t} = \left(\mu_t^p - \alpha + \sigma^2 - \sigma_t^p \sigma\right) dt + \left(\sigma_t^p - \sigma\right) dZ_t. \tag{B.19}$$

Comparing (B.18) and (B.19) yields:

$$\sigma_t^p = \sigma + \eta_t \sigma^\eta \left(\eta_t\right) \frac{q'\left(\eta_t\right)}{q(\eta_t)}, \tag{B.20}$$

and

$$\mu_t^p = \alpha + \eta_t \mu^\eta \left(\eta_t\right) \frac{q'\left(\eta_t\right)}{q(\eta_t)} + \frac{\eta_t^2 \sigma^\eta \left(\eta_t\right)^2}{2} \frac{q''\left(\eta_t\right)}{q(\eta_t)} + \sigma \sigma_t^\eta \eta_t \frac{q'\left(\eta_t\right)}{q(\eta_t)}. \tag{B.21}$$

From (B.16) in Proposition 2, we have that

$$\sigma^\eta \left(\eta_t\right) = \frac{x_t \eta_t - \eta_t}{\eta_t} \sigma_t^p, \tag{B.22}$$

where we used $x_t \eta_t = \psi_t$ from (34). Therefore, by (B.20) and (B.22) we have

$$\sigma_t^p = \frac{\sigma}{1 - \left(x_t \eta_t - \eta_t\right) \dfrac{q'\left(\eta_t\right)}{q(\eta_t)}}. \tag{B.23}$$

■

B.1 Price of Capital and the Optimal Leverages

We derive a closed formed solution for leverage multiple and a first-order differential equation for the price of capital. The following summarize the main results of the paper.

Proposition 5 *The equilibrium domain consists of two intervals $[0, \eta^\psi)$, where $\psi(\eta) < 1$, and $[\eta^\psi, 1]$ where $\psi(\eta) = 1$. The capital price $q(\eta)$ in our equilibrium is given by:*

$$q(0) = l \cdot \frac{1 - i(q(0))}{\rho} \tag{B.24}$$

and

$$q(\eta) = \frac{1 - i(q(\eta))}{\rho}, \quad \forall \eta \in [\eta^\psi, 1] \tag{B.25}$$

[5]Note that in equation (B.19), we use the 'true' process for γ_t^O given in (12).

The following procedures can be used to compute $\psi(\eta)$ and $q'(\eta)$ from $(\eta, q(\eta))$ to $(\eta^{\psi}, 0)$ when $\eta < \eta^{\psi}$:

1. *Find ψ that satisfies*

$$\rho q(\eta) = \psi + (1-\psi)l - i(q)\left(\psi + (1-\psi)l\right) \tag{B.26}$$

2. *Compute $q(\eta)$ where $q(\eta)$ is given by the solution of the equation:*

$$\frac{(1-l)(1-i(q(\eta)))}{q(\eta)} + (1-l)\Lambda^O\left(i(q(\eta))\right) + \delta^H - \delta^O + \frac{\alpha^O - \alpha}{\sigma}\sigma^p(\eta) = \left(\frac{\psi}{\eta} - \frac{1-\psi}{1-\eta}\right)\sigma^p(\eta)^2, \tag{B.27}$$

where $\sigma^p(\eta)$ is given by (B.23). From (B.27), $\sigma^p(\eta)$ can also be expressed as:

$$\sigma^p(\eta) = \frac{\frac{\alpha^O-\alpha}{\sigma} + \sqrt{\left(\frac{\alpha^O-\alpha}{\sigma}\right)^2 + 4\left(\frac{\psi}{\eta} - \frac{1-\psi}{1-\eta}\right)\left(\frac{(1-l)(1-i(q))}{q(\eta)} + (1-l)\Lambda^O(i(q)) + \delta^H - \delta^O\right)}}{2\left(\frac{\psi}{\eta} - \frac{1-\psi}{1-\eta}\right)}, \tag{B.28}$$

where $q = q(\eta)$.

Proof of Proposition 5. From the good market equilibrium condition (24), we have

$$k_t^O\left(\gamma_t^O - \iota_t^O\gamma_t^O\right) + k_t^H\left(\gamma^H - \iota_t^H\gamma^H\right) = c_t^O + c_t^H, \quad \forall t \in [0, \infty). \tag{B.29}$$

Observing that $c_t^O = \rho w_t^O$ and $c_t^H = \rho w_t^H$ in equilibrium, we divide (B.29) by $p_t K_t$ and obtain[6]

$$\rho q = \psi + (1-\psi)l - i(q)\left(\psi + (1-\psi)l\right). \tag{B.30}$$

Since $\mathbb{E}_t^O\left(dr_t^{Ok}\right) - r_t = x_t^H\left(\sigma_t^p\right)^2$ from the optimists' optimal portfolio decision, we obtain

$$\frac{1-i(q(\eta))}{q(\eta)} + \Lambda^O(i(q(\eta))) - \delta^O + \mu_t^p + \frac{\alpha^O - \alpha}{\sigma}\sigma^p(\eta) - r_t = \left(\frac{\psi}{\eta}\right)\sigma^p(\eta)^2. \tag{B.31}$$

For households, from Proposition 1, it must be the case where

$$l \cdot \frac{1-i(q(\eta))}{q(\eta)} + l \cdot \Lambda^O(i(q(\eta))) - \delta^H + \mu_t^p - r_t \leq \left(\frac{1-\psi}{1-\eta}\right)\sigma^p(\eta)^2. \tag{B.32}$$

[6] Since $\iota_t^H(q_t) = \iota_t^O(q_t)$, we use the notation $\iota(q(\eta))$ to denote that $\iota(\cdot)$ is a function of normalized asset price q, which in turn depends on the state variable η.

with equality when $\psi < 1$. Finally, by subtracting (B.32) from (B.31), we get

$$\frac{(1-l)(1-i(q(\eta)))}{q(\eta)}+(1-l)\Lambda^O\left(i(q(\eta))\right)+\delta^H-\delta^O+\frac{\alpha^O-\alpha}{\sigma}\sigma^p(\eta)=\left(\frac{\psi}{\eta}-\frac{1-\psi}{1-\eta}\right)\sigma^p(\eta)^2, \tag{B.33}$$

when $\psi_t < 1$, i.e., households hold some physical capital.

■

B.2 Stationary Distribution

We now compute the stationary (i.e., ergodic) distribution of the wealth share of optimists, η_t. Given the η_t process in equation (35), the Kolmogorov forward equation (KFE) is given by

$$0=-\frac{\partial}{\partial\eta}\left(\mu^\eta(\eta)\eta\cdot d(\eta)\right)+\frac{1}{2}\frac{\partial^2}{\partial\eta^2}\left(\left(\sigma^\eta(\eta)\eta\right)^2 d(\eta)\right). \tag{B.34}$$

where $d(\eta)$ denotes the distribution. With the transformation $D(\eta)\equiv\left(\sigma^\eta(\eta)\eta\right)^2\cdot d(\eta)$, equation (B.34) can be written as

$$\frac{D'(\eta)}{D(\eta)}=2\frac{\mu^\eta(\eta)\eta}{\left(\sigma^\eta(\eta)\eta\right)^2}, \tag{B.35}$$

which can be solved easily by integrating both sides of (B.35).

Deriving $\mu^\eta(0)$ and $\sigma^\eta(0)$ in (39) The asymptotic solution of drift $\mu^\eta(\eta)$ and volatility $\sigma^\eta(\eta)$ of η, when $\eta\to 0$ can be computed in a similar way to Brunnermeier and Sannikov (2014). Asymptotically, let us assume that as $\eta\to 0$, $q(\eta)\to q_0$ where q_0 is the equilibrium level of normalized capital price when households hold the entire aggregate wealth. Let as assume $\psi(\eta)\simeq\psi_0\eta$ when $\eta\sim 0$. Then we know $\sigma_t^p\to\sigma$ (from (37)), $\mu_t^p\to\alpha$ (from (36)), and $\sigma^\eta\to(\psi_0-1)\sigma$ thereafter. Following steps are needed to calculate $\mu^\eta(0)$ and $\sigma^\eta(0)$:

Step 1 From (B.24), i.e., $\rho q_0 = l(1-\iota_o)$ we obtain q_0 and $\iota_0=\iota(q_0)=\frac{q_0^2-1}{2k}$.

Step 2 From (35), we know that as $\eta\to 0$, $\mu^\eta(\eta)\to\mu^\eta(0)$, where μ_0 can be written as

$$\mu^\eta(0)=((\psi_0-1)\sigma)^2-(\psi_0-1)\frac{\alpha^O-\alpha}{\cancel{\sigma}}\cancel{\sigma}+\frac{1-\iota_0}{q_0}+(\delta^H-\delta^O)+(1-l)\Lambda^O(\iota_0)-\rho.$$

Step 3 From (B.27), we know

$$\frac{(1-l)(1-\iota_0)}{q_0} + (1-l)\Lambda^O(\iota_0) + (\delta^H - \delta^O) + \frac{\alpha^O - \alpha}{\cancel{\sigma}}\cancel{\sigma} = (\psi_0 - 1)\sigma^2$$

as

$$\left(\frac{\psi}{\eta} - \frac{1-\psi}{1-\eta}\right) \to \psi_0 - 1.$$

Step 4 Therefore from the above 3 steps, we calculate μ_0 and ψ_0, from which we calculate $\sigma^\eta(0^+) = (\psi_0 - 1)\sigma$.

$$\mu^\eta(0) \equiv \lim_{\eta\to 0} \mu^\eta(\eta) = \mu_0$$
$$\sigma^\eta(0) \equiv \lim_{\eta\to 0} \sigma^\eta(\eta) = (\psi_0 - 1)\sigma$$

