# Structured sparsity

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2021-03-24

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#### Structured sparsity

- Main paper: Kowalski and Torrésani<sup>1</sup>
- Same year, same authors: Kowalski and Torrésani<sup>2</sup>
- Same techniques: Siedenburg and Doerfler<sup>3</sup>

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<sup>1</sup>Matthian Knoudda and Brann Tomissas. "Sparsity and particinism Mind norms produce implies layed models with opposition." In: Signal Image and Vision Processing 3 (Sept. 2009). soc. 15. 1007/s13100-000-0001-1. "Statistics Nouddal and Brann Tomissas." Societies Sparsity from Mind Norm Mind Matthian Statistics Sparsity Spa

<sup>&</sup>lt;sup>1</sup>Matthieu Kowalski and Bruno Torrésani. "Sparsity and persistence: Mixed norms provide simple signal models with dependent coefficients". In: *Signal Image and Video Processing* 3 (Sept. 2009). DOI: 10.1007/s11760-008-0076-1.

<sup>&</sup>lt;sup>2</sup>Matthieu Kowalski and Bruno Torrésani. "Structured Sparsity: from Mixed Norms to Structured Shrinkage". In: *SPARS09-Signal Processing with Adaptive Sparse Structured Representations* 53 (Apr. 2009).

<sup>&</sup>lt;sup>3</sup>Kai Siedenburg and Monika Doerfler. "Structured sparsity for audio signals". In: *Proc. of the 14th International Conference on Digital Audio Effects (DAFx-11), Paris, France.* Jan. 2011.

## **Sparsity**

Sparsity seems to be a particularly efficient guiding principle in view of a number of tasks such as signal compression, denoising, image deblurring, blind source separation, ... The guiding principle may be summarized as follows: for most signal classes, it is possible to find a basis or a dictionary of elementary building blocks (or atoms) with respect to which all (or most) signals in the class may be expanded, so that when the expansion is truncated in a suitable way, high precision approximations are obtained even when very few terms are retained.<sup>4</sup>

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-Sparsity

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<sup>&</sup>lt;sup>4</sup>Kowalski and Torrésani, 2009.

### LASSO shrinkage

Tibshirani $^5$  proposed a new technique for linear regression: LASSO (least absolute shrinkage and selection operator). It "minimizes the usual sum of squared errors, with a bound on the sum of the absolute values of the coefficients."



Figure: Lasso is an acronym and also an analogy for cattle ranchers<sup>7</sup>

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LASSO shrinkage

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Figure: Lasso is an acronym and also an analogy for cattle ranchers?

Robert Thisbirani. "Regression shrinkage selection via the LASSO". In: Journal Royal Statistical Society Series B 73 (June 2011), pp. 273–282. DOI:

n-yen-ryenancemo.utilisticota.edu; =11882.18880.EXB1 Trevor Hastie, Robert Tibshirani, and Martin Walnevright. Statistical Learning o ethy: The Lesso and Generalizations. Chapman & Hall/CRC, 2015. Inter-71.2169.

<sup>&</sup>lt;sup>5</sup>Robert Tibshirani. "Regression shrinkage selection via the LASSO". In: *Journal of the Royal Statistical Society Series B* 73 (June 2011), pp. 273–282. DOI: 10.2307/41262671

<sup>6</sup>https://statweb.stanford.edu/~tibs/lasso.html

<sup>&</sup>lt;sup>7</sup>Trevor Hastie, Robert Tibshirani, and Martin Wainwright. *Statistical Learning with Sparsity: The Lasso and Generalizations*. Chapman & Hall/CRC, 2015. ISBN: 1498712169

#### Similar ideas – basis pursuit

Basis pursuit (BP) is a principle for decomposing a signal into an "optimal" superposition of dictionary elements, where optimal means having the smallest l1 norm of coefficients among all such decompositions<sup>8</sup>

Least Squares Optimization with L1 Regularization: Estimating Least Squares parameters subject to an L1 penalty was presented and popularized independently under the names Least Absolute Selection and Shrinkage Operator (LASSO) in Tibshirani<sup>10</sup> and Basis Pursuit Denoising in Chen. Donoho, and Saunders<sup>11</sup>

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Mallat and Zhang, 1993.

