

1 **A Markovian model of evolving world input-output network**

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6 Abstract

7 The initial theoretical connections between Leontief input-output models and Markov
8 chains were established back in 1950s. However, considering the wide variety of
9 mathematical properties of Markov chains, so far there has not been a full
10 investigation of evolving world economic networks with Markov chain formalism. In
11 this work, using the recently available world input-output database, we investigated
12 the evolution of the world economic network from 1995 to 2011 through analysis of a
13 time series of finite Markov chains. We assessed different aspects of this evolving
14 system via different known properties of the Markov chains such as mixing time,
15 Kemeny constant, steady state probabilities and perturbation analysis of the transition
16 matrices.

17 First, we showed how the time series of mixing times and Kemeny constants could be
18 used as an aggregate *index of globalization*.

19 Next, we focused on the steady state probabilities as a measure of *structural power of*
20 *the economies* that are comparable to GDP shares of economies as the traditional
21 index of economies welfare.

22 Further, we introduced two measures of systemic risk, called *systemic influence* and
23 *systemic fragility*, where the former is the ratio of number of influenced nodes to the
24 total number of nodes, caused by a shock in the activity of a node and the latter is
25 based on the number of times a specific economic node is affected by a shock in the
26 activity of any of the other nodes.

27 Finally, focusing on Kemeny constant as a global indicator of monetary flow across
28 the network, we showed that there is a paradoxical effect of a change in activity levels
29 of economic nodes on the overall flow of the world economic network. While the
30 economic slowdown of the majority of nodes with high structural power results to a

31 slower average monetary flow over the network, there are some nodes, where their
32 slowdowns improve the overall quality of the network in terms of connectivity and the
33 average flow of the money.

34

35 **Introduction**

36 The mathematical beauty of computational algebraic methods such as Markov chains
37 is that they are domain free. This means that having a proper size of observed data and
38 enough computational power they fit very well into many application domains, while
39 unlike many domain specific models, they do not ask for domain specific prior-
40 knowledge. For example, they assume that the rules of interactions among agents
41 (being economic agents or drivers in a transportation network or words in a spoken
42 language), are embedded in the traces of their real interactions, while in traditional
43 rule based or agent based simulations, one needs to specify features and the rules of
44 interactions among those agents beforehand. On the other hand, algebraic methods are
45 data demanding and because of this, Markov chains for example that were introduced
46 in 1906 [1], did not get that much of attention before the advent of computers in 1950s
47 and finally in the late 1990s, Markov chains were applied in large scale problems such
48 as in PageRank algorithm in Google search engine [2]. In principle, the same
49 argument holds for the recently successful field of “*representation learning*” or the
50 so-called “*deep learning*”, where having large amount of data set along with a series
51 of stacked algebraic operators one can come up with highly sophisticated hierarchical
52 representations of complex phenomena [3].

53 In this work our focus is on Markov chains and their applications on evolving
54 economic networks. A Markov chain is a data driven formalism to its underlying

55 dynamical system, where we only need some real observations and usually no prior
56 rules of interactions among the agents or the states of that system. Nevertheless, with
57 this formalism one can benefit from the many interesting mathematical properties of
58 Markov chains such as their steady state probability distribution [4], Kemeny constant
59 [5], recurrence time and mixing time [6], mean first passage times [7] and the
60 sensitivity analysis of the underlying networks through perturbation of the transition
61 matrix [8-10]. Of course, one should be very careful with the prior assumptions in a
62 Markov chain such as its structuralist view to the problem, the issues of memory, the
63 linearity of the operator, the assumptions about closed-ness of the state space in
64 discrete chains, etc.

65 In the domain of economic and financial applications, especially after the financial
66 crisis of 2008, the notions of *networked economy, complexity and systemic risk* are
67 gaining increasing importance [11-16]. Comparing to classical economic models,
68 which are mainly based on the assumptions of independent agents, network based
69 economics is focused on the interaction between agents.

70 Nevertheless, networks are not new topics in economics. For example, one can refer
71 to the works of Leontief on the so-called, input-output tables [17] within 1940s, for
72 which he won a Nobel Prize in economics. An input-output table in fact is a network,
73 where nodes are the segments of an economy (i.e. different industries within a
74 country) and the edges are the monetary flows of goods within these nodes. Input-
75 output tables can be seen as a system of equations where the solution (if exists) is
76 considered as the equilibrium price of products in order to keep the economic network
77 stable.

78 Related to the our work, Solow in 1952 [18] discussed the connections between
79 Leontief input-output models and Markov chain formalism, where he investigated the

80 required conditions for finding a stable solution (i.e. balanced prices) for the
81 underlying system of equations. Further, the authors in [19] modeled input-output
82 models as absorbing Markov chains based on either the flow of materials or the flow
83 of money.

84 In this work, based on the recently available data set, called World-Input-Output-
85 Database (WIOD) [20], we investigate several other properties of Markov chains on a
86 time varying global economic network.

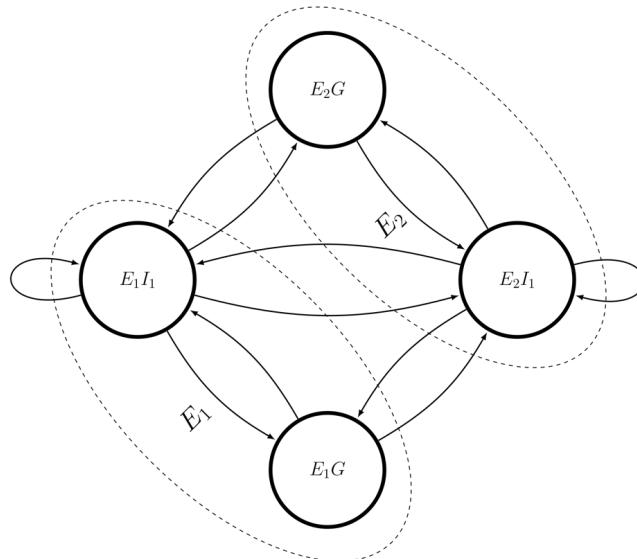
87 In the next section, we briefly describe the data set we used in this work. Next, we
88 describe the proposed Markovian model and those properties we applied to analyze
89 the global economic network. Finally, we show the results and discuss the potential
90 future directions.

