25.2(1)

$(\vec{x},\vec{y}) = x_1 y_1 + 2x_2 y_2$, а) $n=2$; б) $n=3$

а) 1) $(\vec{x},\vec{y}) = (\vec{y},\vec{x})$

2) $(\alpha\vec{x}'+\beta\vec{x}'',\vec{y}) = (\alpha x_1'+\beta x_1'')y_1 + 2(\alpha x_2'+\beta x_2'')y_2 = \alpha(x_1'+2x_2'y_2)+$

$+\beta(x_1''y_1+2x_2''y_2) = \alpha(\vec{x}',\vec{y})+\beta(\vec{x}'',\vec{y})$

3) $(\vec{x},\vec{x}) = x_1^2+2x_2^2 > 0 \quad \forall \vec{x}\neq\vec{0}$

Т.е. в $n=2$ может

б) $\exists\, \vec{x} = \|0\ 0\ 1\|^T \neq \vec{0}:\ (\vec{x},\vec{x}) = 0 \Rightarrow$ не может

25.7.

$(f,g) = \int_{-1}^{1} f(t)g(t)\,dt$

1) $(f,g) = (g,f)$

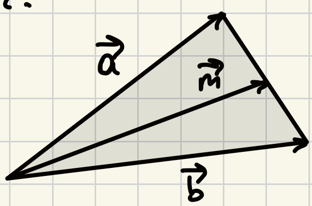
2) $(\alpha f'+\beta f'', g) = \int_{-1}^{1}(\alpha f'(t)+\beta f''(t))g(t)dt = \alpha\int_{-1}^{1} f'(t)g(t)dt + \beta\int_{-1}^{1} f''(t)g(t)\cdot$

$\cdot dt = \alpha(f',g)+\beta(f'',g)$

3) $(f,f) = \int_{-1}^{1} f^2(t)\,dt > 0 \quad \forall f \not\equiv 0$

Т.е. это скалярное произведение

25.17.



$\vec{m} = \frac{1}{2}(\vec{a}+\vec{b})$; $|\vec{m}| = m$, $|\vec{a}| = a$, $|\vec{b}| = b$

$m = \frac{1}{2}\sqrt{(\vec{a}+\vec{b},\vec{a}+\vec{b})} = \frac{1}{2}\sqrt{a^2+2(\vec{a},\vec{b})+b^2} \leq \frac{1}{2}\sqrt{a^2+2ab+b^2} =$

$= \frac{1}{2}(a+b) \leq \frac{1}{2}\cdot 2\max(a,b) = \max(a,b)$

равенство достигается при $(\vec{a},\vec{b}) = ab$ и $a=b$, но тогда не треуг. ■

25.23.

$|\vec{x}|^2 = (\vec{x},\vec{x}) = \sum_{i=1}^{n} x_i^2 = \vec{x}^T\vec{x}$, но $(\vec{x},\vec{x}) = \vec{x}^T\Gamma\vec{x} \Rightarrow \Gamma = E \Rightarrow$ базис ортонорм.

25.25(2)

$$\|-1\ -1\ 1\|^T,\ \|0\ 1\ 3\|^T,\ \Gamma = \begin{Vmatrix} 1 & 2 & 3 \\ 2 & 5 & 8 \\ 3 & 8 & 14 \end{Vmatrix}$$

$$\|-1\ -1\ 1\| \begin{Vmatrix} 1 & 2 & 3 \\ 2 & 5 & 8 \\ 3 & 8 & 14 \end{Vmatrix} \begin{Vmatrix} 0 \\ 1 \\ 3 \end{Vmatrix} = \|0\ 1\ 3\| \begin{Vmatrix} 0 \\ 1 \\ 3 \end{Vmatrix} = 10$$