<sup>&</sup>lt;sup>8</sup>Mallat and Zhang, 1993.

<sup>&</sup>lt;sup>9</sup>Mark Schmidt. Least Squares Optimization with L1-Norm Regularization. 2005.

URL: https://www.cs.ubc.ca/~schmidtm/Documents/2005\_Notes\_Lasso.pdf

<sup>&</sup>lt;sup>10</sup>Tibshirani, "Regression shrinkage selection via the LASSO".

<sup>&</sup>lt;sup>11</sup>S.S. Chen, D. Donoho, and Michael Saunders. "Atomic decomposition by basis pursuit[J]". In: *Siam Review* 43 (Jan. 2001), pp. 33–61.

<sup>&</sup>quot;Misk Schmidt: Later Square Opperization with L1-worm Ingularization. 200

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"5.S. Ohen, D. Donoho, and Michael Saunders." "Atomic decomposition by basi

#### L1 and L2 norm

#### Problem to solve:

- Given  $\mathbf{y} \in \mathbb{R}^T$  be a noisy representation of a signal  $\mathbf{s} \in \mathbb{R}^T$
- Let  $\mathcal{D}$  denote a fixed dictionary for  $\mathbb{R}^T$ , and denote by  $A \in \mathbb{R}^{T \times N}$  the matrix whose columns are the vectors from dictionary  $\mathcal{D}$
- ullet We assume s has a sparse expansion in  ${\mathcal D}$
- We want to estimate s from y

$$\hat{\mathbf{x}} = \operatornamewithlimits{argmin}_{\mathbf{x} \in \mathbb{R}^N} \|\mathbf{y} - A\mathbf{x}\|_2^2 + \lambda \|\mathbf{x}\|_1$$
 • LASSO approach:

- $A\hat{\mathbf{x}}$  is the estimate of  $\mathbf{y}$ ,  $\lambda$  is a fixed parameter
- L1 norm is the  $||\cdot||_1$  expression
- Generally, Lp norm is  $||\cdot||_p^p$ , so L2 norm is  $||\cdot||_2^2$
- These are loss functions for the approximation. Can apply a mixed norm in two dimensions  $\mathcal{P}_{p,q}$ , e.g., time and frequency

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L1 and L2 norm

#### L1 and L2 norm

Given y ∈ ℝ<sup>T</sup> be a noisy representation of a signal s ∈ ℝ<sup>T</sup>
 Let D denote a fixed dictionary for ℝ<sup>T</sup>, and denote by A ∈ ℝ<sup>T×N</sup> the matrix whose columns are the vectors from dictionary D

We assume s has a sparse expansion in D
 We want to estimate s from y

 $\hat{\mathbf{x}} = \underset{\mathbf{x} \in \mathbb{R}^N}{\operatorname{approach}} \|\mathbf{y} - A\mathbf{x}\|_2^2 + \lambda \|$ 

 $\phi$   $A\hat{x}$  is the estimate of y,  $\lambda$  is a fixed parameter  $\phi$  L1 norm is the  $||\cdot||_1$  expression

L1 norm is the ||·||<sub>1</sub> expression
 Generally, Lp norm is ||·||<sup>ρ</sup><sub>θ</sub>, so L2 norm is ||·||<sup>2</sup><sub>2</sub>

Generally, Lp norm is ||·||<sup>a</sup><sub>p</sub>, so L2 norm is ||·||<sup>a</sup><sub>2</sub>
 These are loss functions for the approximation. Can apply a mixed norm in two dimensions P<sub>B,B</sub>, e.g., time and frequency

#### Mixed norm in time and frequency

In the context of time-frequency dictionaries, a natural step beyond classical sparsity approaches is the introduction of sparsity criteria which take into account the two-dimensionality of the time-frequency representations used. Mixed norms on the coefficient arrays make it possible to enforce sparsity in one domain and diversity and persistence in the other domain. 12

