A New LMI-Based Approach to Relaxed Quadratic Stabilization of T–S Fuzzy Control Systems

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***Abstract*—This paper proposes a new quadratic stabilization condition for Takagi–Sugeno (T–S) fuzzy control systems. The condition is represented in the form of linear matrix inequalities (LMIs) and is shown to be less conservative than some relaxed quadratic stabilization conditions published recently in the literature. A rigorous theoretic proof is given to show that the proposed condition can include previous results as special cases. In comparison with conventional conditions, the proposed condition is not only suitable for designing fuzzy state feedback controllers but also convenient for fuzzy static output feedback controller design. The latter design work is quite hard for T–S fuzzy control systems. Based on the LMI-based conditions derived, one can easily synthesize controllers for stabilizing T–S fuzzy control systems. Since only a set of LMIs is involved, the controller design is quite simple and numerically tractable. Finally, the validity and applicability of the proposed approach are successfully demonstrated in the control of a continuous-time nonlinear system.**

***Index Terms*—Fuzzy feedback controller, linear matrix inequality, relaxed quadratic stability, stabilization, Takagi–Sugeno (T–S) fuzzy systems.**

I. Introduction

FUZZY control technique represents a means of collecting human knowledge and expertise. It has been applied to various industrial fields [12], [17]. Although the method has been practically successful, it has proved extremely difficult to develop a general analysis and design theory for conventional fuzzy control systems. Recently, based on Takagi–Sugeno (T–S) fuzzy model [13], [14], there have appeared in the literature a great number of results concerning stability analysis and design [1]–[3], [10], [14]–[17].

Reference [10] proposed a so-called relaxed stabilization condition and then applied the condition to design fuzzy controllers for T–S fuzzy systems. In [4], an interesting quadratic stabilization condition was reported to release the conservatism of the condition of [10] by collecting the interactions in a single matrix. Very recently, a more relaxed stabilization condition was proposed in [8]. It admits more freedom in guaranteeing the stability of T–S fuzzy control systems than the one of [4].

In this paper, a new LMI-based stabilization condition is obtained by relaxing the results in [4], [8], and [10]. A rigorous proof is given to show that the stabilization condition can include the interesting results published recently in [4], [8], and [10] as special cases. By a numerical example, one can see that the new stabilization condition significantly relaxes the conservativeness of existing ones. The present condition is also very useful for fuzzy controller design. Once the condition is feasible, a state feedback controller for stabilizing T–S fuzzy systems can be easily obtained. Unlike conventional approaches [4], [8], [10] which are mainly suitable for designing fuzzy state feedback controllers, the approach used in this paper can be also applied to solve the fuzzy static output feedback stabilization problems. In practical, fuzzy static output feedback control is very useful and more realistic since it can be easily implemented with low cost. However, the problem is essentially hard for T–S fuzzy systems and, thus, rarely studied in the literature [5], [6]. By the proposed approach, a fuzzy static output feedback controller for stabilizing T–S fuzzy systems can be synthesized with ease.

The paper is organized as follows: Some results published recently are briefly reviewed in Section II for a clear overview of related research and further comparison with our contributions. Section III contains main results of the paper, in which both continuous-time and discrete-time cases are investigated. The comparison of relaxation between main results and existing ones mentioned in Section II is also made in this section. Section IV presents a numerical example which illustrates the validity and applicability of the proposed approach in the control of a continuous-time nonlinear system. The conclusion is located in Section V.

In this paper, "continuous-time fuzzy systems" and "discrete-time fuzzy systems" are abbreviated as CFS and DFS, respectively. Other notations used are fair standard. For example, $X < 0$ (or $X \leq 0$) means the matrix X is symmetric and negative definite (or symmetric and negative semidefinite). X^T denotes the transpose of X. The symbol I (or I_m) represents the identity matrix with appropriate dimension (or dimension $m \times m$).

II. Preliminaries

A. T–S Fuzzy Control Systems

T–S fuzzy control system [13], [14] is one of the most popular and promising research platforms in the model-based fuzzy control. A general class of nonlinear systems can be represented by the T–S fuzzy model which is described by a set of fuzzy IF–THEN rules. The ith rule of the T–S fuzzy model has the following form:

$$\begin{aligned}&\text{Plant Rule } i: \text{If } z_1(t) \text{ is } M_{i1} \text{ and} \ldots \text{and } z_s(t) \text{ is } M_{is}\\ &\qquad \text{Then } \nabla x(t) = A_i x(t) + B_i u(t)\end{aligned} \tag{1}$$

Manuscript received June 17, 2003; revised March 20, 2005 and September 12, 2005. This work was supported by National Science Council of Taiwan, R.O.C., under Grant NSC-93-2213-E-151-006.

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Digital Object Identifier 10.1109/TFUZZ.2006.876331

where ∇ represents an operator. For CFS cases $\nabla x(t)$ means $\dot{x}(t)$ and for DFS cases $\nabla x(t)$ represents $x(t+1)$. In (1), M_{ij} $(i=1,2,\ldots,r,\ j=1,2,\ldots,s)$ is the fuzzy set and r is the number of IF–THEN rules. $z_i(t)$, $i=1,2,\ldots,s$ are the premise variables. $x \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}^m$ is the input vector. Assume $A_i \in \mathbb{R}^{n\times n}$ and $B_i \in \mathbb{R}^{n\times m}$. Given a pair of $(x(t), u(t))$, the final output of the fuzzy system is inferred as follows:

$$\nabla x(t) = \sum_{i=1}^{r} h_i\left(z(t)\right)\left(A_i x(t) + B_i u(t)\right) \tag{2}$$

where $h_i(z(t)) = (w_i(z(t))/\sum_{i=1}^{r} w_i(z(t)))$, $w_i(z(t)) = \prod_{j=1}^{s} M_{ij}(z_j(t))$, $M_{ij}(z_j(t))$ is the grade of membership of $z_j(t)$ in M_{ij}, and $w_i(z(t))$ represents the weight of the ith rule. It is easy to check that $h_i(z(t)) \geq 0$, $i=1,2,\ldots,r$, and $\sum_{i=1}^{r} h_i(z(t)) = 1$. Let the controller be

Controller Rule i : If $z_1(t)$ is M_{i1} and … and $z_s(t)$ is M_{is}

$$\text{Then } u(t) = -K_i x(t) \tag{3}$$

where $i=1,2,\ldots,r$. The designed fuzzy controller shares the same fuzzy sets in the premise parts with the plant and has local linear controllers in the consequent parts. The output of the fuzzy state feedback controller is given by

$$u(t) = -\sum_{i=1}^{r} h_i\left(z(t)\right) K_i x(t). \tag{4}$$

By substituting (4) into (2), the closed-loop fuzzy system can be represented as

$$\nabla x(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i\left(z(t)\right) h_j\left(z(t)\right)\left(A_i - B_i K_j\right)x(t). \tag{5}$$

Definition 1 [12]: The fuzzy system (2) is said to be *quadratically stabilizable* if there exists a controller as in (4) such that the closed-loop system (5) is quadratically stable.

B. Basic Stabilization Conditions

In this subsection, two basic stabilization conditions for CFS and DFS are reviewed in Theorems 1 and 2, respectively. In the subsequent discussion, the origin $x=0$ is assumed to be the only equilibrium point of the fuzzy control system.

Theorem 1 [4]: The equilibrium of the *CFS* of (2) is quadratically stabilizable via the fuzzy controller (4) if there exist matrices $Q>0$, N_i, $i=1,2,\ldots,r$, and Z_{ij}, $i,j=1,2,\ldots,r$ such that

$$QA_i^T + A_iQ - N_i^TB_i^T - B_iN_i < Z_{ii}, \qquad i=1,2,\ldots,r \tag{6}$$

$$\begin{aligned} &QA_i^T + A_iQ + QA_j^T + A_jQ - N_j^TB_i^T - B_iN_j \\ &\quad - N_i^TB_j^T - B_jN_i \leq 2Z_{ij}, \\ &\qquad i=1,2,\ldots,r-1, \quad j=i+1,\ldots,r \end{aligned} \tag{7}$$

$$\begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1r} \\ Z_{12} & Z_{22} & \cdots & Z_{2r} \\ \vdots & \vdots & \ddots & \vdots \\ Z_{1r} & Z_{2r} & \cdots & Z_{rr} \end{bmatrix} < 0. \tag{8}$$

Moreover, in this case, the fuzzy local feedback gains are $K_i = N_iQ^{-1}$, $i=1,2,\ldots,r$.

Theorem 2 [4]: The equilibrium of the DFS of (2) is quadratically stabilizable via the fuzzy controller (4) if there exist matrices $Q>0$, N_i, $i=1,2,\ldots,r$, and Z_{ij}, $i,j=1,2,\ldots,r$ such that (9)–(11), as shown at the bottom of the page, hold. Moreover, in this case, the fuzzy local feedback gains are $K_i = N_iQ^{-1}$, $i=1,2,\ldots,r$.

It has been shown in [4] that Theorems 1 and 2 are more relaxed than the famous relaxed stabilization conditions proposed in [10] and [11].