25.26 (6)

$$\vec{c}_{18} = \|3\ 1\|^T \quad \vec{c}_{12} = \|1\ 1\|^T \quad A_{58} = \begin{Vmatrix} 1 & -2 \\ -2 & 5 \end{Vmatrix}$$

$$\angle(\vec{c}_{18}, \vec{c}_{12}) = \arccos\left(\frac{(\vec{c}_{18}, \vec{c}_{12})}{|\vec{c}_{18}||\vec{c}_{12}|}\right) =$$

$$(\vec{c}_{18}, \vec{c}_{12}) = \|3\ 1\| \begin{Vmatrix} 1 & -2 \\ -2 & 5 \end{Vmatrix} \begin{Vmatrix} 1 \\ 1 \end{Vmatrix} = \|1\ -1\| \begin{Vmatrix} 1 \\ 1 \end{Vmatrix} = 0$$

$$= \arccos(0) = \pi/2$$

25.37.

$$\vec{e}_1^{\,2} = \int_{-1}^{1} dt = 2; \quad \vec{e}_2^{\,2} = \int_{-1}^{1} t^2 dt = \frac{2}{3}; \quad \vec{e}_3^{\,2} = \int_{-1}^{1} t^4 dt = \frac{2}{5}$$

$$(\vec{e}_1, \vec{e}_2) = \int_{-1}^{1} t\,dt = 0; \quad (\vec{e}_1, \vec{e}_3) = \int_{-1}^{1} t^2 dt = \frac{2}{3}; \quad (\vec{e}_2, \vec{e}_3) = \int_{-1}^{1} t^3 dt = 0$$

$$\Gamma = \|(\vec{e}_i, \vec{e}_j)\| = \begin{Vmatrix} 2 & 0 & 2/3 \\ 0 & 2/3 & 0 \\ 2/3 & 0 & 2/5 \end{Vmatrix}$$

26.13(3)

$$\mathcal{L} = \langle \underbrace{\|3\ -15\ 9\ 1\|^T}_{\vec{e}_1},\ \underbrace{\|3\ -6\ -3\ 2\|^T}_{\vec{e}_2} \rangle \qquad \Gamma = E$$

а) $\mathcal{L}^{\perp} = \{\vec{y} \mid \forall \vec{x} \in \mathcal{L} \hookrightarrow \vec{y}^T \vec{x} = 0\}$

$$\begin{cases} (\vec{e}_1, \vec{y}) = 0 \\ (\vec{e}_2, \vec{y}) = 0 \end{cases} \Leftrightarrow \begin{Vmatrix} 3 & -15 & 9 & 1 \\ 3 & -6 & -3 & 2 \end{Vmatrix} \vec{y} = \vec{0}$$

б) $$\begin{Vmatrix} 3 & -15 & 9 & 1 \\ 3 & -6 & -3 & 2 \end{Vmatrix} \sim \begin{Vmatrix} 9 & 0 & -33 & 8 \\ 0 & 9 & -12 & 1 \end{Vmatrix} \Rightarrow \Phi = \begin{Vmatrix} 33 & -8 \\ 12 & -1 \\ 9 & 0 \\ 0 & 9 \end{Vmatrix}$$

Т.е базис в $\mathcal{L}^{\perp}$ – $\|33\ 12\ 9\ 0\|^T$, $\|-8\ -1\ 0\ 9\|^T$

26.14(3)

$$\mathcal{L}: \underbrace{\begin{Vmatrix} 1 & 3 & -1 & 2 \\ 2 & -1 & 3 & 5 \\ 1 & 10 & -6 & 1 \end{Vmatrix}}_{A} \vec{\xi} = \vec{0}$$

$$\begin{Vmatrix} 1 & 3 & -1 & 2 \\ 2 & -1 & 3 & 5 \\ 1 & 10 & -6 & 1 \end{Vmatrix} \underset{\text{III}-3\text{I}}{\sim} \begin{Vmatrix} 1 & 3 & -1 & 2 \\ 2 & -1 & 3 & 5 \\ -2 & 1 & -3 & -5 \end{Vmatrix} \sim \begin{Vmatrix} 1 & 3 & -1 & 2 \\ 2 & -1 & 3 & 5 \end{Vmatrix} \Rightarrow \text{базис в } \mathcal{L}^{\perp} - \begin{Vmatrix} 1 \\ 3 \\ -1 \\ 2 \end{Vmatrix}, \begin{Vmatrix} 2 \\ -1 \\ 3 \\ 5 \end{Vmatrix}$$

