

Higher-Order Forward Guidance

Marc Dordal i Carreras
HKUST

Seung Joo Lee
Oxford - Saïd

Macroeconomics Seminar

CREi-UPF

April 16, 2024

“The Federal Reserve, ... affirmed today its **readiness to serve** as a source of liquidity **to support the economic and financial system**”

- Alan Greenspan, 1987 (Black Monday)

“Within our mandate, the ECB is **ready to do whatever it takes** to preserve the euro. And believe me, **it will be enough.**”

- Mario Draghi, 2012 (European Crisis)

Observation

Modern central banks: expanding their roles beyond the conventional policy rate setting

Unconventional policy interventions (e.g. forward guidance): more prevalent

Forward guidance:

- **Odyssean guidance:** Communication about future policy commitment

*“The Central Bank **commits to maintaining** the current interest rate at its present low level **until the unemployment rate falls below 5 percent**, irrespective of fluctuations in inflation.”*

- **Delphic guidance:** Communication about forecasts of future macroeconomic performance

*“The Central Bank **anticipates that the prevailing economic conditions** will necessitate an upward adjustment in interest rates in the foreseeable future.”*

How does it work, exactly?

- First-order effects (levels): “Interest rates will stay low” → **intertemporal substitution channel**: many works in the literature (Eggertsson et al. (2003), Campbell et al. (2012, 2019), Del Negro et al. (2013), McKay et al. (2016), Caballero and Farhi (2017))
- **Second-order effects (volatility)**: Reduce uncertainty, avoid worst-case scenarios, “whatever it takes” → **precautionary savings channel**

Big Question (Uncertainty Management)

How does the central bank manage economic uncertainty at the zero lower bound (ZLB)? Is it possible? Desirable?

Uncertainty (i.e., volatility): an important source of business cycle fluctuations

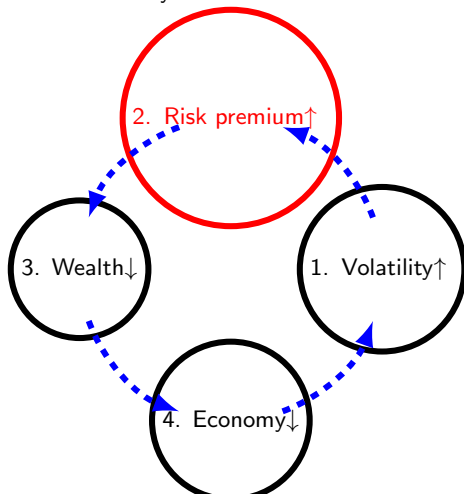
- The literature: Bloom (2009), Ludvigson et al. (2015)
- Finance: risk-premium \propto volatility² (e.g., Merton (1971))
- **VAR analysis:** financial and real volatility ▶ VAR analysis

This paper: forward guidance with a focus on strategic uncertainty management and coordination. We show it is possible for central banks to pick an equilibrium where:

- During the ZLB (now): reduce aggregate volatility (and risk premium) for aggregate demand↑
- But after the ZLB (future): less stabilization
- Welfare-enhancing overall

Non-linear Two-Agent New-Keynesian (TANK) model with a stock market + portfolio choice

- **Build** a parsimonious New-Keynesian framework where: [▶ Explain](#)



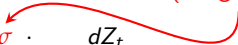
Model

Identical **capitalists** and **hand-to-mouth workers** (Two types of agents)

- **Capitalists:** consumption - portfolio decision (between stock and bond)
- **Workers:** supply labor to firms (hand-to-mouth)

Fundamental risk
(Exogenous)

1. Technology

$$\frac{dA_t}{A_t} = \underbrace{g}_{\text{Growth}} \cdot dt + \underbrace{\sigma}_{\text{Aggregate shock}} \cdot \underbrace{dZ_t}_{\text{Aggregate shock}}$$


