Customer Management in Gaming Environments: A Dynamic Structural Analysis

Seung Yoon Lee, K. Sudhir, Kosuke Uetake*

Yale School of Management

seungyoon.lee@yale.edu, k.sudhir@yale.edu, kosuke.uetake@yale.edu

Draft: July 8, 2024

Job Market Paper

(Click here for the latest version)

Gaming and gamified environments possess unique characteristics that influence how consumers spend time and money. The core of the customer management problem in such environments is to ensure that the challenge is difficult enough to motivate agents to spend time and exert effort, yet not so challenging that they get discouraged and quit. In gaming, firms also monetize players' motivation to exert effort by providing them an opportunity to purchase ability-enhancing items that increase their performance; this increases player retention, while simultaneously driving future monetization as players reach higher levels – creating a two-way positive feedback loop between retention and monetization. In this paper, we develop a dynamic structural model of consumer response for gaming environments. The model accommodates key features that are common in such environments: (i) increasing levels of difficulty that result in player attrition, retaining only the most able players, and (ii) opportunity to purchase in-game items that improve players' ability to win. The dilemma for game designers is to manage the dynamics between retention and monetization for players with widely ranging ability levels and gameplay preferences. Estimates reveal three latent segments of players: *premium enthusiasts* who derive enjoyment from play itself and have the lowest price sensitivity; and *win-seekers* and *progress-seekers* who both find playing the game itself costly and have higher price sensitivity– the former primarily values immediate rewards, while the latter also values level-up rewards. We investigate three types of personalized interventions: targeted discounts from the monetization front; dynamic difficulty adjustment (DDA) from the retention front; and game environment change through DDA – enabling firms to harness the positive feedback loop between retention and monetization.

Key words: gaming environments, retention, monetization, CRM, targeting, dynamic structural model

---

* We thank the game company (which prefers to remain anonymous) for the data and hosting the first author as an intern. The first author is deeply indebted to Jiwoong Shin and Ryungha Oh for their unstinting support and insights; she thanks Joonhwi Joo, Fei Teng, Ankit Sisodia, and Minkyung Kim for sharing resources and providing helpful feedback. The authors thank Soheil Ghili, Vineet Kumar, and the participants at the Yale Quantitative Marketing Brown Bag Seminar and the 2023 AMA-Sheth Foundation Doctoral Consortium for their helpful comments.

1. Introduction

The gaming industry continues to gain in importance both in terms of share of consumer time and money. As of March 2024, an estimated 59% of consumers spend more than an hour a day on video games, with 32% claiming to spend more than 5 hours a day.[1] Worldwide revenues for the video games market in 2023 is estimated at $250 billion and projected to grow over 8% a year. Revenue per user for mobile games is estimated at $53 in 2023.[2] The advancement of virtual and augmented reality technologies is expected to further accelerate consumer spending and time in virtual environments.

Despite its massive size in ongoing revenue and user growth, there has been relatively limited research on engagement and monetization strategies for gaming environments. The context of gaming environments we study is particularly interesting because firms often monetize players' motivation to exert effort by providing them an opportunity to purchase in-game items that enhance players' ability to win, so that players can endogenously adjust their level of difficulty at a monetary cost. The core of the customer management problem in such environments is to ensure that the game is difficult enough to encourage players to purchase in-game items for progression, yet not so challenging that they do not see the value in purchasing and quit the game.

Our paper is the first to formalize a unique aspect of the retention-monetization dynamics in the context of gaming environment—the two-way positive feedback between retention and monetization. Environments involving a production process, such as gaming, are distinct from conventional retail settings because purchases not only accumulate as a state variable, but also directly enhance agent performance (production). As players' ability to win increases, their retention improves, and their willingness to pay (consumption) for the ability-enhancing items also rises as they encounter higher levels of game difficulty (game environment change). This creates a production and consumption loop in the game environment.

[1] https://www.statista.com/forecasts/997154/hours-spent-on-playing-video-games-per-week-in-the-us

[2] https://www.statista.com/outlook/dmo/digital-media/video-games/worldwide

This conceptual framework allows us to raise novel questions around managing the dynamic interaction between customer retention and monetization. How can the firm better monetize and retain players who have already reached their frontier levels, the points where the challenge exceeds their ability? Should higher-ability players be promoted faster through levels below their ability thresholds? The dilemma for game designers is to manage the retention and monetization in creating an engaging game experience in real-time for players with widely ranging ability levels and gameplay preferences. Namely, the design of interventions should dynamically accommodate players with different levels of ability and gameplay preferences in response to changing game environments.

To answer those questions, this paper develops a dynamic structural model of consumer response for gaming environments, accounting for heterogeneous player ability. In each period, a player decides between three choices: continue playing, purchase in-game items before continuing to play, or exit the game platform permanently. The model captures key aspects that are common in such environments: (i) the level promotion design that increases in difficulty, resulting in player attrition and retaining only the high-performing players, (ii) the option to purchase in-game items that enhance players' effective win probabilities, so that players can endogenously balance their level of challenge at a cost depending on their ability levels, and (iii) short-term rewards for immediate success that sustain interest in the game even as they seek long-term rewards of reaching the next level. Building such a structural model allows us to investigate a range of questions relating to the design of game environments.

We estimate the model using data from individual play and purchase choices in a free-to-play single-player mobile golf game. Our comprehensive dataset includes detailed match-level information on players' actions, environments, rewards, and progression throughout their entire gaming experience. We also obtain detailed records of players' in-game item purchase transactions, which allows us to examine the relationship between the timing of in-game purchases and their impact on player performance. In most settings, individual ability is considered an unobserved variable.

Our game environment context and detailed data enable us to treat player ability as an observed variable by estimating it from the player lifetime gameplay records. This allows us to account for wide varying heterogeneity in player ability and design personalized interventions for player retention and monetization.

Our estimation strategy extends and adapts the two-step estimation framework in Chung et al. (2014). First, we estimate the player production function to obtain player ability estimates and incorporate player ability heterogeneity in the first stage estimation of the conditional choice probability (CCP). We accommodate latent class heterogeneity and use the expectation-maximization (EM) algorithm within the two-step framework, following the approach in Arcidiacono and Miller (2011). We estimate the structural parameters in the second stage estimating the value function for each ability and latent segment type. Estimates reveal three latent segments of players: (i) *premium enthusiasts*, the smallest share of players who spend the most and do not find playing the game costly but rather enjoy spending time in the game, (ii) *win-seekers* who have the second lowest price sensitivity but find playing the game itself costly, and primarily values immediate wins more than long-term level-up rewards, and (iii) *progress-seekers*, the largest share of players who have the highest price sensitivity and find playing the game itself costly to play, but receive higher utility from level-up rewards than *win-seekers*.

Compared to standard retail environments, the key difference in gaming and gamified online environments lies in the dynamic modification of the product (the game). In these environments, firms have the opportunity to dynamically change and offer products in a personalized manner (e.g., Leung et al. 2023, Ascarza et al. 2023). We design three levers of personalized interventions to enhance player retention and monetization dynamics, illustrating how firms can manage the positive feedback loop between retention and monetization. Within this feedback cycle, the first two counterfactuals involve direct interventions on player monetization and retention fronts. The final counterfactual intervenes by changing the game environment to optimize the relationship between player production and consumption.

The first intervention targets player monetization by offering discounts at players' frontier levels – the point where they typically exit the game. In doing so, we demonstrate the positive feedback mechanism between retention and monetization – discounts increase player purchases, which increases player win probabilities and retention, while simulatenously increasing player level progression which can bring additional monetization opportunities as players face greater challenges. While a 70% discount was found optimal, increasing total profit by 6.44%, we find substantial heterogeneity among player segments. Cannibalization was highest among *premium enthusiasts*, who would have made purchases even without the discounts. *Progress-seekers* exhibited the strongest effect for the positive feedback loop, with 6.20% of players making additional non-discount purchases following the discount. However, a notable caveat is that while these discounts can increase player level progression and profitability, they may also interact with the firm's level promotion scheme in a way that could decrease overall retention (i.e., the total number of games played). As players purchase items and perform better, they advance through levels more quickly, reaching their exit points sooner. Thus, while targeted discounts can generate short-term profit gains, they may inadvertently shorten the overall engagement period for some players. This reduction in overall retention and potential loss in long-term profits is most evident among *premium enthusiasts*, who derive enjoyment from the act of playing itself.

Second, we investigate how retention-driven interventions, through dynamic difficulty adjustment (DDA) at player level frontiers, impact the feedback loop between retention and monetization. DDA is a common strategy used by game developers to continuously tailor game challenges and features to match the players' abilities and preferences in real-time, aiming to keep players engaged by ensuring the game is neither too easy nor too difficult. Therefore, understanding player heterogeneity in both observed and unobserved domains is critical to ensure DDA effectiveness. In addition to providing evidence of the positive feedback loop from retention to monetization, we highlight the importance of incorporating ability and unobserved preference heterogeneity in DDA design. To assess the incremental impact of incorporating player ability and gameplay preference heterogeneity in designing timing of interventions, we compare three different frontier level targeting strategies: (i) ability-based targeting, (ii) gameplay preference-based targeting, and (iii) targeting using both ability and gameplay preferences. We show that accounting for both ability and unobserved gameplay preference heterogeneity leads to more than a twofold increase in profits compared to using only ability or preference heterogeneity. By accounting for individual differences in ability and preferences, DDA can be more precisely tailored, thereby enhancing its effectiveness in retaining players and fostering their continued monetization.

Our final counterfactual continues to build on the key concept of DDA and examine the effect of changing the game progression speed by appropriately increasing player win probability at early levels for high ability players. By accelerating early level progression for these players, we seek to optimize the player production and consumption dynamics, providing more challenging game content sooner. Overall, accelerating early level progression for high ability players increases total profit and player retention, but we find heterogeneous effects across latent segments, even among the players with the same ability level. The firm gains in profit from players who find playing the game itself more costly but receive higher utility from level progression (*progress-seekers*) by 10%, but loses in profit from players who find playing the game itself more enjoyable (*premium enthusiasts*) and players who receive higher utility from immediate winning (*win-seekers*). This underscores the importance of accounting for unobserved preference heterogeneity in targeted intervention design.

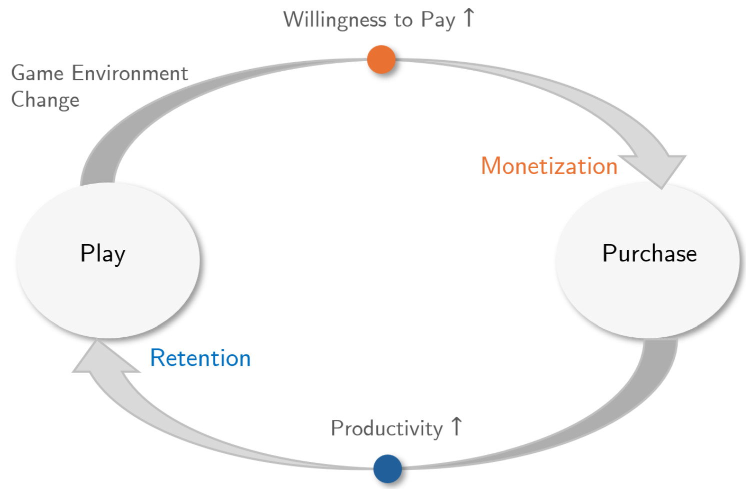
2. Related Literature

Our paper contributes to several streams of literature. First, we contribute to the growing literature on the video games market. The first stream of papers in this literature studies consumer demand for console video games (e.g., Clements and Ohashi 2005, Nair 2007, Derdenger and Kumar 2013, Derdenger 2014, Haviv et al. 2020). The shift in consumer focus to the online and mobile gaming landscape has enabled researchers to collect and utilize detailed usage records. This has given rise to the second stream of empirical papers that studies user engagment and gameplay behavior (e.g., Huang et al. 2019, Nevskaya and Albuquerque 2019, Rutz et al. 2019, Zhao et al. 2022, Castelo-Branco and Manchanda 2023, Chen 2023), monetization strategy and welfare (Appel et al. 2020, Huang et al. 2021b, Runge et al. 2022, Amano and Simonov 2023, Ascarza et al. 2023, Haenlein et al. 2023, Joo and Chiong 2023, Wang et al. 2023), and analytical papers studying game and in-game product design (e.g., Guo et al. 2019, Jiao et al. 2021, Chen et al. 2021, Li et al. 2023a, Li et al. 2023b, Mai and Hu 2023, Miao and Jain 2024, Sheng et al. 2024). Despite of the significant and growing market size of the gaming industry, there is a lack of research documenting consumer spending behavior on in-game items, which are the most common and major driving source of revenue in the gaming market. Although few studies have empirically studied in-game retention and monetization design involving in-app purchases (e.g., Huang et al. 2021b, Runge et al. 2022, Amano and Simonov 2023, Ascarza et al. 2023, Joo and Chiong 2023, Wang et al. 2023), our paper is first to articulate the key conceptual idea inherent in such gaming environments with a structural model: that purchases (consumption) enables greater performance (production) and makes future engagement possible – the two-way positive feedback mechanism between retention and monetization.

Traditionally, monetization (purchase) leading to retention and continued engagement (repeat purchase) has been extensively studied in the literature on customer loyalty. In this literature, loyalty is generally recognized as an asset that accumulates (Aaker 1992), in which purchases are influenced by previous purchase histories or past awareness (Wernerfelt 1991), alternatively referred as state dependence (e.g., Guadagni and Little 1983, Erdem 1996, Keane 1997, Seetharaman et al. 1999, Dubé et al. 2010). A large body of studies have investigated the relationship between customer loyalty and firm profitability (e.g., Reichheld and Teal 1996, Reinartz and Kumar 2000, Kumar and Shah 2004, Helgesen 2006, Shin and Sudhir 2010, Lei et al. 2024).