Proof of Proposition 4. We know from the above Step 1 that q_0, ι_0, $\Lambda^O(\iota_0)$ are determined. By defining

$$\Box_0 \equiv \frac{1}{\sigma^2}\left[(1-l)\frac{1-\iota_0}{q_0} + (\delta^H - \delta^O) + (1-l)\Lambda^O(\iota_0)\right],$$

we obtain from Step 3

$$\psi_0 - 1 = \frac{\alpha^O - \alpha}{\sigma^2} + \Box_0.$$

From Step 2 and defining

$$\Delta_0 \equiv \frac{1-\iota_0}{q_0} + (\delta^H - \delta^O) + (1-l)\Lambda^O(\iota_0) - \rho,$$

it follows that

$$\begin{aligned}
\mu^\eta(0^+) &= (\psi_0 - 1)^2\sigma^2 - (\psi_0 - 1)(\alpha^O - \alpha) + \Delta_0 \\
&= \left[\frac{\alpha^O - \alpha}{\sigma^2} + \Box_0\right]^2 \sigma^2 - \left[\frac{\alpha^O - \alpha}{\sigma^2} + \Box_0\right](\alpha^O - \alpha) + \Delta_0 \\
&= \Box_0(\alpha^O - \alpha) + \Box_0^2\sigma^2 + \Delta_0
\end{aligned} \tag{B.36}$$

and

$$\sigma^\eta(0^+) = (\psi_0 - 1)\sigma = \frac{\alpha^O - \alpha}{\sigma} + \Box_0\sigma. \tag{B.37}$$

For a net worth trap to be realized, we need to have $\tilde{D}_0 \equiv \frac{2\mu^\eta(0^+)}{(\sigma^\eta)^2(0^+)} < 1$, i.e., $2\mu^\eta(0^+) < \sigma^\eta(0^+)^2$, which with equations (B.36) and (B.37 can be written as

$$2\left[\Box_0(\alpha^O - \alpha) + \Box_0^2\sigma^2 + \Delta_0\right] < \left(\frac{\alpha^O - \alpha}{\sigma} + \Box_0\sigma\right)^2$$

which leads to

$$\left(\frac{\alpha^O - \alpha}{\sigma}\right)^2 > \Box_0^2\sigma^2 + 2\Delta_0.$$

Therefore, if $\alpha^O - \alpha > \sigma\sqrt{\Box_0^2\sigma^2 + 2\Delta_0}$, we have a net worth trap.

Pessimism Interestingly, we can see that if experts are pessimistic enough, i.e., if

$$\alpha^O - \alpha < -\sigma\sqrt{\Box_0^2\sigma^2 + 2\Delta_0},$$

a net worth trap arises. The reason is different from the case of optimistic experts, however: pessimism of more productive users of capital (i.e., intermediaries) make them less willing to catch profitable (long) trades, even when the market risk premium is high enough. It can eventually drain their net worth in the long run.

■

Proof of Corollary 1. From (37), we obtain

$$\lim_{\eta \to 0} \sigma_t^p = \sigma.$$

From L'Hôpital's rule, we also obtain that:

$$\lim_{\eta \to 0} \sigma_t^\eta = \lim_{\eta \to 0} \frac{\psi_t - \eta_t}{\eta_t}\sigma_t^p = \lim_{\eta \to 0} \left(\psi_t'(\eta) - 1\right)\sigma \equiv \sigma^\eta(0^+).$$

The derivative of ψ increases with optimism as seen in Figure A1b, and, in the limit, the drift μ^η and the volatility σ^η of η do not depend on η. Since now η in the limit (i.e., $\eta \sim 0$) follows a geometric Brownian motion with constant volatility $\sigma^\eta(0^+)$, its skewness is given by:

$$\left(\exp\left(\left(\sigma^\eta(0^+)\right)^2\right) + 2\right)\sqrt{\exp\left(\left(\sigma^\eta(0^+)\right)^2\right) - 1}$$

which is always positive, and increasing in $\sigma^\eta(0^+)$. Therefore, as $\eta \to 0$, the skewness of the stationary distribution $d(\eta)$ increases when optimism $\alpha^O - \alpha$ increases.

■

B.3 Welfare derivations

From the goods market clearing condition (B.26), i.e.,

$$k_t^O(\gamma^O - \iota^O\gamma^O) + k_t^P(\gamma^P - \iota^P\gamma^P) = C_t$$

where C_t is aggregate consumption, and $\frac{C_t}{W_t} = \frac{c_t^H}{w_t^H} = \rho$, we obtain $c_t^h = \rho(1-\eta_t)W_t = (1-\eta_t)K_t\gamma_t^O \cdot (1-\iota_t)A(\psi_t)$. Then we obtain the decomposition of welfare in (41).

For the capital effect, we obtain

$$\underbrace{\mathbb{E}_0\left[\int_0^\infty e^{-\rho t}\log K_t dt\right]}_{\text{Capital effect}} = \frac{\log K_0}{\rho} + \frac{1}{\rho}\mathbb{E}_0\left[\int_0^\infty e^{-\rho t}\left(\Lambda^O\left(i_t^O\right) - \delta^O - (1-\psi_t)(1-l)\Lambda^O\left(i_t^O\right)\right)dt\right].$$

For the wealth effect, from (35) we obtain

$$\begin{aligned} d\log(1-\eta_t) &= \frac{-d\eta_t}{1-\eta_t} + \frac{1}{2}\frac{(d\eta_t)^2}{(1-\eta_t)^2} \\ &= \left(-\frac{\eta_t}{1-\eta_t}\mu_t^\eta + \frac{1}{2}\frac{\eta_t^2}{(1-\eta_t)^2}(\sigma_t^\eta)^2\right)dt - \frac{\eta_t}{1-\eta_t}\sigma_t^\eta dZ_t. \end{aligned}$$

Therefore,

$$\underbrace{\mathbb{E}_0\left[\int_0^\infty e^{-\rho t}\log(1-\eta_t)dt\right]}_{\text{Wealth effect}} = \frac{\log(1-\eta_0)}{\rho} + \frac{1}{\rho}\mathbb{E}_0\left[\int_0^\infty e^{-\rho t}\left(-\frac{\eta_t}{1-\eta_t}\mu_t^\eta + \frac{1}{2}\left(\frac{\eta_t}{1-\eta_t}\right)^2(\sigma_t^\eta)^2\right)dt\right]$$

where under $\delta^H = \delta^O = 0$, we use

$$\mu_t^\eta = \left(\frac{\psi_t - \eta_t}{\eta_t}\sigma_t^p\right)^2 - \frac{\psi_t - \eta_t}{\eta_t}\frac{\alpha^O - \alpha}{\sigma}\sigma_t^p + \frac{1-\iota_t^O}{q_t} + (1-l)(1-\psi_t)\Lambda^O(i_t^O) - \rho,$$

and

$$\sigma_t^\eta = \frac{\psi_t - \eta_t}{\eta_t}\sigma_t^p.$$

Investment and misallocation effects can be calculated based on simulations.

Online Appendix for
Optimism, Net Worth Trap, and Asset Returns

GOUTHAM GOPALAKRISHNA SEUNG JOO LEE THEOFANIS PAPAMICHALIS

Appendix C Quantitative Analysis

In this section, we perform a quantitative analysis to test a couple of basic model predictions. Section C.1 provides a cross-sectional asset pricing test based on a newly constructed macro-optimism measure, and Section C.2 focuses on return predictability problems, especially about whether the macro-optimism measure explains conditional momentum concentrated in a crisis.

C.1 Factor Model

Our model predicts that asset returns are affected by a two-way feedback loop between optimism and the wealth share of experts, stemming from a rich interaction between financial friction and optimism. As seen in above Section 2.5.3, a fluctuation in optimism amplifies asset price moves, so can be a priced factor independent of the wealth share of experts.

The prediction naturally leads to a factor pricing model with a role for both optimism, and more generally, sentiment, and the intermediary risk capacity as a proxy for the wealth share. In that spirit, we propose a three factor model with macro-optimism along with the aggregate wealth and the intermediary wealth. The intermediary wealth is proxied by the intermediary capital ratio from He et al. (2017). That is, the capital ratio, denoted by η_t is the fraction of total assets held in equity by the bank holding companies in the US:

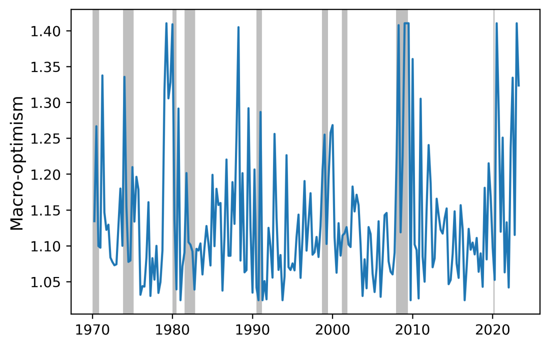
$$\eta_t \equiv \frac{\text{Equity}_t}{\text{Assets}_t} \tag{C.1}$$

The aggregate wealth W_t is proxied by the traditional market factor. The third novel factor, macro-optimism, is based on The Survey of Professional Forecasters and empirically measured as

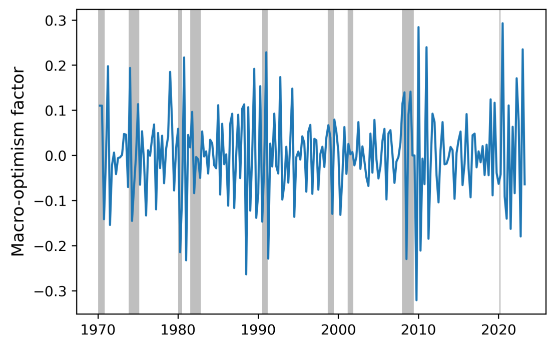
$$O_t = \frac{f_{75}}{|f_{50}|} \tag{C.2}$$

where f_i denotes the i^{th} percentile analyst forecast of quarter-on-quarter GDP growth rate for the $(T+2)^{th}$ quarter ahead at date T. In constructing our optimism measure given in (C.2), we assume that a right tail portion of forecasters in terms of their forecasts of quarter-on-quarter GDP growth rate corresponds to optimists in our model. Dividing by median $|f_{50}|$ in (C.2) provides us the necessary comparability over time.

We define our optimism 'factor' o_t as an innovation term in $\Delta \log O_t$, i.e., the percent change of O_t, as it describes a fluctuation in our optimism measure O_t. Figure A3b illustrates the time series of our optimism factor o_t.



