Managerial Incentives, Financial Innovation, and Risk-Management Policy*

Son Ku Kim† Seoul National University

Seung Joo Lee‡ U.C. Berkeley

Sheridan Titman§ McCombs School of Business

April 19, 2022

Abstract

This paper examines risk management issues from the perspective of a firm run by an effort and risk-averse manager. We show that when shareholders observe manager's risk choice, but not the effort, the optimal compensation contract will direct managers to expose the firm to less risk than they would in the full information environment (e.g., execute costly hedges). Innovations in risk management technology, e.g., the introduction of a futures market, always improves the efficiency of the manager's compensation contract when the risk choice can be observed, and this efficiency gain continues to hold under some circumstances when the manager's risk choice cannot be observed by shareholders. In other cases, however, due to the incentive problems associated with the hedging choice, financial innovation can lower welfare.

Keywords: Agency, Risk management, Hedging

*We thank Stephen Buser, Marc Dordal i Carreras, Kent Daniel, Sudipto Dasgupta, Peter DeMarzo, Nicolae Gârleanu, Yuriy Gorodnichenko, Scott Gibson, Mark Grinblatt, Seung uk Jang, Junekey Jeon, Jin Yong Jung, Shachar Kariv, Chris Shannon, Bill Wilhelm, and seminar participants at Berkeley for helpful discussions.

†sonkukim@snu.ac.kr

‡seungjoo@berkeley.edu

§titman@mail.utexas.edu

1 Introduction

U.S. corporations spend substantial resources on assessing and managing their exposures to various sources of risk not only for their own survival but also for the stability of the entire financial system. As we experienced in the Global Financial Crisis (2008) and the subsequent Great Recession, each corporation's imprudent risk choices lead to its own collapse, while imposing tremendous negative externalities on the aggregate economy. A new set of regulations to properly manage the individual firm's risk exposures and minimize the contagion of risks popped up, including Basel I, II and III accords, afterwords, and interests in benefits of risk management and necessary regulations soared up. Benefits of risk management and the regulations have been one of the most important topics in academic economics and finance and have been described in a number of academic and professional articles.[1]

However, in most cases until recently, those articles have ignored the fact that risk management choices are made not by the value-maximizing firms, but by self-interested managers. In contrast, popular press and the policy institutions are quite concerned with issues relating to the incentives of managers to "properly" manage risk, focusing on a number of highly visible cases where it appears, at least ex post, that managers were using derivatives to speculate rather than hedge.[2] For example, Ben Bernanke, a former chair of the Federal Reserve, stated following the Global Financial Crisis that "compensation practices at some banking organizations have led to misaligned incentives and excessive risk-taking, contributing to bank losses and financial instability.".[3] This feature extends even beyond financial institutions. In a study of gold mining industry, for instance, Tufano (1996) found that the managerial incentives were the most important determinant of corporate derivatives choices.

This paper examines firms' risk management choices within an agency framework, where managers (agents) control not only the effort they expend, which affects the firm's average cash flow, but also the riskiness of the cash flow by choosing real investment projects as well as their positions in derivatives.[4] We assume that shareholders (principals) are unable to directly assess the extent to

[1]For classical literatures, see Smith and Stulz (1985), DeMarzo and Duffie (1991), DeMarzo and Duffie (1995), and Froot et al. (1993) among others. Geczy et al. (1997) examined implications of these theories empirically. Draghi et al. (2003), in particular, emphasized the role of unintended or unanticipated accumulation of large risks as the breeding conditions for financial crises. One interesting point they make is that implicit guarantees that governments extend to banks and other financial institutions might lead to over-accumulation of risks in the system, which aligns with our moral hazard point of view. More recently, Farhi and Tirole (2012) studied banks' collective moral hazard issues in the environment where government's bail-out policies are possible and anticipated.

[2]During 1994 and 1995, there were two well known firms, Barings and Metallgesellschaft, that either went bankrupt or nearly went bankrupt as a result of speculative transactions in financial derivatives. And the Global Financial Crisis (2008) illustrated how big corporations (suck as AIG, Lehman Brothers, etc) had been speculating imprudently using derivatives while being ignorant of their own risk exposures.

[3]Fed press release (2009): https://www.federalreserve.gov/newsevents/pressreleases/bcreg20091022a.htm.

[4]Most agency papers are concerned with the agent's effort choices but not with activities that affect risk exposures,

which the managers are acting in their own interests rather than the shareholders' (e.g., they may be effort averse or equivalently consume unproductive perquisites).

Our analysis indicates that financial innovations (e.g., the introduction of a new derivative contract) that provide managers with greater flexibility to control risk can mitigate some agency problems and thereby improve firm values. However, this added flexibility can create other agency problems if the managers' use of these derivatives cannot be effectively controlled. When this is the case, financial innovation can reduce welfare.

To understand how manager's access to derivative markets affects costs arising from the agency relation, we divide firms' risks into two different components: hedgeable risks and non-hedgeable risks. Basically, a firm's hedgeable risks are related to some market observables such as interest rates, commodity prices, and exchanges rates on which derivative contracts can be written, whereas non-hedgeable risks are risks that cannot be traded in the derivatives market. The total amount of a firm's hedgeable risk depends not only on those market observables but also on the firm's endogenous exposure to those observables (e.g., how cash flows are influenced by changes in oil prices). The firm's hedgeable risk exposure can be amended by its manager's derivative transactions, while its non-hedgeable risks are mainly determined by the manager's real project choices.

In a benchmark case where there is no derivative market, the manager affects the firm's risks only through real investment choices. If the firm's risk exposure to hedgeable risks is observed by both the manager and the shareholders in the benchmark case, the typical dual-agency framework à la Hirshleifer and Suh (1992) applies. Specifically, the manager generally does not choose a real investment project (non-hedgeable risk) that would be preferred by shareholders. Therefore, the manager's optimal compensation contract must be designed not only to incentivize him to take the right effort level but also to induce him to choose the right project from the principal's perspective. We show that the optimal compensation contract either rewards or penalizes output variances depending on whether the manager must be rewarded or deterred from taking riskier projects.

In reality, communication between shareholders and the manager is costly when the firm's risk exposure is observed only by the manager and not by shareholders.[5] Then, what the uninformed shareholders can do at best is to design a wage contract without using any information on the firm's risk exposure, directly or indirectly, and let the informed manager decide both effort and project choice based on the observed risk exposure of the firm. Due to this asymmetric information about the firm's risk exposure, there would inevitably be a welfare loss compared with the benchmark

e.g., Harris and Raviv (1979), Holmström (1979), Shavell (1979), and Grossman and Hart (1983). Other papers consider only the agent's project or risk choice problem, e.g., Ross (1974), Lambert (1986), and Hirshleifer and Thakor (1992). Important exceptions are Hirshleifer and Suh (1992), Sung (1995), Palomino and Prat (2003),Ross (2004), DeMarzo et al. (2013), Hébert (2018) among others, which simultaneously examine the agent's risk and effort choices in either a single-period framework or a continuous time agency framework.

[5]This point was discussed by Laffont and Martimort (1997).

case.

Our analysis indicates that the introduction of a derivative market always improves welfare when the optimal contract in the absence of a derivative market discourages risk taking. Intuitively, this makes sense since the agent in this case will voluntarily choose to hedge completely given the contract that would be optimal in the absence of the hedging opportunity. As we show, in this case there is no cost that arises from the hedgeable risk exposure being unobservable.

However, there will be costs associated with the introduction of derivative contracts when the optimal compensation contract in the absence of a derivative market rewards the agent for taking risk. In this case, the compensation contract needs to be altered to induce the agent to use financial derivatives to hedge, not speculate. As we show, in this case, to incentivize the manager to hedge, the compensation contract must penalize a (both positive and negative) sample covariance (realized covariance) between the market observable and the output (e.g., covariance between oil prices and profits of the firm)[6]. However, revising the compensation contract in this way can be costly because it imposes additional risk on the risk-averse manager. Indeed, in some cases, the costs associated with inducing optimal hedging exceeds the informational benefit of hedging, which implies that the principal should optimally restrict the agent's access to the derivative market.

Since the motivation for hedging suggested by our model requires that the firm's risk exposure to be unobservable by the principal, it makes sense to ask if the agent can be induced to truthfully reveal this information. As we show, when costless communication between the principal and agent is possible, a contract that induces truthful revelation can be designed, without a derivative market, that is at least as good as the contract that requires hedging in the derivatives market. This indicates that if the agency relation is interpreted as a relation between the central headquarters of a firm (the principal) and a division head (the agent), the optimal compensation contract will not necessarily require the division head to hedge since it may be possible to do just as well with a compensation contract that is contingent on information communicated from the agent to the principal. However, if we view the relation between the CEO of a firm (the agent) and a diffuse group of shareholders (the principal), then the assumption of free communication is probably untenable and the optimal contract will probably allow the CEO to engage in derivative transactions.

There have been growing literatures that tackle the issue of agency relationships and the agent's risk choices. On theory side, Hirshleifer and Suh (1992), Sung (1995), Palomino and Prat (2003), DeMarzo et al. (2013) among others analyzed the case where the agent chooses the parametric risk of the output distribution. Hirshleifer and Suh (1992) especially analyzed the model in which a risk-averse agent chooses not only his effort level but also the firm's real project which is relevant for the

[6]Under the incentive contract that penalizes a sample covariance between the hedgeable risk and output, the agent would be tempted to reduce a covariance by engaging in hedging in the derivative market and reducing an exposure of the firm's cash flow to the market observables.

risk level. They showed that principal and agent will not generally agree on the firm's risk level, and agent's compensation contract should be revised in a way that it is more concave (convex) compared with the contract that would optimally be designed without the incentive problem associated with his risk choice, when the principal prefers a lower (higher) risk level than the agent. However, they did not derive the optimal compensation contract in a general framework, and furthermore, did not analyze the effects of a derivative market. DeMarzo et al. (2013) concluded that principal has to pay a large bonus if a firm survives, in order to disincentivize an agent from putting the firm at 'disaster' risk. It's in line with our Proposition 5 that the principal punishes covariance between the output and the hedgeable risk to induce an agent to fully hedge in the derivative market. Among papers that deal with nonparametric risk choices (Makarov and Plantin (2015), Hébert (2018), Barron et al. (2020)), Barron et al. (2020) considered the case where a 'Mean preserving spread (MPS)' risk can be added by an agent without cost and figured out that the agent's indirect utility must be concave at optimum to discourage the agent's additional risk-taking. Our paper contributes to the literature by analyzing the effect of a derivative market on the efficiency of agency relation in lights with how shareholders alter the manager's compensation contract to induce hedging. Empirical literatures (Guay (1999), Rajgopal and Shevlin (2002), Coles et al. (2006) among others) found that the higher a vega (sensitivity of CEO's wealth to the stock volatility) is, the riskier stock returns and firms' project choices become, with more focused investments and higher leverage ratios. It confirms our view that the convexity of manager's compensation contract is a major driving force behind his choice over risk.[7]

The rest of the paper is organized as follows. In Section 2, we formulate the basic model. In Section 3, as a benchmark case, we consider the case in which there is no derivative market. Thus, the manager's risk choices are carried out only through his real investment choices. In Section 4, we consider the case in which there is a derivative market. In Section 5, we also consider the case in which there is free communication between the principal and the agent. Concluding remarks are provided in Section 6, and the proofs of the Lemmas and Propositions as well as omitted derivations are all given in the Appendix.

2 The Basic Model

We consider a two-person single-period agency model in which a risk-averse agent works for a risk-neutral principal. The principal can be thought of as firm's shareholders, and the agent can be

[7] Another closely related paper to ours is DeMarzo and Duffie (1995) which also examines how risk management reduces uncertainty about the manager's unobservable action. However, in contrast to our analysis, in DeMarzo and Duffie (1995), differences in actions are due to differences in ability rather than differences in effort.

regarded as firm's top manager or CEO. Alternatively, we can think of the principal as CEO and the agent as a head of one of firm's subordinate divisions. Hereafter, we use the terms 'agent' and 'manager' interchangeably.

After his wage contract is finalized, agent takes three kinds of actions, $a_1 \in [0, \infty)$, $a_2 \in [\underline{a}_2, \overline{a}_2]$, and $a_3 \in (-\infty, +\infty)$. Agent's first action is a productive effort which increases an expected output, that is, a high effort generates an output level that first-order stochastically dominates the output level generated by a low effort. The agent's second action is his project choice. Projects have different risk levels with more risky projects having higher expected outputs. The third action is his choice in the derivatives market. We assume that the set of projects available to the agent is bounded, i.e., $a_2 \in [\underline{a}_2, \overline{a}_2]$, while the agent can choose any position in the derivatives market, i.e., $a_3 \in (-\infty, +\infty)$.

After the agent chooses a_1, a_2, and a_3, the firm's output, x, is realized and publicly observable without cost. Thus, an output x can be used in the manager's wage contract that is denoted by w. The output is determined not only by the agent's choice of (a_1, a_2, a_3) but also by the state of nature, (η, θ). For simplicity, we assume that the output function exhibits the following additively separable form:

$$x = \phi(a_1, a_2) + (R - a_3)\eta + a_2\theta. \tag{1}$$

An expected output, $\phi(a_1, a_2)$, is a function of both a_1 and a_2, whereas the agent's derivatives choice, a_3, does not directly affect it. The firm's risk is decomposed into two components, η and θ, where $\eta \sim N(0, 1)$ represents one unit of the firm's <u>hedgeable</u> risks and $\theta \sim N(0, 1)$ represents one unit of the firm's <u>non-hedgeable</u> risks. As denoted by equation (1), the firm's total non-hedgeable risks are determined by the manager's project choice, a_2, whereas the firm's hedgeable risks are determined by market variables such as commodity prices, interest rates, and exchange rates which become publicly <u>observable</u> after the agent chooses a_1, a_2, and a_3.[8] Accordingly, we assume η is observable at the end of the period and thus can also be used in the manager's wage contract if necessary. In the above equation, $R \sim N(R_m, \sigma_R^2)$ denotes the firm's innate exposure to the hedgeable risks such as the amount of oil underground for a drilling company. We assume that the manager can observe the true value of R after the contract is signed but before he chooses a_1, a_2, and a_3. In contrast, the principal knows only its distribution. We assume neither a management effort (a_1) nor a project choice (a_2) affects R, the firm's innate exposure to the hedgeable risks.[9] However, the firm's final risk exposure can be manipulated by the manager's transaction a_3 in the

[8]In fact, if the relevant derivative market observable is denoted as p, then $\eta = p - \overline{p}$ where $\overline{p}$ is the expected value of p.

[9]In general, a firm's risk exposure is very much dependent on the production project undertaken. However, even if we allow the firm's risk exposure to be affected by the manager's project choice (a_2), most results in this paper will not change qualitatively.

derivative market. If $a_3 = 0$, the manager does not trade derivatives. The manager hedges, i.e., reduces risk, as long as $|R - a_3| < |R|$ and minimizes risk by setting $a_3 = R$. On the other hand, if $|R - a_3| > |R|$, the manager speculates in the derivative market.

In addition, we make the following assumptions:

Assumption 1 The agent's preferences on wealth and productive effort are additively separable :

$$U(w, a_1, a_2, a_3) = u(w) - v(a_1), \quad u' > 0, u'' < 0,$$

where v, the agent's disutility of exerting productive effort, has the properties $v' > 0, v'' > 0, \quad \forall a_1$.

Assumption 2 $\dfrac{\partial \phi}{\partial a_1}(a_1, a_2) \equiv \phi_1(a_1, a_2) > 0, \ \dfrac{\partial^2 \phi}{\partial a_1^2}(a_1, a_2) \equiv \phi_{11}(a_1, a_2) < 0.$

Assumption 3 $\dfrac{\partial \phi}{\partial a_2}(a_1, a_2) \equiv \phi_2(a_1, a_2) > 0, \ \phi_{22}(a_1, a_2) < 0, \ \phi_2(a_1, \underline{a}_2) = \infty, \ and \ \phi_2(a_1, \overline{a}_2) = 0.$

Assumption 4 $0 < \underline{a}_2 < \overline{a}_2 < \infty.$

Assumption 1 implies that the agent is risk-averse and effort-averse, and the agent's project and derivatives choices have no direct effect on his utility. Assumptions 2 and 3 indicate that a_1 affects the expected output with a usual property of decreasing marginal increase in output, while a higher a_2 raises the expected output in the similar way and also the output variability. Especially, since expected value of output as well as its variability increases in a_2, there is an assumed trade-off between return and risk.[10]. Assumption 4 states that there is neither a completely safe project nor a project with unbounded risk.

3 When There Is No Derivative Market

3.1 The Principal Knows the Firm's Exposure to the Hedgeable Risks

In this section, we consider a benchmark case in which the firm has no access to a derivative market, and the principal also knows the true value of the firm's innate risk exposure, R. We thus specify $a_3 = 0$ so that the production function in equation (1) reduces to

$$x = \phi(a_1, a_2) + R\eta + a_2\theta. \tag{2}$$

[10] As noted from equation (1), reducing the firm's non-hedgeable risks requires the firm to sacrifice a part of an expected output. This trade-off guarantees the existence of an optimal project choice a_2 in our agency setting.

Since there is no derivative market, the manager's incentive problem arises only in inducing (a_1, a_2). As R and η are observable and thus contractible, $y \equiv x - R\eta$ is a sufficient statistic for (x, η) in assessing (a_1, a_2). Therefore, the principal uses y as a contractual variable to induce (a_1, a_2), and the above equation can be expressed as

$$y = \phi(a_1, a_2) + a_2\theta. \tag{3}$$

In general, designing a contract to induce agent's project choice (a_2) as well as the effort choice (a_1) should be different from designing a contract that only induces the agent's effort choice (a_1).[11] Thus, to study how the existence of an additional incentive problem associated with the agent's project choice a_2 affects the agent's wage contract, we first consider a typical standard agency case in which the agent's project choice, a_2, is observable, or equivalently, the principal selects the project level a_2 by himself and mandates it.

The optimal compensation contract $w(\cdot)$, in this case, is found by solving for the contract which maximizes the combined utilities of the principal and agent subject to the restriction that the agent's effort a_1 is chosen to maximize his utility given the contract.[12]

$$\begin{aligned} \max_{a_1, a_2, w(\cdot)} \phi(a_1, a_2) - \int w(y) f(y|a_1, a_2, a_3 = 0)dy + \lambda \Big[\int u(w(y)) f(y|a_1, a_2, a_3 = 0)dy - v(a_1) \Big] \quad \text{s.t.} \\ (i) \;\; a_1 \in \arg\max_{a_1'} \int u(w(y)) f(y|a_1', a_2, a_3 = 0)dy - v(a_1'), \;\; \forall a_1', \\ (ii) \;\; w(y) \geq k, \;\; \forall y, \end{aligned} \tag{4}$$

where $f(y|a_1, a_2, a_3 = 0)$ denotes a probability density function of y given the agent's three actions, and λ denotes the weight placed on the agent's utility in the joint optimization. As shown, the combined utilities of the principal and the agent are maximized subject to the agent's incentive compatibility constraint which specifies that the agent chooses his effort for his own optimization, and his limited liability constraint which specifies that the agent receives at least k, the subsistence level of utility.[13]

[11] Hirshleifer and Suh (1992) call this issue as a 'dual-agency problem'.

[12] This yields a mathematically equivalent solution to a model where the principal maximizes his utility subject to an optimizing agent receiving his reservation utility level. Our purpose here is to analyze the overall efficiency implication of financial market innovations and thus we choose to fix λ, which usually is an endogenous Lagrange multiplier in the latter case.