Our research contributes broadly to the existing CRM literature by recognizing a two-way feedback relationship between retention and monetization in the context of gaming environment. The gaming environment is unique because purchases in this setting not only accumulate as a state variable, but directly lead to an increase in player performance (production). This in turn increases player retention, while simultaneously increasing players' willingness to pay (consumption) for the

Figure 1 The Production and Consumption Loop in the Gaming Environment



next item as players reach higher levels of game difficulty (game environment change). We illustrate our conceptual framework in Figure 1.

Our framework extends to empirical settings that involve a production process, where agents exert effort to achieve performance goals. This includes various contexts such as salesforce management, digital health, and online education. For example, an agent's achievement state in relation to sales compensation rewards can motivate production (effort), which in turn increases monetization for the firm (Chung et al. 2014), and sales training (increasing production) can be used to manage salesforce retention and performance (Chung et al. 2021). Temporary incentives to curb smartphone addiction improve users' self-control ability (production of habit formation) and can have lasting effects in their well-being (Allcott et al. 2022). Premium app version adoption (consumption of $) can help increase future user engagement in mHealth applications (Jiang et al. 2023). Training learners (increasing production) to follow self-regulated learning strategies can increase learning outcomes (Santhanam et al. 2008). Our paper presents a comprehensive framework for understanding the retention and monetization dynamics present in settings with game-like environments.

Compared to standard retail environments, gaming environments enable the collection of more detailed data on the ongoing engagement and monetization of users. While other digital and freemium products such as Dropbox also have consumer usage data, the difference in gaming and gamified online environments is the dynamic modification of the product (game). Within this class of products, firms have the opportunity dynamically change and offer products in a personalized manner. The personalization principles developed in the context of gamified systems claim that accounting for individual heterogeneity can increase engagement and task outcomes (Liu et al. 2017). Such has been documented in gamified settings such as e-training and online learning environments (e.g., Santhanam et al. 2016, Huang et al. 2021a, Huang et al. 2023, Leung et al. 2023).

In the context of gaming, dynamic difficulty adjustment (DDA) is a widely used method by game designers to adaptively modify game challenges and features real-time based on players' abilities and preferences (e.g., Hunicke 2005, Xue et al. 2017, Zohaib 2018, Huang et al. 2019, Zhao et al. 2022). To our knowledge, Ascarza et al. (2023) is the first marketing paper that studies DDA interventions with implications for games that use an in-app purchase monetization model. Using a field experiment, the authors suggest evidence consistent in showing the value of designing personalized difficulty interventions for player retention and monetization.

While one approach to personalizing the redesign is through experimental variations, structural models can complement experimental approaches by providing a theoretical understanding of the dynamics and in cost-effectively assessing which sets of design interventions are likely to yield the best payoffs. Additionally, the presence of unobserved heterogeneity at the player level (e.g., price sensitivity, motivation sensitivity) complicates the implementation of personalized interventions based solely on observables. Our model framework allows us to dynamically redesign consumer engagement and monetization in a personalized manner.

3. Data and Empirical Setting

This section describes the game, provides details of the data, and presents model-free evidence that informs our model development.

3.1. Description of the Game

Our empirical setting is a popular free-to-play single-player mobile golf game with over 2 million registered users. In this game, players engage in one-hole game matches, where the objective is to complete the hole with fewer shots than the opponent. Each game lasts around three to five minutes, and players are assigned matches by the game platform once entering the game. Players accumulate points from winning the match (and lose points from losing), and collection of these points is required to unlock higher levels in the game. The game design ensures a sequential progression where higher levels demand the accumulation of more points to be unlocked. The game has a total of 11 levels.

The level-progression system of the game is designed to increase difficulty through several design features. We report the points system of the game in Table 1. The expected points for each game given a player's win probability are calculated based on the game's win-loss points schedule, which varies by level. We illustrate the level difficulty system embedded within this points system using levels 6 to 11 as examples in Figure 2. In the figure, each line represents a different level, demonstrating that as players advance to higher levels, the expected points for a given win probability generally decrease under the same win rate. Figure 3a helps further illustrates this point by highlighting that as the level increases, the win probability required to at least break even (i.e., zero expected points) also increases, requiring players to have higher win rates at subsequent levels. Finally, higher levels require higher points accumulation criterion to level up, making progression increasingly demanding, as shown in Figure 3b. Overall, the points system of the game is designed to discriminate on player ability, ensuring that only the most able players progress to the top levels.

Players can enhance their win rates by purchasing in-game items. The in-app purchases consist of durable ability-enhancers (i.e., golf clubs) that allow players to improve their win rates. For example, a paid golf club enables greater range and ball guide precision when taking a shot. By incurring a monetary cost, players can increase their chances of progressing through the levels that they might not have had otherwise. By allowing players to self-select the balance between the need to win and the cost of purchasing ability enhancers, the firm can effectively monetize across different player segments of different abilities and price sensitivity. The most popular in-game item offerings cost \$9.99 (generating around 60% of revenue), and around 90% of total durable item

Table 1 Points and Level Progression Design of the Game

Level	Win	Lose	Total Available Points	Cumulative Points Collection
1	+4	-1	25	-
2	+6	-2	75	25
3	+8	-3	125	100
4	+10	-4	175	225
5	+12	-7	225	400
6	+14	-10	300	625
7	+16	-13	375	925
8	+18	-16	450	1300
9	+20	-20	550	1750
10	+22	-24	700	2300
11	+24	-36	900	3000
Final Lvl Clear				3900

transactions are generated from product offerings between \$9.99 and \$19.99. Higher-priced offerings provide more quantities of golf clubs that can help enhance player win rates. While the game also features in-app advertising, the firm primarily generates revenue through in-app purchases.

The game offers a suitable setting for a single-agent model. From the player perspective, it is primarily a single-player experience, in that players do not strategically select which opponent to play against, but rather get assigned by the platform.[3] The progression system of the game incentivizes players to continually improve their skills to maintain or increase their win rates and progress through the levels, inducing a dynamic and forward-looking behavior.

3.2. Data

We leverage a unique dataset that consists of granular match-level information about players' actions, environment, rewards, and progression throughout the player's lifetime from initiation to exit. This includes detailed observations of players' play and exit decisions, in-game item usage, and the gaming environments they encounter, including the opponents they face. We also obtain

[3] In the game, around 40% of gameplay matches consist of bots. To ensure the game is neither too easy nor too difficult, players are typically matched with opponents whose Elo scores, the rating system used by the company for calculating the relative skill levels of players, are slightly below their own. In our model, we control out the effect of the opponent and focus on the player's actions and decisions as a single agent.

Figure 2 **Game Points Design: Expected points by Win Probability by Level**

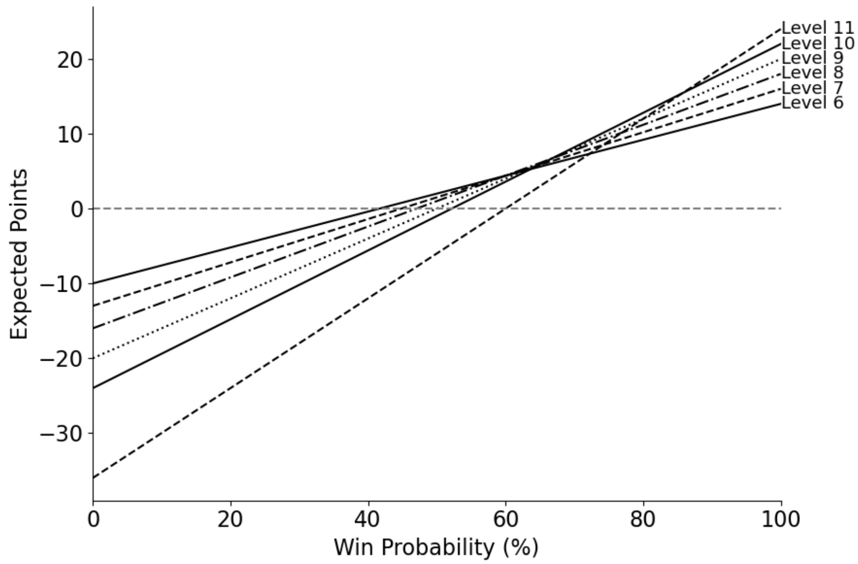
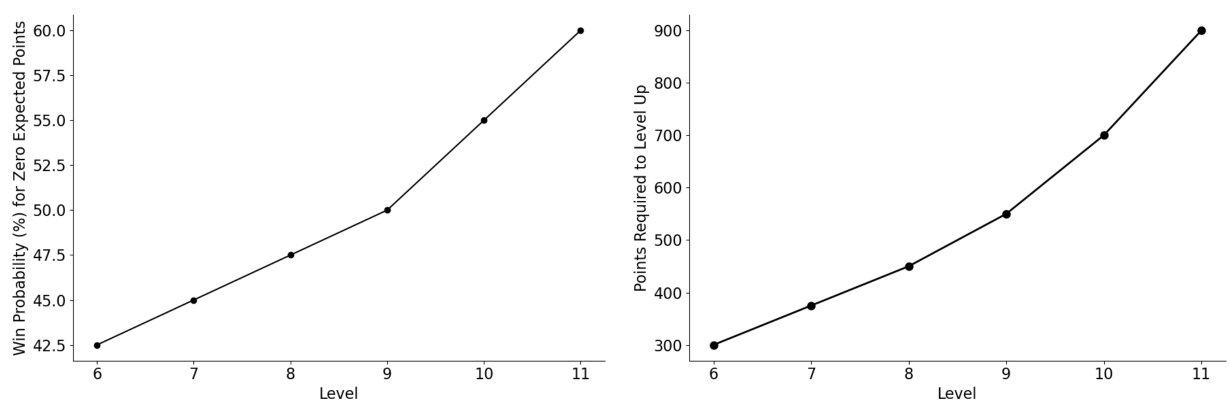


Figure 3 **Game Points and Level-Progression Design**

(a) Win Probability for Zero Expected Points by Level **(b) Points Collection Requirement to Level-Up**



detailed records of players' in-game item purchase transactions ($), which allows us to examine the relationship between the timing of in-game purchases and player performance.

Our analysis focuses on a random sample of 4163 players spanning a 15-month period, from October 2021 to January 2023. We construct the sample as follows. First, we take a random 10% sample of players who meet the criteria of having valid play records in the data period. Second, players who have completed at least level 5 (i.e., collected at least 15% of total points available) are retained. This criterion ensures that our analysis focuses on individuals who exhibit a sufficient level of engagement within the game environment, while excluding less committed players who frequently download the game but discontinue usage shortly thereafter. Finally, we exclude outliers with within-level gameplays above 1.5 times the interquartile range. The final dataset for analysis includes around 750,000 match records.

Table 2 contains summary statistics of our player sample. A median player engages in 274 games throughout their lifetime, and the median exit level is 7.[4] The average game play duration, measured by the number of days from first play to exit, is 106.21 days. Finally, given our sample, which includes players with a minimum progression to level 6, 22.36% have purchased in-game items at least once. While the median player makes no purchases, there is significant heterogeneity in the player purchase behavior at the higher end of the distribution, with total transactions ranging between 14 and 41 for players in the 99th percentile and above.

Table 2 Descriptive Statistics of Players

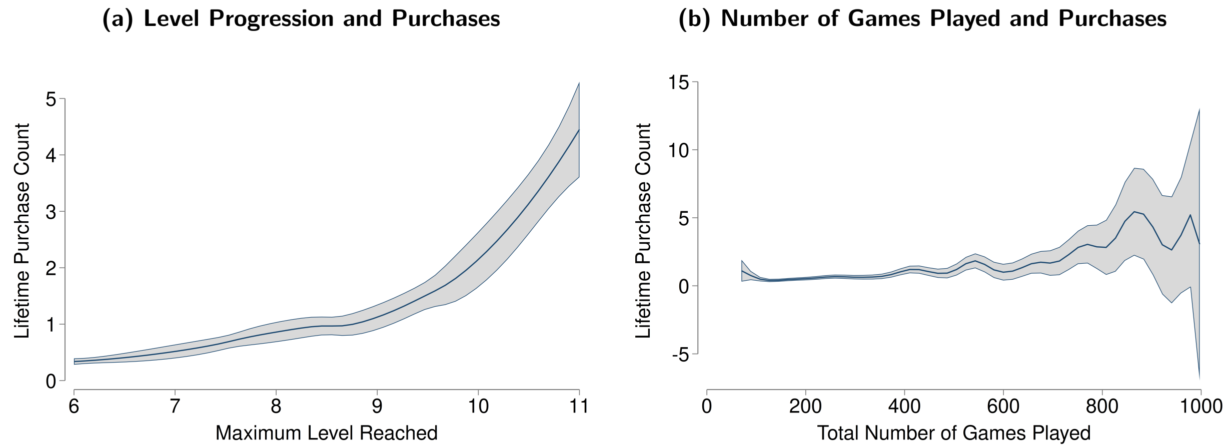
	Mean	SD	Median	75th	90th	99th	Max
Engagement							
Total Number of Games Played	331.57	225.18	274	406	583	1203	3211
Maximum Level Reached	7.41	1.54	7	9	10	11	11
Game Duration (*Days from First Play to Exit*)	106.21	98.55	73	151	259	405	454
Monetization							
In-app Purchase Player Share	22.36%						
Total Number of Purchase	0.88	2.69	0	0	3	13	41

3.3. Model-Free Evidence

We begin this section by providing descriptive evidence of the positive relationship between player retention and monetization. We plot players' final level progression state with their lifetime number of purchases in Figure 4a. We show that players who have reached higher levels are also those who spend more. We similarly shown in Figure 4b that players who play more are generally those who make more purchases. Such positive relationship between player level progression and retention on monetization provides suggestive evidence of the feedback loop between retention and monetization, but is difficult to rationalize without a formal model.

[4] To define player exit beyond our data period, we apply the two-week churn condition.

