

Joint Secure Max-Min Fairness Precoding and Antenna Selection under Coarse Quantization

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Abstract—We propose a joint secure max-min fairness precoding and antenna selection algorithm for multi-user systems constrained by limited active antennas, low-resolution quantizers, and imperfect channel knowledge. We first formulate an optimization problem which is non-convex and non-smooth. Then, we cast it as a tractable problem, allowing us to leverage the available channel knowledge through averaging spectral efficiency and use a Lagrangian method to handle the antenna selection constraint. We identify the first-order optimality condition for a precoder and solve the condition by employing a generalized power iteration method, which achieves a superior sub-optimal solution. Simulations validate the proposed algorithm.

Index Terms—Max-min fairness, low-resolution digital-to-analog converter, antenna selection, physical layer security, imperfect channel state information at the transmitter.

I. INTRODUCTION

The rapid advancement of diverse wireless technologies for 6G has significantly increased the number of connected devices. As a result, providing adequate data rates to ensure the required quality of service for all users has become essential [1]. Simultaneously, the expansion of communication networks leads to increased energy consumption, highlighting the necessity for technologies that can reduce this demand [2]. Moreover, the open communication environment renders legitimate transmissions vulnerable to unauthorized access, such as eavesdropping [3]. Therefore, to meet future requirements, there is a need for communication techniques that are fair, secure, and capable of managing power consumption.

A comprehensive overview of fairness-related aspects was provided in [1]. In [4], it was shown that the complexity of designing max-min fairness (MMF) linear transceivers escalated as the number of users increased in multiple-input multiple-output (MIMO) systems. Accordingly, the design of MMF

precoders across multi-user scenarios has been extensively studied in [5], [6]. With respect to security, conventional cryptographic security approaches are limited by their high computational demands [3]. In this regard, physical-layer security has emerged as a promising solution. In [7], sub-optimal secure precoding algorithms were proposed to maximize the secure energy efficiency (EE) and secrecy spectral efficiency (SE) in a multiple-input single-output (MISO) system.

Recent studies have considered security or fairness within EE systems. In [8], a MMF EE precoding algorithm was proposed for integrated sensing and communications in satellite systems, using low-resolution (LR) digital-to-analog converters (DACs) to address power supply limitations. In [9], the potential of utilizing artificial noise in massive MIMO systems with LR DACs was explored to achieve secure transmission. In addition, since antenna selection in quantized systems demonstrated its improved EE and SE [2], [10], jointly considering antenna selection with fairness and security is a desirable research direction. Moreover, in quantized systems, acquiring perfect channel state information at the transmitter (CSIT) is impractical [11]. This underscores the need for robust precoding methods under imperfect CSIT.

In this work, we propose a joint secure MMF precoding and antenna selection method incorporating the effects of LR DACs under imperfect CSIT. To this end, we formulate a secure MMF optimization problem subject to an antenna selection constraint, for which it is infeasible to obtain a direct optimal solution. To address this infeasibility, we first reformulate the problem into an equivalent formulation, derive a tractable lower bound enabling the utilization of limited channel information, incorporate the antenna constraint via a Lagrangian method [12], and approximate the non-smooth problem into a smooth form. Subsequently, we derive the first-order optimality condition and interpret it as an eigenvalue problem. We adopt a generalized power iteration (GPI) method to obtain a superior sub-optimal precoder. Simulations confirm higher minimum secrecy SE than other benchmark schemes.

Notation: \mathbf{A} is a matrix and \mathbf{a} is a column vector. $(\cdot)^T$, $(\cdot)^{-1}$, and $(\cdot)^H$ denote the transpose, matrix inverse, and Hermitian. $\mathcal{CN}(\mu, \sigma^2)$ is a complex Gaussian distribution with mean μ and variance σ^2 . $\mathbf{0}_{M \times N}$ is a zero matrix of $M \times N$ size, $\text{tr}(\cdot)$ is a trace operator, and \mathbf{I}_N is an identity matrix of size $N \times N$.

II. SYSTEM MODEL

We consider a multi-user MISO system¹, where an access

¹A direct extension to a multi-antenna user scenario with single stream per user is also possible.

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point (AP) with N antennas serves K single-antenna users. We define the set of users as $\mathcal{K} = \{1, \dots, K\}$. The AP is equipped with LR DACs. We constrain the maximum number of active transmit antennas to $M \leq N$. We assume that, at the beginning of every transmission block, the transmitter selects at most M antennas, and there exists a single passive eavesdropper with a single antenna. The resulting active antenna set is denoted by \mathcal{A} . At the AP, a precoded signal is $\mathbf{x} = \sqrt{P}\mathbf{F}\mathbf{s} \in \mathbb{C}^N$, where $\mathbf{F} = [\mathbf{f}_1, \dots, \mathbf{f}_K] \in \mathbb{C}^{N \times K}$ is a precoding matrix, $\mathbf{s} = [s_1, \dots, s_K]^T \in \mathbb{C}^K$ is the vector of user symbols following $\mathbf{s} \sim \mathcal{CN}(\mathbf{0}_{K \times 1}, \mathbf{I}_K)$, and P is the maximum transmission power.

