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№1

a) $f(x|\theta) = \frac{x}{\theta^2} \cdot e^{\frac{-x^2}{2\theta^2}}$

$$L(\theta|x) = \log \prod_{t=1}^{n} \frac{x_t}{\theta^2} \cdot e^{\frac{-x_t^2}{2\theta^2}} =$$

$$= \sum_t \log x_t - \sum_t \log \theta^2 + \sum_t \log e^{\frac{-x_t^2}{2\theta^2}} =$$

$$= \sum_t \log x_t - 2n \log \theta - \sum_t \frac{x_t^2}{2\theta^2}$$

$$\frac{d}{d\theta} L(\theta|x) = -\frac{2n}{\theta} + \frac{1}{\theta^3} \sum_t x_t^2 = 0$$

$$\hat{\theta} = \sqrt{\frac{\sum_t x_t^2}{2n}}$$

b) $f(x|\alpha,\theta) = \alpha\,\theta^{-\alpha}\cdot x^{\alpha-1}\cdot \exp\{-(\frac{x}{\theta})^{\alpha}\}$

$L(\alpha,\theta|x) = \log \prod_{t=1}^{n} \alpha\,\theta^{-\alpha}\cdot x_t^{\alpha-1}\cdot \exp\{-(\frac{x_t}{\theta})^{\alpha}\} =$

$= \sum_t \log\alpha + \sum_t \log\theta^{-\alpha} + \sum_t \log x_t^{(\alpha-1)} -$

$-\sum_t (\frac{x_t}{\theta})^{\alpha} =$

$= n\cdot\log\alpha - \alpha\cdot n\cdot\log\theta + (\alpha-1)\sum_t \log x_t -$

$-\sum_t (\frac{x_t}{\theta})^{\alpha}$

$$\frac{d}{d\theta} L(\alpha,\theta|x) = -\frac{\alpha n}{\theta} + \frac{\alpha}{\theta^{\alpha+1}}\sum_t x_t^{\alpha} = 0$$

$$\frac{\alpha n}{\theta} = \frac{\alpha}{\theta^{\alpha+1}}\sum_t x_t^{\alpha}$$

$$\hat{\theta} = \left(\frac{\sum_t x_t^{\alpha}}{n}\right)^{\frac{1}{\alpha}}$$

c) $\frac{1}{\theta}$ is a decreasing function. To maximize this function θ should be the smallest value that $\geq x_t$.

Hence, $\hat{\theta} = \max(x_1, \dots, x_n)$

$\sqrt{2}$

a) $p(C|x) = \dfrac{p(x|C) \cdot P(C)}{p(x)}$

$P(C_1) = P(C_2) = 0.5$

$p(C_1|x{=}4) = \dfrac{P(x=4|C_1) \cdot P(C_1)}{p(x=4)} =$

$= \dfrac{P(C_1)}{8 \cdot p(x=4)}$

$p(C_2|x=4) = \dfrac{P(x=4|C_2) \cdot P(C_2)}{p(x=4)} =$

$= \dfrac{(4-2) P(C_2)}{9\, p(x=4)} = \dfrac{2\, P(C_2)}{9\, p(x=4)}$

$\frac{2}{9} > \frac{1}{8}$, $p(C_2|x=4) > p(C_1|x=4)$

$\underline{class = 2}$

b) $P(C_1) = 0.7$ $P(C_2) = 0.3$

$p(C_1|x=6) = \dfrac{P(x=6|C_1) \cdot p(C_1)}{p(x=6)} = \dfrac{7}{80\, p(x=6)}$

$p(C_2|x=6) = \dfrac{P(x=6|C_2) \cdot p(C_2)}{p(x=6)} = \dfrac{1}{15\, p(x=6)}$

$\frac{7}{80} > \frac{1}{15}$, $p(C_1|x=6) > p(C_2|x=6)$

$\underline{class = 1}$

c) $P(x \in [5-a, 5+a]) = \int_{5-a}^{5+a} p(x) dx =$

$$= \int_{5-a}^{5+a} 0.5 \, dx = 2a$$

Hence, the optimal value of $a = 0.5$

Q3:

1) As the covariance is class-independent, the discriminant:

$$g_i(x) = -\frac{1}{2}\log|S_i| - \frac{1}{2}(x-m_i)^T S_i^{-1}(x-m_i) + \log \hat{P}(C_i)$$

Confusion matrix:

23	7
7	63

Preision = 23/(23+7) = 0.767
Recall = 23/(23+7) = 0.767
F-score = 2*0.767*0.767/(0.767+0.767) = 0.767

2) As the covariance is class-dependent, the discriminant:

$$g_i(x) = -\frac{1}{2}(x-m_i)^T S^{-1}(x-m_i) + \log \hat{P}(C_i)$$

Confusion matrix:

28	10
2	60

Preision = 28/(28+10) = 0.737
Recall = 28/(28+2) = 0.933
F-score = 2*0. 737*0. 933/(0.737+0. 933) = 0.823

3) As the covariance is a diagonal matrix, the discriminant:

$$g_i(x) = -\frac{1}{2}\sum_{j=1}^{d}(\frac{x_j^t - m_{ij}}{s_j})^2 + \log \hat{P}(C_i)$$

Confusion matrix:

28	13
2	57

Preision = 28/(28+13) = 0.683
Recall = 28/(28+2) = 0.933
F-score = 2*0. 683*0. 933/(0.683+0. 933) = 0.789