Figure 4 Relationship Between Player Retention and Monetization



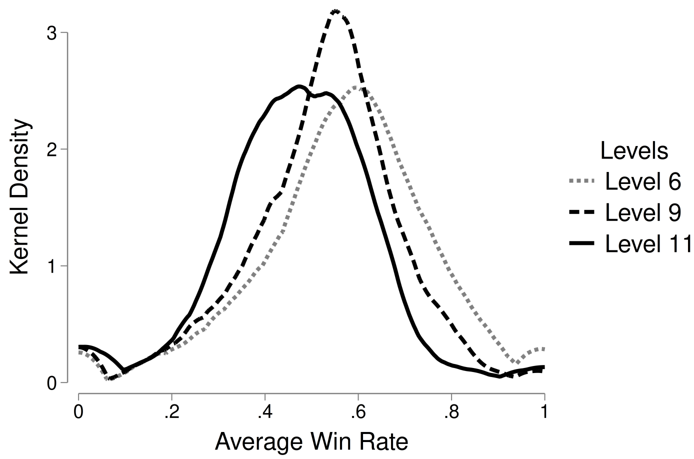
Note: The solid line is a kernel-weighted second-degree polynomial regression using a Gaussian kernel, and the shaded area is the 95% confidence interval.

Next, we present three key features of the data that inform our model development. First, we provide evidence of substantial heterogeneity in player win probability within and across levels. Second, we show the relationship between players' production rate (win probability) and their purchase and exit decisions. Third, we demonstrate the dynamics in the timing of players' purchase and exit decisions in relation to their level completion status.

In the gaming environment, there exists substantial heterogeneity in the player rate of production, indicating their ability to progress through the game levels. We present the player differences in average win rate across players for different levels in Figure 5. First, players have generally lower win rates at higher levels, reflecting the increased level difficulty designed into the game. Within each level, there exist significant differences in player win probabilities, providing suggestive evidence of the varied abilities of players even at the same level. We report the Gini indices of player average win rate within each level in Table 3. We note that even small variations in win rates result in considerable differences in expected points rewards and player progression speed, as shown previously in Figure 2.

We next examine the relationship between players' win probability and their purchase and exit decisions, as illustrated in Figure 6. To prevent reverse causality (i.e., players purchasing items which then increase their win probability), we plot player purchase and exit decision against their

Figure 5 Average Win Rate Distribution of Players By Levels



Note: The lines represent kernel density estimates using the Epanechnikov kernel.

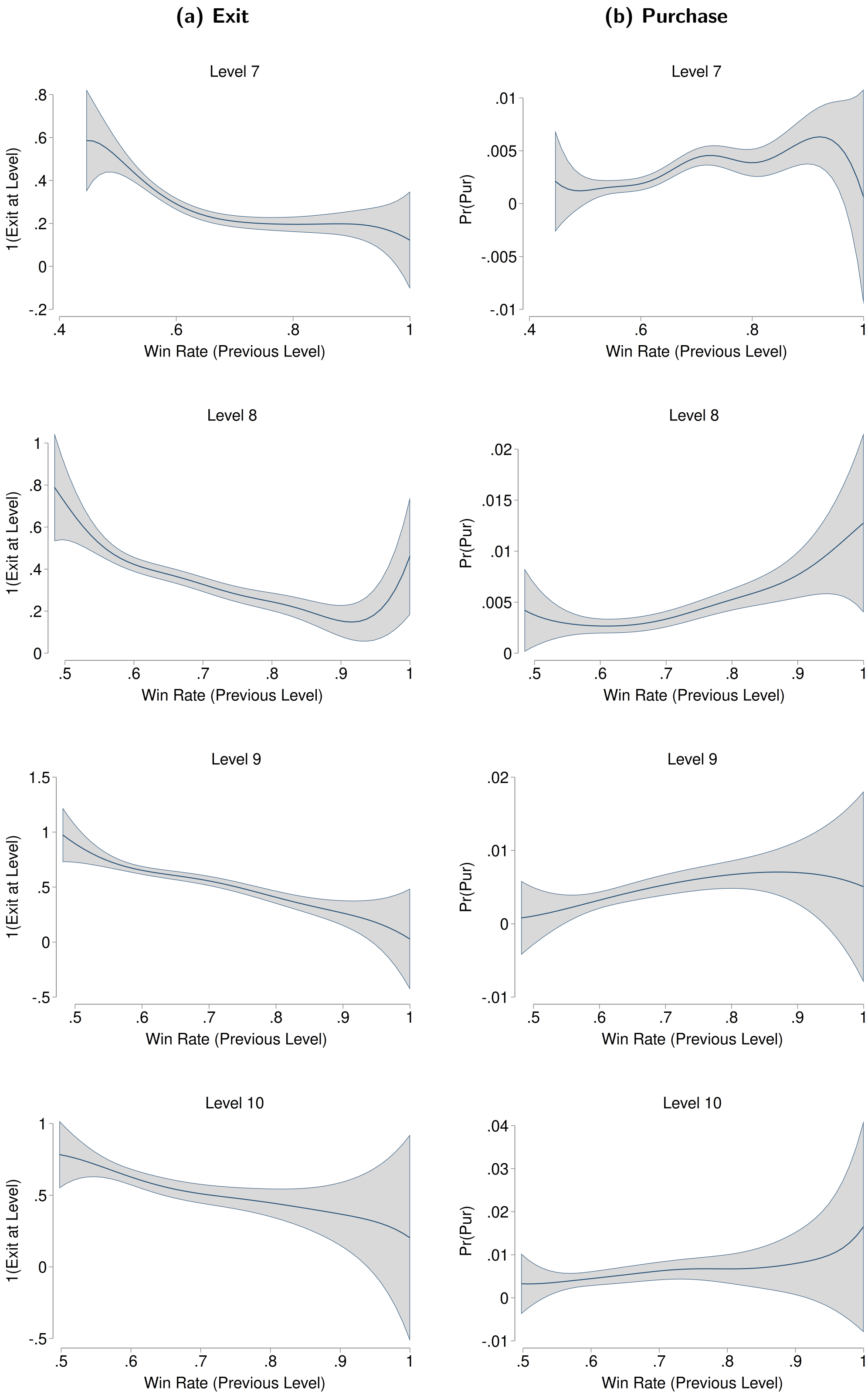
Table 3 Gini Indices of Player Average Win Rate by Level

Level	Gini Index
6	0.18
7	0.18
8	0.13
9	0.18
10	0.17
11	0.20

win rates from the previous level. Within each level, we find that players with higher win probabilities are more likely to continue playing and make in-game purchases as they gain positive continuation value in the gaming experience. Conversely, players with lower win probabilities are more likely to exit the game and purchase less, as their lower production rates can reduce their engagement and the perceived value of the game. This relationship underscores the importance of maintaining a balance in-game difficulty to retain players and encourage in-game spending.

Finally, we show the relationship between the timing of players' purchase and exit decisions and their level completion status. Within each level, player exits are more likely to occur at the beginning when players face increased difficulty after level-up. The durable nature of items encourages upfront purchases, typically around 20-30% into a level, when players believe they have a chance to progress. This suggests that within a level, players are more likely to purchase in-game items early on to enhance their rate of progression.

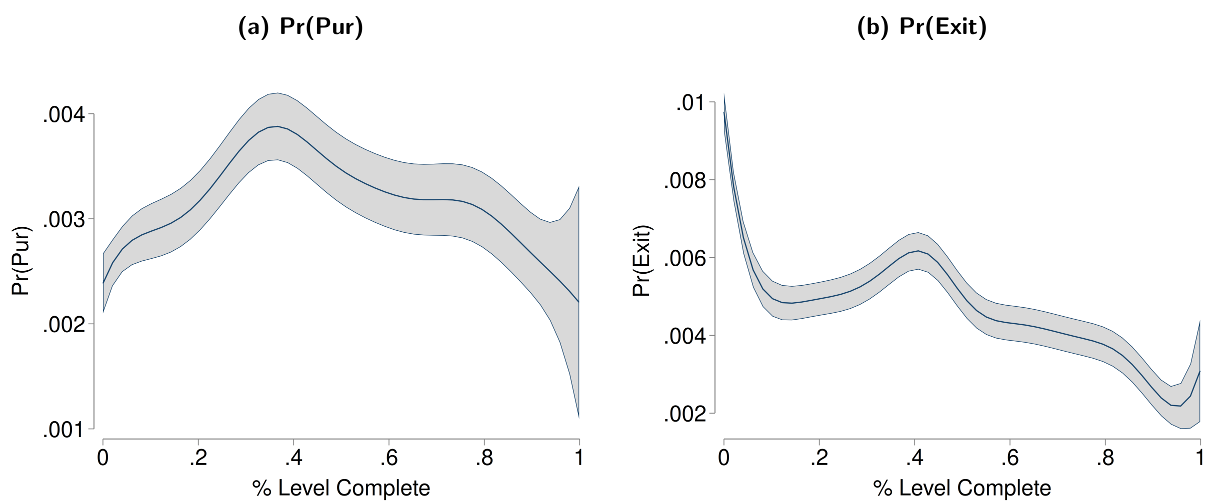
Figure 6 The Production and Consumption Loop: Player Exit and Purchase by Win Rate



Note: The solid line is a kernel-weighted second-degree polynomial regression using a Gaussian kernel, and the shaded area is the 95% confidence interval.

Overall, these patterns inform key features underlying the structural model – (i) ability heterogeneity; (ii) how player exit and purchase decisions relate to ability; and (iii) the timing of player purchase and exit. We highlight the importance of understanding player heterogeneity in ability, and timing interventions by ability and level progression states to manage the dynamics between retention and monetization dynamics.

Figure 7 Player Exit and Purchase by % Level Complete



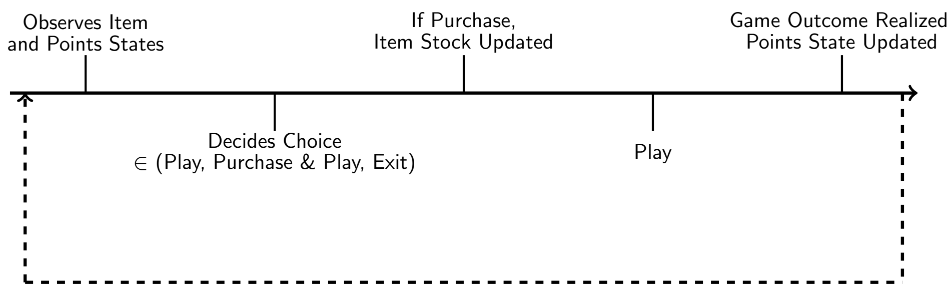
Note: The solid line is a kernel-weighted second-degree polynomial regression using a Gaussian kernel, and the shaded area is the 95% confidence interval.

4. Model

Based on the model-free evidence, we develop a dynamic model of player action in the gaming environment. Time indexed by t is discrete denoting each game. At the beginning of each period t, player i decides whether to (1) play, (2) make an in-app purchase and play, or (3) exit the game permanently. Figure 8 describes the timing of the model. Before entering a game t, player i observes the current item and progression (i.e., accumulated points) state in the game. The player decides on the action choice between playing, purchasing an item and playing, and exiting the game permanently. If the player's decision was to incur the purchase of in-game items, the item stock is updated before play. The player then enters the game, and idiosyncratic game outcome shock is realized. The realized game outcome affects the player's progression points state in the next period. The model repeats every period over an infinite horizon. After the player completes all levels, the player can continue to play the unlocked stages, which is a pattern we observe for the majority of players in the game.

The game's point system and progression design induce the player's dynamic forward-looking behavior. A player choosing the action has to be concerned not just with the current payoff but with the effect of that choice on all future rewards and play costs.

Figure 8 **Model Timeline**



4.1. Player Ability and State Variables

To model player progression and item purchase decisions, we assume that the player has a time-invariant ability type α_i. Players can improve their ability over the course of the game by making in-app purchases. To represent player states in the gaming environment, we parsimoniously track two key state variables. First is player item stock k_{it}, which tracks the number of in-app purchases a player incurred to improve their ability. It represents the vertical quality metric of the player's in-game item state. Second is player progression points stock z_{it}. The points accumulation state (z_{it}) has a one-to-one mapping to the level (ℓ_{it}), given the progression design Ψ of the game, reflecting the player's current progression state in the game.

4.2. Player Production Function

The player winning outcome $W_{it} \in \{0,1\}$ is a function of player ability type α_i, item k_{it}, and level states ℓ_{it}, such that

$$W_{it} = \alpha_i + \delta_1 k_{it} + \ell(z_{it}) + \delta_2 k_{it} \cdot \ell(z_{it}) + \xi_{it}. \tag{1}$$

The player production function accommodates the following key characteristics to model the gameplay outcome. First, the player's win probability increases with the player's ability α_i. Second, items k_{it} enhance player productivity. Third, the player's win probability decreases with increasing level difficulty ℓ_{it}, which is determined by the point accumulation state z_{it}. We allow for item and level interaction to account for the diminishing effect of items at higher levels. Finally, the idiosyncratic shock ξ_{it} affects the outcome of the game.

4.3. Flow Utility

Player i in period t receives the following per-period utility based on his or her choice of actions $d_{it} \in \{1,2,3\}$: whether to play the game without incurring in-app purchase ($d_{it} = 1$), whether to make in-app purchase and play the game ($d_{it} = 2$), or whether to permanently exit the game platform ($d_{it} = 3$).

$$u(S_{it}, d_{it}) = \begin{cases} \theta E[r_{it}(\alpha_i, s_{it}(d_{it}))] + c_m \cdot \mathbf{1}_{\{d_{it}=2\}} + c_p + R \cdot \mathbf{1}_{\text{LEVEL UP}(z_{it})} \cdot L_{\ell(z_{it})} + \epsilon_{it}, & \text{if } d_{it} \in \{1,2\}, \\ 0 & \text{otherwise (exit).} \end{cases}$$

Players derive utility from immediate rewards r_{it} (i.e., expected number of points gained from the current period play), and one-time rewards R for reaching a new level ℓ at time t.The per-period player points reward r_{it} is determined by the game outcome (i.e., win or loss) and the winning and losing points reward design of the game, $(\psi_v^\ell,\, \psi_d^\ell) \in \Psi$, which differs by each level ℓ. That is,

$$r_{it} = W_{it} \cdot \psi_v^\ell + (1 - W_{it}) \cdot \psi_d^\ell \tag{2}$$

and

$$E[r_{it}] = \Pr(W_{it} = 1) \cdot \psi_v^\ell + \Pr(W_{it} = 0) \cdot \psi_d^\ell \tag{3}$$

To model the increasing utility from leveling up at higher levels, we adjust R with respect to the level achievement criterion (i.e., the number of points required to level up), i.e., $L_{\ell(z_{it})}$. Since the level-up points criterion increases monotonically with levels in the current game design, players receive greater rewards for achieving higher levels. The idiosyncratic shock ϵ_{it} follows an extreme value distribution.