Then, \mathbf{x} is quantized at the DACs. To linearly approximate the quantization, we adopt the additive quantization noise model (AQNM) [13]. The quantized \mathbf{x} becomes

$$Q(\mathbf{x}) \approx \mathbf{x}_q = \sqrt{P}\Phi_{\alpha_{AP}}\mathbf{F}\mathbf{s} + \mathbf{q}_{AP}, \quad (1)$$

where $Q(\cdot)$ is a scalar quantizer function, $\mathbf{q}_{AP} \in \mathbb{C}^N$ is a quantization noise vector, and $\Phi_{\alpha_{AP}} = \text{diag}(\alpha_{AP,1}, \dots, \alpha_{AP,N}) \in \mathbb{C}^{N \times N}$ is a diagonal matrix of quantization loss. The loss of the n th DAC, $\alpha_{AP,n} \in (0, 1)$, is defined as $\alpha_{AP,n} = 1 - \beta_{AP,n}$, where $\beta_{AP,n}$ is a normalized mean squared quantization error with $\beta_{AP,n} = \frac{\mathbb{E}[|x - Q_n(x)|^2]}{\mathbb{E}[|x|^2]}$ [13]. The values of $\beta_{AP,n}$ are associated with quantization bits $b_{\text{DAC},n}$ (refer to [14] for specific values of $\beta_{AP,n}$). Here, \mathbf{q}_{AP} is uncorrelated with \mathbf{x} and has zero mean and covariance $\mathbf{R}_{q_{AP}} = \Phi_{\alpha_{AP}}\Phi_{\beta_{AP}}\text{diag}(\mathbb{E}[\mathbf{x}\mathbf{x}^H])$, where $\Phi_{\beta_{AP}} = \text{diag}(\beta_{AP,1}, \dots, \beta_{AP,N})$ [13], [14]. We assume \mathbf{q}_{AP} follows $\mathcal{CN}(\mathbf{0}_{N \times 1}, \mathbf{R}_{q_{AP}})$ [2], [13], which leads to the worst case SE in terms of the noise distribution [15].

The received signal vector is $\mathbf{y} = \mathbf{H}^H\mathbf{x}_q + \mathbf{n}$, where $\mathbf{n} \sim \mathcal{CN}(\mathbf{0}_{K \times 1}, \sigma^2\mathbf{I}_K)$ is an additive white Gaussian noise vector, $\mathbf{H} = [\mathbf{h}_1, \dots, \mathbf{h}_K] \in \mathbb{C}^{N \times K}$ is a channel matrix, $\mathbf{h}_k \sim \mathcal{CN}(\mathbf{0}_{N \times 1}, \mathbf{R}_k)$ is the channel vector between the AP and user k , and $\mathbf{R}_k = \mathbb{E}[\mathbf{h}_k\mathbf{h}_k^H] \in \mathbb{C}^{N \times N}$ is the spatial channel covariance matrix. We assume that the channel is estimated at every channel coherence time. We consider imperfect CSIT. Accordingly, the AP has access to the estimated channel vector $\hat{\mathbf{h}}_k = \mathbf{h}_k - \phi_k$ and the channel error covariance \mathbf{R}_k^ϕ , where ϕ_k is the channel estimation error vector. Additionally, having perfect knowledge of the wiretap channel information is practically infeasible. Thus, we assume that the AP has access only to the wiretap spatial channel covariance matrix \mathbf{R}^e of the wiretap channel vector $\mathbf{g} \in \mathbb{C}^N$.

III. MAX-MIN FAIRNESS PROBLEM FORMULATION

Considering antenna selection, the received signal becomes

$$\tilde{\mathbf{y}} = \mathbf{H}_{\mathcal{A}}^H\mathbf{x}_{q,\mathcal{A}} + \mathbf{n} = \mathbf{H}_{\mathcal{A}}^H(\sqrt{P}\Phi_{\alpha_{AP},\mathcal{A}}\mathbf{F}_{\mathcal{A}}\mathbf{s} + \mathbf{q}_{AP,\mathcal{A}}) + \mathbf{n}, \quad (2)$$

where $\mathbf{F}_{\mathcal{A}} = [\mathbf{f}_{\mathcal{A},1}, \dots, \mathbf{f}_{\mathcal{A},K}] \in \mathbb{C}^{|\mathcal{A}| \times K}$ is the reduced precoding matrix obtained by discarding the zero rows of \mathbf{F} , $\mathbf{f}_{\mathcal{A},i}$ comprises only components of \mathbf{f}_i that correspond to the indices of \mathcal{A} , and $\mathbf{H}_{\mathcal{A}} = [\mathbf{h}_{\mathcal{A},1}, \dots, \mathbf{h}_{\mathcal{A},K}] \in \mathbb{C}^{|\mathcal{A}| \times K}$, $\Phi_{\alpha_{AP},\mathcal{A}} \in \mathbb{C}^{|\mathcal{A}| \times |\mathcal{A}|}$, and $\mathbf{q}_{AP,\mathcal{A}} \in \mathbb{C}^{|\mathcal{A}|}$ denote the reduced sub-matrices and sub-vector obtained by removing the elements associated with inactive antennas. The SE of user k is

$$R_k(\mathbf{F}, \mathcal{A}) = \log_2 \left(1 + \frac{P|\mathbf{h}_{\mathcal{A},k}^H \Phi_{\alpha_{AP},\mathcal{A}} \mathbf{f}_{\mathcal{A},k}|^2}{P \sum_{i=1, i \neq k}^K |\mathbf{h}_{\mathcal{A},k}^H \Phi_{\alpha_{AP},\mathcal{A}} \mathbf{f}_{\mathcal{A},i}|^2 + \mathbf{h}_{\mathcal{A},k}^H \mathbf{R}_{q_{AP},\mathcal{A}} \mathbf{h}_{\mathcal{A},k} + \sigma^2} \right), \quad (3)$$

where $\mathbf{R}_{q_{AP},\mathcal{A}}$ is the covariance matrix of $\mathbf{q}_{AP,\mathcal{A}}$. Similarly, the SE of the eavesdropper for the message of user k is