(a) Macro-optimism O_t defined in (C.2)



(b) Innovation of $\Delta \log O_t$ defined in (C.2)

Figure A3: Macro-optimism O_t and $\Delta \log O_t$'s innovation, which we define as our optimism factor o_t: The shaded regions represent NBER recessionary periods.

We run a usual two-stage Fama-McBeth regression with the three factors $f_t := [M_t, \eta_t, o_t]'$, where M_t is the market excess return. First, we regress the time series excess return of each asset i on the three factors to estimate corresponding betas $\hat{\beta}_{i,t}$ using the following regression equation:

$$R^e_{i,t} = a_i + \beta'_{i,f} f_t + v_{i,t} \tag{C.3}$$

In the second stage regression, the estimated betas, i.e., $\{\beta_{i,f}\}_i$, are used to obtain the risk prices for each of the factors by the following regression

$$E[R^e_{i,t}] = \alpha_i + \hat{\beta}'_{i,f} \lambda_f + \varepsilon_i \tag{C.4}$$

where $E[R^e_{i,t}]$ is the excess return on the asset i, and λ_f is the risk price of the corresponding factor $f \in f_t$. We adjust the standard errors of the estimated λ_f in order to account for the fact that regressors $\hat{\beta}_{i,f}$ are estimated quantities.

Test Assets: To test the ability of macro-optimism in pricing the cross-section of asset returns above and beyond the intermediary factor and the market factor, we use the following test assets from the period 1970Q1 till 2022Q4: 25 size and book-to-market portfolios, 25 size and momentum sorted portfolios, 10 long-term reversal portfolios, 25 profitability and investment portfolios, and 10 maturity sorted US treasury bond portfolios from CRSP Fama bond dataset with maturities in six month intervals up to five years. Using a large number of test assets passes the criticism by Lewellen et al. (2010). In addition to the equity and bond portfolios used above, we extend the analysis to include other asset classes with the following test assets from 1970Q1 till 2012Q4 (due to data availability) - 18 option portfolios, 20 CDS portfolios, and 12 FX portfolios used in He et al. (2017).

Table A1 presents the results from the first-stage regression using the equity and bond portfolios, as well as HKM test assets including momentum and long-term reversal portfolios as test assets. The table displays the following statistics for the two different asset classes: average excess returns, standard deviations of the excess returns in each asset class, mean and standard deviation of the factor exposures of the excess returns to the factors f_t. We consider two models for each asset classes: a two-factor model with the market and the intermediary factors, and a three-factor model with the market, intermediary, and macro-optimism factors. The regression (C.3) controls for the price-dividend ratio, and cyclically adjusted price-earnings ratio (CAPE) obtained from Robert Shiller's website. The first two variables are commonly used to predict asset returns. Consistent with He et al. (2017), we see that there is a large variation in the intermediary factor exposure in both asset classes.[7] In addition, exposures to the macro-optimism factor also feature a considerable variability in both asset classes. That is, the standard deviation of β_O is around 7.5 times its mean, indicating that the excess returns of test assets have sufficient variation in its exposure to the macro-optimism factor in (C.2).

C.1.1 Cross-sectional Asset Pricing Test

Table A2 presents the risk price estimates for both the two-factor and the three-factor models along with Shanken-corrected t-stats. The risk price of the macro-optimism factor o_t in the three-factor model is positive and statistically significant in both equity and bond

[7]Later, we control for the cay measure of Lettau and Ludvigson (2001), and the capital share of Lettau et al. (2019). Since the capital share data are available until 2013Q4, we drop this from our main regression. However, as shown in the robustness section of Appendix C.3, the results are unchanged by the inclusion of those control variables.

	Equities and Bonds		HKM + Momentum	
	Two-factor	Three-factor	Two-factor	Three-factor
Mean excess return	1.88	1.88	1.38	1.38
Std. excess return	0.84	0.84	1.32	1.32
Mean β_M	0.9	0.9	0.55	0.55
Std β_M	0.37	0.37	0.46	0.46
Mean β_η	0.08	0.08	0.07	0.0
Std β_η	0.11	0.11	0.13	0.13
Mean β_O	-	0.004	-	-0.01
Std β_O	-	0.03	-	0.04
Assets	95	95	129	129
Quarters	211	211	211	171
Controls	Yes	Yes	Yes	Yes

Table A1: Expected returns and risk exposures. Equity assets include 25 size and book-to-market portfolios, 25 size and momeutum portfolios, 10 long-term reversal portfolios, and 25 profitability and investment portfolios. Bond portfolios include 10 maturity sorted portfolios from CRSP Fama bond portfolio dataset. HKM assets include 25 size and book-to-market portfolios, 10 U.S. Govt. bond portfolios, 10 U.S. corporate bond portfolios, 18 option portfolios, 20 CDS portfolios, and 12 foreign exchange portfolios taken from He et al. (2017). HKM+Momentum includes 25 size and momentum portfolios and 10 long-term reversal portfolios in addition to the HKM assets. Mean and standard deviation of the excess return is the difference in mean return and risk free rate of the corresponding test assets. The frequency is quarterly and time period is from 1970Q1 till 2022Q4. The mean and std. of betas ($\beta_M, \beta_\eta, \beta_O$) measure the average and standard deviation of the exposure of excess return to the market factor, the intermediary capital ratio, and the macro-optimism factor respectively.

portfolios. The inclusion of our macro-optimism factor increases the adjusted R-squared significantly by 10% in the case of equity and portfolios. The intermediary factor, although has a positive price of risk estimate, is not statistically significant.[8]

Figure A4a displays the actual average excess returns against the predicted average excess returns for equity and bond portfolios under the two-factor model. The predicted excess returns is from the two-factor model with the market and the intermediary factors. Figure A4b displays the same with predicted excess returns from the three-factor model with macro-optimism. The predicted returns from the three-factor model aligns better along

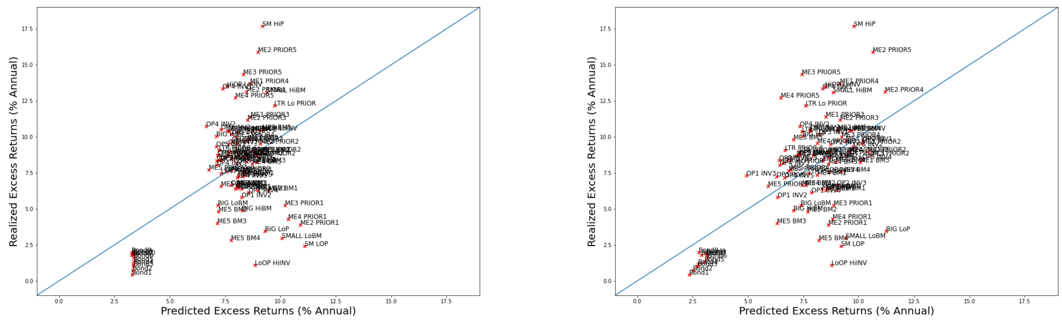
[8] He et al. (2017) document that the intermediary risk factor is not significant with 25 size and book-to-market portfolios and 10 momentum sorted portfolios.

	Equities and Bonds		HKM+Momentum	
	Two-factor	Three-factor	Two-factor	Three-factor
Market	0.01	0.01	0.02	0.02
t-stat Shanken	(1.17)	(1.20)	(1.59)	(1.50)
Intermediary	0.02	0.02	0.06	0.07
t-stat Shanken	(1.08)	(0.75)	(2.86)	(2.68)
Macro-optimism	-	0.1	-	0.08
t-stat Shanken	-	(2.88)	-	(2.06)
MAPE %	2.22	2.08	2.83	2.28
Adj. R2	0.22	0.32	0.45	0.61
Assets	95	95	129	129
Quarters	211	211	171	171

Table A2: Risk price estimates for equities and US government bond portfolios. Equity and bond test assets include 25 size and book-to-market portfolios, 25 size and momentum portfolios, 10 long-term reversal portfolios, and 25 profitability and investment sorted portfolios, 10 maturity sorted US government bond portfolios taken from the CRSP Fama bond portfolio dataset. HKM assets include 25 size and book-to-market portfolios, 10 U.S. Govt. bond portfolios, 10 U.S. corporate bond portfolios, 18 option portfolios, 20 CDS portfolios, and 12 foreign exchange portfolios taken from HKM2017. In addition, it also includes 25 size and momentum portfolios and 10 long-term reversal portfolios. The data is at quarterly frequency from 1970Q1 till 2022Q4. The factors are market, intermediary capital ratio, and macro-optimism. The macro-optimism factor o_t is computed as innovation in the growth rate in O_t, the 75th percentile of GDP forecast scaled by median forecast by analysts. The forecast data is taken from The Survey of Professional Forecasters. The standard errors of risk price estimates are adjusted for generated regressor problem using Shanken correction. MAPE denotes the mean absolute pricing error in annualized terms.

the 45-degree line compared to the predicted returns from the two-factor model without the optimism factor. This observation can be confirmed by Table A2 that shows that the mean absolute pricing error (MAPE) from the three-factor model is 14 basis points lower than the error from the two-factor model.

Table A2 also presents the risk price estimates from the two-factor and the three-factor models with HKM test assets. HKM assets include 25 size and book-to-market portfolios, 10 U.S. Govt. bond portfolios, 10 U.S. corporate bond portfolios, 18 option portfolios, 20 CDS portfolios, and 12 foreign exchange portfolios taken from He et al. (2017). In addition, it also includes 25 size and momentum portfolios and 10 long-term reversal portfolios. The risk price of the macro-optimism factor is positive and statistically significant in both cases.



(a) Pricing error in two-factor model.

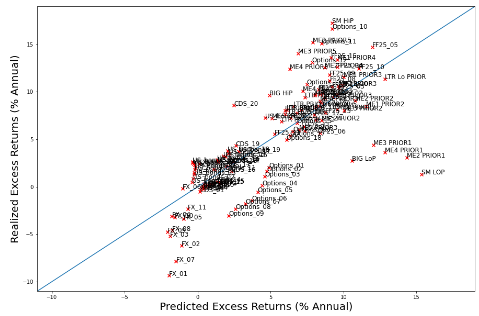
(b) Pricing error in three-factor model.

