№7.25(3,8) $F_{1,2}(\pm 1, 0)$, $P\left(\sqrt{3}, \frac{\sqrt{3}}{2}\right)$

3) $L(x,y) = \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. $L(x_p, y_p) = 0 \Rightarrow$

$\Rightarrow \frac{3}{a^2} + \frac{3}{4b^2} = 1$

$F_1P + F_2P = 2a \Rightarrow \sqrt{(1-\sqrt{3})^2 + \frac{3}{4}} + \sqrt{(-1-\sqrt{3})^2 + \frac{3}{4}} = \sqrt{\frac{19}{4} - 2\sqrt{3}} + \sqrt{\frac{19}{4} + 2\sqrt{3}} =$

$= \frac{1}{2}\left(\sqrt{19-8\sqrt{3}} + \sqrt{19+8\sqrt{3}}\right) = \frac{1}{2}\sqrt{19-8\sqrt{3}+19+8\sqrt{3}+2\sqrt{19^2-3\cdot 64}}$

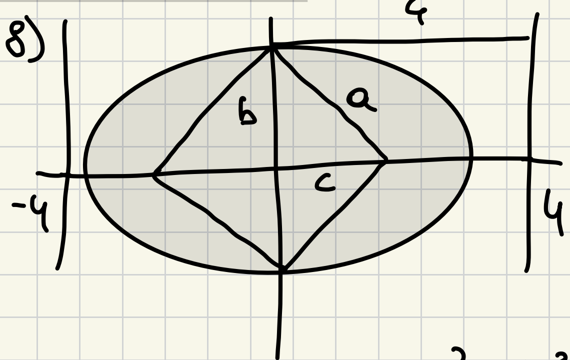
$= \frac{1}{2}\sqrt{38+2\cdot 13} = \frac{1}{2}\cdot 8 = 4 \Rightarrow a = 2 \Rightarrow$

$\Rightarrow \frac{3}{4} + \frac{3}{4}\frac{1}{b^2} = 1 \Rightarrow b^2 = 3$

$(x-x_1)(x-x_2) = x^2 - (x_1+x_2)x + x_1x_2$

$a^2 \geq b^2 \Rightarrow L(x,y) = \frac{x^2}{4} + \frac{y^2}{3} = 1$

8)



$\frac{a}{4} = \varepsilon = \frac{c}{a} \Rightarrow a^2 = 4c$

$a \cdot \cos 45^\circ = c \Rightarrow$

$\Rightarrow a\frac{\sqrt{2}}{2} = a^2\frac{1}{4} \Rightarrow a = 2\sqrt{2}$

$b^2 = a^2 - c^2 = a^2 - \frac{a^4}{16} = 4$

$a^2 \geq b^2 \Rightarrow$ иск.: $\frac{x^2}{8} + \frac{y^2}{4} = 1$

№7.28 хорды: $x + 2y = C$. концы хорд:

$(x_1,y_1), (x_2,y_2)$: $\frac{(C-2y)^2}{25} + \frac{y^2}{9} = 1 \Rightarrow$ по Т Виета

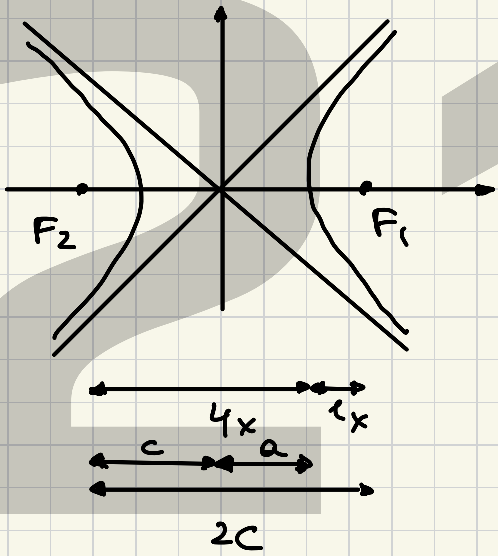
$\frac{1}{2}(y_1+y_2) = \frac{18}{61}C \Rightarrow \frac{1}{2}(x_1+x_2) = \frac{25}{61}C$. $(0;0)$ - центр