26.15(4)

$$\mathcal{L}: \begin{Vmatrix} 5 & 24 & -7 & -3 \\ -1 & -2 & 7 & 3 \end{Vmatrix} \vec{\xi} = \vec{0}$$

$$\begin{Vmatrix} 5 & 24 & -7 & -3 \\ -1 & -2 & 7 & 3 \end{Vmatrix} \sim \begin{Vmatrix} -7 & 0 & 77 & 33 \\ -1 & -2 & 7 & 3 \end{Vmatrix} \sim \begin{Vmatrix} -7 & 0 & 77 & 33 \\ 0 & -7 & -14 & -6 \end{Vmatrix} \Rightarrow \Phi = \begin{Vmatrix} 77 & 33 \\ -14 & -6 \\ 7 & 0 \\ 0 & 7 \end{Vmatrix} \sim \begin{Vmatrix} 11 & 33 \\ -2 & -6 \\ 1 & 0 \\ 0 & 7 \end{Vmatrix}$$

$$\mathcal{L}^{\perp}: \begin{Vmatrix} 11 & -2 & 1 & 0 \\ 33 & -6 & 0 & 7 \end{Vmatrix} \vec{\xi} = \vec{0}$$

26.16(1)

$$\mathcal{L} = A\vec{\xi} = \vec{0} \qquad A = \begin{Vmatrix} 1 & 2 & 1 \end{Vmatrix} \qquad \Gamma = \begin{Vmatrix} 1 & 2 & 0 \\ 2 & 5 & -2 \\ 0 & -2 & 5 \end{Vmatrix}$$

$$\Phi = \begin{Vmatrix} -2 & -1 \\ 1 & 0 \\ 0 & 1 \end{Vmatrix} \Rightarrow \mathcal{L} = \langle \overset{\vec{e}_1}{\begin{Vmatrix} -2 & 1 & 0 \end{Vmatrix}^T}, \overset{\vec{e}_2}{\begin{Vmatrix} -1 & 0 & 1 \end{Vmatrix}^T} \rangle$$

$$\mathcal{L}^{\perp}: \begin{cases} (\vec{e}_1, \vec{y}) = 0 \\ (\vec{e}_2, \vec{y}) = 0 \end{cases} \Leftrightarrow \begin{cases} \vec{e}_1^{\,T} \Gamma \vec{y} = \begin{Vmatrix} -2 & 1 & 0 \end{Vmatrix} \begin{Vmatrix} 1 & 2 & 0 \\ 2 & 5 & -2 \\ 0 & -2 & 5 \end{Vmatrix} \vec{y} = \begin{Vmatrix} 0 & 1 & -2 \end{Vmatrix} \vec{y} = 0 \\ \vec{e}_2^{\,T} \Gamma \vec{y} = \begin{Vmatrix} -1 & 0 & 1 \end{Vmatrix} \begin{Vmatrix} 1 & 2 & 0 \\ 2 & 5 & -2 \\ 0 & -2 & 5 \end{Vmatrix} \vec{y} = \begin{Vmatrix} -1 & -4 & 5 \end{Vmatrix} \vec{y} = 0 \end{cases}$$

$$\mathcal{L}^{\perp}: \begin{Vmatrix} 0 & 1 & -2 \\ -1 & -4 & 5 \end{Vmatrix} \vec{y} = \vec{0}$$

$$\begin{Vmatrix} 0 & 1 & -2 \\ -1 & -4 & 5 \end{Vmatrix} \sim \begin{Vmatrix} 1 & 0 & 3 \\ 0 & 1 & -2 \end{Vmatrix} \to \Phi' = \begin{Vmatrix} -3 & 2 & 1 \end{Vmatrix}^T - \text{базис в } \mathcal{L}^{\perp}$$