Figure A4: Pricing errors on equity and bond portfolios: Realized excess returns versus predicted excess returns using the two-factor model with market and intermediary factors. The data is at quarterly frequency and from 1970Q1 till 2022Q4.

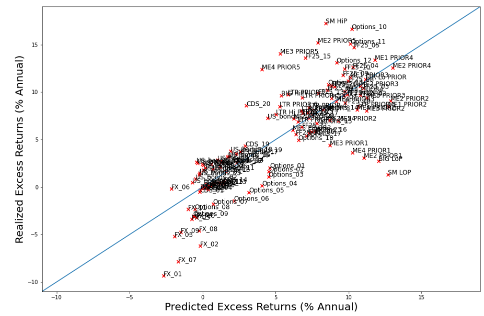
The adjusted R-squared increases by 16% and the mean absolute pricing error decreases by 55 basis points under the three-factor model. Figure A5 also displays the average realized excess returns vs. the average predicted excess returns for HKM+Momemtum assets. The predicted returns in the three-factor model more line up towards the 45-degree line than in the two-factor model, which is consistent with the reduction in pricing error shown in the table A2. Overall, the evidence for the macro-optimism factor in pricing the cross-section of asset returns above and beyond the intermediary and the market factor is strong for equity, bond, and other asset classes.

		Macro-optimism			
		(1)	(2)	(3)	(3)-(1)
Intermediary	(1)	2.60	3.28	5.29	2.68
	(2)	6.53	3.95	7.51	0.97
	(3)	8.95	9.79	9.63	0.67
	(3)-(1)	6.35	6.51	4.34	-

Table A3: Average excess returns. The table reports the annualized mean excess return on equity and bond portfolios double-sorted on their exposures to the intermediary factor and the macro-optimism factor using the three-factor model. The data is at quarterly frequency from 1970Q1 till 2022Q4. The intermediary factor is from HKM2017, and the macro-optimism factor is computed from the growth rate of 75th percentile GDP projection, scaled by the median projection.



(a) Pricing error in two-factor model.



(b) Pricing error in three-factor model.

Figure A5: Pricing errors on HKM+Momentum portfolios. Realized excess returns versus predicted excess returns using the two-factor model with the market and the intermediary factor in panel (a), and the three-factor model with the market, intermediary, and macro-optimism factors in panel (b). The data is quarterly and from 1970Q1 till 2012Q4.

C.1.2 Two-way Sorted Portfolios

To further verify the positive risk price of macro-optimism, we independently double sort the test assets based on their exposures to the intermediary factor and the macro-optimism factor, controlling for the market factor, price-dividend ratio, cyclically adjusted earnings ratio (CAPE), and the cay variable of Lettau and Ludvigson (2001). A rolling-window of 60 periods is used to estimate the betas and for sorting the assets. Table (A3) presents the average excess returns for three-by-three double-sorted portfolios. The high-minus-low return of the portfolios sorted with respect to the macro-optimism beta, i.e., $\beta_{i,o}$, is consistently positive in all three tertiles as seen in the last column of table (A3). The economic magnitude of the high-minus-low returns is also large with up to 2.68% in annualized terms. Thus, the macro-optimism factor has pricing power beyond the market and the intermediary factor.

C.2 Time Series Return Predictability

One prediction of the model is that optimism exacerbates financial instability, exacerbating the amplification of endogenous risk σ_t^p, and leads to a time varying risk premium on the capital stock. In particular, the risk premium increases in crisis as a result of the feedback loop between optimism and the wealth share of productive intermediaries. Since a static comparison in Section 2.5 shows that larger optimism leads to persistently higher risk premium with higher occupancy time (even with a possibility of net worth trap when optimism degree is high enough) in crisis, we test whether our optimism factor O_t constructed in (C.2) can *additionally* predict the conditional excess momentum concentrated in a crisis.

To study the time-series return predictability patterns of asset returns, as a first step we regress the excess return on the S&P500 on its lagged excess return controlling for recession periods.

$$r^e_{t+h} = \alpha(h) + \beta_1(h) r^e_t + \beta_2(h) \times r^e_t \times 1_{Rec} + \epsilon_{t+h} \tag{C.5}$$

The recession indicator 1_{Rec} takes a value 1 during the NBER recessionary months, and 0 otherwise. The coefficient $\beta_2(h)$ measures the excess conditional momentum. Figure A6 displays the empirical auto-correlation coefficients. Empirically, the aggregate stock return exhibits a strong momentum in the shorter horizons, and reversal over horizon 5 months. This effect is stronger in the crisis period which can be seen in the left panel, corroborating with Cujean and Hasler (2017) who find that the return predictability is concentrated during recessionary periods.[9] To compute the model-implied correlation coefficients, we first construct the excess return over a period of length Δ based on the following definition:[10]

$$R^e_t = \int_{t-\Delta}^{t} \left(\frac{d(q_u K_u) + (\hat{A}_u - \iota_u) K_u du}{q_u K_u} - r_{f,u} du \right) \tag{C.6}$$

We then simulate the model 1000 times for 5000 years and compute the average slope cofficients from the following regression

$$R^e_{t+h} = \alpha(h) + \beta_{1,model}(h) R^e_t + \beta_{2,model}(h) \times R^e_t \times 1_{crisis} + \epsilon_{t+h} \tag{C.7}$$

If the sign of coefficients is positive (negative), the return exhibits momentum (reversal). The bottom two graphs in the Figure A6 display the model-implied correlation coefficients. The model captures the 'excess' short-term momentum and long-term momentum

[9] According to Cujean and Hasler (2017), this time-series momentum, documented by Moskowitz et al. (2012), is concentrated during bad times and crashes after market rebounds. Daniela and Moskowitz (2016) documents these momentum crashes in cross-section, which are contemporaneous with market rebounds.

[10] The scaled aggregate productivity $\hat{A}_t := \frac{A_t}{\gamma_O}$ is given by $\hat{A}_t = A(\psi_t) = \psi_t + l(1 - \psi_t)$.

crash quite well.[11] After a series of negative shocks, the economy is dragged into a crisis period with depressed wealth share of optimists and high expected returns. An increase in risk premium during the recapitalization phase leads to the conditional momentum in asset returns. The optimists recapitalize until the economy transitions back to the efficient region where the expected returns are low, leading to momentum crash. While the empirical conditional momentum crashes after a few lags, the model implied conditional momentum persists for several periods. We next turn to analyse the role of macro level optimism in the excess conditional momentum pattern.

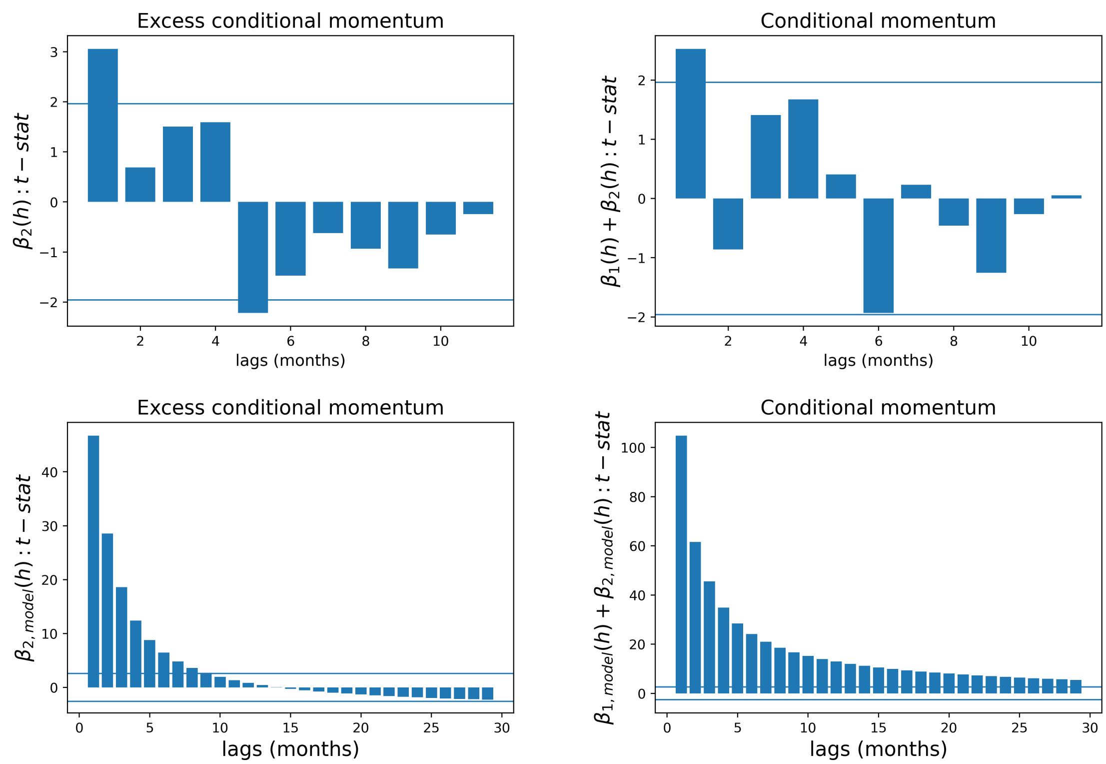


Figure A6: Time series return predictability. Top two panels present the empirical autocorrelation coefficients from regressing the excess return on the S&P500 on its lagged excess return, as shown in the equation (C.5). The data is at monthly frequency from 1945 till 2022. The bottom two panels present the model implied autocorrelation coefficients from the regression (C.7). The model is simulated 1000 times for 5000 years at a monthly frequency. Each correlation coefficient represents the average value across simulations.

[11] We define the 'excess' momentum/reversal to be the momentum/reversal in crisis period relative to the normal period.

