Higher-Order Forward Guidance

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Motivation

Big Question

Forward guidance — How does it work, exactly?

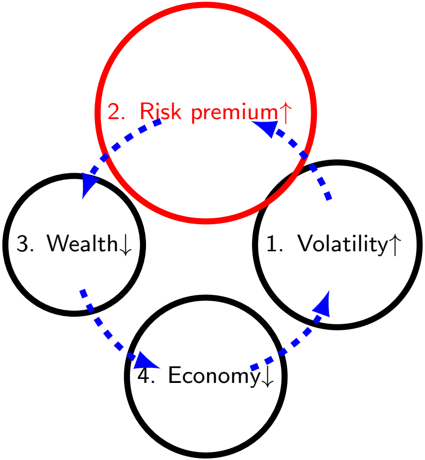
- First-order effects (level): "Interest rates will stay low" $\longrightarrow$ intertemporal substitution channel (aggregate demand↑): e.g., Eggertsson et al. (2003), McKay et al. (2016)
- Second-order effects (volatility): reduce uncertainty, avoid worst-case scenarios, "whatever it takes" $\longrightarrow$ precautionary savings channel (aggregate demand↑)

This paper: focus on central bank's strategic uncertainty management and coordination. Possible for central banks to pick an equilibrium where:

- During the ZLB (now): reduce aggregate volatility (and risk premium). Then aggregate demand↑
- But central banks now create uncertainty about where the economy ends up after the ZLB (future): commit less stabilization after the ZLB
- Welfare-enhancing overall

Non-linear Two-Agent New Keynesian (TANK) model with rigid prices

- With an aggregate stock market + (standard) portfolio choice problem
- Characterize the gap economy (log-deviation from the flexible price economy), e.g., stock price gap



Output and asset price gaps

A non-linear IS equation (in contrast to textbook linearized one)

Fundamental volatility

$$d\hat{Q}_t = \left(i_t - \underbrace{\left(r^n - \frac{1}{2}\underbrace{\boxed{(\sigma + \sigma_t^q)^2}}_{\text{rp}_t} + \frac{1}{2}\underbrace{\boxed{\sigma^2}}_{\text{rp}^n} \right)}_{\equiv r_t^T} \right) dt + \sigma_t^q \, dZ_t$$

Stock price (gap)

$$= (i_t - r_t^T)dt + \sigma_t^q dZ_t$$

$$\sigma_t^q \uparrow \longrightarrow \text{rp}_t \uparrow \longrightarrow \hat{Q}_t \downarrow \longrightarrow \hat{Y}_t \downarrow$$

What is r_t^T?: a risk-adjusted natural rate of interest ($\sigma_t^q \uparrow \longrightarrow r_t^T \downarrow$)

$$r_t^T \equiv r^n - \frac{1}{2}\hat{rp}_t, \quad \hat{rp}_t = \underbrace{\text{rp}_t - \text{rp}_t^n}_{\text{risk-premium gap}}$$

Monetary policy outside the ZLB

Outside the ZLB: can we stabilize the business cycle? Can we prevent the volatility feedback loop?

- **Yes**: Lee and Dordal i Carreras (2025, Job Market Paper)

- Under a risk-premium targeting rule:

$$\begin{aligned} i_t &= r_t^T + \phi_q \hat{Q}_t \\ &= r^n - \frac{1}{2}\, \hat{rp}_t + \phi_q \hat{Q}_t \end{aligned}$$

Risk premium (targeting)

- With $\phi_q > 0$ (i.e., Taylor principle) $\longrightarrow$ $\hat{Q}_t = \sigma_t^q = 0$ for $\forall t$ (unique equilibrium)

At the ZLB, the volatility feedback loop reappears:

$$\begin{aligned} d\hat{Q}_t &= -r_t^T dt + \sigma_t^q dZ_t \\ &= -\left[r^n - \frac{1}{2}(\sigma + \sigma_t^q)^2 + \frac{1}{2}\sigma^2 \right] dt + \sigma_t^q\, dZ_t \end{aligned}$$

ZLB from fundamental volatility shock

Thought experiment: fundamental volatility $\sigma\uparrow$: $\bar{\sigma}$ on $[0, T]$ (e.g., Werning (2012)) and comes back to $\underline{\sigma}$ with $\bar{\sigma} > \underline{\sigma}$