$$R_k^e(\mathbf{F}, \mathcal{A}) = \log_2 \left(1 + \frac{P|\mathbf{g}_{\mathcal{A}}^H \Phi_{\alpha_{AP},\mathcal{A}} \mathbf{f}_{\mathcal{A},k}|^2}{P \sum_{i=1, i \neq k}^K |\mathbf{g}_{\mathcal{A}}^H \Phi_{\alpha_{AP},\mathcal{A}} \mathbf{f}_{\mathcal{A},i}|^2 + \mathbf{g}_{\mathcal{A}}^H \mathbf{R}_{q_{AP},\mathcal{A}} \mathbf{g}_{\mathcal{A}} + \sigma_e^2} \right), \quad (4)$$

where σ_e^2 is a noise variance at the eavesdropper and $\mathbf{g}_{\mathcal{A}} \in \mathbb{C}^{|\mathcal{A}|}$ denotes the reduced sub-vector of \mathbf{g} , analogously to $\mathbf{f}_{\mathcal{A},i}$. Moreover, we consider that users are treated with different priorities, which are controlled by priority weights w_k [16].

Now we formulate a secure MMF precoding and antenna selection optimization problem with $[x]^+ = \max[x, 0]$ as

$$\text{maximize}_{\mathbf{F}, \mathcal{A}} \min_{k \in \mathcal{K}} \left(w_k [R_k(\mathbf{F}, \mathcal{A}) - R_k^e(\mathbf{F}, \mathcal{A})]^+ \right) \quad (5)$$

$$\text{subject to} \quad \text{tr} \left(\mathbb{E} [\mathbf{x}_{q,\mathcal{A}} \mathbf{x}_{q,\mathcal{A}}^H] \right) \leq P, \quad (6)$$

$$\sum_{n=1}^N \mathbb{1}_{\{n \in \mathcal{A}\}} \leq M. \quad (7)$$

where (6) is the transmit power constraint, (7) is the antenna selection constraint, and $\mathbb{1}_{\{a\}}$ is the indicator function defined as $\mathbb{1}_{\{a\}} = 0$ when a is false and $\mathbb{1}_{\{a\}} = 1$ otherwise. Given the complexity of the problem, it is infeasible to obtain a direct optimal solution due to various factors: non-convexity, non-smoothness of the min and indicator function, imperfect CSIT, and the combinatorial complexity of optimal antenna selection.

To address these challenges, we first exploit the correspondence between antenna inactivity and the norm of each row of \mathbf{F} . If the norm of the n th row $\tilde{\mathbf{f}}_n$ is zero, the corresponding antenna is considered to be inactive. Hence, we have the following equality: $\sum_{n=1}^N \mathbb{1}_{\{n \in \mathcal{A}\}} = \sum_{n=1}^N \mathbb{1}_{\{\|\tilde{\mathbf{f}}_n\|^2 > 0\}}$. Here, $\|\cdot\|$ is the L2 norm. Based on this fact, we can reformulate the optimization problem without explicitly optimizing \mathcal{A} as

$$\text{maximize}_{\mathbf{F}} \min_{k \in \mathcal{K}} \left(w_k [R_k(\mathbf{F}) - R_k^e(\mathbf{F})]^+ \right), \quad (8)$$

$$\text{subject to} \quad \text{tr} \left(\mathbb{E} [\mathbf{x}_q \mathbf{x}_q^H] \right) \leq P, \quad (9)$$

$$\sum_{n=1}^N \mathbb{1}_{\{\|\tilde{\mathbf{f}}_n\|^2 > 0\}} \leq M. \quad (10)$$

We note that $R_k(\mathbf{F})$ and $R_k^e(\mathbf{F})$ are defined with full matrices \mathbf{F} , \mathbf{H} , and $\Phi_{\alpha_{AP}}$. This reformulated problem is equivalent to the original problem because (10) forces the number of active antennas to be no more than M . Accordingly, by solving the reformulated problem with respect to \mathbf{F} , we can maximize minimum secrecy SE with at most M active antennas.

In the absence of perfect CSIT, it is impossible to accurately determine the instantaneous downlink SE based on \mathbf{h}_k , and this uncertainty can result in transmissions at undecodable rates for users [17]. However, if the transmitter can perform channel coding over a long sequence of channel states, transmitting messages at the ergodic rate ensures the k th user can successfully decode the messages [18]. Therefore, disregarding $[\cdot]^+$, we reformulate the problem for ergodic SE as

$$\text{maximize}_{\mathbf{F}} \min_{k \in \mathcal{K}} \left(\mathbb{E} [w_k R_k(\mathbf{F}) - w_k R_k^e(\mathbf{F})] \right) \quad (11)$$

$$\text{subject to} \quad (9), (10).$$

Given that the AP can utilize $\hat{\mathbf{h}}_k$ and \mathbf{R}_k^ϕ , the average legitimate SE, $\bar{R}_k(\mathbf{F}) = \mathbb{E}[R_k(\mathbf{F})]$, is lower bounded as

$$\bar{R}_k(\mathbf{F}) = \mathbb{E}_{\hat{\mathbf{h}}_k} \left[\mathbb{E}_{\phi_k} \left[\log_2 \left(1 + \frac{P |\hat{\mathbf{h}}_k^H \Phi_{\alpha_{AP}} \mathbf{f}_k|^2}{\Theta_{\hat{\mathbf{h}}_k} + \mathbf{h}_k^H \mathbf{R}_{q_{AP}} \mathbf{h}_k + \sigma^2} \right) \right] \hat{\mathbf{h}}_k \right] \stackrel{(a)}{\geq} \mathbb{E}_{\hat{\mathbf{h}}_k} [R_k^{\text{lb}}(\mathbf{F})] = \bar{R}_k^{\text{lb}}(\mathbf{F}), \quad (12)$$

