RESEARCH STATEMENT

SEUNG UK JANG

1. Introduction

My primary research interest is about complex manifolds and holomorphic automorphisms on them. To elaborate, here is some context for my research interests.

1.1. **Complex Manifolds.**

Most of my research interests center on the study of compact Kähler manifolds X with zero first Chern class $c_1(X) = 0$, following the ideas in Beauville's study [Bea83]. The benefit of such manifold follows from Yau's theorem on Calabi conjecture [Yau78]: such a manifold X has a Ricci-flat (Kähler) metric ω. Ricci-flat metrics allows us to control the volume form $\omega^{\dim X}$ *as a differential form*, which constrasts with general compact Kähler manifolds which only lets us control its cohomology.

Central to Beauville's study of compact Kähler manifolds X with $c_1(X) = 0$ is that, there exists a finite cover $\widetilde{X}$ of X which is given by a product of (1) complex tori, (2) some *irreducible holomorphic symplectic* (IHS) manifolds, and (3) some *Calabi–Yau* (CY) manifolds. Therefore, once one demands to study the dynamics on X, then a good starting point would be to study what happens on each factor.

Some initial studies were made for the surface case $\dim_{\mathbb{C}} X = 2$, which reduces to K3 surfaces. This is the case when IHS and CY coincides, and studies initiated by Cantat [Can01] and McMullen [McM02] suggest the similarity between holomorphic dynamics of K3 surfaces and diffeomorphisms of real 2-dimensional surfaces. Methods of this vein, together with tools specific for complex manifolds, hints us on the study of dynamics over higher dimensional IHS or CY manifolds.

1.2. **Green currents.**

In a number of studies by Sibony, Dinh, Thélin, Fornæss, Hubbard, and more authors, one of the key analytic methods for studying holomorphic dynamics is the *Green current*, as initiated by Bedford–Lyubich–Smillie [BLS93] and Cantat [Can01] for surfaces and generalized by Dinh and Sibony [DS10] for higher dimensions. To brief, Green currents are positive closed currents that, once pullbacked by the dynamics, gives a constant times that current.

But Green currents are more than just 'eigencurrents.' They interact with other dynamics-relevant tools like Pesin theory, complex geometry motivated by algebraic methods, and study of cohomologies. Some results of de Thélin and Dinh [DTD12] indicate the relation between Green currents and the stable and unstable leaves in Pesin theory. Not only that, Green currents take some special places within the cohomology, which allows us to incorporate some algebro-geometric viewpoint for their study.

1.3. **Tropicalization.** A *tropicalization* of a projective complex manifold is a trick to sketch a complex algebraic variety by logarithmically rescaling the absolute values of coordinates. Applying this, to an n-dimensional variety, we get a piecewise linear n-dimensional simplicial complex. Furthermore any algebraic dynamics induces a piecewise linear map on that real complex as well.

This helps us to visually understand holomorphic dynamics, as recently considered by Filip [Fil19], Spalding and Veselov [SV20], and more. (More traditional account of this may be seen with monomial maps, cf. [Can09, §2.8].) Tropicalized holomorphic maps often have some combinatorial nature, which leads to another interesting aspect of this viewpoint.

Akin to tropical picture of the holomorphic dynamics is the (Berkovich) *analytification* of algebraic maps on varieties defined over non-archimedean fields. One success, due to Favre and Rivera-Letelier [FRL04][FRL10], adapted the potential theory on complex dynamics in non-archimedean contexts to study canonical invariant measures of rational maps in one variable, suggesting potential interactions between non-archimedean dynamics and complex dynamics.

2. Kummer Rigidity

One way to assemble complex manifolds with a Ricci-flat metric and the technique of Green current gives the following conjecture on IHS manifolds, motivated by analogous results on surfaces by Cantat and Dupont [CD20] or Filip and Tosatti [FT21].

