**Analysis - HW4** Jianxiang Fan [jianxiang.fan@colorado.edu](mailto:jianxiang.fan@colorado.edu)

1. **Order of hypothesis classes in terms of complexity:**

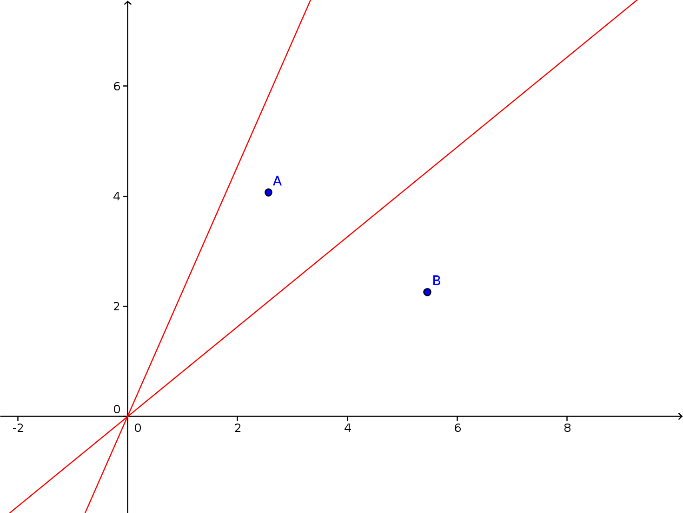
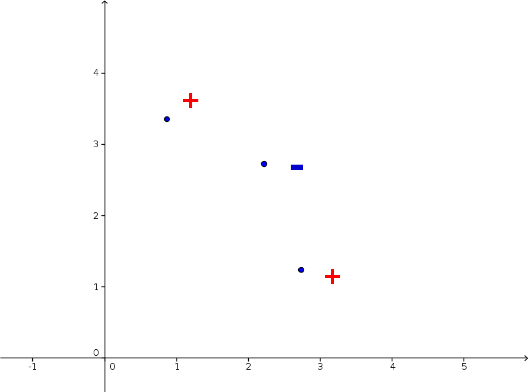
Hyperplanes through the origin < arbitrary hyperplanes < axis-aligned rectangles

***Experimental result of Rademacher Complexity (20 random points in [-20, 20] x [-20, 20]):***

Axis-aligned rectangles: 0.573000 Hyperplanes (origin): 0.400500 Arbitrary hyperplanes: 0.528100

***VC-Dimension analysis:***

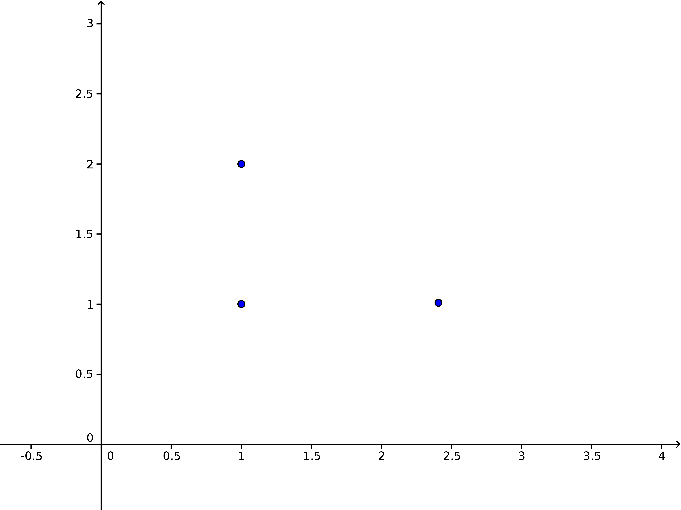
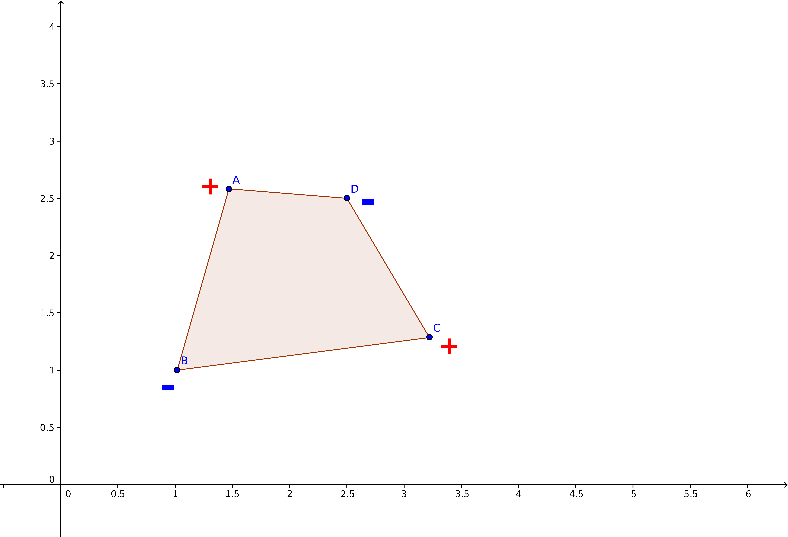
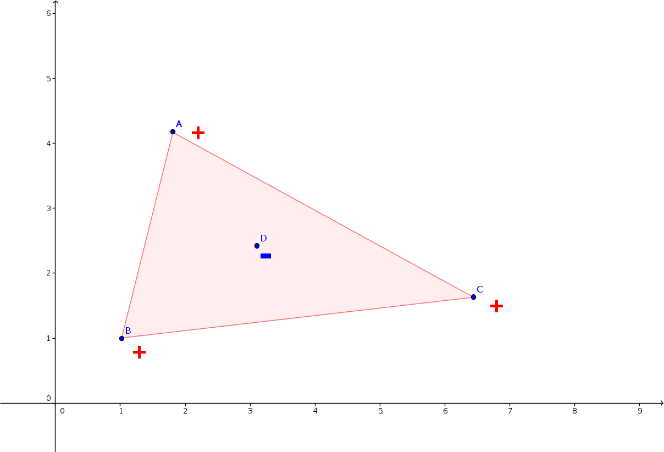
**(a) Hyperplanes through the origin: VC-Dim = 2.** Two points can be shattered (a). Three points: If we let the left and right ones be positive, and the middle one be negative, (b) no hypothesis can realize this labeling.

