Problem Set 2

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1. (a) Space-efficient implementation of Householder QR

```
function [ A ] = myQR( A )
    [m,n] = size(A);
    for k = 1:n;
        x = A(k:m,k);
        if x(1) == 0
            s = 1;
        else
            s = sign(x(1));
        end
        u = s*norm(x)*eye(m-k+1,1) + x;
        u = u / u(1);
        gam = 2 / norm(u)^2;
        A(k:m,k:n) = A(k:m,k:n) - (gam*u)*(u'*A(k:m,k:n));
        A(k+1:m,k) = u(2:m-k+1);
    end
end
```

(b) $\hat{Q}^T\mathbf{b}$

```
function [ b ] = myQb( A, b )
    [m,n] = size(A);
    for k=1:n
        u = eye(m-k+1,1);
        u(2:m-k+1) = A(k+1:m,k);
        gam = 2/norm(u)^2;
        b(k:m) = b(k:m) - gam*u*(u'*b(k:m));
    end
    b = b(1:n);
end
```

(c) Least-squares solver

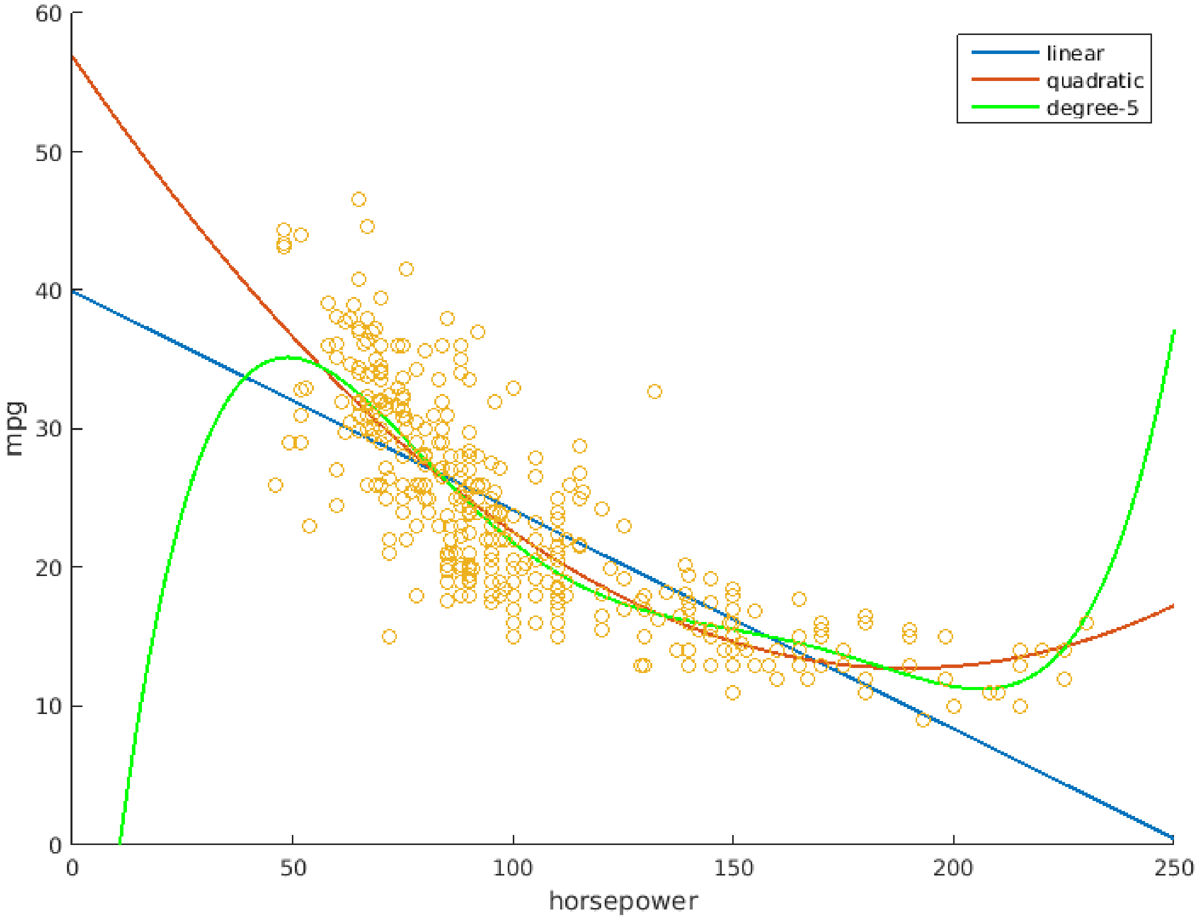
```
function [ x ] = myLSsolve( A, b )
    [m,n] = size(A);
    A = myQR(A);
    c = myQb(A, b);
```

```
    R = triu(A);
    x = R(1:n,:)\c;
end
```