Sparsity of coefficients may be enforced by  $\ell^1$ -regression, also known as the Lasso [2]. Given a noisy observation y = s + ein  $\mathbb{C}^L$  it finds

$$\hat{c} = \arg\min_{c \in \mathbb{R}^n} \frac{1}{2} ||y - \Phi c||_2^2 + \lambda \Psi(c)$$
 (2)

with penalty term  $\Psi(\cdot) = \|\cdot\|_1$  and  $\lambda > 0$ . Since the sequence  $c_{k,i}$  is ordered along two dimensions for Gabor frames, the  $\ell^1$ prior  $\Psi$  in (2) may be replaced by a two-dimensional mixed norm  $\ell^{p,q}$  which acts differently on groups (indexed by q in the sequel, may be either time or frequency) and their members (indexed by

$$\Psi(c) = \|c\|_{p,q} = \left(\sum_{g} \left(\sum_{m} |c_{g,m}|^{p}\right)^{q/p}\right)^{1/q} \tag{3}$$

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☐ Mixed norm in time and frequency

Mixed norm in time and frequency

dictionaries, a natural step beyond introduction of sparsity criteria

coefficient arrays make it possible to

 $\Phi(z) = \|z\|_{p,q} = \left(\sum \left(\sum |c_{p,m}|^p\right)^{p/p}\right)^{1/q}$ 

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<sup>13</sup>Kai Siedenburg and Monika Doerfler. "Structured sparsity for audio signals". 1

<sup>&</sup>lt;sup>12</sup>Kai Siedenburg and Monika Doerfler. "Structured sparsity for audio signals". In: Proc. of the 14th International Conference on Digital Audio Effects (DAFx-11), Paris, France. Jan. 2011.

### Elitist and Group LASSO

Two new and strikingly different shrinkage operators, E-LASSO and G-LASSO:

- ① Elitist-LASSO is the solution  $\widehat{\mathbf{x}}$  of problem  $\mathcal{P}_{1,2}$ , using  $l_{1,2}$  coefficient penalty. Each coefficient is shrunk individually, but the corresponding threshold depends on its 1D neighborhood; can be understood as an elitist group shrinkage, since most members of a given group are thresholded, and only the *emerging* (or best) coefficients of each group remain, or "only the best coefficients are chosen per group"  $^{13}$
- ② Group-LASSO is the solution  $\widehat{\mathbf{x}}$  of problem  $\mathcal{P}_{2,1}$ , using  $l_{2,1}$  coefficient penalty. A 1D group of coefficients is either globally retained or discarded. This may be understood as a united group shrinkage, since the same threshold applies to all members of a given group; "one keeps entire groups of coefficients, namely the most energetic groups"

 $\ensuremath{\mathsf{E}\text{-}\mathsf{LASSO}}$  promotes sparsity within groups of coefficients instead of sparsity across groups like  $\ensuremath{\mathsf{G}\text{-}\mathsf{LASSO}}$ 

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-Elitist and Group LASSO

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Thousaks and Torrisons, 2009.

<sup>&</sup>lt;sup>13</sup>Kowalski and Torrésani, 2009.

## **Group-LASSO**

Group-LASSO:<sup>14</sup> solve the problem of finding signifiance maps (i.e. locations of significant coefficients) in the transform domain

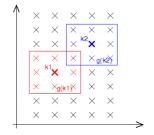


Figure: Group-LASSO regression on two overlapping groups<sup>15</sup>

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└─Group-LASSO

Figure: Group-LASSO regression on two overlapping groups<sup>18</sup>

Asttheu Kowaldo' and Bruso Toniscas: "Sparsity and persistence: Mixed nor
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Astribies Kowaldo' and Bruso Toniscas: "Structured Sparsity from Mixed No
contracted Shrinkage," in: 370/15/05-Signal Processing with Autgries Spanse

<sup>&</sup>lt;sup>14</sup>Matthieu Kowalski and Bruno Torrésani. "Sparsity and persistence: Mixed norms provide simple signal models with dependent coefficients". In: *Signal Image and Video Processing* 3 (Sept. 2009). DOI: 10.1007/s11760-008-0076-1.

<sup>&</sup>lt;sup>15</sup>Matthieu Kowalski and Bruno Torrésani. "Structured Sparsity: from Mixed Norms to Structured Shrinkage". In: *SPARS09-Signal Processing with Adaptive Sparse Structured Representations* 53 (Apr. 2009).