[13] The limited liability constraint is introduced to guarantee the existence of optimal solution for $w(y)$. This is needed because we assume that the signal is normally distributed. For details about this 'unpleasantness', see Mirrlees (1974) and Jewitt et al. (2008).

Subject to some technical assumptions,[14] the above maximization problem reduces to:

$$\max_{a_1,a_2,w(\cdot)} \phi(a_1,a_2) - \int w(y)f(y|a_1,a_2,a_3=0)dy + \lambda\Big[\int u(w(y))f(y|a_1,a_2,a_3=0)dy - v(a_1)\Big] \quad \text{s.t.}$$
$$(i)\ \ \int u(w(y))f_1(y|a_1,a_2,a_3=0)dy - v'(a_1) = 0,$$
$$(ii)\ \ w(y) \geq k,\ \ \forall y, \tag{5}$$

where f_1 denotes the first derivative of f taken with respect to a_1.

To find solution $(a_1^P, a_2^P, w^P(y|a_1^P, a_2^P))$ for the above program, we first derive an optimal contract for an arbitrarily given action combination (a_1, a_2). Let $w^P(y|a_1, a_2)$ be a contract which optimally motivates the agent to take a particular level of a_1 when an arbitrary level of a_2 is chosen by the principal. By solving the Euler equation of the above program after fixing (a_1, a_2), we derive that $w^P(y|a_1, a_2)$ must satisfy,

$$\frac{1}{u'(w^P(y|a_1,a_2))} = \lambda + \mu_1(a_1,a_2)\frac{f_1}{f}(y|a_1,a_2,a_3=0), \tag{6}$$

for almost every y for which equation (6) has a solution $w^P(y|a_1, a_2) \geq k$, and otherwise $w^P(y|a_1, a_2) = k$. In equation (6), $\mu_1(a_1, a_2)$ denotes the optimized Lagrange multiplier for the agent's incentive constraint associated with a_1 when the second action is pinned down at a_2. Since $f(y|a_1, a_2, a_3 = 0)$ is a normal density function with mean $\phi(a_1, a_2)$ and variance a_2^2, equation (6) is reduced to:

$$\frac{1}{u'(w^P(y|a_1,a_2))} = \lambda + \mu_1(a_1,a_2)\frac{[y-\phi(a_1,a_2)]}{a_2^2}\phi_1(a_1,a_2). \tag{7}$$

Before analyzing the optimal contract, we first define

$$SW(a_1,a_2) \equiv \phi(a_1,a_2) - C(a_1,a_2) - \lambda v(a_1), \tag{8}$$

which denotes the joint benefits when $w^P(y|a_1, a_2)$ is designed and a_2 is instructed by the principal where

$$C(a_1,a_2) \equiv \int [w^P(y|a_1,a_2) - \lambda u(w^P(y|a_1,a_2))]f(y|a_1,a_2,a_3=0)dy \tag{9}$$

[14] We assume that the first-order approach is valid. Grossman and Hart (1983) and Rogerson (1985) show that MLRP and CDFC are sufficient for the validity of the first-order approach when the signal space is of one dimension. Jewitt (1988) finds less restrictive conditions for the validity of the first-order approach, which are based on the agent's risk preferences as well as the distribution function of the signal. Sinclair-Desgagné (1994) shows that more generalized version of MLRP and CDFC in a multi-dimensional space are sufficient for the validity of the first-order approach when the signal space is of multiple dimensions. For more recent treatments, see Conlon (2009), Jung and Kim (2015) among others.

represents the efficiency loss of this case compared with the full information case. In other words, $C(a_1, a_2)$ measures the agency cost arising from inducing the agent to take that particular a_1 when a_2 is chosen by the principal. Before we proceed, we make the following additional assumption.

Assumption 5 $\phi_{12}(a_1, a_2) \cdot a_2 < \phi_1(a_1, a_2), \forall(a_1, a_2)$.

If $\phi_{12}(a_1, a_2)$ is positive and decreasing in a_2, and $\phi_1(a_1, \underline{a}_2) \simeq 0$, $\underline{a}_2$ is close to 0, then Assumption 5 holds as we see in Figure 1. As the manager raises a risk level a_2, an increase in effort a_1 (for example, more intense portfolio management) yields a higher increase in expected output $\phi(a_1, a_2)$. Thus it is likely that $\phi_1(a_1, a_2)$ increases in a_2 and $\phi_{12}(a_1, a_2) > 0$ holds. This complementarity gets weaker as the project becomes risker, since corporations can't efficiently allocate their attention over their high risk balance sheets, as the global financial crisis (GFC) showed clearly.

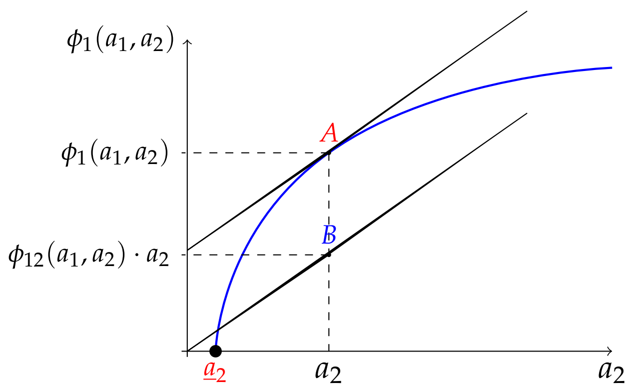


Figure 1: Illustration of the Assumption 5

We start our analysis with the following lemma, which was previously provided in Kim (1995).

Lemma 1 $C(a_1, a_2^0) < C(a_1, a_2^1)$ *for any given* a_1 *if* $a_2^0 < a_2^1$.

Since principal can dictate the agent's project choice, agency problem arises only in inducing a_1. Lemma 1 says under the Assumption 5, when the agent's project choice can be instructed by the principal, the agency cost associated with motivating the agent to take any given a_1 will be reduced if the principal chooses a less risky project. A lowered risk improves the efficiency of an agency relationship by providing a more precise signal y about the agent's effort, a_1, which in turn enables the principal to design contract inducing a particular a_1 in a less costly way. If $\phi_{12}(a_1, a_2)$ is large enough to break Assumption 5, then lower a_2 might lower $\phi_1(a_1, a_2)$ a lot, which in turn makes harder for the principal to give the proper incentive for the action a_1 and raise the incentive cost $C(a_1, a_2)$. Assumption 5 guarantees that this incentive drawback is lower than the informational rent from lower a_2, so principal wants a lower level of non-hedgeable risk a_2.

Lemma 1 suggests firms should take all zero net present value projects that reduce their risks. However, given the trade-off relation between return and risk, i.e., $\phi_2 > 0$, the exact level of a_2 that the principal prefers will be determined by the loss in expected return as well as the benefit from more precise signals. Let a_2^P be the project that is most preferred by the principal, and a_1^P the agent's optimal effort choice for the above program when a_2^P is chosen by the principal. Then, as we prove in Appendix, from the above optimization we obtain that $(a_1^P, a_2^P, w^P(\cdot))$ should satisfy

$$\int [y - w^P(y) + \lambda u(w^P(y))] f_2(y|a_1^P, a_2^P, a_3 = 0) dy + \mu_1(a_1^P, a_2^P) \int u(w^P(y)) f_{12}(y|a_1^P, a_2^P, a_3 = 0) dy = 0, \tag{10}$$

where $w^P(\cdot) = w^P(\cdot|a_1^P, a_2^P)$, f_2 denotes the first derivative of f with respect to a_2 and f_{12} is the second derivative with respect to a_1 and a_2. We obtain the optimal contract $w^P(y|a_1^P, a_2^P)$ satisfies,

$$\frac{1}{u'(w^P(y|a_1^P, a_2^P))} = \lambda + \mu_1(a_1^P, a_2^P) \frac{[y - \phi(a_1^P, a_2^P)]}{(a_2^P)^2} \phi_1(a_1^P, a_2^P), \tag{11}$$

for y satisfying $w^P(y|a_1^P, a_2^P) \geq k$ and $w^P(y|a_1^P, a_2^P) = k$ otherwise.

The above result assumes shareholders are fully aware of the projects chosen by the manager. However, it is not realistic, so we also turn our eyes to the manager's incentive to increase or decrease the firm's risk when the project choice is not fully revealed to the shareholders. Specifically, we ask whether the manager will voluntarily choose the project that would be chosen by informed shareholders, i.e., a_2^P. If the answer to this question is no, then the moral-hazard problem arises not only in motivating a_1 but also in incentivizing a_2. As a result, there are costs associated with the project choice being unobservable, and the optimal wage contract must be modified from the contract, $w^P(y|a_1^P, a_2^P)$, in equation (11).

To formally analyze this issue, we denote $a_2^A(a_2^P)$ as a solution for

$$a_2^A(a_2^P) \ \in \arg\max_{a_2} \int u(w^P(y|a_1^P, a_2^P)) f(y|a_1^P, a_2, a_3 = 0) dy. \tag{12}$$

Thus, $a_2^A(a_2^P)$ represents the project choice that the agent would take under $w^P(y|a_1^P, a_2^P)$ described in equation (11) when a_2 is not actually enforceable. Thus, our previous question, "Will the agent voluntarily choose a_2^P when $w^P(y|a_1^P, a_2^P)$ is designed?", is equivalent to the question, "Will $a_2^A(a_2^P)$ be equal to a_2^P?"

As previously shown, the principal balances two considerations when he directs the agent to take a certain project: the informational benefits from risk reduction and the lower mean return associated with lower risk. However, the risk level to be chosen by the agent primarily depends on his <u>indirect</u> risk preferences induced by contract $w^P(y|a_1^P, a_2^P)$, i.e., the curvature of $u(w^P(y|a_1^P, a_2^P))$

with respect to y, and the effect that a trade-off between return and risk would have on his utility *via* $w^P(y|a_1^P, a_2^P)$.

In general, the curvature of the agent's indirect utility function depends on the distribution of the random state variable and utility function itself. To see how different utility functions affect this curvature differently, consider the case where the agent has constant relative risk aversion with degree $1-t$, where $t<1$ ($u(w)=\frac{1}{t}w^t, t<1$). We obtain from equation (11) that

$$w^P(y|a_1^P, a_2^P) = \Big(\lambda + \mu_1(a_1^P, a_2^P)\Big[\frac{y-\phi(a_1^P, a_2^P)}{(a_2^P)^2}\Big]\phi_1(a_1^P, a_2^P)\Big)^{\frac{1}{1-t}}, \tag{13}$$

and the agent's indirect utility under this wage contract is

$$u(w^P(y|a_1^P, a_2^P)) = \frac{1}{t}\Big(\lambda + \mu_1(a_1^P, a_2^P)\Big[\frac{y-\phi(a_1^P, a_2^P)}{(a_2^P)^2}\Big]\phi_1(a_1^P, a_2^P)\Big)^{\frac{t}{1-t}}. \tag{14}$$

The above equation shows that the agent's indirect utility becomes strictly concave in y if $t<\frac{1}{2}$, linear if $t=\frac{1}{2}$, and convex if $t>\frac{1}{2}$ for y satisfying $w^P(y|a_1^P, a_2^P) \geq k$. If we assume $w^P(y|a_1^P, a_2^P)=k$ for sufficiently low y, as far as the agent's induced risk preferences are concerned, $u(w^P(y|a_1^P, a_2^P))$ makes the agent risk-loving if $t \geq \frac{1}{2}$. Furthermore, since the compensation contract $w^P(y|a_1^P, a_2^P)$ is positively related to the absolute output level (i.e., $\mu_1(a_1^P, a_2^P) > 0$),[15] if $t \geq \frac{1}{2}$, the agent is induced to take the most risky project, i.e., $a_2^A(a_2^P) = \bar{a}_2$ when $w^P(y|a_1^P, a_2^P)$ is designed even if $\phi_2(a_1, \bar{a}_2) = 0$ by Assumption 3. However, in this case, principal prefers to have a firm's risk level a_2 lower than $\bar{a}_2$. This is because, from his standpoint, the informational benefits from risk reduction are still substantial, while the costs of risk reduction are zero at $\bar{a}_2$ (i.e., $\phi_2(\bar{a}_2)=0$). Thus, $a_2^P < a_2^A(a_2^P)$ in this case. In other words, the principal prefers less risk than the agent under $w^P(y|a_1^P, a_2^P)$.

On the other hand, in the case of t being close to $-\infty$ (i.e., the agent is extremely risk-averse), the agent's indirect utility mandates him to choose a lower level of risk than what principal prefers ($a_2^A(a_2^P) < a_2^P$) even though a lower a_2 yields on average a lower output level.[16]

Incentive problems associated with project choice (a_2), in general, exist in all cases except those where both of the following conditions are satisfied: (i) the agent's indirect utility is sufficiently concave and (ii) there is no trade-off between return and risk, i.e., $\phi_2 = 0$, $\forall a_2$. Under these conditions, both the principal and the agent agree that the firm should choose the least risky project, i.e., $a_2 = \underline{a}_2$, and there is no efficiency loss due to the existence of the manager's unobservable project choice. However, when either the agent's induced risk preferences are convex, or the trade-off between return and risk exists as assumed in Assumption 3, the principal and the agent will not

[15] For this issue, see Holmström (1979), Jewitt (1988) among others.

[16] This issue has been dealt with in literatures including Hirshleifer and Suh (1992), Guay (1999), Ross (2004) among others.

generally agree on the firm's optimal project choice, and the compensation contract, $w^P(y|a_1^P, a_2^P)$, described in equation (11) will no longer be optimal.

In this situation, the principal must determine the optimal compensation contract by solving the following optimization problem:

$$\begin{aligned}
\max_{a_1,a_2,w(\cdot)} \phi(a_1,a_2) - &\int w(y) f(y|a_1,a_2,a_3=0)dy + \lambda\Big[\int u(w(y))f(y|a_1,a_2,a_3=0)dy - v(a_1)\Big] \quad \text{s.t.} \\
&(i)\;\; a_1 \in \arg\max_{a_1'} \int u(w(y))f(y|a_1',a_2,a_3=0)dy - v(a_1'),\;\; \forall a_1', \\
&(ii)\;\; a_2 \in \arg\max_{a_2'} \int u(w(y))f(y|a_1,a_2',a_3=0)dy - v(a_1),\;\; \forall a_2', \\
&(iii)\;\; w(y) \geq k,\;\; \forall y,
\end{aligned} \tag{15}$$

In the above optimization we accounts for the fact that the agent selects a_2 to maximize his own expected utility. If an interior solution for (a_1, a_2) exists and the first-order approach is valid, the above maximization problem can be expressed as:

$$\begin{aligned}
\max_{a_1,a_2,w(\cdot)} \phi(a_1,a_2) - &\int w(y) f(y|a_1,a_2,a_3=0)dy + \lambda\Big[\int u(w(y))f(y|a_1,a_2,a_3=0)dy - v(a_1)\Big] \quad \text{s.t.} \\
&(i)\;\; \int u(w(y))f_1(y|a_1,a_2,a_3=0)dy - v'(a_1) = 0, \\
&(ii)\;\; \int u(w(y))f_2(y|a_1,a_2,a_3=0)dy = 0, \\
&(iii)\;\; w(y) \geq k,\;\; \forall y.
\end{aligned} \tag{16}$$

Let (a_1^*, a_2^*) be the optimal action combination for the above program. Then, by solving the Euler equation, we obtain that the optimal wage contract, $w^*(y)$, satisfies,

$$\frac{1}{u'(w^*(y))} = \lambda + \mu_1^* \frac{f_1}{f}(y|a_1^*, a_2^*, a_3=0) + \mu_2^* \frac{f_2}{f}(y|a_1^*, a_2^*, a_3=0), \tag{17}$$

for almost every y for which equation (17) has a solution $w^*(y) \geq k$, and otherwise $w^*(y) = k$. μ_1^* and μ_2^* are the optimized Lagrange multipliers for both incentive constraints, respectively.

Since $f(y|a_1^*, a_2^*, a_3 = 0)$ is a normal density function with mean $\phi(a_1^*, a_2^*)$ and variance $(a_2^*)^2$, we have from equation (17) that:

$$\frac{1}{u'(w^*(y))} = \lambda + (\mu_1^*\phi_1^* + \mu_2^*\phi_2^*)\frac{y - \phi(a_1^*, a_2^*)}{(a_2^*)^2} + \mu_2^* \frac{1}{a_2^*}\Big(\frac{(y-\phi(a_1^*, a_2^*))^2}{(a_2^*)^2} - 1\Big), \tag{18}$$

for y satisfying $w^*(y) \geq k$ and $w^*(y) = k$ otherwise. In the above equation, $\phi_i^* \equiv \phi_i(a_1^*, a_2^*), i = 1, 2$.

We call $w^*(y)$ as an 'optimal dual-agency contract' à la Hirshleifer and Suh (1992).

Compared with equation (11), equation (18) shows when both a_1 and a_2 are not observable, the optimal wage contract is based not only on the absolute output y, but also on its (standardized) deviation from the expected level, $\frac{[y-\phi(a_1^*,a_2^*)]^2}{(a_2^*)^2}$. Since $[y-\phi(a_1^*,a_2^*)]^2$ is a sample variance of a single observation with mean zero and variance $(a_2^*)^2$, the term $\frac{[y-\phi(a_1^*,a_2^*)]^2}{(a_2^*)^2}$ in equation (18) can be regarded a standardized output deviation. Note that $\frac{[y-\phi(a_1^*,a_2^*)]^2}{(a_2^*)^2}-1$ is a sufficient statistic for the project choice a_2, while $\frac{y-\phi(a_1^*,a_2^*)}{(a_2^*)^2}$ is a sufficient statistic for the productive effort a_1. By including the sample variance as a contractual parameter, the principal effectively motivates the agent to take the appropriate level of a_2, i.e., a_2^*.

The optimal dual agency contract is characterized in the following propositions.

Proposition 1 $\mu_1^*\phi_1^*+\mu_2^*\phi_2^*>0$.

Proposition 1 implies, holding the cash flow variance constant, the manager's payout increases when the firm's cash flow increases, which implies that the manager is rewarded for a higher effort. However, this does not necessarily mean that the contracted payout is monotonically increasing in the output level. For example, if $\mu_2^*<0$ in equation (18), the agent can be paid less when the output level is very high.

Thus, a more interesting question has to do with the relation between the agent's rewards and the output deviation, i.e., the sign of μ_2^*.

Proposition 2 *If the principal prefers a less risky project than the agent under $w^P(y|a_1^P,a_2^P)$ in equation (11), i.e., $a_2^P<a_2^A(a_2^P)$, then the optimal dual agency contract will penalize the agent for having unusual output deviation from the expected level, i.e., $\mu_2^*<0$ for $w^*(y)$ in equation (18). If the principal prefers a more risky project than the agent under $w^P(y|a_1^P,a_2^P)$, i.e., $a_2^P>a_2^A(a_2^P)$, then the optimal dual agency contract will reward the agent for having unusual output deviation, i.e., $\mu_2^*>0$ for $w^*(y)$ in equation (18).*

If the principal prefers a lower risk level than the agent under the contract $w^P(y|a_1^P,a_2^P)$, the contract will be revised in a way that motivates the agent to reduce risk. This can be done by setting $\mu_2^*<0$ in equation (18) which penalizes the agent for the unusual output deviation and makes the agent act as if he is more risk-averse. On the other hand, if the principal prefers a higher risk than the agent when $w^P(y|a_1^P,a_2^P)$ is designed, the contract is revised to motivate the agent to increase risk. This is done by setting $\mu_2^*>0$ in equation (18) which rewards the agent for unusual output deviation and makes the agent act as though he is less risk-averse. As discussed earlier, the later case is more likely to occur when the manager is very risk-averse and when the firm's investment opportunities offer a non-trivial trade-off between return and risk.

