Homework 2

Seven Lewis

4/12/24

1. True or False

1) The set $\{(1,1),(2,2),(3,3),(2,3)\}$ is a function.
False
2) Every regular language can be recongized by a deterministic finite automaton (DFA).
True
3) The fuction $f:\mathbb{R}\to\mathbb{R}$ defined by $f(x)=x^2+1$ is surjective.
False
4) Nondeterminist finite automaton (NFA) are more powerful than DFAs in terms of the types of languages they can recongize.
False
5) The function $f:\mathbb{R}\to\mathbb{R}$ defined by $f(x)=x^3-x$ is injective.
False
6) Every NFA can be converted into an equivalent DFA
True
7) Let R be the relation on the set $\{1,2,3,4\}$ such that $R=\{(1,2),(2,3),(3,1)\}$. The relation R is both reflexive and symmetric.
False
8) A given string from a regular expression can always be converted to a NFA.
True
9) The relation $R=\{(a,b)|a$ is younger than $b\}$ on the set of all people. R is Reflexive.
False
10) A DFA can use empty string (ε) transitions.
False

2.

Prove using mathematical induction that for all integers $n \geq 1$:

$$1 + 4 + 7 + \cdots + (3n - 2) = \frac{n(3n-1)}{2}$$

Proof:

Basis Step: $n = 1$

$$(3(1) - 2) = \frac{(1)(3(1)-1)}{2}$$

$1 = 1$ ✓ True.

Inductive Step: $n = k + 1$

Assume $1 + 4 + 7 + \cdots + (3k - 2) = \dfrac{k(3k-1)}{2}$

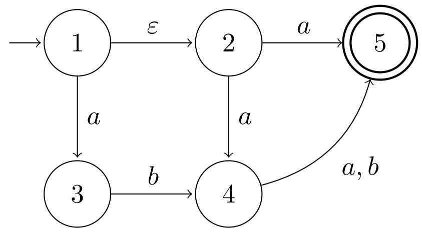
Observe that:

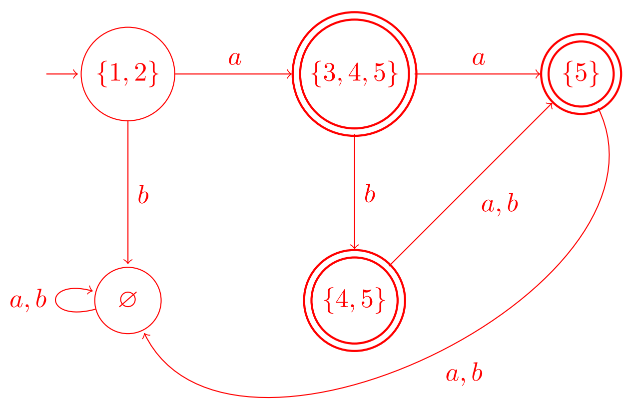
$$\begin{aligned}
1 + 4 + 7 + \cdots + (3(k+1) - 2) &= \\
1 + 4 + 7 + \cdots + (3k - 2) + (3(k+1) - 2) &= \\
\frac{k(3k-1)}{2} + (3(k+1) - 2) &= \\
\frac{k(3k-1) + 2(3(k+1) - 2)}{2} &= \\
\frac{(3k^2 - k) + (6k + 2)}{2} &= \\
\frac{(k+1)(3k+2)}{2} &= \frac{(k+1)(3(k+1)-1)}{2}
\end{aligned}$$

Therefore, $1 + 4 + 7 + \cdots + (3(n+1) - 2) = \dfrac{(n+1)(3(n+1)-1)}{2}$ for all integers $n \geq 1$.

3.

Convert the following NFA to a DFA.





4.

Convert the following regular expression to an NFA: $bb(ab)^*aa$