If the player decides to purchase in-game items ($d_{it} = 2$), he or she incurs a monetary cost c_m. The decision to purchase updates the player's current item stock state, which increases the chance of winning including the immediate period and all future play sequences of the game. We normalize the exit value as 0.

4.4. State Transitions

The state variables item stock k_{it} and points stock z_{it} evolves deterministically as follows:

$$k_{it} = \begin{cases} \min(k_{it} + 1, \overline{K}) & \text{if } d_{it} = 2 \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

The player's item stock k_{it} increments by one with each purchase decision. Players can make multiple purchases over their lifetime. $\overline{K}$ is the maximum item stock, representing the maximum item quality upgrade level.

$$z_{it} = \begin{cases} \min(z_{it} + \psi_v^\ell, \overline{\psi}^L) & \text{if } W_{it} = 1 \\ \max(z_{it} - \psi_d^\ell, \underline{\psi}^\ell) & \text{otherwise} \end{cases} \tag{5}$$

Upon realization of the game outcome, the player's points stock state z_{it} evolves deterministically following the game's points progression design Ψ. Here, $\overline{\psi}^L$ is the maximum points state of the game at the final level and $\underline{\psi}^\ell$ is the points criterion to unlock level ℓ. This ensures that once a player has reached a certain level by accumulating enough points, they cannot fall back below that level's threshold, ensuring that once a level is unlocked, it remains unlocked. In other words, z_{it} is incremented by ψ_v^ℓ when winning the game, but cannot exceed $\overline{\psi}^L$. Similarly, z_{it} is reduced by ψ_d^ℓ but not below the minimum threshold $\underline{\psi}^\ell$.

4.5. Bellman Equation

A player chooses action d_{it} that maximizes the expected discounted sum of utilities given the game design Ψ, the state variables and their transitions, and the idiosyncratic shock ϵ in each period. The Bellman equation can be written as

$$V(k,z,\alpha,\epsilon;\Theta,\Psi) = \max\{0, \max_{d\in\{1,2\}}\{U(k',z,\alpha,\epsilon;\Theta,\Psi) + \beta\mathbf{E}[(V(k'',z',\alpha,\epsilon';\Theta,\Psi)|k',z,\alpha]\}\} \tag{6}$$

where the idiosyncratic shock ϵ follows a Type-I extreme value, and the discount factor β is set to 0.9.[5] The player continues to play the game if the expected continuation value is greater than 0, the normalized outside value of exit.

5. Estimation

We estimate the model using two-step estimation (Hotz and Miller 1993). In the first step, we estimate the production function and the conditional choice probabilities (CCPs) of player action choice as a flexible function of state variables. The key assumption in the two-step estimation is that the first-stage CCPs represent the agent's optimal action probability given the state variables. In the second step, we estimate the structural parameters that rationalize the first stage policy estimates.

5.1. Step 1: Estimating CCPs

In the first stage CCP estimation, we estimate a flexible mapping between observable states and player action probability. The relevant state variables in our model are item and points stock states, $S_{it} = \{k_{it}, z_{it}\}$. We estimate the two-step procedure using the player state space starting from level 6, both for computational efficiency in the state space and practical reasons. The early levels 1-5 consist of beginner level tutorials and short level length, with these five levels accounting for only 15% of total points collection available in the game. We discretize the points state for each level

[5] $\beta = 0.9$ gives the best fit compared to β= 0.95, 0.99, and 0.999. In our context, setting lower discount factors greatly underestimates player exit probabilities and hence overestimates player purchase probabilities. Average inter-play time gap (in day) is around 2 days, and average lifetime play duration is 106 days.

from 6 to 11 into 10 increments and additional transition states to track level-up bonus (a total of 65), and item stock state to evolve deterministically up to 25 transactions. This leaves us with a total of 1690 state combinations.

Typically, player ability is treated as an unobserved variable. In order to incorporate rich player heterogeneity in win rates as shown in the model-free evidence, we instead treat player ability as an observed variable by estimating it from player lifetime gameplay records. Because our setting is a game, where we observe every player action, environment, and outcome, we can estimate player ability α_i as an individual fixed effects parameter from Equation 1, the player production function, controlling for the effects of items and game environments on player win rates. By incorporating player ability as an observed variable, we can directly include it in our first stage policy estimation. This enables us to account for rich observed heterogeneity in player ability levels in predicting player action. For the first stage estimation, we normalize player ability score as a continuous variable between 0 and 1, with 1 representing the highest ability level.

Given the state variables and observed ability heterogeneity of the players, we estimate the player action policy using a flexible multinomial logistic regression. We account for player unobserved heterogeneity in the first stage CCP estimation through persistent latent segments and estimate heterogeneous policy functions using the EM algorithm (Arcidiacono and Miller 2011, Chung et al. 2014). We assume that player i belongs to one of G segments $g \in \{1, ..., G\}$ with segment probabilities $q_i = \{q_{i1}, ...q_{iG}\}$. Let π_g denote the population probability of being in segment g. We iteratively maximize the log likelihood in Equation 7,

$$\sum_{i=1}^{N} \sum_{g=1}^{G} \sum_{t=1}^{T} q_{ig} \ln[\mathcal{L}(d_{it}|S_{it}, \alpha_i, g, \beta_g)], \tag{7}$$

where

$$q_{ig} = \frac{\pi_g \prod_{t=1}^{T} \mathcal{L}(d_{it}|S_{it}, \alpha_i, g, \beta_g)}{\sum_{g=1}^{G} \pi_g \prod_{t=1}^{T} \mathcal{L}(d_{it}|S_{it}, \alpha_i, g, \beta_g)}, \tag{8}$$

and $\mathcal{L}(d_{it}|S_{it},\alpha_i,g,\beta_g)$ is the choice probability of taking action $d_{it}=j$ for segment type g,

$$\mathcal{L}(d_{it}|S_{it},\alpha_i,g,\beta_g)=\frac{e^{\beta_g^j[S_{it},\alpha_i]}}{\sum_j e^{\beta_g^j[S_{it},\alpha_i]}}. \tag{9}$$

The EM algorithm begins by setting initial values for β_g, and π_g.

(a) Compute $q_{ig}^{(m+1)}$ using Equation (2) with $\beta_g^{(m)}$ and $\pi^{(m)}$.

(b) Update population shares

$$\pi_g^{(m+1)}=\frac{1}{N}\sum_{i=1}^{N}q_{ig}^{(m+1)}.$$

(c) Update $\beta_g^{(m+1)}$ for each segment g by maximizing Equation (1) with $q_{ig}^{(m+1)}$ and $\pi_g^{(m+1)}$.

We iterate steps (a)-(c) until convergence. We initialize β_g by randomly partitioning the players into G segments and maximizing the log-likelihood, and population shares to be $1/G$.

From this iterative estimation step, we obtain segment-specific policy function parameters, along with the population segment probability estimates π. We use the segment-level policy functions to obtain structural parameters of each segment, which we describe in the next section. A caveat with the two-step estimation is that the first-stage policy function estimates can be biased if the state variables in the policy function are correlated with the first-stage errors. Our approach, which leverages rich observed and unobserved player heterogeneity, helps mitigate this issue.

5.2. Step 2: Structural Parameter Estimation

The key idea of the two-step estimation is to represent the value function in terms of the policy function estimated in first stage, which reflects the player's optimal actions. That is, $V(\cdot;\theta)=h(P(\cdot;\theta),\theta)$ and $P(\cdot;\theta)=g(V(\cdot;\theta),\theta)$. Given our discrete state space, we can solve the value function as a system of linear equations,

$$V(S_{it},\alpha_i,g;\Theta,\Psi)=(I-\beta F)^{-1}\left\{\sum_{d_{it}\in\{1,2,3\}}P(S_{it},\alpha_i,g;\Theta,\Psi)\cdot[u(S_{it},\alpha_i,g,d_{it};\Theta,\Psi)+E[\epsilon|d_{it}]\right\} \tag{10}$$

where F is the matrices of transition probabilities corresponding to action d_{it}. The Type I extreme value assumption of the error term allows us to solve the value function analytically, such that $E[\epsilon|d_{it}]=\gamma-\ln(P(\cdot;\Theta))$. γ is the Euler's constant.

Furthermore, we can express the player's choice probability of action d_{it} under the structural parameters of our model in closed-form using the distribution assumption of the errors as follows:

$$\Pr(d_{it}|S'_{it}, \alpha_i \epsilon; \Theta, \Psi) = \frac{\exp\left(u(d_{it}, S'_{it}, \alpha_i, \epsilon; \Theta, \Psi) + \beta E_{S''_{it}, \alpha_i, \epsilon'; \Theta, \Psi | d_{it}, \epsilon} V(S''_{it}, \alpha_i, \epsilon')\right)}{\sum_{\tilde{d}_{it} \in \{1,2,3\}} \exp\left(u(\tilde{d}_{it}, S'_{it}, \alpha_i, \epsilon; \Theta, \Psi) + \beta E_{S''_{it}, \alpha_i, \epsilon'; \Theta, \Psi | \tilde{d}_{it}, \epsilon} V(S''_{it}, \alpha_i, \epsilon')\right)}. \tag{11}$$

We construct the moment equality estimator using the following moment condition, where $\hat{Pr}$ is the optimal policy estimated from the first stage, and $\tilde{Pr}$ is the policy informed by the model parameters. We minimize equation 12, the distance between optimal policy and the model choice probabilities, weighted by player segment probabilities q_{ig}.

$$\begin{aligned} \sum_{i=1}^{N} \sum_{t=1}^{T} q_{ig} \Big[& (\hat{Pr}(d_{it} = 2 | S_{it}, \alpha_i, g, \epsilon; \beta_g) - \tilde{Pr}(d_{it} = 2 | S_{it}, \alpha_i, g, \epsilon; \Theta_g, \Psi))^2 \\ & + (\hat{Pr}(d_{it} = 3 | S_{it}, \alpha_i, g, \epsilon; \beta_g) - \tilde{Pr}(d_{it} = 3 | S_{it}, \alpha_i, g, \epsilon; \Theta_g, \Psi))^2 \Big] \end{aligned} \tag{12}$$

For the second stage model estimation, we discretize player ability type variable into 20 bins and estimate the value function for each ability type and segment. To compute standard errors, we generated 500 bootstrap datasets following Bajari et al. (2007). For each bootstrapped dataset, we estimate both the first and the second stage to account for the estimation errors from the first stage policy estimation.

5.3. Identification

There are a few challenges in identifying the dynamic structural model of game playing. First, the intrinsic player ability is not observed. Our long panel of gameplay records and the variations in environments, item quality, and winning outcomes across and within players allows for the identification of player ability through player fixed effects in equation 1. The average gameplay records used for estimating the production function for each player is 171. While it is theoretically possible to identify item-ability substitutability or complementarity (i.e., the interaction between item and ability in the production function), doing so is practically infeasible given our individual-specific measure of ability. The trade-off we make for capturing this level of granularity in individual heterogeneity is the inability to separately identify the effects of player learning and item-ability interactions. We follow Kasahara and Shimotsu (2009) for the identification of unobserved finite mixture heterogeneity.

Second, the parameters in the flow utility function needs to be identified by the revealed preference argument. In particular, the mapping between the differences in conditional value functions and the conditional choice probabilities, and the normalized exit value form the basis for our model identification. With the exit value normalized to 0, the cost of play parameter c_p is identified from the two play decisions. The utility from points reward θ is identified from the play versus play and purchase decision, as the item state is updated upon purchase decision before play, changing the win probability and the expected value of points. The reward from level-up R and the cost of purchase c_m are identified from the intertemporal linkage of states, and the costs and rewards from play.

Lastly, we fix the discount factor to be 0.9.

6. Results

We discuss our results in the following order: 1) player production function and ability estimates, 2) first stage policy estimates, and 3) second stage structural parameter estimates.

6.1. Production Function and Ability Estimates

We estimate the production function using a linear probability model of player item and level states with individual fixed effects, controlling out the opponent effect.[6] We use the estimated individual fixed effects as the measure of player ability. The production function, shown in Table 4, accomodates the following three key features. First, win probability is higher for higher ability players. Second, player win probability increases with item. Specifically, one additional item purchase at level 6 increases player win probability by around 2.6 percentage points.[7] We allow for

[6] For the second stage structural parameter estimation, we control out the effect of opponents using the median value of the opponent Elo scores – the rating system used by the company for calculating the relative skill levels of players – at each level.

[7] To provide additional evidence of the effect of item purchase on player win probability, we conduct a more localized before-and-after analysis of player win rates, comparing the five games before and after the purchase incident in Appendix A. The measured effect size largely aligns with the item effect measured in our production function.

the diminishing effect of item as levels increase. Third, player win probability decreases with level. Notably, the game becomes significantly more difficult at level 9, with win probability dropping by around 10 percentage points compared to the previous level. This increasing difficulty at higher levels is a common feature in gaming environments.

