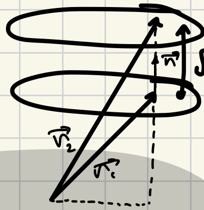
6.11 (3, 5, 9)

3)

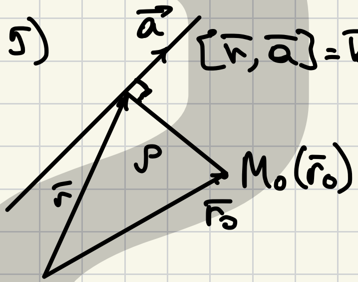


$(\bar r, \bar n) = D_1$,

$(\bar r, \bar n) = D_2$

$$\rho = \left| \left(\vec r, \frac{\bar n}{|\bar n|}\right) - \left(\bar r, \frac{\bar n}{|\bar n|}\right) \right| = \frac{|D_1 - D_2|}{|\bar n|}$$

5)



$[\bar r, \bar a] = \bar b$

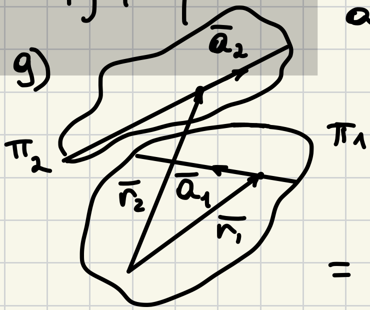
$$[\bar r_0 + \bar \rho, \bar a] = \bar b$$

$$[\bar \rho, \bar a] = \bar b - [\bar r_0, \bar a]$$

$$\bar \rho = \frac{[\bar a, \bar b - [\bar r_0, \bar a]]}{a^2} \Rightarrow$$

$$\Rightarrow |\bar \rho| = \left| \frac{[\bar a, \bar b - [\bar r_0, \bar a]]}{a^2} \right|$$

9)



$$\bar n = [\bar a_1, \bar a_2].$$

$$\rho = \frac{|(\bar r_1, \bar n) - (\bar r_2, \bar n)|}{|\bar n|} = \frac{|(\bar r_1 - \bar r_2, \bar n)|}{|\bar n|} =$$

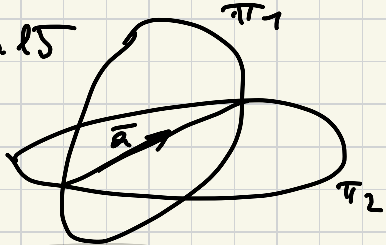
$$= \frac{\left|\left(\frac{[\bar a_1, \bar b_1]}{a_1^2} - \frac{[\bar a_2, \bar b_2]}{a_2^2}, \bar n\right)\right|}{|\bar n|} = \frac{1}{n} \left| \frac{1}{a_1^2}\left([\bar a_1, \bar b_1], [\bar a_1, \bar a_2]\right) - \right.$$

$$\left. - \frac{1}{a_2^2}\left([\bar a_2, \bar b_2], [\bar a_1, \bar a_2]\right) \right| = \frac{1}{n} \left| \frac{1}{a_1^2}\left(a_2, [\bar a_1, [\bar a_1, \bar b_1]]\right) - \right.$$

$$\left. - \frac{1}{a_2^2}\left(\bar a_1, [\bar a_2, [\bar a_2, \bar b_2]]\right) \right| = \frac{1}{n} \left| \frac{1}{a_1^2}\left(\bar a_2, -\bar b_1 a_1^2\right) - \frac{1}{a_2^2}\left(\bar a_1, \right.\right.$$

$$\left.\left. - \bar b_2 a_2^2\right) \right| = \frac{1}{n} \left| (\bar a_2, \bar b_1) + (\bar a_1, \bar b_2) \right| = \frac{|(\bar a_2, \bar b_1) + (\bar a_1, \bar b_2)|}{|[\bar a_1, \bar a_2]|}$$

6.15



$A_i x + B_i y + C_i z + D_i = 0$, $i = 1, 2$

$\bar n_i = \begin{Vmatrix} A_i \\ B_i \\ C_i \end{Vmatrix}$ – соответствующ. вектора

