Active Taylor Rules Still Breed Sunspots: Sunspot Volatility, Risk-Premium, and the Business Cycle

Seung Joo Lee
Oxford - Saïd
(Visiting Princeton)

Marc Dordal i Carreras
HKUST

May 28, 2023

Research interest: macroeconomics (macro-finance, monetary), asset pricing, and contract theory. Among my macro works:

With Marc Dordal Carreras (HKUST):

- A Unified Theory of the Term-Structure and Monetary Stabilization
- Active Taylor Rules Still Breed Sunspots: Sunspot Volatility, Risk-Premium, and the Business Cycle (Job Market Paper)
- A Higher-Order Forward Guidance
- Empirical Estimation of Bond Market Segmentation (with Anna Carruthers)
- Entangled AD-AS through firm entry (with Zhenghua Qi)

With other people:

- Heterogeneous Beliefs, Risk Amplification, and Asset Returns (with Goutham Gopalakrishna (EFPL and SFI) and Theofanis Papamichalis (Cambridge))
- Risky Growth with Short-Term Debt (with Artur Doshchyn (Oxford))

"One of the central and most widely shared ideas in the academic finance literature is the importance of time variation in the **risk premiums** (or expected returns) on a wide range of assets. At the same time, canonical macro models in the New Keynesian genre of the sort that are often used to inform monetary policy tend to exhibit little or no meaningful risk premium variation."

Jeremy Stein, Governor of the Federal Reserve System (2014)

"Monetary policy should not be **the first line of defense** - is not the first line of defense on **financial stability**. We look to more appropriate tools in the first instance, as a first line of defense. And those would be regulation, supervision, high-capital, high-liquidity stress testing, all of those things, macroprudential tools"

Jerome Powell, Chair of the Federal Reserve (2020)

Big Question (Is it possible?)

One monetary tool (i_t) $\Longrightarrow$ (i) inflation, (ii) output, and (iii) financial stability

1. To study a monetary policy's financial stability concern, turn our eyes to the first statement by Stein (2014) and reframe it into:

Big Question (Is it possible?)

One monetary tool (i_t) $\Longrightarrow$ (i) inflation, (ii) output, and (iii) risk-premium

2. Canonical finance: risk-premium $\propto$ volatility^2 (e.g., Merton (1971))
   - Usually overlooked in a textbook macroeconomic model
   - **Reason**: log-linearized $\Longrightarrow$ no price of risk ($\simeq$ risk-premium)
3. We study these components seriously in monetary frameworks
   - Need analytical global solutions

Standard non-linear New-Keynesian model

1. **Show**: proper accounting of a price of risk changes dynamics

$\underline{\text{Aggregate volatility}}\uparrow \Longleftrightarrow \underline{\text{precautionary saving}}\uparrow \Longleftrightarrow \underline{\text{aggregate demand}}\downarrow$

- **Conventional Taylor rules** $\Longrightarrow$ $\exists$new indeterminacy (aggregate volatility)
- **Sunspot equilibria**: $\exists$sunspot in aggregate volatility: driving business cycles

Standard non-linear New-Keynesian model

1. **Show**: proper accounting of a price of risk changes dynamics

$$\underline{\text{Aggregate volatility}}\uparrow \iff \underline{\text{precautionary saving}}\uparrow \iff \underline{\text{aggregate demand}}\downarrow$$

- **Conventional Taylor rules** $\Longrightarrow$ $\exists$new indeterminacy (aggregate volatility)
- **Sunspot equilibria**: $\exists$sunspot in aggregate volatility: driving business cycles

Non-linear New-Keynesian model with a stock market + portfolio

2. **Build** a parsimonious New-Keynesian framework where: Explain

$$\underline{\text{Stock volatility}}\uparrow \iff \underline{\text{risk-premium}}\uparrow \iff \underline{\text{wealth}}\downarrow \iff \underline{\text{aggregate demand}}\downarrow$$

- Asset price as endogenous shifter in aggregate demand (and vice-versa)
- **VAR analysis**: financial vs real volatility VAR analysis

Isomorphic structure between two frameworks

- **Conventional Taylor rules** $\Longrightarrow$ **Sunspot equilibria** (in stock market volatility): (endogenous) stock market volatility and risk-premium driven business cycle

- **Risk-premium targeting** in a specific way $\Longrightarrow$ determinacy again

Takeaway (**Ultra-divine coincidence**)

One monetary tool (i_t) $\Longrightarrow$ (i) inflation, (ii) output, and (iii) risk-premium

- Generalization of the Taylor rule in a risk-centric environment with risk-premium
- Aggregate wealth management of the monetary policy

Remember: no bubble $\Longrightarrow$ only fundamental asset pricing Literature review

A non-linear textbook New-Keynesian model (demand block)

The representative household's problem (given B_0):

$$\max_{\{B_t, C_t, L_t\}_{t\geq 0}} \mathbb{E}_0 \int_0^\infty e^{-\rho t} \left[\log C_t - \frac{L_t^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}} \right] dt \quad \text{s.t.} \quad \dot{B}_t = i_t B_t - \bar{p} C_t + w_t L_t + D_t$$

where

- B_t: nominal bond holding
- D_t includes fiscal transfer + profits of the intermediate sector
- Rigid price: $p_t = \bar{p}$ for $\forall t$ (demand-determined)

