Max-Min Fairness Precoding with Low Resolution DACs

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***Abstract*—We propose a max-min fairness (MMF) precoding algorithm for a multi-user multiple-input multiple-output (MU-MIMO) downlink system with different user weights, considering the quantization effects of low-resolution digital-to-analog converters (DACs). To find a precoder, we first formulate a problem, which jointly considers MMF and quantization. This joint problem presents some challenges that make finding a direct solution infeasible: non-smoothness and non-convexity. To address the non-smoothness, we use LogSumExp method to approximate the problem into a smooth problem. Subsequently, we derive the first-order optimality condition and cast it as a nonlinear eigenvalue problem. Lastly, we employ a generalized power iteration method to determine the superior stationary point. Simulation result demonstrates the superiority of our MMF precoding algorithm.**

***Index Terms*—Max-min fairness, low-resolution digital-to-analog converter, precoding, generalized power iteration**

I. Introduction

As various wireless technologies are developed for 6G, technological advancement has significantly increased the number of connected devices. As a result, providing appropriate data rates is crucial to guarantee a required quality of service (QoS) for all users. Along with this, technology to reduce energy consumption is needed as the scale of communication increases. One approach to enhance energy efficiency is through the use of low-resolution quantizers, as they significantly reduce the energy consumption of radio frequency (RF) chains, which are the primary contributors to overall power consumption [1]. Consequently, it is crucial to jointly consider fairness and the impact of low-resolution quantization to meet future communication requirements. Although many studies have explored low-resolution digital-to-analog converters (DACs) and max-min fairness (MMF) individually, there has been limited research that simultaneously considers both. Thus, this paper proposes a precoding algorithm that incorporates low-resolution DACs and MMF.

II. System Model

We consider a downlink single-cell MU-MIMO system where an access point (AP) is equipped with N_T antennas and serves K_U single-antenna users. The user set is denoted as $\mathcal{K} = \{1, \ldots, K_U\}$. The AP has low-resolution DACs, and users have infinite-bit analog-to-digital converter (ADCs). Additionally, we assume that all users have different user weights. At the AP, the transmit signal $\mathbf{x}_T \in \mathbb{C}^{N_T \times 1}$ is quantized by the DACs. To linearly approximate the quantization process, we adopt an additive quantization noise model (AQNM) in [2]. Then, the quantized transmission signal $\mathbf{x}_q$ is expressed as: $Q(\mathbf{x}) \approx \mathbf{x}_q = \alpha_{\mathsf{AP}}\sqrt{P}\mathbf{F}_T\mathbf{s} + \mathbf{q}_{\mathsf{AP}}$, where $\mathbf{F}_T = [\mathbf{f}_1, ..., \mathbf{f}_{\mathrm{K_U}}] \in \mathbb{C}^{N_T \times K_U}$ is a precoding matrix, $\mathbf{s} = [s_1, ..., s_{K_U}]^\mathsf{T}$ is a user symbols vector following $\mathbf{s} \sim \mathcal{CN}(\mathbf{0}_{K_U \times 1}, \mathbf{I}_{K_U})$, P represents the maximum transmission power, $Q(\cdot)$ is a scalar quantizer function, and $\mathbf{q}_{\mathsf{AP}} \in \mathbb{C}^{N_T}$ is a quantization noise vector. $\alpha_{\mathsf{AP}} \in (0, 1)$ is a quantization loss, and it is defined by $\alpha_{\mathsf{AP}} = 1 - \beta_{\mathsf{AP}}$ where β_{AP} is a normalized mean squared quantization error [2]. The quantization noise $\mathbf{q}_{\mathsf{AP}} \sim \mathcal{CN}(\mathbf{0}_{N_T \times 1}, \mathbf{R}_{\mathbf{q}_{\mathsf{AP}}\mathbf{q}_{\mathsf{AP}}})$ is uncorrelated with $\mathbf{x}$ and the covariance $\mathbf{R}_{\mathbf{q}_{\mathsf{AP}}\mathbf{q}_{\mathsf{AP}}}$ is computed as $\mathbf{R}_{\mathbf{q}_{\mathsf{AP}}\mathbf{q}_{\mathsf{AP}}} = \beta_{\mathsf{AP}}\alpha_{\mathsf{AP}}\mathrm{diag}(P\mathbf{F}\mathbf{F}^\mathsf{H})$ [2]. Subsequently, The signal that user k_u receives is represented as