$\vec a \parallel [\bar n_1, \bar n_2]$. Пусть $n_i = \begin{Vmatrix} A_i \\ B_i \\ C_i \end{Vmatrix}_{e^*}$

От замены $\bar e \to \bar e^*$ вектор $[\bar n_1, \bar n_2]$ не поменяется.

$$[n_1, n_2] = \begin{vmatrix} [\bar e_2^*, \bar e_3^*] & [\bar e_3^*, \bar e_1^*] & [\bar e_1^*, \bar e_2^*] \\ A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \end{vmatrix} =$$

$$= \begin{vmatrix} \frac{[\bar e_2^*, \bar e_3^*]}{(\bar e_1^*, \bar e_2^*, \bar e_3^*)} & \frac{[\bar e_3^*, \bar e_1^*]}{(\bar e_1^*, \bar e_2^*, \bar e_3^*)} & \frac{[\bar e_1^*, \bar e_2^*]}{(\bar e_1^*, \bar e_2^*, \bar e_3^*)} \\ A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \end{vmatrix} (\bar e_1^*, \bar e_2^*, \bar e_3^*) =$$

$$= \begin{vmatrix} \bar e_1 & \bar e_2 & \bar e_3 \\ A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \end{vmatrix} (e_1^*, e_2^*, e_3^*)$$ – коллинеарен

$$\begin{vmatrix} \bar e_1 & \bar e_2 & \bar e_3 \\ A_1 & B_1 & C_1 \\ A_2 & B_2 & C_3 \end{vmatrix}$$ $\Rightarrow$ $\bar a$ можно так находить $\boxtimes$

№6.18(1)

1) $A(1, 3, 1)$

$$\begin{cases} x+y-z+2=0, & \bar n_1 = \left\| \begin{matrix} 1 \\ 1 \\ -1 \end{matrix} \right\|_e \\ 2x+3y+z=0; & \bar n_2 = \left\| \begin{matrix} 2 \\ 3 \\ 1 \end{matrix} \right\|_e \end{cases}$$

– сопутствующий (не обязательно нормальный) / $\bar a \parallel [\bar n_1, \bar n_2] \parallel$

$$\left\| \begin{vmatrix} \bar e_1 & \bar e_2 & \bar e_3 \\ 1 & 1 & -1 \\ 2 & 3 & 1 \end{vmatrix} \right\| \Rightarrow \bar a \begin{pmatrix} 4 \\ 3 \\ 1 \end{pmatrix}_e \Rightarrow \text{иск. } \frac{x-1}{4} = \frac{y-3}{3} = \frac{z-1}{1}$$

6.21(2)

2) $x+y-2z+1=0$ и $6z-3x-3y-3=0$

$\Leftrightarrow -2z+x+y+1=0$

$\Rightarrow$ совпадают.

6.68(2)

2) $x=4+t,\ y=1-t,\ z=5+4t$ и $x=-3-3t,\ y=8+3t,\ z=1$

$$\bar a_1 = \begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix},\ \bar a_2 = \begin{pmatrix} -3 \\ 3 \\ 0 \end{pmatrix} \Rightarrow \bar a_1 \nparallel \bar a_2$$

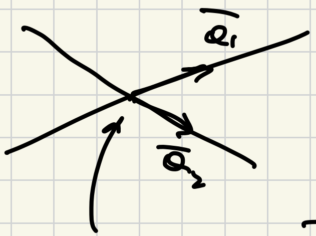
$$\bar r_1 = \begin{pmatrix} 4 \\ 1 \\ 5 \end{pmatrix},\ \bar r_2 = \begin{pmatrix} -3 \\ 8 \\ 1 \end{pmatrix}$$

скрещ. $\Leftrightarrow (\bar r_2 - \bar r_1, \bar a_1, \bar a_2) = 0 = \begin{vmatrix} 7 & -7 & 4 \\ 1 & -1 & 4 \\ -3 & 3 & 0 \end{vmatrix} = \begin{vmatrix} 7 & -7 & 4 \\ 7 & -7 & 4 \\ 6 & -6 & 0 \end{vmatrix} = 0 \Rightarrow$

$\Rightarrow$ пересекаются.