Conjecture 1. *Let X be an irreducible holomorphic symplectic manifold that admits a holomorphic automorphism $f\colon X \to X$ of positive entropy. Suppose f has a measure μ of maximal entropy in the volume class (i.e., $\mu \ll \mathrm{vol}_X$). Then X normalizes a finite quotient $\mathbb{T}/\Gamma$ of a complex torus and f lifts an affine-linear map $A\colon \mathbb{T}/\Gamma \to \mathbb{T}/\Gamma$.*

One partial result towards the conjecture is as follows.

Theorem 2 ([Jan21])**.** *Conjecture 1 holds if X is projective and μ equals to the volume measure vol_X.*

I am currently working on weakening the assumption $\mu = \mathrm{vol}_X$ to $\mu \ll \mathrm{vol}_X$.

2.1. **Background.** The question is motivated by the study of dynamical systems whose measure of maximal entropy (m.m.e.) is in the volume class. That is, we think of compact manifolds X and a diffeomorphism $f\colon X \to X$ that preserves a measure μ of the form $\mu = \rho(x)\, d\mathrm{vol}_X(x)$ and the entropy (complexity of iterating f) detected by μ equals to the topological entropy of f.

Usually they come from local homogeneous structures: for instance, a result of Besson, Courtois and Gallot [BCG95] for the geodesic flow of negatively curved manifold (of dimension ≥ 3) suggests this. They studied extremal problems for the 'entropy of metrics' and conlcuded that locally symmetric metrics are having a role on that, as motivated by a conjecture of Katok [Kat88]. But if it comes to complex dynamics, this usually means that the automorphism comes from a torus; torus is a typical way to keep the homogeneous structure, complex structure, and *compactness* all at once.

One of the first results of this sort are results of Zdunik [Zdu90], and Berteloot and Dupont [BD05], where they studied rational maps $f\colon \mathbb{P}^k \to \mathbb{P}^k$ whose m.m.e. is absolutely continuous to the volume, to conclude that the map is a (dynamical) factor of an affine-linear map $\mathbb{C}^k \to \mathbb{C}^k$.

Later, Cantat and Dupont [CD20], and Filip and Tosatti [FT21] focus on surface automorphisms $f\colon X \to X$, for X either a projective or K3 surface and whose m.m.e. is absolutely continuous to the volume. Their conclusions were that $f\colon X \to X$ is a Kummer example. That is, X is the K3 surface obtained by normalizing the quotient $\mathbb{T}^2/\{\pm 1\}$ of 2D complex torus $\mathbb{T}^2$, and f lifts an affine-linear map on the quotient $\mathbb{T}^2/\{\pm 1\}$.

The method of Green currents was utilized in these works [CD20][FT21] on surfaces. By the hyperbolic action, induced by the dynamics, on the 2nd cohomology group $H^2(X)$ one obtains two eigenclasses expanded and contracted by the dynamics, represented by unstable and stable Green currents respectively. It turned out that they are volumes along unstable and stable leaves (respectively), but also metrics on there.

2.2. **The Irreducible Holomorphic Symplectic Case.** The method of Green currents for surfaces is easily brought to the irreducible holomorphic symplectic (IHS) manifolds, i.e., compact Kähler simply connected manifolds that carry a nondegenerate holomorphic 2-form. Study of their cohomology rings [Ver95] and their dynamical nature [Ogu09] readily tells the existence of Green currents, of bidegree $(1, 1)$. This leads us to guess that the arguments on surfaces may hold in the same way for IHS manifolds, giving Conjecture 1.

The existence of $(1,1)$-Green currents on IHS manifolds gives a much better information than what holomorphic automorphisms on compact Kähler manifolds generally give. This contrasts with previous studies of Dinh and Sibony [DS10] or de Thélin and Dinh [DTD12], where they needed to start with Green currents with higher degrees.

However, there are some parts where the arguments on surfaces do not generalize well on IHS manifolds. The difficulties are sketched below.

2.2.1. *Singularities.* For surfaces, the studies of singularities and maps contracting them are extensively studied, and nowadays became a known construction for surfaces, projective or not. The analogous machinary is not trivial for the IHS case; for *status quo* this requires some (complex) algebraic geometry to establish.