with

$$R_k^{\text{lb}}(\mathbf{F}) = \log_2 \left(1 + \frac{P |\hat{\mathbf{h}}_k^H \Phi_{\alpha_{AP}} \mathbf{f}_k|^2}{\Theta_{\hat{\mathbf{h}}_k} + \Omega_{\phi_k} + \mathbb{E}_{\phi_k} [\Psi_k] + \sigma^2} \right), \quad (13)$$

where $\Theta_{\hat{\mathbf{h}}_k} = P \sum_{i=1, i \neq k}^K |\hat{\mathbf{h}}_k^H \Phi_{\alpha_{AP}} \mathbf{f}_i|^2$, $\Theta_{\hat{\mathbf{h}}_k} = P \sum_{i=1, i \neq k}^K |\hat{\mathbf{h}}_k^H \Phi_{\alpha_{AP}} \mathbf{f}_i|^2$, $\Omega_{\phi_k} = P \sum_{i=1}^K \mathbf{f}_i^H \Phi_{\alpha_{AP}}^H \mathbf{R}_k^\phi \Phi_{\alpha_{AP}} \mathbf{f}_i$, and $\Psi_k = \hat{\mathbf{h}}_k^H \mathbf{R}_{q_{AP}} \hat{\mathbf{h}}_k + \phi_k^H \mathbf{R}_{q_{AP}} \phi_k$. Here, (a) applies the concept of generalized mutual information, treating all error terms as independent Gaussian noise [19], and applies Jensen's inequality. The average wiretap SE, $\bar{R}_k^e(\mathbf{F}) = \mathbb{E}[R_k^e(\mathbf{F})]$, can be transformed as

$$\bar{R}_k^e(\mathbf{F}) \stackrel{(c)}{\approx} \log_2 \left(1 + \frac{P \mathbf{f}_k^H \Phi_{\alpha_{AP}}^H \mathbf{R}^e \Phi_{\alpha_{AP}} \mathbf{f}_k}{\Omega_g + \mathbb{E}_g [\mathbf{g}^H \mathbf{R}_{q_{AP}} \mathbf{g}] + \sigma_e^2} \right) = \bar{R}_k^e(\mathbf{F}), \quad (14)$$

where $\Omega_g = P \sum_{i=1, i \neq k}^K \mathbf{f}_i^H \Phi_{\alpha_{AP}}^H \mathbf{R}^e \Phi_{\alpha_{AP}} \mathbf{f}_i$. Here, (c) follows from Lemma 1 in [20].

Form (12) and (14), we obtain the following approximate lower bound for the objective function in (11):

$$\min_{k \in \mathcal{K}} (w_k \bar{R}_k(\mathbf{F}) - w_k \bar{R}_k^e(\mathbf{F})) \gtrsim \min_{k \in \mathcal{K}} (w_k \bar{R}_k^{\text{lb}}(\mathbf{F}) - w_k \bar{R}_k^e(\mathbf{F})).$$

This approximate lower bound becomes tighter when the channel estimation error decreases and the AP utilizes a more number of active antennas [20]. Since the AP has estimated channel information, we use $R_k^{\text{lb}}(\mathbf{F})$ instead of $\bar{R}_k^{\text{lb}}(\mathbf{F})$. This short-term approach enables rapid adaptation to varying channel conditions, achieving ergodic SE through averaging over all estimated channel states [18], [21].

We redefine the optimization problem using the inequality relation of the SEs, and then incorporate the antenna constraint into the objective function using the Lagrangian method [12]:

$$\underset{\mathbf{F}}{\text{maximize}} \min_{k \in \mathcal{K}} [\Upsilon_k(\mathbf{F})] - \mu \sum_{n=1}^N \mathbb{1}_{\{\|\tilde{\mathbf{f}}_n\|^2 > 0\}} + \mu M, \quad (15)$$

subject to (9), $\mu \geq 0$,

where $\Upsilon_k(\mathbf{F}) = w_k R_k^{\text{lb}}(\mathbf{F}) - w_k \bar{R}_k^e(\mathbf{F})$ and μ is a Lagrange multiplier. Because the term μM is constant and independent of \mathbf{F} , we will omit it from (15) in the sequel. To transform this optimization problem into a tractable form, we first define a new precoding matrix $\mathbf{V} = [\mathbf{v}_1, \dots, \mathbf{v}_K]$, where $\mathbf{v}_k = \Phi_{\alpha_{AP}}^{1/2} \mathbf{f}_k$. Then, we reformulate $\mathbb{E}_{\phi_k} [\Psi_k]$ and $\mathbb{E}_g [\mathbf{g}^H \mathbf{R}_{q_{AP}} \mathbf{g}]$ as in [2]:

$$\mathbb{E}_{\phi_k} [\Psi_k] = P \sum_{i=1}^K \left(\mathbf{v}_i^H \Phi_{\beta_{AP}} \text{diag}(\hat{\mathbf{h}}_k \hat{\mathbf{h}}_k^H) \mathbf{v}_i + \mathbf{v}_i^H \Phi_{\beta_{AP}} \text{diag}(\mathbf{R}_k^\phi) \mathbf{v}_i \right), \quad (16)$$

$$\mathbb{E}_g [\mathbf{g}^H \mathbf{R}_{q_{AP}} \mathbf{g}] = P \sum_{i=1}^K \mathbf{v}_i^H \Phi_{\beta_{AP}} \text{diag}(\mathbf{R}^e) \mathbf{v}_i. \quad (17)$$

