

# Do Cost-of-Living Shocks Pass Through to Wages?

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## Abstract

We develop a tractable New Keynesian model where firms post wages and workers search on the job, motivated by microeconomic evidence on wage setting. Because firms set wages to avoid costly turnover, the rate that workers quit their jobs features prominently in the model's wage Phillips curve, matching U.S. evidence that wage growth tightly correlates with workers' quit rate. We then examine the response of wages to cost-of-living shocks, i.e., shocks that raise the price of household's consumption goods but do not affect the marginal product of labor. Such shocks pass through to wages only to the extent that higher cost of living improves worker's outside options, such as competing jobs or unemployment, relative to their current job. However, higher cost of living lowers real wages at all jobs evenly, and unemployment is rarely a credible outside option. Cost-of-living shocks thus have little to no effect on relative outside options and therefore wages. We conclude that wage posting and on-the-job search, which are prevalent in labor markets such as the United States, limit the scope for pass through from prices to wages and elevate voluntary quits as the primary predictor of nominal wage growth.

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# 1 Introduction

In the economic recovery following the COVID pandemic, economies throughout the world experienced both rapid price inflation and rapid nominal wage growth. This experience has generated interest in the relationship between price and wage growth and raised concerns among policymakers of a wage-price spiral. However, mainstream tools for analyzing the relationship between price inflation and wage growth assume wage setting mechanisms that are counterfactual for economies such as the United States, namely that wages are set unilaterally by unions representing workers. In many advanced economies, union membership has declined dramatically, and evidence suggests that in the United States wage posting is the most common, if not dominant, method of wage determination.<sup>1</sup> In light of this evidence and the recent experience of inflation, we ask, *when firms set both prices and wages, through what mechanisms do workers' wages respond to shocks to cost of living, and how large is this response?*

To answer this, we extend the wage posting model in Bloesch and Larsen (2023) into a Dynamic, Stochastic General Equilibrium (DSGE) environment where workers search on the job, and firms set both prices and wages subject to nominal rigidities in the form of standard, convex adjustment costs. Since hiring is costly, firms are incentivized to pay sufficiently high wages to quickly fill vacancies and prevent workers from quitting. The threat of workers quitting into unemployment is low, so wages are primarily determined by firms competing for already-employed workers. We analytically derive the firm's wage Phillips curve, and show how it can be written as a simple relationship between nominal wage growth and log deviations in the quit rate and the unemployment rate alone. Calibrated to match U.S. data on worker flows, our model predicts that fluctuations in quits are the most important in predicting wage growth, while unemployment, the forcing variable in standard sticky-wage models such as Galí (2011), has almost no weight. Estimating this reduced form Phillips curve on US data, we find empirically that quits dominates unemployment in predicting wage growth, providing a validation of the model.

We then consider how wages respond to a cost-of-living shock: i.e., a shock that raises the cost of households' consumption bundle without affecting the marginal product of labor. To model a pure cost-of-living shock, we assume that workers consume two goods: a labor-intensive services bundle (e.g., haircuts) and an endowment good (e.g., unprepared food or energy). Negative shocks to the quantity of endowment good thus raise the price of workers' consumption basket without affecting the marginal product of labor, allowing us to study how a "pure" cost-of-living shock passes through to wages. This cost-of-living shock raises firms' optimal wage only if it affects turnover costs by making workers harder to recruit or more likely to quit. In our benchmark model, we show that a higher price level has no effect on these probabilities: since a higher price level changes the real wages of all jobs proportionally, cost-of-living shocks do not make

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<sup>1</sup>See Hall and Krueger (2012); Lachowska et al. (2022); Di Addario et al. (2023).

workers more likely to quit or harder to recruit from other firms; and if unemployed workers' purchasing power is also equally eroded by higher prices, then a higher price level leaves the relative desirability of employment and unemployment unchanged. Since the probability that a worker quits or is recruited is unchanged, higher cost of living has no effect on wages.

We then consider an extension where a higher cost of living *does* affect the probability that a worker quits or is recruited, namely that unemployment benefits are indexed to inflation while nominal wages are not.<sup>2</sup> Increases in the cost of living now make unemployment relatively more desirable, making unemployed workers more difficult to recruit and employed workers more likely to quit into unemployment. We show, however, that the presence of on-the-job search renders pass through from cost-of-living shocks to wages quantitatively small, as competition for already employed workers continues to dominate firm wage setting decisions even when the relative desirability of unemployment improves. Thus, a quantitatively realistic amount of on-the-job search severely limits the pass through of cost-of-living shocks to wages even when a higher cost of living makes employment relatively less attractive.

While stylized, our model is consistent with a range of recent microeconomic evidence on how wages are determined. Our model captures the result in [Jäger et al. \(2020\)](#) that wages are insensitive to the flow value of unemployment benefits, even for workers who were hired directly from unemployment. This feature arises for two reasons. First, in our model calibrated to U.S. data, the value of unemployment is significantly below the value of employment, so even sizable changes in the flow value of unemployment benefits do not make unemployment a credible outside option. Second, because firms post wages rather than bargain, all workers are paid the same regardless of their previous employment status. This common wage policy is further supported by the finding in [Di Addario et al. \(2023\)](#) that workers' prior employer has small effects on workers' current wages in a large majority of occupations, as well as results in [Hall and Krueger \(2012\)](#) and [Lachowska et al. \(2022\)](#) that most workers in the United States face wage posting rather than bargaining. Lastly, our model features finite elasticities of hiring and separations rates with respect to firms wage policies (i.e., when a firm raises its wages, workers join the firm more quickly and leave the firm more slowly), as has been extensively documented in the monopsony literature such as in [Bassier et al. \(2022\)](#) and [Datta \(2023\)](#).

While prior research has modeled this relationship between job-to-job mobility and wage growth,<sup>3</sup> our setting provides a tractability advantage: if firms are ex-ante identical and adjust prices and wages subject to [Rotemberg \(1982\)](#) pricing frictions, then our model features a symmetric equilibrium with a single wage alongside endogenous worker flows between firms (and unemployment). This outcome is compatible with on-the-job search due to the presence of id-

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<sup>2</sup>This is economically similar but notationally simpler than assuming there are direct utility benefits from leisure, provided that the elasticity of substitution between leisure and consumption is not one; see Appendix [A.1](#).

<sup>3</sup>See e.g., [Moscarini and Postel-Vinay \(2016\)](#) and [Birinci et al. \(2022\)](#), which assume a three-party bargaining protocol to determine wages. [Faccini and Melosi \(2023\)](#) study the relationship of job-to-job mobility and price inflation.

iosyncratic, worker-specific preference shocks over workplaces, so that workers will sometimes choose to switch jobs even when firms offer identical wages.

Other recent studies have explored supply shocks and the response of wages. We differ from [Lorenzoni and Werning \(2023a,b\)](#) where workers set wages via unions and [Gagliardone and Gertler \(2023\)](#) where workers bargain and wages are rigid in real terms. Unlike these and other papers which study oil or other shocks which affect the marginal product of labor, we study a shock which only affects workers' cost of living and focus on understanding whether the pass through of cost-of-living shocks to wages amplifies inflationary shocks in the modern U.S. economy. Given this focus, we also abstract from assuming *ad hoc* real wage rigidity, which mechanically generates pass through from cost-of-living changes to wages, noting that there is little evidence to suggest this type of indexation is widely used in the United States at present.<sup>4</sup> Similarly, while there is a long tradition of modelling nominal wage rigidity in New Keynesian models by assuming workers are unionized following the tractable approach in [Erceg et al. \(2000\)](#), we refrain from assuming that workers are unionized, noting that only 11.3% of U.S. workers were unionized as of 2022 ([Shierholz et al., 2023](#)); we also show that doing so is important for our results, as assuming unions set wages does imply pass through from prices to wages in response to a cost-of-living shock.

There is also a large macro-labor literature that embeds search frameworks and labor market frictions in DSGE models to study implications for the business cycle. [de la Barrera i Bardalet \(2023\)](#) develops a similar model of labor market monopsony with on-the-job search and finds that increased monopsony power flattens the wage Phillips curve. [Moscarini and Postel-Vinay \(2023\)](#) study a model economy where workers search on the job and bargain when they receive an outside offer, resulting in a wage Phillips curve in which the distribution of wages and misallocation of workers matters for wage growth; we assume wage-posting and study a simpler setting without misallocation, where workers and firms are homogenous in their productivity, and study the implications for pass through from cost-of-living shocks to prices. Recent work by [Pilossoph and Ryngaert \(2023\)](#) find that inflation raises the rate at which workers search for job opportunities, and [Pilossoph et al. \(2023\)](#) study the partial equilibrium effect on workers wages; while we study a general equilibrium model which abstracts from this mechanism, we present an extension in the appendix where on-the-job search intensity is increasing in the price level, suggesting the general equilibrium effects on the aggregate wage are small in practice in our setting.<sup>5</sup>

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<sup>4</sup>Evidence on the use of cost of living adjustments (COLAs) comes from studies of large union contracts, which now cover a small share of U.S. employment. Even within unionized workers, the share covered by contracts with COLAs has shrunk dramatically since the 1970s. See e.g., [Christiano et al. \(2016\)](#), footnote 4, for discussion.

<sup>5</sup>Appendix E finds that an increase in on-the-job search has only modest effects on wage growth. While a greater threat of worker separation incentivizes firms to raise wages when prices rise, there is an important offsetting general equilibrium effect: a greater number of searchers lowers labor market tightness, making it easier for firms to replace departing workers. The net result is that wages respond minimally in general equilibrium when workers search on

Our results suggest that in a setting such as the United States where few workers operate under collective bargaining agreements with cost-of-living adjustments, and where firms' wage setting decision reflects competition for already-employed rather than for unemployed workers, the ability for workers to reclaim real wages in response to a supply shock that raises their cost of living is limited. There is thus little scope for supply-shock induced wage-price spirals fueled by workers' ability to command higher nominal wages in response to higher nominal prices.

**Layout** Section 2 presents stylized facts from U.S. data, demonstrating the tight correlation between quits and wage inflation that motivates our model's assumptions of wage posting and on-the-job search. Section 3 presents our benchmark dynamic New Keynesian model with on-the-job search and wage posting firms, while Section 4 shows that our model's structural wage Phillips curve matches the empirical wage Phillips curve estimated from the data in Section 2. Section 5 demonstrates that our wage-posting model with on-the-job search implies little scope for pass through from prices to wages: specifically, Section 5.1 demonstrates this analytically, comparing our benchmark model to union wage setting models commonly used in the literature and in which changes in prices do pass through to wages. Section 5.2 demonstrates this result quantitatively in our model, and also shows that monetary policy shocks cause wages and prices to comove. Section 5.3 works through an extension of our baseline model, in which our cost-of-living shock makes unemployment more desirable and causes firms to raise wages, and shows that our assumption of on-the-job search renders this channel quantitatively small. Section 6 concludes.

## 2 Stylized Facts on Quits in the Wage Phillips Curve

Before proceeding to our formal framework, we document that wage growth, measured using the employment cost index, is strongly correlated with the quits rate, and that it is more strongly correlated with wage growth than the unemployment rate is. We will show that our simple, calibrated model of wage posting and on-the-job search captures this fact.

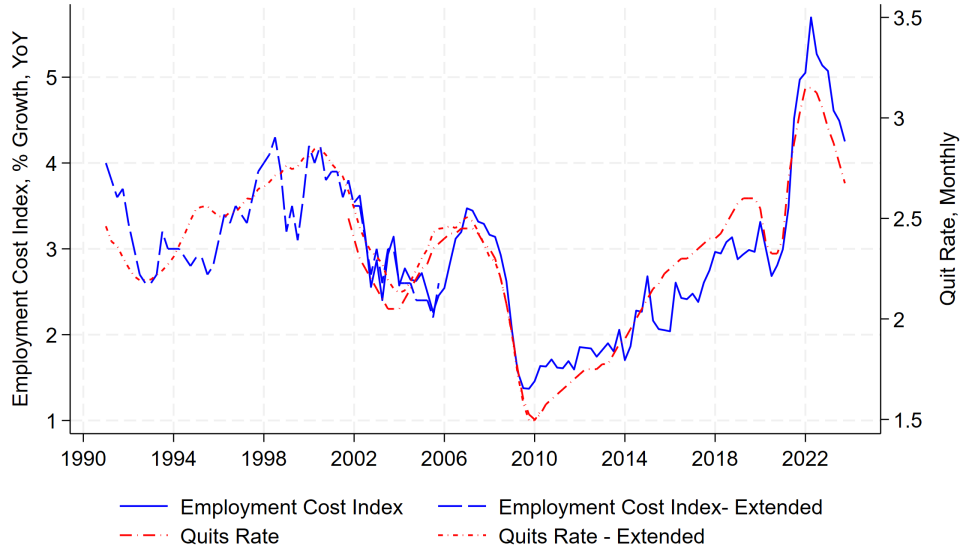
Specifically, Figure 1 plots the relationship between the four quarter moving average of the quit rate, which is measured as quits per hundred employees from the Job Openings and Labor Turnover Survey (JOLTS), and the four quarter growth in the employment cost index. This figure shows that the time series result documented by, e.g., Faberman and Justiniano (2015) and Moscarini and Postel-Vinay (2017), that nominal wage growth is well-predicted by job-to-job transitions, extends to the recent period of COVID shock and recovery. Note that while the figure plots the behavior of quits, most quits are job-to-job transitions, which is why we discuss Figure

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the job more frequently in response to higher cost of living.

1 as documenting the tight correlation between job-to-job transitions and wage inflation.<sup>6</sup> Given that quits is mostly a measure of job-to-job transitions, capturing this fact requires a model where workers quit to change jobs, rather than just quit into unemployment, motivating the inclusion of job-to-job search in our model of wage growth over the business cycle; we will show later that including a realistic quantity of on-the-job search (i.e., by calibrating our model to U.S. data) has important implications for the pass through of cost-of-living shocks to nominal wages.

Figure 1: Wage Growth and Quits



*Notes:* There is a strong correlation between quits, measured here as the four quarter moving average of quits per hundred employees from the Job Openings and Labor Turnover Survey (JOLTS), and year-over-year wage growth. While quits includes multiple kinds of labor market transitions, in practice variation in quits is largely driven by job-to-job transitions.

To estimate the empirical Phillips curve more formally, we combine versions of the employment cost wage data and quits data over time. For the wage data, from 2010Q3-2023, we use the private sector quit rate JTS1000QUR from the St. Louis Federal Reserve FRED database, aggregated by averaging at the quarterly level. Prior to 2000, we use the quarterly private sector quits rate from [Faberman and Justiniano \(2015\)](#). Between 2001q1 and 2010q2, we use the average of these two series. Similar for the employment cost index, we use the employment cost index wages and salaries series for private industry workers ECIWAG from FRED beginning in 2005. Prior to 2001, we use the SIC industry basis of the employment cost index for private industry

<sup>6</sup>The empirical measure of quits include various labor market transitions: job-to-job transitions without a period of non-employment, job-to-job transitions with a period of non-employment, and voluntary quits into non-employment. [Qiu \(2022\)](#) shows finds that only 3% of workers transition from employment to non-participation each month (most of which appear voluntary) and [Elsby et al. \(2010\)](#) find that only 16% of workers who quit enter a period of unemployment.

wages and salaries, series ECS20002I, from the Bureau of Labor Statistics. From 2001-2005, we take the average of these two wage series.

Table 1: Time Series Regression of Wage Growth on Labor Market Variables, 1990Q4-2023

VARIABLES	(1) ECI	(2) ECI	(3) ECI	(4) ECI
Unemployment Rate	-0.0220*** (0.0035)	0.0010 (0.0046)	0.0068 (0.0048)	
Quit Rate		0.0465*** (0.0080)	0.0475*** (0.0081)	0.0466*** (0.0096)
$\log(U) - \log(U^*)$				0.0010 (0.0052)
Observations	135	135	119	135
Include 2020-2023	Y	Y	N	Y

Standard errors in parentheses

\*\*\*  $p < 0.01$ , \*\*  $p < 0.05$ , \*  $p < 0.1$

*Notes:* Results from a regression of wage growth measured using the Employment Cost Index (ECI) on unemployment and the quit rate, with quarterly data, as specified in equation (1). While Column 1 shows that a regression of wage growth on unemployment alone yields a familiar negative sign, including quits flips the sign and reduces the significance to below conventional levels as seen in Column 2. Columns 3 and 4 demonstrates that this result is robust to dropping the COVID pandemic and recovery, and also to measuring unemployment in log deviations from its natural rate as estimated by the CBO,  $\log(U) - \log(U^*)$ .

While unemployment is also (negatively) correlated with wage growth, in practice quits are so strongly correlated with wage growth that it dwarfs the value of unemployment as a predictive measure. Formally, in quarter  $t$ , letting  $W_t$  be the nominal wage,  $Q_t$  be quits, and  $U_t$  be the unemployment rate, we estimate the following regression:

$$4 \times (\log W_t - \log W_{t-1}) = \hat{\beta}_0 + \hat{\beta}_Q \log Q_t + \hat{\beta}_U \log U_t + \varepsilon_t. \quad (1)$$

Table 1 reports the results: the empirical estimate  $\hat{\beta}_Q$  is much larger than  $\hat{\beta}_U$ .<sup>7</sup> Indeed,  $\hat{\beta}_U$  is not generally significant at conventional levels and is not of the expected sign once we include quits. These results are robust to the inclusion of the COVID pandemic and recent recovery.

<sup>7</sup>Column 1 should be interpreted as “for a 100% increase in the unemployment rate (i.e., double the unemployment), there is an annualized 2.2% point decrease in gross wage growth”. Likewise, Column 2 implies that 100% increase in the quits rate (i.e., double the quits) would result in 4.65% point increase in wage growth.



In the following section, we will develop a model that is capable of matching the empirical correlations in equation (1): specifically, we will write down a structural wage Phillips curve of the same form of (1), where we will have both  $\hat{\beta}_Q > \hat{\beta}_U$  and  $\hat{\beta}_U$  small but positive when using both quits and unemployment as our measures of labor market tightness in the wage Phillips curve.

### 3 Model

This section builds a model where firms post wages and workers search on the job, and calibrates that model to U.S. Data. We will then go on to use the model to provide a structural foundation for the empirical OLS regression (1), finding that our calibration implies structural coefficients for  $\beta_Q$  and  $\beta_U$  that are consistent with the empirical coefficients  $\hat{\beta}_Q$  and  $\hat{\beta}_U$ . Finally, we will then show that this model implies that there is little scope for pass through from prices to wages in response to a cost-of-living shock.

In laying out the model, we first describe the problem of a firm posting wages in the presence of recruiting costs and on-the-job search. When deciding whether to raise wages, the firm trades off between a higher wage bill and lower turnover costs. Lower turnover costs come from the fact that a higher wage increases the probability that the firm recruits a particular searcher, regardless of whether that searcher is already employed or unemployed (the recruiting rate), while also lowering the probability that incumbent workers leave (the separation rate). Because the firm's problem does not depend directly on the price level in partial equilibrium, increases in workers' cost of living can only affect wages through their effects on these recruiting or separation rates.

We then describe the solution to the worker's problem, which determines firms' recruiting and separation rates. Since a change in the price level affects the real wages offered by all firms proportionally, changes in the price level can relatively improve workers' outside option, and raise wages, only if changes in the price level make unemployment relatively more attractive. If this is the case, this can lead to pass through from cost-of-living to wages, as firms must now offer a higher wage to retain the same number of workers as before. However, these considerations are quantitatively small when (i) most workers already vastly prefer a job to unemployment and/or when (ii) most searching workers already have a job, rendering the value of unemployment irrelevant when considering whether to accept a new job offer. Moreover, this channel need not exist at all if changes in the price level do not affect the desirability of unemployment, as in our benchmark model described below.



**Structure** There are two goods in the economy: an endowment good  $X_t$  and services  $Y_t$ . They are combined into an aggregate consumption good,  $C_t$ , according to the CES function

$$C_t = \left( \alpha_Y^\eta Y_t^{\frac{\eta-1}{\eta}} + \alpha_X^\eta X_t^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}, \quad (2)$$

with corresponding aggregate price index<sup>8</sup>

$$P_t = \left( \alpha_Y P_{y,t}^{1-\eta} + \alpha_X P_{x,t}^{1-\eta} \right)^{\frac{1}{1-\eta}}. \quad (3)$$

Workers are hired by firms to produce services  $Y_t$ , so that their real wage is determined by the nominal wage offered in that sector divided by the aggregate price level  $P_t$ . The total amount of endowment good  $X_t = 1$  is given.

**Cost-of-Living Shock** Our “pure” cost of living shock is a decline in the endowment good  $X_t$  which raises its price,  $P_{x,t}$ , and hence the price level  $P_t$  in (3). This is a pure cost of living shock in the sense that it raises the cost of living for workers without affecting their marginal products, unlike an oil shock, for example, which affects both. The point of considering such a shock is not to downplay the role or importance of oil shocks to many modern economies, but to highlight how these shocks propagate and question whether a “wage price spiral” amplifies their effects on the price level.

**Firm’s Wage-Posting Problem** We now turn to the determination of the nominal wage. We assume that perfectly-competitive retailers bundle service types  $j$  according to a standard Dixit-Stiglitz production function with an associated ideal price index:

$$\begin{aligned} Y_t &= \left( \int \left( Y_t^j \right)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}}, \\ P_{y,t} &= \left( \int \left( P_{y,t}^j \right)^{1-\epsilon} dj \right)^{\frac{1}{1-\epsilon}}, \end{aligned}$$

yielding product demand for variety  $j$ :

$$\frac{Y_t^j}{Y_t} = \left( \frac{P_{y,t}^j}{P_{y,t}} \right)^{-\epsilon}. \quad (4)$$

The firm  $j$  produces only with labor according to  $Y_t^j = N_{jt}$ . Firm  $j$  sets nominal wages  $W_{jt}$  each period, which is assumed to be the same for all workers in the firm, including new hires. Workers

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<sup>8</sup>We assume  $\alpha_X + \alpha_Y = 1$  with  $\alpha_X > 0$  and  $\alpha_Y > 0$  as usual.

separate from firm  $j$  with probability  $S(W_{jt}|\{W_{kt}\}_{k \neq j})$  each period, with  $S'(W_{jt}|\{W_{kt}\}_{k \neq j}) < 0$ : firms retain a higher share of workers each period by paying a higher wage, given other firms' wages. The firm can recruit workers by posting vacancies  $V_{jt}$ , and the probability that a vacancy successfully results in a hire is  $R(W_{jt}|\{W_{kt}\}_{k \neq j})$ , with  $R'(W_{jt}|\{W_{kt}\}_{k \neq j}) > 0$ .<sup>9</sup> The firm pays a convex, per-vacancy hiring cost,  $c \left( \frac{V_{jt}}{N_{j,t-1}} \right)^\chi W_t$ , to post  $V_t$  vacancies, where  $W_t$  is the aggregate wage,  $c > 0$  and  $\chi \geq 0$ . Finally, the firm is also subject to price and wage adjustment frictions à la **Rotemberg (1982)**.

Given this, each firm  $j$  maximizes the present discounted value of profits, solving

$$\begin{aligned} \max_{\{P_{y,t}^j\}, \{Y_t^j\}, \{N_{jt}\}, \{W_{jt}\}, \{V_t^j\}} \quad & \sum_{t=0}^{\infty} \left( \frac{1}{1+\rho} \right)^t \left( P_{y,t}^j Y_t^j - W_{jt} N_{jt} - c \left( \frac{V_{jt}}{N_{j,t-1}} \right)^\chi V_{jt} W_t - \frac{\psi}{2} \left( \frac{P_{y,t}^j}{P_{y,t-1}^j} - 1 \right)^2 Y_t^j P_{y,t}^j \right. \\ & \left. - \frac{\psi^w}{2} \left( \frac{W_{jt}}{W_{j,t-1}} - 1 \right)^2 W_{jt} N_{jt} \right) \end{aligned} \quad (5)$$

subject to the law of motion for employment

$$N_{jt} = (1 - S(W_{jt}))N_{j,t-1} + V_{jt}R(W_{jt}) \quad (6)$$

and the product demand equation (4). From inspecting equations (5) and (6), we can observe that the service sector firm chooses the wage (and other choice variables) taking as given the choices of other service sector firms (embodied in the price index and aggregate output of the service sector), parameters, and the separation and recruiting rates  $S(\cdot)$  and  $R(\cdot)$ . Note that since the vacancy-posting cost is denominated in labor (i.e., priced by the aggregate wage  $W_t$ ), the aggregate price level  $P_t$  does not appear directly in (5). Thus, in partial equilibrium, the only way that changes in the price level can impact the firm's wage setting decision is through changes in  $S(\cdot)$  and  $R(\cdot)$ , which will be determined by the solution to workers' optimization problem described in Section 3.2.

### 3.1 The Symmetric Equilibrium Features a Nonlinear Wage Phillips Curve

To make this relationship between the separation and recruiting rates and the firm's choice of wage clearer, we derive a wage Phillips curve from the firm's first order conditions, assuming for the moment that a symmetric equilibrium, where all firms offer the same aggregate wage  $W_t$ , exists. Under this assumption, the wage Phillips curve expresses nominal wage growth as exclusively a function of aggregate, endogenous labor market variables: vacancies, employment,

<sup>9</sup>How retention and separation functions  $R(W_{jt}|\{W_{kt}\}_{k \neq j})$  and  $S(W_{jt}|\{W_{kt}\}_{k \neq j})$  depend on wages set by other service firms will be derived after we describe households' and workers' problems in Section 3.2. We write  $R(\cdot)$  and  $S(\cdot)$  solely as functions of  $W_{jt}$  set by firm  $j$  solely for readability.

recruiting and separation rates, and recruiting and separation elasticities, again with no direct role for aggregate price index  $P_t$ .

Denote  $\varepsilon_{R,W}$  and  $\varepsilon_{S,W}$  as the elasticities of the recruiting function  $R(W_{jt})$  and the separation function  $S(W_{jt})$  with respect to the wage  $W_{jt}$ . Then in any symmetric equilibrium where  $W_{jt} = W_t$ ,  $N_{jt} = N_t$ ,  $V_{jt} = V_t$ ,  $P_t^j = P_t$ , and  $Y_t^j = Y_t$ , the wage Phillips curve characterizing nominal wage growth curve is given by:

$$\begin{aligned} \psi^w (\Pi_t^w - 1) \Pi_t^w + 1 = & c(1 + \chi) \left( \frac{V_t}{N_{t-1}} \right)^\chi \left[ \frac{V_t}{N_t} \varepsilon_{R,W_t} + (-\varepsilon_{S,W_t}) \frac{N_{t-1}}{N_t} \frac{S(W_t)}{R(W_t)} \right] \\ & + \frac{1}{1 + \rho} \psi^w (\Pi_{t+1}^w - 1) (\Pi_{t+1}^w)^2 \frac{N_{t+1}}{N_t}. \end{aligned} \quad (7)$$

Thus, in any symmetric equilibrium where firms solve an optimization problem of the form of (5), the wage Phillips curve will be a function of the current and expected future paths of the job vacancy rate  $V_t$ , employment  $N_t$ , the recruiting and separation rates  $R(W_t)$  and  $S(W_t)$ , and their elasticities, denoted  $\varepsilon_{R,W_t} > 0$  and  $\varepsilon_{S,W_t} < 0$  following conventions in the monopsony literature; see e.g. Bloesch and Larsen (2023).<sup>10</sup>

**Interpretation** Taking each term step by step, this wage Phillips curve captures how competition for workers affects firms' optimal wage growth. The first term  $(V_t/N_{t-1})^\chi$  captures the convex cost of posting vacancies: since firms must post vacancies to attract workers, higher marginal vacancy posting costs raises the value of both recruiting a worker the firm has matched with as well as retaining existing workers. If getting a worker in the door is more valuable, then firms will want to pay higher wages. The next term, within brackets, includes the recruiting elasticity term  $\varepsilon_{R,W_t}$ , which captures how sensitive the probability of hiring a matched worker is to the wage. If this recruiting elasticity is elevated, the workers' acceptance probability will be more sensitive to the wage, increasing the incentive at the margin for a firm to raise its wage. This  $\varepsilon_{R,W_t}$  is multiplied by the number of vacancies  $V_t$ . Next is the separation elasticity term  $\varepsilon_{S,W_t}$ . This elasticity is negative, so the negative of the separation elasticity is positive. A more steeply negative separation elasticity means that workers' likelihood of quitting is more sensitive to wages, so the more negative this value is, the greater the incentive to raise wages at the margin. Lastly, we have the separation rate  $S(W_t)$  and recruiting rate  $R(W_t)$ . A higher separation rate indicates that workers have more opportunities to quit, increasing pressure for firms to raise wages. Analogously, when the recruiting rate  $R(W_t)$  is higher, workers are easier to hire, lowering the pressure for firms to raise wages.

