BRL Seminar

(2023. 08. 22. Tue)

TSMixer: An all-MLP Architecture for Time Series Forecasting

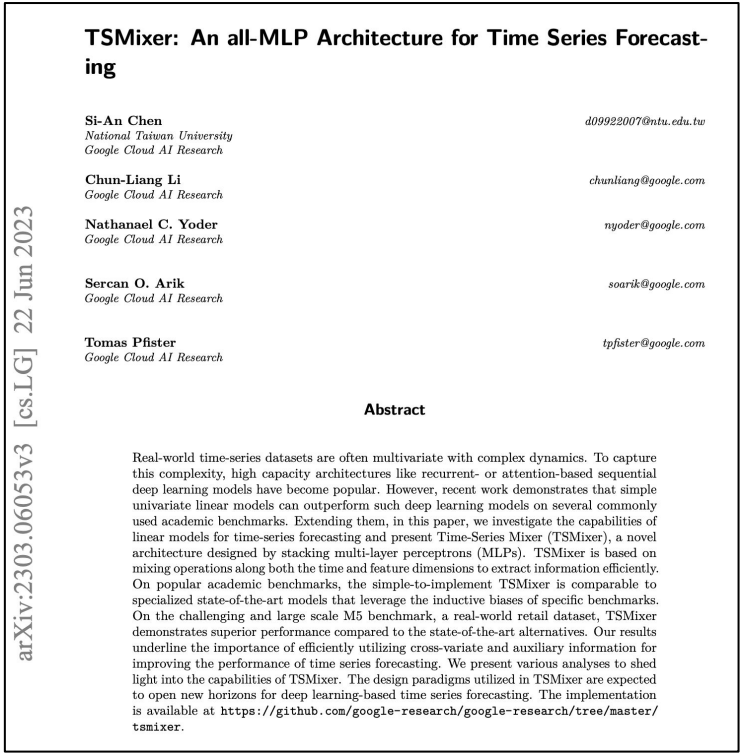
통합과정 6학기 이승한

Paper

TSMixer: An all-MLP Architecture for Time Series Forecasting

(TMLR under review)

- https://arxiv.org/pdf/2303.06053.pdf
- https://github.com/google-research/google-research/tree/master/tsmixer



TSMixer: An all-MLP Architecture for Time Series Forecasting

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Abstract

Real-world time-series datasets are often multivariate with complex dynamics. To capture this complexity, high capacity architectures like recurrent- or attention-based sequential deep learning models have become popular. However, recent work demonstrates that simple univariate linear models can outperform such deep learning models on several commonly used academic benchmarks. Extending them, in this paper, we investigate the capabilities of linear models for time-series forecasting and present Time-Series Mixer (TSMixer), a novel architecture designed by stacking multi-layer perceptrons (MLPs). TSMixer is based on mixing operations along both the time and feature dimensions to extract information efficiently. On popular academic benchmarks, the simple-to-implement TSMixer is comparable to specialized state-of-the-art models that leverage the inductive biases of specific benchmarks. On the challenging and large scale M5 benchmark, a real-world retail dataset, TSMixer demonstrates superior performance compared to the state-of-the-art alternatives. Our results underline the importance of efficiently utilizing cross-variate and auxiliary information for improving the performance of time series forecasting. We present various analyses to shed light into the capabilities of TSMixer. The design paradigms utilized in TSMixer are expected to open new horizons for deep learning-based time series forecasting. The implementation is available at `https://github.com/google-research/google-research/tree/master/tsmixer`.

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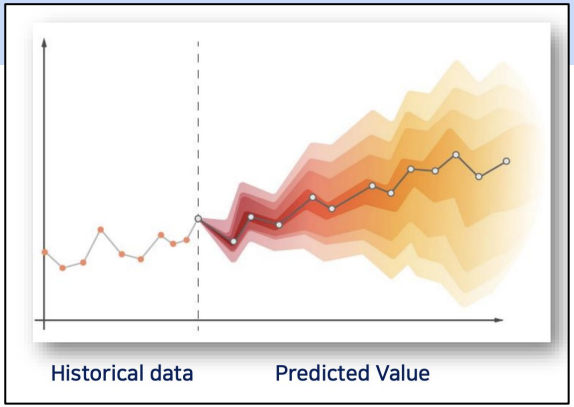
1. Preliminaries

1. Preliminaries

1. Long-term Time Series Forecasting (LTSF) task
2. Channel Dependence vs. Independence

1. Preliminaries



1. Long-term Time Series Forecasting (LTSF) task

Depends on the **“LENGTH of prediction target”**

- **LONG : Long-term Time Series Forecasting** (Widely-used task)
- SHORT : Short-term Time Series Forecasting

Datasets	Weather	Traffic	Electricity	ILI	ETTh1	ETTh2	ETTm1	ETTm2
Features	21	862	321	7	7	7	7	7
Timesteps	52696	17544	26304	966	17420	17420	69680	69680

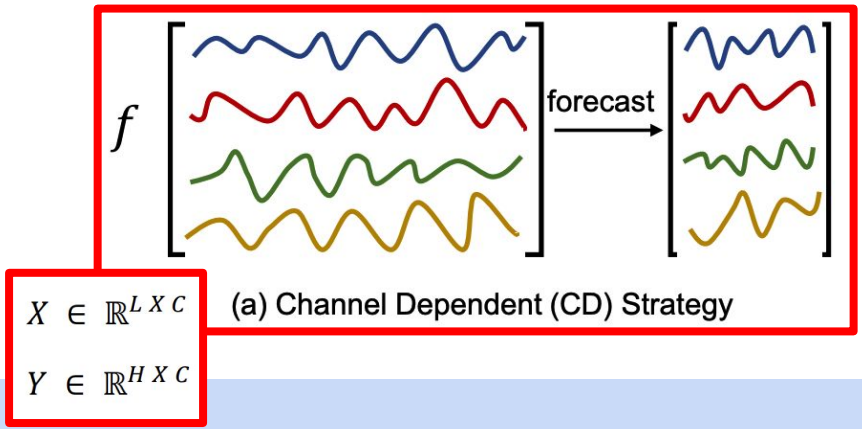
Table 2: Statistics of popular datasets for benchmark.

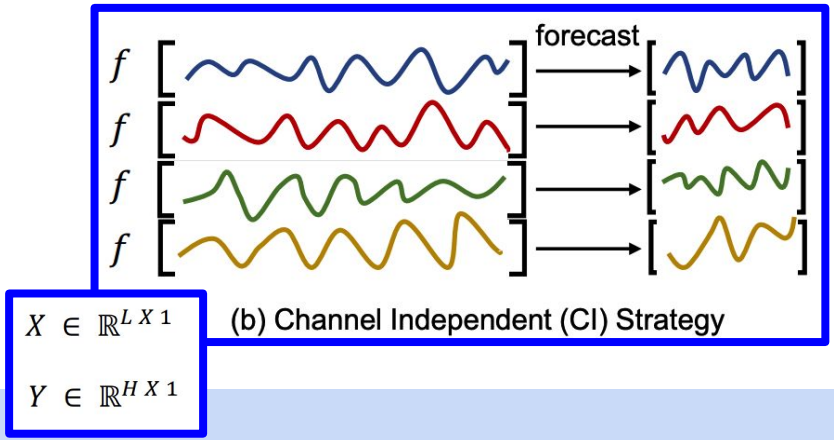
1. Preliminaries

2. Channel Dependence vs. Independence

Does it captures correlation between the channels (dimensions) ?

- **CD (Channel Dependence) : YES** !
- **CI (Channel Independence) : NO** !





1. Preliminaries

2. Channel Dependence vs. Independence

Does it captures correlation between the channels (dimensions) ?

- **CD (Channel Dependence) : YES** !
- **CI (Channel Independence) : NO** !

C : Number of channels

$$\min_f \frac{1}{N} \sum_{i=1}^{N} \ell(f(X^{(i)}), Y^{(i)})$$

Loss function of CD

$$\min_f \frac{1}{NC} \sum_{i=1}^{N} \sum_{c=1}^{C} \ell(f(x_c^{(i)}), y_c^{(i)})$$

Loss function of CI

Intuitively, CD seems better!

2. Recent Trends in LTSF

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Datasets	Weather	Traffic	Electricity	ILI	ETTh1	ETTh2	ETTm1	ETTm2
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- **Informer** (2021)
- **Autoformer** (2022)
- **FEDformer** (2022)

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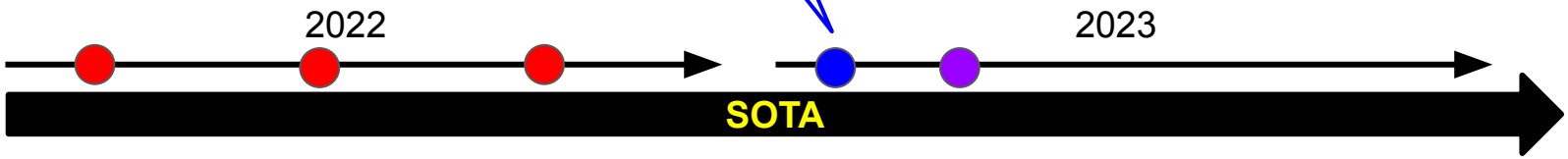
Are Transformers Effective for Time Series Forecasting?

