Joint Precoding and Combining for Quantized Full-Duplex MU-MIMO Systems

Seunghyeong Yoo, Seokjun Park, and Jinseok Choi*
Electrical Engineering, Ulsan National Institute of Science and Technology, Ulsan, South Korea.
*Electrical Engineering, Korea Advanced Institute of Science and Technology, Daejeon, South Korea.
Email: {seunghy, seokjunpark}@unist.ac.kr, jinseok@kaist.ac.kr*

***Abstract*—We consider a full-duplex (FD) multi-user multiple-input multiple-output (MU-MIMO) system with low-resolution quantizers at an access point (AP). In the considered FD system, there are main bottlenecks: self-interference (SI), co-channel interference (CCI), and quantization errors. In this paper, we propose a novel precoding and combining method to maximize the sum spectral efficiency (SE) by incorporating the effect of the quantization errors as well as the SI and CCI. Since the beamformers are intertwined with the quantization errors, SI, and CCI, it is highly challenging to solve the sum SE maximization problem. To address the challenges, we convert the problem into the Rayleigh quotient form. Then, we derive the first-order optimality condition with interpreting it as a generalized eigenvalue problem by leveraging the principle of the Rayleigh quotient problem. Accordingly, we adopt a power iteration method for identifying the leading eigenvector: the best local optimal precoding solution. Consequently, we propose an alternating algorithm to jointly optimize the precoder and combiner. Simulations validate the proposed algorithm.**

***Index Terms*—Full-duplex, precoding, combining, low-resolution quantization, and alternating method.**

I. Introduction

Wireless networks face challenges to improve spectral efficiency (SE) and overcome limited battery life of mobile devices, because of the increasing traffic demand and heavy computations on smart devices [1]. Regarding the traffic demand, one can meet the demand by using full-duplex (FD) operation to increase SE. Unlike half-duplex (HD), FD permits simultaneous transmission and reception, potentially doubling the SE compared to HD [1]. For energy efficiency, using low-resolution analog-to-digital converters (ADCs) and digital-to-analog converters (DACs) reduces power consumption [2]. Therefore it is important to develop a novel transmission method for the FD systems with lowresolution ADCs and DACs.

Unlike the HD system, there are key constraints that degrade the potential of the FD systems such as the self-interference (SI) in the uplink (UL) and co-channel interference (CCI) in the downlink (DL) systems. Since the SI and CCI are coupled with FD communications, DL and UL beamforming vectors are required to be jointly designed by incorporating the SI and CCI. [3] explored the performance of the FD system in terms of both the SE and energy efficiency (EE). Low-complexity beamforming designs were developed based on a sequential convex approximation method for maximizing the SE and EE. A SI cancellation method was also investigated in [4] by introducing a time-domain transmit beamforming method for self-interference cancellation. The proposed method can be directly implemented at the circuit level which is considered as a feasible solution with current radio frequency circuits. In [5], the efficient algorithms for DL and UL channels were proposed by first converting DL channel to the dual UL channel and solving the sum SE optimization problem. In [1], adaptive cancellation schemes such as analog and digital cancellation were proposed to overcome this interference, but residual SI and CCI still remained.

Regarding full-duplex systems with low-resolution quantization, there has been limited effort on optimizing the considered system. In [6], the system model including low-resolution quantization was presented and analyzed, focusing on maximum ratio combining (MRC) and maximum ratio transmission (MRT) with perfect channel state information (CSI). Since only conventional linear beamformers were used to analyze the FD systems with low-resolution quantizers in [6], designing the state-of-the-art beamforming method for such a system is still questionable. In addition, it is highly challenging to solve the problem considering quantization error caused by low-resolution quantizers coupled with SI and CCI.

In this paper, we explore a novel beamforming strategy at the access point (AP) for maximizing the sum SE of DL and UL users. To resolve these challenges of SI, CCI, and quantization errors intertwined with beamformers and each other, we first reformulate a sum SE optimization problem as the Rayleigh quotient form. Then, the first-optimality condition of this problem is derived to utilize the principle of a Rayleigh quotient problem. Accordingly, we cast the optimality condition into a form of the generalized eigenvalue problem. Then we adopt a generalized power iteration (GPI) method [7] to identify the principal eigenvector for finding the best local optimal solution. For a UL combiner, a linear minimum mean square error (MMSE) beamformer is constructed for the derived precoder. Consequently, we propose an efficient algorithm that jointly design the GPI-based FD quantized precoder and quantized MMSE combiner in an alternating manner. Via simulations, we show that the proposed algorithm achieves the higher SE compared to the conventional methods.

II. System model

We consider a quantized FD MU-MIMO system where a AP serves $K_{\sf D}$ DL users and $K_{\sf U}$ UL users with n_t transmit and n_r

receive antennas, with low-resolution DACs and ADCs. Due to the interference caused by the AP and UL user transmission, we consider SI and CCI. In DL, the uncoded data streams $d_{\mathsf{D},k}$, $k \in \{1, \ldots, K_{\mathsf{D}}\}$ are coded through an encoder to yield $s_{\mathsf{D},k}$, which denotes the encoded DL data stream. We assume that $s_{\mathsf{D},k} \sim \mathcal{CN}(0, P_{\mathsf{D}})$, where P_{D} denotes a maximum the AP transmit power. We can represent $s_{\mathsf{D},k}$ as an encoded data stream vector $\mathbf{s}_{\mathsf{D}} = [s_{\mathsf{D},1}, \ldots, s_{\mathsf{D},K_{\mathsf{D}}}] \in \mathbb{C}^{K_{\mathsf{D}}}$, i.e.,

$$\mathbf{x}_{\mathsf{D}} = \mathbf{W}\mathbf{s}_{\mathsf{D}}, \tag{1}$$

where $\mathbf{x}_{\mathsf{D}} \in \mathbb{C}^{n_t}$ denotes a precoded symbol vector, and $\mathbf{W} \in \mathbb{C}^{n_t \times K_{\mathsf{D}}}$ represents a precoding matrix which consists of the precoding vectors $\mathbf{w}_k \in \mathbb{C}^{n_t}$, $k \in \{1, \ldots, K_{\mathsf{D}}\}$. Then, $\mathbf{x}_{\mathsf{D}}$ is quantized at the DACs. The AP employs the DACs with $b_{\mathsf{DAC},n}$-bit resolution where $b_{\mathsf{DAC},n}$ represents the number of quantization bits of the DAC pair at n-th transmit antenna. We adopt an additive quantization noise model (AQNM) [8] for approximately modeling the quantization process in a linear form. After applying the AQNM, $\mathbf{x}_{\mathsf{D}}$ is represented as

$$Q(\mathbf{x}_{\mathsf{D}}) \approx \mathbf{x}_{\mathsf{D,q}} = \mathbf{\Phi}_{\alpha_{\mathsf{DAC}}}\mathbf{x}_{\mathsf{D}} + \mathbf{q}_{\mathsf{DAC}}, \tag{2}$$

where $Q(\cdot)$ is a scalar quantizer which applies for each real and imaginary part, $\mathbf{x}_{\mathsf{D,q}} \in \mathbb{C}^{n_t}$ denotes a quantized $\mathbf{x}_{\mathsf{D}}$, $\mathbf{\Phi}_{\alpha_{\mathsf{DAC}}} \in \mathbb{C}^{n_t \times n_t}$ is a diagonal matrix of the DAC quantization loss, $\mathbf{\Phi}_{\alpha_{\mathsf{DAC}}} = \mathrm{diag}\{\alpha_{\mathsf{DAC},1}, \ldots, \alpha_{\mathsf{DAC},n_t}\}$, and $\mathbf{q}_{\mathsf{DAC}} \in \mathbb{C}^{n_t}$ denotes a DAC quantization noise vector, where $\mathbf{q}_{\mathsf{DAC}} \sim \mathcal{CN}(\mathbf{0}_{n_t \times 1}, \mathbf{R}_{\mathbf{q}_{\mathsf{DAC}}})$. The quantization loss of the n-th DAC $\alpha_{\mathsf{DAC},n} \in (0,1)$ is determined as $1 - \beta_{\mathsf{DAC},n}$ which is a normalized mean squared quantization error with $\beta_{\mathsf{DAC},n} = \frac{\mathbb{E}[|x - Q_n(x)|]^2}{\mathbb{E}[|x|]^2}$ [8]. $\beta_{\mathsf{DAC},n}$ depends on $b_{\mathsf{DAC},n}$. We can compute a covariance matrix of $\mathbf{q}_{\mathsf{DAC}}$ as $\mathbf{R}_{\mathbf{q}_{\mathsf{DAC}}} = \mathbf{\Phi}_{\alpha_{\mathsf{DAC}}}\mathbf{\Phi}_{\beta_{\mathsf{DAC}}}\mathrm{diag}\{\mathbb{E}[\mathbf{x}_{\mathsf{D}}\mathbf{x}_{\mathsf{D}}^{\mathsf{H}}]\}$.