C.2.1 Impact of Optimism

To study the impact of macro-optimism on conditional return predictability patterns empirically, we run the following regression:

$$r^e_{t+h} = \alpha(h) + \beta_1(h)r^e_t + \beta_2(h) \times r^e_t \times 1_{Rec} + \beta_3(h) \times r^e_t \times 1_{Rec} \times 1_o + \epsilon_{t+h} \tag{C.8}$$

where 1_d is a dummy variable that takes a value of 1 if the macro-optimism based on The Survey of Professional Forecasters O_t that is defined in (C.2) in Section C.1 is above its median. The coefficient $\beta_3(h)$ measures the excess conditional momentum when optimism is high. We run a similar regression in our model with optimism proxied by the 'net leverage' defined by

$$x^{net}_t = x_t - x^{REE}_t, \tag{C.9}$$

which captures the component of the optimists' total leverage attributed to optimism. x^{REE}_t is the leverage of experts under the rational expectations equilibrium. While the optimism of experts about the expected technological growth, i.e., $\alpha^O - \alpha$ is constant in our model, it triggers a feedback loop between the experts' optimism and their wealth share, impacting their leverage choices. It implies that the net leverage, a component of leverages generated by the optimism, is time-varying. In the model regression, the dummy variable 1_o takes a value 1 if the net conditional leverage of optimists is larger than its median value in each simulated path.[12]

Figure A7 presents the results from the regression (C.8). The model successfully captures the excess conditional momentum during high optimism periods, which is evident in the data. Higher optimism increases the vulnerability of the economy in stochastic steady state. Thus, an adverse shock might lead to a crisis with high expected returns on capital. With stronger optimism, the expectation error increases the likelihood of a net worth trap leading to an elevated risk premium. That is, the feedback loop between the net worth and capital misallocation keeps the conditional risk premium higher for a long time, leading to even higher excess conditional momentum. After a few months, the momentum crashes since positive shocks erode the expectation errors eventually helping the experts to recapitalize. The economy thus transitions into the efficient region, where the actual risk premium falls.

[12]The reason we use conditional leverage is because in our model, optimism affects equilibrium quantities only during crisis, which is out of the stochastic steady state.

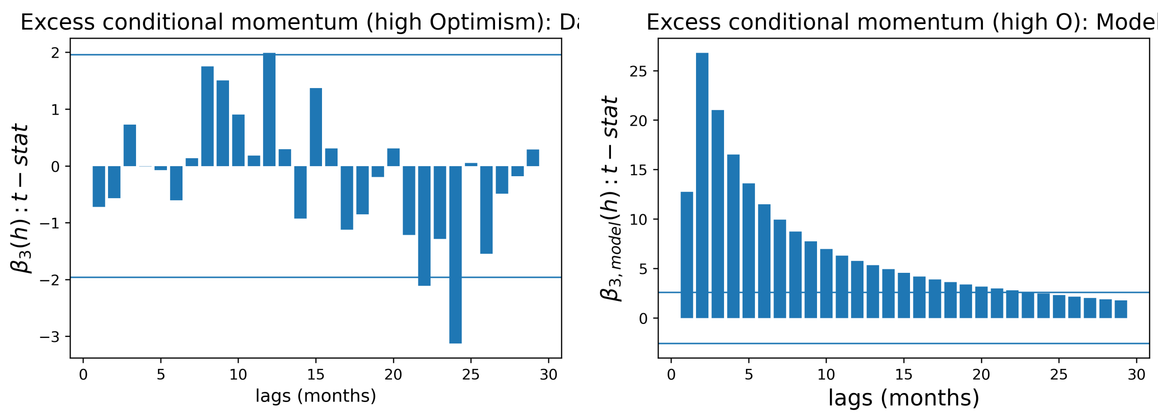


Figure A7: Time series return predictability. The left panel presents empirical autocorrelation coefficients from regressing the excess return on S&P500 on its lagged excess return, as shown in equation (C.8). The data is at monthly frequency from 1945 till 2022. Macro-optimism is computed from the definition (C.2). The macro-optimism data is available at a quarterly frequency and hence interpolated to get monthly values. The left panel presents the conditional t-stats when the optimism is high ($\beta_3(h)$). The right panels presents the model-implied conditional t-stats when the optimism is high ($\beta_{3,model}(h)$).

C.3 Robustness check

	HKM		HKM+Momentum	
	Two-factor	Three-factor	Two-factor	Three-factor
Mean excess return	0.85	0.85	1.2	1.2
Std excess return	1.31	1.31	1.32	1.32
Mean β_M	0.46	0.46	0.62	0.62
Std β_M	0.46	0.46	0.48	0.48
Mean β_η	0.03	0.03	0.04	0.04
Std β_η	0.09	0.09	0.1	0.09
Mean β_O	-	-0.02	-	-0.03
Std β_O	-	0.04	-	0.06
Assets	94	94	129	129
Quarters	195	195	195	195
Controls	Yes	Yes	Yes	Yes

Table A4: Expected returns and risk exposures- Robustness check. HKM assets include 25 size and book-to-market portfolios, 10 U.S. Govt. bond portfolios, 10 U.S. corporate bond portfolios, 18 option portfolios, 20 CDS portfolios, and 12 foreign exchange portfolios taken from He et al. (2017). HKM+Momentum includes 25 size and momentum portfolios and 10 long-term reversal portfolios in addition to the HKM assets. Mean and std. of excess return is the difference in mean return and risk free rate of the corresponding test assets. The frequency is quarterly and time period is from 1970Q1 till 2012Q4. The mean and std. of betas ($\beta_W, \beta_\eta, \beta_O$) measure the average and standard deviation of exposure of the excess return to market factor, intermediary capital ratio, and macro-optimism measure respectively. Controls include price-dividend ratio, cyclically adjusted earnings ratio (CAPE), cay, and capital share risk.

	HKM		HKM+Momentum	
	Two-factor	Three-factor	Two-factor	Three-factor
Market	0.02	0.01	0.02	0.02
	(1.6)	(1.32)	(1.83)	(1.36)
Intermediary	0.09	0.10	0.05	0.07
	(4.48)	(3.64)	(3.01)	(2.47)
Macro-optimism	-	0.06	-	0.09
	-	(1.68)	-	(2.89)
MAPE %	1.7	1.49	2.36	1.95
Adj. R2	0.82	0.86	0.60	0.74
Assets	94	94	129	129
Quarters	195	195	195	195

Table A5: Risk price estimates for HKM and HKM+Momentum portfolios- Robustness check. HKM assets include 25 size and book-to-market portfolios, 10 U.S. Govt. bond portfolios, 10 U.S. corporate bond portfolios, 18 option portfolios, 20 CDS portfolios, and 12 foreign exchange portfolios taken from He et al. (2017). HKM+Momentum includes 25 size and momentum portfolios and 10 long-term reversal portfolios in addition to the HKM assets. The data is at quarterly frequency from 1970Q1 till 2012Q4. The factors are market, intermediary capital ratio, and macro-optimism. The macro-optimism factor is computed as growth rate in the inter-quartile dispersion of GDP forecast scaled by median forecast by analysts. The forecast data is taken from The Survey of Professional Forecasters. The standard errors of risk price estimates are adjusted for generated regressor problem using Shanken correction. MAPE denotes the mean absolute pricing error in annualized terms.

Appendix D General Models

In this section, we illustrate that the concept of net worth trap is not specific to our model specification. In that purpose, we provide a slightly different model specification of behavioral biases and prove that a net worth trap arises in multiple situations, including complete and incomplete markets.

D.1 Complete Market Models

There are two agents in the economy indexed by $j \in i, h$. i is intermediary, while h is household. There is one capital in the economy that evolves as

$$dk_t^a = k_t^a \left(\left(\mu^a + \Phi(\iota_t^a) \right) dt + \sigma^a dZ_t^a \right) \tag{D.1}$$

Agent i is optimistic and believes that the growth rate of capital is $\mu^O > \mu^a$ (similar to Basak (2000) and our Section 2). Later, we consider pessimism cases ($\mu^O < \mu^a$) as well. Thus, the perceived capital process of the agent is given by

$$dk_t^a = k_t^a \left(\left(\mu^O + \Phi(\iota_t^a) \right) dt + \sigma^a \widehat{dZ_t^a} \right) \tag{D.2}$$

where $\widehat{dZ_t^a} := dZ_t - \frac{\mu^O - \mu^a}{\sigma^a} dt$. The price of the capital is given by

$$\frac{dq_t^a}{q_t^a} = \mu_t^{qa} dt + \sigma_t^{qa,a} dZ_t^a \tag{D.3}$$

where both μ_t^{qa} and $\sigma_t^{qa,a}$ are endogenous. The perceived price of capital is

$$\frac{dq_t^a}{q_t^a} = \left(\mu_t^{qa} dt + \frac{\mu^O - \mu^a}{\sigma^a} \sigma_t^{qa,a} \right) dt + \sigma_t^{qa,a} \widehat{dZ_t^a} \tag{D.4}$$

In contrast to our main model in Section 2, there is no technological growth and the TFP is constant and the same across two types. Capital k_t^a produces $y_t^a = \alpha^a k_t^a$ for both types of agents, the actual return process is computed as

$$dr_t^{ka} = \frac{d(q_t k_t^a)}{q_t k_t^a} + \underbrace{\frac{\alpha^a - \iota^a}{q_t^a} dt}_{\text{Dividend yield}} = \underbrace{\left(\mu_t^{qa} + \mu^a + \Phi(\iota_t^a) + \sigma^a \sigma_t^{qa,a} + \frac{\alpha^a - \iota^a}{q_t^a} \right)}_{\equiv r_t^{ka}} dt + (\sigma^a + \sigma_t^{qa,a}) dZ_t^a$$

The agent i uses the perceived capital process (D.2) and the perceived price process (D.4) to compute the return process as

$$\begin{aligned} dr_t^{ka} &= \frac{d(q_t k_t^a)}{q_t k_t^a} + \frac{\alpha^a - \iota^a}{q_t^a} dt \\ &= \underbrace{\left(\mu_t^{qa} + \mu^a + \Phi(\iota_t^a) + \sigma^a \sigma_t^{qa,a} + \frac{\alpha^a - \iota^a}{q_t^a} + \frac{\mu^O - \mu^a}{\sigma^a}(\sigma^a + \sigma_t^{qa,a}) \right)}_{\equiv \widehat{r_t^{ka}}} dt + (\sigma^a + \sigma_t^{qa,a}) \widehat{dZ_t^a} \end{aligned}$$

The household h is rational and uses the correct return processes (D.1) and (D.3). The problem of optimist i is to maximize its lifetime utility of consumption

$$U_t^i = \sup_{C_t^i, w_t^{i,a}, \iota^a} \widehat{\mathbb{E}}_t \left[\int_t^\infty f(C_s^i, U_s^i) ds \right], \tag{D.5}$$

satisfying the budget constraint:

$$\frac{dn_t^i}{n_t^i} = \underbrace{\left(r_t + w_t^{i,a} \left(\widehat{r_t^{ka}} - r_t \right) - c_t^i \right)}_{\equiv \widehat{\mu_t^{ni}}} dt + \underbrace{\left(w_t^{i,a} \left(\sigma^a + \sigma_t^{qa,a} \right) \right)}_{\equiv \sigma_t^{ni,a}} \widehat{dZ_t^a}, \tag{D.6}$$

where $C_t^i = c_t^i n_t^i$ and $w_t^{i,a} := \frac{q_t^a k_t^a}{n_t^i}$ The actual dynamics features the mean of (D.6) as

$$\mu^{ni} = (1 - w_t^{i,a}) r_t + w_t^{ia} r_t^{ka} - c_t^i.$$

Preference following Duffie and Epstein (1992), we use the following preference with unit elasticity of intertemporal substitution (EIS) for both types:

$$f(C, U) = \rho \cdot [(1 - \gamma) U] \cdot \left(\log C - \frac{1}{1 - \gamma} \log((1 - \gamma) U) \right) \tag{D.7}$$

where γ is risk aversion. After we write equilibrium conditions with general γ^i and γ^h, we will assume that $\gamma^i = \gamma^h \to 1$ (i.e., log-utility) as we do in Section 2.

