№4 (2,4)

2) $2i\left(\frac{1}{2}+\frac{\sqrt3}{2}i\right)\left(-\frac{1}{2}+\frac{\sqrt3}{2}i\right)=2i(-z\bar z)=-2i|z|^2=-2i,\ z=\frac12+\frac{\sqrt3}{2}i,\ |z|=1$

4) $\frac{13+12i}{6i-8}+\frac{(1+2i)^2}{2+i}=\frac{(13+12i)(-8-6i)}{64+36}+\frac{(1+2i)^2(2-i)}{4+1}=\frac{-104-78i-96i+72}{100}+$

$+\frac{(1+4i-4)(2-i)}{5}=\frac{1}{100}\left(-32-174i+20(2-i+8i+4-8+4i)\right)=\frac{1}{100}\cdot$

$\cdot\left(-32-174i+40-20i+160i-80+80i\right)=\frac{1}{100}(-72+46i)=\frac{-18}{25}+\frac{23}{50}i$

№13 (3) $z=-5-2\sqrt6 i$ $|z|^2=5^2+(2\sqrt6)^2=25+24=49 \Rightarrow |z|=7$

№15 (2,3)

2) $|z-i|<|z+i| \Leftrightarrow |z-i|^2<|z+i|^2$

$z=x+iy:\ x^2+(y-1)^2<x^2+(y+1)^2 \Leftrightarrow 0<4y$ или $y>0$

искомое – полуплоскость $y>0$

3) $|z+2i-1|\le 2 \Leftrightarrow |z+2i-1|^2\le 4$

$z=x+iy:\ (x-1)^2+(y+2)^2\le 4$ – круг с центром $1-2i$ радиусом 2.

№18 (5)

$z=\sin(\pi/9)-i\cos(\pi/9)=\cos\left(\frac{\pi}{9}-\frac{\pi}{2}\right)+i\sin\left(\frac{\pi}{9}-\frac{\pi}{2}\right)\Rightarrow \arg z=-\frac{7\pi}{18}+2\pi=\frac{29}{18}\pi$

№30 (5)

$z=\frac{(1+i)^9}{(1-i)^7}=\frac{\sqrt2^9}{\sqrt2^7}e^{i\left(\frac{9\pi}{4}-\left(-\frac{7\pi}{4}\right)\right)}=2e^{i4\pi}=2$

№31 (1)

$z=(\sqrt3-i)^{100}=2^{100}\cos\frac{-100\pi}{6}+2^{100}i\sin\frac{-100\pi}{6}=2^{100}\cos\frac{4\pi}{3}+2^{100}i\sin\frac{4\pi}{3}$

№32 (4,7)

4) $z^8=1+i=\sqrt2\exp\left(i\left(\frac{\pi}{4}+2\pi k\right)\right)\Rightarrow z=2^{1/16}\exp\left(i\left(\frac{\pi}{32}+\frac{\pi k}{4}\right)\right)$, $k=0,\dots 7$

7) $z^6=-64=64e^{i\pi(1+2k)} \Rightarrow z=2\exp\left(\frac{1}{6}i\pi(1+2k)\right),\ k=0,\dots 5$

$z_1=\sqrt{3}+i,\ z_2=2i,\ z_3=-\sqrt{3}+i,\ z_4=-\sqrt{3}-i,\ z_5=-2i,\ z_6=\sqrt{3}-i$

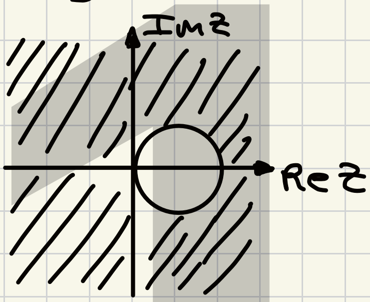
T.1. $z=x+iy$

$$Re\left(\frac{(2-i)(x-iy)}{x^2+y^2}-\frac{(1-2i)(x+iy)}{x^2+y^2}\right)-Im\left(\frac{(2+i)(x-iy)}{x^2+y^2}-\frac{(1+2i)(x+iy)}{x^2+y^2}\right)\le 2$$

$$2x-y-x-2y-(-2y+x-y-2x)\le 2(x^2+y^2)$$

$$x\le x^2+y^2$$

$$\left(x-\frac{1}{2}\right)^2+y^2\ge\frac{1}{4}$$




№3(2,4)