Assume projectivity. For a projective IHS, the sum S of Green currents represent a big and nef class, but not a Kähler class. As a way to remedy this, one can think of a rational map $\Phi\colon X \dashrightarrow Y \subset \mathbb{P}^N$ onto a normal variety that contracts the *null locus* of S, the union of all varieties where S fails to be Kähler [BCL14]. This map is guaranteed to be an isomorphism outside of the null locus of S.

Unlike the surface case, this rational map is not suitable to pushforward S to a Kähler metric on Y, nor to induce a holomorphic automorphism on Y from $f\colon X \to X$. This is mostly because it is not certain whether the null locus of S has a reasonable image in Y, of codimension ≥ 2.

One can still get around this, however, because there is an indirect construction [CT15] on X that gives a Ricci-flat metric outside the null locus of S, cohomologously related with S. So instead of constructing a concrete variety that makes S Kähler, we rather carry a metric on an open subset of X that simulates the role.

2.2.2. *Metric-ness.* According to de Thélin and Dinh [DTD12], if (un)stable leaves have dimension s and if we have a (s,s)-Green current, the current restricts to a volume on each (un)stable leaf. Analogously, it would be reasonable to think that a $(1,1)$-Green current behaves like a metric along (un)stable leaves.

This guess is not as obvious as it seems, as (un)stable Green current along a (un)stable leaf is not guaranteed to be absolutely continuous with respect to a Euclidean metric on the leaf. Some careful study on the singular part (rel the Euclidean metric) is thus required to address this concern.

However, when m.m.e. is *equal* to the volume, then one has a more direct study of the Ricci-flat metric outside the null locus of the sum of Green currents. By some cohomology calculus and Jensen's inequality, I was able to establish some *metrics* on stable and unstable leaves that dialates along the dynamics, and established the result in [Jan21].

2.2.3. *Uniform Hyperbolicity.* One good consequence of the assumptions of Conjecture 1 is that the Lyapunov spectrum of the system has only 2 values of opposites signs. This does not readily give that the system is Anosov (i.e., uniformly hyperbolic), but decently close to that. This virtually tells that the dynamics on stable or unstable leaves are not only linear [JV02][KS17] but also just scalings followed by a unitary transformation.

The proposed technique is Zimmer's amenable reductions [Zim84], which requires invariant metrics to be never singular along any curve on the base manifold. But because the descriptions of the stable or unstable metrics are based on some noncommutative data, it is hard to cancel out the dynamical effects on these data. This was not an issue for the surface cases [CD20][FT21] where the commutativity allowed one to cancel out the dynamical effects.

2.2.4. *Assembly and Prospectives.* Once all three points above are dealt, the matter is nothing but showing that the contraction map $\Phi\colon X \dashrightarrow Y$ forces the normal variety Y to be flat along the regular locus, therefore showing that Y is a torus quotient [GKP16]. This flatness makes use of a flatness result of Benoist, Foulon and Labourie [BFL92] and gives a shorter account of flatness than [FT21].

My prospectives of improving [Jan21], that assumes projectivity and m.m.e. equal to the volume, to the more general Conjecture 1 are that one can drop the assumptions of [Jan21] and still get through the hurdles mentioned above. For instance, weakening m.m.e. to be just in the volume class will be done based on a more careful analysis of the behavior of Green currents along stable or unstable leaves. Weakening the projectivity assumption seems to require better complex geometric results on contracting a given IHS manifold to a torus quotient.

These improvements will let us know that having metrics (Green $(1,1)$-currents) and volume (the power of Green currents) distinguished is not loosing much from the surface case; everything generalizes well for IHS manifolds.

2.3. **The Calabi–Yau Case.** The complex tori and IHS manifolds in Beauville's factors [Bea83] are well-known, in terms of their holomorphic dynamics. The other factor, Calabi–Yau manifold, is yet relatively obscure in that perspective.