2. Hand-to-mouth workers: supply labor + solves the following problem:

$$\max_{C_t^w, N_t^w} \frac{\left(\frac{C_t^w}{A_t}\right)^{1-\varphi}}{1-\varphi} - \frac{(N_t^w)^{1+\chi_0}}{1+\chi_0} \quad \text{s.t.} \quad \bar{p}C_t^w = w_t N_t^w$$

- Hand-to-mouth assumption can be relaxed, without changing implications

3. Firms: Dixit-Stiglitz production using labor + perfectly rigid prices ($\pi_t = 0$)

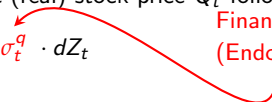
4. Financial market: zero net-supplied risk-free bond + stock (index) market

Capitalists: standard portfolio and consumption decisions (very simple)

1. Total financial wealth $a_t = \bar{p}A_tQ_t$, where (real) stock price Q_t follows:

$$\frac{dQ_t}{Q_t} = \mu_t^q \cdot dt + \sigma_t^q \cdot dZ_t$$

Financial risk
(Endogenous)



- μ_t^q and σ_t^q are both endogenous (to be determined)

2. Each solves the following optimization (standard)

$$\max_{C_t, \theta_t} \mathbb{E}_0 \int_0^\infty e^{-\rho t} \log C_t dt \quad \text{s.t.}$$

$$da_t = (a_t(i_t + \theta_t(i_t^m - i_t)) - \bar{p}C_t)dt + \theta_t a_t(\sigma + \sigma_t^q)dZ_t$$

- Aggregate consumption of capitalists \propto aggregate financial wealth

$$C_t = \rho A_t Q_t$$

- Equilibrium **risk-premium** is determined by the total risk

$$i_t^m - i_t \equiv rp_t = (\sigma + \sigma_t^q)^2$$

Dividend yield: dividend yield = ρ , as in Caballero and Simsek (2020)

- A positive feedback loop between asset price \Longleftrightarrow dividend (output)

Determination of nominal stock return dI_t^m

$$\begin{aligned}
 dI_t^m &= \left[\underbrace{\rho}_{\text{Dividend yield}} + \underbrace{g + \mu_t^q + \overbrace{\sigma\sigma_t^q}^{\text{Covariance}}}_{\text{Capital gain}} \right] dt + \underbrace{(\sigma + \sigma_t^q)}_{\text{Risk term}} dZ_t \\
 &= \underbrace{i_t^m}_{\text{Drift}} = \underbrace{i_t}_{\text{Monetary policy}} + \underbrace{(\sigma + \sigma_t^q)^2}_{\text{Risk-premium}}
 \end{aligned}$$

Equilibrium with rigid prices ($\pi_t = 0, \forall t$)

Flexible price economy as benchmark: the 'natural' consumption of capitalists $C_t^n = \rho A_t Q_t^n$ follows

$$\begin{aligned}\frac{dC_t^n}{C_t^n} &\equiv \frac{d(A_t Q_t^n)}{A_t Q_t^n} = (r^n - \rho + \sigma^2) dt + \sigma dZ_t \\ &= g dt + \sigma dZ_t = \frac{dA_t}{A_t}\end{aligned}$$

where $r^n = \rho + g - \sigma^2$ is the 'natural' rate of interest

Define **asset price gap**

$$\hat{Q}_t = \ln \frac{Q_t}{Q_t^n}, \quad 0 = \underbrace{\text{Var}_t \left(\frac{dQ_t^n}{Q_t^n} \right)}_{\text{Benchmark volatility}}, \quad \underbrace{\left(\overset{\text{Endogenous}}{\sigma_t^q} \right)^2 dt}_{\text{Actual volatility}} = \text{Var}_t \left(\frac{dQ_t}{Q_t} \right)$$

which is proportional to **output gap**

$$\hat{Y}_t = \ln \left(\frac{Y_t}{Y_t^n} \right) \longrightarrow \hat{Y}_t = \underbrace{\zeta}_{>0} \cdot \hat{Q}_t$$

