§15 №24 (9, 15)

9) $y = \sin^2 x \sin 2x = \frac{1-\cos 2x}{2} \sin 2x = \frac{1}{2}\sin 2x - \frac{1}{4}\sin 4x$

$$y^{(n)} = \frac{1}{2}\sin^{(n)} 2x - \frac{1}{4}\sin^{(n)} 4x = 2^{n-1}\sin\left(2x + \frac{\pi n}{2}\right) -$$

$$- 4^{n-1}\sin\left(4x + \frac{\pi n}{2}\right)$$

15) $y = \frac{3-2x^2}{2x^2+3x-2} = \frac{-2x^2-3x+2+3x+1}{2x^2+3x-2} = -1 +$

$$+ \frac{3x+1}{(x+2)(2x-1)} = -1 + \frac{x+2+2x-1}{(x+2)(2x-1)} = -1 + \frac{1}{2x-1} + \frac{1}{x+2}$$

$$y^{(n)} = (-1)^n n!\left(2^n (2x-1)^{-1-n} + (x+2)^{-1-n}\right)$$

№25 (3, 7, 10)

3) $y = (3-2x)^2 e^{2-3x}$

$$y^{(n)}(x) = \sum_{k=0}^{n} C_n^k \left((3-2x)^2\right)^{(k)} \left(e^{2-3x}\right)^{(n-k)}$$

$$\left((3-2x)^2\right)^{(k)} = 0, \quad k > 2 \quad \Bigg| \Rightarrow$$

$$\Rightarrow y^{(n)}(x) = C_n^0 (3-2x)^2 \left(e^{2-3x}\right)^{(n)} + C_n^1 \cdot (-2) 2 (3-2x) \cdot$$

$$\cdot \left(e^{2-3x}\right)^{(n-1)} + C_n^2 \cdot 8 \left(e^{2-3x}\right)^{(n-2)} = (3-2x)^2 (-3)^n \cdot$$

$$\cdot e^{2-3x} - 4n(3-2x)(-3)^{n-1} e^{2-3x} + 4n(n-1)(-3)^{n-2} \cdot$$

$$\cdot e^{2-3x} = (-3)^{n-2} e^{2-3x} \left(9(9-12x+4x^2) + 12n(3-2x) +\right.$$

$$\left. + 4n(n-1)\right) = (-3)^{n-2} e^{2-3x}\left(81 - 108x + 36x^2 + 36n - 24nx + 4n^2 - 4n\right) =$$

$$= (-3)^{n-2} e^{2-3x}\left(36x^2 - 12(9+2n)x + 4n^2 + 32n + 81\right)$$

7) $y = x\ln(x^2-3x+2)$

$$y^{(n)} = \sum_{k=0}^{n} C_n^k\, x^{(k)} (\ln(x^2-3x+2))^{(n-k)}$$

$$x^{(k)} = 0,\ k>1 \quad \Big| \Rightarrow$$

$$\Rightarrow y^{(n)} = x(\ln(x^2-3x+2))^{(n)} + n(\ln(x^2-3x+2))^{(n-1)}$$

$$(\ln(x^2-3x+2))' = \frac{2x-3}{(x^2-3x+2)} = \frac{x-2+x-1}{(x-2)(x-1)} =$$

$$= \frac{1}{x-1} + \frac{1}{x-2}$$

$$(\ln(x^2-3x+2))^{(n-1+1)} = ((\ln(x^2-3x+2))')^{(n-1)} =$$

$$= \left(\frac{1}{x-1}\right)^{(n-1)} + \left(\frac{1}{x-2}\right)^{(n-1)} = (-1)^{n-1}(n-1)!\,((x-1)^{-n} +$$