The next section describes the household and workers' optimization problems, which de-

<sup>10</sup>Appendix A.2 derives the firm's first-order conditions in (5), including the price Phillips curve and wage Phillips curve (7).

termine the recruiting and separation functions faced by firms. Having done so, we can then log-linearize and simplify the above (7) to evaluate the model's ability to match the empirical wage Phillips curve discussed in Section 2. We do this in Section 4, before turning to the implications of our benchmark model for pass-through in Section 5.

### 3.2 Households and Workers

This section derives the household and worker block of the model. We deviate from the standard assumption in the New Keynesian literature of perfect consumption insurance within the household by assuming that households only imperfectly insure the consumption of workers who are unemployed, consistent with evidence that unemployed workers consume less than employed workers (see e.g., [Chodorow-Reich and Karabarbounis \(2016\)](#)). We assume that workers themselves choose whether to take a particular job offer, and make employment decisions based on relative wages and consumption levels, in addition to idiosyncratic firm-specific preference shocks. Workers' mobility decisions aggregate up into the firms' recruiting and separation functions. Households smooth aggregate consumption within the household over time, yielding a standard Euler equation, making the labor block easy to integrate into a standard New Keynesian setting.

**Frictional Markets** Workers and firms match according to random search in a frictional market. As mentioned above, each firm  $j$  posts  $V_{jt}$  vacancies, and aggregate vacancies are  $V_t = \int V_{jt} dj$ . Each period, employed workers can search on the job with some constant, exogenous probability  $\lambda_{EE} \in (0, 1)$ , and unemployed workers can always search.<sup>11</sup> The unemployment rate is defined as  $U_t$ , so the total mass of searchers  $S_t$  is  $S_t = \lambda_{EE}(1 - U_t) + U_t$ . Matching is random and follows a constant returns to scale matching function  $M_t(V_t, S_t)$ ,

$$M_t(V_t, S_t) = \frac{S_t V_t}{(S_t^\nu + V_t^\nu)^{\frac{1}{\nu}}},$$

with  $\nu = 2$  following the literature. Labor market tightness is  $\theta_t = \frac{V_t}{S_t}$ . The job finding rate for workers is  $f(\theta_t) = \frac{M_t}{S_t}$  is increasing in tightness  $\theta_t$ , and the probability that a vacancy is matched with a worker  $g(\theta_t) = \frac{M_t}{V_t}$  is a decreasing function of tightness  $\theta$ . The share of searchers who are employed is  $\phi_{E,t} = \frac{\lambda_{EE}(1-U_t)}{S_t}$ , and the share of searchers who are unemployed is  $\phi_{U,t} = 1 - \phi_{E,t} =$

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<sup>11</sup>This simplifying assumption mechanically shuts down the possibility that on-the-job search intensity increases with the price level ([Pilossoph and Ryngaert, 2023](#)); Appendix E relaxes this assumption and shows that the scope for pass-through from cost-of-living shocks to wages remains small in our model even if workers search harder as prices rise.

$\frac{U_t}{S_t}$ . The job finding rate for workers  $f(\theta_t)$  and vacancy-filling rate  $g(\theta_t)$  are given by

$$f(\theta_t) = \frac{\theta_t}{(1 + \theta_t^\nu)^{\frac{1}{\nu}}}, \quad g(\theta_t) = \frac{1}{(1 + \theta_t^\nu)^{\frac{1}{\nu}}}.$$

**Households** A representative household has a unit mass  $i \in [0, 1]$  of members who can work. Households seek to maximize the discounted present value of their members' utility, which is log in consumption. Without loss of generality, assume that unemployed household members must each have the same consumption level,  $C_t^u$ .<sup>12</sup> Then letting  $C_t(i, j(i))$  denote the consumption of worker  $i$  in state  $j(i)$ , where  $j(i)$  indicates the firm  $i$  is employed at, the household's objective function becomes

$$\sum_{t=0}^{\infty} \left( \frac{1}{1 + \rho} \right)^t \left[ U_t \ln(C_t^u) + \int_0^{1-U_t} \ln(C_t(i, j(i))) di \right].$$

The household is allowed to choose  $C_t^u$  (effectively, an unemployment benefit) and also a linear tax/subsidy on employed workers, who consume their income each period:

$$C_t(i, j) = \tau_t \frac{W_{jt}}{P_t}$$

subject to the following budget constraint: letting  $D_t$  be nominal dividend payments from services firms (who profit from monopoly and monopsony power) and perfectly competitive goods firms (who receive the endowment  $X_t$  and sell it, rebating the proceeds to households),  $B_t$  be nominal bond holdings in zero net supply paying nominal interest rate  $i_t$ , and finally letting  $\bar{W}_t \equiv \frac{1}{1-U_t} \int_0^{1-U_t} W_{j(i)t} di$  be the average wage of employed workers, the budget constraint is

$$U_t C_t^u = \frac{D_t}{P_t} - \frac{B_t}{P_t} + \frac{(1 + i_{t-1})B_{t-1}}{P_t} + (1 - \tau_t)(1 - U_t) \frac{\bar{W}_t}{P_t}. \quad (8)$$

To make further progress in delivering a tractable model with households' standard consumption Euler equation, we impose an *ad hoc* consumption sharing rule within the household requiring that unemployed workers' consumption must be a *constant* fraction of employed workers' average consumption:

$$\frac{\bar{C}_t^e}{C_t^u} = \xi, \quad (9)$$

where  $\xi \geq 1$  and  $\bar{C}_t^e \equiv \frac{1}{1-U_t} \int_0^{1-U_t} C(i, j(i)) di$  is the average consumption of employed. This rule allows us to capture the fact that the ratio of unemployed and employed consumption is relatively constant over the business cycle (Chodorow-Reich and Karabarbounis, 2016). Moreover, it

<sup>12</sup>This is not restrictive, as given our other assumptions the household will always choose to equalize consumption across unemployed agents due to diminishing marginal utility of consumption.

can be thought of as the result of a household facing an incentive-insurance trade-off: by insuring unemployed workers less and making unemployment relatively worse (lower  $\xi$ ), the household encourages workers to take jobs and become employed by taking consumption away from unemployed workers with higher marginal utility of consumption. Note that in Section 5.3, we will study extension of this model in which the household implements a different unemployment insurance scheme in which the consumption ratio is not held constant.

### 3.2.1 Symmetric Equilibrium Features a Standard Euler Equation

In a symmetric equilibrium where all firms set the same wage, and so  $W_{jt} = W_t$ , the household's problem under constraints (8) and (9) simplifies to choosing aggregate consumption  $C_t$  and bond holdings  $B_t$  to maximize

$$\sum_{t=0}^{\infty} \left( \frac{1}{1+\rho} \right)^t \ln \left( \frac{C_t}{(1-U_t)\xi + U_t} \right)$$

subject to the simplified budget constraint

$$C_t = \frac{D_t}{P_t} - \frac{B_t}{P_t} + \frac{(1+i_{t-1,t})B_{t-1}}{P_t} + (1-U_t)W_t.$$

Optimization then yields the standard consumption Euler equation with log-utility given by:

$$C_t^{-1} = \frac{1}{1+\rho} \frac{1+i_{t,t+1}}{\Pi_{t+1}} C_{t+1}^{-1}. \quad (10)$$

**Workers** Workers get utility from consumption and an idiosyncratic preference draw  $\iota$ .  $\iota$  represents how much workers like their current job at firm  $j$ , which is redrawn every period and is i.i.d. Workers draw a similar preference shock each period during unemployment (note that the household does not take the idiosyncratic preference shocks into account when solving the problem described above). Workers are myopic and consider their utility only one period at a time, which for worker  $i$  in state  $j$  is given by<sup>13</sup>

$$\mathcal{V}_t(i, j) = \ln(C_t(i, j)) + \iota_{ijt}.$$

Workers are allowed to search on the job with probability  $\lambda^{EE}$ , and conditional on searching, are matched with a vacancy with probability  $f(\theta)$ . Workers are allowed to consider unemployment with probability  $\lambda^{EU}$ . Consider a worker  $i$  currently employed at firm  $j$  who successfully matches

<sup>13</sup>The absence of utility from leisure here, which may be greater in unemployment, can be viewed as a simplifying assumption: we can introduce leisure without changing the results provided that the elasticity of substitution between leisure and consumption is one. See Section 5.3.2 and accompanying Appendix A.1 for further discussion on how assuming a different elasticity affects the results.

with firm  $k$ 's vacancy. She will move to firm  $k$  only if  $\mathcal{V}_t(i, k) \geq \mathcal{V}_t(i, j)$ . Let us define  $s_{jk}(W_{jt}, W_{kt})$  as the probability that the worker is poached from firm  $j$  to firm  $k$ .

We assume that  $\iota$  follows a Type-1 extreme value distribution with variance  $\gamma^{-1}$  for tractability. Following the consumption sharing rule in (8) and (9),  $s_{jk}(W_{jt}, W_{kt})$  is given by

$$s_{jk}(W_{jt}, W_{kt}) = \frac{\left(\tau_t \frac{W_{kt}}{P_t}\right)^\gamma}{\left(\tau_t \frac{W_{kt}}{P_t}\right)^\gamma + \left(\tau_t \frac{W_{jt}}{P_t}\right)^\gamma} = \frac{W_{kt}^\gamma}{W_{kt}^\gamma + W_{jt}^\gamma}, \quad (11)$$

which is decreasing in  $W_{jt}$ : if firm  $j$  pays a higher wage, workers are less likely to be poached. Notice also that the probability a worker switches jobs is only a function of the relative *nominal* wage. The worker takes as given the internal tax rate set by the household  $\tau_t$  and the price level  $P_t$ , both of which are unchanged regardless of which job the worker chooses.

Now consider a worker who is deciding whether to quit into unemployment. Let the average wage of employed workers in worker  $i$ 's household be  $\bar{W}_t$ , which determines consumption in unemployment through  $C_t^u = \frac{\bar{C}_t^e}{\xi} = \tau_t \frac{\bar{W}_t}{\xi P_t}$ . Since a worker  $i$  who is currently employed at firm  $j$  quits into unemployment only if  $\mathcal{V}(i, j) \geq \mathcal{V}(i, \text{unemployed})$ , thus the probability that a worker voluntarily quits into unemployment  $s_{ju}(W_{jt})$  is given by

$$s_{ju}(W_{jt}) = \frac{\left(\frac{1}{\xi} \tau \frac{\bar{W}_t}{P_t}\right)^\gamma}{\left(\frac{1}{\xi} \tau \frac{\bar{W}_t}{P_t}\right)^\gamma + \left(\tau \frac{W_{jt}}{P_t}\right)^\gamma} = \frac{\left(\frac{\bar{W}_t}{\xi}\right)^\gamma}{\left(\frac{\bar{W}_t}{\xi}\right)^\gamma + W_{jt}^\gamma}, \quad (12)$$

which is decreasing in  $W_{jt}$  but does not depend on the price level  $P_t$ .

These individual transition probabilities aggregate up into the firm's separation rate  $S(W_{jt})$ : each period, a share of workers  $s \in (0, 1)$  exogenously separate while the remainder  $(1 - s)$  endogenously separate if they receive an opportunity that they prefer to their current job (either another job, or the chance to exit to unemployment). Recalling that  $f(\theta_t)\lambda_{EE}$  denotes the probability that a particular employed worker is allowed to search on the job and matches to another firm, and that  $\lambda_{EU}$  denote the probabilities that an employed worker is allowed to consider quitting into unemployment, the separation rate is written as

$$S(W_{jt}) \equiv S(W_{jt} | \{W_{kt}\}_{k \neq j}) = s + (1 - s) \left[ \lambda_{EE} f(\theta_t) \int s_{jk}(W_{jt}, W_{kt}) z(W_{kt}) dk + \lambda_{EU} s_{ju}(W_{jt}) \right], \quad (13)$$

where  $z(W_{kt})$  is an endogenous density function of outside posted wages. Note that  $S(\cdot)$  is a decreasing function of  $W_{jt}$ , i.e.  $S'(W_{jt}) < 0$ , since all of its components are decreasing in  $W_{jt}$ ; in other words, the firm's separation rate falls as the wage rises.

Analogously to the individual separation probabilities, there are probabilities that a matched worker is recruited into the firm conditional on whether the worker is employed or unemployed.



Consider a worker employed at firm  $k$  that encounters firm  $j$ 's vacancy. The probability that firm  $j$  successfully poaches the worker  $r(W_{jt}, W_{kt})$  is:

$$r_{kj}(W_{kt}, W_{jt}) = \frac{\left(\tau_t \frac{W_{jt}}{P_t}\right)^\gamma}{\left(\tau_t \frac{W_{kt}}{P_t}\right)^\gamma + \left(\tau_t \frac{W_{jt}}{P_t}\right)^\gamma} = \frac{W_{jt}^\gamma}{W_{kt}^\gamma + W_{jt}^\gamma}, \quad (14)$$

which is increasing in  $W_{jt}$  and is a function of relative wages.

Now consider an unemployed worker who is matched with firm  $j$ 's vacancy. The probability that the worker takes the job with firm  $j$  is defined as  $r_{uj}(W_{jt})$  and is equal to

$$r_{uj}(W_{jt}) = \frac{\left(\tau_t \frac{W_{jt}}{P_t}\right)^\gamma}{\left(\frac{1}{\xi} \tau_t \frac{W_t}{P_t}\right)^\gamma + \left(\tau_t \frac{W_{jt}}{P_t}\right)^\gamma} = \frac{W_{jt}^\gamma}{\left(\frac{W_t}{\xi}\right)^\gamma + W_{jt}^\gamma}, \quad (15)$$

which is increasing in  $W_{jt}$ .

We can use (14) and (15) to write firm  $j$ 's recruiting rate, defined as the share of vacancies that successfully result in hiring a worker is the following. Recalling that  $g(\theta_t)$  denotes the probability that a vacancy is matched with a worker, and that  $\phi_{E,t}$  and  $\phi_{U,t}$  denote the share of searchers who are employed and unemployed, respectively, we can write the recruiting rate as:

$$R(W_{jt}) \equiv R(W_{jt} | \{W_{kt}\}_{k \neq j}) = g(\theta_t) \left[ \phi_{E,t} \int_k r_{kj}(W_{kt}, W_{jt}) \omega(W_{kt}) dk + \phi_{U,t} r_{uj}(W_{jt}) \right]. \quad (16)$$

where  $\omega(W_{kt})$  is the distribution of wages that workers are currently employed at. The recruiting rate  $R(W_{jt})$  is an increasing function because all of its components  $r_{kj}$  and  $r_{uj}$  are also increasing in  $W_{jt}$ . In other words, a higher wage improves the firms odds of recruiting workers through its vacancies.

### 3.2.2 Symmetric Equilibrium Features Simple Separation and Recruiting Functions

In a symmetric equilibrium where all the firms set the same wage, i.e.,  $W_{jt} = W_t$  for  $\forall j$ , both  $S(\cdot)$  and  $R(\cdot)$  becomes functions of tightness  $\theta_t$  and simplify from (13) and (16) to

$$S_t = s + (1-s) \left( \lambda_{EE} f(\theta_t) \frac{1}{2} + \lambda_{EU} \left( \frac{1}{1 + \xi^\gamma} \right) \right) \quad (17)$$

$$R_t = g(\theta_t) \left( \phi_{E,t} \frac{1}{2} + \phi_{U,t} \left( \frac{\xi^\gamma}{1 + \xi^\gamma} \right) \right). \quad (18)$$

where the aggregate wage  $W_t$  does not appear on the right hand side of (17) and (18). Therefore, in a symmetric equilibrium, both separation and recruiting rates  $S_t$  and  $R_t$  become independent of aggregate wage  $W_t$ . This is because all the competing firms set the same wage level, and

the relative desirability of employment over unemployment is independent of the wage due to household's consumption sharing rule (9).

The reason for the absence of the price level  $P_t$  in the separation rate formula (17) is similar: fundamentally the price level is irrelevant to a worker considering choosing between two different nominal wage offers. Also driving this result is the fact that we have assumed the price level is irrelevant for workers considering choosing between working and *unemployment*. This is because the households' consumption sharing rule (9) fixes the relative consumption of employed and unemployed workers at  $\xi$ , which naturally appears in equations (17) and (18) above: the higher real consumption ratio  $\xi$  is on average, the more likely unemployed workers are to prefer the state of employment to that of unemployment, so  $S_t$  decreases with  $\xi$  and  $R_t$  increases with  $\xi$  all else equal. Note that relaxing our assumption that consumption ratio  $\xi$  is constant will not change the result about pass through: the issue is that the price level,  $P_t$ , does not affect both  $S_t$  and  $R_t$  in equilibrium. Fixing unemployment benefits at some nominal level, for example, would still result in the relative attractiveness of employment and unemployment being insensitive to the price level by the same logic that applies to employed workers choosing between nominal wages at two different jobs.

As we will show in Section 5.1, there is no pass-through at all in our benchmark case where the relative consumption of employed and unemployed workers is fixed at  $\xi$ . However, the assumption that the desirability unemployment (formally, the probability of preferring a job offer at aggregate wage  $W_t$  to the unemployment state) is constant and completely independent of the aggregate price level is strong; this would not be true if, for example, we had assumed that the representative household insures unemployed households by guaranteeing them some constant, real unemployment benefit  $b$  (i.e., if unemployment benefits are perfectly indexed to inflation), or if we had assumed that workers derive some utility from leisure, as well as consumption, and that leisure utility is systematically higher while unemployed. We discuss the former case in Section 5.3; Appendix A.1 discusses the worker's problem with leisure, which has similar implications but requires more burdensome notation.<sup>14</sup>

### 3.3 Equilibrium Selection

We can close the model with a simple Taylor rule, with a potentially persistent policy shock  $\varepsilon_{i,t}$ :

$$1 + i_t = \Pi_{Y,t}^{\phi_\pi} (1 + \varepsilon_{i,t}) \quad (19)$$

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<sup>14</sup>This is because we must take a stance on the elasticity of substitution between leisure and consumption; Appendix A.1 demonstrates that if leisure and consumption have an elasticity of substitution of one, then changes in the price level have no effect on the relative desirability of employment for a given nominal wage, as in the benchmark case with fixed consumption ratios.

and solve for a symmetric equilibrium. Our symmetric equilibrium consists of sequences of all endogenous prices and quantities satisfying that: (i) firms choose identical sequences such that  $W_{jt} = W_t$ ,  $N_{jt} = N_t$ ,  $V_{jt} = V_t$ ,  $P_{jt}^Y = P_t^Y$ , (ii) workers and households maximize utility, (iii) firms maximize profits, (iv) product markets clear, and (v) labor market flows add up.

We linearize these necessary conditions in a symmetric equilibrium around a non-stochastic steady state, and solve for the unique solution. While there is a unique, symmetric equilibrium (for our given parameter values) we cannot rule out and leave unexplored the possibility of non-symmetric equilibria where *ex ante* identical firms choose different wages. The fact that we have one wage in equilibrium, while still having worker flows between unemployment and various firms due to idiosyncratic shocks, buys us a highly-tractable dynamic model with on-the-job search.

### 3.4 Calibration

We choose standard values for most parameters, and choose other parameters governing the labor search block of the model to match the U.S. data: Table 2 lists the model’s calibrated parameters, some of which are chosen to target moments in U.S. data given in Table 3.

Specifically, we calibrate the model to match U.S. data on labor market flows during the period 2015-2019 to capture the approximately full-employment conditions that existed prior to the COVID shock. Data on the unemployment rate and separation rate come from the Bureau of Labor Statistics (BLS) and the Job Openings and Labor Turnover Survey (JOLTS). We set  $\xi = 2$ , which is higher than in Chodorow-Reich and Karabarbounis (2016) but closer to what maximizes steady-state utility for the household in our setting; the results are largely insensitive to changing this parameter.

## 4 Quits in the Wage Phillips Curve

To write down a wage Phillips curve of the same form as equation (1), note that we can define quits  $Q_t$  as all separations  $S_t$  less the exogenous separations  $s$ :

$$Q_t = S_t - s,$$

so quits in the model captures both voluntary job-to-job quits and voluntary quits from employment into unemployment.

**Linearized wage Phillips curve** Based on our characterization of the wage Phillips curve (7) of Section 3.1, this section shows how our wage-posting model based on a quantitatively-realistic

Table 2: Parameters in the Monthly Benchmark New Keynesian Model

Parameter	Value	Meaning	Reason
$\lambda_{EE}$	.14	OTJ search probability	Match EE rates
$\lambda_{EU}$	.30	Opportunity to quit probability	Match voluntary EU rate, <a href="#">Qiu (2022)</a>
$\xi$	2	Consumption ratio: $C_t^e/C_t^u$	See Notes below
$s$	.01	Exogenous separation rate	Match JOLTS monthly separation Rate
$\gamma$	6	Variance <sup>-1</sup> of idiosyncratic preferences	Match $\varepsilon_{R,W} - \varepsilon_{S,W}$
$\epsilon$	10	Elasticity of substitution of services	
$\psi$	100	Services price adjustment cost	
$\psi^w$	100	Wage adjustment cost	
$\eta$	1	Services/endowment good EOS	
$\alpha_X$	.2	Endowment good's share in CES Utility	
$\chi$	1	Convexity in vacancy posting costs	<a href="#">Bloesch and Larsen (2023)</a>
$c$	30	Hiring cost shifter	Targeting $U$
$\rho$	.004	Discount Rate	Monthly model

Table 3: Selected Model Moments and Data in Steady State

Targeted Moment	Meaning	Model	Data	Source
$U$	Unemployment rate	.044	.044	BLS
$S$	Monthly separation rate	.036	.036	JOLTS
$\varepsilon_{R,W} - \varepsilon_{S,W}$	Recruiting minus separation elasticities	4.4	4.2	<a href="#">Bassier et al. (2022)</a>

*Notes:* We calibrate the model to match labor market flows of the U.s. economy during 2015-2019 to capture the approximately full employment conditions that existed prior to the COVID shock. Data on the unemployment rate and separation rate come from the Bureau of Labor Statistics (BLS) and the Job Openings and Labor Turnover Survey (JOLTS). We set  $\xi = 2$ , higher than in [Chodorow-Reich and Karabarbounis \(2016\)](#) but closer to what maximizes steady-state utility for the household in our setting; the results are largely insensitive to changing this parameter.

amount of on-the-job search can match key descriptive facts described in Section 2.

To that purpose, in Appendix [A.3](#), we log-linearize the model's wage Phillips curve (7) and express wage growth as a function of vacancy and unemployment rates first as follows:<sup>15</sup>

$$\check{\Pi}_t^w = \phi_V \check{V}_t + \phi_U \check{U}_t + \frac{1}{1+\rho} \check{\Pi}_{t+1}^w. \quad (20)$$

Since the quits rate itself is a function of vacancies and unemployment, the wage Phillips curve (20) can in turn be written in terms of quits  $\check{Q}_t$  and unemployment  $\check{U}_t$  as follows:<sup>16</sup>

$$\check{\Pi}_t^w = \beta_Q \check{Q}_t + \beta_U \check{U}_t + \frac{1}{1+\rho} \check{\Pi}_{t+1}^w \quad (21)$$

<sup>15</sup>Appendix [D](#) explores how the on-the-job search probability  $\lambda_{EE}$  affects the coefficients  $\phi_V$  and  $\phi_U$  in (20).

<sup>16</sup>For this part of the derivation, i.e., expressing  $\check{Q}_t$  in terms of  $\check{V}_t$  and  $\check{U}_t$ , see Appendix [A.3.1](#).

for some positive  $\beta_Q > 0$  and  $\beta_U$  of indeterminate sign which depends on the calibration (both coefficients are functions of deep parameters and steady state values). In our benchmark calibration given in Table 2, we find  $\beta_Q = .0246$  and  $\beta_U = .0009$ . The model thus captures both the fact that  $\beta_Q > \beta_U$  and that  $\beta_U > 0$ , which we observe from our wage Phillips curve regression in Table 1.

Note, however, that the sign of  $\beta_U$  in equilibrium turns out to be sensitive to the choice of convexity of vacancy-opening cost  $\chi$  in the model: for  $\chi = 0$ , or a linear cost of posting vacancy, the model generally delivers a negative coefficient on unemployment, though the coefficient on quits remains larger in magnitude.<sup>17</sup> We conclude this section by noting that the model is broadly consistent with the wage Phillips curve describing the modern U.S. economy. In the next question, we show that the model will also imply that there is very little scope for pass through from a cost-of-living shock to wages in the modern U.S. economy.

## 5 Implications for Pass Through from Prices to Wages

This section studies the effects of a cost-of-living shock as defined in Section 3, considering the following thought experiment: what happens to nominal wages when an unanticipated, temporary, negative shock to  $X_0$  raises the price level at  $t = 0$ ? In each case, we will consider what happens when monetary policy holds  $N_t$  fixed.

Section 5.1 compares analytical results for baseline wage posting model of Section 3, in which there is no increase in wages in response to the shock, to other standard models of labor supply used in the DSGE literature: specifically, a model with neoclassical labor supply and a model where unions set wages (Erceg et al., 2000). Section 5.2 demonstrates quantitatively both that cost-of-living shocks don't move wages, and that monetary policy shocks cause job-to-job quits and wages to comove, as in the data. Finally, Section 5.3 presents quantitative results for pass through in an extension to the model in which increases in the price level make unemployment more attractive for a given nominal wage, demonstrating that quantitatively on-the-job search mutes the pass through of prices to wages in response to a cost-of-living shock.

### 5.1 Analytical Results

We begin this section by demonstrating that in the baseline model of Section 3, there is no pass-through from the cost of living shock, which raises the overall price level, to wages. We then show how this differs from both the case of a standard neoclassical labor supply model, and also

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<sup>17</sup>With  $\chi = 0$  and fixing other parameters, we obtain

$$\Pi_t^w = 2.13 \times 10^{-2} \check{Q}_t - 1.1 \times 10^{-3} \check{U}_t + \frac{1}{1+\rho} \check{\Pi}_{t+1}^w.$$

from a model where unions set wages as in [Erceg et al. \(2000\)](#). In these two cases, we show that the value of  $\eta$ , the elasticity of substitution between endowment good  $X_t$  and services produced with labor,  $Y_t$ , in equation (2), matters: when  $\eta < 1$ , and the goods are not good substitutes, then wages rise in response to the cost of living shock.