We denote the optimized joint benefits in this case as

$$SW(a_1^*, a_2^*, a_3 = 0) \equiv \phi(a_1^*, a_2^*) - C(a_1^*, a_2^*) - \lambda v(a_1^*), \tag{19}$$

where

$$C(a_1^*, a_2^*) \equiv \int [w^*(y) - \lambda u(w^*(y))] f(y|a_1^*, a_2^*, a_3 = 0) dy \tag{20}$$

denotes the agency cost arising from inducing (a_1^*, a_2^*) when a_3 is fixed at 0 and R is observable.

3.2 The Principal Does Not Know the Firm's Risk Exposure

We now consider the case in which the firm's exposure to hedgeable risks, R, is observed only by the agent but not by the principal. Thus, the wage contract cannot explicitly include $y \equiv x - R\eta$ as a contractual variable. Furthermore, we rule out the possibility of any communication between principal and the agent that allows the agent to reveal R.[17] We will later consider the case in which communication between the principal and the agent is allowed without cost, and thereby the principal can design a truth-telling mechanism freely.

If principal does not observe R, the compensation contract must be based on (x, η), i.e., $w = w(x, \eta)$. The principal's maximization program in this case is thus:[18]

$$\begin{aligned}
&\max_{a_1(\cdot), a_2(\cdot), w(\cdot)} \int_R \int_{x,\eta} [x - w(x, \eta)] g(x, \eta | a_1(R), a_2(R), R) h(R) dx d\eta dR \\
&\quad + \lambda \int_R \Big(\int_{x,\eta} u(w(x, \eta)) g(x, \eta | a_1(R), a_2(R), R) dx d\eta - v(a_1(R)) \Big) h(R) dR \quad \text{s.t.} \\
&\quad (i) \ \ a_1(R) \in \arg\max_{a_1} \int_{x,\eta} u(w(x, \eta)) g(x, \eta | a_1, a_2(R), R) dx d\eta - v(a_1), \forall R, \\
&\quad (ii) \ \ a_2(R) \in \arg\max_{a_2} \int_{x,\eta} u(w(x, \eta)) g(x, \eta | a_1(R), a_2, R) dx d\eta, \forall R, \\
&\quad (iii) \ \ w(x, \eta) \geq k, \ \ \forall (x, \eta),
\end{aligned} \tag{21}$$

where

$$g(x, \eta | a_1, a_2, R) = \frac{1}{2\pi a_2} \exp\Big(-\frac{1}{2} \Big[\frac{(x - \phi(a_1, a_2) - R\eta)^2}{a_2^2} + \eta^2 \Big] \Big) \tag{22}$$

denotes a joint probability density function of (x, η) given (a_1, a_2, R) and $h(R)$ denotes the proba-

[17] In general, communication between principals and agents are likely to be very costly, especially when actually the principal stands for multiple shareholders. For a more detailed discussion of communication costs, see Laffont and Martimort (1997).

[18] In this case as agent is the only one seeing the realized value of R, his actions a_1, a_2 both depend on R, given the contract $w(x, \eta)$.

bility density function of R.

Let $(a_1^N(R), a_2^N(R), w^N(x,\eta))$ be the optimal solution for the above program. The optimized joint benefit in this case is denoted as:

$$SW^N \equiv \int_R [\phi(a_1^N(R), a_2^N(R)) - C^N(a_1^N(R), a_2^N(R)) - \lambda v(a_1^N(R))]h(R)dR, \tag{23}$$

where

$$C^N(a_1^N(R), a_2^N(R)) \equiv \int_{x,\eta} [w^N(x,\eta) - \lambda u(w^N(x,\eta))]g(x,\eta|a_1^N(R), a_2^N(R), R)dxd\eta \tag{24}$$

denotes the agency cost arising from inducing $(a_1^N(R), a_2^N(R))$ given a realized value of R.

Proposition 3 *When there is no derivative market and the communication between the principal and the agent is not possible, the principal's inability to observe the firm's risk exposure reduces welfare, i.e.,*

$$SW^N < SW(a_1^*, a_2^*, a_3 = 0).$$

Intuitively, when the principal observes the true value of the firm's risk exposure, R, this information can be used to design a wage contract that eliminates the influence of the hedgeable risks, i.e., $w = w^*(y \equiv x - R\eta)$. However, if R is not observable and cannot be communicated, this is impossible.

4 When Managers Can Trade Derivatives[19]

This section considers how the introduction of an opportunity to transact derivative contracts (i.e., when a_3 is not fixed at 0) affects the optimal contract and firm's efficiency. Continuing from Section 3.2, we assume that a manager's project choice, a_2, is not observable. Moreover, we assume that neither a manager's derivatives choice, a_3, nor firm's risk exposure, R, can be observed by or communicated to the principal.

Since a firm's exposure to hedgeable risks, R, is assumed to be known to the agent before he takes actions (a_1, a_2, a_3), agent's choice of a_3 can be characterized as his choice of $b \equiv R - a_3$. Then given a wage contract, principal can rationally anticipate the agent's choice of $b = R - a_3$. We denote $\hat{b}$ as the principal's belief of agent's choice of $R - a_3$, and define $z(\hat{b}) \equiv x - \hat{b}\eta$ as a variable that is possibly included in the wage contract, i.e., $w(z(\hat{b}))$ is one possible contract. In order for the principal's beliefs to be consistent, it must be the case that the agent actually chooses a_3 satisfying $b \equiv R - a_3 = \hat{b}$ given the contract (possibly $w(z(\hat{b}))$).

[19]This section benefited a lot from the detailed discussion with Seung uk Jang at the University of Chicago.

Thus, since

$$z(\hat{b}) \equiv x - \hat{b}\eta = \phi(a_1, a_2) + (b - \hat{b})\eta + a_2\theta, \tag{25}$$

if the principal designs a contract $w(z(\hat{b}))$ and if the agent actually chooses a_3 satisfying $b = R - a_3 = \hat{b}$, then

$$z(\hat{b}) = \phi(a_1, a_2) + a_2\theta = y. \tag{26}$$

Note that a maximum level of joint benefits that can be obtained in this case is $SW(a_1^*, a_2^*, a_3 = 0)$ in equation (19).[20] Thus, we first consider the case in which the principal designs the contract the same as $w^*(y)$ described in equation (18) but based on $z(\hat{b})$ instead of $y \equiv x - R\eta$, and examine whether agent chooses $b \equiv R - a_3 = \hat{b}$ under $w^*(z(\hat{b}))$. If this is indeed the case, there is no welfare loss associated with R (and a_3) being unobservable, and this informational benefit is one of the main advantages of letting the agent transact in the derivative market.

Suppose that the principal designs $w^*(z(\hat{b}))$ satisfying

$$\frac{1}{u'(w^*(z(\hat{b})))} = \lambda + (\mu_1^*\phi_1^* + \mu_2^*\phi_2^*)\frac{z(\hat{b}) - \phi(a_1^*, a_2^*)}{(a_2^*)^2} + \mu_2^*\frac{1}{a_2^*}\Big(\frac{(z(\hat{b}) - \phi(a_1^*, a_2^*))^2}{(a_2^*)^2} - 1\Big), \tag{27}$$

for $z(\hat{b})$ satisfying $w^*(z(\hat{b})) \geq k$ and $w^*(z(\hat{b})) = k$ otherwise. Since $w^*(z(\hat{b}))$ in equation (27) has the same contractual form as $w^*(y)$ in equation (18), we can easily see that the agent will take (a_1^*, a_2^*) under $w^*(z(\hat{b}))$ if he chooses a_3 satisfying $b \equiv R - a_3 = \hat{b}$. But, the question is "Will the agent always choose a_3 satisfying $b = \hat{b}$ when $w^*(z(\hat{b}))$ is designed?". Before we proceed, we make an additional assumption which we illustrate in Appendix the importance in proving the following Lemma 2.

Assumption 6 (Jewitt (1988)) $u\Big((u')^{-1}\Big(\frac{1}{z}\Big)\Big)$ is increasing and concave in $\forall z > 0$.

The following Lemma 2 provides an answer to the above question.

Lemma 2 *[Speculation and Hedging with $w^*(z(\hat{b}))$]*
(1) If $\underline{\mu_2^* < 0}$ for the contract, $w^*(z(\hat{b}))$, described in equation (27) for any given $\hat{b}$,[21] *then under the Assumption 6, the manager will choose a_3 such that $b = \hat{b}$ when the contract $w^*(z(\hat{b}))$ is offered when $\lambda > 0$ is large enough.*
(2) If $\underline{\mu_2^* > 0}$ for $w^*(z(\hat{b}))$ in equation (27) for any given $\hat{b}$, then the manager will take a_3 such that $|R - a_3| = \infty$ when $w^*(z(\hat{b}))$ is offered.

[20]Given the contract $w(z(\hat{b}))$, if there is no incentive problem associated with $b = R - a_3$ so the agent voluntarily chooses a_3 such that $R - a_3 = \hat{b}$, then we get the maximum joint benefit $SW(a_1^*, a_2^*, a_3 = 0)$.

[21]One can easily see that if $\mu_2^* < 0$ in $w^*(z(\hat{b}))$ for any given $\hat{b}$, then $\mu_2^* < 0$ in $w^*(z(\hat{b}))$ for all $\hat{b}$. This is because the principal's anticipating different $\hat{b}$ does not change the functional form of $w^*(\cdot)$.

Given $\underline{\mu_2^* < 0}$, Assumption 6 guarantees the agent's indirect utility on normalized output level[22] is concave when limited liability does not bind. An agent with concave indirect utility chooses $b = \hat{b}$ to minimize its own risk exposure. Limited liability constraint, however, distorts this condition around the points where it binds, since the indirect utility becomes very convex at those points. This additional channel might induce the agent to increase risk instead of engaging in perfect hedging. Our premise that λ is large enough assures us the first concavity effect dominates the second convexity effect asymptotically[23] and the agent chooses $b = \hat{b}$. From now on we assume λ to be large enough whenever $\mu_2^* < 0$ in equation (27).

From Lemma 2, we directly obtain the following proposition:

Proposition 4 *When λ is large enough, $\underline{\text{if } \mu_2^* < 0}$ for $w^*(z(\hat{b}))$ described in equation (27) for any given $\hat{b}$, then the level of $b \equiv R - a_3$ to be induced is a matter of indifference as long as it is bounded, i.e., $|b| < \infty$. For example, If $\mu_2^* < 0$ for $w^*(z(0))$ in equation (27), then the manager will choose $(a_1^*, a_2^*, a_3 = R$ (i.e., $b = 0$)) when $w^*(z(0))$ is designed. Therefore, optimized joint benefits are the same as $SW(a_1^*, a_2^*, a_3 = 0)$ in equation (19), implying that the firm's welfare with a derivative market in this case is the same as it is in the case where the risk exposure is observed by the principal.*[24]

Proposition 4 is quite intuitive. If $\underline{\mu_2^* < 0}$ for $w^*(z(\hat{b}))$ in equation (27), the agent is induced to engage in perfect hedging to minimize the variance of $z(\hat{b})$, because his indirect utility given that the contract $w^*(z(\hat{b}))$ is generically concave. Since $\mu_1^*\phi_1^* + \mu_2^*\phi_2^* > 0$ means a higher $z(\hat{b})$ yields the higher compensation $w^*(z(\hat{b}))$ given its squared deviation from the average of $z(\hat{b})$, if $u((u')^{-1}(\frac{1}{z}))$ is 'too' convex, an agent might want to increase risk despite the existence of $\frac{\mu_2^*}{a_2^*}(\frac{(z(\hat{b})-\phi(a_1^*,a_2^*))^2}{(a_2^*)^2} - 1)$ term in equation (27). The role of Assumption 6 is to make sure it never happens, and the agent eliminates, rather than adds, risks in a derivative market by choosing $b = \hat{b}$. In this case, the contract can be designed as if $R - a_3$ is observable, and it allows the principal and the agent to achieve the level of welfare $SW(a_1^*, a_2^*, a_3 = 0)$ that could be achieved when R is observable. We will discuss more thoroughly about this informational gain from the agent's derivative transaction later.

---

[22] In Appendix, we define a normalized output $\tilde{x}$ (with respect to non-hedgeable risk θ) level as

$$\tilde{x} \equiv \frac{z(\hat{b}) - \phi(a_1^*, a_2^*)}{a_2^*}.$$

In fact, $z(\hat{b}) \sim N(\phi(a_1^*, a_2^*), (a_2^*)^2 + (b - \hat{b})^2)$ holds.

[23] As λ goes up, the compensation level goes up and it becomes less likely for the limited liability constraint to bind. However this will increase the convexity at the points at which the indirect utility hits the constraint. Our proof strategy based on order analysis makes sure the first effect is stronger than the second effect asymptotically.

[24] Thus an introduction of a derivative market in this case improves welfare compared with the case where the principal does not observe the firm's risk exposure R and communication between the principal and the agent is prohibitively costly.

However, this is not possible if $\underline{\mu_2^* > 0}$ for $w^*(z(\hat{b}))$ in equation (27), since the agent will speculate infinitely in this case, i.e., choose a_3 such that $|R - a_3| = \infty$. This is because, as shown from equation (27), manager will be paid an infinite amount when $z(\hat{b}) = x - \hat{b}\eta$ is either positive or negative infinity if $\mu_2^* > 0$ for $w^*(z(\hat{b}))$. Given that it is impossible to design a wage contract $w^*(z(\hat{b}))$ based on the belief $\hat{b} = \infty$, the principal has to either alter the wage contract to ensure $|R - a_3| < \infty$ or retain the optimal contract without a derivative market, $w^N(x, \eta)$ and prohibit the manager from engaging in derivative transactions, if possible.

To derive an optimal contract in the presence of derivative market when $\mu_2^* > 0$ for $w(z(\hat{b}))$ in equation (27), we first consider the case in which the principal decides to design a contract that ensures a finite a_3 when $\mu_2^* > 0$ for $w^*(z(\hat{b}))$ in equation (27). Actually, there is a class of compensation contracts that ensure a finite a_3, depending on which a_3 is to be induced. Let (a_1^o, a_2^o, a_3^o), where $|b^o \equiv R - a_3^o| < \infty$, be the optimal action combination and $\underline{w^o(z(\hat{b}), \eta)}$ be the wage contract which optimally induces that action combination where $\hat{b} = b^o \equiv R - a_3^o$. We denote the optimized joint benefits in this case as

$$SW(a_1^o, a_2^o, a_3^o) \equiv \phi(a_1^o, a_2^o) - C(a_1^o, a_2^o, b^o) - \lambda v(a_1^o), \tag{28}$$

where

$$C(a_1^o, a_2^o, b^o) \equiv \int [w^o(z(\hat{b}), \eta) - \lambda u(w^o(z(\hat{b}), \eta))] g(z(\hat{b}), \eta | a_1^o, a_2^o, b^o) dz d\eta \tag{29}$$

denotes the agency cost arising from inducing (a_1^o, a_2^o, a_3^o) when there is a derivative market and $\mu_2^* > 0$ for $w^*(z(\hat{b}))$ in equation (27).

Lemma 3 *If $w^o(z(\hat{b}), \eta)$ is an optimal contract that actually induces (a_1^o, a_2^o, a_3^o) where $\hat{b} = R - a_3^o \equiv b^o \neq 0$, then $w^o(z(0), \eta) \equiv w^o(x, \eta)$[25] is also an optimal contract which induces $(a_1^o, a_2^o, a_3 = R)$. Therefore,*

$$SW(a_1^o, a_2^o, a_3^o) = SW(a_1^o, a_2^o, a_3 = R).$$

Lemma 3 indicates that when the principal has to design a compensation contract to guarantee the agent's choice of a_3 satisfying $|R - a_3| < \infty$ due to the fact that $\mu_2^* > 0$ for $w^*(z(\hat{b}))$, the level of a_3 to be induced by $w^o(z(\hat{b}), \eta)$ is a matter of indifference as long as it is finite. This is because, as shown in equation (25), the agent's derivative choice, a_3, is additively separable from his other two productive action choices, (a_1, a_2), in determining the output level, x, and not only the output level but also the derivative market variables, η, are observable (thus contractible).

From Lemma 3, we know that the wage contract which induces the agent to hedge completely ($b = 0$), i.e., $w^o(z(0), \eta) \equiv w^o(x, \eta)$, is one of the optimal contracts among the wage contracts

[25] Note that $z(0) = x$ from equation (25).

that ensure $|R - a_3| < \infty$ when $\mu_2^* > 0$ for $w^*(z(\hat{b}))$. Thus, without loss of generality, we only characterize $w^o(x, \eta)$ that induces $b = 0$ (or $a_3 = R$). Actually, $w^o(x, \eta)$ is obtained by solving a maximization problem that is similar to equation (16) with an added requirement that a contract induces the manager to take $a_3 = R$.

Given that the agent's choosing a_3 is equivalent to his choosing $b = R - a_3$, new optimal contract, $w^o(x, \eta)$, inducing the agent to take $(a_1^o, a_2^o, b = 0)$ when $\mu_2^* > 0$ for $w^*(z(\hat{b}))$ in equation (27), must satisfy the following maximization problem, given (a_1^o, a_2^o):

$$\begin{aligned}
&\max_{w(\cdot)} \int [x - w(x,\eta)] g(x,\eta|a_1^o, a_2^o, b=0) dx d\eta + \lambda \Big[\int u(w(x,\eta)) g(x,\eta|a_1^o, a_2^o, b=0) dx d\eta - v(a_1^o) \Big] \quad \text{s.t.} \\
&\qquad (i) \;\; \int u(w(x,\eta)) g_1(x,\eta|a_1^o, a_2^o, b=0) dx d\eta - v'(a_1^o) = 0, \\
&\qquad (ii) \;\; \int u(w(x,\eta)) g_2(x,\eta|a_1^o, a_2^o, b=0) dx d\eta = 0, \\
&\qquad (iii) \;\; b = 0 \in \arg\max_{b'} \int u(w(x,\eta)) g(x,\eta|a_1^o, a_2^o, b') dx d\eta, \; \forall b', \\
&\qquad (iv) \;\; w(x,\eta) \geq k, \;\; \forall (x,\eta).
\end{aligned} \tag{30}$$

Note that we use the first-order approach for incentive constraints associated with a_1 and a_2, while we do not use the same approach for the incentive constraint associated with b. The following Lemma 4 demonstrates the reason we cannot use the first-order approach for the incentive constraint of b.