The Gromov–Yomdin theorem on compact Kähler manifolds [Gro03][Yom87] lets us to study cohomology actions first when we want to know about holomorphic dynamics on compact Kähler manifolds. However, it is still worthwhile to think on, for the Calabi–Yau case, what will happen about machinaries studied for the IHS case. Examples are Lyapunov spectra, Green currents, and their behaviors along stable or unstable leaves.

The first question to raise is whether we have a good example to start with, stated as follows.

Problem 3. Let X be a smooth 3-fold in $\mathbb{P}^1 \times \mathbb{P}^1 \times \mathbb{P}^1 \times \mathbb{P}^1$ with quad-degree $(2,2,2,2)$. (This is an example of a Calabi–Yau 3-fold.) Do we have an explicit birational automorphism on X whose (1) Lyapunov spectra, (2) Green currents, and (3) subresonant polynomials along stable and unstable leaves can be computed?

Proposed methodologies to tackle this are not restricted to classical complex geometry. I am looking forward to see how tropicalizations can be used to study this. For instance, I can think of a family of 3-folds in $\mathbb{P}^1 \times \mathbb{P}^1 \times \mathbb{P}^1 \times \mathbb{P}^1$,

$$X_A \colon x^2 + y^2 + z^2 + w^2 = xyzw + A,$$

where $A \in \mathbb{C} \setminus \{0, \pm 4\}$ is a parameter. Then tropicalizing this will give us a 3-dimensional real (infinite) complex together with four Vieta involutions $\sigma_x, \sigma_y, \sigma_z, \sigma_w$ acting on it. We can then start with studying the group action by $\Gamma = \langle \sigma_x, \sigma_y, \sigma_z, \sigma_w \rangle$, in the tropicalization of X_A, say from finding where are the fixed points of the involutions $\sigma_\bullet$.

This direction of study was motivated from an example study (Theorem 5) below, which gave me a success that degenerated piecewise-linear dynamics does analyze to an interesting picture and does help us to understand what is going on with the original holomorphic dynamics. If the family X_A recovers some known 3-dimensional real dynamics, say reflections on the 3-dimensional hyperbolic space $\mathbb{H}^3$, then the study of such reflection group will hint us about—or even directly compute—the dynamical data mentioned in the above Problem 3.

2.4. **The Non-archimedean Case.** There are some recent studies for dynamics on analytification of (low-dimensional) affine spaces over a valuated field, using methods that comes from complex dynamics, as described in the note [Jon15]. Motivated from there, one may raise the following questions regarding general polynomial maps $\mathbb{A}^2_{\mathrm{Berk}} \to \mathbb{A}^2_{\mathrm{Berk}}$ over a general valuated field.

Problem 4.
(1) How far does the Bedford–Lyubich–Smillie theory [BLS93] for polynomial maps $\mathbb{C}^2 \to \mathbb{C}^2$ generalize to polynomial maps $\mathbb{A}^2_{\mathrm{Berk}} \to \mathbb{A}^2_{\mathrm{Berk}}$?
(2) How can one understand the polynomial actions $\mathbb{A}^n_{\mathrm{Berk}} \to \mathbb{A}^n_{\mathrm{Berk}}$, in the light of Payne's tropicalization theorem [Pay09]?

To sketch how (1) and (2) are related, I find Payne's tropicalization theorem [Pay09], relevant:

$$\mathbb{A}^2_{\mathrm{Berk}} = \varprojlim_{\iota} \mathbf{Trop}(\mathbb{A}^2, \iota),$$

where $\iota\colon \mathbb{A}^2 \to \mathbb{A}^k$, $(x,y) \mapsto (f_1,\dots,f_k)$ are embeddings of the affine plane, directed by equivariant morphisms. By an invertible polynomial map $p\colon \mathbb{A}^2 \to \mathbb{A}^2$, one can then view the induced map $p^{an}\colon \mathbb{A}^2_{\text{Berk}} \to \mathbb{A}^2_{\text{Berk}}$ as a compatible family of maps $p^{an}_\iota\colon \mathbf{Trop}(\mathbb{A}^2,\iota) \to \mathbf{Trop}(\mathbb{A}^2,\iota\circ p)$ that builds up the limit.