The representative household's problem (given B_0):

$$\max_{\{B_t, C_t, L_t\}_{t\geq 0}} \mathbb{E}_0 \int_0^\infty e^{-\rho t} \left[\log C_t - \frac{L_t^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}} \right] dt \quad \text{s.t.} \quad \dot{B}_t = i_t B_t - \bar{p} C_t + w_t L_t + D_t$$

where

- B_t: nominal bond holding
- D_t includes fiscal transfer + profits of the intermediate sector
- Rigid price: $p_t = \bar{p}$ for $\forall t$ (demand-determined)

1. A non-linear Euler equation (in contrast to textbook log-linearized one)

Endogenous volatility

$$\mathbb{E}_t \left(\frac{dC_t}{C_t} \right) = (i_t - \rho) dt + \mathrm{Var}_t \left(\frac{dC_t}{C_t} \right)$$

Endogenous drift

2. (Aggregate) business cycle volatility↑ ⇒ precautionary saving↑ ⇒ recession now (thus the drift↑)

Problem: both **variance** and **drift** are endogenous, is monetary policy i_t (Taylor rule) enough for stabilization?

Firm i: face monopolistic competition à la Dixit-Stiglitz with $Y_t^i = A_t L_t^i$ and

$$\frac{dA_t}{A_t} = g dt + \underbrace{\sigma}_{\text{Fundamental risk}} dZ_t$$

- dZ_t: aggregate Brownian motion (i.e., only risk source)
- (g, σ) are exogenous

Flexible price economy (benchmark): the 'natural' output Y_t^n follows

$$\frac{dY_t^n}{Y_t^n} = \left(r^n - \rho + \sigma^2 \right) dt + \sigma dZ_t$$
$$= gdt + \sigma dZ_t = \frac{dA_t}{A_t}$$

where $r^n = \rho + g - \sigma^2$ is the 'natural' rate of interest

With

$$\hat{Y}_t = \ln \frac{Y_t}{Y_t^n}, \quad \underbrace{\left(\sigma \right)^2 dt = \mathrm{Var}_t \left(\frac{dY_t^n}{Y_t^n} \right)}_{\text{Benchmark volatility}}, \quad \underbrace{\left(\sigma + \sigma_t^s \right)^2 dt = \mathrm{Var}_t \left(\frac{dY_t}{Y_t} \right)}_{\text{Actual volatility}}$$

Exogenous

Endogenous

With

$$\hat{Y}_t = \ln \frac{Y_t}{Y_t^n}, \quad \underbrace{(\sigma)^2 dt = \mathrm{Var}_t\left(\frac{dY_t^n}{Y_t^n}\right)}_{\text{Benchmark volatility}}, \quad \underbrace{(\sigma + \sigma_t^s)^2 dt = \mathrm{Var}_t\left(\frac{dY_t}{Y_t}\right)}_{\text{Actual volatility}}$$

Exogenous

Endogenous

A non-linear IS equation (in contrast to textbook linearized one)

$$d\hat{Y}_t = \left(i_t - \underbrace{\left(r^n \overbrace{- \frac{1}{2}(\sigma + \sigma_t^s)^2 + \frac{1}{2}\sigma^2}^{\text{New terms}} \right)}_{\equiv r_t^T} \right) dt + \sigma_t^s \, dZ_t \tag{1}$$

- What is r_t^T?: a risk-adjusted natural rate of interest ($\sigma_t^s \uparrow \Longrightarrow r_t^T \downarrow$)

$$r_t^T \equiv r^n - \frac{1}{2}(\sigma + \sigma_t^s)^2 + \frac{1}{2}\sigma^2$$

Big Question: Taylor rule $\underline{i_t = r^n + \phi_y \hat{Y}_t \text{ for } \phi_y > 0}$ $\Rightarrow$ **full stabilization**?

Up to a first-order (no volatility feedback): Blanchard and Kahn (1980)

- $\phi_y > 0$: Taylor principle $\Longrightarrow$ $\hat{Y}_t = 0$ (unique equilibrium)

- What about the higher-order economy?

Big Question: Taylor rule $\underline{i_t = r^n + \phi_y \hat{Y}_t \text{ for } \phi_y > 0}$ $\Rightarrow$ **full stabilization**?

Up to a first-order (no volatility feedback): Blanchard and Kahn (1980)

- $\phi_y > 0$: Taylor principle $\Longrightarrow$ $\hat{Y}_t = 0$ (unique equilibrium)

- What about the higher-order economy?

Proposition (Fundamental Indeterminacy)

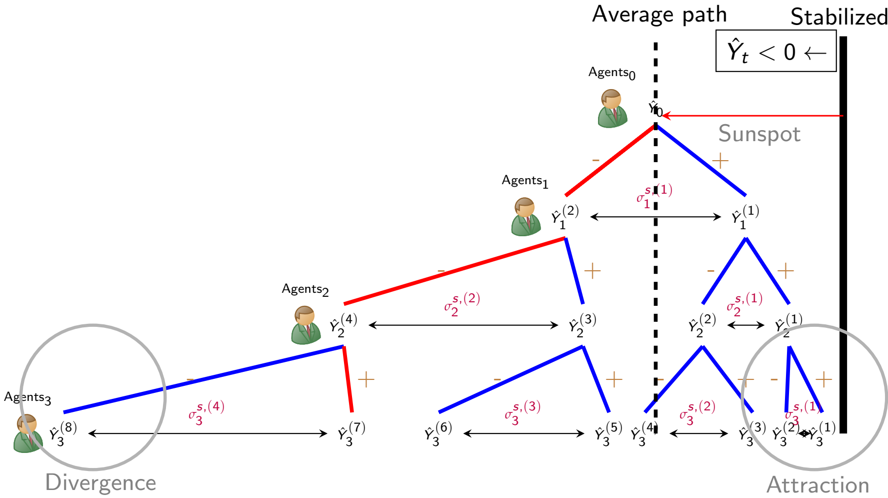
For any $\phi_y > 0$:

$\exists$a rational expectations equilibrium that supports a sunspot $\sigma_0^s > 0$ satisfying:

1. $\mathbb{E}_t\left(\hat{Y}_s\right) = \hat{Y}_t$ for $\forall s > t$ (martingale)
2. $\sigma_t^s \overset{a.s}{\rightarrow} \sigma_\infty^s = 0$ and $\hat{Y}_t \overset{a.s}{\rightarrow} 0$ (almost sure stabilization)
3. $\mathbb{E}_0(\max_{t\geq 0}(\sigma_t^s)^2) = \infty$ (0^+-possibility divergence)

Aggregate volatility↑ possible through the intertemporal coordination of agents

Key: construct a path-dependent intertemporal consumption (demand) strategy



- Stabilized as **attractor**: $\sigma_t^s \xrightarrow{a.s} \sigma_\infty^s = 0$ and $\hat{Y}_t \xrightarrow{a.s} 0$ **but** $\mathbb{E}_0(\max_{t\geq 0}(\sigma_t^s)^2) = \infty$

1. An endogenous aggregate risk arises and drives the business cycle.

2. Sunspots in $\{\sigma_t^s\}$ act similarly to **animal spirit**?

3. New monetary policy

$$i_t = r^n + \phi_y \hat{Y}_t - \boxed{\frac{1}{2}\left((\sigma + \sigma_t^s)^2 - \sigma^2\right)}$$

Aggregate volatility targeting?
Animal spirit targeting?

- Restores a **determinacy** and **stabilization**, but what does it mean?

Next: open the stock market, and relate these terms to the **risk-premium**

The model with a stock market + portfolio decision

Standard demand-determined environment

$\sigma_t^s\uparrow \Longrightarrow$ precautionary saving$\uparrow \Longrightarrow$ consumption (output)$\downarrow$

We can build a **theoretical framework with explicit stock markets** where

Financial volatility$\uparrow \Longrightarrow$ risk-premium$\uparrow \Longrightarrow$ wealth$\downarrow \Longrightarrow$ output$\downarrow$

- Wealth-dependent aggregate demand
- Now, sticky price so $\underline{\pi_t \neq 0}$: Phillips curve à la Calvo (1983)

Skip the detail

Identical **capitalists** and **hand-to-mouth workers** (Two types of agents)

- **Capitalists**: consumption - portfolio decision (between stock and bond)
- **Workers**: supply labors to firms (hand-to-mouth)

Fundamental risk (Exogenous)

1. **Technology**

$$\frac{dA_t}{A_t} = \underbrace{g}_{\text{Growth}} \cdot dt + \sigma \cdot \underbrace{dZ_t}_{\text{Aggregate shock}}$$

2. **Hand-to-mouth workers**: supply labors + solves the following problem

$$\max_{C_t^w, N_t^w} \frac{\left(\frac{C_t^w}{A_t}\right)^{1-\varphi}}{1-\varphi} - \frac{(N_t^w)^{1+\chi_0}}{1+\chi_0} \quad \text{s.t.} \quad p_t C_t^w = w_t N_t^w$$

- Hand-to-mouth assumption can be relaxed, without changing implications

3. **Firms**: production using labors + pricing à la Calvo (1983)

4. **Financial market**: zero net-supplied risk-free bond + stock (index) market

Capitalists: standard portfolio and consumption decisions (very simple)

1. Total financial wealth $a_t = p_t A_t Q_t$, where (real) stock price Q_t follows:

$$\frac{dQ_t}{Q_t} = \mu_t^q \cdot dt + \sigma_t^q \cdot dZ_t$$

Financial risk (Endogenous)

- μ_t^q and σ_t^q are both endogenous (to be determined)

2. Each solves the following optimization (standard)

$$\max_{C_t, \theta_t} \mathbb{E}_0 \int_0^\infty e^{-\rho t} \log C_t dt \quad \text{s.t.}$$

$$da_t = (a_t(i_t + \theta_t(i_t^m - i_t)) - p_t C_t)dt + \theta_t a_t(\sigma + \sigma_t^q)dZ_t$$

- Aggregate consumption of capitalists $\propto$ aggregate financial wealth

$$C_t = \rho A_t Q_t$$

- Equilibrium **risk-premium** is determined by the total risk

$$i_t^m - i_t \equiv \text{rp}_t = \boxed{(\sigma + \sigma_t^q)^2}$$

Dividend yield: dividend yield$= \rho$, as in Caballero and Simsek (2020)

- A positive feedback loop between asset price $\Longleftrightarrow$ dividend (output)

Determination of nominal stock return $d\mathbf{I}_t^m$

$$d\mathbf{I}_t^m = \underbrace{\Big[\underbrace{\rho}_{\text{Dividend yield}} + \underbrace{\underbrace{\pi_t}_{\text{Inflation}} + g + \mu_t^q + \overbrace{\sigma\sigma_t^q}^{\text{Covariance}}}_{\text{Capital gain}} \Big]}_{= \underbrace{i_t^m}_{\text{Drift}} = \underbrace{i_t}_{\text{Monetary policy}} + \underbrace{(\sigma+\sigma_t^q)^2}_{\text{Risk-premium}}} dt + \underbrace{(\sigma + \sigma_t^q)}_{\text{Risk term}} dZ_t$$

- Close the model with supply-side (Phillips curve) and $\{i_t\}$ rule

Flexible price economy allocations (benchmark)