In UL, the data symbols $s_{\mathsf{U},k}$, $k \in \{1, \ldots, K_{\mathsf{U}}\}$ are transmitted from UL users. Then the received signal is

$$\mathbf{r}_{\mathsf{U}} = \mathbf{H}_{\mathsf{U}}\mathbf{s}_{\mathsf{U}} + \mathbf{G}_{\mathsf{SI}}\mathbf{x}_{\mathsf{D,q}} + \mathbf{n}_{\mathsf{U}}, \tag{3}$$

where $\mathbf{r}_{\mathsf{U}} \in \mathbb{C}^{n_r}$ is a received symbol vector from UL users, $\mathbf{H}_{\mathsf{U}} = [\mathbf{h}_{\mathsf{U},1}, \ldots, \mathbf{h}_{\mathsf{U},K_{\mathsf{U}}}] \in \mathbb{C}^{n_r \times K_{\mathsf{U}}}$ is a channel matrix from the UL users to the AP, and $\mathbf{s}_{\mathsf{U}} = [s_{\mathsf{U},1}, \ldots, s_{\mathsf{U},K_{\mathsf{U}}}] \in \mathbb{C}^{K_{\mathsf{U}}}$, $s_{\mathsf{U},k} \sim \mathcal{CN}(0, P_{\mathsf{U}})$ denotes a transmitted signal from UL users, $\mathbf{G}_{\mathsf{SI}} \in \mathbb{C}^{n_r \times n_t}$ is a SI channel matrix, where $\mathbf{G}_{\mathsf{SI}} \sim \mathcal{CN}(0, \rho_{\mathsf{SI}}\sigma_{\mathsf{SI}}^2)$. $\mathbf{n}_{\mathsf{U}} \in \mathbb{C}^{n_r}$ is an additive white Gaussian noise (AWGN) vector at the AP, which has the distribution $\mathbf{n}_{\mathsf{U}} \sim \mathcal{CN}(0, \sigma_{\mathsf{U}}^2\mathbf{I}_{n_r})$ and P_{U} denotes a maximum transmit power at UL user. Here, σ_{SI}^2 includes the large-scale fading of the channel and ρ_{SI} denotes the SI cancellation capability. Then, $\mathbf{r}_{\mathsf{U}}$ is quantized at the ADCs after passing through the radio-frequency chain in the AP. Likewise, the AP employs $b_{\mathsf{ADC},k}$-bit-ADC. Then the quantized $\mathbf{r}_{\mathsf{U}}$ is

$$Q(\mathbf{r}_{\mathsf{U}}) \approx \mathbf{r}_{\mathsf{U,q}} = \mathbf{\Phi}_{\alpha_{\mathsf{ADC}}}\mathbf{r}_{\mathsf{U}} + \mathbf{q}_{\mathsf{ADC}}, \tag{4}$$

where $\mathbf{\Phi}_{\alpha_{\mathsf{ADC}}} \in \mathbb{C}^{n_r \times n_r}$ is a diagonal matrix of ADC quantization loss, and $\mathbf{q}_{\mathsf{ADC}} \sim \mathcal{CN}(\mathbf{0}_{n_r \times 1}, \mathbf{R}_{\mathbf{q}_{\mathsf{ADC}}})$ is a ADC quantization noise vector. The quantization parameters are defined in the similar way as DAC quantization. We can obtain then the covariance matrix of $\mathbf{q}_{\mathsf{ADC}}$ as $\mathbf{R}_{\mathbf{q}_{\mathsf{ADC}}} = \mathbf{\Phi}_{\alpha_{\mathsf{ADC}}}\mathbf{\Phi}_{\beta_{\mathsf{ADC}}}\mathrm{diag}\{\mathbb{E}[\mathbf{r}_{\mathsf{U}}\mathbf{r}_{\mathsf{U}}^{\mathsf{H}}]\}$.

Now, we represent the baseband signals in DL and UL. With CCI, the received baseband signal at DL user k is given by

$$\begin{aligned} y_{\mathsf{D},k} = {} & \mathbf{h}_{\mathsf{D},k}^{\mathsf{H}}\mathbf{\Phi}_{\alpha_{\mathsf{DAC}}}\mathbf{w}_k s_{\mathsf{D},k} + \sum_{i=1, i\neq k}^{K_{\mathsf{D}}} \mathbf{h}_{\mathsf{D},k}^{\mathsf{H}}\mathbf{\Phi}_{\alpha_{\mathsf{DAC}}}\mathbf{w}_i s_{\mathsf{D},i} \\ & + \mathbf{h}_{\mathsf{D},k}^{\mathsf{H}}\mathbf{q}_{\mathsf{DAC}} + \mathbf{g}_{\mathsf{CCI},k}^{\mathsf{H}}\mathbf{s}_{\mathsf{U}} + n_{\mathsf{D},k}, \end{aligned} \tag{5}$$

where $\mathbf{h}_{\mathsf{D},k} \in \mathbb{C}^{n_t}$ is a DL channel vector from the AP to the k-th DL users, $\mathbf{g}_{\mathsf{CCI},k}$ is a column vector of CCI channel matrix $\mathbf{G}_{\mathsf{CCI}} \sim \mathcal{CN}(0, \sigma_{\mathsf{CCI}}^2)$, and $n_{\mathsf{D},k} \sim \mathcal{CN}(0, \sigma_{\mathsf{D}}^2)$ denotes the AWGN of DL user k. In UL, after passing through the combiner $\mathbf{F}$, $\mathbf{r}_{\mathsf{U,q}}$ becomes $\mathbf{y}_{\mathsf{U}} = [y_{\mathsf{U},1}, \ldots, y_{\mathsf{U},K_{\mathsf{U}}}]^{\mathsf{T}}$ where

$$\begin{aligned} y_{\mathsf{U},k} = {} & \mathbf{f}_k^{\mathsf{H}}\mathbf{\Phi}_{\alpha_{\mathsf{ADC}}}\mathbf{h}_{\mathsf{U},k}s_{\mathsf{U},k} + \sum_{i=1, i\neq k}^{K_{\mathsf{U}}} \mathbf{f}_k^{\mathsf{H}}\mathbf{\Phi}_{\alpha_{\mathsf{ADC}}}\mathbf{h}_{\mathsf{U},i}s_{\mathsf{U},i} \\ & + \sum_{i=1}^{K_{\mathsf{D}}} \mathbf{f}_k^{\mathsf{H}}\mathbf{\Phi}_{\alpha_{\mathsf{ADC}}}\mathbf{G}_{\mathsf{SI}}\mathbf{\Phi}_{\alpha_{\mathsf{DAC}}}\mathbf{w}_i s_{\mathsf{D},i} + \mathbf{f}_k^{\mathsf{H}}\mathbf{\Phi}_{\alpha_{\mathsf{ADC}}}\mathbf{G}_{\mathsf{SI}}\mathbf{q}_{\mathsf{DAC}} \\ & + \mathbf{f}_k^{\mathsf{H}}\mathbf{\Phi}_{\alpha_{\mathsf{ADC}}}\mathbf{n}_{\mathsf{U}} + \mathbf{f}_k^{\mathsf{H}}\mathbf{q}_{\mathsf{ADC}}. \end{aligned} \tag{6}$$

Here, $\mathbf{f}_k \in \mathbb{C}^{n_r}$ is a column vector of $\mathbf{F} \in \mathbb{C}^{n_r \times K_{\mathsf{U}}}$.