- $\bar{r} \equiv r^n(\underline{\sigma}) = \rho + g - \underline{\sigma}^2 > 0$: no ZLB before, $t < 0$, or after, $t > T$
- $\underline{r} \equiv r^n(\bar{\sigma}) = \rho + g - \bar{\sigma}^2 < 0$: ZLB binds for $0 \leq t \leq T$

Assume: perfect stabilization (i.e., $\hat{Q}_t = 0$) is achievable outside ZLB, i.e.,

$$i_t = \bar{r} - \frac{1}{2}\hat{rp}_t + \phi_q \hat{Q}_t, \quad \text{with } \phi_q > 0$$

Result: perfect stabilization of risk-premia gap (i.e., excess uncertainty) inside the ZLB

- Recursive argument: full stabilization at T implies $\hat{Q}_T = 0 \longrightarrow \sigma^q_{T-\mathrm{d}t} = 0$, and so on (so $\hat{rp}_t = 0$ for $\forall t$)

ZLB path (full stabilization after T)

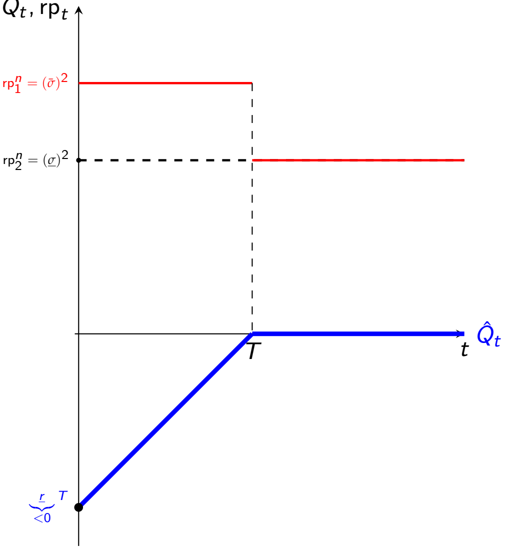


Figure: ZLB dynamics (Benchmark)

Traditional forward guidance (keep $\underline{i_t = 0}$ until $\hat{T}^{\text{TFG}} > T$) ⏩ Details

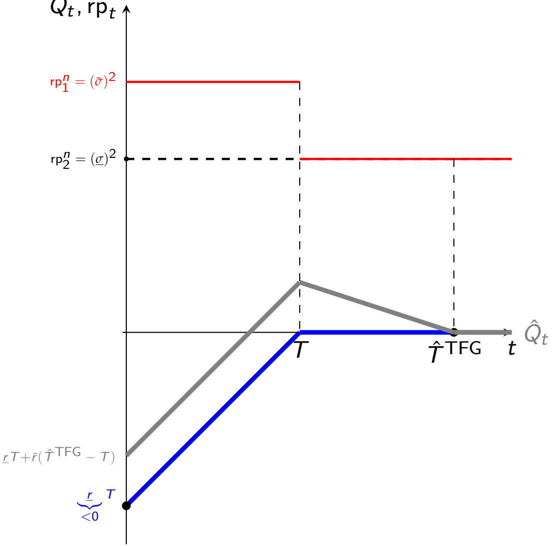


Figure: ZLB dynamics with forward guidance until $\hat{T}^{\text{TFG}} > T$

Alternative forward guidance policies

Big Question

Can we do even better than the traditional forward guidance?

What if we reduce aggregate uncertainty via $\sigma_t^q < 0$?

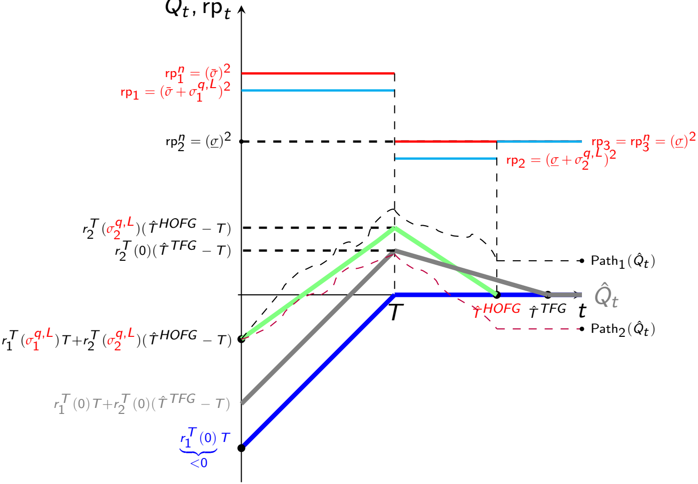
- Then $\mathrm{rp}_t = \left(\bar{\sigma} + \sigma_t^q\right)^2 < \mathrm{rp}_t^n$, raising stock prices and aggregate demand

But how?

