2(3) $f = \ln(x^2+y)$ в $(0;1)$

$\frac{\partial f}{\partial x} = \frac{2x}{x^2+y}$; $\frac{\partial f}{\partial y} = \frac{1}{x^2+y}$ - непрерывны в окр-ти $(0;1) \Rightarrow \frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$

$$\frac{\partial^2 f}{\partial x \partial y}(0;1) = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = \frac{-x^2 \cdot 2x}{(x^2+y)^2}\Big|_{(0;1)} = 0 = \frac{\partial^2 f}{\partial y \partial x}$$

$$\frac{\partial^2 f}{\partial x^2}(0;1) = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right)\Big|_{(0;1)} = \frac{2(x^2+y) - 2x \cdot 2x}{(x^2+y)^2}\Big|_{(0;1)} = \frac{2y-2x^2}{(x^2+y^2)^2}\Big|_{(0;1)} = 2$$

$$\frac{\partial^2 f}{\partial y^2}(0;1) = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial y}\right)\Big|_{(0,1)} = -\frac{1}{(x^2+y)^2}\Big|_{(0;1)} = -1$$

$$d^2 f = 2dx^2 - dy^2$$

4. $f = \begin{cases} xy(x^2-y^2)/(x^2+y^2), & x^2+y^2 \neq 0 \\ 0, & x^2+y^2 = 0 \end{cases}$

$$\frac{\partial f}{\partial x} = \left(\frac{xy(x^2-y^2)}{x^2+y^2}\right)'_x = \frac{(3yx^2-y^3)(x^2+y^2) - 2x^2y(x^2-y^2)}{(x^2+y^2)^2} = \frac{x^4y+4x^2y^3-y^5}{(x^2+y^2)^2}$$

$$\frac{\partial f}{\partial x}(0;0) = \lim_{x \to 0} \frac{f(x,0)-f(0,0)}{x} = 0$$

$$\frac{\partial f}{\partial y} = \frac{x^5-y^4x-4y^2x^3}{(x^2+y^2)^2}$$ (симметрично $\frac{\partial f}{\partial x}$, но с минусом)

$$\frac{\partial f}{\partial y}(0;0) = \lim_{y \to 0} \frac{f(0,y)-f(0,0)}{y} = 0$$

$$\frac{\partial^2 f}{\partial x \partial y}(0;0) = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right)\Big|_{(0;0)} = \lim_{y \to 0} \frac{\frac{\partial f}{\partial x}(0;y) - \frac{\partial f}{\partial x}(0;0)}{y} = \lim_{y \to 0} \frac{-y^5}{y^5} = -1$$

$$\frac{\partial^2 f}{\partial y \partial x}(0;0) = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right)\Big|_{(0;0)} = \lim_{x \to 0} \frac{\frac{\partial f}{\partial y}(x;0) - \frac{\partial f}{\partial y}(0;0)}{x} = \lim_{x \to 0} \frac{x^5}{x^5} = 1$$

15(2) $f = e^{x^2/y}$ в т. $(1;1)$

$\frac{\partial f}{\partial x} = e^{x^2/y} \cdot \frac{2x}{y}$; $\frac{\partial f}{\partial y} = e^{x^2/y}\left(-\frac{x^2}{y^2}\right)$ - непр. в окр-ти $(1;1) \Rightarrow \frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$

$$\frac{\partial^2 f}{\partial x^2}(1;1) = \left(e^{\frac{x^2}{y}} \cdot \frac{4x^2}{y^2} + \frac{2}{y} e^{x^2/y}\right)\Big|_{(1;1)} = 6e$$

$$\frac{\partial^2 f}{\partial x \partial y}(1;1) = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right)(1;1) = \left(e^{x^2/y}\left(-\frac{x^2}{y^2} \cdot \frac{2x}{y}\right) - \frac{2x}{y^2} e^{x^2/y}\right)\Big|_{(1;1)} = -4e$$

$$\frac{\partial^2 f}{\partial y^2}(1;1) = \left(e^{x^2/y}\frac{x^4}{y^4} + \frac{2x^2}{y^3}e^{x^2/y}\right)\Bigg|_{(1;1)} = 3e$$

$$d^2 f = 6e\,dx^2 - 8e\,dx\,dy + 3e\,dy^2$$

71(2,4)