Lemma 4 *If $w^*(z(0))$ in equation (27) is designed, the agent will be indifferent between taking b and taking $-b$, $\forall b$.*

Lemma 4 shows, if $w^*(z(0))$ is designed and offered, agent's expected utility becomes symmetric around $b = 0$ (i.e., $a_3 = R$) in the space of b (i.e., in the space of a_3). We know:

$$\int u(w^*(z(0))) g(z(0), \eta | a_1^o, a_2^o, b) dz d\eta \tag{31}$$

is continuous and differentiable in b, Lemma 4 implies:

$$\int u(w^*(z(0)) g_3(z(0), \eta | a_1^o, a_2^o, b = 0) dz d\eta = 0, \tag{32}$$

where g_3 denotes the first derivative of g taken with respect to b. Thus $w^*(z(0))$ is the solution of the following optimization without the incentive constraint of b.[26]

$$\max_{w(\cdot)} \int [x - w(x,\eta)] g(x,\eta|a_1^o, a_2^o, b=0) dx d\eta + \lambda \Big[\int u(w(x,\eta)) g(x,\eta|a_1^o, a_2^o, b=0) dx d\eta - v(a_1^o) \Big] \quad \text{s.t.}$$
$$\begin{aligned}
&(i) \;\; \int u(w(x,\eta)) g_1(x,\eta|a_1^o, a_2^o, b=0) dx d\eta - v'(a_1^o) = 0,\\
&(ii) \;\; \int u(w(x,\eta)) g_2(x,\eta|a_1^o, a_2^o, b=0) dx d\eta = 0,\\
&(iii) \;\; w(x,\eta) \geq k, \;\; \forall (x,\eta).
\end{aligned} \tag{34}$$

This implies that if we use the first-order approach for the incentive constraint associated with b in the above program, we always end up with getting $w^*(z(0))$ in equation (27) as an optimal contract for the above program. However, since $\mu_2^* > 0$ for $w^*(z(0))$, we can easily see from Lemma 2 that this contract incentivizes an agent to take $b = \pm\infty$ instead of taking a stipulated $b = 0$.[27] Therefore, we have to explicitly include the incentive constraint for b which does not rely only on the first-order condition at $b = 0$.

Following Grossman and Hart (1983), we state formally the optimization problem as follows, taking the optimal (a_1^o, a_2^o) as given.[28]

$$\max_{w(\cdot)} \int [x - w(x,\eta)] g(x,\eta|a_1^o, a_2^o, b=0) dx d\eta + \lambda \Big[\int u(w(x,\eta)) g(x,\eta|a_1^o, a_2^o, b=0) dx d\eta - v(a_1^o) \Big] \quad \text{s.t.}$$
$$\begin{aligned}
&(i) \;\; \int u(w(x,\eta)) g_1(x,\eta|a_1^o, a_2^o, b=0) dx d\eta - v'(a_1^o) = 0,\\
&(ii) \;\; \int u(w(x,\eta)) g_2(x,\eta|a_1^o, a_2^o, b=0) dx d\eta = 0,\\
&(iii) \;\; \int u(w(x,\eta)) [g(x,\eta|a_1^o, a_2^o, b=0) - g(x,\eta|a_1^o, a_2^o, b)] dx d\eta \geq 0, \; \forall b,\\
&(iv) \;\; w(x,\eta) \geq k, \;\; \forall (x,\eta).
\end{aligned} \tag{35}$$

Note that incentive constraints for all b are taken into account to make sure the agent's expected

[26] Since for $b = 0$, the likelihood ratios can be represented as

$$\begin{aligned}
&\frac{g_1}{g}(x,\eta|a_1,a_2,b=0) = \frac{x - \phi(a_1,a_2)}{a_2^2}\phi_1(a_1,a_2), \;\; \frac{g_3}{g}(x,\eta|a_1,a_2,b=0) = \frac{(x-\phi(a_1,a_2))\eta}{a_2^2},\\
&\frac{g_2}{g}(x,\eta|a_1,a_2,b=0) = -\frac{1}{a_2} + \frac{x - \phi(a_1,a_2)}{a_2^2}\phi_2(a_1,a_2) + \frac{(x-\phi(a_1,a_2))^2}{a_2^3}.
\end{aligned} \tag{33}$$

Here, $w^*(z(0))$ becomes the solution of the equation (34) without the incentive constraint of b.

[27] The same technical problem does not arise from the incentive constraint of a_2, which also determines the firm's risks. This is because reducing risk through a_2 is costly, while doing it through a_3 is not. Thus, the manager's expected utility is not symmetric in the space of a_2.

[28] Here we solve the reduced problem where we fix the optimal (a_1^o, a_2^o) and try to find the optimal contract $w(x,\eta)$ that induces $b = 0$.

indirect utility is at maximum when he chooses $b = 0$ instead of other $b > 0$ or $b < 0$.

By solving the Euler equation which is derived from the above program, an optimal contract, $w^o(x, \eta)$, satisfies[29]

$$\begin{aligned}\frac{1}{u'(w^o(x,\eta))} =&\lambda + (\mu_1^o\phi_1^o + \mu_2^o\phi_2^o)\frac{x-\phi(a_1^o,a_2^o)}{(a_2^o)^2} + \mu_2^o\frac{1}{a_2^o}\Big(\frac{(x-\phi(a_1^o,a_2^o))^2}{(a_2^o)^2} - 1\Big) \\ &+ \underbrace{\int \mu_4^o(b)\Big[1 - \frac{g(x,\eta|b)}{g(x,\eta|b=0)}\Big]db}_{\text{Additional term to equation (27)}},\end{aligned} \tag{36}$$

for (x, η) satisfying $w^o(x, \eta) \geq k$ and $w^o(x, \eta) = k$ otherwise. In equation (36), $\phi_i^o \equiv \phi_i(a_1^o, a_2^o), i = 1, 2$, and μ_1^o, μ_2^o, and $\mu_4^o(b)$ are the optimized Lagrange multipliers associated with the first, second, and third constraints (for specific b) in the above optimization program, respectively.[30]

As shown in the Appendix, we obtain the following proposition from equation (36).

Proposition 5 *[Hedging through Punishment]*
If $\mu_2^* > 0$ for $w^*(z(0))$ described in equation (27), then the principal can motivate the manager to hedge completely by designing a new compensation contract, $w^o(x, \eta)$ in equation (36), which (i) satisfies $w^o(x, \eta) = w^o(x, -\eta)$ for all x, η, and (ii) penalizes the manager for having any (positive or negative) sample covariance between the output, x, and derivative market observables, η (i.e., penalizing manager for having high $(x - \phi)^2\eta^2$). To be specific, given realized (x, η), a higher sample covariance $(x - \phi)^2\eta^2$ yields a lower wage $w^o(x, \eta)$, while given the output x and sample covariance $(x - \phi)^2\eta^2$, a higher η raises the wage $w^o(x, \eta)$.

Proposition 5 can be understood in the following way: the production function $x = \phi(a_1^o, a_2^o) + a_2\theta + b\eta$ gives us the relation $b = Cov(x, \eta) = \mathbb{E}[(x - \phi(a_1^o, a_2^o))\eta]$. It implies that if the agent takes $b = 0$, a statistical covariance between output x and hedgeable risk η disappears, whereas any other $b \neq 0$ generates non-zero population covariances. Since $b = 0$ generates $x = \phi(a_1^o, a_2^o) + a_2\theta$, which is independent of η, $w^o(x, \eta) = w^o(x, -\eta)$ for all x, η is ensured to minimize the amount of risk imposed on the agent, as η becomes irrelevant in inducing (a_1^o, a_2^o) and has a symmetric distribution around 0.

At optimum, by punishing the covariance between x and η,[31] shareholders effectively incentivize the manager to engage in full hedging and take $b = 0$. As our framework is one-period setting, any positive or negative sample covariance $\widehat{Cov} = (x - \phi(a_1^o, a_2^o))\eta = b\eta^2 + a_2\theta\eta$, instead of a population covariance, is punished by the principal through a lower compensation $w^o(x, \eta)$. If the magnitude of realized covariance $\widehat{Cov} = (x - \phi(a_1^o, a_2^o))\eta = b\eta^2 + a_2\theta\eta$ is large not from the manager's speculation

[29] We suppress the dependence of distribution g and likelihood ratios on (a_1^o, a_2^o) since we already fixed (a_1^o, a_2^o).
[30] For general reference, see Luenberger (1969).
[31] It is possible since η is observable at the end of the period and thus contracts can be written upon it.

($b \neq 0$) but from a high realized market observable, η, then the principal takes it into account and raises $w^o(x,\eta)$. In contrast, given realized output and market observables (x,η), a bigger realization of $\widehat{Cov}$ is likely to be generated by $b \neq 0$ with a bigger $|b|$, thus the agent is punished and her wage income $w^o(x,\eta)$ decreases.

Designing new contract including this covariance term is not, however, costless compared with $w^N(x,\eta)$ in equation (24), the optimal contract in case where there is no derivative market, since it exposes the agent to additional risks. As we show below, if this cost is relatively high compared to the informational gain that principal gets through the agent's derivative transaction, an introduction of derivative markets can actually reduce the welfare.

Proposition 6 *If $\mu_2^* > 0$ for $w^*(z(0))$ in equation (27), the introduction of a derivative market will reduce the firm's welfare compared with SW^N in equation (23) when the amount of uncertainty about the firm's risk exposure, σ_R^2, is small.*

As we have shown in Proposition 4, in the presence of derivative markets, the optimal contract must be altered from $w^N(x,\eta)$. In cases where the manager voluntarily chooses to hedge after the derivative market is introduced, given their original optimal dual agency contract (i.e., $\mu_2^* < 0$ for $w^*(y)$ in equation (18) or equivalently $\mu_2^* < 0$ for $w^*(z(0))$ in equation (27)), the compensation contract remains mainly unchanged from $w^*(y)$ except for being based on $z(0) = x$ rather than y, and welfare unambiguously increases from the informational gain generated by an opportunity of the manager to hedge in the derivative market and completely eliminate the firm's risk exposure. However, when $w^*(z(0))$ with $\mu_2^* > 0$ induces the manager to speculate in the derivative market, shareholders must revise the manager's contract to $w^o(x,\eta)$ to provide the manager with an incentive to hedge, which imposes additional risks on the risk-averse agent's side and incurs the cost out of it. Thus, there are costs and benefits associated with derivative trading that the principal must consider.

Altering the wage contract to ensure that the agent hedges rather than speculates is costly since it needs to consider the agent's additional incentive problem associated with a_3 by exposing him to an additional risk, the market observables, η, whereas the principal gets some informational benefits as now she does not have to know about the firm's risk exposure R, as the agent is induced to eliminate any hedgeable risk ($a_3 = R$) under $w^o(x,\eta)$. On the other hand, when there is no derivative market, an inability to observe R from principal's perspective causes welfare loss because now she should design $w^N(x,\eta)$ instead of $w^*(y)$.[32] To illustrate these costs and benefits more precisely, we use equation (19), equation (23), and equation (28), and decompose the welfare change in the following

[32]Of course, the principal can always design $w^o(x,\eta)$ instead of $w^N(x,\eta)$ when there is no derivative market. However, $w^o(x,\eta)$ will perform poorly without the derivative market.

way.

$$SW(a_1^o, a_2^o, a_3 = R) - SW^N = (SW(a_1^*, a_2^*, a_3 = 0) - SW^N) - (SW(a_1^*, a_2^*, a_3 = 0) - SW(a_1^o, a_2^o, R)). \tag{37}$$

The first part in the right-hand side represents the welfare loss due to the principal's inability to observe the firm's risk exposure when there is no derivative market (or equivalently informational gains from the introduction of a derivative market). The second part represents welfare loss due to an additional incentive problem associated with the agent's derivative choices when a derivative market is introduced and the agent speculates under $w^*(z(0))$ in equation (27).

Note that no expectation with respect to R is taken for joint benefits $SW(a_1^*, a_2^*, a_3 = 0)$ and $SW(a_1^o, a_2^o, a_3 = R)$, since both of them are independent of R. When there is no derivative market and the firm's risk exposure, R, is observed by the principal as well as the manager, joint benefits, $SW(a_1^*, a_2^*, a_3 = 0)$, are obviously independent of the R's realization because (a_1^*, a_2^*) are independent of R. Likewise, when $w^o(x, \eta)$ is designed in the presence of a derivative market, joint benefits $SW(a_1^o, a_2^o, a_3 = R)$ are independent of R as agent is always induced to take $b = R - a_3 = 0$ no matter what R is realized. However, in calculating joint benefits SW^N, an expectation with respect to R is taken, implying that the distribution of R affects the level of SW^N.

The above discussion implies that informational gains from the agent's derivative transaction declines as the amount of uncertainty around the firm's risk exposure R falls. On the other hand, the cost of controlling the additional incentive problem associated with a_3 (or equivalently $b = R - a_3$) is independent of the firm's risk exposure R and thus σ_R^2. For instance, even if R is known to the principal (i.e., $\sigma_R^2 = 0$), a moral hazard problem associated with inducing $b = 0$ still remains to the same degree. Therefore, the amount of uncertainty on R is indeed a matter of indifference in incentivizing the agent's choice of b.

As a result, if the uncertainty around R is small enough, contractual costs dominate informational gains when a derivative market is introduced, and shareholders would be better off by prohibiting the manager from trading derivatives at all. In sum, recent financial innovations sometimes have potentials to hurt the efficiency of firms through its effects on agency relationships.

5 The Truth-Telling Mechanism

Up to this point we have assumed that there is no communication between principal and agent after compensation contract is written, due to high communication costs. We now relax this assumption and consider the case where the agent can costlessly communicate the firm's risk exposure R to the principal, and receive a payoff that is contingent on the communicated risk exposure as well as the output.

As we will show below, for the case where $\mu_2^* < 0$ for $w^*(z(0))$ in equation (27), a contract that is similar to $w^*(z(0))$ can be designed to induce the agent to truthfully reveal the firm's risk exposure R. In other words, there is no loss associated with the risk exposure being unobservable and thus no gain from the introduction of derivative market. The intuition is the same as the one for why the agent would voluntarily hedge under $w^*(z(0))$ with $\mu_2^* < 0$. Essentially, the truth-telling contract allows the agent to make a side bet with the principal and if the agent hedges in the derivative market with the contract $w^*(z(0))$, he will truthfully reveal what he observes (true R) to minimize the additional risk associated with this side bet.

However, when $\mu_2^* > 0$ for $w^*(z(0))$, a contract similar to $w^*(z(0))$ does not induce truth-telling since the agent wants to add more risks, as he does by engaging in speculation in derivative markets. Again, a new contract must be designed to induce him to reveal the truth.

Suppose the principal does not know the firm's innate risk exposure R and there is no derivative market (i.e., a_3 is again fixed at 0). Since the agent observes R before he takes (a_1, a_2) and communication regarding R is freely allowed, the principal can design a truth-telling mechanism, $w(x, r, \eta)$, without incurring cost where r represents the value of R reported by the agent. Let $(a_1^T(R)a_2^T(R))$ be agent's optimal action combination after observing R and $w^T(x, r, \eta)$ be the wage contract that optimally induces $(a_1^T(R), a_2^T(R))$ with the agent telling the truth. Knowing that $r = R, \forall R$, under $w^T(x, r, \eta)$, we denote optimized joint benefits in this case as

$$SW^T \equiv \int [\phi(a_1^T(R), a_2^T(R)) - C(a_1^T(R), a_2^T(R)) - \lambda v(a_1^T(R))]h(R)dR, \tag{38}$$

where

$$C(a_1^T(R), a_2^T(R)) \equiv \int [w^T(x, R, \eta) - \lambda u(w^T(x, R, \eta))]g(x, \eta | a_1^T(R), a_2^T(R))dxd\eta \tag{39}$$

denotes the agency cost arising from inducing $(a_1^T(R), a_2^T(R))$ through $w^T(x, r, \eta)$ when R is realized. In the above equation, $g(x, \eta | a_1^T(R), a_2^T(R))$ denotes the joint density function of (x, η) given that $(a_1^T(R), a_2^T(R))$ is chosen.

Since $SW(a_1^*, a_2^*, a_3 = 0)$ in equation (19) is the maximum level of joint benefits that SW^T in equation (38) can attain, we first consider the case in which principal designs a wage contract, $w^*(y_r)$, that is the same as $w^*(y)$ in equation (17) except that it is based on $y_r \equiv x - r\eta$ instead of $y \equiv x - R\eta$. That is, $w^*(y_r)$ satisfies

$$\frac{1}{u'(w^*(y_r))} = \lambda + (\mu_1^*\phi_1^* + \mu_2^*\phi_2^*)\frac{(y_r - \phi(a_1^*, a_2^*))}{(a_2^*)^2} + \mu_2^*\frac{1}{a_2^*}\Big(\frac{(y_r - \phi(a_1^*, a_2^*))^2}{(a_2^*)^2} - 1\Big), \tag{40}$$

for y_r satisfying $w^*(y_r) \geq k$ and $w^*(y_r) = k$ otherwise. We call $w^*(y_r)$ a 'full-trust contract' as

$w^*(y_r)$ simply is based on the agent's report instead of the realized R in equation (40).

Note that

$$y_r \equiv x - r\eta = \phi(a_1, a_2) + (R - r)\eta + a_2\theta. \tag{41}$$

Since, from equation (25),

$$z(0) = x = \phi(a_1, a_2) + (R - a_3)\eta + a_2\theta, \tag{42}$$

the principal's problem of designing a truth-telling mechanism based on y_r when there is no derivative market is equivalent to his problem of designing an incentive scheme based on $z(0)$ to induce $b = 0$ ($a_3 = R$) when there is a derivative market. As a result, as is the case for $w^*(z(0))$ in Lemma 2, we directly obtain following results for $w^*(y_r)$.

Lemma 5 *[Speculation and Hedging with $w^*(y_r)$]*
(1) If $\underline{\mu_2^* < 0}$ for the wage contract, $w^*(y_r)$, described in equation (40), then the manager will always report truly, i.e., $r = R, \forall R$, when $w^*(y_r)$ is offered when λ is large enough.
(2) If $\underline{\mu_2^* > 0}$ for $w^*(y_r)$, then the manager will report r such that $|R - r| = \infty$ when $w^*(y_r)$ is offered.

From Lemma 5, we obtain the following propositions.

Proposition 7 *When there is no derivative market and communication between principal and agent is costless, then $w^*(y_r)$ described in equation (40) is the optimal truth-telling mechanism for the firm's hidden risk exposure, R, if $\mu_2^* < 0$ for $w^*(y_r)$. In this case,*
(1) The principal's inability to observe R does not reduce the firm's welfare (i.e., no adverse selection), and
(2) An introduction of a derivative market does not improve the firm's welfare compared with SW^T in equation (38).

Proposition 7 along with Propositions 3 and 4 reaffirms the benefits from a derivative market are actually informational gains as the agent engages in the perfect hedging in the derivative market. Because we assumed initially the principal and agent cannot communicate with each other from the huge cost, these benefits are actually associated with saving communication costs[33] that would realistically incur when principal has to design a truth-telling contract that induces the agent to reveal his exact information about the firm's risk exposure R. When the communication between principal and agent becomes free, the principal, by designing $w^*(y_r)$, can easily reproduce the same results as when he exactly knows the firm's innate risk exposure if $\mu_2^* < 0$ for $w^*(y_r)$. However, in reality, the costs associated with communicating this information and updating the compensation

[33] Because principal and agent do not need to communicate about the realized R, since the agent eliminates this innate risk R through derivative transactions ($a_3 = R$).

contract may be greater than the hedging cost. As shown in equation (41), allowing the agent to choose a_3 in derivative transactions is observationally equivalent to allowing him to freely report the firm's realized risk exposure R.

On the other hand, if $\mu_2^* > 0$ for $w^*(y_r)$, the agent does not reveal truth under $w^*(y_r)$, and the principal has to redesign a truth-telling mechanism, $w^T(x, r, \eta)$ different from $w^*(y_r)$.