$$(x-2)^{-n})$$

Итак, $y^{(n)} = x(-1)^{n-1}(n-1)!((x-1)^{-n} + (x-2)^{-n}) +$

$$+ (-1)^{n-2} n(n-2)!\,((x-1)^{1-n} + (x-2)^{1-n}) = (-1)^{n-2}(n-2)! \cdot$$

$$\cdot (-x(n-1)(x-1)^{-n} - x(n-1)(x-2)^{-n} + n(x-1)^{1-n} + n(x-2)^{1-n}) =$$

$$= (-1)^{n-2}(n-2)!\,((-nx+x+nx-n)(x-1)^{-n} + (-xn+x+nx-2n)(x-2)^{-n}) =$$

$$= (-1)^{n-2}(n-2)!\,((x-n)(x-1)^{-n} + (x-2n)(x-2)^{-n}),\ n>1$$

$$= \ln(x^2-3x+2) + \frac{x(2x-3)}{x^2-3x+2},\ n=1$$

10) $y = (x^2+x)\cos^2 x = (x^2+x)\frac{1+\cos 2x}{2}$

$$y^{(n)} = \sum_{k=0}^{n} C_n^k (x^2+x)^{(k)} \left(\frac{1+\cos 2x}{2}\right)^{(n-k)}$$

$$(x^2+x)^{(k)} = 0,\ k>2 \quad \Big| \Rightarrow$$

$$\Rightarrow y^{(n)} = (x^2+x)\left(\frac{1+\cos 2x}{2}\right)^{(n)} + n(2x+1)\left(\frac{1+\cos 2x}{2}\right)^{(n-1)} +$$

$$+ n(n-1)\left(\frac{1+\cos 2x}{2}\right)^{n-2} = (x^2+x)\,2^{n-1}\cos\left(2x+\frac{\pi n}{2}\right) +$$

$$+ n(2x+1)\,2^{n-2}\cos\left(2x+\frac{\pi(n-1)}{2}\right) + 2^{n-3}n(n-1)\cos\left(2x+\frac{\pi(n-2)}{2}\right)$$

$$= 2^{n-3}\left(\left(4x^2+4x-n^2+n\right)\cos\left(2x+\frac{\pi n}{2}\right) + 2n(2x+1)\cdot\right.$$

$$\left.\cdot\sin\left(2x+\frac{\pi n}{2}\right)\right), \quad n>2$$

$$= 1+\cos 2x - (2x+1)\sin(2x) - 2(x^2+x)\cos 2x, \quad n=2$$

$$= (2x+1)\left(\frac{1+\cos 2x}{2}\right) - (x^2+x)\sin 2x, \quad n=1$$

№26(2) $y = \frac{x^2}{\sqrt{1-2x}}$

$$y^{(n)} = \sum C_n^k (x^2)^{(k)}\left((1-2x)^{-1/2}\right)^{(n-k)}$$

$$(x^2)^{(k)} = 0, \quad k>2$$

$$\Rightarrow y^{(n)} =$$

$$= x^2\left((1-2x)^{-1/2}\right)^{(n)} + 2x\left((1-2x)^{-1/2}\right)^{(n-1)} + 2\cdot$$

$$\cdot\left((1-2x)^{-1/2}\right)^{(n-2)} = x^2(-2)^n\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\dots(1-2x)^{-1/2-n} +$$

$$+ 2x(-2)^{n-1}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\dots(1-2x)^{1/2-n} + (-2)^{n-2}\left(-\frac{1}{2}\right)\cdot$$

$$\cdot\left(-\frac{3}{2}\right)\dots(1-2x)^{3/2-n} = x^2(2n-1)!!\,(1-2x)^{-1/2-n} +$$

$$+ 2x(2n-3)!!\,(1-2x)^{1/2-n} + (2n-5)!!\,(1-2x)^{3/2-n} =$$

$$= (2n-5)!!\,(1-2x)^{-1/2-n}\left(x^2(2n-1)(2n-3) + 2x(2n-3)\cdot\right.$$

$$\left.\cdot(1-2x) + (1-2x)^2\right) = (2n-5)!!\,(1-2x)^{-1/2-n}\left(x^2(4n^2-8n+\right.$$

$$\left.+3-8n+12+4) + (4n-6-4)x+1\right) = (2n-5)!!\,(1-2x)^{-1/2-n}\cdot$$

$\cdot\left((4n^2-16n+19)x^2+(4n-10)x+1\right),\ n>2$

$= \dfrac{3x^2-4x+2}{(1-2x)^{5/2}},\ n=2$

$= \dfrac{2x-3x^2}{(1-2x)^{3/2}},\ n=1$

§9 №50(1,2)

