Introduction to Econometrics 2: Recitation 11

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Failure of Conditional Independence

When is it violated?

- In the regressional framework of $y_i(d) = \mu(X_i, d) + \epsilon_i(d)$, the error terms is not independent of $D_i|X_i$.
- The conditional independence assumption can be broken because:
  - Participants **self-select based on expected benefit**: In a job training program for plumbing, those who are more healthy are likely to join. If health is not perfectly observed, we risk breaking the conditional independence assumption
  - Participants may be **selected, consciously or not, to join**: Think of the clinical trial where participation is voluntary. In such case, individuals who are more risk-loving are more likely to join, something not readily observed
  - There may be **equilibrium effects**: A tuition subsidy program that intends to increase the number of people entering college may have a spillover effect by increasing supply of college graduates at the labor market, leading to a decrease in college premium. This may induce students to enter college less.

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What happens?

- Mathematically, what happens is that when we calculate $E[Y_i|D_i = 1, x] - E[Y_i|D_i = 0, x]$, we end up with

$$\begin{aligned} E[Y_i|D_i = 1, x] - E[Y_i|D_i = 0.x] &= \mu(x.1) - \mu(x, 0) + E[\epsilon_i(1)|1, x] - E[\epsilon_i(0)|0, x] \\ &= TE + E[\epsilon_i(1)|1, x] - E[\epsilon_i(0)|0, x] \end{aligned}$$

- The error term no longer can be erased from the equation since CIA assumption is not applicable.
- The difference between the error term is the **selection bias** (Also appears in Angrist, Pischke 2009).
- This also means that the error terms and the u_i in $D_i = 1(u_i < p(x_i))$ can covary. The result is that the treatment effect estimated from here can be inaccurate.
- There are two possible solutions. Old method relies on **Heckman correction**. Recent focus is on IV to derive **marginal treatment effects** and **localized average treatment effect**.

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Heckman Correction

- Suppose that we are in the situation where we have a DGP

$$Y_i = \max\{X_i\beta + \sigma\eta_i, 0\}, \eta_i \sim N(0,1)$$

  So we only see Y_i if $X_i\beta + \sigma\eta_i > 0$. We observe D_i, specified as

$$D_i = \begin{cases} 1 & \text{if } \eta > -\frac{X_i\beta}{\sigma} \\ 0 & \text{otherwise} \end{cases}$$

- Then, for the observed sample, we are likely to have an η_i that is positively selected - a nonrandom error leading to a bias
- The size of the bias is $\frac{\phi(X_i\beta/\sigma)}{\Phi(X_i\beta/\sigma)} = IMR$ (shown in the recitation notes)
- The β/σ parameters can be estimated by regressing D_i on X_i
- Include estimated IMR in the observed sample and regress

$$Y_i = X_i\beta + \gamma\hat{f}(x) + \epsilon_i$$

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Instrumental Variables

- However, Heckman Correction assumes linearity and normality of the error terms.
- So we look into the UV, where we assume that there exist variables Z_i that affect the treatment D_i but not the outcomes (on its own).
- It should satisfy
  - **Relevancy**: It effects the propensity score $p(x, z) = \Pr(D_i = 1|X_i = x, Z_i = z)$
  - **Validity**: Distribution of the counterfactual outcomes and u_i does not depend on $Z_i|X_i$. To put it in mathematical notation,

$$(Y_i(1), Y_i(0), u_i) \perp\!\!\!\perp Z_i|X_i$$

  - **Full support**: The support of $p(x, z)$ conditional on x extends to all of $[0, 1]$. This implies that change in Z induces large variations in the propensity score.

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Identification

- So what concerns do we run into in terms of identification?
- We can identify $p(x, z)$ by calculating $\Pr(D_i = 1|X_i = x, Z_i = z)$.
- We can also write

$$\begin{aligned} E[Y_i|D_i = 1, X_i = x, Z_i = z] &= \mu(x, 1) + E[\epsilon_i(1)|u_i < p(x, z), x, z] \\ &= \mu(x, 1) + E[\epsilon_i(1)|u_i < p(x, z), x] \ (\because \text{validity}) \end{aligned}$$