Accordingly, we derive the SE of the DL and UL users. The SE of k-th DL user is represented as (7) on the top of next page, where $I_{\mathsf{CCI},k} = ||\mathbf{g}_{\mathsf{CCI},k}||^2 P_{\mathsf{U}}$. Similarly, the SE of k-th UL user is represented as (8) on the top of the next page, where $I_{\mathsf{SI},k} = \sum_{i=1}^{K_{\mathsf{D}}} |\mathbf{f}_k^{\mathsf{H}}\mathbf{\Phi}_{\alpha_{\mathsf{ADC}}}\mathbf{G}_{\mathsf{SI}}\mathbf{\Phi}_{\alpha_{\mathsf{DAC}}}\mathbf{w}_i|^2 P_{\mathsf{D}} + \mathbf{f}_k^{\mathsf{H}}\mathbf{\Phi}_{\alpha_{\mathsf{ADC}}}\mathbf{G}_{\mathsf{SI}}\mathbf{R}_{\mathbf{q}_{\mathsf{DAC}}}\mathbf{G}_{\mathsf{SI}}^{\mathsf{H}}\mathbf{\Phi}_{\alpha_{\mathsf{ADC}}}^{\mathsf{H}}\mathbf{f}_k$. Therefore, to maximize the sum SE, the optimization problem is formulated as

$$\underset{\mathbf{W},\mathbf{F}}{\mathrm{maximize}} \sum_{k=1}^{K_{\mathsf{D}}} R_{\mathsf{D},k} + \sum_{k=1}^{K_{\mathsf{U}}} R_{\mathsf{U},k} \tag{9}$$

$$\text{subject to } \mathrm{Tr}\left(\mathbb{E}\left[\mathbf{x}_{\mathsf{D,q}}\mathbf{x}_{\mathsf{D,q}}^{\mathsf{H}}\right]\right) \leq P_{\mathsf{D}}, \tag{10}$$

where the constraint condition is the transmit power constraint at the AP. In the following section, we reformulate this maximization problem to propose an effective precoding method.

III. Proposed Method

We assume that the AP and users have the perfect CSI. To convert (9) and (10) into tractable forms, we first simplify (10). The covariance matrix of $\mathbf{q}_{\mathsf{DAC}}$ is derived as

$$\mathbf{R}_{\mathbf{q}_{\mathsf{DAC}}} = \mathbf{\Phi}_{\alpha_{\mathsf{DAC}}}\mathbf{\Phi}_{\beta_{\mathsf{DAC}}}\mathrm{diag}\{P_{\mathsf{D}}\mathbf{W}\mathbf{W}^{\mathsf{H}}\}. \tag{11}$$

Using (11), the power constraint in (10) is reformulated as

$$\begin{aligned} \mathrm{Tr}\left(\mathbb{E}\left[\mathbf{x}_{\mathsf{D,q}}\mathbf{x}_{\mathsf{D,q}}^{\mathsf{H}}\right]\right) & \overset{(a)}{=} \mathrm{Tr}\Big(P_{\mathsf{D}}\mathbf{\Phi}_{\alpha_{\mathsf{DAC}}}\mathbf{W}\mathbf{W}^{\mathsf{H}}\mathbf{\Phi}_{\alpha_{\mathsf{DAC}}}^{\mathsf{H}} \\ & \quad + \mathbf{\Phi}_{\alpha_{\mathsf{DAC}}}(\mathbf{I}_{n_t} - \mathbf{\Phi}_{\alpha_{\mathsf{DAC}}})\mathrm{diag}\{P_{\mathsf{D}}\mathbf{W}\mathbf{W}^{\mathsf{H}}\}\Big) \\ & = \mathrm{Tr}\left(P_{\mathsf{D}}\mathbf{\Phi}_{\alpha_{\mathsf{DAC}}}\mathbf{W}\mathbf{W}^{\mathsf{H}}\right) < P_{\mathsf{D}}, \end{aligned} \tag{12}$$

where (a) comes from the relationship of between $\mathbf{\Phi}_{\alpha_{\mathsf{DAC}}}$ and $\mathbf{\Phi}_{\beta_{\mathsf{DAC}}}$, i.e.,$\mathbf{\Phi}_{\alpha_{\mathsf{DAC}}} + \mathbf{\Phi}_{\beta_{\mathsf{DAC}}} = \mathbf{I}_{n_t}$. Thus, we can obtain the reformulated transmit power constraint as $\mathrm{Tr}\left(\mathbf{\Phi}_{\alpha_{\mathsf{DAC}}}\mathbf{W}\mathbf{W}^{\mathsf{H}}\right) < 1$.

$$R_{\mathsf{D},k} = \log_2\left(1+\frac{|\mathbf{h}_{\mathsf{D},k}^{\mathsf{H}}\boldsymbol{\Phi}_{\alpha_{\mathsf{DAC}}}\mathbf{w}_k|^2 P_{\mathsf{D}}}{\sum_{i=1,i\neq k}^{K_{\mathsf{D}}}|\mathbf{h}_{\mathsf{D},k}^{\mathsf{H}}\boldsymbol{\Phi}_{\alpha_{\mathsf{DAC}}}\mathbf{w}_i|^2 P_{\mathsf{D}}+\mathbf{h}_{\mathsf{D},k}^{\mathsf{H}}\mathbf{R}_{\mathbf{q}_{\mathsf{DAC}}}\mathbf{h}_{\mathsf{D},k}+I_{\mathsf{CCI},k}+\sigma_{\mathsf{D}}^2}\right), \tag{7}$$

$$R_{\mathsf{U},k} = \log_2\left(1+\frac{|\mathbf{f}_k^{\mathsf{H}}\boldsymbol{\Phi}_{\alpha_{\mathsf{ADC}}}\mathbf{h}_{\mathsf{U},k}|^2 P_{\mathsf{U}}}{\sum_{i=1,i\neq k}^{K_{\mathsf{U}}}|\mathbf{f}_k^{\mathsf{H}}\boldsymbol{\Phi}_{\alpha_{\mathsf{ADC}}}\mathbf{h}_{\mathsf{U},i}|^2 P_{\mathsf{U}}+\mathbf{f}_k^{\mathsf{H}}\mathbf{R}_{\mathbf{q}_{\mathsf{ADC}}}\mathbf{f}_k+I_{\mathsf{SI},k}+||\mathbf{f}_k^{\mathsf{H}}\boldsymbol{\Phi}_{\alpha_{\mathsf{ADC}}}||^2\sigma_{\mathsf{U}}^2}\right), \tag{8}$$

To obtain DAC quantization error covariance term, we reorganize the partial term of (7) as

$$\mathbf{h}_{\mathsf{D},k}^{\mathsf{H}}\mathbf{R}_{\mathbf{q}_{\mathsf{DAC}}}\mathbf{h}_{\mathsf{D},k}=P_{\mathsf{D}}\sum_{i=1}^{K_{\mathsf{D}}}\mathbf{w}_i^{\mathsf{H}}\boldsymbol{\Phi}_{\alpha_{\mathsf{DAC}}}\boldsymbol{\Phi}_{\beta_{\mathsf{DAC}}}\mathrm{diag}\{\mathbf{h}_{\mathsf{D},k}\mathbf{h}_{\mathsf{D},k}^{\mathsf{H}}\}\mathbf{w}_i. \tag{13}$$

