A Structural Model of Consumer Utility Generation in Virtual Environments

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Virtual environments, such as gaming, offer rich opportunities for personalization due to their controlled and dynamic nature, but also pose challenges as they interact with heterogeneous consumer motivations to spend time and money. These two decisions of time and money are dynamically interlinked: money (purchase "tools") spent changes the utility from the time spent. In this paper, we build a structural model of player behavior in gaming environments, where players dynamically and heterogeneously optimize their choices of time and money as inputs. We use the model to personalize promotions and the gaming environment to improve player retention and monetization. The model generalizes dynamic durable goods purchase models (where only purchases are made) and dynamic models of effort/time response (as in incentive compensation models); this makes our model suitable for novel virtual and gamified environments requiring both time/effort and money inputs (e.g., digital learning/health habits, gamified loyalty programs). Estimates using data from a single player golf game reveal three latent segments of players: *premium enthusiasts* who derive enjoyment from play itself and most willing to purchase tools; and *win-seekers* and *progress-seekers* who both find playing the game itself costly and have higher price sensitivity– the former primarily values immediate rewards, while the latter also values level-up rewards. We use the model to generate real-time personalization policies on who to target and when during gameplay with (i) giving tools; and (ii) dynamic difficulty adjustment of the game. Our results demonstrate dynamic complementarities and substitution in time and money inputs as players heterogeneously and dynamically interact with the gaming environment.

Key words: gaming environments, retention, monetization, personalization, dynamic structural model

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1. Introduction

The gaming industry continues to gain in importance both in terms of share of consumer time and money. As of March 2024, an estimated 59% of consumers spend more than an hour a day on video games, with 32% claiming to spend more than 5 hours a day.[1] Worldwide revenues for the video games market in 2023 is estimated at $250 billion and projected to grow over 8% a year. Revenue per user for mobile games is estimated at $53 in 2023.[2] The advancement of virtual and augmented reality technologies is expected to further accelerate consumer spending and time in virtual environments.

The gaming and gamified environments present a unique time and money problem because the consumer utility generation process is *active* involving inputs of time and money that *dynamically interact* in response to changing game environments. In conventional settings involving consumer decisions to spend time and money (e.g., subscriptions, plan purchases) where consumers "purchase then consume", consumers typically receive a fixed utility from the consumption of goods or services in a passive process. In contrast, money spent by players in gaming/gamified environments changes the utility of the time they invest — in essence, money transforms the "product" (e.g., game) itself in real-time. Purchases are not merely transactions; they directly enhance agent performance and alter the gaming experience. As players invest money to acquire tools or enhancements, their ability to win increases, which in turn increases the value of the time they spend playing. This can create dynamic complementarities between time and money inputs within certain regions of the game design space: as players' performance improves due to their purchases, they derive more utility from their time investment, leading to increased retention and a higher willingness to spend additional money as they progress to higher levels of game difficulty in a positive feedback mechanism.

In this paper, we develop a dynamic structural model that captures the active process of consumer utility generation in gaming environments, where players dynamically and heterogeneously optimize

[1] https://www.statista.com/forecasts/997154/hours-spent-on-playing-video-games-per-week-in-the-us

[2] https://www.statista.com/outlook/dmo/digital-media/video-games/worldwide

their choices of time (play or quit decisions) and monetary inputs (purchasing tools) in response to the game environment. The model captures key aspects that are common in such environments: (i) the level progression design that increases in difficulty, resulting in player attrition and retaining only the high-performing players, (ii) the opportunity to purchase tools that enhance players' effective win probabilities, so that players can endogenously balance their level of challenge at a cost, and (iii) short-term rewards for immediate success that sustain interest in the game even as they seek long-term rewards of reaching the next level. Our model generalizes existing frameworks that have typically focused on either monetary purchases, as in dynamic durable goods models, or time and effort responses, such as in incentive compensation models. This makes the model applicable not only to gaming but also to other gamified environments such as digital learning platforms, digital health applications, and loyalty programs that require active user require active user participation in effort and monetary investment that reduces the cost of effort.

We use the model to identify the *target* and *timing* to generate real-time personalization policies that maximize profits. The managerial challenge is twofold: first, the presence of rich player heterogeneity in observed abilities and unobserved gameplay preferences (e.g., time and money costs, reward valuation) complicates the identification of the right targets for these interventions. Second, this heterogeneity in player decisions dynamically interacts with the evolving game environment, further complicating the process of determining the optimal timing for each intervention. The model allows us to predict when and which players are likely to quit and assess how interventions, such as discounts and dynamic difficulty adjustments, impact players' dynamic input decisions of time and money. For example, the model helps to determine whether offering discounts will create dynamic complementarities—where time and money inputs reinforce each other—or lead to substitution, where the increased ability to win expedites player's level progression and exit, potentially cannibalizing future profits. Since discounts can act as both dynamic complements and substitutes depending on the timing and state of the game environment–even for the same player– the model is needed to generate the hypotheses to pinpoint the optimal timing for each intervention.

We estimate the model using data from individual play and purchase choices in a free-to-play single-player mobile golf game. Our comprehensive dataset includes detailed match-level information on players' actions, environments, rewards, and progression throughout their entire gaming experience. We also obtain detailed records of players' in-game tool purchase transactions, which allows us to examine the relationship between the timing of tool purchases and their impact on player performance. In most settings, individual ability is considered an unobserved variable. Our game environment context and detailed data enable us to treat player ability as an observed variable by estimating it from the player lifetime gameplay records. This allows us to account for wide varying heterogeneity in player ability and design personalized interventions for player retention and monetization.

Our estimation strategy extends and adapts the two-step estimation framework in Chung et al. (2014). First, we estimate the player win probability function to obtain player ability estimates and incorporate player ability heterogeneity in the first stage estimation of the conditional choice probability (CCP). We accommodate latent class heterogeneity and use the expectation-maximization (EM) algorithm within the two-step framework, following the approach in Arcidiacono and Miller (2011). We estimate the structural parameters in the second stage estimating the value function for each ability and latent segment type. Estimates reveal three latent segments of players: (i) *premium enthusiasts*, the smallest share of players who spend the most and do not find playing the game costly but rather enjoy spending time in the game, (ii) *win-seekers* who have the second lowest price sensitivity but find playing the game itself costly, and primarily values immediate wins more than long-term level-up rewards, and (iii) *progress-seekers*, the largest share of players who have the highest price sensitivity and find playing the game itself costly to play, but receive higher utility from level-up rewards than *win-seekers.*

Our first counterfactual investigates the dynamic interaction between players' time (play/exit) and money (purchase tools) choices. Specifically, we offer discounts at players' "give-up" levels – the point where they typically exit the game, a timing likely to induce potential dynamic complementarities between early and late tool purchases: discounts boost tool purchases, which enhance win probabilities and retention, while also accelerating level progression, opening up further monetization opportunities as players face greater challenges. While a 70% discount was found optimal, increasing total profit by 6.44%, we find substantial heterogeneity among player segments. The positive feedack mechanism induced by dynamic complementarities between money and time (and early and later tool purchases) were greatest among *progress-seekers*, with 6.20% of players making additional non-discount purchases following the discount. However, a key caveat is that while these discounts can enhance player progression and short-term profitability, they may also act as dynamic substitutes by reducing overall retention. As players purchase items and perform better, they advance through levels more quickly, reaching their exit points sooner. Thus, while targeted discounts can generate short-term profit gains, they may inadvertently shorten the overall engagement period for some players. Discount cannibalization from targeting timing inaccuracy (i.e., offering discounts too early) was greatest among *premium enthusiasts*, who derive enjoyment from the act of playing itself.

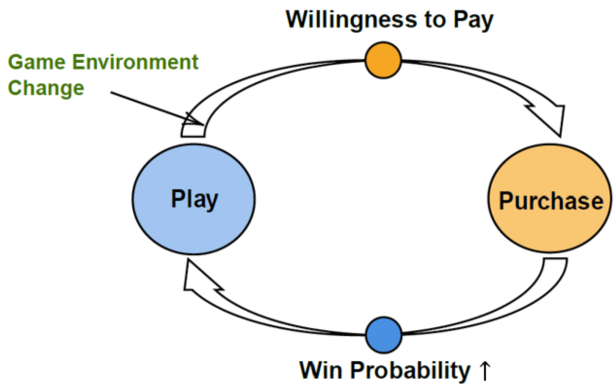
Second, we investigate the dynamic interlinkages in reducing player time spent at early levels on future purchases by personalizing the game progression speed. Building on the concept of dynamic difficulty adjustment (DDA), we examine the effect of changing the game progression speed by appropriately increasing player win probability at early levels for high ability players. By accelerating early level progression for these players, we seek to optimize the player play and purchase dynamics, providing more challenging game content sooner. Overall, accelerating early level progression for high ability players increases total profit and player retention, but we find heterogeneous effects across latent segments, even among the players with the same ability level. The firm gains in profit from players who find playing the game itself more costly but receive higher utility from level progression (*progress-seekers*) by 10%, but loses in profit from players who find playing the game itself more enjoyable (*premium enthusiasts*) and players who receive higher utility from immediate winning (*win-seekers*). Thus, accelerating game difficulty to match player performance increases firm profits by preventing early exits for players with high time costs who value progression, but can hurt profits from those who enjoy spending time in the game itself or value immediate wins more.

2. Related Literature

Our paper contributes to several streams of literature. First, we contribute to the growing literature on the video games market. The first stream of papers in this literature studies consumer demand for console video games (e.g., Clements and Ohashi 2005, Nair 2007, Derdenger and Kumar 2013, Derdenger 2014, Haviv et al. 2020). The shift in consumer focus to the online and mobile gaming landscape has enabled researchers to collect and utilize detailed usage records. This has given rise to the second stream of empirical papers that studies user engagment and gameplay behavior (e.g., Huang et al. 2019, Nevskaya and Albuquerque 2019, Rutz et al. 2019, Zhao et al. 2022, Castelo-Branco and Manchanda 2023, Chen 2023), monetization strategy and welfare (Appel et al. 2020, Huang et al. 2021b, Runge et al. 2022, Ascarza et al. 2023, Haenlein et al. 2023, Joo and Chiong 2023, Wang et al. 2023, Amano and Simonov 2024), and analytical papers studying game and in-game product monetization (e.g., Guo et al. 2019, Jiao et al. 2021, Chen et al. 2021, Li et al. 2023a, Li et al. 2023b, Mai and Hu 2023, Harutyunyan and Koca 2024, Miao and Jain 2024, Sheng et al. 2024).

Despite of the significant and growing market size of the gaming industry, there is a lack of research documenting consumer spending behavior on in-game items, which are the most common and major driving source of revenue in the gaming market. Our work is related to the small but growing literature that empirically investigate in-game retention and monetization design involving in-app purchases (e.g., Huang et al. 2021b, Runge et al. 2022, Ascarza et al. 2023, Joo and Chiong 2023, Wang et al. 2023, Amano and Simonov 2024). To the best of our knowledge, Amano and Simonov (2024) is the first structural paper that examines player play and purchase dynamics addressing the welfare implications of loot box spending and game designs for regular and high-spending players. Our paper builds on and extends this literature by focusing on performance-enhancing tools, which play a central role in player retention and monetization strategies. This makes our approach distinct, as it emphasizes the *active* and *dynamic* process of consumer utility generation, driven by these tools. Our model incorporates rich nonlinear interactions between player ability and unobserved preference parameters to assess the dynamic impact of personalized interventions and game features – the dynamic complementarities and substitutions between time and money inputs. We illustrate our conceptual framework in Figure 1.