(d)

```
data = csvread('auto.csv');
b = data(:,1);
a = data(:,2);
[m,~] = size(a);
x = 0.1:0.1:250;
hold on;

% Linear
A = ones(m,2);
A(:,2) = a;
t = myLSsolve(A,b);
y = t(1) + t(2) * x;
plot(x,y,'LineWidth',1.5);
E1 = norm(b-A*t)^2

% Quadratic
A = ones(m,3);
A(:,2) = a;
A(:,3) = a.^2;
t = myLSsolve(A,b);
y = t(1) + t(2) * x + t(3) * x.^2;
plot(x,y,'LineWidth',1.5);
E2 = norm(b-A*t)^2

% Degree-5
A = ones(m,6);
for i = 1:5
    A(:,i+1) = a.^i;
end
t = myLSsolve(A,b);
y = t(1)+t(2)*x+t(3)*x.^2+t(4)*x.^3+t(5)*x.^4+t(6)*x.^5;
plot(x,y,'g','LineWidth',1.5);
E3 = norm(b-A*t)^2

scatter(a,b);
axis([0, 250, 0, 60]);
legend('linear','quadratic','degree-5');
xlabel('horsepower');
ylabel('mpg');
hold off;
```

Least-squares error:
Linear: 9.3859e+03
Quadratic: 7.4420e+03
Degree-5: 7.2234e+03



2. (a)
$$QA = A - \gamma \mathbf{u}\mathbf{u}^T A = A - (\gamma \mathbf{u})(\mathbf{u}^T A)$$
$\mathbf{u} \in \mathbb{R}^{m\times 1}$, so $\gamma\mathbf{u}$ takes m flops.
$\mathbf{u}^T \in \mathbb{R}^{1\times m}$, $A \in \mathbb{R}^{m\times n}$, so $\mathbf{u}^T A$ takes $2mn$ flops.
$\gamma\mathbf{u} \in \mathbb{R}^{m\times 1}$, $\mathbf{u}^T A \in \mathbb{R}^{1\times n}$, so $(\gamma\mathbf{u})(\mathbf{u}^T A)$ takes mn flops
$A \in \mathbb{R}^{m\times n}$, $(\gamma\mathbf{u})(\mathbf{u}^T A) \in \mathbb{R}^{m\times n}$, so $A - (\gamma\mathbf{u})(\mathbf{u}^T A)$ takes mn flops.
Totally: $m + 2mn + mn + mn = 4mn + m = O(4mn)$

(b) When considering Householder QR's time efficiency, the most significant step in the loop is
$$A_{k:m,k:n} = A_{k:m,k:n} - \gamma_k \mathbf{u}_k(\mathbf{u_k}^T A_{k:m,k:n})$$

which takes approximately $4(m-k+1)(n-k+1)$ flops from the result of (a). Time efficiency of Householder QR in Problem 1:

$$
\begin{aligned}
&\sum_{k=1}^{n} 4(m-k+1)(n-k+1) \\
&= 4\sum_{k=1}^{n}(m-n+n-k+1)(n-k+1) \\
&= 4\sum_{i=1}^{n}(m-n+i)i \\
&= 4\sum_{i=1}^{n}(m-n+i)i \\
&= 4(m-n)\sum_{i=1}^{n} i + 4\sum_{i=1}^{n} i^2 \\
&= 4(m-n)\frac{n(1+n)}{2} + 4\frac{n(n+1)(2n+1)}{6} \\
&= n(n+1)(2m - \frac{2}{3}n + \frac{2}{3}) \\
&= O(2mn^2)
\end{aligned}
$$