[Network Embedding]

Skip-Gram

Project: Skip-Gram Implementation with Python

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20.01.17(Fri)

Goal

"Implement Skip-Gram Model using Random Walk"

INPUT: (One-Hot Encoded) Vertice



Latent Representation of input vector (Embedded Vector)

OUTPUT: Probability Distribution of Vertices

Contents

1

Introduction

Brief overview of Skip-Gram & Random Walk

2

Implementation

- 1) Import Dataset & Libraries
- 2) Define Functions(Random Walk, Softmax, Feed Forward, Back Propagation)
- 3) Skip Gram

3

Result

Visualization of Network



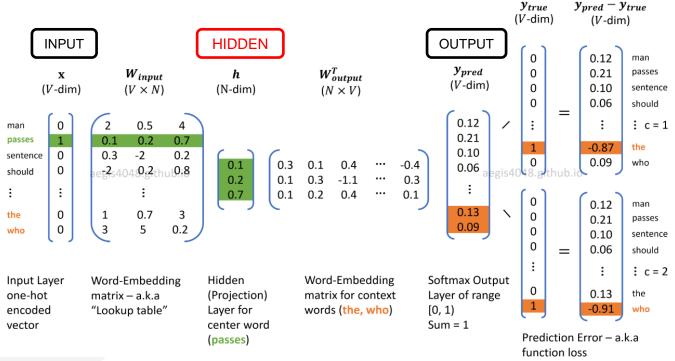






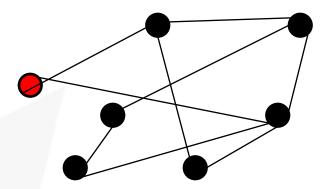
Brief overview of **Skip-Gram** & **Random Walk**

1. Skip-Gram

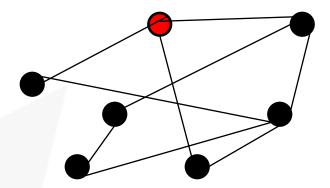


Predict Context Words given One Word

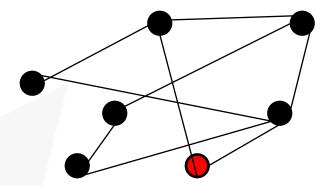
2. Random Walk



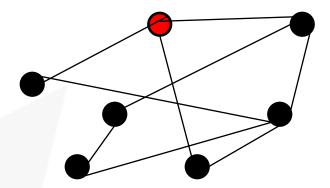
2. Random Walk



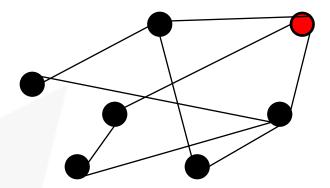
2. Random Walk



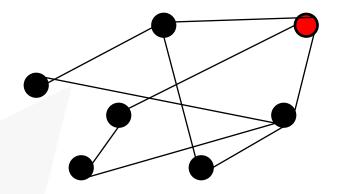
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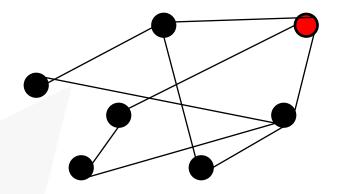


2. Random Walk



- 1. Local exploration is easy to parallelize!
- 2. No need for global recomputation (enable online learning)

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- 2. No need for global recomputation (enable online learning)



2. Random Walk

Original

34 Vertices

Ex) walk length = 9

Random Walk

10 Vertices



2. Random Walk

Original

34 Vertices

Random Walk

Ex) walk length = 9

10 Vertices



Implement Skip Gram from these 10 vertices!



2. Random Walk

Original

34 Vertices

Random Walk

Ex) walk length = 9

10 Vertices



Implement Skip Gram from these 10 vertices!