A non-linear IS equation (in contrast to textbook linearized one)

$$d\hat{Q}_t = \left(i_t - \underbrace{\left(r^n - \frac{1}{2}(\sigma + \sigma_t^q)^2 + \frac{1}{2}\sigma^2 \right)}_{\equiv r_t^T} \right) dt + \sigma_t^q dZ_t \quad (1)$$

$$= (i_t - r_t^T) dt + \sigma_t^q dZ_t \quad (2)$$

What is r_t^T ?: a **risk-adjusted** natural rate of interest ($\sigma_t^q \uparrow \rightarrow r_t^T \downarrow$)

$$\begin{aligned} r_t^T &\equiv r^n - \frac{1}{2}(\sigma + \sigma_t^q)^2 + \frac{1}{2}\sigma^2 \\ &= r^n - \frac{1}{2}\hat{r}p_t \end{aligned}$$

where $\hat{r}p_t = \underbrace{rp_t - rp_t^n}_{\text{risk-premium gap}}$

Outside the ZLB: Can we stabilize this economy? Prevent the volatility feedback loop? **Yes!:** Lee and Dordal i Carreras (2024)

- Under a risk-premium targeting rule: $i_t = r_t^T + \phi_q \hat{Q}_t$. With $\phi_q > 0$ (i.e., Taylor principle) $\rightarrow \hat{Q}_t = 0$ for $\forall t$ (unique equilibrium)

$$d\hat{Q}_t = (i_t - r_t^T) dt + \sigma_t^q dZ_t \quad \underbrace{\quad}_{\text{Under risk-premium targeting}} \quad \phi_q \hat{Q}_t dt + \sigma_t^q dZ_t$$

Then,

$$\mathbb{E}_t (d\hat{Q}_t) = \phi_q \hat{Q}_t$$

- If $\hat{Q}_t \neq 0$, then $\mathbb{E}_t (\hat{Q}_\infty)$ blows up $\rightarrow \hat{Q}_t = 0$ for $\forall t$ as unique equilibrium (Blanchard and Kahn (1980)) $\rightarrow \sigma_t^q = 0$ for $\forall t$ (i.e., zero excess volatility)
- Outside the ZLB, uncertainty can be eliminated by traditional means

At the ZLB, the precautionary feedback loop reappears:

$$d\hat{Q}_t = -r_t^T dt + \sigma_t^q dZ_t = - \left(r^n - \frac{1}{2}(\sigma + \sigma_t^q)^2 + \frac{1}{2}\sigma^2 \right) dt + \sigma_t^q dZ_t$$

Thought experiment: fundamental volatility $\sigma \uparrow$: from $\underline{\sigma}$ to $\bar{\sigma}$ on $[0, T]$ (e.g., Werning (2012)) and comes back to $\underline{\sigma}$ with

- $\bar{r} \equiv r^n(\underline{\sigma}) = \rho + g - \underline{\sigma}^2 > 0$: no ZLB before, $t < 0$, or after, $t > T$
- $\underline{r} \equiv r^n(\bar{\sigma}) = \rho + g - \bar{\sigma}^2 < 0$: ZLB binds for $0 \leq t \leq T$

Assume: perfect stabilization (i.e., $\hat{Q}_t = 0$) is achievable outside ZLB, i.e.,

$$i_t = \bar{r} - \frac{1}{2} \hat{r} p_t + \phi_q \hat{Q}_t, \quad \text{with } \phi_q > 0$$

Result: perfect stabilization of risk-premia gap (i.e., excess uncertainty) inside the ZLB as well

- **Recursive argument:** Full stabilization at T implies $\hat{Q}_T = 0 \longrightarrow \sigma_{T-dt}^q = 0$, and so on (so $\hat{r} p_t = 0$ for $\forall t$)

ZLB path (full stabilization after T)

