Managerial Incentives, Financial Innovation, and Risk-Management Policies*

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Abstract

This paper examines risk management issues from the moral-hazard perspective of a firm run by an effort and risk-averse manager. When shareholders observe manager's risk choice, but not the effort, the optimal compensation contract directs managers to expose the firm to less risk than they would in the full information environment (e.g., execute costly hedges), as less risk makes the firm's cash flow a sharper signal about the hidden effort. Innovations in the risk management technology, e.g., the introduction of a new derivative market, always improves the efficiency of the manager's compensation contract when the risk choice can be observed, and this efficiency gain continues to hold under some circumstances when the manager's risk choice cannot be observed by shareholders. In other cases, however, due to the incentive problems associated with the hedging choice, financial innovation can lower welfare.

Keywords: Agency, Risk management, Hedging

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1 Introduction

U.S. corporations spend substantial resources on assessing and managing their exposures to various sources of risk not only for their own survival but also for the stability of the entire financial system. The global financial crisis (GFC) and the subsequent Great Recession taught us an important lesson that each corporation's imprudent risk choices lead to its own collapse, while imposing tremendous negative externalities on the aggregate economy. A new set of regulations for properly managing individual firms' risk exposures and minimizing the contagion of risks were introduced afterwords, including Basel I, II and III accords, and interests in benefits of risk management and necessary regulations soared up. Indeed, risk management's various implications have been one of the most important topics in academic economics and finance and documented in a number of academic and professional articles.[1]

However, in most cases until recently, those articles have ignored the fact that risk management choices are made not by the *value-maximizing firms (or shareholders)*, but by *self-interested managers*. In contrast, popular press and policy institutions have been quite concerned with issues relating to the incentives of managers to 'properly' manage companies' risk, focusing on a number of highly visible cases where it appears, at least ex post, that managers were using derivatives to speculate rather than hedge.[2] Ben Bernanke, a former chair of the Federal Reserve, for instance stated following the global financial crisis (GFC) that "compensation practices at some banking organizations have led to misaligned incentives and excessive risk-taking, contributing to bank losses and financial instability.".[3] This feature extends even beyond financial institutions. In a study of gold mining industry, for instance, Tufano (1996) found that the managerial incentives were the most important determinant of corporate derivatives choices.

This paper examines firms' risk management choices within an agency framework, where the manager (equivalently, agent) controls not only the effort he expends, which affects the firm's average cash flow, but also its riskiness by choosing real investment projects as well as positions in financial derivatives.[4] We assume

[1]For classical works, see Smith and Stulz (1985), DeMarzo and Duffie (1991), DeMarzo and Duffie (1995), and Froot et al. (1993) among others. Geczy et al. (1997) examined implications of these theories empirically. Draghi et al. (2003), in particular, emphasized the role of unintended or unanticipated accumulation of large risks as pre-conditions for financial crises. One point of theirs is that implicit guarantees that governments extend to banks and other financial institutions might lead to the over-accumulation of risks in the system, which aligns with our moral hazard point of view. In contexts of banks and the government intervention, Farhi and Tirole (2012) studied banks' collective moral hazard issues in the environment where government's bail-out policies are possible and anticipated. The more recent collapses of small and regional US banks (e.g., Silicon Valley Bank (SVB)) sparked many debates about proper roles of governments in both ex-ante and ex-post regulation of risks.

[2]For example, in 1994 and 1995, there were two firms, *Barings* and *Metallgesellschaft*, that either went bankrupt or nearly went bankrupt as a result of their speculative transactions in financial derivatives. And the global financial crisis (GFC) in 2008 illustrated how big corporations (e.g., AIG, Lehman Brothers, etc) had been speculating imprudently using derivatives while being ignorant of their own risk exposures.

[3]Fed press release (2009): https://www.federalreserve.gov/newsevents/pressreleases/bcreg20091022a.htm.

[4]Most papers about the principal-agent problem are concerned with the agent's effort choices but not with activities that affect the project's risk exposures, e.g., Harris and Raviv (1979), Holmström (1979), Shavell (1979), and Grossman and Hart (1983). Other

that shareholders (equivalently, principal) are unable to directly assess the extent to which the managers are acting in their own interests rather than the shareholders' (e.g., managers may be effort averse or equivalently consume unproductive perquisites).

Our analysis indicates that financial innovations (e.g., the introduction of a new derivative contract) that provide managers with greater flexibility to control risks can mitigate some agency problems and thereby improve firm values. However, this added flexibility might lead to other agency problems if the manager's use of derivatives cannot be effectively controlled (e.g., the manager speculates infinitely in derivative markets). When this is the case, financial innovation can reduce welfare, and it is better to ban the manager from transacting financial derivatives.

To understand how managers' access to derivative markets affects costs arising from the agency relation, we divide firm's risks into two different components: *hedgeable risks* and *non-hedgeable risks*. Basically, a company's hedgeable risks are related to some market observables such as interest rates, commodity prices, and exchanges rates on which derivative contracts can be written, whereas non-hedgeable risks are in nature individual risks that cannot be traded in the derivatives market. The total amount of a firm's hedgeable risk depends not only on those market observables but also on the firm's endogenous exposure to those observables (e.g., how cash flows are influenced by changes in oil prices). The firm's hedgeable risk exposure can be amended by its manager's derivative transactions, while its non-hedgeable risks are mainly determined by the manager's real project choices. An important source of asymmetric information in our framework is the firm's hedgeable risk exposure, as we assume that it is observed only by the manager.

In the benchmark case where no derivative transaction is allowed, the manager manipulates the firm's risks only through his project choices. In a more simplistic setting where the firm's risk exposure to hedgeable risks is observed by both shareholders and the manager, the typical dual-agency framework à la Hirshleifer and Suh (1992) applies. Specifically, the manager generally would not choose a real investment project (i.e., non-hedgeable risk) that would be preferred by shareholders. Therefore, the manager's optimal compensation contract must be designed not only to incentivize him to take the right effort level but also to induce him to choose the right project risk level from the principal's perspective. We show that the optimal compensation contract either rewards or penalizes the output variances depending on whether the manager must be rewarded or deterred from taking riskier projects.

In reality, communication between shareholders and the manager is costly when the firm's risk exposure

papers consider only the agent's risky project choice problem, e.g., Ross (1974), Lambert (1986), and Hirshleifer and Thakor (1992). Important exceptions are Hirshleifer and Suh (1992), Sung (1995), Palomino and Prat (2003), Ross (2004), DeMarzo et al. (2013), Hébert (2018) among others, which simultaneously examine the agent's risk and effort choices in either a single-period framework or a continuous time agency framework.

is observed only by the manager.[5] What the uninformed shareholders can do at best in this case is to design a compensation contract without relying on any information on the firm's risk exposure, directly or indirectly, and let the informed manager decide both effort and project choice based on the observed risk exposure of the firm. Due to this asymmetric information about the firm's risk exposure, there would inevitably be some welfare loss compared with the more simplistic case where the firm's risk exposure is observed by both parties.

When derivative transactions are allowed and the manager can manipulate the firm's level of hedgeable risks, welfare is improved when the optimal contract in the absence of derivative transaction (i.e., the first benchmark case) *discourages* risk-taking. Intuitively, the manager in this case will voluntarily choose to hedge completely, given the contract that would be optimal in the absence of the hedging opportunity. In this case, there is no welfare cost that arises from the hedgeable risk exposure being unobservable to shareholders, as the manager eliminates it through hedging activities anyway. Here, derivative markets allow both parties to eliminate asymmetric information and thus coordinate on better risk-sharing.

However, when the optimal contract in the absence of derivative market (i.e., the first benchmark case) *rewards* the manager's risk-taking, there are additional agency costs associated with the manager's potential derivative transactions, as the manager would speculate infinitely in derivative markets, given the contract. In this case, the incentive contract needs to be altered to induce the manager to use financial derivatives to *hedge*, not *speculate*. As we show, to incentivize the manager to hedge (i.e., reduce the firm's risk-exposure to hedgeable risks), the compensation contract must penalize a (both positive and negative) sample covariance (i.e., realized covariance) between the market observable and the output (e.g., covariance between oil prices and profits of the firm)[6]. However, revising the compensation contract in this way can be costly because it imposes additional risk on the risk-averse manager. Indeed, in some cases, the costs associated with inducing optimal hedging exceeds the informational benefit of hedging, which implies that the shareholders should *optimally* restrict the agent's access to the derivative market.

Since the motivation for hedging suggested by our model (i.e., informational benefit of hedging) requires that the firm's risk exposure to be unobservable by the principal, it makes sense to ask if the agent can be induced to truthfully reveal this information. As we show, when costless communication between principal and agent is possible, a contract that induces truthful revelation can be designed, without a derivative market, that is at least as good as the contract inducing hedging in the derivatives market. This result indicates that

[5]For example, this point about potential communication was discussed by Laffont and Martimort (1997).

[6]Under the contract that penalizes a sample covariance between the hedgeable risks and output, the manager would be tempted to engage in hedging transactions and reduce the firm's exposure to market observables. Under complete hedging, the firm no longer has an exposure to the hedgeable risks, and the population covariance between output and hedgeable risks becomes 0.

if the agency relation is interpreted as between a firm's central headquarters (principal) and a division head (agent), the optimal incentive contract will not necessarily require the division head to hedge through derivative transaction, since it may be possible for shareholders to elicit the firm's observed risk-exposure from the manager and do just as well with a compensation contract that is contingent on that communicated information. However, if we view the relation between the CEO of a firm (the agent) and a diffuse group of shareholders (the principal), then the assumption of free communication is probably untenable and the optimal contract will probably allow the CEO to engage in derivative transactions.

There has been a growing literature that tackle joint issues of principal-agent problems and risk choices. On theory side, for example, Hirshleifer and Suh (1992), Sung (1995), Palomino and Prat (2003), DeMarzo et al. (2013) among others analyzed cases where the agent chooses the parametric risk of the output distribution. Hirshleifer and Suh (1992) especially analyzed the model in which a risk-averse agent chooses not only his effort level but also the firm's real project which is relevant for the risk level. They showed that principal and agent will not generally agree on the firm's risk level, and agent's compensation contract should be revised in a way that it is more concave (convex) compared with the contract that would optimally be designed without the incentive problem associated with his risk choice, when the principal prefers a lower (higher) risk level than the agent. However, they did not derive the optimal compensation contract in a general framework as we do, and furthermore, did not analyze the effects of a derivative market. DeMarzo et al. (2013) concluded that the principal has to pay a large bonus if a firm survives, in order to disincentivize the agent from putting the firm at 'disaster' risk. It's in line with our Proposition 10 that the principal punishes covariance between the output and the hedgeable risk to induce the agent to fully hedge through derivative transactions.

Among papers that deal with nonparametric risk choices: e.g., Makarov and Plantin (2015), Hébert (2018), Barron et al. (2020), Barron et al. (2020) considered the case where a 'mean-preserving spread (MPS)' risk can be added by the agent without cost and figured out that the agent's indirect utility must be concave at optimum to discourage his additional risk-taking. Our paper contributes to the literature by analyzing the effect of a derivative market on the efficiency of agency relations in lights with how shareholders should alter the manager's compensation contract to induce hedging. The empirical literature, e.g., Guay (1999), Rajgopal and Shevlin (2002), Coles et al. (2006) among others, found that the higher a vega (i.e., sensitivity of CEO's wealth to the stock market volatility) is, the riskier stock returns and firms' project choices become, with more focused investments and higher leverage ratios. It confirms our view that the convexity of manager's compensation contract is a major driving force behind his choice over risk.[7]

[7] Another closely related paper to ours is DeMarzo and Duffie (1995), which studies the ways that risk management reduces uncertainty about the manager's unobservable actions. However, in contrast to our analysis, in DeMarzo and Duffie (1995), differences in actions are due to differences in ability rather than differences in effort.

The paper is organized in the following way. In Section 2, we provide a simple benchmark model without the manager's real investment (i.e., project) choice to illustrate our main mechanisms more clearly: how the derivative transaction possibility affects the efficiency of the agency relation. In Section 3, we formulate our model specification. In Section 4, as a benchmark case, we consider the case in which there is no derivative market. Thus, the manager's risk choices are carried out only through his real investment choices. In Section 5, we consider the case in which there is a derivative market. As whether the manager can carry out the real investment (i.e., project) or not affects how the introduction of derivative market affects the agency efficiency, we study this issue in depth by comparing Section 2 and 5. In Section 6, we consider the case in which there is free communication between the principal and the agent. Concluding remarks are provided in Section 7, and the proofs of the Lemmas and Propositions as well as omitted derivations are all given in the Appendix.

2 The Simpler Model: 2-actions

In Section 2, we provide a simpler framework with 2 actions that illustrates the paper's main mechanisms. From Section 3, we provide a full-fledged version of the model with 3 different actions, and compare different predictions coming out of Sections 2 and 3.

We consider a two-persons single-period agency framework in which a risk averse agent works for a risk neutral principal. The principal can be thought of as firm's shareholders, and the agent can be regarded as firm's top manager or CEO. Alternatively, we can think of the principal as CEO and the agent as a head of one of firm's subordinate divisions. Hereafter, we use the terms 'agent' and 'manager' interchangeably.

After his wage contract is finalized, agent takes two kinds of actions, $a_1 \in [0, \infty)$ and $a_d \in (-\infty, +\infty)$. Agent's first action a_1 is a productive effort which increases an expected output, that is, a high effort generates an output level that first-order stochastically dominates the output level generated by a low effort. The agent's action a_d is his derivative choice.

After the agent chooses a_1 and a_d, the firm's output, x, is realized and publicly observable without cost. Thus, an output x can be used in the manager's wage contract that is denoted by w. The output is determined not only by the agent's choice of (a_1, a_d) but also by the state of nature, (η, θ). For simplicity, we assume that the output function exhibits the following additively separable form:

$$x = \phi(a_1) + \sigma\theta + (R - a_d)\eta. \tag{1}$$

An expected output, $\phi(a_1)$, is a function of a_1, whereas the agent's derivatives choice, a_d, does not directly affect it. The firm's risk can be decomposed into two components, η and θ, where $\eta \sim N(0,1)$ represents one unit of the firm's hedgeable risks, and $\theta \sim N(0,1)$ represents one unit of the firm's non-hedgeable risks. For simplicity, we assume that η and θ are uncorrelated. As denoted by (1), the firm's total non-hedgeable risk is fixed at σ, whereas the firm's hedgeable risks are determined by market variables such as commodity prices, interest rates, and exchange rates which become publicly observable after the agent chooses a_1 and a_d.[8] Accordingly, we assume η is observable at the end of the period, and thus can also be used in the manager's wage contract if necessary. In the above equation, $R \sim N(R_m, \sigma_R^2)$ denotes the firm's innate exposure to the hedgeable risks (e.g., the amount of oil underground for a drilling company). We assume that the manager can observe the true value of R after the contract is signed but before he chooses a_1 and a_d. In contrast, the principal knows only its distribution. We assume that the management effort a_1 does not affect R, the firm's innate exposure to the hedgeable risks. However, the firm's final risk exposure can be manipulated by the manager's transaction a_d in the derivative market. If $a_d = 0$, the manager does not trade derivatives. The manager hedges, i.e., reduces risk, as long as $|R - a_d| < |R|$ and minimizes risk by setting $a_d = R$. On the other hand, if $|R - a_d| > |R|$, the manager speculates in the derivative market.

In addition, we make the following assumptions:

Assumption 1 The agent's preferences on wealth and productive effort are additively separable :

$$U(w, a_1, a_d) = u(w) - v(a_1), \quad u' > 0, u'' < 0,$$

where v, the agent's disutility of exerting productive effort, has the properties $v' > 0, v'' > 0, \ \forall a_1$.

Assumption 2 $\frac{\partial \phi}{\partial a_1}(a_1) \equiv \phi_1(a_1) > 0, \ \frac{\partial^2 \phi}{\partial a_1^2}(a_1) \equiv \phi_{11}(a_1) < 0.$

Assumption 3 If the agent's different action choices (i.e., different combinations of (a_1, a_d)) yield the same efficiency, then the principal always prefers the minimum risk case (i.e., (a_1, a_d) that yields the lowest value of $|R - a_d|$).

Assumption 1 implies that the agent is risk-averse and effort-averse, and the agent's derivatives choices have no direct effect on his utility.[9] Assumption 2 indicate that a_1 affects the expected output with a usual property of decreasing marginal increase in output. Finally, Assumption 3 is our tie-breaking rule when two different combinations of actions result in the same welfare measure.

[8] In fact, if the relevant derivative market observable is denoted as p, then $\eta = p - \overline{p}$ where $\overline{p}$ is the expected value of p.

[9] For the derivative choice a_d, we assume that direct hedging costs (e.g., option premia) are negligible compared with the nominal amounts of firms' cash flows. Therefore, we assume away the cost for the derivative choice a_d.

2.1 Benchmark Case: $\eta \equiv 0$

Throughout Section 2.1, we consider a simpler case where there is no hedgeable risk (i.e., $\eta \equiv 0$),[10] therefore the output x can be written as:

$$x = \phi(a_1) + \sigma\theta. \tag{2}$$

Note that the firm's risk-exposure R does not affect x with $\eta \equiv 0$, therefore we do not have any asymmetric information between shareholders and manager in this case. The optimal wage contract $w(\cdot)$, in this case, is found by solving for the contract which maximizes the combined utilities of the principal and agent subject to the restriction that the agent's effort a_1 is chosen to maximize his utility given the contract.[11]

$$\begin{aligned} &\max_{a_1, w(\cdot)} \phi(a_1) - \int w(x) f(y|a_1) dy + \lambda \left(\int u(w(x)) f(x|a_1) dy - v(a_1) \right) \quad \text{s.t.} \\ &\qquad (i) \;\; a_1 \in \arg\max_{a_1'} \int u(w(x)) f(x|a_1') dy - v(a_1'), \;\; \forall a_1', \\ &\qquad (ii) \;\; w(y) \geq k, \;\; \forall y, \end{aligned} \tag{3}$$

where $f(x|a_1)$ denotes a probability density function of $x \sim N\left(\phi(a_1), \sigma^2\right)$ given the agent's action a_1, and λ denotes the weight placed on the agent's utility in the joint optimization. As shown, the combined utilities of the principal and the agent are maximized subject to the agent's incentive compatibility constraint which specifies that the agent chooses his effort for his own optimization, and his limited liability constraint which specifies that the agent receives at least k, the subsistence level of utility.[12]

Following the literature (see e.g., Kim (1995)), in finding the optimal solution $(a_1^*, w^*(x|a_1^*))$ for the above program (3), we first derive an optimal contract for an arbitrarily given action a_1. Let $w^*(x)$ be a contract which optimally motivates the agent to take a particular level of a_1. Subject to some technical assumptions,[13],

[10] Equivalently, we can assume that there is no information asymmetri in R between shareholders and manager, and there is no active derivative market (i.e., $a_3 = 0$). As η is contractible, in this case shareholders can write the wage contracts in $y = x - R\eta$, as we do in Section 4.

[11] This yields a mathematically equivalent solution to a model where the principal maximizes his utility subject to an optimizing agent receiving his reservation utility level as in Holmström (1979). Our purpose here is to analyze the overall efficiency implication of financial market innovations and thus we choose to fix λ, which usually is an endogenous Lagrange multiplier in the latter case.

[12] The limited liability constraint is introduced to guarantee the existence of optimal solution for $w(x)$. This is needed because we assume that the signal is normally distributed. For details about this 'unpleasantness', see Mirrlees (1974) and Jewitt et al. (2008).

[13] We assume that the first-order approach is valid. Grossman and Hart (1983) and Rogerson (1985) show that MLRP and CDFC are sufficient for the validity of the first-order approach when the signal space is of one dimension. Jewitt (1988) finds less restrictive conditions for the validity of the first-order approach, based on the agent's risk preferences as well as the distribution function of the signal. Sinclair-Desgagné (1994) shows that more general versions of MLRP and CDFC in a multi-dimensional space are sufficient for the validity of the first-order approach when the signal space is of multiple dimensions. For more recent treatments along this line, see Conlon (2009) and Jung and Kim (2015) among others. Recently, Jung et al. (2022) justifies the first-order approach when the technology follows normal distributions, which corresponds to our problem in (3).

by solving the Euler equation of the above program after fixing a_1, we derive that $w^*(x|a_1)$ must satisfy

$$\frac{1}{u'(w^*(x|a_1))} = \lambda + \mu_1(a_1)\frac{f_1}{f}(x|a_1), \tag{4}$$

for almost every x for which (3) has a solution $w^*(x|a_1) \geq k$, and otherwise $w^*(x|a_1) = k$. In equation (4), $\mu_1(a_1)$ denotes the optimized Lagrange multiplier for the agent's incentive compatibility constraint associated with a_1. Since $x \sim N\left(\phi(a_1), \sigma^2\right)$, (4) is reduced to:

$$\frac{1}{u'(w^*(x|a_1))} = \lambda + \mu_1(a_1)\frac{x-\phi(a_1)}{\sigma^2}\phi_1(a_1). \tag{5}$$

Before analyzing the optimal contract $w^*(\cdot)$, we first define the social welfare SW^* as a function of a_1 in the following way:

$$SW^*(a_1) \equiv \phi(a_1) - C^*(a_1) - \lambda v(a_1), \tag{6}$$

which denotes the joint benefits when $w^*(y|a_1)$ is designed where

$$C^*(a_1) \equiv \int \left(w^*(x|a_1) - \lambda u(w^*(x|a_1))\right) f(x|a_1)dy \tag{7}$$

represents the efficiency loss of this case compared with the full information case, given the contract $w^*(\cdot|a_1)$. In other words, $C^*(a_1)$ measures the agency cost arising from inducing the agent to take particular action a_1. Finally, the optimal action a_1^* can be found by

$$a_1^* \in \arg\max_{a_1'} SW^*(a_1), \tag{8}$$

and we simplify notations, thus $w^*(x) \equiv w^*(x|a_1^*)$. The optimal surplus in this case will be given by $SW^* \equiv SW^*(a_1^*)$.

Given the optimal contract $w^*(\cdot)$, the agent's indirect utility $V(\cdot)$ is defined as $V(x) \equiv u(w^*(x))$. We know from Rothschild and Stiglitz (1970) that if $V(\cdot)$ is convex (concave), then the agent wants to raise (reduce) a level of overall risk to the output x if possible. It turns out that this property governs how the agent chooses his derivative choices when the derivative market opens up in the next Section 2.1. In general, the curvature of the agent's indirect utility function depends on the distribution of the random state variable and utility function itself. To see how different utility functions affect this curvature differently, consider the case where the agent has constant relative risk aversion with degree $1-t$, where $t < 1$ ($u(w) = \frac{1}{t}w^t, t < 1$). We obtain from equation (5) that

$$w^*(x) = \left(\lambda + \mu_1(a_1^*)\left(\frac{x-\phi(a_1^*)}{\sigma^2}\right)\phi_1(a_1^*)\right)^{\frac{1}{1-t}}, \tag{9}$$

and the agent's indirect utility under this wage contract is

$$V(x) \equiv u(w^*(x)) = \frac{1}{t}\left(\lambda + \mu_1(a_1^*)\left(\frac{x-\phi(a_1^*)}{\sigma^2}\right)\phi_1(a_1^*)\right)^{\frac{t}{1-t}}. \tag{10}$$

The above equation shows that the agent's indirect utility $V(\cdot)$ becomes strictly concave in x[14] if $t < \frac{1}{2}$, linear if $t = \frac{1}{2}$, and convex if $t > \frac{1}{2}$ for x satisfying $w^*(x) \geq k$. If we assume $w^*(x) = k$ for sufficiently low x, as far as the agent's induced risk preferences are concerned, the agent acts as if he is risk-loving if and only if $t \geq \frac{1}{2}$.

2.2 With derivative markets: R is observable to the agent only and $\eta \sim N(0,1)$

Now, we go back to the model's original specification in Section 2: the hedgeable risk $\eta \sim N(0,1)$ (which is contractible) is there, and the firm's innate risk-exposure R can be observed only by the agent. And now, the agent has an access to the derivative market, which allows him to manipulate a level of a_d through derivative transactions. One can expect that as there is an asymmetry of information between two parties (i.e., principal and agent) about the value of risk-exposure R, the efficiency of the agency relationship would be generically hurt, even if η is contractible. In this Section 2.2, we study this issue in depth.

We divide all possible contingencies into two different cases, depending on whether the agent's indirect utility function $V(x)$ in the absence of η shock (i.e., hedgeable risk) is convex or concave in the output x.