Figure 1 The Play and Purchase Loop in the Gaming Environment



Thus, our framework extends and generalizes empirical settings that involve a dynamic effort/time response, where agents exert effort to achieve performance goals. This includes various contexts such as salesforce management, digital health, and online education. For example, an agent's achievement state in relation to sales compensation rewards can motivate performance (effort), which in turn increases monetization for the firm (Chung et al. 2014), and sales training can be used to manage salesforce retention and performance (Chung et al. 2021). Temporary incentives to curb smartphone addiction improve users' self-control ability and can have lasting effects in their well-being (Allcott et al. 2022). Premium app version adoption (purchases) can help increase future user engagement in mHealth applications (Jiang et al. 2023). Training learners to follow self-regulated learning strategies can increase learning outcomes (Santhanam et al. 2008). Our paper presents a comprehensive framework for gamified and game-like settings that involve inputs of time/effort by integrating the purchase component.

Compared to standard retail environments, gaming environments enable the collection of more detailed data on the ongoing engagement and monetization of users. While other digital and freemium products such as Dropbox also have consumer usage data, the difference in gaming and gamified online environments is the dynamic modification of the product (game). Within this class of products, firms have the opportunity dynamically change and offer products in a personalized manner. The personalization principles developed in the context of gamified systems claim that accounting for individual heterogeneity can increase engagement and task outcomes (Liu et al. 2017). Such has been documented in gamified settings such as e-training and online learning environments (e.g., Santhanam et al. 2016, Huang et al. 2021a, Huang et al. 2023, Leung et al. 2023).

In the context of gaming, dynamic difficulty adjustment (DDA) is a widely used method by game designers to adaptively modify game challenges and features real-time based on players' abilities and preferences (e.g., Hunicke 2005, Xue et al. 2017, Zohaib 2018, Huang et al. 2019, Zhao et al. 2022). To our knowledge, Ascarza et al. (2023) is the first marketing paper that studies DDA interventions with implications for games that use an in-app purchase monetization model. Using a field experiment, the authors suggest evidence consistent in showing the value of designing personalized difficulty interventions for player retention and monetization.

While A/B testing approaches can validate our personalization policies, a structural model is needed to address the rich interactions between multidimensional heterogeneity (e.g., ability, price sensitivity, value of progression) and the dynamic nature of treatment effects. The combination of these factors, along with their dynamic incremental impacts with response to the changing game environments, scales beyond the capabilities of A/B testing. By providing a theoretical understanding of the dynamics, structural models can cost-effectively assess and inform which sets of design interventions are likely to yield the best payoffs. Our model framework allows us to dynamically redesign consumer engagement and monetization in a personalized and scalable manner.

3. Data and Empirical Setting

This section describes the game, provides details of the data, and presents model-free evidence that informs our model development.

3.1. Description of the Game

Our empirical setting is a popular free-to-play single-player mobile golf game with over 2 million registered users. In this game, players engage in one-hole game matches, where the objective is to complete the hole with fewer shots than the opponent. Each game lasts around three to five minutes, and players are assigned matches by the game platform once entering the game. Players accumulate points from winning the match (and lose points from losing), and collection of these points is required to unlock higher levels in the game. The game design ensures a sequential progression where higher levels demand the accumulation of more points to be unlocked. The game has a total of 11 levels.

The level-progression system of the game is designed to increase difficulty through several design features. We report the points system of the game in Table 1. The expected points for each game given a player's win probability are calculated based on the game's win-loss points schedule, which varies by level. We illustrate the level difficulty system embedded within this points system using levels 6 to 11 as examples in Figure 2. In the figure, each line represents a different level, demonstrating that as players advance to higher levels, the expected points for a given win probability generally decrease under the same win rate. Figure 3a helps further illustrates this point by highlighting that as the level increases, the win probability required to at least break even (i.e., zero expected points) also increases, requiring players to have higher win rates at subsequent levels. Finally, higher levels require higher points accumulation criterion to level up, making progression increasingly demanding, as shown in Figure 3b. Overall, the points system of the game is designed to discriminate on player ability, ensuring that only the most able players progress to the top levels.

Players can enhance their win rates by purchasing tools. The in-app purchases consist of durable ability enhancers (i.e., golf clubs) that allow players to improve their win rates. For example, a paid golf club enables greater range and ball guide precision when taking a shot. By incurring a monetary cost, players can increase their chances of progressing through the levels that they might not have had otherwise. By allowing players to self-select the balance between the need to win and

Table 1 Points and Level Progression Design of the Game

Level	Win	Lose	Total Available Points	Cumulative Points Collection
1	+4	-1	25	-
2	+6	-2	75	25
3	+8	-3	125	100
4	+10	-4	175	225
5	+12	-7	225	400
6	+14	-10	300	625
7	+16	-13	375	925
8	+18	-16	450	1300
9	+20	-20	550	1750
10	+22	-24	700	2300
11	+24	-36	900	3000
Final Lvl Clear				3900

the cost of purchasing ability enhancers, the firm can effectively monetize across different player segments of different abilities and price sensitivity. The most popular in-game tool offerings cost \$9.99 (generating around 60% of revenue), and around 90% of total durable tool transactions are generated from product offerings between \$9.99 and \$19.99. Higher-priced offerings provide more quantities of golf clubs that can help enhance player win rates. While the game also features in-app advertising, the firm primarily generates revenue through in-app purchases.

The game offers a suitable setting for a single-agent model. From the player perspective, it is effectively a single-player experience, in that in the game of golf, player's performance –the number of shots players take to complete a hole – is independent of the opponent's actions, making the opponent an exogenous factor influencing the game outcomes. The players do not strategically select which opponent to play against, but rather get assigned by the platform.[3] The progression system of the game incentivizes players to continually improve their skills to maintain or increase their win rates and progress through the levels, inducing a dynamic and forward-looking behavior.

[3] In the game, approximately 40% of gameplay matches involve bots, but players cannot distinguish whether their opponents are real players or computer-simulated. The competitor ability increases by each level due to selection, and players are randomly matched with opponents of average ability corresponding to their level. In our model estimation, we control out the effect of the opponent as an exogenous factor to winning outcomes.

Figure 2 **Game Points Design: Expected points by Win Probability by Level**

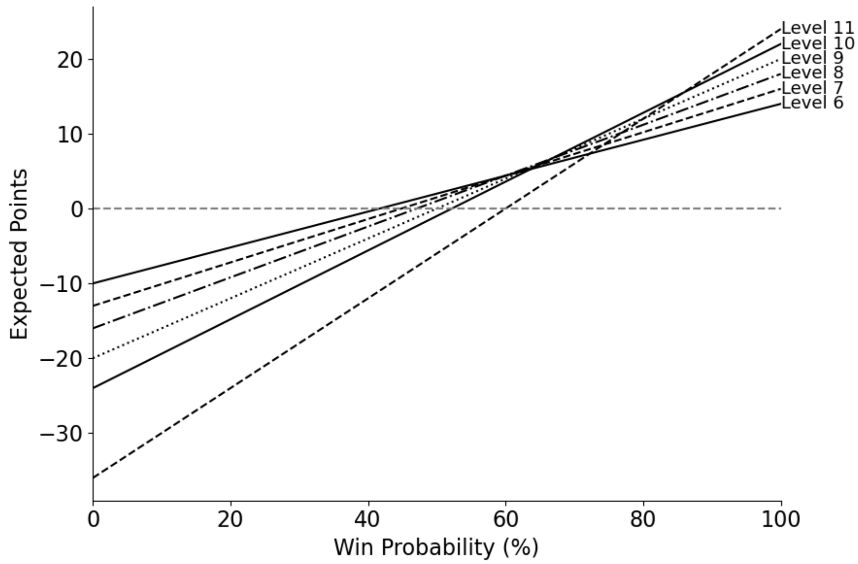
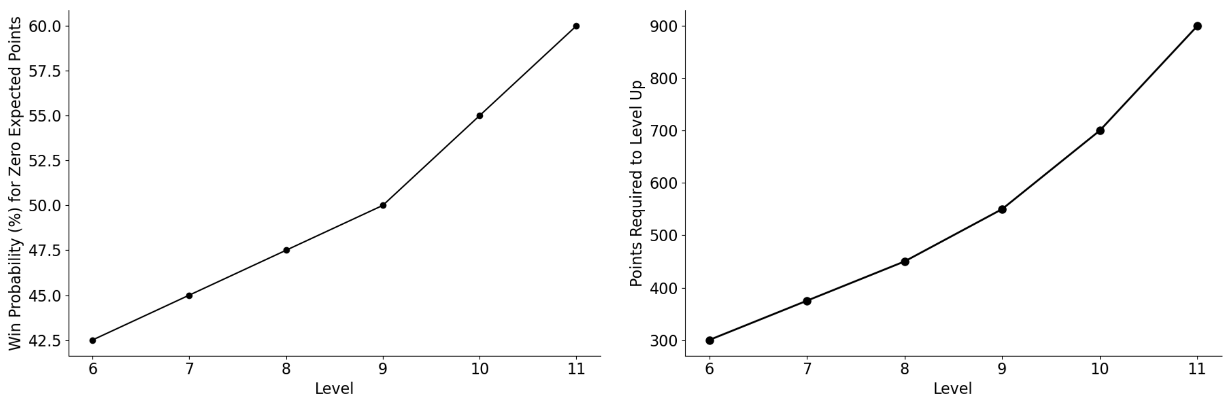


Figure 3 **Game Points and Level-Progression Design**

(a) Win Probability for Zero Expected Points by Level **(b) Points Collection Requirement to Level-Up**



3.2. Data

We leverage a comprehensive dataset comprising every player action within the game environment, along with environmental data spanning each player's lifetime from initiation to exit. This includes detailed observations of players' decisions to play or quit, progression, rewards, in-game tool usage, and the gaming environments they encounter, including the opponents they face. Additionally, we have detailed records of players' in-game tool purchase transactions, enabling us to examine the relationship between the timing of in-game purchases, game environment conditions, and player performance.

Our analysis focuses on a random sample of 4163 players spanning a 15-month period, from October 2021 to January 2023. We construct the sample as follows. First, we take a random 10% sample of players who meet the criteria of having valid play records in the data period. Second, players who have completed at least level 5 (i.e., collected at least 15% of total points available) are retained. This criterion ensures that our analysis focuses on individuals who exhibit a sufficient level of engagement within the game environment, while excluding less committed players who frequently download the game but discontinue usage shortly thereafter. Finally, we exclude outliers with within-level gameplays above 1.5 times the interquartile range. The final dataset for analysis includes around 750,000 match records.

Table 2 contains summary statistics of our player sample. A median player engages in 274 games throughout their lifetime, and the median exit level is 7.[4] The average game play duration, measured by the number of days from first play to exit, is 106.21 days. Finally, given our sample, which includes players with a minimum progression to level 6, 22.36% have purchased tools at least once. While the median player makes no purchases, there is significant heterogeneity in the player purchase behavior at the higher end of the distribution, with total transactions ranging between 14 and 41 for players in the 99th percentile and above.

Table 2 Descriptive Statistics of Players

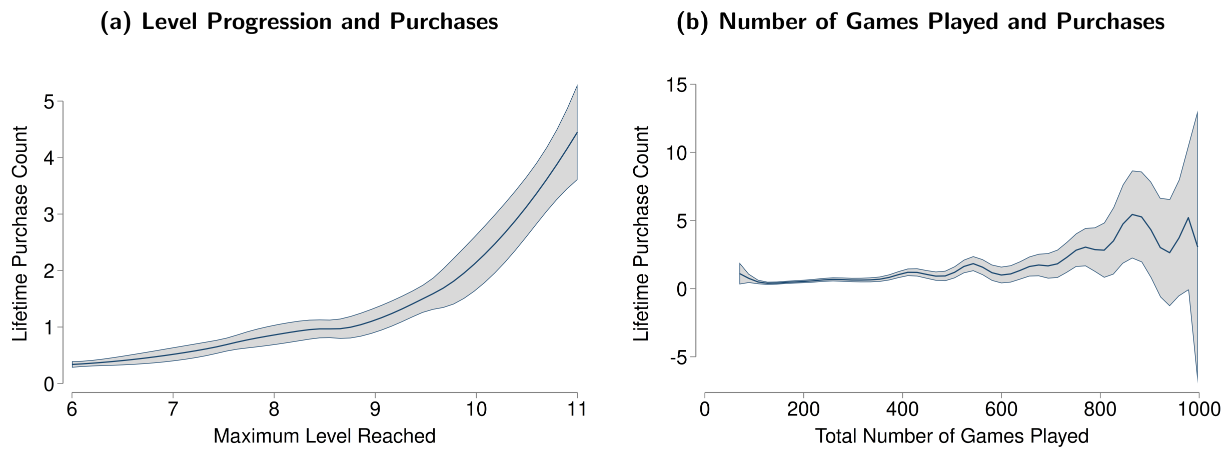
	Mean	SD	Median	75th	90th	99th	Max
Time							
Total Number of Games Played	331.57	225.18	274	406	583	1203	3211
Maximum Level Reached	7.41	1.54	7	9	10	11	11
Game Duration (*Days from First Play to Exit*)	106.21	98.55	73	151	259	405	454
Money							
In-app Purchase Player Share	22.36%						
Total Number of Purchase	0.88	2.69	0	0	3	13	41

[4] To define player exit beyond our data period, we apply the two-week churn condition.

