№14.4 (5,6)

5) $\begin{vmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{vmatrix} = \cos^2\alpha - (-\sin\alpha)\sin\alpha = 1$

6) $\begin{vmatrix} 13547 & 13647 \\ 28423 & 28523 \end{vmatrix} = \begin{vmatrix} 13547 & 100 \\ 28423 & 100 \end{vmatrix} = (13547 - 28423)\cdot$

$\cdot 100 = -1487600$

№14.7 (4,6,11)

4) $\begin{vmatrix} 2 & 2 & -1 \\ 2 & -1 & 2 \\ -1 & 2 & 2 \end{vmatrix} = 2\cdot(-2-4) - 2(4+2) - 1(4-1) = -27$

6) $\begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix} = 1\cdot(15-16) - 2(10-12) + 3(8-9) = 0$

11) $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = bc^2 - b^2c - ac^2 + a^2c + ab^2 - a^2b \;\circleddash$

симметрич. 3 степени $a=b$ решение =>

=> $\circleddash\; (a-b)(b-c)(c-a)$

№14.10 (1)*

1) $\left| \left\| \begin{matrix} 3 & 0 & 0 \\ 2 & 6 & 4 \\ -2 & -3 & -1 \end{matrix} \right\| - \lambda E \right| = 0;$

$\begin{vmatrix} 3-\lambda & 0 & 0 \\ 2 & 6-\lambda & 4 \\ -2 & -3 & -1-\lambda \end{vmatrix} = 0;$

$(3-\lambda)\big((6-\lambda)(-1-\lambda) - (-3)\cdot 4\big) = 0;$

$3-\lambda = 0$ или $(6-\lambda)(-1-\lambda) + 12 = 0;$

$\lambda = 3$ или $\lambda^2 - 5\lambda + 6 = 0;$

$\lambda=3$ или $\lambda=2$ или $\lambda=3$;

<u>Ответ:</u> 2; 3.

№15.2 (1, 3)

1) $3\begin{Vmatrix}1 & 2\\ 1 & 2\end{Vmatrix}-\begin{Vmatrix}3 & 2\\ 3 & 2\end{Vmatrix}-4\begin{Vmatrix}0 & 1\\ 0 & 1\end{Vmatrix}=\begin{Vmatrix}3 & 6\\ 3 & 6\end{Vmatrix}-\begin{Vmatrix}3 & 2\\ 3 & 2\end{Vmatrix}-$

$-\begin{Vmatrix}0 & 4\\ 0 & 4\end{Vmatrix}=\begin{Vmatrix}3-3-0 & 6-2-4\\ 3-3-0 & 6-2-4\end{Vmatrix}=\begin{Vmatrix}0 & 0\\ 0 & 0\end{Vmatrix}$

3) $2\begin{Vmatrix}1 & 8 & 7 & -15\\ 1 & -5 & -6 & 11\end{Vmatrix}-\begin{Vmatrix}5 & 24 & -7 & -1\\ -1 & -2 & 7 & 3\end{Vmatrix}=$

$=\begin{Vmatrix}2-5 & 16-24 & 14-(-7) & -30-(-1)\\ 2-(-1) & -10-(-2) & -12-7 & 22-3\end{Vmatrix}=$

$=\begin{Vmatrix}-3 & -8 & 21 & -29\\ 3 & -8 & -19 & 19\end{Vmatrix}$

№15.5 (1, 2, 11, 14)

1) $\begin{Vmatrix}2 & -3 & 0\end{Vmatrix}\begin{Vmatrix}4\\ 3\\ 1\end{Vmatrix}=\begin{Vmatrix}8-9+0\end{Vmatrix}=\begin{Vmatrix}-1\end{Vmatrix}=-1$

2) $\begin{Vmatrix}4\\ 3\\ 1\end{Vmatrix}\begin{Vmatrix}2 & -3 & 0\end{Vmatrix}=\begin{Vmatrix}8 & -12 & 0\\ 6 & -9 & 0\\ 2 & -3 & 0\end{Vmatrix}$

11) $\begin{Vmatrix}\lambda_1 & & 0\\ & \ddots & \\ 0 & & \lambda_n\end{Vmatrix}\begin{Vmatrix}0 & & \lambda_1\\ & \cdot^{\cdot^{\cdot}} & \\ \lambda_n & & 0\end{Vmatrix}=\begin{Vmatrix}0 & & & \lambda_1^2\\ & & \lambda_2^2 & \\ & \cdot^{\cdot^{\cdot}} & & \\ \lambda_n^2 & & & 0\end{Vmatrix}$

14) $\begin{Vmatrix}0 & 0 & 1\\ 0 & 1 & 0\\ 1 & 0 & 0\end{Vmatrix}\begin{Vmatrix}0 & 0 & 1\\ 0 & 1 & 0\\ 1 & 0 & 0\end{Vmatrix}=\begin{Vmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{Vmatrix}$

№15.11 (3, 4, 7)