1) $x^2=\bar{o}(x) \Rightarrow \lim\limits_{x\to x_0}\dfrac{x^2}{x}=\lim\limits_{x\to x_0} x=0 \Rightarrow x_0=0 \Rightarrow$

$\Rightarrow\ x^2=\bar{o}(x),\ x\to 0$

2) $x=\bar{o}(x^2) \Rightarrow \lim\limits_{x\to x_0}\dfrac{x}{x^2}=\lim\limits_{x\to x_0}\dfrac{1}{x}=0 \Rightarrow x_0=\infty \Rightarrow$

$\Rightarrow\ x=\bar{o}(x^2),\ x\to\infty$

№51(1) $x\to 0,\ n\in\mathbb{N},\ k\in\mathbb{N},\ n\geq k$

1) $\bar{o}(x^n)+\bar{o}(x^k)=\bar{o}(x^k)$ - Показать

Рассм. $\lim\limits_{x\to 0}\dfrac{\bar{o}(x^n)+\bar{o}(x^k)}{x^k}=\lim\limits_{x\to 0}\bar{o}(x^{n-k})+\bar{o}(1)=$

$=\lim\limits_{x\to 0}\bar{o}(x^{n-k})$, т.к. $n-k\geq 0$, то $\lim\limits_{x\to 0}\bar{o}(x^{n-k})=0\Rightarrow$

$\Rightarrow\ \bar{o}(x^n)+\bar{o}(x^k)=\bar{o}(x^k)$

Т.3 Д-ть: при $x\to 0$ верно $f(x)=\bar{o}(g(x))$ и

$g(x)\sim h(x) \Rightarrow f(x)=\bar{o}(h(x)),\ x\to 0.$

$g(x)\sim h(x) \Rightarrow h(x)=g(x)+\bar{o}(g(x)),\ x\to 0$

Рассмотрим $\lim\limits_{x\to 0}\dfrac{f(x)}{h(x)}=\lim\limits_{x\to 0}\dfrac{f(x)}{g(x)+\bar{o}(g(x))}=$

$=\lim\limits_{x\to 0}\dfrac{f(x)/g(x)}{1+\bar{o}(g(x))/g(x)}=\lim\limits_{x\to 0}\dfrac{f(x)}{g(x)}=0$, т.к. $f(x)=\bar{o}(g(x))\Rightarrow$

$\Rightarrow f(x)=\bar{o}(h(x)),\ x\to 0$

Т.4. $x\to 0$

$$(2x-3x^4+\bar{o}(x^4))(1-x+2x^2-x^3+\bar{o}(x^3)) =$$

$$= 2x-2x^2+4x^3-2x^4-3x^4+\bar{o}(x^4) = 2x-2x^2+4x^3-5x^4+$$

$$+\bar{o}(x^4),\ x\to 0$$

§18 №2(6)

$$6)\ \ln\frac{1+2x}{1-x} = \ln|1+2x| - \ln|1-x| = 2x - \frac{(2x)^2}{2} + \frac{(2x)^3}{3} + \ldots +$$

$$+\frac{(-1)^{n-1}(2x)^n}{n} - \left(-x - \frac{(-x)^2}{2} - \frac{(-x)^3}{3} - \ldots + \frac{(-1)^{2n-1}x^n}{n}\right) + \bar{o}(x^n) =$$

$$= x(2+1) - x^2\left(\frac{2^2}{2} - \frac{1}{2}\right) + x^3\left(\frac{2^3}{3} + \frac{1}{3}\right) - x^4\left(\frac{2^4}{4} - \frac{1}{4}\right) + \ldots +$$

$$+ x^n\left(\frac{2^n}{n} + (-1)^{n-1}\frac{1}{n}\right) + \bar{o}(x^n) = \sum_{k=1}^{n}\left(\frac{(-1)^{k-1}2^k+1}{k}\right)x^k + \bar{o}(x^n),$$