3.3. Model-Free Evidence

We begin this section by providing descriptive evidence of the positive relationship between player retention (time) and monetization (money). We plot players' final level progression state with their lifetime number of purchases in Figure 4a. We show that players who have reached higher levels are also those who spend more. We similarly shown in Figure 4b that players who play more are generally those who make more purchases.

Figure 4 Relationship Between Player Retention and Monetization

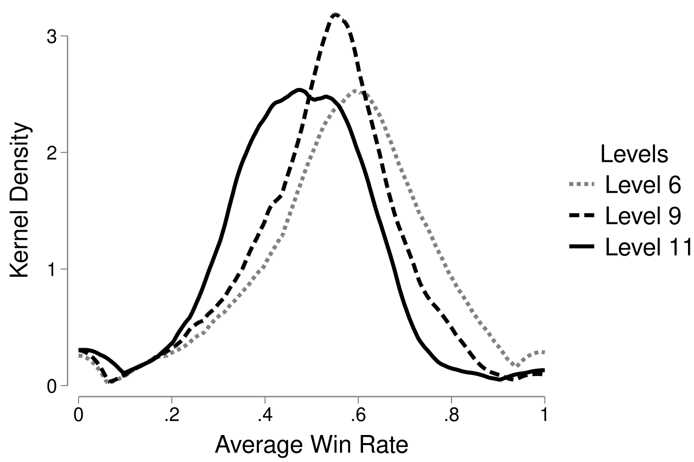


Note: The solid line is a kernel-weighted second-degree polynomial regression using a Gaussian kernel, and the shaded area is the 95% confidence interval.

Next, we present three key features of the data that inform our model development. First, we provide evidence of substantial heterogeneity in player win probability within and across levels. Second, we show the relationship between players' win rates and their purchase and exit decisions. Third, we demonstrate the dynamics in the timing of players' purchase and exit decisions in relation to their level completion status.

In the gaming environment, there exists substantial heterogeneity in player win rates, indicating their ability to progress through the game levels. We present the player differences in average win rate across players for different levels in Figure 5. First, players have generally lower win rates at higher levels, reflecting the increased level difficulty designed into the game. Within each level, there exist significant differences in player win probabilities, providing suggestive evidence of the varied abilities of players even at the same level. We report the Gini indices of player average win rate within each level in Table 3. We note that even small variations in win rates result in considerable differences in expected points rewards and player progression speed, as shown previously in Figure 2.

Figure 5 Average Win Rate Distribution of Players By Levels



Note: The lines represent kernel density estimates using the Epanechnikov kernel.

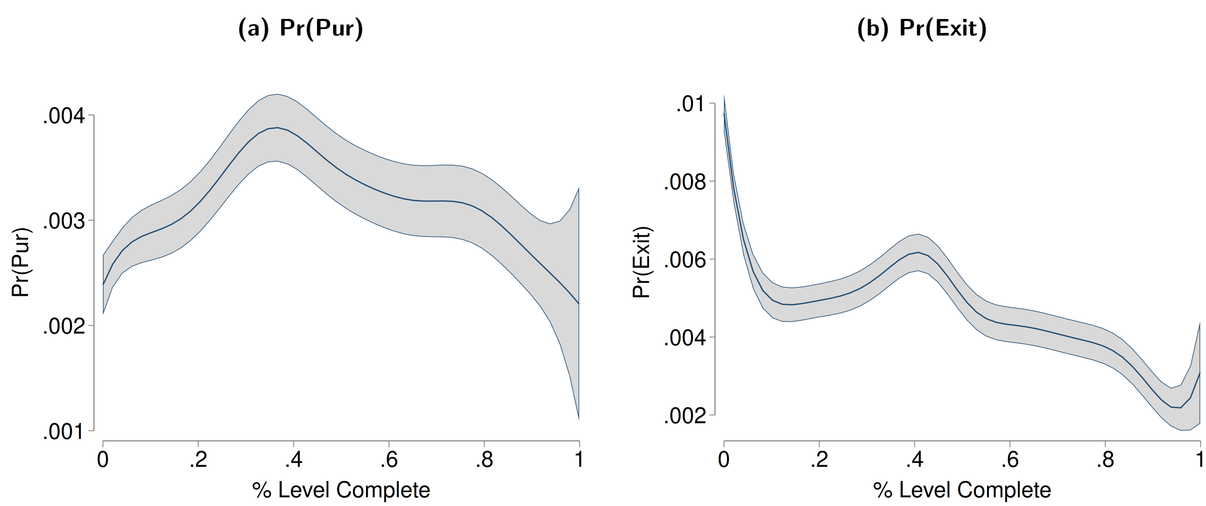
Table 3 Gini Indices of Player Average Win Rate by Level

Level	Gini Index
6	0.18
7	0.18
8	0.13
9	0.18
10	0.17
11	0.20

We next examine the relationship between players' win probability and their purchase and exit decisions, as illustrated in Figure 7. To prevent reverse causality (i.e., players purchasing tools which then increase their win probability), we plot player purchase and exit decision against their win rates from the previous level. Within each level, we find that players with higher win probabilities are more likely to continue playing and make in-game purchases as they gain positive continuation value in the gaming experience. Conversely, players with lower win rates are more likely to exit the game and purchase less, as utility decreases with lower win rates, reducing player perceived value of the game. This relationship underscores the importance of maintaining a balance in-game difficulty to retain players and encourage in-game spending.

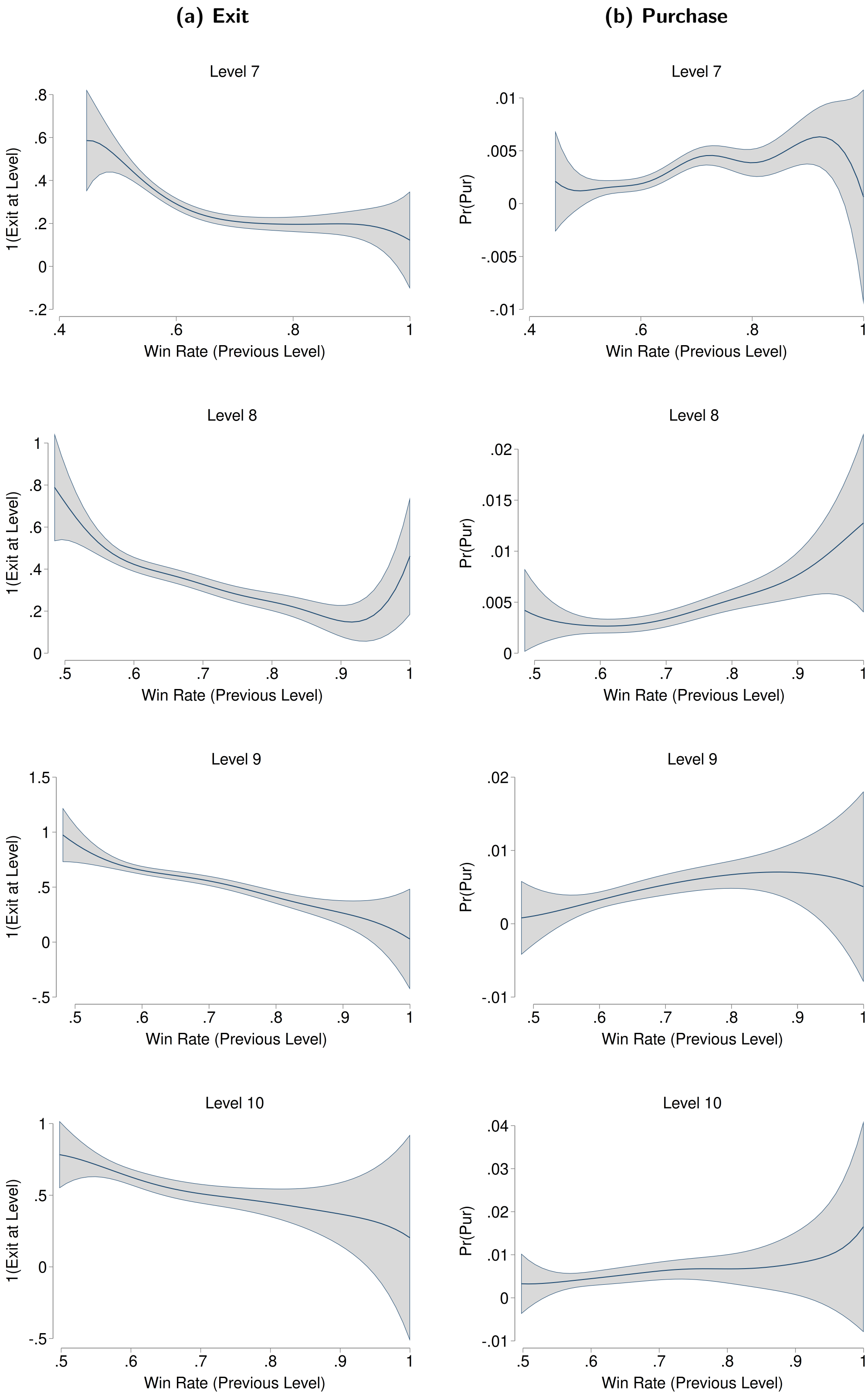
Finally, we show the relationship between the timing of players' purchase and exit decisions and their level completion status in Figure 6. Within each level, player exits are more likely to occur at the beginning when players face increased difficulty after level-up. The durable nature of tools encourages upfront purchases, typically around 20-30% into a level, when players believe they have a chance to progress. This suggests that within a level, players are more likely to purchase tools early on to enhance their rate of progression.

Figure 6 **Player Exit and Purchase by % Level Complete**



Note: The solid line is a kernel-weighted second-degree polynomial regression using a Gaussian kernel, and the shaded area is the 95% confidence interval.

Figure 7 Player Exit and Purchase by Win Rate



Note: The solid line is a kernel-weighted second-degree polynomial regression using a Gaussian kernel, and the shaded area is the 95% confidence interval.