- Define $K_1(p(x, z)) = E[\epsilon_i(1)|u_i < p(x, z), X_i = x]$ to be some unknown function of the conditional expectation of $\epsilon_i(1)$.
- We can also work similarly to define

$$E[Y_i|D_i = 0, X_i = x, Z_i = z] = \mu(x, 0) + E[\epsilon_i(0)|u_i > p(x, z), X_i = x]$$

and $K_0(p(x, z)) = E[\epsilon_i(0)|u_i > p(x, z), X_i = x]$

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Identification

- Note that the left hand sides for both sets of equations can be identified by naively taking an average of Y_i's conditional on some the treated (untreated) for the group $X_i = x, Z_i = z$.
- The estimated result breaks down into

$$\mu(x,1) - \mu(x,0) + \underbrace{K_1(p(x,z)) - K_0(p(x,z))}_{\Delta(p(x,z))} = TE(x) + \Delta(p(x,z))$$

- $\Delta(p(x,z))$ is the control function that stands for the selection bias.
- By relevance, change in z affects the value of $p(x,z)$ for a fixed x, allowing us to identify how $\Delta(p(x,z))$ *changes*.
- However, we do not know what the *exact value* of $\Delta(p(x,z))$ is for the initial value of z we started.
- Therefore, the true parameter of interest, TE, cannot be uncovered.

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Identification

- So how can we progress on? Note that the K_d functions are conditional expectations of $\epsilon_i(d)$ on $X_i = x$ and selection rule $u_i(<,>)p(x,z)$.
- In particular, we can get that

$$
\begin{aligned}
K_1(1) &= E[\epsilon_i(1)|u_i < 1, X_i = x] \\
&= E[\epsilon_i(1)|X_i = x] \ (\because \text{Everyone is treated}) \\
&= 0 \\
K_0(0) &= E[\epsilon_i(1)|u_i > 0, X_i = x] \\
&= E[\epsilon_i(1)|X_i = x] \ (\because \text{No one is treated}) \\
&= 0
\end{aligned}
$$

- Heckman and Vytlacil show that given a continuous instrument z, we can do a much better job of identifying the treatment effect.

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Marginal Treatment Effects

- The **marginal treatment effect** at $p(x, z) = p$ is defined as the treatment effect on individuals whose $u_i = p(x, z)$. We can write

$$MTE(p) = E[Y_i(1) - Y_i(0)|u_i = p]$$

- Heckman and Vytlacil show that

$$MTE(p) = \frac{\partial E[Y_i|p(x, z) = p]}{\partial p}$$

- This is done by
  1. Estimate $p(x, z) = \Pr(D_i = 1|X_i = x.Z_i = z)$
  2. Regress Y_i on the estimated $p(x, z)$ in a flexible setting - preferably not just linearly but with some nonlinearities
  3. Take a derivative with respect to p. (or local linear estimator)
  4. For treatment effects, evaluate the $E[Y_i|p(x, z), x]$ at $p(x, z) = 1$ and $p(x, z) = 0$ and identify the difference. (You can obtain $E[Y_i|\cdot]$ by getting the predicted values).

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Marginal Treatment Effects

- Intuitively, what is going on with MTE is as follows
- By changing p slightly by dp, we are able to identify the marginal compliers who move from not being treated to being treated.
- We are finding out how their outcome changes as they move from non-participation to participation into the treatment

Math Behind MTE

- Define
$$G(p) = E[Y_i \cdot 1(p(x,z) = p)]$$
- $Y_i = D_i Y_i(1) + (1 - D_i) Y_i(0) = 1(u_i < p(x,z)) Y_i(1) + 1(u_i > p(x,z)) Y_i(0)$, so we can rewrite the above as

$$\begin{aligned} G(p) &= E[Y_i(1) \cdot 1(u_i < p) \cdot 1(p(x,z) = p)] + E[Y_i(0) \cdot 1(u_i > p) \cdot 1(p(x,z) = p)] \\ &= G_1(p) + G_0(p) \end{aligned}$$

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Math Behind MTE

- By the validity condition and the fact that $u_i \sim U[0,1]$ (and thus $f(u_i) = 1$), we can write

$$\begin{aligned} E[Y_i(1) \cdot 1(u_i < p) \cdot 1(p(x,z) = p)] &= E[Y_i(1) \cdot 1(u_i < p)] \Pr(p(x,z) = p) \\ &= \int_0^p E[Y_i(1) | u = t] dt \Pr(p(x,z) = p) \end{aligned}$$

  And similarly,

$$E[Y_i(0) \cdot 1(u_i > p) \cdot 1(p(x,z) = p)] = \int_p^1 E[Y_i(0) | u = t] dt \Pr(p(x,z) = p)$$