2.2.1 When the agent's indirect utility $V(x)$ is concave in output x

Note that $V(x)$ is the agent's indirect utility function in the absence of the hedgeable risk η. Let us think of a situation where shareholders offer the same contract $w^*(\cdot)$ to the manager, which was the optimal contract in the benchmark scenario of Section 2.1. Since the manager is risk-averse in this case, he wants to minimize a level of risk contained in the output x. As he observes R , the firm's innate risk-exposure, then his optimal strategy is to eliminate this risk-exposure R entirely, by choosing $a_d = R$.[15] Then no efficiency loss out of the information asymmetry between shareholders and manager is realized, as the risk-exposure is voluntarily eliminated by the manager, and thus no longer affects the output (i.e., signal) x. Also, the optimal contract

[14] We use the well-known property that the optimized Lagrange multiplier $\mu_1(a_1^*)$ is positive. For the proof of $\mu_1(a_1^*) > 0$, see Holmström (1979), Jewitt (1988), and Jung and Kim (2015) among others.

[15] Since $x = \phi(a_1) + \sigma\theta + (R - a_d)\eta$, the agent can minimize the risk by choosing $a_d = R$.

as well as the optimal action stays the same at $w^*(\cdot)$ and a_1^* in (5) and (8).

Therefore, the introduction of derivative markets allow the agent to eliminate the asymmetric information. The social welfare in this case would be the same as in equation (6) of Section 2.1, and there is no role of R in determining optimal contracts and actions of the agent. It is possible since the agent given the contract $w^*(\cdot)$ in equation (5) acts as if he is risk-averse, and eliminates the risk-exposure R he observes through derivative transactions.

This result is summarized by the following Proposition 1.

Proposition 1 *When the agent's indirect utility $V(x)$ in the absence of the hedgeable risk η becomes concave in output x, then the agent always chooses $a_d = R$ (i.e., complete hedging) in derivative markets, eliminating the welfare loss out of the asymmetric information in the risk-exposure R. The surplus remains at SW^*.*

For cases of constant relative risk aversion with degree $1-t$ (i.e., $u(w) = \frac{1}{t}w^t$, $t < 1$), for example, the manager whose preference shows a higher risk aversion than $t = \frac{1}{2}$ case (i.e., $t < \frac{1}{2}$) features a concave $V(\cdot)$, thereby belonging to the case of the above Proposition 1.

2.2.2 When the agent's indirect utility $V(x)$ is convex in output x

Let us imagine the same situation where shareholders offer the same contract $w^*(\cdot)$ in (9) to the manager. Now the manager with convex $V(x)$ acts as if he is risk-loving, thereby would choose to increase the risk in output x as much as possible, choosing $a_d = \infty$, i.e., infinite speculation, if he can transact derivatives in derivative markets, summarized by the following Lemma 1.

Lemma 1 *When the agent's indirect utility $V(x)$ in the absence of the hedgeable risk η becomes convex in output x, then the agent always chooses $a_d = \infty$ (i.e., complete speculation) in derivative markets, given contract $w^*(\cdot)$.*

In this case, we ask what shareholders should do to deal with the manager's infinite risk-taking given the contract $w^*(\cdot)$ in (5). Maybe, shareholders can alter the contract in a way that it induces the agent to choose finite risk-taking. In this case, as we deviate from the efficient risk-sharing of Section 2.1, we might incur the welfare loss to some degree, even if shareholders can design a contract that induces the agent to engage in perfect hedging (i.e., $a_d = R$).

Or shareholders can shut down derivative market access, so the manager can choose $a_d \neq 0$. In this case, some welfare loss will be realized due to the fact that the firm's innate risk-exposure R is observed only by the manager.

Case 1: shutting down derivative markets Imagine, due to the manager's infinite risk-taking (i.e., $b = \pm\infty$) given $w^*(x)$ as we see in Lemma 1, shareholders decide not to allow the manager to transact in derivative markets, so $a_d = 0$, regardless of the level of R that the agent observes before taking a_1.

In this case, there is asymmetric information about R between shareholders and manager, affecting the efficiency of their agency relationship in a negative way. As shareholders do not observe R, the compensation contract must be based on (x, η), i.e., $w = w(x, \eta)$. The principal's maximization program is thus:[16]

$$SW^N \equiv \max_{a_1(\cdot), w(\cdot) \geq k} \int_R [\int_{x,\eta} (x - w(x,\eta) + \lambda u(w(x,\eta))\, g(x,\eta|a_1(R), R)h(R)dxd\eta - \lambda v(a_1(R))dR \quad \text{s.t.}$$
$$(i) \;\; a_1(R) \in \arg\max_{a_1} \int_{x,\eta} u(w(x,\eta))g(x,\eta|a_1,R)dxd\eta - v(a_1), \forall R, \tag{11}$$

where

$$g(x,\eta|a_1,a_2,R) = \frac{1}{2\pi a_2} \exp\left(-\frac{1}{2}\left(\frac{(x-\phi(a_1,a_2)-R\eta)^2}{a_2^2} + \eta^2\right)\right) \tag{12}$$

denotes a joint probability density function of (x, η) given (a_1, a_2, R) and $h(R)$ denotes the probability density function of R. SW^N is defined as the optimized surplus in this case.

For each R, let $\{a_1^N(R), w^N(x,\eta)\}$ be the solution for the above optimization program in (14). If we let $\mu_1(R)$ be the optimized Lagrange multiplier attached to the incentive constraint in $a_1(R)$, the optimal contract $w^N(x,\eta)$ can be written as[17]

$$\frac{1}{u'(w^N(x,\eta))} = \lambda + \int_R \mu_1(R) \left[\frac{g_1(x,\eta|a_1^N(R),R)}{\int_{R'} g(x,\eta|a_1^N(R'),R')h(R')dR'}\right] h(R)dR \tag{13}$$

when $w(x,\eta) \geq k$ and otherwise $w(x,\eta) = k$.

Note that SW^N in this case is lower than SW^* in Section 2.1, as now we have an asymmetry of information between principal and agent about R, and the principal cannot use R in designing contracts for the agent. This is summarized by the following Proposition 2.

Proposition 2 *When there is no derivative market and the communication between the principal and the agent is not possible, the principal's inability to observe the firm's risk exposure reduces welfare, i.e.,*

$$SW^N < SW^*.$$

Intuitively, when the principal observes the true value of the firm's risk exposure, R, this information can be used to

[16] In this case, since the agent is the only one seeing the realized value of R, his action a_1 would depend on observed R, given the contract $w(x,\eta)$.

[17] We provide the derivation for equation (16) in Appendix.

design a wage contract that eliminates the influence of the hedgeable risks, i.e., $w = w^*(y \equiv x - R\eta)$. However, if R is not observable and cannot be communicated, this is impossible.

Case 2: designing a new optimal contract in the presence of derivative markets As we know that $w^*(\cdot)$ in (5) induces infinite speculation, i.e., $a_d = \infty$, we need to design a new optimal contract that induces finite derivative transaction from the manager.

As the manager observes R before choosing a_1 and a_d, the agent's choice of a_d can be characterized as his choice of $b \equiv R - a_d$. Imagine the principal wants to induce the action a_1^o and $\hat{b} = R - a_d^o$ with some contract $w^o(x, \eta)$, i.e., given contract $w^o(x, \eta)$, the agent will choose a_1^o and a_d^o such that $b = R - a_d^o = \hat{b}$, satisfying

$$\hat{b} \in \arg\max_b \mathbb{E}\left[u\left(w^o\left(\underbrace{\phi\left(a_1^o\right) + \sigma\theta + b\eta}_{\equiv x}, \eta\right)\right)\right] - v(a_1^o), \tag{14}$$

which can be translated into

$$0 \in \arg\max_b \mathbb{E}\left[u\left(w^o\left(\underbrace{\phi\left(a_1^o\right) + \sigma\theta + \hat{b}\eta + b\eta}_{\equiv x + \hat{b}\eta}, \eta\right)\right)\right] - v(a_1^o), \tag{15}$$

which implies: if the principal can induce $b = \hat{b}$ with $w^o(x, \eta)$, then she can always induce $b = 0$ with contract $w^o(x + \hat{b}\eta, \eta)$.[18] And due to Assumption 3, we can focus on the case where shareholders try to induce $b = 0$ (i.e., perfect hedging).

Given that the agent's choosing a_d given his private information R is equivalent to his choosing $b = R - a_d$, a new optimal contract, $w^o(x, \eta)$, inducing the agent to take $(a_1^o, b = 0)$ must solve the following optimization problem:[19]

$$\begin{aligned} \max_{w(\cdot) \geq k} \int (x - w(x, \eta)) g(x, \eta | a_1^o, b = 0) dx d\eta + \lambda \left(\int u(w(x, \eta)) g(x, \eta | a_1^o, b = 0) dx d\eta - v(a_1^o) \right) \quad \text{s.t.} \\ (i) \quad \int u(w(x, \eta)) g_1(x, \eta | a_1^o, b = 0) dx d\eta - v'(a_1^o) = 0, \\ (ii) \quad b = 0 \in \arg\max_{b'} \int u(w(x, \eta)) g(x, \eta | a_1^o, b') dx d\eta, \ \forall b. \end{aligned} \tag{16}$$

Note from the above (16) that we take the *optimal* a_1^o as given, and rely on the first-order approach for the incentive constraint associated with the action a_1.[20] However, we do not use the same approach for the in-

[18] As the principal is risk-neutral and the optimal contract does not depend on the level $\hat{b}$ of $b \equiv R - a_d$ that the principal induces from the agent, the social welfare becomes unaffected by the level of $\hat{b}$.

[19] Here the distribution $g(x, \eta | a_1, b)$ is of the joint normal distribution of (x, η) implied by equation (1).

[20] $g_1(x, \eta | a_1, b)$ is defined as a partial derivative of $g(x, \eta | a_1, b)$ with respect to a_1. Likewise we define $g_b(x, \eta | a_1, b)$ as $g(x, \eta | a_1, b)$'s

centive compatibility constraint associated with the hedging choice b. The following Lemma 2 demonstrates the reason we cannot rely on the first-order approach for the incentive compatibility around b.

Lemma 2 *If $w^*(x)$ in equation (5) is designed, the agent will be indifferent between taking b and taking $-b$, $\forall b$.*

Lemma 2 shows that, if $w^*(x)$, the optimal contract in the absence of the hedgeable risk η, is designed and offered, the manager's expected utility becomes symmetric around $b = 0$ (i.e., $a_d = R$) in the space of b (i.e., in the space of a_d). As we know:

$$\int u(w^*(x))g(x, \eta|a_1^o, b)dzd\eta \tag{17}$$

is continuous and differentiable in b, Lemma 2 implies:

$$\int u(w^*(x))g_b(x, \eta|a_1^o, b = 0)dzd\eta = 0. \tag{18}$$

Since $(w^*(x), a_1^*)$ is the solution of the optimization in (3) without the incentive constraint of b,[21] if we use the first-order approach for the incentive constraint associated with b in the above program (16), we always end up with getting $w^*(x)$ in (5) as an optimal contract. However, we can easily see from Lemma 2 that this contract incentivizes an agent to take $b = \pm\infty$ instead of taking a stipulated $b = 0$. Therefore, we have to explicitly include the incentive constraint for b which does not rely only on the first-order condition at $b = 0$.

Without relying on the first-order approach, we follow Grossman and Hart (1983), replacing the incentive constraint for b (i.e., (ii) in (16)) with:

$$\int u(w(x, \eta)) \left(g(x, \eta|a_1^o, b = 0) - g(x, \eta|a_1^o, b)\right) dxd\eta \geq 0, \ \forall b, \tag{20}$$

which implies that the manager's indirect utility is maximized when he takes $b = 0$ (i.e., $a_d = R$).

Now we state formally the optimization problem of choosing the optimal contract $w^o(\cdot)$ given a_1^o as:

$$\begin{aligned} SW^o \equiv \max_{w(\cdot)\geq k} \int (x - w(x, \eta))g(x, \eta|a_1^o, b = 0)dxd\eta + \lambda \left(\int u(w(x, \eta))g(x, \eta|a_1^o, b = 0)dxd\eta - v(a_1^o) \right) \quad \text{s.t.} \\ (i) \quad \int u(w(x, \eta))g_1(x, \eta|a_1^o, b = 0)dxd\eta - v'(a_1^o) = 0, \\ (ii) \quad \int u(w(x, \eta)) \left(g(x, \eta|a_1^o, b = 0) - g(x, \eta|a_1^o, b)\right) dxd\eta \geq 0, \ \forall b, \end{aligned} \tag{21}$$

partial derivative with respect to b.

[21] Since for $b = 0$, the likelihood ratios can be represented as

$$\frac{g_1}{g}(x, \eta|a_1, b = 0) = \frac{x - \phi(a_1)}{\sigma^2}\phi_1(a_1), \ \ \frac{g_b}{g}(x, \eta|a_1, b = 0) = \frac{(x - \phi(a_1))\eta}{\sigma^2}, \tag{19}$$

we see that $w^*(x)$ becomes the solution of the equation (3) without the incentive constraint of b.

where we define SW^o as the optimized surplus in this case. The first-order condition of the above program (21) yields the optimal contract, $w^o(x,\eta)$, that satisfies

$$\frac{1}{u'(w^o(x,\eta))} = \lambda + \mu_1^o \frac{x-\phi(a_1^o)}{\sigma^2}\phi_1(a_1^o) + \underbrace{\int \mu_b^o(b)\left(1-\frac{g(x,\eta|a_1^o,b)}{g(x,\eta|a_1^o,b=0)}\right)db}_{\text{Additional term to (5)}}, \tag{22}$$

for (x,η) satisfying $w^o(x,\eta) \geq k$ and $w^o(x,\eta) = k$ otherwise. In (22), μ_1^o and $\mu_b^o(b)$ are the optimized Lagrange multipliers associated with the first constraint (i.e., (i)) and the second constraint for a particular b (i.e., (ii)) in the above optimization program (21), respectively.[22]

As shown in the Appendix, we obtain the following proposition from (22).

Proposition 3 *When the agent's indirect utility $V(x)$ in the absence of the hedgeable risk η becomes convex in output x, then the principal can induce the manager to hedge completely by designing a new contract, $w^o(x,\eta)$ in equation (22), which (i) satisfies $w^o(x,\eta) = w^o(x,-\eta)$ for all x,η, and (ii) penalizes the manager for having any (positive or negative) sample covariance between the output, x, and derivative market observables, η (i.e., penalizing manager for having high $(x-\phi(a_1^o))^2\eta^2$). To be specific, given the realized (x,η), a higher sample covariance $(x-\phi(a_1^o))^2\eta^2$ reduces $w^o(x,\eta)$, while given the output x and sample covariance $(x-\phi(a_1^o))^2\eta^2$, a higher η raises the wage $w^o(x,\eta)$.*

Proposition 3 can be understood as follows: output $x = \phi(a_1^o) + \sigma\theta + b\eta$ implies $b = Cov(x,\eta) \equiv \mathbb{E}((x - \phi(a_1^o))\eta)$. If the agent takes $b=0$, a statistical (i.e., population) covariance between output x and hedgeable risk η disappears, whereas any other $b \neq 0$ generates non-zero population covariance. Since $b=0$, which the manager chooses given $w^o(x,\eta)$, generates $x = \phi(a_1^o) + \sigma\theta$, which is independent of η, $w^o(x,\eta) = w^o(x,-\eta)$ for all x,η is ensured to minimize the amount of risk imposed on the agent, as η is irrelevant in inducing a_1^o now (as x does not depend on η) and has a symmetric distribution around 0.

At optimum, by punishing the covariance between x and η,[23] shareholders effectively induce the manager to engage in full hedging (i.e., $b=0$). As our model is in one-period, any positive or negative realized sample covariance $\widehat{Cov} = (x-\phi(a_1^o))\eta = b\eta^2 + \sigma\theta\eta$, instead of a population covariance, is punished by the principal through a lower compensation $w^o(x,\eta)$. If the realized sample covariance $\widehat{Cov} = (x-\phi(a_1^o))\eta = b\eta^2 + \sigma\theta\eta$ is large, not because of the manager's speculation (i.e., $b \neq 0$) but from a high realized market observable, $|\eta|$, then the principal takes it into account and raises $w^o(x,\eta)$. In contrast, given realized output and market observables (x,η), a bigger realization of $\widehat{Cov}$ is likely to be generated by $b \neq 0$ with a bigger $|b|$, thus the agent is punished and her wage income $w^o(x,\eta)$ falls.

[22] For general reference about the variational approach to the optimization (21), see e.g., Luenberger (1969).

[23] It is possible since η is observable at the end of the period and thus contracts can be written upon it.

Note that our social welfare in this case SW^o would be lower than the benchmark level SW^*, as we impose an additional incentive compatibility constraint in (16) compared with (3). This is summarized by the following Proposition.

Proposition 4 *When the agent's indirect utility $V(x)$ in the absence of the hedgeable risk η becomes convex in output x, the introduction of a derivative market will reduce the firm's welfare SW^o from SW^* in Section 2.1. Therefore,*

$$SW^o < SW^*.$$

Comparing SW^N and SW^o From Propositions 2 and 4, we observe that in cases where the agent infinitely speculates given the optimal $w^*(x)$ from Section 2.1, i.e., when $V(x)$ is convex in output x, the social welfare (i.e., SW^N or SW^o) is reduced as either (i) the principal does not allow the manager to transact in derivative markets; or (ii) alter the optimal contract in a way that the manager voluntarily hedges, which distorts from the original risk-sharing in (5) and hurts the efficiency.

The following Proposition 5 allows us to compare social welfares in two options, i.e., SW^N and SW^o in some cases.

Proposition 5 *When the agent's indirect utility $V(x)$ in the absence of the hedgeable risk η becomes convex in output x, the introduction of a derivative market will reduce the firm's welfare SW^o compared with SW^N in equation (21) when the amount of uncertainty about the firm's risk exposure, σ_R^2, is small.*

As we illustrate in Proposition 3, if the agent can transact in derivative markets and manipulate a_d, the optimal contract must be altered from $w^N(x, \eta)$ in equation (13) to $w^o(x, \eta)$ in equation (22). In cases where the manager voluntarily chooses to hedge after the derivative market is introduced given his original optimal contract $w^*(\cdot)$ (i.e., $V(x)$ is <u>concave</u> in x), the compensation contract remains unchanged from $w^*(x)$, and the welfare unambiguously increases by the informational gain generated by an opportunity of the manager to hedge in the derivative market and eliminate the firm's risk exposure R.[24] However, when <u>convex</u> $V(x)$ given $w^*(x)$ induces the manager to speculate in the derivative market, shareholders must revise the manager's contract to $w^o(x, \eta)$ to provide the manager with an incentive to hedge, which imposes additional risks on the risk-averse manager's side and incurs the cost out of it. Thus, there are costs and benefits associated with derivative trading that the principal must consider.

Note that no expectation with respect to R is taken for joint benefits SW^* and SW^o, since both of them are independent of R. When there is no hedgeable risk, i.e., $\eta \equiv 0$, then joint benefits, SW^*, become obviously

[24]In cases where $V(x)$ is concave, the social welfare is given by SW^*, which is greater than SW^N by Proposition 4.

independent of the R's realization because the optimal action a_1^* is independent of R. Similarly, when $w^o(x, \eta)$ is designed in the presence of derivative markets, the joint benefits SW^o are independent of R as agent is always induced to take $b = R - a_d = 0$ no matter what R is realized. However, in calculating joint benefits SW^N, an expectation with respect to R is taken, implying that the distribution of R affects the level of SW^N. Generally, as σ_R^2 incrreases, the degree of asymmetric information between two parties rises, reducing SW^N.

The above discussion implies that informational gains from the manager's derivative transaction declines as the amount of uncertainty around the firm's risk exposure R falls. On the other hand, the cost of controlling the additional incentive problem associated with a_d (or equivalently $b = R - a_d$) is independent of the firm's risk exposure R and thus σ_R^2. For instance, even if R is known to the principal (i.e., $\sigma_R^2 = 0$), the moral hazard problem associated with inducing $b = 0$ still remains to the same degree. Therefore, σ_R^2 is indeed a matter of indifference in incentivizing the agent's choice of b, and we can conclude that as $\sigma_R^2 \to 0$, we definitely would have $SW^o < SW^N$, as SW^o is unaffected while $SW^N \to SW^*$ which is greater than SW^o.

Remark We conclude Section 2 noting that sometimes, it is better for shareholders to shut down the manager's access to derivative markets, due to the agency problem around his hedging choices. In Section 6, we consider possible communication between shareholders and manager about the value of R that the manager observes, and study whether this communication can potentially replaces a role of derivative transactions in light with our agency problems.

3 The Model: 3-actions

From Section 3, we provide a full-fledged model with the agent's 3 different actions, including project choice. The setting is the same as in Section 2, except that after his wage contract is finalized, the agent takes three kinds of actions, $a_1 \in [0, \infty)$, $a_2 \in [\underline{a}_2, \overline{a}_2]$, and $a_3 \in (-\infty, +\infty)$. Agent's first action a_1 is a productive effort which increases an expected output as before, that is, a high effort generates an output level that first-order stochastically dominates the output level generated by a low effort. The third action a_3 is his choice in the derivatives market.[25] The agent's second action a_2 is his (real) project choice. These projects have different risk levels with more risky projects having higher expected outputs. We assume that a set of projects available to the agent is bounded, i.e., $a_2 \in [\underline{a}_2, \overline{a}_2]$, while the agent can choose any position in the derivatives market, i.e., $a_3 \in (-\infty, +\infty)$ as in Section 2.

[25]We use the notation a_3 instead of a_d in Section 2 in order to illustrate different roles of our main model's 3 different actions.

After the agent chooses a_1, a_2, and a_3, the firm's output, x, is realized and publicly observable without cost. Thereby, an output x can be used in the manager's wage contract that is denoted by w. The output is determined not only by the agent's choice of (a_1, a_2, a_3) but also by the state of nature, (η, θ). For simplicity, we assume that the output function exhibits the following additively separable form:

$$x = \phi(a_1, a_2) + a_2\theta + (R - a_3)\eta. \tag{23}$$

Equation (23) looks like equation (1), except that (i) the agent's project choice a_2 affects the expected output level $\phi(a_1, a_2)$; (ii) the firm's level of non-hedgeable risk is not fixed *a priori*, but determined by the agent's project choice a_2. Now, an expected output, $\phi(a_1, a_2)$, is a function of both a_1 and a_2, whereas the agent's derivatives choice, a_3, does not directly affect it. As in (1), we assume that (i) $\eta \sim N(0, 1)$ and $\theta \sim N(0, 1)$ are uncorrelated; and (ii) η is observable at the end of the contracting period, and thereby can be used in the manager's wage contract if necessary. As in Section 2, the manager can observe the true value of R after the contract is signed but before he chooses a_1, a_2, and a_3, while shareholders do not observe its value at all, knowing its distribution $R \sim N(R_m, \sigma_R^2)$ only. We assume neither a management effort a_1 nor a project choice a_2 affects R, the firm's innate exposure to the hedgeable risks.[26] However, the firm's final risk exposure can be manipulated by the manager's transaction a_3 in the derivative market. If $a_3 = 0$, the manager does not trade derivatives. The manager hedges, i.e., reduces risk, as long as $|R - a_3| < |R|$ and minimizes risk by setting $a_3 = R$. On the other hand, if $|R - a_3| > |R|$, the manager speculates in the derivative market.

In addition, we make the following assumptions:

Assumption 4 The agent's preferences on wealth and productive effort are additively separable :

$$U(w, a_1, a_2, a_3) = u(w) - v(a_1), \quad u' > 0, u'' < 0,$$

where v, the agent's disutility of exerting productive effort, has the properties $v' > 0, v'' > 0, \ \forall a_1$.

Assumption 5 $\dfrac{\partial \phi}{\partial a_1}(a_1, a_2) \equiv \phi_1(a_1, a_2) > 0, \ \dfrac{\partial^2 \phi}{\partial a_1^2}(a_1, a_2) \equiv \phi_{11}(a_1, a_2) < 0.$

Assumption 6 $\dfrac{\partial \phi}{\partial a_2}(a_1, a_2) \equiv \phi_2(a_1, a_2) > 0, \ \phi_{22}(a_1, a_2) < 0, \ \phi_2(a_1, \underline{a}_2) = \infty, \ \textit{and} \ \ \phi_2(a_1, \overline{a}_2) = 0.$

Assumption 7 $0 < \underline{a}_2 < \overline{a}_2 < \infty$.

[26] In general, a firm's risk exposure can be very much dependent on the production project undertaken. However, even if we allow the firm's risk exposure to be affected by the manager's project choice a_2, most results in this paper will not change qualitatively.

Assumption 8 $\phi_{12}(a_1, a_2) \cdot a_2 < \phi_1(a_1, a_2)$ *for* $\forall (a_1, a_2)$.

Assumption 9 If different action choices (i.e., different combinations of (a_1, a_2, a_3)) yield the same efficiency, then the principal always prefers the minimum risk case (i.e., (a_1, a_2, a_3) that yields the lowest variance of x).

Assumption 4 implies that the agent is risk-averse and effort-averse, and the agent's project and derivatives choices have no direct effect on his utility.[27] Assumptions 5 and 6 indicate that a_1 affects the expected output with a usual property of decreasing marginal increase in output, while a higher a_2 raises the expected output in the similar way and also the output variability. Especially, since expected value of output as well as its variability increases in a_2, there is an assumed trade-off between return and risk.[28]. Assumption 7 states that there is neither a completely safe project nor a project with unbounded risk. Assumption 9 is our tie-breaking rule when two different combinations of actions result in the same welfare measure.

