Recitation 10: Big data and quasi-experiments in econometrics

Seung-hun Lee

Columbia University
Undergraduate Introduction to Econometrics Recitation

December 1st, 2022

Big data

Big data

- It can simply mean data with large observations or large number of control variables.
- In general instances, atypical data such as text data, and satellite imagery are referred to as big data.
- In this class, we focus on the instance where there are many control variables, specifically on what to do if there are many control variables relative to the number of observations.
- Additionally, we focus on trying to get the best way to predict a result that is currently not in the dataset.

Characterizing MSPE

- Regression format is

$$Y_i^* = \beta_1 X_{1i}^* + ... + \beta_k X_{ki}^* + u_i^*$$

- X_{1i}^* is the “standardized" version of X_{1i}'s, Y_i^* the “demeaned" version.
- Note that we CANNOT have a constant term β_0 here because if we demean β_0, which is the same for all i's, they vanish.
- MSPE: the expected value of the squared error made by predicting Y for an observation not in the dataset.

$$MSPE = E[Y^{OS} - \hat{Y}^{OS}]^2$$

  - $\hat{Y}^{OS}$: obtained from the coefficients of β's made from the in-sample
  - Y^{OS} : realized value of Y outside of the sample

Characterizing MSPE

- With these notations, we can define the prediction error as

$$Y_i^{OS} - \hat{Y}_i^{OS} = (\beta_1 - \hat{\beta}_1)X_{1i}^{OS} + ... + (\beta_k - \hat{\beta}_k)X_{ki}^{OS} + u_i^{OS}$$

- Define $\sigma_u^2 = E[u_i^{OS}]^2$, then we can write MSPE as

$$MSPE = \sigma_u^2 + E[(\beta_1 - \hat{\beta}_1)X_{1i}^{OS} + ... + (\beta_k - \hat{\beta}_k)X_{ki}^{OS}]$$

- Oracle prediction: the smallest possible MSPE, σ_u^2
- However, we cannot predict β's perfectly.
- The more predictors we have, we generally end up having larger MSPE $\rightarrow$ need to reduce X's.
- Need Ridge, LASSO, and Principal Component method comes in

Methods

- Goal: Find other estimators that does not increase the MSPE compared to the rate in which the same rises in OLS estimators.
- Idea: Reduce the σ_u^2, the variance from the residual sums of squares, at the expense of introducing a small bit of bias.
- How: Providing a penalty for having a model with large number of regressors (what we formally call 'shrinkage').

Ridge

- Minimize a 'penalized' sum squared of residuals

$$\hat{\beta}_{Ridge} = \arg \min_{\beta_1,..,\beta_k} \left[\sum_{i=1}^{n} (Y_i - \beta_1 X_{1i} - .... - \beta_k X_{ki})^2 + \lambda_{Ridge} \sum_{j=1}^{k} \beta_j^2 \right]$$

- $\lambda_{Ridge} \sum_{j=1}^{k} \beta_j^2$: penalty for complexity.
- Introduce bias so that the variance term σ_u^2 will be reduced.
- Variance and the bias in MSPE moves in a trade-off relation
- Ridge estimator minimizes MSPE by reducing the variance term to the extent that the bias term does not rise too drastically.

LASSO

- Penalty term takes a different form:

$$\hat{\beta}_{LASSO} = \arg \min_{\beta_1,\ldots,\beta_k} \left[\sum_{i=1}^{n} (Y_i - \beta_1 X_{1i} - .... - \beta_k X_{ki})^2 + \lambda_{LASSO} \sum_{j=1}^{k} |\beta_j| \right]$$

- The difference between the two lies in the degree of shrinkage.
  - When the OLS estimates are small, the LASSO shrinks those estimates all the way to 0.
  - Ridge also shrinks those coefficients close to 0, they do not exactly set them to 0.

Principal component

- You are using a linear combination of some subset of k variables so that you end up with $p < k$ number of regressors ('collapsing' the model)
- Solve the following problem to get the j'th principal component PC_j

$$\max var\left(\sum_{i=1}^{K} a_{ji}X_i\right) \text{ s.t. } \sum_{i=1}^{k} a_{ji}^2 = 1$$

  with another condition being that $corr(PC_j, PC_{j-1}) = 0$
- Solve the *maximization*: We want the X's to explain more of the variation
- $\sum_{i=1}^{k} a_{ji}^2 = 1$: Regularization method
- $corr(PC_j, PC_{j-1}) = 0$: We want to minimize the overlapping amount of information across different principal components.

quasi-experiments in econometrics

Experiments in Economics

Motivation

- You categorize some individuals under treatment group and controlled group and compare the differences across groups and times.
- You can do this in lab setting (randomized control trials) or find an exogenous event (natural experiments)
- They provide a conceptual benchmark for assessing observational studies and can solve many validity threats in regular regressions.
- Do note that they have a validity threat of their own