(window size = 2)



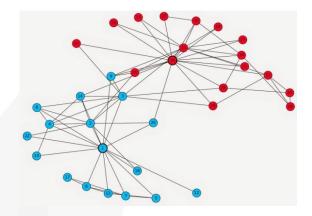




- 1) Import Dataset & Libraries
- 2) Define Functions(Random Walk, Softmax, Feed Forward, Back Propagation)
- 3) Skip Gram



[Data Overview]

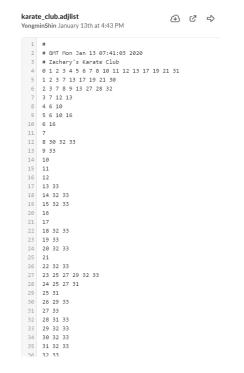


Karate Graph

Network Graph with

34 vertices (labeled 0 or 1)

Implementation



[1. adjacency list]

```
karate_club.edgelist
YongminShin January 13th at 4:43 PM
  1 01 {}
  2 0 2 {}
  3 0 3 {}
   4 0 4 {}
  5 0 5 {}
  6 06 {}
  7 07 {}
  8 08 {}
  9 0 10 {}
 10 0 11 {}
 11 0 12 {}
 12 0 13 {}
 13 0 17 {}
 14 0 19 {}
 15 0 21 {}
 16 0 31 {}
 17 1 2 {}
 18 1 3 {}
 19 1 7 {}
 20 1 13 {}
 21 1 17 {}
 22 1 19 {}
 23 1 21 {}
 24 1 30 {}
 25 2 3 {}
 26 2 7 {}
 27 28 {}
 28 2 9 {}
 29 2 13 {}
 30 2 27 {}
 31 2 28 {}
 32 2 32 {}
 34 3 12 {}
 35 3 13 {}
```

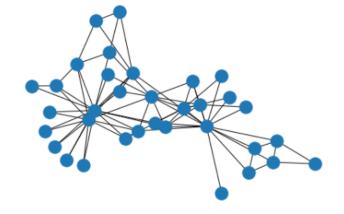
[2. edge list]

36 4 6 {}



1. Import Dataset

```
In [1]:
             import networkx as nx
            import matplotlib.pyplot as plt
             import numpy as np
             import random
             import pandas as pd
            from random import shuffle
             from copy import copy
             %matplotlib inline
In [2]:
             edge = pd.read csv('karate club.edgelist', sep=' ', names=['x','v','w'])
In [3]:
             edge.head()
Out [3]:
            x y w
         0 0 1 {}
         1 0 2 {}
         2 0 3 {}
         3 0 4 {}
         4 0 5 {}
```





1) Adjacency Matrix

2). Input Word Vector (One-Hot encoded)



1) Adjacency Matrix

```
In [5]:
             A = nx.to numpy matrix(graph, nodelist=sorted(graph.nodes()))
In [6]:
Out[6]: matrix([[0., 1., 1., ..., 1., 0., 0.],
                [1., 0., 1., ..., 0., 0., 0.],
                [1., 1., 0., ..., 0., 1., 0.],
                                                      1 in adjacent vertices,
                [1., 0., 0., ..., 0., 1., 1.],
                                                      0 otherwise
                [0., 0., 1., ..., 1., 0., 1.],
                [0., 0., 0., ..., 1., 1., 0.]])
        2). Input Word Vector (One-Hot encoded)
In [7]:
             OH = np.identity(34)
In [8]:
            OH
Out[8]: array([[1., 0., 0., ..., 0., 0., 0.]
               [0., 1., 0., ..., 0., 0., 0.],
               [0., 0., 1., ..., 0., 0., 0.],
                                                      Shape: 34 x 34
               [0., 0., 0., ..., 1., 0., 0.],
               [0., 0., 0., ..., 0., 1., 0.],
               [0., 0., 0., ..., 0., 0., 1.]])
```



1) Adjacency Matrix

2). Input Word Vector (One-Hot encoded)

Implementation

for Random Walk!



- each row : one vertex
- By finding the index of **NON-ZERO** values

for Input Vector of every vertex



(Random Walk, Softmax, Feed Forward, Back Propagation)

1). Random Walk

2) softmax

Implementation

https://medium.com/@jonathan hui/machine-learning-summary-algorithm-d75c64963800



(Random Walk, Softmax, Feed Forward, Back Propagation)

2) softmax

Row 0	0	1	1	0	0		1	1
Row 1	1	0	0	1	0		1	0
							1	
Row 32	1	1	0	0	1	1	0	0
Row 33	1	1	1	0	0		0	0



(Random Walk, Softmax, Feed Forward, Back Propagation)