**Pass Through in Our Baseline Wage Posting Model** In our baseline model, there is no pass-through regardless of  $\eta$ . A cost of living shock, i.e., a temporary, unanticipated drop in  $X_0$ , changes wage inflation only if it affects the labor market variables discussed above, and these are unchanged in equilibrium. We show this quantitatively in Section 5.2.1 under our benchmark calibration, and analyze the more general case rigorously in Appendix C.3.

**Pass Through with Sticky Prices and Flexible Wages** Given the consumption and price aggregators (2) and (3), let us assume that the households maximize

$$\sum_{t=0}^{\infty} \beta^t \left( \log C_t - \frac{1}{1 + \frac{1}{\nu}} N_t^{1 + \frac{1}{\nu}} \right)$$

given the usual budget constraint

$$C_t = \frac{D_t}{P_t} - \frac{B_t}{P_t} + \frac{(1 + i_{t-1,t})B_{t-1}}{P_t} + W_t N_t.$$

so that labor  $N_t$  is hired in a spot market and there is no unemployment. When central banks stabilize  $N_t = N$  under sticky prices, we can prove that in response to  $X_0$  shock at  $t = 0$ , there is pass-through, and wages rise, only if  $\eta < 1$ , i.e., the elasticity of substitution between  $X_t$  and  $Y_t$  is relatively weak. We prove this result in Appendix C.1.<sup>18</sup>

**Pass Through if Unions Set Wages** Given the consumption and price aggregators (2) and (3), let us assume that households now supply multiple types of labor; unions set wages for each type to maximize household utility subject to facing CES demand for each type from a “labor packer” which packages each labor type  $N_t(i)$  into aggregate labor  $N_t = \left( \int_0^1 N_t(i)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}$  which is purchased at wage  $W_t$  by services firms—and in our setting, combined with  $X_t$  to form consumption  $C_t$ . Wages are sticky as in [Erceg et al. \(2000\)](#) and [Galí et al. \(2012\)](#) as unions only occasionally receive the chance to reset their wage. The household maximizes

$$\sum_{t=0}^{\infty} \beta^t \left( \log C_t - \int_0^1 \frac{1}{1 + \frac{1}{\nu}} N_t(i)^{1 + \frac{1}{\nu}} di \right),$$

<sup>18</sup>Appendix C.1 also analyzes the case of a general consumption utility with the elasticity of intertemporal substitution  $\sigma$  in this neoclassical labor supply model.

given the same budget constraint above. Even in this case, we can prove that in response to  $X_0$  shock at  $t = 0$ , there is pass-through, so that wages rise, only if  $\eta < 1$ . We prove this result in Appendix C.2.

**Discussion** Even in a perfectly competitive labor market, we note that workers' wages can respond to a cost-of-living shock even when their productivity is unaffected. The sign and magnitude of the response depends on the strength of income and substitution effects, governed by the elasticity of intertemporal substitution of consumption utility, and wealth effects as well when workers are endowed both with leisure and the good  $X_t$ . The strength of those wealth effects are governed by  $\eta$ , which is why in the log-utility case considered here, where income and substitution effects cancel with each other,  $\eta$  becomes the determining factor; see Appendix C.1. When  $\eta < 1$ , a cost of living shock as described generates positive wealth effects, thereby making the household want to work less, so firms must raise wages if  $N_t$  is to be stabilized at its pre-shock level (if  $\eta > 1$ , the opposite logic will hold: workers will want to work more, and wages will actually fall in response to the shock).

In contrast, the household consumption rule (9) in our baseline model effectively eliminates wealth effects on labor supply by keeping the relative desirability of employment over unemployment at the same level in response to the shock, so that the sign of  $\eta$  no longer matters. We will show quantitatively in Section 5.3 that even if we alter the household's consumption sharing rule so that increases in the price level decrease the desire to work, and cause firms to raise wages in equilibrium, the effect is small with a quantitatively-realistic amount of on-the-job search.

## 5.2 Response of Wages to Cost-of-living and Monetary Policy Shocks: Baseline Model

This section analyzes the response of the baseline model described in Section 3 in response to both cost-of-living shocks (which don't move wages) and monetary policy shocks (which do move wages). We use the calibration Presented in Table 2, except where noted, including the simplifying assumption  $\eta = 1$ , in light of the analytical finding that this corner case is not important for determining pass through in our model when monetary policy stabilizes total labor  $N_t$  in the service sector, as shown above in Section 5.1 (and accompanying Appendix C.3).

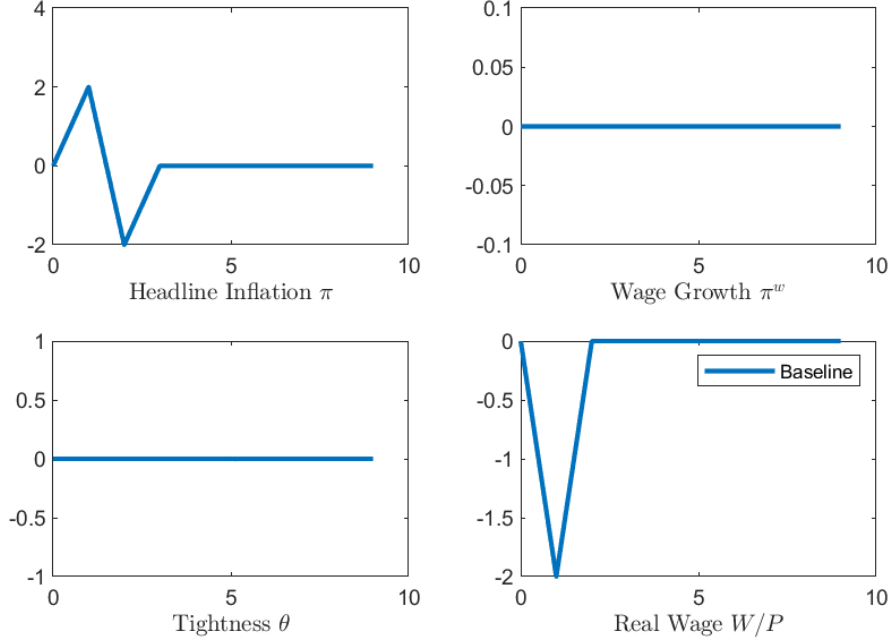
### 5.2.1 Cost-of-Living Shocks

We subject the economy to a 10% quantity shock of the endowment good  $X_t$  from  $X_t = 1$ . Given the assumption of a unit elasticity between services and goods in final aggregation, i.e.,  $\eta = 1$ , in (2) and (3), this implies a 10% relative price shock to good  $X_t$  and an increase in the overall price index  $P_t$ . Additionally, we assume that the central bank leaves nominal interest rates fixed: given the household's Euler equation (10) and that fact that  $\eta = 1$  implies constant expenditure



shares across services and goods, this experiment effectively holds aggregate demand for services constant; see Appendix B for details, which works through this same experiment in a simplified 2-period version of the model and provides analytical results.<sup>19</sup>

Figure 2: Impulse Response to a 10% Negative Shock to Supply of Endowment Good



*Notes:* This figure presents the effects of a decreased supply of the endowment good  $X_t$  under a nominal interest rate peg, which we identify as a pure cost-of-living shock. Given the assumption of a unit elasticity between services and goods in final aggregation, this implies a 10% relative price shock to good  $X_t$  and an increase in the overall price index  $P_t$ . Given the household's Euler equation and constant expenditure shares, the nominal interest rate peg experiment effectively holds aggregate demand for services constant (see Appendix B). Since the shock does not affect the relative attractiveness of unemployment and working, the recruiting and separation elasticities faced by firms are also unchanged as discussed in Section 3.2.2: the result is no change in vacancy posting, no change in tightness, and no change in the *nominal* wage, which causes *real* wages to fall as shown in the last panel.

Note that our nonlinear wage Phillips curve (7) in Section 3.1 expresses wage inflation as exclusively a function of aggregate, endogenous labor market variables: vacancies, employment, recruiting and separation rates, and recruiting and separation elasticities, with no direct role for aggregate price index  $P_t$ . This is the case for our linearized wage Phillips curve (21) as well

<sup>19</sup>Therefore, monetary policy in this environment effectively stabilizes  $N_t$ , as assumed in Section 5.1 unless firms charge higher service price  $P_{y,t}$  in response to a cost-of-living shock, which is not the case. As seen in our price Phillips curve (A.11) in Appendix A that depends on wage inflation and the same set of labor market variables, the aggregate price level does not affect the service price unless it affects equilibrium wage or those labor market variables, which is not the case in our baseline model.

where wage inflation is driven by fluctuations in quits rate and unemployment rate only. In this environment, unless the cost-of-living shock affects those labor market outcomes in equilibrium, there is no effect on wages.

Note that a crucial underlying mechanism is that due to household's consumption sharing scheme (9), the cost of living shock does not affect the relative attractiveness of unemployment and working, thereby not changing the recruiting and separation elasticities faced by firms as well: in general equilibrium, there is no change in vacancy posting, no change in tightness, and no change in the *nominal* wage, which causes *real* wages to fall as shown in the last panel of Figure 2. In Section 5.3, we relax this assumption and assume that a higher aggregate price level changes those elasticities by changing the relative desirability of unemployment and employment. In that case, we will have the pass-through.

### 5.2.2 Monetary Policy Shock

We consider an expansionary monetary policy shock, subjecting the economy to a one period, 1 percentage point decrease in nominal interest rates, with a monthly persistence of 0.8. On impact, both wage growth and employment-to-employment transitions rise, as seen in Figure 3. Lower nominal interest rates increase demand for consumption, which increases demand for labor. Firms then post more vacancies, increasing opportunities for workers to find other jobs, which raises job-to-job transitions, while also increasing competition among firms for workers, leading to higher wages. Appendix B.1 provides analytical results about employment-to-employment transitions and wage growth based on a 2-period version of the model.

This result demonstrates that our model can rationalize comovements between quits rate and wage growth, documented in Figure 1 and the literature, e.g., Faberman and Justiniano (2015); Moscarini and Postel-Vinay (2017), through demand shocks like monetary policy shocks.

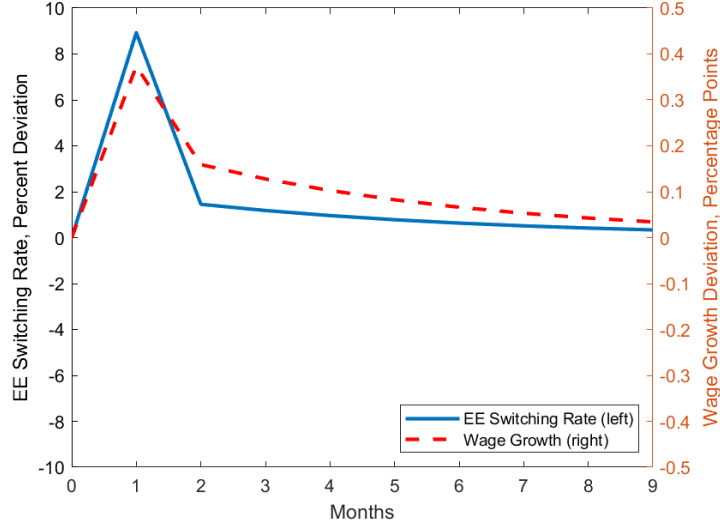
## 5.3 Extension: Cost-of-Living Shocks with Inflation-Indexed UI

This section revisits the experiment of Section 5.2.1 while relaxing the assumption that the relative desirability of unemployment and employment is held fixed by the household, allowing the relative desirability of unemployment to rise along with the price level. We will show how on-the-job search mutes the pass through from wages to prices in this variant of the model.

### 5.3.1 Separation and Recruiting Rates

We now assume that households no longer fix the ratio of consumption between employed and unemployed workers, but instead guarantee unemployed workers some inflation-indexed quantity of consumption,  $b$ . For a given nominal wage, an increase in the price level now raises the

Figure 3: Expansionary 1% Decrease in the Policy Rate



*Notes:* This figure plots the effects of a 1% decrease in nominal interest rates in our benchmark model. Both nominal wage growth and employment-to-employment transitions increase as lower nominal interest rates increase demand for consumption, which increases demand for labor. Firms post more vacancies, increasing opportunities for workers to find other jobs, which raises job-to-job transitions, while also increasing competition for workers, which raises wages. This result demonstrates that the model can rationalize comovements between quits and wage growth, documented in e.g., [Faberman and Justiniano \(2015\)](#); [Moscarini and Postel-Vinay \(2017\)](#), through demand shocks like monetary policy shocks.

relative consumption of unemployed agents, making unemployment more desirable. To see this, note that the probability that a worker separates from employment to unemployment is now

$$s_{ju}(W_{jt}) = \frac{b^\gamma}{\left(\frac{W_{jt}}{P_t}\right)^\gamma + b^\gamma}.$$

The separation rate  $s_{ju}$  from employment to unemployment now depends on the price level: at a given nominal wage, higher prices makes unemployment attractive. Similarly, the new recruiting function from unemployment is

$$r_{uj}(W_{jt}) = \frac{\left(\frac{W_{jt}}{P_t}\right)^\gamma}{\left(\frac{W_{jt}}{P_t}\right)^\gamma + b^\gamma},$$

where now we see that a higher price level makes recruiting from unemployment more difficult at a given nominal wage, by the same logic.

In a symmetric equilibrium where  $W_{jt} = W_t$  for  $\forall j$ , the separation and recruiting rates become

$$S_t = s + (1 - s) \left( \lambda_{EE} f(\theta_t) \frac{1}{2} + \lambda_{EU} \frac{b^\gamma}{\left(\frac{W_t}{P_t}\right)^\gamma + b^\gamma} \right),$$

and

$$R_t = g(\theta_t) \left( \phi_{E,t} \frac{1}{2} + \phi_{U,t} \frac{\left(\frac{W_t}{P_t}\right)^\gamma}{\left(\frac{W_t}{P_t}\right)^\gamma + b^\gamma} \right).$$

Unlike the benchmark case represented by (17) and (18), the price level  $P_t$  affects the recruiting and separation rates via the probability of quitting into unemployment and the probability of successfully recruiting unemployed workers.

All the other model equations (i.e. the firm's problem and the Taylor rule) remain unchanged; Appendix A.4 shows how we can derive an Euler equation in this setting which is identical to that used above, given appropriate assumptions on the representative household's optimization problem. We calibrate the model with a choice for unemployment benefit  $b$  instead of  $\xi$ ; we set  $b = 0.4$  which results in a steady-state consumption ratio for employed to unemployed agents of 2, so that this moment is the same at the steady state as in the benchmark model with  $\xi = 2$ .<sup>20</sup>

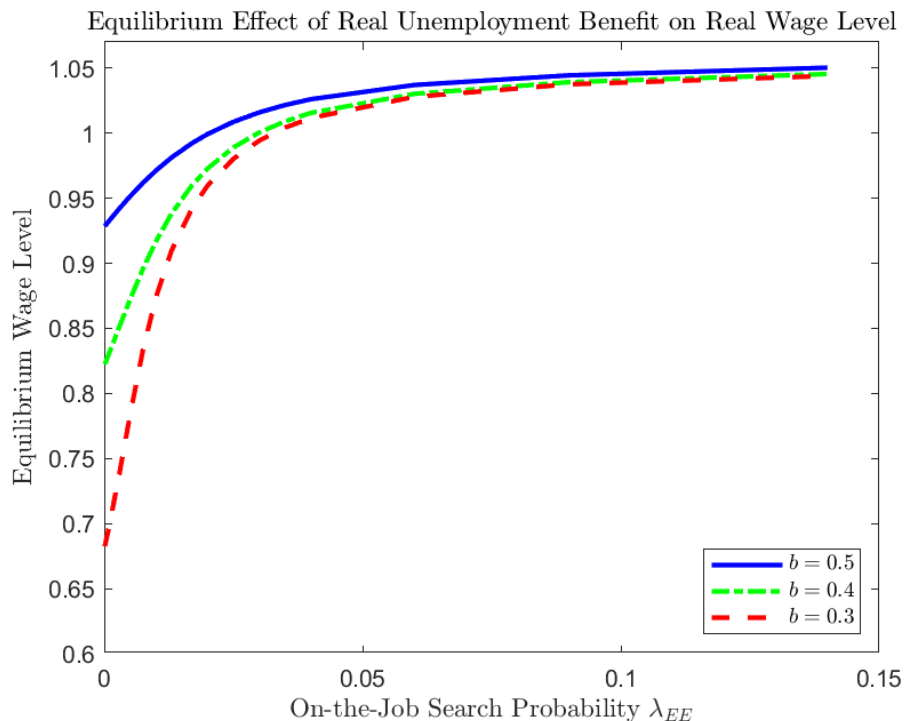
### 5.3.2 Result and the Role of On-the-Job Search $\lambda_{EE}$

First, we demonstrate that incorporating on-the-job search helps the model capture the empirical fact that changes in unemployment benefits do not seem to affect workers wages much in practice, even for new workers who are hired out of unemployment as shown by Jäger et al. (2020). Figure 4 demonstrates that in a model without on-the-job search, where  $\lambda_{EE} \rightarrow 0$ , changing unemployment benefit  $b$  has large effects on the equilibrium real wage, seen by examining the gaps between the blue solid line and dashed red line, for example. At our value of  $\lambda = .14$  calibrated to U.S. data, we see that the same changes in  $b$  have almost no change in the equilibrium real wage offered by firms, as in the data. Thus, beyond the fact that incorporating on-the-job search is important to capture the fact that quits in Figure 1 are mostly job-to-job quits, rather than quits into nonemployment, on-the-job search helps capture the near irrelevance of unemployment benefits for the wage.<sup>21</sup>

<sup>20</sup>Recall from the discussion in Table 2 that this calibrated consumption ratio  $\xi$  of 2:1 is higher than in Chodorow-Reich and Karabarbounis (2016) but closer to what maximizes the households' steady-state utility. While results in the benchmark model are insensitive to changing  $\xi$ , modifying the model to allow for pass-through from cost of living shocks to wages as we do here implies changes in  $b$  matter: lowering  $b$  might raise or reduce the pass-through of cost of living to wages depending on  $\lambda_{EE}$ , but does not affect the result that on-the-job search  $\lambda_{EE}$  mutes this pass-through. Quantitatively, changes in  $b$  do not affect the level of pass-through much.

<sup>21</sup>We can show qualitatively identical results for the effect of unemployment benefits on the wage in the baseline model by varying  $\xi$  instead of  $b$ .

Figure 4: On-the-Job Search Mutes the Effect of Changing Unemployment Benefits on Real Wages

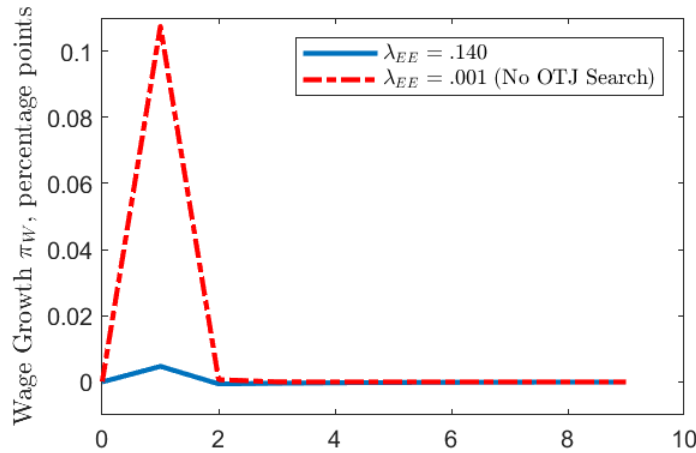


*Notes:* In the model with fixed real unemployment benefits described in Section 5.3, eliminating the role of on-the-job search and sending  $\lambda_{EE} \rightarrow 0$ , means that changes in unemployment benefit  $b$  have large effects on the equilibrium real wage (denominated in the price of aggregate consumption  $P$ ), seen by examining the gaps between the blue solid line and dashed red line, for example. This is because without on-the-job search, firms set wages mostly considering the problem of recruiting unemployed workers, which makes the level of  $b$  important in their wage-setting problem. At our value of  $\lambda = .14$  calibrated to U.S. data, where firms mostly recruit from other firms, we see that the same changes in  $b$  have almost no change in the equilibrium real wage offered, as in the data: the three lines lie on top of each other at this point.

Figure 5 presents the impulse response function of wage growth in the log-linearized model: in the solid blue line, which follows our benchmark calibration, we see that the effect on wage growth is quantitatively small. Intuitively, this is because the increase in the desirability of unemployment is not quantitatively relevant to firms who worry mainly about the risk of losing their workers to other firms, and recruiting workers on the job, than about quits to unemployment and/or recruiting unemployed workers, which our baseline model calibrated to U.S. data assumes is relatively uncommon.

To illustrate the importance of on-the-job search in delivering this result, we also estimate the impulse response function in the version of the model without on-the-job search, where the probability of being allowed to search on the job  $\lambda_{EE}$  is nearly zero, given by the red dashed line, finding that the response of wages becomes considerably larger. When  $\lambda_{EE}$  is low, firms' main concern when deciding wages becomes attracting unemployed workers into employment and discouraging quits to unemployment, since workers almost never have the chance to leave for to join another firm. Thus, firms raise wage more aggressively in response to a cost-of-living shock.<sup>22</sup>

Figure 5: On-the-Job Search Mutes the Pass-Through of Cost of Living Shocks to Wages



*Notes:* This Figure presents the effects of a decreased supply of the endowment good  $X_t$  under a nominal interest rate peg, as in Figure 2, in a variant of the benchmark model where increased cost of living raises the desirability of unemployment as described in Section 5.3. While there is now some pass through from the cost-of-living shock to wages, on-the-job search significantly dampens this result, seen by comparing the results in calibrations where the on-the-job search probability,  $\lambda_{EE}$  is calibrated to match U.S. data (the solid blue line) to a calibration where workers are almost never allowed to search on the job (the dashed red line).

<sup>22</sup>Appendix B demonstrates this result analytically in a simplified 2-period version of the quantitative model.

## 6 Conclusion

This paper developed a DSGE model of wage determination with labor market frictions where firms both set prices and post wages, subject to nominal rigidities in price and wage setting, and workers search on the job. Calibrated to match U.S. data on worker flows, we showed that the model implies a simple, linear wage Phillips curve expressing nominal wage growth as a function of log deviations of quits and unemployment from their long-run natural (steady state) values that is quantitatively in line with results for recent U.S. aggregate data.

We then studied the propagation of cost-of-living shocks in the model economy. Because firms set wages to avoid costly turnover, such shocks pass through to wages only to the extent that higher cost of living improves worker's outside options, such as competing jobs or unemployment, relative to their current job. As higher cost of living lowers real wages at all jobs evenly, and unemployment is rarely a credible outside option, we found that cost-of-living shocks have little to no effect on relative outside options and therefore wages.

While stylized, our model is consistent with a range of recent microeconomic evidence on how wages are determined, including the result in [Jäger et al. \(2020\)](#) that wages are insensitive to the flow value of unemployment benefits, and direct evidence on the preponderance of wage posting ([Hall and Krueger, 2012](#); [Lachowska et al., 2022](#); [Di Addario et al., 2023](#)). Admittedly, our simple model does abstract from the fact that there are a minority of unionized workers in the United States, and workers with automatic COLAs, for whom prices would pass through into prices. However, our results suggest that in a setting such as the United States where few workers operate under collective bargaining agreements with cost-of-living adjustments, and where firms' wage setting decision reflects competition for already-employed rather than for unemployed workers, the ability for most workers to reclaim real wages in response to a supply shock that raises their cost of living is limited. We conclude that there is little scope for supply-shock induced wage-price spirals specifically fueled by workers' ability to command higher nominal wages in response to higher nominal prices.

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## A Derivations and Proofs

### A.1 Worker's Problem with Utility From Leisure

This section reviews the worker's problem in Bloesch and Larsen (2023) also used in the paper's main body, deriving the probability a worker chooses a particular job  $j$  over outside offer  $k$  or unemployment. We then show that allowing for utility from leisure, as well as consumption, will not generally overturn the result that the price level does not affect the worker's optimal choice unless the elasticity of substitution between leisure and consumption is different from one.

**Discrete choice with Type-1 extreme value preference draws** Suppose worker  $i$  in state  $j$  (which could be working at firm  $j$ , for example), gets utility  $\mathcal{U}(ijt)$  plus a draw  $\iota_{ijt}$  that is distributed type-1 extreme value:

$$\mathcal{V}_t(i, j) = \mathcal{U}(ijt) + \iota_{ijt}$$

Let  $\iota_{ijt}$  have variance  $\frac{1}{\gamma}$ . Then given options two states  $j$  and  $k$ , the probability that the worker chooses  $j$  is

$$\frac{\exp(\gamma \mathcal{U}(ijt))}{\exp(\gamma \mathcal{U}(ijt)) + \exp(\gamma \mathcal{U}(ikt))}.$$

Suppose now that utility  $\mathcal{U}$  is a function of log consumption:  $\mathcal{U}(ijt) = \log(C_t(i, j))$ . This is the case in the main text. Then the probability of choosing  $j$  is

$$\frac{C_t(i, j)^\gamma}{C_t(i, j)^\gamma + C_t(i, k)^\gamma}.$$

**Case with a more general utility function** Consider now the more general form

$$\mathcal{V}(ijt) = \log(U(C_t(i, j), \ell_t(i, j))) + \iota_{ijt}$$

where  $\ell_t(i, j)$  is the leisure  $i$  gets in state  $j$  at time  $t$ , which nests the above case. For simplicity, denote utility while unemployed by  $U(C_t(i, u), \ell_t(i, u))$ , and while employed by  $U(C_t(i, e), \ell_t(i, e))$ ; then the probability of an unemployed worker taking a job when matched is now:

$$\frac{1}{1 + \left( \frac{U(C_t(i, u), \ell_t(i, u))}{U(C_t(i, e), \ell_t(i, e))} \right)^\gamma} \quad (\text{A.1})$$

**Proposition 1** *In partial equilibrium (i.e. holding all other equilibrium prices and quantities fixed) the probability that an unemployed worker takes a job in our general setting, (A.1), is invariant to changes in the price level  $P_t$  if and only if  $\frac{\partial}{\partial P_t} \frac{U(C_t(i, u), \ell_t(i, u))}{U(C_t(i, e), \ell_t(i, e))} = 0$ .*

**CES preference** To make progress, consider the case with CES preferences:  $U = \left( aC^{\frac{\rho-1}{\rho}} + (1-a)\ell^{\frac{\rho-1}{\rho}} \right)^{\frac{\rho}{\rho-1}}$  where  $\rho$  is the elasticity of substitution. We write  $U(C, \ell) = U\left(\frac{I}{P}, \ell\right)$ , imposing  $C = \frac{I}{P}$  for both types, who differ only in the nominal spending  $I$  (i.e.  $I_e$  for employed and  $I_u$  for unemployed)<sup>1</sup> Noting constant returns to scale (CRS) yields

$$\frac{U\left(\frac{I_t(i,u)}{P_t}, \ell_t(i,u)\right)}{U\left(\frac{I_t(i,e)}{P_t}, \ell_t(i,e)\right)} = \frac{U(I_t(i,u), P_t \ell_t(i,u))}{U(I_t(i,e), P_t \ell_t(i,e))},$$

and using the property of CES functions:  $\frac{\partial}{\partial P} U(I, P\ell) = (1-a)U(\cdot)^{\frac{1}{\rho}}(P\ell)^{-\frac{1}{\rho}}I$ , we can show:

$$\begin{aligned} \frac{\partial}{\partial P_t} \frac{U\left(\frac{I_t(i,u)}{P_t}, \ell_t(i,u)\right)}{U\left(\frac{I_t(i,e)}{P_t}, \ell_t(i,e)\right)} &= \frac{(1-a)P_t^{\frac{1}{\rho}}}{U(I_t(i,e), P_t \ell_t(i,e))} \\ &\cdot \left[ U(I_t(i,u), P_t \ell_t(i,u))^{\frac{1}{\rho}} \ell_t(i,u)^{1-\frac{1}{\rho}} - U(I_t(i,e), P_t \ell_t(i,e))^{\frac{1}{\rho}} \ell_t(i,e)^{1-\frac{1}{\rho}} \frac{U(I_t(i,u), P_t \ell_t(i,u))}{U(I_t(i,e), P_t \ell_t(i,e))} \right] \end{aligned}$$

which becomes 0 when  $\rho \rightarrow 1$ , i.e. the Cobb-Douglas case. Therefore, under the unit elasticity of substitution between consumption and leisure, Proposition 1 still holds.