симм. линии $\Rightarrow$ середина любой хорды $\Rightarrow (0;0) \in$

иск. прямой => иск. прямая: $18x-25y=0$

№ 7.38 (4, 7)

4) $b=1$



$$\frac{2c-(c+a)}{c+a}=\frac{c-a}{c+a}=\frac{1}{4}\Rightarrow$$

$$\Rightarrow 4c-4a=c+a\Rightarrow$$

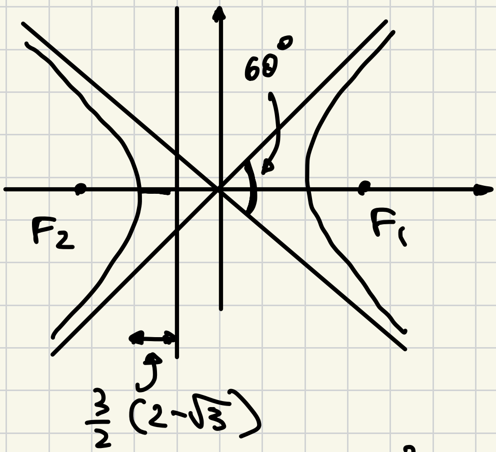
$$\Rightarrow \varepsilon=\frac{c}{a}=\frac{5}{3}$$

$$c^2=b^2+a^2=1+a^2$$

$$\frac{25}{9}a^2=1+a^2\Rightarrow a^2=\frac{9}{16}\Rightarrow$$

$\Rightarrow$ иск. $\frac{16x^2}{9}-y^2=1$

7)



$$\frac{3}{2}(2-\sqrt{3})=a-\frac{a}{\varepsilon}$$

$\frac{bx}{a}=y$ - асимптота

$$\frac{b}{a}=\mathrm{tg}\,30^\circ=\frac{\sqrt{3}}{3}\Rightarrow$$

$$\Rightarrow b=\frac{\sqrt{3}}{3}a$$

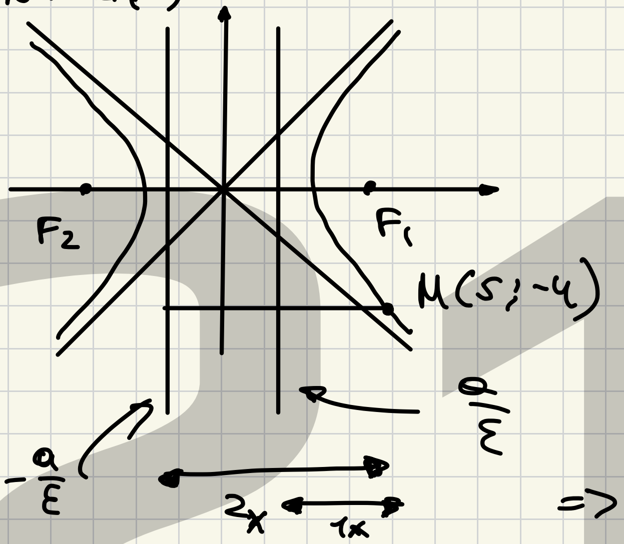
$$c^2=b^2+a^2=a^2\left(\frac{1}{3}+1\right)=\frac{4}{3}a^2$$

$$\frac{c^2}{a^2}=\varepsilon^2=\frac{4}{3}\Rightarrow\varepsilon=\frac{2}{\sqrt{3}}\Rightarrow$$

$$\Rightarrow a-\frac{a}{\varepsilon}=a\left(1-\frac{\sqrt{3}}{2}\right)=\frac{3}{2}(2-\sqrt{3})\Rightarrow a=3\Rightarrow b=\sqrt{3}\Rightarrow$$