Similarly, $\mathbf{f}_k^{\mathsf{H}}\mathbf{R}_{\mathbf{q}_{\mathsf{ADC}}}\mathbf{f}_k$ and $I_{\mathsf{SI},k}$ are formulated as

$$\begin{aligned}\mathbf{f}_k^{\mathsf{H}}\mathbf{R}_{\mathbf{q}_{\mathsf{ADC}}}\mathbf{f}_k &= P_{\mathsf{U}}\sum_{i=1}^{K_{\mathsf{U}}}\mathbf{h}_{\mathsf{U},i}^{\mathsf{H}}\boldsymbol{\Phi}_{\alpha_{\mathsf{ADC}}}\boldsymbol{\Phi}_{\beta_{\mathsf{ADC}}}\mathrm{diag}\left\{\mathbf{f}_k\mathbf{f}_k^{\mathsf{H}}\right\}\mathbf{h}_{\mathsf{U},i}\\ &+P_{\mathsf{D}}\sum_{i=1}^{K_{\mathsf{D}}}\mathbf{w}_i^{\mathsf{H}}\boldsymbol{\Phi}_{\alpha_{\mathsf{DAC}}}^{\mathsf{H}}\mathbf{G}_{\mathsf{SI}}^{\mathsf{H}}\boldsymbol{\Phi}_{\alpha_{\mathsf{ADC}}}\boldsymbol{\Phi}_{\beta_{\mathsf{ADC}}}\mathrm{diag}\{\mathbf{f}_k\mathbf{f}_k^{\mathsf{H}}\}\mathbf{G}_{\mathsf{SI}}\boldsymbol{\Phi}_{\alpha_{\mathsf{DAC}}}\mathbf{w}_i\\ &+P_{\mathsf{D}}\sum_{i=1}^{K_{\mathsf{D}}}\mathbf{w}_i^{\mathsf{H}}\mathrm{diag}\{\mathbf{G}_{\mathsf{SI}}^{\mathsf{H}}\boldsymbol{\Phi}_{\alpha_{\mathsf{ADC}}}\boldsymbol{\Phi}_{\beta_{\mathsf{ADC}}}\mathrm{diag}\{\mathbf{f}_k\mathbf{f}_k^{\mathsf{H}}\}\mathbf{G}_{\mathsf{SI}}\}\boldsymbol{\Phi}_{\alpha_{\mathsf{DAC}}}\boldsymbol{\Phi}_{\beta_{\mathsf{DAC}}}\mathbf{w}_i\\ &+\sigma_{\mathsf{U}}^2\mathbf{f}_k^{\mathsf{H}}\boldsymbol{\Phi}_{\alpha_{\mathsf{ADC}}}\boldsymbol{\Phi}_{\beta_{\mathsf{ADC}}}\mathbf{f}_k,\end{aligned} \tag{14}$$

$$\begin{aligned}I_{\mathsf{SI},k} &= P_{\mathsf{D}}\sum_{i=1}^{K_{\mathsf{D}}}|\mathbf{f}_k^{\mathsf{H}}\boldsymbol{\Phi}_{\alpha_{\mathsf{ADC}}}\mathbf{G}_{\mathsf{SI}}\boldsymbol{\Phi}_{\alpha_{\mathsf{DAC}}}\mathbf{w}_i|^2\\ &+P_{\mathsf{D}}\sum_{i=1}^{K_{\mathsf{D}}}\mathbf{w}_i^{\mathsf{H}}\boldsymbol{\Phi}_{\alpha_{\mathsf{DAC}}}\boldsymbol{\Phi}_{\beta_{\mathsf{DAC}}}\mathrm{diag}\left\{\mathbf{G}_{\mathsf{SI}}^{\mathsf{H}}\boldsymbol{\Phi}_{\alpha_{\mathsf{ADC}}}^{\mathsf{H}}\mathbf{f}_k\mathbf{f}_k^{\mathsf{H}}\boldsymbol{\Phi}_{\alpha_{\mathsf{ADC}}}\mathbf{G}_{\mathsf{SI}}\right\}\mathbf{w}_i.\end{aligned} \tag{15}$$

Let $\tilde{q}_{\mathsf{ADC},k}=\mathbf{f}_k^{\mathsf{H}}\mathbf{R}_{\mathbf{q}_{\mathsf{ADC}}\mathbf{q}_{\mathsf{ADC}}}\mathbf{f}_k/P_{\mathsf{U}}$ and $\tilde{I}_{\mathsf{SI},k}=I_{\mathsf{SI},k}/P_{\mathsf{U}}$. Accordingly, the reformulated SEs of k-th DL and UL user are (16) and (17) which are denoted on the top of the next page, where $P=P_{\mathsf{U}}/P_{\mathsf{D}}$. Thus, the optimization problem is rewritten as

$$\underset{\mathbf{W},\mathbf{F}}{\text{maximize}}\ \sum_{k=1}^{K_{\mathsf{D}}}\gamma_{\mathsf{D},k}+\sum_{k=1}^{K_{\mathsf{U}}}\gamma_{\mathsf{U},k} \tag{18}$$

$$\text{subject to } \mathrm{Tr}\left(\boldsymbol{\Phi}_{\alpha_{\mathsf{DAC}}}\mathbf{W}\mathbf{W}^{\mathsf{H}}\right)<1. \tag{19}$$

Now, we transform $\gamma_{\mathsf{D},k}$ and $\gamma_{\mathsf{U},k}$ into Rayleigh quotient forms. Let the weighted precoding vector of DL user k as $\mathbf{v}_k=\boldsymbol{\Phi}_{\alpha_{\mathsf{DAC}}}^{1/2}\mathbf{w}_k$. Then, $\bar{\mathbf{v}}=\left[\mathbf{v}_1^{\mathsf{T}},\mathbf{v}_2^{\mathsf{T}},\ldots,\mathbf{v}_{K_{\mathsf{D}}}^{\mathsf{T}}\right]^{\mathsf{T}}\in\mathbb{C}^{n_tK_{\mathsf{D}}\times 1}$. We assume $\|\bar{\mathbf{v}}\|^2=1$ which indicates that the AP uses its maximum transmit power for maximizing SE. Let $\mathbf{M}_k$ as

$$\mathbf{M}_k=(\boldsymbol{\Phi}_{\alpha_{\mathsf{DAC}}}^{1/2})^{\mathsf{H}}\mathbf{h}_{\mathsf{D},k}\mathbf{h}_{\mathsf{D},k}^{\mathsf{H}}\boldsymbol{\Phi}_{\alpha_{\mathsf{DAC}}}^{1/2}+\boldsymbol{\Phi}_{\beta_{\mathsf{DAC}}}\mathrm{diag}\{\mathbf{h}_{\mathsf{D},k}\mathbf{h}_{\mathsf{D},k}^{\mathsf{H}}\}. \tag{20}$$

Then, we can represent $\gamma_{\mathsf{D},k}$ as

$$\gamma_{\mathsf{D},k}=\log_2\left(\frac{\bar{\mathbf{v}}^{\mathsf{H}}\mathbf{A}_k\bar{\mathbf{v}}}{\bar{\mathbf{v}}^{\mathsf{H}}\mathbf{B}_k\bar{\mathbf{v}}}\right), \tag{21}$$

where $\mathbf{A}_k\mathbb{C}^{n_tK_{\mathsf{D}}\times n_tK_{\mathsf{D}}}$ and $\mathbf{B}_k\in\mathbb{C}^{n_tK_{\mathsf{D}}\times n_tK_{\mathsf{D}}}$ are as

$$\mathbf{A}_k=\mathrm{blkdiag}(\mathbf{M}_k,\ldots,\mathbf{M}_k)+\left(||\mathbf{g}_{\mathsf{CCI},k}||^2P+\frac{\sigma_{\mathsf{D}}^2}{P_{\mathsf{D}}}\right)\mathbf{I}_{n_tK_{\mathsf{D}}} \tag{22}$$

$$\begin{aligned}&\mathbf{B}_k=\\ &\mathbf{A}_k-\mathrm{blkdiag}\left(\mathbf{0}_{n_t},\ldots,(\boldsymbol{\Phi}_{\alpha_{\mathsf{DAC}}}^{1/2})^{\mathsf{H}}\mathbf{h}_{\mathsf{D},k}\mathbf{h}_{\mathsf{D},k}^{\mathsf{H}}\boldsymbol{\Phi}_{\alpha_{\mathsf{DAC}}}^{1/2},\mathbf{0}_{n_t},\ldots,\mathbf{0}_{n_t}\right).\end{aligned} \tag{23}$$