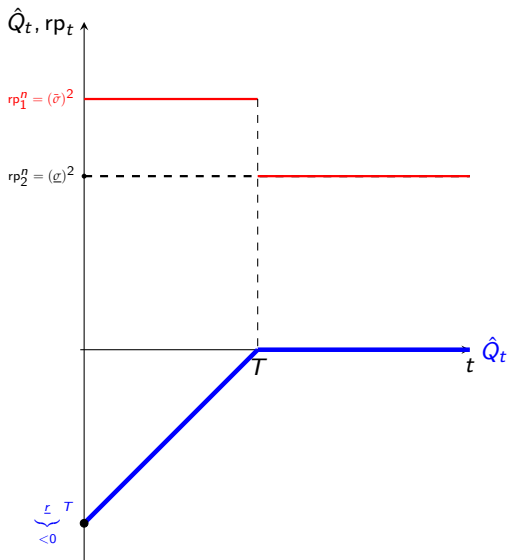


Figure: ZLB dynamics (Benchmark)

Assume:

- Central bank commits to keep $i_t = 0$ until $\hat{T}^{\text{TFG}} \geq T$ (Odyssean guidance)
- Perfect stabilization (i.e., $\hat{Q}_t = 0$) afterwards, i.e., for $t > \hat{T}^{\text{TFG}}$
- By similar arguments: risk-premium gap stabilization beforehand, $t \leq \hat{T}^{\text{TFG}}$ (no excess volatility while $i_t = 0$)

Problem: Minimize smooth quadratic welfare loss

$$\begin{aligned} \min_{\hat{T}^{\text{TFG}}} \mathbb{L}^Q(\{\hat{Q}\}_{t \geq 0}) &\equiv \mathbb{E}_0 \int_0^\infty e^{-\rho t} (\hat{Q}_t)^2 dt \\ \text{s.t. } \hat{Q}_0 &= \underbrace{\underline{r}}_{<0} T + \underbrace{\bar{r}}_{>0} (\hat{T}^{\text{TFG}} - T) \end{aligned}$$

- Smoothing the ZLB costs over time (i.e., welfare enhancing)

Traditional forward guidance (keep $i_t = 0$ until $\hat{T}^{\text{TFG}} > T$)

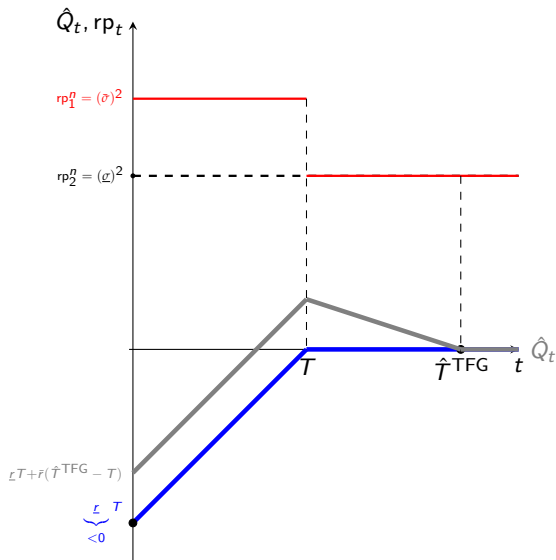


Figure: ZLB dynamics with forward guidance until $\hat{T}^{\text{TFG}} > T$

Big Question

Can we do even **better** than the traditional forward guidance?

Welfare losses are driven by

- High aggregate uncertainty (and risk premium) levels, $(\bar{\sigma} + \sigma_t^q)$, due to high fundamental uncertainty: $\bar{\sigma} > \underline{\sigma}$
- Excess endogenous volatility $\sigma_t^q = 0$ by future stabilization

Can we reduce aggregate uncertainty via $\sigma_t^q < 0$?

- **Yes!** Intuition for $\sigma_t^q < 0$: the real wealth $A_t Q_t$ must respond *less than* proportionally to A_t shocks
- How? Rigid prices \rightarrow demand-determined production (and hence, wealth)
- Policy challenge: the central bank *must convince* households to behave this way \rightarrow *higher-order forward guidance*

Recall an economic mechanism in the ZLB and forward guidance

1. Central bank achieves perfect stabilization: $\hat{Q}_t = \hat{r}p_t = 0, \forall t \geq \hat{T}$



2. $\hat{Q}_{\hat{T}} = 0$ guarantees $\sigma_t^q = \sigma^{q,n} = 0, rp_t = rp^n$ for $t \leq \hat{T}$

Still if rp^n is too high, might want to push $\{\sigma_t^q, rp_t\}$ down for $\hat{Q}_t \uparrow$?

