№8.1(2,5,6)

2) $\frac{3x}{36} - \frac{3y}{12} = \frac{x}{12} - \frac{y}{4} = 1$

5) $xy = 8$

$(x'-y')(x'+y') = 8$

$x'^2 - y'^2 = 8$

$x'x'_0 - y'y'_0 = 8$

$\begin{cases} x = x' - y' \\ y = x' + y' \end{cases} \Rightarrow \begin{cases} x' = \frac{1}{2}(x+y) \\ y' = \frac{1}{2}(y-x) \end{cases} \Rightarrow \begin{cases} x'_0 = 3 \\ y'_0 = -1 \end{cases}$

$\frac{3}{2}(x+y) + \frac{1}{2}(y-x) = 8$

$x + 2y = 8$

6) $3y = 3(x + 3/2) \Rightarrow y = x + 3/2$

№8.3(2)

2) $\begin{cases} Ax + By + C = 0 \\ \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \end{cases}$

$A = 0 \Rightarrow -\frac{C}{B} = b$

$A \neq 0 \Rightarrow \frac{A^2x^2}{a^2} - \frac{A^2y^2}{b^2} = A^2 \Rightarrow$

$\Rightarrow \frac{(C-By)^2}{a^2} - \frac{A^2}{b^2}y^2 = A^2$

$b^2(C^2 - 2BCy + B^2y^2) - (aA)^2y^2 = (Aa)^2b^2$

$\mathcal{D} = (2BCb^2)^2 - 4((Bb)^2 - (Aa)^2)(b^2C^2 - (Aa)^2b^2) =$

$= B^2C^2b^4 - (Bb)^2b^2C^2 + (Aa)^2(Bb^2)^2 + (Aa)^2b^2C^2 +$

$- (Aa)^4b^2 = (Aa)^2(Bb)^2b^2 + (Aa)^2b^2C^2 - (Aa)^4b^2 = 0$

$Aa^2b^2 \neq 0 \Rightarrow (Bb)^2 + C^2 - (Aa)^2 = 0$

$(Aa)^2 - (Bb)^2 = C^2$, $C \neq 0$, т.к.

при $C = 0$ $(Aa)^2 = (Bb)^2$ – асимптоты,

а не касат.

8.7(3) $A = 1$, $B = -2$, C – произв.

$25 - 64 = C^2 \Rightarrow C^2 < 0$ ✗! $\Rightarrow$ гиперб.

8.3(1) Аналогично $\mathcal{D} = (2BCb^2)^2 - 4((Bb)^2 + (Aa)^2) \cdot$

$\cdot (b^2C^2 - (Aa)^2b^2) = 0$

$-(Bb^2)^2(Aa)^2 + A^2a^2b^2C^2 - (Aa)^4b^2 = 0$

$(Aa)^2 + (Bb)^2 = C^2$

8.9(1,3)

1) Кас. к эллипсу парал. $3x + 4y + 5 = 0$

$\frac{28}{27} \cdot 3^3 + \frac{7}{9} \cdot 4^2 = C^2 = 28 + 7 \cdot \frac{16}{9} = 7\left(\frac{52}{9}\right) = \frac{7^2 \cdot 4}{9} \Rightarrow$

$\Rightarrow C = \pm \frac{14}{3}$

иск. $\rho = \max\left\{\frac{|5 \pm \frac{14}{3}|}{5}\right\} = \max\left\{|1 \pm \frac{14}{15}|\right\} = 1 - \frac{14}{15} = \frac{1}{15}$

ур-е касат. $3x+4y+\frac{14}{3}=k\left(\frac{27}{28}xx_0+\frac{9}{7}yy_0-1\right)=0$

$k=-\frac{14}{3} \Rightarrow x_0=3:\left(\frac{14}{3}\cdot\frac{27}{28}\right)=\frac{2}{3};\ y_0=-\frac{2}{3}.$

3) кас. к гиперб. $\parallel 12x+5y-6=0$:

$\frac{19}{6}\cdot 12^2-\frac{19}{5}25=C^2 \Rightarrow C=\pm 19 \Rightarrow$

$\Rightarrow p=\max\left\{\frac{|-6\pm 19|}{\sqrt{12^2+5^2}}\right\}=\frac{19-6}{13}=1$