Ailing Zeng[1]*, Muxi Chen[1]*, Lei Zhang[2], Qiang Xu[1]
[1]The Chinese University of Hong Kong
[2]International Digital Economy Academy (IDEA)
{alzeng, mxchen21, qxu}@cse.cuhk.edu.hk
{leizhang}@idea.edu.cn



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2023 : **Simple** / **SSL** algorithms

- **Linear models** (2023)
- **PatchTST** (2023)

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Datasets	Weather	Traffic	Electricity	ILI	ETTh1	ETTh2	ETTm1	ETTm2
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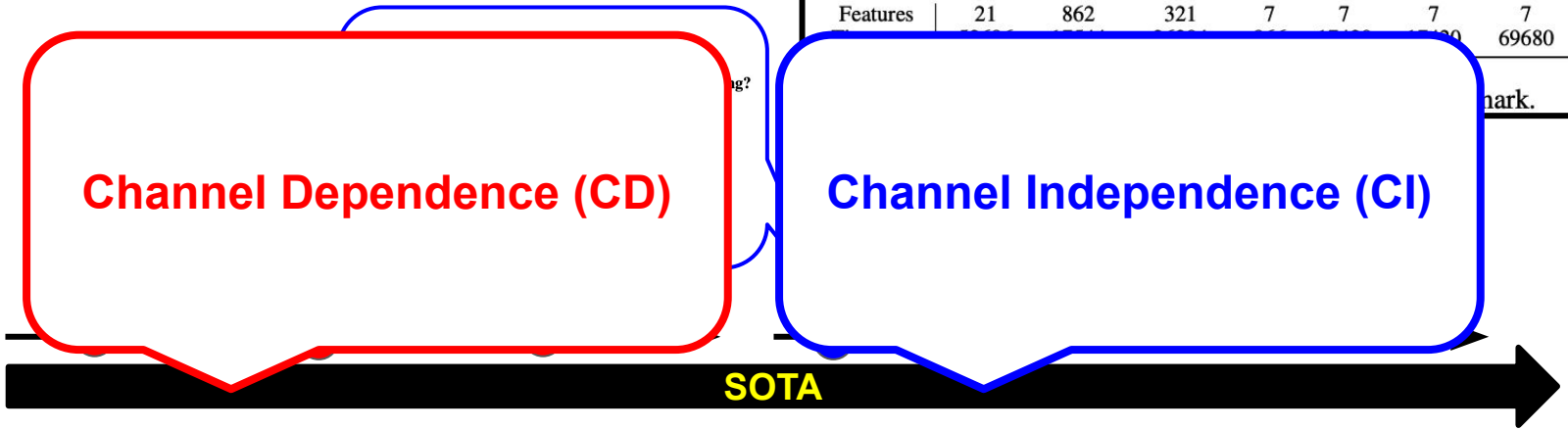

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Are Transformers Effective for Time Series Forecasting? AAAI, 2023

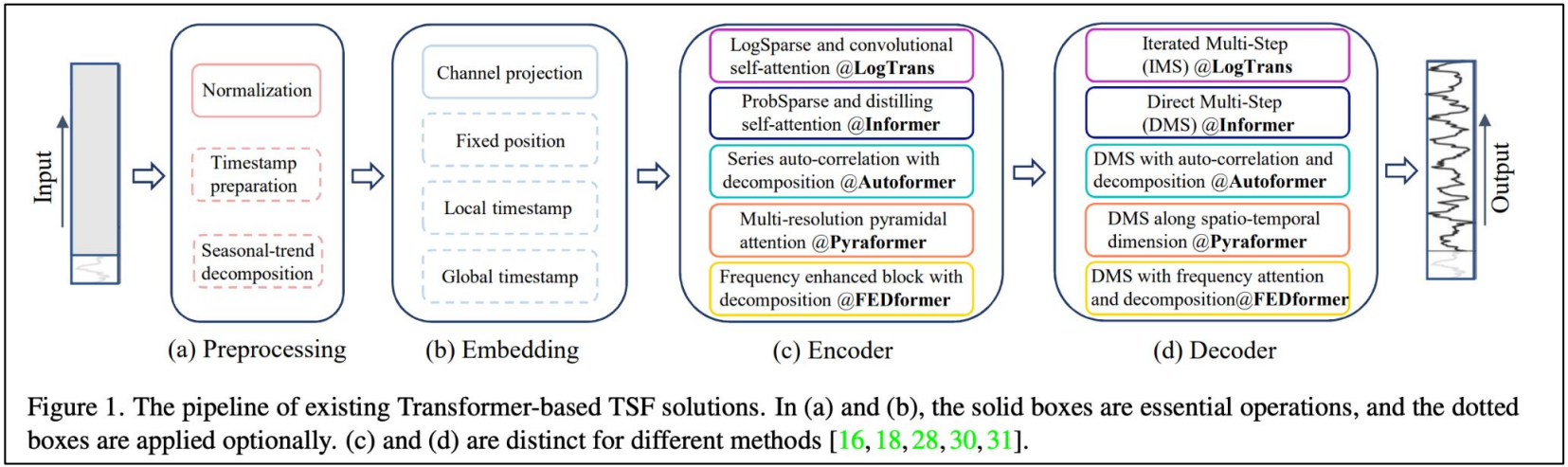


Figure 1. The pipeline of existing Transformer-based TSF solutions. In (a) and (b), the solid boxes are essential operations, and the dotted boxes are applied optionally. (c) and (d) are distinct for different methods [16, 18, 28, 30, 31].

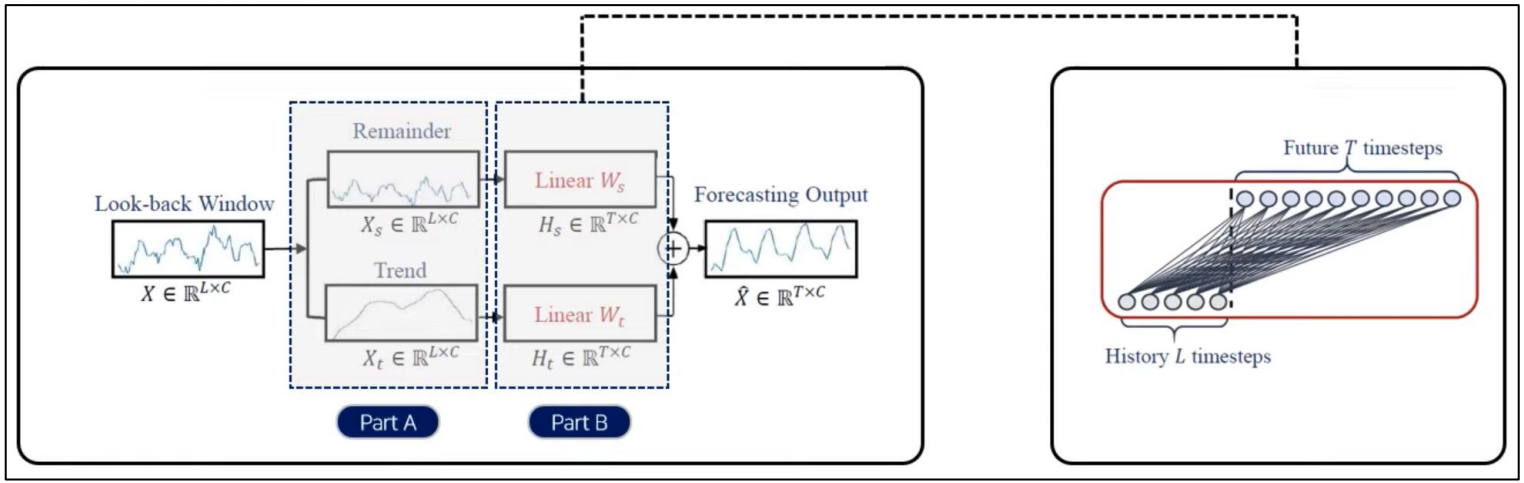
TOO complicated!

2. Recent Trends in LTSF

2023 : **Simple** / SSL algorithms

- **Linear models** (2023)
- PatchTST (2023)

Are Transformers Effective for Time Series Forecasting? AAAI, 2023



VERY SIMPLE!

https://www.youtube.com/watch?v=J5PI5a_mXfE

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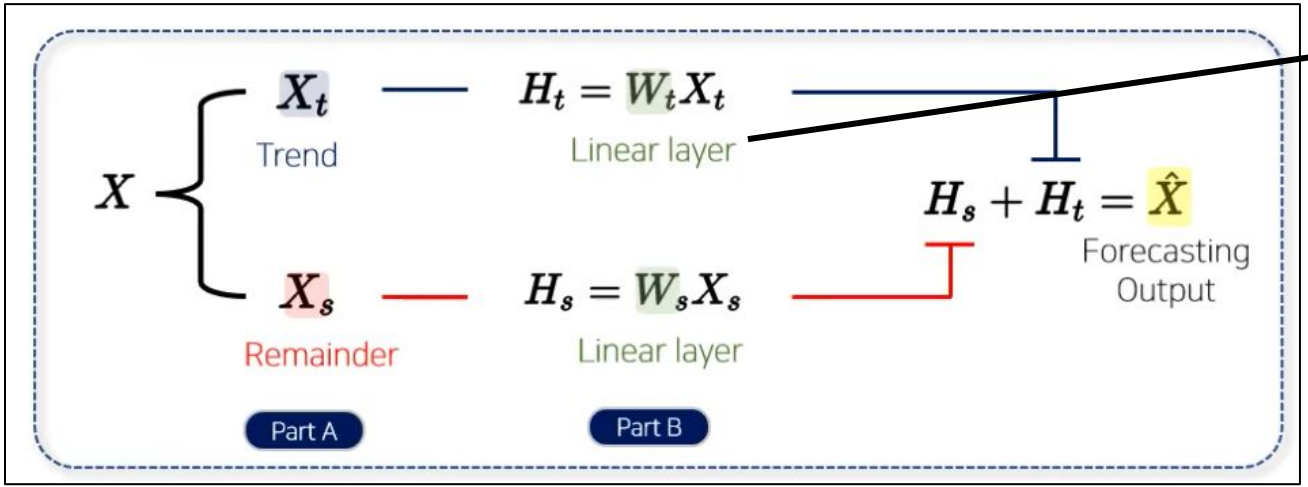

Shared for all channels!

(= Channel Independence)

VERY SIMPLE!

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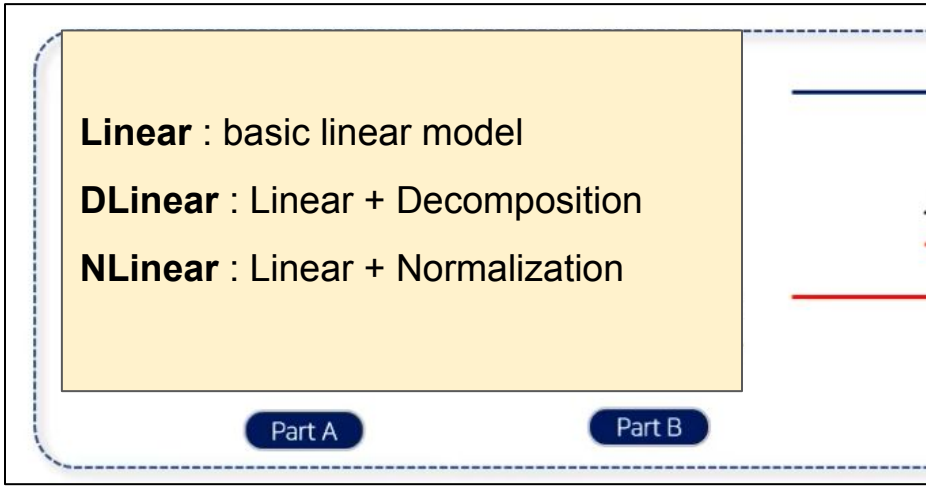
2. Recent Trends in LTSF