Now, the power constraint in (9) can be rewritten as $\text{tr}(\mathbf{P} \mathbf{V} \mathbf{V}^H) \leq P$. Since utilizing the maximum power is the optimal transmission strategy, we set $\text{tr}(\mathbf{V} \mathbf{V}^H) = \|\bar{\mathbf{v}}\|^2 = 1$, where $\bar{\mathbf{v}} = \text{vec}(\mathbf{V})$. Here, $\text{vec}(\cdot)$ is a vectorization operator. Leveraging (16), (17), and $\|\bar{\mathbf{v}}\|^2 = 1$, we reformulate (13) and (14) into the form of a Rayleigh quotient:

$$R_k^{\text{lb}}(\bar{\mathbf{v}}) = \log_2 \left(\frac{\bar{\mathbf{v}}^H \mathbf{A}_k \bar{\mathbf{v}}}{\bar{\mathbf{v}}^H \mathbf{B}_k \bar{\mathbf{v}}} \right), \quad \bar{R}_k^e(\bar{\mathbf{v}}) = \log_2 \left(\frac{\bar{\mathbf{v}}^H \mathbf{A}_k^e \bar{\mathbf{v}}}{\bar{\mathbf{v}}^H \mathbf{B}_k^e \bar{\mathbf{v}}} \right), \quad (18)$$

where

$$\mathbf{A}_k = \mathbf{I}_K \otimes \Phi + \mathbf{I}_{NK} \frac{\sigma^2}{P}, \quad \mathbf{A}_k^e = \mathbf{I}_K \otimes \Phi^e + \mathbf{I}_{NK} \frac{\sigma^2}{P}, \quad (19)$$

$$\mathbf{B}_k = \mathbf{A}_k - \text{diag}\{\mathbf{e}_k^K\} \otimes (\Phi_{\alpha_{AP}}^{1/2})^H \hat{\mathbf{h}}_k \hat{\mathbf{h}}_k^H \Phi_{\alpha_{AP}}^{1/2}, \quad (20)$$

$$\mathbf{B}_k^e = \mathbf{A}_k^e - \text{diag}\{\mathbf{e}_k^K\} \otimes (\Phi_{\alpha_{AP}}^{1/2})^H \mathbf{R}^e \Phi_{\alpha_{AP}}^{1/2}, \quad (21)$$

with $\Phi^e = (\Phi_{\alpha_{AP}}^{1/2})^H \mathbf{R}^e \Phi_{\alpha_{AP}}^{1/2} + \Phi_{\beta_{AP}} \text{diag}(\mathbf{R}^e)$, $\Phi = (\Phi_{\alpha_{AP}}^{1/2})^H \hat{\mathbf{h}}_k \hat{\mathbf{h}}_k^H \Phi_{\alpha_{AP}}^{1/2} + \Phi_{\beta_{AP}} \text{diag}(\hat{\mathbf{h}}_k \hat{\mathbf{h}}_k^H) + (\Phi_{\alpha_{AP}}^{1/2})^H \mathbf{R}_k^\phi \Phi_{\alpha_{AP}}^{1/2} + \Phi_{\beta_{AP}} \text{diag}(\mathbf{R}_k^\phi)$, \otimes is the Kronecker product, and \mathbf{e}_k^N is the k th standard basis for $1 \leq k \leq N$ in N -dimensional space.

The problem in (15) still requires addressing the inherent non-convexity and the non-smoothness. To handle the non-smoothness, we utilize the LogSumExp (LSE) approximation method [22]: $\min_{k \in \mathcal{K}} \{x_k\} \approx -\gamma \ln(\sum_{k \in \mathcal{K}} \exp(-\gamma^{-1}(x_k)))$, where $\gamma > 0$ is a tunable parameter. Because γ directly affects both system performance and convergence, identifying an appropriate value in advance is important. Now, to smooth the indicator function, we apply the approximation approach in [2] which exploits the relationship between \mathbf{F} and \mathcal{A} mentioned earlier (refer to [2] for more detail):

$$\mathbb{1}_{\{\|\tilde{\mathbf{f}}_n\|^2 > 0\}} \approx \log_2 \left(1 + \rho^{-1} \left\| \frac{\tilde{\mathbf{v}}_n}{\sqrt{\mathbf{A}_{AP,n}}} \right\|^2 \right)^{w_\rho} = \log_2 (\bar{\mathbf{v}}^H \mathbf{E}_n \bar{\mathbf{v}})^{w_\rho}, \quad (22)$$

where $\tilde{\mathbf{v}}_n$ is the n th row in the precoding matrix \mathbf{V} , $\rho > 0$ is a sufficiently small value, $w_\rho = 1/\log_2(1 + \rho^{-1})$, $\mathbf{E}_n = \mathbf{I}_{NK} + \rho^{-1}(\mathbf{I}_K \otimes \tilde{\mathbf{e}}_u \otimes \tilde{\mathbf{e}}_u^H)$ as $\|\bar{\mathbf{v}}\|^2 = 1$, and $\tilde{\mathbf{e}}_u = \Phi_{\alpha_{AP}}^{-1/2} \mathbf{e}_u^N$.