- $\sigma_t^{q,n} = 0$, Q_t^n, $N_{W,t}^n$, C_t^n, r^n (natural rate), rp^n (natural risk-premium)

Gap economy (log deviation from the flexible price economy)

- With asset price gap $\hat{Q}_t \equiv \ln \frac{Q_t}{Q_t^n} = \hat{C}_t$ and π_t

Proposition (Dynamic IS)

A dynamic gap economy can be described with the following equations:

1. $\mathbb{E}_t d\pi_t = (\rho\pi_t - \kappa\hat{Q}_t)dt$ with $\kappa > 0$
2. $d\hat{Q}_t = (i_t - \pi_t - \boxed{r_t^T})dt + \sigma_t^q \, dZ_t$ where $r_t^T = r^n - \frac{1}{2}(\mathrm{rp}_t - \mathrm{rp}^n)$

$$\equiv r^n - \frac{1}{2}\hat{\mathrm{rp}}_t$$

where $\mathrm{rp}_t = (\sigma + \sigma_t^q)^2$ and $\mathrm{rp}^n = \sigma^2 \implies \hat{\mathrm{rp}}_t \equiv \mathrm{rp}_t - \mathrm{rp}^n$

Isomorphism: the same mathematical structure

Go back

Now, with asset (stock) price gap $\hat{Q}_t$:

Real volatility

$$d\hat{Q}_t = \left(i_t - \pi_t - \underbrace{\left(r^n - \frac{1}{2}\underbrace{\boxed{(\sigma + \sigma_t^q)^2}}_{\mathrm{rp}_t} + \frac{1}{2}\underbrace{\boxed{\sigma^2}}_{\mathrm{rp}^n} \right)}_{\equiv r_t^T} \right) dt + \sigma_t^q \, dZ_t \qquad (2)$$

Here

$$\sigma_t^q \uparrow \implies \mathrm{rp}_t \uparrow \implies \hat{Q}_t \downarrow \implies \hat{Y}_t \downarrow$$

More intuitions

Monetary policy: Taylor rule to Bernanke and Gertler (2000) rule

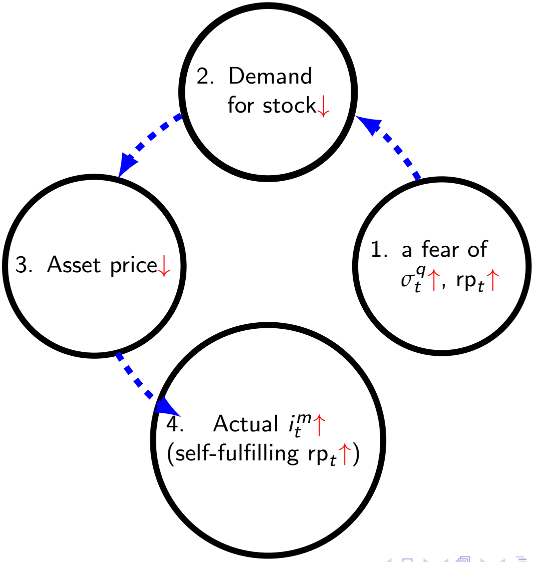
$$i_t = r^n + \phi_\pi \pi_t + \phi_y \underbrace{\hat{y}_t}_{=\zeta \hat{Q}_t}$$

$$= r^n + \phi_\pi \pi_t + \phi_q \hat{Q}_t, \quad \text{where} \quad \underbrace{\phi \equiv \phi_q + \frac{\kappa(\phi_\pi - 1)}{\rho} > 0}_{\text{Taylor principle}}$$

Simulation

Multiple equilibria (risk-premium sunspot)

- **How?**: countercyclical risk-premium with conventional Taylor rules



Assume $\underline{\sigma_0^q > 0}$ for some reason (initial disruption)

- The same **martingale equilibrium** Mathematical explanation Tree diagram

Is a sunspot $\sigma_0^q \neq 0$ supported by a rational expectations equilibrium? : with Bernanke and Gertler (2000) rule

Assume $\sigma_0^q > 0$ for some reason (initial disruption)

- The same **martingale equilibrium** ⏩ Mathematical explanation ⏩ Tree diagram

Proposition (Fundamental Indeterminacy)

For any $\phi > 0$:

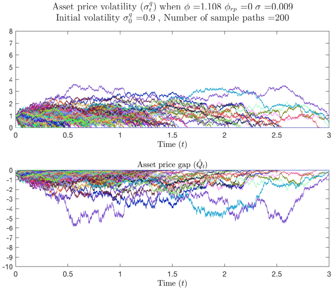
$\exists$a rational expectations equilibrium that supports a sunspot $\sigma_0^q > 0$ satisfying:

1. $\sigma_t^q \overset{a.s}{\to} \sigma_\infty^q = 0$, $\hat{Q}_t \overset{a.s}{\to} 0$, and $\pi_t \overset{a.s}{\to} 0$ (almost sure stabilization)
2. $\mathbb{E}_0(\max_{t\geq 0}(\sigma_t^q)^2) = \infty$ (0^+-possibility divergence)

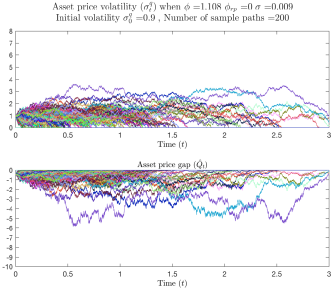
1. (Almost surely) stabilized in the long run after sunspot $\sigma_0^q > 0$