In the same way, let $\mathbf{N}_k$ as

$$\begin{aligned}\mathbf{N}_k=\frac{1}{P}\Bigg(&(\boldsymbol{\Phi}_{\alpha_{\mathsf{DAC}}}^{1/2})^{\mathsf{H}}\mathbf{G}_{\mathsf{SI}}^{\mathsf{H}}\boldsymbol{\Phi}_{\alpha_{\mathsf{ADC}}}\boldsymbol{\Phi}_{\beta_{\mathsf{ADC}}}\mathrm{diag}\{\mathbf{f}_k\mathbf{f}_k^{\mathsf{H}}\}\mathbf{G}_{\mathsf{SI}}\boldsymbol{\Phi}_{\alpha_{\mathsf{DAC}}}^{1/2}\\ &+\boldsymbol{\Phi}_{\beta_{\mathsf{DAC}}}\mathrm{diag}\{\mathbf{G}_{\mathsf{SI}}^{\mathsf{H}}\boldsymbol{\Phi}_{\alpha_{\mathsf{ADC}}}\boldsymbol{\Phi}_{\beta_{\mathsf{ADC}}}\mathrm{diag}\{\mathbf{f}_k\mathbf{f}_k^{\mathsf{H}}\}\mathbf{G}_{\mathsf{SI}}\}\\ &+(\boldsymbol{\Phi}_{\alpha_{\mathsf{DAC}}}^{1/2})^{\mathsf{H}}\mathbf{G}_{\mathsf{SI}}^{\mathsf{H}}\boldsymbol{\Phi}_{\alpha_{\mathsf{ADC}}}^{\mathsf{H}}\mathbf{f}_k\mathbf{f}_k^{\mathsf{H}}\boldsymbol{\Phi}_{\alpha_{\mathsf{ADC}}}\mathbf{G}_{\mathsf{SI}}\boldsymbol{\Phi}_{\alpha_{\mathsf{DAC}}}^{1/2}\\ &+\boldsymbol{\Phi}_{\beta_{\mathsf{DAC}}}\mathrm{diag}\left\{\mathbf{G}_{\mathsf{SI}}^{\mathsf{H}}\boldsymbol{\Phi}_{\alpha_{\mathsf{ADC}}}^{\mathsf{H}}\mathbf{f}_k\mathbf{f}_k^{\mathsf{H}}\boldsymbol{\Phi}_{\alpha_{\mathsf{ADC}}}\mathbf{G}_{\mathsf{SI}}\right\}\Bigg).\end{aligned} \tag{24}$$

Thus, we can rewrite $\gamma_{\mathsf{U},k}$ as

$$\gamma_{\mathsf{U},k}=\log_2\left(\frac{\bar{\mathbf{v}}^{\mathsf{H}}\mathbf{C}_k\bar{\mathbf{v}}}{\bar{\mathbf{v}}^{\mathsf{H}}\mathbf{D}_k\bar{\mathbf{v}}}\right), \tag{25}$$

where $\mathbf{C}_k\mathbb{C}^{n_tK_{\mathsf{D}}\times n_tK_{\mathsf{D}}}$ and $\mathbf{D}_k\in\mathbb{C}^{n_tK_{\mathsf{D}}\times n_tK_{\mathsf{D}}}$ are as

$$\begin{aligned}&\mathbf{C}_k=\mathrm{blkdiag}(\mathbf{N}_k,\ldots,\mathbf{N}_k)+\chi\mathbf{I}_{n_tK_{\mathsf{D}}},\\ &\mathbf{D}_k=\mathbf{C}_k-\mathrm{blkdiag}\left(\mathbf{0}_{n_t},\ldots,|\mathbf{f}_k^{\mathsf{H}}\boldsymbol{\Phi}_{\alpha_{\mathsf{ADC}}}\mathbf{h}_{\mathsf{U},k}|^2\mathbf{I}_{n_t},\mathbf{0}_{n_t},\ldots,\mathbf{0}_{n_t}\right),\end{aligned}$$

here, χ is denoted as

$$\begin{aligned}\chi=&|\mathbf{f}_k^{\mathsf{H}}\boldsymbol{\Phi}_{\alpha_{\mathsf{ADC}}}\mathbf{h}_{\mathsf{U},k}|^2+\sum_{i=1}^{K_{\mathsf{U}}}\mathbf{h}_{\mathsf{U},i}^{\mathsf{H}}\boldsymbol{\Phi}_{\alpha_{\mathsf{ADC}}}\boldsymbol{\Phi}_{\beta_{\mathsf{ADC}}}\mathrm{diag}\{\mathbf{f}_k\mathbf{f}_k^{\mathsf{H}}\}\mathbf{h}_{\mathsf{U},i}\\ &+\left(\mathbf{f}_k^{\mathsf{H}}\boldsymbol{\Phi}_{\alpha_{\mathsf{ADC}}}\boldsymbol{\Phi}_{\beta_{\mathsf{ADC}}}\mathbf{f}_k+||\mathbf{f}_k^{\mathsf{H}}\boldsymbol{\Phi}_{\alpha_{\mathsf{ADC}}}||^2\right)\frac{\sigma_{\mathsf{U}}^2}{P_{\mathsf{U}}}.\end{aligned} \tag{26}$$

Accordingly, based on (21) and (25), the optimization problem in (18) is reformulated as

$$\underset{\bar{\mathbf{v}},\mathbf{F}}{\text{maximize}}\ \sum_{k=1}^{K_{\mathsf{D}}}\log_2\left(\frac{\bar{\mathbf{v}}^{\mathsf{H}}\mathbf{A}_k\bar{\mathbf{v}}}{\bar{\mathbf{v}}^{\mathsf{H}}\mathbf{B}_k\bar{\mathbf{v}}}\right)+\sum_{k=1}^{K_{\mathsf{U}}}\log_2\left(\frac{\bar{\mathbf{v}}^{\mathsf{H}}\mathbf{C}_k\bar{\mathbf{v}}}{\bar{\mathbf{v}}^{\mathsf{H}}\mathbf{D}_k\bar{\mathbf{v}}}\right) \tag{27}$$

$$\text{subject to } ||\bar{\mathbf{v}}||=1. \tag{28}$$

We now derive the first-order optimality condition of (27) with respect to $\bar{\mathbf{v}}$