If $\phi_{12}(a_1, a_2)$ is positive and decreasing in a_2, and $\phi_1(a_1, \underline{a}_2) \simeq 0$, $\underline{a}_2$ is close to 0, then Assumption 8 holds as we see in Figure 1. As the manager raises a project risk level a_2, an increase in effort a_1 results in a *higher* increase in expected output $\phi(a_1, a_2)$, i.e., $\phi_{12}(a_1, a_2)$ is positive.[29] We assume this complementarity between a_1 and a_2 becomes weaker as the project becomes riskier, i.e., a_2 increases.

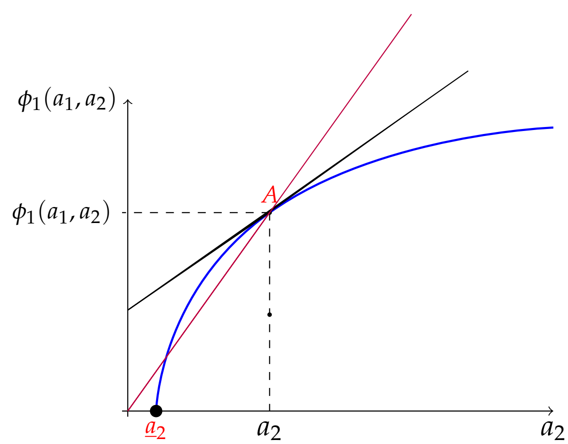

Figure 1: Illustration of the Assumption 8

[27]For the project choice a_2, we have an indirect effect of a_2 on the agent's utility through its effect on the cash flow x in (23). For the derivative choice a_3, we assume that direct hedging costs (e.g., option premia) are negligible compared with the nominal amounts of firms' cash flows. Therefore, we assume away the cost for the derivative choice a_3.

[28]As noted from equation (23), reducing the firm's non-hedgeable risks requires the firm to sacrifice a part of an expected output. This trade-off guarantees the existence of an optimal project choice a_2 in our agency setting.

[29]For example, if we regard the action a_1 as managing the project in a day-to-day basis, it is natural to assume when the manager takes additional unit of project risk a_2, a role of action a_1 in generating output becomes more important, i.e., $\phi_1(a_1, a_2)$ rises.

4 When There Is No Derivative Market

4.1 The Principal Knows the Firm's Exposure to the Hedgeable Risks

In this section, we consider a benchmark case in which the firm has no access to a derivative market, and the principal also knows the true value of the firm's innate risk exposure, R. We thus specify $a_3 = 0$ so that the production function in equation (23) reduces to

$$x = \phi(a_1, a_2) + R\eta + a_2\theta. \tag{24}$$

Since there is no derivative market, the manager's incentive problem arises only in inducing (a_1, a_2). As R and η are observable and thus contractible, $y \equiv x - R\eta$ is a sufficient statistic for (x, η) in assessing (a_1, a_2). Therefore, the principal uses y as a contractual variable to induce (a_1, a_2), and the above equation can be expressed as

$$y = \phi(a_1, a_2) + a_2\theta. \tag{25}$$

In general, designing a contract to induce agent's project choice a_2 as well as the effort choice a_1 should be different from designing a contract that only induces the agent's effort choice (a_1).[30] Thus, to study how the existence of an additional incentive problem associated with the agent's project choice a_2 affects the agent's wage contract, we first consider a typical standard agency case in which the agent's project choice, a_2, is observable, or equivalently, the principal selects the project level a_2 by himself and mandates it.

The optimal compensation contract $w(\cdot)$, in this case, is found similarly to the above Section 2.1 by solving for the contract which maximizes the combined utilities of the principal and agent subject to the restriction that the agent's effort a_1 is chosen to maximize his utility given the contract.

$$\begin{aligned} \max_{a_1, a_2, w(\cdot)} \phi(a_1, a_2) - &\int w(y) f(y|a_1, a_2) dy + \lambda \left(\int u(w(y)) f(y|a_1, a_2) dy - v(a_1) \right) \quad \text{s.t.} \\ &(i) \;\; a_1 \in \arg\max_{a_1'} \int u(w(y)) f(y|a_1', a_2) dy - v(a_1'), \;\; \forall a_1', \\ &(ii) \;\; w(y) \geq k, \;\; \forall y, \end{aligned} \tag{26}$$

where $f(y|a_1, a_2)$ denotes a probability density function of y given the agent's three actions, and λ denotes the weight placed on the agent's utility in the joint optimization. As shown, the combined utilities of the principal and the agent are maximized subject to the agent's incentive compatibility constraint which specifies that the agent chooses his effort for his own optimization, and his limited liability constraint which specifies that the

[30] Hirshleifer and Suh (1992) call this issue as a 'dual-agency problem'.

agent receives at least k, the subsistence level of utility.

Subject to some technical assumptions,[31] the above maximization problem reduces to:

$$\max_{\substack{a_1,a_2 \\ w(\cdot)\geq k}} \phi(a_1,a_2) - \int w(y)f(y|a_1,a_2)dy + \lambda\left(\int u(w(y))f(y|a_1,a_2)dy - v(a_1)\right) \quad \text{s.t.}$$
$$(i) \quad \int u(w(y))f_1(y|a_1,a_2)dy - v'(a_1) = 0, \tag{27}$$

where f_1 denotes the first derivative of f taken with respect to a_1.

To find solution $(a_1^P, a_2^P, w^P(y|a_1^P, a_2^P))$ for the above program, we first derive an optimal contract for an arbitrarily given action combination (a_1, a_2). Let $w^P(y|a_1, a_2)$ be a contract which optimally motivates the agent to take a particular level of a_1 when an arbitrary level of a_2 is chosen by the principal. By solving the Euler equation of the above program after fixing (a_1, a_2), we derive that $w^P(y|a_1, a_2)$ must satisfy

$$\frac{1}{u'(w^P(y|a_1,a_2))} = \lambda + \mu_1(a_1,a_2)\frac{f_1}{f}(y|a_1,a_2), \tag{28}$$

for almost every y for which equation (28) has a solution $w^P(y|a_1, a_2) \geq k$, and otherwise $w^P(y|a_1, a_2) = k$. In equation (28), $\mu_1(a_1, a_2)$ denotes the optimized Lagrange multiplier for the agent's incentive constraint associated with a_1 when the second action is pinned down at a_2. Since $f(y|a_1, a_2, a_3 = 0)$ is a normal density function with mean $\phi(a_1, a_2)$ and variance a_2^2, (28) is reduced to:

$$\frac{1}{u'(w^P(y|a_1,a_2))} = \lambda + \mu_1(a_1,a_2)\frac{y-\phi(a_1,a_2)}{a_2^2}\phi_1(a_1,a_2). \tag{29}$$

Before analyzing the optimal contract, we first define given (a_1, a_2):

$$SW^P(a_1,a_2) \equiv \phi(a_1,a_2) - C^P(a_1,a_2) - \lambda v(a_1), \tag{30}$$

which denotes the joint benefits when $w^P(y|a_1, a_2)$ is designed and a_2 is instructed by the principal where

$$C^P(a_1,a_2) \equiv \int \left(w^P(y|a_1,a_2) - \lambda u(w^P(y|a_1,a_2))\right) f(y|a_1,a_2)dy \tag{31}$$

represents the efficiency loss of this case compared with the full information case. In other words, $C(a_1, a_2)$

[31] We assume that the first-order approach is valid. Grossman and Hart (1983) and Rogerson (1985) show that MLRP and CDFC are sufficient for the validity of the first-order approach when the signal space is of one dimension. Jewitt (1988) finds less restrictive conditions for the validity of the first-order approach, based on the agent's risk preferences as well as the distribution function of the signal. Sinclair-Desgagné (1994) shows that more general versions of MLRP and CDFC in a multi-dimensional space are sufficient for the validity of the first-order approach when the signal space is of multiple dimensions. For more recent treatments along this line, see Conlon (2009) and Jung and Kim (2015) among others. Recently, Jung et al. (2022) justifies the first-order approach when the technology follows normal distributions, which corresponds to our problem in (26).

measures the agency cost arising from inducing the agent to take that particular a_1 when a_2 is chosen by the principal.

We start our analysis with the following lemma, which was previously provided in Kim (1995).

Lemma 3 *$C^P(a_1, a_2^0) < C^P(a_1, a_2^1)$ for any given a_1 if $a_2^0 < a_2^1$.*

Since the principal dictates the agent's project choice a_2 here, an agency problem arises only in inducing a_1. Lemma 3 says: under Assumption 8, when the agent's project choice a_2 can be instructed by the principal, the agency cost associated with motivating the agent to take any given action a_1, i.e., $C^P(a_1, a_2)$, is reduced if the principal chooses a less risky project. A lowered risk a_2 improves the efficiency of the agency relationship by providing a more precise signal y about the agent's effort, a_1, which in turn enables the principal to design contract inducing a particular a_1 in a less costly way. If $\phi_{12}(a_1, a_2)$ is large enough to break Assumption 8, then lower a_2 might lower $\phi_1(a_1, a_2)$ a lot, which in turn makes harder for the principal to give the proper incentive for the action a_1 and raise the incentive cost $C^P(a_1, a_2)$. Assumption 8 guarantees that this incentive drawback is lower than the informational rent from lower a_2, so principal wants a lower level of non-hedgeable risk a_2.

Lemma 3 suggests firms should take all zero net present value projects that reduce their risks. However, given the trade-off relation between return and risk, i.e., $\phi_2 > 0$, the exact level of a_2 that the principal prefers will be determined by the loss in expected return as well as the benefit from more precise signals. Let a_2^P be the project that is most preferred by the principal, and a_1^P the agent's optimal effort choice for the above program when a_2^P is chosen by the principal. Then, as we prove in Appendix, from the above optimization we obtain that $(a_1^P, a_2^P, w^P(\cdot))$ should satisfy

$$\int \Big(y - w^P(y) + \lambda u(w^P(y)) \Big) f_2(y|a_1^P, a_2^P) dy + \mu_1(a_1^P, a_2^P) \int u(w^P(y)) f_{12}(y|a_1^P, a_2^P) dy = 0, \tag{32}$$

where $w^P(\cdot) = w^P(\cdot|a_1^P, a_2^P)$, f_2 denotes the first derivative of f with respect to a_2 and f_{12} is the second derivative with respect to a_1 and a_2. We obtain the optimal contract $w^P(y|a_1^P, a_2^P)$ satisfies,

$$\frac{1}{u'(w^P(y|a_1^P, a_2^P))} = \lambda + \mu_1(a_1^P, a_2^P) \frac{y - \phi(a_1^P, a_2^P)}{(a_2^P)^2} \phi_1(a_1^P, a_2^P), \tag{33}$$

for y satisfying $w^P(y|a_1^P, a_2^P) \geq k$ and $w^P(y|a_1^P, a_2^P) = k$ otherwise.

The above result assumes shareholders are fully aware of the projects chosen by the manager. However, it is not realistic, so we also turn our eyes to the manager's incentive to increase or decrease the firm's risk when the project choice is not fully revealed to the shareholders. Specifically, we ask whether the manager will voluntarily choose the project that would be chosen by informed shareholders, i.e., a_2^P. If the answer to this question is no, then the moral-hazard problem arises not only in motivating a_1 but also in incentivizing a_2. As a result, there are costs associated with the project choice being unobservable, and the optimal wage contract must be modified from the contract, $w^P(y|a_1^P, a_2^P)$, in equation (33).

To formally analyze this issue, we denote $a_2^A(a_2^P)$ as a solution for

$$a_2^A(a_2^P) \in \arg\max_{a_2} \int u(w^P(y|a_1^P, a_2^P)) f(y|a_1^P, a_2) dy. \tag{34}$$

Thus, $a_2^A(a_2^P)$ represents the project choice that the agent would take under $w^P(y|a_1^P, a_2^P)$ described in equation (33) when a_2 is not actually enforceable. Thus, our previous question, "Will the agent voluntarily choose a_2^P when $w^P(y|a_1^P, a_2^P)$ is designed?", is equivalent to the question, "Will $a_2^A(a_2^P)$ be equal to a_2^P?"

As previously shown, the principal balances two considerations when he directs the agent to take a certain project: the informational benefits from risk reduction and the lower mean return associated with lower risk. However, the risk level to be chosen by the agent primarily depends on his indirect risk preferences induced by contract $w^P(y|a_1^P, a_2^P)$, i.e., the curvature of $u(w^P(y|a_1^P, a_2^P))$ with respect to y, and the effect that a trade-off between return and risk would have on his utility *via* $w^P(y|a_1^P, a_2^P)$.

In general, the curvature of the agent's indirect utility function depends on the distribution of the random state variable and utility function itself. To see how different utility functions affect this curvature differently, we again consider the case where the agent has constant relative risk aversion with degree $1-t$ as we did in Section 2.1, where $t < 1$ (i.e., $u(w) = \frac{1}{t}w^t, t < 1$). We obtain from equation (33) that

$$w^P(y|a_1^P, a_2^P) = \left(\lambda + \mu_1(a_1^P, a_2^P)\left(\frac{y - \phi(a_1^P, a_2^P)}{(a_2^P)^2}\right)\phi_1(a_1^P, a_2^P)\right)^{\frac{1}{1-t}}, \tag{35}$$

and the agent's indirect utility under this wage contract is

$$u(w^P(y|a_1^P, a_2^P)) = \frac{1}{t}\left(\lambda + \mu_1(a_1^P, a_2^P)\left(\frac{y - \phi(a_1^P, a_2^P)}{(a_2^P)^2}\right)\phi_1(a_1^P, a_2^P)\right)^{\frac{t}{1-t}}. \tag{36}$$

The above equation shows that the agent's indirect utility becomes strictly concave in y if $t < \frac{1}{2}$, linear if $t = \frac{1}{2}$, and convex if $t > \frac{1}{2}$ for y satisfying $w^P(y|a_1^P, a_2^P) \geq k$. If we assume $w^P(y|a_1^P, a_2^P) = k$ for sufficiently low y, as far as the agent's induced risk preferences are concerned, $u(w^P(y|a_1^P, a_2^P))$ makes the agent risk-loving if $t \geq \frac{1}{2}$. Furthermore, since the compensation contract $w^P(y|a_1^P, a_2^P)$ is positively related to the absolute output level (i.e., $\mu_1(a_1^P, a_2^P) > 0$),[32] if $t \geq \frac{1}{2}$, the agent is induced to take the most risky project, i.e., $a_2^A(a_2^P) = \overline{a}_2$ when $w^P(y|a_1^P, a_2^P)$ is designed even if $\phi_2(a_1, \overline{a}_2) = 0$ by Assumption 6. However, in this case, principal prefers to

[32] For the proof of $\mu_1(a_1^P, a_2^P) > 0$, see Holmström (1979), Jewitt (1988), and Jung and Kim (2015) among others.

have a firm's risk level a_2 lower than $\bar{a}_2$. This is because, from his standpoint, the informational benefits from risk reduction are still substantial, while the costs of risk reduction are zero at $\bar{a}_2$ (i.e., $\phi_2(\bar{a}_2) = 0$). Thus, $a_2^P < a_2^A(a_2^P)$ in this case. In other words, the principal prefers less risk than the agent under $w^P(y|a_1^P, a_2^P)$.

On the other hand, in the case of t being close to $-\infty$ (i.e., the agent is extremely risk-averse), the agent's indirect utility mandates him to choose a lower level of risk than what principal prefers ($a_2^A(a_2^P) < a_2^P$) even though a lower a_2 yields on average a lower output level.[33]

Incentive problems associated with project choice a_2, in general, exist in all cases except those where both of the following conditions are satisfied: (i) the agent's indirect utility is sufficiently concave and (ii) there is no trade-off between return and risk, i.e., $\phi_2 = 0$, $\forall a_2$. Under these conditions, both the principal and the agent agree that the firm should choose the least risky project, i.e., $a_2 = \underline{a}_2$, and there is no efficiency loss due to the existence of the manager's unobservable project choice. However, when either the agent's induced risk preferences are convex, or the trade-off between return and risk exists as assumed in Assumption 6, the principal and the agent will not generally agree on the firm's optimal project choice, and the compensation contract, $w^P(y|a_1^P, a_2^P)$, described in equation (33) will no longer be optimal.

In this situation, the principal must determine the optimal compensation contract by solving the following optimization problem:

$$\begin{aligned} &\max_{\substack{a_1, a_2 \\ w(\cdot) \geq k}} \phi(a_1, a_2) - \int w(y) f(y|a_1, a_2) dy + \lambda \left(\int u(w(y)) f(y|a_1, a_2) dy - v(a_1) \right) \quad \text{s.t.} \\ &\qquad (i) \;\; (a_1, a_2) \in \arg\max_{a_1', a_2'} \int u(w(y)) f(y|a_1', a_2') dy - v(a_1'), \;\; \forall a_1', a_2'. \end{aligned} \tag{37}$$

In the above optimization (37), we accounts for the fact that the agent selects a_2 to maximize his own expected utility. If an interior solution for (a_1, a_2) exists and the first-order approach is valid, the above maximization problem can be expressed as:

$$\begin{aligned} &\max_{\substack{a_1, a_2 \\ w(\cdot) \geq k}} \phi(a_1, a_2) - \int w(y) f(y|a_1, a_2) dy + \lambda \left(\int u(w(y)) f(y|a_1, a_2) dy - v(a_1) \right) \quad \text{s.t.} \\ &\qquad (i) \;\; \int u(w(y)) f_1(y|a_1, a_2) dy - v'(a_1) = 0, \\ &\qquad (ii) \;\; \int u(w(y)) f_2(y|a_1, a_2) dy = 0, \end{aligned} \tag{38}$$

Let (a_1^*, a_2^*) be the optimal action combination for the above program. Then, by solving the Euler equation,

[33] This issue has been dealt with in the literature: see Hirshleifer and Suh (1992), Guay (1999), and Ross (2004) among others.

we obtain that the optimal wage contract, $w^*(y)$, satisfies,

$$\frac{1}{u'(w^*(y))} = \lambda + \mu_1^* \frac{f_1}{f}(y|a_1^*, a_2^*) + \mu_2^* \frac{f_2}{f}(y|a_1^*, a_2^*), \tag{39}$$

for almost every y for which equation (39) has a solution $w^*(y) \geq k$, and otherwise $w^*(y) = k$. μ_1^* and μ_2^* are the optimized Lagrange multipliers for both incentive constraints, respectively.

Since $f(y|a_1^*, a_2^*)$ is a normal distribution with mean $\phi(a_1^*, a_2^*)$ and variance $(a_2^*)^2$, we have from (39) that:

$$\frac{1}{u'(w^*(y))} = \lambda + \mu_1^* \underbrace{\frac{y - \phi(a_1^*, a_2^*)}{(a_2^*)^2} \phi_1^*}_{\equiv SS_1} + \mu_2^* \left[\underbrace{\frac{y - \phi(a_1^*, a_2^*)}{(a_2^*)^2} \phi_2^*}_{\equiv SS_2^1} + \underbrace{\frac{1}{a_2^*} \left(\frac{(y - \phi(a_1^*, a_2^*))^2}{(a_2^*)^2} - 1 \right)}_{\equiv SS_2^2} \right], \tag{40}$$

where we define SS_1, $SS_2 \equiv SS_2^1 + SS_2^2$ as sufficient statistics for unobservbable action a_1 and project choice a_2, respectively. Compared with (33), (40) shows that when both a_1 and a_2 are not observable, the optimal wage contract is based not only on the absolute output y, but also on its (standardized) deviation from the expected level, $\frac{(y-\phi(a_1^*, a_2^*))^2}{(a_2^*)^2}$. Since $(y - \phi(a_1^*, a_2^*))^2$ is a sample (i.e., realized) variance of a single observation with mean zero and variance $(a_2^*)^2$, the term $\frac{(y-\phi(a_1^*, a_2^*))^2}{(a_2^*)^2}$ in (40) can be regarded a standardized output deviation. Note that SS_2, the sufficient statistic for the project choice a_2, can be now decomposed into two parts: SS_2^1 and SS_2^2. SS_2^1 takes account of the effects that an increase in a_2 has on the mean cash flow $\phi(a_1, a_2)$,[34] while SS_2^2 is about how an increase in a_2 affects the signal y's volatility. By including the sample variance as a contractual parameter, the principal effectively motivates the agent to take the appropriate level of a_2, i.e., a_2^*. (40) can be written in a simpler way as

$$\frac{1}{u'(w^*(y))} = \lambda + (\mu_1^* \phi_1^* + \mu_2^* \phi_2^*) \frac{y - \phi(a_1^*, a_2^*)}{(a_2^*)^2} + \mu_2^* \frac{1}{a_2^*} \left(\frac{(y - \phi(a_1^*, a_2^*))^2}{(a_2^*)^2} - 1 \right), \tag{41}$$

for y satisfying $w^*(y) \geq k$ and $w^*(y) = k$ otherwise. In the above equation, $\phi_i^* \equiv \phi_i(a_1^*, a_2^*), i = 1, 2$. We call $w^*(y)$ as an *optimal dual-agency contract* à la Hirshleifer and Suh (1992).

The optimal dual agency contract is characterized in the following propositions.

Proposition 6 $\mu_1^* \phi_1^* + \mu_2^* \phi_2^* > 0$.

Proposition 6 implies, holding the cash flow variance constant, the manager's payout increases when the firm's cash flow increases, which implies that the manager is rewarded for a higher effort. However, this does not necessarily mean that the contracted payout is monotonically increasing in the output level. For

[34] This term is present since we assume the risk-return trade-off in a_2, i.e., $\phi(a_1, a_2)$ is increasing in a_2.

example, if $\mu_2^* < 0$ in equation (41), the agent can be paid less when the output level is very high.

Thus, a more interesting question has to do with the relation between the agent's rewards and the output deviation, i.e., the sign of μ_2^*.

Proposition 7 *If the principal prefers a less risky project than the agent under $w^P(y|a_1^P, a_2^P)$ in equation (33), i.e., $a_2^P < a_2^A(a_2^P)$, then the optimal dual agency contract will <u>penalize</u> the agent for having unusual output deviation from the expected level, i.e., $\mu_2^* < 0$ for $w^*(y)$ in equation (41). If the principal prefers a more risky project than the agent under $w^P(y|a_1^P, a_2^P)$, i.e., $a_2^P > a_2^A(a_2^P)$, then the optimal dual agency contract will <u>reward</u> the agent for having unusual output deviation, i.e., $\mu_2^* > 0$ for $w^*(y)$ in equation (41).*

If the principal prefers a lower level of project risk than the agent under the contract $w^P(y|a_1^P, a_2^P)$, the contract will be revised in a way that motivates the agent to reduce risk. This can be done by setting $\underline{\mu_2^* < 0}$ in equation (41) which penalizes the agent for the unusual output deviation and makes the agent act as if he is more risk-averse. On the other hand, if the principal prefers a higher risk than the agent when $w^P(y|a_1^P, a_2^P)$ is designed, the contract is revised to motivate the agent to increase risk. This can be done by setting $\underline{\mu_2^* > 0}$ in equation (41) which rewards the agent for unusual output deviation and makes the agent act as though he is less risk-averse. As discussed earlier, the later case is more likely to occur when the manager is more risk averse and when the firm's investment opportunities offer a non-trivial trade-off between return and risk.[35]

We denote the optimized joint benefits in this case as

$$SW^*(a_1^*, a_2^*) \equiv \phi(a_1^*, a_2^*) - C^*(a_1^*, a_2^*) - \lambda v(a_1^*), \tag{42}$$

where

$$C^*(a_1^*, a_2^*) \equiv \int (w^*(y) - \lambda u(w^*(y))) f(y|a_1^*, a_2^*) dy \tag{43}$$

denotes the agency cost arising from inducing (a_1^*, a_2^*) when a_3 is fixed at 0 and R is observable.

4.2 The Principal Does Not Know the Firm's Risk Exposure

We now consider the case where the firm's exposure to hedgeable risks, R, is observed only by the agent but not by the principal. Thus, the wage contract cannot explicitly include $y \equiv x - R\eta$ as a contractual variable. Furthermore, we rule out the possibility of any communication between principal and the agent that allows

[35]For example, in cases of constant relative risk aversion with degree $1 - t$, it is more likely that $\underline{\mu_2^* > 0}$ when $1 - t$ is higher (i.e., t is lower).

the agent to reveal R.[36] We will later consider the case in which communication between the principal and the agent is allowed without cost, and thereby the principal can design a truth-telling mechanism freely.