## A.2 Firm's Problem and Derivation of the wage Phillips curve in (7)

The firm's problem is:<sup>2</sup>

$$\begin{aligned} \max_{\{P_{y,t}^j\}, \{N_{jt}\}, \{W_{jt}\}, \{V_{j,t}\}} \sum_{t=0}^{\infty} \left( \frac{1}{1+\rho} \right)^t &\left( P_{y,t}^j Y_t^j - W_{jt} N_{jt} - c \left( \frac{V_{j,t}}{N_{j,t-1}} \right)^{\chi} V_{j,t} \mathbf{W}_t - \frac{\psi}{2} \left( \frac{P_{y,t}^j}{P_{y,t-1}^j} - 1 \right)^2 Y_t^j P_{y,t}^j \right. \\ &\left. - \frac{\psi^w}{2} \left( \frac{W_{jt}}{W_{j,t-1}} - 1 \right)^2 W_{jt} N_{jt} \right) \end{aligned} \quad (\text{A.2})$$

subject to

$$N_{jt} = (1 - S(W_{jt}))N_{j,t-1} + R(W_{jt})V_{j,t}. \quad (\text{A.3})$$

We will have that physical output is produced with labor with the linear production:  $Y_t^j = A_t^j N_{jt}$ , and Dixit-Stiglitz demand, so  $\frac{Y_t^j}{Y_t} = \left( \frac{P_{y,t}^j}{P_{y,t}} \right)^{-\epsilon}$ , hence  $N_{jt} = \left( \frac{P_{y,t}^j}{P_{y,t}} \right)^{-\epsilon} \frac{Y_t}{A_t^j}$  with  $\epsilon > 1$ . The Lagrangian

<sup>1</sup>Here, the result does not depend on the case where we impose a tax-and-transfer scheme to keep  $I_e/I_u = C_e/C_u$  constant over the business cycle as in Section 3.2.

<sup>2</sup>Note that we assume that vacancy costs are denominated in labor; see Bloesch and Weber (2023) for microfoundations. We also use the aggregate wage  $W_t$  rather than the firm-specific wage  $W_{jt}$  to simplify the firm's wage setting problem and its interpretation.

then can be written as:

$$\begin{aligned}\mathcal{L} = \sum_{t=0}^{\infty} \left( \frac{1}{1+\rho} \right)^t & \left( (P_{y,t}^j)^{1-\epsilon} (P_{y,t})^\epsilon Y_t - W_{jt} \left( \frac{P_{y,t}^j}{P_{y,t}} \right)^{-\epsilon} \frac{Y_t}{A_t^j} - c (V_{j,t})^{1+\chi} \left( \frac{P_{y,t-1}^j}{P_{y,t-1}} \right)^{\epsilon\chi} \left( \frac{Y_{t-1}}{A_{t-1}^j} \right)^{-\chi} W_t \right. \\ & - \frac{\psi}{2} \left( \frac{P_{y,t}^j}{P_{y,t-1}^j} - 1 \right)^2 (P_{y,t}^j)^{1-\epsilon} (P_t)^\epsilon Y_t - \frac{\psi^w}{2} \left( \frac{W_{jt}}{W_{j,t-1}} - 1 \right)^2 W_{jt} \left( \frac{P_{y,t}^j}{P_{y,t}} \right)^{-\epsilon} \frac{Y_t}{A_t^j} \\ & \left. + \lambda_t^j \left[ - \left( \frac{P_{y,t}^j}{P_{y,t}} \right)^{-\epsilon} \frac{Y_t}{A_t^j} + V_{j,t} R(W_{jt}) + (1 - S(W_{jt})) \left( \frac{P_{y,t-1}^j}{P_{y,t-1}} \right)^{-\epsilon} \frac{Y_{t-1}}{A_{t-1}^j} \right] \right).\end{aligned}$$

The first order conditions are:

$$\begin{aligned}\mathcal{L}_{W_{jt}} = & - \left( \frac{P_{y,t}^j}{P_{y,t}} \right)^{-\epsilon} \frac{Y_t}{A_t^j} + \lambda_t^j \left( V_{j,t} R'(W_{jt}) - S'(W_{jt}) \left( \frac{P_{y,t-1}^j}{P_{y,t-1}} \right)^{-\epsilon} \frac{Y_{t-1}}{A_{t-1}^j} \right) - \frac{\psi^w}{2} \left( \frac{W_{jt}}{W_{j,t-1}} - 1 \right)^2 \left( \frac{P_{y,t}^j}{P_{y,t}} \right)^{-\epsilon} \frac{Y_t}{A_t^j} \\ & - \psi^w \left( \frac{W_{jt}}{W_{j,t-1}} - 1 \right) \frac{1}{W_{j,t-1}} W_t N_{jt} + \frac{1}{1+\rho} \psi^w \left( \frac{W_{j,t+1}}{W_{jt}} - 1 \right) \frac{W_{t+1}^j}{(W_{jt})^2} W_{j,t+1} N_{j,t+1} = 0. \quad (\text{A.4})\end{aligned}$$

and

$$\mathcal{L}_{V_{j,t}} = -c(1+\chi)(V_{j,t})^\chi \left( \frac{P_{y,t-1}^j}{P_{y,t-1}} \right)^{\epsilon\chi} \left( \frac{Y_{t-1}}{A_{t-1}^j} \right)^{-\chi} W_t + \lambda_t^j R(W_{jt}) = 0. \quad (\text{A.5})$$

$$\begin{aligned}\mathcal{L}_{P_{y,t}^j} = & (1-\epsilon) \left( \frac{P_{y,t}^j}{P_{y,t}} \right)^{-\epsilon} Y_t + \epsilon W_{jt} (P_{y,t}^j)^{-1} \left( \frac{P_{y,t}^j}{P_{y,t}} \right)^{-\epsilon} \frac{Y_t}{A_t^j} - \frac{c\epsilon}{1+\rho} \chi (V_{j,t+1})^{1+\chi} (P_{y,t}^j)^{-1} \left( \frac{P_{y,t}^j}{P_{y,t}} \right)^{\epsilon\chi} \left( \frac{Y_t}{A_t^j} \right)^{-\chi} W_{t+1} \\ & - \psi \left( \frac{P_{y,t}^j}{P_{y,t-1}^j} - 1 \right) \frac{1}{P_{y,t-1}^j} (P_{y,t}^j)^{1-\epsilon} P_{y,t}^\epsilon Y_t - (1-\epsilon) \frac{\psi}{2} \left( \frac{P_{y,t}^j}{P_{y,t-1}^j} - 1 \right)^2 \left( \frac{P_{y,t}^j}{P_{y,t}} \right)^{-\epsilon} Y_t \\ & + \frac{1}{1+\rho} \psi \left( \frac{P_{t+1}^j}{P_t^j} - 1 \right) \frac{P_{t+1}^j}{(P_{y,t}^j)^2} (P_{t+1}^j)^{1-\epsilon} P_{t+1}^\epsilon Y_{t+1} \\ & + \epsilon \frac{\psi^w}{2} \left( \frac{W_{jt}}{W_{j,t-1}} - 1 \right)^2 \frac{W_{jt}}{P_{y,t}} \left( \frac{P_{y,t}^j}{P_{y,t}} \right)^{-\epsilon-1} \frac{Y_t}{A_t^j} \\ & + \lambda_t^j \epsilon (P_{y,t}^j)^{-1} \left( \frac{P_{y,t}^j}{P_{y,t}} \right)^{-\epsilon} \frac{Y_t}{A_t^j} - \frac{1}{1+\rho} \lambda_{t+1}^j \epsilon (1 - S(W_{j,t+1})) (P_{y,t}^j)^{-1} \left( \frac{P_{y,t}^j}{P_{y,t}} \right)^{-\epsilon} \frac{Y_t}{A_t^j} = 0. \quad (\text{A.6})\end{aligned}$$

**Equilibrium** We focus on one particular equilibrium where  $P_{y,t}^j = P_{y,t}$ ,  $V_{j,t} = V_t$ ,  $W_{j,t} = W_t$ ,  $A_t^j = A_t \forall j$ . Then we can summarize the above equations as follows:

**FOC on Wages in (A.4):**

$$\begin{aligned} & -N_t + \lambda_t (V_t R'(W_t) - N_{t-1} S'(W_t)) - \psi^w (\Pi_t^w - 1) \Pi_t^w N_t - \frac{\psi^w}{2} \underbrace{(\Pi_t^w - 1)^2 N_t}_{\simeq 0} \\ & + \frac{1}{1+\rho} \psi^w (\Pi_{t+1}^w - 1) (\Pi_{t+1}^w)^2 N_{t+1} = 0, \end{aligned}$$

where we define the aggregate wage inflation  $\Pi_t^w = \frac{W_t}{W_{t-1}}$  and approximate with  $(\Pi_t^w - 1)^2 \simeq 0$ . Multiplying the second term by  $\frac{W_t P_{y,t}}{W_t P_{y,t}}$  yields:

$$-N_t + \frac{\lambda_t}{P_{y,t}} \left( \frac{W_t}{P_{y,t}} \right)^{-1} (V_t R'(W_t) W_t - N_{t-1} S'(W_t) W_t) - \psi^w (\Pi_t^w - 1) \Pi_t^w N_t + \frac{1}{1+\rho} \psi^w (\Pi_{t+1}^w - 1) (\Pi_{t+1}^w)^2 N_{t+1} = 0 \quad (\text{A.7})$$

This is important so that we have the real wage and real Lagrange multiplier, i.e.,  $\frac{\lambda_t}{P_{y,t}}$  in our equilibrium equations

**FOC on vacancies in (A.5):**

$$-c(1+\chi) V_t^\chi \left( \frac{Y_{t-1}}{A_{t-1}} \right)^{-\chi} W_t + \lambda_t R(W_t) = 0.$$

Dividing both sides by the price level gives and replacing  $\frac{Y_{t-1}}{A_{t-1}} = N_{t-1}$ :

$$-c(1+\chi) \left( \frac{V_t}{N_{t-1}} \right)^\chi \frac{W_t}{P_{y,t}} + \frac{\lambda_t}{P_{y,t}} R(W_t) = 0. \quad (\text{A.8})$$

Plugging in (A.8) into (A.7) and rearranging gives:

$$\begin{aligned} N_t + \psi^w (\Pi_t^w - 1) \Pi_t^w N_t &= c(1+\chi) \left( \frac{V_t}{N_{t-1}} \right)^\chi \frac{1}{R(W_t)} (V_t R'(W_t) W_t - N_{t-1} S'(W_t) W_t) \\ &+ \frac{1}{1+\rho} \psi^w (\Pi_{t+1}^w - 1) (\Pi_{t+1}^w)^2 N_{t+1} \\ &= c(1+\chi) \left( \frac{V_t}{N_{t-1}} \right)^\chi \left( V_t \underbrace{\frac{R'(W_t) W_t}{R(W_t)}}_{\equiv \varepsilon_{R, W_t}} - N_{t-1} \frac{S(W_t)}{R(W_t)} \underbrace{\frac{S'(W_t) W_t}{S(W_t)}}_{\equiv \varepsilon_{S, W_t}} \right) \\ &+ \frac{1}{1+\rho} \psi^w (\Pi_{t+1}^w - 1) (\Pi_{t+1}^w)^2 N_{t+1}. \end{aligned}$$

Dividing by  $N_t$  in both sides, we obtain

$$\psi^w (\Pi_t^w - 1) \Pi_t^w + 1 = c(1 + \chi) \left( \frac{V_t}{N_{t-1}} \right)^\chi \left( \frac{V_t}{N_t} \varepsilon_{R, W_t} - \frac{N_{t-1}}{N_t} \frac{S(W_t)}{R(W_t)} \varepsilon_{S, W_t} \right) + \frac{1}{1 + \rho} \psi^w (\Pi_{t+1}^w - 1) (\Pi_{t+1}^w)^2 \frac{N_{t+1}}{N_t}, \quad (\text{A.9})$$

which is the wage Phillips curve in our model.

**FOC on pricing in (A.6):**

$$\begin{aligned} (1 - \epsilon) + \epsilon \frac{W_t}{P_{y,t}} A_t^{-1} - \frac{1}{1 + \rho} c \epsilon \chi (V_{t+1})^{1+\chi} \frac{P_{y,t+1}}{P_{y,t}} \frac{W_{t+1}}{P_{y,t+1}} Y_t^{-1-\chi} A_t^\chi + \epsilon \frac{\psi^w}{2} \underbrace{(\Pi_t^w - 1)^2}_{\approx 0} \frac{W_t}{P_{y,t}} N_t \\ - \psi \left( \frac{P_{y,t}}{P_{y,t-1}} - 1 \right) \frac{P_{y,t}}{P_{y,t-1}} - (1 - \epsilon) \underbrace{\frac{\psi}{2} \left( \frac{P_{y,t}}{P_{y,t-1}} - 1 \right)^2}_{\approx 0} + \frac{1}{1 + \rho} \psi \left( \frac{P_{y,t+1}}{P_{y,t}} - 1 \right) \left( \frac{P_{y,t+1}}{P_{y,t}} \right)^2 \frac{Y_{t+1}}{Y_t} \\ + \frac{\lambda_t}{P_{y,t}} \epsilon A_t^{-1} - \frac{1}{1 + \rho} \frac{\lambda_{t+1}}{P_{y,t+1}} \frac{P_{y,t+1}}{P_{y,t}} \epsilon (1 - S(W_{t+1})) A_t^{-1} = 0, \end{aligned} \quad (\text{A.10})$$

where we use  $(\frac{P_{y,t+1}}{P_{y,t}} - 1)^2 \approx 0$  as above. If we define the service inflation  $\frac{P_{y,t}}{P_{y,t-1}} = \Pi_{Y,t}$ , (A.10) can be written as

$$\begin{aligned} (1 - \epsilon) + \epsilon \frac{W_t}{P_{y,t}} A_t^{-1} - \frac{1}{1 + \rho} c \epsilon \chi \left( \frac{V_{t+1}}{N_t} \right)^{1+\chi} \Pi_{Y,t+1} A_t^{-1} \frac{W_{t+1}}{P_{y,t+1}} - \psi (\Pi_{Y,t} - 1) \Pi_{Y,t} \\ + \frac{1}{1 + \rho} \psi (\Pi_{Y,t+1} - 1) (\Pi_{Y,t+1})^2 \frac{Y_{t+1}}{Y_t} + \frac{\lambda_t}{P_{y,t}} \epsilon A_t^{-1} - \frac{1}{1 + \rho} \frac{\lambda_{t+1}}{P_{y,t+1}} \Pi_{Y,t+1} \epsilon (1 - S(W_{t+1})) A_t^{-1} = 0. \end{aligned}$$

Dividing both sides by  $-\epsilon$  yields:

$$\begin{aligned} \frac{\epsilon - 1}{\epsilon} - \frac{W_t}{P_{y,t}} A_t^{-1} + \frac{1}{1 + \rho} c \chi \left( \frac{V_{t+1}}{N_t} \right)^{1+\chi} \Pi_{Y,t+1} A_t^{-1} \frac{W_{t+1}}{P_{y,t+1}} + \frac{\psi}{\epsilon} (\Pi_{Y,t} - 1) \Pi_{Y,t} \\ - \frac{1}{1 + \rho} \frac{\psi}{\epsilon} (\Pi_{Y,t+1} - 1) \Pi_{Y,t+1}^2 \frac{Y_{t+1}}{Y_t} - \frac{\lambda_t}{P_{y,t}} A_t^{-1} + \frac{1}{1 + \rho} \frac{\lambda_{t+1}}{P_{y,t+1}} \Pi_{Y,t+1} (1 - S(W_{t+1})) A_t^{-1} = 0. \end{aligned}$$

Further rearranging gives:

$$\begin{aligned} \frac{\psi}{\epsilon} (\Pi_{Y,t} - 1) \Pi_{Y,t} + \frac{\epsilon - 1}{\epsilon} = \frac{W_t}{P_{y,t}} A_t^{-1} + \frac{1}{1 + \rho} \frac{\psi}{\epsilon} (\Pi_{Y,t+1} - 1) \Pi_{Y,t+1}^2 \frac{Y_{t+1}}{Y_t} \\ + A_t^{-1} \left( -\frac{c \chi}{1 + \rho} \left( \frac{V_{t+1}}{N_t} \right)^{1+\chi} \Pi_{Y,t+1} \frac{W_{t+1}}{P_{y,t+1}} + \frac{\lambda_t}{P_{y,t}} - \frac{1}{1 + \rho} \frac{\lambda_{t+1}}{P_{y,t+1}} \Pi_{Y,t+1} (1 - S(W_{t+1})) \right). \end{aligned}$$

Multiplying both sides again by  $Y_t$  with  $\frac{Y_t}{A_t} = N_t$  gives:



$$\begin{aligned} \frac{\psi}{\epsilon}(\Pi_{Y,t} - 1)\Pi_{Y,t}Y_t + \frac{\epsilon - 1}{\epsilon}Y_t = \frac{W_t}{P_{y,t}}N_t + \frac{1}{1 + \rho} \frac{\psi}{\epsilon}(\Pi_{Y,t+1} - 1)\Pi_{Y,t+1}^2Y_{t+1} \\ + N_t \left( \frac{-c\chi}{1 + \rho} \left( \frac{V_{t+1}}{N_t} \right)^{1+\chi} \Pi_{Y,t+1} \frac{W_{t+1}}{P_{y,t+1}} + \frac{\lambda_t}{P_{y,t}} - \frac{1}{1 + \rho} \frac{\lambda_{t+1}}{P_{y,t+1}} \Pi_{Y,t+1}(1 - S(W_{t+1})) \right) \end{aligned} \quad (\text{A.11})$$

which is our price Phillips curve.

### A.3 Linearized wage Phillips curve

**A Log-Linear wage Phillips curve** We log-linearize the wage Phillips curve in (7), except we leave in the second order term,  $\frac{\psi^w}{2} (\Pi_t^w - 1)^2$ , which we dropped when we derive (7).

$$\begin{aligned} \frac{\psi^w}{2} (\Pi_t^w - 1)^2 + \psi^w (\Pi_t^w - 1) \Pi_t^w + 1 = c(1 + \chi) \left( \frac{V_t}{N_{t-1}} \right)^\chi \left( \frac{V_t}{N_t} \varepsilon_{R,W_t} - \frac{N_{t-1}}{N_t} \frac{S(W_t)}{R(W_t)} \varepsilon_{S,W_t} \right) \\ + \frac{1}{1 + \rho} \psi^w (\Pi_{t+1}^w - 1) (\Pi_{t+1}^w)^2 \frac{N_{t+1}}{N_t}. \end{aligned} \quad (\text{A.12})$$

It is worth pointing out that equation (A.12) would hold even if we added other factors of production (e.g., we could have Cobb-Douglass production with capital or some other inputs including oil), and is unaffected by the presence of price rigidities (e.g., if we had flexible or price rigidity à la Rotemberg (1982), (A.12) would be the same).

To ease interpretation, we rewrite this using  $T_t \equiv \frac{V_t}{N_{t-1}}$  and  $g_t \equiv \frac{N_t}{N_{t-1}}$ :

$$\begin{aligned} 0 = \frac{\psi^w}{2} (\Pi_t^w - 1)^2 + \psi^w (\Pi_t^w - 1) \Pi_t^w + 1 - c(1 + \chi) T_t^\chi g_t^{-1} \left( T_t \varepsilon_{R,W_t} - \frac{S(W_t)}{R(W_t)} \varepsilon_{S,W_t} \right) \\ - \frac{1}{1 + \rho} \psi^w (\Pi_{t+1}^w - 1) (\Pi_{t+1}^w)^2 g_{t+1}. \end{aligned} \quad (\text{A.13})$$

We can suppress the dependence of  $S(\cdot)$  and  $R(\cdot)$  on  $W_t$  (since we know that in equilibrium,  $S_t$  and  $R_t$  are not functions of the aggregate wage  $W_t$ ): we rewrite (A.13) as:

$$0 = F(\ln(\Pi_t^w), \ln(\Pi_{t+1}^w), \ln(S_t), \ln(R_t), \ln(\varepsilon_{R,t}), \ln(\varepsilon_{S,t}), \ln(T_t), \ln(g_t), \ln(g_{t+1})),$$

and take a linear approximation around a zero wage-inflation steady state with variables  $\ln(\Pi_t^w)$ ,  $\ln(\Pi_{t+1}^w)$ ,  $\ln(S_t)$ ,  $\ln(R_t)$ ,  $\ln(\varepsilon_{R,t})$ ,  $\ln(\varepsilon_{S,t})$ ,  $\ln(T_t)$ ,  $\ln(g_t)$ , and  $\ln(g_{t+1})$ . We first calculate derivatives

of  $F(\cdot)$  with respect to each variable as follows:

$$\begin{aligned}
F_{\ln(\Pi_t^w)} &= \psi^w \Pi_t^w (2(\Pi_t^w - 1) + \Pi_t^w) \\
F_{\ln(\Pi_{t+1}^w)} &= -\frac{\psi^w g_{t+1}}{1 + \rho} (\Pi_{t+1}^w (\Pi_{t+1}^w)^2 + (\Pi_{t+1}^w - 1) 2\Pi_{t+1}^w) \\
F_{\ln(S_t)} &= c(1 + \chi) T_t^\chi g_t^{-1} \frac{S_t}{R_t} \varepsilon_{S,t} \\
F_{\ln(R_t)} &= -c(1 + \chi) T_t^\chi g_t^{-1} \frac{S_t}{R_t} \varepsilon_{S,t} \\
F_{\ln(\varepsilon_{R,t})} &= -c(1 + \chi) T_t^{\chi+1} g_t^{-1} \varepsilon_{R,t} \\
F_{\ln(\varepsilon_{S,t})} &= c(1 + \chi) T_t^\chi g_t^{-1} \frac{S_t}{R_t} \varepsilon_{S,t} \\
F_{\ln(\mathbf{T}_t)} &= -c(1 + \chi) g_t^{-1} \left( (1 + \chi) T_t^{\chi+1} \varepsilon_{R,t} - \chi T_t^\chi \frac{S_t}{R_t} \varepsilon_{S,t} \right) \\
F_{\ln(\mathbf{g}_t)} &= c(1 + \chi) T_t^\chi g_t^{-1} \left( T_t \varepsilon_{R,t} - \frac{S_t}{R_t} \varepsilon_{S,t} \right) \\
F_{\ln(\mathbf{g}_{t+1})} &= -\frac{1}{1 + \rho} \psi^w (\Pi_{t+1}^w - 1) (\Pi_{t+1}^w)^2 g_{t+1},
\end{aligned}$$

which at the steady state with zero wage inflation can be written as

$$\begin{aligned}
F_{\ln(\Pi_t^w)} &= \psi^w \\
F_{\ln(\Pi_{t+1}^w)} &= -\frac{\psi^w}{1 + \rho} \\
F_{\ln(S_t)} &= c(1 + \chi) T^\chi g^{-1} \frac{S}{R} \varepsilon_S = c(1 + \chi) \frac{T^{\chi+1}}{g} \varepsilon_S \\
F_{\ln(R_t)} &= -c(1 + \chi) T^\chi g^{-1} \frac{S_t}{R_t} \varepsilon_{S,t} = -c(1 + \chi) \frac{T^{\chi+1}}{g} \varepsilon_S \\
F_{\ln(\varepsilon_{R,t})} &= -c(1 + \chi) \frac{T^{\chi+1}}{g} \varepsilon_R \\
F_{\ln(\varepsilon_{S,t})} &= c(1 + \chi) \frac{T^{\chi+1}}{g} \varepsilon_S \\
F_{\ln(\mathbf{T}_t)} &= -c(1 + \chi) \frac{T^{\chi+1}}{g} ((1 + \chi) \varepsilon_R - \chi \varepsilon_S) \\
F_{\ln(\mathbf{g}_t)} &= c(1 + \chi) \frac{T^{\chi+1}}{g} (\varepsilon_R - \varepsilon_S) > 0 \\
F_{\ln(\mathbf{g}_{t+1})} &= 0
\end{aligned}$$

where we made use of the fact that  $T = \frac{V}{N} = \frac{S}{R}$  in steady state, which we obtain from (A.3). We can see that the assumption above that  $\frac{\psi^w}{2} (\Pi_t^w - 1)^2 \approx 0$  is correct in the sense that it drops out in our first-order approximation. We also find that in a zero inflation steady-state, there is no

role for expectations of future employment growth in our first-order approximation.

Let the magenta terms above be collected as  $\kappa \equiv c(1 + \chi) \frac{T^{\chi+1}}{g}$ . Then the first order approximation of  $F(\cdot)$  around its steady state is given by<sup>3</sup>

$$0 = \psi^w \check{\Pi}_t^w - \frac{\psi^w}{1 + \rho} \check{\Pi}_{t+1}^w + \kappa \varepsilon_S (\check{S}_t - \check{R}_t) + \kappa (\varepsilon_S \check{\varepsilon}_{S,t} - \varepsilon_R \check{\varepsilon}_{R,t}) + \kappa (\chi \varepsilon_S - (1 + \chi) \varepsilon_R) \check{T}_t + \kappa (\varepsilon_R - \varepsilon_S) \check{g}_t,$$

which we can rewrite as

$$\check{\Pi}_t^w = \underbrace{-\frac{\kappa(\varepsilon_R - \varepsilon_S)}{\psi^w} \check{g}_t}_{\text{Employment Growth}} + \underbrace{\frac{1}{1 + \rho} \check{\Pi}_{t+1}^w}_{\text{Expectations}} - \underbrace{\frac{\kappa \varepsilon_S}{\psi^w} (\check{S}_t - \check{R}_t) - \frac{\kappa}{\psi^w} (\varepsilon_S \check{\varepsilon}_{S,t} - \varepsilon_R \check{\varepsilon}_{R,t}) - \frac{\kappa(\chi \varepsilon_S - (1 + \chi) \varepsilon_R)}{\psi^w} \check{T}_t}_{\text{Three Labor Market "Tightness" Terms}} \quad (\text{A.14})$$

Note that the law of motion for employment in (A.3) that the firm faces implies:

$$g_t = (1 - S_t) + R_t T_t \quad (\text{A.15})$$

Log linearizing (A.15) yields:

$$\frac{1}{S} \check{g}_t = \check{R}_t + \check{T}_t - \check{S}_t,$$

which leads to

$$\check{S}_t - \check{R}_t = \check{T}_t - \frac{1}{S} \check{g}_t \quad (\text{A.16})$$

Plugging (A.16) into the log-linear wage Phillips curve in (A.14) and assuming  $g=1$ , we obtain

$$\check{\Pi}_t^w = \underbrace{\frac{\kappa(-\varepsilon_R + \frac{1+S}{S} \varepsilon_S)}{\psi^w} \check{g}_t}_{\text{Employment Growth}} + \underbrace{\frac{\kappa}{\psi^w} (\varepsilon_R \check{\varepsilon}_{R,t} - \varepsilon_S \check{\varepsilon}_{S,t}) + \frac{\kappa(1 + \chi)(\varepsilon_R - \varepsilon_S)}{\psi^w} \check{T}_t}_{\text{Two Labor Market "Tightness" Terms}} + \underbrace{\frac{1}{1 + \rho} \check{\Pi}_{t+1}^w}_{\text{Expectations}}. \quad (\text{A.17})$$

where  $\kappa \equiv c(1 + \chi) T^{\chi+1}$ . From (A.17), we observe that stronger monopsony, i.e., a lower  $\varepsilon_R - \varepsilon_S$ , flattens the wage Phillips curve, which is documented in de la Barrera i Bardalet (2023). We summarize this in the following Proposition 2.

