Recitation 10

Seung-hun Lee

Columbia University

Note on the final project

- Approach the TA's for advice: regression methods, data, or ideas
- Also, refer to my recitation notes for Diff-in-Diffs and Regression discontinuity methods - I explain how you can use it in a natural experiment context
- Data: Household surveys are the safe place to start but country level data could work depending on your idea
- But first, **be clear about what kind of question you want to explore**: are you looking for a correlation/causal relation? Are you trying to predict something?
- Who to look for (topic-wise) - all three of us cover basic grounds in most topics, but for specialized stuff...
  - Me: Development/Public/Labor econ (applied microeconomics in general). If you are trying to study health-care markets in the US, I am the wrong person to ask

Experiments in Economics

Motivation

- You categorize some individuals under treatment group and controlled group and compare the differences across groups and times.
- You can do this in lab setting (randomized control trials) or find an exogenous event (natural experiments)
- They provide a conceptual benchmark for assessing observational studies and can solve many validity threats in regular regressions.
- Do note that they have a validity threat of their own

Potential Outcome Framework

Setup

- Start by assuming that the treatment effect is identical for everyone (**constant treatment effect** assumption)
- Let $Y_{i,t}$ be the **observed** outcome variable for individual i at time t
- X_{it} be the treatment variable: It is 1 if individual i is treated and 0 if otherwise.
- Let $Y_{it}(0)$ denote a **potential** outcome if subject i is not treated at time t and $Y_{it}(1)$ be the same if i is treated.
- Then Y_{it} can be split into

$$\begin{aligned} Y_{it} &= Y_{it}(1)X_{it} + Y_{it}(0)(1 - X_{it}) \\ &= Y_{it}(0) + (Y_{it}(1) - Y_{it}(0))X_{it} \end{aligned}$$

Potential Outcome Framework

Two takeaways

- **Fundamental problem of missing data**: We can only observe at most one of $Y_{it}(1)$ and $Y_{it}(0)$, since individual i cannot be treated and untreated simultaneously at time t
  - For us to make statements about the treatment effect, we need to be sure about what the missing outcome looks like.
  - Perfect randomization, randomization conditional on observables, instrumental variables on treatment variable X_{it}...
- **Average treatment effect**: Average treatment effect is defined as

$$ATE = E[Y_{it}(1) - Y_{it}(0)] = \frac{1}{N}\sum_{i=1}^{N}(Y_{it}(1) - Y_{it}(0))$$

which is obtained from the coefficient of the X_{it} variable of the regression

$$Y_{it} = \beta_0 + \beta_1 X_{it} + u_{it}$$

Average Treatment Effect

Regressional form

- We will assume that the experiment is **perfectly randomized**
  - The treated individuals and controlled individuals are identical except for treatment status, or that $E[u_{it}|X_{it}] = 0$ for all possible X_{it} values.
- Derive the expected value of Y_{it} separately for the treated and controlled individuals. For the controlled ($X_{it} = 0$, so that $Y_{it} = Y_{it}(0)$):

$$E[Y_{it}|X_{it} = 0] = E[Y_{it}(0)] = \beta_0$$

  and for the treated, ($X_{it} = 1$, so that $Y_{it} = Y_{it}(1)$)

$$E[Y_{it}|X_{it} = 1] = E[Y_{it}(1)] = \beta_0 + \beta_1$$

Average Treatment Effect

Regressional form

- Then, the average treatment effect can be characterized as

$$ATE = E[Y_{it}(1) - Y_{it}(0)] = (\beta_0 + \beta_1) - \beta_0 = \beta_1$$

  where subtraction among $E[Y_{it}|X_{it} = 1]$ and $E[Y_{it}|X_{it} = 0]$ is possible since we are assuming perfect randomization.
- Under perfect randomization, identifying the average treatment effect is equivalent to obtaining the β_1 coefficient through an OLS process.
- However this is possible in rare circumstances...

