№15.11 (2, 4)

2) Докажем по индукции, что $\begin{Vmatrix}1&1&1\\0&0&0\\0&0&0\end{Vmatrix}^n = \begin{Vmatrix}1&1&1\\0&0&0\\0&0&0\end{Vmatrix}$

База:

$$\begin{Vmatrix}1&1&1\\0&0&0\\0&0&0\end{Vmatrix}^2 = \begin{Vmatrix}1&1&1\\0&0&0\\0&0&0\end{Vmatrix}\begin{Vmatrix}1&1&1\\0&0&0\\0&0&0\end{Vmatrix} = \begin{Vmatrix}1&1&1\\0&0&0\\0&0&0\end{Vmatrix}$$

Шаг:

$$\begin{Vmatrix}1&1&1\\0&0&0\\0&0&0\end{Vmatrix}^n = \begin{Vmatrix}1&1&1\\0&0&0\\0&0&0\end{Vmatrix}^{n-1}\begin{Vmatrix}1&1&1\\0&0&0\\0&0&0\end{Vmatrix} = \begin{Vmatrix}1&1&1\\0&0&0\\0&0&0\end{Vmatrix}^2 = \begin{Vmatrix}1&1&1\\0&0&0\\0&0&0\end{Vmatrix} \blacksquare$$

4) $\begin{Vmatrix}1&-1\\1&-1\end{Vmatrix}^n$

$$\begin{Vmatrix}1&-1\\1&-1\end{Vmatrix}^2 = \begin{Vmatrix}0&0\\0&0\end{Vmatrix} = 0 \Rightarrow$$

$$\Rightarrow \forall n \geq 2 \quad \begin{Vmatrix}1&-1\\1&-1\end{Vmatrix}^n = \begin{Vmatrix}0&0\\0&0\end{Vmatrix} = 0$$

№15.18 (2)

$$A = \begin{Vmatrix}0&-1\\1&0\end{Vmatrix} \quad B = \begin{Vmatrix}1&-1\\1&1\end{Vmatrix}$$

$$AB = \begin{Vmatrix}-1&-1\\1&-1\end{Vmatrix}; \quad BA = \begin{Vmatrix}-1&-1\\1&-1\end{Vmatrix}$$

$$[A, B] = AB - BA = \begin{Vmatrix}0&0\\0&0\end{Vmatrix} = 0$$

№15.22 (2,5)

2) $f(A) = (A-E)^2 = \begin{Vmatrix} 0 & 1 \\ 0 & 0 \end{Vmatrix}^2 = \begin{Vmatrix} 0 & 0 \\ 0 & 0 \end{Vmatrix} = 0$

5) $A^2 = \begin{Vmatrix} -1 & 1 & 1 \\ -5 & 21 & 17 \\ 6 & -26 & -21 \end{Vmatrix}^2 = \begin{Vmatrix} 2 & -6 & -5 \\ 2 & -6 & -5 \\ -2 & 6 & 5 \end{Vmatrix}$

$f(A) = A^2 + A + E = \begin{Vmatrix} 2 & -6 & -5 \\ 2 & -6 & -5 \\ -2 & 6 & 5 \end{Vmatrix} + \begin{Vmatrix} -1 & 1 & 1 \\ -5 & 21 & 17 \\ 6 & -26 & -21 \end{Vmatrix} + \begin{Vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{Vmatrix} =$

$= \begin{Vmatrix} 2 & -5 & -4 \\ -3 & 16 & 12 \\ 4 & -20 & -15 \end{Vmatrix}$

№15.23 (1)

$f(A) = A(A-E) = \begin{Vmatrix} -1 & 4 & 3 \\ -2 & 5 & 3 \\ 2 & -4 & -2 \end{Vmatrix} \begin{Vmatrix} -2 & 4 & 3 \\ -2 & 4 & 3 \\ 2 & -4 & -3 \end{Vmatrix} = \begin{Vmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{Vmatrix} = 0$

№15.24 (1,3,4)

1) $(A+B)^2 = A^2 + AB + BA + B^2 \not\equiv A^2 + 2AB + B^2$ $(\exists A,B : AB \neq BA)$

3) $(A+B)(A-B) = A^2 - AB + BA - B^2 \not\equiv A^2 - B^2$ $(\exists A,B : AB \neq BA)$

4) $(A+E)^3 = (A+E)(A+E)(A+E) = (A^2 + AE + EA + E^2)\cdot$

$\cdot (A+E) = (A^2 + 2A + E)(A+E) = A^3 + 2A^2 + EA + EA^2 + 2AE +$

$EA = AE = A, \; E^2 = E$

$+ EA + E^2 = A^3 + 3A^2 + 3A + E \Rightarrow (A+E)^3 \equiv A^3 + 3A^2 + 3A + E$

№15.45 (2,7,9)

2) $\begin{Vmatrix} 2 & -1 & 0 \\ 0 & 2 & -1 \\ -1 & -1 & 1 \end{Vmatrix}^{-1} = A^{-1}$

$\det A = 2 - 1 = 1$

$M_{11} = 1 \quad M_{12} = 1 \quad M_{13} = 2$

$M_{21} = 1 \quad M_{22} = 2 \quad M_{23} = 3$

$M_{31} = 1 \quad M_{32} = 2 \quad M_{33} = 4$

$A^{-1} = \begin{Vmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 2 & 3 & 4 \end{Vmatrix}$

7) $\begin{Vmatrix} 1 & 2 & 0 \\ 2 & 5 & -2 \\ 0 & -2 & 5 \end{Vmatrix}^{-1} = A^{-1}$