91 **Materials and methods**

92 **World input output network**

93 The World-Input-Output-Database (WIOD) represents a network of two types of
94 nodes. The first type of node, I , corresponds to a specific industrial sector within an
95 economy. Each industry, based on some inputs from other industries, produces some
96 products and sells them to other intermediate industries and final consumers that are
97 call households and the governments. These households, together with the
98 government of each economy, represent an additional kind of node, G . This node
99 participates in the money flow through the network by consumption of final products,
100 and by receiving money consisting of taxes and value added coming from the
101 corresponding industries working in that economy. This process can be visualized in a
102 weighted digraph structure, where an industrial sector j of a specific economy i is

103 defined as $E_i I_j$. Further, we assign one node for the governments and households
 104 within each economy that from now on we refer to by $E_i G$. In this manner, each input
 105 output table is represented as a closed system, which makes it suitable for Markov
 106 Chain formalism.
 107 Fig 1 shows a schematic view of a closed network with two economies, each with one
 108 unique industry and one node representing the government and the households
 109 together. As it is shown in this figure, in WIOD data set, due to aggregation of flows
 110 within industries, there are explicit self-loops for industry nodes. Further, from now
 111 on we assume that the edges are representing the flow of money. As we will show
 112 later, it is also possible to easily aggregate the flows over each economy in order to
 113 come up with the measures at the level of economies.



114
 115 **Fig 1. A schematic view of a closed economic network**
 116 There are two economies and one industry within each economy, and one node for the
 117 government and households within each economy. The edges represent the flow of
 118 money between nodes.

119

120

121 In the WIOD that we used in this study, there are 35 industries within 41 economies
122 (27 EU countries and 13 major economies in other regions) plus the rest of the world
123 (RoW) as one economy. A complete list of industries and economies can be found in
124 [20]. Considering the 35 industries within each economy plus one node for each
125 government and households (together in one node), there are 1476 nodes for each
126 year. While the flows (i.e. the edges and their values) change from year to year, the
127 same structure repeats for 17 years from 1995 to 2011, which makes it suitable for
128 trend analysis. WIOD is a valuable data set that has been used in several recent
129 studies, including identification of global value chains and trade fragmentation
130 [21,22] and global environmental accounting in ecology and resources management
131 [23]. From a network analytics point of view recently there has been a work on this
132 data set, where several network based measures such as different centrality measures
133 and clustering measures of the world economic network have been studied [24]. In
134 this work we applied several properties of Markov chains on this time varying
135 network.

136 **The proposed Markovian model of the world economic**

137 **network**

138 As mentioned before, the formalism of Input Output (IO) models by Markov chain
139 has an old history back to 1952 [18] and more recently to [19] who modeled an open
140 IO models as absorbing Markov chains. In this work, a closed IO network is studied,
141 which can be translated naturally to a regular Markov chain with no absorbing states.
142 More explicitly, an IO table of a specific year in the WIOD is an asymmetric non-
143 negative squared matrix W whose elements w_{ij} correspond to the flow of money from

144 a node j to a node i . A stochastic matrix T can be directly associated to W by column
145 normalization, thus a specific element t_{ij} of T is defined by:

146
$$t_{ij} = \frac{w_{ij}}{\sum_i w_{ij}} \quad (1)$$

147 The elements t_{ij} can be interpreted either as the relative flow of money between nodes
148 or as the probability for a random walker to move from one node to another. Since
149 there is only one table per year, these probabilities are the annual average values. As a
150 result, for each year, we assume a single discrete time-homogeneous Markov chain
151 model with a corresponding stochastic matrix T_y with $y \in \{1995, \dots, 2011\}$.

152 Our interest in this work is mainly on the dynamics of the world economic network
153 over time, where the T_y matrices are changing for each year. Therefore, we are facing
154 a time inhomogeneous Markov chain. However for every year y the stochastic matrix
155 is well defined and its properties can be used to characterize the global economic
156 network and follow its evolution through 17 years. In particular three specific
157 properties have been chosen:

158 **Steady State Vector:** is the first eigenvector of T defined by:

159
$$\pi = T\pi \quad (2)$$

160 One can easily estimate this vector using power iteration method, starting with any
161 initial random vector. This vector is sometimes called *Eigenvector centrality*, however
162 this terminology has not been used in this work. In our case, π is a normalized one-
163 dimensional vector with the same size as the number of nodes in the global economic
164 network (i.e. 1476 nodes). Its values can be interpreted as the expected long-term
165 relative amount of money within each government or each industry. As we will see
166 later, while it is common in network studies to use centrality measures for ranking of

167 the nodes (e.g. PageRank algorithm [2]), here these values are highly comparable to
168 annual GDP shares of the economic nodes. It is important to note that since the
169 underlying dynamical system in our global economic network cannot be explained
170 with one fixed transition matrix, one cannot claim that the global economic network
171 reaches the steady state within the scope of one year. But at the same time, taking
172 stationary probabilities as a kind of *structural property* of each node, the comparisons
173 of their values over time reveals interesting features of this evolving global economic
174 network.

175 **Mixing time:** It can be measured as the average number of steps that a Markov chain
176 takes from any random initial state in order to reach its steady state [6]. Mixing time is
177 a very good global measure, which shows how connected the network is. In principle,
178 if a chain has more local loops or disconnected regions that is difficult to enter or
179 leave, mixing time will be longer. In the context of global economic network this can
180 be considered as an index of globalization, where higher values of mixing times
181 shows less connected network and vice versa. In this work, the mixing time of each
182 year's transition matrix is calculated through the average number of iterations in the
183 power iteration method.

184 **Kemeny constant:** Similar to mixing time, this is another global measure of the
185 Markov chains, which shows the average expected time from any given state (node)
186 to a random state (node). Interestingly, this value is constant over different states of a
187 given Markov chain and therefore it can be considered as an intrinsic feature of a
188 chain. Similar to mixing time, this constant can be a good indicator of the connectivity
189 of the underlying network. Therefore, as a hypothesis we expect that corresponding
190 Kemeny constants of different Markov chains for different years should form a
191 decreasing pattern over time, which indicates a faster flow of money and more

192 development of the global economic network within the years 1995 to 2011. Along
193 the same line, there is another interesting property of Markov chains, called Mean
194 First Passage Time (MFPT) [7], which indicates the expected time for a Markov chain
195 to transit from specific node to another specific node. In the context of economic
196 networks, this measure can be used to analyze the inter-relationships between two
197 specific industries within or across a value chain. However, we did not use this
198 measure in this work.