   Meantime: **crisis** with financial volatility (risk-premium)↑, asset price↓, and business cycle↓

2. $\mathbb{E}_0(\max_t(\sigma_t^q)^2) = \infty$: an $\epsilon \to 0$ possibility of ∞-severity crisis ($\sigma_t^q \to \infty$)
   - $\exists$big crisis that supports $\sigma_0^q > 0$ (e.g., Martin (2012) in asset pricing contexts)



(a) With $\phi_\pi = 1.5$

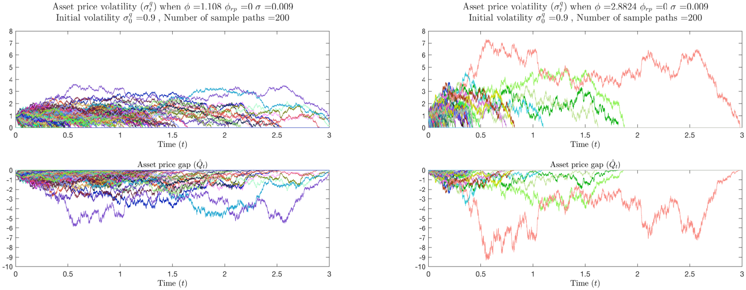


(a) With $\phi_\pi = 1.5$

(b) With $\phi_\pi = 2.5$.

Figure: $\{\sigma_t^q, \hat{Q}_t\}$ dynamics when $\sigma^{q,n} = 0$ and $\sigma_0^q = 0.9$, with reasonable calibration

- As monetary policy responsiveness $\phi\uparrow$
  Stabilization speed$\uparrow$, $\exists$more severe crisis sample path
- $\sigma_t^q\uparrow$ by $\sigma \Longrightarrow 2 - 10\%\downarrow$ in Q_t (depending on monetary responsiveness)



(a) With $\phi_\pi = 1.5$ (b) With $\phi_\pi = 2.5$.

Figure: $\{\sigma_t^q, \hat{Q}_t\}$ dynamics when $\sigma^{q,n} = 0$ and $\sigma_0^q = 0.9$, with reasonable calibration

- As monetary policy responsiveness $\phi\uparrow$

  Stabilization speed$\uparrow$, $\exists$more severe crisis sample path
- $\sigma_t^q\uparrow$ by $\sigma \implies 2-10\%\downarrow$ in Q_t (depending on monetary responsiveness)

Opposite case: with initial sunspot $\sigma_0^q < 0$

- Explains **boom** phase

  Financial volatility (risk-premium)$\downarrow$, asset price$\uparrow$ and business cycle$\uparrow$

New monetary policy $\Longrightarrow$ financial + macro stabilities $\hat{Q}_t = \pi_t = \hat{rp}_t = 0$

$$i_t = r^n + \phi_\pi \pi_t + \phi_q \hat{Q}_t - \overbrace{\frac{1}{2}\,\hat{rp}_t}^{\text{New targeting}} \quad , \quad \text{where} \quad \underbrace{\phi \equiv \phi_q + \frac{\kappa(\phi_\pi - 1)}{\rho} > 0}_{\text{Taylor principle}}$$

Sharp

restores a determinacy with:

Takeaway (Ultra-divine coincidence)

One monetary tool (i_t) $\Longrightarrow$ (i) inflation, (ii) output, and (iii) risk-premium

Sharpness

Leading to:

Ito term   Ito term

$$\boxed{\underbrace{i_t + \mathrm{rp}_t}_{=i_t^m} - \frac{1}{2}\mathrm{rp}_t} = \boxed{\underbrace{r^n + \mathrm{rp}^n}_{=i_t^{m,n}} - \frac{1}{2}\mathrm{rp}^n} + \underbrace{\phi_\pi \pi_t + \phi_q \hat{Q}_t}_{\text{Business cycle targeting}}$$

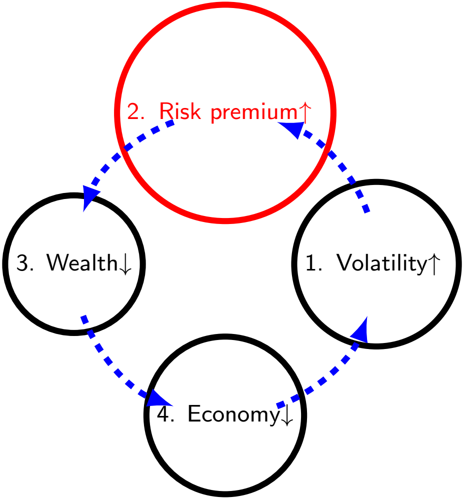
$$\rho + \frac{\mathbb{E}_t(d\log a_t)}{dt} \qquad \rho + \frac{\mathbb{E}_t(d\log a_t^n)}{dt}$$

- i_t^m, not i_t, follows a Taylor rule?
- A rate of change of log-wealth follows a Taylor rule both in standard model (without risk-premium) and our framework (with risk-premium)

Thank you very much!