For a given μ , the optimization problem in (15) is reformulated using (18), (22), and the LSE method as

$$\underset{\bar{\mathbf{v}}}{\text{maximize}} \quad \log_2 \lambda(\bar{\mathbf{v}}), \quad (23)$$

where

$$\lambda(\bar{\mathbf{v}}) = \left(\sum_{k=1}^K d_k(\bar{\mathbf{v}}) \right)^{-\gamma \ln 2} \left(\prod_{n=1}^N \bar{\mathbf{v}}^H \mathbf{E}_n \bar{\mathbf{v}} \right)^{-\mu w_\rho}, \quad (24)$$

with $d_k(\bar{\mathbf{v}}) = \left(\frac{\bar{\mathbf{v}}^H \mathbf{A}_k \bar{\mathbf{v}}}{\bar{\mathbf{v}}^H \mathbf{B}_k \bar{\mathbf{v}}} \right)^{-\bar{w}} \left(\frac{\bar{\mathbf{v}}^H \mathbf{A}_k^e \bar{\mathbf{v}}}{\bar{\mathbf{v}}^H \mathbf{B}_k^e \bar{\mathbf{v}}} \right)^{\bar{w}}$ and $\bar{w} = \frac{w_k}{\gamma \ln 2}$. The power constraint is disregarded, as it will be satisfied through normalization within the proposed algorithm. Afterwards, we derive the optimality condition for obtaining the superior $\bar{\mathbf{v}}$.

Lemma 1. *The first-order optimality condition of the problem (23) for given μ is satisfied if the following holds:*

$$\mathbf{B}_{\text{KKT}}^{-1}(\bar{\mathbf{v}}) \mathbf{A}_{\text{KKT}}(\bar{\mathbf{v}}) \bar{\mathbf{v}} = \lambda(\bar{\mathbf{v}}) \bar{\mathbf{v}}, \quad (25)$$

where

$$\mathbf{A}_{\text{KKT}}(\bar{\mathbf{v}}) = \lambda_{\text{num}}(\bar{\mathbf{v}}) \left(\sum_{k=1}^K w_k d_k(\bar{\mathbf{v}}) \left(\frac{\mathbf{A}_k}{\bar{\mathbf{v}}^H \mathbf{A}_k \bar{\mathbf{v}}} + \frac{\mathbf{B}_k^e}{\bar{\mathbf{v}}^H \mathbf{B}_k^e \bar{\mathbf{v}}} \right) \right), \quad (26)$$

$$\mathbf{B}_{\text{KKT}}(\bar{\mathbf{v}}) = \lambda_{\text{den}}(\bar{\mathbf{v}}) \left(\sum_{k=1}^K w_k d_k(\bar{\mathbf{v}}) \left(\frac{\mathbf{B}_k}{\bar{\mathbf{v}}^H \mathbf{B}_k \bar{\mathbf{v}}} + \frac{\mathbf{A}_k^e}{\bar{\mathbf{v}}^H \mathbf{A}_k^e \bar{\mathbf{v}}} \right) + \mu w_p \sum_{i=1}^K d_i(\bar{\mathbf{v}}) \sum_{n=1}^N \left(\frac{\mathbf{E}_n}{\bar{\mathbf{v}}^H \mathbf{E}_n \bar{\mathbf{v}}} \right) \right). \quad (27)$$

Here, $\lambda_{\text{num}}(\bar{\mathbf{v}})$ and $\lambda_{\text{den}}(\bar{\mathbf{v}})$ can be arbitrary functions of $\bar{\mathbf{v}}$, which satisfy $\lambda(\bar{\mathbf{v}}) = \lambda_{\text{num}}(\bar{\mathbf{v}}) / \lambda_{\text{den}}(\bar{\mathbf{v}})$.

Proof. We take the partial derivative of $L(\bar{\mathbf{v}}) = \log_2 \lambda(\bar{\mathbf{v}})$ as

$$\frac{\partial L(\bar{\mathbf{v}})}{\partial \bar{\mathbf{v}}^H} = \frac{1}{\lambda(\bar{\mathbf{v}}) \ln 2} \frac{\partial \lambda(\bar{\mathbf{v}})}{\partial \bar{\mathbf{v}}^H}. \quad (28)$$

Then, we derive $\partial \lambda(\bar{\mathbf{v}}) / \partial \bar{\mathbf{v}}^H$ and set it to zero:

$$\frac{\partial \lambda(\bar{\mathbf{v}})}{\partial \bar{\mathbf{v}}^H} = \lambda(\bar{\mathbf{v}}) \left(\sum_{k=1}^K \left(\frac{d_k(\bar{\mathbf{v}}) L_k(\bar{\mathbf{v}})}{\sum_{i \in \mathcal{K}} d_i(\bar{\mathbf{v}})} \right) - \mu w_p \sum_{n=1}^N \frac{\mathbf{E}_n \bar{\mathbf{v}}}{\bar{\mathbf{v}}^H \mathbf{E}_n \bar{\mathbf{v}}} \right) = 0, \quad (29)$$

where $L_k(\bar{\mathbf{v}}) = w_k \left(\frac{\mathbf{A}_k \bar{\mathbf{v}}}{\bar{\mathbf{v}}^H \mathbf{A}_k \bar{\mathbf{v}}} - \frac{\mathbf{B}_k \bar{\mathbf{v}}}{\bar{\mathbf{v}}^H \mathbf{B}_k \bar{\mathbf{v}}} - \frac{\mathbf{A}_k^e \bar{\mathbf{v}}}{\bar{\mathbf{v}}^H \mathbf{A}_k^e \bar{\mathbf{v}}} + \frac{\mathbf{B}_k^e \bar{\mathbf{v}}}{\bar{\mathbf{v}}^H \mathbf{B}_k^e \bar{\mathbf{v}}} \right)$. We can express (29) as $\mathbf{A}_{\text{KKT}}(\bar{\mathbf{v}}) \bar{\mathbf{v}} = \lambda(\bar{\mathbf{v}}) \mathbf{B}_{\text{KKT}}(\bar{\mathbf{v}}) \bar{\mathbf{v}}$. Since $\mathbf{B}_{\text{KKT}}(\bar{\mathbf{v}})$ is generally invertible, this equation is equivalent to the first-order KKT condition. This completes the proof. ■

