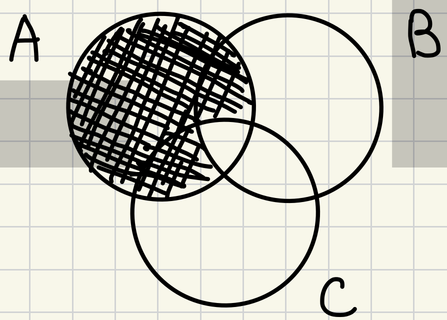
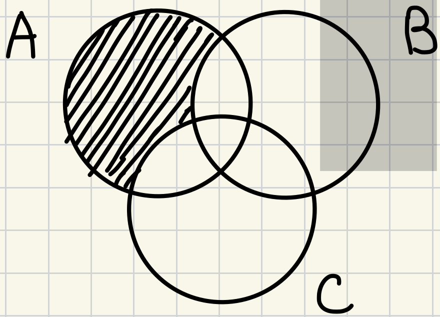
Д/З 2.

1. $(A\setminus B)\cap((A\cup B)\setminus(A\cap B)) = A\setminus B$?

$f(x) = a\bar b\,((a\vee b)\wedge\overline{(ab)}) = a\bar b\,((a\vee b)\wedge(\bar a\vee\bar b)) = a\bar b\,(a\bar a\vee a\bar b\vee b\bar a\vee b\bar b) =$
$= a\bar b\,(a\bar b\vee b\bar a) = a\bar b\wedge a\bar b\vee a\bar b\wedge b\bar a =$
$= a\bar b\vee a\bar b\wedge b\bar a = a\bar b = f_{A\setminus B}(x)$ => верно.

2. $((A\setminus B)\cup(A\setminus C))\cap(A\setminus(B\cap C)) = A\setminus(B\cup C)$?



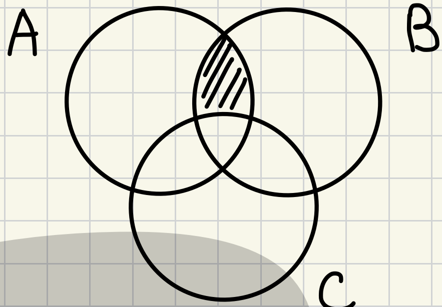


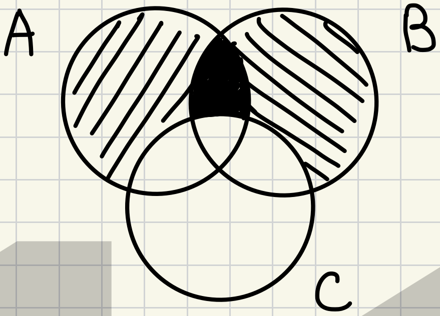
$f_{((A\setminus B)\cup(A\setminus C))\cap(A\setminus(B\cap C))}(x) = f_{(A\setminus B)\cup(A\setminus C)}(x)\, f_{A\setminus(B\cap C)}(x) =$
$= (a\bar b\vee a\bar c)(a\wedge\overline{(b\wedge c)}) = (a\bar b\vee a\bar c)(a\wedge(\bar b\vee\bar c)) = (a\bar b\vee a\bar c)\wedge$
$\wedge(a\bar b\vee a\bar c) = (a\bar b\vee a\bar c) = a(\bar b\vee\bar c) = a\overline{(b\wedge c)}$

$f_{A\setminus(B\cup C)} = a\overline{(b\vee c)}$

$a\overline{(b\wedge c)} = 1$ при $a=1,\ b=1,\ c=0$
$a\overline{(b\vee c)} = 0$ при $a=1,\ b=1,\ c=0$ => неверно.

3. $(A\cap B)\setminus C = (A\setminus C)\cap(B\setminus C)$?

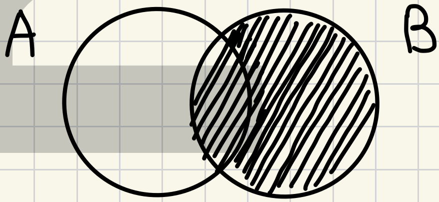




$f_{(A\cap B)\setminus C}(x) = (ab)\bar{c} = ab\bar{c}$

$f_{(A\setminus C)\cap(B\setminus C)}(x) = a\bar{c} \wedge b\bar{c} = ab\bar{c}$ $\Big| \Rightarrow$ верно.

4. $(A\cup B)\setminus(A\setminus B) \subseteq B$? $\forall\, A, B$



$f_{(A\cup B)\setminus(A\setminus B)} = (a\vee b)\wedge$

$\wedge\, \overline{(a\bar{b})} = (a\vee b)\wedge(\bar{a}\vee b) =$

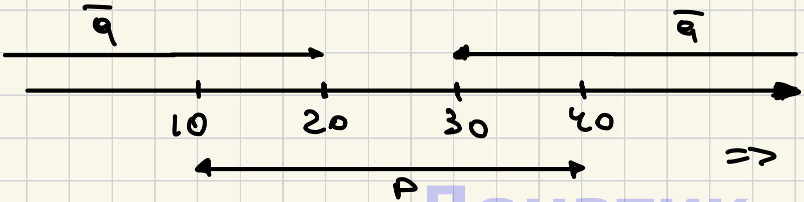
$= a\bar{a} \vee ab \vee \bar{a}b \vee b\bar{b} = b(a\vee\bar{a}) =$

$= b$ $\quad f_B(x) = b \Rightarrow$ верно.