4. Model

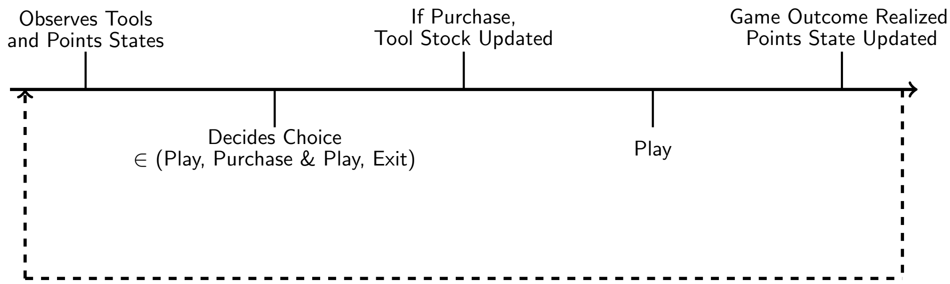
Based on the model-free evidence, we develop a dynamic model of player action in the gaming environment. Time indexed by t is discrete denoting each game. At the beginning of each period t, player i decides whether to (1) play, (2) make a purchase and play, or (3) exit the game permanently. To represent player states in the gaming environment, we parsimoniously track two key state variables $S_{it} = \{k_{it}, z_{it}\}$. First is player tool stock k_{it}, which tracks the number of in-app purchases a player incurred to improve their ability. It represents the vertical quality metric of the player's tools. Second is player progression points stock z_{it}. The points accumulation state (z_{it}) has a one-to-one mapping to the level (ℓ_{it}), given the progression design Ψ of the game, reflecting the player's current progression state in the game. Figure 8 describes the timing of the model. Before entering a game t, player i observes the current tools and progression (i.e., accumulated points) state in the game. The player decides on the action choice between playing, purchasing a tool and playing, and exiting the game permanently. If the player's decision was to purchase a tool, the tool stock is updated before play. The player then enters the game, and idiosyncratic game outcome shock is realized. The realized game outcome affects the player's progression points state in the next period. The model repeats every period over an infinite horizon. After the player completes all levels, the player can continue to play the unlocked stages, which is a pattern we observe for the majority of players in the game.

The game's point system and progression design induce the player's dynamic forward-looking behavior. A player choosing the action has to be concerned not just with the current payoff but with the effect of that choice on all future rewards and play costs.

4.1. Player Utility Generation Process

Player i in period t receives the following per-period utility based on his or her choice of actions $d_{it} \in \{1, 2, 3\}$: whether to play the game without incurring in-app purchase ($d_{it} = 1$), whether to make in-app purchase and play the game ($d_{it} = 2$), or whether to permanently exit the game platform ($d_{it} = 3$).

Figure 8 **Model Timeline**



$$u(S_{it}, d_{it}) = \begin{cases} \theta E[r_{it}(\alpha_i, S_{it}(d_{it}))] + c_m \cdot \mathbf{1}_{\{d_{it}=2\}} + c_p + R \cdot \mathbf{1}_{\text{LEVEL UP}(z_{it})} \cdot L_{\ell(z_{it})} + \epsilon_{it}, & \text{if } d_{it} \in \{1,2\}, \\ 0 & \text{otherwise (exit).} \end{cases}$$

Players derive utility from immediate rewards r_{it} (i.e., expected number of points gained from the current period play), and one-time rewards R for reaching a new level ℓ. The per-period player points reward r_{it} is determined by the game winning outcome $W_{it} \in \{0,1\}$ and the winning and losing points reward design of the game, $(\psi_v^\ell,\, \psi_d^\ell) \in \Psi$, which differs by each level ℓ. That is,

$$r_{it} = W_{it} \cdot \psi_v^\ell + (1 - W_{it}) \cdot \psi_d^\ell \tag{1}$$

and

$$E[r_{it}] = \Pr(W_{it} = 1) \cdot \psi_v^\ell + \Pr(W_{it} = 0) \cdot \psi_d^\ell \tag{2}$$

To model the increasing utility from leveling up at higher levels, we adjust R with respect to the level achievement criterion (i.e., the number of points required to level up), $L_{\ell(z_{it})}$. Since the level-up points criterion increases monotonically with levels in the current game design, players receive greater rewards for achieving higher levels.

If the player decides to purchase tools ($d_{it} = 2$), he or she incurs a monetary cost c_m. The decision to purchase updates the player's current tool stock state, which increases the chance of winning including the immediate period and all future play sequences of the game. The idiosyncratic shock ϵ_{it} follows an extreme value distribution. We normalize the exit value as 0.

4.1.1. Player Win Probability Function The player winning outcome $W_{it} \in \{0,1\}$ is a function of player level ℓ_{it}, tools k_{it}, and ability type α_i, such that

$$\Pr(W_{it}=1) = \ell(z_{it}) + \alpha_i + \delta_1 k_{it} + \delta_2 k_{it} \cdot \ell(z_{it}) + \xi_{it}. \tag{3}$$

The player win probability function accommodates the following key characteristics to model the gameplay outcome. First, the player's win probability decreases with level difficulty ℓ_{it}, which is determined by the point accumulation state z_{it}. First, the player's win probability increases with tools k_{it} and player's innate ability α_i. We allow for tools and level interaction to account for the diminishing effect of tools at higher levels. Finally, the idiosyncratic shock ξ_{it} affects the outcome of the game.

4.2. State Transitions

The state variables tool stock k_{it} and points stock z_{it} evolves deterministically as follows:

$$k_{it} = \begin{cases} \min(k_{it}+1, \overline{K}) & \text{if } d_{it} = 2 \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

The player's tool stock k_{it} increments by one with each purchase decision. Players can make multiple purchases over their lifetime. $\overline{K}$ is the maximum tool stock, representing the maximum tool quality upgrade level.

$$z_{it} = \begin{cases} \min(z_{it} + \psi_v^\ell, \overline{\psi}^L) & \text{if } W_{it} = 1 \\ \max(z_{it} - \psi_d^\ell, \underline{\psi}^\ell) & \text{otherwise} \end{cases} \tag{5}$$

Upon realization of the game outcome, the player's points stock state z_{it} evolves deterministically following the game's points progression design Ψ. Here, $\overline{\psi}^L$ is the maximum points state of the game at the final level and $\underline{\psi}^\ell$ is the points criterion to unlock level ℓ. This ensures that once a player has reached a certain level by accumulating enough points, they cannot fall back below that level's threshold, ensuring that once a level is unlocked, it remains unlocked. In other words, z_{it} is incremented by ψ_v^ℓ when winning the game, but cannot exceed $\overline{\psi}^L$. Similarly, z_{it} is reduced by ψ_d^ℓ but not below the minimum threshold $\underline{\psi}^\ell$.

4.3. Bellman Equation

A player chooses action d_{it} that maximizes the expected discounted sum of utilities given the game design Ψ, the state variables and their transitions, and the idiosyncratic shock ϵ in each period. The Bellman equation can be written as

$$V(k,z,\alpha,\epsilon;\Theta,\Psi) = \max\{\epsilon_{3it}, \max_{d\in\{1,2\}}\{U(k',z,\alpha,\epsilon;\Theta,\Psi) + \beta\mathbf{E}[V(k'',z',\alpha,\epsilon';\Theta,\Psi)|k',z,\alpha]\}\} \quad (6)$$

where the idiosyncratic shock ϵ follows a Type-I extreme value, and the discount factor β is set to 0.9.[5] The player continues to play the game if the expected continuation value is greater than 0, the normalized outside value of exit.

5. Estimation

We estimate the model using two-step estimation (Hotz and Miller 1993). In the first step, we estimate the player win probability function and the conditional choice probabilities (CCPs) of player action choice as a flexible function of state variables. The key assumption in the two-step estimation is that the first-stage CCPs represent the agent's optimal action probability given the state variables. In the second step, we estimate the structural parameters that rationalize the first stage policy estimates.

5.1. Step 1: Estimating CCPs

In the first stage CCP estimation, we estimate a flexible mapping between observable states and player action probability. The relevant state variables in our model are tool and points stock states, $S_{it} = \{k_{it}, z_{it}\}$. We estimate the two-step procedure using the player state space starting from level 6, both for computational efficiency in the state space and practical reasons. The early levels 1-5 consist of beginner level tutorials and short level length, with these five levels accounting for only 15% of total points collection available in the game. We discretize the points state for each level

[5] $\beta = 0.9$ gives the best fit compared to β= 0.95, 0.99, and 0.999. In our context, setting lower discount factors greatly underestimates player exit probabilities and hence overestimates player purchase probabilities. Average inter-play time gap (in day) is around 2 days, and average lifetime play duration is 106 days.

from 6 to 11 into 10 increments and additional transition states to track level-up bonus (a total of 65), and tool stock state to evolve deterministically up to 25 transactions. This leaves us with a total of 1690 state combinations.

Typically, player ability is treated as an unobserved variable. In order to incorporate rich player heterogeneity in win rates as shown in the model-free evidence, we instead treat player ability as an observed variable by estimating it from player lifetime gameplay records. Because our setting is a game, where we observe every player action, environment, and outcome, we can estimate player ability α_i as an individual fixed effects parameter from Equation 3, the player win probability function, controlling for the effects of tools and game environments on player win rates. By incorporating player ability as an observed variable, we can directly include it in our first stage policy estimation. This enables us to account for rich observed heterogeneity in player ability levels in predicting player action. For the first stage estimation, we normalize player ability score as a continuous variable between 0 and 1, with 1 representing the highest ability level.

Given the state variables and observed ability heterogeneity of the players, we estimate the player action policy using a flexible multinomial logistic regression. We account for player unobserved heterogeneity in the first stage CCP estimation through persistent latent segments and estimate heterogeneous policy functions using the EM algorithm (Arcidiacono and Miller 2011, Chung et al. 2014). We assume that player i belongs to one of G segments $g \in \{1, ..., G\}$ with segment probabilities $q_i = \{q_{i1}, ...q_{iG}\}$. Let π_g denote the population probability of being in segment g. We iteratively maximize the log likelihood in Equation 7,

$$\sum_{i=1}^{N} \sum_{g=1}^{G} \sum_{t=1}^{T} q_{ig} \ln[\mathcal{L}(d_{it}|S_{it}, \alpha_i, g, \beta_g)], \tag{7}$$

where

$$q_{ig} = \frac{\pi_g \prod_{t=1}^{T} \mathcal{L}(d_{it}|S_{it}, \alpha_i, g, \beta_g)}{\sum_{g=1}^{G} \pi_g \prod_{t=1}^{T} \mathcal{L}(d_{it}|S_{it}, \alpha_i, g, \beta_g)}, \tag{8}$$

and $\mathcal{L}(d_{it}|S_{it},\alpha_i,g,\beta_g)$ is the choice probability of taking action $d_{it}=j$ for segment type g,

$$\mathcal{L}(d_{it}|S_{it},\alpha_i,g,\beta_g)=\frac{e^{\beta_g^j[S_{it},\alpha_i]}}{\sum_j e^{\beta_g^j[S_{it},\alpha_i]}}. \tag{9}$$

The EM algorithm begins by setting initial values for β_g, and π_g.

(a) Compute $q_{ig}^{(m+1)}$ using Equation (2) with $\beta_g^{(m)}$ and $\pi^{(m)}$.

(b) Update population shares

$$\pi_g^{(m+1)}=\frac{1}{N}\sum_{i=1}^{N}q_{ig}^{(m+1)}.$$

(c) Update $\beta_g^{(m+1)}$ for each segment g by maximizing Equation (1) with $q_{ig}^{(m+1)}$ and $\pi_g^{(m+1)}$.

We iterate steps (a)-(c) until convergence. We initialize β_g by randomly partitioning the players into G segments and maximizing the log-likelihood, and population shares to be $1/G$.

From this iterative estimation step, we obtain segment-specific policy function parameters, along with the population segment probability estimates π. We use the segment-level policy functions to obtain structural parameters of each segment, which we describe in the next section. A caveat with the two-step estimation is that the first-stage policy function estimates can be biased if the state variables in the policy function are correlated with the first-stage errors. Our approach, which leverages rich observed and unobserved player heterogeneity, helps mitigate this issue.

5.2. Step 2: Structural Parameter Estimation

The key idea of the two-step estimation is to represent the value function in terms of the policy function estimated in first stage, which reflects the player's optimal actions. That is, $V(\cdot;\theta)=h(P(\cdot;\theta),\theta)$ and $P(\cdot;\theta)=g(V(\cdot;\theta),\theta)$. Given our discrete state space, we can solve the value function as a system of linear equations,

$$V(S_{it},\alpha_i,g;\Theta,\Psi)=(I-\beta F)^{-1}\left\{\sum_{d_{it}\in\{1,2,3\}}P(S_{it},\alpha_i,g;\Theta,\Psi)\cdot[u(S_{it},\alpha_i,g,d_{it};\Theta,\Psi)+E[\epsilon|d_{it}]\right\} \tag{10}$$

where F is the matrices of transition probabilities corresponding to action d_{it}. The Type I extreme value assumption of the error term allows us to solve the value function analytically, such that $E[\epsilon|d_{it}]=\gamma-\ln(P(\cdot;\Theta))$. γ is the Euler's constant.