3) $$\begin{Vmatrix} 0&1&0&0\\ 0&0&1&0\\ 0&0&0&1\\ 0&0&0&0 \end{Vmatrix}^3 = \begin{Vmatrix} 0&0&1&0\\ 0&0&0&1\\ 0&0&0&0\\ 0&0&0&0 \end{Vmatrix} \cdot \begin{Vmatrix} 0&1&0&0\\ 0&0&1&0\\ 0&0&0&1\\ 0&0&0&0 \end{Vmatrix} =$$

$$= \begin{Vmatrix} 0&0&0&1\\ 0&0&0&0\\ 0&0&0&0\\ 0&0&0&0 \end{Vmatrix}$$

4) $$\begin{Vmatrix} 1&-1\\ 1&-1 \end{Vmatrix}^n = \begin{Vmatrix} 1&-1\\ 1&-1 \end{Vmatrix} \begin{Vmatrix} 1&-1\\ 1&-1 \end{Vmatrix} \begin{Vmatrix} 1&-1\\ 1&-1 \end{Vmatrix}^{n-2} =$$

$$= \begin{Vmatrix} 0&0\\ 0&0 \end{Vmatrix} \begin{Vmatrix} 1&-1\\ 1&-1 \end{Vmatrix}^{n-2} = \begin{Vmatrix} 0&0\\ 0&0 \end{Vmatrix}$$

$$\begin{Vmatrix} 1&-1\\ 1&-1 \end{Vmatrix}^n = \begin{cases} \begin{Vmatrix} 0&0\\ 0&0 \end{Vmatrix}, & n \geq 2 \\ \begin{Vmatrix} 1&-1\\ 1&-1 \end{Vmatrix}, & n = 1 \end{cases}$$

7) $$\begin{Vmatrix} \cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha \end{Vmatrix}^n = \begin{Vmatrix} \cos n\alpha & -\sin n\alpha\\ \sin n\alpha & \cos n\alpha \end{Vmatrix}$$

Докажем по мат. индукции

1) $$\begin{Vmatrix} \cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha \end{Vmatrix}^1 = \begin{Vmatrix} \cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha \end{Vmatrix}$$

2) верно для n

3) $$\begin{Vmatrix} \cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha \end{Vmatrix}^{n+1} = \begin{Vmatrix} \cos n\alpha & -\sin n\alpha\\ \sin n\alpha & \cos n\alpha \end{Vmatrix} \cdot$$

$$\cdot \begin{Vmatrix} \cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha \end{Vmatrix} = \begin{Vmatrix} \cos n\alpha\cos\alpha - \sin n\alpha \sin\alpha & \cos n\alpha(-\sin\alpha) + (-\sin\alpha)\cos n\alpha \\ \sin n\alpha\cos\alpha + \cos n\alpha \sin\alpha & \sin n\alpha(-\sin\alpha) + \cos n\alpha\cos\alpha \end{Vmatrix} =$$

$$= \begin{Vmatrix} \cos(n+1)\alpha & -\sin(n+1)\alpha \\ \sin(n+1)\alpha & \cos(n+1)\alpha \end{Vmatrix} \blacksquare \Rightarrow$$

$$\Rightarrow \begin{Vmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{Vmatrix}^n = \begin{Vmatrix} \cos n\alpha & -\sin n\alpha \\ \sin n\alpha & \cos n\alpha \end{Vmatrix}$$

№ 15.22 (1)

1) $f(t) = t^2 - 2t + 1 = 0$; $A = \begin{Vmatrix} 1 & 1 \\ 1 & 1 \end{Vmatrix}$

$$f(A) = \begin{Vmatrix} 2 & 2 \\ 2 & 2 \end{Vmatrix} - 2\begin{Vmatrix} 1 & 1 \\ 1 & 1 \end{Vmatrix} + 1 \cdot E = \begin{Vmatrix} 2 & 2 \\ 2 & 2 \end{Vmatrix} -$$

$$- \begin{Vmatrix} 2 & 2 \\ 2 & 2 \end{Vmatrix} + 1 \cdot \begin{Vmatrix} 1 & 0 \\ 0 & 1 \end{Vmatrix} = \begin{Vmatrix} 1 & 0 \\ 0 & 1 \end{Vmatrix}$$

Т. 1. $\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31}$

$+ a_{13}a_{21}a_{32} - a_{13}a_{22}a_{31} - a_{12}a_{21}a_{33} -$

$- a_{11}a_{23}a_{32} = t_1 + t_2 + t_3 - t_4 - t_5 - t_6$. Если все одного знака, то $t_1 t_4 < 0$; $t_2 t_5 < 0$;

$$t_3 t_6 < 0 \Rightarrow \begin{cases} a_{11}a_{33}a_{13}a_{31}a_{22}^2 < 0 \\ a_{23}a_{21}a_{31}a_{33}a_{12}^2 < 0 \\ a_{13}a_{11}a_{21}a_{23}a_{32}^2 < 0 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} a_{11}a_{33}a_{13}a_{31} < 0 & (1) \\ a_{23}a_{21}a_{31}a_{33} < 0 & (2) \\ a_{13}a_{11}a_{21}a_{23} < 0 & (3) \end{cases}$$