Lemma 1. *The first-order optimality condition of the Rayleigh quotient form in* (27) *is denoted as*

$$\mathbf{B}_{\mathsf{KKT}}^{-1}(\bar{\mathbf{v}})\mathbf{A}_{\mathsf{KKT}}(\bar{\mathbf{v}})\bar{\mathbf{v}}=\lambda(\bar{\mathbf{v}})\bar{\mathbf{v}}, \tag{29}$$

$$\gamma_{\mathsf{D},k} = \log_2\left(1+\frac{|\mathbf{h}_{\mathsf{D},k}^{\mathsf{H}}\mathbf{\Phi}_{\alpha_{\mathsf{DAC}}}\mathbf{w}_k|^2}{\sum_{i=1,i\neq k}^{K_{\mathsf{D}}}|\mathbf{h}_{\mathsf{D},k}^{\mathsf{H}}\mathbf{\Phi}_{\alpha_{\mathsf{DAC}}}\mathbf{w}_i|^2+\sum_{i=1}^{K_{\mathsf{D}}}\mathbf{w}_i^{\mathsf{H}}\mathbf{\Phi}_{\alpha_{\mathsf{DAC}}}\mathbf{\Phi}_{\beta_{\mathsf{DAC}}}\mathrm{diag}\left\{\mathbf{h}_{\mathsf{D},k}\mathbf{h}_{\mathsf{D},k}^{\mathsf{H}}\right\}\mathbf{w}_i+||\mathbf{g}_{\mathsf{CCI},k}||^2P+\frac{\sigma_{\mathsf{D}}^2}{P_{\mathsf{D}}}}\right), \tag{16}$$

$$\gamma_{\mathsf{U},k} = \log_2\left(1+\frac{|\mathbf{f}_k^{\mathsf{H}}\mathbf{\Phi}_{\alpha_{\mathsf{ADC}}}\mathbf{h}_{\mathsf{U},k}|^2}{\sum_{i=1,i\neq k}^{K_{\mathsf{U}}}|\mathbf{f}_k^{\mathsf{H}}\mathbf{\Phi}_{\alpha_{\mathsf{ADC}}}\mathbf{h}_{\mathsf{U},i}|^2+\tilde{q}_{\mathsf{ADC},k}+\tilde{I}_{\mathsf{SI},k}+||\mathbf{f}_k^{\mathsf{H}}\mathbf{\Phi}_{\alpha_{\mathsf{ADC}}}||^2\frac{\sigma_{\mathsf{U}}^2}{P_{\mathsf{U}}}}\right). \tag{17}$$

where $\lambda(\bar{\mathbf{v}}) = \lambda_{\mathsf{num}}(\bar{\mathbf{v}})/\lambda_{\mathsf{den}}(\bar{\mathbf{v}})$,

$$\lambda_{\mathsf{num}}(\bar{\mathbf{v}}) = \prod_{k=1}^{K_{\mathsf{D}}}\left(\bar{\mathbf{v}}^{\mathsf{H}}\mathbf{A}_k\bar{\mathbf{v}}\right)\prod_{k=1}^{K_{\mathsf{U}}}\left(\bar{\mathbf{v}}^{\mathsf{H}}\mathbf{C}_k\bar{\mathbf{v}}\right), \tag{30}$$

$$\lambda_{\mathsf{den}}(\bar{\mathbf{v}}) = \prod_{k=1}^{K_{\mathsf{D}}}\left(\bar{\mathbf{v}}^{\mathsf{H}}\mathbf{B}_k\bar{\mathbf{v}}\right)\prod_{k=1}^{K_{\mathsf{U}}}\left(\bar{\mathbf{v}}^{\mathsf{H}}\mathbf{D}_k\bar{\mathbf{v}}\right), \tag{31}$$

$$\mathbf{A}_{\mathsf{KKT}}(\bar{\mathbf{v}})=\left[\sum_{k=1}^{\mathsf{K_D}}\left(\frac{\mathbf{A}_k}{\bar{\mathbf{v}}^{\mathsf{H}}\mathbf{A}_k\bar{\mathbf{v}}}\right)+\sum_{k=1}^{\mathsf{K_U}}\left(\frac{\mathbf{C}_k}{\bar{\mathbf{v}}^{\mathsf{H}}\mathbf{C}_k\bar{\mathbf{v}}}\right)\right]\lambda_{\mathsf{num}}(\bar{\mathbf{v}}), \tag{32}$$

$$\mathbf{B}_{\mathsf{KKT}}(\bar{\mathbf{v}})=\left[\sum_{k=1}^{\mathsf{K_D}}\left(\frac{\mathbf{B}_k}{\bar{\mathbf{v}}^{\mathsf{H}}\mathbf{B}_k\bar{\mathbf{v}}}\right)+\sum_{k=1}^{\mathsf{K_U}}\left(\frac{\mathbf{D}_k}{\bar{\mathbf{v}}^{\mathsf{H}}\mathbf{D}_k\bar{\mathbf{v}}}\right)\right]\lambda_{\mathsf{den}}(\bar{\mathbf{v}}). \tag{33}$$

Proof. Since the reformulated problem assume $\|\bar{\mathbf{v}}\| = 1$ and it is invariant up to the scaling of $\bar{\mathbf{v}}$, the power constraint can be ignored. Then the Lagrangian of (27) is

$$\mathcal{L}(\bar{\mathbf{v}}) = \log_2\prod_{k=1}^{K_{\mathsf{D}}}\left(\frac{\bar{\mathbf{v}}^{\mathsf{H}}\mathbf{A}_k\bar{\mathbf{v}}}{\bar{\mathbf{v}}^{\mathsf{H}}\mathbf{B}_k\bar{\mathbf{v}}}\right)+\log_2\prod_{k=1}^{K_{\mathsf{U}}}\left(\frac{\bar{\mathbf{v}}^{\mathsf{H}}\mathbf{C}_k\bar{\mathbf{v}}}{\bar{\mathbf{v}}^{\mathsf{H}}\mathbf{D}_k\bar{\mathbf{v}}}\right). \tag{34}$$

We denote the objective function in (34) as $\mathcal{L}(\bar{\mathbf{v}}) = \log_2\lambda(\bar{\mathbf{v}}) = \mathcal{L}_{\mathsf{D}}(\bar{\mathbf{v}})+\mathcal{L}_{\mathsf{U}}(\bar{\mathbf{v}}) = \log_2\lambda_{\mathsf{D}}(\bar{\mathbf{v}})+\log_2\lambda_{\mathsf{U}}(\bar{\mathbf{v}})$. Then, a derivative of $\mathcal{L}(\bar{\mathbf{w}})$ is represented as

$$\frac{\partial\mathcal{L}(\bar{\mathbf{v}})}{\partial\bar{\mathbf{v}}^{\mathsf{H}}} = \frac{1}{\lambda_{\mathsf{D}}(\bar{\mathbf{v}})\ln 2}\frac{\partial\lambda_{\mathsf{D}}(\bar{\mathbf{v}})}{\partial\bar{\mathbf{v}}^{\mathsf{H}}}+\frac{1}{\lambda_{\mathsf{U}}(\bar{\mathbf{v}})\ln 2}\frac{\partial\lambda_{\mathsf{U}}(\bar{\mathbf{v}})}{\partial\bar{\mathbf{v}}^{\mathsf{H}}}. \tag{35}$$

We derive $\partial\lambda_{\mathsf{D}}(\bar{\mathbf{v}})/\partial\bar{\mathbf{v}}^{\mathsf{H}} = \lambda_{\mathsf{D}}(\bar{\mathbf{v}})\sum_{k=1}^{K_{\mathsf{D}}}\left(\frac{\mathbf{A}_k\bar{\mathbf{v}}}{\bar{\mathbf{v}}^{\mathsf{H}}\mathbf{A}_k\bar{\mathbf{v}}}-\frac{\mathbf{B}_k\bar{\mathbf{v}}}{\bar{\mathbf{v}}^{\mathsf{H}}\mathbf{B}_k\bar{\mathbf{v}}}\right)$ and $\partial\lambda_{\mathsf{U}}(\bar{\mathbf{v}})/\partial\bar{\mathbf{v}}^{\mathsf{H}}=\lambda_{\mathsf{U}}(\bar{\mathbf{v}})\sum_{k=1}^{K_{\mathsf{U}}}\left(\frac{\mathbf{C}_k\bar{\mathbf{v}}}{\bar{\mathbf{v}}^{\mathsf{H}}\mathbf{C}_k\bar{\mathbf{v}}}-\frac{\mathbf{D}_k\bar{\mathbf{v}}}{\bar{\mathbf{v}}^{\mathsf{H}}\mathbf{D}_k\bar{\mathbf{v}}}\right)$. Let the first term and second term in (35) as $\partial\mathcal{L}_{\mathsf{D}}(\bar{\mathbf{v}})/\partial\bar{\mathbf{v}}^{\mathsf{H}}$ and $\partial\mathcal{L}_{\mathsf{U}}(\bar{\mathbf{v}})/\partial\bar{\mathbf{v}}^{\mathsf{H}}$, respectively. The derivatives are