**b**

**a**

**(b)** **Arbitrary hyperplanes: VC-Dim = 3.** Three points can be shattered (c). Four points: (Case 1) The four points can make up a quadrangle (d): let one opposite pair be positive and another pair be negative, no hypothesis can realize this labeling. (Case 2) The four points can’t make up a quadrangle: Let the interior point be negative and the others be positive (e), no hypothesis can realize this labeling.

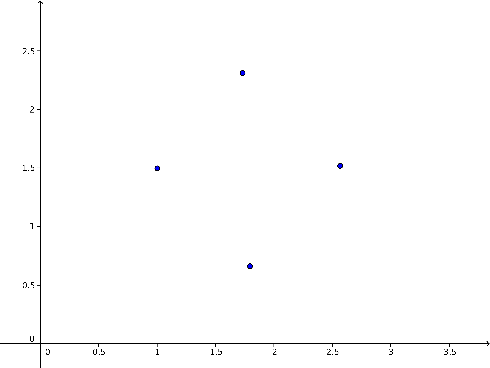
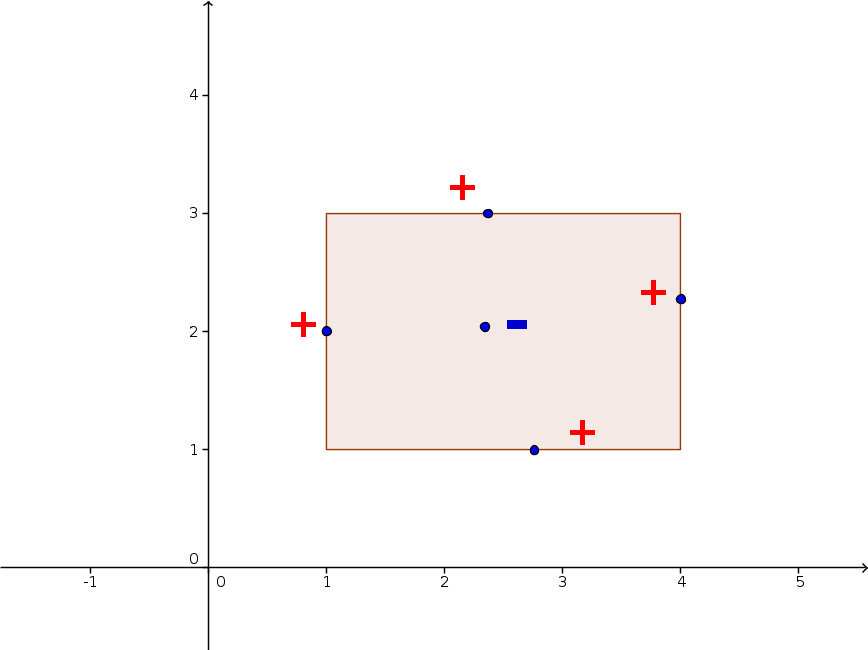
  

**e**

**d**

**c**

**Axis-aligned rectangles: VC-Dim = 4.** Four points can be shattered (f). Five points: If we construct the minimal axis-aligned rectangle containing these points, assign a negative label to this interior point and a positive label to each of the remaining four points (g), no hypothesis can realize this labeling..

**f**

**g**

1. **Prove that your frequency correctly classifies any training set**

Frequency:

For any

If then

If then

1. **Classifying real numbers**

Choose four points x = 1, 2, 3, 4. They can't be shattered by this sine classifier.

Proof:

Suppose there is an labeling these four points as +, -, -, -, thus:

Since and ,

we have , then .

While and ,

we have , then .

These two results lead to a contradiction, so this classifier can’t realize this labeling.

Although the sine classifier can’t shatter these four points, this conclusion is not related to its VC dimension. Actually the VC-dimension of the family of sine functions is infinite, as we can see in question 2.