This assignment is due by 11:55pm on Friday, October 14th. Submit your solutions as a **single pdf** to Moodle. Minimal credit will be given for incomplete solutions or solutions that do not provide details on how the solution is found. You are encouraged to discuss the problems with your classmates, but all work (analysis and code) must be your own.

1. In this problem you will prove several elementary facts about eigenvalues and eigenvectors. Spend a bit of time trying to prove the statements yourself, but if your are unsuccessful you may look up the result in any textbook or online source and then write up the proof in **your own words**. If you do the latter then you should cite any source that you use. Unless stated otherwise, you should assume that $A \in \mathbb{C}^{n\times n}$, $\mathbf{v} \in \mathbb{C}^n$, and $\lambda \in \mathbb{C}$.

   (a) Prove that if $(\lambda, \mathbf{v})$ is an eigenpair of A then $(c\lambda + d, \mathbf{v})$ is an eigenpair of the matrix $cA + dI$, where here c and d are scalars.

   (b) Prove that if λ is an eigenvalue of A then λ^2 is an eigenvalue of A^2.

   (c) Prove that if $\lambda \neq 0$ is an eigenvalue of A then $1/\lambda$ is an eigenvalue of A^{-1}.

   (d) Prove that if A is real and $\lambda = a + bi$ is an eigenvalue with associated eigenvector $\mathbf{v}$ then $\bar{\lambda} = a - ib$ is also an eigenvalue of A with associated eigenvector $\bar{\mathbf{v}}$ (where here the bar indicates complex conjugation). In other words, if A is real then complex eigenvalues come in conjugate pairs.

   (e) Prove that if A is Hermitian (i.e. $A^* = A$) then all of it's eigenvalues are real.

2. Write a function `hess()`, in your favorite language, that takes a real matrix A and returns a unitarily similar upper Hessenberg matrix H, i.e. $H = Q^*AQ$. Your function should use the method based on Householder reflectors discussed in class. Note that there is no need to assemble or return the Q. Turn in a print-out of your code with this assignment. Also, apply your function to matrix P below and give the resulting H with entries rounded to three decimal places.

$$P = \begin{bmatrix} 0.63 & 0.18 & 0.14 \\ 0.26 & 0.65 & 0.31 \\ 0.11 & 0.17 & 0.55 \end{bmatrix}$$