№ 5.1. $\bar r = \bar r_1 + \bar a_1 t$; $\bar r = \bar r_2 + \bar a_2 t$

1) пересекаются $\Rightarrow \bar a_1 \nparallel \bar a_2$

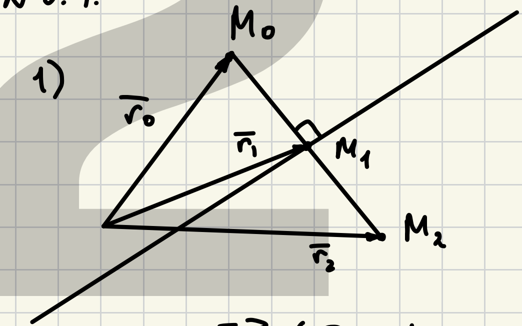
2) параллельны $\Rightarrow \bar a_1 \parallel \bar a_2$

не совпадают $\Rightarrow \vec a_1$ или $\vec a_2 \nparallel \bar r_1 - \bar r_2$

3) совпадают $\Rightarrow \bar a_1 \parallel \vec a_2 \parallel \bar r_1 - \bar r_2$

№ 5.4.

1)



$(\bar r, \bar n) = D$

$$\begin{cases} \bar r_0 - \bar r_1 = \lambda \bar n \\ (\bar r_1, \bar n) = D \end{cases} \Rightarrow$$

$\Rightarrow (r_0 - \lambda \bar n, \bar n) = D \Rightarrow (\bar r_0, \bar n) - D = \lambda(\bar n, \bar n) \Rightarrow$

$\Rightarrow \lambda = \dfrac{(\bar r_0, \bar n) - D}{n^2} \Rightarrow \bar r_1 = \bar r_0 - \lambda \bar n = \bar r_0 - \dfrac{(\bar r_0, \bar n) - D}{n^2} \bar n$

$\bar r_0 + \bar r_2 = 2\bar r_1 \Rightarrow \bar r_2 = 2\bar r_1 - \bar r_0 = \bar r_0 - 2\dfrac{(\bar r_0, \bar n) - D}{n^2} \bar n$

№ 5.8. $A(-3,4)$

1) $x - 2y + 5 = 0$; $-3 - 2\cdot 4 + d = 0 \Rightarrow d = 11 \Rightarrow x - 2y + 11 = 0$

2) $\dfrac{x-1}{2} = \dfrac{y+2}{3} \Rightarrow \dfrac{x+3}{2} = \dfrac{y-4}{3}$

5) $x = 3 + t$, $y = 4 - 7t$. При $t = 0$ $x = -3$, $y = -4 \Rightarrow$

$\Rightarrow x = -3 + t$, $4 - 7t$. Коэф. при t сохраняются, т.к.

в виде $ax + by + c = 0$ $a_1 = a_2$, $b_1 = b_2$ (получаем из $7x + y = -c =$
$= const$)

№ 5.11 $ax-4y=6$; $x-ay=3$ $\Rightarrow$ $\bar{n}_1 = \begin{Vmatrix} a \\ -4 \end{Vmatrix}$, $\bar{n}_2 = \begin{Vmatrix} 1 \\ -a \end{Vmatrix}$

1) пересекаются $\Rightarrow$ $\bar{n}_1 \not\parallel \bar{n}_2 \Rightarrow |[\bar{n}_1, \bar{n}_2]| \neq 0 \Rightarrow$

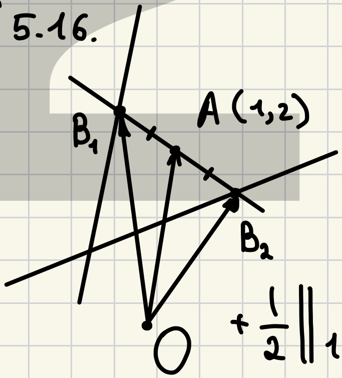
$\Rightarrow \begin{vmatrix} a & -4 \\ 1 & -a \end{vmatrix} \neq 0 \Rightarrow a^2 \neq 4 \Rightarrow a \neq \pm 2$.

2) параллельные $\Rightarrow |[\bar{n}_1, \bar{n}_2]| = 0 \Rightarrow a = \pm 2$.

не совпадают $\Rightarrow \frac{a}{2} \neq 1 \Rightarrow a \neq 2 \Rightarrow a = -2$

3) из 2) $\Rightarrow a = 2$

№ 5.16.