2) $f(x;y) = \operatorname{arctg}\frac{1+x}{1+y}$ в окр-ти $(0;0)$

$$\frac{\partial f}{\partial x} = \frac{1}{1+\left(\frac{1+x}{1+y}\right)^2}\frac{1}{1+y} = \frac{1+y}{(1+y)^2+(1+x)^2};\quad \frac{\partial f}{\partial x}(0;0) = \frac{1}{2}$$

$$\frac{\partial f}{\partial y} = \frac{1}{1+\left(\frac{1+x}{1+y}\right)^2}\left(-\frac{1+x}{(1+y)^2}\right) = \frac{-(1+x)}{(1+x)^2+(1+y)^2};\quad \frac{\partial f}{\partial y}(0;0) = -\frac{1}{2}$$

$$\frac{\partial^2 f}{\partial x^2} = -(1+y)\frac{2(1+x)}{((1+x)^2+(1+y)^2)^2};\quad \frac{\partial^2 f}{\partial x^2}(0;0) = -\frac{1}{2}$$

$$\frac{\partial^2 f}{\partial y^2} = (1+x)\frac{2(1+y)}{((1+x)^2+(1+y)^2)^2};\quad \frac{\partial^2 f}{\partial y^2}(0;0) = \frac{1}{2}$$

$$\frac{\partial^2 f}{\partial x\partial y} = \frac{(1+x)^2+(1+y)^2-2(1+y)^2}{((1+x)^2+(1+y)^2)^2};\quad \frac{\partial^2 f}{\partial x\partial y}(0;0) = 0$$

$$f(x,y) = \frac{\pi}{4} + \frac{1}{2}x - \frac{1}{2}y + \frac{1}{2!}\left(-\frac{1}{2}x^2 + \frac{1}{2}y^2\right) + \tilde{o}(\rho^2) = \frac{\pi}{4} + \frac{1}{2}x - \frac{1}{2}y - \frac{1}{4}x^2 + \frac{1}{4}y^2 + \tilde{o}(\rho^2),\ \rho\to 0$$

4) $f(x,y) = \operatorname{arctg}(x^2y - 2e^{x-1})$ в окр-ти $(1;3)$

$$\frac{\partial f}{\partial x}(1;3) = \frac{2xy - 2e^{x-1}}{1+(x^2y-2e^{x-1})^2} = \frac{6-2}{2} = 2;\quad \frac{\partial f}{\partial y}(1;3) = \frac{x^2}{1+(x^2y-2e^{x-1})^2}\Bigg|_{(1;3)} = \frac{1}{2}$$

$$\frac{\partial^2 f}{\partial x^2}(1;3) = \left(\frac{(2y-2e^{x-1})(1+(x^2y-2e^{x-1})^2) - 2(x^2y-2e^{x-1})(2xy-2e^{x-1})}{(1+(x^2y-2e^{x-1})^2)^2}\right)\Bigg|_{(1;3)} = -6$$

$$\frac{\partial^2 f}{\partial y^2}(1;3) = \frac{-x^2\cdot x^2}{(1+(x^2y-2e^{x-1})^2)^2}\,2(x^2y-2e^{x-1}) = -\frac{1}{2}$$

$$\frac{\partial^2 f}{\partial x\partial y} = \frac{\partial^2 f}{\partial y\partial x}\ \left(\text{т.к. } \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \text{ непр. в окр-ти } (1;3)\right) = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right)(1;3) =$$

$$= \frac{2x(1+(x^2y-2e^{x-1})^2) - x^2\cdot 2(x^2y-2e^{x-1})(2xy-2e^{x-1})}{(1+(x^2y-2e^{x-1})^2)^2}\Bigg|_{(1;3)} = -1$$

$$f(x,y) = \frac{\pi}{4} + 2(x-1) + \frac{1}{2}(y-3) + \frac{1}{2!}\left(-6(x-1)^2 - \frac{1}{4}(y-3)^2 - 2(x-1)(y-3)\right) +$$

$$+ \tilde{o}(\rho^2) = \frac{\pi}{4} + 2(x-1) + \frac{1}{2}(y-3) - 3(x-1)^2 - \frac{1}{4}(y-3)^2 - (x-1)(y-3) + \tilde{o}(\rho^2),\ \rho \to 0$$

74(4,5) до $\tilde{o}(\rho^4)$

4) $f = \sin x / \cos y = \dfrac{x - \frac{x^3}{6} + \tilde{o}(x^4)}{1 - \frac{1}{2}y^2 + \tilde{o}(y^3)} = \left(x - \frac{1}{6}x^3 + \tilde{o}(x^4)\right)\left(1 + \frac{1}{2}y^2 + \tilde{o}(y^3)\right) =$

$= 1 + \frac{1}{2}xy^2 - \frac{1}{6}x^3 + \tilde{o}(\rho^4),\ \rho \to 0$

5) $f = e^x \sin y = \left(1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \tilde{o}(x^3)\right)\left(y - \frac{1}{3}y^3 + \tilde{o}(y^4)\right) = y + xy - \frac{1}{6}y^3 + \frac{1}{2}x^2y - \frac{1}{6}xy^3 + \frac{1}{6}x^3y + \tilde{o}(\rho^4),\ \rho \to 0$