

$$17.1(3) \quad \begin{cases} 2x_1 + x_2 - x_3 = 2 \\ 3x_1 + 2x_2 - 2x_3 = 3 \\ x_1 + x_3 = 3 \end{cases} \quad \left| \begin{array}{ccc|c} 2 & 1 & -1 & 2 \\ 3 & 1 & -2 & 3 \\ 1 & 0 & 1 & 3 \end{array} \right| - \text{матрица}$$

$$\Delta = \begin{vmatrix} 2 & 1 & -1 \\ 3 & 1 & -2 \\ 1 & 0 & 1 \end{vmatrix} = 2 - 5 + 1 = -2 \quad \Delta_1 = \begin{vmatrix} 2 & 1 & -1 \\ 3 & 1 & -2 \\ 3 & 0 & 1 \end{vmatrix} = 2 - 9 + 3 = -4$$

$$\Delta_2 = \begin{vmatrix} 2 & 2 & -1 \\ 3 & 3 & -2 \\ 1 & 3 & 1 \end{vmatrix} = 18 - 10 - 6 = 2 \quad \Delta_3 = \begin{vmatrix} 2 & 1 & 2 \\ 3 & 1 & 3 \\ 1 & 0 & 3 \end{vmatrix} = 6 - 6 - 2 = -2$$

$$x_i = \frac{\Delta_i}{\Delta} \Rightarrow \text{решение } (x_1, x_2, x_3) = (2, -1, 1)$$

18.1 (2, 10)

$$2) x_1 - x_2 + 2x_3 = 0 \quad \parallel 1 -1 2 \parallel - \text{матрица коэффициентов}$$

$$\tilde{\Phi} = \begin{vmatrix} 1 & -2 \\ 1 & 0 \\ 0 & 1 \end{vmatrix} \Rightarrow \begin{vmatrix} x_1 & x_2 & x_3 \end{vmatrix}^T = \begin{vmatrix} 1 & -2 \\ 1 & 0 \\ 0 & 1 \end{vmatrix} \begin{vmatrix} c_1 \\ c_2 \end{vmatrix}, c_1, c_2 \in \mathbb{R}$$

$$10) \begin{cases} x_1 + x_2 + x_3 - x_4 = 0 \\ 3x_1 + 2x_2 + x_3 - x_5 = 0 \end{cases} \quad \left| \begin{array}{ccccc|c} 1 & 1 & 1 & -1 & 0 \\ 3 & 2 & 1 & 0 & -1 \\ 1 & 2 & 3 & 4 & 5 \end{array} \right| \sim \left| \begin{array}{ccccc|c} 1 & 0 & -1 & -1 & -1 \\ 0 & 1 & -3 & -2 & -1 \\ 4 & 5 & 1 & 2 & 3 \end{array} \right|$$

$$\tilde{\tilde{\Phi}} = \begin{vmatrix} 1 & 1 & 1 \\ 3 & 2 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}^4 \quad \tilde{\Phi} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} \Rightarrow \vec{x} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} c_1 \\ c_2 \\ c_3 \end{vmatrix}$$

19.6 (4, 21, 23)

$$4) \left| \begin{array}{ccc|c} 0 & 1 & 2 & 0 \\ -1 & 0 & -2 & 1 \\ -2 & 2 & 0 & 2 \end{array} \right| \sim \left| \begin{array}{ccc|c} 1 & 0 & 2 & -1 \\ 0 & 1 & 2 & 0 \\ 0 & 2 & 4 & 0 \end{array} \right| \sim \left| \begin{array}{ccc|c} 1 & 0 & 2 & -1 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right| \Rightarrow \vec{x} = \begin{pmatrix} -2 \\ -2 \\ 1 \end{pmatrix} + \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}, c_1, c_2 \in \mathbb{R}$$

$$21) \left| \begin{array}{ccc|c} 2 & -1 & 2 & 1 \\ 1 & 1 & 1 & 2 \\ -1 & 1 & 1 & 0 \end{array} \right| \sim \left| \begin{array}{ccc|c} 2 & -1 & 2 & 1 \\ 0 & 2 & 8 & -4 \\ 0 & 1 & 4 & -2 \end{array} \right| \sim \left| \begin{array}{ccc|c} 3 & 0 & 9 & -3 \\ 0 & 2 & 8 & -4 \\ 0 & 1 & 4 & -2 \end{array} \right| \sim \left| \begin{array}{ccc|c} 3 & 0 & 9 & -3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right| \sim \left| \begin{array}{ccc|c} 1 & 0 & 3 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 4 & -2 \end{array} \right|$$

$$\sim \left| \begin{array}{ccc|c} 1 & 0 & 3 & -1 \\ 0 & 1 & 4 & -2 \\ 0 & 0 & 0 & 0 \end{array} \right| \Rightarrow \tilde{\Phi} = \begin{vmatrix} -3 & 1 \\ -4 & 2 \\ 0 & 0 \end{vmatrix} \Rightarrow \vec{x} = \begin{pmatrix} -3 \\ -4 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, c_1, c_2 \in \mathbb{R}$$