C. Relaxed Stabilization Conditions

The basic stabilization conditions stated in the Section II-B may be conservative since all Z_{ij}, $i=1,2,\ldots,r-1$, $j=i+1,\ldots,r$ are required to be symmetric. To release the conservatism, the following relaxed stabilization conditions were reported recently in [8], in which each Z_{ij} is not required to be symmetric.

Theorem 3 [8], [18]: The equilibrium of the *CFS* of (2) is quadratically stabilizable via the fuzzy controller (4) if there exist matrices $Q>0$, N_i, Z_{ii}, $i=1,2,\ldots,r$ and $Z_{ji}=Z_{ij}^T$, $i=1,2,\ldots,r-1$, $j=i+1,\ldots,r$ such that

$$QA_i^T + A_iQ - N_i^TB_i^T - B_iN_i < Z_{ii},\ i=1,2,\ldots,r \tag{12}$$

$$\begin{aligned} &QA_i^T + A_iQ + QA_j^T + A_jQ - N_j^TB_i^T - B_iN_j - N_i^TB_j^T \\ &\quad - B_jN_i \leq Z_{ij} + Z_{ij}^T, \\ &\qquad i=1,2,\ldots,r-1, \quad j=i+1,\ldots,r \end{aligned} \tag{13}$$

$$\begin{bmatrix} -Q - Z_{ii} & QA_i^T - N_i^TB_i^T \\ A_iQ - B_iN_i & -Q \end{bmatrix} < 0, \quad i=1,2,\ldots,r \tag{9}$$

$$\begin{aligned} &\begin{bmatrix} -2Q - 2Z_{ij} & QA_i^T + QA_j^T - N_i^TB_j^T - N_j^TB_i^T \\ A_iQ + A_jQ - B_iN_j - B_jN_i & -2Q \end{bmatrix} \leq 0, \\ &i=1,2,\ldots,r-1, \quad j=i+1,\ldots,r \end{aligned} \tag{10}$$

$$\begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1r} \\ Z_{12} & Z_{22} & \cdots & Z_{2r} \\ \vdots & \vdots & \ddots & \vdots \\ Z_{1r} & Z_{2r} & \cdots & Z_{rr} \end{bmatrix} < 0 \tag{11}$$

$$\begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1r} \\ Z_{21} & Z_{22} & \cdots & Z_{2r} \\ \vdots & \vdots & \ddots & \vdots \\ Z_{r1} & Z_{r2} & \cdots & Z_{rr} \end{bmatrix} < 0. \tag{14}$$

Moreover, in this case, the fuzzy local feedback gains are $K_i = N_i Q^{-1}$, $i = 1, 2, \ldots, r$.

Theorem 4 [8], [18]: The equilibrium of the *DFS* of (2) is quadratically stabilizable via the fuzzy controller (4) if there exist matrices $Q > 0$, N_i, Z_{ii}, $i = 1, 2, \ldots, r$ and $Z_{ji} = Z_{ij}^T$, $i = 1, 2, \ldots, r-1$, $j = i+1, \ldots, r$ such that (15)–(17), as shown at the bottom of the page, hold. Moreover, in this case, the fuzzy local feedback gains are $K_i = N_i Q^{-1}$, $i = 1, 2, \ldots, r$.

Remark 1: When this paper was submitted, [18] was not yet available in the open literature. However, its conference paper [8] had been cited in our initial submission. For completing the references, the paper [18] was added in reference section when this paper was revised. ■

Obviously, if set $Z_{ij} = Z_{ij}^T$, Theorems 3 and 4 reduce to Theorems 1 and 2. Thus Theorems 1 and 2 are particular cases of Theorems 3 and 4, respectively. In next section, new conditions that are more relaxed than Theorems 3 and 4 will be proposed. Actually, Theorems 3 and 4 are shown to be particular cases of the proposed conditions.

III. Main Results

In this section, two kinds of stabilization conditions are presented. One is for fuzzy state feedback stabilization problem. The other one is for fuzzy static output feedback stabilization problem. The latter problem is essentially hard for T–S fuzzy models but very useful in practice. To the best of authors' knowledge, very little effort has been paid to this problem for T–S fuzzy control systems.

A. LMI-Based Stabilization Condition for Fuzzy State Feedback Control

In this subsection, the state feedback case is considered firstly. New stabilization conditions for CFS and DFS are derived and expressed in terms of a set of LMIs. These conditions are more relaxed than Theorems 1–4. It can be shown that Theorems 1–4 are our special cases.

Theorem 5: The equilibrium of the CFS of (2) is quadratically stabilizable via the fuzzy controller (4) if there exist matrices $Q > 0$; N_i, $i = 1, 2, \ldots, r$; Y_{iii}, $i = 1, 2, \ldots, r$; $Y_{jii} = Y_{iij}^T$ and Y_{iji}, $i = 1, 2, \ldots, r$, $j \neq i$, $j = 1, 2, \ldots, r$ and $Y_{ij\ell} = Y_{\ell ji}^T$, $Y_{i\ell j} = Y_{j\ell i}^T$, $Y_{ji\ell} = Y_{\ell ij}^T$, $i = 1, 2, \ldots, r-2$, $j = i+1, \ldots, r-1$, $\ell = j+1, \ldots, r$ satisfying the following LMIs:

$$QA_i^T + A_iQ - N_i^TB_i^T - B_iN_i < Y_{iii},\ i = 1, 2, \ldots, r \tag{18}$$

$$\begin{aligned} &2QA_i^T + QA_j^T + 2A_iQ + A_jQ - (N_i + N_j)^TB_i^T \\ &\quad - N_i^TB_j^T - B_i(N_i + N_j) - B_jN_i \leq Y_{iij} + Y_{iji} + Y_{iij}^T, \\ &\quad i = 1, 2, \ldots, r, \quad j \neq i, \quad j = 1, 2, \ldots, r \end{aligned} \tag{19}$$

$$\begin{aligned} &2Q(A_i + A_j + A_\ell)^T - (N_j + N_\ell)^TB_i^T - (N_i + N_\ell)^TB_j^T \\ &\quad - (N_i + N_j)^TB_\ell^T + 2(A_i + A_j + A_\ell)Q - B_i(N_j + N_\ell) \\ &\quad - B_j(N_i + N_\ell) - B_\ell(N_i + N_j) \leq Y_{ij\ell} + Y_{i\ell j} \\ &\quad + Y_{ji\ell} + Y_{ij\ell}^T + Y_{i\ell j}^T + Y_{ji\ell}^T, \quad i = 1, 2, \ldots, r-2, \\ &\quad j = i+1, \ldots, r-1, \quad \ell = j+1, \ldots, r \end{aligned} \tag{20}$$

$$\begin{bmatrix} Y_{1i1} & Y_{1i2} & \cdots & Y_{1ir} \\ Y_{2i1} & Y_{2i2} & \cdots & Y_{2ir} \\ \vdots & \vdots & \ddots & \vdots \\ Y_{ri1} & Y_{ri2} & \cdots & Y_{rir} \end{bmatrix} \leq 0, \qquad i = 1, 2, \ldots, r. \tag{21}$$

Moreover, in this case, the fuzzy local state feedback gains are $K_j = N_j Q^{-1}$, $j = 1, 2, \ldots, r$.

Proof: For simplicity, we will use h_i, h_j, and h_ℓ in place of $h_i(z(t))$, $h_j(z(t))$, and $h_\ell(z(t))$, respectively. Consider a candidate of quadratic function $V(x(t)) = x^T(t)Px(t)$. The equilibrium of CFS (5) is quadratically stable if

$$\begin{aligned} \dot{V}(x(t)) = x^T(t) &\left\{ \sum_{i=1}^r \sum_{j=1}^r h_ih_j(A_i - B_iK_j)^TP \right. \\ &\left. + P\sum_{i=1}^r \sum_{j=1}^r h_ih_j(A_i - B_iK_j) \right\} x(t) < 0 \quad \forall x(t) \neq 0. \end{aligned} \tag{22}$$

From (22), the equilibrium of CFS (5) is quadratically stable if

$$\sum_{i=1}^r \sum_{j=1}^r h_ih_j \left((A_i - B_iK_j)^TP + P(A_i - B_iK_j) \right) < 0. \tag{23}$$

$$\begin{bmatrix} -Q - Y_{ii} & QA_i^T - N_i^TB_i^T \\ A_iQ - B_iN_i & -Q \end{bmatrix} < 0, \quad i = 1, 2, \ldots, r \tag{15}$$

$$\begin{aligned} &\begin{bmatrix} -2Q - Z_{ij} - Z_{ij}^T & QA_i^T + QA_j^T - N_i^TB_j^T - N_j^TB_i^T \\ A_iQ + A_jQ - B_iN_j - B_jN_i & -2Q \end{bmatrix} \leq 0, \\ &i = 1, 2, \ldots, r-1, \quad j = i+1, \ldots, r \end{aligned} \tag{16}$$

$$\begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1r} \\ Z_{21} & Z_{22} & \cdots & Z_{2r} \\ \vdots & \vdots & \ddots & \vdots \\ Z_{r1} & Z_{r2} & \cdots & Z_{rr} \end{bmatrix} < 0 \tag{17}$$