- Nominal rigidities $\longrightarrow$ demand-determined production (and hence, wealth)
- Policy challenge: the central bank *must convince* households to "coordinate" on this particular equilibrium $\longrightarrow$ *higher-order forward guidance*
- Give up perfect stabilization in the future (no stabilization at all)
- Imagine the central bank pegs the policy rate at $i_t = \bar{r}$ after zero rate periods

Central bank picks $\hat{T}^{HOFG}$ and $\{\sigma_t^q\}$

Details



Proposition (Optimal commitment path)

At optimum, $\sigma_1^{q,L} < 0 = \sigma_1^{q,n}$, $\sigma_2^{q,L} < 0 = \sigma_2^{q,n}$, and $\hat{T}^{HOFG} < \hat{T}^{TFG}$

Optimal policy

Proposition (Optimal forward guidance policy)

Optimal higher-order forward guidance (HOFG) always results in an equal or lower expected quadratic loss than the traditional guidance policy

Proof

With $(\sigma_1^{q,L}, \sigma_2^{q,L}, \hat{T}^{\text{HOFG}}) = (0, 0, \hat{T}^{\text{TFG}})$, solutions coincide

Remarks:

- Alternative higher-order forward guidance policy implementations **are** possible
- This paper shows HOFG dominates TFG in a simple setting

Optimal policy: stochastic stabilization ▶▶ Details

Extension: still higher-order forward guidance (HOFG) policy, now with stochastic stabilization after $\hat{T}^{\text{HOFG}}$. Return to stabilization with νdt probability after $\hat{T}^{\text{HOFG}}$

- Central bank commits to stabilizing the economy after $\hat{T}^{HOFG}$ with some probability. Expected stabilization after $1/\nu$ quarters
- $\nu = 0$: the above higher-order forward guidance
- $\nu = \infty$: the traditional forward guidance policy

Big discontinuity:

$$\lim_{\nu \to +\infty^-} \mathbb{L}^{Q,*}\left(\{\hat{Q}_t\}_{t\geq 0}, \nu\right) < \underbrace{\mathbb{L}^{Q,*}\left(\{\hat{Q}_t\}_{t\geq 0}, \nu = \infty\right)}_{\text{Traditional forward guidance}}$$

- Slight probability that stabilization might not happen $\longrightarrow$ HOFG possible
- Welfare↑ (i.e., loss↓) as ν↑ but $\nu \neq \infty$

Policy implication

Real World Example (Covid-19 and the Federal Reserve)

Flexible Average Inflation Targeting (FAIT) (2020)

- Commitment to delaying stabilization – by allowing inflation to "moderately" overshoot its target after periods of persistent undershooting at the ZLB
- "Moderate" overshooting of the business cycle now is allowed: nudging agents toward a favorable equilibrium with lower volatility

HOFG equilibrium $\longrightarrow$ supported by fiscal policy as a unique equilibrium ⏩ Details

- Zero transfer along the equilibrium path (out-of-equilibrium threat)
- Draghi's "whatever it takes" speech $\longrightarrow$ lowers periphery yields without actual expenditures, coordinating agents to an equilibrium with lower risk premium (Acharya et al., 2019)

Welfare comparisons

$T = 20$ quarters ZLB spell

Loss function $\mathbb{L}$ as the (conditional) quadratic output loss per quarter:

$$\mathbb{L}^{Y}_{\text{Per-period}} \equiv \rho \int_0^{\infty} e^{-\rho t} \mathbb{E}_0 \left(\hat{Y}_t^2 \right) \approx \zeta^2 \cdot \rho \int_0^{\infty} e^{-\rho t} \frac{1}{s} \sum_{i=1}^{s} \left(\hat{Q}_t^{(i)} \right)^2 dt$$

Policy	**No guidance**	**Traditional**	**Higher-Order** (no stochastic stabilization)	**Higher-Order** (*with* stoch. stab., $\nu = 1$)
$\sigma_1^{q,L}$	0	0	-1.27%	-4.13%
$\sigma_2^{q,L}$	0	0	-0.24%	-3.79%
$\hat{T}$	20	25.27	25.09	24.68
$\mathbb{L}^{Y}_{\text{Per-period}}$	7%	1.93%	1.81%	1.69%