If principal does not observe R, the compensation contract must be based on (x, η), i.e., $w = w(x, \eta)$. The principal's maximization program in this case is thus:[37]

$$
\begin{aligned}
&\max_{\substack{a_1(\cdot),a_2(\cdot)\\ w(\cdot)\geq k}} \int_R \int_{x,\eta} (x - w(x,\eta))\, g(x,\eta|a_1(R),a_2(R),R)h(R)dxd\eta dR \\
&\qquad + \lambda \int_R \left(\int_{x,\eta} u(w(x,\eta)) g(x,\eta|a_1(R),a_2(R),R) dxd\eta - v(a_1(R)) \right) h(R)dR \quad \text{s.t.} \\
&\qquad (i) \;\; (a_1(R),a_2(R)) \in \arg\max_{a_1,a_2} \int_{x,\eta} u(w(x,\eta)) g(x,\eta|a_1,a_2,R)dxd\eta - v(a_1), \forall R,
\end{aligned}
\tag{44}
$$

where

$$
g(x,\eta|a_1,a_2,R) = \frac{1}{2\pi a_2} \exp\left(-\frac{1}{2} \left(\frac{(x-\phi(a_1,a_2)-R\eta)^2}{a_2^2} + \eta^2 \right) \right) \tag{45}
$$

denotes a joint probability density function of (x, η) given (a_1, a_2, R) and $h(R)$ denotes the probability density function of R.

For each R, let $(a_1^N(R), a_2^N(R), w^N(x,\eta))$ be the solution for the above optimization program in (44). If we let $\mu_1(R)$, $\mu_2(R)$ be Lagrange multipliers attached to incentive constraints in $a_1(R)$ and $a_2(R)$, respectively, the optimal contract $w^N(x, \eta)$ can be written as[38]

$$
\begin{aligned}
\frac{1}{u'(w(x,\eta))} =& \lambda + \int_R \mu_1(R) \left[\frac{g_1(x,\eta|a_1^N(R),a_2^N(R),R)}{\int_{R'} g(x,\eta|a_1^N(R'),a_2^N(R'),R')h(R')dR'} \right] h(R)dR \\
&+ \int_R \mu_2(R) \left[\frac{g_2(x,\eta|a_1^N(R),a_2^N(R),R)}{\int_{R'} g(x,\eta|a_1^N(R'),a_2^N(R'),R')h(R')dR'} \right] h(R)dR,
\end{aligned}
\tag{46}
$$

when $w(x, \eta) \geq k$ and otherwise $w(x, \eta) = k$.

The optimized joint benefit in this case is denoted as:

$$
SW^N \equiv \int_R \left(\phi(a_1^N(R),a_2^N(R)) - C^N(a_1^N(R),a_2^N(R)) - \lambda v(a_1^N(R)) \right) h(R)dR, \tag{47}
$$

[36]In general, communication between principals and agents are likely to be very costly, especially when actually the principal stands for multiple shareholders. For a more detailed discussion of communication costs, see Laffont and Martimort (1997).

[37]In this case, since the agent is the only one seeing the realized value of R, his actions a_1, a_2 both depend on R, given the contract $w(x, \eta)$.

[38]We provide the derivation for equation (46) in Appendix.

where

$$C^N(a_1^N(R), a_2^N(R)) \equiv \int_{x,\eta} \left(w^N(x,\eta) - \lambda u(w^N(x,\eta)) \right) g(x,\eta | a_1^N(R), a_2^N(R), R) dx d\eta \tag{48}$$

denotes the agency cost arising from inducing $(a_1^N(R), a_2^N(R))$ given a realized value of R.

Proposition 8 *When there is no derivative market and the communication between the principal and the agent is not possible, the principal's inability to observe the firm's risk exposure reduces welfare, i.e.,*

$$SW^N < SW^*(a_1^*, a_2^*).$$

Intuitively, when the principal observes the true value of the firm's risk exposure, R, this information can be used to design a wage contract that eliminates the influence of the hedgeable risks, i.e., $w = w^*(y \equiv x - R\eta)$*. However, if R is not observable and cannot be communicated, this is impossible.*

5 When Managers Can Trade Derivatives

This section considers how the introduction of an opportunity to transact derivative contracts (i.e., when a_3 is not fixed at 0) affects the optimal contract and firm's efficiency. Continuing from Section 4.2, we assume that a manager's project choice, a_2, is not observable. Moreover, we assume that neither a manager's derivatives choice, a_3, nor firm's risk exposure, R, can be observed by or communicated to the principal.

Now the logic becomes similar to Section 2.2.2: since the firm's exposure to hedgeable risks, R, is observed by the agent before he takes actions (a_1, a_2, a_3), the agent's choice of a_3 can be characterized as his choice of $b \equiv R - a_3$. Then given a compensation contract, the principal can rationally anticipate the agent's choice of $b = R - a_3$. We denote the principal's prediction of agent's choice of $R - a_3$ by $\hat{b}$, and define $z(\hat{b}) \equiv x - \hat{b}\eta$ as a variable that is possibly included in the wage contract, i.e., $w(z(\hat{b}))$ is one possible contract. In order for the principal's beliefs to be consistent,[39] it must be the case that the agent actually chooses a_3 satisfying $b \equiv R - a_3 = \hat{b}$ given the contract.

Thus, since

$$z(\hat{b}) \equiv x - \hat{b}\eta = \phi(a_1, a_2) + (b - \hat{b})\eta + a_2\theta, \tag{49}$$

if the principal designs a contract $w(z(\hat{b}))$ and if the agent actually chooses a_3 satisfying $b = R - a_3 = \hat{b}$, then

$$z(\hat{b}) = \phi(a_1, a_2) + a_2\theta = y. \tag{50}$$

[39] Basically, as the principal predicts the agent with the risk-exposure R to choose $\hat{b} = R - a_3$, the contract that relies on $\hat{b}$ must induce the agent to take $b = \hat{b}$.

Note that a maximum level of joint benefits that can be obtained in this case is $SW(a_1^*, a_2^*, a_3 = 0)$ in equation (42).[40] Thus, we first consider the case in which the principal designs the contract the same as $w^*(y)$ described in equation (41) but based on $z(\hat{b})$ instead of $y \equiv x - R\eta$, and examine whether agent chooses $b \equiv R - a_3 = \hat{b}$ under $w^*(z(\hat{b}))$. If this is indeed the case, there is no welfare loss associated with R (and a_3) being unobservable, and this informational benefit is one of the main advantages of letting the agent transact in the derivative market.

Suppose that the principal designs $w^*(z(\hat{b}))$ satisfying

$$\frac{1}{u'(w^*(z(\hat{b})))} = \lambda + (\mu_1^*\phi_1^* + \mu_2^*\phi_2^*)\frac{z(\hat{b}) - \phi(a_1^*, a_2^*)}{(a_2^*)^2} + \mu_2^*\frac{1}{a_2^*}\left(\frac{\left(z(\hat{b}) - \phi(a_1^*, a_2^*)\right)^2}{(a_2^*)^2} - 1\right), \tag{51}$$

for $z(\hat{b})$ satisfying $w^*(z(\hat{b})) \geq k$ and $w^*(z(\hat{b})) = k$ otherwise. Since $w^*(z(\hat{b}))$ in equation (51) has the same contractual form as $w^*(y)$ in equation (41),[41] we can easily see that the agent will take (a_1^*, a_2^*) under $w^*(z(\hat{b}))$ *if* he chooses a_3 satisfying $b \equiv R - a_3 = \hat{b}$. But, the real question is: "Will the agent always choose a_3 satisfying $b = \hat{b}$ when $w^*(z(\hat{b}))$ is designed and offered?".

The following Lemma 4 provides an answer to the above question.

Lemma 4 [Speculation and Hedging with $w^*(z(\hat{b}))$]
(1) *If $\mu_2^* < 0$ for the contract, $w^*(z(\hat{b}))$, described in equation (51) for any given $\hat{b}$,*[42] *then the manager will choose a_3 such that $b = \hat{b}$ when the contract $w^*(z(\hat{b}))$ is offered.*
(2) *If $\mu_2^* > 0$ for $w^*(z(\hat{b}))$ in equation (51) for any given $\hat{b}$, then the manager will take a_3 such that $|R - a_3| = \infty$ when $w^*(z(\hat{b}))$ is offered.*

From Lemma 4, we directly obtain the following proposition:

Proposition 9 *If $\mu_2^* < 0$ for $w^*(z(\hat{b}))$ described in (51) for any given $\hat{b}$, then the level of $b \equiv R - a_3$ to be induced is a matter of indifference as long as it is bounded, i.e., $|b| < \infty$. For example, If $\mu_2^* < 0$ for $w^*(z(0))$ in (51), then the manager will choose $a_1^*, a_2^*, a_3 = R$ (i.e., $b = 0$) when $w^*(z(0))$ is offered. Therefore, the optimized joint benefits in this case are the same as $SW^*(a_1^*, a_2^*)$ in (42), implying that the firm's welfare with a derivative market will be the same as it is in the case where the risk exposure is observed by the principal.*[43]

[40] Given the contract $w(z(\hat{b}))$, if there is no incentive problem associated with $b = R - a_3$ so the agent voluntarily chooses a_3 such that $R - a_3 = \hat{b}$, then we get the maximum joint benefit $SW(a_1^*, a_2^*, a_3 = 0)$.

[41] Note that $\mu_1^*, \mu_1^*, a_1^*, a_2^*$ in (41) and (51) are endogenous variables characterized by solving the optimization in (37).

[42] One can easily see that if $\mu_2^* < 0$ in $w^*(z(\hat{b}))$ for any given $\hat{b}$, then $\mu_2^* < 0$ in $w^*(z(\hat{b}))$ for all $\hat{b}$. This is because the principal's anticipating different $\hat{b}$ does not change the functional form of $w^*(\cdot)$.

[43] Therefore, the introduction of derivative markets in this case improves the welfare compared with the case where the principal

Proposition 9 is quite intuitive. If $\underline{\mu_2^* < 0}$ for $w^*(z(\hat{b}))$ in equation (51), the agent is induced to engage in perfect hedging to minimize the variance of $z(\hat{b})$. Intuitively, the contract $w^*(z(\hat{b}))$ with $\mu_2^* < 0$ induces the agent to sacrifice expected payoffs to lower risk.[44] If the risk can be reduced through a channel that does not decrease the expected payoff (e.g., here a_3 does not have risk-return trade-off.), then agent will clearly do so. In addition, $\mu_1^*\phi_1^* + \mu_2^*\phi_2^* > 0$ means a higher $z(\hat{b})$ yields the higher compensation $w^*(z(\hat{b}))$ given its squared deviation from the average of $z(\hat{b})$.

In this case, the optimal contract can be designed *as if* $R - a_3$ is observable to the principal, and it allows the principal and the agent to achieve the welfare $SW^*(a_1^*, a_2^*)$ that could be achieved when the risk-exposure R is observable. We will discuss more thoroughly about this *informational gain* from the manager's derivative transaction later.[45]

However, this is not possible if $\underline{\mu_2^* > 0}$ for $w^*(z(\hat{b}))$ in equation (51), since the agent will speculate infinitely, i.e., choose a_3 such that $|R - a_3| = \infty$. This is because, as shown from equation (51), manager will be paid an infinite amount when $z(\hat{b}) = x - \hat{b}\eta$ is either positive or negative infinity if $\mu_2^* > 0$ for $w^*(z(\hat{b}))$. Given that it is impossible to design a wage contract $w^*(z(\hat{b}))$ based on the belief $\hat{b} = \infty$, the principal has to either alter the wage contract to ensure $|R - a_3| < \infty$ or retain the optimal contract without a derivative market, $w^N(x, \eta)$ and prohibit the manager from engaging in derivative transactions, if possible.

Comparison with Section 2: different implications? In Section 2 without the project choice (i.e., action a_1 and derivative transactions a_d only), we start from the benchmark case without the hedgeable risk η, which reduces to the canonical principal-agent model (e.g., Holmström (1979)). Given the optimal contract $w^*(x)$ in this benchmark scenario, which results in the agent's indirect utility $V(x)$ as a function of x, our findings suggest that (i) if $V(x)$ is concave (convex) in x, then the manager would choose to perfectly hedge (infinitely speculate) when the derivative markets open up and he can maniuplate a_d; (ii) $V(x)$ is concave (convex) when the agent's utility function features higher (lower) risk-aversion. Therefore, less risk-averse manager is more likely to speculate given the contract $w^*(\cdot)$ when the derivative market opens up.

Now, we have completely the opposite prediction: (i) the agent with $\mu_2^* > 0$ is speculating infinitely when

does not observe the firm's risk-exposure R and the communication between the principal and the agent is prohibitively costly (i.e., $SW^*(a_1^*, a_2^*) > SW^N$ in Proposition 8). In practice, benefits derivative markets provide to firms are multi-dimensional (e.g., firms can prevent themselves from going bankrupt through proper hedging. In this paper, we focus on the new channel, in which derivative markets can eliminate the informational asymmetry between shareholders and the manager about firms' innate risk-exposure, only if the manager properly hedges in the derivative market.

[44] Since the optimal contract $w^*(z(\hat{b}))$ features $\mu_2^* < 0$ when $a_2^P < a_2^A(a_2^P)$ holds.

[45] Derivative markets exists for many reasons (e.g., allowing market participants to hedge against various risks, thereby preventing bankruptcies). We focus on the new channel through which the derivative market affects the efficiency: it eliminates informational asymmetry about the firm's risk-exposure, thereby making shareholders less vulnerable to adverse selection issues around the risk exposure.

derivative markets open up; and (ii) the principal offers a contract with $\mu_2^* > 0$ since the principal prefers a higher level of project risk than the agent, which implies that the agent is generally more risk-averse. When the manager is more risk-averse, shareholders try to incentivize the manager to raise the project risk level, as a higher level of project risk yields higher average output, so reward a higher level of output variance (i.e., $\mu_2^* > 0$) in the compensation contract. When the derivative markets open up, then the originally more risk-averse manager will speculate infinitely, choosing $a_3 = \pm\infty$ due to the additional incentive effect from $\mu_2^* > 0$.

It can be understood as a side effect of inducing the project risk-taking which is productive (i.e., $\phi_2(a_1, a_2) > 0$) through incentive contracts, since the incentive contract would make the manager to speculate infinitely in the presence of derivative transaction opportunities, as he acts as if he is effectively risk-loving under the contract with $\mu_2^* > 0$.

New optimal contract with $\mu_2^* > 0$ in the presence of derivative markets In deriving the optimal contract in the presence of derivative markets when $\mu_2^* > 0$ for $w(z(\hat{b}))$ in (51), we first consider the case in which the principal designs a contract that ensures a finite a_3 when $\mu_2^* > 0$ for $w^*(z(\hat{b}))$ in (51). Let (a_1^o, a_2^o, a_3^o), where $|b^o \equiv R - a_3^o| < \infty$, be the optimal action combination and $\underline{w^o(z(\hat{b}), \eta)}$ be the wage contract which optimally induces that action combination (a_1^o, a_2^o, a_3^o) where $\hat{b} = b^o \equiv R - a_3^o$. We denote the optimized joint benefits in this case as

$$SW^o(a_1^o, a_2^o, a_3^o) \equiv \phi(a_1^o, a_2^o) - C^o(a_1^o, a_2^o, b^o) - \lambda v(a_1^o), \tag{52}$$

where

$$C^o(a_1^o, a_2^o, b^o) \equiv \int [w^o(z(\hat{b}), \eta) - \lambda u(w^o(z(\hat{b}), \eta))] g(z(\hat{b}), \eta | a_1^o, a_2^o, b^o) dz d\eta \tag{53}$$

denotes the agency cost arising from inducing (a_1^o, a_2^o, a_3^o) when there is a derivative market and $\mu_2^* > 0$ for $w^*(z(\hat{b}))$ in equation (51).

Lemma 5 *If $w^o(z(\hat{b}), \eta)$ is an optimal contract that actually induces (a_1^o, a_2^o, a_3^o) where $\hat{b} = R - a_3^o \equiv b^o \neq 0$, then $w^o(z(0), \eta) \equiv w^o(x, \eta)$[46] is also an optimal contract which induces $(a_1^o, a_2^o, a_3 = R)$. Therefore,*

$$SW^o(a_1^o, a_2^o, a_3^o) = SW^o(a_1^o, a_2^o, a_3 = R).$$

In this case, due to Assumption 9, the principal chooses to induce complete hedging from the agent (i.e., $a_3 = R$).

Lemma 5 indicates that when the principal has to design a compensation contract to guarantee the agent's

[46] Note that $z(0) = x$ from equation (49).

choice of a_3 satisfying $|R-a_3|<\infty$ due to the fact that $\mu_2^*>0$ for $w^*(z(\hat{b}))$, the level of a_3 to be induced by $w^o(z(\hat{b}),\eta)$ is a matter of indifference as long as it is finite and the efficiency is concerned. This is because, as shown in equation (49), the agent's derivative choice, a_3, is additively separable from his other two productive action choices, (a_1,a_2), in determining the output level, x, and not only the output level but also the derivative market variables, η, are observable (thus contractible). This feature allows the principal to always eliminate or add hedgeable risks to the agent's compensation, making a level of remaining hedgeable risks irrelevant. With our tie-breaking rule in Assumption 9, from now on we focus on the case where $a_3=R$ (i.e., complete hedging) is induced.

From Lemma 5, we know that the wage contract which induces the agent to hedge completely, $b=0$, i.e., $w^o(z(0),\eta)\equiv w^o(x,\eta)$, is one of the optimal contracts among the wage contracts that ensure $|R-a_3=\hat{b}|<\infty$ when $\mu_2^*>0$ for $w^*(z(\hat{b}))$. Thus, with Assumption 9, we characterize $w^o(x,\eta)$ that induces $b=0$ (i.e., $a_3=R$) as it minimizes the cash flow x's risk. Actually, $w^o(x,\eta)$ is obtained by solving a maximization problem that is similar to (38) with an added requirement that a contract induces the manager to take $a_3=R$.

Given that the agent's choosing a_3 given his private information R is equivalent to his choosing $b=R-a_3$, a new optimal contract, $w^o(x,\eta)$, inducing the agent to take $(a_1^o,a_2^o,b=0)$ when $\mu_2^*>0$ for $w^*(z(\hat{b}))$ in (51), must solve the following optimization problem:[47,48]

$$\begin{aligned}
&\max_{w(\cdot)\geq k}\int(x-w(x,\eta))g(x,\eta|a_1^o,a_2^o,b=0)dxd\eta+\lambda\left(\int u(w(x,\eta))g(x,\eta|a_1^o,a_2^o,b=0)dxd\eta-v(a_1^o)\right)\quad\text{s.t.}\\
&\qquad(i)\quad\int u(w(x,\eta))g_1(x,\eta|a_1^o,a_2^o,b=0)dxd\eta-v'(a_1^o)=0,\\
&\qquad(ii)\quad\int u(w(x,\eta))g_2(x,\eta|a_1^o,a_2^o,b=0)dxd\eta=0,\\
&\qquad(iii)\quad b=0\in\arg\max_{b'}\int u(w(x,\eta))g(x,\eta|a_1^o,a_2^o,b')dxd\eta,\ \forall b.
\end{aligned}\tag{54}$$

Note that we take the optimal (a_1^o,a_2^o) as given, and rely on the first-order approach for incentive constraints associated with the action a_1 and the project choice a_2. However, we do not use the same approach for the incentive compatibility constraint associated with the hedging choice b. The following Lemma 6 demonstrates the reason we cannot rely on the first-order approach for the incentive compatibility around b.

Lemma 6 *If $w^*(z(0))$ in (51) is designed, the agent will be indifferent between taking b and taking $-b$, $\forall b$.*

Lemma 6 shows that, if $w^*(z(0))$ is designed and offered, the manager's expected utility becomes symmetric

[47] Here the distribution $g(x,\eta|a_1,a_2,b)$ is of the same form as (45) with b in the position of R in (45).

[48] The analysis of the new optimal contract $w^o(x,\eta)$ follows closely to our treatment without project choice a_2 in Section 2.2. There is no specific role of a_2^o in deriving a new optimal contract $w^o(x,\eta)$ that induces perfect hedging from the agent.

around $b = 0$ (i.e., $a_3 = R$) in the space of b (i.e., in the space of a_3). As we know:

$$\int u(w^*(z(0)))g(z(0), \eta | a_1^o, a_2^o, b) dz d\eta \tag{55}$$

is continuous and differentiable in b, Lemma 6 implies:

$$\int u(w^*(z(0))g_3(z(0), \eta | a_1^o, a_2^o, b = 0) dz d\eta = 0, \tag{56}$$

where $g_3(\cdot|\cdot)$ denotes the first derivative of $g(\cdot|\cdot)$ taken with respect to b. Since $(w^*(z(0)), a_1^*, a_2^*)$ is the solution of the following optimization in (58) which is without the incentive constraint of b:[49]

$$\begin{aligned} \max_{a_1^o, a_2^o, w(\cdot) \geq k} \int (x - w(x,\eta)) g(x, \eta | a_1^o, a_2^o, b = 0) dx d\eta + \lambda \left(\int u(w(x,\eta)) g(x, \eta | a_1^o, a_2^o, b = 0) dx d\eta - v(a_1^o) \right) \quad \text{s.t.} \\ (i) \quad \int u(w(x,\eta)) g_1(x, \eta | a_1^o, a_2^o, b = 0) dx d\eta - v'(a_1^o) = 0, \\ (ii) \quad \int u(w(x,\eta)) g_2(x, \eta | a_1^o, a_2^o, b = 0) dx d\eta = 0, \end{aligned} \tag{58}$$

if we use the first-order approach for the incentive constraint associated with b in the above program (54), we always end up with getting $w^*(z(0))$ in (51) as an optimal contract for the above program (54). However, since $\mu_2^* > 0$ for $w^*(z(0))$, we can easily see from Lemma 4 that this contract incentivizes an agent to take $b = \pm\infty$ instead of taking a stipulated $b = 0$.[50] Therefore, we have to explicitly include the incentive constraint for b which does not rely only on the first-order condition at $b = 0$.

Without relying on the first-order approach, we follow Grossman and Hart (1983), replacing the incentive constraint for b (i.e., (iii) in (54)) with:

$$\int u(w(x,\eta)) \left(g(x, \eta | a_1^o, a_2^o, b = 0) - g(x, \eta | a_1^o, a_2^o, b) \right) dx d\eta \geq 0, \ \forall b, \tag{59}$$

which implies that the manager's indirect utility is maximized when he takes $b = 0$ (i.e., $a_3 = R$).

[49]Since for $b = 0$, the likelihood ratios can be represented as

$$\begin{aligned} &\frac{g_1}{g}(x, \eta | a_1, a_2, b = 0) = \frac{x - \phi(a_1, a_2)}{a_2^2} \phi_1(a_1, a_2), \quad \frac{g_3}{g}(x, \eta | a_1, a_2, b = 0) = \frac{(x - \phi(a_1, a_2))\eta}{a_2^2}, \\ &\frac{g_2}{g}(x, \eta | a_1, a_2, b = 0) = -\frac{1}{a_2} + \frac{x - \phi(a_1, a_2)}{a_2^2} \phi_2(a_1, a_2) + \frac{(x - \phi(a_1, a_2))^2}{a_2^3}, \end{aligned} \tag{57}$$

we see that $w^*(z(0))$ becomes the solution of the equation (58) without the incentive constraint of b.

[50]Note that the same technical problem about the first-order approach does not arise in the incentive constraint about the project choice a_2, which also determines the firm's risks. This is because reducing risk through a_2 is costly in terms of return, while doing it through a_3 is not. Thus, the manager's expected utility is not symmetric in the space of a_2.

Now we state formally the optimization problem of choosing the optimal contract $w^o(\cdot)$ given (a_1^o, a_2^o) as:

$$
\begin{aligned}
&\max_{w(\cdot)\geq k} \int (x - w(x,\eta))g(x,\eta|a_1^o, a_2^o, b=0)dxd\eta + \lambda\left(\int u(w(x,\eta))g(x,\eta|a_1^o,a_2^o,b=0)dxd\eta - v(a_1^o)\right) \quad \text{s.t.}\\
&\qquad (i) \quad \int u(w(x,\eta))g_1(x,\eta|a_1^o,a_2^o,b=0)dxd\eta - v'(a_1^o) = 0,\\
&\qquad (ii) \quad \int u(w(x,\eta))g_2(x,\eta|a_1^o,a_2^o,b=0)dxd\eta = 0,\\
&\qquad (iii) \quad \int u(w(x,\eta))\left(g(x,\eta|a_1^o,a_2^o,b=0) - g(x,\eta|a_1^o,a_2^o,b)\right)dxd\eta \geq 0, \ \forall b,
\end{aligned}
\tag{60}
$$

Note that the set of incentive constraints for all b (i.e., (59)) are taken into account to make sure the agent's expected indirect utility is at maximum when he chooses $b = 0$ instead of other $b > 0$ or $b < 0$.