#### Tonal/transient separation with Group-LASSO

 $WMDCT^{16}\,+\,Group\text{-}LASSO-\text{``audioshrink''}^{17}$ 

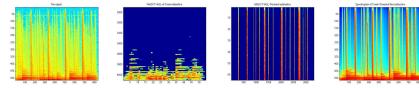


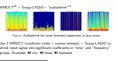
Figure: Audioshrink for tonal/transient separation in jazz music

Use 2 WMDCT transforms (wide + narrow window) + Group-LASSO to shrink input signal into significant coefficients in "time" and "frequency" groups. Example:  $\blacktriangleleft$  mix,  $\blacktriangleleft$  tonal,  $\blacktriangleleft$  transient

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-Tonal/transient separation with Group-LASSO



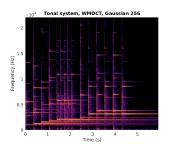
Fonal/transient separation with Group-LASSO

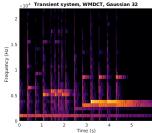
Jan. 2011. ops://homepage.univie.ac.at/moniks.doerfler/Struckudio.h //ltfat.github.io/doc/demos/demo\_mudioshrink.html

<sup>&</sup>lt;sup>16</sup>Kai Siedenburg and Monika Doerfler. "Structured sparsity for audio signals". In: *Proc. of the 14th International Conference on Digital Audio Effects (DAFx-11), Paris, France.* Jan. 2011.

<sup>17</sup>https://homepage.univie.ac.at/monika.doerfler/StrucAudio.html, https://ltfat.github.io/doc/demos/demo\_audioshrink.html

#### WMDCT tonal/transient dictionaries





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Figure: WMDCT + Group-LASSO shrink tonal/transient separation<sup>18</sup>

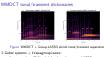
#### 2 Gabor systems + franagrouplasso:

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franagrouplasso(F,f,lambda) solves the group LASSO regression problem in the time-frequency domain: minimize a functional of the synthesis coefficients defined as the sum of half the  $l_2$  norm of the approximation error and the mixed  $l_1/l_2$  norm of the coefficient sequence, with a penalization coefficient lambda.

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-WMDCT tonal/transient dictionaries



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"https://ltfs.githab.io/scs/fransg/fransgrouplesso.html, styte/flts.githab.io/scs/fransgrouplesso.html, styte/flts.githab.io/scs/fransgrou

<sup>18</sup>https://ltfat.github.io/doc/frames/franagrouplasso.html, https://ltfat.github.io/doc/demos/demo\_audioshrink.html

#### Lasso comparison for tonal/transient separation

Tonal layer is expected to be sparsely represented in the frequency domain, with emergent frequencies that may evolve slowly with time (i.e. almost horizontal lines of large MDCT coefficients). Lasso choices:

- E-LASSO (sparse within group) with the time label as group label
- G-LASSO (sparse across groups) with the frequency label as group label - bad choice because of the slow evolution in time of frequencies

Transient layer is expected to be sparse in time, but spread out in the frequency domain. Lasso choices:

- E-LASSO (sparse within group) with the frequency label as group label
- G-LASSO (sparse across groups) with the time label as group label **good choice** because transients are sharply time-localized

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Lasso comparison for tonal/transient separation

Lasso comparison for tonal/transient separation

Tonal layer is expected to be sparsely represented in the frequency domain, with emergent frequencies that may evolve slowly with time (i.e. almost

- E-LASSO (sparse within group) with the time label as group label
   G-LASSO (sparse across groups) with the frequency label as group
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  - label

    G-LASSO (sparse across groups) with the time label as group label
  - good choice because transients are sharply time-localized

#### Transform to dictionary

Mallat and Zhang<sup>19</sup> proposed a novel sparse signal expansion scheme based on the selection of a small subset of functions from a general overcomplete dictionary of functions. Chen, Donoho and Saunders<sup>20</sup> published their influential paper on the Basis Pursuit, and the two works signalled the beginning of a fundamental move from transforms to dictionaries for sparse signal representation ... The terminological change enclosed the idea that a signal was allowed to have more than one description in the representation domain, and that selecting the best one depended on the task<sup>21</sup>

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☐ Transform to dictionary

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PS.G. Mallat and Zhifeng Zhang. \*Matching Pursuits with Time-Frequency

<sup>&</sup>lt;sup>19</sup>S.G. Mallat and Zhifeng Zhang. "Matching Pursuits with Time-Frequency Dictionaries". In: Trans. Sig. Proc. 41.12 (Dec. 1993), pp. 3397–3415. ISSN: 1053-587X, DOI: 10.1109/78.258082.