Are Transformers Effective for Time Series Forecasting? AAAI,



Outperforms previous SOTA ! Are Transfor

Proposed | SOTA

Methods		IMP.	Linear*		NLinear*		DLinear*		FEDformer	
Metric		MSE	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE
Electricity	96	27.40%	**0.140**	**0.237**	0.141	**0.237**	**0.140**	**0.237**	0.193	0.308
	192	23.88%	**0.153**	0.250	0.154	**0.248**	**0.153**	0.249	0.201	0.315
	336	21.02%	**0.169**	0.268	0.171	**0.265**	**0.169**	0.267	0.214	0.329
	720	17.47%	**0.203**	0.301	0.210	**0.297**	**0.203**	0.301	0.246	0.355
Exchange	96	45.27%	0.082	0.207	0.089	0.208	**0.081**	0.203	0.148	0.278
	192	42.06%	0.167	0.304	0.180	0.300	**0.157**	0.293	0.271	0.380
	336	33.69%	0.328	0.432	0.331	0.415	**0.305**	0.414	0.460	0.500
	720	46.19%	0.964	0.750	1.033	0.780	**0.643**	**0.601**	1.195	0.841
Traffic	96	30.15%	**0.410**	0.282	**0.410**	**0.279**	**0.410**	0.282	0.587	0.366
	192	29.96%	**0.423**	0.287	**0.423**	**0.284**	**0.423**	0.287	0.604	0.373
	336	29.95%	0.436	0.295	**0.435**	**0.290**	0.436	0.296	0.621	0.383
	720	25.87%	0.466	0.315	**0.464**	**0.307**	0.466	0.315	0.626	0.382
Weather	96	18.89%	**0.176**	0.236	0.182	**0.232**	**0.176**	0.237	0.217	0.296
	192	21.01%	**0.218**	0.276	0.225	**0.269**	0.220	0.282	0.276	0.336
	336	22.71%	**0.262**	0.312	0.271	**0.301**	0.265	0.319	0.339	0.380
	720	19.85%	0.326	0.365	0.338	**0.348**	**0.323**	0.362	0.403	0.428
ILI	24	47.86%	1.947	0.985	**1.683**	**0.858**	2.215	1.081	3.228	1.260
	36	36.43%	2.182	1.036	**1.703**	**0.859**	1.963	0.963	2.679	1.080
	48	34.43%	2.256	1.060	**1.719**	**0.884**	2.130	1.024	2.622	1.078
	60	34.33%	2.390	1.104	**1.819**	**0.917**	2.368	1.096	2.857	1.157
ETTh1	96	0.80%	0.375	0.397	**0.374**	**0.394**	0.375	0.399	0.376	0.419
	192	3.57%	0.418	0.429	0.408	**0.415**	**0.405**	0.416	0.420	0.448
	336	6.54%	0.479	0.476	**0.429**	**0.427**	0.439	0.443	0.459	0.465
	720	13.04%	0.624	0.592	**0.440**	**0.453**	0.472	0.490	0.506	0.507
ETTh2	96	19.94%	0.288	0.352	**0.277**	**0.338**	0.289	0.353	0.346	0.388
	192	19.81%	0.377	0.413	**0.344**	**0.381**	0.383	0.418	0.429	0.439
	336	25.93%	0.452	0.461	**0.357**	**0.400**	0.448	0.465	0.496	0.487
	720	14.25%	0.698	0.595	**0.394**	**0.436**	0.605	0.551	0.463	0.474
ETTm1	96	21.10%	0.308	0.352	0.306	0.348	**0.299**	**0.343**	0.379	0.419
	192	21.36%	0.340	0.369	0.349	0.375	**0.335**	**0.365**	0.426	0.441
	336	17.07%	0.376	0.393	0.375	0.388	**0.369**	**0.386**	0.445	0.459
	720	21.73%	0.440	0.435	0.433	0.422	**0.425**	**0.421**	0.543	0.490
ETTm2	96	17.73%	0.168	0.262	**0.167**	**0.255**	**0.167**	0.260	0.203	0.287
	192	17.84%	0.232	0.308	**0.221**	**0.293**	0.224	0.303	0.269	0.328
	336	15.69%	0.320	0.373	**0.274**	**0.327**	0.281	0.342	0.325	0.366
	720	12.58%	0.413	0.435	**0.368**	**0.384**	0.397	0.421	0.421	0.415

- Methods* are implemented by us; Other results are from FEDformer [31].

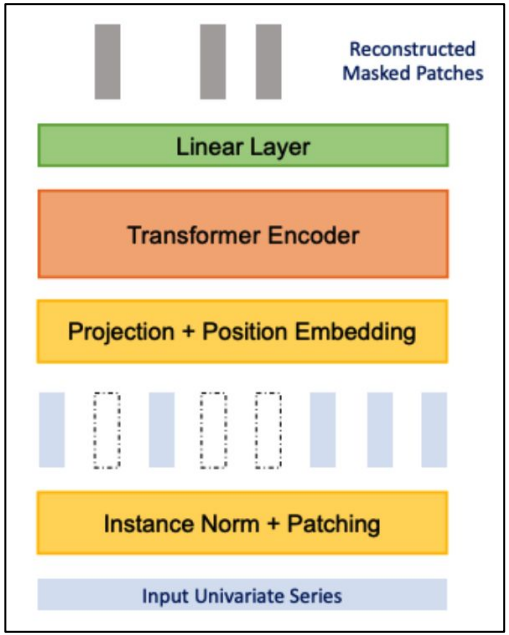
https://www.youtube.com/watch?v=J5PI5a_mXfE

2. Recent Trends in LTSF

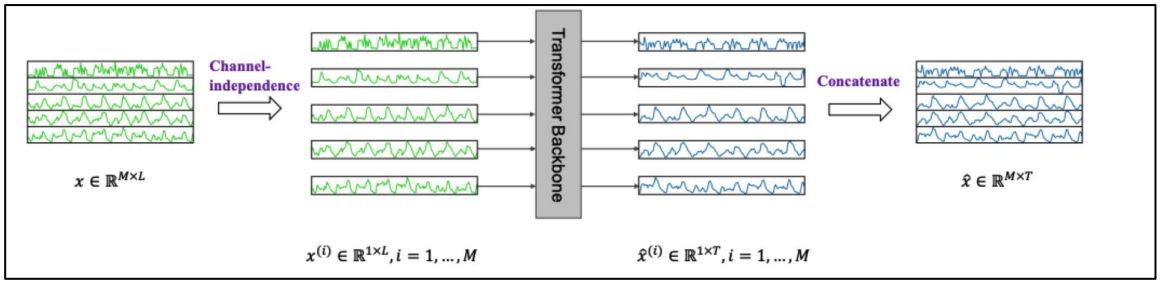
2023 : Simple / **SSL** algorithms

- Linear models (2023)
- **PatchTST (2023)**

A Time Series is Worth 64 Words : Long-term Forecasting with Transformers, ICLR 2023



PatchTST



Key Properties

(1) Channel Independence

(2) Patching time series (N time steps = 1 patch)

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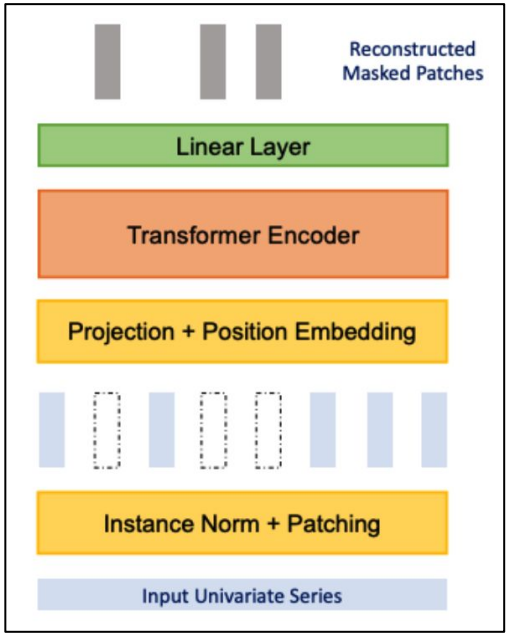
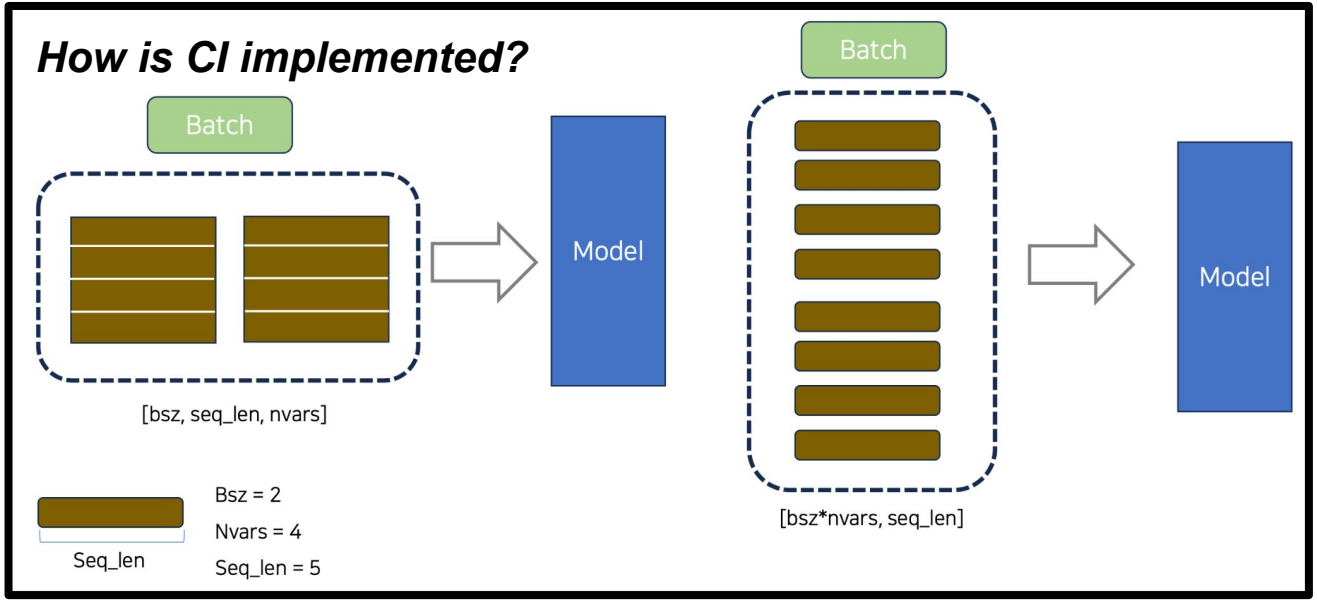