The first-order condition of the above program (60) yields the optimal contract, $w^o(x,\eta)$, that satisfies[51]

$$
\begin{aligned}
\frac{1}{u'(w^o(x,\eta))} =& \lambda + (\mu_1^o\phi_1^o + \mu_2^o\phi_2^o)\frac{x-\phi(a_1^o,a_2^o)}{(a_2^o)^2} + \mu_2^o\frac{1}{a_2^o}\left(\frac{(x-\phi(a_1^o,a_2^o))^2}{(a_2^o)^2} - 1\right)\\
&+ \underbrace{\int \mu_4^o(b)\left(1 - \frac{g(x,\eta|b)}{g(x,\eta|b=0)}\right)db}_{\text{Additional term to (51)}},
\end{aligned}
\tag{61}
$$

for (x,η) satisfying $w^o(x,\eta) \geq k$ and $w^o(x,\eta) = k$ otherwise. In equation (61), $\phi_i^o \equiv \phi_i(a_1^o,a_2^o), i = 1,2$, and μ_1^o, μ_2^o, and $\mu_4^o(b)$ are the optimized Lagrange multipliers associated with the first, second, and third constraints (for specific b) in the above optimization program (60), respectively.

As shown in the Appendix, we obtain the following proposition from (61).

Proposition 10 [Hedging through Punishment]
If $\mu_2^* > 0$ for $w^*(z(0))$ described in equation (51), then the principal can motivate the manager to hedge completely by designing a new compensation contract, $w^o(x,\eta)$ in equation (61), which (i) satisfies $w^o(x,\eta) = w^o(x,-\eta)$ for all x,η, and (ii) penalizes the manager for having any (positive or negative) sample covariance between the output, x, and derivative market observables, η (i.e., penalizing manager for having high $(x-\phi)^2\eta^2$). To be specific, given realized (x,η), a higher sample covariance $(x-\phi)^2\eta^2$ yields a lower wage $w^o(x,\eta)$, while given the output x and sample covariance $(x-\phi)^2\eta^2$, a higher η raises the wage $w^o(x,\eta)$.

Proposition 10 can be understood in the following way: the production function $x = \phi(a_1^o,a_2^o) + a_2\theta + b\eta$ gives us the relation $b = Cov(x,\eta) = \mathbb{E}((x-\phi(a_1^o,a_2^o))\eta)$. It implies that if the agent takes $b = 0$, a statistical covariance between output x and hedgeable risk η disappears, whereas any other $b \neq 0$ generates non-zero population covariances. Since $b = 0$ generates $x = \phi(a_1^o,a_2^o) + a_2\theta$, which is independent of η, $w^o(x,\eta) =$

[51] We suppress the dependence of distribution g and likelihood ratios on (a_1^o,a_2^o).

$w^o(x, -\eta)$ for all x, η is ensured to minimize the amount of risk imposed on the agent, as η becomes irrelevant in inducing (a_1^o, a_2^o) and has a symmetric distribution around 0.

At optimum, by punishing the covariance between x and η,[52] shareholders effectively incentivize the manager to engage in full hedging and take $b = 0$. As our framework is in one-period setting, any positive or negative realized sample covariance $\widehat{Cov} = (x - \phi(a_1^o, a_2^o))\eta = b\eta^2 + a_2\theta\eta$, instead of a population covariance, is punished by the principal through a lower compensation $w^o(x, \eta)$. If the realized sample covariance $\widehat{Cov} = (x - \phi(a_1^o, a_2^o))\eta = b\eta^2 + a_2\theta\eta$ is large, not because of the manager's speculation ($b \neq 0$) but from a high realized market observable, $|\eta|$, then the principal takes it into account and raises $w^o(x, \eta)$. In contrast, given realized output and market observables (x, η), a bigger realization of $\widehat{Cov}$ is likely to be generated by $b \neq 0$ with a bigger $|b|$, thus the agent is punished and her wage income $w^o(x, \eta)$ falls.

Designing the new optimal contract including this covariance punishment is not, however, costless compared with $w^N(x, \eta)$ in equation (48), the optimal contract in case where there is no derivative market, since it exposes the agent to additional risks. As we show below, if this cost is relatively high compared to the informational gain that principal gets through the agent's derivative transaction, an introduction of derivative markets can actually reduce the welfare.

Proposition 11 *If $\mu_2^* > 0$ for $w^*(z(0))$ in equation (51), the introduction of a derivative market will reduce the firm's welfare compared with SW^N in equation (47) when the amount of uncertainty about the firm's risk exposure, σ_R^2, is small.*

The logic is similar to Proposition 5, except that whether the manager would speculate depend on the sign of μ_2^* here. As we show in Proposition 9, the optimal contract must be altered from $w^N(x, \eta)$ in the presence of derivative markets. In cases where the manager voluntarily chooses to hedge after the derivative market is introduced given his original optimal dual-agency contract (i.e., $\mu_2^* < 0$ for $w^*(y)$ in (41) or equivalently $\mu_2^* < 0$ for $w^*(z(0))$ in (51)), the compensation contract remains mainly unchanged from $w^*(y)$ while being based on $z(0) = x$ rather than y, and the welfare unambiguously increases by the informational gain generated by an opportunity of the manager to hedge in the derivative market and eliminate the firm's risk exposure R. However, when $w^*(z(0))$ with $\mu_2^* > 0$ induces the manager to speculate in the derivative market, shareholders must revise the manager's contract to $w^o(x, \eta)$ to provide the manager with an incentive to hedge, which imposes additional risks on the risk-averse manager's side and incurs the cost out of it. Thus, there are costs and benefits associated with derivative trading that the principal must consider.

Altering the wage contract to ensure that the agent hedges rather than speculates is costly since it needs

[52] It is possible since η is observable at the end of the period and thus contracts can be written upon it.

to consider the agent's additional incentive problem associated with a_3 by exposing him to the additional risk: market observables η, whereas the principal gets informational benefits as now she does not have to know about the firm's risk exposure R as the agent eliminates any hedgeable risk (i.e., $b = 0$ or $a_3 = R$) under $w^o(x, \eta)$. On the other hand, when there is no derivative market, principal's inability to observe R causes welfare loss because now she should offer $w^N(x, \eta)$ instead of $w^*(y)$.[53] To illustrate these costs and benefits more precisely, we use equation (42), equation (47), and equation (52), and decompose the welfare change in the following way.

$$SW^o(a_1^o, a_2^o) - SW^N = \left(SW^*(a_1^*, a_2^*) - SW^N\right) - \left(SW^*(a_1^*, a_2^*) - SW^o(a_1^o, a_2^o, R)\right). \tag{62}$$

The first part in the right-hand side represents the welfare loss due to the principal's inability to observe the firm's risk exposure when there is no derivative market (or equivalently informational gains from the introduction of a derivative market). The second part represents welfare loss due to the additional incentive problem associated with the manager's derivative choices when the derivative market is introduced and the manager speculates under $w^*(z(0))$ in equation (51).

Note that no expectation with respect to R is taken for joint benefits $SW^*(a_1^*, a_2^*)$ and $SW^o(a_1^o, a_2^o, a_3 = R)$, since both of them are independent of R. When there is no derivative market and the firm's risk exposure, R, is observed by the principal as well as the manager, joint benefits, $SW^*(a_1^*, a_2^*)$, are obviously independent of the R's realization because (a_1^*, a_2^*) are independent of R. Likewise, when $w^o(x, \eta)$ is designed in the presence of derivative markets, the joint benefits $SW^o(a_1^o, a_2^o, a_3 = R)$ are independent of R as agent is always induced to take $b = R - a_3 = 0$ no matter what R is realized. However, in calculating joint benefits SW^N, an expectation with respect to R is taken, implying that the distribution of R affects the level of SW^N.

The above discussion implies that informational gains from the manager's derivative transaction declines as the amount of uncertainty around the firm's risk exposure R falls. On the other hand, the cost of controlling the additional incentive problem associated with a_3 (or equivalently $b = R - a_3$) is independent of the firm's risk exposure R and thus σ_R^2. For instance, even if R is known to the principal (i.e., $\sigma_R^2 = 0$), the moral hazard problem associated with inducing $b = 0$ still remains to the same degree. Therefore, the amount of uncertainty on R is indeed a matter of indifference in incentivizing the agent's choice of b.

As a result, if the uncertainty around R is small enough, the contractual cost dominates informational gains when derivative markets are introduced, and shareholders would be better off by prohibiting the manager from trading derivatives at all. Therefore, recent financial innovations have potentials to hurt the

[53] Of course, principal can always design $w^o(x, \eta)$ instead of $w^N(x, \eta)$ when there is no derivative market. However, $w^o(x, \eta)$ will perform poorly without the derivative market.

efficiency of firms through its effects on agency relationships.

6 The Truth-Telling Mechanism

Up to this point we have assumed that there is no communication between principal and agent after compensation contract is written, due to high communication costs. We now relax this assumption and consider the case where the agent can costlessly communicate the firm's risk exposure R to the principal, and receive a payoff that is contingent on the communicated risk exposure as well as the output and hedgeable risks.

As we will show below, for the case where $\mu_2^* < 0$ for $w^*(z(0))$ in equation (51), a contract that is similar to $w^*(z(0))$ can be designed to induce the agent to truthfully reveal the firm's risk exposure R. In other words, there is no loss associated with the risk exposure being unobservable and thus no gain from the introduction of derivative market. The intuition is the same as the one for why the manager would voluntarily hedge under $w^*(z(0))$ with $\mu_2^* < 0$. Essentially, the truth-telling contract allows the agent to make a *side bet* with the principal and if the agent hedges in the derivative market with the contract $w^*(z(0))$, he will truthfully reveal what he observes (true R) to minimize the additional risk associated with this side bet.

However, when $\mu_2^* > 0$ for $w^*(z(0))$ in equation (51), a contract similar to $w^*(z(0))$ does not induce truth-telling since the manager wants to add more risks, as he does by engaging in speculation in derivative markets. Again, a new contract must be designed to induce him to reveal the truth.

Suppose the principal does not know the firm's innate risk exposure R and there is no derivative market (i.e., a_3 is again fixed at 0). Since the agent observes R before he takes (a_1, a_2) and communication regarding R is freely allowed, the principal can design a truth-telling mechanism, $w(x, r, \eta)$, without incurring cost where r represents the value of R reported by the agent. Let $(a_1^T(R), a_2^T(R))$ be agent's optimal action combination after observing R and $w^T(x, r, \eta)$ be the wage contract that optimally induces $(a_1^T(R), a_2^T(R))$ with the agent telling the truth. Knowing that $r = R, \forall R$, under $w^T(x, r, \eta)$, we denote optimized joint benefits in this case as

$$SW^T \equiv \int \Big(\phi(a_1^T(R), a_2^T(R)) - C^T(a_1^T(R), a_2^T(R)) - \lambda v(a_1^T(R))\Big) h(R) dR, \tag{63}$$

where

$$C^T(a_1^T(R), a_2^T(R)) \equiv \int \Big(w^T(x, R, \eta) - \lambda u(w^T(x, R, \eta))\Big) g(x, \eta | a_1^T(R), a_2^T(R)) dx d\eta \tag{64}$$

denotes the agency cost arising from inducing $(a_1^T(R), a_2^T(R))$ through $w^T(x, r, \eta)$ when R is realized. In the above equation, $g(x, \eta | a_1^T(R), a_2^T(R))$ denotes the joint density function of (x, η) given that $(a_1^T(R), a_2^T(R))$ is chosen by the manager when a_3 is fixed at 0.

Since $SW^*(a_1^*, a_2^*)$ in (42) is the maximum level of joint benefits that SW^T can attain, we first consider the case in which principal designs a wage contract, $w^*(y_r)$, that is the same as $w^*(y)$ in (39) except that it is based on $y_r \equiv x - r\eta$ instead of $y \equiv x - R\eta$. That is, $w^*(y_r)$ satisfies

$$\frac{1}{u'(w^*(y_r))} = \lambda + (\mu_1^*\phi_1^* + \mu_2^*\phi_2^*)\frac{y_r - \phi(a_1^*, a_2^*)}{(a_2^*)^2} + \mu_2^*\frac{1}{a_2^*}\left(\frac{(y_r - \phi(a_1^*, a_2^*))^2}{(a_2^*)^2} - 1\right), \tag{65}$$

for y_r satisfying $w^*(y_r) \geq k$ and $w^*(y_r) = k$ otherwise. We call $w^*(y_r)$ the *full-trust contract* as $w^*(y_r)$ simply is based on the agent's report instead of the realized R in (65).

Note that

$$y_r \equiv x - r\eta = \phi(a_1, a_2) + (R - r)\eta + a_2\theta. \tag{66}$$

Since, from (49),

$$z(0) = x = \phi(a_1, a_2) + (R - a_3)\eta + a_2\theta, \tag{67}$$

the principal's problem of designing a truth-telling mechanism based on y_r when there is no derivative market is equivalent to his problem of designing an incentive scheme based on $z(0)$ to induce $b = 0$ (i.e., $a_3 = R$) when derivative transactions are allowed. As a result, as is the case for $w^*(z(0))$ in Lemma 4, we directly obtain following results for $w^*(y_r)$.

Lemma 7 [Speculation and Hedging with $w^*(y_r)$]

(1) *If $\underline{\mu_2^* < 0}$ for the wage contract, $w^*(y_r)$, described in equation (65), then the manager will always report truly, i.e., $r = R, \forall R$, when $w^*(y_r)$ is offered.*

(2) *If $\underline{\mu_2^* > 0}$ for $w^*(y_r)$, then the manager will report r such that $|R - r| = \infty$ when $w^*(y_r)$ is offered.*

From Lemma 7, we obtain the following propositions.

Proposition 12 *When there is no derivative market and communication between principal and agent is costless, then $w^*(y_r)$ described in equation (65) is the optimal truth-telling mechanism for the firm's hidden risk exposure, R, if $\underline{\mu_2^* < 0}$ for $w^*(y_r)$. In this case:*

(1) The principal's inability to observe R does not reduce the firm's welfare (i.e., no adverse selection).

(2) An introduction of a derivative market does not improve the firm's welfare compared with SW^T in (63).

Proposition 12 along with Propositions 8 and 9 reaffirms that the benefits that derivative markets bring are actually informational gains, as the agent engages in the perfect hedging in the derivative market. Because we assumed initially that principal and agent cannot communicate with each other due to the huge cost, these benefits are actually associated with saving communication costs[54] that would realistically incur when principal has to design a truth-telling contract that induces the agent to reveal his exact information about the firm's risk exposure R. When the communication between principal and agent becomes free, the principal, by designing $w^*(y_r)$, can easily reproduce the same results as when he exactly knows the firm's innate risk exposure if $\mu_2^* < 0$ for $w^*(y_r)$. However, in reality, the costs associated with communicating this information and updating the compensation contract based on the reavealed R may be greater than the hedging cost. As shown in (66), allowing the manager to choose a_3 in derivative transactions is observationally equivalent to allowing him to freely report the firm's realized risk exposure R.

On the other hand, if $\mu_2^* > 0$ for $w^*(y_r)$, the manager does not report the true R under $w^*(y_r)$, and shareholders have to redesign a truth-telling mechanism, $w^T(x, r, \eta)$ different from $w^*(y_r)$.

Proposition 13 *When $\underline{\mu_2^* > 0}$ for $w^*(y_r)$ described in equation (65), the introduction of a derivative market does not improve on the firm's efficiency when communication between the principal and the agent is freely allowed, and it actually lowers the firm's efficiency if σ_R^2 is very small.*

As explained in Proposition 11, when communication between shareholders and manager is not available and $\mu_2^* > 0$ for $w^*(y_r)$ in (51), the manager's opportunity to transact derivatives may or may not improve the firm's welfare compared to the case without the derivative market depending on the size of uncertainty σ_R^2 on the exposure R.

If communication becomes free between principal and agent however, an access to the derivative market actually reduces the firm's welfare when σ_R^2 is small enough. It is because both $w^o(x, \eta)$ in (61) and $w^N(x, \eta)$ in (48)[55] are actually truth-telling contracts. Therefore, when there is no derivative market, the principal can at least design either $w^o(x, \eta)$ or $w^N(x, \eta)$ under free communication depending on which of two gives the better welfare.[56] As shown in Proposition 11, the principal prefers designing $w^N(x, \eta)$ to $w^o(x, \eta)$ if σ_R^2 is very small. The optimal truth-telling mechanism, $w^T(x, r, \eta)$, thus performs <u>weakly</u> better than $w^N(x, \eta)$, which yields <u>strictly</u> larger welfare than $w^o(x, \eta)$. However, after the derivative market is introduced, the principal has to shift from $w^T(x, r, \eta)$ to $w^o(x, \eta)$ because there now exists an incentive problem associated with a_3.

In summary, when communication between shareholders and manager becomes free, the manager's access to derivative market transactions does not change the firm's welfare if $\mu_2^* < 0$, and even lowers it if $\mu_2^* > 0$.

[54] In the presence of derivative markets, principal and agent do not need to communicate about the realized R, since the agent can eliminate this innate risk R through derivative transactions (i.e., $a_3 = R$).

[55] Note that both $w^o(x, \eta)$ and $w^N(x, \eta)$ do not depend on the reported value of R, so we regard both two contracts as truth-telling mechanism.

[56] The optimal truth-telling mechanism in this case may not be $w^o(x, \eta)$ or $w^N(x, \eta)$.

7 Conclusion

As different kinds of financial crises hit the world economy, interests in (i) risk management and (ii) welfare implications of new developments in financial markets have increased dramatically. Since then, academics and policymakers proposed a number of explanations for the economy-wide effects of innovations in financial markets, different ways to manage corporate risks, and rationales for more strict regulations. However, up to our knowledge, few prior works have seriously attempted to connect this trend with the managerial incentive aspects. In line with the former Federal Reserve chairman Ben Bernanke's emphasis on the role of 'compensation structures' in many banking sector corporations in generating excessive risks[57], this agency aspect should be taken into account more seriously when we try to understand the implication of financial market innovations in relation to risk-management issues.

Many corporations in *Wall street* and the *Main Street* have used various forms of compensation contracts to motivate their workers to engage in innovations. However, under certain circumstances, as illustrated by the shadow banking industries during the global financial crisis (GFC), managers of those companies speculate rather than hedge, if compensation contracts are not modified to account for unobservable opportunities to alter the firm's risk exposures. When compensation contracts do need to be modified, some efficiency gains associated with the introduction of hedging instruments is lessened because the new contractual form may less efficiently share risk between the manager and shareholders and induce the efforts. Indeed, our analysis suggests that in some circumstances, the incentive costs associated with keeping the agent from speculating in the derivative market is sufficiently large that firms would be better off by prohibiting derivative transactions. This point is understood in line with the last two decade's hard works of governments, central banks, and international institutions to curb the degree of risk-taking through regulations on incentive compensation schemes. For example, 2010's 'Dodd-Frank Wall Street Reform and Consumer Protection Act', section 956, mandates that the agencies must prohibit all covered institutions from establishing or maintaining incentive-based compensation arrangements that encourage inappropriate risk-taking.[58]

We stress that while we focused on an agency relationship, improving the information content of a firm's cash flows by reducing the risk can improve firm values in a number of different ways. For example, a lower risk provides better information about the managers' *abilities* as well as their *effort* levels, which allows shareholders to better match managers with appropriate positions. In addition, more informative cash flows

[57] "The Federal Reserve is working to ensure that compensation packages appropriately tie rewards to longer-term performance and do not create undue risk for the firm or the financial system.": https://www.nytimes.com/2009/10/23/business/23pay.html

[58] Federal Reserve's 06.21.2010 joint press said, "Last month, the Federal Reserve delivered assessments to the firms that included analysis of current compensation practices and areas requiring prompt attention. Firms are submitting plans to the Federal Reserve outlining steps and timelines for addressing outstanding issues to ensure that incentive compensation plans do not encourage excessive risk-taking."

are likely to improve the informational efficiency of firms' share prices. With more informative stock prices, capitals are allocated more efficiently and managers have less incentive to take short-sighted actions and in other ways expend resources signalling their firms' values.

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Appendix. Derivations and Proofs

Derivation of equation (13): With Lagrange multiplier $\mu_1(R)$ attached to the incentive constraint for action $a_1(R)$ for any given R, we set up the Lagrangian for the optimization in (11) as follows:

$$\begin{aligned}\mathcal{L} = &\int_R \left[\int_{x,\eta} (x - w(x,\eta) + \lambda u(w(x,\eta)))\, g(x,\eta|a_1(R),R)dxd\eta - v(a_1(R))\right] h(R)dR \\ &+ \int_R \mu_1(R) \left[\int_{x,\eta} u(w(x,\eta))g_1(x,\eta|a_1(R),R)dxd\eta - v'(a_1(R))\right] h(R)dR\end{aligned} \tag{A1}$$

from which we get the following first-order Euler equation about $w(\cdot,\cdot)$:

$$(-1+\lambda u'(w(x,\eta))) \int_R g(x,\eta|a_1(R),R)h(R)dR + u'(w(x,\eta)) \left[\int_R \mu_1(R)g_1(x,\eta|a_1(R),R)\cdot h(R)dR\right] = 0, \tag{A2}$$

which derives the optimal contract $w^N(x,\eta)$ in equation (13). ■

Proof of Proposition 2: Consider the principal's following alternative maximization program:

$$\begin{aligned}\max_{a_1(\cdot),w(\cdot)\geq k} &\int_R \int_{x,\eta} (x - w(x,R,\eta))\, g(x,\eta|a_1(R),R)h(R)dxd\eta dR \\ &+ \lambda \int_R \left(\int_{x,\eta} u(w(x,R,\eta))g(x,\eta|a_1(R),R)dxd\eta - v(a_1(R))\right) h(R)dR \quad \text{s.t.} \\ &(i) \quad \int_{x,\eta} u(w(x,R,\eta))g_1(x,\eta|a_1(R),R)dxd\eta - v'(a_1(R)) = 0, \forall R.\end{aligned} \tag{A3}$$

Note that the above program is different from the original program (11) in that here contract can be written on the realized value of R. If we let the Lagrange multipliers to the constraint be $\mu_1(R)h(R)$, we get the following optimal contractual form:

$$\frac{1}{u'(w(x,R,\eta))} = \lambda + \mu_1(R)\frac{\underbrace{x - R\eta}_{\equiv y} - \phi(a_1(R))}{\sigma^2}\phi_1(a_1(R)), \tag{A4}$$

when $w(x,R,\eta) \geq k$. The above equation (A4) implies that the optimal contract only depends on $y \equiv x - R\eta$ and the solution $\{a_1(R), w(x,R,\eta)\}$ becomes $\{a_1^*, w^*(y) \equiv w^*(x - R\eta)\}$ in equation (5) in Section 2.1. By comparing equation (A4) with the program in (11) when he does not know R, one can easily see that the set of wage contracts, $\{w(x,R,\eta)\}$, satisfying the incentive constraints for a given action $a_1(R)$ in the above program (A3), always contains the set of wage contracts that would be available when the principal does not know R, $\{w(x,\eta)\}$, satisfying the incentive constraints for the same action. Therefore, we have

$$SW^N \leq SW^*. \tag{A5}$$

However, one can easily see that $w^*(y) = w^*(x - R\eta)$ which is a unique solution for the wage contract of the above program (A3) is not in the set of $\{w(x, \eta)\}$. As a result, we finally derive

$$SW^N < SW^*. \tag{A6}$$

Proof of Lemma 2: Given $w^*(x)$ described in equation (5) is designed,[1] if the agent takes (a_1^o, b) under $w^*(x)$, then his expected utility becomes:

$$\int u(w^*(x))g(x, \eta | a_1^o, b)dzd\eta - v(a_1^o) = \int u(w^*(x))q(x|a_1^o, b, \eta)l(\eta)dzd\eta - v(a_1^o), \tag{A7}$$

where $q(\cdot)$ denotes the conditional density function of x given (a_1^o, b, η) and $l(\cdot)$ denotes the density function of $\eta \sim N(0, 1)$.

Now, suppose the agent takes $(a_1^o, -b)$ under $w^*(x)$. Then, his expected utility becomes:

$$\int u(w^*(x))g(x, \eta | a_1^o, -b)dzd\eta - v(a_1^o) = \int u(w^*(x))q(x|a_1^o, -b, \eta)l(\eta)dzd\eta - v(a_1^o). \tag{A8}$$

Since

$$q(x|a_1^o, b, \eta) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(x - \phi(a_1^o) - b\eta)^2}{2\sigma^2}\right), \tag{A9}$$

we have

$$q(x|a_1^o, b, \eta) = q(x|a_1, -b, -\eta). \tag{A10}$$

Since $\eta \sim N(0, 1)$ is symmetrically distributed around 0 and $l(\eta) = l(-\eta)$, $\forall \eta$, we finally have

$$\int u(w^*(x))g(x, \eta | a_1^o, b)dzd\eta - v(a_1^o) = \int u(w^*(x))g(x, \eta | a_1^o, -b)dzd\eta - v(a_1^o). \tag{A11}$$

■

Proof of Proposition 3: To prove this proposition, we start with the following Lemma 8.

Lemma 8 *When the agent's indirect utility $V(x)$ in the absence of the hedgeable risk η is convex in output x, then the optimal contract $w^o(x, \eta)$ guaranteeing that the agent takes $a_1^o, a_d^o = R$ (i.e., $b = 0$), i.e., $w^o(x, \eta)$ in equation (22), must satisfy*

(1) $\mu_b^o(b) \neq 0$ (> 0) for a positive Borel-measure of b.[2]

(2) $w^o(x, \eta) = w^o(x, -\eta)$ for all x, η and $\mu_b^o(b) = \mu_b^o(-b)$ for all b.

