For these problems, you should justify your answers. You do not need to provide a rigorous mathematical proof, but rather an informal argument.

Problem 1. Determine whether the following statements are logically equivalent without using a truth table.

$$P \wedge (Q \vee \sim Q) \text{ and } (\sim P) \Rightarrow (Q \wedge \sim Q)$$

Solution.

Let's first look at $P \wedge (Q \vee \sim Q)$:

A logical AND is true only when both arguments are true. But $(Q \vee \sim Q)$ is always true, so this statement only depends upon the logical value of P.

Now $(\sim P) \Rightarrow (Q \wedge \sim Q)$:

Because $(Q \vee \sim Q)$ is always false, the implication statement will be true when $\sim P$ is false and false when $\sim P$ is true. This is the same as saying it is logically equivalent to P.

Because both statements are logically equivalent to P, and P is logically equivalent to itself. Thus, they are logically equivalent.

□

Problem 2. Complete the following truth table.

P	Q	R	$\sim Q$	$\sim Q \vee R$	$P \Rightarrow (\sim Q \vee R)$
T	T	T	F	T	T
T	T	F	F	F	F
T	F	T	T	T	T
T	F	F	T	T	T
F	T	T	F	T	T
F	T	F	F	F	T
F	F	T	T	T	T
F	F	F	T	T	T

Problem 3. Use a truth table to verify the distributive law:

$$P \wedge (Q \vee R) = (P \wedge Q) \vee (P \wedge R)$$

P	Q	R	$Q \vee R$	$P \wedge (Q \vee R)$	$(P \wedge Q) \vee (P \wedge R)$
T	T	T	T	T	T
T	T	F	T	T	T
T	F	T	T	T	T
T	F	F	F	F	F
F	T	T	T	F	F
F	T	F	T	F	F
F	F	T	T	F	F
F	F	F	F	F	F

Problem 4. Use a truth table to verify the following is a **tautology** – i.e., it is *always* true.

$$[(P \Rightarrow Q) \wedge (Q \Rightarrow R)] \Rightarrow (P \Rightarrow R)$$

P	Q	R	$P \Rightarrow Q$	$Q \Rightarrow R$	$P \Rightarrow R$	$[(P \Rightarrow Q) \wedge (Q \Rightarrow R)] \Rightarrow (P \Rightarrow R)$
T	T	T	T	T	T	T
T	T	F	T	F	F	T
T	F	T	F	T	T	T
T	F	F	F	T	F	T
F	T	T	T	T	T	T
F	T	F	T	F	T	T
F	F	T	T	T	T	T
F	F	F	T	T	T	T