Solution Based on the preference in (D.7), we guess and verify the value function as:

$$V^j(\xi_t^j, n_t^j) = \frac{(n_t^j \xi_t^j)^{1-\gamma^j}}{1 - \gamma^j},$$

for both intermediaries and households, i.e., $j = i, h$, where ξ_t^j depends on the aggregate state of the economy explained later. Its dynamics is given by

$$\frac{d\xi_t^i}{\xi_t^i} = \mu_t^{\xi,i} dt + \sigma_t^{\xi i,a} dZ_t^a \tag{D.8}$$

where $\mu_t^{\xi,i}, \sigma_t^{\xi i,a}$ will be determined in equilibrium. We write the Hamilton-Jacobi-Bellman (HJB) equation as:

$$0 = \max_{w_t^{i,a}, \iota^a, c_t^i} \left\{ \frac{f^i(c_t^i n_t^i, U_t^i)}{(\xi_t^i n_t^i)^{(1-\gamma^i)}} + \mu^{\xi i} + \mu_t^{ni} - \frac{\gamma^i}{2}(\sigma_t^{ni,a})^2 - \frac{\gamma^i}{2}(\sigma_t^{\xi i,a})^2 + (1-\gamma^i)\sigma_t^{\xi i,a}\sigma_t^{ni,a} \right\}. \tag{D.9}$$

We have the following market clearing conditions, with the fact that both types $j = i, h$ choose the same investment ratio ι_t^a which will be confirmed in (D.14) and (D.17):

$$k_t^a = k_t^{i,a} + k_t^{h,a} \tag{D.10}$$

$$c_t^i n_t^i + c_t^h n_t^h = (a^a - \iota_t^a)(k_t^{i,a} + k_t^{h,a}) \tag{D.11}$$

Rewriting the HJB equation (D.9), we obtain

$$\begin{aligned} 0 = \max_{w_t^{i,a}, \iota^a, c_t^i} \Bigg\{ & \rho \log(c_t^i) - \rho \log(\xi_t^i) + \mu_t^{i,\xi} + r_t - c_t^i + w_t^{i,a}\left(\widehat{r_t^{ka}}(\iota^a) - r_t\right) - \frac{\gamma^i}{2}\left(w_t^{i,a}\right)^2 (\sigma^a + \sigma_t^{qa,a})^2 \\ & - \frac{\gamma}{2}\left(\sigma_t^{i,\xi,a}\right)^2 + (1-\gamma^i) w_t^{i,a} \sigma_t^{i,\xi,a} (\sigma^a + \sigma_t^{qa,a}) \Bigg\} \end{aligned} \tag{D.12}$$

The first order conditions with respect to $c_t^i, \iota_t^a, w_t^{i,a}$ are

$$c_t^i = \rho^i \tag{D.13}$$

$$\Phi'(\iota_t^a) = \frac{1}{q_t^a} \tag{D.14}$$

$$0 = (\widehat{r_t^{ka}} - r_t) - \gamma^i w_t^{i,a} (\sigma^a + \sigma_t^{qa,a})^2 + \underbrace{(1-\gamma^i)\sigma_t^{i,\xi,a}(\sigma^a + \sigma_t^{qa,a}))}_{\text{Hedging term}} \tag{D.15}$$

where the last hedging term will disappear under our log-preference, i.e., $\gamma^i = 1$.

The optimization problem and the first order conditions of agent h are identical except that as the agent h uses the correct expectation, r_t^{ka} replaces $\widehat{r_t^{ka}}$. That is,

$$c_t^h = \rho^h \tag{D.16}$$

$$\Phi'(\iota_t^a) = \frac{1}{q_t^a} \tag{D.17}$$

$$0 = (r_t^{ka} - r_t) - \gamma^h w_t^{h,a} \left(\sigma^a + \sigma_t^{qa,a}\right)^2 + (1-\gamma^h)\sigma_t^{h,\xi,a} \left(\sigma^a + \sigma_t^{qa,a}\right) \tag{D.18}$$

Markov equilibrium: As we do in Section 2.4.2, we can construct a Markov equilibrium in one state variable: $\eta_t := \frac{n_t^i}{n_t}$, which denotes the share of wealth held by i. The market clearing conditions can be written as

$$w_t^{i,a}\eta_t + w_t^{h,a}(1-\eta_t) = 1 \tag{D.19}$$

$$c_t^i\eta_t + c_t^h(1-\eta_t) = \frac{a-\iota_t^a}{q_t} \tag{D.20}$$

Dynamics of state variable: We assume that each type $j = i, h$'s net worth n_t^j follows

$$\frac{dn_t^j}{n_t^j} = \mu_t^{nj}dt + \sigma_t^{nj,a}dZ_t^a$$

where both μ_t^{nj} and $\sigma_t^{nj,a}$ are endogenous. We then describe the dynamics of the economy's total wealth n_t as follows:

$$\begin{aligned} dn_t &= dn_t^i + dn_t^h \\ &= \underbrace{\left(\eta_t\mu_t^{ni} + (1-\eta_t)\mu_t^{nh}\right)}_{\equiv\mu_t^n} n_t dt + \underbrace{\left(\eta_t\sigma_t^{ni,a} + (1-\eta_t)\sigma_t^{nh,a}\right)}_{\equiv\sigma_t^{n,a}} n_t dZ_t^a \end{aligned} \tag{D.21}$$

And we obtain

$$\begin{aligned} \frac{d\eta_t}{\eta_t} &= \frac{dn_t^i}{n_t^i} - \frac{dn_t}{n_t} + \left(\frac{dn_t}{n_t}\right)^2 - \frac{dn_t^i}{n_t^i}\cdot\frac{dn_t}{n_t} \\ &= \left((1-\eta_t)(\mu_t^{n,i} - \mu_t^{n,h}) + (\sigma_t^{n,a})^2 - \sigma_t^{ni,a}\sigma_t^{n,a}\right)dt + (1-\eta_t)\left(\sigma_t^{ni,a} - \sigma_t^{nh,a}\right)dZ_t^a \end{aligned}$$

A more intuitive way of rewriting the dynamics of η_t is as follows. From $\eta_t = \frac{n_t^i}{q_t^a k_t^a}$, we

obtain
$$\frac{d(q_t^a k_t^a)}{q_t^a k_t^a} = \underbrace{\left(\mu^a + \Phi(\iota_t^a) + \mu_t^{qa} + \sigma^a \sigma_t^{qa,a}\right)}_{\equiv \mu_t^{qk}} dt + (\sigma^a + \sigma_t^{qa,a}) dZ_t^a,$$

which leads to

$$\begin{aligned}\frac{d\eta_t}{\eta_t} &= \frac{dn_t^i}{n_t^i} - \frac{d(q_t^a k_t^a)}{q_t^a k_t^a} + \left(\frac{d(q_t^a k_t^a)}{q_t^a k_t^a}\right)^2 - \frac{dn_t^i}{n_t^i} \cdot \frac{d(q_t^a k_t^a)}{q_t^a k_t^a} \\ &= \left(r_t + w_t^{i,a}\left(\underbrace{r_t^{ka}}_{\text{True expected return}} - r\right) - c_t^i - \mu_t^{qk} + (\sigma^a + \sigma_t^{qa,a})^2 \left(1 - w_t^{i,a}\right)\right) dt + (w_t^{ia} - 1)(\sigma^a + \sigma_t^{qa,a}) dZ_t^a\end{aligned}$$

Further simplification: log-preference Let us assume the case of log utility with $\gamma^i = \gamma^h = 1$. Then, we have

$$w_t^{i,a} = \frac{\widehat{r_t^{ka}} - r_t}{(\sigma^a + \sigma_t^{qa,a})^2} = \frac{r_t^{ka} - r_t}{(\sigma^a + \sigma_t^{qa,a})^2} + \frac{\frac{\mu^O - \mu^a}{\sigma^a}\cancel{(\sigma^a + \sigma_t^{qa,a})}}{(\sigma^a + \sigma_t^{qa,a})^{\cancel{2}}}$$

leading to

$$r_t^{ka} - r_t = w_t^{i,a} \left(\sigma^a + \sigma_t^{qa,a}\right)^2 - \frac{\mu^O - \mu^a}{\sigma^a} \left(\sigma^a + \sigma_t^{qa,a}\right) \tag{D.22}$$

Plugging (D.22) in the drift of η_t above, we obtain

$$\mu_t^\eta = r_t + \left(w_t^{i,a}(\sigma^a + \sigma_t^{qa,a})\right)^2 - \underbrace{w_t^{i,a} \frac{\mu^O - \mu^a}{\sigma^a} \left(\sigma^a + \sigma_t^{qa,a}\right)}_{\text{Expectation error}} + \left(\sigma^a + \sigma_t^{qa,a}\right)^2 (1 - w_t^{i,a}) - c_t^i - \mu_t^{qk} \tag{D.23}$$

which contains a term stemming from the expectation error of optimistic intermediaries i. For $\mu^O > \mu^a$ case, the error comes from the fact that the agent takes portfolio decisions based on a higher perceived risk premium, but gains a lower risk premium.[13]

Note that we have

$$\begin{aligned} r_t - \mu_t^{qk} &= \frac{\alpha^a - \iota_t^a}{q_t^a} - \left(r_t^{ka} - r_t\right) \\ &= \frac{\alpha^a - \iota_t^a}{q_t^a} - w_t^{i,a} \left(\sigma^a + \sigma_t^{qa,a}\right)^2 + \frac{\mu^O - \mu^a}{\sigma^a} \left(\sigma^a + \sigma_t^{qa,a}\right)\end{aligned}$$

[13]In **?**, we illustrated that this term with $\mu^O > \mu^a$ leads to a potential net worth trap.