$5+4t_1 = 1 = z_0 \Rightarrow t_1 = -1 \Rightarrow -3-3t_2 = 4-1 \Rightarrow$

$\Rightarrow t_2 = -2 \Rightarrow (x_0, y_0, z_0) = (3, 2, 1)$



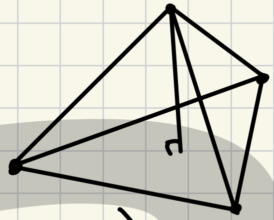
(x_0, y_0, z_0)

$|\bar a_1| = |\bar a_2| = \sqrt{18} \Rightarrow$ бис-сы:

$$\bar r = \bar r_0 + (\bar a_1 \pm \bar a_2)t = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \mp 3 \\ -1 \pm 3 \\ 4 \pm 0 \end{pmatrix} t \Rightarrow$$

$$\Rightarrow \text{иск.: } \bar r = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} + \begin{pmatrix} -2 \\ 2 \\ 4 \end{pmatrix} t;\ \bar r = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} + \begin{pmatrix} 4 \\ -4 \\ 4 \end{pmatrix} t.$$

6.74



$D(2,1,-4)$

$C(-1,-3,5)$

$A(-1,-3,1)$ $B(5,3,8)$

1) $(ABC): (\overline{AX},\overline{AB},\overline{AC})=0$

$$\begin{vmatrix} x+1 & y+3 & z-1\\ 6 & 6 & 7\\ 0 & 0 & 4\end{vmatrix} = 4(6(x+1)-6(y+3))=0$$

$$x-y-2=0 \Rightarrow \rho(D,(ABC)) = \frac{|2-1-2|}{\sqrt{1+1}} = \frac{\sqrt2}{2}$$

2) $\rho(C,(AB)) = \dfrac{|[\overline{AC},\overline{AB}]|}{|\overline{AB}|} = \dfrac{\left\|\begin{matrix} i & j & k\\ 0 & 0 & 4\\ 6 & 6 & 7\end{matrix}\right\|}{\sqrt{6^2+6^2+7^2}} = \dfrac{\sqrt{24^2+24^2}}{\sqrt{121}} = \dfrac{24}{11}\sqrt2$

3) $\rho = \dfrac{(\overline{DC},\overline{AD},\overline{BC})}{|[\overline{AD},\overline{BC}]|} = \dfrac{1}{\left|\left\|\begin{matrix} i & j & k\\ 3 & 4 & -5\\ -6 & -6 & -3\end{matrix}\right\|\right|}\left|\begin{matrix} -3 & -4 & 9\\ 3 & 4 & -5\\ -6 & -6 & -3\end{matrix}\right| = \dfrac{24}{\sqrt{42^2+38^2+6^2}} = \dfrac{8\sqrt{41}}{123}$

4) $\cos\angle(\overline{AD},\overline{BC}) = \left|\dfrac{(\overline{AD},\overline{BC})}{|\overline{AD}||\overline{BC}|}\right| = \left|\dfrac{-18-24+15}{\sqrt{3^2+4^2+5^2}\sqrt{6^2+6^2+3^2}}\right| =$

$$= \frac{27}{\sqrt{50}\ \sqrt{81}} = \frac{3\sqrt2}{10}$$

5) $\cos\angle(\overline{AD},(ABC)) = |\sin\angle(\overline{AD},\overline{n}_{ABC})| = \sqrt{1-\cos^2\angle(\overline{AD},\overline{n}_{ABC})} =$

$$= \sqrt{1-\left(\frac{(\overline{AD},\overline{n}_{ABC})}{|\overline{AD}||\overline{n}_{ABC}|}\right)^2} = \sqrt{1-\left(\frac{3-4}{\sqrt{50}\sqrt2}\right)^2} = \sqrt{\frac{99}{100}} \Rightarrow$$

$$\Rightarrow \sin\angle(\overline{AD},(ABC)) = \frac{1}{10} \Rightarrow \angle(\overline{AD},(ABC)) = \arcsin\frac{1}{10}$$