26.27(4,5)

$$\mathcal{L} = \langle \vec{a}_1, \ldots \vec{a}_k \rangle; \ \vec{\xi}' - \text{на } \mathcal{L}; \ \vec{\xi}'' - \text{на } \mathcal{L}^{\perp}$$

4) $\vec{a}_1 = \|3\ -2\ 1\ 1\|^T$; $\vec{a}_2 = \|1\ 0\ -1\ 1\|^T$; $\vec{\xi} = \|2\ -1\ 3\ -2\|^T$

$$A = \left\| \begin{matrix} 3 & 1 \\ -2 & 0 \\ 1 & -1 \\ 1 & 1 \end{matrix} \right\| \qquad \vec{\xi}' = A(A^TA)^{-1}A^T\vec{\xi}$$

$$A^TA = \left\| \begin{matrix} 3 & -2 & 1 & 1 \\ 1 & 0 & -1 & 1 \end{matrix} \right\| \left\| \begin{matrix} 3 & 1 \\ -2 & 0 \\ 1 & -1 \\ 1 & 1 \end{matrix} \right\| = \left\| \begin{matrix} 15 & 3 \\ 3 & 3 \end{matrix} \right\|$$

$$\left\| \begin{matrix} 15 & 3 \\ 3 & 3 \end{matrix} \middle| \begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix} \right\| \sim \left\| \begin{matrix} 12 & 0 \\ 0 & 12 \end{matrix} \middle| \begin{matrix} 1 & -1 \\ -1 & 5 \end{matrix} \right\| \to (A^TA)^{-1} = \frac{1}{12} \left\| \begin{matrix} 1 & -1 \\ -1 & 5 \end{matrix} \right\|$$

$$\left\| \begin{matrix} 3 & 1 \\ -2 & 0 \\ 1 & -1 \\ 1 & 1 \end{matrix} \right\| \left\| \begin{matrix} 1 & -1 \\ -1 & 5 \end{matrix} \right\| \left\| \begin{matrix} 3 & -2 & 1 & 1 \\ 1 & 0 & -1 & 1 \end{matrix} \right\| \left\| \begin{matrix} 2 \\ -1 \\ 3 \\ -2 \end{matrix} \right\| = \left\| \begin{matrix} 2 & 2 \\ -2 & 2 \\ 2 & -6 \\ 0 & 4 \end{matrix} \right\| \left\| \begin{matrix} 9 \\ -3 \end{matrix} \right\| = \left\| \begin{matrix} 12 \\ -24 \\ 36 \\ -12 \end{matrix} \right\|$$

$$\vec{\xi}' = \frac{1}{12} \left\| \begin{matrix} 12 \\ -24 \\ 36 \\ -12 \end{matrix} \right\| = \|1\ -2\ 3\ -1\|^T$$

$$\vec{\xi}'' = \vec{\xi} - \vec{\xi}' = \|1\ 1\ 0\ -1\|^T$$

5) $\vec{a}_1 = \|2\ 3\ 0\ 1\|^T$, $\vec{a}_2 = \|0\ 5\ -2\ -1\|^T$, $\vec{\xi} = \|6\ 0\ 4\ 2\|^T$

$$\vec{b}_1 = \vec{a}_1;\ \vec{b}_2 = \vec{a}_2 - \frac{(\vec{a}_2, \vec{b}_1)}{(\vec{b}_1, \vec{b}_1)}\vec{b}_1 = \|0\ 5\ -2\ -1\|^T - \frac{15-1}{4+9+1}\|2\ 3\ 0\ 1\|^T = \|-2\ 2\ -2\ -2\|$$

$$\vec{\xi}' = \beta_1\vec{b}_1 + \beta_2\vec{b}_2 =$$

$$\beta_1 = \frac{(\vec{\xi}, \vec{b}_1)}{(\vec{b}_1, \vec{b}_1)} = \frac{12+2}{4+9+1} = 1;\ \beta_2 = \frac{(\vec{\xi}, \vec{b}_2)}{(\vec{b}_2, \vec{b}_2)} = \frac{-12-8-4}{4+4+4+4} = -\frac{3}{2}$$