Let $P = Q^{-1}$, $N_j = K_j Q$. Premultiply and postmultiply (23) by Q, we have the first equation shown at bottom of page. If (18)–(20) are feasible

$$\begin{aligned}\Lambda_c <& \sum_{i=1}^{r} h_i^3 Y_{iii} + \sum_{i=1}^{r}\sum_{\substack{j=1\\ j\neq i}}^{r} h_i^2 h_j \left(Y_{iij}+Y_{iji}+Y_{iij}^T\right)\\ &+\sum_{i=1}^{r-2}\sum_{j=i+1}^{r-1}\sum_{\ell=j+1}^{r} h_i h_j h_\ell\\ &\times\left(Y_{ij\ell}+Y_{i\ell j}+Y_{ji\ell}+Y_{ij\ell}^T+Y_{i\ell j}^T+Y_{ji\ell}^T\right)\\ =& h_1\begin{bmatrix}h_1I\\h_2I\\ \vdots\\ h_rI\end{bmatrix}^T\begin{bmatrix}Y_{111}&Y_{112}&\cdots&Y_{11r}\\Y_{211}&Y_{212}&\cdots&Y_{21r}\\ \vdots&\vdots&\ddots&\vdots\\ Y_{r11}&Y_{r12}&\cdots&Y_{r1r}\end{bmatrix}\begin{bmatrix}h_1I\\h_2I\\ \vdots\\ h_rI\end{bmatrix}\\ &+h_2\begin{bmatrix}h_1I\\h_2I\\ \vdots\\ h_rI\end{bmatrix}^T\begin{bmatrix}Y_{121}&Y_{122}&\cdots&Y_{12r}\\Y_{221}&Y_{222}&\cdots&Y_{22r}\\ \vdots&\vdots&\ddots&\vdots\\ Y_{r21}&Y_{r22}&\cdots&Y_{r2r}\end{bmatrix}\begin{bmatrix}h_1I\\h_2I\\ \vdots\\ h_rI\end{bmatrix}\\ &+\cdots+h_r\begin{bmatrix}h_1I\\h_2I\\ \vdots\\ h_rI\end{bmatrix}^T\begin{bmatrix}Y_{1r1}&Y_{1r2}&\cdots&Y_{1rr}\\Y_{2r1}&Y_{2r2}&\cdots&Y_{2rr}\\ \vdots&\vdots&\ddots&\vdots\\ Y_{rr1}&Y_{rr2}&\cdots&Y_{rrr}\end{bmatrix}\begin{bmatrix}h_1I\\h_2I\\ \vdots\\ h_rI\end{bmatrix}\\ =&\begin{bmatrix}h_1I\\h_2I\\ \vdots\\ h_rI\end{bmatrix}^T\left(\sum_{i=1}^{r}h_i\begin{bmatrix}Y_{1i1}&Y_{1i2}&\cdots&Y_{1ir}\\Y_{2i1}&Y_{2i2}&\cdots&Y_{2ir}\\ \vdots&\vdots&\ddots&\vdots\\ Y_{ri1}&Y_{ri2}&\cdots&Y_{rir}\end{bmatrix}\right)\begin{bmatrix}h_1I\\h_2I\\ \vdots\\ h_rI\end{bmatrix}.\end{aligned}$$

Thus, if (21) holds, $\Lambda_c < 0$. In other words, the equilibrium of the CFS of (2) is quadratically stabilizable via the fuzzy controller (4). □

Theorem 6: The equilibrium of the *DFS* of (2) is quadratically stabilizable via the fuzzy controller (4) if there exist matrices $Q > 0$; $N_i, i = 1,2,\ldots,r$; $Y_{iii}, i = 1,2,\ldots,r$; $Y_{jii} = Y_{iij}^T$ and Y_{iji}, $i = 1,2,\ldots,r$, $j \neq i$, $j = 1,2,\ldots,r$ and $Y_{ij\ell} = Y_{\ell ji}^T$, $Y_{i\ell j} = Y_{j\ell i}^T$, $Y_{ji\ell} = Y_{\ell ij}^T$, $i = 1,2,\ldots,r-2$, $j = i+1,\ldots,r-1$, $\ell = j+1,\ldots,r$ satisfying the LMIs shown in (24)–(27) at the bottom of the page. Moreover, in this case, the fuzzy local feedback gains are $K_j = N_j Q^{-1}$, $j = 1,2,\ldots,r$.

$$\begin{aligned}&\sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j\left(QA_i^T + A_iQ - N_j^TB_i^T - B_iN_j\right)\\ &= \left(\sum_{i=1}^{r} h_i\right)\sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j\left(QA_i^T + A_iQ - N_j^TB_i^T - B_iN_j\right)\\ &= \sum_{i=1}^{r} h_i^3\left(QA_i^T + A_iQ - N_i^TB_i^T - B_iN_i\right)\\ &\quad+\sum_{i=1}^{r}\sum_{\substack{j=1\\ j\neq i}}^{r} h_i^2h_j\left(2QA_i^T + QA_j^T + 2A_iQ + A_jQ - (N_i+N_j)^TB_i^T - N_i^TB_j^T - B_i(N_i+N_j) - B_jN_i\right)\\ &\quad+\sum_{i=1}^{r-2}\sum_{j=i+1}^{r-1}\sum_{\ell=j+1}^{r} h_ih_jh_\ell\begin{pmatrix}2Q(A_i+A_j+A_\ell)^T - (N_j+N_\ell)^TB_i^T - (N_i+N_\ell)^TB_j^T - (N_i+N_j)^TB_\ell^T\\ +2(A_i+A_j+A_\ell)Q - B_i(N_j+N_\ell) - B_j(N_i+N_\ell) - B_\ell(N_i+N_j)\end{pmatrix}\\ &\triangleq \Lambda_c\end{aligned}$$

$$\begin{bmatrix}-Q & QA_i^T - N_i^TB_i^T\\ A_iQ - B_iN_i & -Q\end{bmatrix} < Y_{iii},\quad i = 1,2,\ldots,r \tag{24}$$

$$\begin{aligned}&\begin{bmatrix}-3Q & 2QA_i^T+QA_j^T-(N_i+N_j)^TB_i^T-N_i^TB_j^T\\ 2A_iQ+A_jQ-B_i(N_i+N_j)-B_jN_i & -3Q\end{bmatrix} \le Y_{iij}+Y_{iji}+Y_{iij}^T,\\ &i = 1,2,\ldots,r,\quad j\neq i,\quad j = 1,2,\ldots,r\end{aligned} \tag{25}$$

$$\begin{aligned}&\left[\begin{array}{c|c}-6Q & \begin{matrix}2Q(A_i+A_j+A_\ell)^T - (N_j+N_\ell)^TB_i^T\\ -(N_i+N_\ell)^TB_j^T - (N_i+N_j)^TB_\ell^T\end{matrix}\\ \hline \begin{matrix}2(A_i+A_j+A_\ell)Q - B_i(N_j+N_\ell)\\ -B_j(N_i+N_\ell) - B_\ell(N_i+N_j)\end{matrix} & -6Q\end{array}\right] \le Y_{ij\ell}+Y_{i\ell j}+Y_{ji\ell}+Y_{ij\ell}^T+Y_{i\ell j}^T+Y_{ji\ell}^T,\\ &i = 1,2,\ldots,r-2,\quad j = i+1,\ldots,r-1,\quad \ell = j+1,\ldots,r\end{aligned} \tag{26}$$

$$\begin{bmatrix}Y_{1i1}&Y_{1i2}&\cdots&Y_{1ir}\\Y_{2i1}&Y_{2i2}&\cdots&Y_{2ir}\\ \vdots&\vdots&\ddots&\vdots\\ Y_{ri1}&Y_{ri2}&\cdots&Y_{rir}\end{bmatrix} \le 0,\qquad i = 1,2,\ldots,r \tag{27}$$

Proof: Consider a candidate of quadratic function $V(x(t)) = x^T(t)Px(t)$, where $P > 0$. The equilibrium of DFS (5) is quadratically stable if

$$\begin{aligned} &V(x(t+1)) - V(x(t)) \\ &= x^T(t)\left\{\sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j (A_i - B_i K_j)^T \right. \\ &\qquad \left. \times P \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j (A_i - B_i K_j) - P\right\} x(t) < 0 \\ &\forall x(t) \neq 0 \end{aligned} \tag{28}$$

or if

$$\sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j (A_i - B_i K_j)^T P \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j (A_i - B_i K_j) - P < 0 \tag{29}$$

which is equivalent to

$$\sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j \begin{bmatrix} -P & (A_i - B_i K_j)^T P \\ P(A_i - B_i K_j) & -P \end{bmatrix} < 0. \tag{30}$$

Let $P = Q^{-1}$, $N_j = K_j Q$. Premultiply and postmultiply the left-hand side of (30) by $\mathrm{diag}(Q, Q)$, we have the equation shown at the bottom of the page. If (24)–(26) are feasible

$$\begin{aligned} \Lambda_d &< \sum_{i=1}^{r} h_i^3 Y_{iii} + \sum_{i=1}^{r}\sum_{\substack{j=1 \\ j\neq i}}^{r} h_i^2 h_j \left(Y_{iij} + Y_{iji} + Y_{iij}^T\right) \\ &\quad + \sum_{i=1}^{r-2}\sum_{j=i+1}^{r-1}\sum_{\ell=j+1}^{r} h_i h_j h_\ell \\ &\quad \times \left(Y_{ij\ell} + Y_{i\ell j} + Y_{ji\ell} + Y_{ij\ell}^T + Y_{i\ell j}^T + Y_{ji\ell}^T\right) \\ &= \begin{bmatrix} h_1 I \\ h_2 I \\ \vdots \\ h_r I \end{bmatrix}^T \left(\sum_{i=1}^{r} h_i \begin{bmatrix} Y_{1i1} & Y_{1i2} & \cdots & Y_{1ir} \\ Y_{2i1} & Y_{2i2} & \cdots & Y_{2ir} \\ \vdots & \vdots & \ddots & \vdots \\ Y_{ri1} & Y_{ri2} & \cdots & Y_{rir} \end{bmatrix}\right) \begin{bmatrix} h_1 I \\ h_2 I \\ \vdots \\ h_r I \end{bmatrix}. \end{aligned}$$

If (27) holds, $\Lambda_d < 0$, which implies that the equilibrium of the *DFS* of (2) is quadratically stabilizable via the fuzzy controller (4). □

In what follows, the relationships among Theorems 5 and 6 and the previous works (Theorems 3 and 4) are discussed. It will be shown that the conditions suggested herein include the previous ones as special cases.