Table 4 Linear Probability Model: Production Function Estimates

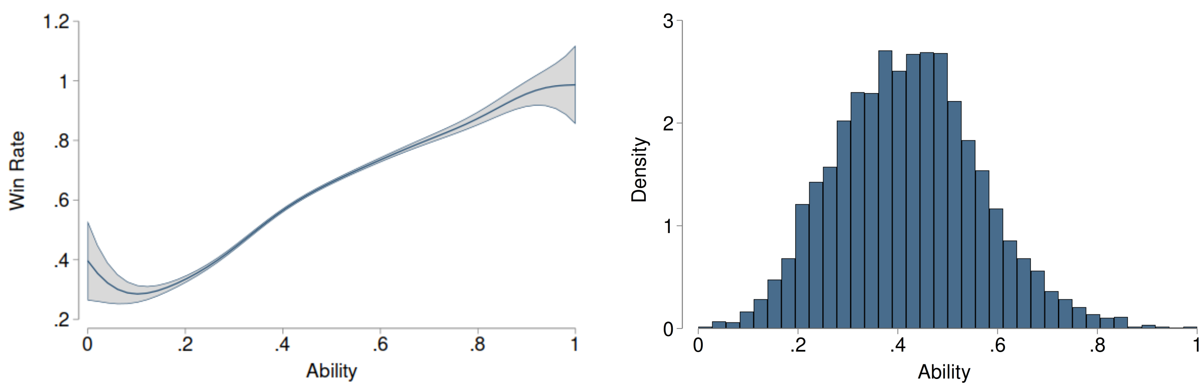
	(1)
	Win
item stock	0.02623^{***}
	(0.00157)
lvl:6	-0.08207^{***}
	(0.00146)
lvl:7	-0.06441^{***}
	(0.00187)
lvl:8	-0.07087^{***}
	(0.00199)
lvl:9	-0.17077^{***}
	(0.00228)
lvl:10	-0.17392^{***}
	(0.00296)
lvl:11	-0.23899^{***}
	(0.00384)
lvl:6 $\times$ item stock	-0.00473^{***}
	(0.00136)
lvl:7 $\times$ item stock	-0.00699^{***}
	(0.00141)
lvl:8 $\times$ item stock	-0.01082^{***}
	(0.00140)
lvl:9 $\times$ item stock	-0.01151^{***}
	(0.00143)
lvl:10 $\times$ item stock	-0.01430^{***}
	(0.00148)
lvl:11 $\times$ item stock	-0.01505^{***}
	(0.00147)
opponent elo score	-0.24202^{***}
	(0.00277)
Observations	1,168,880
Individual FE	Y
Adjusted R^2	0.049

Note: Level 5 used as baseline. Robust standard errors in parentheses; *** <0.01, ** p<0.05, * p<0.1

To demonstrate the reliability of our ability measure, we present the relationship between player average win probability in Figure 9a. There is a clear positive relationship, with higher ability players exhibiting higher win rates. Figure 9b displays the distribution of the player ability estimates, highlighting significant heterogeneity in our player sample.

Figure 9 Player Ability Estimates

(a) Average Win Rate and Ability: Level 6 Example **(b) Distribution of Player Ability**



Note: The solid line in Figure 9a is a kernel-weighted second-degree polynomial regression using a Gaussian kernel, and the shaded area is the 95% confidence interval.

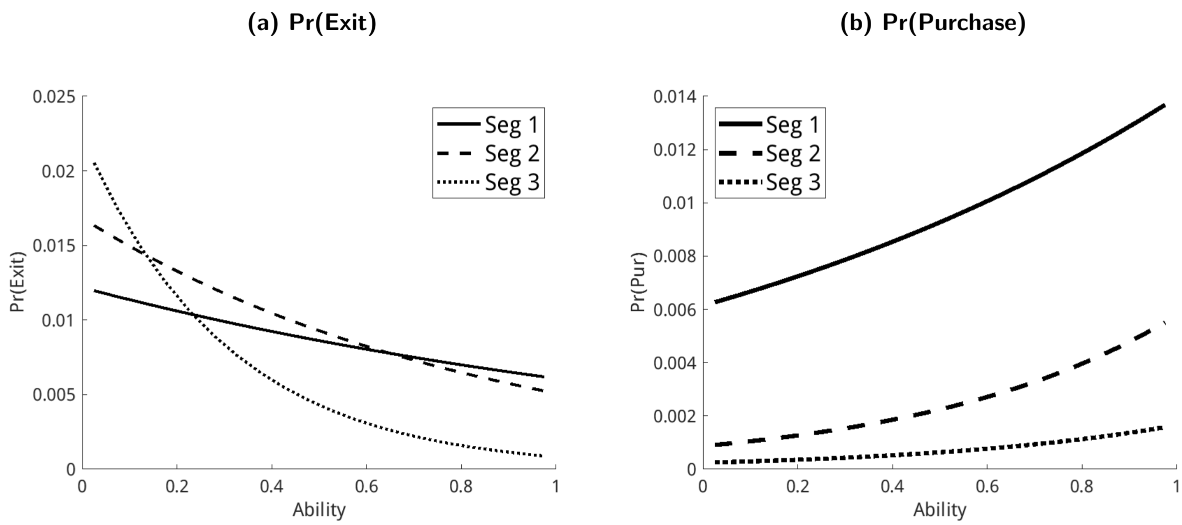
6.2. First Stage Policy Estimates

Given the ability estimates of players, we estimate the first stage policy estimation of player action probabilities using a flexible multinomial logistic regression, accounting for both observed and unobserved heterogeneity. We find that the three-segment model best fits the data based on the AIC and BIC criterion.[8] We report the first stage policy estimates for each segment in Table 5. Segment 1 is the smallest, comprising 7% of the players, followed by Segment 2 with 23%, and Segment 3 with 70%. We report some illustrative features of the policy with respect to player ability in Figure 10. Consistent with our model-free evidence, the probability of exit decreases with player ability, while the probability of purchase increases with ability across all segments. Segment 1 has the highest purchase rates and generally the lowest exit rates, indicating that these players

[8] The AIC for the two- and three-segment models are 76764.81 and 76299.59, respectively; the BIC values are 77041.41 and 76714.49, respectively.

are the most engaged and likely to spend money in the game. Segment 3, the largest segment, has the lowest purchase rates. While Segment 3 shows a relatively flat decrease in exit probability across abilities, Segment 3 exhibits a steeper decline among lower ability players.

Figure 10 Player Action Policy by Ability Type



Note: Policy visualization based on example state: (Level 6, Level Completion 20-30%, and Item Stock 0)

To gain deeper insights on the segment characteristics, we report descriptive statistics in Table 6 and visualize the survival rate and the ability distribution of each segment in Figure 11. The descriptive evidence further provides support that the smallest share of players, Segment 1, comprises of high-spending individuals who also stays longer in the game and progress to higher levels, followed by Segment 2. Segment 3 has the lowest average player ability and spends the least. Because this segment has greater share of low ability players, it also has higher exit rates, especially at the early levels.

6.3. Structural Parameter Estimates

Table 7 reports the estimates of the structural parameters of the model, which further reveal important differences in the game play preferences among the three segments.

Segment 1 represents the smallest share of players (7%) but those who spend the most, which is reflected in their lower price sensitivity for ability enhancers (c_m).[9] Despite their low sensitivity to

[9] Since price is invariant, we normalized the price to 1; so c_m is the disutility for paying that unit price.

Table 5 First Stage Estimates: Player Action Policy Function

		Seg 1	Seg 2	Seg 3
Segment Probability		0.0692	0.2287	0.7021
SE		0.0046	0.0048	0.0046
Choice				
Purchase	ability	1.0078**	1.8555***	1.9965***
		(0.3696)	(0.3389)	(0.3636)
	item stock	0.2119***	0.4950***	1.8689***
		(0.0185)	(0.0163)	(0.0478)
	item stockˆ2	-0.0069***	-0.0128***	-0.1999***
		(0.0010)	(0.0007)	(0.0086)
	pct lvl complete	2.6695***	-0.9826**	1.2923***
		(0.4361)	(0.4249)	(0.4825)
	ability × pct lvl complete	-0.7362	0.1418	-0.3467
		(0.7061)	(0.6362)	(0.7756)
	pct lvl completeˆ2	-2.7620***	0.2979	-1.8515***
		(0.4350)	(0.4454)	(0.4203)
	lvl:7	-0.0348	-0.2065**	-0.2914***
		(0.1038)	(0.1016)	(0.0872)
	lvl:8	0.1223	-0.4376***	-0.5923***
		(0.1009)	(0.1042)	(0.0908)
	lvl:9	-0.3773***	-0.6725***	-0.7774***
		(0.1139)	(0.1089)	(0.0983)
	lvl:10	-0.6498***	-1.3472***	-0.8373***
		(0.1268)	(0.1435)	(0.1201)
	lvl:11	-0.9534***	-1.5476***	-1.0228***
		(0.1431)	(0.1598)	(0.1534)
	cons	-5.5698***	-6.8071***	-8.5193***
		(0.1687)	(0.1457)	(0.1699)
Exit	ability	0.7717**	-3.0300***	-3.6848***
		(0.3469)	(0.2113)	(0.1987)
	item stock	-0.1393***	-0.2053***	-0.0152
		(0.0286)	(0.0219)	(0.0262)
	item stockˆ2	0.0056***	0.0087***	0.0089***
		(0.0013)	(0.0009)	(0.0034)
	pct lvl complete	-0.9030*	-1.7382***	-1.3912***
		(0.5159)	(0.2657)	(0.2639)
	ability × pct lvl complete	-5.8553***	7.3085***	1.4232***
		(0.9142)	(0.5172)	(0.4862)
	pct lvl completeˆ2	2.8489***	-2.1943***	-0.0362
		(0.4691)	(0.2832)	(0.2415)
	lvl:7	-0.8087***	-0.3470***	-0.0847*
		(0.0877)	(0.0532)	(0.0500)
	lvl:8	-0.6385***	-0.3837***	0.0199
		(0.1128)	(0.0568)	(0.0520)
	lvl:9	-0.5340***	-0.1310**	0.3191***
		(0.1140)	(0.0640)	(0.0549)
	lvl:10	-0.9494***	0.0801	0.3248***
		(0.1757)	(0.1075)	(0.0785)
	lvl:11	-0.7838***	-0.6067***	0.3303***
		(0.1840)	(0.1916)	(0.1036)
	cons	-4.3423***	-3.4950***	-3.4308***
		(0.1257)	(0.0748)	(0.0701)

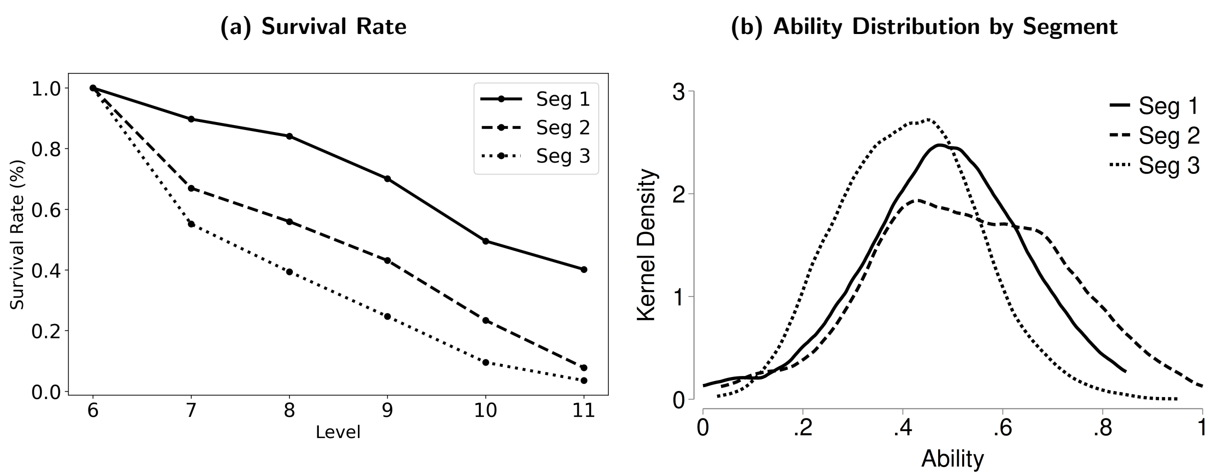
Note: Robust standard errors in parentheses; *** $p<0.01$, ** $p<0.05$, * $p<0.1$

Table 6 Descriptive Characteristics of Segment

Seg 1 (2.6%) N=107	Mean	Median	75Q	90Q	99Q
Avg Total No. Purchase	6.8785	5	9	14	24
Avg Purchase Rate	0.0219	0.0158	0.0260	0.0365	0.0789
Avg Level Reached	9.3364	9	11	11	11
Avg Total No. of Games Played	470.66	314	639	1021	1980
Ability Score	0.4823	0.4860	0.5912	0.6861	0.8398
Pay-to-Win Player Share			100.00%		
Seg 2 (5.2%) N=218	**Mean**	**Median**	**75Q**	**90Q**	**99Q**
Avg Total No. Purchase	2.7661	1	3	7	25
Avg Purchase Rate	0.0120	0	0.0120	0.0328	0.0829
Avg Level Reached	7.9725	8	9	10	11
Avg Total No. of Games Played	263.68	164	444	673	916
Ability Score	0.5330	0.5193	0.6684	0.7812	0.9282
Pay-to-Win Player Share			68.35%		
Seg 3 (92.2%) N=3,838	**Mean**	**Median**	**75Q**	**90Q**	**99Q**
Avg Total No. Purchase	0.2507	0	0	1	5
Avg Purchase Rate	0.0017	0	0	0.0024	0.0333
Avg Level Reached	7.3233	7	8	9	11
Avg Total No. of Games Played	166.74	112	227	387	887
Ability Score	0.4129	0.4117	0.5056	0.5882	0.7556
Pay-to-Win Player Share			10.76%		

Note: The descriptive statistics are based on deterministic segment assignment (i.e., the maximum of probabilistic segment probabilities estimated in the first stage).

Figure 11 Descriptive Characteristics of Segment: Survival Rate and Ability Distribution



immediate points rewards (θ), they derive significant utility from progressing through game levels, as indicated by their high level-up reward (R). This segment's positive (near zero) cost of play (c_p) suggests that these players do not find playing the game costly, but rather enjoy spending time in the game. We henceforth label this group as *premium enthusiasts*.