(1), (3) $\Rightarrow a_{33}a_{31}$ и $a_{21}a_{23}$ одного знака, т.к. при домножении на $a_{11}a_{13}$ оба меньне нуля $\Rightarrow a_{33}a_{31}\cdot a_{21}a_{23} > 0$ – противоречит ② нер-ву $\Rightarrow$ не могут быть одного знака ■

Т. 2. A – ? : $\forall B$ $A\cdot B = B\cdot A$. $\exists A = \begin{Vmatrix} a & b \\ c & d \end{Vmatrix}$, $B = \begin{Vmatrix} e & f \\ g & h \end{Vmatrix}$. Тогда
$$\begin{cases} \cancel{ae}+bg = \cancel{ea}+fc \\ af+bh = eb+fd \\ ce+dg = ga+hc \\ cf+\cancel{hd} = gb+\cancel{hd} \end{cases} \Leftrightarrow$$
$$\Leftrightarrow \begin{cases} bg = fc \\ f(a-d) = b(e-h) \\ g(a-d) = c(e-h) \end{cases}.$$
Равенство $bg = fc$ очевидно не может выполняться $\forall g, f$ если b и $c \neq 0 \Rightarrow b = c = 0 \Rightarrow$

$\Rightarrow f(a-d) = g(a-d) = 0 \Rightarrow a = d \Rightarrow$

$\Rightarrow$ искомые матрицы вида $\begin{Vmatrix} t & 0 \\ 0 & t \end{Vmatrix}$, где t – любое число.

№17.1 (2,4)

2) $\begin{cases} 3x+5y = 2, \\ 5x+9y = 4; \end{cases}$

$$\|A|b\| = \left\|\begin{array}{cc|c} 3 & 5 & 2 \\ 5 & 9 & 4 \end{array}\right\| \sim \left\|\begin{array}{cc|c} 15 & 25 & 10 \\ 15 & 27 & 12 \end{array}\right\| = \left\|\begin{array}{cc|c} 3 & 5 & 2 \\ 0 & 2 & 2 \end{array}\right\|$$

$2y = 2 \Rightarrow y = 1$ $\quad 3x+5\cdot 1 = 2 \Rightarrow x = -1$.

Ответ: $(-1;1)$.

3) $\begin{cases} y+3z=-1, \\ 2x+3y+5z=3, \\ 3x+5y+7z=6; \end{cases}$

$$\|A|b\| = \left\| \begin{array}{ccc|c} 0 & 1 & 3 & -1 \\ 2 & 3 & 5 & 3 \\ 3 & 5 & 7 & 6 \end{array} \right\| \sim \left\| \begin{array}{ccc|c} 0 & 1 & 3 & -1 \\ 6 & 9 & 15 & 9 \\ 6 & 10 & 14 & 12 \end{array} \right\| \sim$$

$$\sim \left\| \begin{array}{ccc|c} 0 & 1 & 3 & -1 \\ 6 & 9 & 15 & 9 \\ 0 & 1 & -1 & 3 \end{array} \right\| \sim \left\| \begin{array}{ccc|c} 0 & 1 & 3 & -1 \\ 6 & 9 & 15 & 9 \\ 0 & 0 & -4 & 4 \end{array} \right\|$$

$-4z=4 \Rightarrow z=-1;\ y+3\cdot(-1)=-1 \Rightarrow y=2;$

$6x+9\cdot2+15\cdot(-1)=9 \Rightarrow x=1$

Ответ: $(1;2;-1)$.

№17.2(4)

4) $\left\| \begin{array}{ccc|c} 1 & -3 & -1 & -4 \\ -2 & 7 & 2 & 10 \\ 3 & 2 & -4 & 9 \end{array} \right\| \Leftrightarrow \begin{cases} x-3y-z=-4, \\ -2x+7y+2z=10, \\ 3x+2y-4z=9; \end{cases}$

$$\Delta = \begin{vmatrix} 1 & -3 & -1 \\ -2 & 7 & 2 \\ 3 & 2 & -4 \end{vmatrix} = 1(-28-4)+3(8-6)-(-4-21) =$$
$$= -32+6+25=-1$$

$$\Delta x = \begin{vmatrix} -4 & -3 & -1 \\ 10 & 7 & 2 \\ 9 & 2 & -4 \end{vmatrix} = -4(-28-4)-(-3)(-40-18)-1\cdot$$
$$\cdot(20-9\cdot7)=128-174+43=-3$$

$$\Delta y = \begin{vmatrix} 1 & -4 & -1 \\ -2 & 10 & 2 \\ 3 & 9 & -4 \end{vmatrix} = (-40-18)+4(8-6)-(-18-30)=$$
$$=-58+8+48=-2$$

$$\Delta_z = \begin{vmatrix} 1 & -3 & -4 \\ -2 & 7 & 10 \\ 3 & 2 & 9 \end{vmatrix} = (63-20)+3(-18-30)-4\cdot(-4-21) = 43-144+100 = -1$$

$\Delta \neq 0 \Rightarrow x = \dfrac{\Delta x}{\Delta} = 3$; $y = \dfrac{\Delta y}{\Delta} = 2$; $z = \dfrac{\Delta z}{\Delta} = 1$

<u>Ответ:</u> $(3; 2; 1)$.