Problem Set 5

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1. (a) Recursive version:

$$\begin{cases} \hat{G}\,\hat{\mathbf{x}} = \hat{\mathbf{b}} \\ \hat{\mathbf{h}}^T\,\hat{\mathbf{x}} + g_{nn}\,x_n = b_n \end{cases}$$

$$\begin{cases} \hat{G}\,\hat{\mathbf{x}} = \hat{\mathbf{b}} \\ x_n = (b_n - \hat{\mathbf{h}}^T\,\hat{\mathbf{x}})/g_{nn} \end{cases}$$

   (b) Non-recursive version (overwrite **b** to storage the result):

```
for i = 1, ..., n
    if g_ii = 0
        set error flag and exit
    for j = 1, ..., i-1
        b_i <- b_i - g_ij b_j
    b_i <- b_i / g_ii
```

   (c) Since the inner loop is over j, using the ith row of matrix G in contiguous order, the implementation would be more suitable for row-oriented storage scheme.

2. (a) Pseudocode (overwrite **b** to storage the result):

```
for i = 1, ..., k
    if g_ii = 0
        set error flag and exit
for i = k+1, ..., n
    if g_ii = 0
        set error flag and exit
    for j = k+1, ..., i-1
        b_i <- b_i - g_ij b_j
    b_i <- b_i / g_ii
```

   Flops:

$$\sum_{i=k+1}^{n}\sum_{j=k+1}^{i-1} 2 = 2\sum_{i=k+1}^{n}(i-1-k) = 2\sum_{j=0}^{n-1-k} j = (n-k-1)(n-k) \approx (n-k)^2$$

(b) Since $\mathbf{e}_k$ has $k-1$ leading zeros, the flops to solve $LU\mathbf{x}_k = \mathbf{e}_k$ is roughly $(n-k+1)^2 + n^2$.

Total flops to compute A^{-1}:

$$\begin{aligned}
&\sum_{k=1}^{n}((n-k+1)^2 + n^2) + \frac{2}{3}n^3 \\
&= \sum_{i=1}^{n} i^2 + n^3 + \frac{2}{3}n^3 \\
&\approx \frac{1}{3}n^3 + n^3 + \frac{2}{3}n^3 \\
&= 2n^3
\end{aligned}$$