Table 7 Structural Parameter Estimates

	Segment 1 *Premium Enthusiasts* (7.0%)	Segment 2 *Win-Seekers* (23.0%)	Segment 3 *Progress-Seekers* (70.0%)
θ	0.0003	0.0050	0.0050
	(0.0002)	(0.0004)	(0.0003)
c_m	-4.4089	-5.2775	-6.4997
	(0.0333)	(0.0386)	(0.0448)
c_p	0.0002	-0.0395	-0.0288
	(0.0020)	(0.0036)	(0.0026)
R	0.0039	0.0016	0.0036
	(0.0006)	(0.0009)	(0.0014)

Note: standard errors are shown in parentheses.

Segment 2 players, representing share of 23% of players, receive a greater utility from immediate points reward (θ) and have the second lowest price sensitivity for ability enhancers (c_m), below the *premium enthusiasts*. They however have the highest cost of play (c_p), indicating that they find playing the game more of a chore. This suggests that these players are less likely to continue playing without high enough points reward from current game, and their utility from level-up (R) is the lowest among the segments. We label this group as *win-seekers*.

Segment 3 constitutes the largest group with 70% of the players. These players have the same utility from immediate points reward (θ) as *win-seekers*, but their price sensitivity for enhancers (c_m) is the highest. They also find playing the game costly (c_p), but receive higher utility from level-up rewards (R) than *win-seekers*. We label them as *progress-seekers*.

To assess our model fit, we generate a representative sample of 20,000 individuals from the segment, ability, and initial item stock state distribution, simulating a play sequence for each player until they exit or finish the game; the sequences average about 200 games per player. We report the model fit in Table 8. In general, the model performs reasonably in matching the targeted moments, total play count (retention) and total purchase (monetization), and in accounting for the heterogeneity across the three latent segments. Hence, we conclude that the model can reasonably match player action and play behavior observed in data, especially with respect to different insights and predictions across the latent segments.

Table 8 Model Fit

	Real Data			Model Simulation		
	Seg 1	Seg 2	Seg 3	Seg 1	Seg 2	Seg 3
Average Lifetime Play Count	231.35	147.07	185.14	243.15	166.73	196.51
Average Level Reached	7.69	7.15	7.47	8.92	7.79	7.97
Average Total Purchase	2.77	0.70	0.29	2.95	0.89	0.31
Average Purchase Rate (per 100 plays)	0.97	0.33	0.19	1.2	0.53	0.15

7. Counterfactuals

The primary challenge for game designers is managing the retention and monetization for players with wide ranging ability and gameplay preferences (e.g., price sensitivity for ability enhancers, reward valuation, play costs). In this section, we investigate three levers of personalized interventions aimed at enhancing player retention-monetization dynamics. First, we directly intervene on the monetization front by inducing player purchases through targeted discounts at players' frontier levels of gameplay, the point at which the player exit the game. In doing so, we provide evidence of the positive feedback mechanism between retention and monetization. Second, we directly intervene on the retention front by dynamically adjusting the game difficulty (DDA) to match player ability and gameplay preference at these frontier levels, ensuring that the challenges are engaging but not discouraging that they induce player exit. We show the importance of incorporating both ability and preference heterogeneity into the player frontier targeting strategies. Finally, we optimize the player production and consumption dynamics by changing the game environment through DDA. Specifically, we make early level progression easier for high ability players, enabling them to advance faster to more challenging levels. To perform counterfactual analysis, we generate a representative sample of 20,000 individuals as we did for the model fit analysis earlier.

7.1. Monetization Through Targeted Discounts: Illustration of the Positive Feedback Mechanism

In this section, we investigate how interventions from the monetization front, specifically targeted discounts, impact the dynamics of player retention and monetization. The key idea of the production and consumption loop is follows: when players receive discounts–specifically at their frontier levels of gameplay where players exit the game–they are more likely to make in-game purchases. These purchases, while directly increasing firm revenue, also indirectly enhance player level progression by increasing player win probability (production). This subsequently enables future monetization (consumption of $) opportunities for the firm as players progress to higher levels.

To assess the effects of discounts on player monetization and retention dynamics, we first define player frontier levels as the median exit levels of players for each ability and segment type combination. Then, at the start of each level, we applied one-time discounts on item purchase to players whose current level matched their identified frontier level. We varied the discount depths from 10% to 90%, as shown in Table 9. In all cases, we find that providing targeted discounts increases player level progression. That is, players who redeem discounts on item purchase experience an increased win probability, which in turn helps them to progress to higher levels in the game.

We visualize the impact on firm profit and player retention in Figure 12a and Figure 12b. Our analysis reveals that a 70% discount is optimal for maximizing total profit by 6.44%, with the largest profit increase generated from the *progress-seekers*, who have high price sensitivity but gain higher utility from level progression, by 13.71%. Conversely, providing discounts to *premium enthusiasts*, a segment of players who has lowest price sensitivity and derives enjoyment from play itself, results in the lowest and sometimes even negative changes in profit. This group also has an optimal discount depth slightly below that of other segments, at 60%.

However, a notable caveat is that while these discounts can increase player level progression and profitability, they may also interact with the firm's level promotion scheme in a way that could decrease overall retention (i.e., total number of games played). As players purchase items and perform better, they may advance through levels more quickly, reaching their exit points faster.

Table 9 Effect of Price Discount Promotion at Frontier Level

Discount Depth	10%	20%	30%	40%	50%	60%	70%	80%	90%	1 Free Item
% Change in Level Progression										
Total	0.03	0.03	0.04	0.05	0.08	0.11	0.17	0.22	0.30	0.74
Seg										
1: *Premium Enthusiasts*	0.00	0.00	0.02	0.02	0.05	0.06	0.10	0.13	0.13	0.28
2: *Win-seekers*	0.04	0.04	0.05	0.06	0.09	0.12	0.18	0.22	0.34	0.74
3: *Progress-Seekers*	0.02	0.03	0.04	0.05	0.08	0.11	0.18	0.22	0.30	0.79
Monetization (% Change in Profit)										
Total	1.35	1.87	2.75	3.71	4.88	5.75	6.44	5.86	4.16	0.70
Seg										
1: *Premium Enthusiasts*	-0.11	-0.00	0.25	0.41	0.61	0.86	0.80	0.62	0.12	-0.64
2: *Win-seekers*	0.75	1.07	1.51	2.00	2.87	3.59	4.40	4.09	3.47	1.19
3: *Progress-Seekers*	3.28	4.40	6.28	8.47	10.84	12.44	13.71	12.48	8.62	1.49
Retention (% Change in Avg Total Number of Games Played)										
Total	0.05	0.04	0.05	0.10	0.16	0.19	0.26	0.29	0.35	0.76
Seg										
1: *Premium Enthusiasts*	-0.09	-0.09	-0.09	-0.11	-0.12	-0.18	-0.23	-0.22	-0.29	-0.55
2: *Win-seekers*	0.10	0.09	0.11	0.06	0.13	0.23	0.29	0.25	0.57	0.89
3: *Progress-Seekers*	0.05	0.04	0.04	0.13	0.20	0.22	0.31	0.35	0.36	0.88

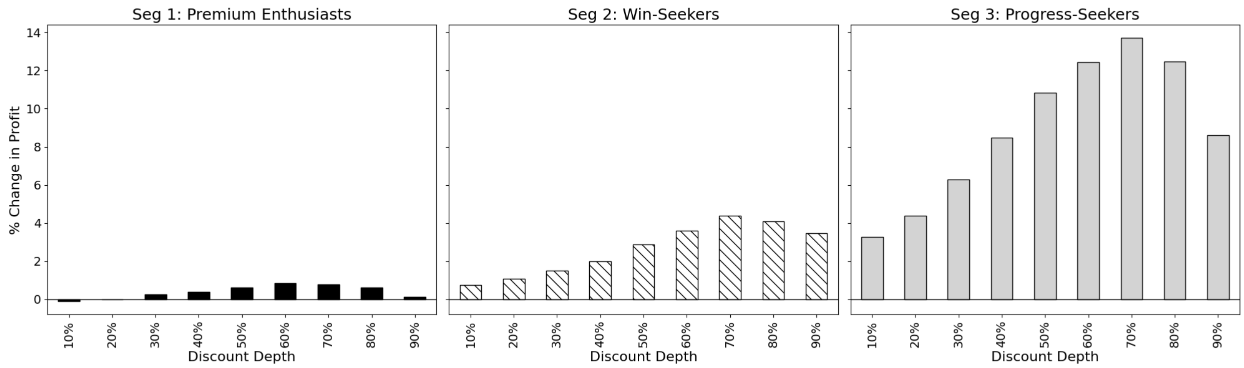
This accelerated progression can result in a decreased total number of games played, as players finish the game sooner. Thus, while targeted discounts can drive short-term gains in profit, they may inadvertently shorten the overall engagement period for some players. We see such decrease in overall retention greatest among *premium enthusiasts*, those who gain utility from play itself.

To examine the sources of the change in profit, we decompose the profit change from the discount intervention into two main components: (i) the direct effect of the discounts, and (ii) the feedback mechanism between monetization and retention. To quantify the contribution of each component, we focus on the 70% discount depth intervention case, which has been shown optimal for total profit, and identify players who have redeemed the discount. Then, we compare the incremental change in the number of purchases after the discount intervention with the original design in the absence of discounts.

We report the incremental profit decomposition from discounts in Table 10. Our analysis shows that 86.69% of players made purchases directly due to the discount, while the firm's profit was

Figure 12 Impact of Price Discounts on Firm Profit and Retention

(a) Monetization (% Change in Profit): Price Discount Promotion at Players' Level Frontier



(b) Retention (% Change in Avg Total No. of Games Played): Price Discount at Players' Level Frontier

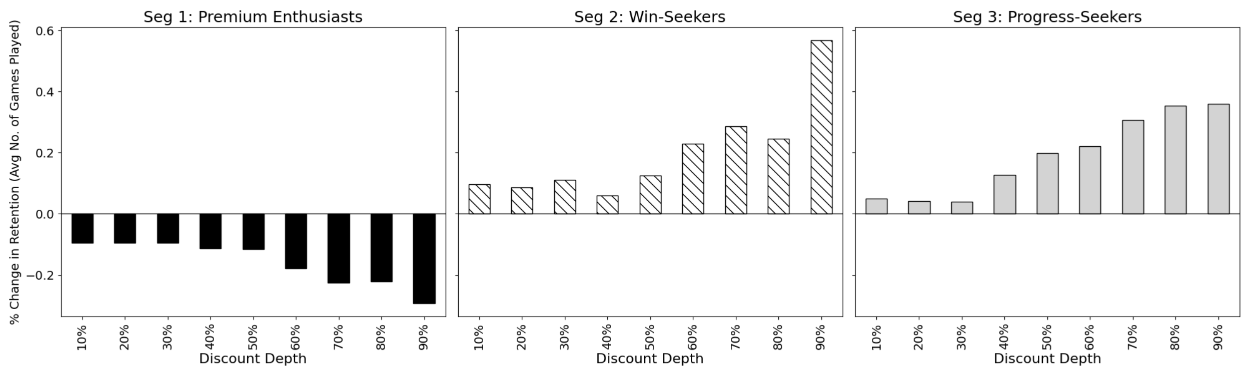


Table 10 Effect of 70% Discount at Frontier: % Share Among Players Who Redeemed the Discount

	Mechanism			
	Direct Discount Effect		Production and Consumption Loop	
Incremental Effect on Item Purchase	+1 (Discount Purchase)	0 (Cannibalization)	>1	<0
Total	86.69	6.58	6.20	0.53
Seg				
1: *Premium Enthusiasts*	87.22	9.44	2.22	1.11
2: *Win-seekers*	86.00	8.07	4.12	0.16
3: *Progress-Seekers*	86.86	5.81	6.73	0.59

cannibalized by 6.58% of players, where the discount replaced a planned full-price purchase. Cannibalization was greatest among *premium enthusiasts*, who would have made purchases anyway in the absence of discounts. Importantly, 6.20% of players, primarily from *progress-seekers*, exhibited a positive feedback loop, making additional non-discount purchases following the discount. However, a small percentage (0.53%) of players reduced their purchasing behavior after redeeming the discount, suggesting potential long-term drawbacks due to changes in level progression speed as discussed earlier.

Finally, to isolate the effects of discounts and provide direct evidence of the positive feedback mechanism between retention and monetization, we offered a free item to players at their frontier levels. We report the effects of providing one free item in the last column of Table 9. Overall, we detect evidence of the feedback mechanism between retention and monetization. Providing players with an additional item increases their rate of production, which in turn increases their level progression and retention, leading to increase in profits. However, we find heterogeneous effects across player segments. The positive feedback loop is found dominant in *win-seekers* and *progress-seekers*, while we find such feedback loop between retention and monetization going in reverse direction for *premium enthusiasts*. This underscores the importance of incorporating unobserved preference heterogeneity in targeted intervention design.

7.2. Retention Through Dynamic Difficulty Adjustment (DDA)

Next, we investigate how retention-driven interventions, through dynamic difficulty adjustment (DDA)[10] at player-level frontiers, impact the feedback loop between retention and monetization. DDA involves modifying the game's difficulty in real-time to maintain an optimal level of challenge. This approach aims to keep players engaged by ensuring the game is neither too easy nor too difficult. Thus, understanding player heterogeneity in both observed and unobserved domains is critical to ensure effectiveness of DDA interventions. In this section, we demonstrate the value of incorporating player ability and gameplay preference heterogeneity in designing the timing of difficulty adjustments. Specifically, we compare three different frontier targeting strategies: (i) ability-based targeting, (ii) gameplay preference-based targeting, and (iii) targeting using both ability and gameplay preferences.

[10] Dynamic difficulty adjustment (DDA) is a widely used method by game companies to tailor gameplay experiences to individual players with different skill levels. For example, Candy Crush Saga adjusts the difficulty of levels based on the player performance. Clash of Clans reduces the time spent on lower-level tasks for high-ability players.