In Lemma 1, (25) belongs to a class of nonlinear eigenvalue problem with eigenvector-dependency (NEPv) [23]. Accordingly, $\bar{\mathbf{v}}$ is considered an eigenvector of $\mathbf{B}_{\text{KKT}}^{-1}(\bar{\mathbf{v}}) \mathbf{A}_{\text{KKT}}(\bar{\mathbf{v}})$, and $\lambda(\bar{\mathbf{v}})$ is a corresponding eigenvalue. Here, $\mathbf{A}_{\text{KKT}}(\bar{\mathbf{v}})$ is composed of the terms that include the desired signal as well as interference and penalty terms, whereas $\mathbf{B}_{\text{KKT}}(\bar{\mathbf{v}})$ comprises only interference and penalty terms. This indicates that maximizing $L(\bar{\mathbf{v}})$ is equivalent to finding the principal eigenvector of $\mathbf{B}_{\text{KKT}}^{-1}(\bar{\mathbf{v}}) \mathbf{A}_{\text{KKT}}(\bar{\mathbf{v}})$. In addition, the resulting solution maximizes $\mathbf{A}_{\text{KKT}}(\bar{\mathbf{v}})$ while minimizing $\mathbf{B}_{\text{KKT}}(\bar{\mathbf{v}})$, making the proposed algorithm almost unlikely to converge to a local minimum. Based on this interpretation, we propose Algorithm 1 to obtain the superior sub-optimal precoder $\bar{\mathbf{v}}^*$.

Algorithm 1 aims to determine $\bar{\mathbf{v}}^*$ by applying the GPI method [17] to the eigenvalue problem defined in (25). For a given μ , Algorithm 1 begins by initializing the precoder $\bar{\mathbf{v}}_0$. Then, it iteratively updates $\bar{\mathbf{v}}_t$ by computing $\mathbf{A}_{\text{KKT}}(\bar{\mathbf{v}}_{t-1})$ and $\mathbf{B}_{\text{KKT}}(\bar{\mathbf{v}}_{t-1})$, calculating $\mathbf{B}_{\text{KKT}}^{-1}(\bar{\mathbf{v}}_{t-1}) \mathbf{A}_{\text{KKT}}(\bar{\mathbf{v}}_{t-1}) \bar{\mathbf{v}}_{t-1}$, and normalizing the result to obtain $\bar{\mathbf{v}}_t$. After obtaining the precoding vector, it is crucial to select an appropriate active set that satisfies $n_r \leq M$, where n_r represents the number of selected antennas. This requires adjusting μ , which serves as the penalty weight for antenna selection. To this end, we first determine the set \mathcal{A} that satisfies the condition $\frac{\|\bar{\mathbf{v}}_{t,i}\|^2}{\|\bar{\mathbf{v}}_{t,\max}\|^2} > \tau$, where $\tau > 0$ is an antenna selection threshold, $\bar{\mathbf{v}}_{t,\max}$ is the row vector with the maximum norm in \mathbf{V}_t , and $\bar{\mathbf{v}}_{t,i}$ is the i th row vector of \mathbf{V}_t . If $n_r > M$, μ is increased by $\Delta\mu$, and the same process is repeated until the antenna constraint is satisfied. The optimal solution to (8) is achieved with M active antennas obviously. Accordingly, when the iteration ends with $n_r < M$, we augment \mathcal{A} by adding $(M - n_r)$ antennas with the largest geometric mean (GM) channel vectors among the remaining

Algorithm 1: Proposed Secure MMF Precoding and Antenna Selection

```

1 initialize:  $\bar{\mathbf{v}}_0$ 
2 Set  $n_r = N$  and  $\mu = \mu_{\min}$ 
3 while  $n_r > M$  do
4   Set the iteration count  $t = 1$ 
5   while  $\|\bar{\mathbf{v}}_t - \bar{\mathbf{v}}_{t-1}\| > \epsilon_t$  &  $t \leq t_m$  do
6     Build matrix  $\mathbf{A}_{\text{KKT}}(\bar{\mathbf{v}}_{t-1})$  in (26)
7     Build matrix  $\mathbf{B}_{\text{KKT}}(\bar{\mathbf{v}}_{t-1})$  in (27)
8      $\bar{\mathbf{v}}_t \leftarrow \frac{\mathbf{B}_{\text{KKT}}^{-1}(\bar{\mathbf{v}}_{t-1}) \mathbf{A}_{\text{KKT}}(\bar{\mathbf{v}}_{t-1}) \bar{\mathbf{v}}_{t-1}}{\|\mathbf{B}_{\text{KKT}}^{-1}(\bar{\mathbf{v}}_{t-1}) \mathbf{A}_{\text{KKT}}(\bar{\mathbf{v}}_{t-1}) \bar{\mathbf{v}}_{t-1}\|}$ 
9      $t \leftarrow t + 1$ 
10  Select active antenna set:  $\mathcal{A} = \{i \mid \frac{\|\bar{\mathbf{v}}_{t,i}\|^2}{\|\bar{\mathbf{v}}_{t,\max}\|^2} > \tau\}$ 
11   $n_r \leftarrow |\mathcal{A}|$ 
12  Increment penalty parameter:  $\mu \leftarrow \mu + \Delta\mu$ 
13 if  $n_r < M$  then
14   Augment  $\mathcal{A}$  by adding  $M - n_r$  antennas with
    largest geometric mean channel vectors
15 Do lines 4-9 with  $\mathcal{A}$  and  $\mu = 0$  to obtain  $\bar{\mathbf{v}}^*$ 
16 return  $\bar{\mathbf{v}}^*$ .
```