- Thus achieve $\sigma_t^q < \sigma^{q,n} = 0, rp_t < rp^n \implies \hat{Q}_t \uparrow$ at the ZLB

Take **contrapositive** to the above (necessary condition):

$\neg 2. \sigma_t^q < \sigma^{q,n} = 0, rp_t < rp^n$ for $t \leq \hat{T}$



$\neg 1. \hat{Q}_{\hat{T}} \neq 0$. Central bank commits not to perfectly stabilize the economy after \hat{T}

- Giving up **future** financial stability $\implies rp_t \downarrow$ and $\hat{Q}_t \uparrow$ **now** (at the ZLB)

Assume:

- Central bank can commit to keep $i_t = 0$ until $\hat{T}^{HOFG} \geq T$
- No stabilization (i.e., $\hat{Q}_t = \hat{Q}_{\hat{T}^{HOFG}}$) guaranteed afterwards, $t \geq \hat{T}^{HOFG}$
- Pick $\{\sigma_t^q\}$ for $t < \hat{T}^{HOFG}$

Problem: Minimize smooth quadratic welfare loss

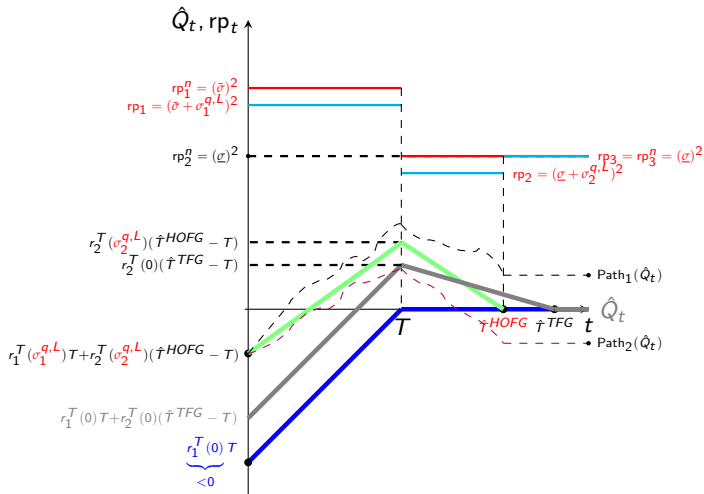
$$\min_{\sigma_1^{q,L}, \sigma_2^{q,L}, \hat{T}^{HOFG}} \mathbb{L}^Q(\{\hat{Q}_t\}_{t \geq 0}) \equiv \mathbb{E}_0 \int_0^\infty e^{-\rho t} (\hat{Q}_t)^2 dt,$$

$$\text{s.t. } \begin{cases} d\hat{Q}_t = -\underbrace{r_1^T (\sigma_1^{q,L})}_{<0} dt + \sigma_1^{q,L} dZ_t, & \text{for } t < T, \\ d\hat{Q}_t = -\underbrace{r_2^T (\sigma_2^{q,L})}_{>0} dt + \sigma_2^{q,L} dZ_t, & \text{for } T \leq t < \hat{T}^{HOFG}, \\ d\hat{Q}_t = 0, & \text{for } t \geq \hat{T}^{HOFG}, \end{cases}$$

with

$$\hat{Q}_0 = \underbrace{r_1^T (\sigma_1^{q,L})}_{<0} T + \underbrace{r_2^T (\sigma_2^{q,L})}_{>0} (\hat{T}^{HOFG} - T)$$

Central bank picks \hat{T}^{HOFG} and $\{\sigma_t^q\}$



Proposition (Optimal commitment path)

