This assignment is due by 11:55pm on Friday, October 7th. Minimal credit will be given for incomplete solutions or solutions that do not provide details on how the solution is found. You are encouraged to discuss the problems with your classmates, but all work (analysis and code) must be your own.

1. (a) Write a function that implements *row-oriented LU* factorization with partial row pivoting *without physically swapping the rows in memory* and stores L and U over A. You will need an auxiliary vector of row indices, call it $\mathbf{p}$, to keep track of swaps. Submit a printout of your code and demonstrate that your method works by printing the overwritten A and row index vector $\mathbf{p}$ obtained when calling your function on the 4×4 Hilbert matrix.

   (b) Write a function that uses your row index vector $\mathbf{p}$ and your overwritten LU decomposition from (a) to solve $A\mathbf{x} = \mathbf{b}$ using forward and back substitution. Submit a printout of your code and demonstrate that your method works by printing the solution obtained when solving $A\mathbf{x} = \mathbf{b}$ where A is the 4×4 Hilbert matrix and $\mathbf{b} = [3/2 \;\; 5/6 \;\; 7/12 \;\; 9/20]^T$

2. In this exercise you will assess the backward stability of *LU* factorization with and without partial pivoting.

   (a) Modify your code from Problem 1 to accept a flag that turns partial pivoting on and off.

   (b) Calculate the *LU* factorization of random matrices without pivoting for several choices of n (e.g. $n = 40, 80, 160$), and record $\|\hat{L}\|_\infty$, $\|\hat{U}\|_\infty$, and the norm of the backward error $\|E\|_\infty = \|\hat{L}\hat{U} - A\|_\infty$. On the same matrices compute the LU factorization with partial pivoting and record $\|\hat{L}\|_\infty$, $\|\hat{U}\|_\infty$, and the norm of the backward error $\|E\|_\infty = \|\hat{L}\hat{U} - \hat{P}A\|_\infty$. Discuss the effectiveness of both methods, in terms of stability, and comment on the effect of partial pivoting. Note that to be really confident in your numbers, you might want to perform the tests for many random matrices (50 should do it) and average the results. Turn in your table of computed norms along with your discussion.

   (c) To demonstrate the weakness of the LU factorization without pivoting, give it a matrix for which one of the pivots is guaranteed to be small. The easiest way to do this is to use matrices whose $(1, 1)$ entry is tiny (say, on the order of 10^{-13}). Repeat the experiments from (a) using such matrices. Turn in your table of computed norms along with your discussion.