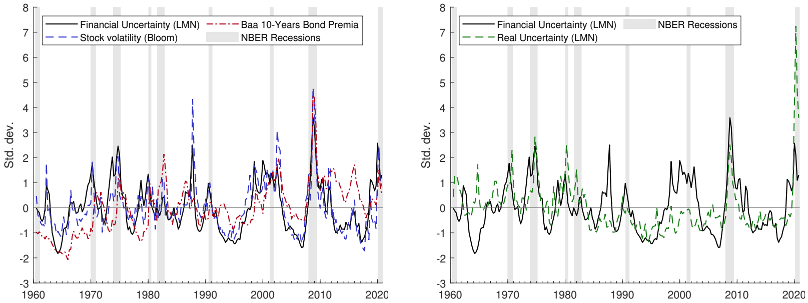
(Appendix)

Go back



- **1** $\rightarrow$ **2** comes from "non-linearity (not linearizing)"
- **2** $\rightarrow$ **3** comes from "portfolio decision" of each investor and externality
- **3** $\rightarrow$ **4** comes from the fact wealth drives aggregate demand
- **4** $\rightarrow$ **1** where business cycle has its own volatility (self-sustaining)

Go back



(a) Financial Uncertainty series

(b) Financial vs. Real Uncertainty

Figure: Common measures of the financial volatility (left) and real vs. financial uncertainty decomposed by Ludvigson et al. (2015) (right)

The correlation between series is remarkably high and they all display positive spikes at the beginning and/or initial months following NBER-dated recessions

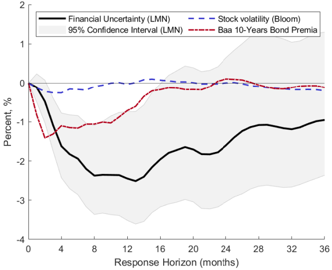
- Many of past recessions are, in nature, financial

In a similar manner to Bloom (2009), Ludvigson et al. (2015):

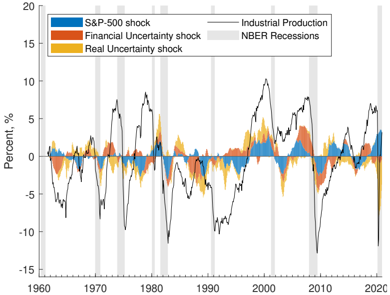
$$\text{VAR-11 order:} \quad \begin{bmatrix} \log\left(\text{Industrial Production}\right) \\ \log\left(\text{Employment}\right) \\ \log\left(\text{Real Consumption}\right) \\ \log\left(\text{CPI}\right) \\ \log\left(\text{Wages}\right) \\ \text{Hours} \\ \text{Real Uncertainty (LMN)} \\ \text{Fed Funds Rate} \\ \log\left(\text{M2}\right) \\ \log\left(\text{S\&P-500 Index}\right) \\ \text{Financial Uncertainty (LMN)} \end{bmatrix} \tag{3}$$

Financial uncertainty (LMN) is also replaced by the stock price volatility (following Bloom (2009)) and Baa 10-years bond premia

Vector Autoregression (VAR) analysis



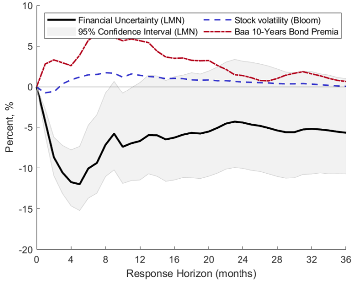
(a) Response: Industrial Production



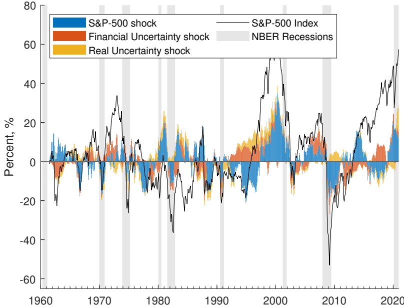
(b) Industrial Production

Figure: Impulse-response of IP to one std.dev shock in financial uncertainty measures (left) and the historical decomposition of IP to various attributes (right)

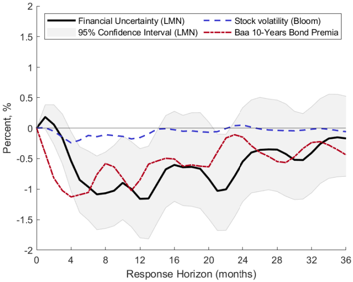
1. IP falls by 2.5% after one standard deviation spike in the Ludvigson et al. (2015)'s financial uncertainty measure
   - Financial uncertainty has been important in driving IP boom-bust patterns
2. Other graphs: IRF and historical decomposition of S&P 500 S&P500, and FFR (monetary policy) FFR, FEVD FEVD



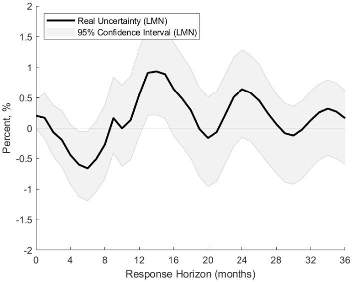
(a) Response: S&P-500 Index



(b) S&P-500 Index



(a) Shock: Financial Uncertainty



(b) Shock: Real Uncertainty

With 3 different financial uncertainty measures: Ludvigson et al. (2015), Bloom (2009), Baa 10-years bond premia (left)

(i) Industrial Production

Horizon	Fin. Uncert. (LMN)	Real Uncert. (LMN)	Stock Vol. (Bloom)	Baa 10-Yr Premia
h=1	0	0.30	0.21	0.12
h=6	1.27	3.37	2.98	1.36
h=12	4.28	4.38	3.16	1.94
h=36	3.24	1.67	1.98	0.64

(ii) S&P-500 Index

Horizon	Fin. Uncert. (LMN)	Real Uncert. (LMN)	Stock Vol. (Bloom)	Baa 10-Yr Premia
h=1	0.11	0.08	0.39	0.06
h=6	3.30	0.25	3.26	0.62
h=12	4.77	0.54	10.03	2.16
h=36	6.50	0.91	12.16	2.40