2) softmax

Implementation

Row 0	0	1	1	0	0		1	1
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							1	
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(input) 1 - 32



(Random Walk, Softmax, Feed Forward, Back Propagation)

```
1). Random Walk

In [9]: 1 def random_step(i,w):
    walk_list = []
    walk_list.append(i)
    for k in range(w):
        ad = np.nonzero(A[i])[1] # i와 인접한 vertex 들의 list
        rand = random.cho ce(ad) # 그 list중 랜덤하게 하나 고르기
        walk_list.append(rand)
        i = rand
        return walk_list

In [78]: 1 random_step(3,10)

Out [78]: [3, 2, 1, 21, 0, 21, 1, 0, 21, 1, 19]
```

2) softmax

Implementation

Row 0	0	1	1	0	0		1	1
Row 1	1	0	0	1	0		1	0
							1	
Row 32	1	1	0	0	1	1	0	0
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							1	
Row 32	1	1	0	0	1	1	0	0
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(input) 1 - 32 - 0



(Random Walk, Softmax, Feed Forward, Back Propagation)

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Implementation

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							1	:
Row 32	1	1	0	0	1	1	0	0
Row 33	1	1	1	0	0		0	0
1	ı	ı	ı		1			•

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							1	
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(input) 1 - 32 - 0 - 2



(Random Walk, Softmax, Feed Forward, Back Propagation)

1). Random Walk

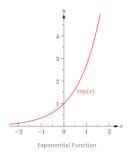
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In [78]:
             random step(3,10)
```

2) softmax

Out [78]: [3, 2, 1, 21, 0, 21, 1, 0, 21, 1, 19]

```
In [93]:
              def softmax(x):
                c = np.max(x)
                b = x-c
                exp x = np.exp(b)
                sum exp x = np.sum(exp x)
                y = \exp x / \sup \exp x
                return y
```

Implementation



The problem arise when x(i) is too small or too large. Suppose each x(i) is very small negative number, $\exp(x(i))$ will be close to 0, since all the x(i)are very small the denominator of softmax function will be close to 0 and result will be not defined. This is called **underflow**. If x(i) is very large $\exp(x(i))$ will be very large number, may exceed the computational limit. This is called overflow.

$$softmax(x)_{i} = \frac{e^{x_{i}}}{\sum_{j=1}^{n} e^{x_{j}}}$$

$$= \frac{e^{x'_{i}}}{\sum e^{x'_{j}}}$$
Avoid overflow or underflow
$$m = \max(x)$$

$$x_{j} \to x_{j} - m = x'_{j}$$

https://medium.com/@jonathan hui/machine-learning-summary-algorithm-d75c64963800 https://medium.com/@ravish1729/analysis-of-softmax-function-ad058d6a564d



(Random Walk, Softmax, Feed Forward, Back Propagation)

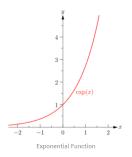
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$$softmax(x)_{i} = \frac{e^{x_{i}}}{\sum_{j=1}^{n} e^{x_{j}}}$$

$$= \frac{e^{x_{i}'}}{\sum e^{x_{j}'}}$$
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(Random Walk, Softmax, Feed Forward, Back Propagation)

3) Feed Forward

4) Back Propagation

```
def backprop(input_word,w1,w2,lr,h,y_pred,index,window_size):
    front = input_word[index-window_size : index]
    back = input_word[index+1 : index+window_size+1]
    window_OH = np.concatenate([front,back])

# output -> hidden
for j in range(w2.shape[1]):
    adjust = (y_pred-window_OH)[:,j].sum()*h
    w2[:,j] -= -lr*adjust

# hidden -> input
adjust2 = ((y_pred-window_OH).sum(axis=0)*w2).T
    w1-= lr*adjust2
return w1,w2
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(Random Walk, Softmax, Feed Forward, Back Propagation)