**Proposition 2** *The wage Phillips curve in (A.17) becomes flatter as the recruiting elasticity net of the separation elasticity,  $\varepsilon_R - \varepsilon_S$ , falls.*

**Further Simplification** Plugging (A.15) into (A.14) yields:

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<sup>3</sup>We let  $\check{X}_t \equiv \ln X_t - \ln X$  for any  $X_t$ . If  $X_t < 0$ , then we let  $\check{X}_t \equiv \frac{X_t - X}{X}$ .

$$\begin{aligned}\check{\Pi}_t^w = & \frac{\kappa}{\psi^w} \left( -S(\varepsilon_R - \varepsilon_S) (\check{T}_t + \check{R}_t - \check{S}_t) - \varepsilon_S (\check{S}_t - \check{R}_t) + (\varepsilon_R \check{e}_{R,t} - \varepsilon_S \check{e}_{S,t}) + (\varepsilon_R + \chi(\varepsilon_R - \varepsilon_S)) \check{T}_t \right) \\ & + \frac{1}{1+\rho} \check{\Pi}_{t+1}^w,\end{aligned}\tag{A.18}$$

which leads to

$$\check{\Pi}_t^w = \frac{\kappa}{\psi^w} \left[ \underbrace{(-\varepsilon_S + S(\varepsilon_R - \varepsilon_S))}_{>0} (\check{S}_t - \check{R}_t) + \underbrace{(\varepsilon_R + (\chi - S)(\varepsilon_R - \varepsilon_S))}_{>0} \check{T}_t + (\varepsilon_R \check{e}_{R,t} - \varepsilon_S \check{e}_{S,t}) \right] + \frac{1}{1+\rho} \check{\Pi}_{t+1}^w,\tag{A.19}$$

### A.3.1 Reduced Form Log-Linear Wage Phillips Curve in Only Quits and Unemployment

Estimating this regression (21) in the data via OLS shows that the regression puts more weight on quits than on unemployment, as documented in Table 1. As explained in Section 2, the regression yields a surprising empirical result for the sign of the coefficient on unemployment: replacing vacancies with quits, the sign on unemployment flips, and becomes positive. In other words, holding quits constant, a higher unemployment rate is correlated with *higher* wage growth!

Intriguingly, our model's benchmark calibration actually captures this result: when  $\chi = 1$ , i.e., firms' vacancy costs are convex, we find a much larger coefficient on quits than on unemployment, where the coefficient on unemployment is relatively small and positive. In showing this, our strategy is to first express the above (A.19) into the following form:

$$\check{\Pi}_t^w = \phi_V \check{V}_t + \phi_U \check{U}_t + \frac{1}{1+\rho} \check{\Pi}_{t+1}^w\tag{A.20}$$

for some  $\phi_V$  and  $\phi_U$ , which are complex collections of model parameters and steady-state values. And then we use the fact that quits, which can also be decomposed into deviations of vacancy and unemployment, can be viewed as an imperfect proxy for "true" tightness since higher tightness leads to a higher rate of quits. A higher unemployment (vacancy) rate lowers (raises) tightness, and thus reduces (raises) quits. If  $\check{Q}_t \equiv g_{Q,V} \check{V}_t + g_{Q,U} \check{U}_t$  where  $g_{Q,U} < 0$  is of magnitude large enough, then equation (A.20) becomes

$$\Pi_t^w = \underbrace{\frac{\phi_V}{g_{Q,V}}}_{\equiv \beta_Q > 0} \check{Q}_t + \underbrace{\left( \phi_U - \phi_V \frac{g_{Q,U}}{g_{Q,V}} \right)}_{\equiv \beta_U} \check{U}_t + \frac{1}{1+\rho} \check{\Pi}_{t+1}^w,\tag{A.21}$$

possibly yielding a positive  $\beta_U$ .

**Derivation** To begin to simplify the wage Phillips curve (A.19), we first reproduce it here:

$$\check{\Pi}_t^w = \frac{\kappa}{\psi^w} \left[ \underbrace{(-\varepsilon_S + S(\varepsilon_R - \varepsilon_S))}_{>0} (\check{S}_t - \check{R}_t) + \underbrace{(\varepsilon_R + (\chi - S)(\varepsilon_R - \varepsilon_S))}_{>0} \check{T}_t + (\varepsilon_R \check{\varepsilon}_{R,t} - \varepsilon_S \check{\varepsilon}_{S,t}) \right] + \frac{1}{1+\rho} \check{\Pi}_{t+1}^w,$$

where  $\kappa \equiv c(1+\chi)T^{\chi+1} > 0$ ; we can sign the coefficients assuming convexity parameter  $\chi$  is not much smaller than steady state separations  $S$ ,<sup>4</sup> recalling both that the steady state separation rate  $S$  is quite small and that  $\varepsilon_R > 0$  and  $\varepsilon_S < 0$ .

We will now show how to decompose all of the following right-hand-side variables,  $\check{Q}_t$ ,  $\check{T}_t$ ,  $\check{R}_t$ ,  $\check{\varepsilon}_{R,t}$ , and  $\check{\varepsilon}_{S,t}$  into vacancy and unemployment deviations. The tightness term,  $T_t = \frac{V_t}{N_{t-1}}$ , is simple: in log deviations from steady state, it becomes

$$\check{T}_t = \check{V}_t + \frac{U}{1-U} \check{U}_t.$$

As for the rest, we will show that we can write the decompositions as follows:

$$1. \check{R}_t \equiv g_{R,V} \check{V}_t + g_{R,U} \check{U}_t$$

**Derivation:** Recall that the recruiting function is

$$R_t = g(\theta_t) \left( \phi_{E,t} \frac{1}{2} + \phi_{U,t} \left( \frac{\xi^\gamma}{1 + \xi^\gamma} \right) \right).$$

For practical purposes we define  $\mathcal{C} \equiv \frac{\xi^\gamma}{1 + \xi^\gamma}$ , which is increasing in the ratio of consumption for employed to unemployed workers. Then we obtain:

$$g_{R,V} = -\frac{\theta^2}{1 + \theta^2},$$

and

$$g_{R,U} = \frac{\theta^2}{1 + \theta^2} \cdot \frac{U(1 - \lambda_{EE})}{\lambda_{EE}(1 - U) + U} + \frac{0.5\phi_E}{0.5\phi_E + \mathcal{C}\phi_U} \cdot \frac{U}{1 - U} \cdot \frac{\lambda_{EE}\phi_E - \lambda_{EE} - \phi_E}{\lambda_{EE}} + \frac{\mathcal{C}\phi_U}{0.5\phi_E + \mathcal{C}\phi_U} \cdot (1 - \phi_U(1 - \lambda_{EE})).$$

$$2. \check{Q}_t \equiv g_{Q,V} \check{V}_t + g_{Q,U} \check{U}_t$$

**Derivation:** Recall that the quit function  $Q_t = S_t - s$  is given by

$$Q_t = (1 - s) \left( \lambda_{EE} f(\theta_t) \frac{1}{2} + \lambda_{EU} \left( \frac{1}{1 + \xi^\gamma} \right) \right)$$

---

<sup>4</sup>As  $\chi = 1$  and  $S = 3.6\%$ ,  $\chi > S$  at our steady state.

Then, we obtain:

$$g_{Q,V} = \frac{0.5\lambda_{EE}f}{0.5\lambda_{EE}f + \lambda_{EU}(1-C)} \cdot \frac{1}{1+\theta^2},$$

and

$$g_{Q,U} = -\frac{0.5\lambda_{EE}f}{0.5\lambda_{EE}f + \lambda_{EU}(1-C)} \cdot \frac{1}{1+\theta^2} \cdot \frac{U(1-\lambda_{EE})}{\lambda_{EE}(1-U) + U}.$$

$$3. \check{\varepsilon}_{R,t} = g_{\varepsilon_{R,U}} \check{U}_t$$

**Derivation:** Note that in equilibrium,  $\varepsilon_{R,t}$  is given by

$$\varepsilon_{R,t} = \frac{g(\theta_t) \gamma \left( \frac{\phi_{E,t}}{4} + \phi_{U,t} \frac{\xi^{-\gamma}}{(1+\xi^{-\gamma})^2} \right)}{g(\theta_t) \left( 0.5\phi_{E,t} + \left( \frac{\xi^\gamma}{1+\xi^\gamma} \right) \phi_{U,t} \right)} = \frac{\gamma \left( \frac{\phi_{E,t}}{4} + \phi_{U,t} C(1-C) \right)}{0.5\phi_{E,t} + \phi_{U,t} C},$$

from which we obtain

$$g_{\varepsilon_{R,U}} = \left( \frac{0.25\phi_E}{0.25\phi_E + C(1-C)\phi_U} - \frac{0.5\phi_E}{0.5\phi_E + C\phi_U} \right) \frac{U}{1-U} \frac{\lambda_{EE}\phi_E - \lambda_{EE} - \phi_E}{\lambda_{EE}} + \left( \frac{C(1-C)\phi_U}{0.25\phi_E + C(1-C)\phi_U} - \frac{C\phi_U}{0.5\phi_E + C\phi_U} \right) (1 - \phi_U(1 - \lambda_{EE})).$$

$$4. \check{\varepsilon}_{S,t} = g_{\varepsilon_{S,V}} \check{V}_t + g_{\varepsilon_{S,U}} \check{U}_t$$

**Derivation:** Note that in equilibrium  $\varepsilon_{S,t}$  is given by

$$\varepsilon_{S,t} = \frac{-(1-s)\gamma \left( f(\theta_t)\lambda_{EE}\frac{1}{4} + \lambda_{EU}\frac{\xi^{-\gamma}}{(1+\xi^{-\gamma})^2} \right)}{s + (1-s) \left( 0.5 \cdot \lambda_{EE}f(\theta_t) + \left( 1 - \frac{\xi^\gamma}{1+\xi^\gamma} \right) \lambda_{EU} \right)} = \frac{-(1-s)\gamma \left( f(\theta_t)\lambda_{EE}\frac{1}{4} + C(1-C)\lambda_{EU} \right)}{s + (1-s) \left( 0.5 \cdot \lambda_{EE}f(\theta_t) + (1-C)\lambda_{EU} \right)},$$

from which we obtain

$$g_{\varepsilon_{S,V}} = \left( \frac{0.25\lambda_{EE}f}{0.25\lambda_{EE}f + C(1-C)\lambda_{EU}} - \frac{0.5(1-s)\lambda_{EE}f}{s + (1-s)(0.5\lambda_{EE}f + (1-C)\lambda_{EU})} \right) \frac{1}{1+\theta^2},$$

and

$$g_{\varepsilon_{S,U}} = -\left( \frac{0.25\lambda_{EE}f}{0.25\lambda_{EE}f + C(1-C)\lambda_{EU}} - \frac{0.5(1-s)\lambda_{EE}f}{s + (1-s)(0.5\lambda_{EE}f + (1-C)\lambda_{EU})} \right) \frac{1}{1+\theta^2} \frac{U(1-\lambda_{EE})}{\lambda_{EE}(1-U) + U}.$$

**Decomposing Wage Growth into Vacancies and Unemployment** Combining these results, we can plug in and rewrite the wage Phillips curve just in terms of vacancies and unemployment. Let  $\Delta_1 \equiv -\varepsilon_S + S(\varepsilon_R - \varepsilon_S)$  and let  $\Lambda_1 \equiv \varepsilon_R + (\chi - S)(\varepsilon_R - \varepsilon_S)$ . Then the wage Phillips curve

(A.19) can be written as:

$$\begin{aligned}\check{\Pi}_t^w = & \underbrace{\frac{\kappa}{\psi^w} [\Lambda_1 + \Delta_1 (g_{S,V} - g_{R,V}) - \varepsilon_S g_{\varepsilon_S,V}]}_{\equiv \phi_V > 0} \check{V}_t \\ & + \underbrace{\frac{\kappa}{\psi^w} \left[ \frac{U}{1-U} \Lambda_1 + \Delta_1 (g_{S,U} - g_{R,U}) + \varepsilon_R g_{\varepsilon_R,U} - \varepsilon_S g_{\varepsilon_S,U} \right]}_{\equiv \phi_U < 0} \check{U}_t + \frac{1}{1+\rho} \check{\Pi}_{t+1}^w.\end{aligned}\quad (\text{A.22})$$

Under our calibration in Table 2, quantitatively (A.22) becomes

$$\check{\Pi}_t^w = 10^{-2} \times (1.83 \check{V}_t - 0.3 \check{U}_t) + \frac{1}{1+\rho} \check{\Pi}_{t+1}^w \quad (\text{A.23})$$

**Decomposing Wage Growth into Quits and Unemployment:** First, note  $\check{Q}_t \equiv g_{Q,V} \check{V}_t + g_{Q,U} \check{U}_t$  yields

$$\check{V}_t = \frac{1}{g_{Q,V}} \check{Q}_t - \frac{g_{Q,U}}{g_{Q,V}} \check{U}_t,$$

which with (A.23) yields:

$$\begin{aligned}\Pi_t^w = & \underbrace{\frac{\phi_V}{g_{Q,V}}}_{\equiv \beta_Q > 0} \check{Q}_t + \underbrace{\left( \phi_U - \phi_V \frac{g_{Q,U}}{g_{Q,V}} \right)}_{\equiv \beta_U} \check{U}_t + \frac{1}{1+\rho} \check{\Pi}_{t+1}^w \\ = & 10^{-2} \times (2.46 \check{S}_t + 0.0916 \check{U}_t) + \frac{1}{1+\rho} \check{\Pi}_{t+1}^w\end{aligned}\quad (\text{A.24})$$

where  $\beta_Q$  dominates  $\beta_U$  in magnitude under our calibration, and  $\beta_U$  becomes positive. Thus, we prove equation (21).

**A Simpler Wage Phillips Curve With No On-the-job Search** Here, we argue that convex vacancy costs and on-the-job search combine to make vacancies more important in the wage Phillips curve, i.e.,  $|\phi_V|$  is significantly bigger than  $|\phi_U|$ , than when  $\chi \approx 0$  and  $\lambda_{EE} = 0$ , the case where  $\check{V}_t - \check{U}_t$ , i.e.,  $\left( \frac{\check{V}_t}{\check{U}_t} \right)$ , becomes a sufficient statistic that explains wage growth.<sup>5</sup> In doing this, we will assume that  $s \simeq 0$  and  $\mathcal{C} \equiv \frac{\xi^\gamma}{1+\xi^\gamma} \simeq 1$ , both of which hold approximately under our calibration.

First, we demonstrate that as we eliminate OTJ search and let  $\lambda_{EE} \rightarrow 0$ , the decomposition of the wage Phillips curve into  $\check{V}_t$  and  $\check{U}_t$  in (A.22) simplifies considerably. The first term  $\check{T}_t$  remains:

$$\lim_{\lambda_{EE} \rightarrow 0} \check{T}_t = \check{V}_t + \frac{U}{1-U} \check{U}_t.$$

As for the rest, we will show that we can write the decompositions as follows:

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<sup>5</sup>In other words, when  $\chi \approx 0$  and  $\lambda_{EE} = 0$  with very small  $s$ ,  $\phi_V \simeq \phi_U$ .



1.  $\lim_{\lambda_{EE} \rightarrow 0} \check{R}_t = -\frac{\theta^2}{1+\theta^2} (\check{V}_t - \check{U}_t)$  since  $\phi_U \rightarrow 1$  (and  $\phi_E \rightarrow 0$ ) as we shut down on-the-job search, i.e.,  $\lambda_{EE} \rightarrow 0$ .
2.  $\lim_{\lambda_{EE} \rightarrow 0} \check{S}_t = 0$
3.  $\lim_{\lambda_{EE} \rightarrow 0} \check{\varepsilon}_{R,t} = 0$ .
4.  $\lim_{\lambda_{EE} \rightarrow 0} \check{\varepsilon}_{S,t} = 0$

Which means that the wage Phillips curve simplifies to:

$$\lim_{\lambda_{EE} \rightarrow 0} \check{\Pi}_t^w = \frac{\kappa}{\psi^w} \left[ \underbrace{(-\varepsilon_S + S(\varepsilon_R - \varepsilon_S))}_{\equiv \Delta_1} \frac{\theta^2}{1+\theta^2} (\check{V}_t - \check{U}_t) + \underbrace{(\varepsilon_R + (\chi - S)(\varepsilon_R - \varepsilon_S))}_{\equiv \Lambda_1} \check{T}_t \right] + \frac{1}{1+\rho} \check{\Pi}_{t+1}^w$$

Noting that:

$$\lim_{\lambda_{EE} \rightarrow 0} \theta_t = \frac{V_t}{U_t}$$

So we have further:

$$\lim_{\lambda_{EE} \rightarrow 0} \check{\Pi}_t^w = \frac{\kappa}{\psi^w} \left[ \underbrace{(-\varepsilon_S + S(\varepsilon_R - \varepsilon_S))}_{\equiv \Delta_1} \frac{\theta^2}{1+\theta^2} \check{\theta}_t + \underbrace{(\varepsilon_R + (\chi - S)(\varepsilon_R - \varepsilon_S))}_{\equiv \Lambda_1} \check{T}_t \right] + \frac{1}{1+\rho} \check{\Pi}_{t+1}^w$$

As we further assume that  $s \simeq 0$  and  $\mathcal{C} \simeq 1$ , we can find the following results for the steady-state values underlying  $\Lambda_1$  and  $\Delta_1$ :

$$\begin{aligned} \lim_{\lambda_{EE} \rightarrow 0} S &= s + (1-s)\lambda_{EU}(1-\mathcal{C}) = 0 \\ \lim_{\lambda_{EE} \rightarrow 0} \varepsilon_R &= \gamma(1-\mathcal{C}) = 0 \\ \lim_{\lambda_{EE} \rightarrow 0} \varepsilon_S &= \frac{-(1-s)\gamma(\mathcal{C}(1-\mathcal{C})\lambda_{EU})}{s + (1-s)(1-\mathcal{C})\lambda_{EU}} = -\gamma \end{aligned}$$

Which implies that  $\Delta_1 = -\gamma$  and  $\Lambda_1 = \gamma\chi$ . So with  $\chi = 0$ , our wage Phillips curve in terms of  $\check{V}_t$  and  $\check{U}_t$  simplifies to:

$$\lim_{\lambda_{EE} \rightarrow 0} \check{\Pi}_t^w = \frac{\kappa}{\psi^w} \gamma \left( \frac{\theta^2}{1+\theta^2} \right) \check{\theta}_t + \frac{1}{1+\rho} \check{\Pi}_{t+1}^w$$

.

Therefore, the wage Phillips curve can be written entirely in terms of market tightness  $\theta_t$  when there is no on-the-job search, as in [Gagliardone and Gertler \(2023\)](#).

In sum, as the exogenous separation rate  $s \rightarrow 0$ , the consumption ratio  $\zeta \rightarrow \infty$  or  $\mathcal{C} \rightarrow 1$  (so unemployed workers always take jobs) and  $\lambda_{EE} \rightarrow 0$ , we have that  $S \rightarrow 0$ ,  $\varepsilon_R \rightarrow 0$ , and  $\varepsilon_S \rightarrow -\gamma$ . Then there is complete weight on  $\theta$  in the wage Phillips curve, and if  $\lambda_{EE} = 0$ ,  $\theta_t = \frac{V_t}{U_t}$ . In

contrast, in our setting where  $\chi = 1$ , which implies a convex vacancy cost, and  $\lambda_{EE} > 0$ , we see that  $|\phi_V|$  is much higher than  $|\phi_U|$  as seen in (A.23), and  $\beta_S$  is much higher than  $\beta_U$  as seen in (A.24).

#### A.4 Euler Equation With Fixed Real Unemployment Benefits

This section shows how the assumptions in Section 5.3.2 can be made consistent with the standard Euler equation of the household, given appropriate assumptions on how the household reallocates consumption. Recall the goal in Section 5.3.2 was to modify the model so that the desirability of unemployment varied with the price level; here, we show one way to make that model consistent with the standard Euler equation (10) used throughout the main text.

Suppose that when unemployed, household members are guaranteed some quantity  $b$  of real consumption goods and receive no other income (e.g., some nominal unemployment benefit perfectly indexed to inflation). When employed, they receive a nominal wage  $W_t$ . The household takes  $b$ , market wages  $W_t$ , and the price level  $P_t$  as given, but can smooth all members consumption by choosing a proportional “top-up” each period, multiplying each type of worker’s income by  $1 + \tau_t$ . This yields consumption levels

$$\begin{aligned} C_t^u &= b(1 + \tau_t) \\ C_t^e &= \frac{W_t}{P_t}(1 + \tau_t), \end{aligned}$$

and total consumption

$$C_t = U_t b(1 + \tau_t) + (1 - U_t) \frac{W_t}{P_t}(1 + \tau_t). \quad (\text{A.25})$$

Making the top-up proportional and identical in both states  $u$  and  $e$  implies that as the household smooths consumption, it does not affect the relative attractiveness of unemployment and employment: the  $1 + \tau_t$  terms cancel out separation and recruiting probabilities from unemployment  $s_{ju}(W_{jt})$  and  $r_{uj}(W_{jt})$  presented in Section 5.3.2.

Continuing on to derive the Euler equation, the household maximizes

$$\begin{aligned} & \sum_{t=0}^{\infty} \left( \frac{1}{1 + \rho} \right)^t (U_t \ln(C_t^u) + (1 - U_t) \ln(C_t^e)) \\ &= \sum_{t=0}^{\infty} \left( \frac{1}{1 + \rho} \right)^t \left( U_t \ln(b(1 + \tau_t)) + (1 - U_t) \ln\left(\frac{W_t}{P_t}(1 + \tau_t)\right) \right) \\ &= \sum_{t=0}^{\infty} \left( \frac{1}{1 + \rho} \right)^t \left( U_t \ln(b) + (1 - U_t) \ln\left(\frac{W_t}{P_t}\right) + \ln(1 + \tau_t) \right). \end{aligned}$$

The household's budget constraint can be written as:

$$(1 + \tau_t) \left( U_t b + (1 - U_t) \frac{W_t}{P_t} \right) + \frac{B_t}{P_t} = \frac{D_t}{P_t} + (1 - U_t) \frac{W_t}{P_t} + \frac{(1 + i_{t-1,t}) B_{t-1}}{P_t}.$$

The household's Lagrangian function is given by:

$$\begin{aligned} \mathcal{L} = \sum_{t=0}^{\infty} \left( \frac{1}{1 + \rho} \right)^t & \left( U_t \ln(b) + (1 - U_t) \ln \left( \frac{W_t}{P_t} \right) + \ln(1 + \tau_t) \right. \\ & \left. + \lambda_t \left[ -(1 + \tau_t) \left( U_t b + (1 - U_t) \frac{W_t}{P_t} \right) - \frac{B_t}{P_t} + \frac{D_t}{P_t} + \frac{(1 + i_{t-1,t}) B_{t-1}}{P_t} \right] \right). \end{aligned}$$

The household's only choice variables are  $\tau_t$  and  $B_t$ . The first order conditions are

$$\mathcal{L}_{\tau_t} = 0 : \frac{1}{1 + \tau_t} = \lambda_t \left( U_t b + (1 - U_t) \frac{W_t}{P_t} \right),$$

$$\mathcal{L}_{B_t} = 0 : \frac{\lambda_t}{P_t} = \left( \frac{1}{1 + \rho} \right) \lambda_{t+1} \frac{(1 + i_{t,t+1}) B_t}{P_{t+1}}.$$

Plugging in the expression for aggregate consumption in equation (A.25) into the first order condition on  $\tau_t$  yields the standard Euler equation used in the main text:

$$C_t^{-1} = \frac{1}{1 + \rho} \frac{1 + i_{t,t+1}}{\pi_{t,t+1}} C_{t+1}^{-1}.$$

## B A Simpler Two-Period Model

In this section, we build a simple two-period general equilibrium model that illustrates the following two features in a sharper way:

1. When the employed and unemployed share consumption risks according to  $\frac{C_t^e}{C_t^u} = \zeta$ , i.e., the unemployed receives the consumption expenditure that is  $\zeta^{-1}$  times that of employed workers, the cost of living shock does not affect wage and labor market outcomes in general.
2. When the unemployed benefit  $b_t$  is in real terms, which workers compare with real wage  $\frac{W_t}{P_t}$  in deciding whether to join the workforce or not, a cost of living shock generates a positive wage response. This wage response becomes more muted as  $\lambda_{EE}$ , the on-the-job search probability, increases.

We consider 3 different points in time:  $t = 0, 1, 2$ . At  $t = 0$ , the economy is at its steady-state: the number of employed is  $\bar{N}$ , that of unemployed is  $\bar{U} = 1 - \bar{N}$ . At  $t = 2$ , the economy gets back to the steady state, regardless of what happens at the interim period,  $t = 1$ .

**Demand block** The policy rate is given by  $i_t = \rho$  for  $t = 0, 1$  (i.e., pegged) so the households' Euler equation under log-preference implies the intertemporal equalization of consumption expenditures, given by

$$P_0 C_0 = P_1 C_1 = P_2 C_2, \quad (\text{B.1})$$

where  $P_t$  is the price aggregator (of endowment good  $X_t$  and service good  $Y_t$  which is produced by firms) at time  $t$ , and  $C_t$  is the corresponding consumption aggregator. Under the unit elasticity of substitution between goods  $X_t$  and  $Y_t$ , i.e.,  $\eta = 1$  in our dynamic general equilibrium model, the households' expenditures on  $X_t$  and  $Y_t$  goods become proportional, implying

$$\frac{P_{X,t} X_t}{\alpha_X} = \frac{P_{Y,t} Y_t}{\alpha_Y} = P_t C_t \quad (\text{B.2})$$

for all  $t = 0, 1, 2$ . From (B.1) and (B.2), we obtain:

$$P_{Y,0} Y_0 = P_{Y,1} Y_1 = P_{Y,2} Y_2 \quad (\text{B.3})$$

in equilibrium. We further assume the full price rigidity for the service good sector for tractability purposes:  $P_{Y,0} = P_{Y,1} = P_{Y,2} = \bar{P}_Y$ ,<sup>6</sup> which implies  $Y_0 = Y_1 = Y_2 = \bar{Y}$  where  $\bar{Y}$  is the steady-state level of service output. Therefore, the service output  $Y_1$  at the interim period  $t = 1$  is always at the steady state level  $\bar{Y}$ , regardless of shocks realized at  $t = 1$ . It is because the economy

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<sup>6</sup>We will characterize the flexible price case later as a separate case.

is demand-determined, and the household always insures their perfect consumption smoothing under pegged monetary policy.

