

For these problems, you should justify your answers. You do not need to provide a rigorous mathematical proof, but rather an informal argument.

Problem 1. The English alphabet has 26 letters and 5 vowels. How many lists of 8 English letters are there that...

- (i) contain no vowels, if letters can be repeated?
- (ii) contain no vowels, if letters cannot be repeated?
- (iii) start with a vowel, if letters can be repeated?
- (iv) start with a vowel, if letters cannot be repeated?
- (v) contain at least one vowel, if letters can be repeated?
- (vi) contain exactly one vowel, if letters cannot be repeated?
- (vii) start with z and contain at least one vowel, if letters cannot be repeated?

Solution.

$$(i) (26 - 5)^8 = 37,822,859,361$$

$$(ii) \frac{(26-5)!}{(25-5-8)!} = 106,661,318,400$$

$$(iii) 5 \cdot 26^7 = 40,159,050,880$$

$$(iv) 5 \cdot \frac{(26-1)!}{(26-1-7)!} = 12,113,640,000$$

$$(v) 26^8 - (26 - 5)^8 = 171,004,205,215$$

$$(vi) 5 \cdot 8 \cdot \frac{(26-5)!}{(26-5-7)!} = 23,442,048,000$$

$$(vii) \frac{(26-1)!}{(26-1-7)!} - \frac{(26-1-5)!}{(26-1-5-7)!} = 2,032,027,200$$

□

Problem 2. How many hands from a standard 52-card deck contain a full house? (A full house means three cards of one value and two cards of a second value.)

Solution.

$$\frac{13}{(13-2)!} \cdot \binom{4}{3} \cdot \binom{4}{2} = 1872$$

□

Problem 3. A classroom has 2 rows of 8 desks. There are 14 students, 5 of whom always sit in the front row, and 4 of whom always sit in the back row. In how many ways can the students be seated?

Solution.

pick 5 seats in the front · pick 4 seats in the back · place rest (including empty seats)

$$\left(\binom{8}{5} \cdot 5!\right) \cdot \left(\binom{8}{4} \cdot 4!\right) \cdot \left(\binom{7}{2} \cdot 5!\right) = 28,449,792,000$$

□