$x\to 0$

№3(5)

$$5)\ \frac{x}{\sqrt[3]{9-6x+x^2}} = x(3-x)^{-\frac{2}{3}} = 3^{-\frac{2}{3}}x\left(1-\frac{x}{3}\right)^{-\frac{2}{3}} =$$

$$= 3^{-\frac{2}{3}}x\left(\sum_{k=0}^{n-1} C^k_{-\frac{2}{3}}\left(-\frac{x}{3}\right)^k + \bar{o}(x^{n-1})\right) = 3^{-\frac{2}{3}}\sum_{k=0}^{n-1} C^k_{-\frac{2}{3}}(-1)^k 3^{-k} x^{k+1} +$$

$$+\bar{o}(x^n) = \sum_{k=1}^{n} C^{k-1}_{-\frac{2}{3}}(-1)^{k-1} 3^{\frac{1}{3}-k} x^k + \bar{o}(x^n),\ x\to 0$$

№4(7)

$$7)\ \frac{x^2+4x-1}{x^2+2x-3} = 1 + \frac{2x+2}{x^2+2x-3} = 1 + \frac{2x-2+4}{(x-1)(x+3)} = 1 + \frac{2}{x+3} +$$

$$+4\cdot\frac{1}{4}\left(\frac{1}{x-1} - \frac{1}{x+3}\right) = 1 + \frac{1}{x-1} + \frac{1}{x+3} = 1 - \sum_{k=0}^{n} x^k + 3\sum_{k=0}^{n}\frac{(-1)^k}{3^k}x^k +$$

$$+\bar{o}(x^n) = 1 + \sum_{k=0}^{n} x^k\left[\frac{(-1)^k}{3^{k-1}} - 1\right] + \bar{o}(x^n),\ x\to 0$$

№5(3)

3) $x\sin^2 2x = x\,\frac{1-\cos 4x}{2} = \frac{x}{2}\left(1-\sum_{k=0}^{n-1}(-1)^k\frac{(4x)^{2k}}{(2k)!}+\bar{o}(x^{2n-1})\right)=$

$=\frac{x}{2}-\frac12\sum_{k=0}^{n-1}(-1)^k\frac{4^{2k}}{(2k)!}x^{2k+1}+\bar o(x^{2n}) = \frac{x}{2}-\frac{x}{2}+\sum_{k=1}^{n-1}(-1)^k\frac{2^{4k-1}}{(2k)!}x^{2k+1}+$

$+\bar o(x^{2n}) = \sum_{k=1}^{n-1}\frac{(-1)^{k-1}2^{4k-1}}{(2k)!}x^{2k+1}+\bar o(x^{2n}),\ x\to 0$

№2(4)

4) $(2x+1)\sqrt{1-x} = (2x+1)\left[\sum_{k=0}^{n} C_{\frac12}^k(-1)^k x^k+\bar o(x^n)\right]=$

$=\sum_{k=0}^{n-1}C_{\frac12}^k(-1)^kx^k\cdot 2x+\sum_{k=0}^{n}C_{\frac12}^k(-1)^kx^k+\bar o(x^n)=$

$=\sum_{k=1}^{n}C_{\frac12}^{k-1}(-1)^{k-1}2x^k+\sum_{k=0}^{n}C_{\frac12}^k(-1)^kx^k+\bar o(x^n)=$

$=1+\sum_{k=1}^{n}(-1)^k\left[2C_{\frac12}^k - C_{\frac12}^{k-1}\right]x^k+\bar o(x^n)\ =$

$2C_{1/2}^k - C_{1/2}^{k-1} = \frac{2\cdot\frac12(\frac12-1)(\frac12-2)\dots(\frac12-k)}{k!} -$

$-\frac{\frac{k}{2}(\frac12-1)\dots(\frac12-k+1)}{k(k-1)!} = \frac{1}{k!}\left(\frac12-1\right)\left(\frac12-2\right)\dots\left(\frac12-k+1\right)\cdot$

$\cdot\left[\left(\frac12-k\right)-\frac{k}{2}\right] = 2C_{1/2}^k\left[\frac12-\frac{3k}{2}\right]\frac{1}{\frac12-k} = C_{1/2}^k\,\frac{1-3k}{\frac12-k}=$