$$\frac{\partial\mathcal{L}_{\mathsf{D}}(\bar{\mathbf{v}})}{\partial\bar{\mathbf{v}}^{\mathsf{H}}} = \frac{1}{\ln 2}\sum_{k=1}^{K_{\mathsf{D}}}\left(\frac{\mathbf{A}_k\bar{\mathbf{v}}}{\bar{\mathbf{v}}^{\mathsf{H}}\mathbf{A}_k\bar{\mathbf{v}}}-\frac{\mathbf{B}_k\bar{\mathbf{v}}}{\bar{\mathbf{v}}^{\mathsf{H}}\mathbf{B}_k\bar{\mathbf{v}}}\right), \tag{36}$$

$$\frac{\partial\mathcal{L}_{\mathsf{U}}(\bar{\mathbf{v}})}{\partial\bar{\mathbf{v}}^{\mathsf{H}}} = \frac{1}{\ln 2}\sum_{k=1}^{K_{\mathsf{U}}}\left(\frac{\mathbf{C}_k\bar{\mathbf{v}}}{\bar{\mathbf{v}}^{\mathsf{H}}\mathbf{C}_k\bar{\mathbf{v}}}-\frac{\mathbf{D}_k\bar{\mathbf{v}}}{\bar{\mathbf{v}}^{\mathsf{H}}\mathbf{D}_k\bar{\mathbf{v}}}\right). \tag{37}$$

To find the stationary condition, we set it equal to zero, i.e., $\partial\mathcal{L}(\bar{\mathbf{v}})/\partial\bar{\mathbf{v}}^{\mathsf{H}} = 0$. Based on the (36) and (37), the first-order optimality condition is formulated as $\frac{\partial\mathcal{L}_{\mathsf{D}}(\bar{\mathbf{v}})}{\partial\bar{\mathbf{v}}^{\mathsf{H}}}+\frac{\partial\mathcal{L}_{\mathsf{U}}(\bar{\mathbf{v}})}{\partial\bar{\mathbf{v}}^{\mathsf{H}}} = 0$. Therefore, we can derive a necessary condition of the first-order optimality condition with proper reorganization as

$$\mathbf{A}_{\mathsf{KKT}}(\bar{\mathbf{v}})\bar{\mathbf{v}} = \lambda(\bar{\mathbf{v}})\mathbf{B}_{\mathsf{KKT}}(\bar{\mathbf{v}})\bar{\mathbf{v}}. \tag{38}$$

This completes the proof. ∎

Algorithm 1: Quantized GPI for FD (Q-GPI-FD)

initialize: $\bar{\mathbf{u}}^{(0)}, \bar{\mathbf{v}}^{(0)}, \mathbf{f}^{(0)}$
while $||\bar{\mathbf{u}}^{(t)}-\bar{\mathbf{u}}^{(t-1)}|| > \varepsilon_{\mathbf{u}}$ and $||\mathbf{f}^{(t)}-\mathbf{f}^{(t-1)}|| > \varepsilon_{\mathbf{f}}$ or $t \leq t_{\mathsf{max}}$ **do**
    while $||\bar{\mathbf{v}}^{(n)}-\bar{\mathbf{v}}^{(n-1)}|| > \varepsilon_{\mathbf{v}}$ or $n \leq n_{\mathsf{max}}$ **do**
        Build matrix $\mathbf{A}_{\mathsf{KKT}}(\bar{\mathbf{v}}^{(n-1)})$ in (32)
        Build matrix $\mathbf{B}_{\mathsf{KKT}}(\bar{\mathbf{v}}^{(n-1)})$ in (33)
        Compute $\bar{\mathbf{v}}^{(n)} = \mathbf{B}_{\mathsf{KKT}}^{-1}(\bar{\mathbf{v}}^{(n-1)})\mathbf{A}_{\mathsf{KKT}}(\bar{\mathbf{v}}^{(n-1)})\bar{\mathbf{v}}^{(n-1)}$.
        Normalize $\bar{\mathbf{v}}^{(n)} = \bar{\mathbf{v}}^{(n)}/||\bar{\mathbf{v}}^{(n)}||$.
        $n \leftarrow n+1$.
    $\bar{\mathbf{u}}^{(t)} \leftarrow \bar{\mathbf{v}}^{(n)}$
    Build MMSE filter vector $\mathbf{f}^{(t)}$ with $\bar{\mathbf{v}} \leftarrow \bar{\mathbf{u}}^{(t)}$.
    $t \leftarrow t+1$.
$\bar{\mathbf{v}}^{\star} \leftarrow \bar{\mathbf{u}}^{(t)}$ **and** $\mathbf{f}^{\star} \leftarrow \mathbf{f}^{(t)}$
return $\mathbf{V}^{\star}=\left[\mathbf{v}_1^{\star}, \mathbf{v}_2^{\star}, \ldots, \mathbf{v}_{K_{\mathsf{D}}}^{\star}\right]$ **and** $\mathbf{F}^{\star}=\left[\mathbf{f}_1^{\star}, \mathbf{f}_2^{\star}, \ldots, \mathbf{f}_{K_{\mathsf{U}}}^{\star}\right]$

The necessary condition (38) is considered as a generalized eigenvalue problem regarding the matrices $\mathbf{A}_{\mathsf{KKT}}(\bar{\mathbf{v}})$ and $\mathbf{B}_{\mathsf{KKT}}(\bar{\mathbf{v}})$, where $\bar{\mathbf{v}}$ and $\lambda(\bar{\mathbf{v}})$ are the corresponding eigenvector and eigenvalue of $\mathbf{B}_{\mathsf{KKT}}^{-1}(\bar{\mathbf{v}})\mathbf{A}_{\mathsf{KKT}}(\bar{\mathbf{v}})$, respectively. The stationary point can exist when it satisfies (38). In addition, we note that the eigenvalue $\lambda(\bar{\mathbf{v}})$ is equivalent to the objective function in (27). Consequently, among the stationary points, the principal eigenvector $\bar{\mathbf{v}}$ maximizes the objective function (34). In other words, we can obtain the best local optimal solution by finding the leading eigenvector of (38) because any eigenvector of (38) is one of the stationary points of (27).

We can obtain the leading eigenvector of (29) by adopting the generalized power iteration-based method. We first initialize the stacked precoding vector $\bar{\mathbf{v}}^{(0)}$ using the MRT and combining vector $\mathbf{f}^{(0)}$ using MRC. At each iteration n, we build the matrices according to (32) and (33). Then, we compute $\bar{\mathbf{v}}^{(n)}$ and normalize it. Consequently, we update $\bar{\mathbf{u}}^{(t)} = [\mathbf{u}_1^{(t)\mathsf{T}}, \ldots, \mathbf{u}_{K_{\mathsf{D}}}^{(t)\mathsf{T}}]^{\mathsf{T}}$ with $\bar{\mathbf{v}}^{(n)}$. For the derived precoder, we build the MMSE combiner $\mathbf{f}^{(t)}$ by using $\bar{\mathbf{u}}^{(t)}$; let $\mathbf{z}_k^{(t)}$ be a sum of UL noise and interference before passing through $\mathbf{F}$:

$$\mathbf{z}_k^{(t)} = \sum_{i=1,i\neq k}^{K_{\mathsf{U}}}\mathbf{\Phi}_{\alpha_{\mathsf{ADC}}}\mathbf{h}_{\mathsf{U},i}s_{\mathsf{U},i}+\sum_{i=1}^{K_{\mathsf{D}}}\mathbf{\Phi}_{\alpha_{\mathsf{ADC}}}\mathbf{G}_{\mathsf{SI}}\mathbf{\Phi}_{\alpha_{\mathsf{DAC}}}^{1/2}\mathbf{u}_i^{(t)}s_{\mathsf{D},i} + \mathbf{\Phi}_{\alpha_{\mathsf{ADC}}}\mathbf{G}_{\mathsf{SI}}\mathbf{q}_{\mathsf{DAC}}+\mathbf{\Phi}_{\alpha_{\mathsf{ADC}}}\mathbf{n}_{\mathsf{U}}+\mathbf{q}_{\mathsf{ADC}}. \tag{39}$$

Thus, the MMSE combiner is computed as

$$\mathbf{f}_k^{(t)} = \mathbf{K}_{\mathbf{z}_k^{(t)}}^{-1} \mathbf{h}_{\mathsf{U},k}, \tag{40}$$

where the covariance matrix is $\mathbf{K}_{\mathbf{z}_k} = \mathbb{E}\left[\mathbf{z}_k \mathbf{z}_k^{\mathsf{H}}\right]$. We repeat this alternating process as described in Algorithm 1.