Average Treatment Effect: No constant treatment effects

Approach

- Define $\beta_{1i} = Y_{it}(1) - Y_{it}(0)$ and let $\beta_1 = E[\beta_{1i}]$.
- The potential outcome framework can be formally written as

$$\begin{aligned} Y_i &= Y_i(1)X_i + Y_i(0)(1 - X_i) \\ &= Y_i(0) + (Y_i(1) - Y_i(0))X_i \\ &= E[Y_i(0)] + (Y_i(1) - Y_i(0))X_i + Y_i(0) - E[Y_i(0)] \\ &= \beta_0 + \beta_{1i}X_i + u_i \end{aligned}$$

- Furthermore,

$$Y_i = \beta_0 + \beta_1 X_i + \underbrace{(\beta_{1i} - \beta_1)X_i + u_i}_{=v_i}$$

Average Treatment Effect: No constant treatment effects

Approach

- As long as we have perfect randomization, even $E[v_i|X_i] = 0$ will hold.

$$\begin{aligned} E[v_i|X_i] &= E[(\beta_{1i} - \beta_1)X_i + u_i|X_i] \\ &= E[(\beta_{1i} - \beta_1)X_i|X_i] + E[u_i|X_i] \\ &= E[(\beta_{1i} - \beta_1)|X_i]X_i + E[u_i|X_i] \\ &= 0 \end{aligned}$$

- With perfect randomization, OLS gives an unbiased estimate of the average treatment effect, whether we assume constant treatment effects or not.

Validity Threats

Validity threats

- We cannot automatically subtract $E[Y_{it}|X_{it}=1]$ and $E[Y_{it}|X_{it}=0]$ under **imperfect randomization**.
- If the **attrition rate is nonrandom** - if it differs by a treatment status - we end up with a biased estimate of the treatment effect.
- In terms of experimental procedure, **failure to comply to experiment protocol** and other **experimental effects** that rises through peculiar behaviors of the experimental and the experiment subject could also serve as a validity threat.
- Constant treatment effect assumption that we have imposed on ourselves may not be accurate.
- There are external validity threats when the sample is not representative, is not replicable in other settings, and the results may have general equilibrium effects.

Validity Threats

Addressing validity threats

- If the treated and the controlled differ in some observed characteristics, we can simply include control variables Z_{it}
  - This gives us ATE conditional on observables and identify treatment effects for people with different treatment effects
- Instrumental variable approach: If there exists a Z_{it} variable that influences the treatment status X_{it} and uncorrelated with u_{it}, we can use Z_{it} to instrument X_{it} and run 2SLS regression
  - The predicted value of X_{it}: The probability of being treated.
  - 2SLS estimates the causal effect for those whose value of X_{it} is influenced by Z_{it}, putting more weight on those more likely to be treated.
  - This effectively identifies the treatment effect 'localized' for those more likely to be treated
  - We call this **localized average treatment effect** (LATE).

IV in treatment effects

Regression

- Let Z_{it} be an instrument for X_{it} and apply 2SLS method in this manner

$$X_{it} = \pi_0 + \pi_{1i} Z_{it} + e_{it} \text{ (First stage)}$$
$$Y_{it} = \beta_0 + \beta_{1i} X_{it} + u_{it} \text{ (Equation of interst)}$$

- Obtain $\hat{X}_{it} = \hat{\pi_0} + \hat{\pi}_{1i} X_{it}$ - the predicted probability of being treated.
- Put $\hat{X}_{it}$ in place of X_{it} in the equation of interest.
- If β_{1i}, π_{1i} are independent of (u_{it}, v_{it}, Z_{it}), $E(\pi_{1i}) \neq 0$, then

$$\hat{\beta}_1 \xrightarrow{p} \frac{E(\beta_{1i}\pi_{1i})}{E(\pi_{1i})} \text{ (Refer to appendix 13.2)}$$

- Ultimately, $\hat{\beta}_1$ can be written as

$$\hat{\beta}_1 \xrightarrow{p} E[\beta_{1i}] + \frac{cov(\beta_{1i}, \pi_{1i})}{E(\pi_{1i})} = \beta_1 + \frac{cov(\beta_{1i}, \pi_{1i})}{E(\pi_{1i})}$$

- Treatment effect is large for individuals for whom the effect of the instrument is large.