$=C_{1/2}^k\,\frac{2-6k}{1-2k} \Rightarrow \quad = 1+\sum_{k=1}^{n}(-1)^kC_{1/2}^k\frac{2-6k}{1-2k}x^k+\bar o(x^n),$
$x\to 0$

№14(3)

3) $x_0=-2$, $x\ln(2-3x+x^2)$. $t:=x-x_0=x+2 \Rightarrow$

$\Rightarrow x\ln(2-3x+x^2) = (t-2)\ln(2-3t+6+t^2-4t+4) = (t-2)\cdot$

в окрестности $t_0=0$

$$\ln(12-7t+t^2) = (t-2)\left[\ln(6-t)+\ln(2-t)\right] =$$

$$= (t-2)\left[\ln 6 - \sum_{k=1}^{n} \frac{t^k}{6^k k} + \ln 2 - \sum_{k=1}^{n} \frac{t^k}{2^k k} + \bar{o}(t^n)\right] =$$

$$= (t-2)\ln 12 - \sum_{k=1}^{n} \frac{t^k}{2^k k}\left[\frac{1}{3^k}+1\right] + \bar{o}(t^n) =$$

$$= x\ln 12 - \sum_{k=1}^{n} \frac{(x+2)^k}{2^k k}\,\frac{1+3^k}{3^k} + \bar{o}((x+2)^n) =$$

$$= x\ln 12 - \sum_{k=1}^{n} \frac{(1+3^k)}{6^k k}(x+2)^k + \bar{o}((x+2)^n) =$$

$$= x\ln 12 - \frac{4}{6}(x+2) - \sum_{k=2}^{n} \frac{(1+3^k)}{6^k k}(x+2)^k + \bar{o}((x+2)^n) =$$

$$= -\frac{4}{3} + \left(\ln 12 - \frac{2}{3}\right)x - \sum_{k=2}^{n} \frac{(1+3^k)}{6^k k}(x-2)^k + \bar{o}((x+2)^n),\quad x\to -2$$

№20(б) x_0 до $\bar{o}((x-x_0)^{2n+1})$

б) $x_0=1$, $x(x-2)2^{x^2-2x-1}$. $t := x-x_0 = x-1 \Rightarrow$

$$\Rightarrow (t^2-1)2^{t^2-2} = \frac{1}{4}(t^2-1)2^{t^2} = \frac{1}{4}t^2 2^{t^2} - \frac{1}{4}2^{t^2} =$$

$$= \frac{1}{4}t^2 e^{t^2\ln 2} - \frac{1}{4}e^{t^2\ln 2} = \frac{1}{4}t^2 \sum_{k=0}^{n-1} \frac{t^{2k}(\ln 2)^k}{k!} - \frac{1}{4}\sum_{k=0}^{n} \frac{t^{2k}(\ln 2)^k}{k!} +$$

$$+ \bar{o}(t^{2n+1}) = \frac{1}{4}t^2 \sum_{k=1}^{n} \frac{t^{2k-2}(\ln 2)^{k-1}}{(k-1)!} - \frac{1}{4}\sum_{k=1}^{n} \frac{t^{2k}(\ln 2)^k}{k!} -$$

$$- \frac{1}{4} + \bar{o}(t^{2n+1}) = -\frac{1}{4} + \frac{1}{4}\sum_{k=1}^{n}\left[\frac{(\ln 2)^{k-1}}{(k-1)!} - \frac{(\ln 2)^k}{k!}\right]t^{2k} + \bar{o}(t^{2n+1})$$

$$= -\frac{1}{4} + \frac{1}{4}\sum_{k=1}^{n} \frac{(\ln 2)^{k-1}}{k!}(k-\ln 2)(x-1)^{2k} + \bar{o}((x-1)^{2n+1}),\quad x\to 1$$

№30(1)

1) $x^3|x| + \cos^2 x$ (=)

$f(x) = x^3|x|$

$f' = 4x^3 \operatorname{sgn} x, x \neq 0.$ $\lim\limits_{x\to 0} \frac{x^3|x|}{x} = 0$, $x = 0$