Proposition 8 *If $\mu_2^* > 0$ for $w^*(y_r)$ described in equation (40), the introduction of a derivative market does not improve on the firm's efficiency when communication between the principal and the agent is freely allowed, and it actually lowers the firm's efficiency if σ_R^2 is very small.*

As explained in Proposition 6, if communication is not available with $\mu_2^* > 0$ in equation (27), an opportunity to transact derivatives may or may not improve the firm's welfare compared to the case without the derivative market depending on the size of uncertainty σ_R^2 on the exposure R.

If communication is free between principal and agent, however, an access to the derivative market actually reduces the firm's welfare when σ_R^2 is small enough. It is because both $w^o(x, \eta)$ in equation (36) and $w^N(x, \eta)$ in equation (24)[34] are actually truth-telling contracts. Thus, when there is no derivative market, the principal can at least design either $w^o(x, \eta)$ or $w^N(x, \eta)$ under free communication depending on which gives the better result.[35] As shown in Proposition 6, the principal prefers designing $w^N(x, \eta)$ to $w^o(x, \eta)$ if σ_R^2 is very small. The optimal truth-telling mechanism, $w^T(x, r, \eta)$, thus performs <u>weakly</u> better than $w^N(x, \eta)$, which yields <u>strictly</u> larger welfare than $w^o(x, \eta)$. However, after the derivative market is introduced, the principal has to shift from $w^T(x, r, \eta)$ to $w^o(x, \eta)$ because there now exists an incentive problem associated with a_3.

In sum, when communication between the principal and agent is free, the agent's access to the derivative transactions does not change the firm's welfare if $\mu_2^* < 0$, and even lowers it if $\mu_2^* > 0$.

6 Conclusion

As the last financial crisis hit the world, interests in risk management and effects of new developments in financial markets have increased dramatically. Since then, academics and policymakers proposed a number of explanations for the economy-wide effects of new innovations in financial markets, efficient ways to manage risks, and rationales for more strict regulations. However, up to our knowledge, few have seriously attempted to connect this trend with the managerial incentive aspects. In line with the former Federal Reserve chairperson Ben Bernanke's emphasis on the

[34]Note both $w^o(x, \eta)$ and $w^N(x, \eta)$ do not depend on the reported value of R, so we regard both two contracts as truth-telling mechanism.

[35]The optimal truth-telling mechanism in this case may not be $w^o(x, \eta)$ or $w^N(x, \eta)$.

role of compensation structures in many banking corporations in generating excessive risks[36], this agency aspect must be taken more seriously whenever we try to understand the effect of financial innovation in conjunction with the risk-management issues.

Many corporations in both Wall street and main streets have used various forms of compensation contracts to motivate their workers and incentivize them to engage in more innovations. However, under certain circumstances, as illustrated by shadow banking industries during the Global Financial Crisis (2008), managers of the firms speculate rather than hedge, if compensation contracts are not modified to account for unobservable opportunities to alter the firm's risk exposures. When compensation contracts do need to be modified, some efficiency gains associated with the introduction of hedging instruments is lessened because the new contractual form may less efficiently share risk between the manager and shareholders and induce efforts. Indeed, our analysis suggests that in some circumstances, the incentive costs associated with keeping the agent from speculating through derivative transactions is sufficiently large that firms would be better off by prohibiting derivative transactions. This point is understood in line with the last decade's hard works of governments, central banks, and international institutions to curb the degree of risk-taking through regulations on incentive compensation schemes. For example, 2010's 'Dodd-Frank Wall Street Reform and Consumer Protection Act, section 956, mandates that agencies must prohibit all covered institutions from establishing or maintaining incentive-based compensation arrangements that encourage inappropriate risk-taking.[37]

We stress that while we focused on an agency relationship, improving the information content of a firm's cash flows by reducing the risk can improve firm values in a number of different ways. For example, a lower risk provides better information about the managers' abilities as well as their effort levels, which allows shareholders to better match managers with appropriate positions. In addition, more informative cash flows are likely to improve the informational efficiency of firms' share prices. With more informative stock prices, capitals are allocated more efficiently and managers have less incentive to take short-sighted actions and in other ways expend resources signalling their firms' values.

[36]"The Federal Reserve is working to ensure that compensation packages appropriately tie rewards to longer-term performance and do not create undue risk for the firm or the financial system.", said Ben Bernanke, the former Fed chair. https://www.nytimes.com/2009/10/23/business/23pay.html

[37]"Last month, the Federal Reserve delivered assessments to the firms that included analysis of current compensation practices and areas requiring prompt attention. Firms are submitting plans to the Federal Reserve outlining steps and timelines for addressing outstanding issues to ensure that incentive compensation plans do not encourage excessive risk-taking.", said Federal Reserve's 06.21.2010 joint press.

References

The federal reserve press release. https://www.federalreserve.gov/newsevents/pressreleases/bcreg20091022a.htm, 2009.

Martin N. Baily, Andrew B. Bernard, John Y. Campbell, Douglas W. Diamond, Darrell Duffie, Kenneth R. French, Anil K Kashyap, Frederic S. Mishkin, Raghuram G. Rajan, David S. Scharfstein, Robert J. Shiller, Hyun Song Shin, Matthew J. Slaughter, Jeremy C. Stein, and René M. Stulz. Regulation of executive compensation in financial services", squam lake working group on financial regulation.

George P. Baker. Incentive contracts and performance measurement. *Journal of Political Economy*, 100(3):598–614, 1992.

Daniel Barron, George Georgiadis, and Jeroen Swinkels. Optimal contracts with a risk-taking agent. *Theoretical Economics*, 15(2):715–761, 2020.

Jeffrey L. Coles, Naveen D Daniel, and Lalitha Naveen. Managerial incentives and risk-taking. *Journal of Financial Economics*, 79:431–468, 2006.

John R. Conlon. Two new conditions supporting the first-order approach to multisignal principal-agent problems. *Econometrica*, 77(1):249–278, 2009.

Peter DeMarzo and Darrell Duffie. Corporate financial hedging with proprietary information. *Journal of Economic Theory*, 53:261–286, 1991.

Peter DeMarzo and Darrell Duffie. Corporate incentives for hedging and hedge accounting. *Review of Financial Studies*, 8:743–771, 1995.

Peter DeMarzo, Dmitry Livdan, and Alexei Tchistyi. Risking other people's money : Gambling, limited liability, and optimal incentives. *Working Paper*, 2013.

Mario Draghi, Francesco Giavazzi, and Robert C. Merton. Transparency, risk management and international financial fragility. *NBER Working Paper 9806*, 2003.

Emmanuel Farhi and Jean Tirole. Collective moral hazard, maturity mismatch, and systemic bailouts. *American Economic Review*, 102(1):60–93, 2012.

Kenneth A. Froot, David S. Scharfstein, and Jeremy Stein. Risk management: Coordinating corporate investment and financing policies. *Journal of Finance*, 48(5):1629–1658, 1993.

Luis Garicano and Luis Rayo. Why organizations fail : Models and cases. *Journal of Economic Literature*, 54(1):137–92, 2016.

Christopher Geczy, Bernadette A. Minton, and Catherine Schrand. Why firms use currency derivatives. *Journal of Finance*, 52(4):1323–1354, 1997.

Sanford J. Grossman and Oliver Hart. An analysis of the principal-agent problem. *Econometrica*, 51: 7–45, 1983.

Wayne R. Guay. The sensitivity of ceo wealth to equity risk : an analysis of the magnitude and determinants. *Journal of Financial Economics*, 53:43–71, 1999.

Milton Harris and Artur Raviv. Optimal incentive contracts with imperfect information. *Journal of Economic Theory*, 20(2):231–259, 1979.

David Hirshleifer and Yoon Suh. Risk, managerial effort, and project choice. *Journal of Financial Intermediation*, 2(3):308–345, 1992.

David Hirshleifer and Anjan V. Thakor. Managerial conservatism, project choice and debt. *The Review of Financial Studies*, 5(3):437–470, 1992.

Bengt Holmström. Moral hazard and observability. *The Bell Journal of Economics*, 10(1):74–91, 1979.

Bengt Holmström and Paul Milgrom. Aggregation and linearity in provision of intertemporal incentives. *Econometrica*, 55:303–328, 1987.

Benjamin Hébert. Moral hazard and the optimality of debt. *Review of Economic Studies*, 85(4):2214–2252, 2018.

Ian Jewitt. Justifying the first-order approach to principal-agent problems. *Econometrica*, 56(5): 1177–1190, 1988.

Ian Jewitt, Ohad Kadan, and Jeroen M. Swinkels. Moral hazard with bounded payments. *Journal of Economic Theory*, 143(1):59–82, 2008.

Jin Yong Jung and Son Ku Kim. Information space conditions for the first-order approach in agency problems. *Journal of Economic Theory*, 160:243–279, 2015.

Son Ku Kim. Efficiency of an information system in an agency model. *Econometrica*, 63(1):89–102, 1995.

Son Ku Kim and Yoon Suh. Ranking of accounting information systems for management control. *Journal of Accounting Research*, 29(2):386–396, 1991.

Jean-Jacques Laffont and David Martimort. Collusion under asymmetric information. *Econometrica*, 65(4):875–912, 1997.

Richard A. Lambert. Long term contracts and moral hazard. *The Bell Journal of Economics*, 14(2): 441–452, 1983.

Richard A. Lambert. Executive effort and the selection of risky projects. *Rand Journal of Economics*, 17(1):77–88, 1986.

D. G. Luenberger. *Optimization by Vector Space Methods*. John Wiley & Sons, Inc. 27, 28, 30, 1969.

Igor Makarov and Guillaume Plantin. Rewarding trading skills without inducing gambling. *Journal of Finance*, 70:925–962, 2015.

James A. Mirrlees. Notes on welfare economics, information, and uncertainty. *Essays on Economic Behavior Under Uncertainty: E. Balch, D. McFadden, and H. Wu, eds., North-Holland, Amsterdam.*, 1974.

Frederic. Palomino and Andrea Prat. Risk-taking and optimal contracts for money manegers. *RAND Journal of Economics*, 34:113–137, 2003.

Roy Radner. Repeated principal-agent games with discounting. *Econometrica*, 53(5):1173–1198, 1985.

Shivaram Rajgopal and Terry Shevlin. Empirical evidence on the relation between stock option compensation and risk taking. *Journal of Accounting and Economics*, 33(2):145–171, 2002.

William Rogerson. The first order approach to principal-agent problem. *Econometrica*, 53:1357–1368, 1985.

Steve Ross. On the economic theory of agency and the principle of similarity. *Essays on Economic Behavior Under Uncertainty: E. Balch, D. McFadden, and H. Wu, eds., North-Holland, Amsterdam.*, 1974.

Steve Ross. Compensation, incentives, and the duality of the risk-aversion and riskiness. *Journal of Finance*, 59:207–225, 2004.

Michael Rothschild and Joseph E. Stiglitz. Increasing risk i: A definition. *Journal of Economic Theory*, 2(3):225–243, 1970.

Steven Shavell. Risk sharing and incentives in the principal and agent relationship. *The Bell Journal of Economics*, 10(1):55–73, 1979.

Bernanrd Sinclair-Desgagné. The first-order approach to multi-signal principal-agent problems. *Econometrica*, 62(2):459–465, 1994.

Clifford W. Smith and Rene M. Stulz. The determinants of firms' hedging policies. *The Journal of Financial and Quantitative Analysis*, 20(4):391–405, 1985.

Jaeyoung Sung. Ilinearity with project selection and controllable diffusion rate in continuous-time principal-agent problems. *The RAND Journal of Economics*, 26(5):720–743, 1995.

Peter Tufano. Who manages risk? an empirical examination of risk management practices in the gold mining industry. *The Journal of Finance*, 51(4):1097–1137, 1996.

Appendix. Derivations and Proofs

Proof of Lemma 1: We know from $y \sim N(\phi(a_1, a_2), a_2^2)$ that

$$\frac{y-\phi(a_1,a_2)}{a_2} \sim N(0,1), \quad \frac{f_1}{f}(y|a_1,a_2,a_3=0) = \frac{y-\phi(a_1,a_2)}{a_2^2}\phi_1(a_1,a_2) \sim N(0, \frac{\phi_1(a_1,a_2)^2}{a_2^2}). \tag{A1}$$

So if $\frac{\phi_1(a_1,a_2)}{a_2}$ is decreasing in a_2, for any pair $a_2^0 < a_2^1$, $\frac{f_1}{f}(y|a_1,a_2^0,a_3=0)$'s distribution is mean-preserving spread (MPS) of that of $\frac{f_1}{f}(y|a_1,a_2^1,a_3=0)$. Assumption 5 guarantees this condition holds, and Kim (1995) proves $C(a_1,a_2^0) < C(a_1,a_2^1)$ for $\forall a_1$.
■

Derivation of equation (10): Given $a_1 = a_1^P$ (fixed), $\phi_2(a_1^P,a_2^P) = C_2(a_1^P,a_2^P)$ holds at optimum. We can write $C_2(a_1^P,a_2^P)$ in the following way:

$$\begin{aligned}\phi_2(a_1^P,a_2^P) = C_2(a_1^P,a_2^P) = &\int [w^P(y|a_1^P,a_2^P) - \lambda u(w^P(y|a_1^P,a_2^P))] f_2(y|a_1^P,a_2^P)dy \\ &+ \int \Big[\frac{\partial w^P}{\partial a_2}(y|a_1^P,a_2^P) - \lambda u'(w^P(y|a_1^P,a_2^P))\frac{\partial w^P}{\partial a_2}(y|a_1^P,a_2^P)\Big] f(y|a_1^P,a_2^P)dy,\end{aligned} \tag{A2}$$

where we know the following equation is satisfied:[1]

$$\begin{aligned}\frac{\partial w^P}{\partial a_2}(y|a_1^P,a_2^P) - \lambda u'(w^P(y|a_1^P,a_2^P))\frac{\partial w^P}{\partial a_2}(y|a_1^P,a_2^P) &= \frac{\partial w^P}{\partial a_2}(y|a_1^P,a_2^P)\Big(1-\lambda u'(w^P(y|a_1^P,a_2^P))\Big) \\ &= \frac{\partial w^P}{\partial a_2}(y|a_1^P,a_2^P)\cdot \mu_1(a_1^P,a_2^P)\frac{f_1}{f}(y|a_1^P,a_2^P)u'(w^P(y|a_1^P,a_2^P)).\end{aligned} \tag{A3}$$

Thus by plugging equation (A3) into equation (A2), we obtain

$$\begin{aligned}&\int \Big[\frac{\partial w^P}{\partial a_2}(y|a_1^P,a_2^P) - \lambda u'(w^P(y|a_1^P,a_2^P))\frac{\partial w^P}{\partial a_2}(y|a_1^P,a_2^P)\Big] f(y|a_1^P,a_2^P)dy \\ &= \mu_1(a_1^P,a_2^P)\int \frac{\partial w^P}{\partial a_2}(y|a_1^P,a_2^P) f_1(y|a_1^P,a_2^P)u'(w^P(y|a_1^P,a_2^P))dy.\end{aligned} \tag{A4}$$

When $w^P(y|a_1^P,a_2)$ is designed for $\forall a_2$, it should satisfy the such incentive constraint (where $w^P(y|a_1^P,a_2 = a_2^P) \equiv w^P(y|a_1^P,a_2^P)$) as

$$\int u(w^P(y|a_1^P,a_2))f_1(y|a_1^P,a_2)dy = v'(a_1^P). \tag{A5}$$

[1]The second equality below holds even in the region where the limited liability constraint binds and $w^P(y|a_1^P,a_2^P) = k$ as its derivative with respect to a_2 is 0, except on measure 0. A small change in a_2 leads to only a small change in the region of a binding limited liability.

We get the following by differentiating both side of equation (A5) by a_2 at $a_2 = a_2^P$:

$$\int u'(w^P(y|a_1^P, a_2^P)) \frac{\partial w^P}{\partial a_2}(y|a_1^P, a_2^P) f_1(y|a_1^P, a_2^P) dy = -\int u(w^P(y|a_1^P, a_2^P) f_{12}(y|a_1^P, a_2^P) dy. \tag{A6}$$

Plugging equation (A6) into equation (A2), we get the following equation (10).[2]

$$\begin{aligned}\phi_2(a_1^P, a_2^P) = \int y f_2(y|a_1^P, a_2^P) dy = \int [w^P(y|a_1^P, a_2^P) - \lambda u(w^P(y|a_1^P, a_2^P))] f_2(y|a_1^P, a_2^P) dy \\ - \mu_1(a_1^P, a_2^P) \int u(w^P(y|a_1^P, a_2^P) f_{12}(y|a_1^P, a_2^P) dy.\end{aligned} \tag{A9}$$

■

Proof of Proposition 1: Assume to the contrary that $\mu_1^* \phi_1^* + \mu_2^* \phi_2^* \leq 0$. Then, pick up any two levels of y: y_1 and y_2, such that

$$y_1 < y_2, \text{ and } \frac{y_1 + y_2}{2} = \phi(a_1^*, a_2^*). \tag{A10}$$

That is, y_1 and y_2 are located at the same distance from the mean value $\phi(a_1^*, a_2^*)$. If $\mu_1^* \phi_1^* + \mu_2^* \phi_2^* \leq 0$, we have from equation (18) that

$$w^*(y_1) \geq w^*(y_2), \text{ and } u(w^*(y_1)) \geq u(w^*(y_2)). \tag{A11}$$

Since $f_1(y_1|a_1^*, a_2^*, a_3 = 0) = -f_1(y_2|a_1^*, a_2^*, a_3 = 0) < 0$ for any y_1 and y_2 satisfying equation (A10), we have:

$$\int u(w^*(y)) f_1(y|a_1^*, a_2^*, a_3 = 0) dy \leq 0, \text{ and } \int u(w^*(y)) f_1(y|a_1^*, a_2^*, a_3 = 0) dy - v'(a_1^*) < 0. \tag{A12}$$

Therefore, there is a contradiction.

■

Proof of Proposition 2:

[2] We can derive it using the envelope theorem. We regard the principal's optimization as the one in which given a fixed a_2, we find optimal $a_1, w(\cdot)$ that maximizes joint utility of the principal and agent under the incentive constraint for a_1 and limited liability constraint. A principal solves the following optimization.

$$\begin{aligned}SW(a_2) = \min_{\mu_1} \max_{w(\cdot), a_1} L(a_2) \equiv \phi(a_1, a_2) - \int w(y) f(y|a_1, a_2) dy + \lambda \Big[\int u(w(y)) f(y|a_1, a_2) dy - v(a_1) \Big] \\ + \mu_1 \Big[\int u(w(y)) f_1(y|a_1, a_2) dy - v'(a_1) \Big]\end{aligned} \tag{A7}$$

As $(a_1^P, w^P(\cdot|a_1^P, a_2^P))$ are the solution given a_2^P, $SW'(a_2^P) = 0$ must hold, which turns out to be the same as equation (10). Thus an envelope theorem yields equation (10), where $\mu_1(a_1^P, a_2^P)$ are the endogenous Lagrange multiplier for incentive constraint for a_1 at a_1^P given a_2^P. Thus, we obtain

$$SW'(a_2^P) = \int [y - w^P(y) + \lambda u(w^P(y))] f_2(y|a_1^P, a_2^P, a_3 = 0) dy + \mu_1(a_1^P, a_2^P) \int u(w^P(y)) f_{12}(y|a_1^P, a_2^P, a_3 = 0) dy = 0. \tag{A8}$$

(Case 1) $\underline{\mu_2^* > 0 \quad \text{if} \quad a_2^A(a_2^P) < a_2^P}$: Compare the following two optimizations:[3]

$$
\begin{aligned}
\max_{a_1,a_2,w(\cdot)} & \int (y-w(y))f(y|a_1,a_2,a_3=0)dy + \lambda\Big[\int u(w(y))f(y|a_1,a_2,a_3=0)dy - v(a_1)\Big] \quad \text{s.t.} \\
& (i) \ \int u(w(y))f_1(y|a_1,a_2,a_3=0)dy - v'(a_1) = 0, \\
& (ii) \ \int u(w(y))f_2(y|a_1,a_2,a_3=0)dy = 0, \\
& (iii) \ w(y) \geq k, \ \ \forall y,
\end{aligned}
\tag{A13}
$$

and

$$
\begin{aligned}
\max_{a_1,a_2,w(\cdot)} & \int (y-w(y))f(y|a_1,a_2,a_3=0)dy + \lambda\Big[\int u(w(y))f(y|a_1,a_2,a_3=0)dy - v(a_1)\Big] \quad \text{s.t.} \\
& (i) \ \int u(w(y))f_1(y|a_1,a_2,a_3=0)dy - v'(a_1) = 0, \\
& (ii) \ \int u(w(y))f_2(y|a_1,a_2,a_3=0)dy \geq 0, \\
& (iii) \ w(y) \geq k, \ \ \forall y,
\end{aligned}
\tag{A14}
$$

where the incentive constraint associated with the non-hedgeable risk choice a_2 takes the form of inequality in the latter program, instead of equality as in the former optimization program.

