Problem 1. Let $\mathcal{C}$ be the unit circle, and let A be a collection of 20 evenly spaced points on $\mathcal{C}$. A straight segment connecting two points of A is called a *chord*. A *chord* that goes through the center of the circle is called a *diameter*; diameters connect opposite points of A. How many ways are there to...

(i) Draw chords connecting 10 distinct pairs of points from A?
(ii) Draw chords connecting 10 distinct pairs of points from A such that at least two of the chords are diameters?
(iii) Draw chords connecting 10 distinct pairs of points from A such that exactly half of the chords are diameters?

Solution.

(i) $\prod_{i=1}^{10} \binom{2 \cdot i}{2} = 3628800$

(ii) all possibilities − (no cord diameters + exacty one cord diameters)
$\prod_{i=1}^{10} \binom{2 \cdot i}{2} - (\binom{20}{1}\binom{18}{1})$

□

Problem 2. How many binary strings of length 10 contain somewhere within them the string `10001`?

Solution.

There are 6 different placements of `10001` with two positions that risk double count. We can fill the rest. $6 \cdot 2^5$ is the result before accounting for doubly count strings.

It can be seen that `1000110001` is the only string that will be counted twice and it happens in position 1 and 6.

Thus, $6 \cdot 5^2 - 1$.

THIS IS WRONG, you can double count when `100010001?`

□

Problem 3. Let $X = \{1, 2, 3, \ldots, n\}$.

(i) Determine the cardinality of the set

$$Z_k = \{A \in \mathscr{P}(X) \colon |A| = k\}$$

for $0 \leq k \leq n$.

(ii) Find a combinatorial proof showing that $\displaystyle\sum_{k=0}^{n} k\binom{n}{k} = n2^{n-1}$.

Hint: The parts of this problem are related.

Solution. □