Problem 1. Consider the function $f\colon [0, 2\pi] \to [-1, 1]$ given by $f(x) = \cos x$. Determine each of the following sets.

(i) $f([0, \pi])$
(ii) $f(\{\pi\})$
(iii) $f\left((0, \frac{\pi}{2})\right)$
(iv) $f((0, \pi)$
(v) $f^{-1}(\{-1, 1\})$
(vi) $f^{-1}(\{0, 1\}) = \{0, \pi, 2\pi\}$
(vii) $f^{-1}((-1, 0))$
(viii) $f^{-1}(\{0\})$

Solution. □

Problem 2. Consider $f\colon A \to B$.

(i) Prove f is injective if and only if $X = f^{-1}(f(X))$ for all $X \subseteq A$.
(ii) Prove f is surjective if and only if $Y = f(f^{-1}(Y))$ for all $Y \subseteq B$.

Solution. □