CSCI 301 Homework 1

Name W#

If your work is handwritten, please make sure your handwriting is clean and readable. Unreadable homework will be returned ungraded.

1. True or False (10pts)
   1. Every element of a subset is also an element of the original set.
   2. The statement (p → q) is false only when p is true, and q is false.
   3. The empty set is a subset of every set.
   4. If A ⊆ B and B ⊆ A, then A and B are equal.
   5. The negation of the statement (p V q) is logically equivalent to (~p ∧ ~q).
   6. The intersection of two sets is always non-empty.
   7. If sets A and B are disjoint, they have no elements in common.
   8. The converse of the statement (p → q) is logically equivalent to (q → p).
   9. The union of a set and its complement is the universal set.
   10. Here is a list data structure: [1, 2, 2, 3]. When you convert this list to a set, the cardinality of that set is 4.
2. (5pts) A survey was taken by 200 students on what courses they have taken. The results show that 90 students took CS, 110 took Mathematics, 60 took Physics, 20 took both CS and Math, 20 took both CS and Physics, and 30 took both Math and Physics. How many students have taken course in all three areas, including Mathematics, CS, and Physics? Please demonstrate your stepwise analysis. (You can assume that all students took at least one of the courses)
3. (5pts) A detective has interviewed four witnesses of a crime. From the stories of the witnesses the detective has concluded that:
   1. If the butler is lying, then so is the handyman.
   2. The cook and the butler cannot both be telling the truth.
   3. The cook and the gardener are not both lying.
   4. If the butler is telling the truth, then the gardener is lying.

For each of the four witnesses, can the detective determine whether that person is telling the truth or lying? Use the truth table to explain your reasoning. Please use the following notations in your reasoning –

**B**: The butler is telling the truth.

**C**: The cooking is telling the truth.

**G**: The gardener is telling the truth. **H**: The handyman is telling the truth.

1. (5pts) Prove if n ∈ Z, then n2+5n-3 is odd.
2. (5pts) Suppose a, b ∈ Z where Z denotes all integers. Try to prove the following statement using **contraposition**:

If ab is even, then a and b are not both odd.

**Submission**

Please submit a PDF file to Canvas.