№3.2(2)

2) $[\bar a-\bar b+\frac{\bar c}{2}, -\bar a+2\bar b-5\bar c] = [\bar a,-\bar a]+[\bar a,2\bar b]-[\bar a,5\bar c]+[\bar b,\bar a]+[\bar b,2\bar b]+[\bar b,5\bar c]+[\frac{\bar c}{2},-\bar a]+[\frac{\bar c}{2},2\bar b]+[\frac{\bar c}{2},-5\bar c] = 2[\bar a,\bar b]-5[\bar a,\bar c]-[\bar a,\bar b]+5[\bar b,\bar c]+[\bar a,\frac{\bar c}{2}]-[\bar b,\bar c] = [\bar a,\bar b]-\frac{9}{2}[\bar a,\bar c]+4[\bar b,\bar c]$

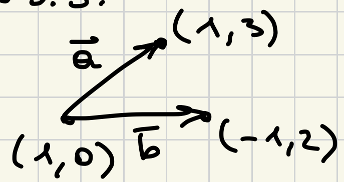
№3.6. $\bar a=[\bar b,\bar c] \Rightarrow (\bar a,\bar b)=([\bar b,\bar c],\bar b)=0,$ $(\bar a,\bar c)=([\bar b,\bar c],\bar c)=0 \Rightarrow \angle(\bar a,\bar b)=\angle(\bar a,\bar c)=\frac{\pi}{2}$

Аналогично с другими => попарно перпен.

$(\bar a,\bar a)=(\bar a,\bar b,\bar c)=(\bar b,\bar b)=(\bar c,\bar c)\Rightarrow|\bar a|=|\bar b|=|\bar c| \Rightarrow |\bar a|=|[\bar b,\bar c]|=|\bar b||\bar c|\sin\frac{\pi}{2}=|\bar a|^2 \Rightarrow$

$\Rightarrow |\bar a|=1=|\bar b|=|\bar c|$. Либо $\vec a=\vec b=\vec c=\vec 0$.

№3.9.



$S=[\bar a,\bar b]=[3\bar e_2,-2\bar e_1+2\bar e_2] =$

$= 6[\bar e_2,-\bar e_1]+6[\bar e_2,\bar e_2] =$

$= 6[e_1,e_2]=6\cdot3\cdot2\sin30^\circ=18$

№3.12 $\boxed{\Rightarrow}$

$$\begin{cases}[\bar a,\bar b]=[\bar b,\bar c]\\ [\bar b,\bar c]=[\bar c,\bar a]\\ [\bar c,\bar a]=[\bar a,\bar b]\end{cases} \Leftrightarrow \begin{cases}[\bar a+\bar c,\bar b]=\bar 0\\ [\bar b+\bar a,\bar c]=\bar 0\\ [\bar c+\bar b,\bar a]=\bar 0\end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} [\bar a+\bar b+\bar c, \bar b] = [\bar b,\bar b]=\bar 0 \\ [\bar b+\bar a+\bar c, \bar c] = [\bar c,\bar c]=\bar 0 \\ [\bar c+\bar b+\bar a, \bar a]=[\bar a,\bar a]=\bar 0\end{cases} \Rightarrow$$

$\Rightarrow$ либо $\bar a+\bar b+\bar c \parallel \bar b,\ \parallel \bar c,\ \parallel \bar a \Rightarrow$

$\Rightarrow \bar a \parallel \bar b \parallel \bar c$, что неверно по условию $\Rightarrow$

$\Rightarrow \bar a+\bar b+\bar c=\bar 0$ ∎

$\boxed{\Leftarrow}$ $\bar a+\bar b=-\bar c$

$$[\bar b,\bar c] = -[\bar b, \bar a+\bar b] = -[\bar b,\bar a]-[\bar b,\bar b] =$$
$= [\bar a,\bar b]$ аналогично с остальными ∎

№ 3.13(1, 2)

1) $$([\vec a,\vec b],[\vec a,\vec b]) = (\vec a, [\vec b, [\vec a,\vec b]]) =$$
$$=(\vec a, \vec a(\vec b,\vec b)-\vec b(\vec a,\vec b)) = (\vec a,\vec a)(\vec b,\vec b)-(\vec a,\vec b)\cdot$$
$$\cdot(\vec a,\vec b) = \begin{vmatrix} (\bar a,\bar a) & (\bar a,\bar b) \\ (\bar a,\bar b) & (\bar b,\bar b)\end{vmatrix}$$