$$= \vec{b}_1 - \frac{3}{2}\vec{b}_2 = \|2\ 3\ 0\ 1\|^T - \frac{3}{2}\|-2\ 2\ -2\ -2\| = \|5\ 0\ 3\ 4\|^T$$

$$\vec{\xi}'' = \vec{\xi} - \vec{\xi}' = \|1\ 0\ 1\ -2\|^T$$

26.42(5,6)

5) $\|1\ 2\ 3\|^T$, $\|2\ 1\ 1\|^T$, $\|6\ -7\ -2\|^T$

$$\vec{b}_1 = \|1\ 2\ 3\|^T$$

$$\vec{b}_2 = \vec{a}_2 - \frac{(\vec{a}_2, \vec{b}_1)}{(\vec{b}_1, \vec{b}_1)}\vec{b}_1 = \|2\ 1\ 1\|^T - \frac{2+2+3}{1+4+9}\|1\ 2\ 3\|^T = \|3/2\ 0\ -1/2\|^T$$

$$\vec{b}_3 = \vec{a}_3 - \frac{(\vec{a}_3, \vec{b}_1)}{(\vec{b}_1, \vec{b}_1)}\vec{b}_1 - \frac{(\vec{a}_3, \vec{b}_2)}{(\vec{b}_2, \vec{b}_2)}\vec{b}_2 = \|6\ -7\ -2\|^T - \frac{6-14-6}{1+4+9}\|1\ 2\ 3\|^T - \frac{9+1}{9/4+1/4}\|\tfrac{3}{2}\ 0\ -\tfrac{1}{2}\|^T =$$

$$= \|1\ -5\ 3\|^T$$

б) $\|1\ 2\ 1\ 2\|^T,\ \|4\ 0\ 4\ 1\|^T,\ \|1\ 13\ -1\ -3\|^T$

$\vec{b}_1 = \|1\ 2\ 1\ 2\|^T$

$\vec{b}_2 = \vec{a}_2 - \dfrac{(\vec{a}_2,\vec{b}_1)}{(\vec{b}_1,\vec{b}_1)}\vec{b}_1 = \|4\ 0\ 4\ 1\|^T - \dfrac{4+4+2}{1+4+1+4}\|1\ 2\ 1\ 2\|^T = \|3\ -2\ 3\ -1\|^T$

$\vec{b}_3 = \vec{a}_3 - \dfrac{(\vec{a}_3,\vec{b}_1)}{(\vec{b}_1,\vec{b}_1)}\vec{b}_1 - \dfrac{(\vec{a}_3,\vec{b}_2)}{(\vec{b}_2,\vec{b}_2)}\vec{b}_2 = \|1\ 13\ -1\ -3\|^T - \dfrac{1+26-1-6}{1+4+1+4}\|1\ 2\ 1\ 2\|^T -$

$- \dfrac{3-26-3+3}{9+4+9+1}\|3\ -2\ 3\ -1\|^T = \|2\ 7\ 0\ -8\|^T$

26.44 (2)

$\|1\ 2\ 0\|^T,\ \|2\ 0\ 3\|^T,\ \|1\ 8\ 6\|^T;\quad \Gamma = \left\|\begin{matrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{matrix}\right\|$

$\vec{b}_1 = \|1\ 2\ 0\|^T$

$(\vec{a}_2,\vec{b}_1) = \|2\ 0\ 3\|\left\|\begin{matrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{matrix}\right\|\|1\ 2\ 0\|^T = 14$

$\begin{array}{rrr} 1 & 8 & 6 \\ -4 & -8 & 0 \\ -2 & 4 & -6 \\ \hline -5 & 4 & 0 \end{array}$

$(\vec{b}_1,\vec{b}_1) = \|1\ 2\ 0\|\left\|\begin{matrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{matrix}\right\|\|1\ 2\ 0\|^T = 14$