[1] The output x is given by $x = \phi(a_1^o) + \sigma\theta + b\eta$ given a_1^o and $b = R - a_d$.
[2] We already know $\mu_4^o(b) \geq 0$ for every b (almost surely), since it is derived from the inequality constraint at each b.

Proof. (1) $\underline{\mu_b^o(b) \neq 0 \text{ for a positive Borel-measure of } b}$: Assume $\mu_b^o(b) = 0$, *a.s.* Then the optimal contract $w^o(x,\eta)$ in (22) can be written as

$$\frac{1}{u'(w^o(x,\eta))} = \lambda + \mu_1^o \frac{x - \phi(a_1^o)}{\sigma^2} \phi_1(a_1^o), \tag{A12}$$

for (x,η) satisfying $w^o(x,\eta) \geq k$ and $w^o(x,\eta) = k$.

Because we already know $(w^o(x,\eta), \mu_1^o, a_1^o)$ becomes $(w^*(x), \mu_1^*, a_1^*)$ in this case and $V(x)$ (i.e., the agent's indirect utility given $w^*(x)$) is convex in x by assumption, $(w^o(x,\eta), \mu_1^o)$ will induce $b = \pm\infty$ instead of $b = 0$ from the agent, a contradiction to the constraint (ii) in the optimization (16).

(3) $\underline{w^o(x,\eta) = w^o(x,-\eta) \text{ for all } x, \eta \text{ and } \mu_4^o(b) = \mu_4^o(-b) \text{ for all } b}$: We first see:[3,4]

$$g(x,\eta|b) = \frac{1}{2\pi\sigma} \exp\left(-\frac{1}{2}\frac{(x - \phi(a_1^o) - b\eta)^2}{\sigma^2} - \frac{1}{2}\eta^2\right), \tag{A14}$$

where

$$\frac{g(x,\eta|b)}{g(x,\eta|b=0)} = \exp\left(\frac{b\eta(x - \phi(a_1^o))}{\sigma^2}\right) \exp\left(-\frac{b^2\eta^2}{2\sigma^2}\right). \tag{A15}$$

From (A13), (A14), and (A15), we observe that $g(x,\eta|b=0)$ and $g_1(x,\eta|b=0)$ are both even with η where g_1 is a partial derivative of g with respect to a_1: i.e., (i) $g(x,-\eta|b=0) = g(x,\eta|b=0)$; (ii) $g_1(x,-\eta|b=0) = g_1(x,\eta|b=0)$. Also from (A14), we acknowledge:

$$g(x,-\eta|b) = g(x,\eta|-b), \;\; \forall (x,\eta,b). \tag{A16}$$

Our strategy is to prove that: (i) if $w^o(x,\eta)$ is an optimal contract, then $w^o(x,-\eta)$ satisfies all the constraints in (16); (ii) Related to (i), if $w^o(x,\eta)$ is an optimal contract, then $w^o(x,-\eta)$ also becomes an optimal contract; and (iii) $\mu_b^o(-b) = \mu_b^o(b)$ for $\forall b$ at the optimum.

Step 1. If $w^o(x,\eta)$ is an optimal contract, then $w^o(x,-\eta)$ satisfies all the constraints in (16).

(i) As $w^o(x,\eta)$ is optimal, note that it satisfies all of the constraints in (16). We start from the incentive compatibility

[3] We suppress a_1^o in $g(x,\eta|a_1^o,b)$ in (16) to make our expressions simpler.

[4] Note that $g(x,\eta|a_1,b)$ yields the following likelihood ratios:

$$\frac{g_1}{g}(x,\eta|a_1,b) = \frac{x - b\eta - \phi(a_1)}{\sigma^2}\phi_1(a_1), \;\; \frac{g_b}{g}(x,\eta|a_1,b) = \frac{(x - b\eta - \phi(a_1))\eta}{\sigma^2}. \tag{A13}$$

in action a_1: based on that $g_1(x,\eta|b=0)$ is even in η,

$$\begin{aligned}\int u(w^o(x,-\eta))g_1(x,\eta|b=0)dxd\eta - v'(a_1^o) &= \int u(w^o(x,-\eta))g_1(x,-\eta|b=0)dxd\eta - v'(a_1^o)\\ &= \int u(w^o(x,\eta))g_1(x,\eta|b=0)dxd\eta - v'(a_1^o) = 0,\end{aligned}$$

where we use the change of variable (i.e., $-\eta$ to η) in the second equality.

(ii) Incentive compatibility in *after-hedging* risk exposure b: as $w^o(x,\eta)$ is optimal, we know it satisfies

$$\int u(w^o(x,\eta))\left(g(x,\eta|b=0) - g(x,\eta|b)\right)dxd\eta \geq 0,\ \underline{\forall b}. \tag{A17}$$

From (A16) and that $g(x,\eta|b=0)$ is even in η, we obtain for $\forall b$,

$$\begin{aligned}\int u(w^o(x,-\eta))\left(g(x,\eta|b=0) - g(x,\eta|b)\right)dxd\eta &= \int u(w^o(x,-\eta))\left(g(x,-\eta|b=0) - g(x,-\eta|-b)\right)dxd\eta\\ &= \int u(w^o(x,\eta))\left(g(x,\eta|b=0) - g(x,\eta|-b)\right)dxd\eta \geq 0,\end{aligned} \tag{A18}$$

where the first equality is from (A16) and the second equality is from the change of variable (i.e., $-\eta$ to η). Thus, we proved that if $w^o(x,\eta)$ is an optimal contract, then $w^o(x,-\eta)$ satisfies all the constraints in (16).

Step 2. Next, if $w^o(x,\eta)$ is an optimal contract, then $w^o(x,-\eta)$ also becomes an optimal contract.

From the above **Step 1**, $w^o(x,-\eta)$ satisfies all the constraints in (16). It is sufficient to show that $w^o(x,-\eta)$ achieves the same efficiency as $w^o(x,\eta)$. It follows from:

$$\begin{aligned}&\int(x-w^o(x,-\eta))g(x,\eta|b=0)dxd\eta + \lambda\left(\int u(w^o(x,-\eta))g(x,\eta|b=0)dxd\eta - v(a_1^o)\right)\\ &= \int(x-w^o(x,-\eta))g(x,-\eta|b=0)dxd\eta + \lambda\left(\int u(w^o(x,-\eta))g(x,-\eta|b=0)dxd\eta - v(a_1^o)\right)\\ &= \int(x-w^o(x,\eta))g(x,\eta|b=0)dxd\eta + \lambda\left(\int u(w^o(x,\eta))g(x,\eta|b=0)dxd\eta - v(a_1^o)\right),\end{aligned} \tag{A19}$$

where the first equality is from that $g(x,\eta|b=0)$ is symmetric in η, and the second equality is from the change of variable (i.e., $-\eta$ to η). Therefore, if $w^o(x,\eta)$ is an optimal contract, then $w^o(x,-\eta)$ becomes an optimal contract and we obtain $w^o(x,-\eta) = w^o(x,\eta)$.[5]

Step 3. $\mu_b^o(-b) = \mu_b^o(b)$ for $\forall b$.

[5]We implicitly assume that the optimal contract is unique in this environment, following the literature (e.g., Jewitt et al. (2008)).

Note from the Lagrange duality theorem (see e.g., Luenberger (1969)) that the optimal solution $(\mu_1^o, \{\mu_b^o(b)\}, w^o(\cdot))$ is the one that solves $\min_{\mu_1,\{\mu_b(\cdot)\}} \max_{w(\cdot)} \mathcal{L}$ where $\mathcal{L}$ is given by

$$\begin{aligned}\mathcal{L} \equiv \int (x - w(x,\eta))g(x,\eta|b=0)dxd\eta + \lambda \left(\int u(w(x,\eta))g(x,\eta|b=0)dxd\eta - v(a_1^o) \right) \\ + \mu_1 \left(\int u(w(x,\eta))g_1(x,\eta|b=0)dxd\eta - v'(a_1^o) \right) + \int_b \mu_b(b) \left(\int u(w(x,\eta)) \left(g(x,\eta|b=0) - g(x,\eta|b) \right) dxd\eta \right) db,\end{aligned}$$

while satisfying $\mu_b^o(b) \geq 0$ for $\forall b$ and the following complementary slackness condition at the optimum:

$$\mu_b^o(b) \left(\int u(w^o(x,\eta)) \left(g(x,\eta|b=0) - g(x,\eta|b) \right) dxd\eta \right) = 0, \quad \forall b. \tag{A20}$$

The last term in the above Lagrangian $\mathcal{L}$ given the optimal contract $w^o(x,\eta)$ can be written as

$$\begin{aligned}&\int_b \mu_4(b) \left(\int u(w^o(x,\eta)) \left(g(x,\eta|b=0) - g(x,\eta|b) \right) dxd\eta \right) db \\ &\quad = \int_b \mu_4(-b) \left(\int u(w^o(x,-\eta)) \left(g(x,\eta|b=0) - g(x,\eta|-b) \right) dxd\eta \right) db,\end{aligned} \tag{A21}$$

where we use the change of variable (i.e., b to $-b$) and $w^o(x,-\eta) = w^o(x,\eta)$. Now with (A16) and that $g(x,\eta|b=0)$ is even in η, we know:

$$\begin{aligned}\int u(w^o(x,-\eta)) \left(g(x,\eta|b=0) - g(x,\eta|-b) \right) dxd\eta &= \int u(w^o(x,-\eta)) \left(g(x,-\eta|b=0) - g(x,-\eta|b) \right) dxd\eta \\ &= \int u(w^o(x,\eta)) \left(g(x,\eta|b=0) - g(x,\eta|b) \right) dxd\eta,\end{aligned} \tag{A22}$$

where we use the change of variable (i.e., $-\eta$ to η) for the second equality. With (A21) and (A22), the last term in Lagrangian $\mathcal{L}$ can be therefore written as

$$\begin{aligned}&\int_b \mu_4(b) \left(\int u(w^o(x,\eta)) \left(g(x,\eta|b=0) - g(x,\eta|b) \right) dxd\eta \right) db \\ &\quad = \int_b \mu_4(-b) \left(\int u(w^o(x,\eta)) \left(g(x,\eta|b=0) - g(x,\eta|b) \right) dxd\eta \right) db.\end{aligned} \tag{A23}$$

Plugging in (A23) to the original Lagrangian $\mathcal{L}$ yields $\mu_4^o(-b) = \mu_4^o(b)$.

<u>**Step 4**</u>. We have:

$$\int u(w^o(x,\eta))g(x,\eta|b)dxd\eta = \int u(w^o(x,\eta))g(x,\eta|-b)dxd\eta, \tag{A24}$$

which implies that the agent's indirect utility is symmetric in b around $b = 0$.

It follows from:

$$\begin{aligned}\int u(w^o(x,\eta))g(x,\eta|-b)dxd\eta &= \int u(w^o(x,\eta))g(x,-\eta|b)dxd\eta = \int u(w^o(x,-\eta))g(x,-\eta|b)dxd\eta \\ &= \int u(w^o(x,\eta))g(x,\eta|b)dxd\eta,\end{aligned} \tag{A25}$$

where we use (A16) in the first equality, $w^o(x,-\eta) = w^o(x,\eta)$ in the second, and and the change of variable (i.e., $-\eta$ to η) in the third equality. ■

Proof of Proposition 3: Given the optimal action a_1^o, we define $\widehat{Cov} \equiv (x-\phi(a_1^o))\eta$.[6] Since

$$\exp\left(\frac{b\eta(x-\phi(a_1^o))}{\sigma^2}\right) = \exp\left(\frac{b}{\sigma^2}\widehat{Cov}\right) = \sum_{k=0}^{\infty}\frac{1}{k!}\frac{b^k}{\sigma^{2k}}\widehat{Cov}^k, \tag{A26}$$

From equation (A15), we obtain

$$\frac{g(x,\eta|b)}{g(x,\eta|b=0)} = \left(\sum_{k=0}^{\infty}\frac{1}{k!}\frac{b^k}{\sigma^{2k}}\widehat{Cov}^k\right)\exp\left(-\frac{b^2\eta^2}{2\sigma^2}\right), \tag{A27}$$

and therefore, we attain

$$\begin{aligned}\int \mu_4^o(b)\left(1-\frac{g(x,\eta|b)}{g(x,\eta|b=0)}\right)db &= \int \mu_4^o(b)db - \int \mu_4^o(b)\left(\sum_{k=0}^{\infty}\frac{1}{k!}\frac{b^k}{\sigma^{2k}}\widehat{Cov}^k\right)\exp\left(-\frac{b^2\eta^2}{2\sigma^2}\right)db \\ &= \int \mu_4^o(b)db - \sum_{k=0}^{\infty}\left(\frac{1}{k!}\frac{1}{\sigma^{2k}}\underbrace{\left(\int \mu_4^o(b)b^k\exp\left(-\frac{b^2\eta^2}{2\sigma^2}\right)db\right)}_{\equiv C_k(\eta)}\right)\widehat{Cov}^k.\end{aligned} \tag{A28}$$

When k is odd, the coefficient $C_k(\eta)$ becomes 0 for $\forall\eta$, since $\mu_4^o(b) = \mu_4^o(-b)$ for all b from Lemma 8 implies

$$C_{k:odd}(\eta) = \int \mu_4^o(b)b^k\exp\left(-\frac{b^2\eta^2}{2\sigma^2}\right)db = \int_{b\geq 0}\left(\underbrace{\mu_4^o(b)-\mu_4^o(-b)}_{=0}\right)b^k\exp\left(-\frac{b^2\eta^2}{2\sigma^2}\right)db = 0. \tag{A29}$$

When k is even, the coefficient $C_k(\eta)$ becomes strictly positive for $\forall\eta$, since $\mu_4^o(b) \neq 0$ for the non-zero measure of b from Lemma 8 implies

$$\begin{aligned}C_{k:even}(\eta) = \int \mu_4^o(b)b^k\exp\left(-\frac{b^2\eta^2}{2\sigma^2}\right)db &= \int_{b\geq 0}(\mu_4^o(b)+\mu_4^o(-b))b^k\exp\left(-\frac{b^2\eta^2}{2\sigma^2}\right)db \\ &= 2\int_{b\geq 0}\mu_4^o(b)b^k\exp\left(-\frac{b^2\eta^2}{2\sigma^2}\right)db > 0.\end{aligned} \tag{A30}$$

[6]This is a realized value of sample covariance between x and η, as our framework is in single-period setting.

Therefore, (A28) can be written as

$$\int \mu_4^o(b)\left(1-\frac{g(x,\eta|b)}{g(x,\eta|b=0)}\right)db = \int \mu_4^o(b)db - 2\sum_{k:even}^{\infty}\left(\frac{1}{k!}\frac{1}{\sigma^{2k}}\left(\int_{b\geq 0}\mu_4^o(b)b^k \exp\left(-\frac{b^2\eta^2}{2\sigma^2}\right)db\right)\right)\widehat{Cov}^k. \tag{A31}$$

Finally, we can plug the expression (A31) into our optimal contact $w^o(x,\eta)$ in (22) when $w^o(x,\eta)\geq k$ and obtain

$$\frac{1}{u'(w^o(x,\eta))} = \lambda + \mu_1^o\frac{x-\phi(a_1^o)}{\sigma^2}\phi_1(a_1^o) + \underbrace{\int \mu_4^o(b)db}_{>0} - 2\sum_{k:\text{even}}^{\infty}\underbrace{\frac{1}{k!}\frac{1}{\sigma^{2k}}\underbrace{\left(\int_{b\geq 0}\mu_4^o(b)b^k\exp\left(-\frac{b^2\eta^2}{2\sigma^2}\right)db\right)}_{\equiv C_{k:\text{even}}(\eta)>0}}_{\equiv D_{k:\text{even}}(\eta)>0}\widehat{Cov}^k. \tag{A32}$$

Since $D_{k:\text{even}}(\eta) > 0$ for all even numbers k, given (x,η) a higher $\widehat{Cov}$ results in a lower compensation $w^o(x,\eta)$. Also as $D_{k:\text{even}}(\eta) > 0$ decreases in η^2, given $(x,\widehat{Cov})$, a higher η^2 results in a higher $w^o(x,\eta)$. In sum the principal punishes a sample covariance $|\widehat{Cov}|$ but becomes lenient when a high $|\widehat{Cov}|$ comes from the high η realization, not from the agent's speculation activity ($b \neq 0$). ■

Proof of Lemma 3: We know from $y \sim N(\phi(a_1,a_2),a_2^2)$ that

$$\frac{y-\phi(a_1,a_2)}{a_2} \sim N(0,1), \quad \frac{f_1}{f}(y|a_1,a_2) = \frac{y-\phi(a_1,a_2)}{a_2^2}\phi_1(a_1,a_2) \sim N\left(0,\frac{\phi_1(a_1,a_2)^2}{a_2^2}\right). \tag{A33}$$

Therefore, we observe that if $\frac{\phi_1(a_1,a_2)}{a_2}$ is decreasing in a_2, for any pair $a_2^0 < a_2^1$, $\frac{f_1}{f}(y|a_1,a_2^0)$'s distribution is mean-preserving spread (MPS) of that of $\frac{f_1}{f}(y|a_1,a_2^1)$. Assumption 8 guarantees that this condition holds, and the following Lemma 9, a slightly changed form of Kim (1995), proves $C(a_1,a_2^0) < C(a_1,a_2^1)$ for $\forall a_1$.

Lemma 9 *For given action a_1 and technology $h(\cdot|a_1)$, let the solution of the following optimization problem be $w_h(\cdot)$:*

$$\begin{aligned}&\max_{w(\cdot)}\int (y-w(y))h(y|a_1)dy + \lambda\left(\int u(w(y))h(y|a_1)dy - v(a_1)\right) \quad s.t.\\ &\qquad (i)\ \int u(w(y))h_1(y|a_1)dy - v'(a_1) = 0,\\ &\qquad (ii)\ w(y)\geq k, \forall y.\end{aligned} \tag{A34}$$

For two different technologies $h = f$, g such that $\frac{f_1}{f}(y|a_1)$ is a mean-preserving spread of $\frac{g_1}{g}(y|a_1)$, we have:

$$C_f(a_1) \equiv \int \left(w_f(y) - \lambda u(w_f(y))\right) f(y|a_1)dy < \int \left(w_g(y) - \lambda u(w_g(y))\right) g(y|a_1)dy \equiv C_g(a_1). \tag{A35}$$

Proof. We know that the solution of (A34) would be given as

$$\frac{1}{u'(w_h(y))} = \max\left\{\lambda + \mu_h \frac{h_1}{h}(y|a_1), \frac{1}{u'(k)}\right\}, \tag{A36}$$

where μ_h is the Lagrange multiplier attached to the incentive constraint for the given a_1. If we define $q_h \equiv \lambda + \mu_h \frac{h_1}{h}(y|a_1)$, we can rewrite the optimal contract $w_h(\cdot)$ as a function of q_h so that $w_h(y) = r(q_h)$ where $r(\cdot) = (\frac{1}{u'})^{-1}(\cdot)$ is increasing and does not rely on the technology h. Therefore, (A36) can be written as

$$u'(r(q_h))q_h = 1, \tag{A37}$$

if $q_h \geq u(k)^{-1}$ and otherwise $r(q_h) = k$. Now, we obtain

$$\begin{aligned} \mathbb{E}_h\left(u(r(q_h))q_h\right) &= \int u(r(q_h)) \cdot q_h \cdot h(y|a_1)dy = \int u(r(q_h))\left[\lambda + \mu_h \frac{h_1}{h}(y|a_1)\right] h(y|a_1)dy \\ &= \lambda \underbrace{\int u(r(q_h))h(y|a_1)dy}_{\equiv B_h} + \mu_h \underbrace{\int u(r(q_h))h_1(y|a_1)dy}_{=v'(a_1)} = \lambda B_h + \mu_h v'(a_1), \end{aligned} \tag{A38}$$

where we used the fact that $r(q_h)$ satisfies the agent's incentive constraint in a_1. Following Kim (1995), we define

$$\psi(q) \equiv r(q) - u(r(q))q, \tag{A39}$$

which is concave in $\forall q$, since: (i) with $q \geq u(k)^{-1}$, we have $\psi'(q) = \cancel{r'(q)} - \cancel{u'(r(q))r'(q)q} - u(r(q)) = -u(r(q))$ as $u'(r(q))q = 1$ and $\psi''(q) = -u'(r(q))r'(q) < 0$; (ii) with $q < u(k)^{-1}$, we have $r(q) = k$ so $\psi(q)$ becomes linear.[7] Now we can introduce two different technologies $f(\cdot|a_1)$ and $g(\cdot|a_1)$ such that $\frac{f_1}{f}(y|a_1)$ is a mean-preserving spread of $\frac{g_1}{g}(y|a_1)$, and define

$$\bar{q} \equiv \lambda + \mu_f \frac{g_1}{g}(y|a_1), \tag{A40}$$

which is possibly different from q_g as μ_f is possibly different from μ_g. As $\psi(q)$ is globally concave, we obtain

$$\begin{aligned} \mathbb{E}_g\left(\psi(\bar{q})\right) - \mathbb{E}_g\left(\psi(q_g)\right) &\leq \mathbb{E}_g\left(\psi'(q_g)(\bar{q} - q_g)\right) = \mathbb{E}_g\left(-u(r(q_g))(\mu_f - \mu_g)\frac{g_1}{g}\right) \\ &= (\mu_g - \mu_f)\int u(r(q_g))g_1(y|a_1)dy = (\mu_g - \mu_f)v'(a_1) \\ &= \left(\mathbb{E}_g\left(u(r(q_g))q_g\right) - \lambda B_g\right) - \left(\mathbb{E}_f\left(u(r(q_f))q_f\right) - \lambda B_f\right), \end{aligned} \tag{A41}$$

[7] We see that $\psi(q)$ is continuously differentiable at all points including $q = u(k)^{-1}$.

where we used (A38). Finally, it leads to the following agency cost comparison:

$$\begin{aligned} C_g(a_1) - C_f(a_1) &= \mathbb{E}_g\left(r(q_g) - \lambda B_g\right) - \mathbb{E}_f\left(r(q_f) - \lambda B_f\right) = \mathbb{E}_g(r(q_g)) - \mathbb{E}_f(r(q_f)) - (\lambda B_g - \lambda B_f) \\ &= \mathbb{E}_g(\psi(q_g)) + \mathbb{E}_g(u(r(q_g))q_g) - \mathbb{E}_f(\psi(q_f)) - \mathbb{E}_f(u(r(q_f))q_f) - (\lambda B_g - \lambda B_f) \\ &\geq \cancel{\mathbb{E}_g(\psi(q_g))} - \mathbb{E}_f(\psi(q_f)) + \mathbb{E}_g(\psi(\bar{q})) - \cancel{\mathbb{E}_g(\psi(q_g))} = \mathbb{E}_g(\psi(\bar{q})) - \mathbb{E}_f(\psi(q_f)) \\ &= \int \psi\left(\lambda + \mu_f \frac{g_1}{g}(y|a_1)\right) g(y|a_1)dy - \int \psi\left(\lambda + \mu_f \frac{f_1}{f}(y|a_1)\right) f(y|a_1)dy \end{aligned} \tag{A42}$$

where we used (A41) in the above (A42)'s inequality part. Finally, if $\frac{f_1}{f}(y|a_1)$ is a mean-preserving spread of $\frac{g_1}{g}(y|a_1)$, then (A42) with Rothschild and Stiglitz (1970) implies $C_g(a_1) \geq C_f(a_1)$, as $\mu_f > 0$ and $\psi(\cdot)$ is globally concave.

■

Finally, with $f(y|a_1) \equiv f(y|a_1, a_2^0)$ and $g(y|a_1) \equiv f(y|a_1, a_2^1)$ in our specification, Lemma 9 proves Lemma 3.