2) Д-ть: $[\bar a,[\bar b,\bar c]] = \bar b(\bar a,\bar c)-\bar c(\bar a,\bar b)$

Выберем правый О.Н.Б. $\{\bar e_1,\bar e_2,\bar e_3\}$ такой, что $\bar a = a_1\bar e_1+a_2\bar e_2+a_3\bar e_3$; $\bar b=b_1\bar e_1+b_2\bar e_2$; $\bar c=c_1\bar e_1$.

$$[\bar a,[\bar b,\bar c]] = [\bar a, b_2c_1[\bar e_2,\bar e_1]] = [\bar a, -b_2c_1\bar e_3] =$$
$$= -a_1b_2c_1[\bar e_1,\bar e_3]-a_2b_2c_1[\bar e_2,\bar e_3] =$$

$= a_1 b_2 c_1 \bar{e}_2 - a_2 b_2 c_1 \bar{e}_1$

$\bar{b}(\bar{a},\bar{c}) - \bar{c}(\bar{a},\bar{b}) = (b_1\bar{e}_1 + b_2\bar{e}_2)\cdot a_1 c_1 -$

$- c_1\bar{e}_1(a_1 b_1 + a_2 b_2) = a_1 b_1 c_1 \bar{e}_1 + a_1 b_2 c_1 \bar{e}_2 -$

$- a_1 b_1 c_1 \bar{e}_1 - a_2 b_2 c_1 \bar{e}_1 = a_1 b_2 c_1 \bar{e}_2 - a_2 b_2 c_1 e_1$

Л.ч. = П.ч. ■

№ 3.19(2)

$(\bar{a},\bar{b},\bar{c}) = \begin{vmatrix} 3 & 5 & 1 \\ 4 & 0 & -1 \\ 2 & 1 & 1 \end{vmatrix} = 3 - 5(4+2) + 4 = -23$

№ 3.21

В базисе $\{\bar{a},\bar{b},\bar{c}\}$ смешанное произв.:

$\begin{vmatrix} 1 & 2 & \lambda \\ 4 & 5 & 6 \\ 7 & 8 & \lambda^2 \end{vmatrix} (\bar{a},\bar{b},\bar{c}) = 0$

$(\bar{a},\bar{b},\bar{c}) \neq 0 \Rightarrow 5\lambda^2 - 48 - 2(4\lambda^2 - 42) + \lambda(32-35) =$

$= -3\lambda^2 - 3\lambda + 36 = 0$

$\lambda^2 + \lambda - 12 = (\lambda+4)(\lambda-3) = 0$

$\lambda = -4$ или $\lambda = 3$.

№3.24.

$(\bar{e}_1,\bar{e}_2,\bar{e}_3)^2 = |G_e| = \begin{vmatrix} (\bar{e}_1,\bar{e}_1) & (\bar{e}_1,\bar{e}_2) & (\bar{e}_1,\bar{e}_3) \\ (\bar{e}_2,\bar{e}_1) & (\bar{e}_2,\bar{e}_2) & (\bar{e}_2,\bar{e}_3) \\ (\bar{e}_3,\bar{e}_1) & (\bar{e}_3,\bar{e}_2) & (\bar{e}_3,\bar{e}_3) \end{vmatrix} =$

$$= \begin{vmatrix}1&-1&1\\-1&4&-2\\1&-2&2\end{vmatrix} = 8-4+(-2+2)+(2-4)=2 \Rightarrow$$

$$\Rightarrow (\bar e_1,\bar e_2,\bar e_3)=\sqrt{2}$$

$$V_{пар} = \left| \begin{vmatrix}-1&0&2\\1&1&3\\2&-1&1\end{vmatrix} (\bar e_1,\bar e_2 \bar e_3)\right| = \left|\left(-(1+3)+2(-1-2)\right)\sqrt{2}\right| =$$

$$= 10\sqrt{2}$$

№ 3.26 (3)

$$[[\bar a,\bar b],[\bar c,\bar d]] = \bar c([\bar a,\bar b],\bar d) -$$

$$- \bar d([\bar a,\bar b],\bar c) = \bar c(\bar a,\bar b,\bar d)-\bar d(\bar a,\bar b,\bar c)$$

$$[[\bar a,\bar b],[\bar c,\bar d]] = -[[\bar c,\bar d],[\bar a,\bar b]] =$$

$$= - \bar a([\bar c,\bar d],\bar b) - \bar b([\bar c,\bar d],\bar a) = -\bar a(\bar c,\bar d,\bar b) -$$