Furthermore, we can express the player's choice probability of action d_{it} under the structural parameters of our model in closed-form using the distribution assumption of the errors as follows:

$$\Pr(d_{it}|S'_{it},\alpha_i,\epsilon;\Theta,\Psi)=\frac{\exp\left(u(d_{it},S'_{it},\alpha_i;\Theta,\Psi)+\beta E_{S''_{it},\alpha_i,\epsilon';\Theta,\Psi|d_{it},\epsilon}V(S''_{it},\alpha_i,\epsilon')\right)}{\sum_{\tilde{d}_{it}\in\{1,2,3\}}\exp\left(u(\tilde{d}_{it},S'_{it},\alpha_i;\Theta,\Psi)+\beta E_{S''_{it},\alpha_i,\epsilon';\Theta,\Psi|\tilde{d}_{it},\epsilon}V(S''_{it},\alpha_i,\epsilon')\right)}. \quad (11)$$

We construct the moment equality estimator using the following moment condition, where $\hat{Pr}$ is the optimal policy estimated from the first stage, and $\tilde{Pr}$ is the policy informed by the model parameters (Pesendorfer and Schmidt-Dengler 2008). We minimize equation 12, the distance between optimal policy and the model choice probabilities, weighted by player segment probabilities q_{ig}.

$$\sum_{i=1}^{N}\sum_{t=1}^{T}q_{ig}\Big[(\hat{Pr}(d_{it}=2|S_{it},\alpha_i,g,\epsilon;\beta_g)-\tilde{Pr}(d_{it}=2|S_{it},\alpha_i,g,\epsilon;\Theta_g,\Psi))^2$$
$$+(\hat{Pr}(d_{it}=3|S_{it},\alpha_i,g,\epsilon;\beta_g)-\tilde{Pr}(d_{it}=3|S_{it},\alpha_i,g,\epsilon;\Theta_g,\Psi))^2\Big] \quad (12)$$

For the second stage model estimation, we discretize player ability type variable into 20 bins and estimate the value function for each ability type and segment. To compute standard errors, we generated 500 bootstrap datasets following Bajari et al. (2007). For each bootstrapped dataset, we estimate both the first and the second stage to account for the estimation errors from the first stage policy estimation.

5.3. Identification

There are a few challenges in identifying the dynamic structural model of game playing. First, the intrinsic player ability is not observed. Our long panel of gameplay records and the variations in environments, tool stock, and winning outcomes across and within players allows for the identification of player ability through player fixed effects in equation 3. The average gameplay records used for estimating the win probability function for each player is 171. While it is theoretically possible to identify tools-ability substitutability or complementarity (i.e., the interaction between tools and ability in the win probability function), doing so is practically infeasible given our individual-specific measure of ability. The trade-off we make for capturing this level of granularity in individual heterogeneity is the inability to separately identify the effects of player learning and tools-ability interactions. We follow Kasahara and Shimotsu (2009) for the identification of unobserved finite mixture heterogeneity.

Second, the parameters in the flow utility function needs to be identified by the revealed preference argument. In particular, the exogeneity of the current shock (i.e., current state variables are affected by past shocks but remain exogenous to current shock), the mapping between the differences in conditional value functions and the conditional choice probabilities, and the normalized exit value form the basis for our model identification. We fix the discount factor to 0.9. With the exit value normalized to 0, the cost of play parameter c_p is identified from the exit decision. With c_p identified, the utility from points reward θ is identified from the variation in player win rates (i.e., due to within and across player differences in tools, level, and ability) and the exit decision. The disutility of purchase parameter c_m is identified from the play versus play and purchase decision, as the tool purchase changes the win probability and the expected value of points in the current and future gameplays. The reward from level-up R is identified from player behavior (i.e., exit/purchase decision) at the threshold between the level-up points, as the level-up bonus in non-level-up periods does not affect current utility, only future utility.

6. Results

We discuss our results in the following order: 1) player win probability function and ability estimates, 2) first stage policy estimates, and 3) second stage structural parameter estimates.

6.1. Player Win Probability Function and Ability Estimates

We estimate the player win probability function using a linear probability model of player tool and level states with individual fixed effects, controlling out the opponent effect.[6] We use the estimated individual fixed effects as the measure of player ability. The estimated player win probability function, shown in Table 4, accomodates the following three key features. First, win probability is

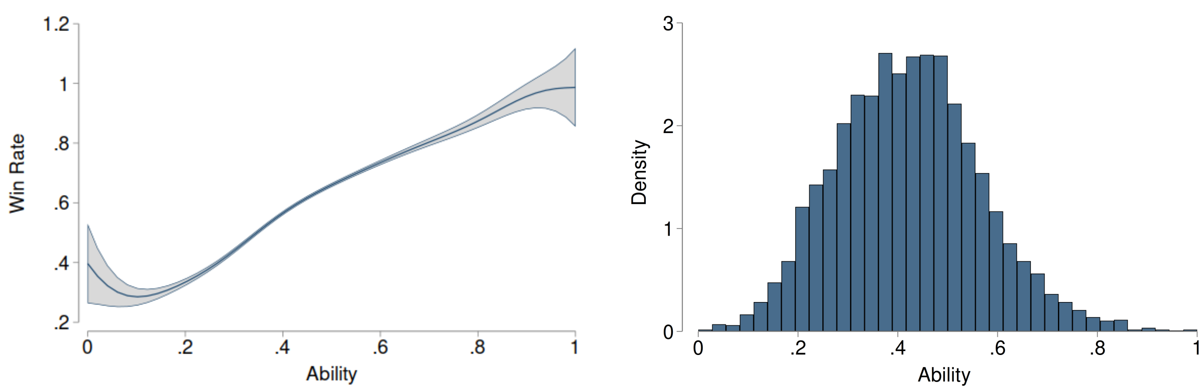
[6] For the second stage structural parameter estimation, we control out the effect of opponents using the median value of the opponent Elo scores – the rating system used by the company for calculating the relative skill levels of players – at each level.

higher for higher ability players. Second, player win probability increases with tools. Specifically, one additional tool purchase at level 6 increases player win probability by around 2.6 percentage points.[7] We allow for the diminishing effect of tools as levels increase. Third, player win probability decreases with level. Notably, the game becomes significantly more difficult at level 9, with win probability dropping by around 10 percentage points compared to the previous level. This increasing difficulty at higher levels is a common feature in gaming environments.

To demonstrate the reliability of our ability measure, we present the relationship between player average win probability and ability in Figure 9a. There is a clear positive relationship, with higher ability players exhibiting higher win rates. Figure 9b displays the distribution of the player ability estimates, highlighting significant heterogeneity in our player sample.

Figure 9 Player Ability Estimates

(a) Average Win Rate and Ability: Level 6 Example **(b) Distribution of Player Ability**



Note: The solid line in Figure 9a is a kernel-weighted second-degree polynomial regression using a Gaussian kernel, and the shaded area is the 95% confidence interval.

[7] To provide additional evidence of the effect of tool purchase on player win probability, we conduct a more localized before-and-after analysis of player win rates, comparing the five games before and after the purchase incident in Appendix A. The measured effect size largely aligns with the incremental effect of tool measured in our win probability function.

Table 4 Linear Probability Model: Production Function Estimates

	Win
tool stock	0.02623***
	(0.00157)
lvl:6	-0.08207***
	(0.00146)
lvl:7	-0.06441***
	(0.00187)
lvl:8	-0.07087***
	(0.00199)
lvl:9	-0.17077***
	(0.00228)
lvl:10	-0.17392***
	(0.00296)
lvl:11	-0.23899***
	(0.00384)
lvl:6 × tool stock	-0.00473***
	(0.00136)
lvl:7 × tool stock	-0.00699***
	(0.00141)
lvl:8 × tool stock	-0.01082***
	(0.00140)
lvl:9 × tool stock	-0.01151***
	(0.00143)
lvl:10 × tool stock	-0.01430***
	(0.00148)
lvl:11 × tool stock	-0.01505***
	(0.00147)
opponent elo score	-0.24202***
	(0.00277)
Observations	1,168,880
Individual FE	Y
Adjusted R^2	0.049

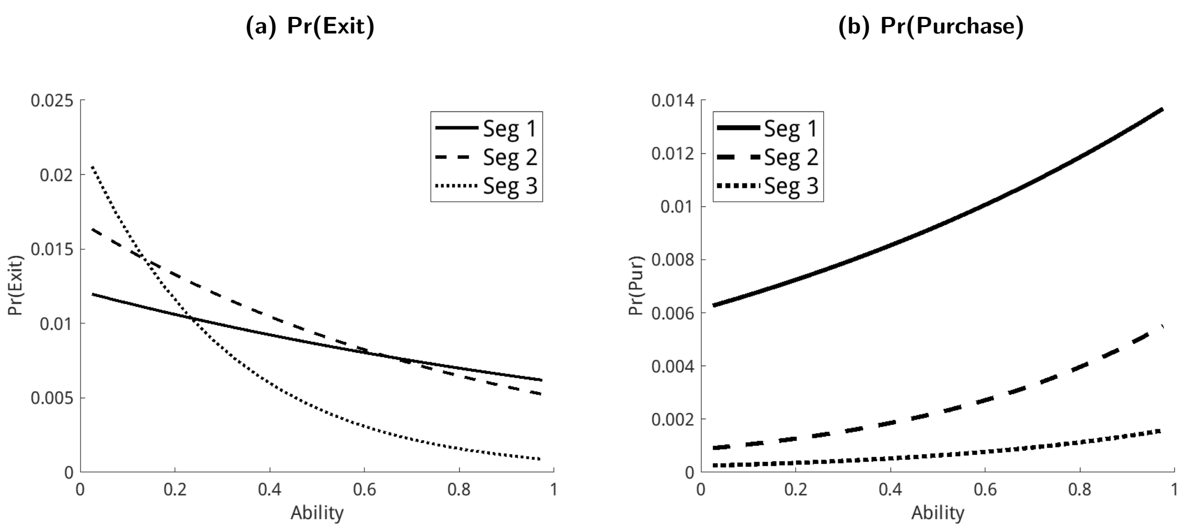
Note: Level 5 used as baseline. Robust standard errors in parentheses; *** p<0.01, ** p<0.05, * p<0.1

6.2. First Stage Policy Estimates

Given the ability estimates of players, we estimate the first stage policy estimation of player action probabilities using a flexible multinomial logistic regression, accounting for both observed and unobserved heterogeneity. We find that the three-segment model best fits the data based on the

AIC and BIC criterion.[8] We report the first stage policy estimates for each segment in Table 5. Segment 1 is the smallest, comprising 7% of the players, followed by Segment 2 with 23%, and Segment 3 with 70%. We report some illustrative features of the policy with respect to player ability in Figure 10. Consistent with our model-free evidence, the probability of exit decreases with player ability, while the probability of purchase increases with ability across all segments. Segment 1 has the highest purchase rates and generally the lowest exit rates, indicating that these players are the most engaged and likely to spend money in the game. Segment 3, the largest segment, has the lowest purchase rates. While Segment 3 shows a relatively flat decrease in exit probability across abilities, Segment 3 exhibits a steeper decline among lower ability players.

Figure 10 Player Action Policy by Ability Type



Note: Policy visualization based on example state: (Level 6, Level Completion 20-30%, and Item Stock 0)

To gain deeper insights on the segment characteristics, we report descriptive statistics in Table 6 and visualize the survival rate and the ability distribution of each segment in Figure 11. The descriptive evidence further provides support that the smallest share of players, Segment 1, comprises of high-spending individuals who also stays longer in the game and progress to higher levels, followed by Segment 2. Segment 3 has the lowest average player ability and spends the least.