IV. Numerical Results

For simulations, small-scale fading channels for DL, UL, SI, and CCI are generated based on the one-ring channel model [9]. We also utilize the ITU-R model that considers an indoor non-line-of-sight path-loss (PL) model [10]. We set a carrier frequency as 5.2 GHz, 10 MHz bandwidth, and 5 dB noise figure. Then, the distance PL coefficient is 31 and the noise power spectral density of the AP and DL users is -174 dBm/Hz. Users are randomly generated around the AP within the radius of $d = 50$ m with a minimum distance of $d = 5$ m between each user and the AP. We also assume that $\sigma_{\mathsf{SI}}^2 = 1$ based on [11]. We evaluate the following algorithms: (1) Q-GPI-FD (proposed), (2) quantized MRT (Q-MRT), (3) quantized zero forcing (Q-ZF), and (4) quantized regularized ZF (Q-RZF). The benchmarks also use the alternating optimization with the MMSE combiner in (40).

In Fig. 1, we evaluate the sum SE versus SI cancellation capability ρ_{SI} for $n_t = n_r = 6$, $K_{\mathsf{D}} = K_{\mathsf{U}} = 3$, $b_{\mathsf{DAC}} = b_{\mathsf{ADC}} = 4$, $P_{\mathsf{D}} = 27$ dBm, and $P_{\mathsf{U}} = 10$ dBm. We also assume that PL of the CCI channel is 97 dB so that $\sigma_{\mathsf{CCI}}^2 = -97$ dB [5]. As shown in Fig. 1, Q-FD-GPI achieves the highest SE than the baseline algorithms. With the increase of SI in case of the lowering cancellation capability regime, the sum SE decreases as the sum SE of UL primarily diminishes.

In Fig. 2, we plot the sum SE versus the number of the AP antennas with the equal simulation setting in Fig. 1. Fig. 2 shows that the proposed method also outperforms for the considered antennas. In addition, Q-ZF and Q-RZF obtains higher performance gain than Q-MRT when the number of AP antennas is sufficient to cancel the interference. In this regard, the proposed full-duplex method with low-resolution regime plays key role in future wireless communication.

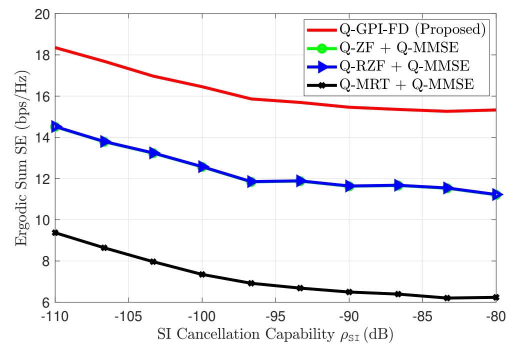

Fig. 1. The SE with variation of SI cancellation capability.

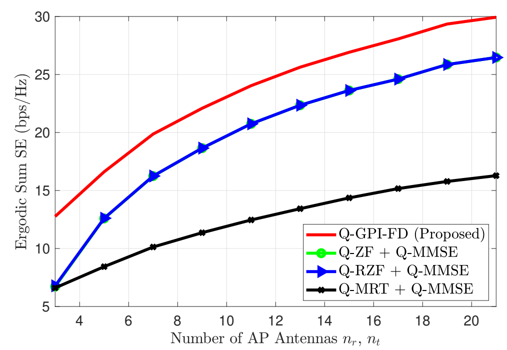

Fig. 2. The SE versus the number of AP antennas n_t and n_r

V. Conclusion

In this paper, we developed joint beamforming design to maximize the sum SE in the FD MU-MIMO system under coarse quantization. The formulated optimization problem involved the SI, CCI, and quantization errors, complicating the optimization problem to solve. We transformed the problem into a more tractable form and derived the first-order optimality condition. Then, we proposed an alternating algorithm based on power iteration based algorithm considering the MMSE combiner in UL. Via simulation results, we validated the performance of the proposed method. Therefore, the proposed FD method with low-resolution regime can play a key role in future wireless communications.

Acknowledgment

This work was supported by the National Research Foundation of Korea (NRF) grant funded by the Korea government (MSIT) (No. 2021R1C1C1004438).

References

[1] R. Li, Y. Chen, G. Y. Li, and G. Liu, "Full-Duplex Cellular Networks," *IEEE Commun. Mag.*, vol. 55, no. 4, pp. 184–191, 2017.

[2] J. Zhang, L. Dai, Z. He, B. Ai, and O. A. Dobre, "Mixed-ADC/DAC Multipair Massive MIMO Relaying Systems: Performance Analysis and Power Optimization," *IEEE Trans. Commun.*, vol. 67, no. 1, pp. 140–153, 2018.

[3] D. Nguyen, L.-N. Tran, P. Pirinen, and M. Latva-aho, "Precoding for Full Duplex Multiuser MIMO Systems: Spectral and Energy Efficiency Maximization," *IEEE Trans. Signal Process.*, vol. 61, no. 16, pp. 4038–4050, 2013.

[4] Y. Hua, P. Liang, Y. Ma, A. C. Cirik, and Q. Gao, "A method for broadband full-duplex MIMO radio," *IEEE Signal Process. Lett.*, vol. 19, no. 12, pp. 793–796, 2012.

[5] J. Kim, W. Choi, and H. Park, "Beamforming for Full-Duplex Multiuser MIMO Systems," *IEEE Trans. Veh. Technol.*, vol. 66, no. 3, pp. 2423–2432, 2016.

[6] J. Dai, J. Liu, J. Wang, J. Zhao, C. Cheng, and J.-Y. Wang, "Achievable Rates for Full-Duplex Massive MIMO Systems With Low-Resolution ADCs/DACs," *IEEE Access*, vol. 7, pp. 24 343–24 353, 2019.

[7] J. Choi, N. Lee, S.-N. Hong, and G. Caire, "Joint User Selection, Power Allocation, and Precoding Design With Imperfect CSIT for Multi-Cell MU-MIMO Downlink Systems," *IEEE Trans. Wireless Commun.*, vol. 19, no. 1, pp. 162–176, 2019.

[8] A. K. Fletcher, S. Rangan, V. K. Goyal, and K. Ramchandran, "Robust Predictive Quantization: Analysis and Design Via Convex Optimization," *IEEE J. Sel. Topics Signal Process.*, vol. 1, no. 4, pp. 618–632, 2007.

[9] A. Adhikary, J. Nam, J.-Y. Ahn, and G. Caire, "Joint Spatial Division and Multiplexing—The Large-Scale Array Regime," *IEEE Trans. Inf. Theory*, vol. 59, no. 10, pp. 6441–6463, 2013.

[10] X. Zhao, S. Geng, and B. M. Coulibaly, "Path-loss model including LOS-NLOS transition regions for indoor corridors at 5 GHz [wireless corner]," *IEEE Antennas Propag. Mag.*, vol. 55, no. 3, pp. 217–223, 2013.

[11] D. Bharadia and S. Katti, "Full Duplex MIMO Radios." in *NSDI*, vol. 14, 2014, pp. 359–372.