- Still, traditional forward guidance too strong: e.g., McKay et al. (2016)
- HOFG with $\nu \to \infty$ but $\nu \neq \infty$ most effective

Thank you very much!
(Appendix)

Model structure

Go back

Identical **capitalists** and **hand-to-mouth workers** (two types of agents)

- **Capitalists**: consumption - portfolio decision (between stock and bond)
- **Workers**: supply labor to firms (hand-to-mouth)

1. **Technology**

Fundamental risk (Exogenous)

$$\frac{dA_t}{A_t} = \underbrace{g}_{\text{Growth}} \cdot dt + \sigma \cdot \underbrace{dZ_t}_{\text{Aggregate shock}}$$

2. **Hand-to-mouth workers**: solves the following problem:

$$\max_{C_t^w, N_t^w} \frac{\left(\frac{C_t^w}{A_t}\right)^{1-\varphi}}{1-\varphi} - \frac{\left(N_t^w\right)^{1+\chi_0}}{1+\chi_0} \quad \text{s.t.} \quad \bar{p} C_t^w = w_t N_t^w$$

- Hand-to-mouth assumption can be relaxed, without changing implications

3. **Firms**: Dixit-Stiglitz production using labor + perfectly rigid prices ($\pi_t = 0$)

4. **Financial market**: zero net-supplied risk-free bond + stock (index) market – pooling firm profits

Capitalists ⏩ Go back

Capitalists: standard portfolio and consumption decisions (very simple)

1. Stock market valuation $= \bar{p} A_t Q_t$, where (real) stock price Q_t follows:

$$\frac{dQ_t}{Q_t} = \mu_t^q \cdot dt + \sigma_t^q \cdot dZ_t$$

Financial risk (Endogenous)

- μ_t^q and σ_t^q are both endogenous (to be determined)

2. Each solves the following optimization (standard)

$$\max_{C_t, \theta_t} \mathbb{E}_0 \int_0^\infty e^{-\rho t} \log C_t dt \quad \text{s.t.}$$

$$da_t = (a_t(i_t + \theta_t(i_t^m - i_t)) - \bar{p} C_t)dt + \theta_t a_t(\sigma + \sigma_t^q)dZ_t$$

- Aggregate consumption of capitalists $\propto$ aggregate financial wealth

$$C_t = \rho A_t Q_t$$

- Equilibrium risk-premium is determined by the total risk

$$i_t^m - i_t \equiv \text{rp}_t = \boxed{(\sigma + \sigma_t^q)^2}$$

Equilibrium with rigid prices ($\pi_t = 0, \forall t$) Go back

Flexible price economy as benchmark: 'natural' consumption of capitalists $C_t^n = \rho A_t Q_t^n$ follows

$$\frac{dC_t^n}{C_t^n} \equiv \frac{d\left(A_t Q_t^n\right)}{A_t Q_t^n} = \left(r^n - \rho + \sigma^2\right) dt + \sigma dZ_t$$
$$= g dt + \sigma dZ_t = \frac{dA_t}{A_t}$$

where $r^n = \rho + g - \sigma^2$ is the 'natural' rate of interest

Define **asset price gap**

$$\hat{Q}_t = \ln \frac{Q_t}{Q_t^n}, \quad \underbrace{0 = \mathrm{Var}_t\left(\frac{dQ_t^n}{Q_t^n}\right)}_{\text{Benchmark volatility}}, \quad \underbrace{\left(\underset{\text{Endogenous}}{\sigma_t^q}\right)^2 dt = \mathrm{Var}_t\left(\frac{dQ_t}{Q_t}\right)}_{\text{Actual volatility}}$$

which is proportional to **output gap** up to a first order

$$\hat{Y}_t = \ln\left(\frac{Y_t}{Y_t^n}\right) \longrightarrow \hat{Y}_t = \underbrace{\zeta}_{>0} \cdot \hat{Q}_t$$

Other equilibrium conditions ⏩ Go back

Dividend yield: dividend yield$= \rho$, as in Caballero and Simsek (2020)

- A positive feedback loop between asset price $\Longleftrightarrow$ dividend (output)

Determination of nominal stock return $d\mathbf{I}_t^m$

$$d\mathbf{I}_t^m = \underbrace{[\ \underbrace{\rho}_{\text{Dividend yield}} + \underbrace{g + \mu_t^q + \overbrace{\sigma\sigma_t^q}^{\text{Covariance}}}_{\text{Capital gain}}\ ]}_{= \underbrace{i_t^m}_{\text{Drift}} = \underbrace{i_t}_{\text{Monetary policy}} + \underbrace{(\sigma+\sigma_t^q)^2}_{\text{Risk-premium}}} dt + \underbrace{(\sigma + \sigma_t^q)}_{\text{Risk term}} dZ_t$$