[8] The AIC for the two- and three-segment models are 76764.81 and 76299.59, respectively; the BIC values are 77041.41 and 76714.49, respectively.

Table 5 First Stage Estimates: Player Action Policy Function

		Seg 1	Seg 2	Seg 3
Segment Probability		0.0692	0.2287	0.7021
SE		0.0046	0.0048	0.0046
Choice				
Purchase	ability	1.0078**	1.8555***	1.9965***
		(0.3696)	(0.3389)	(0.3636)
	tool stock	0.2119***	0.4950***	1.8689***
		(0.0185)	(0.0163)	(0.0478)
	tool stockˆ2	-0.0069***	-0.0128***	-0.1999***
		(0.0010)	(0.0007)	(0.0086)
	pct lvl complete	2.6695***	-0.9826**	1.2923***
		(0.4361)	(0.4249)	(0.4825)
	ability × pct lvl complete	-0.7362	0.1418	-0.3467
		(0.7061)	(0.6362)	(0.7756)
	pct lvl completeˆ2	-2.7620***	0.2979	-1.8515***
		(0.4350)	(0.4454)	(0.4203)
	lvl:7	-0.0348	-0.2065**	-0.2914***
		(0.1038)	(0.1016)	(0.0872)
	lvl:8	0.1223	-0.4376***	-0.5923***
		(0.1009)	(0.1042)	(0.0908)
	lvl:9	-0.3773***	-0.6725***	-0.7774***
		(0.1139)	(0.1089)	(0.0983)
	lvl:10	-0.6498***	-1.3472***	-0.8373***
		(0.1268)	(0.1435)	(0.1201)
	lvl:11	-0.9534***	-1.5476***	-1.0228***
		(0.1431)	(0.1598)	(0.1534)
	cons	-5.5698***	-6.8071***	-8.5193***
		(0.1687)	(0.1457)	(0.1699)
Exit	ability	0.7717**	-3.0300***	-3.6848***
		(0.3469)	(0.2113)	(0.1987)
	tool stock	-0.1393***	-0.2053***	-0.0152
		(0.0286)	(0.0219)	(0.0262)
	tool stockˆ2	0.0056***	0.0087***	0.0089***
		(0.0013)	(0.0009)	(0.0034)
	pct lvl complete	-0.9030*	-1.7382***	-1.3912***
		(0.5159)	(0.2657)	(0.2639)
	ability × pct lvl complete	-5.8553***	7.3085***	1.4232***
		(0.9142)	(0.5172)	(0.4862)
	pct lvl completeˆ2	2.8489***	-2.1943***	-0.0362
		(0.4691)	(0.2832)	(0.2415)
	lvl:7	-0.8087***	-0.3470***	-0.0847*
		(0.0877)	(0.0532)	(0.0500)
	lvl:8	-0.6385***	-0.3837***	0.0199
		(0.1128)	(0.0568)	(0.0520)
	lvl:9	-0.5340***	-0.1310**	0.3191***
		(0.1140)	(0.0640)	(0.0549)
	lvl:10	-0.9494***	0.0801	0.3248***
		(0.1757)	(0.1075)	(0.0785)
	lvl:11	-0.7838***	-0.6067***	0.3303***
		(0.1840)	(0.1916)	(0.1036)
	cons	-4.3423***	-3.4950***	-3.4308***
		(0.1257)	(0.0748)	(0.0701)

Note: Robust standard errors in parentheses; *** $p<0.01$, ** $p<0.05$, * $p<0.1$

Because this segment has greater share of low ability players, it also has higher exit rates, especially at the early levels.

Table 6 Descriptive Characteristics of Segment

Seg 1 (2.6%) N=107	Mean	Median	75Q	90Q	99Q
Avg Total No. Purchase	6.8785	5	9	14	24
Avg Purchase Rate	0.0219	0.0158	0.0260	0.0365	0.0789
Avg Level Reached	9.3364	9	11	11	11
Avg Total No. of Games Played	470.66	314	639	1021	1980
Ability Score	0.4823	0.4860	0.5912	0.6861	0.8398
Pay-to-Win Player Share			100.00%		
Seg 2 (5.2%) N=218	**Mean**	**Median**	**75Q**	**90Q**	**99Q**
Avg Total No. Purchase	2.7661	1	3	7	25
Avg Purchase Rate	0.0120	0	0.0120	0.0328	0.0829
Avg Level Reached	7.9725	8	9	10	11
Avg Total No. of Games Played	263.68	164	444	673	916
Ability Score	0.5330	0.5193	0.6684	0.7812	0.9282
Pay-to-Win Player Share			68.35%		
Seg 3 (92.2%) N=3,838	**Mean**	**Median**	**75Q**	**90Q**	**99Q**
Avg Total No. Purchase	0.2507	0	0	1	5
Avg Purchase Rate	0.0017	0	0	0.0024	0.0333
Avg Level Reached	7.3233	7	8	9	11
Avg Total No. of Games Played	166.74	112	227	387	887
Ability Score	0.4129	0.4117	0.5056	0.5882	0.7556
Pay-to-Win Player Share			10.76%		

Note: The descriptive statistics are based on deterministic segment assignment (i.e., the maximum of probabilistic segment probabilities estimated in the first stage).

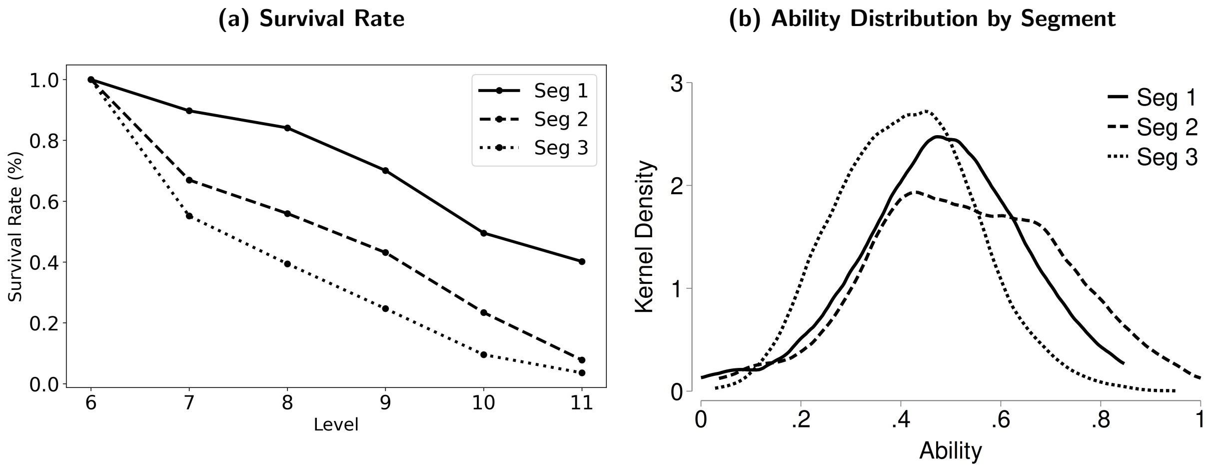
6.3. Structural Parameter Estimates

Table 7 reports the estimates of the structural parameters of the model, which further reveal important differences in the game play preferences among the three segments.

Segment 1 represents the smallest share of players (7%) but those who spend the most, which is reflected in their lower price sensitivity for ability enhancers (c_m).[9] Despite their low sensitivity to immediate points rewards (θ), they derive significant utility from progressing through game levels,

[9] Since price is invariant, we normalized the price to 1; so c_m is the disutility for paying that unit price.

Figure 11 **Descriptive Characteristics of Segment: Survival Rate and Ability Distribution**



as indicated by their high level-up reward (R). This segment's positive (near zero) cost of play (c_p) suggests that these players do not find playing the game costly, but rather enjoy spending time in the game. We henceforth label this group as *premium enthusiasts*.

Table 7 **Structural Parameter Estimates**

	Segment 1	Segment 2	Segment 3
	Premium Enthusiasts	*Win-Seekers*	*Progress-Seekers*
	(7.0%)	(23.0%)	(70.0%)
θ	0.0003	0.0050	0.0050
	(0.0002)	(0.0004)	(0.0003)
c_m	-4.4089	-5.2775	-6.4997
	(0.0333)	(0.0386)	(0.0448)
c_p	0.0002	-0.0395	-0.0288
	(0.0020)	(0.0036)	(0.0026)
R	0.0039	0.0016	0.0036
	(0.0006)	(0.0009)	(0.0014)

Note: standard errors are shown in parentheses.

Segment 2 players, representing share of 23% of players, receive a greater utility from immediate points reward (θ) and have the second lowest price sensitivity for ability enhancers (c_m), below the *premium enthusiasts*. They however have the highest cost of play (c_p), indicating that they find playing the game more of a chore. This suggests that these players are less likely to continue playing without high enough points reward from current game, and their utility from level-up (R) is the lowest among the segments. We label this group as *win-seekers*.

Segment 3 constitutes the largest group with 70% of the players. These players have the same utility from immediate points reward (θ) as *win-seekers*, but their price sensitivity for enhancers (c_m) is the highest. They also find playing the game costly (c_p), but receive higher utility from level-up rewards (R) than *win-seekers*. We label them as *progress-seekers*.

To assess our model fit, we generate a representative sample of 20,000 individuals from the segment, ability, and initial tool stock state distribution, simulating a play sequence for each player until they exit or finish the game; the sequences average about 200 games per player. We report the model fit in Table 8. In general, the model performs reasonably in matching the targeted moments, time (total play count) and money (total purchase), and in accounting for the heterogeneity across the three latent segments. Hence, we conclude that the model can reasonably match player action and play behavior observed in data, especially with respect to different insights and predictions across the latent segments.

Table 8 Model Fit

	Real Data			Model Simulation		
	Seg 1	Seg 2	Seg 3	Seg 1	Seg 2	Seg 3
Average Lifetime Play Count	231.35	147.07	185.14	243.15	166.73	196.51
Average Level Reached	7.69	7.15	7.47	8.92	7.79	7.97
Average Total Purchase	2.77	0.70	0.29	2.95	0.89	0.31
Average Purchase Rate (per 100 plays)	0.97	0.33	0.19	1.2	0.53	0.15

7. Counterfactuals

We use our model to predict and generate real-time personalization policies aimed at increasing profitability by leveraging the dynamic interaction between players' time and purchase decisions. In doing so, we investigate the dynamic interlinkages between time and money decisions in the gaming environment. In the first counterfactual, we examine how firms can leverage discounts to induce dynamic complementarities between players' time and money inputs, as well as between initial discounted purchases and subsequent tool purchases at later stages of the game. Second, we examine whether reducing player time spent at early levels can increase firm profits and identify from which players the firm benefits the most from accelerating game level difficulty. To perform counterfactual analysis, we generate a representative sample of 20,000 individuals as we did for the model fit analysis earlier.

7.1. Investigating Dynamic Interlinkages in Tools Purchase using Discounts

In this section, we investigate how firms can leverage the dynamic complementarities in players' time and money inputs to increase profits. Specifically, we analyze the optimal timing and targeting of discounts at key intervention points, such as when players are likely to quit. The dynamic complementarities can operate as follows: when players are about to quit, specifically at their "give-up" levels, offering discounts on tools can incentivize purchases. These purchases directly boost firm revenue and enhance player performance by increasing their win probability, which motivates players to keep playing and potentially make further purchases as they progress to more challenging levels. Thus, by offering discounts at these "give-up" moments, the firm can induce dynamic complementarities between initial discount purchase and future tool purchases.

To assess the effects of discounts on player time (play/quit) and money (purchase) dynamics, we first need to predict player exit level as the median exit levels of players for each ability and segment type combination. We detail the process in Appendix C and highlight the importance of accounting for both observed ability and unobserved preference heterogeneity. Notably, premium enthusiasts, who enjoy spending time in the game itself, have exit levels at least one level higher than players with similar abilities in other segments. Then, at the start of each level, we gave one-time discounts on tool purchase to players whose current level matched their predicted exit level. We varied the discount depths from 10% to 90%, as shown in Table 9. In all cases, we find that providing targeted discounts increases player level progression. That is, players who redeem discounts on tool purchase experience an increased win probability, which in turn helps them to progress to higher levels in the game.