Potential Outcome Framework

Setup

- Start by assuming that the treatment effect is identical for everyone (**constant treatment effect** assumption)
- Let $Y_{i,t}$ be the **observed** outcome variable for individual i at time t
- X_{it} be the treatment variable: It is 1 if individual i is treated and 0 if otherwise.
- Let $Y_{it}(0)$ denote a **potential** outcome if subject i is not treated at time t and $Y_{it}(1)$ be the same if i is treated.
- Then Y_{it} can be split into

$$\begin{aligned} Y_{it} &= Y_{it}(1)X_{it} + Y_{it}(0)(1 - X_{it}) \\ &= Y_{it}(0) + (Y_{it}(1) - Y_{it}(0))X_{it} \end{aligned}$$

Potential Outcome Framework

Two takeaways

- **Fundamental problem of missing data**: We can only observe at most one of $Y_{it}(1)$ and $Y_{it}(0)$, since individual i cannot be treated and untreated simultaneously at time t
  - For us to make statements about the treatment effect, we need to be sure about what the missing outcome looks like.
  - Perfect randomization, randomization conditional on observables, instrumental variables on treatment variable X_{it}...
- **Average treatment effect**: Average treatment effect is defined as

$$ATE = E[Y_{it}(1) - Y_{it}(0)] = \frac{1}{N}\sum_{i=1}^{N}(Y_{it}(1) - Y_{it}(0))$$

which is obtained from the coefficient of the X_{it} variable of the regression

$$Y_{it} = \beta_0 + \beta_1 X_{it} + u_{it}$$

Average Treatment Effect

Regressional form

- We will assume that the experiment is **perfectly randomized**
  - The treated individuals and controlled individuals are identical except for treatment status, or that $E[u_{it}|X_{it}] = 0$ for all possible X_{it} values.
- Derive the expected value of Y_{it} separately for the treated and controlled individuals. For the controlled ($X_{it} = 0$, so that $Y_{it} = Y_{it}(0)$):

$$E[Y_{it}|X_{it} = 0] = E[Y_{it}(0)] = \beta_0$$

and for the treated, ($X_{it} = 1$, so that $Y_{it} = Y_{it}(1)$)

$$E[Y_{it}|X_{it} = 1] = E[Y_{it}(1)] = \beta_0 + \beta_1$$

Average Treatment Effect

Regressional form

- Then, the average treatment effect can be characterized as

$$ATE = E[Y_{it}(1) - Y_{it}(0)] = (\beta_0 + \beta_1) - \beta_0 = \beta_1$$

  where subtraction among $E[Y_{it}|X_{it} = 1]$ and $E[Y_{it}|X_{it} = 0]$ is possible since we are assuming perfect randomization.
- Under perfect randomization, identifying the average treatment effect is equivalent to obtaining the β_1 coefficient through an OLS process.
- However this is possible in rare circumstances...

Validity Threats

Validity threats

- We cannot automatically subtract $E[Y_{it}|X_{it} = 1]$ and $E[Y_{it}|X_{it} = 0]$ under **imperfect randomization**.
- If the **attrition rate is nonrandom** - if it differs by a treatment status - we end up with a biased estimate of the treatment effect.
- In terms of experimental procedure, **failure to comply to experiment protocol** and other **experimental effects** that rises through peculiar behaviors of the experimental and the experiment subject could also serve as a validity threat.
- Constant treatment effect assumption that we have imposed on ourselves may not be accurate.
- There are external validity threats when the sample is not representative, is not replicable in other settings, and the results may have general equilibrium effects.

Validity Threats: Unconfoundedness

- If the treated and the controlled differ in some observed characteristics, we can simply include control variables W_{it}
- We assume that
  - Unconfoundedness: $(Y_i(1), Y_i(0)) \perp X_i | W_i$
  - Overlap: $0 < \Pr(X_i = 1 | W_i = w) = p(W_i = w) < 1$ (This is the propensity score)
- ATE can be identified using the fact that (separate page)

$$E\left[\frac{X_i Y_i}{p(W_i)}\right] = E[Y_i(1)], E\left[\frac{(1 - X_i) Y_i}{1 - p(W_i)}\right] = E[Y_i(0)] \implies E[Y_i(1) - Y_i(0)] = E\left[\frac{(X_i - p(W_i)) Y_i}{p(W_i)(1 - p(W_i))}\right]$$

  and using the sample analogues (propensity score estimated using probit/logit)
- For ATT, we can use

$$E[Y_i(1) - Y_i(0) | X_i = 1] = E\left[\frac{(X_i - p(W_i)) Y_i}{\rho(1 - p(W_i))}\right]$$

  where ρ is the overall probability of treatment (not controlling for observables)

IV in treatment effects

Regression

- Let Z_{it} be an instrument for X_{it} (An exogenous variation dictating treatment assignment) and apply 2SLS

$$X_{it} = \pi_0 + \pi_{1i} Z_{it} + e_{it} \text{ (First stage)}$$
$$Y_{it} = \beta_0 + \beta_{1i} X_{it} + u_{it} \text{ (Equation of interst)}$$