Traditional forward guidance ⏩ Go back

Assume:

- Central bank commits to keep $i_t = 0$ until $\hat{T}^{\text{TFG}} \geq T$ (i.e., Odyssean guidance)
- Perfect stabilization (i.e., $\hat{Q}_t = 0$) afterwards, i.e., for $t > \hat{T}^{\text{TFG}}$
- From the same arguments, risk-premium gap stabilization beforehand, $t \leq \hat{T}^{\text{TFG}}$ (no excess volatility while $i_t = 0$)

Problem: minimize smooth quadratic welfare loss

$$\min_{\hat{T}^{\text{TFG}}} \mathbb{L}^Q \left(\{\hat{Q}\}_{t\geq 0}\right) \equiv \mathbb{E}_0 \int_0^\infty e^{-\rho t} \left(\hat{Q}_t\right)^2 dt$$
$$\text{s.t. } \hat{Q}_0 = \underbrace{\underline{r}}_{<0} T + \underbrace{\bar{r}}_{>0} \left(\hat{T}^{\text{TFG}} - T\right)$$

- Smoothing the ZLB costs over time (i.e., welfare enhancing)

Higher-order forward guidance ▸▸ Go back

Assume:

- Central bank can commit to keep $i_t = 0$ until $\hat{T}^{HOFG} \geq T$
- No stabilization (i.e., $\hat{Q}_t = \hat{Q}_{\hat{T}^{HOFG}}$) guaranteed afterwards, $t \geq \hat{T}^{HOFG}$
- Pick $\{\sigma_t^q\}$ for $t < \hat{T}^{HOFG}$

Problem: minimize smooth quadratic welfare loss

$$\min_{\sigma_1^{q,L}, \sigma_2^{q,L}, \hat{T}^{HOFG}} \mathbb{L}^Q\left(\{\hat{Q}\}_{t\geq 0}\right) \equiv \mathbb{E}_0 \int_0^\infty e^{-\rho t}\left(\hat{Q}_t\right)^2 dt,$$

$$\text{s.t.} \quad \begin{cases} d\hat{Q}_t = -\underbrace{r_1^T\left(\sigma_1^{q,L}\right)}_{<0} dt + \sigma_1^{q,L} dZ_t, & \text{for } t < T, \\ d\hat{Q}_t = -\underbrace{r_2^T\left(\sigma_2^{q,L}\right)}_{>0} dt + \sigma_2^{q,L} dZ_t, & \text{for } T \leq t < \hat{T}^{HOFG}, \\ d\hat{Q}_t = 0, & \text{for } t \geq \hat{T}^{HOFG}, \end{cases}$$

with

$$\hat{Q}_0 = \underbrace{r_1^T\left(\sigma_1^{q,L}\right)}_{<0} T + \underbrace{r_2^T\left(\sigma_2^{q,L}\right)}_{>0}\left(\hat{T}^{HOFG} - T\right)$$

Higher-order forward guidance with stochastic stabilization

Go back

Change:

- Central bank commits to stabilizing the economy after $\hat{T}^{HOFG}$ with Poisson probability ν: at each point after $\hat{T}^{HOFG}$, $\hat{Q}_t$ becomes 0 with probability νdt

Problem: minimize smooth quadratic welfare loss

$$\min_{\sigma_1^{q,L}, \sigma_2^{q,L}, \hat{T}^{\text{HOFG}}} \quad \mathbb{E}_0 \left[\int_0^{\hat{T}^{\text{HOFG}}} e^{-\rho t} \hat{Q}_t^2 \, dt + \int_{\hat{T}^{\text{HOFG}}}^{\infty} e^{-\rho t} e^{-\nu\left(t - \hat{T}^{\text{HOFG}}\right)} \hat{Q}_t^2 \, dt \right],$$

$$\text{s.t.} \quad \begin{cases} d\hat{Q}_t = -\underbrace{r_1^T\left(\sigma_1^{q,L}\right)}_{<0} dt + \sigma_1^{q,L} dZ_t, & \text{for } t < T, \\ d\hat{Q}_t = -\underbrace{r_2^T\left(\sigma_2^{q,L}\right)}_{>0} dt + \sigma_2^{q,L} dZ_t, & \text{for } T \le t < \hat{T}^{HOFG}, \\ d\hat{Q}_t = 0, & \text{for } t \ge \hat{T}^{HOFG}, \end{cases}$$

with

$$\hat{Q}_0 = \underbrace{r_1^T\left(\sigma_1^{q,L}\right)}_{<0} T + \underbrace{r_2^T\left(\sigma_2^{q,L}\right)}_{>0} \left(\hat{T}^{HOFG} - T\right)$$