199 Calculation of Kemeny constant of a each Markov chain is very straight forward. As
200 shown in [5], the eigenvalues $\lambda_2, \dots, \lambda_n$ of T other than 1 can be used to compute the
201 Kemeny constant as follows:

$$202 \quad K(T) = 1 + \sum_{i=2}^n \frac{1}{1-\lambda_i} \quad (3)$$

203 **Sensitivity analysis of transition matrices**

204 By perturbing the values of the transition matrices, one can analyze the effect of each
205 node on the other nodes. There are many different approaches for perturbation
206 analysis of Markov chains within the literature such as [8-10]. A common way for
207 perturbation analysis is to change the transition probabilities by small random noises,
208 while the sum of these noises is equal to zero. In this way the transition matrix will
209 remain stochastic.

210 However in this work, since we are ultimately interested in defining risk measures
211 attributed to individual economic nodes, we choose a different procedure of
212 perturbation, repeated for all the nodes. As described in [10], we analyze the effect of
213 slowing down the activity level of one specific economic node on all the other nodes
214 by the following procedure.

215 If we want to change the activity of one node by α percent we multiply all the outflow
216 and inflow rates of that node by $1 + \alpha/100$ and then we normalize all the affected
217 columns. After this change, we have a new transition matrix.

218 One interesting property of Markov chain is that since it is a linear operator, if we
219 increase (decrease) the rates of a node by α percent, based on the described procedure,
220 after calculating the new steady state probabilities, the new value of that node will
221 increase (decrease) by α percent. Further, since we assume a closed system, then we
222 have a zero sum game. This means that a decrease (increase) in the π_i will result to
223 decrease or increase of π_j for $j \neq i$ such that $\sum_{i=1}^n \pi_i = 1$.

224 It is important to mention that there is a pre-assumption in this manipulation of the
225 original transition matrix that by slowing down the activity of a node, all of its
226 connected industries redistribute their slack resources to other activities proportionally
227 to the their flow rates. Therefore, here we assume that there is no limit in resources
228 and production capacities or any limits on the absolute flow levels of money
229 (commodity) over the edges of the network.

230 In the perturbation process, there might be nodes (assumingly with not a large
231 structural power) that have effects on many other nodes. Thus instead of considering
232 the total values of these effects, by focusing on the number of nodes that are being
233 affected by the change in the activity of one node, we introduce the two following
234 measures.

235 *Systemic Influence*, which is a measure for each economic node, calculated as the ratio
236 of number of affected nodes (negatively or positively) to the total number of nodes,
237 caused by a change in the activity of that node.

238 *Systemic Fragility*, which is a measure for each economic node, calculated as the ratio
239 of number of times a node is affected (negatively or positively) by a change in the
240 activity of all the other nodes.

241 Another possible sensitivity analysis is to consider the effect of each node on a global
242 measure of the economic network such as Kemeny constant. This type of analysis
243 sometimes leads to unexpected results, where by removing important nodes (in terms
244 of steady state probabilities) the total flow of the network will improve and vice versa
245 [10]. In the next section we will present the results of applying the above-mentioned
246 analyses in to the evolving global economic network.

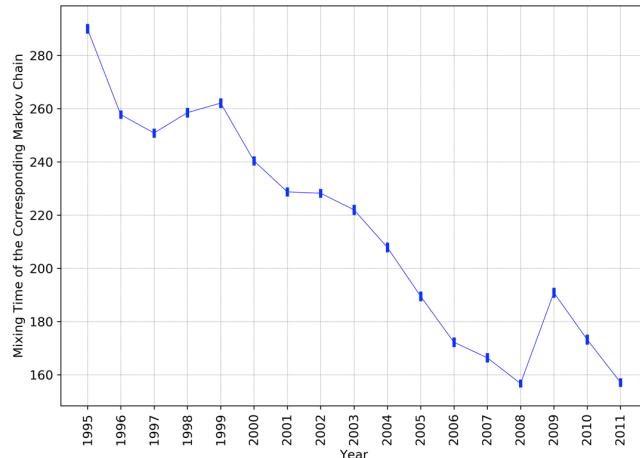
247 **Results**

248 In this section, based on the previously described properties of the Markov chains we
249 show the results of our experiments on the WIOD data set.

250 **The overall patterns of globalization**

251 In this part, we focus on the global features of the underlying network by showing the
252 results of mixing times and Kemeny constants for the years from 1995 to 2011. Fig 2
253 shows the sequence of mixing times for the corresponding Markov chains of each
254 year. We run these iterations several times with the same threshold of termination for
255 all the years and we observed that the mixing times are stable in different runs. This
256 can be seen in the very small error bars around the average values of each year. As we
257 expected the mixing time series has an overall downward pattern from 1995 to 2011,
258 which indicates that during these years the underlying TMs and consequently the
259 world economic network was getting more and more interconnected. Therefore, one
260 could interpret this feature as an *index of globalization*. Further, as it is shown in Fig
261 2, this index reflects the effect of global financial crisis in 2008, which results to a

262 jump in the mixing time in 2009. This implies that the world economic network was
263 less connected in 2009 comparing to 2008. A similar pattern can be seen from 1997
264 to 1999, where we could not argue its underlying reason. Nevertheless, the overall
265 pattern shows a rapid globalization during 1995 to 2011, which seemingly will
266 continue for the next coming years.



267
268 **Fig 2. The sequence of average mixing time of Markov chains as an aggregate**
269 ***index of globalization***

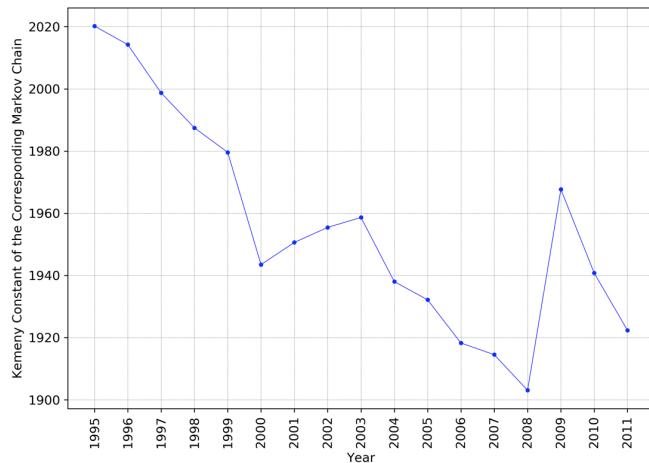
270 Lower values indicate more globally connected network. The error bars represent 3
271 standard deviations.