ур. касат.: $12x+5y-19=k\left(\frac{6}{19}xx_0-\frac{5}{19}yy_0-1\right)=0$

$k=19 \Rightarrow x_0=2;\ y_0=-1.$

№8.13 $\begin{cases}\frac{b^2}{a^2}=3\\ 4a^2-b^2=9\end{cases}\Leftrightarrow\begin{cases}b^2=27\\ a^2=9\end{cases}\Rightarrow$

$\Rightarrow$ ист. $\frac{x^2}{9}-\frac{y^2}{27}=1$

№8.24(2) (x_0,y_0) – т. в которой общая кас. эллипса

2) $\begin{cases}\frac{2x_0}{18}+\frac{\sqrt{7}y_0}{8}=1\\ \frac{x_0^2}{18}+\frac{y_0^2}{8}=1\end{cases}\Rightarrow\begin{cases}y_0=\frac{8}{\sqrt{7}}\left(1-\frac{3}{20}x_0\right)\\ \frac{x_0^2}{18}+\frac{8}{7}\left(1-\frac{3}{20}x_0\right)^2=1\end{cases}\Rightarrow$

$\Rightarrow \frac{x_0^2}{18}+\frac{8}{7}-\frac{8\cdot 3}{7\cdot 10}x_0+\frac{8}{7}\cdot\frac{9}{400}x_0^2=1$

$\left(\frac{1}{18}+\frac{72}{2800}\right)x_0^2-\frac{24}{70}x_0+\frac{1}{7}=0$

$256x_0^2-1080x_0+450=0$

$128x_0^2-540x_0+225=0$

$(32x_0 - 15)(4x_0 - 15) = 0$

$x_0 = \frac{15}{32}$; $x_0 = \frac{15}{4}$

$x_0 = \frac{15}{32}$: $y_0 = \frac{8}{\sqrt{7}}\left(1 - \frac{3}{20}\cdot\frac{15}{32}\right) = \frac{8}{\sqrt{7}}\left(1 - \frac{9}{128}\right) =$

$= \frac{119}{128}\cdot\frac{8}{\sqrt{7}} = \frac{17\sqrt{7}}{16}$

$x_0 = \frac{15}{4}$: $y_0 = \frac{8}{\sqrt{7}}\left(1 - \frac{3}{20}\cdot\frac{15}{4}\right) = \frac{8}{\sqrt{7}}\left(1 - \frac{9}{16}\right) =$

$= \frac{\sqrt{7}}{2}$

1 ур-е кас.: $\frac{x\cdot 15}{32\cdot 18} + \frac{y\cdot 17\sqrt{7}}{8\cdot 16} = 1$

$\frac{5x}{6} + \frac{y\cdot 17\sqrt{7}}{4} - 32 = 0$

$10x + 51\sqrt{7}y - 384 = 0$

2 ур-е кас.: $\frac{x}{18}\cdot\frac{15}{4} + \frac{y}{8}\cdot\frac{\sqrt{7}}{2} = 1$

$\frac{5x}{3} + \frac{\sqrt{7}}{2}y = 8$

$10x + 3\sqrt{7}y - 48 = 0$

№ 8.25 (3)

$\begin{cases} \frac{x_0^2}{4} - y_0^2 = 1 \\ \frac{4x_0}{4} - \sqrt{3}y_0 = 1 \end{cases} \Leftrightarrow \begin{cases} \frac{(1+\sqrt{3}y_0)^2}{4} - y_0^2 = 1 \\ x_0 = 1 + \sqrt{3}y_0 \end{cases} \Rightarrow$

$\Rightarrow 1 + 2\sqrt{3}y_0 + 3y_0^2 - 4y_0^2 = 4 \Rightarrow -3 + 2\sqrt{3}y_0 - y_0^2 = 0$

$(y_0 - \sqrt{3})^2 = 0 \Rightarrow y_0 = \sqrt{3} \Rightarrow x_0 = 4 \Rightarrow$

$\Rightarrow$ иск. $x - \sqrt{3}y - 1 = 0$

8.3(5) $\begin{cases} Ax + By + C = 0 \\ y^2 = 2px \end{cases}$

$A \neq 0 \Rightarrow Ay^2 = 2pAx = 2p(-C - By)$

$Ay^2 + 2pBy + 2pC = 0$

$\mathcal{D} = 4p^2B^2 - 8ApC = 0$

$p^2B^2 = 2ApC$

$pB^2 = 2AC$

№8.28(3,6)

3) $Ax + By + C = 0$: $\begin{cases} 3A^2 + B^2 = C^2 \\ B^2 = 2AC \end{cases} \Leftrightarrow \begin{cases} 3A^2 + 2AC - C^2 = 0 \\ B^2 = 2AC \end{cases} \Leftrightarrow$