2) $\int\frac{(x^2+2)dx}{(x-1)(x+1)^2} =$

$$\frac{(x^2+2)}{(x-1)(x+1)^2}=\frac{A}{x-1}+\frac{Bx+C}{(x+1)^2} \Rightarrow x^2+2=Ax^2+2Ax+A+Bx^2+Cx-Bx-C$$

$$A+B=1;\ 2A+C-B=0;\ A-C=2 \Rightarrow A=\frac{3}{4},\ B=\frac{1}{4},\ C=-\frac{5}{4}$$

$$= \frac{3}{4}\int\frac{dx}{x-1}+\frac{1}{4}\int\frac{x-5}{(x+1)^2}dx=\frac{3}{4}\ln|x-1|+\frac{1}{8}\ln|x+1|^2-\frac{3}{4}(-2)(x+1)^{-1}+C=$$

$$=\frac{1}{4}\ln|(x+1)(x-1)^3|+\frac{3}{2}(x+1)^{-1}+C$$

4) $\int\frac{x^2+1}{x(x-1)^3}dx =$

$$\frac{x^2+1}{x(x-1)^3}=\frac{A}{x}+\frac{Bx^2+Cx+D}{(x-1)^3}\Rightarrow x^2+1=A(x-1)^3+Bx^3+Cx^2+Dx=$$

$$=(A+B)x^3+(-3A+C)x^2+(3A+D)x-A$$

$$A+B=0,\ -3A+C=1,\ 3A+D=0,\ -A=1 \Rightarrow A=-1,\ B=1,\ C=-2,\ D=3$$

$$= -\int\frac{dx}{x}+\int\frac{x^2-2x+3}{(x-1)^3}dx=-\int\frac{dx}{x}+\int\frac{dx}{x-1}+\int\frac{2dx}{(x-1)^3}=-\ln|x|+\ln|x-1|-$$

$$-(x-1)^{-2}+C=-(x-1)^{-2}+\ln\left|\frac{x-1}{x}\right|+C$$

№4(2,5)

2) $\int \frac{dx}{x^3+1} =$

$$\frac{1}{x^3+1} = \frac{A}{x+1} + \frac{Bx+C}{x^2-x+1} \Rightarrow 1 = A(x^2-x+1)+Bx^2+Bx+Cx+C =$$

$$= (A+B)x^2 + (B+C-A)x + A+C \Rightarrow A+B=0,\ B+C-A=0,\ A+C=1$$

$$C=1-A \quad B+1-2A=0 \Rightarrow A=\frac{1}{3},\ B=-\frac{1}{3},\ C=\frac{2}{3}$$

$$= \frac{1}{3}\int\frac{dx}{x+1} + \frac{1}{3}\int\frac{-x+2}{x^2-x+1}dx = \frac{1}{3}\ln|x+1| + \frac{1}{3}\Big[-\frac{1}{2}\int\frac{2x-1}{x^2-x+1}dx +$$

$$+\frac{1}{2}\int\frac{dx}{x^2-x+1}\Big] = \frac{1}{3}\ln|x+1| - \frac{1}{6}\ln|x^2-x+1| + \frac{1}{2}\int\frac{dx}{(x-\frac{1}{2})^2+3/4} =$$

$$= \frac{1}{6}\ln\frac{x^2+2x+1}{x^2-x+1} + \frac{1}{\sqrt{3}}\,\mathrm{arctg}\,\frac{2x-1}{\sqrt{3}}+C$$

5) $\int\frac{x^3+x^2+x+3}{(x+3)(x^2+x+1)}dx =$

$$\frac{x^3+x^2+x+3}{x^3+4x^2+4x+3} = 1-\frac{3x^2+3x}{(x+3)(x^2+x+1)} = 1-\Big[\frac{A}{x+3}+\frac{Bx+C}{x^2+x+1}\Big]$$

$$A(x^2+x+1)+(Bx+C)(x+3) = 3x^2+3x$$

$$(A+B)x^2+(A+3B+C)x+A+3C = 3x^2+3x \Rightarrow A+B=3,\ A+3B+C=3,\ A+$$

$$+3C=0 \Rightarrow A=18/7,\ B=3/7,\ C=-6/7$$

$$= \int dx - \frac{18}{7}\int\frac{dx}{x+3} - \frac{1}{7}\int\frac{3x-6}{x^2+x+1}dx = x-\frac{18}{7}\ln|x+3| - \frac{1}{7}\Big[\frac{3}{2}\int\frac{2x+1}{x^2+x+1}dx -$$

$$-\frac{15}{2}\int\frac{dx}{(x+\frac{1}{2})^2+\frac{3}{4}}\Big] = x-\frac{18}{7}\ln|x+3| - \frac{3}{14}\ln|2x+1| - \frac{5\sqrt{3}}{7}\,\mathrm{arctg}\,\frac{2x+1}{\sqrt{3}}+C$$