$3x+y+2=0$, $4x+y-1=0$

$\bar{OA} = \frac{1}{2}(\bar{OB}_1 + \bar{OB}_2)$

$\begin{Vmatrix} x_A \\ y_A \end{Vmatrix} = \frac{1}{2}\begin{Vmatrix} x_{B_1} \\ y_{B_1} \end{Vmatrix} + \frac{1}{2}\begin{Vmatrix} x_{B_2} \\ y_{B_2} \end{Vmatrix} = \frac{1}{2}\begin{Vmatrix} x_{B_1} \\ -3x_{B_1}-2 \end{Vmatrix} +$

$+ \frac{1}{2}\begin{Vmatrix} x_{B_2} \\ 1-4x_{B_2} \end{Vmatrix} = -\frac{3}{2}\begin{Vmatrix} x_{B_1} \\ x_{B_1} \end{Vmatrix} + \begin{Vmatrix} 0 \\ -1 \end{Vmatrix} - 2\begin{Vmatrix} x_{B_2} \\ x_{B_2} \end{Vmatrix} + \begin{Vmatrix} 0 \\ 1/2 \end{Vmatrix}$

$2\begin{Vmatrix} x_A \\ y_A + 1/2 \end{Vmatrix} = -3\begin{Vmatrix} x_{B_1} \\ x_{B_1} \end{Vmatrix} - 4\begin{Vmatrix} x_{B_2} \\ x_{B_2} \end{Vmatrix} = \begin{Vmatrix} 1 & 1 \\ -3 & -4 \end{Vmatrix} \begin{Vmatrix} x_{B_1} \\ x_{B_2} \end{Vmatrix}$

$\begin{Vmatrix} x_{B_1} \\ x_{B_2} \end{Vmatrix} = \begin{Vmatrix} 1 & 1 \\ -3 & -4 \end{Vmatrix}^{-1} \begin{Vmatrix} 2x_A \\ 2y_A+1 \end{Vmatrix} = \begin{Vmatrix} 4 & 1 \\ -3 & -1 \end{Vmatrix} \begin{Vmatrix} 2x_A \\ 2y_A+1 \end{Vmatrix} =$

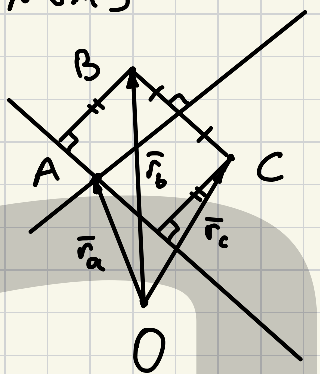
$= \begin{Vmatrix} 8x+2y+1 \\ -6x-2y-1 \end{Vmatrix} = \begin{Vmatrix} 13 \\ -11 \end{Vmatrix}$

$3x_{B_1} + y_{B_1} + 2 = 0 \Rightarrow y_{B_1} = -2 - 3x_{B_1} = -41$

$4x_{B_2} + y_{B_2} - 1 = 0 \Rightarrow y_{B_2} = 1 - 4x_{B_2} = 45$

$\Rightarrow$ искомая прямая: $\frac{x - x_{B_1}}{x_{B_2} - x_{B_1}} = \frac{y - y_{B_1}}{y_{B_2} - y_{B_1}} \Leftrightarrow \frac{x-13}{-24} = \frac{y+41}{86}$

№5.19



$\bar r = \bar r_a + \bar a t$ – ур-е искомой прямой

$\bar r_1 = \dfrac{\bar r_b + \bar r_c}{2} = \bar r_a + \bar a_1 t, \; t \in \mathbb{R}$

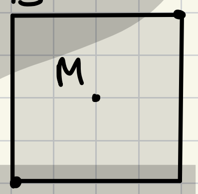
Для $t=1$: $\bar a_1 = \dfrac{\bar r_b + \bar r_c}{2} - \bar r_a = \left\| \begin{matrix} 3 \\ -2 \end{matrix} \right\|$

$\bar a_2 = \bar r_b - \bar r_c = \left\| \begin{matrix} 2 \\ 8 \end{matrix} \right\|$

Две прямые: $\bar r_1 = \left\| \begin{matrix} -1 \\ 5 \end{matrix} \right\| + \left\| \begin{matrix} 3 \\ -2 \end{matrix} \right\| t, \; t \in \mathbb{R}$

$\bar r_2 = \left\| \begin{matrix} -1 \\ 5 \end{matrix} \right\| + \left\| \begin{matrix} 2 \\ 8 \end{matrix} \right\| t, \; t \in \mathbb{R}$