At optimum, $\sigma_1^{q,L} < 0 = \sigma_1^{q,n}$, $\sigma_2^{q,L} < 0 = \sigma_2^{q,n}$, and $\hat{T}^{HOFG} < \hat{T}^{TFG}$

Proposition (Optimal forward guidance policy)

Optimal higher-order forward guidance always results in an equal or lower expected quadratic loss than the traditional guidance policy

Proof

With $(\sigma_1^{q,L}, \sigma_2^{q,L}, \hat{T}^{\text{HOFG}}) = (0, 0, \hat{T}^{\text{TFG}})$, two solutions coincide

Remarks:

- Alternative higher-order forward guidance policy implementations **are** possible
- This paper shows **HOFG** dominates **TFG** in a simple setting

Extension: higher-order forward guidance policy, but instead of no stabilization, now with stochastic stabilization after \hat{T}^{HOFG} : return to stabilization with νdt probability after \hat{T}^{HOFG}

- Central bank commits to stabilizing the economy with some probability. Expected stabilization after $1/\nu$ quarters
- $\nu = 0$: the above higher-order forward guidance
- $\nu = \infty$: the traditional forward guidance policy

Big discontinuity:

$$\lim_{\nu \rightarrow +\infty^-} \mathbb{L}^{Q,*}(\{\hat{Q}_t\}_{t \geq 0}, \nu) < \underbrace{\mathbb{L}^{Q,*}(\{\hat{Q}_t\}_{t \geq 0}, \nu = \infty)}_{\text{Traditional forward guidance}}$$

- Slight probability that stabilization might not happen \rightarrow **HOFG** possible

$T = 20$ quarters ZLB spell

Loss function \mathbb{L} as the (conditional) quadratic output loss per quarter:

$$\mathbb{L}_{\text{Per-period}}^Y \equiv \rho \int_0^\infty e^{-\rho t} \mathbb{E}_0 \left(\hat{Y}_t^2 \right) \approx \zeta^2 \cdot \rho \int_0^\infty e^{-\rho t} \frac{1}{s} \sum_{i=1}^s \left(\hat{Q}_t^{(i)} \right)^2 dt$$

Policy	No guidance	Traditional	Higher-Order (no stochastic stabilization)	Higher-Order (with stoch. stab., $\nu = 1$)
$\sigma_1^{q,L}$	0	0	-1.27%	-4.13%
$\sigma_2^{q,L}$	0	0	-0.24%	-3.79%
\hat{T}	20	25.27	25.09	24.68
$\mathbb{L}_{\text{Per-period}}^Y$	7%	1.93%	1.81%	1.69%

- Still, traditional forward guidance too strong: e.g., [McKay et al. \(2016\)](#)
- **HOFG** with $\nu \rightarrow \infty$ but $\nu \neq \infty$ most effective

Other policy interventions at the ZLB:

- **Remember:** ZLB as the result of capitalist's unwillingness to bear additional risk when the fundamental volatility is high: $\bar{\sigma} > \underline{\sigma}$
- Two possible fiscal interventions:
 - Subsidy on stock returns (equivalently, tax break on capital income)
 - Fiscal redistribution across agent types with different MPCs
- Effectiveness of policies depends on tax burden distribution

Subsidy $\tau \geq 0$ on (expected) stock returns, i_t^m

- **Implementation 1:** financed via lump-sum taxation L_t on capitalists

$$da_t = (a_t (i_t + \theta_t((1 + \tau)i_t^m - i_t)) - \bar{p}C_t - L_t) dt + \theta_t a_t (\bar{\sigma} + \sigma_t^q) dZ_t$$

Solution:

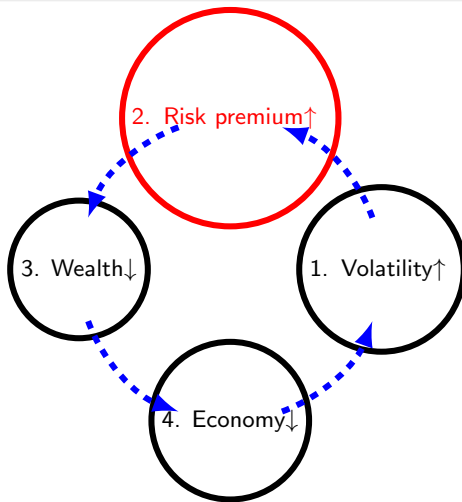
$$i_t^m \downarrow = \frac{i_t + (\bar{\sigma} + \sigma_t^q)^2}{1 + \tau \uparrow}$$

leading to a higher asset price during the ZLB

- **Implementation 2:** financed via lump-sum taxation L_t on hand-to-mouth workers: nullifying the above result

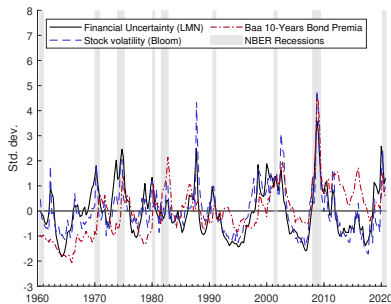
1. Higher-order forward guidance via intertemporal uncertainty management
 - Through central bank's equilibrium selection
2. Traditional forward guidance is welfare-enhancing, but can do better
3. Trade-off between current and future financial stability
 - Credibly “irresponsible” behaviour in the future
4. Central bank credibility still a necessary condition

Thank you very much!
(Appendix)

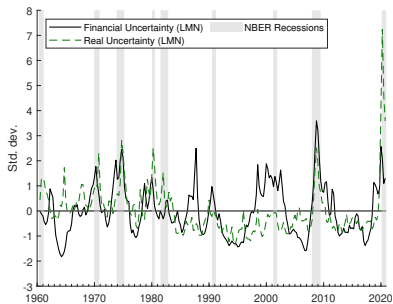


- 1 \rightarrow 2 comes from “non-linearity (not linearizing)”
- 2 \rightarrow 3 comes from “portfolio decision” of each investor and externality
- 3 \rightarrow 4 comes from the fact wealth drives aggregate demand
- 4 \rightarrow 1 where business cycle has its own volatility (self-sustaining)

Financial volatility measures



(a) Financial Uncertainty series



(b) Financial vs. Real Uncertainty

Figure: Common measures of the financial volatility (left) and real vs. financial uncertainty decomposed by [Ludvigson et al. \(2015\)](#) (right)

The correlation between series is remarkably high and they all display positive spikes at the beginning and/or initial months following NBER-dated recessions

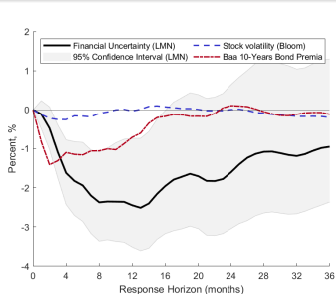
- Many of past recessions are, in nature, financial

In a similar manner to Bloom (2009), Ludvigson et al. (2015):

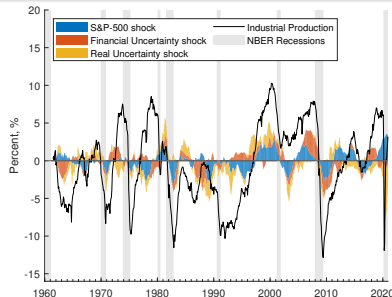
$$\text{VAR-11 order:} \quad \begin{bmatrix} \log(\text{Industrial Production}) \\ \log(\text{Employment}) \\ \log(\text{Real Consumption}) \\ \log(\text{CPI}) \\ \log(\text{Wages}) \\ \text{Hours} \\ \text{Real Uncertainty (LMN)} \\ \text{Fed Funds Rate} \\ \log(\text{M2}) \\ \log(\text{S\&P-500 Index}) \\ \text{Financial Uncertainty (LMN)} \end{bmatrix}$$

Financial uncertainty (LMN) is also replaced by the stock price volatility (following Bloom (2009)) and Baa 10-years bond premia