272

273

274 Similar to mixing times we expected that the Kemeny constant series to show a
275 downward pattern. As presented in [5], we calculated the Kemeny constant of each
276 Markov chain based on the Eigenvalue decomposition of the corresponding matrices.
277 Fig 3 shows that although Kemeny constants have the same overall pattern as the time
278 series of mixing times, including the shock in 2008 and 2009, there is an upward
279 pattern in the values of Kemeny constants within the years 2000 to 2004. As a

280 reminder, we should note that Kemeny constant indicates the average time from any
281 given state (here any industry within any economy) to any random state in the
282 network, where surprisingly this average time is constant independent of the starting
283 point. However, when there is a local loop within the network this average time will
284 increase. In the context of economic network, this might mean that within the years of
285 2000 to 2004, there might have been a creation or reinforcement of some local loops
286 in the global economic network. Nevertheless, Kemeny constant is an aggregated and
287 emergent measure of the underlying dynamics and one needs specific investigations in
288 order to find out the underlying reasons for these macro behaviors.



289
290 **Fig 3. The sequence of Kemeny constants of Markov chains as an aggregate**
291 *index of globalization*

292 Lower values indicate more globally connected networks.

293

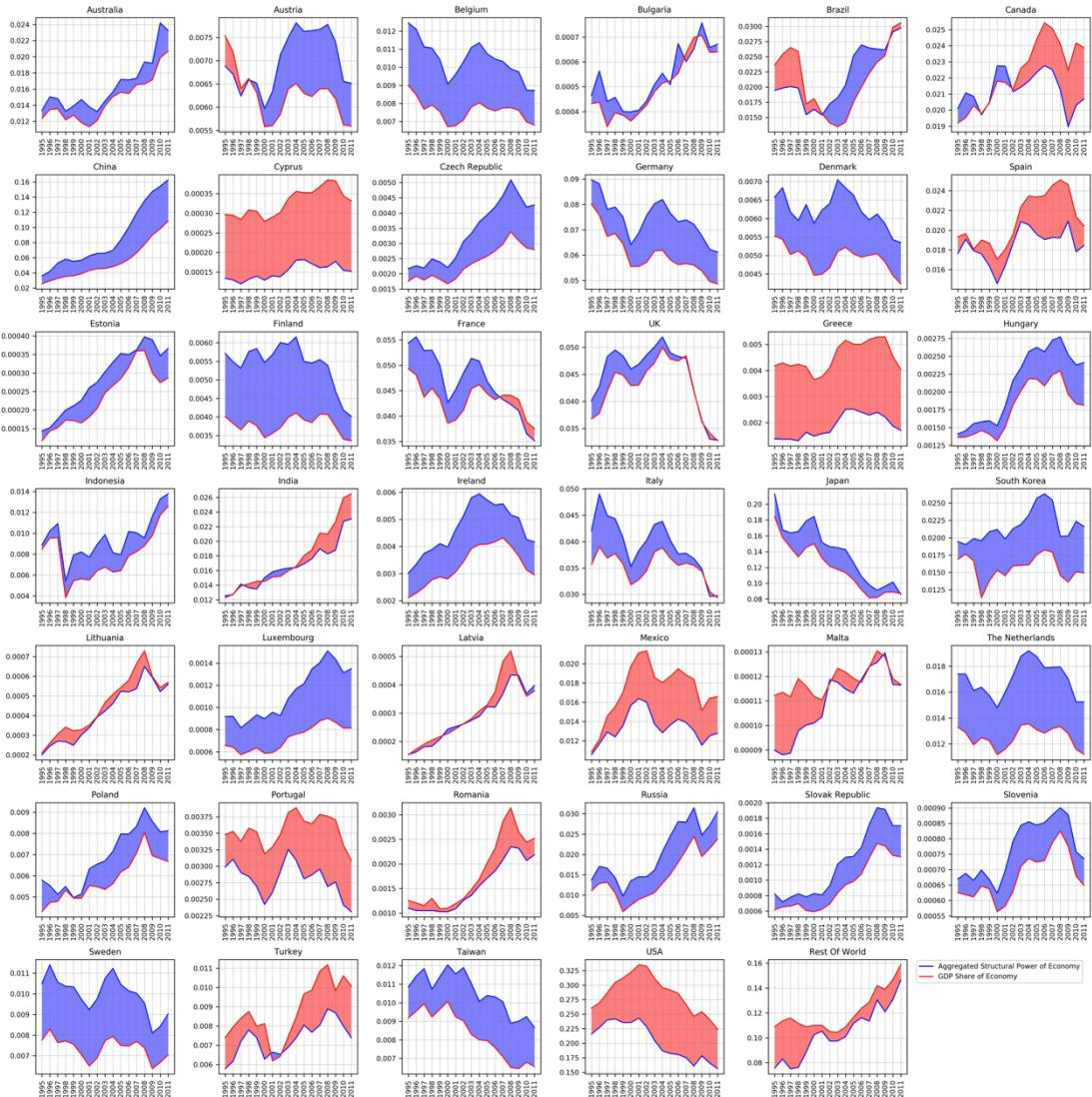
294 In the next section, we focus on the analysis of steady state probability distributions
295 for different years.

296 **Steady state probabilities as a measure of structural power**

297 **of economies**

298 In a stochastic transition matrix, the first Eigenvector, π , shows the steady state
299 probabilities of the underlying dynamical system. As described in the previous
300 section, this can be calculated easily using power iteration method. However, as we
301 discussed before, since we have one unique Markov chain for each year, we cannot
302 claim that the underlying economic system reaches to its steady state within each
303 year. However, the steady state vectors of each year can be interpreted as the
304 *structural power* of each node in the economic network and since the structure of the
305 network (i.e. the number of nodes in the global economic network) is fixed,
306 comparing the time series of π_i^y for each node i at year y reveals interesting results.
307 Further, one can easily calculate different aggregated measures by summing up these
308 steady state values over different categories such as industries or economies. As we
309 will show there is a direct relation between the aggregated values of each economy,
310 called π_E^y and its GDP share at the same time. In principle, GDP as a measure of
311 economy's welfare considers one economy in an isolated set up, while the steady state
312 probabilities are being calculated based on the relationships between all the economic
313 nodes. Therefore, looking at economies in isolation might reveal different results than
314 considering the developments in other economies at the same time. Recently, in this
315 direction there have been interesting works such as [11,15,16] that came up with
316 measures of economic fitness of countries that are fundamentally relational and
317 consequently reveal different features than classical GDP measures.
318 Fig 4 compares two time series of GDP shares of economies with the time series of
319 π_E^y . Each individual plot corresponds to one aggregated economy (industries plus