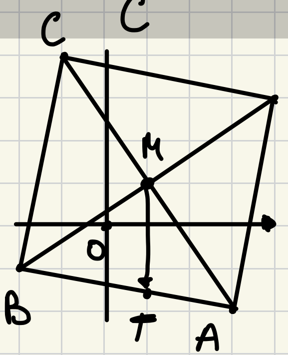
№5.24



$\overline{AC} = 2\overline{AM}$. $\overline{AC}(-4, 6) \Rightarrow C(-1, 4)$

$|\overline{BD}| = |\overline{AC}|$, $\overline{BD} \perp \overline{AC} \Rightarrow x_{BD} = y_{AC}$;

$y_{BD} = -x_{AC}$ с точностью до перестановки точек B и D. Тогда $\overline{BD}(6, 4)$



AB, BC, CD, DA задаются так:

AB: $a\overrightarrow{MB} + (1-a)\overrightarrow{MA} = \overrightarrow{r_{MT}}, \; 0 \le a \le 1$

$\overrightarrow{r_{MT}} = \overrightarrow{r_T} - \overrightarrow{r_M} \Rightarrow \overrightarrow{r_T} = \overrightarrow{r_M} + \overrightarrow{r_{MT}} \Rightarrow \overrightarrow{r_T} =$

$= a\overline{MB} + (1-a)\overline{MA} + \overline{OM} = a(\overline{MB} - \overline{MA}) + \overrightarrow{OM} + \overrightarrow{MA} =$

$= a\left(\dfrac{\overline{DB} - \overline{CA}}{2}\right) + \overline{OM} + \dfrac{\overline{CA}}{2}$

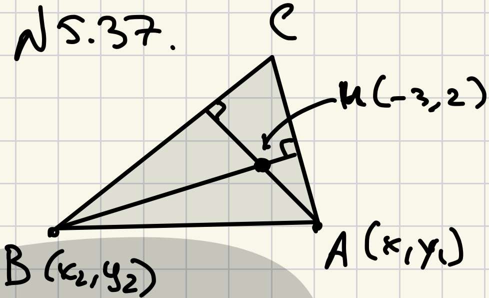
Тогда $\vec r_T = \vec r = \frac{1}{2} a \left\| \begin{matrix} -6-4 \\ -4+6 \end{matrix} \right\| + \left\| \begin{matrix} 1 \\ 1 \end{matrix} \right\| + \left\| \begin{matrix} 2 \\ -3 \end{matrix} \right\| = a \left\| \begin{matrix} -5 \\ 1 \end{matrix} \right\| + \left\| \begin{matrix} 3 \\ -2 \end{matrix} \right\|, \; 0 \le a \le 1$

BC: $\vec r = b \left\| \begin{matrix} -2+3 \\ 3+2 \end{matrix} \right\| + \left\| \begin{matrix} 1 \\ 1 \end{matrix} \right\| + \left\| \begin{matrix} -3 \\ -2 \end{matrix} \right\| = b \left\| \begin{matrix} 1 \\ 5 \end{matrix} \right\| + \left\| \begin{matrix} -2 \\ -1 \end{matrix} \right\|, \; 0 \le b \le 1$

CD: $\vec r = c \left\| \begin{matrix} 3+2 \\ 2-3 \end{matrix} \right\| + \left\| \begin{matrix} 1 \\ 1 \end{matrix} \right\| + \left\| \begin{matrix} 3 \\ 2 \end{matrix} \right\| = c \left\| \begin{matrix} 5 \\ -1 \end{matrix} \right\| + \left\| \begin{matrix} 4 \\ 3 \end{matrix} \right\|, \; 0 \le c \le 1$

DA: $\vec r = d \left\| \begin{matrix} 2-3 \\ -3-2 \end{matrix} \right\| + \left\| \begin{matrix} 1 \\ 1 \end{matrix} \right\| + \left\| \begin{matrix} 2 \\ -3 \end{matrix} \right\| = d \left\| \begin{matrix} -1 \\ -5 \end{matrix} \right\| + \left\| \begin{matrix} 3 \\ -2 \end{matrix} \right\|, \; 0 \le d \le 1$

№5.37.