$$23) \left| \begin{array}{cccc|c} 3 & 1 & -2 & 4 & 1 \\ 2 & -3 & 6 & -5 & 2 \\ 8 & -1 & 2 & 3 & -2 \end{array} \right| \sim \left| \begin{array}{cccc|c} 3I + II & | & 11 & 0 & 0 & 7 & | & 5 \\ 3III - II & | & 2 & -3 & 6 & -5 & | & 2 \\ 22 & 0 & 0 & 14 & & -8 \end{array} \right|$$

I и II л.з. в A и л.з. в \tilde{A} \Rightarrow н.т. Кронекера Капелли несовместна

19.7(2)

$$\left| \begin{array}{ccc|c} -2 & 4 & -3 & 2 \\ 4 & 4 & 2 & 5 \\ 1 & 4 & 2 & 2 \end{array} \right| \sim \left| \begin{array}{ccc|c} I - 2III & | & -4 & -4 & -7 & | & 1 - 14 \\ 4 & 4 & 2 & 5 \\ 1 & 4 & 2 & 2 \end{array} \right| \sim \left| \begin{array}{ccc|c} I + II & | & 0 & 0 & 0 & | & 1 - 9 \\ 4 & 4 & 2 & 5 \\ 1 & 4 & 2 & 2 \end{array} \right|$$

$rg A = rg \tilde{A}$, т.е. при $\lambda = 9$ совместна

$$\left| \begin{array}{ccc|c} 4 & 4 & 7 & 5 \\ 1 & 4 & 2 & 7 \\ 0 & 0 & 0 & 0 \end{array} \right| \sim \left| \begin{array}{ccc|c} I - II & | & 3 & 0 & 5 & | & -2 \\ 4II - I & | & 0 & 12 & 1 & | & 23 \\ 0 & 0 & 0 & 0 \end{array} \right| \sim \left| \begin{array}{ccc|c} 4I & | & 12 & 0 & 20 & | & -8 \\ 0 & 12 & 1 & 23 \\ 0 & 0 & 0 & 0 \end{array} \right| \Rightarrow$$

$$\Rightarrow \Phi = \begin{pmatrix} -20 \\ -1 \\ 12 \end{pmatrix} \Rightarrow \vec{x} = \begin{pmatrix} -20 \\ -1 \\ 12 \end{pmatrix} C + \begin{pmatrix} -8 \\ 23 \\ 0 \end{pmatrix}$$

19.10

$$\left\| \vec{a}_1, \dots, \vec{a}_n \mid \vec{a}_1 + \dots + \vec{a}_n \right\|. \text{ Очев., что } (1, \dots, 1) \text{- решение}$$

19.13

$$\underset{n \times n}{\underset{n \neq 1}{\underset{n \times 1}{A}} \vec{x} = \vec{b}} \quad rg A = n \Rightarrow rg \Phi = n - rg A = 0 \Rightarrow \Phi = \emptyset \Rightarrow$$

\Rightarrow если система совместна общ. решение $X = \Phi \vec{h} + X_0 = X_0$, и.е. однозначное, и.е. у системы не более 1 решения.

19.14 $rg A = m \wedge \vec{b} \Rightarrow rg A = rg \tilde{A} \wedge b \Rightarrow$ н.т. Кронекера Капелли совместна.

$$19.17(2) \quad \Phi = \begin{pmatrix} 1 & 0 \\ 1 & -1 \\ 0 & 1 \\ -4 & 1 \end{pmatrix}$$

$$\underline{1 \text{ способ}} \quad \Phi^+ = \begin{pmatrix} 1 & 1 & 0 & -4 \\ 0 & -1 & 1 & -2 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 1 & -3 \\ 0 & 1 & -1 & 1 \end{pmatrix} \Rightarrow \Phi' = \begin{pmatrix} -1 & 3 \\ 1 & -1 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}. A^T = \Phi' \Rightarrow$$

$$= A = \begin{pmatrix} -1 & 1 & 0 \\ 3 & -1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

$$\underline{2 \text{ способ}} \quad \left| \begin{array}{cc|c} 1 & 0 & x_1 \\ 1 & -1 & x_2 \\ 0 & 1 & x_3 \\ -4 & 1 & x_4 \end{array} \right| \sim \left| \begin{array}{cc|c} 6 & 5x_1 - x_2 + x_4 & | \\ 0 & x_2 - x_1 + x_3 & | \\ 0 & x_3 & | \\ -4 & x_4 & | \end{array} \right| \Rightarrow \text{исл. } \begin{cases} 5x_1 - x_2 + x_4 = 0 \\ -x_1 + x_2 + x_3 = 0 \end{cases}$$