320 households/government), where the x-axis is the year and y-axis is for the GDP shares
321 (red line) and π_E values of each economy (blue lines). As we expected, the two time
322 series are highly correlated. However, the differences between the two time series
323 indicate an interesting aspect of these economies. We think this difference can be
324 considered as a measure of *economic fitness* or the *structural potential* of the
325 economies for further growth. As a hypothesis, we think whenever the GDP share is
326 larger than the aggregated π_E values of the economy, that economy is at risk (for
327 example, the red gaps in Cyprus and Greece) and when the gap is blue, this means
328 that the country has still more potential structural power than what is being produced.
329 An interesting feature of this ratio is that it does not correlate with the overall patterns
330 of the economy in time. For example, while Germany and Japan are loosing their
331 global competitiveness (with downward patterns), still they are not in a risky area (the
332 blue gap). On the other hand, while India and Turkey for example are gaining more
333 competitive powers (with upward patterns), both are at the same time going to the
334 risky area (the red gap).



335

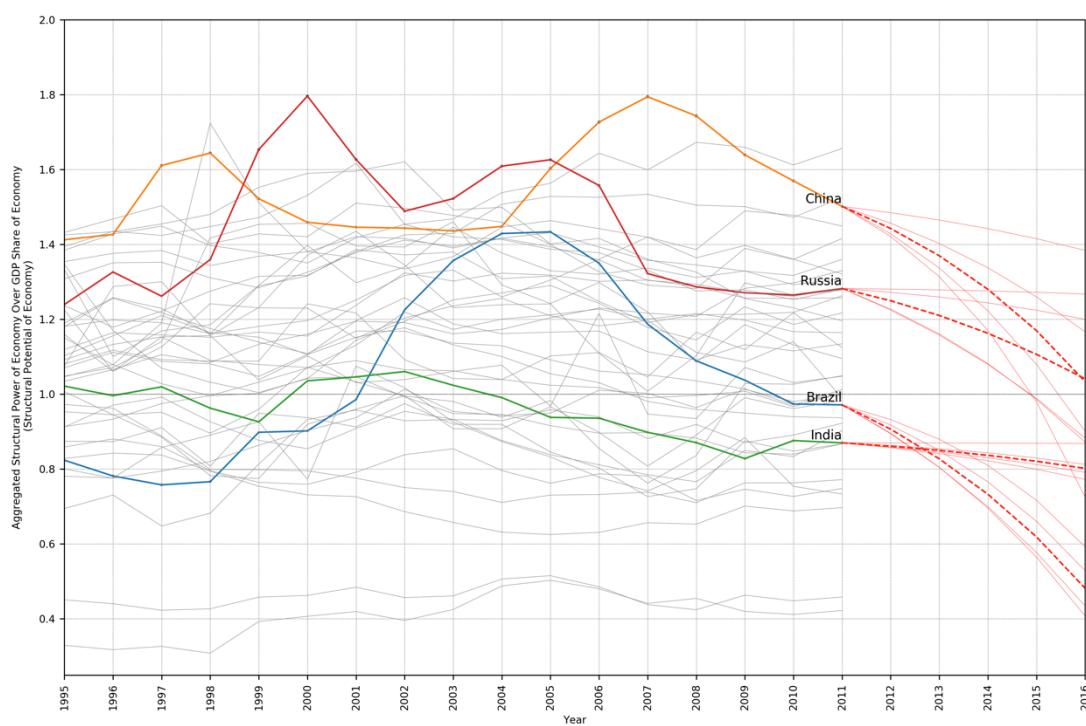
336 **Fig 4. GDP shares of economies (red line) compared with their aggregated
337 structural powers (blue line) over time**

338 The ratio between two time series reveals the *structural potential* of the economies for
339 further growth (blue gaps) or the risk of economic failure (red gaps).

340

341 Within the literature of economic complexity there has been always an interest in
342 predicting the future states of the dynamical systems. Plotting the patterns of the so-
343 called BRIC countries (Brazil, Russia, India and China) together shows an interesting
344 similarity (Fig 5). It seems that all of these countries have passed a curve shape
345 behavior and in 2011 they are slowing down, where the GDP share is getting closer to

346 the aggregated π_E , hypothetically implying less structural potential for further growth.
 347 The red dashed lines are calculated based on the moving average of the first
 348 momentum of each time series with the time lags between 3 to 6 years. The tick
 349 dashed line shows the median prediction.
 350 This result is similar to the results of the recent works published in [16], where the
 351 authors predict the future economic fitness of different economies in comparison to
 352 their GDP per capita.



353
 354 **Fig 5. Predicted trends of structural potential of different economies**
 355
 356 In the next section, we will focus on the sensitivity analysis of the Markov chains in
 357 order to assess the influence of different nodes on each other and to further, identify
 358 those fragile nodes that get affected by shocks in the network. In addition, we analyze
 359 the effect of slowdown in the economic activities of each individual node on the
 360 overall monetary flow of the world economic network.

361 **Sensitivity analysis of Markov chains**

362 Since Markov chain provides a formalism of the underlying dynamical system, as
363 described before, it is then very easy to perform sensitivity analysis by slight changes
364 in the values of the constructed transition matrix.

365 In a drastic scenario, Fig 6 shows the effect of 99% slow down in the electrical and
366 optical equipment industry of China in 1995 and 2011 respectively. As it was
367 expected, comparing to 1995, a change in this industry in 2011 has enormous negative
368 and positive effects on the final shares (based on the new π_t vector) of other industries
369 across the globe. In the depicted diagrams, negative effects are highlighted by red
370 color and positive effects are shown by green color. The size of green or red circles is
371 proportional to the primary values in π_t vector of that economic node. For better
372 visualization purpose, those nodes with less than 1 percent of change in their
373 corresponding structural power ($\pi_{t,i}$) are shown with a small dot. A large orange
374 circle highlights the perturbed industry. Further, it is important to mention that the
375 nodes are arranged in a two dimensional space, based on their similarities in exports
376 related links. This means, closer nodes have similar export patterns.



377

378 **Fig 6. The effect of 99% slowdown of electrical and optical equipment industry**
 379 **of China**

380 Left side shows the shocked network in 1995 the right side shows 2011. The green
 381 (red) color declares an increase (decrease) in the final share (structural power, $\pi_{t,i}$) of
 382 the node as a result of the slow down in the selected industry.