$\exists\ A(x_1,y_1),\ B(x_2,y_2)$

$\exists\ CA \in y=2x;\ B\in y=-x+3 \Rightarrow$

$\Rightarrow A(x_1, 2x_1),\ B(x_2, 3-x_2)$

$\exists\ \overline{a_1}, \overline{a_2}$ напр. $y=2x;\ y=-x+3 \Rightarrow$

$\Rightarrow \overline{a_1}(1;2),\ \overline{a_2}(-1;1).\ \overline{BH}(-3-x_2; -1+x_2),\ \overline{AH}(-3-x_1; 2-2x_1)$

$\overline{BH} \perp \overline{a_1};\ \overline{AH} \perp \overline{a_2} \Leftrightarrow (\overline{BH}, \overline{a_1}) = (\overline{AH}, \overline{a_2}) = 0 \Leftrightarrow$

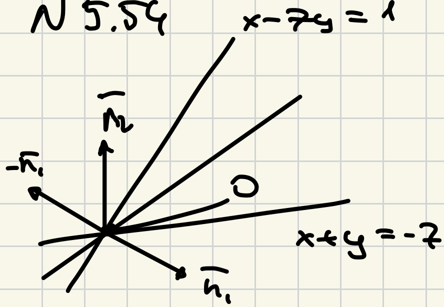
$$\Leftrightarrow \begin{cases} -3-x_2-2+2x_2=0 \\ 3+x_1+2-2x_1=0 \end{cases} \Leftrightarrow \begin{cases} x_2=5 \\ x_1=5 \end{cases} \Rightarrow \begin{cases} A(5,10) \\ B(5,-2) \end{cases}$$

$\Rightarrow AB:\ t\overrightarrow{HB} + (1-t)\overrightarrow{HA} = \vec{r}$ $+\rho H$, $0\le t\le 1$. $\overrightarrow{HB}(8,-4);\ \overrightarrow{HA}(8,8)$

$$\vec{r} = t\left\|\begin{matrix} 8-8 \\ -4-8 \end{matrix}\right\| + \left\|\begin{matrix} 8 \\ 8 \end{matrix}\right\| + \left\|\begin{matrix} -3 \\ 2 \end{matrix}\right\|,\ 0\le t\le 1$$

$$\vec{r} = t\left\|\begin{matrix} 0 \\ -12 \end{matrix}\right\| + \left\|\begin{matrix} 5 \\ 10 \end{matrix}\right\|,\ 0\le t\le 1$$

№5.54



$\overline{n_1}\begin{pmatrix} 1 \\ -7 \end{pmatrix};\ n_2\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$(\overline{n_1}, \overline{n_2}) = 1-7 < 0 \Rightarrow$ бисс. угла - бисс. угла $\angle(\overline{n_1}, -\overline{n_2})$.

направляющ. прямой: $\frac{\overline{n_2}}{|n_2|} + \frac{\overline{n_1}}{|n_1|} =$

$$= \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} + \frac{1}{\sqrt{50}}\begin{pmatrix} 1 \\ -7 \end{pmatrix} \parallel \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \frac{1}{5}\begin{pmatrix} 1 \\ -7 \end{pmatrix} \parallel \begin{pmatrix} 6 \\ -2 \end{pmatrix} \parallel \begin{pmatrix} 3 \\ -1 \end{pmatrix}$$

$$O: \begin{cases} x-7y=1 \\ x+y=-7 \end{cases} \Leftrightarrow O(-6,-1) \Rightarrow \text{кан. ур-ие: } \frac{x+6}{3} = \frac{y+1}{-1}$$

6.1 (1,3,5)

1) $\bar r = \bar r_0 + \bar a u + \bar b v$

$$(\bar r - \bar r_0, \bar a, \bar b) = 0$$

$$(\bar r, \bar a, \bar b) = (\bar r_0, \bar a, \bar b)$$

$$(\bar r, \bar n) = D; \quad \bar n = [\bar a, \bar b], \; D = (\bar r_0, \bar a, \bar b)$$

3) $[\bar r, \bar a] = \bar b$

Если задано $\bar r = \bar r_0 + \bar a t$, то $[\bar r - \bar r_0, \bar a] = \bar 0 \Rightarrow$

$$\Rightarrow [\bar r, \bar a] = [\bar r_0, \bar a] = \bar b$$

Если $\bar r_0 = \dfrac{[\bar a, \bar b]}{|a|^2}$, то $[\bar r_0, \bar a] =$

$$= \frac{1}{a^2}[\bar a, [\bar b, \bar a]] = \frac{1}{a^2}\left(\bar b \cdot a^2 - \bar a \underbrace{\cancel{(\bar a, \bar b)}}_{0}\right) =$$

$$= \bar b \Rightarrow \bar r = \frac{[\bar a, \bar b]}{|\bar a|^2} + \bar a t$$

5) $\begin{cases} (\bar r, \bar n_1) = D_1 \\ (\bar r, \bar n_2) = D_2 \end{cases} \Leftrightarrow \begin{cases} \bar n_2(\bar r, \bar n_1) = D_1 \bar n_2 \\ \bar n_1(\bar r, \bar n_2) = D_2 \bar n_1 \end{cases} \Rightarrow$