Table 9 Effect of Price Discount Promotion at Exit Level

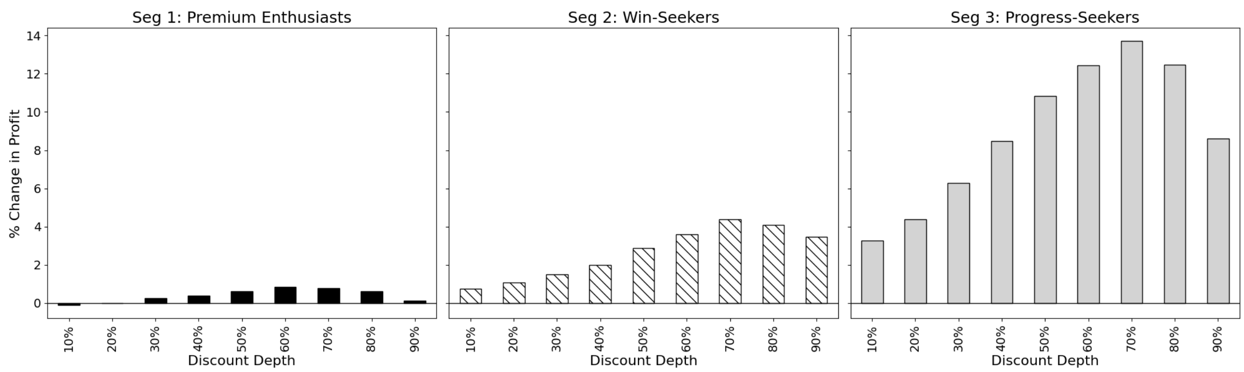
Discount Depth	10%	20%	30%	40%	50%	60%	70%	80%	90%	1 Free Tool
% Change in Level Progression										
Total	0.03	0.03	0.04	0.05	0.08	0.11	0.17	0.22	0.30	0.74
Seg										
1: *Premium Enthusiasts*	0.00	0.00	0.02	0.02	0.05	0.06	0.10	0.13	0.13	0.28
2: *Win-Seekers*	0.04	0.04	0.05	0.06	0.09	0.12	0.18	0.22	0.34	0.74
3: *Progress-Seekers*	0.02	0.03	0.04	0.05	0.08	0.11	0.18	0.22	0.30	0.79
Monetization (% Change in Profit)										
Total	1.35	1.87	2.75	3.71	4.88	5.75	6.44	5.86	4.16	0.70
Seg										
1: *Premium Enthusiasts*	-0.11	-0.00	0.25	0.41	0.61	0.86	0.80	0.62	0.12	-0.64
2: *Win-Seekers*	0.75	1.07	1.51	2.00	2.87	3.59	4.40	4.09	3.47	1.19
3: *Progress-Seekers*	3.28	4.40	6.28	8.47	10.84	12.44	13.71	12.48	8.62	1.49
Retention (% Change in Avg Total Number of Games Played)										
Total	0.05	0.04	0.05	0.10	0.16	0.19	0.26	0.29	0.35	0.76
Seg										
1: *Premium Enthusiasts*	-0.09	-0.09	-0.09	-0.11	-0.12	-0.18	-0.23	-0.22	-0.29	-0.55
2: *Win-Seekers*	0.10	0.09	0.11	0.06	0.13	0.23	0.29	0.25	0.57	0.89
3: *Progress-Seekers*	0.05	0.04	0.04	0.13	0.20	0.22	0.31	0.35	0.36	0.88

We visualize the impact on firm profit and player retention in Figure 12a and Figure 12b. Our analysis reveals that a 70% discount is optimal for maximizing total profit by 6.44%, with the largest profit increase generated from the *progress-seekers*, who have high price sensitivity but gain higher utility from level progression, by 13.71%. Conversely, providing discounts to *premium enthusiasts*, a segment of players who has lowest price sensitivity and derives enjoyment from play itself, results in the lowest and sometimes even negative changes in profit. This group also has an optimal discount depth slightly below that of other segments, at 60%.

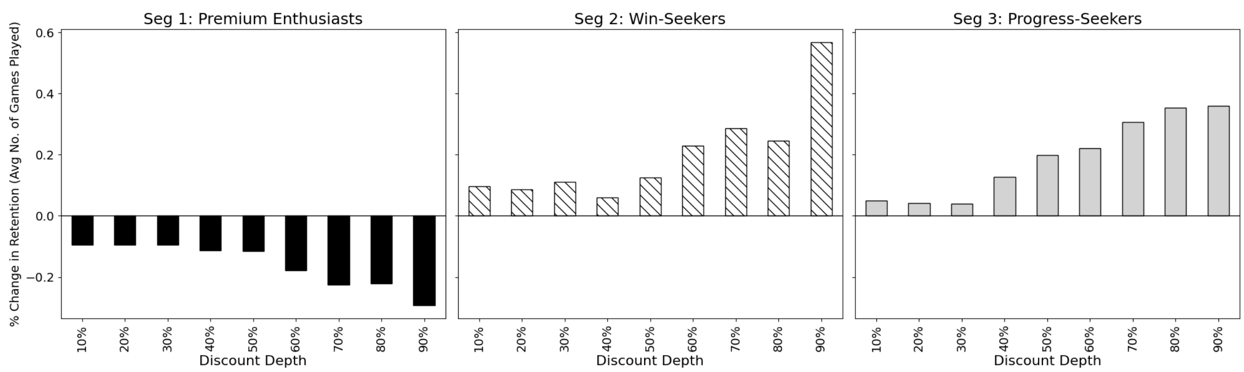
However, a notable caveat is that while these discounts can increase player level progression and profitability, they may also interact with the firm's level promotion scheme in a way that could decrease overall retention (i.e., total number of games played). As players purchase tools and perform better, they may advance through levels more quickly, reaching their exit points faster. This accelerated progression can result in a decreased total number of games played, as players finish the game sooner. Thus, while targeted discounts can drive short-term gains in profit, they

Figure 12 Impact of Price Discounts on Firm Profit and Retention

(a) Monetization (% Change in Profit): Price Discount Promotion at Players' Exit Level



(b) Retention (% Change in Avg Total No. of Games Played): Price Discount at Players' Exit Level



may inadvertently shorten the overall engagement period for some players. We see such dynamic substitution between money and time (total number of games played) greatest among *premium enthusiasts*, those who gain utility from play itself.

To examine the sources of the change in profit, we decompose the profit change from the discount intervention into three components: (i) the direct effect of discount purchase, and (ii) dynamic complementarities between discount purchase and future tool purchases, and (iii) dynamic substitution between discount purchase and future tool purchases. To quantify the contribution of each component, we focus on the 70% discount depth intervention case, which has been shown optimal for total profit, and identify players who have redeemed the discount. Then, we compare the incremental change in the number of purchases after the discount intervention with the original design in the absence of discounts.

Table 10 Effect of 70% Discount at Exit Level: % Share Among Players Who Redeemed the Discount

	Mechanism			
	Discount Purchase	Dynamic Complementarity	Dynamic Substitution	
Incremental Effect on Tool Purchase	+1	>1	0	<0
Total	86.69	6.20	6.58	0.53
Seg				
1: *Premium Enthusiasts*	87.22	2.22	9.44	1.11
2: *Win-Seekers*	86.00	5.77	8.07	0.16
3: *Progress-Seekers*	86.86	6.73	5.81	0.59

We report the incremental profit decomposition from discounts in Table 10. Among players who redeemed a discount, 86.7% made only the discounted purchase with no subsequent purchases. Importantly, 6.20% of players made one or more additional non-discount purchases following the discount. This dynamic complementarity in tool purchases was most pronounced among *progress-seekers*, who have high price sensitivity and value progressing to higher difficulty levels. However, the discount cannibalization due to targeting timing inaccuracy— offering discounts too early—was most pronounced among *premium enthusiasts*, where the discount replaced a planned full-price purchase. This dynamic substitution between discount and later purchases was greatest among *premium enthusiasts*, who would have made purchases anyway in the absence of discounts given their negative time costs and low price sensitivity. Additionally, for a small fraction (0.53%) of players, the dynamic substitution not only cannibalized discounts but led to reduced total number of lifetime purchases, where discounts inadvertently expedited player progression and exits, as previously discussed.

Finally, to isolate the effects of discounts and provide direct evidence of dynamic complementarities/subsitution between discounts and future purchases, we offered a free tool to players at their frontier levels. We report the effects of providing one free tool in the last column of Table 9. Overall, providing players with an additional tool increases their win rates, which in turn increases their level progression and retention, leading to increase in total profits. However, we find heterogeneous effects across player segments. The effects of dynamic complementarity are found dominant in *win-seekers* and *progress-seekers*, while we again detect the opposite effects of dynamic substitution for *premium enthusiasts*. This underscores the importance of incorporating unobserved preference heterogeneity in targeted intervention design.

7.2. Investigating Dynamic Interlinkages in Reducing Time Spent at Early Levels

Next, we investigate how changing the game progression speed affects the player dynamics in future play and purchases. We achieve this by evaluating the key concept of dynamic difficulty adjustment (DDA).[10] Specifically, we examine the impact of accelerating early-level progression for high-ability players.

In the game, players progress up to their own levels of "give-up" points, in which the difficulty level exceeds their winning ability, resulting in exit. For high ability players, such "give-up" points come much later in the game. By facilitating faster level-ups for high-ability players, we seek to optimize the player play and purchase dynamics, providing appropriately challenging game content sooner, thereby enhancing engagement and increasing monetization opportunities.

To implement this policy, we first classify players into four ability quartiles, and target those in the highest quartile. Table 11 shows the targeted segment share of the representative players from model simulations. The average frontier level of high ability players is above level 9 across all segments. To expedite the speed of player level progression, we increase the player win probability by 5 percentage points at early levels.[11]

Table 11 Segment Share of High Ability Players

Segment	Ability	Player Share (%)	Avg Level Reached
1: Premium Enthusiasts	Q4	1.57	9.70
2: Win-Seekers	Q4	2.74	9.13
3: Progress-Seekers	Q4	10.55	9.27

[10] Dynamic difficulty adjustment (DDA) is a widely used method by game companies to tailor gameplay experiences to individual players with different skill levels. For example, Candy Crush Saga adjusts the difficulty of levels based on the player performance. Clash of Clans reduces the time spent on lower-level tasks for high-ability players.

[11] This corresponds to levels 6-8 in our analysis. Practically, the firm can implement this policy by matching the player with less difficult opponents.

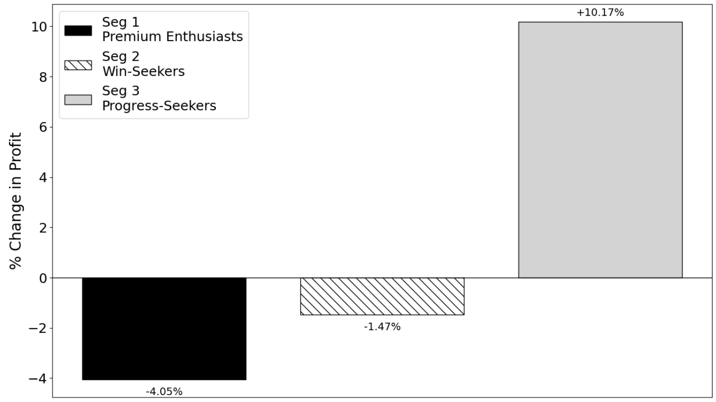
Table 12 reports the retention and monetization results for the accelerated level progression intervention. Overall, we find that expediting early level progression for high ability players increases both total retention by 0.96% (measured by lifetime number of games played) and total profit by 1.29%. In all segments, players on average progress to higher levels and play a greater number of games.