$\Rightarrow$ иск. $\frac{x^2}{9}-\frac{y^2}{3}=1$

№7.41(1)



$$\frac{5-\frac{a}{\varepsilon}}{5+\frac{a}{\varepsilon}} = \frac{1}{2} \Rightarrow$$

$$\Rightarrow 10 - 2\frac{a}{\varepsilon} = 5 + \frac{a}{\varepsilon} \Rightarrow$$

$$\Rightarrow 5 = 3\frac{a}{\varepsilon} = 3\frac{a^2}{c} \Rightarrow c = \frac{3}{5}a^2.$$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \Rightarrow$$

$$\Rightarrow \frac{25}{a^2} - \frac{16}{c^2-a^2} = 1 \Rightarrow$$

$$\Rightarrow \frac{25}{a^2} - \frac{16}{\frac{9}{25}a^4 - a^2} = 1;$$

$$9a^4 - 25a^2 - 16a^2 = \frac{9}{25}a^6 - a^4 \quad \Big| : a^2 \ (a \neq 0)$$

$$\frac{9}{25}a^4 - 10a^2 - 41 = 0$$

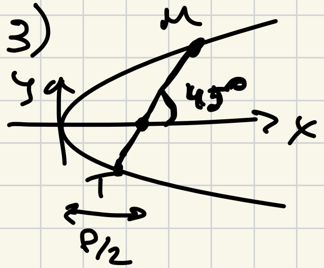
$$a^2 = 5 \text{ или } a^2 = \frac{205}{9} \Rightarrow \varepsilon = \frac{3}{5}a = \frac{3}{5}\sqrt{5};\ \sqrt{\frac{41}{5}}$$

№7.54(1,2,3)

1) $y^2 = 2px \Rightarrow 25 = 2p\cdot 5 \Rightarrow p = \frac{5}{2} \Rightarrow$ иск. $y^2 = 5x$

2) $F(\frac{p}{2}, 0)$ дир.: $x = -p/2 \Rightarrow p = 12 \Rightarrow$ иск. $y^2 = 24x$

3)



$$\begin{cases} x_M + p/2 = (x_M - p/2)\dfrac{1}{\sin 45^\circ} \\ x_T + p/2 = (p/2 - x_T)\dfrac{1}{\sin 45^\circ} \\ x_M - x_T = 18\cos 45^\circ \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} x_M = \frac{p}{2}(3+2\sqrt{2}) \\ x_T = \frac{p}{2}(3-2\sqrt{2}) \\ x_M - x_T = 9\sqrt{2} \end{cases} \Rightarrow p = \frac{9}{2} \Rightarrow$$

$\Rightarrow$ иск. $y^2 = 9x$

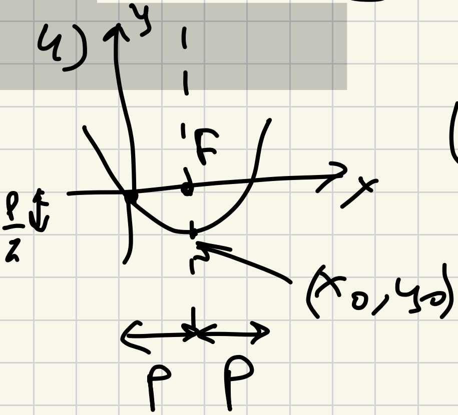
№7.62(2,4)

2) вершина посередине F и дир. $\Rightarrow$

$\Rightarrow$ верш. в т. $(7,5;\ 0)$, $p = 8-7 = 1$

$\Rightarrow$ иск. $y^2 = 2(-x+7,5) = -2x+15$, $x \le 7,5$

4)



$2p = 6 \Rightarrow p = 3$

$(x_0, y_0) = \left(p, -\frac{p}{2}\right) = \left(3, -\frac{3}{2}\right)$

$(x-x_0)^2 = 2p(y-y_0)$

$(x-3)^2 = 6\left(y+\frac{3}{2}\right)$ – иск.