№1(4)

$$\int\frac{\sqrt{x-1}-\sqrt{x+1}}{\sqrt{x-1}+\sqrt{x+1}}dx = -\frac{1}{2}\int(x-1-2\sqrt{x^2-1}+x+1)\,dx = \int(\sqrt{x^2-1}-x)\,dx =$$

$$= -\frac{1}{2}x^2+\int\frac{x^2-1}{\sqrt{x^2-1}}dx =$$

$$\int\frac{x^2-1}{\sqrt{x^2-1}}dx = Q(x)\sqrt{x^2-1}+\lambda\int\frac{dx}{\sqrt{x^2-1}} \ \Big|\ \frac{d}{dx},\quad Q(x)=Ax+B$$

$$\sqrt{x^2-1} = A\sqrt{x^2-1} + (Ax+B)\frac{x}{\sqrt{x^2-1}} + \frac{\lambda}{\sqrt{x^2-1}} \ \Big|\cdot\sqrt{x^2-1}$$

$$x^2-1 = A(x^2-1) + Ax^2 + Bx + \lambda = 2Ax^2 + Bx - A + \lambda \Rightarrow A=\frac{1}{2},\ B=0,\ \lambda=-\frac{1}{2}$$

$$= -\frac{1}{2}x^2 + \frac{1}{2}x\sqrt{x^2-1} - \frac{1}{2}\int\frac{dx}{\sqrt{x^2-1}} = \frac{x}{2}\left(\sqrt{x^2-1}-x\right) - \frac{1}{2}\ln\left|x+\sqrt{x^2-1}\right| + C$$

№2(7)

$$\int\frac{dx}{\sqrt[6]{(x-7)^7(x-5)^5}} = \int\sqrt[6]{\frac{x-5}{x-7}}\,\frac{dx}{(x-7)(x-5)} =$$

$$\frac{x-5}{x-7} = t^6 \Rightarrow 1+\frac{2}{x-7} = t^6 \Rightarrow x = 7+\frac{2}{t^6-1} \Rightarrow dx = \frac{-12t^5}{(t^6-1)^2}dt$$

$$\frac{1}{x-7} = \frac{t^6-1}{2};\quad \frac{1}{x-5} = \frac{t^6-1}{2t^6}$$

$$= \int t\,\frac{(t^6-1)^2}{4t^6}\,\frac{-12t^5}{(t^6-1)^2}\,dt = \int -3\,dt = -3t + C = -3\sqrt[6]{\frac{x-5}{x-7}} + C$$

№18(2)

$$\int\sqrt[3]{1+\sqrt[4]{x}}\,dx = \int(1+x^{1/4})^{1/3}dx$$

$$1+x^{1/4} = t^3 \Rightarrow x = (t^3-1)^4 \Rightarrow dx = 12t^2(t^3-1)^3dt$$

$$= \int 12t^3(t^3-1)^3dt = \int 12t^3(t^9-3t^6+3t^3-1)\,dt = \frac{12}{13}t^{13} -$$

$$-\frac{18}{5}t^{10} + \frac{36}{7}t^7 - 3t^4 + C = \frac{12}{13}(1+x^{1/4})^{\frac{13}{3}} - \frac{18}{5}(1+x^{1/4})^{\frac{10}{3}} + \frac{36}{7}(1+x^{1/4})^{\frac{7}{3}} -$$

$$-3(1+x^{1/4})^{\frac{4}{3}} + C$$

№18(3)

$$\int\frac{\sqrt[3]{1+\sqrt[4]{x}}}{\sqrt{x}}dx = \int x^{-1/2}(1+x^{1/4})^{1/3}dx\ \Big|_{t^3=1+x^{1/4}} = \int(t^3-1)^{-2}t\cdot 12t^2(t^3-1)^3dt =$$

$$= \int 12t^3(t^3-1)\,dt = \frac{12}{7}t^7 - \frac{12}{4}t^4 + C = \frac{12}{7}(1+x^{1/4})^{\frac{7}{3}} - 3(1+x^{1/4})^{\frac{4}{3}} + C$$