PatchTST



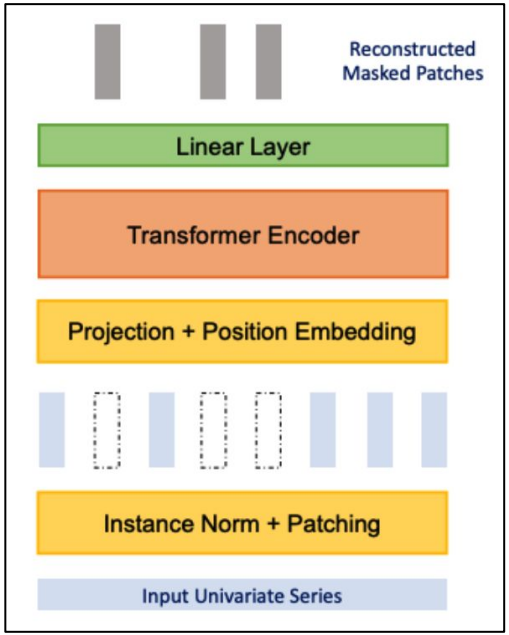
2. Recent Trends in LTSF

2023 : Simple / SSL algorithms
- Linear models (2023)
- PatchTST (2023)

A Time Series is Worth 64 Words : Long-term Forecasting with Tra

[SOTA summary (CI & CD)]

PatchTST > DLinear >>> Transformer-based



PatchTST

Models		PatchTST						DLinear		FEDformer	
		Fine-tuning		Lin. Prob.		Sup.					
Metric		MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE
Weather	96	**0.144**	**0.193**	0.158	0.209	0.152	0.199	0.176	0.237	0.238	0.314
	192	**0.190**	**0.236**	0.203	0.249	0.197	0.243	0.220	0.282	0.275	0.329
	336	**0.244**	**0.280**	0.251	0.285	0.249	0.283	0.265	0.319	0.339	0.377
	720	**0.320**	**0.335**	0.321	0.336	**0.320**	**0.335**	0.323	0.362	0.389	0.409
Traffic	96	**0.352**	**0.244**	0.399	0.294	0.367	0.251	0.410	0.282	0.576	0.359
	192	**0.371**	**0.253**	0.412	0.298	0.385	0.259	0.423	0.287	0.610	0.380
	336	**0.381**	**0.257**	0.425	0.306	0.398	0.265	0.436	0.296	0.608	0.375
	720	**0.425**	**0.282**	0.460	0.323	0.434	0.287	0.466	0.315	0.621	0.375
Electricity	96	**0.126**	**0.221**	0.138	0.237	0.130	**0.222**	0.140	0.237	0.186	0.302
	192	**0.145**	**0.238**	0.156	0.252	0.148	0.240	0.153	0.249	0.197	0.311
	336	**0.164**	**0.256**	0.170	0.265	0.167	0.261	0.169	0.267	0.213	0.328
	720	**0.193**	**0.291**	0.208	0.297	0.202	**0.291**	0.203	0.301	0.233	0.344

3. TSMixer

3. TSMixer

Background

According to the recent trends …

can we say that **Channel Independence** > **Channel Dependence** (?)

TSMixer : ***“No! It is because of the DATASET BIAS!***

3. TSMixer

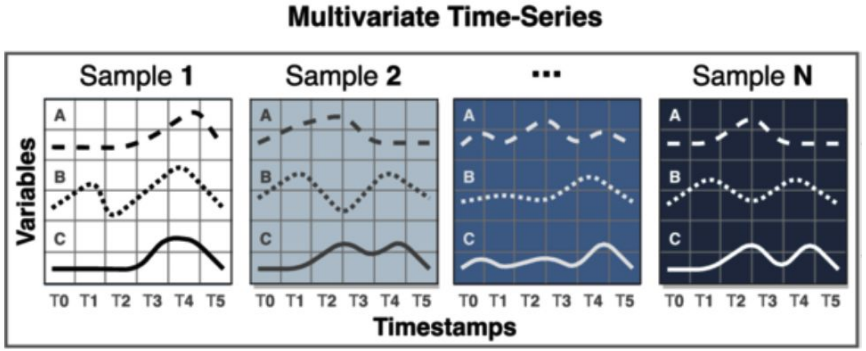
Abstract

- Investigate the **capabilities of linear models** for TS forecasting
- Present **TSMixer**
  - a novel architecture designed by **stacking MLPs**
  - based on mixing operations along both..
    - (1) **Time** dimension
    - (2) **Feature** dimension
  - Simple-to-implement

3. TSMixer

3 Major aspects of forecastability of TS

- **(1) Persistent temporal patterns**
  - trend & seasonal patterns
- **(2) Cross-variate information**
  - correlations between different variables
- **(3) Auxiliary features**
  - comprising static features and future information



3. TSMixer

- (1) Persistent temporal patterns
- (2) Cross-variate information
- (3) Auxiliary features

Categorization of recent TSF models

Table 1: Recent works in time series forecasting. Category I is univariate time series forecasting; Category II is multivariate time series forecasting, and Category III is time series forecasting with auxiliary information. In this work, we propose TSMixer for Category II. We also extend TSMixer to leverage auxiliary information including static and future time-varying features for Category III.

Category	Extrapolating temporal patterns	Consideration of cross-variate information (i.e. multivariateness)	Consideration of auxiliary features	Models
I	✔			ARIMA (Box et al., 1970) N-BEATS (Oreshkin et al., 2020) LTSF-Linear (Zeng et al., 2023) PatchTST (Nie et al., 2023)
II	✔	✔		Informer (Zhou et al., 2021) Autoformer (Wu et al., 2021) Pyraformer (Liu et al., 2022a) FEDformer (Zhou et al., 2022b) NS-Transformer (Liu et al., 2022b) FiLM (Zhou et al., 2022a) **TSMixer** (this work)
III	✔	✔	✔	MQRNN (Wen et al., 2017) DSSM (Rangapuram et al., 2018) DeepAR (Salinas et al., 2020) TFT (Lim et al., 2021) **TSMixer-Ext** (this work)

SOTA method only uses (1)

PROPOSED METHODS

3. TSMixer

2 Key Questions

- (1) Does **cross-variate information** truly provide a **benefit** for TS forecasting?
- (2) When **cross-variate information** is **not beneficial**, can multivariate models still perform **as well as univariate models**?

3. TSMixer

Linear Models for TS Forecasting

- a) Theoretical Insights on the **capacity of linear models**
  - have been overlooked due to its simplicity
- b) **Comparison with DL methods**

Notation

$\boldsymbol{X} \in \mathbb{R}^{L \times C_x}$: input

$\boldsymbol{Y} \in \mathbb{R}^{T \times C_y}$: target

Focus on the case where $(C_y \leq C_x)$

Linear model params : $\boldsymbol{A} \in \mathbb{R}^{T \times L}, \boldsymbol{b} \in \mathbb{R}^{T \times 1}$

- $\hat{\boldsymbol{Y}} = \boldsymbol{A}\boldsymbol{X} \oplus \boldsymbol{b} \in \mathbb{R}^{T \times C_x}$
  - $\oplus$: column-wise addition

3. TSMixer

Linear Models for TS Forecasting

- a) Theoretical Insights on the **capacity of linear models**

Most impactful real-world applications have either ..

- (1) smoothness
- (2) periodicity

Assumption 1) Time series is periodic (Holt, 2004; Zhang & Qi, 2005).

(A) arbitrary **periodic function** $x(t) = x(t-P)$, where $P < L$ is the period

perfect solution :

- $\boldsymbol{A}_{ij} = \begin{cases} 1, & \text{if } j = L - P + (i \bmod P) \\ 0, & \text{otherwise} \end{cases}, \boldsymbol{b}_i = 0..$

(B) affine-transformed periodic sequences, $x(t) = a \cdot x(t-P) + c$ where $a, c \in \mathbb{R}$ are constants

perfect solution :

- $\boldsymbol{A}_{ij} = \begin{cases} a, & \text{if } j = L - P + (i \bmod P) \\ 0, & \text{otherwise} \end{cases}, \boldsymbol{b}_i = c..$

3. TSMixer

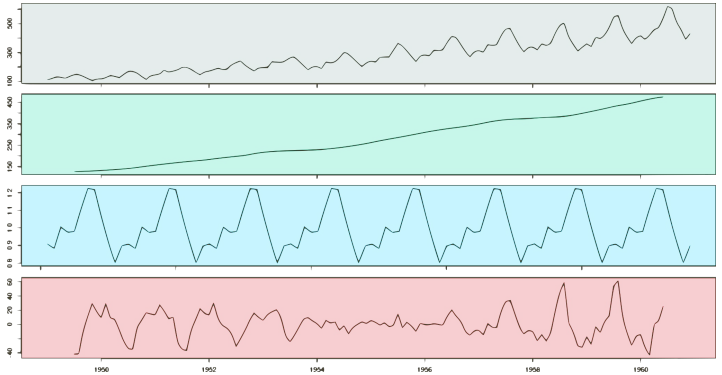
Linear Models for TS Forecasting

- a) Theoretical Insights on the capacity of linear models

Most impactful real-world applications have either ..

- (1) smoothness
- (2) periodicity

https://sthalles.github.io/assets/time-series-decomposition/cover.png



Assumption 2) Time series can be decomposed into a periodic sequence and a sequence with smooth trend

- proof in Appendix A

3. TSMixer

Linear Models for TS Forecasting

- b) Comparison with **DL methods**

Deeper insights into ***why previous DL models tend to overfit the data***

- **Linear models** = **“time-step-dependent”**
  - weights of the mapping are fixed for each time step
- **Recurrent / Attention models** = **"data-dependent"**
  - weights over the input sequence are outputs of a "data-dependent" function

3. TSMixer

Linear Models for TS Forecasting

- b) Comparison with **DL methods**

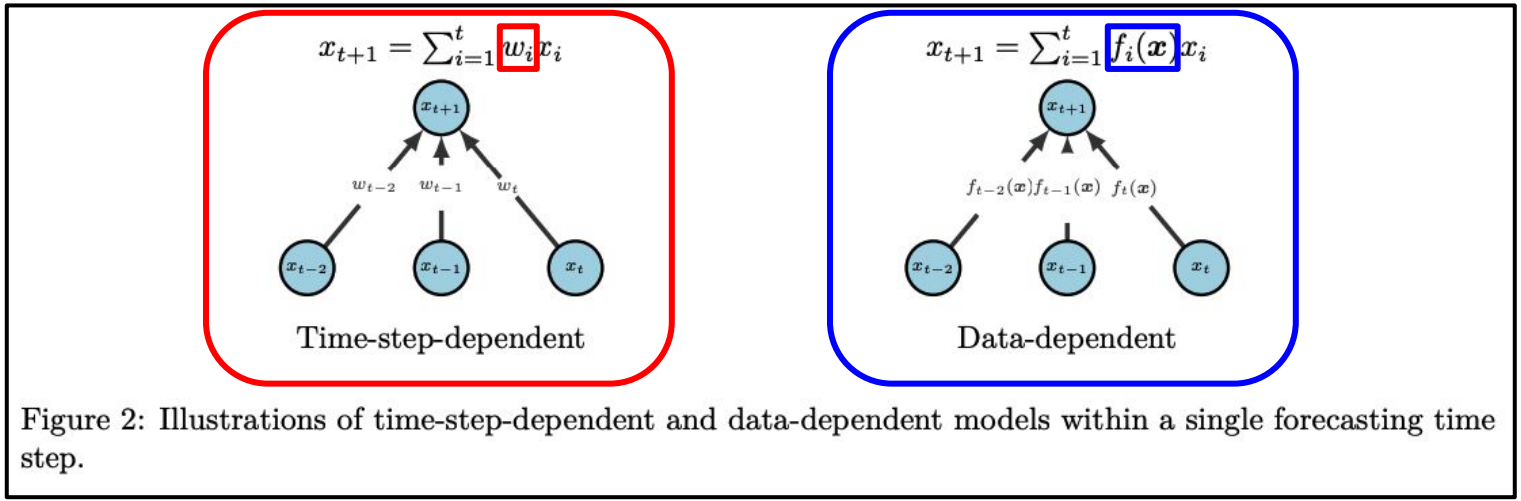


Figure 2: Illustrations of time-step-dependent and data-dependent models within a single forecasting time step.

