Problem 1. For each statement below, (a) write the statement in symbolic logic, (b) write the negation of the statement in symbolic logic, and (c) write the negation of the statement in English.

(i) *If n is a natural number greater than one, then n is divisible by a prime number p.*

(ii) *For every positive real number ε, there is a positive real number δ with the property that $|f(x) - f(a)| < \varepsilon$ whenever $|x - a| < \delta$.*

(iii) For any vector $\vec{v}$, there exist real numbers $c_1, c_2, \ldots, c_n$ such that

$$\vec{v} = c_1\vec{e}_1 + c_2\vec{e}_2 + \cdots + c_n\vec{e}_n.$$

(iv) The set A is an element of the set X, but A is not an element of $\mathscr{P}(X)$.

Solution.

(i)

(a) $\forall n \in \mathbb{N}, \exists p \in \text{primes}, ((n > 1) \Rightarrow (p|n))$

(b) $\exists n \in \mathbb{N}, \forall p \in \text{primes}, ((n > 1) \wedge (p \nmid n))$

(c) There exists some natural number greater than 1 that is not divisible by any prime number.

(ii)

(a)

$\forall f \in \{\text{continuous functions}\}, \forall a \in \mathbb{R}, \forall \varepsilon \in \mathbb{R}, \exists \delta \in \mathbb{R}, \forall x \in \mathbb{R},$

$((\epsilon > 0 \wedge \delta > 0 \wedge (|a - x| < \delta)) \Rightarrow (|f(a) - f(x)| < \varepsilon))$

(b)

$\exists f \in \{\text{continuous functions}\}, \exists a \in \mathbb{R}, \exists \varepsilon \in \mathbb{R}, \forall \delta \in \mathbb{R}, \exists x \in \mathbb{R},$

$(\epsilon > 0 \wedge \delta > 0 \wedge (|a - x| < \delta) \wedge (|f(a) - f(x)| \geq \varepsilon))$

(c)

There exists at least one continuous function that has at least one point where no matter how small you make δ, there will always be some x within δ of a that has $f(x)$ beyond some ε of $f(a)$.

(iii)

$\vec{e_i}$ is the ith column of I_∞.

(a) $\forall \vec{v}, \exists \vec{c} \in \mathbb{R}^\infty, (\vec{v} = \sum_{i=0}^{\infty} c_i \vec{e_i})$

(b) $\exists \vec{v}, \forall \vec{c} \in \mathbb{R}^\infty, (\vec{v} \neq \sum_{i=0}^{\infty} c_i \vec{e_i})$

(c) There exists some vector v where there is no vector $\vec{c}$ times the identity matrix that yields the vector v.

(iv)

(a) $(A \in X) \wedge (A \notin \mathscr{P}(X))$

(b) $(A \notin X) \vee (A \in \mathscr{P}(X))$

(c) A is either not in X or A is in $\mathscr{P}(X)$.

$\square$