23.83(3) $\|A_\varphi\|_e = \begin{Vmatrix}5&-1\\4&1\end{Vmatrix}$ в станд. базисе; $\|A_\psi\|_{A_{45}} = \begin{Vmatrix}-2&0\\1&-2\end{Vmatrix}$ в базисе $\begin{Vmatrix}1\\3\end{Vmatrix}$, $\begin{Vmatrix}-1\\-2\end{Vmatrix}$. $A_{45} = \begin{Vmatrix}1&-1\\3&-2\end{Vmatrix}$

$\|A_\psi\|_e$ в станд. базисе: $\|A_\psi\|_e = S^{-1}_{A_{45}\to e}\|A_\psi\|_{A_{45}}\, S_{A_{45}\to e} =$

$\|e\| = S_{A_{45}\to e}\|A_{45}\| \Rightarrow S_{A_{45}\to e} = \|A_{45}\|^{-1}$, $S^{-1}_{A_{45}\to e} = A_{45}$

$A_{45}^{-1} = \left\|\begin{matrix}1&-1\\3&-2\end{matrix}\middle|\begin{matrix}1&0\\0&1\end{matrix}\right\| \sim^{3I-II} \left\|\begin{matrix}0&-1\\3&-2\end{matrix}\middle|\begin{matrix}3&-1\\0&1\end{matrix}\right\| \sim^{\frac{1}{3}(II-2I)}_{-I} \left\|\begin{matrix}1&0\\0&1\end{matrix}\middle|\begin{matrix}-2&1\\-3&1\end{matrix}\right\|$

$= A_{45}\|A_\psi\|_{A_{45}} A_{45}^{-1} = \begin{Vmatrix}1&-1\\3&-2\end{Vmatrix}\begin{Vmatrix}-2&0\\1&-2\end{Vmatrix}\begin{Vmatrix}-2&1\\-3&1\end{Vmatrix} = \begin{Vmatrix}-3&2\\-8&4\end{Vmatrix}\begin{Vmatrix}-2&1\\-3&1\end{Vmatrix} = \begin{Vmatrix}0&-1\\4&-4\end{Vmatrix}$

$A_{\psi^2} = A_\psi^2 = \begin{Vmatrix}0&-1\\4&-4\end{Vmatrix}\begin{Vmatrix}0&-1\\4&-4\end{Vmatrix} = \begin{Vmatrix}-4&4\\-16&12\end{Vmatrix}$; $A_{\varphi^2} = A_\varphi^2 = \begin{Vmatrix}5&-1\\4&1\end{Vmatrix}\begin{Vmatrix}5&-1\\4&1\end{Vmatrix} = \begin{Vmatrix}21&-6\\24&-3\end{Vmatrix}$

$A_{\varphi^2-\psi^2} = \begin{Vmatrix}21&-6\\24&-3\end{Vmatrix} - \begin{Vmatrix}-4&4\\-16&12\end{Vmatrix} = \begin{Vmatrix}25&-10\\40&-15\end{Vmatrix}$

31.19(2) f - линейная на $\mathcal{P}^{(3)}$, если $\forall\, p,q\in\mathcal{P}^{(3)}$ $\forall\,\alpha\in\mathbb{R}$ ↳

$f(p+q) = f(p)+f(q)$; $f(\alpha p) = \alpha f(p)$.

$f(p+q) = \int_0^1 (p+q)(t^2)\,dt = \int_0^1 (p(t^2)+q(t^2))\,dt = \int_0^1 p(t^2)\,dt + \int_0^1 q(t^2)\,dt = f(p)+f(q) \Rightarrow$

$f(\alpha p) = \int_0^1 \alpha p(t^2)\,dt = \alpha\int_0^1 p(t^2)\,dt = \alpha f(p)$

$\Rightarrow$ f - линейная на $\mathcal{P}^{(3)}$

$f(1) = \int_0^1 dt = 1$; $f(t) = \int_0^1 t^2\,dt = \frac{1}{3}$; $f(t^2) = \int_0^1 t^4\,dt = \frac{1}{5}$; $f(t^3) = \int_0^1 t^6\,dt = \frac{1}{7} \Rightarrow$

$\Rightarrow$ искомая - $\begin{Vmatrix}1 & \frac{1}{3} & \frac{1}{5} & \frac{1}{7}\end{Vmatrix}$

31.35(1) Ǝ в $\mathcal{L}$ базисы e и e' связаны равенством $e' = eS$. Матрица перехода от базиса e^* к базису e'^* в пространстве $\mathcal{L}^*$ будет матрица $(S^{-1})^T$, т.е. $e'^{*T} = e^{*T}(S^{-1})^T$ или $e'^* = S^{-1}e^*$

$\begin{cases}\vec{e}_1{}' = \vec{e}_1 + \vec{e}_2\\ \vec{e}_2{}' = \vec{e}_2 + \vec{e}_3\\ \vec{e}_3{}' = \vec{e}_3\end{cases} \Rightarrow S = \begin{Vmatrix}1&0&0\\1&1&0\\0&1&1\end{Vmatrix}$. $\left\|\begin{matrix}1&0&0\\1&1&0\\0&1&1\end{matrix}\middle|\begin{matrix}1&0&0\\0&1&0\\0&0&1\end{matrix}\right\| \sim_{\substack{II-I\\III-II+I}} \left\|\begin{matrix}1&0&0\\0&1&0\\0&0&1\end{matrix}\middle|\begin{matrix}1&0&0\\-1&1&0\\1&-1&1\end{matrix}\right\| \Rightarrow$

$\Rightarrow e'^* = \begin{Vmatrix}1&0&0\\-1&1&0\\1&-1&1\end{Vmatrix} e' = \begin{Vmatrix}1&0&0\\-1&1&0\\1&-1&1\end{Vmatrix}\begin{Vmatrix}f_1\\f_2\\f_3\end{Vmatrix} = \begin{Vmatrix}f_1\\f_2-f_1\\f_1-f_2+f_3\end{Vmatrix}$

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