Theorem 7: The set of solutions to LMIs (12)–(14) in Theorem 3 is a subset of solutions to LMIs (18)–(21) in Theorem 5.

Proof: Suppose $Q > 0, N_i, Z_{ii}, i = 1, 2, \ldots, r$ and $Z_{ji} = Z_{ij}^T, i = 1, 2, \ldots, r-1, j = i+1, \ldots, r$ is a set of solutions to the LMIs (12)–(14). We will show that such set of solutions is also a set of solutions to the LMIS (18)–(21) with the variables $Y'_{ij\ell}s$ chosen particularly by the following:

$$Y_{iii} = Z_{ii}, \quad i = 1, 2, \ldots, r \tag{31}$$

$$Y_{iij} = Z_{ij}, \quad i = 1, 2, \ldots, r, \quad j \neq i, \quad j = 1, 2, \ldots, r \tag{32}$$

$$Y_{iji} = Z_{ii}, \quad i = 1, 2, \ldots, r, \quad j \neq i, \quad j = 1, 2, \ldots, r \tag{33}$$

$$Y_{ij\ell} = Z_{i\ell}, \quad i = 1, 2, \ldots, r-2, \quad j = i+1, \ldots, r-1, \quad \ell = j+1, \ldots, r \tag{34}$$

$$Y_{i\ell j} = Z_{ij}, \quad i = 1, 2, \ldots, r-2, \quad j = i+1, \ldots, r-1, \quad \ell = j+1, \ldots, r \tag{35}$$

$$Y_{ji\ell} = Z_{j\ell}, \quad i = 1, 2, \ldots, r-2, \quad j = i+1, \ldots, r-1, \quad \ell = j+1, \ldots, r. \tag{36}$$

With (31), the inequality (18) coincides with (12). Let f, g, h be any three integers satisfying

$$1 \leq f < g < h \leq r. \tag{37}$$

The solutions of (13) gives the following three inequalities:

$$\begin{aligned} &QA_f^T + A_f Q + QA_g^T + A_g Q - N_g^T B_f^T - B_f N_g \\ &\quad - N_f^T B_g^T - B_g N_f \leq Z_{fg} + Z_{fg}^T \end{aligned} \tag{38}$$

$$\begin{aligned} &QA_g^T + A_g Q + QA_h^T + A_h Q - N_h^T B_g^T - B_g N_h \\ &\quad - N_g^T B_h^T - B_h N_g \leq Z_{gh} + Z_{gh}^T \end{aligned} \tag{39}$$

$$\begin{aligned} &\sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j \begin{bmatrix} -Q & QA_i^T - N_j^T B_i^T \\ A_i Q - B_i N_j & -Q \end{bmatrix} \\ &= \left(\sum_{i=1}^{r} h_i\right) \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j \begin{bmatrix} -Q & QA_i^T - N_j^T B_i^T \\ A_i Q - B_i N_j & -Q \end{bmatrix} \\ &= \sum_{i=1}^{r} h_i^3 \begin{bmatrix} -Q & QA_i^T - N_i^T B_i^T \\ A_i Q - B_i N_i & -Q \end{bmatrix} \\ &\quad + \sum_{i=1}^{r}\sum_{\substack{j=1 \\ j\neq i}}^{r} h_i^2 h_j \begin{bmatrix} -3Q & 2QA_i^T + QA_j^T - (N_i + N_j)^T B_i^T - N_i^T B_j^T \\ 2A_i Q + A_j Q - B_i(N_i + N_j) - B_j N_i & -3Q \end{bmatrix} \\ &\quad + \sum_{i=1}^{r-2}\sum_{j=i+1}^{r-1}\sum_{\ell=j+1}^{r} h_i h_j h_\ell \left[\begin{array}{c|c} -6Q & \begin{matrix} 2Q(A_i + A_j + A_\ell)^T - (N_j + N_\ell)^T B_i^T \\ -(N_i + N_\ell)^T B_j^T - (N_i + N_j)^T B_\ell^T \end{matrix} \\ \hline \begin{matrix} 2(A_i + A_j + A_\ell)Q - B_i(N_j + N_\ell) \\ -B_j(N_i + N_\ell) - B_\ell(N_i + N_j) \end{matrix} & -6Q \end{array}\right] \\ &\triangleq \Lambda_d \end{aligned}$$

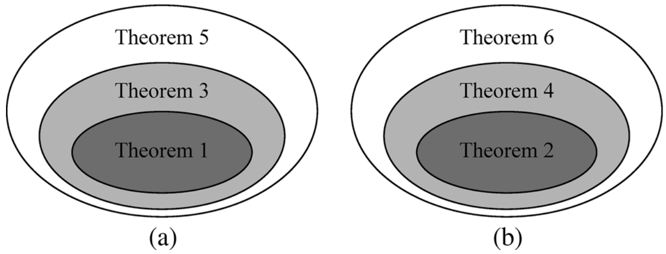

Fig. 1. Relationship among the new conditions and the conventional ones. (a) Continuous cases. (b) Discrete cases.

$$\begin{aligned} &QA_f^T + A_fQ + QA_h^T + A_hQ - N_h^T B_f^T - B_f N_h \\ &\quad - N_f^T B_h^T - B_h N_f \le Z_{fh} + Z_{fh}^T. \end{aligned} \tag{40}$$

Summing (38)–(40) yields

$$\begin{aligned} &2Q\left(A_f^T + A_g^T + A_f^T\right) - (N_g + N_h)^T B_f^T - (N_f + N_h)^T B_g^T \\ &\quad - (N_f + N_g)^T B_h^T + 2(A_f + A_g + A_h)Q \\ &\quad - B_f(N_g + N_h) - B_g(N_f + N_h) - B_h(N_f + N_g) \\ &\quad \le Z_{fh} + Z_{fg} + Z_{gh} + Z_{fh}^T + Z_{fg}^T + Z_{gh}^T. \end{aligned}$$

Replacing the subscripts (f, g, h) with (i, j, ℓ) and using (37), one gets

$$\begin{aligned} &2Q(A_i+A_j+A_\ell)^T - (N_j+N_\ell)^T B_i^T - (N_i+N_\ell)^T B_j^T \\ &\quad - (N_i+N_j)^T B_\ell^T + 2(A_i+A_j+A_\ell)Q - B_i(N_j+N_\ell) \\ &\quad - B_j(N_i+N_\ell) - B_\ell(N_i+N_j) \\ &\quad \le Z_{i\ell}+Z_{ij}+Z_{j\ell}+Z_{i\ell}^T+Z_{ij}^T+Z_{j\ell}^T, \quad i = 1, 2, \ldots, r-2, \\ &\quad j = i+1, \ldots, r-1, \quad \ell = j+1, \ldots, r \end{aligned} \tag{41}$$

which is equal to (20) with the particular choice of (34)–(36). Thus the solutions to (13) are also one set of solutions to (20). In the next section, we will show they are also one set of solutions to (19). In Theorem 3, (13) can be equivalently represented as

$$\begin{aligned} &QA_i^T + A_iQ + QA_j^T + A_jQ - N_j^T B_i^T - B_iN_j - N_i^T B_j^T - B_jN_i \\ &\quad \le Z_{ij} + Z_{ji}, \quad i = 1, 2, \ldots, r, \quad j \ne i, \quad j = 1, 2, \ldots, r \end{aligned} \tag{42}$$

due to $Z_{ji} = Z_{ij}^T$ and the symmetry of (13). Note that (42) is obtained by just duplication of all inequalities in (13). Thus, (42) and (13) have the same set of solutions, Adding (12) to (42) yields

$$\begin{aligned} &2QA_i^T + QA_j^T + 2A_iQ + A_jQ - (N_i + N_j)^T B_i^T - N_i^T B_j^T \\ &\quad - B_i(N_i + N_j) - B_jN_i \le Z_{ij} + Z_{ii} + Z_{ji}, \\ &\quad i = 1, 2, \ldots, r, \quad j \ne i, \quad j = 1, 2, \ldots, r \end{aligned}$$

which is equal to (19) with the particular choice of (32) and (33). With the choice of (31)–(36), the inequality (21) reduces to (14). □

Theorem 8: The set of solutions to LMI's (15)–(17) in Theorem 4 is a subset of solutions to LMIs (24)–(27) in Theorem 6.