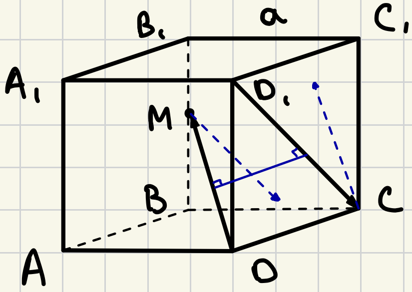
$$- \bar b(\bar c,\bar d,\bar a) = -\bar a(\bar b,\bar c,\bar d) - \bar b(\bar c,\bar d,\bar a)$$

$$\bar d(\bar a,\bar b,\bar c) - \bar c(\bar a,\bar b,\bar d) - \bar a(\bar b,\bar c,\bar d) - \bar b(\bar c,\bar d,\bar a) =$$

$$= -[[\bar a,\bar b],[\bar c,\bar d]] + [[\bar a,\bar b],[\bar c,\bar d]] = 0 \Rightarrow$$

$$\Rightarrow \bar d(\bar a,\bar b,\bar c) = \bar a(\bar b,\bar c,\bar d)+\bar b(\bar c,\bar a,\bar d)+\bar c(\bar a,\bar b,\bar d) \blacksquare$$

№ 3.32



$$V_{\langle \overline{DM},\overline{D_1C},\overline{DD_1}\rangle} = |(\overline{DM},\overline{D_1C},\overline{DD_1})|$$

$$\rho(DM,D_1C) = \frac{V_{\langle \overline{DM},\overline{D_1C},\overline{DD_1}\rangle}}{S_{осн}}$$

$$S_{осн} = |[\overline{DM},\overline{D_1C}]| \Rightarrow$$

$$\rho(DM, D_1C) = \frac{|(\overline{DM}, \overline{D_1C}, \overline{DD_1})|}{|[\overline{DM}, \overline{D_1C}]|}$$

В правом о.к.б. $\{\overline{AD}, \overline{AB}, \overline{AA_1}\}$: $\overline{DM} = -\overline{AD} + \overline{AB} + \frac{2}{3}\overline{AA_1}$; $\overline{D_1C} = \overline{AB} - \overline{AA_1}$; $\overline{DD_1} = \overline{AA_1}$

$$(\overline{DM}, \overline{D_1C}, \overline{DD_1}) = \begin{vmatrix} -1 & 1 & \frac{2}{3} \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{vmatrix} (\overline{AD}, \overline{AB}, \overline{AA_1}) =$$

$$= -a^3$$

$$[\overline{DM}, \overline{D_1C}] = [-\overline{AD} + \overline{AB} + \frac{2}{3}\overline{AA_1}, \overline{AB} - \overline{AA_1}] =$$

$$= -[\overline{AD}, \overline{AB}] + [\overline{AD}, \overline{AA_1}] + [\overline{AB}, \overline{AB}] - \frac{5}{3}[\overline{AB}, \overline{AA_1}] - \frac{2}{3}[\overline{AA_1}, \overline{AA_1}] = (-\overline{AA_1} - \overline{AB} - \frac{5}{3}\overline{AD}) \cdot a$$

$$|[\overline{DM}, \overline{D_1C}]| = a^2\sqrt{1+1+\frac{25}{9}} = a^2\frac{\sqrt{44}}{3} = \frac{\sqrt{43}}{3} \cdot a \Rightarrow$$

$$\Rightarrow \rho(DM, DC) = \frac{a^3}{\frac{\sqrt{43}}{3}a} = \frac{3}{\sqrt{43}}a^2$$

Т.3. $\begin{vmatrix} 1 & 1 & 1 \\ 2 & 0 & 1 \\ 3 & 1 & \lambda \end{vmatrix} = -1 - (2\lambda - 3) + 2 = 4 - 2\lambda \neq 0 \Rightarrow \lambda \neq 2$

Т.4. $[\overline{a}, [\overline{a}, \overline{x}]] = \overline{x} + \overline{a}$

$\vec{a}(\vec{a}, \vec{x}) - \vec{x}a^2 = \overline{x} + \overline{a}$

$\vec{a}((\overline{a}, \overline{x}) - 1) = \overline{x}(1 + a^2) \Rightarrow \overline{x} = \lambda\overline{a} \Rightarrow$

$\Rightarrow [\overline{a}, [\overline{a}, \lambda\overline{a}]] = \overline{0} = \overline{x} + \overline{a} \Rightarrow \overline{x} = -\overline{a}.$