<sup>&</sup>lt;sup>20</sup>S.S. Chen, D. Donoho, and Michael Saunders. "Atomic decomposition by basis pursuit[J]". In: Siam Review 43 (Jan. 2001), pp. 33-61.

<sup>&</sup>lt;sup>21</sup>Ron Rubinstein, Alfred Bruckstein, and Michael Elad. "Dictionaries for Sparse Representation Modeling". In: Proceedings of the IEEE 98 (July 2010), pp. 1045–1057. DOI: 10.1109/JPROC.2010.2040551. URL:

https://www.cs.technion.ac.il/~ronrubin/Publications/dictdesign.pdf. Sevag Hanssian (MUMT 622, Winter 2021)

<sup>&</sup>lt;sup>30</sup>S.S. Chen, D. Donoho, and Michael Saunders. "Atomic decomposition by basis parealt[J]\*, In: Stam Review 43 (Jan. 2001), pp. 33-61.

<sup>28</sup> Ron Rubinstein, Alfred Bruckstein, and Michael Elad. "Dictionaries for Sparse Representation Modeling", In: Proceedings of the IEEE 98 (July 2010), pp. 1045-1

#### Sparsity and entropy

Sparsity and entropy are complementary concepts

Given a signal x, its transform w, and probability mass function p:

- Sparsity, compressibility: the property of concentrating most of the energy of x in few coefficients of w
- Entropy, uncertainty: the property of not concentrating most of the probability mass of x in few atoms of p

For a given signal x, the uncertainty (randomness) of its elements defines the compressibility (compactness) of its coefficients w in a given domain<sup>22</sup> Structured sparsity

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Sparsity and entropy

Sparsity and entropy

Sparsity and entropy are complementary concepts

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Gancario Pastor et al. Mathematics of Sparsity and Entropy: Assoms, Core Functions and Sparse Recovery, 2015, arXiv: 1501.05126 [cs.17].

<sup>&</sup>lt;sup>22</sup>Giancarlo Pastor et al. Mathematics of Sparsity and Entropy: Axioms, Core Functions and Sparse Recovery. 2015. arXiv: 1501.05126 [cs.IT].

#### Time-Frequency Jigsaw Puzzle

- Create time-frequency "super-tiles" by superimposing a large window + small window Gabor analysis
- ② Use Rényi entropy to set coefficients to zero where sound has more entropy than random white noise

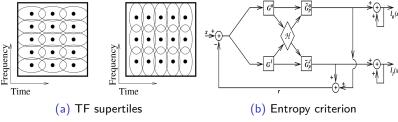


Figure: TF Jigsaw Puzzle tonal/transient separation<sup>23</sup>

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Time-Frequency Jigsaw Puzzle

O Case to time frequency "user either by superimposing a large window is well window discar analyse.

I the control of the con

Fime-Frequency Jigsaw Puzzle

- i.e. good tonal/good transient
- high entropy = indicating sound is poorly represented

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<sup>&</sup>lt;sup>23</sup>Florent Jaillet and Bruno Torrésani. "Time-Frequency Jigsaw Puzzle: adaptive multiwindow and multilayered Gabor expansions". In: *International Journal of Wavelets, Multiresolution and Information Processing* 05 (Mar. 2007). DOI: 10.1142/S0219691307001768

# **TFJigsaw**

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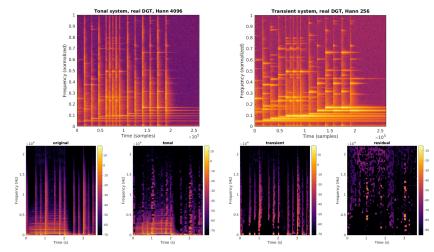


Figure: TF Jigsaw Puzzle tonal/transient separation<sup>24</sup>

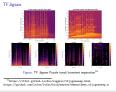
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└─TFJigsaw



<sup>24</sup>https://ltfat.github.io/doc/sigproc/tfjigsawsep.html, https://github.com/ltfat/ltfat/blob/master/demos/demo\_tfjigsawsep.m