Proof: Similar to the proof of Theorem 7. □

Note that Theorems 1 and 2 are special cases of Theorems 3 and 4, thus they are also special cases of Theorems 5 and 6. The relationships among the conditions suggested herein and the conventional ones are depicted in Fig. 1 for continuous-time cases and discrete-time cases, respectively.

The following example illustrates the relaxation of Theorems 5 and 6 in comparison with Theorems 1–4.

Example 1: Consider a CFS defined by the following rules:

$$\begin{aligned} &\text{Plant Rule 1:If } x_1(t) \text{ is } M_1, \\ &\qquad \text{Then } \dot{x}(t) = A_1x(t) + B_1u(t) \\ &\text{Plant Rule 2:If } x_1(t) \text{ is } M_2, \\ &\qquad \text{Then } \dot{x}(t) = A_2x(t) + B_2u(t) \\ &\text{Plant Rule 3:If } x_1(t) \text{ is } M_3, \\ &\qquad \text{Then } \dot{x}(t) = A_3x(t) + B_3u(t) \end{aligned}$$

where $A_1 = \begin{bmatrix} 1.59 & -7.29 \\ 0.01 & 0 \end{bmatrix}$, $B_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $A_2 = \begin{bmatrix} 0.02 & -4.64 \\ 0.35 & 0.21 \end{bmatrix}$, $B_2 = \begin{bmatrix} 8 \\ 0 \end{bmatrix}$, $A_3 = \begin{bmatrix} -a & -4.33 \\ 0 & 0.05 \end{bmatrix}$, $B_3 = \begin{bmatrix} -b+6 \\ -1 \end{bmatrix}$. The parameters a and b are adjusted to compare the relaxation of Theorems 1, 3, and 5. Fig. 2(a)–(c) show the parameter regions where the fuzzy state feedback stabilizing controllers of the above system can be found (Fig. 2(a)–(c) are for Theorems 1, 3, and 5, respectively). In Fig. 2, the mark $\bullet$ denotes the LMI conditions in Theorems 1, 3, and 5 are feasible and $\times$ denotes the LMI conditions are infeasible. From the figures, it can be seen that Theorem 5 shows the most relaxed results.

Remark 2: Reference [19] appeared four months after this paper had been submitted. The reviewer suggested to cite [19] as the paper was revised. In this remark, we are going to show the condition in [19] is a special case of ours, too. For readability, the stabilization condition of [19] ([19, Th. 5]) is restated: The equilibrium of the *CFS* of (2) is quadratically stabilizable via the fuzzy controller (4) if there exist a symmetric positive definite matrix Q, symmetric matrices $T_{ij\alpha}$, R_{ij}, matrices S_{ijh}, and N_i, $i = 1, 2, \ldots, r-1$, $j = i+1, \ldots, r$, $h = 1, 2, \ldots, r$, such that

$$\begin{aligned} &T_{ijh} \ge 0, \ i = 1, 2, \ldots, r-1, \ j = i+1, \ldots, r, \\ &\quad h = 1, 2, \ldots, r \end{aligned} \tag{43}$$

$$\begin{aligned} &\begin{bmatrix} \overline{X}_{11} - Z_{1h} & X_{12h} & \cdots & X_{1rh} \\ X_{21h} & \overline{X}_{22} - Z_{2h} & \cdots & X_{2rh} \\ \vdots & \vdots & \ddots & \vdots \\ X_{r1h} & X_{r2h} & \cdots & \overline{X}_{rr} - Z_{rh} \end{bmatrix} < 0, \\ &h = 1, 2, \ldots, r \end{aligned} \tag{44}$$

where $\overline{X}_{ii} = QA_i^T + A_iQ + N_i^T B_{ui}^T + B_{ui}N_i, i = 1, 2, \ldots, r$; see (45)–(47) at the bottom of the next page. Assume the previous solutions are obtained. Define

$$\begin{aligned} \tilde{X}_{ij} \triangleq 0.5 \big(&QA_i^T + A_iQ + QA_j^T + A_jQ + N_j^T B_{ui}^T \\ &+ B_{ui}N_j + N_i^T B_{uj}^T + B_{uj}N_i\big) \quad \text{for } i < j \end{aligned} \tag{48}$$

and set the variables $Y_{ij\ell}$ by the following way:

$$Y_{iii} = \overline{X}_{ii}, \quad i = 1, 2, \ldots, r \tag{49}$$

$$Y_{iij} = \begin{cases} \tilde{X}_{ij} + T_{iji} + \left(S_{iji} - S_{iji}^T\right) + 0.5R_{ij}, & \text{if } i < j \\ \tilde{X}_{ji} + T_{jii} + \left(S_{jii} - S_{jii}^T\right) + 0.5R_{ji}, & \text{if } i > j \end{cases} \tag{50}$$

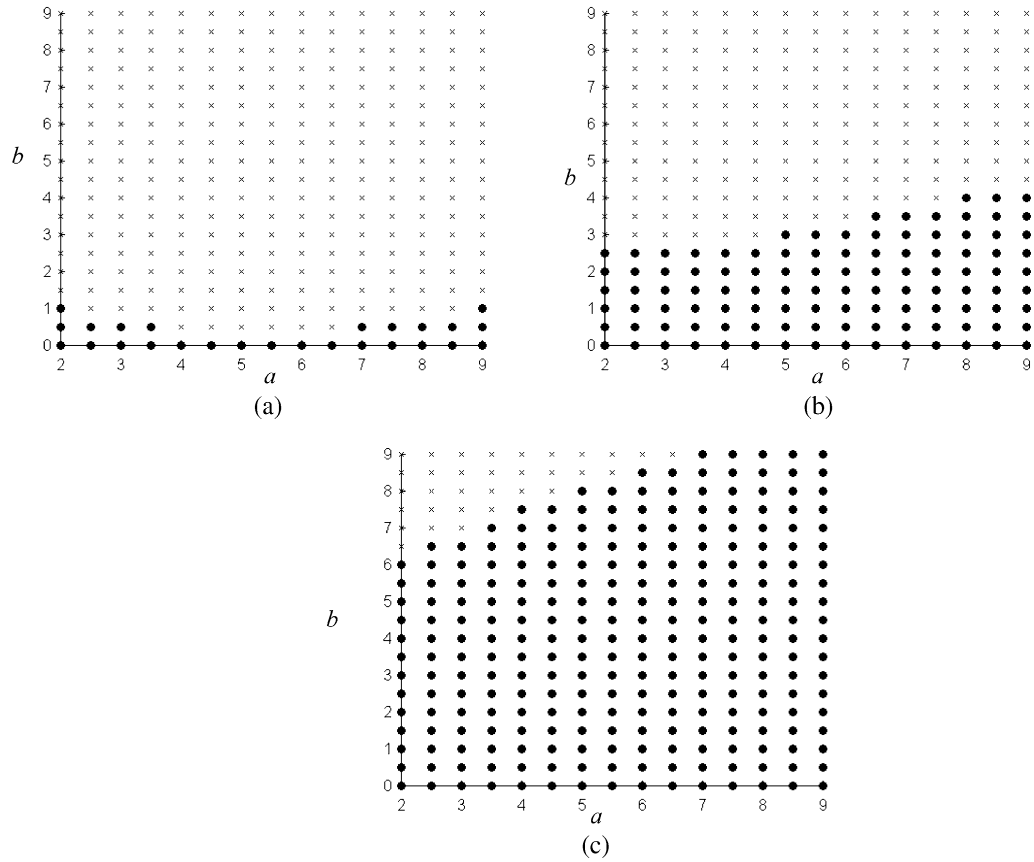

Fig. 2. Stabilization region based on (a) Theorems 1, (b) 3; and (c) 5.

$$Y_{iji} = \begin{cases} \overline{X}_{ii} - R_{ij}, & \text{if } i < j \\ \overline{X}_{ii} - R_{ji}, & \text{if } i > j \end{cases} \tag{51}$$

$$Y_{ij\ell} = \tilde{X}_{i\ell} + T_{i\ell j} + \left(S_{i\ell j} - S_{i\ell j}^T\right), \quad i = 1, 2, \ldots, r-2, \quad j = i+1, \ldots, r-1, \quad \ell = j+1, \ldots, r \tag{52}$$

$$Y_{i\ell j} = \tilde{X}_{ij} + T_{ij\ell} + \left(S_{ij\ell} - S_{ij\ell}^T\right), \quad i = 1, 2, \ldots, r-2, \quad j = i+1, \ldots, r-1, \quad \ell = j+1, \ldots, r \tag{53}$$

$$Y_{ji\ell} = \tilde{X}_{j\ell} + T_{j\ell i} + \left(S_{j\ell i} - S_{j\ell i}^T\right), \quad i = 1, 2, \ldots, r-2, \quad j = i+1, \ldots, r-1, \quad \ell = j+1, \ldots, r. \tag{54}$$

Substituting (49)–(54) into (18)–(20), it can be checked that the above $Y'_{ij\ell}s$ satisfies (18)–(20). With $Y'_{ij\ell}s$ in (49)–(54), the condition (21) also coincides with (44). Obviously, the feasibility of (43)–(47) implies that of (18)–(20). Thus, one can conclude that the solution set of (43)–(47) is a subset of the solution set of (18)–(20). Although only continuous-time cases are discussed, the discrete-time cases can be proved in a similar fashion. ■