5. $P = [10, 40]$; $Q = [20, 30]$

$((x\in A)\to(x\in P))\wedge((x\in Q)\to(x\in A))$

$(\bar{a}\vee p)\wedge(\bar{q}\vee a) \equiv 1 \Rightarrow \begin{cases}\bar{a}\vee p = 1\\ \bar{q}\vee a = 1\end{cases}$



$\left.\begin{matrix} a \Rightarrow p \\ q \Rightarrow a \end{matrix}\right\} = ?$

$\Rightarrow$ 1) $[10; 40]$

2) $[20; 30]$.

6. $A, B, X, Y:\ A \cap X = B \cap X,\ A \cup Y = B \cup Y \overset{?}{\Rightarrow} A \cup (Y \setminus X) = B \cup (Y \setminus X)$

$f_{A \cup (Y \setminus X)}(t) = a \vee y\bar{x} = \neg(\bar{a} \wedge \overline{y\bar{x}}) =$
$= \neg(\bar{a} \wedge (\bar{y} \vee x)) = \neg(\bar{a}\bar{y} \vee \bar{a}x) = (a \vee y) \wedge$
$\wedge \overline{(\bar{a}x)} = (b \vee y) \wedge (a \vee \bar{x}) = ab \vee b\bar{x} \vee ay \vee$
$\vee y\bar{x}$

$f_{B \cup (Y \setminus X)}(t) = b \vee y\bar{x} = \neg(\bar{b} \wedge \overline{y\bar{x}}) =$
$= \neg(\bar{b} \wedge (\bar{y} \vee x)) = \neg(\bar{b}\bar{y} \vee \bar{b}x) = (b \vee y) \wedge$
$\wedge \overline{(\bar{b}x)} = (a \vee y) \wedge (b \vee \bar{x}) = ab \vee b\bar{x} \vee by \vee y\bar{x}$

Если $ay = by$, т.е. $a = b$, то $f_{A \cup (Y \setminus X)}(t) = f_{B \cup (Y \setminus X)}(t)$

Если $a \neq b$, то $ab = 0 \Rightarrow f_{A \cup (Y \setminus X)}(t) =$
$= b\bar{x} \vee ay \vee y\bar{x} = \bar{x}(b \vee y) \vee ay = \bar{x}(a \vee y) \vee$
$\vee ay = a\bar{x} \vee \bar{x}y \vee ay$

$f_{B \cup (Y \setminus X)}(t) = b\bar{x} \vee by \vee y\bar{x}$

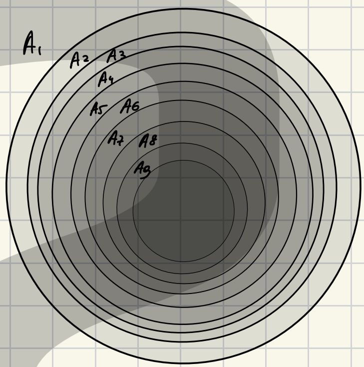
Т.к. $a \neq b$ $a\bar{x} = b\bar{x}$, только если $\bar{x} = 0$,
$ay = by$, только если $y = 0 \Rightarrow$ при $\bar{x} = 0$
или $y = 0$ $f_{A \cup (Y \setminus X)}(t) = f_{B \cup (Y \setminus X)}(t)$.

Если $\bar{x} \neq 0$ и $y \neq 0$, то $y\bar{x} = 1 \Rightarrow f_{A \cup (Y \setminus X)}(t) =$

$f_{B\setminus(Y\setminus X)}(t) \Rightarrow$ л.ч. и п.ч. равны.

7. $A_1 \supseteq A_2 \supseteq A_3 \supseteq \dots \supseteq A_n \supseteq \dots$ – невозраст. посл.

$A_1\setminus A_4 = A_6\setminus A_9$. Д-ть: $A_2\setminus A_7 = A_3\setminus A_8$.



$A_4 \supseteq A_6 \supseteq A_6\setminus A_9 \Rightarrow$

$\Rightarrow A_1\setminus A_4 = \{x \mid (x\in A_1)\wedge(x\notin A_4)\} \subseteq$

$\subseteq \{x \mid (x\in A_1)\wedge(x\notin A_6\setminus A_9)\}$

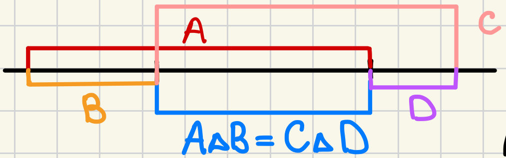
$x\in(A_1\setminus A_4) \Rightarrow (x\in A_1)\wedge(x\notin A_6\setminus A_9)$

т.е. $A_1\setminus A_4 = A_6\setminus A_9 = \varnothing \Rightarrow$

$\Rightarrow A_1 = A_2 = A_3 = A_4$ и $A_6 = A_7 = A_8 = A_9 \Rightarrow$

$\Rightarrow A_2\setminus A_7 = A_3\setminus A_8$ $\blacksquare$

8. A, B, C, D: $A\triangle B = C\triangle D$. $A\cap B \subseteq C$?



Пример, когда не выполняется:

$A = [0;2)$; $B = [0;1) \Rightarrow$

$\Rightarrow A\triangle B = [1;2)$. $C = [1;3]$; $D = [2;3] \Rightarrow$

$\Rightarrow C\triangle D = [1;2) = A\triangle B$. $A\cap B = [0;1) \not\subseteq C \Rightarrow$

$\Rightarrow$ не выполняется.

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