3. TSMixer

Linear Models for TS Forecasting

- b) Comparison with **DL methods**

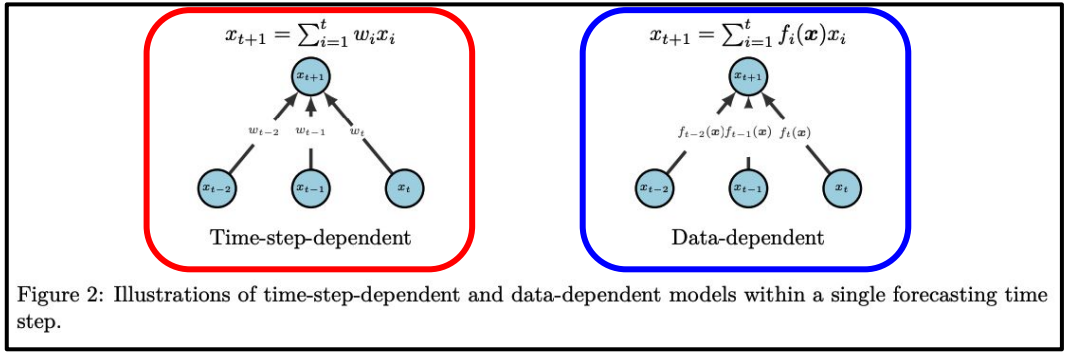


Figure 2: Illustrations of time-step-dependent and data-dependent models within a single forecasting time step.

“Time-step”-dependent linear models

- simple & highly effective in modeling temporal patterns

“Data”-dependent models

- high representational capacity

  (= achieving time-step independence is challenging)
- usually overfit on the data

  (= instead of solely considering the positions)

3. TSMixer

TSMixer Architecture

Propose a natural enhancement by ***stacking linear models with non-linearities***

Use common DL techniques

- (1) normalization
- (2) residual connections

➡ However, this **DOES NOT consider cross-variate information**

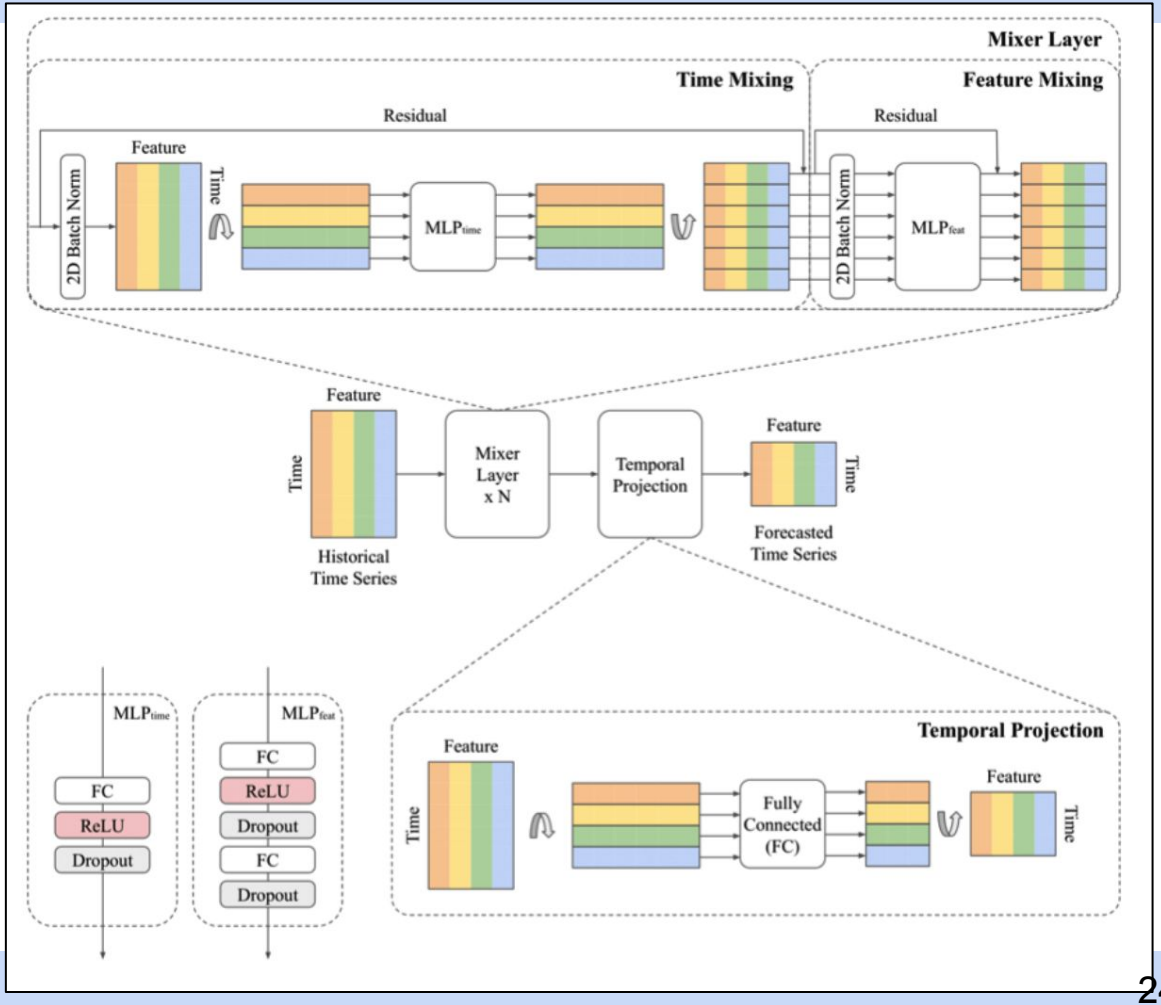
For ***cross-variate information…*** propose the application of MLPs in

- the **time-domain**
- the **feature-domain**

in an alternating manner.

3. TSMixer

TSMixer Architecture



3. TSMixer

TSMixer Architecture

Time-domain MLPs

- shared across all of the features

Feature-domain MLPs

- shared across all of the time steps.

just by transposing!

3. TSMixer

TSMixer Architecture

Time-domain MLPs

- shared across all of the features

Feature-domain MLPs

- shared across all of the time steps.

Interleaving design between these 2 operations

- efficiently utilizes both **TEMPORAL** dependencies & **CROSS-VARIATE** information

  (while limiting computational complexity and model size)
- allows to use a long lookback window
  - parameter growth in only $O(L + C)$, not $O(LC)$ if FC-MLPs were used

3. TSMixer

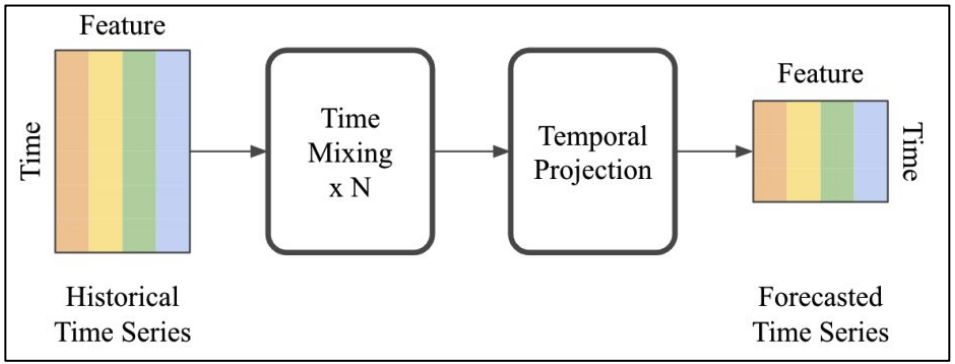
TSMixer Architecture

Time-domain MLPs

- shared across all of the features

TMix-Only : also consider a simplified variant of TSMixer

- only employs time-mixing
- consists of a residual MLP shared across each variate



3. TSMixer

TSMixer Architecture

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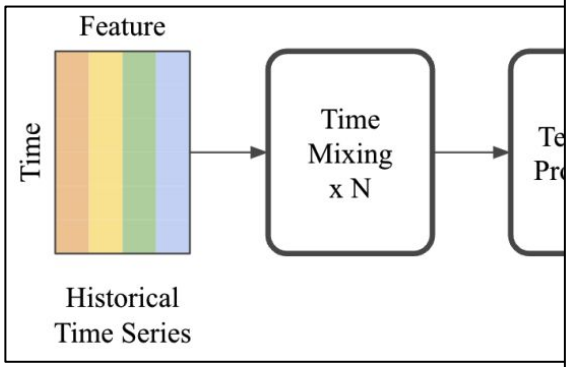

