Introduction to Linear Algebra

Sam Wang

Oct 22, 2017

Contents

1 Vector

Definition

$$v = \begin{bmatrix} v1 \\ v2 \\ v3 \end{bmatrix} \tag{1}$$

$$w = \begin{bmatrix} w1 \\ w2 \\ w3 \end{bmatrix} \tag{2}$$

Linear combination $cv + dw = \begin{bmatrix} c \cdot v1 + d \cdot w1 \\ c \cdot v2 + d \cdot w2 \\ c \cdot v3 + d \cdot w3 \end{bmatrix}$

Dot(inner) product: $v \cdot w = v1 \cdot w1 + v2 \cdot w2 + v3 \cdot w3$

Length: $\|v\| = \sqrt{v \cdot v} = \sqrt{{v_1}^2 + {v_2}^2 + {v_3}^2}$

Unit vector: $u = \frac{v}{\|v\|}$

Perpendicular vectors

$$\|v\|^2 + \|w\|^2 = \|v - w\|^2 \tag{3}$$

$$(v_1^2 + v_2^2) + (w_1^2 + w_2^2) = (v_1 - w_1)^2 + (v_2 - w_2)^2 \tag{4}$$

$$v_1 w_1 + v_2 w_2 = 0 \tag{5}$$

Angle between vectors

$$u = \begin{bmatrix} \cos\beta \\ \sin\beta \end{bmatrix} U = \begin{bmatrix} \cos\alpha \\ \sin\alpha \end{bmatrix} u{\cdot}U = \cos\alpha\cos\beta + \sin\alpha\sin\beta = \cos\beta - \alpha = \cos\theta = \frac{v \cdot w}{\|v\|\,\|w\|} because \cos\theta \leq 1 there \tag{6}$$

06

$$v = \begin{bmatrix} a \\ b \end{bmatrix} w = \begin{bmatrix} b \\ a \end{bmatrix} v{\cdot}w = 2ab \leq \|v\|\,\|w\| = a^2 + b^2 x = a^2, y = b^2 \sqrt{xy} \leq \frac{x+y}{2} therefore geometric.mean \leq ar \tag{7}$$