- Obtain $\hat{X}_{it} = \hat{\pi_0} + \hat{\pi}_{1i} X_{it}$ - the predicted probability of being treated.
- If β_{1i}, π_{1i} are independent of (u_{it}, v_{it}, Z_{it}), $E(\pi_{1i}) \neq 0$, then

$$\hat{\beta}_1 \xrightarrow{p} \frac{E(\beta_{1i}\pi_{1i})}{E(\pi_{1i})}$$

- Ultimately, $\hat{\beta}_1$ can be written as

$$\hat{\beta}_1 \xrightarrow{p} E[\beta_{1i}] + \frac{cov(\beta_{1i}, \pi_{1i})}{E(\pi_{1i})} = \beta_1 + \frac{cov(\beta_{1i}, \pi_{1i})}{E(\pi_{1i})}$$

- Treatment effect is large for individuals for whom the effect of the instrument is large.

Difference in Differences: Regression format

- Assume two time periods (before and after) and two groups (treated and controlled)
- You have the following regression

$$Y_{it} = \beta_0 + \beta_1 X_{it} + \beta_2 D_{it} + \beta_3 X_{it} D_{it} + u_{it}$$

  where $X = 1$ if in treatment, $D = 1$ if after treatment
- You can get the following group averages
  - Treated, After: $E[Y_{it}|X_{it} = 1, D_{it} = 1] = \beta_0 + \beta_1 + \beta_2 + \beta_3$
  - Treated, Before: $E[Y_{it}|X_{it} = 1, D_{it} = 0] = \beta_0 + \beta_1$
  - Controlled, After: $E[Y_{it}|X_{it} = 0, D_{it} = 1] = \beta_0 + \beta_2$
  - Controlled, Before: $E[Y_{it}|X_{it} = 0, D_{it} = 1] = \beta_0$
- Difference-in-differences $\hat{\beta}^{DD} = [\bar{Y}_{\text{treat,after}} - \bar{Y}_{\text{treat,before}}] - [\bar{Y}_{\text{control,after}} - \bar{Y}_{\text{control,before}}]$
  - In the above equation, we would be looking at $\hat{\beta}_3$

Difference in Differences: Regression format

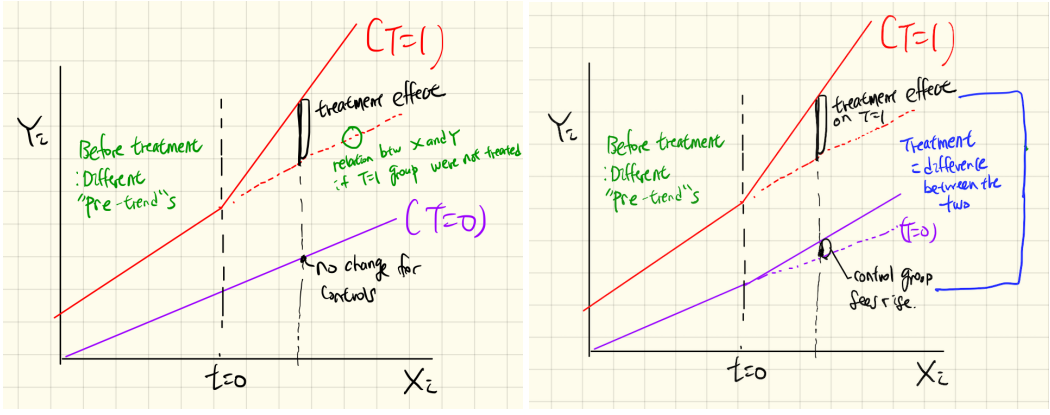
- Other way around is to difference the Y and get

$$\Delta Y_{it} = \beta_0 + \beta_1 X_{it} + u_{it}$$

  where $X = 1$ if in treatment
- You can get the following group averages
  - Outcome difference, treatment: $E[\Delta Y_{it} | X_{it} = 1] = \beta_0 + \beta_1$
  - Outcome differnece, control: $E[\Delta Y_{it} | X_{it} = 0] = \beta_0$
  - In the above equation, we would be looking at $\hat{\beta}_1$ to get the treatment difference
- Can be generalized to many time periods and many groups with TWFE, Event-studies etc (active area of research)

Difference in Differences: In pictures

Figure: DD estimator



(a) No change in control group

(b) Some change in control group

Regression discontinuity

- This applies to case where the treatment status depends on a threshold
- Loosely speaking, we have

$$Y_i = \beta_0 + \beta_1 X_i + u_i$$

  where $X_i = 1$ if i passes threshold for eligibility (0 if otherwise)
- Back out the treatment effect using the difference
  - For those not in the program, $E[Y_i|X_i = 0] = \beta_0$
  - For those in the program, $E[Y_i|X_i = 1] = \beta_0 + \beta_1$
- Thus, β_1 is the treatment effect
- Note: For RD to work, there should be no 'jump' in observables around the threshold except treatment status
- Loss of samples: RD is applied to a narrow bandwidth around the threshold
- There may be cases where threshold is not strict (fuzzy RD)

Regression discontinuity: In pictures