(iii) Fed Funds Rate

Horizon	Fin. Uncert. (LMN)	Real Uncert. (LMN)	Stock Vol. (Bloom)	Baa 10-Yr Premia
h=1	0.01	0.98	0	0.08
h=6	0.42	0.84	3.11	1.66
h=12	1.47	0.91	4.69	2.30
h=36	2.81	2.05	5.02	3.17

Financial uncertainty shocks explain close to:

- 5% of the fluctuations in both IP and S&P-500 series

Real uncertainty explains:

- Additional 2-4% of movements in industrial activity in the medium run

- Financial wealth (e.g., risk-intolerance) and aggregate demand: Mian and Sufi (2014), Caballero and Farhi (2017), Guerrieri and Lacoviello (2017), Caballero and Simsek (2020a, 2020b), Chodorow-Reich et al. (2021), Caballero et al. (2021)
- Financial disruption (volatility) and macroeconomy: Gilchrist and Zakrajšek (2012), Brunnermeir and Sannikov (2014), Guerrieri and Lorenzoni (2017), Di Tella and Hall (2020)

Our paper: a monetary framework that incorporates financial wealth, aggregate financial volatility, risk-premium, and business cycle (all endogenous)

- Monetary policy and financial market disruptions: Bernanke and Gertler (2000), Nisticò (2012), Stein (2012), Cúrdia and Woodford (2016), Cieslak and Vissing-Jorgensen (2020), Galí (2021)

Our paper: a monetary policy's financial targeting (first and second-orders) in the world without bubble + lean against the stock market

- Asset pricing and nominal rigidity: Weber (2015), Gorodnichenko and Weber (2016), Campbell et al. (2020)
- Time-varying risk-premium in New-Keynesian model: Laseen et al. (2015)
- Indeterminacy with an idiosyncratic risk: Acharya and Dogra (2020)

Our paper: an analytical expression of time-varying risk-premium in a monetary model + new indeterminacy in aggregate volatility

Go back

1. Capitalists bear $(\sigma_t + \sigma_t^q)$ amount of risks when investing in stock market
   - Risk-premium $\text{rp}_t = (\sigma_t + \sigma_t^q)^2$
   - Natural risk-premium (in the flexible price economy) $\text{rp}_t^n = (\sigma_t + \underbrace{\sigma_t^{q,n}}_{=0})^2$
2. If a real return on stock investment is different from its natural level (return of stock investment in the flexible price economy), then $\hat{Q}_t$ jumps

Takeaway (Risk-adjusted natural rate)

r_t^T is a real risk-free rate that makes:

stock market's real return (with risk-premium rp_t) = natural economy's (with risk-premium rp_t^n

$$\left(\underbrace{r_t^T}_{\substack{\text{Risk-free rate yielding} \\ \text{equal return on stock}}} + \text{rp}_t\right) - \frac{1}{2}\text{rp}_t = \left(\underbrace{r_t^n}_{\text{Natural rate}} + \text{rp}_t^n\right) - \frac{1}{2}\text{rp}_t^n$$

Ito term

Ito term

Is a sunspot $\sigma_0^q \neq \sigma^{q,n}$ supported by a rational expectations equilibrium? : with Bernanke-Gertler (2000) rule

Go back

Assume $\underline{\sigma_0^q > \sigma^{q,n} = 0}$ for some reason (initial sunspot)

Blanchard and Kahn (1980) does not apply: we construct a rational expectations equilibrium (REE: not diverging on average) supporting an initial sunspot σ_0^q

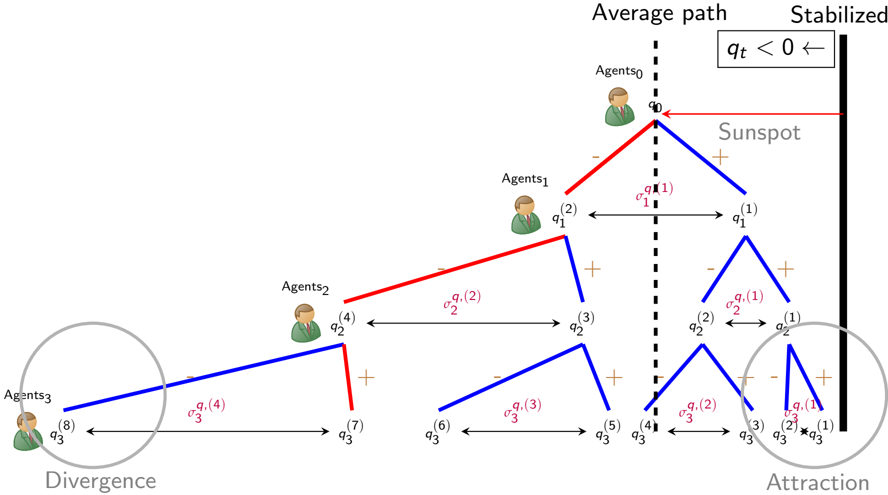
$$d\hat{Q}_t = \left(i_t - \pi_t - \left(r_t^n - \frac{1}{2}(\mathrm{rp}_t - \mathrm{rp}_t^n)\right)\right)dt + \sigma_t^q dZ_t$$
$$= \underbrace{\left((\phi_\pi - 1)\pi_t + \phi_q \hat{Q}_t + \frac{1}{2}(\mathrm{rp}_t - \mathrm{rp}_t^n)\right)}_{=0,\quad \forall t} dt + \sigma_t^q dZ_t$$