I expect that one can initiate the study of Green currents from a cofinal subfamily of $(p^{an}_\iota)_\iota$, namely those embeddings whose coordinate functions $f_i \in \mathcal{O}(\mathbb{A}^2)$ are irreducible polynomials. In that case, because $p\colon \mathbb{A}^2 \to \mathbb{A}^2$ permutes irreducible polynomials, the maps p^{an}_ι behave like permuting the coordinates. From there I can study on where do the Green currents (or potentials) concentrate on, how can I picture the maps p^{an}_ι, and more.

Another note is that there is a connection between non-archimedean rational dynamics to a classical math algorithm, the *Babylonian method* of finding square roots. This method iterates the rational map $T(x) = \frac{1}{2}(x+\frac{a}{x})$ to obtain a convergent sequence to $\pm\sqrt{a}$. The rational map T moreover induces a rational map $T\colon \mathbb{P}^1_{\text{Berk}} \to \mathbb{P}^1_{\text{Berk}}$ on the analytification level, and objects like Green measures of T [FRL10] does have good implication on the behavior of the Babylonian method. Thus the study is not restricted to a pure complex dynamical interest, but rather can be widen to give an invitation towards general math enthusiasts.

3. Tropical Dynamics

There are more projects that I view that tropical methods are useful in their approaches. This will not restrict myself to study the dynamics of a single holomorphic map, but can open up the study of holomorphic group actions on a complex manifold. Indeed, tropical viewpoints let us to study such group actions in reference to some known geometric group actions on real manifolds or simplicial complexes.

3.1. **Tropical Triangle Group Action.** A recent work of Rebelo and Roeder [RR21] exhibits an interest to cubic Markov surfaces

$$S_{ABCD}\colon X^2+Y^2+Z^2+XYZ = AX+BY+CZ+D,$$

together with the group action generated by Vieta involutions $\sigma_X,\sigma_Y,\sigma_Z$, i.e., the action $\Gamma_{ABCD} = \langle \sigma_X,\sigma_Y,\sigma_Z\rangle \curvearrowright S_{ABCD}$. By that they were able to ask about the nature of this group action, say about the Julia set of the group action.

A work in progress of mine illuminates their interest in the view of the tropicalization, which potentially has an implication on the complex algebraic dynamics.

Theorem 5. *Let*

$$S_{ABCD}\colon X^2+Y^2+Z^2+XYZ = AX+BY+CZ+D$$

be a family of cubic Markov surfaces. Consider the Vieta involutions $\sigma_X, \sigma_Y, \sigma_Z$ on each variable. Then there is an open set U of parameters A, B, C, D such that, whenever $(A, B, C, D) \in U$, the tropical action of the group $\langle \sigma_X, \sigma_Y, \sigma_Z \rangle$ on the tropicalization of S_{ABCD} is conjugate to the (∞, ∞, ∞)-triangle reflection action $\Gamma \curvearrowright \mathbb{H}^2$ on the hyperbolic plane.

For instance, this gives clearer ping-pong proof that the Vieta involutions on the complex manifold $S_{ABCD}(\mathbb{C})$ necessarily generates the free product $\mathbb{Z}/2\mathbb{Z} * \mathbb{Z}/2\mathbb{Z} * \mathbb{Z}/2\mathbb{Z}$, when at least one of A, B, C, D is large enough. Furthermore, the result indicates that the locally discrete locus $\mathcal{D} \subset S_{ABCD}$ and the Fatou set $\mathcal{F} \subset S_{ABCD}$ as in [RR21] are nonempty, when at least one of A, B, C, D is large enough.

For the further studies of this vein, I note that methods used to prove Theorem 5 may be applied for other families of complex manifolds, as mentioned in §2.3 above.