$f'' = 12x^2 \operatorname{sgn} x, x \neq 0$ $\lim\limits_{x\to 0} \frac{4x^2|x|}{x} = 0$, $x = 0$

$f''' = 24x \operatorname{sgn} x, x \neq 0$ $\lim\limits_{x\to 0} \frac{12x|x|}{x} = 0$, $x = 0$

$f'''' = 24 \operatorname{sgn} x, x \neq 0$ $\lim\limits_{x\to 0} \frac{24|x|}{x}$ – не сущ. $\Rightarrow n = 3.$

$\cos^2 x = \frac{1}{2} + \frac{1}{2}\cos 2x = \frac{1}{2} + \frac{1}{2}\left(1 - 2x^2 + \bar{o}(x^3)\right) = 1 - x^2 + \bar{o}(x^3)$

(=) $1 - x^2 + \bar{o}(x^3)$, $x \to 0$

№ 38(6) $\bar{o}(x^4)$

6) $\sqrt{\cos x} = \sqrt{1 + (\cos x - 1)} = \sum\limits_{k=0}^{n} C_{\frac{1}{2}}^{k} (\cos x - 1)^k + \bar{o}((\cos x - 1)^n) = \sum\limits_{k=0}^{2} C_{\frac{1}{2}}^{k} \left(\frac{x^4}{4!} - \frac{x^2}{2!}\right)^k + \bar{o}\left(\left(\frac{x^4}{4!} - \frac{x^2}{2!}\right)^2\right) =$

$= 1 + \frac{1}{2}\left(\frac{x^4}{4!} - \frac{x^2}{2!}\right) - \frac{1}{8}\frac{x^4}{(2!)^2} + \bar{o}(x^4) = 1 - \frac{x^2}{4} - \frac{x^4}{96} + \bar{o}(x^4)$, $x \to 0$

№ 39(7) $\bar{o}(x^5)$

7) $(1+x)^{\sin x} = e^{\ln(1+x)\sin x}$ (=)

$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \bar{o}(x^4)$

$\sin x = x - \frac{x^3}{3!} + \bar{o}(x^4)$

$\ln(1+x)\sin x = x^2 - \frac{x^3}{2} + \frac{x^4}{3} - \frac{x^5}{4} - \frac{x^4}{6} + \frac{x^5}{12} + \bar{o}(x^5) =$

$= x^2 - \frac{x^3}{2} + \frac{x^4}{6} - \frac{x^5}{6}$

$$= 1+x^2-\frac{x^3}{2}+\frac{x^4}{6}-\frac{x^5}{6}+\frac{(x^2-x^3/2+x^4/6-x^5/6)^2}{2}+$$

$$+\overline{o}(x^5) = 1+x^2-\frac{x^3}{2}+\frac{2}{3}x^4-\frac{2}{3}x^5+\overline{o}(x^5),\ x\to 0$$

Т. 5. До $\overline{o}(x^6),\ x\to 0$

а) $y = tg\, x = \frac{\sin x}{\cos x} =$

$$\sin x = x-\frac{x^3}{3!}+\frac{x^5}{5!}+\overline{o}(x^6),\ x\to 0$$

$$\cos x = 1-\frac{x^2}{2!}+\frac{x^4}{4!}+\overline{o}(x^5),\ x\to 0$$

$$\frac{1}{\cos x} = \frac{1}{1-(1-\cos x)} = 1+(1-\cos x)+(1-\cos x)^2+$$

$$+\overline{o}((1-\cos x)^2) = 1+\frac{x^2}{2!}-\frac{x^4}{4!}+\left(\frac{x^2}{2!}-\frac{x^4}{4!}\right)^2+\overline{o}(x^5) =$$

$$= 1+\frac{x^2}{2!}-\frac{x^4}{4!}+\frac{x^4}{4}+\overline{o}(x^5) = 1+\frac{x^2}{2!}+\frac{5x^4}{24}+\overline{o}(x^5),\ x\to 0$$

$$= \left(x-\frac{x^3}{3!}+\frac{x^5}{5!}+\overline{o}(x^6)\right)\left(1+\frac{x^2}{2!}+\frac{5x^4}{24}+\overline{o}(x^5)\right) = x+\frac{x^3}{2!}+\frac{5x^5}{24}-$$

$$-\frac{x^3}{3!}+-\frac{x^5}{12}+\frac{x^5}{120} = x+\frac{x^3}{3}+\frac{2x^5}{15}+\overline{o}(x^6),\ x\to 0$$