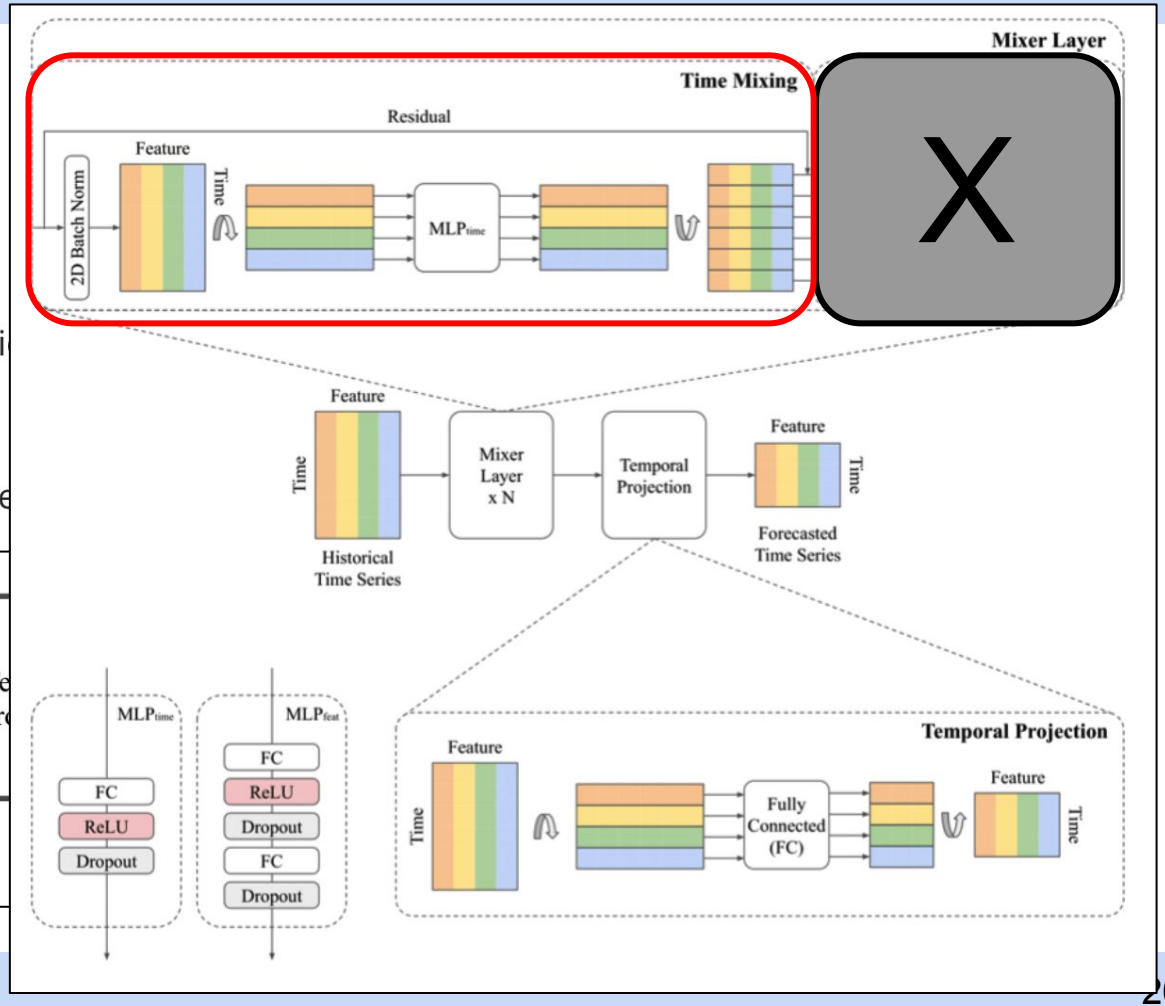

3. TSMixer

TSMixer Architecture

Notation : (N,T,C)

- N : number of TS
- T : length of TS
- C : channel(dimension) of TS

ex) 200 sensors, tracking 5 variates for 300 min.

- N = 200
- T = 300
- C = 5

```
def res_block(inputs, norm_type, activation, dropout, ff_dim):
  """Residual block of TSMixer."""

  norm = (
      layers.LayerNormalization
      if norm_type == 'L'
      else layers.BatchNormalization
  )

  # Temporal Linear
  x = norm(axis=[-2, -1])(inputs)
  x = tf.transpose(x, perm=[0, 2, 1])  # [Batch, Channel, Input Length]
  x = layers.Dense(x.shape[-1], activation=activation)(x)
  x = tf.transpose(x, perm=[0, 2, 1])  # [Batch, Input Length, Channel]
  x = layers.Dropout(dropout)(x)
  res = x + inputs

  # Feature Linear
  x = norm(axis=[-2, -1])(res)
  x = layers.Dense(ff_dim, activation=activation)(
      x
  )  # [Batch, Input Length, FF_Dim]
  x = layers.Dropout(dropout)(x)
  x = layers.Dense(inputs.shape[-1])(x)  # [Batch, Input Length, Channel]
  x = layers.Dropout(dropout)(x)
  return x + res
```

3. TSMixer

TSMixer Architecture

Time-domain weight

(N, T, C)

(N, C, T) x **(T,T)** = (N, C, T)

(N, C, T)

(N, T, C)

Feature-domain weight

(N, T, C) x **(C,D)** = (N, T, D)