$$y_{k_u} = \alpha_{\mathsf{AP}}\sqrt{P}\mathbf{h}_{k_u}^\mathsf{H}\mathbf{f}_{k_u}s_{k_u} + \alpha_{\mathsf{AP}}\sqrt{P}\sum_{i=1, i\neq k_u}^{K_U}\mathbf{h}_{k_u}^\mathsf{H}\mathbf{f}_i s_i + \mathbf{h}_{k_u}^\mathsf{H}\mathbf{q}_{\mathsf{AP}} + n_{k_u}, \tag{1}$$

where n_{k_u} is k_uth element of additive white Gaussian noise vector $\mathbf{n} \sim \mathcal{CN}(\mathbf{0}_{K_U \times 1}, \sigma^2\mathbf{I}_{K_U})$, $\mathbf{h}_{k_u} \sim \mathcal{CN}(\mathbf{0}_{N_T \times 1}, \mathbf{R}_{k_u})$ represent channel vector, and $\mathbf{R}_{k_u} = \mathbb{E}[\mathbf{h}_{k_u}\mathbf{h}_{k_u}^\mathsf{H}] \in \mathbb{C}^{N_T \times N_T}$ is a spatial covariance matrix. Additionally, we assume perfect channel state information (CSI) and a block fading model, which indicates $\mathbf{h}_{k_u}$ as a time invariant channel within a transmission block.

III. Max-Min Fairness Optimization

Using the received signal in (1), the SINR of user k_u is computed as

$$\mathsf{SINR}_{k_u} = \frac{|\alpha_{\mathsf{AP}}\mathbf{h}_{k_u}^\mathsf{H}\mathbf{f}_{k_u}|^2}{\sum_{i=1, i\neq k_u}^{K_U}|\alpha_{\mathsf{AP}}\mathbf{h}_{k_u}^\mathsf{H}\mathbf{f}_i|^2 + \frac{\mathbf{h}_{k_u}^\mathsf{H}\mathbf{R}_{\mathbf{q}_{\mathsf{AP}}\mathbf{q}_{\mathsf{AP}}}\mathbf{h}_{k_u}}{P} + \frac{\sigma^2}{P}}. \tag{2}$$

The SE of user k_u is formulated as: $R_{k_u} = \log_2(1 + \mathsf{SINR}_{k_u})$. After that, we define the optimization MMF problem as

$$\underset{\mathbf{f}_1, \cdots, \mathbf{f}_{K_U}}{\text{maximize}} \quad \min_{k_u \in \mathcal{K}}[w_{k_u}R_{k_u}] \tag{3}$$

$$\text{subject to} \quad \mathrm{tr}\left(\mathbb{E}\left[\mathbf{x}_q\mathbf{x}_q^\mathsf{H}\right]\right) \leq P, \tag{4}$$