**Firm's problem** Firm  $i$ , with its production function  $Y_t^i = N_t^i$ <sup>7</sup>, solves the following optimization at  $t = 1$ , with its number of workers  $N_0 = \bar{N}$  inherited from the previous period:

$$J(\bar{N}) = \max_{V_1^i, W_1^i} \bar{P}_Y N_1^i - W_1^i N_1^i - \kappa(W_1) \cdot V_1^i + \frac{1}{1+\rho} J(N_1^i) \quad (\text{B.4})$$

subject to

$$N_1^i = \bar{N} = (1 - S(W_1^i|W_1))\bar{N} + R(W_1^i|W_1)V_1^i, \quad (\text{B.5})$$

where  $\kappa(W_1)V_1^i$  is a vacancy-creation cost, where  $\kappa(W_1)$  is a function of aggregate wage  $W_1$ . We will later consider two cases:  $\kappa(W_1) = \kappa$  (i.e., constant) and  $\kappa(W_1) = \kappa W_1$  (i.e., linear function).  $S(W_1^i|W_1)$  and  $R(W_1^i|W_1)$  are separation and retaining probabilities, respectively, that depend on the firm's individual wage  $W_1^i$  and the aggregate wage  $W_1$ . We will use the same functional form as in our dynamic general equilibrium model. Note that in (B.4), we do not incorporate nominal wage rigidities for now. Note that due to demand-determined nature,  $N_1 = \bar{N}$  is taken as given by each firm.

Solving (B.4) and (B.5) with  $\mu_1^i$  as the Lagrange multiplier to (B.5) yields the followings:

- For vacancy  $V_1^i$ :

$$\mu_1^i = \frac{\kappa(W_1)}{R(W_1^i|W_1)} \quad (\text{B.6})$$

which implies: the value of each worker is equal to the expected cost of hiring the worker. The creation of one vacancy costs  $\kappa(W_1)$  but each vacancy is filled with probability  $R(W_1^i|W_1)$ . This interpretation is provided in [de la Barrera i Bordalet \(2023\)](#) as well.

- Wage  $W_1^i$ :

$$\begin{aligned} N_1^i &= \frac{\kappa(W_1)}{R(W_1^i|W_1)} \left[ R'(W_1^i|W_1)V_1^i - S'(W_1^i|W_1)\bar{N} \right] \\ &= \frac{\kappa(W_1)}{R(W_1^i|W_1)} \left[ \frac{R(W_1^i|W_1)}{W_1^i} \underbrace{\frac{R'(W_1^i|W_1)W_1^i}{R(W_1^i|W_1)}}_{=\varepsilon_{R,1}} V_1^i - \underbrace{\frac{S'(W_1^i|W_1)W_1^i}{S(W_1^i|W_1)}}_{=\varepsilon_{S,1}} \cdot \frac{S(W_1^i|W_1)}{W_1^i} \bar{N} \right] \end{aligned} \quad (\text{B.7})$$

which becomes

$$N_1^i = \frac{\kappa(W_1)}{W_1^i} \left[ \varepsilon_{R,1} \cdot V_1^i - \varepsilon_{S,1} \cdot \frac{S(W_1^i|W_1)}{R(W_1^i|W_1)} \bar{N} \right]. \quad (\text{B.8})$$

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<sup>7</sup>With production function  $Y_t^i = N_t^i$ , from (B.3), we obtain that  $N_0 = N_1 = N_2 = \bar{N}$ .

Envelope condition:

$$J'(\bar{N}) = (1 - S(W_1^i|W_1))\mu_1^i = (1 - S(W_1^i|W_1))\frac{\kappa(W_1)}{R(W_1^i|W_1)}. \quad (\text{B.9})$$

Later, we will impose the equilibrium condition:  $W_1^i = W_1$  and  $N_1^i = N_1 = \bar{N}$ .

**Search and matching process** For now, we use the same functional forms for  $R(W_1^i|W_1)$  and  $S(W_1^i|W_1)$  as in our dynamic general equilibrium model in Section 3.2. As we stated, we assume employed and unemployed share consumption risks according to  $\frac{C_t^e}{C_t^u} = \xi$ . Therefore, under the equilibrium condition with equal decisions across firms, i.e.,  $W_1^i = W_1$ ,  $N_1^i = N_1$ ,  $V_1^i = V_1$ , the following definitions can be introduced:

- Labor market tightness  $\theta_1$ :

$$\theta_1 = \frac{V_1}{\lambda_{EE}\bar{N} + 1 - \bar{N}} \quad (\text{B.10})$$

where  $\lambda_{EE}$  is the on-the-job search intensity, and we use  $N_0 = \bar{N}$ .

- Retaining probability  $R(W_1^i = W_1|W_1)$ :

$$R(W_1|W_1) = g(\theta_1) \left( \frac{1}{2}\phi_{E,1} + \frac{\xi^\gamma}{1 + \xi^\gamma}\phi_{U,1} \right) \quad (\text{B.11})$$

where  $\phi_{E,1}$  and  $\phi_{U,1} \equiv 1 - \phi_{E,1}$  are fractions of employed (i.e., on-the-job searchers) and unemployed among job seekers, given by

$$\phi_{E,1} = \frac{\lambda_{EE}\bar{N}}{\lambda_{EE}\bar{N} + 1 - \bar{N}}. \quad (\text{B.12})$$

- Separation probability  $S(W_1^i = W_1|W_1)$ :

$$S(W_1|W_1) = \frac{1}{2}\lambda_{EE}f(\theta_1) + \frac{1}{1 + \xi^\gamma}\lambda_{EU} \quad (\text{B.13})$$

where we assume zero automatic separation (i.e.,  $s = 0$  in our dynamic general equilibrium model), and  $\lambda_{EU}$  is the exogenous job-quitting probability.

- Elasticity  $\varepsilon_{R,1}$  and  $\varepsilon_{S,1}$ : from (B.11) and (B.13), we obtain

$$\varepsilon_{R,1} = \gamma \cdot \left( \frac{\frac{1}{4}\phi_{E,1} + \phi_{U,1} \left( \frac{\xi^\gamma}{1 + \xi^\gamma} \right)}{\frac{1}{2}\phi_{E,1} + \left( \frac{\xi^\gamma}{1 + \xi^\gamma} \right) \phi_{U,1}} \right) \simeq \gamma \cdot \left( \frac{\frac{1}{4}\phi_{E,1}}{\frac{1}{2}\phi_{E,1} + \left( \frac{\xi^\gamma}{1 + \xi^\gamma} \right) \phi_{U,1}} \right), \quad (\text{B.14})$$

and

$$\varepsilon_{S,1} = -\gamma \cdot \left( \frac{f(\theta_1)\lambda_{EE}\frac{1}{4} + \lambda_{EU}\frac{\xi^r}{(1+\xi^r)^2}}{0.5\lambda_{EE}f(\theta_1) + \left(\frac{1}{1+\xi^r}\right)\lambda_{EU}} \right) \simeq -\frac{\gamma}{2}. \quad (\text{B.15})$$

where we approximate  $\frac{\lambda_{EU}}{1+\xi^r} \simeq 0$  and  $\frac{\phi_{U,1}\xi^r}{(1+\xi^r)^2} \simeq 0$ , which hold well under our calibration. In (B.14), our approximation is based on that the effect of higher wages in making currently unemployed people choose to work at a firm is small compared with the effect on attracting on-the-job searchers from other firms.

**Equilibrium characterization** Since every firm  $i$  chooses the same decisions in equilibrium, i.e.,  $W_1^i = W_1$ ,  $V_1^i = V_1$ , and  $N_1^i = N_1 = \bar{N}$ , from (B.11) and (B.13), we obtain

$$\begin{aligned} \frac{S(W_1|W_1)\bar{N}}{R(W_1|W_1)} &= \frac{\frac{1}{2}\lambda_{EE} \underbrace{f(\theta_1)}_{=\theta_1 g(\theta_1)} \bar{N} + \frac{1}{1+\xi^r}\lambda_{EU}\bar{N}}{g(\theta_1) \left( \frac{1}{2}\phi_{E,1} + \frac{\xi^r}{1+\xi^r}\phi_{U,1} \right)} \\ &= \frac{\frac{1}{2}\phi_{E,1}g(\theta_1)V_1 + \frac{1}{1+\xi^r}\lambda_{EU}\bar{N}}{g(\theta_1) \left( \frac{1}{2}\phi_{E,1} + \frac{\xi^r}{1+\xi^r}\phi_{U,1} \right)}. \end{aligned} \quad (\text{B.16})$$

We then plug in (B.14), (B.15), and (B.16) to (B.8) to obtain

$$\bar{N} = N_1 = \frac{\kappa(W_1)}{W_1} \left\{ \underbrace{V_1 \left[ \gamma \left( \frac{\frac{1}{2}\phi_{E,1}}{\frac{1}{2}\phi_{E,1} + \frac{\xi^r}{1+\xi^r}\phi_{U,1}} \right) \right]}_{\equiv \varepsilon_{11}} + \underbrace{\frac{\frac{\gamma}{2} \frac{1}{1+\xi^r} \lambda_{EU} \bar{N}}{\left( \frac{1}{2}\phi_{E,1} + \frac{\xi^r}{1+\xi^r}\phi_{U,1} \right) g(\theta_1)}}_{\equiv \varepsilon_{21}} \right\}, \quad (\text{B.17})$$

where  $\varepsilon_{11} + \varepsilon_{21}$  in (B.17) becomes the ‘effective’ labor supply elasticity each firm faces.  $\varepsilon_{11}$  is about the elasticity due to those who are on-the-job search: an increase in wage attracts more on-the-job searchers from other firms and reduce the endogenous separation of current workers, and given other variables, this effect becomes more pronounced with higher measure of on-the-job searchers among job seekers, i.e., higher  $\phi_{E,1}$  (thereby decrease in  $\phi_{U,1}$ ). Eventually in equilibrium, every firm sets the same wage:  $W_1^i = W_1$  for  $\forall i$ .

$\varepsilon_{21}$  is the elasticity attributed to those who quit their jobs to be unemployed: a higher wage deters workers from going to be unemployed. The proportion of those who exit the labor market becomes smaller under a bigger and more competitive job market with higher  $\lambda_{EE}$ , i.e., higher  $\lambda_{EE}$  lowers  $\varepsilon_{21}$  and raises  $\varepsilon_{11}$ .

From (B.5), (B.11), and (B.13), we obtain the labor dynamics as follows:

$$\begin{aligned}
\bar{N} = N_1 &= \left[ 1 - \left( \frac{1}{2} \lambda_{EE} \underbrace{f(\theta_1)}_{=\theta_1 g(\theta_1)} + \frac{1}{1 + \xi^\gamma} \lambda_{EU} \right) \right] \bar{N} + g(\theta_1) \left( \frac{1}{2} \phi_{E,1} + \frac{\xi^\gamma}{1 + \xi^\gamma} \phi_{U,1} \right) V_1 \\
&= \bar{N} - \bar{N} \frac{1}{1 + \xi^\gamma} \lambda_{EU} + g(\theta_1) V_1 \left[ \left\{ \frac{1}{2} \cancel{\phi_{E,1}} + \frac{\xi^\gamma}{1 + \xi^\gamma} \phi_{U,1} \right\} - \left\{ \frac{1}{2} \cancel{\phi_{E,1}} \right\} \right] \\
&= \bar{N} - \bar{N} \frac{1}{1 + \xi^\gamma} \lambda_{EU} + g(\theta_1) V_1 \frac{\xi^\gamma}{1 + \xi^\gamma} \phi_{U,1},
\end{aligned} \tag{B.18}$$

which implies

$$\frac{\bar{N} \frac{1}{1 + \xi^\gamma} \lambda_{EU}}{\lambda_{EE} \bar{N} + 1 - \bar{N}} = f(\theta_1) \frac{\xi^\gamma}{1 + \xi^\gamma} \phi_{U,1}. \tag{B.19}$$

Equations (B.17) and (B.19) constitute our equilibrium, with the condition  $N_1 = Y_1 = \bar{N}$ . We can theoretically elicit equilibrium  $W_1$  and  $V_1$  from those two equations.

**Cost-of-living shock** We assume that the endowment good  $X_t$  drops from its steady state level  $\bar{X}$  to  $X_1 < \bar{X}$  at  $t = 1$  in an unanticipated manner, and see how the business cycle variables adjust at  $t = 1$ . From (B.17) and (B.19), a sudden drop in  $X_1$  from  $\bar{X}$  does not affect the equilibrium levels of  $V_1$  and  $W_1$ , and from the household's Euler equation (B.3),  $N_1 = \bar{N}$  remains the same. From (B.2), the only change is the price of endowment good  $X_t$ , and  $P_{X,1}$  rises satisfying  $P_{X,1} X_1 = \bar{P}_X \bar{X}$ . The following Proposition 3 summarizes this finding.

**Proposition 3** *A cost-of-living shock, i.e., a sudden drop in  $X_1$  from  $\bar{X}$ , does not affect equilibrium labor market outcomes:  $N_1 = \bar{N}$ ,  $W_1$ , and  $V_1$ . The price  $P_{X,1}$  of endowment good  $X_1$  rises so that the expenditure stays the same, i.e.,  $P_{X,1} X_1 = \bar{P}_X \bar{X}$ .*

**Flexible price case** The result in Proposition 3 holds even if firms set their prices fully flexibly. As in our dynamig general equilibrium model, we assume firms are in monopolistic competition, represented by Dixit-Stiglitz aggregator with elasticity of substitution  $\epsilon$ . Then

$$Y_1^i = Y_1 \left( \frac{P_{Y,1}^i}{P_{Y,1}} \right)^{-\epsilon}. \tag{B.20}$$

Each firm  $i$  solves instead the following problem:

$$J(\bar{N}) = \max_{P_{Y,1}^i, N_1^i, V_1^i, W_1^i} P_{Y,1}^i N_1^i - W_1^i N_1^i - \kappa(W_1) \cdot V_1^i + \frac{1}{1 + \rho} J(N_1^i) \tag{B.21}$$

subject to (B.20) and

$$N_1^i = (1 - S(W_1^i | W_1)) \bar{N} + R(W_1^i | W_1) V_1^i. \tag{B.22}$$



The solution to (B.21), with  $W_1^i = W_1$ , will be given by

$$\begin{aligned} P_{Y,1}^i &= P_{Y,1} = \frac{\epsilon}{\epsilon - 1} \left( W_1 + \frac{\kappa(W_1)}{R(W_1|W_1)} - \frac{1}{1 + \rho} J'(N_1^i) \right) \\ &= \frac{\epsilon}{\epsilon - 1} \left( W_1 + \frac{\kappa(W_1)}{R(W_1|W_1)} - \frac{1}{1 + \rho} (1 - S(W_2|W_2)) \frac{\kappa(W_2)}{R(W_2|W_2)} \right) \end{aligned} \quad (\text{B.23})$$

where  $W_2 = \bar{W}$  as the economy gets back to its steady state at  $t = 2$ . The term  $\frac{\kappa(W_1)}{R(W_1|W_1)}$  is a cost of hiring through additional vacancy. If a firm hires at  $t = 1$ , it can reduce hiring at  $t = 2$  by one. The last term  $\frac{1}{1 + \rho} (1 - S(W_2|W_2)) \frac{\kappa(W_2)}{R(W_2|W_2)}$  represents this reduction in future hiring costs.<sup>8</sup>

From (B.3), (B.17), and (B.23), we obtain

$$\begin{aligned} \underbrace{P_{Y,0}}_{=P_Y} \bar{Y} &= P_{Y,1} Y_1 = \frac{\epsilon}{\epsilon - 1} \left[ W_1 + \frac{\kappa(W_1)}{R(W_1|W_1)} - \frac{1}{1 + \rho} (1 - S(W_2|W_2)) \frac{\kappa(W_2)}{R(W_2|W_2)} \right] \\ &\quad \cdot \frac{\kappa(W_1)}{W_1} \left\{ \underbrace{V_1 \left[ \gamma \left( \frac{\frac{1}{2} \phi_{E,1}}{\frac{1}{2} \phi_{E,1} + \frac{\xi^\gamma}{1 + \xi^\gamma} \phi_{U,1}} \right) \right]}_{\equiv \varepsilon_{11}} + \underbrace{\frac{\frac{\gamma}{2} \frac{1}{1 + \xi^\gamma} \lambda_{EU} \bar{N}}{\left( \frac{1}{2} \phi_{E,1} + \frac{\xi^\gamma}{1 + \xi^\gamma} \phi_{U,1} \right) g(\theta_1)}}_{\equiv \varepsilon_{21}} \right\}, \end{aligned} \quad (\text{B.24})$$

which, with (B.19), constitute the flexible price equilibrium. Since (B.19) and (B.24) do not depend on  $X_1$  or  $P_{X,1}$ , a cost-of-living shock, i.e., reduction in  $X_1$  from  $\bar{X}$ , does not affect the labor market equilibrium outcome as in the rigid price case.

**Corollary 1** *Even if the price-setting of firms is fully flexible, a cost-of-living shock, i.e., a sudden drop in  $X_1$  from  $\bar{X}$ , does not affect the equilibrium labor market outcomes:  $N_1 = \bar{N}$ ,  $W_1$ , and  $V_1$ . The price  $P_{X,1}$  of endowment good  $X_1$  rises so that the expenditure stays the same, i.e.,  $P_{X,1} X_1 = \bar{P}_X \bar{X}$ .*

## B.1 Quits rate and wage growth under demand shocks

In this section, we show analytically that a positive demand shock generates positive responses in both on-the-job switching rate  $\frac{1}{2} \lambda_{EE} f(\theta_1)$ <sup>9</sup> and wage growth. As  $f(\cdot)$  is increasing, it is equivalent to a positive correlation between market tightness  $\theta_1$  and wage growth under a demand shock.

We define a positive demand shock that raises  $N_1$  from  $\bar{N}$ , e.g., a reduction in the policy rate at  $t = 1$  will result in a consumption boom, thereby leading to firms' higher labor demand level

<sup>8</sup>The decomposition of marginal costs in equation (B.23) is similarly given in de la Barrera i Bardalet (2023).

<sup>9</sup>Quits rate includes those who voluntarily quit to unemployed as well, which is a small margin compared to the on-the-job switching part.

at  $t = 1$ . We start from our equilibrium conditions: instead of  $\bar{N}$ , we use  $N_1 > \bar{N}$  there:

$$\bar{N} < N_1 = \frac{\kappa(W_1)}{W_1} \left\{ \underbrace{V_1 \left[ \gamma \left( \frac{\frac{1}{2}\phi_{E,1}}{\frac{1}{2}\phi_{E,1} + \frac{\xi^\gamma}{1+\xi^\gamma}\phi_{U,1}} \right) \right]}_{\equiv \varepsilon_{11}} + \underbrace{\frac{\frac{\gamma}{2} \frac{1}{1+\xi^\gamma} \lambda_{EU} \bar{N}}{\left( \frac{1}{2}\phi_{E,1} + \frac{\xi^\gamma}{1+\xi^\gamma}\phi_{U,1} \right) g(\theta_1)}}_{\equiv \varepsilon_{21}} \right\}, \quad (\text{B.25})$$

and

$$\bar{N} < N_1 = \bar{N} - \bar{N} \frac{1}{1+\xi^\gamma} \lambda_{EU} + g(\theta_1) V_1 \frac{\xi^\gamma}{1+\xi^\gamma} \phi_{U,1}. \quad (\text{B.26})$$

We divide into two cases according to different functional forms of  $\kappa(W_1)$ : (i)  $\kappa(W_1) = \kappa$  (i.e., constant), and (ii)  $\kappa(W_1) = \kappa W_1$  (i.e., linear) with nominal wage rigidity.

**Case 1:**  $\kappa(W_1) = \kappa$  In this case, (B.25) becomes:

$$\bar{N} < N_1 = \frac{\kappa}{W_1} \left\{ \underbrace{V_1 \left[ \gamma \left( \frac{\frac{1}{2}\phi_{E,1}}{\frac{1}{2}\phi_{E,1} + \frac{\xi^\gamma}{1+\xi^\gamma}\phi_{U,1}} \right) \right]}_{\equiv \varepsilon_{11}} + \underbrace{\frac{\frac{\gamma}{2} \frac{1}{1+\xi^\gamma} \lambda_{EU} \bar{N}}{\left( \frac{1}{2}\phi_{E,1} + \frac{\xi^\gamma}{1+\xi^\gamma}\phi_{U,1} \right) g(\theta_1)}}_{\equiv \varepsilon_{21}} \right\}. \quad (\text{B.27})$$

In order to get a sharper results, we log-linearize (B.26) and obtain<sup>10</sup>

$$0 < \check{N}_1 = \frac{1}{1+\xi^\gamma} \lambda_{EU} \left( \underbrace{\frac{g'(\bar{\theta}_1) \bar{\theta}_1}{g(\bar{\theta}_1)}}_{\equiv -\varepsilon_{g,\theta}} \check{\theta}_1 + \check{\theta}_1 \right) = \frac{1}{1+\xi^\gamma} \lambda_{EU} \left( 1 - \underbrace{\varepsilon_{g,\theta}}_{<1} \right) \check{\theta}_1, \quad (\text{B.28})$$

where we assume the firm's matching elasticity  $\varepsilon_{g,\theta} \geq 0$  of  $g(\theta_1)$  is less than 1, which holds under various specification.<sup>11</sup> Therefore, from (B.28),  $\check{\theta}_1 > 0$  when  $\check{N}_1 > 0$ , i.e., labor market gets tighter at  $t = 1$ . We then log-linearize (B.27) and use (B.28) to obtain

$$\underbrace{\frac{1}{1+\xi^\gamma} \lambda_{EU} \left( 1 - \underbrace{\varepsilon_{g,\theta}}_{<1} \right)}_{=\check{N}_1} \check{\theta}_1 + \check{W}_1 = \left[ \frac{\bar{\varepsilon}_{11}}{\bar{\varepsilon}_{11} + \bar{\varepsilon}_{21}} + \frac{\bar{\varepsilon}_{11}}{\bar{\varepsilon}_{11} + \bar{\varepsilon}_{21}} \varepsilon_{g,\theta} \right] \check{\theta}_1. \quad (\text{B.29})$$

Since  $\frac{1}{1+\xi^\gamma} \lambda_{EU}$  is small under our calibration,  $\check{\theta}_1 > 0$  from (B.28) implies  $\check{W}_1 > 0$  in (B.29). Thus, we generate a positive correlation between movements in wage and market tightness (on-

<sup>10</sup>We use  $\check{\theta}_1 = \check{V}_1$  as  $\theta_1$  and  $V_1$  are proportional and  $\lambda_{EE} \bar{N} + 1 - \bar{N}$  is constant.

<sup>11</sup>Since  $f(\theta_1) = \theta_1 g(\theta_1)$ ,  $\varepsilon_{f,\theta} \equiv \frac{g'(\bar{\theta}_1) \bar{\theta}_1}{g(\bar{\theta}_1)} = 1 - \varepsilon_{g,\theta} > 0$  under our specification, as  $f(\theta_1)$  is increasing in  $\theta_1$ .

the-job switching rate), which is summarized in the following Proposition 4.

**Proposition 4** *When  $\kappa(W_1) = \kappa$ , i.e.,  $\kappa(W_1)$  is a constant function, both market tightness  $\theta_1$  (on-the-job switching rate  $0.5\lambda_{EE}f(\theta_1)$ ) and wage  $W_1$  rises in response to a positive demand shock.*

**Case 2:  $\kappa(W_1) = \kappa W_1$  with nominal wage stickiness** Now we assume  $\kappa(W_1) = \kappa W_1$  (i.e., linear function) but incorporate nominal wage rigidity à la Rotemberg (1982). Firm  $i$  solves:

$$J(\bar{N}) = \max_{V_1^i, W_1^i} \bar{P}_Y N_1^i - W_1^i N_1^i - \underbrace{\kappa(W_1) \cdot V_1^i}_{\equiv \kappa W_1} - \underbrace{\frac{\psi^W}{2} \left( \frac{W_1^i}{\bar{W}} - 1 \right)^2 \bar{W} N_1^i}_{\text{Wage changing cost}} + \frac{1}{1+\rho} J(N_1^i) \quad (\text{B.30})$$

subject to

$$N_1^i = (1 - S(W_1^i | W_1)) \bar{N} + R(W_1^i | W_1) V_1^i. \quad (\text{B.31})$$

Solving (B.30) subject to (B.31) with  $W_1^i = W_1$  and  $N_1^i = N_1$  yields

$$N_1 \left( 1 + \psi^W \frac{W_1 - \bar{W}}{\bar{W}} \right) = \frac{\kappa \cancel{W}_1}{\cancel{W}_1} \left\{ \underbrace{V_1 \left[ \gamma \left( \frac{\frac{1}{2} \phi_{E,1}}{\frac{1}{2} \phi_{E,1} + \frac{\xi^\gamma}{1+\xi^\gamma} \phi_{U,1}} \right) \right]}_{\equiv \varepsilon_{11}} + \underbrace{\frac{\frac{\gamma}{2} \left( \frac{1}{1+\xi^\gamma} \right) \lambda_{EU} \bar{N}}{\left( \frac{1}{2} \phi_{E,1} + \frac{\xi^\gamma}{1+\xi^\gamma} \phi_{U,1} \right) g(\theta_1)}}_{\equiv \varepsilon_{21}} \right\}, \quad (\text{B.32})$$

We log-linearize (B.32) and use (B.28) to obtain

$$\frac{1}{1+\xi^\gamma} \lambda_{EU} \left( 1 - \underbrace{\varepsilon_{g,\theta}}_{<1} \right) \check{\theta}_1 + \psi^W \check{W}_1 = \left[ \frac{\bar{\varepsilon}_{11}}{\bar{\varepsilon}_{11} + \bar{\varepsilon}_{21}} + \frac{\bar{\varepsilon}_{11}}{\bar{\varepsilon}_{11} + \bar{\varepsilon}_{21}} \varepsilon_{g,\theta} \right] \check{\theta}_1. \quad (\text{B.33})$$

Since  $\frac{1}{1+\xi^\gamma} \lambda_{EU}$  is small under our calibration,  $\check{\theta}_1 > 0$  from (B.28) implies  $\check{W}_1 > 0$  in (B.33) as well. Thus, we generate a positive correlation between movements in wage and market tightness (on-the-job switching rate), which is summarized in the following Proposition 5. Finally, note that **Case 2** (which is the case in our dynamic stochastic general equilibrium model in Section 3, with  $\chi = 0$ ) generate similar results to **Case 1**, where  $\kappa(\cdot)$  is a constant function.

**Proposition 5** *When  $\kappa(W_1) = \kappa W_1$  and firms face nominal wage rigidities à la Rotemberg (1982), both market tightness  $\theta_1$  (on-the-job switching rate  $0.5\lambda_{EE}f(\theta_1)$ ) and wage  $W_1$  rises in response to a positive demand shock.*

Therefore, our simple model generates the results in Section 5.2.2.

## B.2 With real benefits of unemployment

In this section, we assume that unemployed workers consume some inflation-indexed quantity of consumption  $b_1$  at  $t = 1$  as we do in Section 5.3.2. In those cases, all the equilibrium conditions above, i.e., (B.10), (B.11), (B.12), (B.13), (B.14), (B.15), (B.17), (B.19), hold, with

$$c(P_1, W_1) \equiv \frac{\left(\frac{W_1}{P_1}\right)^\gamma}{b_1^\gamma + \left(\frac{W_1}{P_1}\right)^\gamma}.$$

in the position of  $\frac{\bar{\xi}^\gamma}{1+\bar{\xi}^\gamma}$ . Here  $b_1$  is the consumption-equivalent during unemployment, which an unemployed person compares with real wage  $\frac{W_1}{P_1}$  in deciding whether to be back at work.