B. *Fuzzy Static Output Feedback Control Design*

We have proposed an LMI-based design method using fuzzy state feedback control in Section III-A. However, in real-word control problems, the states may not be completely accessible. In such cases, one needs to resort to output feedback design methods. Fuzzy static output feedback control is the most desirable since it can be implemented easily with low cost. Nevertheless, the fuzzy static output feedback stabilization problem of T–S fuzzy systems is rarely investigated because it is quite hard in essence. The idea proposed in this paper is very suitable

$$Z_{jh} = \begin{cases} R_{jh}, & \text{if } j < h \\ R_{hj}, & \text{if } j > h \\ 0, & \text{if } j = h \end{cases} \tag{45}$$

$$X_{ijh} = \begin{cases} 0.5\left(QA_i^T + A_iQ + QA_j^T + A_jQ + N_j^TB_{ui}^T + B_{ui}N_j + N_i^TB_{uj}^T + B_{uj}N_i\right) \\ \quad +T_{ijh} + \left(S_{ijh} - S_{ijh}^T\right) + 0.5W_{ijh}, & \text{if } i < j \\ 0.5\left(QA_j^T + A_jQ + QA_i^T + A_iQ + N_i^TB_{uj}^T + B_{uj}N_i + N_j^TB_{ui}^T + B_{ui}N_j\right) \\ \quad +T_{jih} + \left(S_{jih}^T - S_{jih}\right) + 0.5W_{jih}, & \text{if } i > j \end{cases} \tag{46}$$

$$W_{\ell kh} = \begin{cases} R_{\ell k}, & \text{if } \ell = h \text{ or } k = h \\ 0, & \text{if } \ell \neq h \text{ or } k \neq h \end{cases} \tag{47}$$

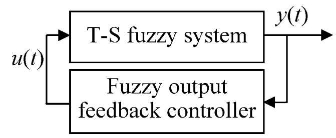

Fig. 3. Schematic of the fuzzy control system.

for dealing with such design problem. Consider the following fuzzy rules:

$$\text{Plant Rule } i: \text{If } z_1(t) \text{ is } M_{i1} \text{ and } \ldots \text{ and } z_s(t) \text{ is } M_{is}$$
$$\text{Then} \begin{cases} \nabla x(t) = A_i x(t) + B_i u(t) \\ y(t) = C_i x(t) \end{cases} \quad (55)$$

where $z_i(t), i = 1, 2, \ldots, s$ are the premise variables which are measurable. $C_i \in \mathbb{R}^{q\times n}$ and $B_i \in \mathbb{R}^{n\times m}$ are the output vector and the input vector, respectively. Assume at least one of B_i is of full-column rank. Given a pair of $(x(t), u(t))$, the final output of the fuzzy system are inferred as follows:

$$\nabla x(t) = \sum_{i=1}^{r} h_i(z(t))(A_i x(t) + B_i u(t)) \quad (56)$$
$$y(t) = \sum_{i=1}^{r} h_i(z(t)) C_i x(t). \quad (57)$$

Let the fuzzy static output feedback controller be

$$\text{Controller Rule } i: \text{If } z_1(t) \text{ is } M_{i1} \text{ and} \ldots \text{and } z_s(t) \text{ is } M_{is}$$
$$\text{Then } u(t) = -F_i y(t) \quad (58)$$

where $i = 1, 2, \ldots, r$. The output of the controller is given by

$$\begin{aligned} u(t) &= -\sum_{i=1}^{r} h_i(z(t)) F_i y(t) \\ &= -\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z(t)) h_j(z(t)) F_i C_j x(t) \end{aligned} \quad (59)$$

The schematic of the fuzzy control system is depicted in Fig. 3. By substituting (59) into (56), the closed-loop fuzzy system can be represented as

$$\nabla x(t) = \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{\ell=1}^{r} h_i(z(t)) h_j(z(t)) h_\ell(z(t)) \times (A_i - B_i F_j C_\ell) x(t). \quad (60)$$

Our goal is to find the local fuzzy static output feedback gains $F_i, i = 1, 2, \ldots, r$, in the consequent parts such that the closed-loop system (60) is quadratically stable. This is called fuzzy static output feedback stabilization problem. In next two theorems, such stabilization problem is solved by using the idea proposed in previous subsection.

Theorem 9: The equilibrium of the *CFS* of (56) is quadratically stabilizable via the fuzzy static output feedback controller (59) if there exist matrices $P > 0$; $N_i, i = 1, 2, \ldots, r$; Y_{iii}, $i = 1, 2, \ldots, r$; $Y_{jii} = Y_{iij}^T$ and $Y_{iji}, i = 1, 2, \ldots, r, j \neq i$, $j = 1, 2, \ldots, r$; $Y_{ij\ell} = Y_{\ell ji}^T$, $Y_{i\ell j} = Y_{j\ell i}^T$, $Y_{ji\ell} = Y_{\ell ij}^T$, $i = 1, 2, \ldots, r-2, j = i+1, \ldots, r-1, \ell = j+1, \ldots, r$, and M satisfy

$$A_i^T P + P A_i - C_i^T N_i^T B_i^T - B_i N_i C_i < Y_{iii}, \quad i = 1, 2, \ldots, r \quad (61)$$

$$\begin{aligned} &2\left(A_i^T P + P A_i\right) + A_j^T P + P A_j - C_j^T N_i^T B_i^T - B_i N_i C_j \\ &\quad - C_i^T N_j^T B_i^T - B_i N_j C_i - C_i^T N_i^T B_j^T - B_j N_i C_i \\ &\quad\quad \leq Y_{iij} + Y_{iji} + Y_{iij}^T, \quad i = 1, 2, \ldots, r, \quad j \neq i, \\ &\quad\quad j = 1, 2, \ldots, r \end{aligned} \quad (62)$$

$$\begin{aligned} &2\left(A_i^T P + A_j^T P + A_\ell^T P + P A_i + P A_j + P A_\ell\right) \\ &\quad - C_\ell^T N_j^T B_i^T - B_i N_j C_\ell - C_j^T N_\ell^T B_i^T - B_i N_\ell C_j \\ &\quad - C_\ell^T N_i^T B_j^T - B_j N_i C_\ell - C_i^T N_\ell^T B_j^T - B_j N_\ell C_i \\ &\quad - C_j^T N_i^T B_\ell^T - B_\ell N_i C_j - C_i^T N_j^T B_\ell^T - B_\ell N_j C_i \\ &\quad\quad \leq Y_{ij\ell} + Y_{i\ell j} + Y_{ji\ell} + Y_{ij\ell}^T + Y_{i\ell j}^T + Y_{ji\ell}^T, \\ &\quad\quad i = 1, 2, \ldots, r-2, \quad j = i+1, \ldots, r-1, \\ &\quad\quad \ell = j+1, \ldots, r \end{aligned} \quad (63)$$

$$\begin{bmatrix} Y_{1i1} & Y_{1i2} & \cdots & Y_{1ir} \\ Y_{2i1} & Y_{2i2} & \cdots & Y_{2ir} \\ \vdots & \vdots & \ddots & \vdots \\ Y_{ri1} & Y_{ri2} & \cdots & Y_{rir} \end{bmatrix} \leq 0, \quad i = 1, 2, \ldots, r \quad (64)$$

$$P B_i = B_i M, \quad i = 1, 2, \ldots, r. \quad (65)$$

Moreover, in this case, the local fuzzy static output feedback gains that stabilizes the system (56) can be chosen as $F_i = M^{-1} N_i, i = 1, 2, \ldots, r$.

Proof: Consider a candidate of quadratic function $V(x(t)) = x^T(t) P x(t)$, where $P > 0$ Then, by (60)

$$\begin{aligned} &\dot{V}(x(t)) \\ &= \dot{x}^T(t) P x(t) + x^T(t) P \dot{x}(t) \\ &= x^T(t) \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{\ell=1}^{r} h_i h_j h_\ell \left((A_i - B_i F_j C_\ell)^T P \right. \\ &\quad \left. + P(A_i - B_i F_j C_\ell)\right) x(t) \\ &= x^T(t) \left\{ \sum_{i=1}^{r} h_i^3 \left((A_i - B_i F_i C_i)^T P + P(A_i - B_i F_i C_i)\right) \right. \\ &\qquad + \sum_{i=1}^{r}\sum_{\substack{j=1 \\ j\neq i}}^{r} h_i^2 h_j \\ &\qquad \times \left((A_i - B_i F_i C_j)^T P + P(A_i - B_i F_i C_j) \right. \\ &\qquad\quad + (A_i - B_i F_j C_i)^T P + P(A_i - B_i F_j C_i) \\ &\qquad\quad \left. + (A_j - B_j F_i C_i)^T P + P(A_j - B_j F_i C_i)\right) \\ &\qquad + \sum_{i=1}^{r-2}\sum_{j=i+1}^{r-1}\sum_{\ell=j+1}^{r} h_i h_j h_\ell \\ &\qquad \times \left((A_i - B_i F_j C_\ell)^T P + P(A_i - B_i F_j C_\ell) \right. \\ &\qquad\quad + (A_i - B_i F_\ell C_j)^T P + P(A_i - B_i F_\ell C_j) \\ &\qquad\quad + (A_j - B_j F_i C_\ell)^T P + P(A_j - B_j F_i C_\ell) \\ &\qquad\quad + (A_j - B_j F_\ell C_i)^T P + P(A_j - B_j F_\ell C_i) \\ &\qquad\quad + (A_\ell - B_\ell F_i C_j)^T P + P(A_\ell - B_\ell F_i C_j) \\ &\qquad\quad + (A_\ell - B_\ell F_j C_i)^T P \\ &\qquad\quad \left.\left. + P(A_\ell - B_\ell F_j C_i)\right)\right\} x(t) \\ &\triangleq x^T(t) \Omega_c x(t). \end{aligned} \quad (66)$$