383

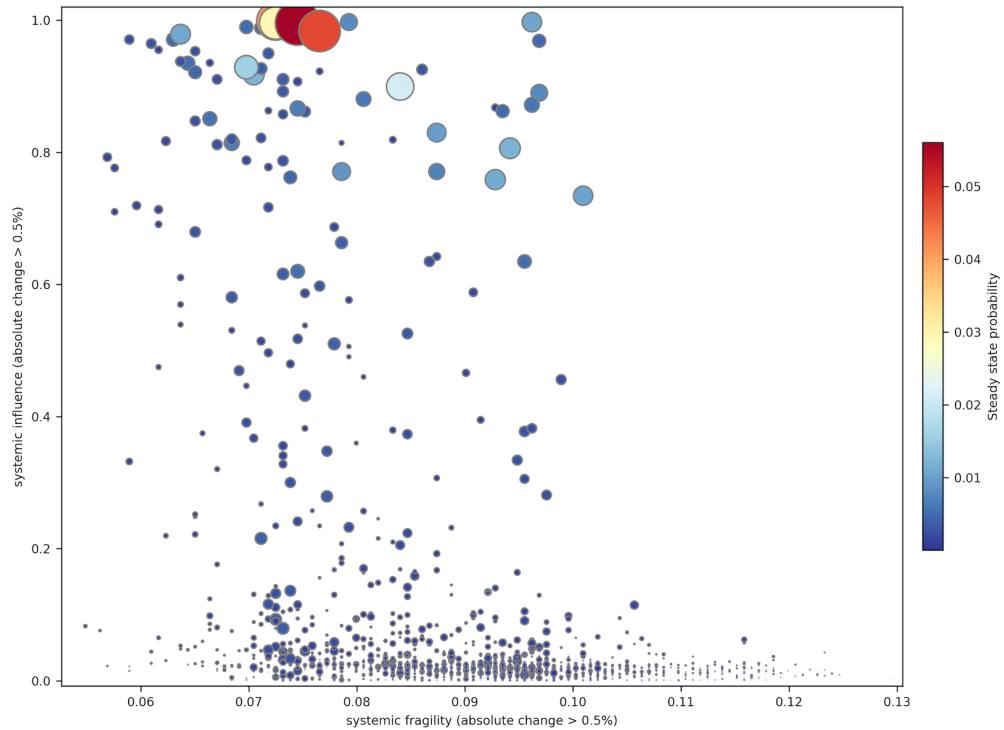
384

385 As we mentioned before, by changing α percent of the activity of node i , the total
 386 absolute amount of positive and negative changes (i.e. redistribution of values in new
 387 π_t) are equal to α percent of $\pi_{t,i}$. Therefore, the global effect of fluctuations in the
 388 activity of each node is the same as the change in its π_i .

389 Thus instead of focusing on the magnitude of changes, two new measures (*Systemic*
 390 *Influence* and *Systemic Fragility*) that were introduced in the previous section are
 391 based on the multitude of changes, happening as a result of a shock in the network.

392 In order to calculate these two measures for all the nodes in the network, we
 393 performed the perturbation procedure, which we described before for all the nodes

394 with $\alpha = -99$. Fig 7 shows the distribution of systemic fragility vs. systemic
 395 influence of each node as a result of 99% percent slowdown of each individual node
 396 for the year 2011. We should note that for the calculation of these two measures we
 397 only considered those absolute changes, which are more than 0.5% of the structural
 398 power of the node itself. The size and color of nodes correspond to their structural
 399 powers (i.e. π_i or Eigen Centralities). As it can be seen although all the nodes with
 400 high structural power have relatively high systemic influence, there are nodes with
 401 high systemic influence, but low structural power. For the case of systemic fragility
 402 there is even less correlations to structural power. While nodes with high structural
 403 power are relatively robust (i.e. low fragility), there is a very wide range of fragility
 404 values for those nodes with low structural power.



405
 406 **Fig 7. Systemic fragility vs. systemic influence of each industry for the year 2011**
 407

408 Table 1 shows the top 10 economic nodes (except Rest Of World) in 2011 with the
 409 highest systemic influences along with their structural power and systemic fragility.

410 **Table 1. Top 10 nodes with the highest systemic influence in 2011**

Rank	Names	Structural power	Systemic Fragility	Systemic Influence
1	China-Government	0.0407637	0.0724932	0.998645
2	Japan-Real Estate Activities	0.00434353	0.0718157	0.997967
3	Brazil-Government	0.0106923	0.096206	0.99729
4	India-Government	0.00788957	0.0792683	0.99729
5	USA-Government	0.0560428	0.0745257	0.995935
6	USA-Real Estate Activities	0.00796444	0.0738482	0.995935
7	Japan-Government	0.0294709	0.0724932	0.995935
8	USA-Retail Trade, Except of Motor Vehicles	0.0049287	0.0731707	0.995257
9	USA-Wholesale Trade and Commission Trade	0.00488901	0.0738482	0.99187
10	USA-Renting of M&Eq and Other Business Activities	0.0116091	0.0718157	0.99187

411
 412 Further, Table 2 shows the top 10 economic nodes in 2011 with the lowest systemic
 413 fragilities. Note that since we are interested in the nodes that can be attributed to a
 414 specific economy we removed those nodes, which were attributed to Rest of the
 415 World (ROW).

416 **Table 2. Top 10 nodes (except rest of the world) with the lowest systemic fragility**
 417 **in 2011**

Rank	Names	Structural power	Systemic Fragility	Systemic Influence
1	UK-Electrical and Optical Equipment	0.000292468	0.054878	0.0826558
2	Finland-Electrical and Optical Equipment	0.000137664	0.0562331	0.0758808
3	Taiwan-Manufacturing; Recycling	3.36E-05	0.0569106	0.0223577
4	Germany-Electrical and Optical Equipment	0.00154004	0.0575881	0.776423
5	Germany-Chemicals and Chemical Products	0.00110951	0.0575881	0.710027
6	Ireland-Machinery	1.32E-05	0.0589431	0.0149051
7	USA-Electrical and Optical Equipment	0.00232597	0.0589431	0.970867
8	Malta-Electrical and Optical Equipment	6.46E-06	0.0589431	0.0216802
9	Germany-Machinery	0.00183113	0.0596206	0.719512