$\Leftrightarrow \begin{cases} \left[\begin{array}{l} A = -C \\ A = \frac{1}{3}C \end{array}\right. \\ B^2 = 2AC \end{cases} \Leftrightarrow \left[\begin{array}{l} \begin{cases} A = -C \\ B^2 = -2C^2 \end{cases} \Rightarrow A = B = C = 0 \text{ - не подх.} \\ \begin{cases} A = \frac{1}{3}C \\ B^2 = \frac{2}{3}C^2 \end{cases} \Rightarrow \begin{cases} A = \frac{1}{3}C \\ B = \pm\sqrt{\frac{2}{3}}C \end{cases} \Rightarrow \end{array}\right.$

$\Rightarrow$ иск. $Ax + By + C = \frac{1}{3}Cx \pm \sqrt{\frac{2}{3}}Cy + C = 0$. $C \neq 0$ -

не подх., т.к. пересекает эллипс $\Rightarrow$ иск.: $\frac{1}{3}x \pm \sqrt{\frac{2}{3}}y + 1 = 0$

или $x \pm \sqrt{6}y + 3 = 0$

6) $Ax+By+C=0$: $\begin{cases} 6A^2+3B^2=C^2 \\ 25A^2-16B^2=C^2 \end{cases} \Leftrightarrow$

$\Leftrightarrow \begin{cases} A^2=B^2 \\ 9A^2=C^2 \end{cases} \Leftrightarrow \begin{cases} A=\pm B \\ A=\pm\frac{C}{3} \end{cases} \Rightarrow \pm\frac{C}{3}x\pm\frac{C}{3}y\pm C=0$

$C\neq 0$, т.к. перпенд. обе оси $\Rightarrow \pm\frac{1}{3}x\pm\frac{1}{3}y\pm 1=0$

или $x\pm y\pm 3=0$

№ 9.1(6)

6) $2x^2+y^2+4x-6y+11=0$;

$2(x+1)^2+(y-3)^2=0$

$2x'^2+y'^2=0$ – пара мнимых прямых

$\begin{cases} x=x'-1 \\ y=y'+3 \end{cases} \Rightarrow S=\begin{Vmatrix} 1 & 0 \\ 0 & 1 \end{Vmatrix}, O'(1,-3)$

№ 9.4(1,3,10)

1) $2x^2-4xy+5y^2+8x-2y+9=0$

$\operatorname{ctg}2\alpha=\frac{A-C}{2B}=\frac{-3}{-4}=\frac{3}{4}=\frac{\cos^2\alpha-\sin^2\alpha}{2\sin\alpha\cos\alpha}$

$3\sin 2\alpha=2\cos^2 2\alpha-2\sin^2\alpha$

$\cos 2\alpha=\frac{3\pm 5}{4}=-\frac{1}{2};2$

$\cos^2 2\alpha=4-4\cos^2\alpha \Rightarrow \cos\alpha=\frac{2}{\sqrt{5}}$, $\sin\alpha=\frac{1}{\sqrt{5}}$.

$$\begin{cases} x = x'\cos\alpha - y'\sin\alpha \\ y = x'\sin\alpha + y'\cos\alpha \end{cases} \qquad \begin{cases} x' = x\cos\alpha + y\sin\alpha \\ y' = -x\sin\alpha + y\cos\alpha \end{cases}$$

$$\frac{2}{5}(2x'-y')^2 - \frac{4}{5}(2x'-y')(x'+2y') + (x'+2y')^2 + \frac{8}{\sqrt{5}}(2x'-y') -$$

$$- \frac{2}{\sqrt{5}}(x'+2y') + 9 = 0$$

$$x'^2\left(\frac{8}{5} - \frac{8}{5} + 1\right) + x'y'\underbrace{\left(-\frac{8}{5} - \frac{12}{5} + 4\right)}_{0} + y'^2\left(\frac{2}{5} + \frac{8}{5} + 4\right) +$$