Substituting in the drift of wealth share (D.23), we obtain

$$\mu_t^\eta = \frac{\alpha^a - \iota_t^a}{q_t^a} + \left((w_t^{i,a} - 1)(\sigma^a + \sigma_t^{qa,a})\right)^2 - (w_t^{i,a} - 1)\frac{\mu^O - \mu^a}{\sigma^a}(\sigma^a + \sigma_t^{qa,a}) - \rho^i$$

Further simplification and net worth trap Assuming no investments, i.e., $\iota_t = 0$, and no discount rate heterogeneity, i.e., $\rho^i = \rho^h = \rho$, we can write the drift μ_t^η as

$$\mu_t^\eta = \frac{\alpha^a}{q_t^a} + \left((w_t^{i,a} - 1)(\sigma^a + \sigma_t^{qa,a})\right)^2 - \rho - \frac{\mu^O - \mu^a}{\sigma^a}(\sigma^a + \sigma_t^{qa,a})\left(w_t^{i,a} - 1\right) \tag{D.24}$$

From good market equilibrium (D.20), we obtain $q_t^a = \frac{\alpha}{\rho}$ for all t, implying $\sigma_t^{qa,a} = 0$ for $\forall t$ as well. From (D.19), we have $1 = w_t^{i,a}\eta_t + w_t^{h,a}(1 - \eta_t)$.

When $\eta_t \to 0$, $w_t^{h,a} \to 1$, which implies

$$r_t^{ka} - r_t = \left(\sigma^a + \underbrace{\sigma_t^{qa,a}}_{=0}\right)^2 = (\sigma^a)^2,$$

leading to

$$w_t^{i,a} = \frac{\mu^O - \mu^a}{(\sigma^a)^2} + 1.$$

Therefore, in the limit $\eta \to 0^+$, we have the followings:[14]

$$\begin{aligned} &q_0 = \frac{\alpha^a}{\rho}, \quad \sigma_0^{qa,a} = 0, \quad w_0^{ia} = \frac{\mu^O - \mu^a}{(\sigma^a)^2} + 1 \\ &\mu_0^\eta = \left((w_0^{ia} - 1)\sigma^a\right)^2 - \frac{\mu^O - \mu^a}{\sigma^a}(w_0^{ia} - 1)\sigma^a = 0 \\ &\sigma_0^\eta = (w_0^{ia} - 1)\sigma^a \end{aligned} \tag{D.25}$$

Ergodic distribution and net worth trap As proven in Section B.2, the asymptotic distribution of our state variable η_t when $\eta_t \sim 0$ is given by

$$d(\eta) \sim \left(\frac{2\mu_0^\eta}{(\sigma_0^\eta)^2} - 1\right)\eta^{\frac{2\mu_0^\eta}{(\sigma_0^\eta)^2} - 2} \tag{D.26}$$

[14]We put subscript 0 to denote that variables are valued at $\eta \to 0^+$ limit.

where the ratio $\tilde{D}_0 := \frac{2\mu_0^\eta}{(\sigma_0^\eta)^2}$ determines the existence of a degenerate distribution around $\eta = 0$. If $\tilde{D}_0 < 2$, the density becomes a Dirac mass around $\eta = 0$. If $\tilde{D}_0 < 1$, the density becomes degenerate. For example, in the above benchmark complete market model with no investment and homogeneous discount rates, i.e., $\rho^h = \rho^i = \rho$, we get $\tilde{D}_0 = 0$ as $\mu_0^\eta = 0$ as seen in (D.25), which implies that we have a net worth trap, i.e., Dirac-delta measure at $\eta \sim 0$.

It is clear in this benchmark complete market case that when $\eta \to 0$ households hold all the capital outstanding in the market. As the dividend yield $\frac{\alpha^a - \iota_t}{q_t^a}$ is the same across different groups of agents, i.e., optimists and households' production and investment efficiencies are exactly the same, optimists' rate of return on capital cannot exceed that of the aggregate capital valuation $q_t^a k_t^a$. Even if optimists carry leverage, i.e., $w_0^{ia} > 1$, it does not affect as $\eta_t \sim 0$, leading to $\tilde{D}_0 = 0$. In that case, we have a degenerate stationary distribution. How exactly?

Observation 1 We calculate that for $\forall t$, from $w_t^{i,a}\eta_t + w_t^{h,a}(1-\eta_t) = 1$,

$$w_t^{i,a} - 1 = \frac{\mu^O - \mu^a}{(\sigma^a)^2}(1 - \eta_t) \tag{D.27}$$

leading to

$$\mu_t^\eta = \left((w_t^{i,a} - 1)\sigma^a\right)^2 - (w_t^{i,a} - 1)\frac{\mu^O - \mu^a}{\sigma^a}\sigma^a = (w_t^{i,a} - 1)\sigma^a \left[\frac{\mu^O - \mu^a}{\sigma^a}(\cancel{1} - \eta_t) - \cancel{\frac{\mu^O - \mu^a}{\sigma^a}}\right]$$
$$= -\eta_t(1 - \eta_t)\left(\frac{\mu^O - \mu^a}{\sigma^a}\right)^2$$

which is always negative for $0 < \eta_t < 1$ when $\mu^O \neq \mu^a$. Therefore, regardless of $\mu^O > \mu^a$ (i.e., optimism) or $\mu^O < \mu^a$ (i.e., pessimism), the distribution is degenerate at $\eta_t = 0$.

Including investments in the benchmark Now, if investment ι_t^a is there, with investment function $\Phi(\iota) = \frac{1}{\kappa}\log(\kappa\iota + 1)$, then for $\forall t$ we obtain

$$q_t^a = \underbrace{\frac{1 + \alpha^a\kappa}{1 + \rho\kappa}}_{\text{Constant}}, \quad \iota_t = \frac{q_t^a - 1}{\kappa}.$$

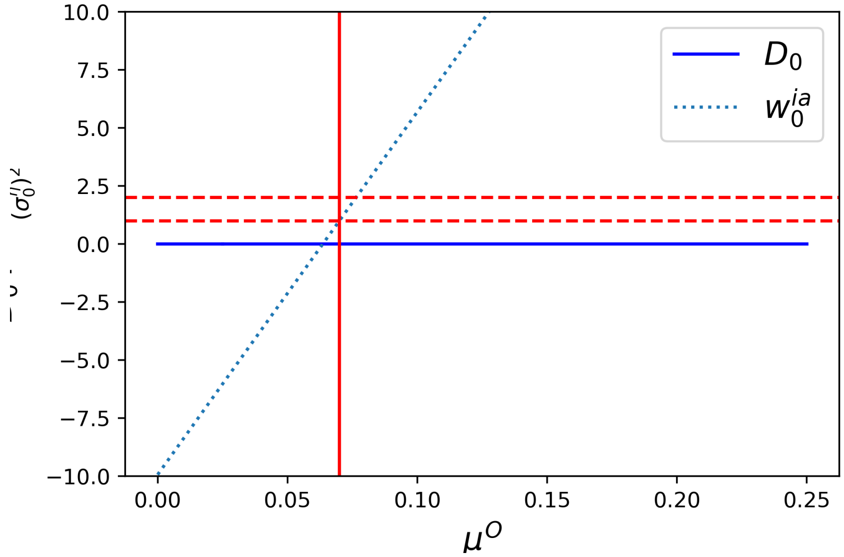


Figure A8: Net worth trap and sentiment. Tail analysis in the case of a complete market model.

As above, in the limit $\eta_t \to 0^+$, we have the followings:

$$q_0 = \frac{1+\alpha^a\kappa}{1+\rho\kappa}, \;\; \iota_0 = \frac{q_0 - 1}{\kappa}, \;\; {\sigma_0}^{qa,a} = 0, \;\; w_0^{ia} = \frac{\alpha^O - \alpha^a}{(\sigma^a)^2} + 1 \tag{D.28}$$

$$\mu_0^\eta = \frac{\alpha^a - \iota_0}{q_0} - \rho + \left((w_0^{ia} - 1)\sigma^a\right)^2 - \frac{\alpha^O - \alpha^a}{\sigma^a}(w_0^{ia} - 1)\sigma^a = 0 \tag{D.29}$$

$$\sigma_0^\eta = (w_0^{ia} - 1)\sigma^a \tag{D.30}$$

leading again to $\tilde{D}_0 = 0$. Even in this case, the above (D.27) and (Observation 1) holds, therefore we obtain a Dirac-delta distribution at $\eta_t = 0$, both when $\mu^O > \mu^a$ (i.e., optimism) and $\mu^O < \mu^a$ (i.e., pessimism).

Figure A8 illustrates $\tilde{D}_0$ measure, which is always 0 regardless of $\mu^O - \mu^a$. We summarize the complete market result as follows:

Observation 2 Even with the investment ι_t^a, the complete market benchmark model always features a net worth trap when $\mu^O \neq \mu^a$, i.e., when an intermediary i is either optimistic or pessimistic.

D.2 Model with Market Incompleteness

Now, we deviate from the above complete market model with $\log$ preference with a slight twist. As in Section 2, intermediary i has an access to superior technology that she uses to transform capital into consumption goods at a higher rate than the rational agent, i.e., now, while intermediary i faces production function $y_t^i = \alpha^a k_t^i$, households have $y_t^h = l \cdot \alpha^a k_t^h$ as their inferior production function.