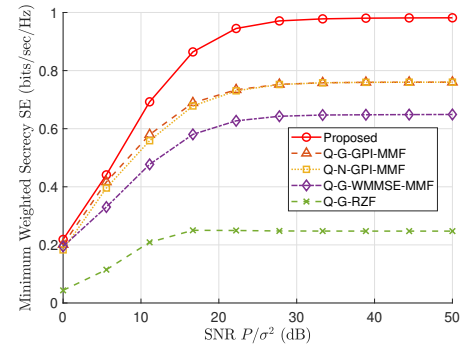


Fig. 1. The minimum weighted secrecy SE versus SNR for $\kappa = 0.1$ channel quality variable and no user priorities, i.e., $w_k = 1$.

antennas because GM-based optimization is often used as a fairness-related metric [24]. The GM of the i th row channel vector $\mathbf{h}_i = [h_{i,1}, \dots, h_{i,K}]^T$ is calculated as $(\prod_{k=1}^K |h_{i,k}|)^{1/K}$. Finally, we set the row vectors corresponding to the inactive antennas to zero, and perform lines 4-9 with $\mu = 0$.

IV. SIMULATION RESULTS

We consider the benchmarks: 1) Q-G-GPI-MMF and 2) Q-N-GPI-MMF: quantization-aware secure MMF GPI precoding algorithms that select the M antennas with the highest channel GM and channel gains, respectively. Q-G-GPI-MMF and Q-N-GPI-MMF are the algorithms that exclude the joint antenna selection in the proposed algorithm. 3) Q-G-WMMSE-MMF: a quantization-aware GM-based MMF algorithm using the weighted minimum mean square error method [25]. 4) Q-G-RZF-MMF: a quantization-aware GM-based MMF algorithm using the regularized zero-forcing method.

We generate the channel by employing the one-ring model in [26]. Considering frequency division duplex (FDD) systems, the error covariance is $\mathbf{R}_k^\phi = \mathbb{E}[\phi_k \phi_k^H] = \mathbf{U}_k \mathbf{\Lambda}_k^{\frac{1}{2}} (2 - 2\sqrt{1 - \kappa^2}) \mathbf{\Lambda}_k^{\frac{1}{2}} \mathbf{U}_k^H$ [17]. Here, $\mathbf{\Lambda}_k$ is a diagonal matrix contain-

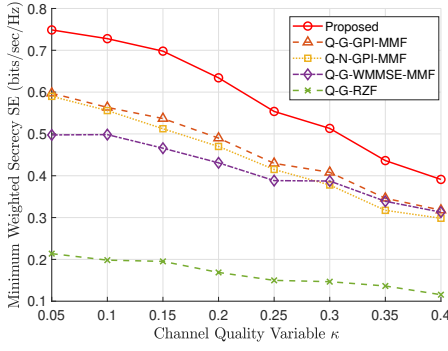


Fig. 2. The minimum weighted secrecy SE versus channel quality variable κ for SNR = 20 dB transmit power and random user priorities w_k .

ing the non-zero eigenvalues of \mathbf{R}_k , \mathbf{U}_k is the corresponding eigenvector matrix, and $\kappa \in [0, 1]$ is a channel quality parameter. We initialize $\bar{\mathbf{v}}_0$ using the maximum ratio transmission. We set $N = 8$, $K = 4$, $M = 4$, $b_{\text{DAC}} = 2$ bits, $\sigma^2 = \sigma_e^2 = 1$, $\mu_{\min} = 0$, $\Delta\mu = 0.1$, $\rho = 10^{-8}$, $t_m = 10$, $\epsilon_t = 0.01$, $\tau = 0.1$, and empirically tune γ for each SNR.

Fig. 1 illustrates that our proposed algorithm outperforms other benchmarks across most signal-to-noise ratio (SNR) ranges, implying that it provides the fairest and most secure communication. The performance improvement of the proposed algorithm from Q-G-GPI-MMF and Q-N-GPI-MMF suggests that jointly optimizing the antenna selection with the secure MMF precoding has a significant impact on communication performance. In addition, the proposed method outperforms the WMMSE-based MMF precoding by incorporating both the joint antenna selection and the security.

Fig. 2 illustrates that our proposed algorithm consistently outperforms other benchmarks over the κ range of 0.05 to 0.4, with randomly generated user priorities w_k . In this scenario, user priorities are randomly generated subject to $\sum_{k=1}^K w_k = K$. This result indicates that the proposed algorithm effectively incorporates the effects of imperfect CSIT. In conclusion, the simulation results demonstrate that the proposed algorithm provides fair and secure communication by selecting the appropriate antenna set and effectively accounting for the effects of imperfect CSIT and LR DACs.

V. CONCLUSION

We proposed a precoding algorithm that considered security and fairness under imperfect CSIT. In addition, to reduce power consumption, we employed antenna selection and LR DACs. Deriving an approximate lower bound of a secrecy rate and transforming the optimization problem into tractable form, we identified the stationary condition, interpreted it as the eigenvalue problem, and obtained the superior sub-optimal precoder. Simulation results demonstrated that the proposed algorithm provided the fairest and most secure communication compared to other benchmarks by jointly optimizing the antenna selection and secure precoding in the MMF framework.

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