- So

$$G(p) = \left(\int_0^p E[Y_i(1) | u = t] dt + \int_p^1 E[Y_i(0) | u = t] dt \right) \Pr(p(x,z) = p)$$

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Math Behind MTE

- Since $E[Y_i \cdot 1(p(x,z) = p)] = E[Y_i | p(x,z) = p] \cdot \Pr(p(x,z) = p)$ this implies that

$$E[Y_i | p(x,z) = p] = \frac{G(p)}{\Pr(p(x,z) = p)} = \int_0^p E[Y_i(1)|u = t]dt + \int_p^1 E[Y_i(0)|u = t]dt$$

- Then, by Leibniz's integral rule

$$\frac{\partial E[Y_i | p(x,z) = p]}{\partial p} = E[Y_i(1)|u = p] - E[Y_i(0)|u = p] = MTE(p)$$

- Hence, the treatment effect can be recovered by

$$TE = \int_0^1 MTE(p)dp = E[Y_i | p(x,z) = 1] - E[Y_i | p(x,z) = 0]$$

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Some Caveats with MTE

- For the above methods to work, we need the Z_i instruments to satisfy
  - Z belongs in the treatment (relevancy): $D_i = 1(u_i < p(X_i, Z_i))$
  - Z not belong in the outcome (exclusivity): $Y_i(d) = \mu(x, d) + \epsilon_i(d)$
  - In other words, we get $(\epsilon_i(1), \epsilon_i(0), u_i) \perp\!\!\!\perp Z_i | X_i$
  - Continuity: $p(x, z)$ changes w.r.t z in a continuous way (This is because we need to be able to take derivatives)
- For the range of $p(x, z)$ available, the above condition allows us to estimate the marginal treatment effect.
- For any work you see that uses marginal treatment effect, there includes a graph that maps marginal treatment effect on the vertical axis and the 'resistance' parameter u_i on the horizontal axis.

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Testing CIA

- With this framework, we can also test if conditional independence assumption holds.
- Recall that conditional independence assumption is satisfied when

$$(\epsilon_i(1), \epsilon_i(0)) \perp\!\!\!\perp u_i | X_i$$

- In cases where this is true, then the outcomes are independent of u_i conditional on X_i. Thus,

$$MTE(x, p) = E[Y_i(1) - Y_i(0) | X_i = x, u_i = p] = E[Y_i(1) - Y_i(0) | X_i = x]$$

- Since $MTE(x, p) = \frac{\partial E[Y_i | X_i = x, p(x,z) = p]}{\partial p}$, the above condition implies that $E[Y_i | X_i = x, p(x, z) = p]$ is linear in p.
- Thus, it is highly recommended to put polynomial terms of $p^k, k = 1, 2, 3, ...$ when you estimate marginal treatment effects.
- Then, test to find whether the nonlinear terms have coefficient zero.

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Examples

- Carneiro, Lee (2009): The paper estimates the impact of attending college on log wage distributions. The paper finds that individuals more likely to attend college (and have low resistance parameters) are more likely to have higher college wage ($Y_i(1)$) over high school wage ($Y_i(0)$). The opposite holds true for people with high school degree. They have a MTE figure (at figure 3) that maps MTE as a function of resistance parameters.
- Johnson, Taylor (2019): The paper shows that causal impact of migration decreases longevity (at least heterogenously). This is even with the consideration that migrants are more likely to be educated and have higher baseline earnings compared to non-migrants. They use the MTE to (with railcar traffic at the town of origin as one of their IVs). They document that those who have lower latent ability (high U_d) suffer more from migrating out, reflected in the downward sloping MTE

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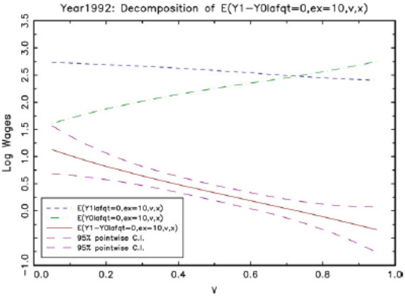
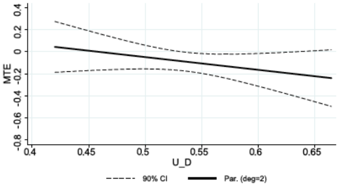


(a) Carneiro, Lee (2009)



(b) Johnson, Taylor (2019)

