For these problems, you should justify your answers. You do not need to provide a rigorous mathematical proof, but rather an informal argument.

Problem 1. Let $A = \mathbb{Z} \times \mathbb{R}$, let $B = \mathbb{R} \times \mathbb{Z}$, and let $C = \{(x, y) : x^2 + 4y^2 \leq 16\}$. For each of the following sets, describe the set in set-builder notation and give a detailed sketch the set in $\mathbb{R}^2$.

(i) $C \cap (A \cup B)$
(ii) $C \cap A \cap B$
(iii) $C - (A \cup B)$
(iv) $C - (A \cap B)$

Solution. $\square$

Problem 2. Let $\mathcal{F} = \{A \subseteq \mathbb{N} : |A| \text{ is finite}\}$. Determine whether each of the following is true or false; justify your answer.

(i) $\mathbb{N} \in \mathcal{F}$.
(ii) If $A \in \mathcal{F}$, then $\overline{A} \in \mathcal{F}$.
(iii) $|\mathcal{F}|$ is finite.
(iv) If $A_1, A_2, \ldots \in \mathcal{F}$, then $\bigcup_{n=1}^{\infty} A_n \in \mathcal{F}$.
(v) If $A_1, A_2, \ldots \in \mathcal{F}$, then $\bigcap_{n=1}^{\infty} A_n \in \mathcal{F}$.

Solution. $\square$

Problem 3. For each $\alpha \in \mathbb{R}$, define $A_\alpha = \{(x, \alpha \cos x) \in \mathbb{R}^2 : -2\pi \leq x \leq 2\pi\}$. Describe the following sets in set-builder notation and draw them in the plane $\mathbb{R}^2$.

(i) $A_{\frac{1}{2}}$
(ii) $\bigcup_{\alpha \in \mathbb{Z}} A_\alpha$
(iii) $\bigcap_{\alpha \in \mathbb{Z}} A_\alpha$
(iv) $\bigcup_{\alpha \in \mathbb{R}} A_\alpha$
(v) $\bigcap_{\alpha \in \mathbb{R}} A_\alpha$

Solution. $\square$