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3) Feed Forward

In [94]:

| def feedforward(input_word,index,w1,w2):
| h=np.matmul(w1.T,input_word[index])
| u=np.matmul(w2.T,h)
| y = softmax(u)
| return h,u,y
```

4) Back Propagation

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(Random Walk, Softmax, Feed Forward, Back Propagation)

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```

$$\mathbf{h} = \mathbf{W}_{(k,\cdot)}^T := \mathbf{v}_{w_I}^T$$

$$u_{c,j} = u_j = \mathbf{v}'_{w_j}^T \cdot \mathbf{h}$$

$$y_{c,j} = \frac{\exp(u_{c,j})}{\sum_{j'=1}^{V} \exp(u_{j'})}$$



(Random Walk, Softmax, Feed Forward, Back Propagation)

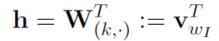
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adjust2 = ((y_pred-window_OH).sum(axis=0)*w2).T
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    return w1,w2
```

Implementation



Calculate the hidden layer

(W:input -> hidden weight) (k:index of input word)

$$u_{c,j} = u_j = \mathbf{v}_{w_j}^{\prime} \cdot \mathbf{h}$$

$$y_{c,j} = \frac{\exp(u_{c,j})}{\sum_{j'=1}^{V} \exp(u_{j'})}$$



(Random Walk, Softmax, Feed Forward, Back Propagation)

In [94]: | def feedforward(input_word,index,w1,w2): | h=np.matmul(w1.T,input_word[index]) | u=np.matmul(w2.T,h) | y = softmax(u) | return h,u,y

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```

Implementation

$$\mathbf{h} = \mathbf{W}_{(k,\cdot)}^T := \mathbf{v}_{w_I}^T$$

$$u_{c,j} = u_j = \mathbf{v}'_{w_j}^T \cdot \mathbf{h}$$

Input of j-th unit on the c-th panel of the output layer (c:# of Multinomial Distributions)

$$y_{c,j} = \frac{\exp(u_{c,j})}{\sum_{j'=1}^{V} \exp(u_{j'})}$$



(Random Walk, Softmax, Feed Forward, Back Propagation)

4) Back Propagation

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Implementation

$$\mathbf{h} = \mathbf{W}_{(k,\cdot)}^T := \mathbf{v}_{w_I}^T$$

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$$y_{c,j} = \frac{\exp(u_{c,j})}{\sum_{j'=1}^{V} \exp(u_{j'})}$$

output of the j-th unit on the c-th panel



(Random Walk, Softmax, Feed Forward, Back Propagation)

3) Feed Forward

4) Back Propagation

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```



(Random Walk, Softmax, Feed Forward, Back Propagation)

3) Feed Forward

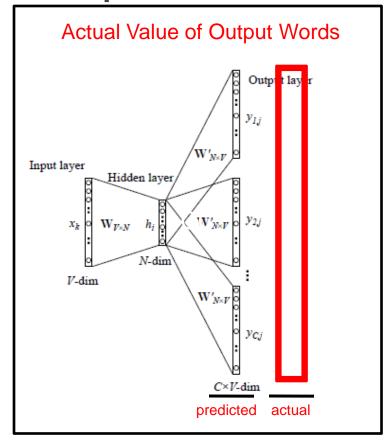
4) Back Propagation

```
def backprop(input word w1 w2 lr h v pred index window size):

front = input_word[index-window_size : index]
back = input_word[index+1 : index+window_size+1]
window_OH = np.concatenate([front,back])

# output -> hidden
for j in range(w2.shaps[1]);
adjust = (y_pred-window_OH)[:,j].sum()*h
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(Random Walk, Softmax, Feed Forward, Back Propagation)

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```

$$\mathbf{v}'_{w_j}^{(\text{new})} = \mathbf{v}'_{w_j}^{(\text{old})} - \eta \cdot \text{EI}_j \cdot \mathbf{h}$$

$$EI_j = \sum_{c=1}^{C} e_{c,j}$$
 for $j = 1, 2, \dots, V$.

$$\mathbf{v}_{w_I}^{\text{(new)}} = \mathbf{v}_{w_I}^{\text{(old)}} - \eta \cdot \mathbf{E}\mathbf{H}^T$$

$$EH_i = \sum_{j=1}^{V} EI_j \cdot w'_{ij}.$$



(Random Walk, Softmax, Feed Forward, Back Propagation)

3) Feed Forward