In view of (65), $B_i = PB_iM^{-1}$, $i = 1,2,\ldots,r$ (nonsingularity of P and full column rank of at least one of B_i imply M is invertible). Replacing B_i, B_j, and B_ℓ in (61)–(63) with PB_iM^{-1}, PB_jM^{-1}, and $PB_\ell M^{-1}$, respectively, and denoting $F_i \triangleq M^{-1}N_i$, $F_j \triangleq M^{-1}N_j$, and $F_\ell \triangleq M^{-1}N_\ell$ therein. Comparing the new resulting conditions with Ω_c of (66) yields

$$\begin{aligned}\Omega_c &< \sum_{i=1}^{r} h_i^3 Y_{iii} + \sum_{i=1}^{r}\sum_{\substack{j=1\\ j\neq i}}^{r} h_i^2 h_j \left(Y_{iij}+Y_{iji}+Y_{iij}^T\right)\\ &\quad+\sum_{i=1}^{r-2}\sum_{j=i+1}^{r-1}\sum_{\ell=j+1}^{r} h_ih_jh_\ell\\ &\quad\times\left(Y_{ij\ell}+Y_{i\ell j}+Y_{ji\ell}+Y_{ij\ell}^T+Y_{i\ell j}^T+Y_{ji\ell}^T\right)\\ &= \begin{bmatrix} h_1I\\ h_2I\\ \vdots\\ h_rI\end{bmatrix}^T \left(\sum_{i=1}^{r} h_i \begin{bmatrix} Y_{1i1} & Y_{1i2} & \cdots & Y_{1ir}\\ Y_{2i1} & Y_{2i2} & \cdots & Y_{2ir}\\ \vdots & \vdots & \ddots & \vdots\\ Y_{ri1} & Y_{ri2} & \cdots & Y_{rir}\end{bmatrix}\right)\begin{bmatrix} h_1I\\ h_2I\\ \vdots\\ h_rI\end{bmatrix}.\end{aligned}$$

Thus, if (64) holds, $\Omega_c < 0$. In other words, the equilibrium of the *CFS* of (56) is quadratically stabilizable via the fuzzy static output feedback controller (59). □

For discrete-time cases, the corresponding result is stated as follows.

Theorem 10: The equilibrium of the *DFS* of (56) is quadratically stabilizable via the fuzzy static output feedback controller (59) if there exist matrices $P > 0$; N_i, $i = 1,2,\ldots,r$; Y_{iii}, $i = 1,2,\ldots,r$; $Y_{jii} = Y_{iij}^T$ and Y_{iji}, $i = 1,2,\ldots,r$, $j \neq i$, $j = 1,2,\ldots,r$; $Y_{ij\ell} = Y_{\ell ji}^T$, $Y_{i\ell j} = Y_{j\ell i}^T$, $Y_{ji\ell} = Y_{\ell ij}^T$, $i = 1,2,\ldots,r-2$, $j = i+1,\ldots,r-1$, $\ell = j+1,\ldots,r$, and M satisfy the following LMIs:

$$\begin{bmatrix} -P & A_i^TP - C_i^TN_i^TB_i^T\\ PA_i - B_iN_iC_i & -P\end{bmatrix} < Y_{iii},\quad i = 1,2,\ldots,r \tag{67}$$

$$\begin{bmatrix} -3P & \begin{matrix}2A_i^TP + A_j^TP - C_j^TN_i^TB_i^T\\ -C_i^TN_j^TB_i^T - C_i^TN_i^TB_j^T\end{matrix}\\ \begin{matrix}2PA_i + PA_j - B_iN_iC_j\\ -B_iN_jC_i - B_jN_iC_i\end{matrix} & -3P\end{bmatrix} \le Y_{iij} + Y_{iji} + Y_{iij}^T,\quad i = 1,2,\ldots,r,\quad j = 1,2,\ldots,r,\quad j\neq i \tag{68}$$

and (69)–(71), as shown at the bottom of the page. Moreover, in this case, the local fuzzy static output feedback gains that stabilize the system (56) can be chosen as $F_i = M^{-1}N_i$, $i = 1,2,\ldots,r$.

Proof: The closed-loop system of (56) is quadratically stable via fuzzy static output feedback (59) if there exist $P > 0$ and F_i, $i = 1,2,\ldots,r$, such that the second equation shown at the bottom of the page holds. Ω_d can be decomposed as shown in the equation at the bottom of the next page. Similar to the proof of continuous-time, one can deduce from (67)–(69) and (71) that

$$\begin{aligned}\Omega_d &< \sum_{i=1}^{r} h_i^3 Y_{iii} + \sum_{i=1}^{r}\sum_{\substack{j=1\\ j\neq i}}^{r} h_i^2 h_j \left(Y_{iij}+Y_{iji}+Y_{iij}^T\right)\\ &\quad+\sum_{i=1}^{r-2}\sum_{j=i+1}^{r-1}\sum_{\ell=j+1}^{r} h_ih_jh_\ell\\ &\quad\times\left(Y_{ij\ell}+Y_{i\ell j}+Y_{ji\ell}+Y_{ij\ell}^T+Y_{i\ell j}^T+Y_{ji\ell}^T\right)\\ &= \begin{bmatrix} h_1I\\ h_2I\\ \vdots\\ h_rI\end{bmatrix}^T \left(\sum_{i=1}^{r} h_i \begin{bmatrix} Y_{1i1} & Y_{1i2} & \cdots & Y_{1ir}\\ Y_{2i1} & Y_{2i2} & \cdots & Y_{2ir}\\ \vdots & \vdots & \ddots & \vdots\\ Y_{ri1} & Y_{ri2} & \cdots & Y_{rir}\end{bmatrix}\right)\begin{bmatrix} h_1I\\ h_2I\\ \vdots\\ h_rI\end{bmatrix}.\end{aligned}$$

If (70) holds, $\Omega_d < 0$, which implies that the equilibrium of the *DFS* of (56) is quadratically stabilizable via the fuzzy static output feedback controller (59). □

$$\begin{bmatrix} -6P & \begin{matrix}2\left(A_i^TP + A_j^TP + A_\ell^TP\right) - C_\ell^TN_j^TB_i^T - C_j^TN_\ell^TB_i^T\\ -C_\ell^TN_i^TB_j^T - C_i^TN_\ell^TB_j^T - C_j^TN_i^TB_\ell^T - C_i^TN_j^TB_\ell^T\end{matrix}\\ \begin{matrix}2(PA_i + PA_j + PA_\ell) - B_iN_jC_\ell - B_iN_\ell C_j\\ -B_jN_iC_\ell - B_jN_\ell C_i - B_\ell N_iC_j - B_\ell N_jC_i\end{matrix} & -6P\end{bmatrix} \le Y_{ij\ell} + Y_{i\ell j} + Y_{ji\ell} + Y_{ij\ell}^T + Y_{i\ell j}^T + Y_{ji\ell}^T,\quad i = 1,2,\ldots,r-2,\ j = i+1,\ldots,r-1,\ \ell = j+1,\ldots,r \tag{69}$$

$$\begin{bmatrix} Y_{1i1} & Y_{1i2} & \cdots & Y_{1ir}\\ Y_{2i1} & Y_{2i2} & \cdots & Y_{2ir}\\ \vdots & \vdots & \ddots & \vdots\\ Y_{ri1} & Y_{ri2} & \cdots & Y_{rir}\end{bmatrix} \le 0,\quad i = 1,2,\ldots,r \tag{70}$$

$$PB_i = B_iM,\quad i = 1,2,\ldots,r \tag{71}$$

$$\Omega_d \triangleq \begin{bmatrix} -P & \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{\ell=1}^{r} h_ih_jh_\ell (A_i - B_iF_jC_\ell)^TP\\ P\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{\ell=1}^{r} h_ih_jh_\ell (A_i - B_iF_jC_\ell) & -P\end{bmatrix} < 0$$

IV. Numerical Examples

In this section, a fuzzy static output feedback controller is designed to stabilize the following complicated nonlinear continuous-time system, which is similar to the example used in [10]

$$\begin{aligned}
\dot{x}_1(t) &= x_1(t)+x_2(t)+\sin x_3(t)-0.1x_4(t)+\left(x_1^2(t)+1\right)u(t)\\
\dot{x}_2(t) &= x_1(t)-2x_2(t)\\
\dot{x}_3(t) &= x_1(t)+x_1^2(t)x_2(t)-0.3x_3(t)\\
\dot{x}_4(t) &= \sin x_3(t)-x_4(t)\\
y_1(t) &= x_2(t)+\left(x_1^2(t)+1\right)x_4(t)\\
y_2(t) &= x_1(t).
\end{aligned} \tag{72}$$

