# Report: Exercises 1

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**Problem 1**

As explained in the homework sheet, one of the methods to estimate the value of Pi (3.141592...) is using the Monte Carlo method. To do so, we can select random points in the unit square and keep a count of how many of them fall into the unit circle (that is if they meet the condition: x*x = y*y <= 1), let's call these points circle_points and the ones that fall into the unit square (equals to the total number of points generated) are called square_points, Any point consists of two coordinates namely, x and y. The number of randomly generated points here is what really makes the difference.

If we have a square with 2r side length, the area of this square is 4r*r. The circle that is in this square has the radius r, the area of the circle is Pi*r*r. The ratio of these two areas would be then Pi/4. For a large number of randomly generated points circle_points/square_points = Pi/4, which means Pi = 4*(circle_points/square_points).

Now what are algorithm should do is:

1. Initialize circle_points, square_points, interval, i. (integers)
2. Generate random x. (floating number with value between 0 and 1)
3. Generate random y. (floating number with value between 0 and 1)
4. Calculate a value = x*x = y*y.
5. If value <= 1 increment the number of circle_points.
6. Increment number of square_points.
7. Increment interval.
8. If interval < i (number of iterations), repeat from step 2.
9. Calculate Pi = 4*(circle_points/square_points).
10. Print out the estimated Pi.

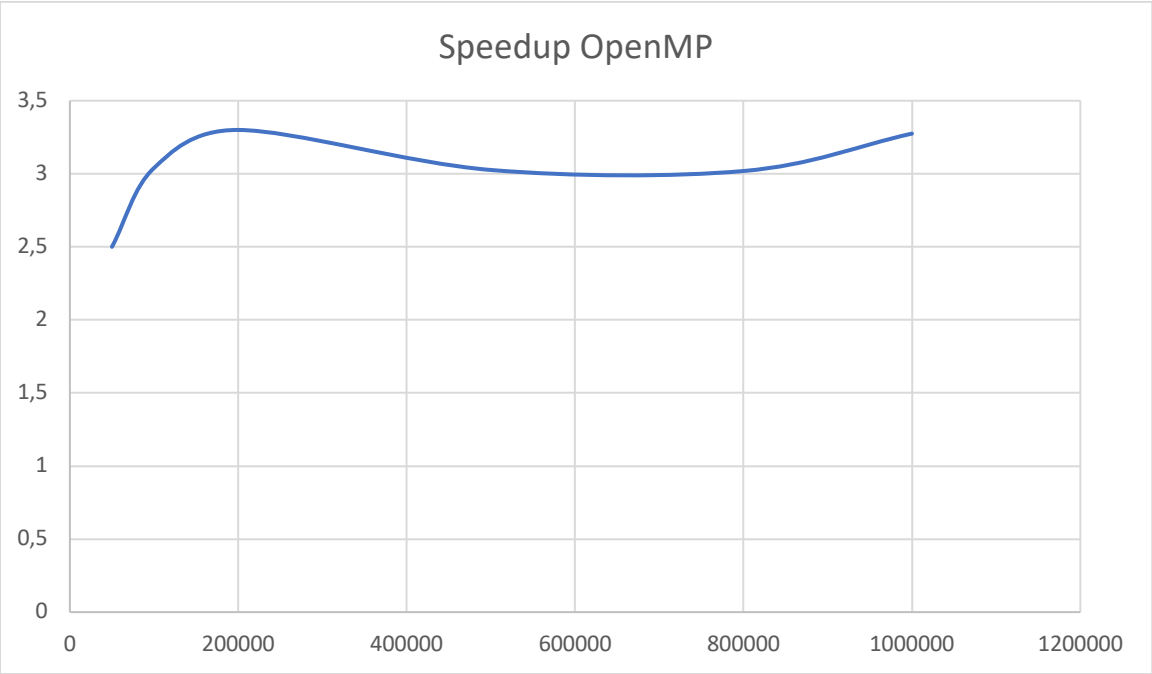
**Problem 2**

Since the program we wrote is an implementation of the Monte Carlo method to estimate Pi using OpenMP, the calculation of each point is independent of others thus the work can be easily divided into separate threads and each thread can calculate a subset of the points. After using OpenMP to parallelize our program, our program can use multiple threads to perform the calculation in parallel, which reduces the overall running time.

1. The program uses omp_set_num_threads() function to set the number of threads to be used.
2. #pragma omp parallel is written to indicate the beginning of the section to be run parallelized.
3. #pragma omp for distributes the loop iterations among threads.
4. The reduction function is used to calculate the sum of all circle_points and square_points among the threads.
5. Pi is calculated using the sum of all circle_points and square_points among the threads.
6. Print out Pi and running time.

*Speedup* = normal runtime / parallel runtime

| i | 50000 | 100000 | 200000 | 500000 | 800000 | 1000000 |
|---|---|---|---|---|---|---|
| Normal runtime | 1 ms | 2 ms | 4 ms | 9 ms | 15 ms | 19 ms |
| Parallel runtime | 0,4 ms | 0,657 ms | 1,211 ms | 2,974 ms | 4,967 ms | 5,801 ms |
| Speedup | 2,5 | 3,0441 | 3,303 | 3,026 | 3,019 | 3,275 |



*Efficiency* = Speedup / No. of threads

We have set the number of threads to 4 in our program.

| i | 50000 | 100000 | 200000 | 500000 | 800000 | 1000000 |
|---|---|---|---|---|---|---|
| Speedup | 2,5 | 3,0441 | 3,303 | 3,026 | 3,019 | 3,275 |
| Efficiency | 0,625 | 0,761 | 0,825 | 0,756 | 0,754 | 0,818 |

Since the efficiency mostly increases depending on the number of points generated, it can be said that the parallelization becomes more effective with larger workload. This may be because as the workload gets larger the overhead becomes less significant.