Table 11 Frontier Targeting Policy for Personalized Interventions

		Variable Used for Frontier Targeting		
		(1) Ability Only	(2) Preference Only	(3) Ability & Preference
Segment	Ability	Frontier Level	Frontier Level	Frontier Level
1: Premium Enthusiasts	Q1	6	9	8
1	Q2	7	9	9
1	Q3	9	9	10
1	Q4	11	9	11
2: Win-Seekers	Q1	6	7	7
2	Q2	7	7	8
2	Q3	9	7	8
2	Q4	11	7	10
3: Progress-Seekers	Q1	6	8	7
3	Q2	7	8	8
3	Q3	9	8	9
3	Q4	11	8	10

To elaborate, we first define player frontier levels as the median exit levels of players for each ability type. Second, we use only the player gameplay preference information (i.e., the latent segment type) to compute the median exit level for each preference type. Third, we implement a fully personalized policy that incorporates both player ability and preference heterogeneity in computing the median exit level of the player. With these computed frontier levels, we provide targeted interventions and reduce difficulty by increasing the effective win rate of 2.5 percentage points when the player reaches his or her frontier.[11] For example, if the player's computed frontier level is 7, we adjust down the level difficulty for the player from level 7 and beyond. We summarize the computed frontier levels for targeted intervention in Table 11. For presentation purposes, we aggregate the frontier-level information for ability types into four quartiles. Notably, the high-spending enjoyer segment has frontier levels at least one level higher than players with similar abilities in other segments.

[11] This specific adjustment is similar to the incremental effect of an item, ensuring the win rate does not increase excessively, which would either hurt the difficulty balance of the current level progression scheme or make the game too easy and boring (which is the effect we do not model). Therefore, we only perform small localized changes in win probability, set at 2.5 pp in our case.

Table 12 Effect of Personalized Timing of Interventions on Retention and Monetization

	Personalized Intervention Policy		
	(1)	(2)	(3)
Variable Used for Frontier Targeting	Ability	Gameplay Preferences	Ability & Gameplay Preferences
Win Probability Adjustment at Frontier (pp)	+ 2.5	+ 2.5	+ 2.5
% Change in Avg Level Reached	+ 1.15	+ 1.09	+ 1.14
Retention (% Change in Avg Play Count)	+ 0.97	+ 0.91	+ 1.09
Monetization (% Change in Profit)	+ 0.28	+ 0.27	+ 0.92

We report the result of the targeted interventions in Table 12. We again provide evidence of the positive feedback mechanism between retention and monetization. Overall, retention efforts to decrease difficulty at the frontier lead to an increase in profit for all targeting policies. Incorporating heterogeneity in player ability and unobserved preference heterogeneity for gameplay each yields 0.97% and 0.91% increase in player retention, which in turn increases firm profit by 0.28% and 0.27%, respectively. The fully personalized policy, which uses both ability and gameplay preferences for targeting, increases player retention by 1.14% and yields the largest increase in profit by 0.92%. The importance of incorporating both players' observed and unobserved heterogeneity is supported by our earlier observation about differences in exit levels by segment, even within similar ability levels. By increasing player retention at their frontier levels, the firm can further reap profits from players who are about to leave with a costless intervention.

7.3. Managing the Production and Consumption Loop: Game Environment Change Through DDA

The first two counterfactuals involved direct interventions on player monetization and retention. Our final counterfactual investigates how changing the game environment affects the dynamics between player production (ability to win) and consumption (willingness to pay for ability enhancers). We achieve this by building on the key concept of DDA: changing the game difficulty to tailor the game progression experience for players with heterogeneous abilities and preferences. Specifically, we examine the impact of accelerating early-level progression for high-ability players.

In the game, players progress up to their own levels of difficulty frontiers. For high ability players, such difficulty frontiers come much later in the game. By facilitating faster level-ups for high-ability players, we seek to optimize the player production and consumption dynamics, providing appropriately challenging game content sooner, thereby enhancing engagement and increasing monetization opportunities.

To implement this policy, we first classify players into four ability quartiles, and target those in the highest quartile. Table 13 shows the targeted segment share of the representative players from model simulations. The average frontier level of high ability players is above level 9 across all segments. To expedite the speed of player level progression, we increase the player win probability by 5 percentage points at early levels.[12]

Table 13 Segment Share of High Ability Players

Segment	Ability	Player Share (%)	Avg Level Reached
1: Premium Enthusiasts	Q4	1.57	9.70
2: Win-Seekers	Q4	2.74	9.13
3: Progress-Seekers	Q4	10.55	9.27

Table 14 reports the retention and monetization results for the accelerated level progression intervention. Overall, we find that expediting early level progression for high ability players increases both total retention by 0.96% (measured by lifetime number of games played) and total profit by 1.29%. In all segments, players on average progress to higher levels and play a greater number of games.

We visualize the change in profit in Figure 13. We show that the firm gains in profit by expediting level-ups by effectively lowering difficulty level at early stages from players who find playing the game itself more costly but receive higher utility from level progression (*progress-seekers*), but loses in profit from players who find playing the game itself more enjoyable (*premium enthusiasts*) and players who receive higher utility from immediate winning (*win-seekers*). This relates to the

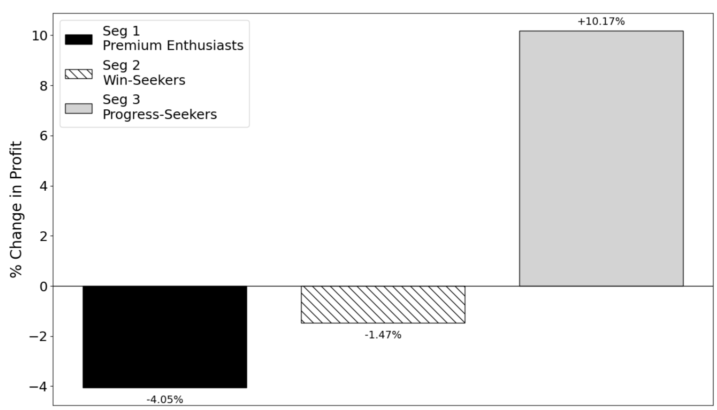
[12] This corresponds to levels 6-8 in our analysis. Practically, the firm can implement this policy by matching the player with less difficult opponents.

Table 14 Accelerating Level Progression for High Ability Players: Effect on Retention and Monetization

	Ability	% Change in Avg Level Reached	Retention (% Change in Avg Play Count)	Monetization (% Change in Profit)
Total	Q4	+ 1.87	+ 0.96	+ 1.29
Seg				
1: *Premium Enthusiasts*	Q4	+ 0.16	- 1.59	- 4.05
2: *Win-Seekers*	Q4	+ 1.50	- 0.07	- 1.47
3: *Progress-Seekers*	Q4	+ 2.22	+ 1.55	+ 10.17

caveat of difficulty level adjustment discussed earlier: While *progress-seekers* benefit from faster advancement, premium *premium enthusiasts* and *win-seekers* who each find the game enjoyable and a higher likelihood of winning rewards at easier levels may find their enjoyment diminished by the quicker pace, leading to earlier exits and reduced long-term engagement. The increase in net total profit is driven by the (*progress-seekers*), with their profit increased by 10.17%. By expediting level progression to higher levels, the firm can generate more profit from the higher-ability players in the *progress-seekers* who possess sufficient ability to advance through the later stages of the game. Because they gain higher utility from level-ups but incur significant play costs, expediting them to higher levels benefits firm profit by reducing the risk of early-stage attrition for these players.

Figure 13 Accelerating Level Progression for High Ability Players: Change in Profit by Segments



8. Conclusion

This paper develops a dynamic structural model of consumer response to gaming environments to address novel questions that broaden the focus of empirical research in customer relationship management. We uncover a unique aspect of the retention-monetization dynamics in the context of the gaming environment—the two-way positive feedback between production and consumption. The positive feedback mechanism allows even giving something for free (item rewards) to benefit the firm in terms of higher player retention and profits.

Unlike standard retail environments, the key difference in gaming environments is the ability to dynamically modify the product – the game itself. This allows firms to personalize and adjust the game in real-time to enhance player engagement and monetization. In gaming, dynamic difficulty adjustment (DDA) is a common technique used by developers to continuously tailor game challenges and features to match the players' abilities and preferences in real-time. Structural models can be extremely helpful in gaining a theoretical understanding of the dynamics and in cost-effectively assessing effectiveness of different intervention strategies. Additionally, the presence of unobserved heterogeneity at the player level (e.g., price sensitivity, motivation sensitivity) complicates the implementation of personalized interventions based solely on observables. Our model framework allows us to dynamically redesign consumer engagement and monetization in a personalized manner. In doing so, we underscore the importance of accounting for ability and unobserved preference heterogeneity in targeted intervention design.

We now discuss the limitations of our paper, which provide promising avenues for future research. First, our model abstracts away from potential complementarity and substitutability between player ability and in-game items. Players can have different intensities in the feedback cycle between production and consumption based on their abilities and the type of item they purchase. Understanding how player ability interacts with different types of in-game items and its impact on player engagement and monetization can inform product design. Second, we consider only durable in-game items, excluding consumables that may have different impacts on player behavior. Consumable items, which are used once and provide temporary benefits, could influence spending patterns and engagement differently than durable items. Investigating the role of consumables in gaming environments could reveal interesting insights for balancing immediate and long-term player engagement and monetization. Third, we address only environments in which in-game items lead to increased performance (performance enhancers). Future research should explore the effects of non-performance enhancing purchases, such as cosmetic or identity-related items (e.g., fashion items, avatars). Understanding how these types of purchases influence player engagement and monetization spillovers to ability-enhancing items could provide a more comprehensive understanding of consumer response in gaming environments.

Gamification has become more mainstream, with gaming features incorporated into many aspects of our lives. Our framework extends broadly to settings in which a gaming layer has been added to benefit from consumer preference for game-like features. By understanding consumer response in these environments and accounting for their observed and unobserved preference heterogeneity, firms can develop targeted interventions to effectively harness the positive feedback loop between retention and monetization.

References

Aaker DA (1992) The value of brand equity. *Journal of business strategy* 13(4):27–32, URL https://www.emerald.com/insight/content/doi/10.1108/eb039503/full/html, publisher: MCB UP Ltd.

Allcott H, Gentzkow M, Song L (2022) Digital addiction. *American Economic Review* 112(7):2424–2463, URL https://www.aeaweb.org/articles?id=10.1257/aer.20210867, publisher: American Economic Association 2014 Broadway, Suite 305, Nashville, TN 37203.

Amano T, Simonov A (2023) A Welfare Analysis of Gambling in Video Games. *Available at SSRN 4355019* URL https://papers.ssrn.com/sol3/papers.cfm?abstract_id=4355019.

Appel G, Libai B, Muller E, Shachar R (2020) On the monetization of mobile apps. *International Journal of research in Marketing* 37(1):93–107, URL https://www.sciencedirect.com/science/article/pii/S0167811619300473, publisher: Elsevier.

Arcidiacono P, Miller RA (2011) Conditional Choice Probability Estimation of Dynamic Discrete Choice Models With Unobserved Heterogeneity. *Econometrica* 79(6):1823–1867, ISSN 0012-9682, URL http://dx.doi.org/10.3982/ECTA7743.

Ascarza E, Netzer O, Runge J (2023) Personalized Game Design for Improved User Retention and Monetization in Freemium Mobile Games. *Available at SSRN 4653319* URL https://papers.ssrn.com/sol3/papers.cfm?abstract_id=4653319.

Bajari P, Benkard CL, Levin J (2007) Estimating Dynamic Models of Imperfect Competition. *Econometrica* 75(5):1331–1370, ISSN 0012-9682, 1468-0262, URL http://dx.doi.org/10.1111/j.1468-0262.2007.00796.x.

Castelo-Branco B, Manchanda P (2023) Is Video Gaming Addictive?: An Empirical Analysis. *An Empirical Analysis (June 1, 2023)* URL https://papers.ssrn.com/sol3/papers.cfm?abstract_id=4714079.

Chen J (2023) Learning and skill set formation: A structural examination of version upgrades, user visibility, and AI strategies. *Production and Operations Management* 32(12):3856–3872, ISSN 1059-1478, 1937-5956, URL http://dx.doi.org/10.1111/poms.14065.

Chen N, Elmachtoub AN, Hamilton ML, Lei X (2021) Loot Box Pricing and Design. *Management Science* 67(8):4809–4825, ISSN 0025-1909, 1526-5501, URL http://dx.doi.org/10.1287/mnsc.2020.3748.

Chung DJ, Kim B, Park BG (2021) The Comprehensive Effects of Sales Force Management: A Dynamic Structural Analysis of Selection, Compensation, and Training. *Management Science* 67(11):7046–7074, ISSN 0025-1909, 1526-5501, URL http://dx.doi.org/10.1287/mnsc.2020.3853.

Chung DJ, Steenburgh T, Sudhir K (2014) Do Bonuses Enhance Sales Productivity? A Dynamic Structural Analysis of Bonus-Based Compensation Plans. *Marketing Science* 33(2):165–187, ISSN 0732-2399, 1526-548X, URL http://dx.doi.org/10.1287/mksc.2013.0815.

Clements MT, Ohashi H (2005) Indirect Network Effects and the Product Cycle: Video Games in the U.S. 1994-2002. *Journal of Industrial Economics* 53(4):515–542, ISSN 0022-1821, 1467-6451, URL http://dx.doi.org/10.1111/j.1467-6451.2005.00268.x.

Derdenger T (2014) Technological tying and the intensity of price competition: An empirical analysis of the video game industry. *Quantitative Marketing and Economics* 12(2):127–165, ISSN 1570-7156, 1573-711X, URL http://dx.doi.org/10.1007/s11129-014-9143-9.

Derdenger T, Kumar V (2013) The Dynamic Effects of Bundling as a Product Strategy. *Marketing Science* 32(6):827–859, ISSN 0732-2399, 1526-548X, URL http://dx.doi.org/10.1287/mksc.2013.0810.

Dubé JP, Hitsch GJ, Rossi PE (2010) State dependence and alternative explanations for consumer inertia. *The RAND Journal of Economics* 41(3):417–445, ISSN 07416261, URL http://dx.doi.org/10.1111/j.1756-2171.2010.00106.x.

Erdem T (1996) A Dynamic Analysis of Market Structure Based on Panel Data. *Marketing Science* 15(4):359–378, ISSN 0732-2399, 1526-548X, URL http://dx.doi.org/10.1287/mksc.15.4.359.