$$+ \frac{x'}{\sqrt{5}}(16-2) + \frac{y'}{\sqrt{5}}(-8-4) + 9 = 0$$

$$x'^2 + 6y'^2 + \frac{14}{\sqrt{5}}x' - \frac{12}{\sqrt{5}}y' + 9 = 0$$

$$5x'^2 + 14\sqrt{5}x + 30y'^2 - 12\sqrt{5}y' + 45 = 0$$

$$(\sqrt{5}x' + 7)^2 + (\sqrt{30}y' - \sqrt{6})^2 - 10 = 0$$

$$5\left(x' + \frac{7}{\sqrt{5}}\right)^2 + 30\left(y' - \frac{1}{\sqrt{5}}\right)^2 = 10$$

$$\begin{cases} x'' = x' + \frac{7}{\sqrt{5}} \\ y'' = y' - \frac{1}{\sqrt{5}} \end{cases} \qquad \begin{cases} x'' = \frac{1}{\sqrt{5}}(2x + y + 7) \\ y'' = \frac{1}{\sqrt{5}}(-x + 2y - 1) \end{cases} \Rightarrow$$

$$\frac{x''^2}{2} + \frac{y''^2}{1/3} = 1 \text{ - эллипс}$$

$$\Rightarrow \begin{cases} x = \frac{2}{\sqrt{5}}x'' - \frac{1}{\sqrt{5}}y'' - 3 \\ y = \frac{1}{\sqrt{5}}x'' + \frac{2}{\sqrt{5}}y'' - 1 \end{cases} \Rightarrow \begin{cases} \bar{e}_1' = \frac{2}{\sqrt{5}}\bar{e}_1 + \frac{1}{\sqrt{5}}\bar{e}_2 \\ \bar{e}_2'' = -\frac{1}{\sqrt{5}}\bar{e}_1 + \frac{?}{\sqrt{5}}\bar{e}_2 \end{cases} O''(-3;-1)$$

3) $9x^2 - 24xy + 16y^2 - 8x + 18y + 4 = 0$

$\operatorname{ctg} 2\varphi = \frac{7}{24} \Rightarrow \sin\varphi = \frac{3}{5}; \cos\varphi = \frac{4}{5}$

$\frac{9}{25}(4x'-3y')^2 - \frac{24}{25}(4x'-3y')(3x'+4y') +$

$+\frac{16}{25}(3x'+4y')^2 - \frac{8}{5}(4x'-3y') +$

$+\frac{18}{5}(3x'+4y') + 4 = 0;$ $\begin{cases} x' = x\cos\alpha + y\sin\alpha \\ y' = -x\sin\alpha + y\cos\alpha \end{cases}$

$25y'^2 + 5x' + 20y' + 4 = 0;$

$25\left(y' + \frac{2}{5}\right)^2 = -5x';$

$\left(y' + \frac{2}{5}\right)^2 = 2 \cdot \frac{1}{10}(-x');$

$\begin{cases} y'' = y' + \frac{2}{5} \\ x'' = -x' \end{cases}$

$y''^2 = \frac{1}{5}x''$ – парабола

$\begin{cases} x'' = -\frac{4}{5}x - \frac{3}{5}y \\ y'' = -\frac{3}{5}x + \frac{4}{5}y + \frac{2}{5} \end{cases} \Rightarrow \begin{cases} x = -\frac{4}{5}x'' + \frac{3}{5}y'' + \frac{6}{25} \\ y = -\frac{3}{5}y'' - \frac{4}{5}x'' - \frac{8}{25} \end{cases}$

$\vec{e_1}\left(-\frac{4}{5}; -\frac{3}{5}\right), \vec{e_2}\left(-\frac{3}{5}; -\frac{4}{5}\right)$ $O\left(\frac{6}{25}; -\frac{8}{25}\right)$

10) $8x^2 + 6xy + 6x + 3y + 1 = 0$

$\operatorname{ctg} 2\varphi = \frac{8-0}{6} = \frac{4}{3}$

$3\cos^2\varphi - 8\sin\varphi\cos\varphi - 3\sin^2\varphi = 0$

$\cos^2\varphi = \frac{1}{9}(1+\cos^2\varphi) \Rightarrow \cos\varphi = \frac{3}{\sqrt{10}}$ $\sin\varphi = \frac{1}{\sqrt{10}}$

$$\frac{8}{10}(3x'-y')^2+\frac{6}{10}(3x'-y')(x'+3y')+$$

$$+\frac{6}{\sqrt{10}}(3x'-y')+\frac{3}{\sqrt{10}}(x'+3y')+1=0$$

$$x'^2\left(\frac{72}{10}+\frac{18}{10}\right)+x'y'\left(-\frac{48}{10}+\frac{48}{10}\right)+y'^2\left(\frac{8}{10}-\frac{18}{10}\right)+$$

$$+x'\left(\frac{18}{\sqrt{10}}+\frac{3}{\sqrt{10}}\right)+y'\left(-\frac{6}{\sqrt{10}}+\frac{9}{\sqrt{10}}\right)+1=0$$

$$9x'^2-y'^2+\frac{21}{\sqrt{10}}x'+\frac{3}{\sqrt{10}}y'+1=0$$

$$9\left(x'+\frac{7}{6\sqrt{10}}\right)^2-\left(y'-\frac{3}{2\sqrt{10}}\right)^2=0$$