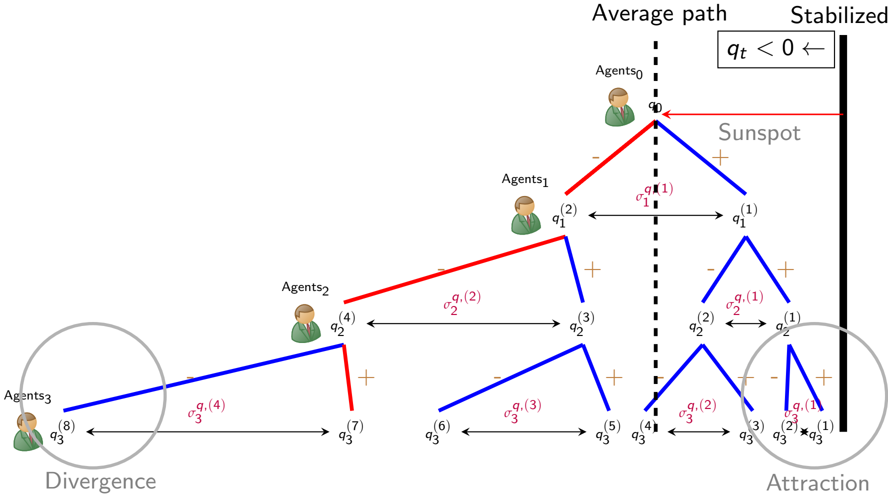
- Called the 'martingale equilibrium': supporting an initial sunspot in financial volatility σ_0^q
- $\{\sigma_t^q\}$ has its own (endogenous) stochastic process, given initial $\sigma_0^q \neq 0$

$$d\sigma_t^q = -\frac{\phi^2(\sigma_t^q)^2}{2(\sigma_t + \sigma_t^q)^3}dt - \phi\frac{\sigma_t^q}{\sigma_t + \sigma_t^q}dZ_t$$

Go back

Again, the same structure





Go back

Asset price $\{q_t\}$ and the conditional volatility $\{\sigma_t^q\}$ are stochastic

- Rational expectations equilibrium (REE): no divergence on expectation
- As q_t approaches the stabilized path, then $\sigma_t^q \downarrow$, and more likely stays there: convergence ($\sigma_t^q \xrightarrow{a.s} \sigma_\infty^q = \sigma^{q.n} = 0$)
- But in the worst scenario σ_t^q diverges (with 0^+-probability)

Go back

What if central bank uses the following alternative rule, where $\phi_{\text{rp}} \neq \frac{1}{2}$?

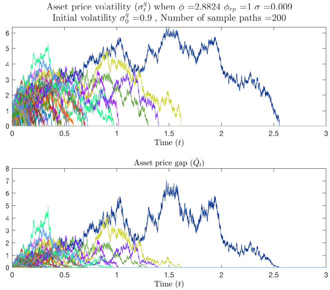
$$i_t = r_t^n + \phi_\pi \pi_t + \phi_q \hat{Q}_t - \boxed{\phi_{\text{rp}}} \hat{rp}_t, \quad \text{where} \quad \phi \equiv \phi_q + \frac{\kappa(\phi_\pi - 1)}{\rho} > 0$$

- Then still $\exists$martingale equilibrium supporting sunspot $\sigma_0^q \neq 0$
- As $|\phi_{\text{rp}} - \frac{1}{2}| \uparrow \implies$ (on average) longer time for σ_t^q to vanish
- Especially, $\phi_{\text{rp}} < 0$ (**Real Bills Doctrine**) is a bad idea Summary Simulation

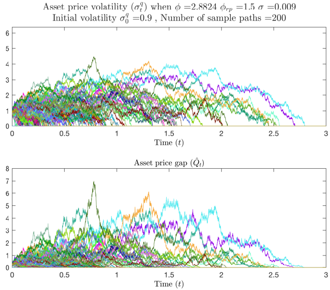
When ϕ_{rp} deviates from $\frac{1}{2}$

Go back

$\phi_{\mathbf{rp}} < 0$ **(Real Bills Doctrine)**	$0 < \phi_{\mathbf{rp}} < \frac{1}{2}$
(i) With $\phi_{\text{rp}}\downarrow$, convergence speed$\downarrow$ and less amplified paths (ii) $\sigma_t^q > \sigma_t^{q,n} = 0$ means a crisis ($\hat{Q}_t < 0$ and $\pi_t < 0$)	(i) With $\phi_{\text{rp}}\uparrow$, convergence speed$\uparrow$ and more amplified paths (ii) $\sigma_t^q > \sigma_t^{q,n} = 0$ means a crisis ($\hat{Q}_t < 0$ and $\pi_t < 0$)
$\phi_{\mathbf{rp}} = \frac{1}{2}$	$\phi_{\mathbf{rp}} > \frac{1}{2}$
No sunspot (Ultra-divine coincidence)	(i) With $\phi_{\text{rp}}\uparrow$, convergence speed$\downarrow$ and less amplified paths (ii) $\sigma_t^q > \sigma_t^{q,n} = 0$ means a boom ($\hat{Q}_t > 0$ and $\pi_t > 0$)
As $\phi\uparrow$, convergence speed$\uparrow$ and $\exists$ more amplified paths	



(a) With $\phi_{\text{rp}} = 1$



(b) With $\phi_{\text{rp}} = 1.5$.

Figure: $\{\sigma_t^q, \hat{Q}_t\}$ dynamics when $\sigma^{q,n} = 0$ and $\sigma_0^q = 0.9$, with varying $\phi_{\text{rp}} > \frac{1}{2}$