$\vec{b}_2 = \vec{a}_2 - \dfrac{(\vec{a}_2,\vec{b}_1)}{(\vec{b}_1,\vec{b}_1)}\vec{b}_1 = \|2\ 0\ 3\|^T - \|1\ 2\ 0\|^T = \|1\ -2\ 3\|^T$

$(\vec{a}_3,\vec{b}_1) = \|1\ 8\ 6\|^T\left\|\begin{matrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{matrix}\right\|\|1\ 2\ 0\|^T = 56$

$(\vec{a}_3,\vec{b}_2) = \|1\ 8\ 6\|^T\left\|\begin{matrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{matrix}\right\|\|1\ -2\ 3\|^T = 24$

$(\vec{b}_2,\vec{b}_2) = \|1\ -2\ 3\|^T\left\|\begin{matrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{matrix}\right\|\|1\ -2\ 3\|^T = 12$

$\vec{b}_3 = \vec{a}_3 - \dfrac{(\vec{a}_3,\vec{b}_1)}{(\vec{b}_1,\vec{b}_1)}\vec{b}_1 - \dfrac{(\vec{a}_3,\vec{b}_2)}{(\vec{b}_2,\vec{b}_2)}\vec{b}_2 = \|1\ 8\ 6\|^T - \dfrac{56}{14}\|1\ 2\ 0\|^T - \dfrac{24}{12}\|1\ -2\ 3\|^T = \|-5\ 4\ 0\|^T$

$(\vec{b}_3,\vec{b}_3) = \|-5\ 4\ 0\|^T\left\|\begin{matrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{matrix}\right\|\|-5\ 4\ 0\|^T = 42$

$\vec{e}_1 = \dfrac{\vec{b}_1}{|\vec{b}_1|} = \dfrac{1}{\sqrt{14}}\|1\ 2\ 0\|^T;\quad \vec{e}_2 = \dfrac{\vec{b}_2}{|\vec{b}_2|} = \dfrac{1}{\sqrt{12}}\|1\ -2\ 3\|^T;\quad \vec{e}_3 = \dfrac{\vec{b}_3}{|\vec{b}_3|} = \dfrac{1}{\sqrt{42}}\|-5\ 4\ 0\|^T$

T.2

$$(f,g) = \int_{-1}^{1} f(t)g(t)\,dt \qquad \vec{a}_1 = \|1\ 0\ 0\|^T;\ \vec{a}_2 = \|0\ 1\ 0\|^T;\ \vec{a}_3 = \|0\ 0\ 1\|^T$$

$$\vec{b}_1 = \|1\ 0\ 0\|$$

$$\vec{b}_2 = \vec{a}_2 - \frac{(\vec{a}_2,\vec{b}_1)}{(\vec{b}_1,\vec{b}_1)}\vec{b}_1 = \vec{a}_2 = \|0\ 1\ 0\|$$

$$(\vec{a}_2,\vec{b}_1) = \int_{-1}^{1} t\,dt = 0$$

$$\vec{b}_3 = \vec{a}_3 - \frac{(\vec{a}_3,\vec{b}_1)}{(\vec{b}_1,\vec{b}_1)}\vec{b}_1 - \frac{(\vec{a}_3,\vec{b}_2)}{(\vec{b}_2,\vec{b}_2)}\vec{b}_2 = \|0\ 0\ 1\|^T - \frac{1}{3}\|0\ 0\ 1\|^T = \|-1/3\ 0\ \ 1\|$$

$$(\vec{a}_3,\vec{b}_1) = \int_{-1}^{1} t^2\,dt = \frac{2}{3}$$

$$(\vec{b}_1,\vec{b}_1) = \int_{-1}^{1} dt = 2$$

$$(\vec{a}_3,\vec{b}_2) = \int_{-1}^{1} t^3\,dt = 0$$

Т.е. искомые: $1,\ t,\ t^2 - \frac{1}{3}$