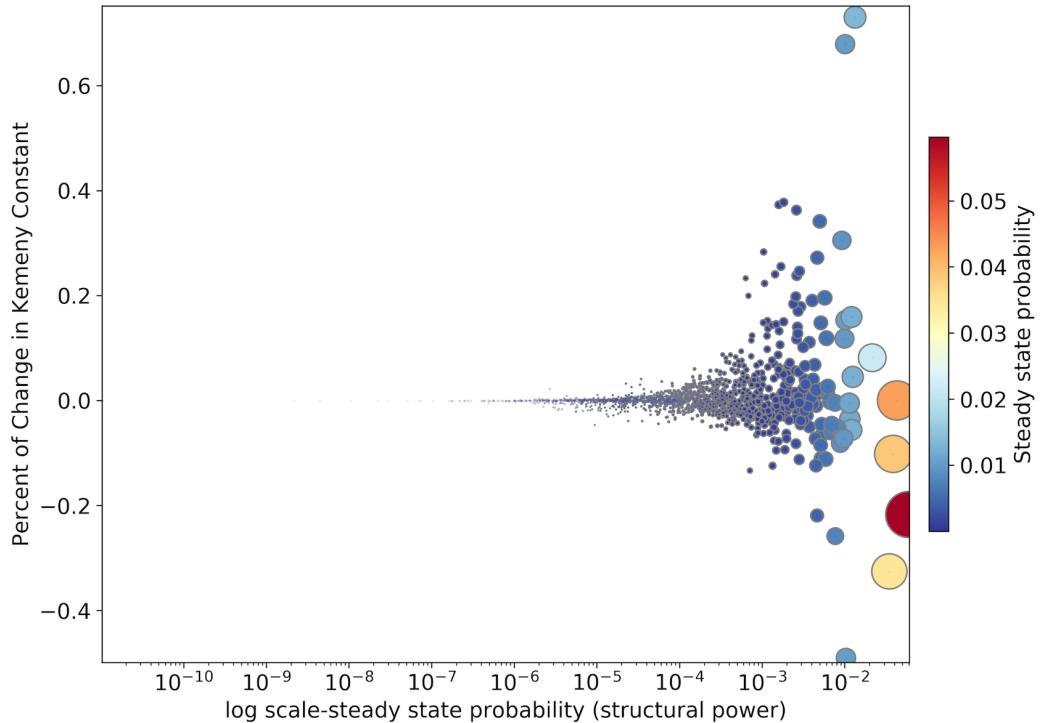
10	Denmark-Chemicals and Chemical Products	7.48E-05	0.0609756	0.0264228
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418

419 As another possible sensitivity analysis, we assessed the role of each individual
 420 economic node to the overall flow of the economic network. As we discussed before,
 421 Kemeny constant and mixing time are two global measures of a Markov chain, where
 422 the lower values show a more globally connected network and faster flow of money.
 423 In [25], the authors introduce a simple procedure to see the effect of removing each
 424 node on the average flow within a network. In the domain of urban traffic network
 425 analysis, this method has been used to analyze the effect of closing a road (or a
 426 junction) on the overall flow of the network, where the results are sometimes
 427 paradoxical. In [25, 26] it has been shown that by removing some nodes with high
 428 structural power (i.e. high level of expected share of traffic) the overall average flow
 429 (in terms of Kemeny constant) will be better. This phenomenon is known as Braess
 430 paradox [27]. This apparently paradoxical result implies that in order to improve the
 431 overall flow of a network, sometimes it is better not to add a new node, but to remove
 432 some.

433 We implemented this procedure to the economic network for all the years from 1995
 434 to 2011, where we calculated the percent of change in the Kemeny constant of the
 435 Markov chain by slowing down the activity of each node by 99%. We manipulated
 436 the transition matrix with the same procedure that we used for the calculation of
 437 systemic influence and systemic fragility. For the year 2011 Fig 8 shows the
 438 relationship between steady state probabilities (i.e. structural power of economic
 439 nodes) and the percent of change in Kemeny constant, caused by the slowdown in the
 440 activity of the node. As we expected, the slowdown of the majority of economic
 441 nodes, leads to higher Kemeny constants (i.e. a slower overall monetary flow across
 442 the network). On the other hand, there are some nodes that the slow downs of their

443 activity decrease the Kemeny constant (i.e. a faster overall monetary flow across the
444 network).



445

446 **Fig 8. Effect of slowing downs the activity of economic nodes on Kemeny**
447 **constant in 2011**

448

449

450 In addition, Table 3 shows top 10 economic nodes with the highest positive effects on
451 Kemeny constant in 2011 along with their corresponding structural power (and their
452 systemic influences as calculated before. Similar to previous tables, we removed those
453 nodes related to Rest of World in the following tables.

454

455 **Table 3. Top 10 nodes (except rest of the world) with the highest positive effects**
 456 **on Kemeny constant in 2011**

Rank	Names	Structural power	Systemic Influence	% of change in Kemeny constant
1	China-Electrical and Optical Equipment	0.0109195	0.978997	0.637486
2	Germany-Transport Equipment	0.00282169	0.811653	0.354232
3	China-Textiles and Textile Products	0.00498553	0.970867	0.2612
4	Germany-Machinery	0.00183113	0.719512	0.255455
5	Germany-Electrical and Optical Equipment	0.00154004	0.776423	0.230203
6	Germany-Chemicals and Chemical Products	0.00110951	0.710027	0.209375
7	Romania-Government	0.000723924	0.0765583	0.203782
8	Russia-Government	0.0104454	0.734417	0.202094
9	USA-Transport Equipment	0.00252037	0.910569	0.191263
10	USA-Coke, Refined Petroleum and Nuclear	0.00264798	0.469512	0.182051

457
 458 Further, Table 4 shows the top 10 economic nodes with the highest negative effects on
 459 Kemeny constant in 2011 along with their corresponding Eigen centralities and
 460 systemic influences.

