Problem Set 7

Name: Jianxiang Fan
Email: jianxiang.fan@colorado.edu

1. (a) Since $(\lambda, \mathbf{v})$ is an eigenpair of A, $A\mathbf{v} = \lambda\mathbf{v}$.

$$\begin{aligned}(cA + dI)\mathbf{v} &= cA\mathbf{v} + d\mathbf{v} \\ &= c\lambda\mathbf{v} + d\mathbf{v} \\ &= (c\lambda + d)\mathbf{v}\end{aligned}$$

$(c\lambda + d,\, \mathbf{v})$ is an eigenpair of $cA + dI$.

(b) Since λ is an eigenvalue of A, there exists a vector $\mathbf{v} \neq \mathbf{0}$ s.t. $A\mathbf{v} = \lambda\mathbf{v}$.

$$\begin{aligned}A^2\mathbf{v} &= A(A\mathbf{v}) \\ &= A(\lambda\mathbf{v}) \\ &= \lambda(A\mathbf{v}) \\ &= \lambda(\lambda\mathbf{v}) \\ &= \lambda^2\mathbf{v}\end{aligned}$$

λ^2 is an eigenvalue of A^2.

(c) Since λ is an eigenvalue of A and $\lambda \neq 0$, there exists a vector $\mathbf{v} \neq \mathbf{0}$ s.t. $A\mathbf{v} = \lambda\mathbf{v}$.

$$\begin{aligned}A^{-1}\mathbf{v} &= A^{-1}(\lambda\mathbf{v})\frac{1}{\lambda} \\ &= A^{-1}A\mathbf{v}\frac{1}{\lambda} \\ &= \frac{1}{\lambda}\mathbf{v}\end{aligned}$$

$1/\lambda$ is an eigenvalue of A^{-1}.

(d) Since $(a + bi, \mathbf{v})$ is an eigenpair of A, $A\mathbf{v} = (a + bi)\mathbf{v}$. A is real, so $\bar{A} = A$

$$\begin{aligned}A\bar{\mathbf{v}} &= \overline{\bar{A}\mathbf{v}} \\ &= \overline{A\mathbf{v}} \\ &= \overline{(a + bi)\mathbf{v}} \\ &= (a - bi)\bar{\mathbf{v}}\end{aligned}$$

$(a - bi,\, \bar{\mathbf{v}})$ is also an eigenpair of A.

(e) $\forall\ (\lambda, \mathbf{v})$ s.t. $A\mathbf{v} = \lambda\mathbf{v}$ ($\mathbf{v} \neq \mathbf{0}$), $\|\mathbf{v}\|_2^2 = \mathbf{v}^*\mathbf{v} \neq 0$

$$\begin{aligned}
\bar{\lambda} &= \frac{\bar{\lambda}\mathbf{v}^*\mathbf{v}}{\|\mathbf{v}\|_2^2} \\
&= \frac{(\lambda\mathbf{v})^*\mathbf{v}}{\|\mathbf{v}\|_2^2} \\
&= \frac{(A\mathbf{v})^*\mathbf{v}}{\|\mathbf{v}\|_2^2} \\
&= \frac{\mathbf{v}^*A^*\mathbf{v}}{\|\mathbf{v}\|_2^2} \\
&= \frac{\mathbf{v}^*(A\mathbf{v})}{\|\mathbf{v}\|_2^2} \quad (A^* = A) \\
&= \frac{\mathbf{v}^*(\lambda\mathbf{v})}{\|\mathbf{v}\|_2^2} \\
&= \frac{\mathbf{v}^*(\lambda\mathbf{v})}{\|\mathbf{v}\|_2^2} \\
&= \lambda
\end{aligned}$$

Since $\bar{\lambda} = \lambda$, λ is real.

2. .

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function [ A ] = hess( A )
[~,n] = size(A);
for k = 1:n-2;
    x = A(k+1:n,k);
    if x(1) == 0
        s = 1;
    else
        s = sign(x(1));
    end
    u = s*norm(x)*eye(size(x)) + x;
    gam = 2 / sumsqr(u);
    A(k+1:n,k:n) = A(k+1:n,k:n) - gam*u*(u'*A(k+1:n,k:n));
    A(1:n, k+1:n) = A(1:n, k+1:n) - gam*(A(1:n, k+1:n)*u)*u';
end
end
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P = [0.63,0.18,0.14;0.26,0.65,0.31;0.11,0.17,0.55];
H = hess(P);
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$$H = \begin{bmatrix} 0.6300 & -0.2203 & 0.0588 \\ -0.2823 & 0.8071 & -0.2012 \\ 0 & -0.0612 & 0.3929 \end{bmatrix}$$