We know $(w^*(y), a_1^*, a_2^*, \mu_1^*, \mu_2^*)$ are the optimal solution for the first program. Let $(\hat{w}(y), \hat{a}_1, \hat{a}_2, \hat{\mu}_1, \hat{\mu}_2)$ be the optimal solution for the second program. We will show that the above two programs are equivalent in that two solutions align perfectly with each other when $a_2^A(a_2^P) < a_2^P$. Then we can directly derive $\mu_2^* \geq 0$ when $a_2^A(a_2^P) < a_2^P$, since $\hat{\mu}_2 \geq 0$ by Kuhn-Tucker theorem.

Assume that the second constraint in the second program is not binding. Then, $\hat{\mu}_2 = 0$, and $\hat{w}(y)$ should satisfy:

$$
\frac{1}{u'(\hat{w}(y))} = \lambda + \hat{\mu}_1 \frac{y - \phi(\hat{a}_1, \hat{a}_2)}{(\hat{a}_2)^2}\phi_1(\hat{a}_1, \hat{a}_2),
\tag{A15}
$$

for y satisfying $\hat{w}(y) \geq k$ and $\hat{w}(y) = k$ otherwise. As we know that the second constraint is not binding, $\hat{a}_2$ becomes the best (from the principal's perspective) a_2, i.e., $\hat{a}_2 = a_2^P$. Then we must have $\hat{a}_1 = a_1^P$ and $\hat{w}(y) = w^P(y|a_1^P, a_2^P)$. Therefore, the fact that the second constraint in the second program is not binding implies

$$
\int u(w^P(y|a_1^P, a_2^P))f_2(y|a_1^P, a_2^P, a_3=0)dy > 0.
\tag{A16}
$$

However, equation (A16) implies $a_2^A(a_2^P) > a_2^P$, which is a contradiction.[4] Thus the second constraint in the second program must be binding, and the above two programs are equivalent so $\mu_2^* = \hat{\mu}_2 \geq 0$. And also, $\mu_2^* \neq 0$, because $\mu_2^* = 0$ implies $a_2^A(a_2^P) = a_2^P$.

[3]Following Rogerson (1985), we replace the incentive constraint with the corresponding inequality constraint, and exploit the fact that a multiplier to the inequality constraint must be non-negative.

[4]We assume that $\int u(w(y|a_2^P))f(y|a_1^P, a_2, a_3=0)dy$ is concave in a_2, which is based on the first-order approach associated with a_2.

(**Case 2**) $\mu_2^* < 0 \quad \text{if} \quad a_2^A(a_2^P) > a_2^P$: This proof easily follows by using the same method in (**Case 1**).[5]

■

Proof of Proposition 3: Consider the principal's maximization program such as:[6]

$$\begin{aligned}
&\max_{a_1(\cdot),a_2(\cdot),w(\cdot)} \int_R \int_{x,\eta} [x - w(x,R,\eta)] g(x,\eta|a_1(R),a_2(R),R)h(R)dxd\eta dR \\
&\quad + \lambda \int_R [\int_{x,\eta} u(w(x,R,\eta))g(x,\eta|a_1(R),a_2(R),R)dxd\eta - v(a_1(R))]h(R)dR \quad \text{s.t.} \\
&\quad (i) \int_{x,\eta} u(w(x,R,\eta))g_1(x,\eta|a_1(R),a_2(R),R)dxd\eta - v'(a_1(R)) = 0, \forall R, \\
&\quad (ii) \int_{x,\eta} u(w(x,R,\eta))g_2(x,\eta|a_1(R),a_2(R),R)dxd\eta = 0, \forall R, \\
&\quad (iii) \; w(x,R,\eta) \geq k, \quad \forall (x,\eta).
\end{aligned} \tag{A20}$$

Note that the above program is different from the original equation (21) in that here contract can be written on the realized value of R. If we let Lagrange multipliers to the constraints (i) and (ii) be

[5] We compare following two optimization programs similar to equation (A13) and equation (A14).

$$\begin{aligned}
&\max_{a_1,a_2,w(\cdot)} \int (y - w(y))f(y|a_1,a_2,a_3=0)dy + \lambda\Big[\int u(w(y))f(y|a_1,a_2,a_3=0)dy - v(a_1)\Big] \quad \text{s.t.} \\
&\quad (i) \int u(w(y))f_1(y|a_1,a_2,a_3=0)dy - v'(a_1) = 0, \\
&\quad (ii) \int u(w(y))f_2(y|a_1,a_2,a_3=0)dy = 0, \\
&\quad (iii) \; w(y) \geq k, \quad \forall y,
\end{aligned} \tag{A17}$$

and

$$\begin{aligned}
&\max_{a_1,a_2,w(\cdot)} \int (y - w(y))f(y|a_1,a_2,a_3=0)dy + \lambda\Big[\int u(w(y))f(y|a_1,a_2,a_3=0)dy - v(a_1)\Big] \quad \text{s.t.} \\
&\quad (i) \int u(w(y))f_1(y|a_1,a_2,a_3=0)dy - v'(a_1) = 0, \\
&\quad (ii) \int u(w(y))f_2(y|a_1,a_2,a_3=0)dy \leq 0, \\
&\quad (iii) \; w(y) \geq k, \quad \forall y,
\end{aligned} \tag{A18}$$

Solutions of two optimization programs must align with each other, and due to the property that the multiplier attached to the incentive constraint associated with a_2 in the second program must be non-positive, we easily conclude $\mu_2^* < 0$ when $a_2^A(a_2^P) > a_2^P$.

[6] $g(x,\eta|a_1,a_2,R)$ yields the following likelihood ratios:

$$\begin{aligned}
&\frac{g_1}{g}(x,\eta|a_1,a_2,R) = \frac{x - R\eta - \phi(a_1,a_2)}{a_2^2}\phi_1(a_1,a_2), \quad \frac{g_3}{g}(x,\eta|a_1,a_2,R) = \frac{(x - R\eta - \phi(a_1,a_2))\eta}{a_2^2}, \\
&\frac{g_2}{g} = -\frac{1}{a_2} + \frac{x - R\eta - \phi(a_1,a_2)}{a_2^2}\phi_2(a_1,a_2) + \frac{(x - R\eta - \phi(a_1,a_2))^2}{a_2^3}.
\end{aligned} \tag{A19}$$

$\mu_1(R)h(R)$ and $\mu_2(R)h(R)$ respectively, we get the following optimal solution.[7]

$$\begin{aligned}\frac{1}{u'(w(x,R,\eta))} &= \lambda + \mu_1(R)\frac{x-R\eta-\phi_R}{a_2(R)^2}\phi_{1,R} + \mu_2(R)\Big[-\frac{1}{a_2(R)} + \frac{x-R\eta-\phi_R}{a_2(R)^2}\phi_{2,R} + \frac{(x-R\eta-\phi_R)^2}{a_2(R)^3}\Big] \\ &= \lambda + (\mu_1(R)\phi_{1,R}+\mu_2(R)\phi_{2,R})\frac{\overbrace{x-R\eta}^{\equiv y}-\phi_R}{a_2(R)^2} + \frac{\mu_2(R)}{a_2(R)}\Big[\frac{(\overbrace{x-R\eta}^{\equiv y}-\phi_R)^2}{a_2(R)^2} - 1\Big],\end{aligned} \tag{A21}$$

when $w(x,R,\eta) \geq k$. The above equation (A21) implies that optimal contract only depends on $y \equiv x - R\eta$ and the solution $(w(x,R,\eta), a_1(R), a_2(R))$ becomes $(a_1^*, a_2^*, w^*(y) \equiv w^*(x-R\eta))$. By comparing the above equation (A20) with the program in equation (21) when he does not know R, one can easily see that the set of wage contracts, $\{w(x,R,\eta)\}$, satisfying the incentive constraints for a given action combination $(a_1(R), a_2(R))$ in the above program always contains the set of wage contracts, $\{w(x,\eta)\}$, satisfying the incentive constraints for the same action combination when the principal does not know R. Therefore, we have

$$SW^N \leq SW(a_1^*, a_2^*, a_3 = 0). \tag{A22}$$

However, one can easily see that $w^*(y) = w^*(x-R\eta)$ which is a unique solution for the wage contract of the above program is not in the set of $\{w(x,\eta)\}$. As a result, we finally derive

$$SW^N < SW(a_1^*, a_2^*, a_3 = 0). \tag{A23}$$

<u>Proof of Lemma 2</u>:

(1) Suppose $\underline{\mu_2^* < 0}$ for $w^*(z(\hat{b}))$ in equation (27). Given $a_1 = a_1^*, a_2 = a_2^*$, $z(\hat{b}) = x - \hat{b}\eta = \phi(a_1^*, a_2^*) + (b-\hat{b})\eta + a_2^*\theta$ holds. With the normalized output $\tilde{x} \equiv \frac{z(\hat{b})-\phi(a_1^*,a_2^*)}{a_2^*} = \frac{b-\hat{b}}{a_2^*}\eta + \theta$, which follows the normal distribution, as η and θ both follow the standard normal and are independent, equation (27) becomes:

$$\begin{aligned}\frac{1}{u'(w^*(z(\hat{b})))} &= \lambda + (\mu_1^*\phi_1^* + \mu_2^*\phi_2^*)\frac{z(\hat{b})-\phi(a_1^*,a_2^*)}{(a_2^*)^2} + \mu_2^*\frac{1}{a_2^*}\Big(\frac{(z(\hat{b})-\phi(a_1^*,a_2^*))^2}{(a_2^*)^2} - 1\Big) \\ &= \underbrace{\lambda - \frac{\mu_2^*}{a_2^*}}_{\equiv\kappa_0>0} + \underbrace{\frac{\mu_1^*\phi_1^* + \mu_2^*\phi_2^*}{a_2^*}}_{\equiv\kappa_1>0}\tilde{x} + \underbrace{\frac{\mu_2^*}{a_2^*}}_{\equiv\kappa_2<0}\tilde{x}^2 = \kappa_0 + \kappa_1\tilde{x} + \kappa_2\tilde{x}^2,\end{aligned} \tag{A24}$$

when $\kappa_0 + \kappa_1\tilde{x} + \kappa_2\tilde{x}^2 \geq \frac{1}{u'(k)}$. Otherwise $w^*(z(\hat{b})) = k$. Let $p \equiv \frac{1}{u'(k)} > 0$. Let $\rho(z) \equiv u((u')^{-1}(\frac{1}{z})), \forall z$ and $\hat{\rho}(\tilde{x}) \equiv \rho(\max\{p, \kappa_0 + \kappa_1\tilde{x} + \kappa_2\tilde{x}^2\})$. Then, $\hat{\rho}(\tilde{x})$ is the agent's indirect utility given $\tilde{x}$, since

$$u(w^*(z(\hat{b}))) = u\Big((u')^{-1}\Big(\frac{1}{\max\Big\{\underbrace{\frac{1}{u'(k)}}_{=p>0}, \kappa_0 + \kappa_1\tilde{x} + \kappa_2\tilde{x}^2\Big\}}\Big)\Big) = \rho(\max\{p, \kappa_0 + \kappa_1\tilde{x} + \kappa_2\tilde{x}^2\}) = \hat{\rho}(\tilde{x}). \tag{A25}$$

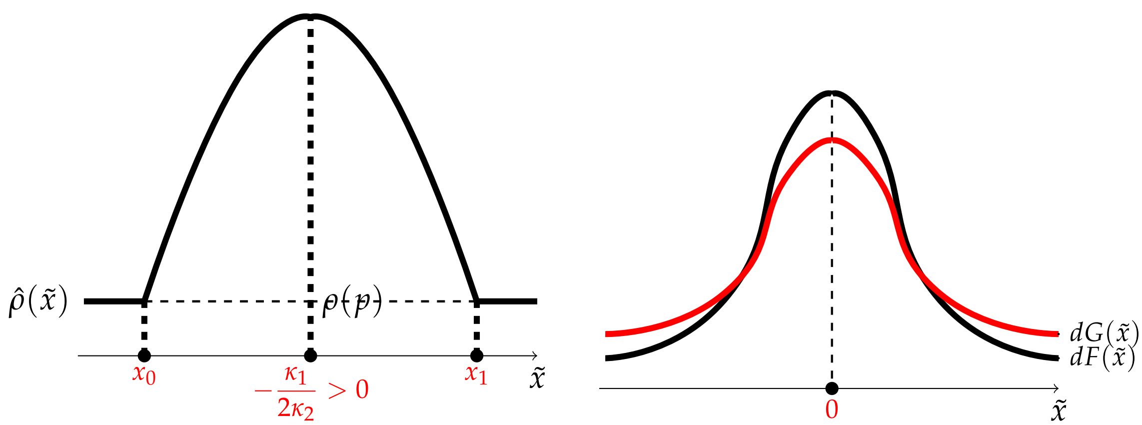
[7] We define $\phi_R \equiv \phi(a_1(R), a_2(R)), \phi_{i,R} \equiv \phi_i(a_1(R), a_2(R))$ for $\forall i = 1,2$, where $a_1(R), a_2(R)$ are optimal actions.

$\hat{\rho}(\tilde{x})$ is symmetric around $-\frac{\kappa_1}{2\kappa_2}$ and concave in $\tilde{x}$ under Assumption 6 when $\kappa_0 + \kappa_1\tilde{x} + \kappa_2\tilde{x}^2 \geq p$:

$$\begin{aligned} \hat{\rho}'(\tilde{x}) &= \underbrace{\rho'(\kappa_0 + \kappa_1\tilde{x} + \kappa_2\tilde{x}^2)}_{>0} \cdot (\kappa_1 + \underbrace{2\kappa_2}_{<0}\,\tilde{x}), \\ \hat{\rho}''(\tilde{x}) &= \underbrace{\rho''(\kappa_0 + \kappa_1\tilde{x} + \kappa_2\tilde{x}^2)}_{<0} \cdot \underbrace{(\kappa_1 + 2\kappa_2\tilde{x})^2}_{>0} + \underbrace{\rho'(\kappa_0 + \kappa_1\tilde{x} + \kappa_2\tilde{x}^2)}_{>0} \cdot \underbrace{2\kappa_2}_{<0} < 0, \end{aligned} \tag{A26}$$

where $\rho''(\kappa_0 + \kappa_1\tilde{x} + \kappa_2\tilde{x}^2) < 0$ from Assumption 6. However at x_0 and x_1 in Figure F.1 where $\hat{\rho}(\tilde{x})$ hits limited liability constraint, we can see $\hat{\rho}(\tilde{x})$ becomes very convex, which creates some subtlety.

We see $\tilde{x}$'s distribution when $b \neq \hat{b}$ is a mean-preserving spread (MPS) of that of $\tilde{x}$ when $b = \hat{b}$. To be more specific, let dF be a distribution of $\tilde{x} = \theta \sim N(0,1)$ when $b = \hat{b}$ and dG be that of $\tilde{x} = \frac{b-\hat{b}}{a_2^*}\eta + \theta$ when $b \neq \hat{b}$. Then à la Rothschild and Stiglitz (1970), we see $dG(\tilde{x})$ is mean-preserving spread of $dF(\tilde{x})$ as seen in Figure F.1. Without limited liability (or equivalently $k = -\infty$), $\mathbb{E}_F(\hat{\rho}(\tilde{x})) > \mathbb{E}_G(\hat{\rho}(\tilde{x}))$ holds as $\hat{\rho}(\tilde{x})$ is globally concave. However, we have now the convexity that pops up around points where a limited liability binds.