Guadagni PM, Little JDC (1983) A Logit Model of Brand Choice Calibrated on Scanner Data. *Marketing Science* 2(3):203–238, ISSN 0732-2399, 1526-548X, URL http://dx.doi.org/10.1287/mksc.2.3.203.

Guo H, Hao L, Mukhopadhyay T, Sun D (2019) Selling Virtual Currency in Digital Games: Implications for Gameplay and Social Welfare. *Information Systems Research* 30(2):430–446, ISSN 1047-7047, 1526-5536, URL http://dx.doi.org/10.1287/isre.2018.0812.

Haenlein M, Libai B, Muller E (2023) Satiation and cross promotion: Selling and swapping users in mobile games. *International Journal of Research in Marketing* 40(2):342–361, URL https://www.sciencedirect.com/science/article/pii/S0167811622000829, publisher: Elsevier.

Haviv A, Huang Y, Li N (2020) Intertemporal Demand Spillover Effects on Video Game Platforms. *Management Science* 66(10):4788–4807, ISSN 0025-1909, 1526-5501, URL http://dx.doi.org/10.1287/mnsc.2019.3414.

Helgesen 2006) Are Loyal Customers Profitable? Customer Satisfaction, Customer (Action) Loyalty and Customer Profitability at the Individual Level. *Journal of Marketing Management* 22(3-4):245–266, ISSN 0267-257X, 1472-1376, URL http://dx.doi.org/10.1362/026725706776861226.

Hotz VJ, Miller RA (1993) Conditional choice probabilities and the estimation of dynamic models. *The Review of Economic Studies* 60(3):497–529, URL https://academic.oup.com/restud/article-abstract/60/3/497/1570375, publisher: Wiley-Blackwell.

Huang N, Wang L, Hong Y, Lin L, Guo X, Chen G (2023) When the Clock Strikes: A Multimethod Investigation of On-the-Hour Effects in Online Learning. *Information Systems Research* isre.2023.1234, ISSN 1047-7047, 1526-5536, URL http://dx.doi.org/10.1287/isre.2023.1234.

Huang N, Zhang J, Burtch G, Li X, Chen P (2021a) Combating Procrastination on Massive Online Open Courses via Optimal Calls to Action. *Information Systems Research* 32(2):301–317, ISSN 1047-7047, 1526-5536, URL http://dx.doi.org/10.1287/isre.2020.0974.

Huang Y, Jasin S, Manchanda P (2019) "Level Up": Leveraging Skill and Engagement to Maximize Player Game-Play in Online Video Games. *Information Systems Research* 30(3):927–947, ISSN 1047-7047, 1526-5536, URL http://dx.doi.org/10.1287/isre.2019.0839.

Huang Y, Lim KH, Lin Z (2021b) Leveraging the Numerosity Effect to Influence Perceived Expensiveness of Virtual Items. *Information Systems Research* 32(1):93–114, ISSN 1047-7047, 1526-5536, URL http://dx.doi.org/10.1287/isre.2020.0955.

Hunicke R (2005) The case for dynamic difficulty adjustment in games. *Proceedings of the 2005 ACM SIGCHI International Conference on Advances in computer entertainment technology*, 429–433 (Valencia Spain: ACM), ISBN 978-1-59593-110-8, URL http://dx.doi.org/10.1145/1178477.1178573.

Jiang Y, Uetake K, Yang N (2023) Engagement Dynamics in mHealth. *Available at SSRN 4298547* URL https://www.canadiancentreforhealtheconomics.ca/wp-content/uploads/2023/04/Jieang-Uetake-Yang_Engagement-Dynamics-in-mHealth.pdf.

Jiao Y, Tang CS, Wang J (2021) Selling Virtual Items in Free-to-Play Games: Transparent Selling vs. Opaque Selling. *Service Science* 13(2):53–76, ISSN 2164-3962, 2164-3970, URL http://dx.doi.org/10.1287/serv.2021.0271.

Joo J, Chiong K (2023) Getting the Most Out of Online A/B Tests Using the Minimax-Regret Criteria. *B Tests Using the Minimax-Regret Criteria (November 14, 2023)* URL https://papers.ssrn.com/sol3/papers.cfm?abstract_id=4323382.

Kasahara H, Shimotsu K (2009) Nonparametric Identification of Finite Mixture Models of Dynamic Discrete Choices. *Econometrica* 77(1):135–175, ISSN 0012-9682, URL http://dx.doi.org/10.3982/ECTA6763.

Keane MP (1997) Modeling Heterogeneity and State Dependence in Consumer Choice Behavior. *Journal of Business & Economic Statistics* 15(3):310–327, ISSN 0735-0015, 1537-2707, URL http://dx.doi.org/10.1080/07350015.1997.10524709.

Kumar V, Shah D (2004) Building and sustaining profitable customer loyalty for the 21st century. *Journal of retailing* 80(4):317–329, URL https://www.sciencedirect.com/science/article/pii/S0022435904000636, publisher: Elsevier.

Lei Y, Shen J, Yang E, Zhai X (2024) On the Profitability of Loyalty. *Marketing Science* Articles in Advance, URL https://scholar.google.com/scholar?hl=en&as_sdt=0%2C7&q=on+the+profitability+of+loyalty&oq=.

Leung ACM, Santhanam R, Kwok RCW, Yue WT (2023) Could Gamification Designs Enhance Online Learning Through Personalization? Lessons from a Field Experiment. *Information Systems Research* 34(1):27–49, ISSN 1047-7047, 1526-5536, URL http://dx.doi.org/10.1287/isre.2022.1123.

Li Y, Ryan CT, Sheng L (2023a) Optimal Sequencing in Single-Player Games. *Management Science* 69(10):6057–6075, ISSN 0025-1909, 1526-5501, URL http://dx.doi.org/10.1287/mnsc.2022.4665.

Li Y, Ryan CT, Sheng L, Wong B (2023b) Optimal World Design in Video Games. *Available at SSRN 4618046* URL https://christopher-thomas-ryan.github.io/papers/world-design.pdf.

Liu D, Santhanam R, Webster J (2017) Toward meaningful engagement. *MIS quarterly* 41(4):1011–1034, URL https://www.jstor.org/stable/26630283, publisher: JSTOR.

Mai Y, Hu B (2023) Optimizing Free-to-Play Multiplayer Games with Premium Subscription. *Management Science* 69(6):3437–3456, ISSN 0025-1909, 1526-5501, URL http://dx.doi.org/10.1287/mnsc.2022.4510.

Miao J, Jain S (2024) Designing Loot Boxes: Implications for Profits and Welfare. *Marketing Science* mksc.2023.0007, ISSN 0732-2399, 1526-548X, URL http://dx.doi.org/10.1287/mksc.2023.0007.

Nair H (2007) Intertemporal price discrimination with forward-looking consumers: Application to the US market for console video-games. *Quantitative Marketing and Economics* 5(3):239–292, ISSN 1570-7156, 1573-711X, URL http://dx.doi.org/10.1007/s11129-007-9026-4.

Nevskaya Y, Albuquerque P (2019) How Should Firms Manage Excessive Product Use? A Continuous-Time Demand Model to Test Reward Schedules, Notifications, and Time Limits. *Journal of Marketing Research* 56(3):379–400, ISSN 0022-2437, 1547-7193, URL http://dx.doi.org/10.1177/0022243718821698.

Reichheld FF, Teal T (1996) The loyalty effect: The hidden force behind growth, profits and lasting. *Harvard Business School Publications* URL https://www.raliance.org/prevention-database/sites/default/files/webform/pdf/pdf-the-loyalty-effect-the-hidden-force-behind-growth-profits-and--frederick-f-reichheld-thomas-t pdf, publisher: Boston.

Reinartz WJ, Kumar V (2000) On the Profitability of Long-Life Customers in a Noncontractual Setting: An Empirical Investigation and Implications for Marketing. *Journal of Marketing* 64(4):17–35, ISSN 0022-2429, 1547-7185, URL http://dx.doi.org/10.1509/jmkg.64.4.17.18077.

Runge J, Levav J, Nair HS (2022) Price promotions and "freemium" app monetization. *Quantitative Marketing and Economics* 20(2):101–139, ISSN 1570-7156, 1573-711X, URL http://dx.doi.org/10.1007/s11129-022-09248-3.

Rutz O, Aravindakshan A, Rubel O (2019) Measuring and forecasting mobile game app engagement. *International Journal of Research in Marketing* 36(2):185–199, URL https://www.sciencedirect.com/science/article/pii/S0167811619300023, publisher: Elsevier.

Santhanam R, Liu D, Shen WCM (2016) Research Note—Gamification of Technology-Mediated Training: Not All Competitions Are the Same. *Information Systems Research* 27(2):453–465, ISSN 1047-7047, 1526-5536, URL http://dx.doi.org/10.1287/isre.2016.0630.

Santhanam R, Sasidharan S, Webster J (2008) Using Self-Regulatory Learning to Enhance E-Learning-Based Information Technology Training. *Information Systems Research* 19(1):26–47, ISSN 1047-7047, 1526-5536, URL http://dx.doi.org/10.1287/isre.1070.0141.

Seetharaman P, Ainslie A, Chintagunta PK (1999) Investigating Household State Dependence Effects across Categories. *Journal of Marketing Research* 36(4):488–500, ISSN 0022-2437, 1547-7193, URL http://dx.doi.org/10.1177/002224379903600407.

Sheng L, Zhao X, Thomas Ryan C (2024) Selling Bonus Actions in Video Games. *Management Science* mnsc.2022.02348, ISSN 0025-1909, 1526-5501, URL http://dx.doi.org/10.1287/mnsc.2022.02348.

Shin J, Sudhir K (2010) A Customer Management Dilemma: When Is It Profitable to Reward One's Own Customers? *Marketing Science* 29(4):671–689, ISSN 0732-2399, 1526-548X, URL http://dx.doi.org/10.1287/mksc.1090.0547.

Wang L, Lowry PB, Luo XR, Li H (2023) Moving Consumers from Free to Fee in Platform-Based Markets: An Empirical Study of Multiplayer Online Battle Arena Games. *Information Systems Research* 34(1):275–296, ISSN 1047-7047, 1526-5536, URL http://dx.doi.org/10.1287/isre.2022.1127.

Wernerfelt B (1991) Brand Loyalty and Market Equilibrium. *Marketing Science* 10(3):229–245, ISSN 0732-2399, 1526-548X, URL http://dx.doi.org/10.1287/mksc.10.3.229.

Xue S, Wu M, Kolen J, Aghdaie N, Zaman KA (2017) Dynamic Difficulty Adjustment for Maximized Engagement in Digital Games. *Proceedings of the 26th International Conference on World Wide Web Companion - WWW '17 Companion*, 465–471 (Perth, Australia: ACM Press), ISBN 978-1-4503-4914-7, URL http://dx.doi.org/10.1145/3041021.3054170.

Zhao Y, Yang S, Shum M, Dutta S (2022) A Dynamic Model of Player Level-Progression Decisions in Online Gaming. *Management Science* 68(11):8062–8082, ISSN 0025-1909, 1526-5501, URL http://dx.doi.org/10.1287/mnsc.2021.4255.

Zohaib M (2018) Dynamic Difficulty Adjustment (DDA) in Computer Games: A Review. *Advances in Human-Computer Interaction* 2018:1–12, ISSN 1687-5893, 1687-5907, URL http://dx.doi.org/10.1155/2018/5681652.

Appendix

A. Effect of Item Purchase on Player Win Probability

We provide additional evidence in estimating the effect of item purchase on player win probability by conducting a localized before-and-after analysis of player win rates. This analysis focuses on player performance immediately before and after an item purchase. Specifically, we compare the win rates of players over the course of five games before and five games after they make an in-game purchase.

The after coefficient in Table A.1 captures the immediate effects of the purchased item on player win probability, controlling for ability, level, and opponent effects. We find that one in-game item transaction translates to around 3.6 percentage points increase in win probability. This effect size is largely consistent with the item effects measured in our production function, which quantifies the overall impact of in-game item purchase on player win probability across their entire match records.

Table A.1 Linear Probability Model: Before and After Purchase (5 Games)

	(1) Win
ability	0.52063***
	(0.02648)
after	0.03554***
	(0.00679)
lvl=7	0.02375**
	(0.01136)
lvl=8	0.02674**
	(0.01129)
lvl=9	-0.07839***
	(0.01148)
lvl=10	-0.05936***
	(0.01279)
lvl=11	-0.10529***
	(0.01271)
opponent elo score	-0.14020***
	(0.01811)
Constant	0.33278***
	(0.01627)
Observations	20241
Adjusted R^2	0.029

Note: Robust standard errors in parentheses; *** <0.01, ** p<0.05, * p<0.1

B. Pay-to-Win and Player Welfare

We evaluate the welfare implications of the pay-to-win business model in gaming environments. To simulate a scenario without pay-to-win, we set the cost of purchase coefficient to infinity so that no purchase occurs. Our result in Table B.1 suggest that pay-to-win models can create a win-win situation for both the firm and the player. Firm benefits from monetizing player engagement and retention. Players benefit from the option to purchase in-game items that can enhance effective win probability of the players, enabling faster progression and greater rewards. Among the three segments, the welfare gain from pay-to-win is greatest for segment 1, the *premium enthusiasts*, at 1.95%. As shown in Figure B.1, which shows player ability distribution at the final level of the game, pay-to-win options allow less able players to progress further than they otherwise would have, leveling the playing field and enhancing player experience in the game.

Table B.1 No Pay-to-Win and Player Welfare

	Current Design			No Pay-to-Win		
	Seg 1	Seg 2	Seg 3	Seg 1	Seg 2	Seg 3
Welfare	5.97	5.68	5.76	5.86 (-1.95%)	5.63 (-0.86%)	5.74 (-0.25%)
Average total games played	243.15	166.73	196.51	228.68	158.33	193.04
Average level reached	8.92	7.79	7.97	8.62	7.67	7.92

Figure B.1 Ability Distribution at the Final Level: Current Desgin vs No Pay-to-Win