Problem Set 6

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1. (a) Row-oriented LU factorization with partial row pivoting

```
function [ A, p ] = myLU( A, pivoting )
[n,~] = size(A);
p = 1:n;
for k=1:n
    if pivoting == 1
        for i=k+1:n
            if A(p(i),k) > A(p(k), k)
                t = p(i);
                p(i) = p(k);
                p(k) = t;
            end
        end
    end
    for i=k+1:n
        A(p(i),k) = A(p(i),k)/A(p(k),k);
        for j=k+1:n
            A(p(i),j) = A(p(i),j) - A(p(i),k) * A(p(k),j);
        end
    end
end
end
```

```
A = hilb(4);
[A, p] = myLU(A, 1);
```

$$p = \begin{bmatrix} 1 & 3 & 4 & 2 \end{bmatrix}$$

$$A = \begin{bmatrix} 1.0000 & 0.5000 & 0.3333 & 0.2500 \\ 0.5000 & 1.0000 & -1.6667 & 0.0006 \\ 0.3333 & 0.0833 & 0.0889 & 0.0833 \\ 0.2500 & 0.9000 & 0.0033 & 0.0054 \end{bmatrix}$$

(b) Solve $A\mathbf{x} = \mathbf{b}$.

```
function [ x ] = solve( A, b )
[n,~] = size(A);
```

```
[A, p] = myLU(A,1);
for i=1:n
    for j=1:i-1
        b(p(i)) = b(p(i)) - A(p(i),j) * b(p(j));
    end
end
for i=n:-1:1
    for j=i+1:n
        b(p(i)) = b(p(i)) - A(p(i),j) * b(p(j));
    end
    b(p(i)) = b(p(i)) / A(p(i), i);
end
x = b(p,:);
end
```

```
A = hilb(4);
b = [3/2; 5/6; 7/12; 9/20];
x = solve(A, b);
```

$$\mathbf{x} = \begin{bmatrix} 1.0000 \\ 1.0000 \\ 0.0000 \\ -0.0000 \end{bmatrix}$$

2. (a) See the code snippet in 1(a).

   (b) i. Without partial pivoting:

n	$\|A\|_\infty$	$\|L\|_\infty$	$\|U\|_\infty$	$\|LU-A\|_\infty$	$\frac{\|LU-A\|_\infty}{\|A\|_\infty}$
40	24.1031	617.5794	2.4889e+03	6.5920e-13	2.7349e-14
80	46.3254	2.0808e+03	3.2513e+04	6.1507e-12	1.3277e-13
160	89.3349	1.7369e+03	6.4817e+04	1.0313e-11	1.1544e-13

From the table, we can see that

$$\|L\|_\infty \|U\|_\infty \neq O(\|A\|_\infty)$$

and

$$\frac{\|LU-A\|_\infty}{\|A\|_\infty} \neq O(\epsilon_{\text{machine}})$$

So the algorithm without partial pivoting is not backward stable.

n	$\|A\|_\infty$	$\|L\|_\infty$	$\|U\|_\infty$	$\|LU - PA\|_\infty$	$\frac{\|LU-PA\|_\infty}{\|A\|_\infty}$
40	24.1031	56.3254	111.5379	1.6730e-14	6.9411e-16
80	46.3254	74.3828	198.4236	2.7268e-14	5.8862e-16
160	89.3349	88.2208	242.2430	8.1456e-14	9.1181e-16

ii. With partial pivoting:

$$\frac{\|LU - PA\|_\infty}{\|A\|_\infty} = O(\epsilon_{\text{machine}})$$

The algorithm with partial pivoting is backward stable.

(c) Let A be a random matrix whose (1,1) entry is always 10^{-13}.

i. Without partial pivoting:

n	$\|A\|_\infty$	$\|L\|_\infty$	$\|U\|_\infty$	$\|LU - A\|_\infty$	$\frac{\|LU-A\|_\infty}{\|A\|_\infty}$
40	23.8694	9.7706e+12	1.0196e+14	0.0081	3.4106e-04
80	46.0338	9.8800e+12	2.2139e+14	0.0169	3.6618e-04
160	89.8639	9.9340e+12	4.6459e+14	0.0334	3.7126e-04

ii. With partial pivoting:

n	$\|A\|_\infty$	$\|L\|_\infty$	$\|U\|_\infty$	$\|LU - PA\|_\infty$	$\frac{\|LU-PA\|_\infty}{\|A\|_\infty}$
40	23.8694	54.4380	109.8358	1.3291e-14	5.5683e-16
80	46.0338	90.3562	144.2268	3.3482e-14	7.2734e-16
160	89.8639	77.5250	240.8489	7.9720e-14	8.8712e-16

Compare these two tables, we can see that the LU factorization without partial pivoting is much less numerically accurate than the one with partial pivoting.