```
In [94]: 1 | def feedforward(input_word,index,w1,w2): | h=np.matmul(w1.T,input_word[index]) | | u=np.matmul(w2.T,h) | | y = softmax(u) | | return h,u,y |
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```

$$\mathbf{v}'_{w_j}^{\text{(new)}} = \mathbf{v}'_{w_j}^{\text{(old)}} - \eta \cdot \text{EI}_j \cdot \mathbf{h}$$

$$\text{EI}_j = \sum_{c=1}^C e_{c,j} \quad \text{for } j = 1, 2, \dots, V.$$

$$\mathbf{v}_{w_I}^{(\text{new})} = \mathbf{v}_{w_I}^{(\text{old})} - \eta \cdot \mathbf{E}\mathbf{H}^T$$

$$EH_i = \sum_{j=1}^{V} EI_j \cdot w'_{ij}.$$



3. Skip Gram

3. Skip-Gram

```
In [96]:

def Skipgram(input_word, reduced_dim, lr, walk_size, window_size,epoch):
    W1 = np.random.random((input_word.shape[0],reduced_dim))
    W2 = np.random.random((reduced_dim, input_word.shape[0]))

for _ in range(epoch):
    input_word = copy(input_word)
    shuffle(input_word)
    for index in range(input_word.shape[0]):
        RW = input_word[random_step(index,walk_size)]
    for i in range(len(RW)):
        h,u,y = feedforward(RW,i,W1,W2)
        W1,W2 = backprop(RW,W1,W2,lr,h,y,i,window_size)

return W1,W2
```

Implementation

[Input Variables]

1. input_word:

Matrix of Input Words (34 x 34 Identity Matrix)

2. reduced dim:

Dimension of the embedded vector

3. lr:

Learning Rate

4. walk size:

Walk length in random walk

5. window_size :

(one-sided) Size of the window from the index

6. epoch:

Walks per vertex



3. Skip Gram

3. Skip-Gram

```
In [96]:

def Skipgram(input_word, reduced_dim, Ir, walk_size, window_size,epoch):
    W1 = np.random.random((input_word.shape[0],reduced_dim))
    W2 = np.random.random((reduced_dim, input_word.shape[0]))

for _ in range(epoch):
    input_word = copy(input_word)
    shuffle(input_word)
    for index in range(input_word.shape[0]):
        RW = input_word[random_step(index,walk_size)]
    for i in range(len(RW)):
        h,u,y = feedforward(RW,i,W1,W2)
        W1,W2 = backprop(RW,W1,W2,Ir,h,y,i,window_size)

return W1,W2
```

Implementation

[Process]

1) Initialize weight

(uniform distribution) (W1 : input – hidden Weight) (W2 : hidden – output Weight)

- 2) Shuffle the words
- 3) Implement a Random Walk
- 4) Feed Forward(with the vertices selected by RW)
- 5) Back Propagation
- 6) Return Weights







3. Result

Visualization of Network



Result

4. Result

```
w1,w2 = Skipgram(OH,reduced_dim=2, Ir=0.02, walk_size=10,window_size=3,epoch=7)

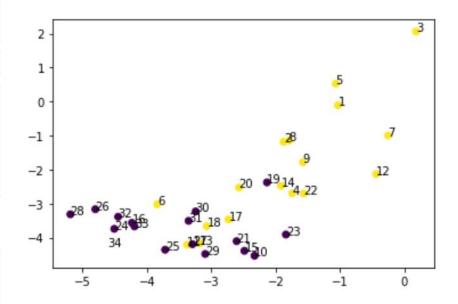
| Emb = np.matmul(OH,w1)

| Emb_df = pd.DataFrame({'X':Emb[:,0], 'Y':Emb[:,1],'Label':range(1,35)})
| blue = [1,2,3,4,5,6,7,8,9,11,12,13,14,17,18,20,22] | red = list(set(range(0,34))-set(blue))
| Emb_df.loc[Emb_df.Label.isin(blue),'Color']=1 | Emb_df.loc[Emb_df.Label.isin(red),'Color']=0
```

Visualization

```
plt.scatter(Emb_df['X'], Emb_df['Y'], c=Emb_df['Color'])

for i,txt in enumerate(Emb_df['Label']):
plt.annotate(txt, (Emb_df['X'][i], Emb_df['Y'][i]))
```



Reference

[1] Bryan Perozzi, Rami Al-Rfou, Steven Skiena: Deepwalk: Online Learning of Social Representations

[2] Xin Rong: word2vec Parameter Learning Explained