461 **Table 4- Top 10 nodes (except rest of the world) with the highest negative effects**
 462 **on Kemeny constant in 2011**

Rank	Names	Structural power	Systemic Influence	% of change in Kemeny constant
1	Brazil-Government	0.0106923	0.99729	-0.448618
2	Japan-Government	0.0294709	0.995935	-0.28818
3	India-Government	0.00788957	0.99729	-0.275446
4	Mexico-Government	0.00473883	0.968835	-0.201355
5	USA-Government	0.0560428	0.995935	-0.201247
6	Greece-Government	0.000652282	0.0623306	-0.133354
7	Finland-Government	0.00127287	0.105014	-0.116016
8	Spain-Government	0.00610205	0.871951	-0.114455
9	China-Government	0.0407637	0.998645	-0.111065

10	Sweden-Government	0.0030026	0.377371	-0.110952
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463

464 Unlike Table 3, in Table 4 all the top nodes are related to governments, where by
 465 slowing down their activities, one can expect to have a better overall flow in the
 466 network (i.e. smaller Kemeny constant). In order to investigate if there are other nodes
 467 from different sectors than government that will show this paradoxical effect, Table 5
 468 shows the top 10 nodes without governments and rest of the world (ROW), whose
 469 economic slowdowns improve the overall flow of money (i.e. smaller Kemeny
 470 constant). While, comparing to values in Table 4, the percentages of changes in Table
 471 5 are much smaller, it is interesting to note that five out of 10 top nodes belong to the
 472 sector of real estate activities.

473

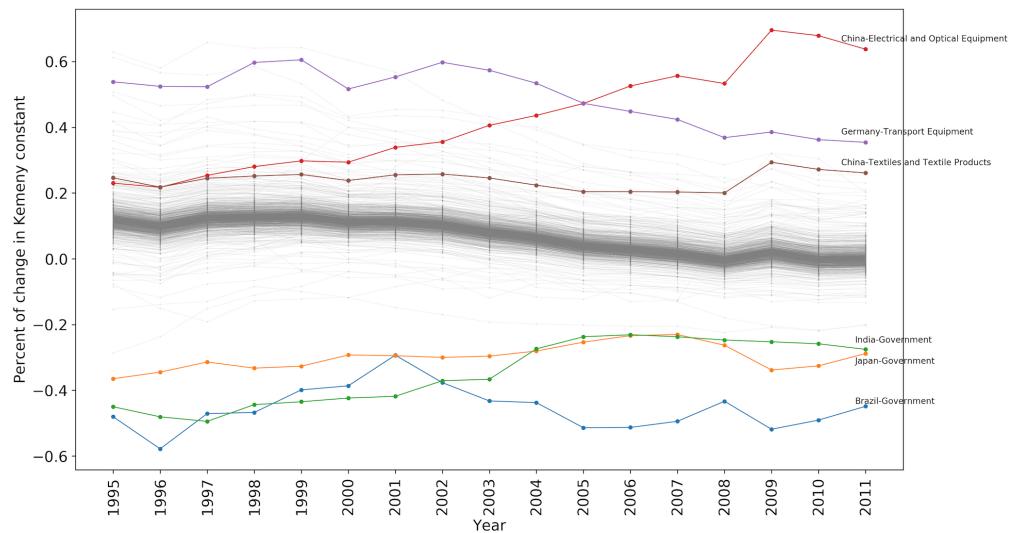
474 **Table 5 - Top 10 nodes (except governments and rest of the world) with the**
 475 **highest negative effects on Kemeny constant in 2011**

Rank	Names	Structural power	Systemic Influence	% of change in Kemeny constant
1	Japan-Real Estate Activities	0.00434353	0.997967	-0.0994393
2	Brazil-Public Admin and Defense; Compulsory So...	0.00153962	0.0264228	-0.0872356
3	India-Agriculture, Hunting, Forestry and Fishing	0.00157759	0.0528455	-0.0769576
4	USA-Real Estate Activities	0.00796444	0.995935	-0.0739146
5	Australia-Real Estate Activities	0.00132178	0.0121951	-0.0690959
6	France-Real Estate Activities	0.00190527	0.00880759	-0.0678286
7	Japan-Public Admin and Defense	0.00371188	0.910569	-0.0646992
8	Brazil-Real Estate Activities	0.000888012	0.0216802	-0.0593976
9	Japan-Renting of M&Eq and Other Business Activities	0.00458614	0.988482	-0.0576155
10	Sweden-Government	0.0030026	0.377371	-0.110952

476

477 The results shown here are based on world economic network in 2011. However, in
 478 the same way as the previous measures, it is possible to analyze the behavior of these

479 measures over time that we leave it to future research. Fig 9 shows that the slowdown
 480 of the activity of economic nodes has most of the times a very little positive effect on
 481 the Kemeny constant of the corresponding year. On the other hand there are few
 482 nodes whose changes have a big impact (either negative or positive) on the overall
 483 flow of the money in the global economic network during the years from 1995 to
 484 2011. It is also interesting to see how the influence of China's electrical and optical
 485 equipment industry has increased during the last decade, which is presumably because
 486 of expansion of information technology across the world.



487
 488 **Fig 9. The paradoxical effect of slowdown in the activity economic nodes (except**
 489 **rest of the world) on the Kemeny constants**

490 Discussions and Conclusions

491 Thanks to the recently available World Input Output Database (WIOD), in this work
 492 we modeled the evolution of world economic network from 1995 to 2011 by a series
 493 of finite state Markov chains. As a result, we were able to analyze different aspects of
 494 the underlying dynamical system, by analyzing different properties of the constructed
 495 Markov chains.

496 We showed that the ratio between the aggregated steady state probabilities of an
497 economy to its GDP-share could be considered as a measure of structural potential of
498 economies for further growth, where the values less than one show a decline in the
499 speed of growth (economic slowdown) and the values more than one show the
500 potential for faster economic growth. Further, we claimed that this ratio could be
501 considered as a risk measure, which is independent of the trend an economy has in
502 comparison to other economies. Therefore, there are economies gaining more
503 structural power with a risky path (i.e. lower structural power than the GDP share)
504 and vice versa.

505 In addition, via perturbation analysis of the underlying transition matrices we
506 introduced two measures of systemic risk, called *systemic influence* and *systemic*
507 *fragility*, which measure the effect of change in the activity of one node (i.e. an
508 industrial sector of an economy) on the structural power of all the other nodes in terms
509 of multitude rather than the magnitude. Further, we showed that the slow down of
510 activities in different nodes has both negative and positive results in terms of Kemeny
511 constant, which is a measure of connectivity of the network. This result, which is
512 paradoxical, needs further investigations.

513 Finally, we should mention that there are similarities between our work and two
514 recent works [15,16], where using a bi-partite economy-product network, they come
515 up with an interesting measure of economic fitness and product complexity. However,
516 we think our approach based on the Markov chain formalism on input-output tables
517 has more advantages. We will investigate these relations in our future research.

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578 **Supporting information**

579 All the data sets and codes used to produce the results of this work can be found at

580 https://sevamoo.github.io/Markovian_IO_SI_PLOSONE/