$\det A = 21 - 20 = 1$

$M_{11} = 21 \quad M_{12} = -10 \quad M_{13} = -4$

$M_{21} = -10 \quad M_{22} = 5 \quad M_{23} = 2$

$M_{31} = -4 \quad M_{32} = 2 \quad M_{33} = 1$

$A^{-1} = \begin{Vmatrix} 21 & -10 & -4 \\ -10 & 5 & 2 \\ -4 & 2 & 1 \end{Vmatrix}$

9) $\begin{Vmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{Vmatrix}^{-1} = A^{-1}$

$\det A = -3-12-12 = -27$

$M_{11} = -3 \quad M_{12} = -6 \quad M_{13} = -6$

$M_{21} = -6 \quad M_{22} = -3 \quad M_{23} = 6$

$M_{31} = -6 \quad M_{32} = 6 \quad M_{33} = -3$

$$A^{-1} = \frac{1}{9}\begin{Vmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{Vmatrix}$$

№15.54 (3)

$$\begin{Vmatrix} 1 & 1 & \dots & 1 \\ 0 & 1 & \dots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \end{Vmatrix}^{-1} = A^{-1}$$

$\det A = 1$

$$i = j \text{ или } i = j+1 \Rightarrow M_{ij} = \begin{vmatrix} 1 & 1 & \dots & 1 \\ 0 & 1 & \dots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \end{vmatrix} = 1$$

В остальных случаях

$$M_{ij} = \begin{vmatrix} 0 & 1 & 1 & \dots & 1 \\ 0 & 0 & 1 & \dots & 1 \\ \vdots & \vdots & & \ddots & \vdots \\ 0 & 0 & & \dots & 1 \end{vmatrix} = 0 \quad \text{или } M_{ij} = \begin{vmatrix} 1 & 1 & \dots & 1 \\ & \ddots & 0 & \vdots \\ \vdots & & \ddots & \vdots \\ 0 & 0 & 0 \dots & 1 \end{vmatrix} = 0$$

$$A^{-1} = \begin{Vmatrix} 1 & -1 & 0 & \dots & 0 \\ 0 & 1 & -1 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \end{Vmatrix}$$

№15.65 (4, 5, 6)

4) $\left\|\begin{matrix} 1 & 2 & 0 \\ 2 & 5 & -2 \\ 0 & -2 & 5 \end{matrix}\right\| X = \left\|\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 0 \end{matrix}\right\|$

$$X = \left\|\begin{matrix} 1 & 2 & 0 \\ 2 & 5 & -2 \\ 0 & -2 & 5 \end{matrix}\right\|^{-1} \left\|\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 0 \end{matrix}\right\| = \left\|\begin{matrix} 21 & -10 & -4 \\ -10 & 5 & 2 \\ -4 & 2 & 1 \end{matrix}\right\| \left\|\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 0 \end{matrix}\right\| =$$

15.45(7)

$$= \left\|\begin{matrix} 21 & -14 & -10 \\ -10 & 7 & 5 \\ -4 & 3 & 2 \end{matrix}\right\|$$

5) $X \left\|\begin{matrix} 2 & 2 & -1 \\ 2 & -1 & 2 \\ -1 & 2 & 2 \end{matrix}\right\| = \left\|\begin{matrix} 5 & 5 & 2 \\ 5 & 8 & -1 \end{matrix}\right\|$

$$XA = B$$

$$\det A = -12 - 12 - 3 = -27$$

$M_{11} = -6 \quad M_{12} = -6 \quad M_{13} = 3$

$M_{21} = -6 \quad M_{22} = 3 \quad M_{23} = -6$

$M_{31} = 3 \quad M_{32} = -6 \quad M_{33} = -6$

$$A^{-1} = \frac{1}{9} \left\|\begin{matrix} 2 & 2 & -1 \\ 2 & -1 & 2 \\ -1 & 2 & 2 \end{matrix}\right\|$$

$$X = BA^{-1} = \left\|\begin{matrix} 5 & 5 & 2 \\ 5 & 8 & -1 \end{matrix}\right\| \frac{1}{9} \left\|\begin{matrix} 2 & 2 & -1 \\ 2 & -1 & 2 \\ -1 & 2 & 2 \end{matrix}\right\| =$$

$$= \frac{1}{9}\begin{Vmatrix} 18 & 9 & 9 \\ 27 & 0 & 9 \end{Vmatrix} = \begin{Vmatrix} 2 & 1 & 1 \\ 3 & 0 & 1 \end{Vmatrix}$$

б) $$X\begin{Vmatrix} 1 & 1 \\ 1 & 1 \end{Vmatrix} = \begin{Vmatrix} 1 & -1 \\ 1 & -1 \end{Vmatrix}$$

$$\exists\ X = \begin{Vmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{Vmatrix}$$

$$X\begin{Vmatrix} 1 & 1 \\ 1 & 1 \end{Vmatrix} = \begin{Vmatrix} x_{11}+x_{12} & x_{11}+x_{12} \\ x_{21}+x_{22} & x_{21}+x_{22} \end{Vmatrix} = \begin{Vmatrix} 1 & -1 \\ 1 & -1 \end{Vmatrix} \Rightarrow$$

$$\Rightarrow \begin{cases} x_{11}+x_{12} = 1 \\ x_{11}+x_{12} = -1 \end{cases} \Rightarrow 1 = -1 \text{ ?!} \Rightarrow \nexists\ X \Rightarrow$$

$\Rightarrow$ нет. реш.