However, sentimental agents cannot sell equity claims to the rational agent. The actual and perceived expected return of a sentimental agent is the same as before, but the actual expected return on capital for rational agent h is given by

$$\begin{aligned} dr_t^{h,ka} &= \frac{d(q_t^a k_t^a)}{q_t^a k_t^a} + \frac{\ell\alpha^a - \iota^a}{q_t^a} dt \\ &= \underbrace{\left(\mu_t^{qa} + \mu^a + \Phi(\iota_t^a) + \sigma^a \sigma_t^{qa,a} + \frac{\ell\alpha^a - \iota^a}{q_t^a} \right)}_{\equiv r_t^{h,ka}} dt + (\sigma^a + \sigma_t^{qa,a}) dZ_t \end{aligned}$$

where $0 < \ell < 1$. The goods market market clearing becomes

$$\left(c_t^i \eta_t + c_t^h (1 - \eta_t) \right) q_t = (w_t^{ia} \eta_t + \ell w_t^{ha} (1 - \eta_t)) \alpha^a - \iota_t^a \tag{D.31}$$

The drift and the volatility of the wealth share process will be the same as before, i.e.,

$$\mu_t^\eta = \frac{\alpha^a - \iota_t^a}{q_t^a} + \left((w_t^{i,a} - 1)(\sigma^a + \sigma_t^{qa,a}) \right)^2 - (w_t^{i,a} - 1) \frac{\mu^O - \mu^a}{\sigma^a} (\sigma^a + \sigma_t^{qa,a}) - \rho^i.$$

with

$$\sigma_t^\eta = (w_t^{i,a} - 1)(\sigma^a + \sigma_t^{qa,a}). \tag{D.32}$$

In our Markov equilibrium, $q_t^a = q(\eta_t)$ for some function $q(\cdot)$. Then

$$dq_t^a = q'(\eta_t) d\eta_t + \frac{1}{2} q''(\eta_t) (d\eta_t)^2 ,$$

leading to

$$\sigma_t^{qa,a} = \frac{q'(\eta_t)}{q(\eta_t)} \eta_t \cdot \left(w_t^{i,a} - 1 \right) (\sigma^a + \sigma_t^{qa,a}) , \tag{D.33}$$

where $\sigma_t^{qa,a} \to 0$ as $\eta_t \to 0$.

Around $\eta \sim 0$ At the limit $\eta_t \to 0^+$, it will be that

$$q_0^a = \frac{1+\ell\alpha^a\kappa}{1+\rho\kappa}, \;\; \iota_0^a = \frac{q_0-1}{\kappa} = \frac{\ell\alpha^a-\rho}{1+\rho\kappa}, \;\; \sigma_0^{qa,a} = 0,$$

leading to

$$w_0^{i,a} = \underbrace{\frac{\alpha^a}{q_0^a(\sigma^a)^2}(1-\ell)}_{\equiv\Delta_0>0} + \frac{\mu^O-\mu^a}{(\sigma^a)^2} + 1 \tag{D.34}$$

$$\mu_0^\eta = \frac{\alpha^a-\iota_0}{q_0} - \rho + \left((w_0^{ia}-1)\sigma^a\right)^2 - \frac{\mu^O-\mu^a}{\cancel{\sigma^a}}(w_0^{ia}-1)\cancel{\sigma_a} \tag{D.35}$$

$$\sigma_0^\eta = (w_0^{ia}-1)\sigma^a \tag{D.36}$$

Note that due to the productivity difference, now

$$\alpha^a - \iota_0^a = \alpha^a - \frac{\ell\alpha^a-\rho}{1+\rho\kappa} = \frac{\alpha^a(1-\ell)+\rho(1+\kappa\alpha^a)}{1+\rho\kappa},$$

which with $\ell < 1$ leads to

$$\Box_0 \equiv \frac{\alpha^a-\iota_0^a}{q_0^a} - \rho = \frac{\alpha^a(1-\ell)+\rho(1+\kappa\alpha^a)}{1+\ell\alpha^a\kappa} - \rho > 0.$$

From equation (D.35), we obtain

$$(w_0^{i,a}-1)^2 = \left(\Delta_0 + \frac{\mu^O-\mu^a}{(\sigma^a)^2}\right)^2,$$

from which we obtain

$$\mu_0^\eta = \Box_0 + \underbrace{\left(\Delta_0 + \frac{\mu^O-\mu^a}{(\sigma^a)^2}\right)^2(\sigma^a)^2}_{=(\sigma_0^\eta)^2} - (\mu^O-\mu^a)\left(\Delta_0 + \frac{\mu^O-\mu^a}{(\sigma^a)^2}\right), \tag{D.37}$$

$$\sigma_0^\eta = \left(\Delta_0 + \frac{\mu^O-\mu^a}{(\sigma^a)^2}\right)\sigma^a. \tag{D.38}$$

In order to generate a net worth trap around $\eta_t \sim 0$, we need to have $\mu_0^\eta < \frac{1}{2}(\sigma_0^\eta)^2$, i.e.,

$$\Box_0 + \frac{1}{2}\left(\Delta_0^2 + \cancel{\frac{2(\mu^O-\mu^a)}{(\sigma^a)^2}\Delta_0} + \frac{(\mu^O-\mu^a)^2}{(\sigma^a)^4}\right)(\sigma^a)^2 - (\mu^O-\mu^a)\left(\cancel{\Delta_0} + \frac{\mu^O-\mu^a}{(\sigma^a)^2}\right) < 0$$

leading to

$$\square_0 + \frac{1}{2}\Delta_0^2(\sigma^a)^2 < \frac{(\mu^O - \mu^a)^2}{2(\sigma^a)^2}$$

Case.1 So in the optimism case ($\mu^O > \mu^a$), we need to have

$$\mu^O - \mu^a > \sqrt{2}\sigma^a\sqrt{\square_0 + \frac{1}{2}\left(\Delta_0\right)^2\left(\sigma^a\right)^2} > 0$$

Case.2 In the pessimism case ($\mu^O < \mu^a$), we need to have

$$\mu^O - \mu^a < -\sqrt{2}\sigma^a\sqrt{\square_0 + \frac{1}{2}\left(\Delta_0\right)^2\left(\sigma^a\right)^2} < 0.$$

Therefore, in both optimism and pessimism cases, we can have a net worth trap if the degree of optimistic or pessimistic belief of intermediaries is strong enough.

Short-sale constraint under pessimism When μ^O is low enough compared with μ^a, i.e.,

$$\mu^O - \mu^a < -\left(\sigma^a\right)^2\left(1 + \Delta_0\right), \tag{D.39}$$

we can see from (D.34) that the optimal $w_0^{i,a}$ is negative. In that case, due to the short sale constraint of capital, the optimal $w_0^{i,a}$ around $\eta_t \sim 0$ will be $w_0^{i,a} = 0$. In that case, we obtain

$$\sigma_0^\eta = -\sigma^a, \;\; \mu_0^\eta = \square_0 + \left(\sigma^a\right)^2 + \left(\mu^O - \mu^a\right).$$

Therefore, in order to have $\tilde{D}_0 < 1$, i.e., $\mu_0^\eta < \frac{1}{2}\left(\sigma_0^\eta\right)^2$, we need to have

$$\mu^O - \mu^a < -\left(\square_0 + \frac{1}{2}\left(\sigma^a\right)^2\right). \tag{D.40}$$

From (D.39) and (D.40), it should be

$$\mu^O - \mu^a < -\max\left\{\left(\sigma^a\right)^2\left(1 + \Delta_0\right), \square_0 + \frac{1}{2}\left(\sigma^a\right)^2\right\}.$$

Even with mild pessimism, the sentimental agent chooses to allocate some fraction of her capital to the risky capital. This is because she has a higher productivity rate in holding capital compared to the rational agent ($l < 1$ in the model). Thus, under mild pessimism, the drift at the limit is positive since $w_0^{ia} > 1$. For excessive pessimism, $w_0^{ia} = 0$ since a higher productivity rate does not compensate the sentimental agent enough to be long in the risky capital. The no-shorting constraint forces the weight to be equal to zero. In that case, the rational household h holds all capital and earns the risk premium, which ends up draining the wealth share of the sentimental intermediary i. Figure A9 depicts that when μ^O is below some threshold, $w_0^{i,a} \geq 0$ constraint starts binding, and $\tilde{D}_0$ actually becomes negative, leading to a Dirac-delta measure at $\eta_t \sim 0$.

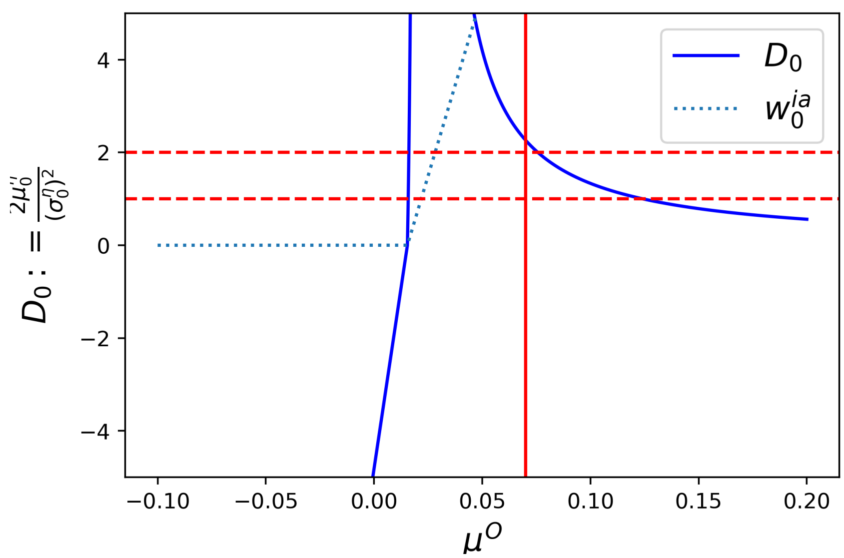


Figure A9: Net worth trap and sentiment. Tail analysis in the case of market incompleteness.