

$$17.1(3) \quad \begin{cases} 2x_1 + x_2 - x_3 = 2 \\ 3x_1 + x_2 - 2x_3 = 3 \\ x_1 + x_3 = 3 \end{cases} \quad \left\| \begin{array}{ccc|c} 2 & 1 & -1 & 2 \\ 3 & 1 & -2 & 3 \\ 1 & 0 & 1 & 3 \end{array} \right\| - \text{расш. матрица}$$

$$\Delta = \begin{vmatrix} 2 & 1 & -1 \\ 3 & 1 & -2 \\ 1 & 0 & 1 \end{vmatrix} = 2 - 5 + 1 = -2 \quad \Delta_1 = \begin{vmatrix} 2 & 1 & -1 \\ 3 & 1 & -2 \\ 3 & 0 & 1 \end{vmatrix} = 2 - 9 + 3 = -4$$

$$\Delta_2 = \begin{vmatrix} 2 & 2 & -1 \\ 3 & 3 & -2 \\ 1 & 3 & 1 \end{vmatrix} = 18 - 10 - 6 = 2 \quad \Delta_3 = \begin{vmatrix} 2 & 1 & 2 \\ 3 & 1 & 3 \\ 1 & 0 & 3 \end{vmatrix} = 6 - 6 - 2 = -2$$

$$x_i = \frac{\Delta_i}{\Delta} \Rightarrow \text{исковое} - (x_1, x_2, x_3) = (2, -1, 1)$$

18.1(2,10)

$$2) x_1 - x_2 + 2x_3 = 0 \quad \|1 \ -1 \ 2\| - \text{матрица коэффициентов}$$

$$\Phi = \left\| \begin{array}{cc} 1 & -2 \\ 1 & 0 \\ 0 & 1 \end{array} \right\| \Rightarrow \|x_1 \ x_2 \ x_3\|^T = \left\| \begin{array}{cc} 1 & -2 \\ 1 & 0 \\ 0 & 1 \end{array} \right\| \left\| \begin{array}{c} c_1 \\ c_2 \end{array} \right\|, c_1, c_2 \in \mathbb{R}$$

$$10) \begin{cases} x_1 + x_2 + x_3 - x_4 = 0 \\ 3x_1 + 2x_2 + x_3 - x_5 = 0 \end{cases} \quad \left\| \begin{array}{ccccc} 1 & 1 & 1 & -1 & 0 \\ 3 & 2 & 1 & 0 & -1 \end{array} \right\| \sim \left\| \begin{array}{ccccc} 1 & 0 & -1 & -1 & -1 \\ 0 & 1 & -3 & -2 & -1 \end{array} \right\|$$

1 2 3 4 5 4 5 1 2 3

$$\tilde{\Phi} = \left\| \begin{array}{ccc} 1 & 1 & 1 \\ 3 & 2 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \right\| \begin{array}{c} 4 \\ 5 \\ 4 \\ 2 \\ 3 \end{array} \quad \Phi = \left\| \begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \\ 3 & 2 & 1 \end{array} \right\| \Rightarrow \vec{x} = \left\| \begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \\ 3 & 2 & 1 \end{array} \right\| \left\| \begin{array}{c} c_1 \\ c_2 \\ c_3 \end{array} \right\|$$

19.6(4,21,23)

$$4) \left\| \begin{array}{ccc|c} 0 & 1 & 2 & 0 \\ -1 & 0 & -2 & 1 \\ -2 & 2 & 0 & 2 \end{array} \right\| \xrightarrow{\text{II} \leftrightarrow \text{I}} \left\| \begin{array}{ccc|c} 1 & 0 & 2 & -1 \\ 0 & 1 & 2 & 0 \\ 0 & 2 & 4 & 0 \end{array} \right\| \sim \left\| \begin{array}{ccc|c} 1 & 0 & 2 & -1 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right\| \Rightarrow \vec{x} = \begin{pmatrix} -2 \\ -2 \\ 1 \end{pmatrix} c + \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}, c \in \mathbb{R}$$

$$21) \left\| \begin{array}{cccc|c} 2 & -1 & 2 & 0 & 1 \\ 1 & 1 & 7 & -3 & 2 \\ -1 & 1 & 1 & -1 & 0 \end{array} \right\| \sim \begin{array}{l} \text{I} + \text{II} \\ \text{II} + \text{III} \\ 2\text{III} + \text{I} \end{array} \left\| \begin{array}{cccc|c} 3 & 0 & 9 & -3 & 3 \\ 0 & 2 & 8 & -4 & 2 \\ 0 & 1 & 4 & -2 & 1 \end{array} \right\| \xrightarrow{\frac{\text{I}}{3}} \left\| \begin{array}{cccc|c} 1 & 0 & 3 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 4 & -2 & 1 \end{array} \right\| \sim$$

