CSCI 5521: Introduction to Machine Learning (Fall 2021)[1]

Homework 1

Due date: Oct 6, 2021 11:59pm

1. (**30 points**) Find the Maximum Likelihood Estimation (MLE) of $\boldsymbol{\theta}$ in the following probabilistic density functions. In each case, consider a random sample of size n. Show your calculation:

   (a) $f(x|\theta) = \frac{x}{\theta^2} \exp\{\frac{-x^2}{2\theta^2}\}, x \geq 0$

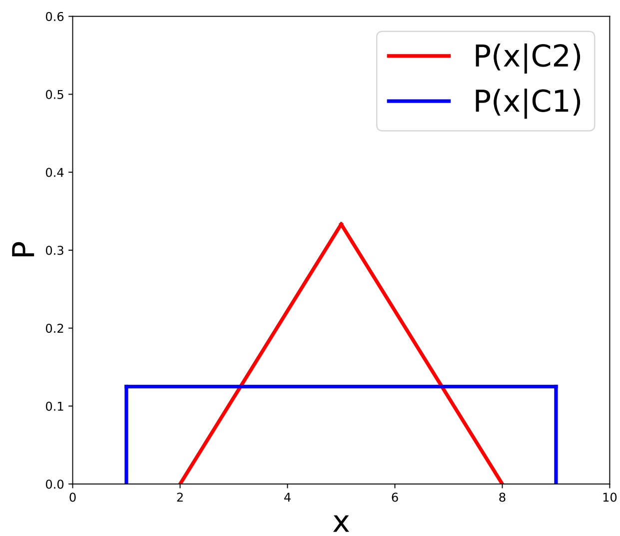
   (b) $f(x|\alpha, \theta) = \alpha\theta^{-\alpha}x^{\alpha-1} \exp\{-(\frac{x}{\theta})^\alpha\}, x \geq 0, \alpha > 0, \theta > 0$

   (c) $f(x|\theta) = \frac{1}{\theta}, 0 \leq x \leq \theta, \theta > 0$ (Hint: You can draw the likelihood function)

2. (**30 points**) We want to build a pattern classifier with continuous attribute using Bayes' Theorem. The object to be classified has one feature, x in the range $1 \leq x < 9$. The conditional probability density functions for each class are listed below:

$$P(x|C_1) = \begin{cases} \frac{1}{8} & if\ 1 \leq x < 9 \\ 0 & otherwise \end{cases}$$

$$P(x|C_2) = \begin{cases} \frac{1}{9}(x-2) & if\ 2 \leq x < 5 \\ \frac{1}{9}(8-x) & if\ 5 \leq x < 8 \\ 0 & otherwise \end{cases}$$



   (a) Assuming equal priors, $P(C_1) = P(C_2) = 0.5$, classify an object with the attribute value $x = 4$.

   (b) Assuming unequal priors, $P(C_1) = 0.7, P(C_2) = 0.3$, classify an object with the attribute value $x = 6$.

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(c) Consider a decision function $\phi(x)$ of the form $\phi(x) = (|x - 5|) - \alpha$ with one free parameter α in the range $0 \leq \alpha \leq 2$. You classify a given input x as class 2 if and only if $\phi(x) < 0$, or equivalently $5 - \alpha < x < 5 + \alpha$, otherwise you choose x as class 1. Assume equal priors, $P(C_1) = P(C_2) = 0.5$, what is the optimal decision boundary - that is, what is the value of α which minimizes the probability of misclassification? What is the resulting probability of misclassification with this optimal value for α? (Hint: take advantage of the symmetry around x = 5.)

3. (**40 points**) In this programming exercise you will first implement the multivariate Gaussian classifiers with two different assumptions as follows:

   - Assume S_1 and S_2 are learned from the data from each class.
   - Assume $S_1 = S_2$ (learned from the data from both classes).

   What is the discriminant function in each case? Show in your report and briefly explain.

   For each assumption, your program should fit two Gaussian distributions to the 2-class training data in `training_data.txt` to learn m_1, m_2, S_1 and S_2 (S_1 and S_2 refer to the same variable for the second assumption). Then, you use this model to classify the test data in `test_data.txt` by comparing $\log P(C_i|x)$ for each class C_i, with $P(C_1) = 0.3$ and $P(C_2) = 0.7$. Each of the data files contains a matrix $M \in \mathbb{R}^{N \times 9}$ with N samples, the first 8 columns include the features (*i.e.* $x \in \mathbb{R}^8$) used for classifying the samples while the last column stores the corresponding class labels (*i.e.* $r \in \{1, 2\}$).

   Report the confusion matrix on the test set for each assumption. Briefly explain the results.

   We further assume that $S_1 = S_2$ and the covariance is a diagonal matrix. Implement the multivariate Gaussian classifier under this assumption, and report the confusion matrix. Briefly explain the results.

   We have provided the skeleton code `MyDiscriminant.py` for implementing the classifiers. It is written in a *scikit-learn* convention, where you have a *fit* function for model training and a *predict* function for generating predictions on given samples. Use Python class `GaussianDiscriminant` for implementing the multivariate Gaussian classifiers under the first two assumptions, and `GaussianDiscriminant_Diagonal` for the third one. To verify your implementation, call the main function `hw1.py`, which automatically generates the confusion matrix for each classifier. Note that you do not need to modify this file.

Submission

- **Things to submit:**

1. hw1_sol.pdf: a document containing all your answers for the written questions (including those in problem 3).
2. `MyDiscriminant.py`: a Python source file containing two python classes for Problem 3, *i.e.,* `GaussianDiscriminant` and `GaussianDiscriminant_Diagonal`. Use the skeleton file `MyDiscriminant.py` found with the data on the class web site, and fill in the missing parts. For each class object, the *fit* function should take the training features and labels as inputs, and update the model parameters. The *predict* function should take the test features as inputs and return the predictions.

- **Submit**: All material must be submitted electronically via Gradescope. **Note that there are two entries for the assignment, *i.e.,* Hw1-Written (for hw1_sol.pdf) and Hw1-Programming (for a zipped file containing the Python code). Please submit your files accordingly.** We will grade the assignment with vanilla Python, and code submitted as iPython notebooks will not be graded.