$9x''^2-y''^2=0$ — пара пересек. пр.

$$\begin{cases} x=\frac{1}{\sqrt{10}}(3x'-y')=\frac{1}{\sqrt{10}}\left(3x''-\frac{7}{2\sqrt{10}}-y''+\frac{3}{2\sqrt{10}}\right) \\ y=\frac{1}{\sqrt{10}}(x'+3y')=\frac{1}{\sqrt{10}}\left(x''-\frac{7}{6\sqrt{10}}+3y''+\frac{9}{2\sqrt{10}}\right) \end{cases}$$

$$\begin{cases} x=\frac{3}{\sqrt{10}}x''-\frac{1}{\sqrt{10}}y''-\frac{1}{2} \\ y=\frac{1}{\sqrt{10}}x''+\frac{3}{\sqrt{10}}y''+\frac{1}{3} \end{cases} \Rightarrow \begin{cases} \bar{e}_1''=\frac{3}{\sqrt{10}}\bar{e}_1+\frac{1}{\sqrt{10}}\bar{e}_2 \\ \bar{e}_2''=-\frac{1}{10}\bar{e}_1+\frac{3}{\sqrt{10}}\bar{e}_2 \end{cases} \quad O\left(-\frac{1}{2};\frac{1}{3}\right)$$

$\sqrt{9.13}$

1) $(3x-4y)^2-5(x+2y-1)^2=1$

$9x^2-24xy+16y^2-5(x^2+4y^2+1+4xy-2x-4y)=1$

$4x^2-44xy-4y^2+10x+20y-6=0$

$$\delta=\begin{vmatrix} 4 & -22 \\ -22 & -4 \end{vmatrix}=-16-22^2<0$$

$$\Delta = \begin{vmatrix} A & B & D \\ B & C & E \\ D & E & F \end{vmatrix} = \begin{vmatrix} 4 & -22 & 5 \\ -22 & 4 & 10 \\ 5 & 10 & -6 \end{vmatrix} \neq 0 \Rightarrow \text{гипербола}$$

№9.15(2) $\lambda(x^2+y^2) - 10xy + x + y + 4 = 0$

$$\delta = \begin{vmatrix} A & B \\ B & C \end{vmatrix} = \begin{vmatrix} \lambda & -5 \\ -5 & \lambda \end{vmatrix} = \lambda^2 - 25$$

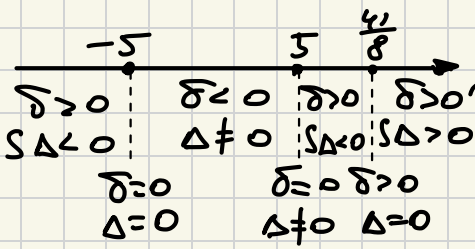
$$\Delta = \begin{vmatrix} A & B & D \\ B & C & E \\ D & E & F \end{vmatrix} = \begin{vmatrix} \lambda & -5 & \frac{1}{2} \\ -5 & \lambda & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 4 \end{vmatrix} =$$

$$= 4\lambda^2 - 5 \cdot \frac{1}{4} + \frac{1}{4} \cdot (-5) - \left(\frac{1}{4}\lambda + \frac{1}{4}\lambda - 100\right) = 0$$

$$8\lambda^2 - \lambda - 205 = 0$$

$$(8\lambda - 41)(\lambda + 5) = 0$$

$S = A + C = 2\lambda$



$\Rightarrow$

$\lambda \in (-\infty; -5)$ – эллипс

$\lambda = -5$ – пара $\parallel$ прямых

$\lambda \in (-5; 5)$ – гипербола

$\lambda = 5$ – парабола

$\lambda \in (5; \frac{41}{8})$ – эллипс

$\lambda = \frac{41}{8}$ – пара мним. пер. пр

$\lambda \in (\frac{41}{8}; +\infty)$ – мнимый эллипс