,

Figure F.1: . The Agent's Indirect Utility $\hat{\rho}(\tilde{x})$ and Mean-Preserving Spread (MPS) when $b \neq \hat{b}$

From our premise that $\underline{\lambda > 0 \text{ is large enough}}$, $\kappa_0 = \lambda - \frac{\mu_2^*}{a_2^*}$ becomes also large enough due to $\mu_2^* < 0$. Now, we calculate the difference in expected value of $\hat{\rho}(\tilde{x})$ with respect to the distributions $dF(\tilde{x})$ and $dG(\tilde{x})$. What we show below is that when $\kappa_0 > 0$ is big enough, $\mathbb{E}_F(\hat{\rho}(\tilde{x})) > \mathbb{E}_G(\hat{\rho}(\tilde{x}))$ holds. We define $\grave{F}(x) \equiv \int^x F(\tilde{x})d\tilde{x}$ and $\grave{G}(x) \equiv \int^x G(\tilde{x})d\tilde{x}$, then we know $\grave{F}(x) < \grave{G}(x)$ for $\forall x$, $\grave{F}(\underline{x}) = \grave{G}(\underline{x})$, and $\grave{F}(\bar{x}) = \grave{G}(\bar{x})$ where $\underline{x}$ and $\bar{x}$ are infimum and supremum of support of distributions, respectively, where for normal distributions, $\underline{x} \to -\infty$ and $\bar{x} \to \infty$. Then we obtain

$$\begin{aligned} \int \hat{\rho}(\tilde{x})(dF(\tilde{x}) - dG(\tilde{x})) &= -\int \hat{\rho}'(\tilde{x})(F(\tilde{x}) - G(\tilde{x}))d\tilde{x} = -\int_{x_0^+}^{x_1^-} \hat{\rho}'(\tilde{x})(F(\tilde{x}) - G(\tilde{x}))d\tilde{x} \\ &= -\hat{\rho}'(\tilde{x})(\grave{F}(\tilde{x}) - \grave{G}(\tilde{x}))\Big|_{x_0^+}^{x_1^-} + \underbrace{\int_{x_0^+}^{x_1^-} \underbrace{\hat{\rho}''(\tilde{x})}_{<0}\underbrace{(\grave{F}(\tilde{x}) - \grave{G}(\tilde{x}))}_{<0}d\tilde{x}}_{>0}, \end{aligned} \tag{A27}$$

where the first term can be written as:

$$
\begin{aligned}
-\hat{\rho}'(\tilde{x})(\grave{F}(\tilde{x})-\grave{G}(\tilde{x}))\Big|_{x_0^+}^{x_1^-} &= \underbrace{-\hat{\rho}'(x_1^-)}_{=\hat{\rho}'(x_0^+)>0}(\grave{F}(x_1)-\grave{G}(x_1))+\hat{\rho}'(x_0^+)(\grave{F}(x_0)-\grave{G}(x_0)) \\
&= \underbrace{\hat{\rho}'(x_0^+)}_{>0}[\underbrace{\grave{F}(x_0)-\grave{G}(x_0)}_{<0}+\underbrace{\grave{F}(x_1)-\grave{G}(x_1)}_{<0}] < 0.
\end{aligned} \tag{A28}
$$

Thus the first term is negative[8] and this feature complicates our analysis. However, we will show that as $\kappa_0 \to \infty$, this first term vanishes quickly, and we can guarantee $\mathbb{E}_F(\hat{\rho}(\tilde{x})) > \mathbb{E}_G(\hat{\rho}(\tilde{x}))$. For that purpose, let us express the explicit formulas for x_0 and x_1 as

$$
\kappa_2 x^2+\kappa_1 x+\kappa_0 = p, \quad x_{0,1} = \frac{-\kappa_1 \pm \sqrt{\kappa_1^2-4\kappa_2(\kappa_0-p)}}{2\kappa_2} = \frac{\kappa_1 \mp \sqrt{\kappa_1^2-4\kappa_2(\kappa_0-p)}}{-2\kappa_2}. \tag{A29}
$$

And $2\kappa_2 x_0+\kappa_1 = \sqrt{\kappa_1^2-4\kappa_2(\kappa_0-p)}$ tells us $\hat{\rho}'(x_0^+) = \rho'(p)(2\kappa_2 x_0+\kappa_1) = \rho'(p)\sqrt{\kappa_1^2-4\kappa_2(\kappa_0-p)}$.[9]

For expositional purposes, we introduce the big-O notation. Since both x_0 and x_1 has order of $\sqrt{\kappa_0}$, we write $x_0 = -O(\sqrt{\kappa_0})$ and $x_1 = O(\sqrt{\kappa_0})$. Then we can write equation (A28) as

$$
\begin{aligned}
-\hat{\rho}'(\tilde{x})(\grave{F}(\tilde{x})-\grave{G}(\tilde{x}))\Big|_{x_0^+}^{x_1^-} &= \rho'(p)\sqrt{\kappa_1^2-4\kappa_2(\kappa_0-p)}[(\grave{F}-\grave{G})(x_0)+(\grave{F}-\grave{G})(x_1)] \\
&= O(\sqrt{\kappa_0})[(\grave{F}-\grave{G})(x_0)+(\grave{F}-\grave{G})(x_1)].
\end{aligned} \tag{A30}
$$

Now we should prove $[(\grave{F}-\grave{G})(x_0)+(\grave{F}-\grave{G})(x_1)]$ decreases in a faster speed than the speed at which $O(\sqrt{\kappa_0})$ increases in a rate of $\sqrt{\kappa_0}$ as $\kappa_0 \to \infty$ so that the first term in equation (A27), a multiple of two, vanishes as $\kappa_0 \to \infty$. For standard normal cumulative distribution function Φ, we know $F(x) = \Phi(x)$, and $G(x) = \Phi(\frac{x}{\sigma})$ for some $\sigma > 1$.[10] Then we write $(\grave{F}-\grave{G})(x) = \int_{-\infty}^{x}(\Phi(\tilde{x})-\Phi(\frac{\tilde{x}}{\sigma}))d\tilde{x}$ and obtain:

$$
(\grave{F}-\grave{G})(x_0)+(\grave{F}-\grave{G})(x_1) = \int_{-\infty}^{-O(\sqrt{\kappa_0})}\Big(\Phi(x)-\Phi\Big(\frac{x}{\sigma}\Big)\Big)dx + \int_{-\infty}^{O(\sqrt{\kappa_0})}\Big(\Phi(x)-\Phi\Big(\frac{x}{\sigma}\Big)\Big)dx. \tag{A31}
$$

We use the error function approximation, $\Phi(x) \sim 1-Ce^{-\frac{x^2}{2}}$ when $x \simeq \infty$ and $\Phi(x) \sim Ce^{-\frac{x^2}{2}}$ when $x \simeq -\infty$ to calculate the two integrals. For the first integral, by using substitution $x = -O(\sqrt{\kappa_0})\cdot v$,

$$
\begin{aligned}
\int_{-\infty}^{-O(\sqrt{\kappa_0})}\Big(\Phi(x)-\Phi\Big(\frac{x}{\sigma}\Big)\Big)dx &= \int_1^{\infty}\Big(\underbrace{\Phi(-O(\sqrt{\kappa_0})\cdot v)}_{\sim Ce^{-\frac{O(\kappa_0)v^2}{2}}} - \underbrace{\Phi\Big(\frac{-O(\sqrt{\kappa_0})\cdot v}{\sigma}\Big)}_{\sim Ce^{-\frac{O(\kappa_0)v^2}{2\sigma^2}}}\Big)dv\cdot O(\sqrt{\kappa_0}) \\
&\sim e^{-O(\kappa_0)}\cdot O(\sqrt{\kappa_0}),
\end{aligned} \tag{A32}
$$

[8]It is related to the fact that $\hat{\rho}(\tilde{x})$ becomes quite convex at the points x_0 and x_1, which induces the agent to bear more risks and choose $b \neq \hat{b}$ (speculation).

[9]We use the property $\kappa_0+\kappa_1 x_0+\kappa_2 x_0^2 = p$.

[10]When $b \neq \hat{b}$, $\sigma^2 = 1+(\frac{b-\hat{b}}{a_2^*})^2 > 1$ as η and θ are both standard normal and independent of each other.

where $\kappa_0 \to \infty$. For the second integral, by using substitution $x = O(\sqrt{\kappa_0}) \cdot v$, we obtain

$$\begin{aligned}\int_{-\infty}^{O(\sqrt{\kappa_0})} \Big(\Phi(x) - \Phi\Big(\frac{x}{\sigma}\Big)\Big)dx &= -\int_{O(\sqrt{\kappa_0})}^{\infty} \Big(\Phi(x) - \Phi\Big(\frac{x}{\sigma}\Big)\Big)dx \\ &= -\int_{1}^{\infty} \Big(\underbrace{\Phi(O(\sqrt{\kappa_0}) \cdot v)}_{\sim 1 - Ce^{-\frac{O(\kappa_0)v^2}{2}}} - \underbrace{\Phi\Big(\frac{O(\sqrt{\kappa_0}) \cdot v}{\sigma}\Big)}_{\sim 1 - Ce^{-\frac{O(\kappa_0)v^2}{2\sigma^2}}}\Big)dv \cdot O(\sqrt{\kappa_0}).\end{aligned} \tag{A33}$$

Thus, the second integral yields the same order in κ_0.[11] Finally we obtain:

$$[(\grave{F} - \grave{G})(x_0) + (\grave{F} - \grave{G})(x_1)] \sim e^{-O(\kappa_0)} \cdot O(\sqrt{\kappa_0}), \tag{A34}$$

with which and $\kappa_0 \to \infty$, we obtain

$$\hat{\rho}'(x_0)[\grave{F}(x_0) - \grave{G}(x_0) + \grave{F}(x_1) - \grave{G}(x_1)] = O(\sqrt{\kappa_0}) \cdot e^{-O(\kappa_0)} \cdot O(\sqrt{\kappa_0}) \to 0, \tag{A35}$$

which proves $\mathbb{E}_F(\hat{\rho}(\tilde{x})) > \mathbb{E}_G(\hat{\rho}(\tilde{x}))$ as $\lambda \to \infty$. Therefore, when λ is large enough, the agent voluntarily chooses $b = \hat{b}$ under $w^*(z(\hat{b}))$ in equation (A24) when $\underline{\mu_2^* < 0}$. ■

(2) If $\underline{\mu_2^* > 0}$, then $\kappa_2 > 0$. Then $\kappa_0 + \kappa_1\tilde{x} + \kappa_2\tilde{x}^2$ yields a minimum at $\tilde{x}_c = -\frac{\kappa_1}{2\kappa_2} < 0$. If a limited liability does not bind at any point, i.e., $\kappa_0 + \kappa_1\tilde{x}_c + \kappa_2\tilde{x}_c^2 > p$[12], then $\hat{\rho}''(\tilde{x}_c) = \rho'(\kappa_0 + \kappa_1\tilde{x}_c + \kappa_2\tilde{x}_c^2) \cdot 2\kappa_2 > 0$ and $\hat{\rho}(\cdot)$ becomes convex around $\tilde{x} = \tilde{x}_c$. In other cases where the limited liability binds around $\tilde{x}_c$, $\hat{\rho}(\tilde{x})$ can be flat around $\tilde{x}_c$. However, as $\tilde{x} \to \pm\infty$, $(\kappa_1 + 2\kappa_2\tilde{x})^2 \to \infty$ so $\hat{\rho}(\tilde{x})$ becomes concave from equation (A26) since $\rho''(\cdot) < 0$ by Assumption 6.

In sum, the agent's indirect utility $\hat{\rho}(\tilde{x})$ as a function of $\tilde{x}$ is symmetric around the point $\tilde{x}_c$, convex around $\tilde{x}_c$, and becomes very concave as $\tilde{x} \to \pm\infty$. In this case the agent chooses $b = \pm\infty$ (or $a_3 = \pm\infty$).

Why? Let $w(\eta, \theta, b|w^*)$ be the real wage that the agent will receive under $w^*(z(\hat{b}))$ when he takes (a_1^*, a_2^*, b) and (η, θ) are realized. Then, by substituting equation (25) into equation (27), we have

$$\frac{1}{u'(w(\eta,\theta,b|w^*))} = \lambda + (\mu_1^*\phi_1^* + \mu_2^*\phi_2^*)\frac{(b-\hat{b})\eta + a_2^*\theta}{(a_2^*)^2} + \mu_2^*\frac{1}{a_2^*}\Big(\frac{((b-\hat{b})\eta + a_2^*\theta)^2}{(a_2^*)^2} - 1\Big). \tag{A36}$$

Therefore, for two different b, say b^0 and b^1, given the realized (η, θ), we have

$$\begin{aligned}\frac{1}{u'(w(\eta,\theta,b^1|w^*))} - \frac{1}{u'(w(\eta,\theta,b^0|w^*))} =& (\mu_1^*\phi_1^* + \mu_2^*\phi_2^*)\frac{(b^1 - b^0)\eta}{(a_2^*)^2} \\ &+ \mu_2^*\frac{1}{a_2^*}\Big(\frac{((b^1-\hat{b})^2 - (b^0-\hat{b})^2)\eta^2 + 2a_2^*(b^1-b^0)\eta\theta]}{(a_2^*)^2}\Big).\end{aligned} \tag{A37}$$

[11]In the first equality above, we use the property $\grave{F}(\bar{x}) = \grave{G}(\bar{x})$.

[12]We defined $p \equiv \frac{1}{u'(k)} > 0$ before, where k is the minimum subsistence level of the compensation.

Assume that $b^1 = +\infty$ or $-\infty$, and $-\infty < b^0 < +\infty$. Since $\mu_2^* > 0$, we have from the above that

$$\frac{1}{u'(w(\eta, \theta, b^1|w^*))} - \frac{1}{u'(w(\eta, \theta, b^0|w^*))} > 0, \quad \forall(\eta, \theta). \tag{A38}$$

Therefore, we have

$$w(\eta, \theta, b^1|w^*) > w(\eta, \theta, b^0|w^*), \quad \forall(\eta, \theta). \tag{A39}$$

which implies that the agent will actually take a_3 satisfying $b = +\infty$ or $-\infty$ when $w^*(z(\hat{b}))$ with $\mu_2^* > 0$ is designed. ■

Proof of Lemma 3: From equation (25), we have $z(\hat{b}) = \phi(a_1, a_2) + (b - \hat{b})\eta + a_2\theta$ for any given (a_1, a_2, b), and $z(0) = \phi(a_1, a_2) + b'\eta + a_2\theta$ for any given (a_1, a_2, b'). Therefore, we obtain

$$z(\hat{b}|a_1, a_2, b) = z(0|a_1, a_2, b'), \quad \text{whenever } \; b' = b - \hat{b}. \tag{A40}$$

Furthermore, if $b' = b - \hat{b}$, two joint density functions of $(z(\hat{b}), \eta)$ and $(z(0), \eta)$ are the same, i.e.,

$$\begin{aligned} g(z(\hat{b}), \eta|a_1, a_2, b) &= \frac{1}{2\pi a_2} \exp\Big(-\frac{1}{2}\Big[\frac{[z(\hat{b}) - \phi(a_1, a_2) - (b - \hat{b})\eta]^2}{a_2^2} + \eta^2\Big]\Big) \\ &= g(z(0), \eta|a_1, a_2, b'), \quad \forall b' = b - \hat{b}. \end{aligned} \tag{A41}$$

Thus, we derive that for $\forall(a_1, a_2, b)$, we have

$$\int u(w^o(z(\hat{b}), \eta))g(z(\hat{b}), \eta|a_1, a_2, b)dzd\eta = \int u(w^o(z(0), \eta))g(z(0), \eta|a_1, a_2, b' = b - \hat{b}))dzd\eta. \tag{A42}$$

Note that the agent is induced to take $(a_1^o, a_2^o, b^o \equiv R - a_3^o = \hat{b})$ under the contract $w^o(z(\hat{b}), \eta)$. Thus, agent will be induced to take $(a_1^o, a_2^o, b' = 0$ (i.e., $a_3 = R$)) under wage contract $w^o(z(0), \eta)$. Moreover, since

$$\int w^o(z(\hat{b}), \eta)g(z(\hat{b}), \eta|a_1^o, a_2^o, b^o)dzd\eta = \int w^o(z(0), \eta)g(z(0), \eta|a_1^o, a_2^o, 0))dzd\eta, \tag{A43}$$

using equation (28) and equation (29), we finally derive:

$$SW(a_1^o, a_2^o, a_3^o) = SW(a_1^o, a_2^o, R).$$

■

Proof of Lemma 4: When $w^*(z(0))$ described in equation (27) is designed, we have

$$z(0|a_1, a_2, b) = x = \phi(a_1, a_2) + b\eta + a_2\theta. \tag{A44}$$

If the agent takes (a_1, a_2, b) under $w^*(z(0))$, then his expected utility is:

$$\int u(w^*(z(0)))g(z(0), \eta|a_1, a_2, b)dzd\eta - v(a_1) = \int u(w^*(z(0)))q(z(0)|a_1, a_2, b, \eta)l(\eta)dzd\eta - v(a_1), \tag{A45}$$

where $q(\cdot)$ denotes the conditional density function of $z(0)$ given (a_1, a_2, b, η) and $l(\cdot)$ denotes the density function of $\eta \sim N(0,1)$.

Now, suppose the agent takes $(a_1, a_2, -b)$ under $w^*(z(0))$. Then, his expected utility becomes:

$$\int u(w^*(z(0)))g(z(0),\eta|a_1,a_2,-b)dzd\eta - v(a_1) = \int u(w^*(z(0)))q(z(0)|a_1,a_2,-b,\eta)l(\eta)dzd\eta - v(a_1). \tag{A46}$$

Since

$$q(z(0)|a_1,a_2,b,\eta) = \frac{1}{\sqrt{2\pi}a_2}\exp\Big(-\frac{(z(0)-\phi(a_1,a_2)-b\eta)^2}{2a_2^2}\Big), \tag{A47}$$

we have

$$q(z(0)|a_1,a_2,b,\eta) = q(z(0)|a_1,a_2,-b,-\eta). \tag{A48}$$

Since $\eta \sim N(0,1)$ is symmetrically distributed around 0 and $l(\eta) = l(-\eta)$, $\forall \eta$, we finally have

$$\int u(w^*(z(0)))g(z(0),\eta|a_1,a_2,b)dzd\eta - v(a_1) = \int u(w^*(z(0)))g(z(0),\eta|a_1,a_2,-b)dzd\eta - v(a_1). \tag{A49}$$

■

Proof of Proposition 5: To prove this proposition, we start with the following lemma.

Lemma A1: *If $\mu_2^* > 0$ for contract $w^*(z(0))$ in equation (27), then the optimal contract $w^o(x,\eta)$ guaranteeing that the agent takes $a_3 = R$, i.e., $w^o(x,\eta)$ in equation (36), must satisfy*

(1) $\mu_2^o \geq 0$

(2) $\mu_4^o(b) \neq 0$ (> 0) for a positive Borel-measure of b.[13]

(3) $\mu_4^o(b) = \mu_4^o(-b)$ for all b and $w^o(x,\eta) = w^o(x,-\eta)$ for all x,η

Proof of Lemma A1:
(1) $\mu_2^o \geq 0$: Assume that $\mu_2^o < 0$, then under the contract $w^1(x,\eta)$ satisfying

$$\frac{1}{u'(w^1(x,\eta))} = \lambda + (\mu_1^o\phi_1^o + \mu_2^o\phi_2^o)\frac{x-\phi(a_1^o,a_2^o)}{(a_2^o)^2} + \frac{\mu_2^o}{a_2^o}\Big(\frac{(x-\phi(a_1^o,a_2^o))^2}{(a_2^o)^2} - 1\Big), \tag{A50}$$

for (x,η) satisfying $w^1(x,\eta) \geq k$ and $w^1(x,\eta) = k$, the agent voluntarily chooses $b = 0$[14] even though we did not consider the constraint (iii) in equation (35). Thus $w^1(x,\eta)$ becomes the solution of equation (35). However, it contradicts with our assumption of $\mu_2^* > 0$ for $w^*(z(0))$ since $(w^1(x,\eta), \mu_1^o, \mu_2^o)$ becomes $(w^*(z(0)), \mu_1^*, \mu_2^*)$ without the constraint (iii) about b.

(2) $\mu_4^o(b) \neq 0$ for a positive Borel-measure of b: Assume $\mu_4^o(b) = 0$ *a.s.* Then optimal contract $w^o(x,\eta)$ becomes:

$$\frac{1}{u'(w^o(x,\eta))} = \lambda + (\mu_1^o\phi_1^o + \mu_2^o\phi_2^o)\frac{x-\phi(a_1^o,a_2^o)}{(a_2^o)^2} + \frac{\mu_2^o}{a_2^o}\Big(\frac{(x-\phi(a_1^o,a_2^o))^2}{(a_2^o)^2} - 1\Big), \tag{A51}$$

[13] We already know $\mu_4^o(b) \geq 0$ for every b (almost surely), since it is derived from the inequality constraint at each b.

[14] We continue to assume that λ is large enough, thus with $\mu_2^o < 0$ the agent engages in a perfect hedging, as discussed in Lemma 2.

for (x,η) satisfying $w^o(x,\eta) \geq k$ and $w^o(x,\eta) = k$.

Because we already know $(w^o(x,\eta), \mu_1^o, \mu_2^o)$ becomes $(w^*(z(0)), \mu_1^*, \mu_2^*)$ in this case and $\mu_2^* > 0$ holds, $(w^o(x,\eta), \mu_1^o, \mu_2^o)$ will induce $b = \pm\infty$ instead of $b = 0$ from the agent, which is contradiction to the constraint (iii) in equation (35).