(N, T, D) x **(D,C)** = (N, T, C)

```python
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```

4. TSMixer-Ext

4. TSMixer-Ext

3 Major aspects of forecastability of TS

- **(1) Persistent temporal patterns**
- **(2) Cross-variate information**
- **(3) Auxiliary features**

Extended TSMixer with **Auxiliary Information**

Real-world scenarios : access to ..

- (1) static features : $\boldsymbol{S} \in \mathbb{R}^{1 \times C_s}$
- (2) future time-varying features : $\boldsymbol{Z} \in \mathbb{R}^{T \times C_z}$

$\rightarrow$ extended to multiple TS, represented by $\boldsymbol{X}^{(i)}{}_{i=1}^{N}$,

- N : number of TS

4. TSMixer-Ext

3 Major aspects of forecastability of TS

- (1) Persistent temporal patterns
- (2) Cross-variate information
- (3) Auxiliary features

Extended TSMixer with **Auxiliary Information**

Long-term forecasting

- (In general) Only consider the historical features & targets on all variables (i.e. $C_x = C_y > 1, C_s = C_z = 0$).
- (In this paper) Also consider the case where auxiliary information is available (i.e $C_s > 0, C_z > 0$).

4. TSMixer-Ext

Architecture

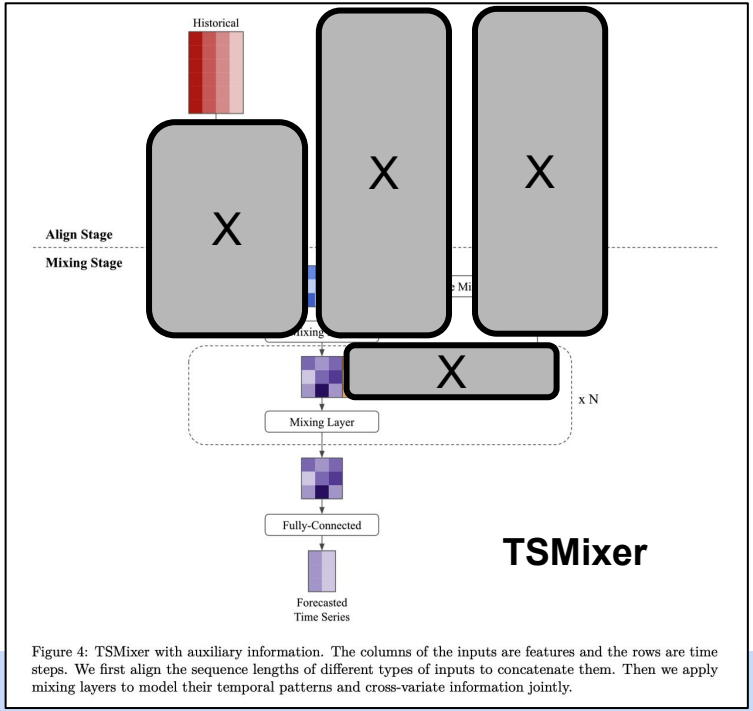
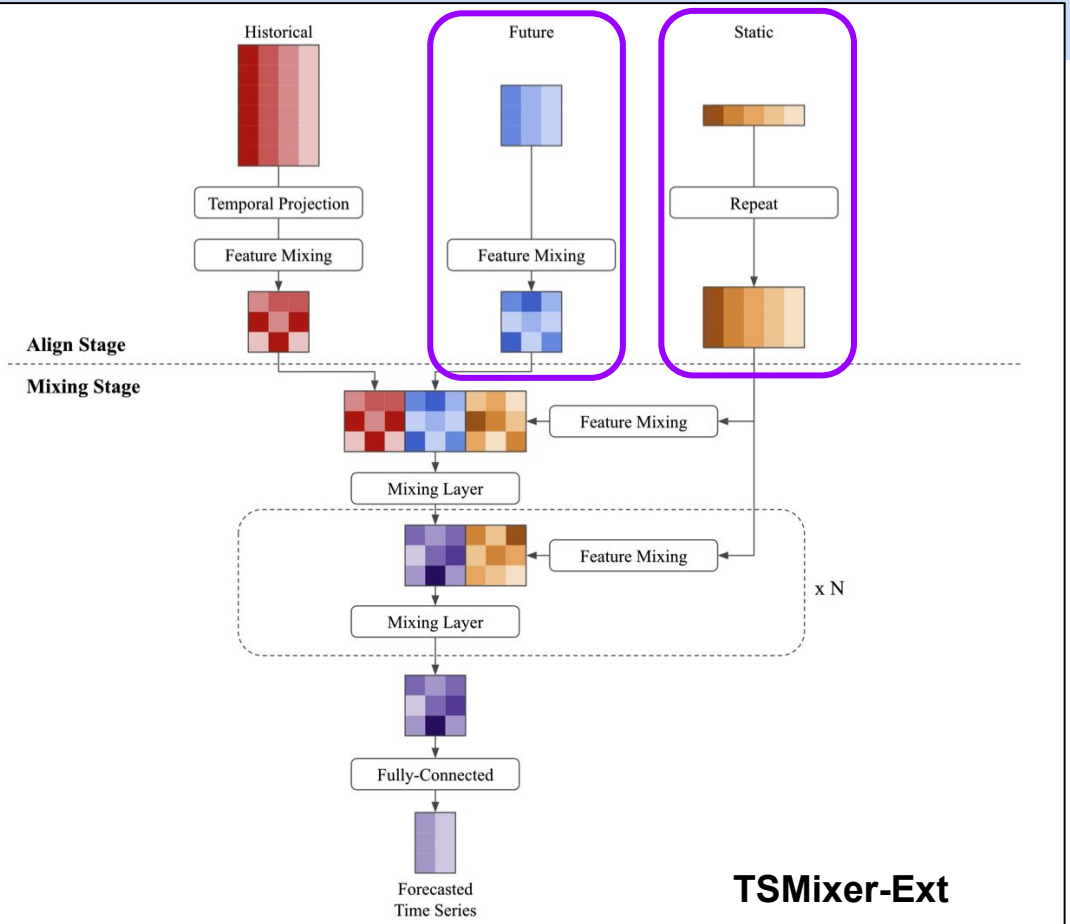


Figure 4: TSMixer with auxiliary information. The columns of the inputs are features and the rows are time steps. We first align the sequence lengths of different types of inputs to concatenate them. Then we apply mixing layers to model their temporal patterns and cross-variate information jointly.



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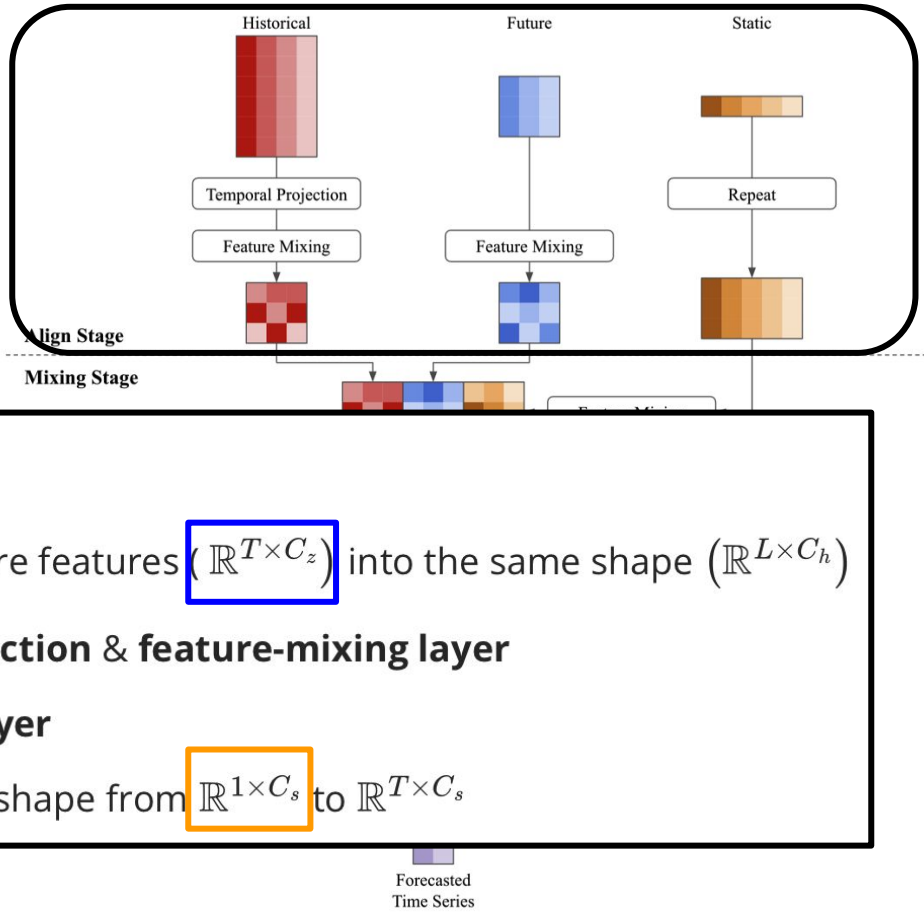
Architecture



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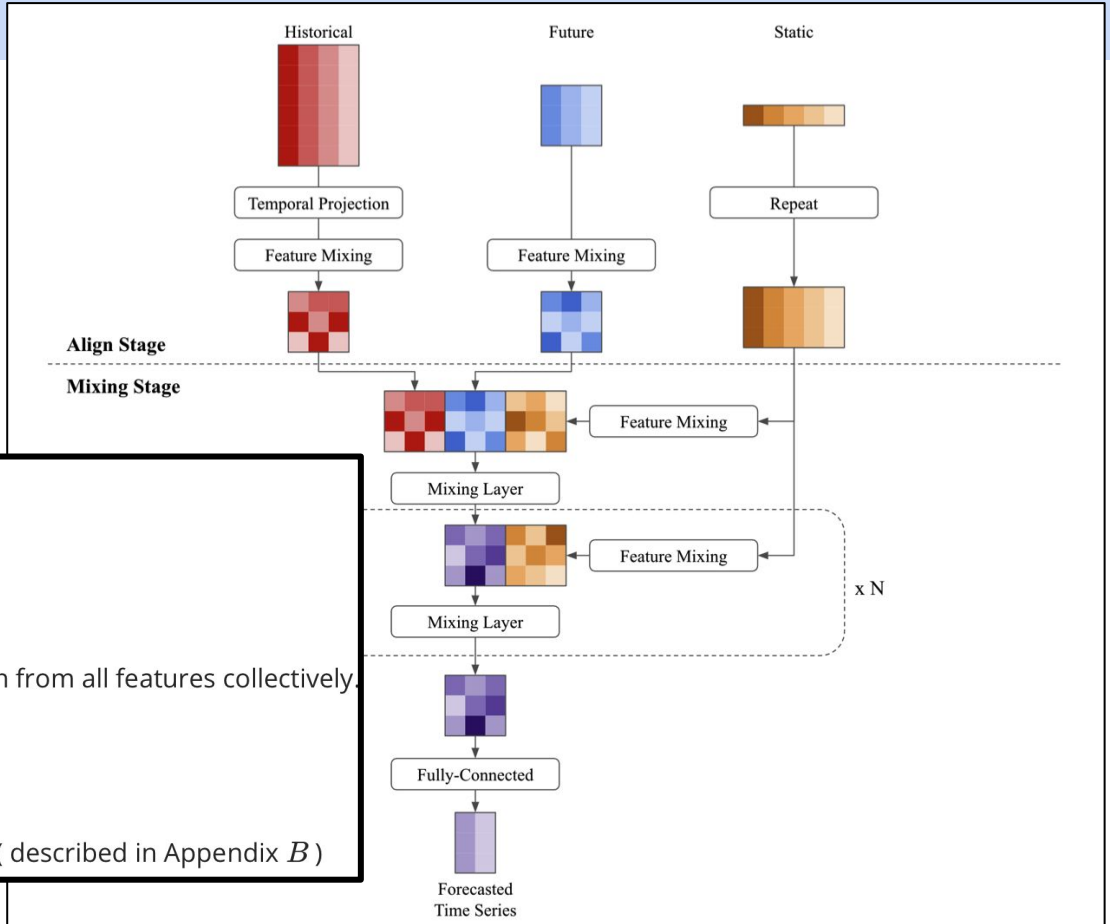
a) Align

aligns historical features $\left(\mathbb{R}^{L \times C_x}\right)$ and future features $\left(\mathbb{R}^{T \times C_z}\right)$ into the same shape $\left(\mathbb{R}^{L \times C_h}\right)$

- [Historical input] apply **temporal projection** & **feature-mixing layer**
- [Future input] apply **feature-mixing layer**
- [Static input] repeat to transform their shape from $\mathbb{R}^{1 \times C_s}$ to $\mathbb{R}^{T \times C_s}$

4. TSMixer-Ext

Architecture



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b) Mixing

Mixing layer

- time-mixing & feature-mixing operations
- leverages temporal patterns and cross-variate information from all features collectively.

FC layer

- to generate outputs for each time step.
- slightly modify mixing layers to better handle M5 dataset (described in Appendix B)

5. Experiments

5. Experiments

Table 2: Statistics of all datasets. Note that Electricity and Traffic can be considered as multivariate time series or multiple univariate time series since all variates share the same physical meaning in the dataset (e.g. electricity consumption at different locations).

	ETTh1/h2	ETTm1/m2	Weather	Electricity	Traffic	M5
# of time series (M)	1	1	1	1	1	30,490
# of variants (C)	7	7	21	321	862	1
Time steps	17,420	699,680	52,696	26,304	17,544	1,942
Granularity	1 hour	15 minutes	10 minutes	1 hour	1 hour	1day
Historical feature (C_x)	0	0	0	0	0	14
Future feature (C_z)	0	0	0	0	0	13
Static feature (C_s)	0	0	0	0	0	6
Data partition (Train/Validation/Test)	12/4/4 (month)		7:2:1			1886/28/28 (day)

5. Experiments

LTSF datasets

Real-world Retail dataset

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Auxiliary Features

single MULTIVARIATE time series

multiple MULTIVARIATE time series

5. Experiments

Real-world Retail dataset

Table 2: Statistics of all datasets. Note that Electricity and Traffic can be considered as multivariate time series ... physical meaning in the dataset (e.g. electr...

	city	Traffic	M5
#		1	30,490
#		862	1
Ti	04	17,544	1,942
Gr	ur	1 hour	1day
Hi		0	14
Fu		0	13
St		0	6
Da			1886/28/28 (day)
(T			

Auxiliary Features

- static features: store locations …
- time-varying features: campaign information ..

=> **more challenging benchmark** to explore the potential benefits of **cross-variate information** and **auxiliary features** (compared to LTSF task)

Auxiliary Features

multiple MULTIVARIATE time series

5. Experiments

(1) LTSF task

Table 3: Eval[redacted] are obtained [redacted] numbers are underlined.

CD

CI

Models		Multivariate Models										Univariate Models					
		TSMixer		TFT		FEDformer*		Autoformer*		Informer*		**TMix-Only**		Linear		PatchTST*	
Metric		MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE
ETTh1	96	0.361	0.392	0.674	0.634	0.376	0.415	0.435	0.446	0.941	0.769	**0.359**	**0.391**	0.368	0.392	0.370	0.400
	192	0.404	0.418	0.858	0.704	0.423	0.446	0.456	0.457	1.007	0.786	**0.402**	0.415	0.404	**0.415**	0.413	0.429
	336	**0.420**	**0.431**	0.900	0.731	0.444	0.462	0.486	0.487	1.038	0.784	0.420	0.434	0.436	0.439	0.422	0.440
	720	0.463	0.472	0.745	0.666	0.469	0.492	0.515	0.517	1.144	0.857	0.453	**0.467**	0.481	0.495	**0.447**	0.468
ETTh2	96	0.274	0.341	0.409	0.505	0.332	0.374	0.332	0.368	1.549	0.952	0.275	0.342	0.297	0.363	**0.274**	**0.337**
	192	0.339	0.385	0.953	0.651	0.407	0.446	0.426	0.434	3.792	1.542	**0.339**	0.386	0.398	0.429	0.341	**0.382**
	336	0.361	0.406	1.006	0.709	0.400	0.447	0.477	0.479	4.215	1.642	0.366	0.413	0.500	0.491	**0.329**	**0.384**
	720	0.445	0.470	1.187	0.816	0.412	0.469	0.453	0.490	3.656	1.619	0.437	0.465	0.795	0.633	**0.379**	**0.422**
ETTm1	96	0.285	0.339	0.752	0.626	0.326	0.390	0.510	0.492	0.626	0.560	**0.284**	**0.338**	0.303	0.346	0.293	0.346
	192	0.327	0.365	0.752	0.649	0.365	0.415	0.514	0.495	0.725	0.619	**0.324**	**0.362**	0.335	0.365	0.333	0.370
	336	**0.356**	**0.382**	0.810	0.674	0.392	0.425	0.510	0.492	1.005	0.741	0.359	0.384	0.365	0.384	0.369	0.392
	720	0.419	**0.414**	0.849	0.695	0.446	0.458	0.527	0.493	1.133	0.845	0.419	0.414	0.419	0.415	**0.416**	0.420
ETTm2	96	0.163	0.252	0.386	0.472	0.180	0.271	0.205	0.293	0.355	0.462	**0.162**	**0.249**	0.170	0.266	0.166	0.256
	192	**0.216**	**0.290**	0.739	0.626	0.252	0.318	0.278	0.336	0.595	0.586	0.220	0.293	0.236	0.317	0.223	0.296
	336	**0.268**	**0.324**	0.477	0.494	0.324	0.364	0.343	0.379	1.270	0.871	0.269	0.326	0.308	0.369	0.274	0.329
	720	0.420	0.422	0.523	0.537	0.410	0.420	0.414	0.419	3.001	1.267	**0.358**	**0.382**	0.435	0.449	0.362	0.385
Weather	96	0.145	0.198	0.441	0.474	0.238	0.314	0.249	0.329	0.354	0.405	**0.145**	**0.196**	0.170	0.229	0.149	0.198
	192	0.191	0.242	0.699	0.599	0.275	0.329	0.325	0.370	0.419	0.434	**0.190**	**0.240**	0.213	0.268	0.194	0.241
	336	0.242	0.280	0.693	0.596	0.339	0.377	0.351	0.391	0.583	0.543	**0.240**	**0.279**	0.257	0.305	0.245	0.282
	720	0.320	0.336	1.038	0.753	0.389	0.409	0.415	0.426	0.916	0.705	0.325	0.339	0.318	0.356	**0.314**	**0.334**
Electricity	96	0.131	0.229	0.295	0.376	0.186	0.302	0.196	0.313	0.304	0.393	0.132	0.225	0.135	0.232	**0.129**	**0.222**
	192	0.151	0.246	0.327	0.397	0.197	0.311	0.211	0.324	0.327	0.417	0.152	0.243	0.149	0.246	**0.147**	**0.240**
	336	**0.161**	0.261	0.298	0.380	0.213	0.328	0.214	0.327	0.333	0.422	0.166	0.260	0.164	0.263	0.163	**0.259**