$$\sim \left\| \begin{array}{cccc|c} 1 & 0 & 3 & -1 & 1 \\ 0 & 1 & 4 & -2 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right\| \Rightarrow \Phi = \left\| \begin{array}{cc} -2 & 1 \\ -4 & 2 \\ 0 & 0 \end{array} \right\| \Rightarrow \vec{x} = \begin{pmatrix} -3 \\ -4 \\ 0 \end{pmatrix} \left\| \begin{array}{c} c_1 \\ c_2 \end{array} \right\| + \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, c_1, c_2 \in \mathbb{R}$$

$$23) \left\| \begin{array}{cccc|c} 3 & 1 & -2 & 4 & 1 \\ 2 & -3 & 6 & -5 & 2 \\ 8 & -1 & 2 & 3 & -2 \end{array} \right\| \sim \begin{array}{l} 3I+II \\ 3III-II \end{array} \left\| \begin{array}{cccc|c} 11 & 0 & 0 & 7 & 5 \\ 2 & -3 & 6 & -5 & 2 \\ 22 & 0 & 0 & 14 & -8 \end{array} \right\|$$

I и II л.з. в A и л.к.з. в $\tilde{A} \Rightarrow$ по Т. Кронекера Канони несовместна

19.7(2)

$$\left\| \begin{array}{ccc|c} -2 & 4 & -3 & \lambda \\ 4 & 4 & 7 & 5 \\ 1 & 4 & 2 & 7 \end{array} \right\| \sim I-2III \left\| \begin{array}{ccc|c} -4 & -4 & -7 & \lambda-14 \\ 4 & 4 & 7 & 5 \\ 1 & 4 & 2 & 7 \end{array} \right\| \sim I+II \left\| \begin{array}{ccc|c} 0 & 0 & 0 & \lambda-9 \\ 4 & 4 & 7 & 5 \\ 1 & 4 & 2 & 7 \end{array} \right\|$$

$\text{rg } A = \text{rg } \tilde{A}$, т.е. при $\lambda=9$ совместна

$$\left\| \begin{array}{ccc|c} 4 & 4 & 7 & 5 \\ 1 & 4 & 2 & 7 \\ 0 & 0 & 0 & 0 \end{array} \right\| \sim I-II \left\| \begin{array}{ccc|c} 3 & 0 & 5 & -2 \\ 0 & 12 & 1 & 23 \\ 0 & 0 & 0 & 0 \end{array} \right\| \sim 4I \left\| \begin{array}{ccc|c} 12 & 0 & 20 & -8 \\ 0 & 12 & 1 & 23 \\ 0 & 0 & 0 & 0 \end{array} \right\| \Rightarrow$$

$$\Rightarrow \Phi = \left\| \begin{array}{c} -20 \\ -1 \\ 12 \end{array} \right\| \Rightarrow \vec{x} = \left\| \begin{array}{c} -20 \\ -1 \\ 12 \end{array} \right\| c + \left\| \begin{array}{c} -8 \\ 23 \\ 0 \end{array} \right\|$$

19.10

$\|\vec{a}_1 \dots \vec{a}_n | \vec{a}_1 + \dots + \vec{a}_n\|$. Очев., что $(1, \dots, 1)$ - решение

19.13

$$A \vec{x} = \vec{b} \quad \text{rg } A = n \Rightarrow \text{rg } \Phi = n - \text{rg } A = 0 \Rightarrow \Phi = 0 \Rightarrow$$

\Rightarrow если система совместна обяз. решение $X = \Phi \vec{h} + X_0 = X_0$, т.е. одно решение, т.е. у системы не более 1 решения

19.14

$\text{rg } A = m \nmid \vec{b} \Rightarrow \text{rg } A = \text{rg } \tilde{A} \nmid \vec{b} \Rightarrow$ по Т. Кронекера Канони совместна

18.17(2)

$$\Phi = \left\| \begin{array}{c} 10 \\ 1-1 \\ 01 \\ -4-1 \end{array} \right\|$$

1 Способ

$$\Phi^T = \left\| \begin{array}{cccc} 1 & 1 & 0 & -4 \\ 0 & -1 & 1 & -1 \end{array} \right\| \sim \left\| \begin{array}{cccc} 1 & 0 & 1 & -3 \\ 0 & 1 & -1 & 1 \end{array} \right\| \Rightarrow \Phi' = \left\| \begin{array}{c} -13 \\ 1-1 \\ 101 \end{array} \right\| \cdot A^T = \Phi' \Rightarrow$$

$$= A = \left\| \begin{array}{ccc} -1 & 1 & 0 \\ 3 & -1 & 0 \end{array} \right\|$$

2 Способ

$$\left\| \begin{array}{cc|c} 1 & 0 & x_1 \\ 1 & -1 & x_2 \\ 0 & 1 & x_3 \\ -4 & -1 & x_4 \end{array} \right\| \sim \left\| \begin{array}{cc|c} 0 & 0 & 5x_1 - x_2 + x_4 \\ 0 & 0 & x_2 - x_1 + x_3 \\ 0 & 1 & x_3 \\ -4 & 1 & x_4 \end{array} \right\| \Rightarrow \text{см. } \begin{cases} 5x_1 - x_2 + x_4 = 0 \\ -x_1 + x_2 + x_3 = 0 \end{cases}$$