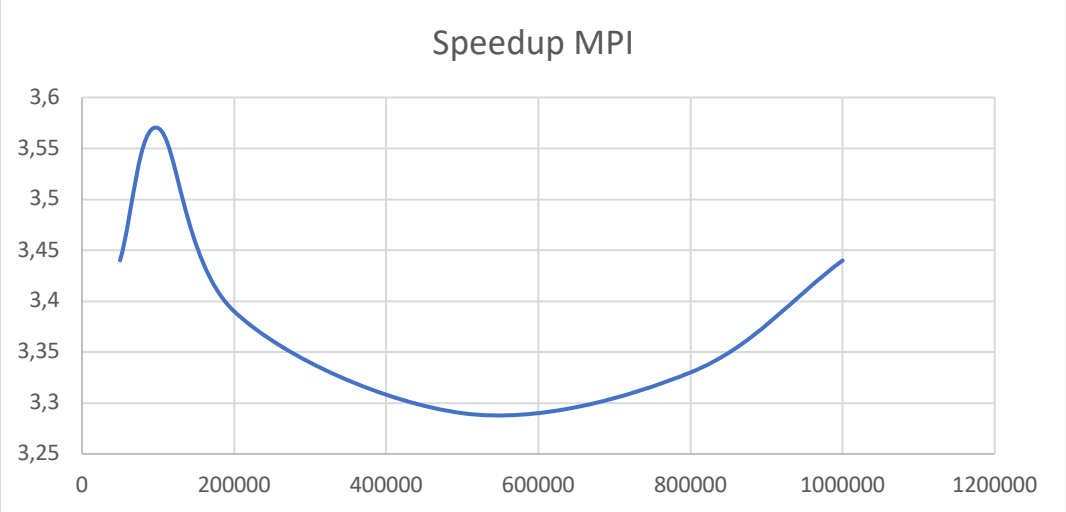
**Problem 3**

Again, we implemented the Monte Carlo method to estimate the value of Pi using MPI (Message Passing Interface).

1. The MPI environment is initialized and the rank and size for the current process are retrieved.
2. The number of points to be generated are divided equally among the processes.
3. Each process generates a set of random points and counts the number of points in the unit circle and the unit square.
4. The process reduces the counts of the circle_points and square_points found in each process using the MPI reduce operation.
5. The root process calculates the final estimation of Pi = 4*(circle_points/square_points)
6. Print out Pi and running time.

*Speedup* = normal runtime / MPI runtime

| i | 50000 | 100000 | 200000 | 500000 | 800000 | 1000000 |
|---|---|---|---|---|---|---|
| Normal runtime | 1 ms | 2 ms | 4 ms | 9 ms | 15 ms | 19 ms |
| MPI runtime | 0,29 ms | 0,56 ms | 1,178 ms | 2,734 ms | 4,495 ms | 5,509 ms |
| Speedup | 3,44 | 3,57 | 3,39 | 3,29 | 3,33 | 3,44 |



*Efficiency* = Speedup / No. of MPI processes

We run our program with 4 MPI processes. (mpirun -np 4)

| i | 50000 | 100000 | 200000 | 500000 | 800000 | 1000000 |
|---|---|---|---|---|---|---|
| Speedup | 3,44 | 3,57 | 3,39 | 3,29 | 3,33 | 3,44 |
| Efficiency | 0,86 | 0,89 | 0,84 | 0,82 | 0,83 | 0,86 |

The efficiency around 0,86 seems alright however since it doesn't seem to be improving maybe there is some overhead.

**Problem 4**

The program implements binomial tree broadcasting in MPI.

1. The MPI environment is initialized and the rank and size for the current process are retrieved.
2. Two binomial tree broadcasts are performed,
   - One in the normal order (each process send the data to its parent until the root is reached)
   - One in reversed order (each process starts from the root and sends data to its child)
3. The running times for the processes are computed using the MPI_Wtime() function.
4. The results are printed for each process.

In the normal order binomial tree broadcast, the running time is O(log2(p)) because Each level of the tree takes almost the same amount of time, and the total running time is proportional to the number of levels, so it is log2(p). while the reverse order binomial tree broadcast, due to the dependency between the send and receive operations, each level introduces additional waiting time as processes need to receive data from their child processes before proceeding and the running time of the reverse order broadcast will be Ω(log2(p)), meaning that it can be worse than log2(p) .

**Problem 5**

In the implementation, the function called scalar_product() takes two vectors of doubles and calculates their scalar product. The computation of the scalar product is parallelized using OpenMP. The result is calculated using the reduction operation that sums up the partial products computed by each thread. To test it out two vectors are initialized in main() and their scalar product is computed using the scalar_product() function. The results are printed out.

Number of threads can be controlled using **omp_set_num_threads()**, but by default it is based on the system's capabilities. In this code, the number of processors is determined by the OpenMP runtime.

In EREW PRAM model, the code is parallelized using OpenMP, allowing multiple threads to execute the loop iterations in parallel.
In this code, The loop is divided among the available threads, with each thread performing a portion of the iterations. Since the loop iterates over the elements of the vectorswith the size of n, the work done by each thread is proportional to n , resulting in O(n) complexity.

**Problem 6**

In this code, the number of processors is set to 4, using **omp_set_num_threads(4)**, which means the computation can be executed in parallel on up to 4 processors .

Complexity of the matrix-vector product computation is O(m * n):

1. The outer loop iterates over the rows of the matrix, and there are 'm' rows.
2. The inner loop iterates over the columns of the matrix and the elements of the vector, and there are 'n' columns.