Note that  $c(P_1, W_1)$  is increasing in  $W_1$  and decreasing in  $P_1$ , where  $P_1$  is total price aggregator of endowment good  $X_1$  and service good  $Y_1$ . Under the rigid service prices, i.e.,  $P_{Y,1} = \bar{P}$ , a cost-of-living shock as described above increases  $P_1$  and lower  $c(P_1, W_1)$ . We ask how the economy's responses to a cost-of-living shock under this specification would differ from the above case where  $c(P_1, W_1) \equiv \frac{\bar{\xi}^\gamma}{1+\bar{\xi}^\gamma}$ . Intuitively, a rise in cost-of-living reduces the relative attractiveness of working compared with being unemployed, resulting in a lower  $c(P_1, W_1)$ . The equilibrium will be represented by

$$\frac{\bar{N} (1 - c(P_1, W_1)) \lambda_{EU}}{\lambda_{EE} \bar{N} + 1 - \bar{N}} = f(\theta_1) c(P_1, W_1) \phi_{U,1}. \quad (\text{B.34})$$

and

$$\bar{N} = N_1 = \frac{\kappa(W_1)}{W_1} \left\{ V_1 \underbrace{\left[ \gamma \left( \frac{\frac{1}{2} \phi_{E,1}}{\frac{1}{2} \phi_{E,1} + c(P_1, W_1) \phi_{U,1}} \right) \right]}_{\equiv \varepsilon_{11}} + \underbrace{\frac{\frac{\gamma}{2} (1 - c(P_1, W_1)) \lambda_{EU} \bar{N}}{(\frac{1}{2} \phi_{E,1} + c(P_1, W_1) \phi_{U,1}) g(\theta_1)}}_{\equiv \varepsilon_{21}} \right\}, \quad (\text{B.35})$$

where we use the fact that output (and labor) remains at the steady state level due to households' perfect consumption smoothing. We assume that at the steady state,  $c(\bar{P}_1, \bar{W}_1) = \bar{c} = \frac{\bar{\xi}^\gamma}{1+\bar{\xi}^\gamma}$ .

We divide into three cases according to different functional forms of  $\kappa(W_1)$ : (i)  $\kappa(W_1) = \kappa \cdot W_1$  (i.e., linear); (ii)  $\kappa(W_1) = \kappa$  (i.e., constant), and (iii) whether we introduce nominal wage rigidity.

**Case 1:**  $\kappa(W_1) = \kappa \cdot W_1$  In this case, (B.35) becomes:

$$\bar{N} = \kappa \left\{ V_1 \underbrace{\left[ \gamma \left( \frac{\frac{1}{2} \phi_{E,1}}{\frac{1}{2} \phi_{E,1} + c(P_1, W_1) \phi_{U,1}} \right) \right]}_{\equiv \varepsilon_{11}} + \underbrace{\frac{\frac{\gamma}{2} (1 - c(P_1, W_1)) \lambda_{EU} \bar{N}}{(\frac{1}{2} \phi_{E,1} + c(P_1, W_1) \phi_{U,1}) g(\theta_1)}}_{\equiv \varepsilon_{21}} \right\}. \quad (\text{B.36})$$

Since (B.34) and (B.36) constitute the equilibrium, an increase in  $P_1$  will lead to an increase in

$W_1$  so that  $c(P_1, W_1) = \bar{c}$ . Then other labor market variables, e.g.,  $V_1$ , remain the same. Therefore, in this case, wage rises to compensate higher costs of living so that real wage stays constant, and real wage rigidity naturally arises as optimal decisions of firms.

**Proposition 6** ( $\kappa(W_1) = \kappa \cdot W_1$ ) *A rise in cost-of-living is exactly compensated by the same rate of increase in wage in equilibrium, and labor market equilibrium outcomes remain the same. The result does not depend on  $\lambda_{EE}$ , the on-the-job search intensity. Therefore, real wage rigidity naturally arises as optimal decisions of firms.*

**Case 2:**  $\kappa(W_1) = \kappa$  In this case, (B.35) becomes

$$\bar{N} = \frac{\kappa}{W_1} \left\{ V_1 \left[ \underbrace{\gamma \left( \frac{\frac{1}{2}\phi_{E,1}}{\frac{1}{2}\phi_{E,1} + c(P_1, W_1)\phi_{U,1}} \right)}_{\equiv \varepsilon_{11}} \right] + \underbrace{\frac{\frac{\gamma}{2}(1 - c(P_1, W_1))\lambda_{EU}\bar{N}}{(\frac{1}{2}\phi_{E,1} + c(P_1, W_1)\phi_{U,1})g(\theta_1)}}_{\equiv \varepsilon_{21}} \right\}. \quad (\text{B.37})$$

If, as in the above case,  $W_1$  rises at the same rate as  $P_1$  so that  $c(P_1, W_1)$  does not change, then (B.37) is not satisfied as its left hand side becomes smaller than  $\bar{N}$ . Thus, we can infer that in this case, the wage response would be generically smaller than the price increase. In order to obtain sharper results, we log-linearize (B.34) and obtain

$$-\frac{\bar{c}}{1 - \bar{c}}\check{c} = \underbrace{\frac{f'(\bar{\theta}_1)\bar{\theta}_1}{f(\bar{\theta}_1)}}_{\equiv \varepsilon_{f,\theta}}\check{\theta}_1 + \check{c} \quad (\text{B.38})$$

with

$$\check{c} = \frac{\bar{c}_P \bar{P}_1}{\bar{c}}\check{P}_1 + \frac{\bar{c}_W \bar{W}_1}{\bar{c}}\check{W}_1. \quad (\text{B.39})$$

Equations (B.38) and (B.39) yield

$$\check{\theta}_1 = -\frac{1}{(1 - \bar{c})\varepsilon_{f,\theta}} \left( \frac{\bar{c}_P \bar{P}_1}{\bar{c}}\check{P}_1 + \frac{\bar{c}_W \bar{W}_1}{\bar{c}}\check{W}_1 \right). \quad (\text{B.40})$$

We also log-linearize (B.37) and obtain<sup>12</sup>

$$0 = -\check{W}_1 + \frac{\bar{\varepsilon}_{11}}{\bar{\varepsilon}_{11} + \bar{\varepsilon}_{21}} \left[ \check{\theta}_1 - \frac{\bar{c}\phi_{U,1}}{\frac{1}{2}\phi_{E,1} + \bar{c}\phi_{U,1}}\check{c} \right] + \frac{\bar{\varepsilon}_{21}}{\bar{\varepsilon}_{11} + \bar{\varepsilon}_{21}} \left[ -\frac{\bar{c}}{1 - \bar{c}}\check{c} - \frac{\bar{c}\phi_{U,1}}{\frac{1}{2}\phi_{E,1} + \bar{c}\phi_{U,1}}\check{c} - \underbrace{\frac{g'(\bar{\theta}_1)\bar{\theta}_1}{g(\bar{\theta}_1)}}_{\equiv \varepsilon_{g,\theta} > 0}\check{\theta}_1 \right]. \quad (\text{B.41})$$

<sup>12</sup>Again, we use  $\check{\theta}_1 = \check{V}_1$  as  $\theta_1$  and  $V_1$  are proportional and  $\lambda_{EE}\bar{N} + 1 - \bar{N}$  is constant.

Therefore, from (B.40) and (B.41), we obtain

$$\begin{aligned} \check{W}_1 & \left[ 1 + \left\{ \frac{\bar{\varepsilon}_{11}}{\bar{\varepsilon}_{11} + \bar{\varepsilon}_{21}} \left( \frac{\bar{c}\phi_{U,1}}{\frac{1}{2}\phi_{E,1} + \bar{c}\phi_{U,1}} + \frac{1}{(1-\bar{c})\varepsilon_{f,\theta}} \right) + \frac{\bar{\varepsilon}_{21}}{\bar{\varepsilon}_{11} + \bar{\varepsilon}_{21}} \left( \frac{\bar{c}}{1-\bar{c}} + \frac{\bar{c}\phi_{U,1}}{\frac{1}{2}\phi_{E,1} + \bar{c}\phi_{U,1}} + \frac{\varepsilon_{g,\theta}}{(1-\bar{c})\varepsilon_{f,\theta}} \right) \right\} \frac{\bar{c}_W \bar{W}_1}{\bar{c}} \right] \\ & = \left\{ \frac{\bar{\varepsilon}_{11}}{\bar{\varepsilon}_{11} + \bar{\varepsilon}_{21}} \left( \frac{\bar{c}\phi_{U,1}}{\frac{1}{2}\phi_{E,1} + \bar{c}\phi_{U,1}} + \frac{1}{(1-\bar{c})\varepsilon_{f,\theta}} \right) + \frac{\bar{\varepsilon}_{21}}{\bar{\varepsilon}_{11} + \bar{\varepsilon}_{21}} \left( \frac{\bar{c}}{1-\bar{c}} + \frac{\bar{c}\phi_{U,1}}{\frac{1}{2}\phi_{E,1} + \bar{c}\phi_{U,1}} + \frac{\varepsilon_{g,\theta}}{(1-\bar{c})\varepsilon_{f,\theta}} \right) \right\} \left( -\frac{\bar{c}_P \bar{P}_1}{\bar{c}} \right) \check{P}_1. \end{aligned}$$

If we define

$$\begin{aligned} d_W & \equiv \frac{\bar{\varepsilon}_{11}}{\bar{\varepsilon}_{11} + \bar{\varepsilon}_{21}} \left( \frac{\bar{c}\phi_{U,1}}{\frac{1}{2}\phi_{E,1} + \bar{c}\phi_{U,1}} + \frac{1}{(1-\bar{c})\varepsilon_{f,\theta}} \right) + \frac{\bar{\varepsilon}_{21}}{\bar{\varepsilon}_{11} + \bar{\varepsilon}_{21}} \left( \frac{\bar{c}}{1-\bar{c}} + \frac{\bar{c}\phi_{U,1}}{\frac{1}{2}\phi_{E,1} + \bar{c}\phi_{U,1}} + \frac{\varepsilon_{g,\theta}}{(1-\bar{c})\varepsilon_{f,\theta}} \right) \\ & = \underbrace{\frac{\bar{c}\phi_{U,1}}{\frac{1}{2}\phi_{E,1} + \bar{c}\phi_{U,1}}}_{\equiv d_{W,1}} + \underbrace{\frac{\bar{\varepsilon}_{11}}{\bar{\varepsilon}_{11} + \bar{\varepsilon}_{21}} \frac{1}{(1-\bar{c})\varepsilon_{f,\theta}} + \frac{\bar{\varepsilon}_{21}}{\bar{\varepsilon}_{11} + \bar{\varepsilon}_{21}} \left( \frac{\bar{c}}{1-\bar{c}} + \frac{\varepsilon_{g,\theta}}{(1-\bar{c})\varepsilon_{f,\theta}} \right)}_{\equiv d_{W,2}} > 0 \end{aligned} \quad (\text{B.42})$$

then because at the steady state we have<sup>13</sup>

$$\frac{\bar{c}_W \bar{W}_1}{\bar{c}} = -\frac{\bar{c}_P \bar{P}_1}{\bar{c}} = \frac{\gamma}{1 + \bar{\zeta}^\gamma} = \gamma(1 - \bar{c}),$$

the wage response  $\check{W}_1$  is given by

$$\check{W}_1 = \frac{d_W}{\frac{1}{\gamma(1-\bar{c})} + d_W} \check{P}_1 < \check{P}_1, \quad (\text{B.43})$$

which is increasing in  $d_W$ . From (B.39) and (B.43),  $\check{\theta}_1 > 0$  follows, i.e., labor market becomes tighter. This result is summarized in the following Proposition 7.

**Proposition 7** *When  $\kappa(W_1) = \kappa$ , i.e.,  $\kappa(W_1)$  is a constant function, wage rises in response to a cost-of-living shock, but the rate of wage increase is lower than that of price aggregator, i.e.,  $\check{W}_1 < \check{P}_1$ . As a result, labor market becomes tighter, i.e.,  $\check{\theta}_1 > 0$ .*

**Role of on-the-job search intensity  $\lambda_{EE}$**  At the steady state,  $\frac{1}{1+\bar{\zeta}^\gamma} \lambda_{EU} \simeq 0$  under our calibration, and  $\frac{\bar{\varepsilon}_{21}}{\bar{\varepsilon}_{11} + \bar{\varepsilon}_{21}} \simeq 0$  with  $\frac{\bar{\varepsilon}_{11}}{\bar{\varepsilon}_{11} + \bar{\varepsilon}_{21}} \simeq 1$ . Then from (B.42),

$$d_W \simeq \underbrace{\frac{\bar{c}\phi_{U,1}}{\frac{1}{2}\phi_{E,1} + \bar{c}\phi_{U,1}}}_{\equiv d_{W,1}} + \underbrace{\frac{1}{(1-\bar{c})\varepsilon_{f,\theta}}}_{\equiv d_{W,2}},$$

which is decreasing in  $\lambda_{EE}$  as  $\phi_{E,1}$  falls and  $\phi_{U,1}$  increases. Therefore, we can see from (B.43) that wage rises less under higher  $\lambda_{EE}$ . This result is summarized by the next Proposition 8.

<sup>13</sup>We assume that at the steady state,  $c(\bar{P}_1, \bar{W}_1) = \bar{c} = \frac{\bar{\zeta}^\gamma}{1+\bar{\zeta}^\gamma}$ .

**Proposition 8** *Equilibrium wage rises less in response to a cost-of-living shock, under higher on-the-job search intensity  $\lambda_{EE}$ .*

**Case 3:**  $\kappa(W_1) = \kappa W_1$  **with nominal wage stickiness** Now we go back to the first **Case 1** where  $\kappa(W_1)$  is linear in  $W_1$ , but incorporate nominal wage rigidity à la **Rotemberg (1982)**. Firm  $i$  solves:

$$J(\bar{N}) = \max_{V_1^i, W_1^i} \bar{P}_Y N_1^i - W_1^i N_1^i - \underbrace{\kappa(W_1)}_{\equiv \kappa W_1} \cdot V_1^i - \underbrace{\frac{\psi^W}{2} \left( \frac{W_1^i}{\bar{W}} - 1 \right)^2 \bar{W} N_1^i}_{\text{Wage changing cost}} + \frac{1}{1+\rho} J(N_1^i) \quad (\text{B.44})$$

subject to

$$N_1^i = (1 - S(W_1^i | W_1)) \bar{N} + R(W_1^i | W_1) V_1^i. \quad (\text{B.45})$$

Solving (B.44) subject to (B.45) with  $W_1^i = W_1$  and  $N_1^i = \bar{N}$  yields

$$\bar{N} \left( 1 + \psi^W \frac{W_1 - \bar{W}}{\bar{W}} \right) = \frac{\kappa \cancel{W_1}}{\cancel{W_1}} \left\{ \underbrace{V_1 \left[ \gamma \left( \frac{\frac{1}{2} \phi_{E,1}}{\frac{1}{2} \phi_{E,1} + c(P_1, W_1) \phi_{U,1}} \right) \right]}_{\equiv \varepsilon_{11}} + \underbrace{\frac{\frac{\gamma}{2} (1 - c(P_1, W_1)) \lambda_{EU} \bar{N}}{(\frac{1}{2} \phi_{E,1} + c(P_1, W_1) \phi_{U,1}) g(\theta_1)}}_{\equiv \varepsilon_{21}} \right\}, \quad (\text{B.46})$$

which in log-linear form becomes

$$\psi^W \check{W}_1 = \frac{\bar{\varepsilon}_{11}}{\bar{\varepsilon}_{11} + \bar{\varepsilon}_{21}} \left[ \check{\theta}_1 - \frac{\bar{c} \phi_{U,1}}{\frac{1}{2} \phi_{E,1} + \bar{c} \phi_{U,1}} \check{c} \right] + \frac{\bar{\varepsilon}_{21}}{\bar{\varepsilon}_{11} + \bar{\varepsilon}_{21}} \left[ -\frac{\bar{c}}{1 - \bar{c}} \check{c} - \frac{\bar{c} \phi_{U,1}}{\frac{1}{2} \phi_{E,1} + \bar{c} \phi_{U,1}} \check{c} - \underbrace{\frac{g'(\bar{\theta}_1) \bar{\theta}_1}{g(\bar{\theta}_1)}}_{\equiv \varepsilon_{g,\theta}} \check{\theta}_1 \right]. \quad (\text{B.47})$$

With (B.40) and (B.47), in equilibrium, the equilibrium wage response  $\check{W}_1$  to cost-of-living shock  $\check{P}_1$  is given by

$$\check{W}_1 = \frac{d_W}{\psi^W \frac{1}{\gamma(1-\bar{c})} + d_W} \check{P}_1 < \check{P}_1, \quad (\text{B.48})$$

and Propositions 7 and 8 holds as well in this case. Again, note that **Case 3** (which is the case in our dynamic stochastic general equilibrium model in Section 3, with  $\chi = 0$ ) generate similar results to **Case 2**, where  $\kappa(\cdot)$  is a constant function.

### B.3 Variable On-the-Job Search Intensity

Following Appendix E, we now assume that on-the-job probability  $\lambda_{EE}$  at  $t = 1$  is following

$$\lambda_{EE}(P_1, W_1) \equiv \bar{\lambda}_{EE} \left( \frac{\bar{W}_1}{\bar{P}_1} \right)^m \left( \frac{W_1}{P_1} \right)^{-m} \quad (\text{B.49})$$

with  $m = 4$ . A cost-of-living shock raises  $\lambda_{EE,1}$ . Now from

$$\phi_{E,1} = \frac{\lambda_{EE}\bar{N}}{\lambda_{EE}\bar{N} + 1 - \bar{N}}, \quad \phi_{U,1} = \frac{1 - \bar{N}}{\lambda_{EE}\bar{N} + 1 - \bar{N}}, \quad \theta_1 = \frac{V_1}{\lambda_{EE}\bar{N} + 1 - \bar{N}}, \quad (\text{B.50})$$

we see higher  $\lambda_{EE,1}$  raises  $\phi_{E,1}$  and lowers  $\phi_{U,1}$ , i.e., more of job seekers are on-the-job searchers. We start from the equilibrium conditions with  $\kappa(W_1) = \kappa$ :<sup>14</sup>

$$N_1 = \frac{\kappa}{W_1} \left\{ \underbrace{(\lambda_{EE}\bar{N} + 1 - \bar{N})}_{=V_1} \underbrace{\theta_1 \left[ \gamma \left( \frac{\frac{1}{2}\phi_{E,1}}{\frac{1}{2}\phi_{E,1} + \frac{\xi^\gamma}{1+\xi^\gamma}\phi_{U,1}} \right) \right]}_{\equiv \varepsilon_{11}} + \underbrace{\frac{\frac{\gamma}{2} \frac{1}{1+\xi^\gamma} \lambda_{EU}\bar{N}}{\left( \frac{1}{2}\phi_{E,1} + \frac{\xi^\gamma}{1+\xi^\gamma}\phi_{U,1} \right) g(\theta_1)}}_{\equiv \varepsilon_{21}} \right\}, \quad (\text{B.51})$$

and

$$\begin{aligned} N_1 &= \bar{N} - \bar{N} \frac{1}{1 + \xi^\gamma} \lambda_{EU} + g(\theta_1) V_1 \frac{\xi^\gamma}{1 + \xi^\gamma} \phi_{U,1} \\ &= \bar{N} - \bar{N} \frac{1}{1 + \xi^\gamma} \lambda_{EU} + f(\theta_1) \frac{\xi^\gamma}{1 + \xi^\gamma} (1 - \bar{N}). \end{aligned} \quad (\text{B.52})$$

**Price stickiness** In contrast to Appendices B.1 and B.2 where we assume fully rigid prices, we assume a flexible form of price stickiness: in contrast to increase in  $W_1$ , service price  $P_{Y,1}$  increases to some degree. More specifically, we assume  $\check{P}_{Y,1} = d_P \check{W}_1$ , with  $d_P > 0$ , where  $\check{P}_{Y,1}$  and  $\check{W}_1$  are log-deviations from the steady state levels.  $d_P = 0$  corresponds to rigid prices.

Since  $P_{Y,1}N_1 = \bar{P}_Y\bar{Y}$  holds due to the household's equal expenditure under pegged monetary policy, we know

$$\check{N}_1 = -\check{P}_{Y,1} = -d_P \check{W}_1 = \frac{1}{1 + \xi^\gamma} \lambda_{EU} \underbrace{\varepsilon_{f,\theta}}_{>0} \check{\theta}_1 \quad (\text{B.53})$$

where the last equality is derived from (B.52). From (B.53), we can see that if we have  $\check{W}_1 > 0$  in equilibrium in response to a cost-of-living shock, i.e.,  $\check{P}_1 > 0$ , then we need to have  $\check{\theta}_1 < 0$ , i.e., labor market becomes less tight. With lower  $\theta_1$ , wage  $\check{W}_1$  rises less in response to  $\check{P}_1 > 0$  in (B.51), as  $\theta_1$  appears in  $\varepsilon_{11}$  and  $g(\theta_1)$  is decreasing in  $\theta_1$ : less tight labor market means that firms need not raise wage as much to attract job seekers and potential leavers.

By log-linearizing (B.50), we obtain

$$\check{\phi}_{E,1} = \check{\phi}_{U,1} \check{\lambda}_{EE}, \quad \check{\phi}_{U,1} = -\check{\phi}_{E,1} \check{\lambda}_{EE} \quad (\text{B.54})$$

with  $\check{\lambda}_{EE} = -m(\check{W}_1 - \check{P}_1)$ . Linearizing (B.51) yields:

<sup>14</sup>From Appendices B.1 and B.2, we know that  $\kappa(W_1) = \kappa$  (i.e., constant) generates similar results to our specification in Section 3 of  $\kappa(W_1) = \kappa W_1$  (i.e., linear) with nominal wage stickiness.



$$\check{N}_1 = -\check{W}_1 + \frac{\bar{\varepsilon}_{11}}{\bar{\varepsilon}_{11} + \bar{\varepsilon}_{21}} [\bar{\phi}_{E,1} \check{\lambda}_{EE} + \check{\theta}_1 + (1 - \chi) \check{\lambda}_{EE}] - \frac{\bar{\varepsilon}_{21}}{\bar{\varepsilon}_{11} + \bar{\varepsilon}_{21}} [\chi \check{\phi}_{E,1} + (1 - \chi) \check{\phi}_{U,1} - \varepsilon_{g,\theta} \check{\theta}_1], \quad (\text{B.55})$$

where

$$\chi \equiv \frac{\frac{1}{2} \bar{\phi}_{E,1}}{\frac{1}{2} \bar{\phi}_{E,1} + \frac{\bar{\xi}^\gamma}{1 + \bar{\xi}^\gamma} \bar{\phi}_{U,1}}.$$

Combining (B.53), (B.54), and (B.55) with  $\check{\lambda}_{EE} = -m(\check{W}_1 - \check{P}_1)$  and approximating  $\frac{\bar{\varepsilon}_{21}}{\bar{\varepsilon}_{11} + \bar{\varepsilon}_{21}} \simeq 0$  with  $\frac{\bar{\varepsilon}_{11}}{\bar{\varepsilon}_{11} + \bar{\varepsilon}_{21}} \simeq 1$  as before, we obtain

$$\check{W}_1 = \frac{m(\bar{\phi}_{EE} + 1 - \chi)}{1 - d_P + m(\bar{\phi}_{EE} + 1 - \chi) + \frac{d_P}{\frac{\lambda_{EU}}{1 + \bar{\xi}^\gamma} \varepsilon_{f,\theta}}} \check{P}_1 > 0. \quad (\text{B.56})$$

**Interpretation** Under fully rigid prices, i.e.,  $d_P = 0$ , then we would have

$$\check{W}_1 = \frac{m(\bar{\phi}_{EE} + 1 - \chi)}{1 + m(\bar{\phi}_{EE} + 1 - \chi)} \check{P}_1 > 0.$$

with  $\check{\theta}_1 = 0$ : no change in tightness. When employees engage in intensified on-the-job searches, firms offer more vacancies so that labor market tightness  $\theta_1$  remains the same: it is because under fully rigid prices, labor demand remains unchanged in response to a cost-of-living shock.

Under sticky prices following (B.53),  $\check{\theta}_1 < 0$  and  $\check{W}_1 > 0$  hold from (B.56). In equilibrium, firms raise service price in response to a cost-of-living shock, leading to lower service and labor demand. Since workers have higher intensity of on-the-job search, it reduces the market tightness  $\theta_1$ . It in turn lowers the incentive of firms to raise wage to attract job seekers, resulting in muted wage responses: this effect is represented by  $\frac{d_P}{\frac{\lambda_{EU}}{1 + \bar{\xi}^\gamma} \varepsilon_{f,\theta}}$ .

On the other hand, a lower labor demand implies the marginal cost of wage increase in terms of wage bills (e.g., \$ increase in wage implies all workers, new hires and incumbents, benefit from it) is lower from each firm's perspective, and raises firms' incentive to raise wage: this effect is represented by  $d_P$  term in (B.56). In effect, the first effect dominates the second effect,<sup>15</sup> and we have muted wage increase under endogenous on-the-job search intensity following (B.49).

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<sup>15</sup>Remember  $\frac{\lambda_{EU}}{1 + \bar{\xi}^\gamma}$  is small.

## C Analytical Results for Pass-Through Across Models

This section analyzes the pass-through of prices to wages in response to a temporary decline in the endowment good  $X_t$ , assuming that monetary policy stabilizes the business cycle holding  $N_t$  fixed. We consider the following variations of the model which alter the labor block in Section 3. We work through the case of (i) a sticky-price, flexible-wage New Keynesian model where workers supply labor in a frictionless market, (ii) a flexible price, sticky-wage New Keynesian model where wages are set by unions as in Erceg et al. (2000); Galí et al. (2012); and (iii) our benchmark model in Section 3.