Table 12 Accelerating Level Progression for High Ability Players: Effect on Retention and Monetization

	Ability	**% Change in Avg Level Reached**	**Retention (% Change in Avg Play Count)**	**Monetization (% Change in Profit)**
Total	Q4	+ 1.87	+ 0.96	+ 1.29
Seg				
1: *Premium Enthusiasts*	Q4	+ 0.16	- 1.59	- 4.05
2: *Win-Seekers*	Q4	+ 1.50	- 0.07	- 1.47
3: *Progress-Seekers*	Q4	+ 2.22	+ 1.55	+ 10.17

We visualize the change in profit in Figure 13. We show that the firm gains in profit by expediting level-ups by effectively lowering difficulty level at early stages from players who find playing the game itself more costly but receive higher utility from level progression (*progress-seekers*), but loses in profit from players who find playing the game itself more enjoyable (*premium enthusiasts*) and players who receive higher utility from immediate winning (*win-seekers*). This relates to the caveat of difficulty level adjustment: while *progress-seekers* benefit from faster advancement, premium *premium enthusiasts* and *win-seekers* who each find the game enjoyable and a higher likelihood of winning rewards at easier levels may find their enjoyment diminished by the quicker pace, leading to earlier exits and reduced long-term engagement. The increase in net total profit is driven by the (*progress-seekers*), with their profit increased by 10.17%. By expediting level progression to higher levels, the firm can generate more profit from the higher-ability players in the *progress-seekers* who possess sufficient ability to advance through the later stages of the game. Because they gain higher utility from level-ups but incur significant play costs, expediting them to higher levels benefits firm profit by reducing the risk of early-stage attrition for these players.

Figure 13 Accelerating Level Progression for High Ability Players: Change in Profit by Segments



8. Conclusion

This paper builds a model of active and dynamic consumer utility generation process in gaming environments, where players heterogeneously and dynamically optimize their input decisions of time and money. Our model addresses key challenges in designing personalized interventions by identifying the optimal target and timing for policies in a setting in which player choices of time and money are dynamically interlinked. Through the evaluation of personalization counterfactuals, we demonstrate how targeted interventions such as discounts and dynamic difficulty adjustments can lead to different dynamic interactions (complementarities/substitution) across and within individuals at different regions of the game design space.

We use the model to predict the timing when players are likely to quit and predicting how interventions affect their subsequent time and money decisions. We demonstrate that discounts can function as dynamic complements—enhancing player engagement and retention by reinforcing time and money inputs—or as substitutes, where improved win probabilities accelerate progression and lead to earlier exits, potentially hurting long-term profitability. These dual effects underscore the importance of player-and-timing-specific personalization, as even the same intervention can yield contrasting outcomes depending on its timing and the player's state within the game environment.

Unlike standard retail environments, the key difference in gaming environments is the ability to dynamically modify the product – the game itself. This allows firms to personalize and adjust the game in real-time to enhance player engagement and monetization. In gaming, dynamic difficulty adjustment (DDA) is a common technique used by developers to continuously tailor game challenges and features to match the players' abilities and preferences in real-time. Structural models can be extremely helpful in gaining a theoretical understanding of the dynamics and in cost-effectively assessing effectiveness of different intervention strategies and game design. We show that accelerating progression for high-ability players increases profits and engagement for those with high time costs who value progression, but it can hurt profits from segments that value the act of playing or immediate wins. This underscores the importance of accounting for unobserved preference heterogeneity in personalizing game experience.

We now discuss the limitations of our paper, which provide promising avenues for future research. First, our model abstracts away from potential complementarity and substitutability between player ability and tools. Players can have different intensities in the feedback cycle between play and purchase based on their abilities and the type of item they purchase. Understanding how player ability interacts with different types of in-game tools and their impact on player engagement and monetization can inform product design. Second, we consider only tools that are durable, excluding consumables that may have different impacts on player behavior. Consumable items, which are used once and provide temporary benefits, could influence spending patterns and engagement differently than durable items. Investigating the role of consumables in gaming environments could reveal interesting insights for balancing immediate and long-term player engagement and monetization. Third, we address only environments in which in-game items lead to increased performance ("tools" - performance enhancers). Future research should explore the effects of non-performance enhancing purchases, such as cosmetic or identity-related items (e.g., fashion items, avatars). Understanding how these types of purchases influence player engagement and monetization spillovers to ability-enhancing items could provide a more comprehensive understanding of consumer response in gaming environments.

Gamification has become more mainstream, with gaming features incorporated into many aspects of our lives. Our framework extends broadly to settings in which active utility generation through performance and purchases (of performance enhancers) co-exist, generalizing the structural features of two disparate types of models in marketing – *dynamic effort/time response* models and *dynamic durable goods choice* models. This allows our model to serve as a valuable tool for generating hypotheses and informing the design of real-time personalization policies in settings that involves time and/or money inputs. By understanding and predicting the likely points of dynamic interaction among players' time and purchase decisions – e.g., flexibly including complementarities between time and money, between early and late monetary inputs, etc– firms can better optimize interventions to effectively harness these dynamics for enhanced engagement and profitability.

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Appendix

A. Effect of Tool Purchase on Player Win Probability

We provide additional evidence in estimating the effect of tools purchase on player win probability by conducting a localized before-and-after analysis of player win rates. This analysis focuses on player performance immediately before and after a tool purchase. Specifically, we compare the win rates of players over the course of five games before and five games after they make an in-game purchase.

The after coefficient in Table A.1 captures the immediate effects of the purchased tool on player win probability, controlling for ability, level, and opponent effects. We find that one tool purchase translates to around 3.6 percentage points increase in win probability. This effect size is largely consistent with the effects of a tool measured in our win probability function, which quantifies the overall impact of tool purchase on player win probability across their entire match records.

Table A.1 Linear Probability Model: Before and After Purchase (5 Games)

	(1) Win
ability	0.52063***
	(0.02648)
after	0.03554***
	(0.00679)
lvl=7	0.02375**
	(0.01136)
lvl=8	0.02674**
	(0.01129)
lvl=9	-0.07839***
	(0.01148)
lvl=10	-0.05936***
	(0.01279)
lvl=11	-0.10529***
	(0.01271)
opponent elo score	-0.14020***
	(0.01811)
Constant	0.33278***
	(0.01627)
Observations	20241
Adjusted R^2	0.029

Note: Robust standard errors in parentheses; *** <0.01, ** p<0.05, * p<0.1

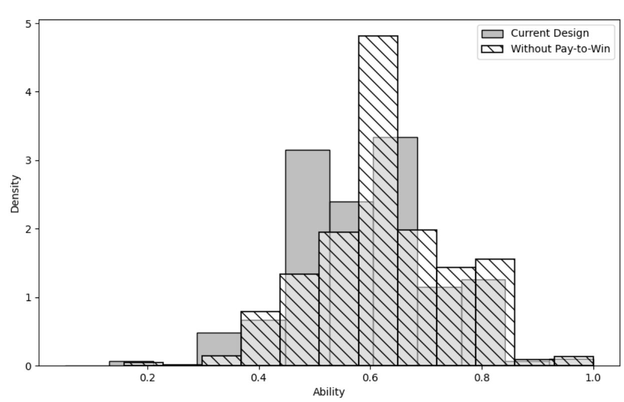
B. Pay-to-Win and Player Welfare

We evaluate the welfare implications of the pay-to-win business model in gaming environments. To simulate a scenario without pay-to-win, we set the cost of purchase coefficient to infinity so that no purchase occurs. Our result in Table B.1 suggest that pay-to-win models can create a win-win situation for both the firm and the player. Firm benefits from monetizing player engagement and retention. Players benefit from the option to purchase tools that can enhance effective win probability of the players, enabling faster progression and greater rewards. Among the three segments, the welfare gain from pay-to-win is greatest for segment 1, the *premium enthusiasts*, at 1.95%. As shown in Figure B.1, which shows player ability distribution at the final level of the game, pay-to-win options allow less able players to progress further than they otherwise would have, leveling the playing field and enhancing player experience in the game.

Table B.1 No Pay-to-Win and Player Welfare

	Current Design			No Pay-to-Win		
	Seg 1	Seg 2	Seg 3	Seg 1	Seg 2	Seg 3
Welfare	5.97	5.68	5.76	5.86 (-1.95%)	5.63 (-0.86%)	5.74 (-0.25%)
Average total games played	243.15	166.73	196.51	228.68	158.33	193.04
Average level reached	8.92	7.79	7.97	8.62	7.67	7.92

Figure B.1 Ability Distribution at the Final Level: Current Desgin vs No Pay-to-Win



C. Predicting Player Exit Levels: Value of Ability and Unobserved Heterogeneity

In this section, we demonstrate the value of incorporating player ability and gameplay preference heterogeneity in predicting the timing of player exit. Specifically, we evaluate three different data strategies for exit prediction: (i) using ability alone, (ii) using gameplay preferences alone, and (iii) using both ability and gameplay preferences.

Table C.1 Frontier Targeting Policy for Personalized Interventions

		Variable Used for Predicting Player Exit		
		(1)	(2)	(3)
		Ability Only	Preference Only	Ability & Preference
Segment	Ability	Exit Level	Exit Level	Exit Level
1: Premium Enthusiasts	Q1	6	9	8
1	Q2	7	9	9
1	Q3	9	9	10
1	Q4	11	9	11
2: Win-Seekers	Q1	6	7	7
2	Q2	7	7	8
2	Q3	9	7	8
2	Q4	11	7	10
3: Progress-Seekers	Q1	6	8	7
3	Q2	7	8	8
3	Q3	9	8	9
3	Q4	11	8	10

To elaborate, we first define player exit levels as the median exit levels of players for each ability type. Second, we use only the player gameplay preference information (i.e., the latent segment type) to compute the median exit level for each preference type. Third, we implement a fully personalized policy that incorporates both player ability and preference heterogeneity in computing the median exit level of the player. With these computed exit levels, we provide targeted interventions and reduce difficulty by increasing the effective win rate of 2.5 percentage points when the player reaches his or her exit level.[12] For example, if the player's computed frontier level is 7, we adjust down the level difficulty for the player from level 7 and beyond. We

[12] This specific adjustment is similar to the incremental effect of a tool, ensuring the win rate does not increase excessively, which would either hurt the difficulty balance of the current level progression scheme or make the game too easy and boring (which is the effect we do not model). Therefore, we only perform small localized changes in win probability, set at 2.5 pp in our case.

summarize the computed exit levels for targeted intervention in Table C.1. For presentation purposes, we aggregate the frontier-level information for ability types into four quartiles. Notably, premium enthusiasts have exit levels at least one level higher than players with similar abilities in other segments.

Table C.2 Effect of Personalized Timing of Interventions on Retention and Monetization

	Personalized Intervention Policy		
	(1)	(2)	(3)
Variable Used for Exit Prediction	Ability	Gameplay Preferences	Ability & Gameplay Preferences
Win Probability Adjustment at Exit Level (pp)	+ 2.5	+ 2.5	+ 2.5
% Change in Avg Level Reached	+ 1.15	+ 1.09	+ 1.14
Retention (% Change in Avg Play Count)	+ 0.97	+ 0.91	+ 1.09
Monetization (% Change in Profit)	+ 0.28	+ 0.27	+ 0.92

We report the result of the targeted interventions in Table C.2. Overall, retention efforts to decrease difficulty at the frontier lead to an increase in profit for all targeting policies. Incorporating heterogeneity in player ability and unobserved preference heterogeneity for gameplay each yields 0.97% and 0.91% increase in player retention, which in turn increases firm profit by 0.28% and 0.27%, respectively. The fully personalized policy, which uses both ability and gameplay preferences for targeting, increases player retention by 1.14% and yields the largest increase in profit by 0.92%. The importance of incorporating both players' observed and unobserved heterogeneity is supported by our earlier observation about differences in exit levels by segment, even within similar ability levels. By increasing player retention at player exit levels, the firm can further reap profits from players who are about to leave with a costless intervention.