

# Homework 2

Seven Lewis

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## 1. True or False

1) The set  $\{(1, 1), (2, 2), (3, 3), (2, 3)\}$  is a function.

False

2) Every regular language can be recognized by a deterministic finite automaton (DFA).

True

3) The function  $f : \mathbb{R} \rightarrow \mathbb{R}$  defined by  $f(x) = x^2 + 1$  is surjective.

False

4) Nondeterministic finite automaton (NFA) are more powerful than DFAs in terms of the types of languages they can recognize.

False

5) The function  $f : \mathbb{R} \rightarrow \mathbb{R}$  defined by  $f(x) = x^3 - x$  is injective.

False

6) Every NFA can be converted into an equivalent DFA

True

7) Let  $R$  be the relation on the set  $\{1, 2, 3, 4\}$  such that  $R = \{(1, 2), (2, 3), (3, 1)\}$ . The relation  $R$  is both reflexive and symmetric.

False

8) A given string from a regular expression can always be converted to a NFA.

True

9) The relation  $R = \{(a, b) | a \text{ is younger than } b\}$  on the set of all people.  $R$  is Reflexive.

False

10) A DFA can use empty string ( $\varepsilon$ ) transitions.

False

**2.**

Prove using mathematical induction that for all integers  $n \geq 1$ :

$$1 + 4 + 7 + \cdots + (3n - 2) = \frac{n(3n - 1)}{2}$$

Proof:

**Basis Step:**  $n = 1$

$$(3(1) - 2) = \frac{(1)(3(1) - 1)}{2}$$

$$1 = 1 \quad \checkmark \text{ True.}$$

**Inductive Step:**  $n = k + 1$

$$\text{Assume } 1 + 4 + 7 + \cdots + (3k - 2) = \frac{k(3k - 1)}{2}$$

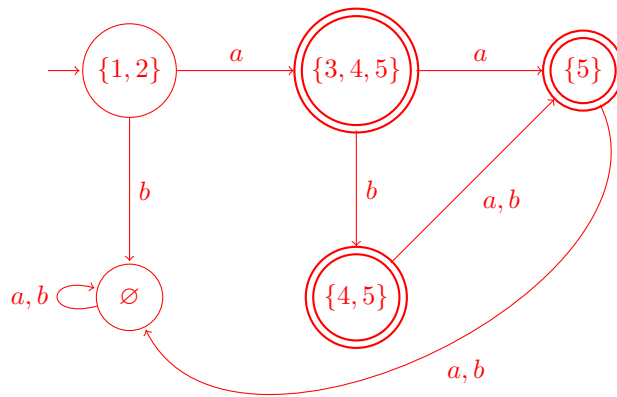
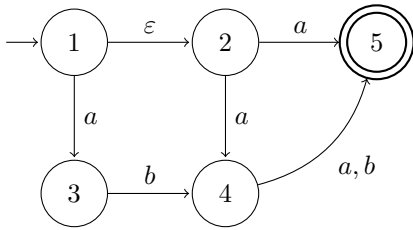
Observe that:

$$\begin{aligned} 1 + 4 + 7 + \cdots + (3(k + 1) - 2) &= \\ 1 + 4 + 7 + \cdots + (3k - 2) + (3(k + 1) - 2) &= \\ \frac{k(3k - 1)}{2} + (3(k + 1) - 2) &= \\ \frac{k(3k - 1) + 2(3(k + 1) - 2)}{2} &= \\ \frac{(3k^2 - k) + (6k + 2)}{2} &= \\ \frac{(k + 1)(3k + 2)}{2} &= \frac{(k + 1)(3(k + 1) - 1)}{2} \end{aligned}$$

Therefore,  $1 + 4 + 7 + \cdots + (3n - 2) = \frac{n(3n - 1)}{2}$  for all integers  $n \geq 1$ .

3.

Convert the following NFA to a DFA.



4.

Convert the following regular expression to an NFA:  $bb(ab)^*aa$