Fiscal policy coordination

Go back

Fiscal authority's monetary reserves F_t

$$dF_t = -\theta_t a_t \tau_t dZ_t, \quad \text{with:} \quad F_0 = F_{0-} - \underbrace{\chi \theta_{0-} a_{0-}}_{\text{Instant subsidy}} \ , \tag{1}$$

Then capitalist's dynamic flow becomes:

$$da_t = \left(a_t \left(i_t + \theta_t (i_t^m - i_t)\right) - \bar{p} C_t\right) dt + \theta_t a_t \left[\left(\sigma_t + \sigma_t^q\right) + \tau_t\right] dZ_t \ , \tag{2}$$

with $\Delta a_0 \equiv a_0 - a_{0^-} = \chi \theta_{0-} a_{0-} + \bar{p} A_{0-} \quad \underbrace{\Delta Q_0}_{\text{Asset price change}}$

Proposition

HOFG equilibrium (with $\sigma_t^{q,*}$) becomes a unique equilibrium under the following rule:

$$\tau_t = \left(\sigma_t^{q,*} - \sigma_t^q\right) \ , \quad \text{and} \quad \chi = \bar{p} A_{0-} \frac{Q_0^* - Q_0}{\theta_{0-} a_{0-}} \ , \tag{3}$$

In this case, $\tau_t = 0$, and $\chi = 0$ on the equilibrium path

Standard New Keynesian Model (Global Approach) Go back

A textbook New Keynesian model with rigid price

From Lee and Dordal i Carreras (2025)

- The representative household's problem (given B_0) is

$$\Gamma_t \equiv \max_{\{B_t\}_{t>0},\{C_t,L_t\}_{t\geq 0}} \mathbb{E}_0 \int_0^\infty e^{-\rho t} \left[\log C_t - \frac{L_t^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}} \right] dt \quad \text{s.t.} \quad \dot{B}_t = i_t B_t - \bar{p} C_t + w_t L_t + D_t$$

where

- B_t: nominal bond holding, D_t includes fiscal transfer + profits
- Rigid price: $p_t = \bar{p}$ for $\forall t$ (i.e., purely demand-determined)

Endogenous volatility

A non-linear Euler equation (in contrast to log-linearized one)

$$\mathbb{E}_t \left(\frac{dC_t}{C_t} \right) = (i_t - \rho) dt + \underbrace{\mathrm{Var}_t \left(\frac{dC_t}{C_t} \right)}_{\text{Precautionary premium}}$$

Endogenous drift

- Aggregate volatility↑ $\Longrightarrow$ precautionary saving↑ $\Longrightarrow$ recession (the drift↑)

Mathematical equivalence: higher-order forward guidance (HOFG) becomes implementable

Defining output gap and excess volatility:

$$\hat{Y}_t = \ln \frac{Y_t}{Y_t^n}, \quad \underbrace{(\sigma)^2 dt = \mathrm{Var}_t\left(\frac{dY_t^n}{Y_t^n}\right)}_{\text{Benchmark volatility}}, \quad \underbrace{\left(\sigma + \sigma_t^s\right)^2 dt = \mathrm{Var}_t\left(\frac{dY_t}{Y_t}\right)}_{\text{Actual volatility}}$$

Then again, a non-linear IS equation written in output:

Fundamental volatility

$$d\hat{Y}_t = \left(i_t - \underbrace{\left(r^n - \frac{1}{2} \underbrace{\boxed{(\sigma + \sigma_t^s)^2}}_{\text{pp}_t} + \frac{1}{2} \underbrace{\boxed{\sigma^2}}_{\text{pp}^n} \right)}_{\equiv r_t^T} \right) dt + \sigma_t^s \, dZ_t$$

$$= (i_t - r_t^T) dt + \sigma_t^s dZ_t$$

$$\sigma_t^s \uparrow \longrightarrow \text{pp}_t \uparrow \longrightarrow \hat{Y}_t \downarrow$$