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where (4) is transmit power constraint and $w_{k_u} \geq 0$ is the predefined weight of user k_u. Non-smoothness and non-convexity of the problem in (3) make it difficult to obtain a direct solution. To address the difficulties, we first reformulate quantization noise covariance and power constraint as: $\mathbf{h}_{k_u}^{\mathsf{H}} \mathbf{R}_{\mathbf{q}_{\mathsf{AP}}\mathbf{q}_{\mathsf{AP}}} \mathbf{h}_{k_u} = P \sum_{i=1}^{K_U} \mathbf{f}_i^{\mathsf{H}} (\alpha_{\mathsf{AP}} \beta_{\mathsf{AP}} \operatorname{diag}(\mathbf{h}_{k_u} \mathbf{h}_{k_u}^{\mathsf{H}})) \mathbf{f}_i$, and $\operatorname{tr}(\mathbb{E}[\mathbf{x}_q \mathbf{x}_q^{\mathsf{H}}]) = \operatorname{tr}(\alpha_{\mathsf{AP}}{}^2 P \sum_{i=1}^{K_U} \mathbf{f}_i \mathbf{f}_i^{\mathsf{H}} + \mathbf{R}_{\mathbf{q}_{\mathsf{AP}}\mathbf{q}_{\mathsf{AP}}}) = \operatorname{tr}(\alpha_{\mathsf{AP}} P \mathbf{F}\mathbf{F}^{\mathsf{H}}) \leq P$. Then, we define a new precoding matrix $\mathbf{V} = [\mathbf{v}_1, \ldots, \mathbf{v}_{K_U}]$ where $\mathbf{v}_{k_u} = \alpha_{\mathsf{AP}}^{1/2} \mathbf{f}_{k_u}$. Using these, we express the modified R_{k_u} in Rayleigh quotient form by converting the precoder matrix into a vector, $\bar{\mathbf{v}} = \operatorname{vec}(\mathbf{V})$, i.e., $\bar{\mathbf{v}} = \left[\mathbf{v}_1^{\mathsf{T}}, \ldots, \mathbf{v}_{K_U}^{\mathsf{T}}\right]^{\mathsf{T}} \in \mathbb{C}^{N_T K_U}$. Subsequently, we assume $\|\bar{\mathbf{v}}\|^2 = 1$ to use maximum transmit power. Then, the optimization problem (3) is redefined as

$$\underset{\bar{\mathbf{v}}}{\text{maximize}} \quad \min_{k_u \in \mathcal{K}} \left(\log_2 \left(\frac{\bar{\mathbf{v}}^{\mathsf{H}} \mathbf{A}_{k_u} \bar{\mathbf{v}}}{\bar{\mathbf{v}}^{\mathsf{H}} \mathbf{B}_{k_u} \bar{\mathbf{v}}} \right)^{w_{k_u}} \right) \tag{5}$$

$$\text{subject to} \quad ||\bar{\mathbf{v}}|| = 1, \tag{6}$$

where

$$\mathbf{A}_{k_u} = \mathbf{I}_{K_U} \otimes \mathbf{\Phi}_t + \mathbf{I}_{N_T K_U} \frac{\sigma^2}{P}, \tag{7}$$

$$\mathbf{B}_{k_u} = \mathbf{A}_{k_u} - \operatorname{diag}\left\{\mathbf{e}_{k_u}^{K_U}\right\} \otimes \alpha_{\mathsf{AP}} \mathbf{h}_{k_u} \mathbf{h}_{k_u}^{\mathsf{H}}, \tag{8}$$

with $\mathbf{\Phi}_t = \alpha_{\mathsf{AP}} \mathbf{h}_{k_u} \mathbf{h}_{k_u}^{\mathsf{H}} + \beta_{\mathsf{AP}} \operatorname{diag}\left(\mathbf{h}_{k_u} \mathbf{h}_{k_u}^{\mathsf{H}}\right)$.

To address non-smooth problem, we first apply LogSumExp method in [3]. The nonsmoothness in (5) can be approximated as: $\min_{k_u \in \mathcal{K}} \{w_{k_u} R_{k_u}\} \approx -\tau \ln(\sum_{k_u \in \mathcal{K}} \exp(w_{k_u} R_{k_u} / -\tau))$.

To solve non-convex difficulty, we first apply the approximation to (5) and derive the first order optimality condition.

Lemma 1. *The first-order optimality condition of the reformulated problem* (5) *is satisfied if the following holds:*

$$\mathbf{B}_{\mathsf{KKT}}^{-1}(\bar{\mathbf{v}}) \mathbf{A}_{\mathsf{KKT}}(\bar{\mathbf{v}}) \bar{\mathbf{v}} = \lambda(\bar{\mathbf{v}}) \bar{\mathbf{v}}, \tag{9}$$

where

$$\mathbf{A}_{\mathsf{KKT}}(\bar{\mathbf{v}}) = \lambda_{\text{num}}(\bar{\mathbf{v}}) \times \sum_{k_u \in \mathcal{K}} \left(\frac{\mathbf{A}_{k_u}}{\bar{\mathbf{v}}^{\mathsf{H}} \mathbf{A}_{k_u} \bar{\mathbf{v}}} \tilde{R}_{k_u}(\bar{\mathbf{v}}) \right), \tag{10}$$