Assume

$$x_1(t) \in [-a \quad a] \quad x_3(t) \in [-b \quad b]$$

where a and b are positive numbers. The nonlinear system (72) is exactly represented by the following T–S fuzzy model:

Plant Rule 1:If $x_1(t)$ is M_1^1 and $x_3(t)$ is M_3^1

$$\text{Then} \begin{cases} \dot{x}(t) = A_1x(t) + B_1u(t) \\ y(t) = C_1x(t) \end{cases}$$

Plant Rule 2:If $x_1(t)$ is M_1^1 and $x_3(t)$ is M_3^2

$$\text{Then} \begin{cases} \dot{x}(t) = A_2x(t) + B_2u(t) \\ y(t) = C_2x(t) \end{cases}$$

Plant Rule 3:If $x_1(t)$ is M_1^2 and $x_3(t)$ is M_3^1

$$\text{Then} \begin{cases} \dot{x}(t) = A_3x(t) + B_3u(t) \\ y(t) = C_3x(t) \end{cases}$$

Plant Rule 4:If $x_1(t)$ is M_1^2 and $x_3(t)$ is M_3^2

$$\text{Then} \begin{cases} \dot{x}(t) = A_4x(t) + B_4u(t) \\ y(t) = C_4x(t) \end{cases}$$

where the premise membership functions and the consequent matrices are as follows:

$$\begin{aligned}
M_1^1(x_1) &= \frac{x_1^2}{a^2}\\
M_1^2(x_1) &= 1 - M_1^1(x_1)\\
M_3^1(x_3) &= \begin{cases} \frac{b\sin x_3 - x_3\sin b}{x_3(b-\sin b)} & x_3 \neq 0 \\ 1 & x_3 = 0 \end{cases}\\
M_3^2(x_3) &= 1 - M_3^1(x_3);
\end{aligned}$$

$$A_1 = \begin{bmatrix} 1 & 1 & 1 & -0.1 \\ 1 & -2 & 0 & 0 \\ 1 & a^2 & -0.3 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix} \quad B_1 = \begin{bmatrix} 1+a^2 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$C_1 = \begin{bmatrix} 0 & 1 & 0 & 1+a^2 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 1 & 1 & (\sin b)/b & -0.1 \\ 1 & -2 & 0 & 0 \\ 1 & a^2 & -0.3 & 0 \\ 0 & 0 & (\sin b)/b & -1 \end{bmatrix}$$

$$B_2 = \begin{bmatrix} 1+a^2 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad C_2 = \begin{bmatrix} 0 & 1 & 0 & 1+a^2 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 1 & 1 & 1 & -0.1 \\ 1 & -2 & 0 & 0 \\ 1 & 0 & -0.3 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix} \quad B_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$C_3 = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

$$A_4 = \begin{bmatrix} 1 & 1 & (\sin b)/b & -0.1 \\ 1 & -2 & 0 & 0 \\ 1 & 0 & -0.3 & 0 \\ 0 & 0 & (\sin b)/b & -1 \end{bmatrix} \quad B_4 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$C_4 = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}.$$

$$\begin{aligned}
\Omega_d &= \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{\ell=1}^{r} h_ih_jh_j \begin{bmatrix} -P & \left(A_i^T - C_\ell^T F_j^T B_i^T\right)P \\ P(A_i - B_iF_jC_\ell) & -P \end{bmatrix}\\
&= \sum_{i=1}^{r} h_i^3 \begin{bmatrix} -P & \left(A_i^T - C_i^T F_i^T B_i^T\right)P \\ P(A_i - B_iF_iC_i) & -P \end{bmatrix}\\
&\quad + \sum_{i=1}^{r}\sum_{\substack{j=1\\ j\neq i}}^{r} h_i^2h_j \left[\begin{array}{c|c} -3P & \begin{matrix}\left(2A_i^T + A_j^T - C_j^T F_i^T B_i^T\right.\\ \left.-C_i^T F_j^T B_i^T - C_i^T F_i^T B_j^T\right)Pc\end{matrix} \\ \hline \begin{matrix} P(2A_i + A_j - B_iF_iC_j \\ -B_iF_jC_i - B_jF_iC_i) \end{matrix} & -3P \end{array}\right]\\
&\quad + \sum_{i=1}^{r-2}\sum_{j=i+1}^{r-1}\sum_{\ell=j+1}^{r} h_ih_jh_\ell\\
&\quad \times \left[\begin{array}{c|c} -6P & \begin{matrix}\left(2\left(A_i^T + A_j^T + A_\ell^T\right) - C_\ell^T F_j^T B_i^T - C_j^T F_\ell^T B_i^T\right.\\ \left.-C_\ell^T F_i^T B_j^T - C_i^T F_\ell^T B_j^T - C_j^T F_i^T B_\ell^T - C_i^T F_j^T B_\ell^T\right)P\end{matrix} \\ \hline \begin{matrix} P\left(2(A_i + A_j + A_\ell) - B_iF_jC_\ell - B_iF_\ell C_j\right. \\ \left.-B_jF_iC_\ell - B_jF_\ell C_i - B_\ell F_iC_j - B_\ell F_jC_i\right) \end{matrix} & -6P \end{array}\right]
\end{aligned}$$

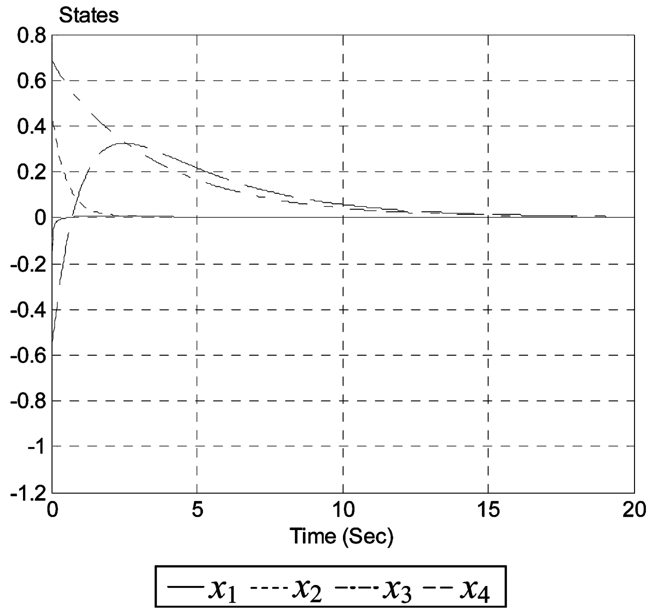

Fig. 4. State response.

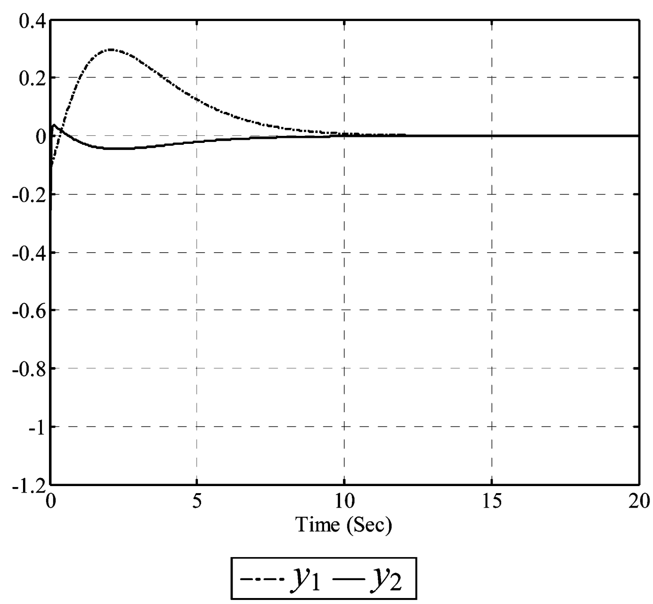

Fig. 5. Output response.

In this simulation, assume $a = 1.4$ and $b = 0.7$. By Theorem 9, we obtain

$$F_1 = [3.7255 \quad 31.6739] \quad F_2 = [3.8230 \quad 32.3984]$$
$$F_3 = [7.5220 \quad 44.0395] \quad F_4 = [7.5611 \quad 44.4575].$$

Figs. 4 and 5 show the state and output response of the closed-loop CFS with initial states $[-1.2\ \ 0.5\ \ 0.7\ \ -0.6]$.

V. Conclusion

A new LMI-based stabilization condition for T–S fuzzy control systems is proposed. The new condition not only relaxed the conservatism of the previous works but also includes them as special cases. If the conditions are feasible, the state feedback controller can be easily constructed by solving a set of LMI's for stabilizing T–S fuzzy control systems. In comparison with conventional approaches, a special feature of the proposed idea is that it can be also applied to solve fuzzy static output feedback stabilization problems which are hard in essence for T–S fuzzy systems but useful and important in practice.

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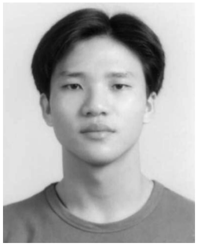

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