	720	0.197	0.293	0.338	0.412	0.233	0.344	0.236	0.342	0.351	0.427	0.200	0.291	0.199	0.297	**0.197**	**0.290**
Traffic	96	0.376	0.264	0.678	0.362	0.576	0.359	0.597	0.371	0.733	0.410	0.370	0.258	0.395	0.274	**0.360**	**0.249**
	192	0.397	0.277	0.664	0.355	0.610	0.380	0.607	0.382	0.777	0.435	0.390	0.268	0.406	0.279	**0.379**	**0.256**
	336	0.413	0.290	0.679	0.354	0.608	0.375	0.623	0.387	0.776	0.434	0.404	0.276	0.416	0.286	**0.392**	**0.264**
	720	0.444	0.306	0.610	0.326	0.621	0.375	0.639	0.395	0.827	0.466	0.443	0.297	0.454	0.308	**0.432**	**0.286**

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5. Experiments

(1) LTSF task

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	720	0.445	0.470	.187	0.816	.412	0.469	0.453	0.490	3.656	1.61	0.437	0.465	.795	0.633	**0.379**	**0.422**
ETTm1	96	0.285	0.339	.752	0.626	.326	0.390	0.510	0.492	0.626	0.56	**0.284**	**0.338**	.303	0.346	0.293	0.346
	192	0.327	0.365	.752	0.649	.365	0.415	0.514	0.495	0.725	0.61	**0.324**	**0.362**	.335	0.365	0.333	0.370
	336	**0.356**	**0.382**	.810	0.674	.392	0.425	0.510	0.492	1.005	0.74	0.359	0.384	.365	0.384	0.369	0.392
	720	0.419	**0.414**	.849	0.695	.446	0.458	0.527	0.493	1.133	0.84	0.419	0.414	.419	0.415	**0.416**	0.420
ETTm2	96	0.163	0.252	.386	0.472	.180	0.271	0.205	0.293	0.355	0.46	**0.162**	**0.249**	.170	0.266	0.166	0.256
	192	**0.216**	**0.290**	.739	0.626	.252	0.318	0.278	0.336	0.595	0.58	0.220	0.293	.236	0.317	0.223	0.296
	336	**0.268**	**0.324**	.477	0.494	.324	0.364	0.343	0.379	1.270	0.87	0.269	0.326	.308	0.369	0.274	0.329
	720	0.420	0.422	.523	0.537	.410	0.420	0.414	0.419	3.001	1.26	**0.358**	**0.382**	.435	0.449	0.362	0.385
Weather	96	0.145	0.198	.441	0.474	.238	0.314	0.249	0.329	0.354	0.40	**0.145**	**0.196**	.170	0.229	0.149	0.198
	192	0.191	0.242	.699	0.599	.275	0.329	0.325	0.370	0.419	0.43	**0.190**	**0.240**	.213	0.268	0.194	0.241
	336	0.242	0.280	.693	0.596	.339	0.377	0.351	0.391	0.583	0.54	**0.240**	**0.279**	.257	0.305	0.245	0.282
	720	0.320	0.336	.038	0.753	.389	0.409	0.415	0.426	0.916	0.70	0.325	0.339	.318	0.356	**0.314**	**0.334**
Electricity	96	0.131	0.229	.295	0.376	.186	0.302	0.196	0.313	0.304	0.39	0.132	0.225	.135	0.232	**0.129**	**0.222**
	192	0.151	0.246	.327	0.397	.197	0.311	0.211	0.324	0.327	0.41	0.152	0.243	.149	0.246	**0.147**	**0.240**
	336	**0.161**	0.261	.298	0.380	.213	0.328	0.214	0.327	0.333	0.42	0.166	0.260	.164	0.263	0.163	**0.259**
	720	0.197	0.293	.338	0.412	.233	0.344	0.236	0.342	0.351	0.42	0.200	0.291	.199	0.297	**0.197**	**0.290**
Traffic	96	0.376	0.264	.678	0.362	.576	0.359	0.597	0.371	0.733	0.41	0.370	0.258	.395	0.274	**0.360**	**0.249**
	192	0.397	0.277	.664	0.355	.610	0.380	0.607	0.382	0.777	0.43	0.390	0.268	.406	0.279	**0.379**	**0.256**
	336	0.413	0.290	.679	0.354	.608	0.375	0.623	0.387	0.776	0.43	0.404	0.276	.416	0.286	**0.392**	**0.264**
	720	0.444	0.306	.610	0.326	.621	0.375	0.639	0.395	0.827	0.46	0.443	0.297	.454	0.308	**0.432**	**0.286**

1 2 3 4 5 6

Table 2: Statistics of all datasets. Note that Electricity and Traffic can be considered as multivariate time series or multiple univariate time series since all variates share the same physical meaning in the dataset (e.g. electricity consumption at different locations).

	ETTh1/h2	ETTm1/m2	Weather	Electricity	Traffic	M5
# of time series (M)	1	1	1	1	1	30,490
# of variants (C)	7	7	21	321	862	1
Time steps	17,420	699,680	52,696	26,304	17,544	1,942
Granularity	1 hour	15 minutes	10 minutes	1 hour	1 hour	1day
Historical feature (C_x)	0	0	0	0	0	14
Future feature (C_z)	0	0	0	0	0	13
Static feature (C_s)	0	0	0	0	0	6
Data partition (Train/Validation/Test)	12/4/4 (month)		7:2:1			1886/28/28 (day)

	CD vs CI	SL vs. SSL	Architecture
1	CD	SL	MLP
2	CD	SSL	Transformer
3	CD	SL	Transformer
4	CI	SL	MLP
5	CI	SL	Linear
6	CI	SSL	Transformer

SL : supervised learning & SSL : self-supervised learning

GOOD & BAD

5. Experiments

(1) LTSF task

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Table 3: Eval… results on the long-term forecasting datasets. The … are obtained … numbers are underlined.

CD

CI

Models																							
Metric																							
ETTh1	96																						
	19																						
	33																						
	72																						
ETTh2	96																						
	19																						
	33																						
	72																						
ETTm1	96																						
	19																						
	33																						
	72																						
ETTm2	96																						
	19																						
	33																						
	72																						
Weather	96																						
	19																						
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1 2 3 4 5 6

CI > CD seems to be better for **LTSF task**!

(& TSOnly > TSMixer)

Then, can we conclude that CI > CD ??

	CD vs CI	SL vs SSL	Architecture
			MLP
			nsformer
			MLP
			Linear
		SSL	Transformer

SL : supervised learning & SSL : self-supervised learning

GOOD & **BAD**

5. Experiments

(2) M5 competition

(w.o Auxiliary Information)

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Table 4: Evaluation on M5 without auxiliary information. We report the mean and standard deviation of WRMSSE across 5 different random seeds. TMix-Only is a univariate variant of TSMixer where only time-mixing is applied. The multivariate models outperforms univariate models with a significant gap.

	Models	Multivariate	Test WRMSSE	Val WRMSSE
CI	Linear		0.983±0.016	1.045±0.018
	PatchTST		0.976±0.014	0.992±0.011
	TMix-Only		0.960±0.041	1.000±0.027
CD	FEDformer	✔	0.804±0.039	0.674±0.014
	TSMixer	✔	**0.737±0.033**	0.605±0.027

5. Experiments

(2) M5 competition

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Summary :

- LTSF : CI > CD
- M5 : CD > CI

Previous works : CI > CD

TSMixer : ***It depends on the dataset!***

5. Experiments

(2) M5 competition

Effect of Auxiliary Information

Models	Auxiliary feature		Test WRMSSE	Val WRMSSE
	Static	Future		
DeepAR	✔	✔	0.789±0.025	0.611±0.007
TFT	✔	✔	0.670±0.020	0.579±0.011
TSMixer-Ext			0.737±0.033	0.000±0.000
	✔		0.657±0.046	0.000±0.000
		✔	0.697±0.028	0.000±0.000
	✔	✔	**0.640**±0.013	0.568±0.009

Table 2: Statistics of all datasets. Note that Electricity and Traffic can be considered as multivariate time series or multiple univariate time series since all variates share the same physical meaning in the dataset (e.g. electricity consumption at different locations).

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Conclusion

1. (Architecture) **Transformer** may not be necessary! (**Linear / MLP** might be enough for TS modeling)

2. (Framework) **Channel Independence** is not always the best! It depends on the dataset!

3. Desirable properties of TSF model

= “**automatically** detects the need for capturing the **cross-variate relationships**”

4. Limitations of TSMixer

- Which model to choose, TSMixer vs. TMix-Only?
- Hyperparameter tuning

  (Different experimental settings for each prediction length & dataset (96, 192, 336, 720))

Thank You!