3.2. **Minimal Exceptional Set.** Another problem may be stated, based on a question due to Serge Cantat.

Problem 6 (Tropical Problem of the Minimal Exceptional Set)**.** Let $\mathbb{TP}^2$ be the tropical projective plane, and let $\mathcal{F}$ be a 1-dimensional foliation with affine-linear local chart maps. Let L be a leaf of $\mathcal{F}$. Does the closure of L in $\mathbb{TP}^2$ necessarily contain a singularity of $\mathcal{F}$?

Because $\mathbb{TP}^2$ appears as a surface with boundaries, we see that this question may be reduced to a study of (piecewise-linear) foliations on real surfaces. Together with the dynamical foliations that may be built on $\mathbb{TP}^2$, perhaps those originating from invertible polynomial maps $p\colon \mathbb{A}^2_{\mathrm{Berk}} \to \mathbb{A}^2_{\mathrm{Berk}}$, I view this problem to be worked out with some concrete example to start.

References

[BCG95] G. Besson, G. Courtois, and S. Gallot, *Entropies et rigidités des espaces localement symétriques de courbure strictement négative*, Geom. Funct. Anal. **5** (1995), no. 5, 731–799. MR 1354289

[BCL14] Sébastien Boucksom, Salvatore Cacciola, and Angelo Felice Lopez, *Augmented base loci and restricted volumes on normal varieties*, Math. Z. **278** (2014), no. 3-4, 979–985. MR 3278900

[BD05] F. Berteloot and C. Dupont, *Une caractérisation des endomorphismes de Lattès par leur mesure de Green*, Comment. Math. Helv. **80** (2005), no. 2, 433–454. MR 2142250

[Bea83] Arnaud Beauville, *Variétés Kähleriennes dont la première classe de Chern est nulle*, J. Differential Geom. **18** (1983), no. 4, 755–782 (1984). MR 730926

[BFL92] Yves Benoist, Patrick Foulon, and François Labourie, *Flots d'Anosov à distributions stable et instable différentiables*, J. Amer. Math. Soc. **5** (1992), no. 1, 33–74. MR 1124979

[BLS93] Eric Bedford, Mikhail Lyubich, and John Smillie, *Polynomial diffeomorphisms of* $\mathbf{C}^2$*. IV. The measure of maximal entropy and laminar currents*, Invent. Math. **112** (1993), no. 1, 77–125. MR 1207478

[Can01] Serge Cantat, *Dynamique des automorphismes des surfaces* $K3$, Acta Math. **187** (2001), no. 1, 1–57. MR 1864630

[Can09] ———, *Bers and Hénon, Painlevé and Schrödinger*, Duke Math. J. **149** (2009), no. 3, 411–460. MR 2553877

[CD20] Serge Cantat and Christophe Dupont, *Automorphisms of surfaces: Kummer rigidity and measure of maximal entropy*, J. Eur. Math. Soc. (JEMS) **22** (2020), no. 4, 1289–1351. MR 4071328

[CT15] Tristan C. Collins and Valentino Tosatti, *Kähler currents and null loci*, Invent. Math. **202** (2015), no. 3, 1167–1198. MR 3425388

[DS10] Tien-Cuong Dinh and Nessim Sibony, *Super-potentials for currents on compact Kähler manifolds and dynamics of automorphisms*, J. Algebraic Geom. **19** (2010), no. 3, 473–529. MR 2629598

[DTD12] Henry De Thélin and Tien-Cuong Dinh, *Dynamics of automorphisms on compact Kähler manifolds*, Adv. Math. **229** (2012), no. 5, 2640–2655. MR 2889139

[Fil19] Simion Filip, *Tropical dynamics of area-preserving maps*, J. Mod. Dyn. **14** (2019), 179–226. MR 3959360

[FRL04] Charles Favre and Juan Rivera-Letelier, *Théorème d'équidistribution de Brolin en dynamique p-adique*, C. R. Math. Acad. Sci. Paris **339** (2004), no. 4, 271–276. MR 2092012