■

Derivation of equation (32):

Given the fixed $a_1 = a_1^P$, $\phi_2(a_1^P, a_2^P) = C_2(a_1^P, a_2^P)$ holds at optimum. We can write $C_2(a_1^P, a_2^P)$ as follows:

$$\begin{aligned} \phi_2(a_1^P, a_2^P) = C_2(a_1^P, a_2^P) &= \int \left(w^P(y|a_1^P, a_2^P) - \lambda u(w^P(y|a_1^P, a_2^P))\right) f_2(y|a_1^P, a_2^P)dy \\ &+ \int \left(\frac{\partial w^P}{\partial a_2}(y|a_1^P, a_2^P) - \lambda u'(w^P(y|a_1^P, a_2^P))\frac{\partial w^P}{\partial a_2}(y|a_1^P, a_2^P)\right) f(y|a_1^P, a_2^P)dy, \end{aligned} \tag{A43}$$

where we know the following equation is satisfied:[8]

$$\begin{aligned} \frac{\partial w^P}{\partial a_2}(y|a_1^P, a_2^P) - \lambda u'(w^P(y|a_1^P, a_2^P))\frac{\partial w^P}{\partial a_2}(y|a_1^P, a_2^P) &= \frac{\partial w^P}{\partial a_2}(y|a_1^P, a_2^P)\left(1 - \lambda u'(w^P(y|a_1^P, a_2^P))\right) \\ &= \frac{\partial w^P}{\partial a_2}(y|a_1^P, a_2^P) \cdot \mu_1(a_1^P, a_2^P)\frac{f_1}{f}(y|a_1^P, a_2^P)u'(w^P(y|a_1^P, a_2^P)). \end{aligned} \tag{A44}$$

Thus by plugging equation (A44) into equation (A43), we obtain

$$\begin{aligned} &\int \left(\frac{\partial w^P}{\partial a_2}(y|a_1^P, a_2^P) - \lambda u'(w^P(y|a_1^P, a_2^P))\frac{\partial w^P}{\partial a_2}(y|a_1^P, a_2^P)\right) f(y|a_1^P, a_2^P)dy \\ &= \mu_1(a_1^P, a_2^P)\int \frac{\partial w^P}{\partial a_2}(y|a_1^P, a_2^P) f_1(y|a_1^P, a_2^P)u'(w^P(y|a_1^P, a_2^P))dy. \end{aligned} \tag{A45}$$

When $w^P(y|a_1^P, a_2)$ is designed for $\forall a_2$, it should satisfy the such incentive constraint (where $w^P(y|a_1^P, a_2 = a_2^P) \equiv$

[8] The second equality below holds even in the region where the limited liability constraint binds and $w^P(y|a_1^P, a_2^P) = k$ as its derivative with respect to a_2 is 0, except on measure 0. A small change in a_2 leads to only a small change in the region of a binding limited liability.

$w^P(y|a_1^P, a_2^P))$ as

$$\int u(w^P(y|a_1^P, a_2))f_1(y|a_1^P, a_2)dy = v'(a_1^P). \tag{A46}$$

We get the following by differentiating both side of equation (A46) by a_2 at $a_2 = a_2^P$:

$$\int u'(w^P(y|a_1^P, a_2^P))\frac{\partial w^P}{\partial a_2}(y|a_1^P, a_2^P)f_1(y|a_1^P, a_2^P)dy = -\int u(w^P(y|a_1^P, a_2^P)f_{12}(y|a_1^P, a_2^P)dy. \tag{A47}$$

Plugging equation (A47) into equation (A43), we get the following equation (32).[9]

$$\begin{aligned}\phi_2(a_1^P, a_2^P) = \int yf_2(y|a_1^P, a_2^P)dy = \int \left(w^P(y|a_1^P, a_2^P) - \lambda u(w^P(y|a_1^P, a_2^P))\right) f_2(y|a_1^P, a_2^P)dy \\ - \mu_1(a_1^P, a_2^P)\int u(w^P(y|a_1^P, a_2^P)f_{12}(y|a_1^P, a_2^P)dy.\end{aligned} \tag{A50}$$

■

Proof of Proposition 6: Assume to the contrary that $\mu_1^*\phi_1^* + \mu_2^*\phi_2^* \leq 0$. Then, pick up any two levels of y: y_1 and y_2, such that

$$y_1 < y_2, \text{ and } \frac{y_1 + y_2}{2} = \phi(a_1^*, a_2^*). \tag{A51}$$

That is, y_1 and y_2 are located at the same distance from the mean value $\phi(a_1^*, a_2^*)$. If $\mu_1^*\phi_1^* + \mu_2^*\phi_2^* \leq 0$, we have from equation (41) that

$$w^*(y_1) \geq w^*(y_2), \text{ and } u(w^*(y_1)) \geq u(w^*(y_2)). \tag{A52}$$

Since $f_1(y_1|a_1^*, a_2^*) = -f_1(y_2|a_1^*, a_2^*) < 0$ for any y_1 and y_2 satisfying equation (A51), we have:

$$\int u(w^*(y))f_1(y|a_1^*, a_2^*)dy \leq 0, \text{ and } \int u(w^*(y))f_1(y|a_1^*, a_2^*)dy - v'(a_1^*) < 0. \tag{A53}$$

Therefore, there is a contradiction.

■

Proof of Proposition 7:

[9] We can derive it using the envelope theorem. We regard the principal's optimization as the one in which given a fixed a_2, we find optimal $a_1, w(\cdot)$ that maximizes joint utility of the principal and agent under the incentive constraint for a_1 and limited liability constraint. A principal solves the following optimization.

$$\begin{aligned}SW(a_2) = \min_{\mu_1} \max_{w(\cdot), a_1} L(a_2) \equiv &\phi(a_1, a_2) - \int w(y)f(y|a_1, a_2)dy + \lambda\left(\int u(w(y))f(y|a_1, a_2)dy - v(a_1)\right) \\ &+ \mu_1\left(\int u(w(y))f_1(y|a_1, a_2)dy - v'(a_1)\right)\end{aligned} \tag{A48}$$

As $(a_1^P, w^P(\cdot|a_1^P, a_2^P))$ are the solution given a_2^P, $SW'(a_2^P) = 0$ must hold, which turns out to be the same as equation (32). Thus an envelope theorem yields equation (32), where $\mu_1(a_1^P, a_2^P)$ are the endogenous Lagrange multiplier for incentive constraint for a_1 at a_1^P given a_2^P. Thus, we obtain

$$SW'(a_2^P) = \int \left(y - w^P(y) + \lambda u(w^P(y)\right) f_2(y|a_1^P, a_2^P)dy + \mu_1(a_1^P, a_2^P)\int u(w^P(y))f_{12}(y|a_1^P, a_2^P)dy = 0. \tag{A49}$$

(**Case 1**) $\underline{\mu_2^* > 0 \quad \text{if} \quad a_2^A(a_2^P) < a_2^P}$: Compare the following two optimizations:[10]

$$\begin{aligned}
\max_{a_1,a_2,w(\cdot)} & \int (y - w(y))f(y|a_1,a_2)dy + \lambda\left(\int u(w(y))f(y|a_1,a_2)dy - v(a_1)\right) \quad \text{s.t.} \\
& (i) \quad \int u(w(y))f_1(y|a_1,a_2)dy - v'(a_1) = 0, \\
& (ii) \quad \int u(w(y))f_2(y|a_1,a_2)dy = 0, \\
& (iii) \quad w(y) \geq k, \quad \forall y,
\end{aligned} \tag{A54}$$

and

$$\begin{aligned}
\max_{a_1,a_2,w(\cdot)} & \int (y - w(y))f(y|a_1,a_2)dy + \lambda\left(\int u(w(y))f(y|a_1,a_2)dy - v(a_1)\right) \quad \text{s.t.} \\
& (i) \quad \int u(w(y))f_1(y|a_1,a_2)dy - v'(a_1) = 0, \\
& (ii) \quad \int u(w(y))f_2(y|a_1,a_2)dy \geq 0, \\
& (iii) \quad w(y) \geq k, \quad \forall y,
\end{aligned} \tag{A55}$$

where the incentive constraint associated with the non-hedgeable risk choice a_2 takes the form of inequality in the latter program, instead of equality in the original optimization program.

We know that $(w^*(y), a_1^*, a_2^*, \mu_1^*, \mu_2^*)$ are the optimal solution for the first program. Let $(\hat{w}(y), \hat{a}_1, \hat{a}_2, \hat{\mu}_1, \hat{\mu}_2)$ be the optimal solution for the second program. We will show that the above two programs are equivalent in that two solutions align perfectly with each other when $a_2^A(a_2^P) < a_2^P$. Then, we can directly derive $\mu_2^* \geq 0$ when $a_2^A(a_2^P) < a_2^P$, since $\hat{\mu}_2 \geq 0$ by Kuhn-Tucker theorem.

Assume that the second constraint in the second program is not binding. Then, $\hat{\mu}_2 = 0$, and $\hat{w}(y)$ should satisfy:

$$\frac{1}{u'(\hat{w}(y))} = \lambda + \hat{\mu}_1 \frac{y - \phi(\hat{a}_1, \hat{a}_2)}{(\hat{a}_2)^2} \phi_1(\hat{a}_1, \hat{a}_2), \tag{A56}$$

for y satisfying $\hat{w}(y) \geq k$ and $\hat{w}(y) = k$ otherwise. As we know that the second constraint is not binding, $\hat{a}_2$ becomes the best (from the principal's perspective) a_2, i.e., $\hat{a}_2 = a_2^P$. Then we must have $\hat{a}_1 = a_1^P$ and $\hat{w}(y) = w^P(y|a_1^P, a_2^P)$. Therefore, the fact that the second constraint in the second program is not binding implies

$$\int u(w^P(y|a_1^P, a_2^P))f_2(y|a_1^P, a_2^P, a_3 = 0)dy > 0. \tag{A57}$$

However, equation (A57) implies $a_2^A(a_2^P) > a_2^P$, which is a contradiction.[11] Thus, the second constraint in the second program must be binding, and the above two programs are equivalent so $\mu_2^* = \hat{\mu}_2 \geq 0$. And also, $\mu_2^* \neq 0$, because $\mu_2^* = 0$ implies $a_2^A(a_2^P) = a_2^P$.

[10]Following Rogerson (1985), we replace the incentive constraint with the corresponding inequality constraint, and exploit the fact that a multiplier to the inequality constraint must be non-negative.

[11]We assume that $\int u(w(y|a_2^P))f(y|a_1^P, a_2, a_3 = 0)dy$ is concave in a_2, which is based on the first-order approach associated with a_2.

(**Case 2**) $\underline{\mu_2^* < 0 \quad \text{if} \quad a_2^A(a_2^P) > a_2^P}$: This part also easily follows by using the same method in (**Case 1**). We compare following two optimization programs similar to equation (A54) and equation (A55).

$$\begin{aligned}
&\max_{a_1,a_2,w(\cdot)} \int (y - w(y)) f(y|a_1,a_2)dy + \lambda \left(\int u(w(y)) f(y|a_1,a_2)dy - v(a_1) \right) \quad \text{s.t.} \\
&\qquad (i) \ \int u(w(y)) f_1(y|a_1,a_2)dy - v'(a_1) = 0, \\
&\qquad (ii) \ \int u(w(y)) f_2(y|a_1,a_2)dy = 0, \\
&\qquad (iii) \ w(y) \geq k, \ \ \forall y,
\end{aligned} \tag{A58}$$

and

$$\begin{aligned}
&\max_{a_1,a_2,w(\cdot)} \int (y - w(y)) f(y|a_1,a_2)dy + \lambda \left(\int u(w(y)) f(y|a_1,a_2)dy - v(a_1) \right) \quad \text{s.t.} \\
&\qquad (i) \ \int u(w(y)) f_1(y|a_1,a_2)dy - v'(a_1) = 0, \\
&\qquad (ii) \ \int u(w(y)) f_2(y|a_1,a_2)dy \leq 0, \\
&\qquad (iii) \ w(y) \geq k, \ \ \forall y,
\end{aligned} \tag{A59}$$

Solutions of the above two optimization programs must be the same, and due to the property that the multiplier attached to the incentive constraint associated with a_2 in the second program must be non-positive, we conclude $\mu_2^* < 0$ when $a_2^A(a_2^P) > a_2^P$.

■

Derivation of equation (46): With the Lagrange multipliers $\mu_1(R)$, $\mu_2(R)$ attached to the incentive constraints for action $a_1(R)$ and the project choice $a_2(R)$ given R, respectively, we can set up the Lagrangian for the optimization in (44) as follows:[12]

$$\begin{aligned}
\mathcal{L} = &\int_R \left[\int_{x,\eta} (x - w(x,\eta) + \lambda u(w(x,\eta)))\, g(x,\eta|a_1(R), a_2(R), R)dxd\eta - v(a_1(R)) \right] h(R)dR \\
&+ \int_R \mu_1(R) \left[\int_{x,\eta} u(w(x,\eta)) g_1(x,\eta|a_1(R), a_2(R), R)dxd\eta - v'(a_1(R)) \right] h(R)dR \\
&+ \int_R \mu_2(R) \left[\int_{x,\eta} u(w(x,\eta)) g_2(x,\eta|a_1(R), a_2(R), R)dxd\eta \right] h(R)dR,
\end{aligned} \tag{A61}$$

[12] Note that $g(x,\eta|a_1,a_2,R)$ yields the following likelihood ratios:

$$\begin{aligned}
&\frac{g_1}{g}(x,\eta|a_1,a_2,R) = \frac{x - R\eta - \phi(a_1,a_2)}{a_2^2}\phi_1(a_1,a_2), \quad \frac{g_3}{g}(x,\eta|a_1,a_2,R) = \frac{(x - R\eta - \phi(a_1,a_2))\eta}{a_2^2}, \\
&\frac{g_2}{g}(x,\eta|a_1,a_2,R) = -\frac{1}{a_2} + \frac{x - R\eta - \phi(a_1,a_2)}{a_2^2}\phi_2(a_1,a_2) + \frac{(x - R\eta - \phi(a_1,a_2))^2}{a_2^3}.
\end{aligned} \tag{A60}$$

from which we get the following first-order Euler equation about $w(\cdot,\cdot)$:

$$\begin{aligned}&(-1+\lambda u'(w(x,\eta)))\int_R g(x,\eta|a_1(R),a_2(R),R)h(R)dR\\ +u'(w(x,\eta))&\left[\int_R \{\mu_1(R)g_1(x,\eta|a_1(R),a_2(R),R)+\mu_2(R)g_2(x,\eta|a_1(R),a_2(R),R)\}\cdot h(R)dR\right]=0,\end{aligned} \tag{A62}$$

which derives (46).

■

Proof of Proposition 8: Proof will be similar to Proposition 2, except that now we have the project choice $a_2(R)$ that depends on the observed R by the manager. Consider the principal's following alternative maximization program:

$$\begin{aligned}\max_{a_1(\cdot),a_2(\cdot),w(\cdot)}&\int_R\int_{x,\eta}(x-w(x,R,\eta))\,g(x,\eta|a_1(R),a_2(R),R)h(R)dxd\eta dR\\ &+\lambda\int_R\left(\int_{x,\eta}u(w(x,R,\eta))g(x,\eta|a_1(R),a_2(R),R)dxd\eta-v(a_1(R))\right)h(R)dR\quad\text{s.t.}\\ &(i)\ \int_{x,\eta}u(w(x,R,\eta))g_1(x,\eta|a_1(R),a_2(R),R)dxd\eta-v'(a_1(R))=0,\forall R,\\ &(ii)\ \int_{x,\eta}u(w(x,R,\eta))g_2(x,\eta|a_1(R),a_2(R),R)dxd\eta=0,\forall R,\\ &(iii)\ w(x,R,\eta)\geq k,\quad\forall(x,\eta).\end{aligned} \tag{A63}$$

Note that the above program is different from the original program (44) in that here contract can be written on the realized value of R. If we let the Lagrange multipliers to the constraints (i) and (ii) be $\mu_1(R)h(R)$ and $\mu_2(R)h(R)$ respectively, we get the following optimal contractual form:[13]

$$\begin{aligned}\frac{1}{u'(w(x,R,\eta))}&=\lambda+\mu_1(R)\frac{x-R\eta-\phi_R}{a_2(R)^2}\phi_{1,R}+\mu_2(R)\Big[-\frac{1}{a_2(R)}+\frac{x-R\eta-\phi_R}{a_2(R)^2}\phi_{2,R}+\frac{(x-R\eta-\phi_R)^2}{a_2(R)^3}\Big]\\ &=\lambda+(\mu_1(R)\phi_{1,R}+\mu_2(R)\phi_{2,R})\frac{\overbrace{x-R\eta}^{\equiv y}-\phi_R}{a_2(R)^2}+\frac{\mu_2(R)}{a_2(R)}\Big[\frac{(\overbrace{x-R\eta}^{\equiv y}-\phi_R)^2}{a_2(R)^2}-1\Big],\end{aligned} \tag{A64}$$

when $w(x,R,\eta)\geq k$. The above equation (A64) implies that optimal contract only depends on $y\equiv x-R\eta$ and the solution $(w(x,R,\eta),a_1(R),a_2(R))$ becomes $(a_1^*,a_2^*,w^*(y)\equiv w^*(x-R\eta))$. By comparing the above equation (A63) with the program in equation (44) when he does not know R, one can easily see that the set of wage contracts, $\{w(x,R,\eta)\}$, satisfying the incentive constraints for a given action combination $(a_1(R),a_2(R))$ in the above program always contains the set of wage contracts that would be available when the principal does not know R, $\{w(x,\eta)\}$,

[13] We define $\phi_R\equiv\phi(a_1(R),a_2(R)),\phi_{i,R}\equiv\phi_i(a_1(R),a_2(R))$ for $\forall i=1,2$, where $\{a_1(R),a_2(R)\}$ are optimal actions for each R.

satisfying the incentive constraints for the same action combination. Therefore, we have

$$SW^N \leq SW^*(a_1^*, a_2^*). \tag{A65}$$

However, one can easily see that $w^*(y) = w^*(x - R\eta)$ which is a unique solution for the wage contract of the above program is not in the set of $\{w(x, \eta)\}$. As a result, we finally derive

$$SW^N < SW^*(a_1^*, a_2^*). \tag{A66}$$

■

Proof of Lemma 4:

(1) Suppose $\underline{\mu_2^* < 0}$ in equation (51) for any given $\hat{b}$. Proposition 7 implies that if the shareholders want their manager to reduce the risk through the project choice (i.e., if $a_2^P < a_2^A(a_2^P)$), the optimal contract in equation (41) features $\mu_2^* < 0$. Note that risk reduction through the real project choice (i.e., lowering a_2) is costly to the manager in the sense that a less risky project generates the lower expected return, and thereby reduces the agent's expected payoff (i.e., $\mu_1^*\phi_1^* + \mu_2^*\phi_2^* > 0$). Thus, the fact that even costly risk reduction is encouraged by $w^*(z(\hat{b}))$ implies that any risk reduction (i.e., reducing the variance of $z(\hat{(b)})$) in the absence of expected return reduction will be taken by the manager under $w^*(z(\hat{b}))$. Risk reduction through derivative transaction is costless to the agent because there is no risk-return trade-off for derivative transaction (i.e., manipulating a_3). Whenever taking further risk reduction is encouraged, therefore, the manager would like to do it through the derivative choices first.

Thus, the manager will choose a_3 so that $b \equiv R - a_3 = \hat{b}$ which minimizes the variance of $z(\hat{b})$, when $w^*(z(\hat{b}))$ with $\mu_2^* < 0$ is designed.

■

(2) Suppose $\underline{\mu_2^* > 0}$ for $w^*(z(\hat{b}))$ in equation (51). Given (a_1^*, a_2^*), $z(\hat{b}) = x - \hat{b}\eta = \phi(a_1^*, a_2^*) + (b - \hat{b})\eta + a_2^*\theta$ holds. Let $w(\eta, \theta, b|w^*)$ be the wage that the manager will receive under $w^*(z(\hat{b}))$ when he takes (a_1^*, a_2^*, b) and (η, θ) are realized. Then, by substituting equation (49) into equation (51), we have

$$\frac{1}{u'(w(\eta, \theta, b|w^*))} = \lambda + (\mu_1^*\phi_1^* + \mu_2^*\phi_2^*)\frac{(b - \hat{b})\eta + a_2^*\theta}{(a_2^*)^2} + \mu_2^*\frac{1}{a_2^*}\left(\frac{((b - \hat{b})\eta + a_2^*\theta)^2}{(a_2^*)^2} - 1\right), \tag{A67}$$

when $w(\eta, \theta, b|w*) \geq k$ and otherwise $w(\eta, \theta, b|w*) = k$. Therefore, for two different b, say b^0 and b^1, given some realized (η, θ), we have

$$\begin{aligned}\frac{1}{u'(w(\eta,\theta,b^1|w^*))}-\frac{1}{u'(w(\eta,\theta,b^0|w^*))}=&(\mu_1^*\phi_1^*+\mu_2^*\phi_2^*)\frac{(b^1-b^0)\eta}{(a_2^*)^2}\\&+\mu_2^*\frac{1}{a_2^*}\left(\frac{\left((b^1-\hat{b})^2-(b^0-\hat{b})^2\right)\eta^2+2a_2^*(b^1-b^0)\eta\theta]}{(a_2^*)^2}\right).\end{aligned}\tag{A68}$$

Assume that $b^1=+\infty$ or $-\infty$, and $-\infty<b^0<+\infty$. Since $\mu_2^*>0$, we have from the above that

$$\frac{1}{u'(w(\eta,\theta,b^1|w^*))}-\frac{1}{u'(w(\eta,\theta,b^0|w^*))}>0,\ \ \forall(\eta,\theta).\tag{A69}$$

Therefore, we have

$$w(\eta,\theta,b^1|w^*)>w(\eta,\theta,b^0|w^*),\ \ \forall(\eta,\theta).\tag{A70}$$

which implies that the agent will actually take a_3 satisfying $b=+\infty$ or $-\infty$ when $w^*(z(\hat{b}))$ with $\mu_2^*>0$ is designed.

■

Proof of Lemma 5: From equation (49), we have $z(\hat{b})=\phi(a_1,a_2)+(b-\hat{b})\eta+a_2\theta$ for any given (a_1,a_2,b), and $z(0)=\phi(a_1,a_2)+b'\eta+a_2\theta$ for any given (a_1,a_2,b'). Therefore, we obtain

$$z(\hat{b}|a_1,a_2,b)=z(0|a_1,a_2,b'),\ \ \text{whenever}\ \ b'=b-\hat{b}.\tag{A71}$$

Furthermore, if $b'=b-\hat{b}$, two joint density functions of $(z(\hat{b}),\eta)$ and $(z(0),\eta)$ are the same, i.e.,

$$\begin{aligned}g(z(\hat{b}),\eta|a_1,a_2,b)&=\frac{1}{2\pi a_2}\exp\left(-\frac{1}{2}\left(\frac{\left(z(\hat{b})-\phi(a_1,a_2)-(b-\hat{b})\eta\right)^2}{a_2^2}+\eta^2\right)\right)\\&=g(z(0),\eta|a_1,a_2,b'),\ \ \forall b'=b-\hat{b}.\end{aligned}\tag{A72}$$

Thus, we derive that for $\forall(a_1,a_2,b)$, we have

$$\int u(w^o(z(\hat{b}),\eta))g(z(\hat{b}),\eta|a_1,a_2,b)dzd\eta=\int u(w^o(z(0),\eta))g(z(0),\eta|a_1,a_2,b'=b-\hat{b}))dzd\eta.\tag{A73}$$

Note that the manager is induced to take $(a_1^o,a_2^o,b^o\equiv R-a_3^o=\hat{b})$ under the contract $w^o(z(\hat{b}),\eta)$. Thus, the manager will be induced to take $(a_1^o,a_2^o,b'=0$ (i.e., $a_3=R$)) under wage contract $w^o(z(0),\eta)$. Moreover, since

$$\int w^o(z(\hat{b}),\eta)g(z(\hat{b}),\eta|a_1^o,a_2^o,b^o)dzd\eta=\int w^o(z(0),\eta)g(z(0),\eta|a_1^o,a_2^o,0))dzd\eta,\tag{A74}$$

using equation (52) and equation (53), we finally derive:

$$SW^o(a_1^o, a_2^o, a_3^o) = SW^o(a_1^o, a_2^o, R).$$

■

Proof of Lemma 6: Proof is almost the same as in Lemma 2. When $w^*(z(0))$ described in equation (51) is designed, we have

$$z(0|a_1, a_2, b) = x = \phi(a_1, a_2) + b\eta + a_2\theta. \tag{A75}$$

If the agent takes (a_1, a_2, b) under $w^*(z(0))$, then his expected utility is:

$$\int u(w^*(z(0)))g(z(0), \eta|a_1, a_2, b)dzd\eta - v(a_1) = \int u(w^*(z(0)))q(z(0)|a_1, a_2, b, \eta)l(\eta)dzd\eta - v(a_1), \tag{A76}$$

where $q(\cdot)$ denotes the conditional density function of $z(0)$ given (a_1, a_2, b, η) and $l(\cdot)$ denotes the density function of $\eta \sim N(0, 1)$.

Now, suppose the agent takes $(a_1, a_2, -b)$ under $w^*(z(0))$. Then, his expected utility becomes:

$$\int u(w^*(z(0)))g(z(0), \eta|a_1, a_2, -b)dzd\eta - v(a_1) = \int u(w^*(z(0)))q(z(0)|a_1, a_2, -b, \eta)l(\eta)dzd\eta - v(a_1). \tag{A77}$$

Since

$$q(z(0)|a_1, a_2, b, \eta) = \frac{1}{\sqrt{2\pi}a_2} \exp\left(-\frac{(z(0) - \phi(a_1, a_2) - b\eta)^2}{2a_2^2}\right), \tag{A78}$$

we have

$$q(z(0)|a_1, a_2, b, \eta) = q(z(0)|a_1, a_2, -b, -\eta). \tag{A79}$$

Since $\eta \sim N(0, 1)$ is symmetrically distributed around 0 and $l(\eta) = l(-\eta)$, $\forall \eta$, we finally have

$$\int u(w^*(z(0)))g(z(0), \eta|a_1, a_2, b)dzd\eta - v(a_1) = \int u(w^*(z(0)))g(z(0), \eta|a_1, a_2, -b)dzd\eta - v(a_1). \tag{A80}$$

■

Proof of Proposition 10: To prove this proposition, we start with the following Lemma 10. Our proof strategy here will be similar to Proposition 3, but now we have the project choice a_2^o chosen by the manager.