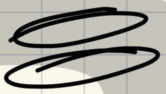
$$\Rightarrow n_2(\bar r, \bar n_1) - \bar n_1(\bar r, \bar n_2) = [\bar r, [\bar n_2, \bar n_1]] = D_1 \bar n_2 - D_2 \bar n_1$$

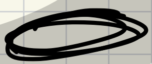
$\bar a = [\bar n_2, \bar n_1] \Rightarrow [\bar r, \bar a] = D_1 \bar n_2 - D_2 \bar n_1$. Из 3) $\Rightarrow$

$$\Rightarrow \bar r = \frac{[[\bar n_2, \bar n_1], D_1 \bar n_2 - D_2 \bar n_1]}{|[\bar n_2, \bar n_1]|^2} + [\bar n_2, \bar n_1] t$$

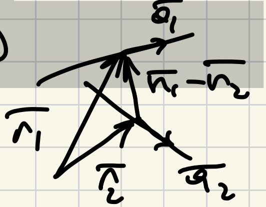
6.2 $(\bar r, \bar n_1) = D_1$; $(\bar r, \bar n_2) = D_2$

1)  $\Leftrightarrow [\bar n_1, \bar n_2] \neq 0$

2)  $\Leftrightarrow \begin{cases} [\bar n_1, \bar n_2] = \bar 0 \\ \dfrac{D_1}{|n_1|} \neq \dfrac{D_2}{|n_2|} \end{cases}$

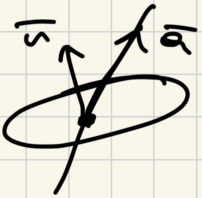
3)  $\Leftrightarrow \begin{cases} [\bar n_1, \bar n_2] = \bar 0 \\ \dfrac{D_1}{|n_1|} = \dfrac{D_2}{|n_2|} \end{cases}$

6.3(2) $\bar r = \bar r_1 + \bar a_1 t$ $\bar r = \bar r_2 + \bar a_2 t$

2) 

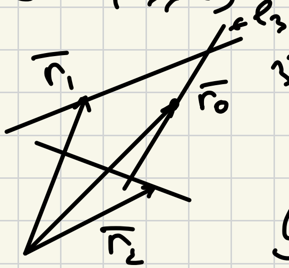
$(\bar r_1 - \bar r_2, \bar a_1, \bar a_2) \neq 0$

6.4(1) $\bar r = \bar r_0 + \bar a t$ $(\bar r, \bar n) = D$

1) 

$\Leftrightarrow (\bar a, \bar n) \neq 0$

6.10(3,4) $\overbrace{\bar r = \bar r_1 + \bar a_1 t}^{l_1}$; $\overbrace{\bar r = \bar r_2 + \bar a_2 t}^{l_2}$, $M_0(\bar r_0)$



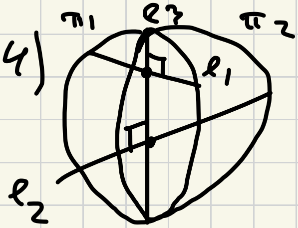
3) $\Pi_1: l_1 \in \Pi_1, l_3 \in \Pi_1$; $\Pi_2: l_2 \in \Pi_2, l_3 \in \Pi_2$

$l_3 = \Pi_1 \cap \Pi_2 \Rightarrow l_3: \begin{cases} (\bar r - \bar r_0, \bar r_0 - \bar r_1, \bar a_1) = 0 \\ (\bar r - \bar r_0, \bar r_0 - \bar r_2, \bar a_2) = 0 \end{cases}$

Условие $\exists$ такой прямой: $\left[\begin{array}{l} (\bar r_2 - \bar r_0, \bar a_2, \bar a_1) \neq 0 \\ (\bar r_1 - \bar r_0, \bar a_2, \bar a_1) \neq 0 \end{array} \right.$

$\leftarrow M_0 \notin \sigma_1$ и $M_0 \notin \sigma_2$: $\sigma_1: l_1 \in \sigma_1, l_2 \parallel \sigma_1$, $\sigma_2: l_2 \in \sigma_2, l_1 \parallel \sigma_1$

4)



$\pi_1 \cap \pi_2 = l_3$

$$\begin{cases} \bar{r} = \bar{r}_1 + p\bar{a}_1 + q[\bar{a}_1, \bar{a}_2] \\ \bar{r} = \bar{r}_2 + t\bar{a}_2 + s[\bar{a}_1, \bar{a}_2] \end{cases}$$