б) $y = arctg\, x$

$$y' = arctg'\, x = \frac{1}{1+x^2} = \sum_{k=0}^{2}(-1)^k x^{2k}+\overline{o}(x^5),\ x\to 0$$

$$y = \int y'dx + C = C+\sum_{k=0}^{2}(-1)^k\int x^{2k}dx+\int\overline{o}(x^5)dx =$$

$$= C+\sum_{k=0}^{2}(-1)^k\frac{x^{2k+1}}{2k+1}+\overline{o}(x^6),\ x\to 0$$

$$y(0) = arctg(0) = 0 \Rightarrow C = 0$$

$$arctg\, x = x-\frac{x^3}{3}+\frac{x^5}{5}+\overline{o}(x^6),\ x\to 0$$

в) $y = \arcsin x$

$$y' = \frac{1}{\sqrt{1-x^2}} = 1 + \sum_{k=1}^{2} \frac{(2k-1)!!}{2^k k!} x^{2k} + \bar{o}(x^4),\ x \to 0$$

$$y = \int y' dx + \arcsin(0) = x + \sum_{k=1}^{3} \frac{(2k-1)!!}{2^k k!} \int x^{2k} dx + \bar{o}(x^6) =$$

$$= x + \frac{x^3}{6} + \frac{3x^5}{40} + \bar{o}(x^6),\ x \to 0$$

2) $y = th\, x = \dfrac{e^x - e^{-x}}{e^x + e^{-x}} = \dfrac{e^{2x}-1}{e^{2x}+1}$

$$e^{2x} - 1 = 2x + \frac{(2x)^2}{2} + \frac{(2x)^3}{3} + \frac{(2x)^4}{4} + \frac{(2x)^5}{5\cdot 2} + \frac{(2x)^6}{6} + \bar{o}(x^6),\ x \to 0$$

$$\frac{1}{e^{2x}+1} = \frac{1}{2 + (e^{2x}-1)} = \frac{1/2}{1 + \left(\frac{e^{2x}-1}{2}\right)} = \frac{1}{2} \sum_{k=0}^{2} (-1)^k \cdot$$

$$\cdot \left(x + x^2 + \frac{4x^3}{3} + 2x^4 + \frac{16}{5}x^5 + \frac{16}{3}x^6\right)^k + \bar{o}(x^6) =$$

$$= \frac{1}{2} - \frac{1}{2}\left(x + x^2 + \frac{4x^3}{3} + 2x^4 + \frac{16}{5}x^5 + \frac{16}{3}x^6\right) + \frac{1}{2} \cdot$$

$$\cdot \left(x^2 + x^4 + \frac{16}{9}x^6 + 2\left(x^3 + \frac{4x^4}{3} + 2x^5 + \frac{16}{5}x^6 + \frac{4}{3}x^5 + 2x^6\right)\right) +$$

$$+ \bar{o}(x^6) = \frac{1}{2} - \frac{1}{2}x + \frac{1}{6}x^3 - \frac{1}{15}x^5 + \bar{o}(x^6)$$

$$th\, x = (e^{2x}-1)\frac{1}{e^{2x}+1} = \left(2x + 2x^2 + \frac{8x^3}{3} + 4x^4 + \frac{32}{5}x^5 +\right.$$

$$\left. + \frac{32}{3}x^6\right)\left(\frac{1}{2} - \frac{1}{2}x + \frac{1}{6}x^3 - \frac{1}{15}x^5\right) + \bar{o}(x^6) =$$

$$= x - \frac{x^3}{3} + \frac{2x^5}{15} + \bar{o}(x^6)$$