Vector Autoregression (VAR) analysis



(a) Response: Industrial Production

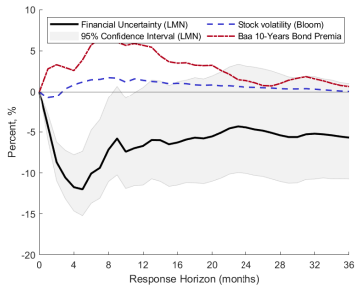


(b) Industrial Production

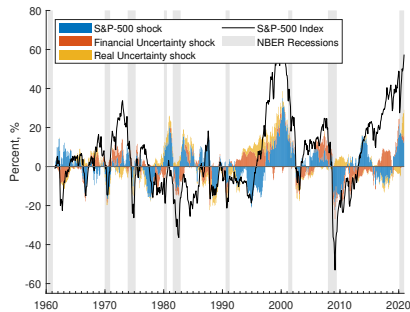
Figure: Impulse-response of IP to one std.dev shock in financial uncertainty measures (left) and the historical decomposition of IP to various attributes (right)

- 1 IP falls by 2.5% after one standard deviation spike in the **Ludvigson et al. (2015)**'s financial uncertainty measure
 - Financial uncertainty has been important in driving IP boom-bust patterns
- 2 Other graphs: IRF and historical decomposition of S&P 500 [▶ S&P500](#), and FFR (monetary policy) [▶ FFR](#), FEVD [▶ FEVD](#)

IRF and historical decomposition of S&P500 index



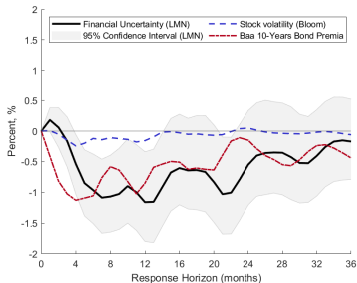
(a) Response: S&P-500 Index



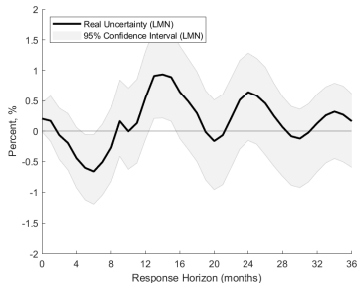
(b) S&P-500 Index

» Go back

IRF of FFR in response to financial and real uncertainty shocks



(a) Shock: Financial Uncertainty



(b) Shock: Real Uncertainty

With 3 different financial uncertainty measures: Ludvigson et al. (2015), Bloom (2009), Baa 10-years bond premia (left)

» Go back

Forecast Error Variance Decomposition (FEVD) of IP, S&P500, FFR

(i) Industrial Production

Horizon	Fin. Uncert. (LMN)	Real Uncert. (LMN)	Stock Vol. (Bloom)	Baa 10-Yr Premia
h=1	0	0.30	0.21	0.12
h=6	1.27	3.37	2.98	1.36
h=12	4.28	4.38	3.16	1.94
h=36	3.24	1.67	1.98	0.64

(ii) S&P-500 Index

Horizon	Fin. Uncert. (LMN)	Real Uncert. (LMN)	Stock Vol. (Bloom)	Baa 10-Yr Premia
h=1	0.11	0.08	0.39	0.06
h=6	3.30	0.25	3.26	0.62
h=12	4.77	0.54	10.03	2.16
h=36	6.50	0.91	12.16	2.40

(iii) Fed Funds Rate

Horizon	Fin. Uncert. (LMN)	Real Uncert. (LMN)	Stock Vol. (Bloom)	Baa 10-Yr Premia
h=1	0.01	0.98	0	0.08
h=6	0.42	0.84	3.11	1.66
h=12	1.47	0.91	4.69	2.30
h=36	2.81	2.05	5.02	3.17

Financial uncertainty shocks explain close to:

- 5% of the fluctuations in both IP and S&P-500 series

Real uncertainty explains:

- Additional 2-4% of movements in industrial activity in the medium run