Lemma 10 *If $\mu_2^* > 0$ for contract $w^*(z(0))$ in equation (51), then the optimal contract $w^o(x, \eta)$ guaranteeing that the agent takes $a_1^o, a_2^o, a_3^o = R$ $(b = 0)$, i.e., $w^o(x, \eta)$ in equation (61), must satisfy*

(1) $\mu_2^o \geq 0$

(2) $\mu_4^o(b) \neq 0$ (> 0) for a positive Borel-measure of b.[14]

(3) $w^o(x,\eta) = w^o(x,-\eta)$ for all x,η and $\mu_4^o(b) = \mu_4^o(-b)$ for all b.

Proof. (1) $\underline{\mu_2^o \geq 0}$: Assume that $\mu_2^o < 0$, then under the contract $w^1(x,\eta)$ satisfying

$$\frac{1}{u'(w^1(x,\eta))} = \lambda + (\mu_1^o \phi_1^o + \mu_2^o \phi_2^o)\frac{x - \phi(a_1^o, a_2^o)}{(a_2^o)^2} + \frac{\mu_2^o}{a_2^o}\left(\frac{(x-\phi(a_1^o,a_2^o))^2}{(a_2^o)^2} - 1\right), \tag{A81}$$

for (x,η) satisfying $w^1(x,\eta) \geq k$ and $w^1(x,\eta) = k$, the agent voluntarily chooses $b = 0$ even though we did not consider the constraint (iii) in (60). Thus $w^1(x,\eta)$ becomes the solution of (60). However, it contradicts with our assumption of $\mu_2^* > 0$ for $w^*(z(0))$ since $(w^1(x,\eta), \mu_1^o, \mu_2^o)$ becomes $(w^*(z(0)), \mu_1^*, \mu_2^*)$ without the incentive constraint (iii) about b.

(2) $\underline{\mu_4^o(b) \neq 0 \text{ for a positive Borel-measure of } b}$: Assume $\mu_4^o(b) = 0\ \ a.s.$ Then optimal contract $w^o(x,\eta)$ becomes:

$$\frac{1}{u'(w^o(x,\eta))} = \lambda + (\mu_1^o \phi_1^o + \mu_2^o \phi_2^o)\frac{x - \phi(a_1^o, a_2^o)}{(a_2^o)^2} + \frac{\mu_2^o}{a_2^o}\left(\frac{(x-\phi(a_1^o,a_2^o))^2}{(a_2^o)^2} - 1\right), \tag{A82}$$

for (x,η) satisfying $w^o(x,\eta) \geq k$ and $w^o(x,\eta) = k$.

Because we already know $(w^o(x,\eta), \mu_1^o, \mu_2^o, a_1^o, a_2^o)$ becomes $(w^*(z(0)), \mu_1^*, \mu_2^*, a_1^*, a_2^*)$ in this case and $\mu_2^* > 0$ holds, $(w^o(x,\eta), \mu_1^o, \mu_2^o)$ will induce $b = \pm\infty$ instead of $b = 0$ from the agent, a contradiction to the constraint (iii) in (60).

(3) $\underline{w^o(x,\eta) = w^o(x,-\eta) \text{ for all } x,\eta \text{ and } \mu_4^o(b) = \mu_4^o(-b) \text{ for all } b}$: We first see:[15]

$$g(x,\eta|b) = \frac{1}{2\pi a_2^o}\exp\left(-\frac{1}{2}\frac{(x-\phi(a_1^o,a_2^o) - b\eta)^2}{(a_2^o)^2} - \frac{1}{2}\eta^2\right), \tag{A83}$$

where

$$\frac{g(x,\eta|b)}{g(x,\eta|b=0)} = \exp\left(\frac{b\eta(x-\phi(a_1^o,a_2^o))}{(a_2^o)^2}\right)\exp\left(-\frac{b^2\eta^2}{2(a_2^o)^2}\right). \tag{A84}$$

From (A60), (A83), and (A84), we observe that $g(x,\eta|b=0)$, $g_1(x,\eta|b=0)$, and $g_2(x,\eta|b=0)$ are all even with η where g_1 and g_2 are partial derivatives of g with respect to a_1 and a_2: i.e., (i) $g(x,-\eta|b=0) = g(x,\eta|b=0)$; (ii) $g_1(x,-\eta|b=0) = g_1(x,\eta|b=0)$; (iii) $g_2(x,-\eta|b=0) = g_2(x,\eta|b=0)$. Also from (A83), we acknowledge:

$$g(x,-\eta|b) = g(x,\eta|-b), \ \ \forall (x,\eta,b). \tag{A85}$$

[14] We already know $\mu_4^o(b) \geq 0$ for every b (almost surely), since it is derived from the inequality constraint at each b.
[15] We suppress a_1^o, a_2^o in $g(x,\eta|a_1^o,a_2^o,b)$ in (60) to make our expressions simpler.

Our strategy is to prove that: (i) if $w^o(x,\eta)$ is an optimal contract, then $w^o(x,-\eta)$ satisfies all the constraints in (60); (ii) Related to (i), if $w^o(x,\eta)$ is an optimal contract, then $w^o(x,-\eta)$ also becomes an optimal contract; and (iii) $\mu_4^o(-b) = \mu_4^o(b)$ for $\forall b$ at the optimum.

Step 1. If $w^o(x,\eta)$ is an optimal contract, then $w^o(x,-\eta)$ satisfies all the constraints in (60).

(i) As $w^o(x,\eta)$ is optimal, note that it satisfies all of the constraints in (60). We start from the incentive compatibility in action a_1: based on that $g_1(x,\eta|b=0)$ is even in η,

$$\begin{aligned}\int u(w^o(x,-\eta))g_1(x,\eta|b=0)dxd\eta - v'(a_1^o) &= \int u(w^o(x,-\eta))g_1(x,-\eta|b=0)dxd\eta - v'(a_1^o) \\ &= \int u(w^o(x,\eta))g_1(x,\eta|b=0)dxd\eta - v'(a_1^o) = 0,\end{aligned}$$

where we use the change of variable (i.e., $-\eta$ to η) in the second equality.

(ii) Incentive compatibility in action a_2: based on that $g_2(x,\eta|b=0)$ is even in η,

$$\int u(w^o(x,-\eta))g_2(x,\eta|b=0)dxd\eta = \int u(w^o(x,-\eta))g_2(x,-\eta|b=0)dxd\eta = \int u(w^o(x,\eta))g_2(x,\eta|b=0)dxd\eta = 0.$$

(iii) Incentive compatibility in *after-hedging* risk exposure b: as $w^o(x,\eta)$ is optimal, we know it satisfies

$$\int u(w^o(x,\eta))\left(g(x,\eta|b=0) - g(x,\eta|b)\right)dxd\eta \geq 0,\ \underline{\forall b}. \tag{A86}$$

From (A85) and that $g(x,\eta|b=0)$ is even in η, we obtain for $\forall b$,

$$\begin{aligned}\int u(w^o(x,-\eta))\left(g(x,\eta|b=0) - g(x,\eta|b)\right)dxd\eta &= \int u(w^o(x,-\eta))\left(g(x,-\eta|b=0) - g(x,-\eta|-b)\right)dxd\eta \\ &= \int u(w^o(x,\eta))\left(g(x,\eta|b=0) - g(x,\eta|-b)\right)dxd\eta \geq 0,\end{aligned} \tag{A87}$$

where the first equality is from (A85) and the second equality is from the change of variable (i.e., $-\eta$ to η). Thus, we proved that if $w^o(x,\eta)$ is an optimal contract, then $w^o(x,-\eta)$ satisfies all the constraints in (60).

Step 2. Next, if $w^o(x,\eta)$ is an optimal contract, then $w^o(x,-\eta)$ also becomes an optimal contract.

From the above **Step 1**, $w^o(x,-\eta)$ satisfies all the constraints in (60). It is sufficient to show that $w^o(x,-\eta)$ achieves the same efficiency as $w^o(x,\eta)$. It follows from:

$$\begin{aligned}
&\int (x - w^o(x,-\eta))g(x,\eta|b=0)dxd\eta + \lambda\left(\int u(w^o(x,-\eta))g(x,\eta|b=0)dxd\eta - v(a_1^o)\right) \\
&\quad = \int (x - w^o(x,-\eta))g(x,-\eta|b=0)dxd\eta + \lambda\left(\int u(w^o(x,-\eta))g(x,-\eta|b=0)dxd\eta - v(a_1^o)\right) \\
&\quad = \int (x - w^o(x,\eta))g(x,\eta|b=0)dxd\eta + \lambda\left(\int u(w^o(x,\eta))g(x,\eta|b=0)dxd\eta - v(a_1^o)\right),
\end{aligned} \tag{A88}$$

where the first equality is from that $g(x,\eta|b=0)$ is symmetric in η, and the second equality is from the change of variable (i.e., $-\eta$ to η). Therefore, if $w^o(x,\eta)$ is an optimal contract, then $w^o(x,-\eta)$ becomes an optimal contract and we obtain $w^o(x,-\eta) = w^o(x,\eta)$.[16]

Step 3. $\mu_4^o(-b) = \mu_4^o(b)$ for $\forall b$.

Note from the Lagrange duality theorem (see e.g., Luenberger (1969)) that the optimal solution $(\mu_1^o, \mu_2^o, \{\mu_4^o(b)\}, w^o(\cdot))$ is the one that solves:

$$\begin{aligned}
\min_{\mu_1,\mu_2,\{\mu_4(\cdot)\}} \max_{w(\cdot)} \mathcal{L} \equiv &\int (x - w(x,\eta))g(x,\eta|b=0)dxd\eta + \lambda\left(\int u(w(x,\eta))g(x,\eta|b=0)dxd\eta - v(a_1^o)\right) \\
&+ \mu_1\left(\int u(w(x,\eta))g_1(x,\eta|b=0)dxd\eta - v'(a_1^o)\right) + \mu_2\left(\int u(w(x,\eta))g_2(x,\eta|b=0)dxd\eta\right) \\
&+ \int_b \mu_4(b)\left(\int u(w(x,\eta))\left(g(x,\eta|b=0) - g(x,\eta|b)\right)dxd\eta\right)db,
\end{aligned} \tag{A89}$$

while satisfying $\mu_4^o(b) \geq 0$ for $\forall b$ and the following complementary slackness condition at the optimum:

$$\mu_4^o(b)\left(\int u(w^o(x,\eta))\left(g(x,\eta|b=0) - g(x,\eta|b)\right)dxd\eta\right) = 0, \;\; \forall b. \tag{A90}$$

The last term in (A89) given the optimal contract $w^o(x,\eta)$ can be written as

$$\begin{aligned}
&\int_b \mu_4(b)\left(\int u(w^o(x,\eta))\left(g(x,\eta|b=0) - g(x,\eta|b)\right)dxd\eta\right)db \\
&\quad = \int_b \mu_4(-b)\left(\int u(w^o(x,-\eta))\left(g(x,\eta|b=0) - g(x,\eta|-b)\right)dxd\eta\right)db,
\end{aligned} \tag{A91}$$

where we use the change of variable (i.e., b to $-b$) and $w^o(x,-\eta) = w^o(x,\eta)$. Now with (A85) and that $g(x,\eta|b=0)$

[16] We implicitly assume that the optimal contract is unique in this environment, following the literature (e.g., Jewitt et al. (2008)).

is even in η, we know:

$$\begin{aligned}\int u(w^o(x,-\eta))\left(g(x,\eta|b=0)-g(x,\eta|-b)\right)dxd\eta &= \int u(w^o(x,-\eta))\left(g(x,-\eta|b=0)-g(x,-\eta|b)\right)dxd\eta \\ &= \int u(w^o(x,\eta))\left(g(x,\eta|b=0)-g(x,\eta|b)\right)dxd\eta,\end{aligned} \tag{A92}$$

where we use the change of variable (i.e., $-\eta$ to η) for the second equality. With (A91) and (A92), the last term in (A89) can be therefore written as

$$\begin{aligned}&\int_b \mu_4(b)\left(\int u(w^o(x,\eta))\left(g(x,\eta|b=0)-g(x,\eta|b)\right)dxd\eta\right)db \\ &\quad = \int_b \mu_4(-b)\left(\int u(w^o(x,\eta))\left(g(x,\eta|b=0)-g(x,\eta|b)\right)dxd\eta\right)db.\end{aligned} \tag{A93}$$

Plugging in (A93) to the original Lagrangian in (A89) yields $\mu_4^o(-b)=\mu_4^o(b)$.

Step 4. We have:

$$\int u(w^o(x,\eta))g(x,\eta|b)dxd\eta = \int u(w^o(x,\eta))g(x,\eta|-b)dxd\eta, \tag{A94}$$

which implies that the agent's indirect utility is symmetric in b around $b=0$.

It follows from:

$$\begin{aligned}\int u(w^o(x,\eta))g(x,\eta|-b)dxd\eta &= \int u(w^o(x,\eta))g(x,-\eta|b)dxd\eta = \int u(w^o(x,-\eta))g(x,-\eta|b)dxd\eta \\ &= \int u(w^o(x,\eta))g(x,\eta|b)dxd\eta,\end{aligned} \tag{A95}$$

where we use (A85) in the first equality, $w^o(x,-\eta)=w^o(x,\eta)$ in the second, and and the change of variable (i.e., $-\eta$ to η) in the third equality. ■

Proof of Proposition 10: Given (a_1^o,a_2^o), we define $\widehat{Cov}\equiv(x-\phi(a_1^o,a_2^o))\eta$.[17] Since

$$\exp\left(\frac{b\eta(x-\phi(a_1^o,a_2^o))}{(a_2^o)^2}\right) = \exp\left(\frac{b}{(a_2^o)^2}\widehat{Cov}\right) = \sum_{k=0}^{\infty}\frac{1}{k!}\frac{b^k}{(a_2^o)^{2k}}\widehat{Cov}^k, \tag{A96}$$

From equation (A84), we obtain

$$\frac{g(x,\eta|b)}{g(x,\eta|b=0)} = \left(\sum_{k=0}^{\infty}\frac{1}{k!}\frac{b^k}{(a_2^o)^{2k}}\widehat{Cov}^k\right)\exp\left(-\frac{b^2\eta^2}{2(a_2^o)^2}\right), \tag{A97}$$

[17] This is a realized value of sample covariance between x and η, as our framework is in single-period setting.

and therefore, we attain

$$\begin{aligned}\int \mu_4^o(b)\left(1-\frac{g(x,\eta|b)}{g(x,\eta|b=0)}\right)db &= \int \mu_4^o(b)db - \int \mu_4^o(b)\left(\sum_{k=0}^{\infty}\frac{1}{k!}\frac{b^k}{(a_2^o)^{2k}}\widehat{Cov}^k\right)\exp\left(-\frac{b^2\eta^2}{2(a_2^o)^2}\right)db \\ &= \int \mu_4^o(b)db - \sum_{k=0}^{\infty}\left(\frac{1}{k!}\frac{1}{(a_2^o)^{2k}}\underbrace{\left(\int \mu_4^o(b)b^k\exp\left(-\frac{b^2\eta^2}{2(a_2^o)^2}\right)db\right)}_{\equiv C_k(\eta)}\right)\widehat{Cov}^k.\end{aligned} \tag{A98}$$

When k is odd, the coefficient $C_k(\eta)$ becomes 0 for $\forall\eta$, since $\mu_4^o(b)=\mu_4^o(-b)$ for all b from Lemma 10 implies

$$C_{k:odd}(\eta) = \int \mu_4^o(b)b^k\exp\left(-\frac{b^2\eta^2}{2(a_2^o)^2}\right)db = \int_{b\geq 0}\left(\underbrace{\mu_4^o(b)-\mu_4^o(-b)}_{=0}\right)b^k\exp\left(-\frac{b^2\eta^2}{2(a_2^o)^2}\right)db = 0. \tag{A99}$$

When k is even, the coefficient $C_k(\eta)$ becomes strictly positive for $\forall\eta$, since $\mu_4^o(b)\neq 0$ for the non-zero measure of b from Lemma 10 implies

$$\begin{aligned}C_{k:even}(\eta) = \int \mu_4^o(b)b^k\exp\left(-\frac{b^2\eta^2}{2(a_2^o)^2}\right)db &= \int_{b\geq 0}(\mu_4^o(b)+\mu_4^o(-b))b^k\exp\left(-\frac{b^2\eta^2}{2(a_2^o)^2}\right)db \\ &= 2\int_{b\geq 0}\mu_4^o(b)b^k\exp\left(-\frac{b^2\eta^2}{2(a_2^o)^2}\right)db > 0.\end{aligned} \tag{A100}$$

Therefore, (A98) becomes:

$$\int \mu_4^o(b)\left(1-\frac{g(x,\eta|b)}{g(x,\eta|b=0)}\right)db = \int \mu_4^o(b)db - 2\sum_{k:even}^{\infty}\left(\frac{1}{k!}\frac{1}{(a_2^o)^{2k}}\left(\int_{b\geq 0}\mu_4^o(b)b^k\exp\left(-\frac{b^2\eta^2}{2(a_2^o)^2}\right)db\right)\right)\widehat{Cov}^k. \tag{A101}$$

Finally, we can plug the expression (A101) into our optimal contact $w^o(x,\eta)$ in (61) when $w^o(x,\eta)\geq k$ and obtain

$$\begin{aligned}\frac{1}{u'(w^o(x,\eta))} =& \lambda + (\mu_1^o\phi_1^o+\mu_2^o\phi_2^o)\frac{x-\phi(a_1^o,a_2^o)}{(a_2^o)^2} + \frac{\mu_2^o}{a_2^o}\left(\frac{(x-\phi(a_1^o,a_2^o))^2}{(a_2^o)^2}-1\right) + \underbrace{\int \mu_4^o(b)db}_{>0} \\ &- 2\sum_{k:\text{even}}^{\infty}\underbrace{\frac{1}{k!}\frac{1}{(a_2^o)^{2k}}\underbrace{\left(\int_{b\geq 0}\mu_4^o(b)b^k\exp\left(-\frac{b^2\eta^2}{2(a_2^o)^2}\right)db\right)}_{\equiv C_{k:\text{even}}(\eta)>0}}_{\equiv D_{k:\text{even}}(\eta)>0}\widehat{Cov}^k.\end{aligned} \tag{A102}$$

Since $D_{k:\text{even}}(\eta)>0$ for all even numbers k, given (x,η) a higher $\widehat{Cov}$ results in a lower compensation $w^o(x,\eta)$. Also as $D_{k:\text{even}}(\eta)>0$ decreases in η^2, given $(x,\widehat{Cov})$, a higher η^2 results in a higher $w^o(x,\eta)$. In sum the principal punishes a sample covariance $|\widehat{Cov}|$ but becomes lenient when a high $|\widehat{Cov}|$ comes from the high η realization, not from the agent's speculation activity ($b\neq 0$).

Note: Let $\rho(b) \equiv \int u(w^o(x,\eta))g(x,\eta|a_1^o,a_2^o,b)dxd\eta - v(a_1^o)$ be the agent's expected indirect utility as a function of b. Then from , we obtain $\rho(b) = \rho(0)$ holds for a positive measure of b[18] and $\rho(b)$ must be symmetric around $b = 0$ from (A94) in Lemma 10. Possible shapes of $\rho(b)$ are provided in the following Figure G.1.

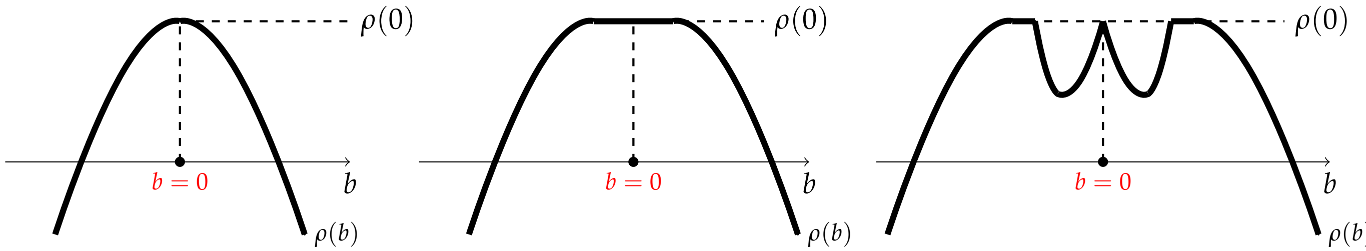

Figure G.1: Agent's Indirect Utility $\rho(b)$ as a function of b

As $b \to \pm\infty$, $\widehat{Cov} \to \pm\infty$ at any realization of (θ,η) since $\widehat{Cov} = b\eta^2 + a_2^o\theta\eta$ and $\eta^2 > 0$. The above optimal contract in (A102) implies: as $b \to \pm\infty$, we have $w(x,\eta) = w(\phi(a_1^o,a_2^o) + b\eta + a_2^o\theta,\eta) < w(\phi(a_1^o,a_2^o) + a_2^o\theta,\eta)$ uniformly on (θ,η).[19] Thus we have $\rho(b) < \rho(0)$ when $b \to \pm\infty$.

■

Proof of Proposition 11: Although we do not explicitly characterize SW^N in (47), we at least see SW^N is a continuous function of σ_R^2. On the other hand, $w^*(y)$ characterized in (41) and $w^o(x,\eta)$ in equation (61) are independent of σ_R^2, and so are $SW^*(a_1^*,a_2^*)$ and $SW^o(a_1^o,a_2^o,a_3^o = R)$. Thus, as the amount of uncertainty on the firm's risk exposure approaches zero (i.e., $\sigma_R^2 \to 0$), we have

$$SW^*(a_1^*,a_2^*) - SW^N \to 0, \tag{A103}$$

since the reason $SW^N < SW^*(a_1^*,a_2^*)$ is that the shareholders do not observe the realized R and this informational asymmetry disappears as $\sigma_R^2 \to 0$. As $SW^*(a_1^*,a_2^*) - SW^o(a_1^o,a_2^o,R) > 0$ remains unchanged as $\sigma_R^2 \to 0$, when σ_R^2 is very small, we have

$$SW^o(a_1^o,a_2^o,a_3^o = R) - SW^N < 0. \tag{A104}$$

■

Proof of Proposition 12: From Lemma 7, we see that $w^*(y_r)$ is a truth-telling mechanism for the firm's hidden risk exposure, R, if $\mu_2^* < 0$ for $w^*(y_r)$. Since $r = R, \forall R$, under $w^*(y_r)$, we have

$$y \equiv x - R\eta = \phi(a_1,a_2) + a_2\theta = y_r. \tag{A105}$$

Furthemore, we have that $w^*(y_r)$ in equation (65) has the same contractual form as $w^*(y)$ in (41). Thus, the optimal

[18]Due to the complementary slackness condition (A90) about the constraint (iii) of the optimization in equation (60), $\mu_4^o(b) > 0$ for a positive measure of b in Lemma 10 means $\rho(b) = \rho(0)$ for a positive measure of b.

[19]Actually $b \to \pm\infty$ also affects the output x in (A102). While terms up to a second-order of the output x enter in the optimal contract in (A102), higher-order terms of $\widehat{Cov}$ clearly dominates the first and second order terms of x.

action combination to be chosen by the agent under $w^*(y_r)$ is (a_1^*, a_2^*), i.e., $(a_1^T(R), a_2^T(R)) = (a_1^*, a_2^*), \forall R$. Therefore, we derive

$$SW^T = SW^*(a_1^*, a_2^*), \tag{A106}$$

and from Proposition 9, we derive that SW^T is the same as the joint benefits that will be obtained under $w^*(z(0))$ when there is a derivative market.

■

Proof of Proposition 13 Note that both non-communication contracts $w^N(x, \eta)$ and $w^o(x, \eta)$ in (61) are truth-telling mechanisms.[20] Therefore, if $\mu_2^* > 0$ for $w^*(y_r)$ in equation (65), we have

$$SW^T \geq max\{SW^N, SW^o(a_1^o, a_2^o, R)\}. \tag{A107}$$

Furthermore, from Proposition 11, we have $SW^N > SW^o(a_1^o, a_2^o, R)$ when σ_R^2 is very small. Thus, we obtain that $SW^T > SW^o(a_1^o, a_2^o, R)$ when σ_R^2 is very small.

■

[20]The principal can design $w^N(x, \eta)$ without using r. Also, by designing $w^o(y_r, \eta)$ as a truth-telling mechanism, he can obtain the same result as $w^o(x, \eta)$ would provide.