As in the paper, we assume throughout that consumption is a CES bundle of services  $Y_t$ , produced with labor, and goods  $X_t$  which households receive as an endowment (equivalently, perfectly competitive firms receive  $X_t$  and sell it for pure profit, rebating the proceeds to households as dividends). We have

$$\begin{aligned} C_t &= \left( \alpha_Y^{\frac{1}{\eta}} Y_t^{\frac{\eta-1}{\eta}} + \alpha_X^{\frac{1}{\eta}} X_t^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}} \\ P_t &= \left( \alpha_Y P_{y,t}^{1-\eta} + \alpha_X P_{x,t}^{1-\eta} \right)^{\frac{1}{1-\eta}} \end{aligned} \quad (\text{C.1})$$

### C.1 Sticky-Price, Flexible Wage New Keynesian Model

We assume here that  $P_{y,t}$  is set subject to some nominal rigidities as in the benchmark model in Section 3 (i.e. Rotemberg adjustment costs), but where firms hire labor in a standard spot market with flexible nominal wage  $W_t$ , so there is no unemployment. The household chooses paths for consumption and labor (and zero net supply nominal bonds) to maximize:

$$\sum_{t=0}^{\infty} \beta^t \left( \frac{\sigma}{\sigma-1} C_t^{\frac{\sigma-1}{\sigma}} - \frac{1}{1+\frac{1}{v}} N_t^{1+\frac{1}{v}} \right)$$

subject to the budget constraint,

$$C_t = \frac{D_t}{P_t} - \frac{B_t}{P_t} + \frac{(1+i_{t-1,t})B_{t-1}}{P_t} + W_t N_t.$$

This yields the following intratemporal optimality condition:

$$N_t^{\frac{1}{v}} = W_t \frac{C_t^{\frac{-1}{\sigma}}}{P_t}$$

So in our model with Neoclassical labor supply, the following decomposition must hold to first order:

$$\begin{aligned}\frac{1}{v}\check{N}_t &= \check{W}_t - \frac{1}{\sigma}\check{C}_t - \check{P}_t \\ \frac{1}{v}\check{N}_t &= \check{W}_t - \frac{1}{\sigma}\underbrace{(\check{P}_t + \check{C}_t)}_{=\widetilde{\check{P}_t\check{C}_t}} + \frac{1-\sigma}{\sigma}\check{P}_t\end{aligned}$$

So that when monetary policy fixes  $\check{N}_t = 0$ , we have

$$\begin{aligned}\check{W}_t &= \frac{1}{\sigma}\widetilde{\check{P}_t\check{C}_t} + \frac{\sigma-1}{\sigma}\check{P}_t \\ \frac{\widetilde{\check{W}_t}}{P_{y,t}} &= \frac{1}{\sigma}\frac{\widetilde{\check{P}_t\check{C}_t}}{P_{y,t}} + \frac{\sigma-1}{\sigma}\frac{\check{P}_t}{P_{y,t}}\end{aligned}\tag{C.2}$$

Now we can write the two right hand side terms as functions of the shock  $X_t$ : first note that CES demand implies

$$\frac{P_t}{P_{y,t}} = \left(\frac{Y_t}{C_t}\right)^{\frac{1}{\eta}} \alpha_y^{-\frac{1}{\eta}}.$$

Under our experiment where monetary policy stabilizes  $N_t$ , and hence  $Y_t$ , from (C.1) we have to first order that  $\check{C}_t = \alpha_x^{\frac{1}{\eta}} \left(\frac{X}{C}\right)^{\frac{\eta-1}{\eta}} \check{X}_t$  and

$$\frac{\widetilde{\check{P}_t\check{C}_t}}{P_{y,t}} = -\frac{1}{\eta} \left( \alpha_x^{\frac{1}{\eta}} \left(\frac{X}{C}\right)^{\frac{\eta-1}{\eta}} \check{X}_t \right)\tag{C.3}$$

so that when  $X_t$  falls, the price of the aggregate consumption bundle in terms of the labor-intensive good,  $\frac{P_t}{P_{y,t}}$  goes up (i.e. we need more units of  $Y$ -good to buy one unit of  $C$ -good). We also have for aggregate spending,

$$\frac{\widetilde{\check{P}_t\check{C}_t}}{P_{y,t}} = \frac{\eta-1}{\eta} \left( \alpha_x^{\frac{1}{\eta}} \left(\frac{X}{C}\right)^{\frac{\eta-1}{\eta}} \check{X}_t \right)$$

So that aggregate nominal spending may either rise, or fall, depending on  $\eta$ . With Cobb-Douglas utility with  $\eta = 1$ , nominal spending is unchanged. Consider the effects of negative shock to  $X_t$  on the wage when monetary policy holds  $N_t$  fixed, and examine equation (C.2):

- We can see when  $\eta = \sigma = 1$ , the wage denoted in units of the service good or numeraire, i.e.,  $\frac{W_t}{P_{y,t}}$  remains unchanged.
- With Cobb-Douglas preferences,  $\eta = 1$ , we see from (C.3) that the relative price still rises, so everything depends on  $\sigma$ : if  $\sigma > 1$ , as is commonly assumed in macro applications, then

there is positive pass through from prices to wages.

- If  $\sigma = 1$ ,  $\eta < 1$  then there is positive pass-through from prices to wages. When it is hard to substitute away from  $X_t$ , and total expenditure rises.

**Discussion:** Even in a perfectly competitive labor market, workers' wages can respond to an increased cost of living even when their productivity is unaffected by the shock. The sign and magnitude of the response depends on the strength of income and substitution effects (governed by  $\sigma$ ) and wealth effects (governed by  $\eta$ ) stemming from a change in  $P_{x,t}X_t$ , where we obtain

$$\frac{\widetilde{P_{x,t}X_t}}{P_{y,t}} = \underbrace{\frac{\eta - 1}{\eta}}_{<0} \underbrace{\check{X}_t}_{<0} > 0 \quad (\text{C.4})$$

when  $\eta < 1$ , so that households' non-labor income from endowment good  $X_t$  increases and so does their wealth, possibly lowering labor supply due to the wealth effect.

In specifications where  $\sigma \geq 1$  and  $\eta < 1$ , the decline in  $X_t$  makes workers prefer leisure; thus if monetary policy is holding leisure (and labor) fixed, the wage must rise in equilibrium.

## C.2 Flexible Price, Sticky Wage New Keynesian Model

We now consider the effect of a temporary fall in  $X_t$  when wages are sticky as in [Erceg et al. \(2000\)](#), again analyzing the shock under the assumption that monetary policy stabilizes aggregate labor output  $N_t$ . Specifically, we assume that households now supply multiple types of labor; unions set wages for each type to maximize household utility subject to facing CES demand for each type from a "labor packer" which packages each labor type  $N_t(i)$  into aggregate labor  $N_t = \left( \int_0^1 N_t(i)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}$  which is purchased at wage  $W_t$  by services firms—and in our setting, combined with  $X_t$  to form consumption  $C_t$ . Wages are sticky because unions only occasionally receive the chance to reset their wage.

Households now maximize the following: specializing to log utility with  $\sigma = 1$ ,

$$\sum_{t=0}^{\infty} \beta^t \left( \log C_t - \int_0^1 \frac{1}{1 + \frac{1}{v}} N_t(i)^{1 + \frac{1}{v}} di \right),$$

subject to the budget constraint,  $C_t = \frac{D_t}{P_t} - \frac{B_t}{P_t} + \frac{(1+i_{t-1})B_{t-1}}{P_t} + W_t N_t$ . Under these assumptions, we can derive the following standard wage Phillips curve (see e.g., [Galí, 2011](#); [Galí et al., 2012](#)):

$$\Pi_t^w = \beta E\{\Pi_{t+1}^w\} + \lambda \left( -\check{W}_t + \widetilde{P_t C_t} + \frac{1}{v} \check{N}_t \right).$$

for some constant  $\lambda > 0$ . Analyzing this case is only harder than the flexible wage case of Section

C.1 because of the presence of the forward-looking term  $\Pi_{t+1}^w$ . To make progress, rewrite this in relative price terms:

$$\check{\Pi}_t^w = \beta E\{\check{\Pi}_{t+1}^w\} + \lambda \left( -\frac{\widetilde{W}_t}{P_{y,t}} + \frac{\widetilde{P_t C_t}}{P_{y,t}} + \frac{1}{v} \check{N}_t \right). \quad (\text{C.5})$$

Consider the household's budget constraint in equilibrium: using the fact that bonds are in zero net supply, and expanding the  $D_t$  term by denoting  $d_t$  the dividends potentially paid by services firms (zero, if prices are flexible and they are perfectly competitive), this is

$$P_t C_t = W_t N_t + P_{x,t} X_t + d_t.$$

How will each term in this equation respond to an  $X_t$  shock? Rewriting, we have:

$$\frac{P_t C_t}{P_{y,t}} = \frac{W_t}{P_{y,t}} N_t + \frac{P_{x,t}}{P_{y,t}} X_t + \frac{d_t}{P_{y,t}}.$$

Now recall that given fixed  $N_t$  CES demand yields:

$$\frac{\widetilde{P_t C_t}}{P_{y,t}} = \frac{\eta - 1}{\eta} \left( \alpha_x^{\frac{1}{\eta}} \left( \frac{X}{C} \right)^{\frac{\eta-1}{\eta}} \check{X}_t \right)$$

If  $\eta < 1$ , then  $\frac{P_0 \check{C}_0}{P_{y,0}}$  rises in response to a negative  $X_0$  shock and  $\frac{P_t \check{C}_t}{P_{y,t}}$  is zero in other periods ( $t > 0$ ) when there is no shock. From (C.4), we see the middle term, in its deviation from steady state, is zero when there is no shock. Thus, we obtain for all  $t > 0$ :

$$0 = \frac{WN}{PC} \frac{\widetilde{W}_t}{P_{y,t}} + \frac{d}{PC} \check{d}_t.$$

If there are no time-varying profits, e.g., if prices are flexible, then we have that  $\check{d}_t = 0$  and thus  $\frac{\widetilde{W}_t}{P_{y,t}} = 0$ . As a result, the forward looking wage Phillips curve (C.5) implies  $\check{\Pi}_t^w = 0$  for all  $t > 0$ , and the wage Phillips curve for the initial period greatly simplifies to

$$\pi_0^w = \lambda (-\check{W}_0 + P_0 \check{C}_0)$$

Given that wage inflation is defined as  $\check{\Pi}_t^w = \check{W}_t - \check{W}_{t-1}$  with  $\check{W}_{-1} = 0$ , we can write

$$\check{W}_0 = \lambda (-\check{W}_0 + P_0 \check{C}_0)$$

Divide by  $P_{y,0}$  to apply our above results for  $\frac{P_0 \check{C}_0}{P_{y,0}}$  and find that when  $\eta < 1$ , the right hand side is positive for a negative  $X$  shock, and we thus have positive pass-through to wages.

**Discussion:** As discussed in above Section C.1, depending on the strength of income, substitution, and wealth effects (governed by  $\eta$ ), wages can either rise or fall in response to the shock. Here for  $\sigma = 1$ , we again find that  $\eta < 1$  implies pass through from prices to wages in response to the  $X_t$  shock. The analysis with sticky wages is not that different from the flexible wage case.

### C.3 Wage Posting Model with On-the-job Search and Nominal Rigidities

This section analyzes the baseline model in Section 3 to elaborate the conditions under which there is no pass-through from prices to wages. In our benchmark model, there is no pass-through from prices to wages: in response to an  $X_t$  shock, when monetary policy perfectly stabilizes  $N_t$ , it also perfectly stabilizes wage inflation. We demonstrate both that this relies on the assumption that vacancy costs are denominated in labor. If vacancy costs are denominated in final goods, then headline inflation passes through into wages, even when monetary policy stabilizes the labor market. The title of this section reflects the fact that it does not matter for the analysis here whether prices or wages are sticky, so long as the presence of nominal rigidities allows monetary authorities to stabilize  $N_t$ .

To demonstrate the role of how adjustment costs are denominated, we generalize the firms problem slightly: let  $P_t^V$  denote the nominal price in which vacancy costs are denominated, and let  $P_t^\psi$  be the nominal price in which wage adjustment costs are denominated (which we will show will not matter). Then firm  $j$  maximizes present-discounted revenues, less costs (abstracting from price adjustment costs, which do not affect the wage Phillips curve), given by

$$\sum_{t=0}^{\infty} \left( \frac{1}{1+\rho} \right)^t \left( P_{y,t}^j Y_t^j - W_t^j N_t^j - c(V_t^j)^{1+\chi} (N_{t-1}^j)^{-\chi} P_t^V - \frac{\psi^w}{2} \left( \frac{W_t^j}{W_{t-1}^j} - 1 \right)^2 N_t P_t^\psi \right),$$

subject to the law of motion for employment,

$$N_t^j = (1 - S(W_t^j)) N_{t-1}^j + V_t^j R(W_t^j)$$

and some production and demand functions for  $Y_t^j$ . Combining the firm's first order conditions for  $V_t^j$  and  $W_t^j$ , and assuming a symmetric equilibrium, yields a nonlinear wage Phillips curve:

$$\begin{aligned} \psi^w (\Pi_t^w - 1) \Pi_t^w P_t^\psi + W_t = P_t^V c(1 + \chi) \left( \frac{V_t}{N_{t-1}} \right)^\chi \left( \frac{V_t}{N_t} \varepsilon_{R, W_t} - \frac{N_{t-1}}{N_t} \frac{S(W_t)}{R(W_t)} \varepsilon_{S, W_t} \right) \\ + \frac{\psi^w}{1 + \rho} (\Pi_{t+1}^w - 1) \Pi_{t+1}^w \frac{N_{t+1}}{N_t} P_{t+1}^\psi. \end{aligned}$$

Gather the labor market tightness terms in  $Z_t \equiv c(1 + \chi) \left( \frac{V_t}{N_{t-1}} \right)^\chi \left( \frac{V_t}{N_t} \varepsilon_{R, W_t} - \frac{N_{t-1}}{N_t} \frac{S(W_t)}{R(W_t)} \varepsilon_{S, W_t} \right)$  and

log-linearize, defining  $\pi_t^V \equiv \frac{P_t^V}{P_{t-1}^V}$ , let  $\check{\omega}_t \equiv \sum_{s=0}^t (\pi_s^w - \pi_s^V)$ , obtaining

$$\check{\pi}_t^w = \frac{Z P^V}{\psi^w P^\psi} \sum_{s=0}^{\infty} \left( \frac{1}{1+\rho} \right)^s (\check{Z}_{t+s} - \check{\omega}_{t+s}).$$

When monetary policy stabilizes employment, and  $\check{N}_t = 0$ , it does follow that  $\check{Z}_t = 0$ , as proved in Section C.3.1 so the wage Phillips curve reduces to:

$$\check{\pi}_t^w = \frac{Z P^V}{\psi^w P^\psi} \sum_{s=0}^{\infty} \left( \frac{1}{1+\rho} \right)^s (-\check{\omega}_{t+s}).$$

If the cost of posting vacancies is denominated in labor, so  $P_t^V = W_t$ , then  $-\check{\omega}_t = 0$  and monetary policy stabilizes wage growth as well as employment.

### C.3.1 Showing That $\check{N}_t = 0$ implies $\check{Z}_t = 0$

To see this result, first note that holding  $N_t = N$  constant implies that the number of searchers  $S = \lambda_{EE}N + \lambda_{EU}(1 - N)$  is constant; the shares appearing in the definitions of the separation and recruiting rates,  $\phi_{E,t}$  and  $\phi_{U,t}$  in equations (17) and (18), are thus also constant. This means the tightness term  $\theta_t$  is constant so long as  $V_t$  is constant. If  $V_t$  and therefore  $\theta_t$  are constant, then the separation rates and elasticities in  $Z_t$  are also held constant. Ergo, all we must do is show that  $V_t$  is constant.

To do so, write the law of motion for employment when  $N_t = N_{t-1} = N$ , plugging in for the separation and recruiting rates, to yield:

$$\begin{aligned} V_t \cdot R_t &= N \cdot S_t \\ V_t g(\theta_t) \left( \phi_E \frac{1}{2} + \phi_U \left( \frac{\xi^\gamma}{1 + \xi^\gamma} \right) \right) &= N \left[ s + (1-s) \left( \lambda_{EE} f(\theta_t) \frac{1}{2} + \lambda_{EU} \left( \frac{1}{1 + \xi^\gamma} \right) \right) \right] \\ V_t g \left( \frac{V_t}{S} \right) \left( \phi_E \frac{1}{2} + \phi_U \left( \frac{\xi^\gamma}{1 + \xi^\gamma} \right) \right) - N f \left( \frac{V_t}{S} \right) (1-s) \frac{\lambda_{EE}}{2} &= N \left[ s + (1-s) \left( \lambda_{EU} \left( \frac{1}{1 + \xi^\gamma} \right) \right) \right] \end{aligned}$$

Now using our definition for  $g$ , rewrite the left hand side in terms of  $f$ :

$$\begin{aligned} S f \left( \frac{V_t}{S} \right) \left( \phi_E \frac{1}{2} + \phi_U \left( \frac{\xi^\gamma}{1 + \xi^\gamma} \right) \right) - N f \left( \frac{V_t}{S} \right) (1-s) \frac{\lambda_{EE}}{2} &= N \left[ s + (1-s) \left( \lambda_{EU} \left( \frac{1}{1 + \xi^\gamma} \right) \right) \right] \\ f \left( \frac{V_t}{S} \right) &= \frac{N \left[ s + (1-s) \left( \lambda_{EU} \left( \frac{1}{1 + \xi^\gamma} \right) \right) \right]}{S \left( \phi_E \frac{1}{2} + \phi_U \left( \frac{\xi^\gamma}{1 + \xi^\gamma} \right) \right) - N(1-s) \frac{\lambda_{EE}}{2}} \end{aligned}$$

Thus, there is a unique solution for  $V_t = V$  for a given  $N$  (the steady state value). So we conclude

that when monetary policy stabilizes  $N_t$ ,  $V_t$  and  $\theta_t$  are also stabilized, and  $Z_t$  is stabilized.

## D Weight on Vacancies and Unemployment

We start from the log-linearized wage Phillips curve (A.22) we derive in Appendix A.3.1, written in vacancy and unemployment rates :

$$\check{\Pi}_t^w = \phi_V \check{V}_t + \phi_U \check{U}_t + \frac{1}{1+\rho} \check{\Pi}_{t+1}^w \quad (\text{D.1})$$

for some constants  $\phi_V > 0$  and  $\phi_U < 0$ . In Appendix C.3, we prove that the absence of aggregate price inflation in the right hand side of the linearized wage Phillips curve (D.1) is stemming from the fact that the vacancy-creating cost is denominated in labor, not the final good. Based on our calibration in Table 2, we now show how varying the probability of being allowed to search on the job,  $\lambda_{EE}$ , affects the predictions of the model for the relative importance of vacancies, as opposed to unemployment, in the wage Phillips curve.

Since both  $\phi_V > 0$  and  $\phi_U < 0$  are complex collections of model parameters and steady-state values, to consider their relative magnitudes, we proceed numerically, and specialize to particular parameter choices. As we already saw in (A.23), we observe that  $\phi_V$  is much larger in magnitude than  $\phi_U$ . This result turns out to stem both from the presence of on-the-job search ( $\lambda_{EE} > 0$ ) and also from the convexity of vacancy costs ( $\chi > 0$ ). Figure D.1 shows how the relative importance of vacancies in explaining wage growth, represented by the ratio of coefficients in (D.1),  $\left| \frac{\phi_V}{\phi_U} \right|$ , increases monotonically in on-the-job search intensity  $\lambda_{EE}$  under the benchmark calibration  $\chi = 1$  and also when  $\chi = 0$ , or a linear cost of posting a vacancy which is commonly assumed in the search literature. The limit case where  $\chi = 0$  and  $\lambda_{EE} \rightarrow 0$  is of particular interest as a benchmark: as Appendix A.3.1 shows, at the limit where  $\lambda_{EE} \rightarrow 0$  and  $\chi \rightarrow 0$ ,  $\left| \frac{\phi_V}{\phi_U} \right|$  converges to one and wage growth becomes solely a function of market tightness  $\theta_t = \frac{V_t}{U_t}$ <sup>16</sup> following the literature: see e.g., [Gagliardone and Gertler \(2023\)](#).

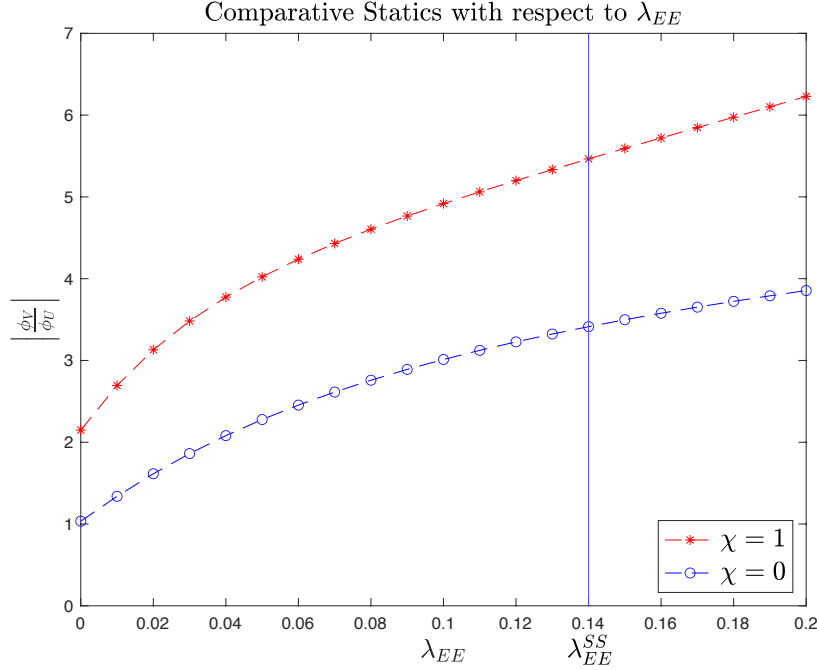
We acknowledge that simply pointing out the coefficient on  $V$  is larger than  $U$  does not technically imply that variations in  $U$  are less important in explaining wage growth: if  $U$  has a much higher variance than  $V$ , it can have a small coefficient while still playing a large role. To show more formally how rising  $\left| \frac{\phi_V}{\phi_U} \right|$  diminishes the importance of unemployment in explaining wage growth, consider the variance decomposition of wage growth in the model under the assumption that we can ignore the inflation expectations term:<sup>17</sup>

<sup>16</sup>With  $\lambda_{EE} = 0$ ,  $\theta_t = \frac{V_t}{S_t} = \frac{V_t}{U_t}$ .

<sup>17</sup>E.g., assuming firms have constant inflation expectations,  $E_t \Pi_{t+1}^w = \Pi_t^w$ , or "adaptive" expectations  $E_t \Pi_{t+1}^w = \Pi_t^w$  (yielding the same expression up to a constant). Alternatively, we can view (D.2) as an approximation when  $\rho$  is high, permitting us to ignore the many covariance cross-terms complicating the expression when solved forward.



Figure D.1: In Economies with More On-the-job Search, Vacancies Matter More in the Wage PC



Notes: The red, starred line plots the effects of a change in on-the-job search intensity  $\lambda_{EE}$ , holding all other model parameters constant at their values in Table 2, on the ratio of the coefficients in equation (D.1):  $\check{\Pi}_t^w = \phi_V \check{V}_t + \phi_U \check{U}_t + \frac{1}{1+\rho} \check{\Pi}_{t+1}^w$ . The blue dotted line repeats the exercise but with  $\chi = 0$ , a linear cost of vacancy posting. The vertical line marks the value for  $\lambda_{EE}$  used in our benchmark calibration. The relative importance of vacancies in explaining wage inflation, compared with unemployment, increases with both  $\lambda_{EE}$  and  $\chi$ .

$$\text{Var}(\check{\Pi}_t^w) = \left(\frac{\phi_V}{\phi_U}\right)^2 \text{Var}(\check{V}_t) + \text{Var}(\check{U}_t) + \underbrace{2\frac{\phi_V}{\phi_U} \text{Cov}(\check{V}_t, \check{U}_t)}_{<0} \quad (\text{D.2})$$

Now consider the exercise in Figure D.1, which increases  $\lambda_{EE}$  holding other parameters constant, raising  $\left|\frac{\phi_V}{\phi_U}\right|$ . Given that the covariance term  $\text{Cov}(\check{V}_t, \check{U}_t)$  in (D.2) is strongly negative (both empirically, and thus also in any reasonably calibrated model), the importance of unemployment in explaining wage growth falls monotonically as we increase the amount of on-the-job search, and convexity of the vacancy costs, in the model.

## E Extension with Variable On-the-Job Search Intensity

Our baseline model features an exogenous, constant on-the-job search probability of  $\lambda_{EE}$ , which we calibrate to match U.S. data. However, it is possible that employed workers may respond to a pure cost-of-living shock by searching more intensely. [Pilossoph and Ryngaert \(2023\)](#) provide evidence that this is indeed the case.

Motivated by their findings, we solve a version of the model where  $\lambda_{EE}$  is assumed to rise along with inflation according to a reduced form, *ad hoc* relationship calibrated to match the results in [Pilossoph and Ryngaert \(2023\)](#). Specifically, we assume:

$$\lambda_{EE,t} = \lambda_{EE,0} \left( \frac{W_t}{P_t} \right)^{-m},$$

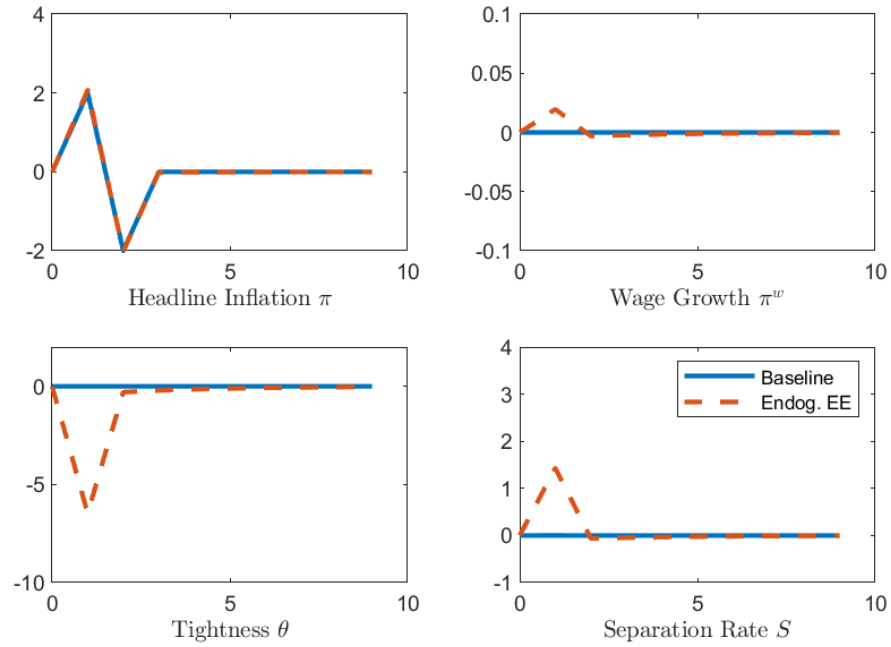
where  $\lambda_{EE,0}$  is chosen to target the same steady-state value for  $\lambda_{EE}$  as in the benchmark model, and  $m = 4$  to match the fact that [Pilossoph and Ryngaert \(2023\)](#) find that in response to a one percentage point increase in inflation expectations (and thus a 1% decline in expected real wages), the probability that an employed worker searches on the job rises by 0.57 percentage points.<sup>18</sup> With a share of 14.9% of employed workers typically searching, this represents a  $(0.0057/0.149) \approx 4$  percent increase in search probability, yielding an elasticity of search probability with respect to expected real wages of -4.

We then revisit the response in the model to a shock to the quantity of the endowment good  $X_t$ . Note that here, there are two contrasting effects of allowing for endogenous on-the-job search probability. In response to the inflationary shock, workers search more which induces firms to raise wages in order to retain workers (more searchers means more workers find jobs they prefer to their current jobs, due to the idiosyncratic preference shocks over workplaces). However, as separation rates rise, so do recruiting rates: with more searchers, tightness falls, and thereby firms can afford to lower wages and still recruit the same number of workers as before. Figure [E.2](#) plots the impulse responses of headline inflation, wage growth, labor market tightness, and the separation rate to the shock to the quantity of endowment good  $X_t$ . We can observe the net effect of the shock is an extremely limited pass-through from cost-of-living to wages: separations and wage growth rise, pushing firms to want to raise wages, but on the other hand tightness  $\theta_t$  falls due to the increasing number of searchers, pushing firms to lower wages. In sum, wages respond positively but modestly in response to the cost of living shock.

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<sup>18</sup>See equation (3) and accompanying Table 3 of [Pilossoph and Ryngaert \(2023\)](#).

Figure E.2: Impulse Response with Endogenous On-the-Job Search



*Notes:* This figure presents the effects of a decreased supply of the endowment good  $X_t$  under a nominal interest rate peg, i.e. the same experiment as in Figure 2, but comparing the benchmark case (solid blue line) of the main text to the case described in Section E where the job-to-job search probability increases along with the price level (dashed red line). In this second model, as on-the-job search rises, separations rise, inducing firms to raise wages to retain workers. But at the same time, tightness  $\theta_t$  falls due to the increasing number of searchers, pushing firms to lower wages. The net effect is the modest increase in wages in the top right panel, so that overall there is very little pass-through from the aggregate price to wages even when the probability of on-the-job search rises in response to lower real wages. Note that the axes are in percent deviations, so the axis for wage growth is comparable to Figure 5.