(3) $\underline{\mu_4^o(b) = \mu_4^o(-b) \text{ for all } b \text{ and } w^o(x,\eta) = w^o(x,-\eta) \text{ for all } x,\eta}$: We first see:

$$g(x,\eta|b) = \frac{1}{2\pi a_2^o} \exp\Big(-\frac{1}{2}\frac{(x-\phi(a_1^o,a_2^o)-b\eta)^2}{(a_2^o)^2} - \frac{1}{2}\eta^2 \Big), \tag{A52}$$

where

$$\begin{aligned} \frac{g(x,\eta|b)}{g(x,\eta|b=0)} &= \exp\Big(-\frac{1}{2}\frac{(x-\phi(a_1^o,a_2^o)-b\eta)^2 - (x-\phi(a_1^o,a_2^o))^2}{(a_2^o)^2} \Big) \\ &= \exp\Big(-\frac{-b\eta(2x-2\phi(a_1^o,a_2^o)-b\eta)}{2(a_2^o)^2} \Big) = \exp\Big(\frac{b\eta(x-\phi(a_1^o,a_2^o))}{(a_2^o)^2} \Big) \exp\Big(-\frac{b^2\eta^2}{2(a_2^o)^2} \Big). \end{aligned} \tag{A53}$$

Our strategy is to prove that the optimal contract $w^o(x,\eta)$ with $\mu_4^o(b) = \mu_4^o(-b)$ for $\forall b$ from equation (36) satisfies the complementary slackness condition of $\forall b$, which by Karush-Kuhn-Tucker theorem (1951) guarantees both necessity and sufficiency of the solution.[15] For that purpose, for a while let us assume $\mu_4^o(b) = \mu_4^o(-b)$ holds. To show $w^o(x,\eta) = w^o(x,-\eta)$ for $\forall x,\eta$, equation (36) and equation (A53) implies it suffices to prove:

$$\begin{aligned} &\int \mu_4^o(b) \exp\Big(\frac{b\eta(x-\phi(a_1^o,a_2^o))}{(a_2^o)^2} \Big) \exp\Big(-\frac{b^2\eta^2}{2(a_2^o)^2} \Big) db \\ &= \int \mu_4^o(b) \exp\Big(\frac{b(-\eta)(x-\phi(a_1^o,a_2^o))}{(a_2^o)^2} \Big) \exp\Big(-\frac{b^2\eta^2}{2(a_2^o)^2} \Big) db. \end{aligned} \tag{A54}$$

Using our assumed symmetry $\mu_4^o(b) = \mu_4^o(-b)$, we prove equation (A54) as follows.

$$\begin{aligned} &\int \mu_4^o(b) \exp\Big(\frac{b(-\eta)(x-\phi(a_1^o,a_2^o))}{(a_2^o)^2} \Big) \exp\Big(-\frac{b^2\eta^2}{2(a_2^o)^2} \Big) db \\ &\underbrace{=}_{\tilde{b}\equiv -b} \int \mu_4^o(-\tilde{b}) \exp\Big(\frac{\tilde{b}\eta(x-\phi(a_1^o,a_2^o))}{(a_2^o)^2} \Big) \exp\Big(-\frac{\tilde{b}^2\eta^2}{2(a_2^o)^2} \Big) d(-\tilde{b}) \\ &= \int \mu_4^o(\tilde{b}) \exp\Big(\frac{\tilde{b}\eta(x-\phi(a_1^o,a_2^o))}{(a_2^o)^2} \Big) \exp\Big(-\frac{\tilde{b}^2\eta^2}{2(a_2^o)^2} \Big) d\tilde{b}. \end{aligned} \tag{A55}$$

Thus we have $w^o(x,\eta) = w^o(x,-\eta)$ for any x,η with $\mu_4^o(b) = \mu_4^o(-b)$ for $\forall b$. Now let us deal with the complementary slackness conditions. For an arbitrary b_1 such that $\mu_4^o(b_1) > 0$, we have:

$$0 = \int u(w^o(x,\eta))[g(x,\eta|b=0) - g(x,\eta|b_1)]dxd\eta, \tag{A56}$$

[15] See Luenberger (1969) for a comprehensive treatment of this issue.

where equation (A53) implies:

$$\begin{aligned} &g(x,\eta|b=0)-g(x,\eta|b_1) \\ &=\frac{1}{2\pi a_2^o}\exp\Big(-\frac{(x-\phi(a_1^o,a_2^o))^2}{2(a_2^o)^2}-\frac{\eta^2}{2}\Big)\Big[1-\exp\Big(\frac{b_1\eta(x-\phi(a_1^o,a_2^o))}{(a_2^o)^2}\Big)\exp\Big(-\frac{b_1^2\eta^2}{2(a_2^o)^2}\Big)\Big]. \end{aligned} \tag{A57}$$

For our purpose of justifying our initial assumption $\mu_4^o(b)=\mu_4^o(-b)$ for $\forall b$, we must prove it does not violate the complementary slackness conditions. we must prove that equation (A56) implies:

$$0=\int u(w^o(x,\eta))[g(x,\eta|b=0)-g(x,\eta|-b_1)]dxd\eta. \tag{A58}$$

With the help of equation (A57), it is sufficient to prove:[16]

$$\begin{aligned} &\int u(w^o(x,\eta))\exp\Big(\frac{b_1\eta(x-\phi(a_1^o,a_2^o))}{(a_2^o)^2}\Big)\exp\Big(-\frac{b_1^2\eta^2}{2(a_2^o)^2}\Big)\exp\Big(-\frac{(x-\phi(a_1^o,a_2^o))^2}{2(a_2^o)^2}-\frac{\eta^2}{2}\Big)dxd\eta \\ &=\int u(w^o(x,\eta))\exp\Big(\frac{-b_1\eta(x-\phi(a_1^o,a_2^o))}{(a_2^o)^2}\Big)\exp\Big(-\frac{b_1^2\eta^2}{2(a_2^o)^2}\Big)\exp\Big(-\frac{(x-\phi(a_1^o,a_2^o))^2}{2(a_2^o)^2}-\frac{\eta^2}{2}\Big)dxd\eta. \end{aligned} \tag{A59}$$

Since $w(x,\eta)=w(x,-\eta)$ for $\forall x,\eta$ holds, we can easily prove equation (A59) in the following way.

$$\begin{aligned} &\int u(w^o(x,\eta))\exp\Big(\frac{-b_1\eta(x-\phi(a_1^o,a_2^o))}{(a_2^o)^2}\Big)\exp\Big(-\frac{b_1^2\eta^2}{2(a_2^o)^2}\Big)\exp\Big(-\frac{(x-\phi(a_1^o,a_2^o))^2}{2(a_2^o)^2}-\frac{\eta^2}{2}\Big)dxd\eta \\ &\underbrace{=}_{\tilde\eta\equiv-\eta}\int u(w^o(x,-\tilde\eta))\exp\Big(\frac{b_1\tilde\eta(x-\phi(a_1^o,a_2^o))}{(a_2^o)^2}\Big)\exp\Big(-\frac{b_1^2\tilde\eta^2}{2(a_2^o)^2}\Big)\exp\Big(-\frac{(x-\phi(a_1^o,a_2^o))^2}{2(a_2^o)^2}-\frac{\tilde\eta^2}{2}\Big)dxd\tilde\eta \\ &\underbrace{=}_{w(x,-\tilde\eta)=w(x,\tilde\eta)}\int u(w^o(x,\tilde\eta))\exp\Big(\frac{b_1\tilde\eta(x-\phi(a_1^o,a_2^o))}{(a_2^o)^2}\Big)\exp\Big(-\frac{b_1^2\tilde\eta^2}{2(a_2^o)^2}\Big)\exp\Big(-\frac{(x-\phi(a_1^o,a_2^o))^2}{2(a_2^o)^2}-\frac{\tilde\eta^2}{2}\Big)dxd\tilde\eta. \end{aligned} \tag{A60}$$

Thus we effectively guarantee that $w^o(x,\eta)$ with $\mu_4^o(b)=\mu_4^o(-b)$ for $\forall b$ is the optimal contract solving equation (35) and satisfies $w^o(x,\eta)=w^o(x,-\eta)$ for all x,η. ∎

Proof of Proposition 5: Given (a_1^o,a_2^o), we define $\widehat{Cov}\equiv(x-\phi(a_1^o,a_2^o))\eta$.[17] We can see

$$\exp\Big(\frac{b\eta(x-\phi(a_1^o,a_2^o))}{(a_2^o)^2}\Big)=\sum_{k=0}^{\infty}\frac{1}{k!}\frac{b^k}{(a_2^o)^{2k}}\widehat{Cov}^k, \tag{A61}$$

and thus with equation (A53), we get

$$\frac{g(x,\eta|b)}{g(x,\eta|b=0)}=\Big(\sum_{k=0}^{\infty}\frac{1}{k!}\frac{b^k}{(a_2^o)^{2k}}\widehat{Cov}^k\Big)\exp\Big(-\frac{b^2\eta^2}{2(a_2^o)^2}\Big). \tag{A62}$$

[16]Thus our proof of equation (A59) actually proves that indirect utility of the agent is symmetric around $b=0$.
[17]This is the value of sample covariance between x and η, as our framework is single-period setting.

We know from equation (36) that the optimal contract $w^o(x,\eta)$ has the following form:

$$\frac{1}{u'(w^o(x,\eta))} = \lambda + (\mu_1^o\phi_1^o + \mu_2^o\phi_2^o)\frac{x-\phi(a_1^o,a_2^o)}{(a_2^o)^2} + \frac{\mu_2^o}{a_2^o}\Big(\frac{(x-\phi(a_1^o,a_2^o))^2}{(a_2^o)^2}-1\Big) + \int \mu_4^o(b)\Big(1-\frac{g(x,\eta|b)}{g(x,\eta|b=0)}\Big)db, \tag{A63}$$

for (x,η) satisfying $w^o(x,\eta)\geq k$ and $w^o(x,\eta)=k$ otherwise, where

$$\int \mu_4^o(b)\Big(1-\frac{g(x,\eta|b)}{g(x,\eta|b=0)}\Big)db = \underbrace{\int \mu_4^o(b)db}_{>0} - \int \mu_4^o(b)\exp\Big(\frac{b}{(a_2^o)^2}\widehat{Cov}\Big)\exp\Big(-\frac{b^2\eta^2}{2(a_2^o)^2}\Big)db. \tag{A64}$$

Plugging equation (A61) into equation (A64), we get

$$\begin{aligned}\int \mu_4^o(b)\exp\Big(\frac{b}{(a_2^o)^2}\widehat{Cov}\Big)\exp\Big(-\frac{b^2\eta^2}{2(a_2^o)^2}\Big)db &= \int \mu_4^o(b)\Big(\sum_{k=0}^{\infty}\frac{1}{k!}\frac{b^k}{(a_2^o)^{2k}}\widehat{Cov}^k\Big)\exp\Big(-\frac{b^2\eta^2}{2(a_2^o)^2}\Big)db \\ &= \sum_{k=0}^{\infty}\Big(\frac{1}{k!}\frac{1}{(a_2^o)^{2k}}\underbrace{\Big(\int \mu_4^o(b)b^k\exp\Big[-\frac{b^2\eta^2}{2(a_2^o)^2}\Big]db\Big)}_{\equiv C_k(\eta)}\Big)\widehat{Cov}^k.\end{aligned} \tag{A65}$$

When k is odd, the coefficient $C_k(\eta)$ becomes 0 for $\forall\eta$, since $\mu_4^o(b)=\mu_4^o(-b)$ for all b implies

$$C_{k:odd}(\eta) = \int \mu_4^o(b)b^k\exp\Big(-\frac{b^2\eta^2}{2(a_2^o)^2}\Big)db = \int_{b\geq 0}\underbrace{(\mu_4^o(b)-\mu_4^o(-b))}_{=0}b^k\exp\Big(-\frac{b^2\eta^2}{2(a_2^o)^2}\Big)db = 0. \tag{A66}$$

When k is even, the coefficient $C_k(\eta)$ becomes strictly positive for $\forall\eta$, since $\mu_4^o(b)\neq 0$ for the non-zero measure of b implies

$$C_{k:even}(\eta) = \int \mu_4^o(b)b^k\exp\Big(-\frac{b^2\eta^2}{2(a_2^o)^2}\Big)db = \int_{b\geq 0}(\mu_4^o(b)+\mu_4^o(-b))b^k\exp\Big(-\frac{b^2\eta^2}{2(a_2^o)^2}\Big)db > 0. \tag{A67}$$

Finally we can plug the expressions equation (A57), equation (A65), and equation (A67) into our optimal contact $w^o(x,\eta)$ in equation (A63) when $w^o(x,\eta)\geq k$ and obtain

$$\begin{aligned}\frac{1}{u'(w^o(x,\eta))} =&\lambda + (\mu_1^o\phi_1^o + \mu_2^o\phi_2^o)\frac{x-\phi(a_1^o,a_2^o)}{(a_2^o)^2} + \frac{\mu_2^o}{a_2^o}\Big(\frac{(x-\phi(a_1^o,a_2^o))^2}{(a_2^o)^2}-1\Big) + \underbrace{\int \mu_4^o(b)db}_{>0} \\ &- \sum_{k:\text{even}}^{\infty}\underbrace{\frac{1}{k!}\frac{1}{(a_2^o)^{2k}}\underbrace{\Big(\int_{b\geq 0}(\mu_4^o(b)+\mu_4^o(-b))b^k\exp\Big[-\frac{b^2\eta^2}{2(a_2^o)^2}\Big]db\Big)}_{\equiv C_{k:\text{even}}(\eta)>0}}_{\equiv D_{k:\text{even}}(\eta)>0}\widehat{Cov}^k.\end{aligned} \tag{A68}$$

Since $D_{k:\text{even}}(\eta)>0$ for all even numbers k, given (x,η), a higher $\widehat{Cov}$ results in a lower compensation $w^o(x,\eta)$. Also as $D_{k:\text{even}}(\eta)>0$ decreases in η^2, given $(x,\widehat{Cov})$, a higher η^2 results in a higher $w^o(x,\eta)$.

In sum the principal punishes a sample covariance $|\widehat{Cov}|$ but becomes lenient when it happens due to high η realization, not from the agent's speculation ($b \neq 0$).

Note: Let $\rho(b) \equiv \int u(w^o(x,\eta))g(x,\eta|a_1^o, a_2^o, b)dxd\eta - v(a_1^o)$ be the agent's expected indirect utility as a function of b. Then due to equation (A59) in (2) and (3) of Lemma A1, $\rho(b) = \rho(0)$ holds for a positive measure of b[18] and $\rho(b)$ must be symmetric around $b = 0$. Possible shapes of $\rho(b)$ are provided in the following Figure F.2.

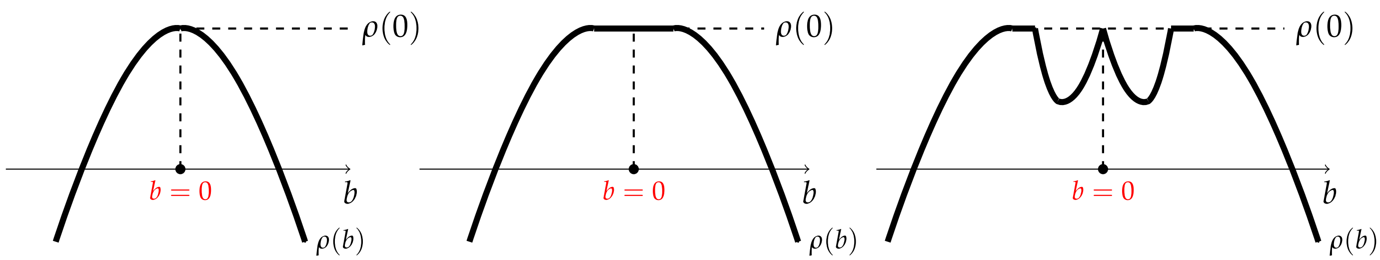


Figure F.2: Agent's Indirect Utility $\rho(b)$ as a function of b

As $b \to \pm\infty$, $\widehat{Cov} \to \pm\infty$ at any realization of (θ, η) since $\widehat{Cov} = b\eta^2 + a_2^o\theta\eta$ and $\eta^2 > 0$, The above argument implies as $b \to \pm\infty$, we have $w(x,\eta) = w(\phi(a_1^o, a_2^o) + b\eta + a_2^o\theta, \eta) < w(\phi(a_1^o, a_2^o) + a_2^o\theta, \eta)$ uniformly on (θ, η).[19] Thus we have $\rho(b) < \rho(0)$ when $b \to \pm\infty$. ■

Proof of Proposition 6: Although we do not explicitly characterize SW^N in equation (23), we at least see SW^N is a continuous function of σ_R^2. On the other hand, $w^*(y)$ characterized in equation (18) and $w^o(x,\eta)$ in equation (36) are independent of σ_R^2, and so are $SW(a_1^*, a_2^*, a_3 = 0)$ and $SW(a_1^o, a_2^o, a_3 = R)$. Thus, as the amount of uncertainty on the firm's risk exposure approaches zero (i.e., $\sigma_R^2 \to 0$), we have

$$SW(a_1^*, a_2^*, a_3 = 0) - SW^N \to 0, \tag{A69}$$

since the reason $SW^N < SW(a_1^*, a_2^*, a_3 = 0)$ is that principal does not observe a realized R and this informational asymmetry disappears as $\sigma_R^2 \to 0$. As $SW(a_1^*, a_2^*, a_3 = 0) - SW(a_1^o, a_2^o, R) > 0$ remains unchanged as $\sigma_R^2 \to 0$, when σ_R^2 is very small, we have

$$SW(a_1^o, a_2^o, R) - SW^N < 0. \tag{A70}$$

■

Proof of Proposition 7: From Lemma 5, we see that $w^*(y_r)$ is a truth-telling mechanism for the firm's hidden risk exposure, R, if $\mu_2^* < 0$ for $w^*(y_r)$. Since $r = R, \forall R$, under $w^*(y_r)$, we have

$$y \equiv x - R\eta = \phi(a_1, a_2) + a_2\theta = y_r. \tag{A71}$$

Furthemore, we have that $w^*(y_r)$ in equation (40) has the same contractual form as $w^*(y)$ in equation (18). Thus, the optimal action combination to be chosen by the agent under $w^*(y_r)$ is (a_1^*, a_2^*), i.e.,

[18] Due to the complementarity slackness condition about the constraint (iii) of the optimization in equation (35).

[19] Actually $b \to \pm\infty$ also affects the output x in equation (A68). While terms up to a second-order of the output x enter in the optimal contract in equation (A68), higher order terms of $\widehat{Cov}$ clearly dominates the first and second order terms of x.

$(a_1^T(R), a_2^T(R)) = (a_1^*, a_2^*), \forall R$. Therefore, we derive

$$SW^T = SW(a_1^*, a_2^*, a_3 = 0), \tag{A72}$$

and from Proposition 4, we derive that SW^T is the same as the joint benefits that will be obtained under $w^*(z(0))$ when there is a derivative market. ■

Proof of Proposition 8 Note that both non-communication contracts $w^N(x, \eta)$ and $w^o(x, \eta)$ in equation (36) are truth-telling mechanisms.[20] Therefore, if $\mu_2^* > 0$ for $w^*(y_r)$ in equation (40), we have

$$SW^T \geq max\{SW^N, SW(a_1^o, a_2^o, R)\}. \tag{A73}$$

Furthermore, from Proposition 6, we have $SW^N > SW(a_1^o, a_2^o, R)$ when σ_R^2 is very small. Thus, we obtain that $SW^T > SW(a_1^o, a_2^o, R)$ when σ_R^2 is very small. ■

[20]The principal can design $w^N(x, \eta)$ without using r. Also, by designing $w^o(y_r, \eta)$ as a truth-telling mechanism, he can obtain the same result as $w^o(x, \eta)$ would provide.