$$\mathbf{B}_{\mathsf{KKT}}(\bar{\mathbf{v}}) = \lambda_{\text{den}}(\bar{\mathbf{v}}) \times \sum_{k_u \in \mathcal{K}} \left(\frac{\mathbf{B}_{k_u}}{\bar{\mathbf{v}}^{\mathsf{H}} \mathbf{B}_{k_u} \bar{\mathbf{v}}} \tilde{R}_{k_u}(\bar{\mathbf{v}}) \right). \tag{11}$$

with $\tilde{R}_{k_u}(\bar{\mathbf{v}}) = \exp\left(-\frac{w_{k_u}}{\tau} \log_2\left(\frac{\bar{\mathbf{v}}^{\mathsf{H}} \mathbf{A}_{k_u} \bar{\mathbf{v}}}{\bar{\mathbf{v}}^{\mathsf{H}} \mathbf{B}_{k_u} \bar{\mathbf{v}}}\right)\right)$. $\lambda_{\text{num}}(\bar{\mathbf{v}})$ *and* $\lambda_{\text{den}}(\bar{\mathbf{v}})$ *can be any functions of* $\bar{\mathbf{v}}$*, which satisfy* $\lambda(\bar{\mathbf{v}}) = \lambda_{\text{num}}(\bar{\mathbf{v}}) / \lambda_{\text{den}}(\bar{\mathbf{v}})$.

Proof. Refer to Appendix of [4]. □

In Lemma 1, we indicate the first-order optimality condition in (9) and consider the condition as a nonlinear eigenvalue problem [5]. Consequently, $\lambda(\bar{\mathbf{v}})$ and $\bar{\mathbf{v}}$ represent an eigenvalue and a corresponding eigenvector of $\mathbf{B}_{\mathsf{KKT}}^{-1}(\bar{\mathbf{v}}) \mathbf{A}_{\mathsf{KKT}}(\bar{\mathbf{v}})$, respectively. In this regard, finding the principal eigenvector is equivalent to maximizing the objective function. In conclusion, we propose a precoding algorithm based generalized power iteration (GPI) to find the principal eigenvector $\bar{\mathbf{v}}^{\star}$: Refer to the algorithm in [4]. In the algorithm, we remove the roof condition imposed by $t_{\max}$.

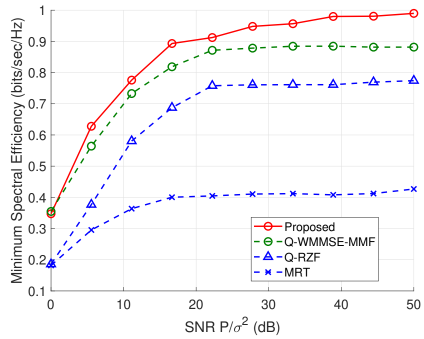

Fig. 1. The minimum SE versus signal-to-noise ratio (SNR) for $N_T = 8$ AP antennas, $K_U = 4$ users, and resolution bit is 2.

IV. Simulation Result

To evaluate our algorithm, we consider following benchmarks: 1) MMF weighted MMSE algorithm with quantization (Q-WMMSE-MMF) [6], 2) regularized zero forcing with quantization (Q-RZF), and 3) maximum ratio transmission (MRT). To model the spatial channel covariance matrix, we utilize the one-ring model [7]. We assume that all users have random weigt with $\sum_{i=1}^{K_U} w_i = K_U$, and we set $\epsilon = 0.01$, $\beta_{\mathsf{AP}} = 0.1175$, $\sigma^2 = 1$, and empirically tune τ.

Fig. 1 illustrates that our algorithm achieves the highest minimum SE values across different SNRs, demonstrating that it provides fair communication quality for all users compared to other benchmarks. This indicates that our algorithm effectively accounts for both MMF and low-resolution DACs.

V. Conclusion

In this paper, we proposed a MMF algorithm that considers low-resolution quantization with random user weights. To solve the associated optimization problem, we approximated the non-smooth problem by using LogSumExp method and derived the first-order optimality condition. Subsequently, the principal eigenvector was determined through the application of the GPI method. In simulations, we showed that our algorithm outperforms other benchmarks.

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