Failure of Conditional Independence

Local Average Treatment Effect

- The marginal treatment effect looks at 'marginal' compliers: It is interested in the small group of individuals who changes treatment status from $D_i = 0$ to $D_i = 1$ as $p(x, z)$ changes slightly due to Z_i
- Instead of changing the propensity score by a tiny bit, we can look at finite differences - like Z_i having just two possible values.
- Then we are going to be looking at a larger group of compliers than in the marginal treatment effect context. The treatment effect for this larger group of compliers is called **local average treatment effect**.
- We will setup the framework as follows:
  - Z_i will be a binary instrumental variable. This of this as eligibility rule.
  - $D_i(z)$ can be characterized as $D_i(z) = 1(u_i < p(x, z))$. Note that as z rises, so will $p(x, z)$. This is the relevance condition
  - Z_i itself has no bearing, at least directly, on the outcome. So $Y_i(d) = \mu(x, d) + \epsilon_i(d)$. So we still have $(\epsilon_i(1), \epsilon_i(0), u_i) \perp\!\!\!\perp Z_i | X_i$.

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Local Average Treatment Effect

- The formal way to define local average treatment effect is as follows

$$LATE(x) = E[Y_i(1) - Y_i(0) | p(x,z) < u_i < p(x,z').X_i = x]$$

- We can identify LATE by using the following (I skip $X_i = x$)

$$\begin{aligned}
E[Y_i|Z_i = z'] - E[Y_i|Z_i = z] &= E[1(u_i < p(z'))Y_i(1) + 1(u_i > p(z'))Y_i(0)|z'] \\
&- E[1(u_i < p(z))Y_i(1) + 1(u_i > p(z))Y_i(0)|z] \\
&= E[1(u_i < p(z'))Y_i(1) + 1(u_i > p(z'))Y_i(0)] \\
&- E[1(u_i < p(z))Y_i(1) + 1(u_i > p(z))Y_i(0)] \\
&= E[(1(u_i < p(z')) - 1(u_i < p(z))Y_i(1) \\
&- (1(u_i > p(z')) - 1(u_i > p(z))Y_i(0)] \\
&= E[(1(u_i < p(z')) - 1(u_i < p(z))(Y_i(1) - Y_i(0))] \\
&= \Pr[1(u_i < p(z')) - 1(u_i < p(z)) = 1] \\
&\times E[Y_i(1) - Y_i(0)|1(u_i < p(z')) - 1(u_i < p(z)) = 1]
\end{aligned}$$

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Local Average Treatment Effect

- We only consider complies and $1(u_i < p(z')) - 1(u_i < p(z)) = 1$ holds iff $p(z) < u_i < p(z')$.

$$E[Y_i|z'] - E[Y_i|z] = \Pr[p(z) < u_i < p(z')]E[Y_i(1) - Y_i(0)|p(z) < u_i < p(z')]$$

- Therefore, we are able to identify LATE as

$$LATE(x, z, z') = \frac{E[Y_i|Z_i = z'] - E[Y_i|Z_i = z]}{\Pr(p(z) < u_i < p(z'))} = \frac{E[Y_i|Z_i = z'] - E[Y_i|Z_i = z]}{p(z') - p(z)}$$

- Or in terms of the propensity score (and by bringing $X_i = x$ back in)

$$LATE(x, p(z), p(z')) = \frac{E[Y_i|p = p(x.z')] - E[Y_i|p = p(x, z)]}{p(x, z') - p(x, z)}$$

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Local Average Treatment Effect

- We can go further: Estimate propensity scores with

$$p(x, z) = E[D_i | X_i = x, Z_i = z]$$

  and get

$$LATE(x, z, z') = \frac{E[Y_i | x, z'] - E[Y_i | x, z]}{E[D_i | x, z'] - E[D_i | x, z]}$$

- To obtain this from the regression, we follow these steps:
  1. Regress D on Z and other covariates X to get $\hat{D} = \hat{p}(x, z)$
  2. Regress Y on other covariates X and $\hat{D}$.

  The LATE estimator can then be obtained here is called the Wald estimator.

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Some Words on Monotonicity Condition

- When we wrote $D_i = 1(u_i < p(x, z))$ and assume that $P(x, z)$ rises with z for fixed x, monotonicity condition implies that D_i can only increase.
- We are ruling out a case where D_i can change from 1 to 0. (Defiers)
- We also throw away information on the always takers and never takers as well.
- Both LATE and MTE can only tells us about compliers while throwing away defiers.
- This condition, along with the validity condition, cannot be tested empirically. Therefore, the applicability of this condition should be argued using intuition or external facts.