[FRL10] ———, *Théorie ergodique des fractions rationnelles sur un corps ultramétrique*, Proc. Lond. Math. Soc. (3) **100** (2010), no. 1, 116–154. MR 2578470

[FT21] Simion Filip and Valentino Tosatti, *Kummer rigidity for K3 surface automorphisms via Ricci-flat metrics*, Amer. J. Math. **143** (2021), no. 5, 1431–1462. MR 4334400

[GKP16] Daniel Greb, Stefan Kebekus, and Thomas Peternell, *Étale fundamental groups of Kawamata log terminal spaces, flat sheaves, and quotients of abelian varieties*, Duke Math. J. **165** (2016), no. 10, 1965–2004. MR 3522654

[Gro03] Mikhaïl Gromov, *On the entropy of holomorphic maps*, Enseign. Math. (2) **49** (2003), no. 3-4, 217–235. MR 2026895

[Jan21] Seung uk Jang, *Kummer rigidity for hyperkähler automorphisms*, 2021.

[Jon15] Mattias Jonsson, *Dynamics of Berkovich spaces in low dimensions*, Berkovich spaces and applications, Lecture Notes in Math., vol. 2119, Springer, Cham, 2015, pp. 205–366. MR 3330767

[JV02] Mattias Jonsson and Dror Varolin, *Stable manifolds of holomorphic diffeomorphisms*, Invent. Math. **149** (2002), no. 2, 409–430. MR 1918677

[Kat88] Anatole Katok, *Four applications of conformal equivalence to geometry and dynamics*, Ergodic Theory Dynam. Systems **8*** (1988), no. Charles Conley Memorial Issue, 139–152. MR 967635

[KS17] Boris Kalinin and Victoria Sadovskaya, *Normal forms for non-uniform contractions*, J. Mod. Dyn. **11** (2017), 341–368. MR 3642250

[McM02] Curtis T. McMullen, *Dynamics on* $K3$ *surfaces: Salem numbers and Siegel disks*, J. Reine Angew. Math. **545** (2002), 201–233. MR 1896103

[Ogu09] Keiji Oguiso, *A remark on dynamical degrees of automorphisms of hyperkähler manifolds*, manuscripta mathematica **130** (2009), no. 1, 101–111.

[Pay09] Sam Payne, *Analytification is the limit of all tropicalizations*, Math. Res. Lett. **16** (2009), no. 3, 543–556. MR 2511632

[RR21] Julio Rebelo and Roland Roeder, *Dynamics of groups of automorphisms of character varieties and fatou/julia decomposition for painlevé 6*, 2021.

[SV20] K. Spalding and A. P. Veselov, *Tropical Markov dynamics and Cayley cubic*, Integrable systems and algebraic geometry. Vol. 1, London Math. Soc. Lecture Note Ser., vol. 458, Cambridge Univ. Press, Cambridge, 2020, pp. 383–394. MR 4421422

[Ver95] Mikhail Sergeevic Verbitsky, *Cohomology of compact hyperkaehler manifolds*, ProQuest LLC, Ann Arbor, MI, 1995, Thesis (Ph.D.)–Harvard University. MR 2692913

[Yau78] Shing Tung Yau, *On the Ricci curvature of a compact Kähler manifold and the complex Monge-Ampère equation. I*, Comm. Pure Appl. Math. **31** (1978), no. 3, 339–411. MR 480350

[Yom87] Y. Yomdin, *Volume growth and entropy*, Israel J. Math. **57** (1987), no. 3, 285–300. MR 889979

[Zdu90] Anna Zdunik, *Parabolic orbifolds and the dimension of the maximal measure for rational maps*, Invent. Math. **99** (1990), no. 3, 627–649. MR 1032883

[Zim84] Robert J. Zimmer, *Ergodic theory and semisimple groups*, Monographs in Mathematics, vol. 81, Birkhäuser Verlag, Basel, 1984. MR 776417