Computer Graphics HW 7

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1. Show the derivation process for obtaining the parallel projection matrix and perspective projection matrix.

- Parallel projection matrix, M_{ortho}

$$M_{ortho} = R(1,1,-1) \cdot S\left(\frac{2}{r-l}, \frac{2}{t-b}, \frac{2}{f-n}\right) \cdot T\left(-\frac{l+r}{2}, -\frac{b+t}{2}, \frac{n+f}{2}\right)$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \frac{2}{r-l} & 0 & 0 & 0 \\ 0 & \frac{2}{t-b} & 0 & 0 \\ 0 & 0 & \frac{2}{f-n} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & -\frac{l+r}{2} \\ 0 & 1 & 0 & -\frac{b+t}{2} \\ 0 & 0 & 1 & \frac{n+f}{2} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \frac{2}{r-l} & 0 & 0 & -\frac{r+1}{r-l} \\ 0 & \frac{2}{t-b} & 0 & -\frac{t+b}{t-b} \\ 0 & 0 & \frac{2}{f-n} & \frac{f+n}{f-n} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{2}{r-l} & 0 & 0 & -\frac{r+1}{r-l} \\ 0 & \frac{2}{t-b} & 0 & -\frac{t+b}{t-b} \\ 0 & 0 & -\frac{2}{f-n} & -\frac{f+n}{f-n} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- Perspective projection matrix, $M_{pers}M_s$

$$M_{pers}M_S = \begin{bmatrix} \frac{2n}{r-l} & 0 & 0 & 0 \\ 0 & \frac{2n}{t-b} & 0 & 0 \\ 0 & 0 & -\frac{f+n}{f-n} & -\frac{2nf}{f-n} \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & \frac{r+l}{2n} & 0 \\ 0 & 1 & \frac{t+b}{2n} & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{2n}{r-l} & 0 & \frac{r+l}{r-l} & 0 \\ 0 & \frac{2n}{t-b} & \frac{t+b}{t-b} & 0 \\ 0 & 0 & -\frac{f+n}{f-n} & -\frac{2nf}{f-n} \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

If the object is symmetric, or in other words, $r = -l$ and $t = -b$, then the perspective matrix will be

$$M_{pers}M_S = \begin{bmatrix} \frac{2n}{r-l} & 0 & 0 & 0 \\ 0 & \frac{2n}{t-b} & 0 & 0 \\ 0 & 0 & -\frac{f+n}{f-n} & -\frac{2nf}{f-n} \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

2. Show how the viewing transformation matrix $M_{view} = \begin{bmatrix} {}^{E}R_W & {}^{E}R_W(-{}^{W}\mathbf{p}_{eye}) \\ \mathbf{0}^{\mathrm{T}} & 1 \end{bmatrix}$ is equivalent to M_{view}^{glm} of glm::lookAtRH(eye, center, up) using the relations $s \equiv U, u \equiv V,\ f \equiv -N$.
   - glm::lookAtRH() takes three arguments, and each represents the location of camera, the target camera watches, and the upvector on a right-hand coordinate system.
   - On a right-hand coordinate system, the unit-vector for x, y and z-axis are U, V, and N respectively.
   - f is the unit vector of depth between the camera, or eye, and the target object. Since the object locates on the negative side of z-axis on a right-hand coordinate

system, $f \equiv -N$.

- s is the unit vector that is orthogonal to both f and the upvector, which means s is the cross product of f and the upvector. Let's assume the upvector as V', which represents a vector on y-axis. Since $f \times upvector \equiv -N \times V' = U,\ s \equiv U$
- u is the unit vector that is orthogonal to both s and f. Because $s \times f \equiv U \times -N = V,\ u \equiv V$ can be shown, and therefore, s, u, and f are unit vectors and orthogonal to each other.
- Through these relations, M_{view} is equivalent to M_{view}^{glm} in that

$$
{}^{E}R_{W} = [U \quad V \quad N]^{\mathrm{T}} = \begin{bmatrix} U_x & U_y & U_z \\ V_x & V_y & V_z \\ N_x & N_y & N_z \end{bmatrix} \equiv \begin{bmatrix} s_x & s_y & s_z \\ u_x & u_y & u_z \\ -f_x & -f_y & -f_z \end{bmatrix} = \begin{bmatrix} s^T \\ u^T \\ -f^T \end{bmatrix} \text{ and}
$$

$$
{}^{E}R_{W}(-{}^{W}\mathbf{p}_{eye}) = \begin{bmatrix} U_x & U_y & U_z \\ V_x & V_y & V_z \\ N_x & N_y & N_z \end{bmatrix}(-{}^{W}\mathbf{p}_{eye}) \equiv \begin{bmatrix} s_x & s_y & s_z \\ u_x & u_y & u_z \\ -f_x & -f_y & -f_z \end{bmatrix}(-{}^{W}\mathbf{p}_{eye})
$$

$$
= \begin{bmatrix} s^T \\ u^T \\ -f^T \end{bmatrix}(-{}^{W}\mathbf{p}_{eye}) = \begin{bmatrix} -s^T \cdot {}^{W}\mathbf{p}_{eye} \\ -u^T \cdot {}^{W}\mathbf{p}_{eye} \\ f^T \cdot {}^{W}\mathbf{p}_{eye} \end{bmatrix}
$$

since U, V and N are 3-dimensional column vectors.