

For these problems, you should justify your answers. You do not need to provide a rigorous mathematical proof, but rather an informal argument.

Problem 1. Consider the sets

$$A = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 6\} \text{ and } B = \{(x, y) \in \mathbb{R}^2 : y \geq x^2\}.$$

For each of the ten sets

$$A, B, A \cup B, A \cap B, A - B, B - A, \overline{A}, \overline{B}, \overline{A \cup B}, \overline{A \cap B}$$

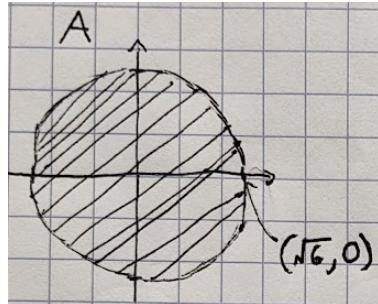
- (i) Shade the relevant portion of the plane.
- (ii) Describe the set in set-builder notation using inequalities.

(Note: Don't try to illustrate too many of the sets in the same picture; give multiple pictures, and consider using colors.)

Solution.

• A

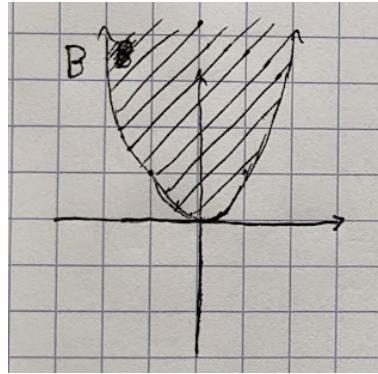
(i)



(ii) $\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 6\}$

• B

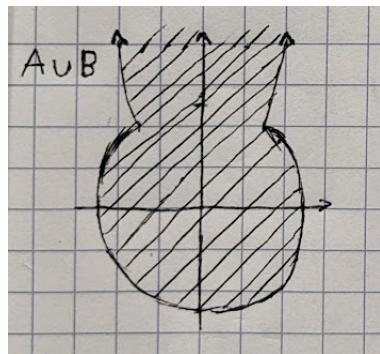
(i)



(ii) $\{(x, y) \in \mathbb{R}^2 : y \geq x^2\}$

• $A \cup B$

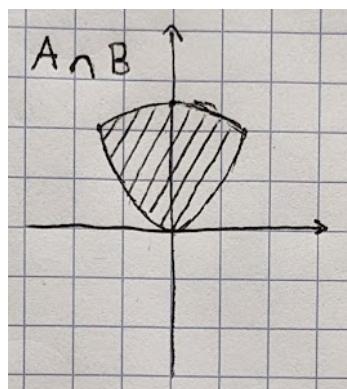
(i)



(ii) $\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 6 \text{ or } y \geq x^2\}$

• $A \cap B$

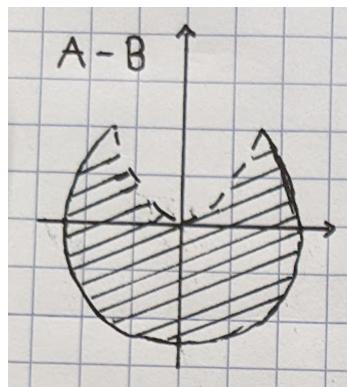
(i)



(ii) $\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 6 \text{ and } y \geq x^2\}$

• $A - B$

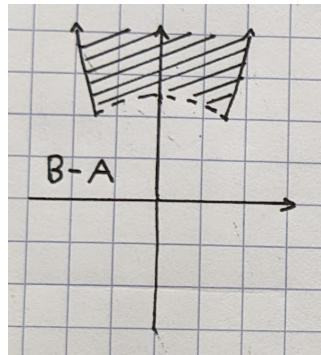
(i)



(ii) $\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 6 \text{ and } y < x^2\}$

• $B - A$

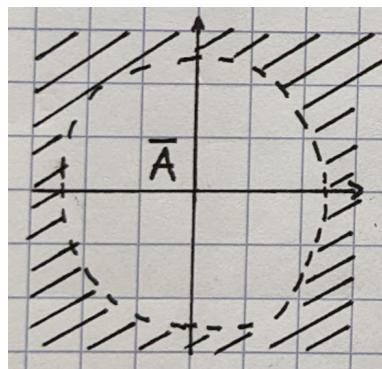
(i)



(ii) $\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 > 6 \text{ and } y \geq x^2\}$

• \bar{A}

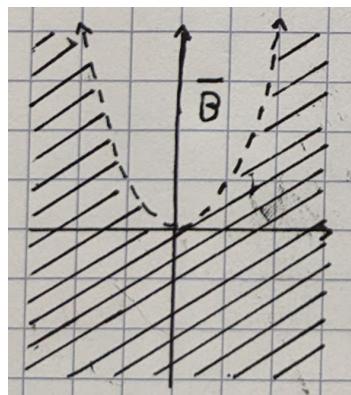
(i)



(ii) $\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 > 6\}$

• \bar{B}

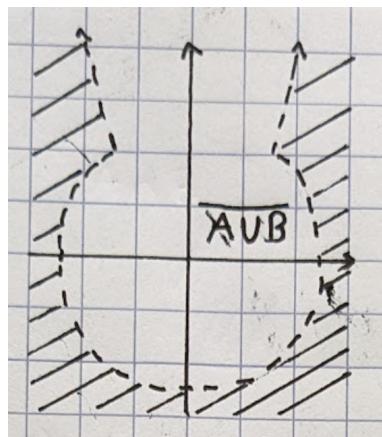
(i)



(ii) $\{(x, y) \in \mathbb{R}^2 : y < x^2\}$

• $\overline{A \cup B}$

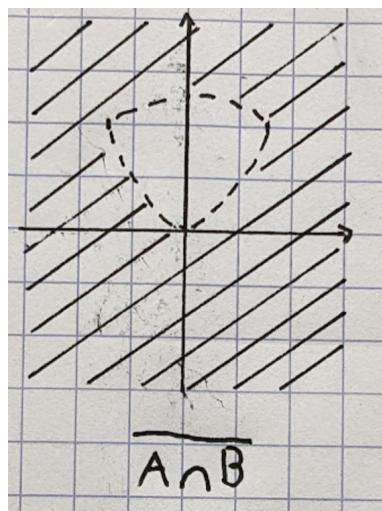
(i)



(ii) $\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 > 6 \text{ and } y < x^2\}$

• $\overline{A \cap B}$

(i)



(ii) $\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 > 6 \text{ or } y < x^2\}$

□

Problem 2. For each $\alpha \in \mathbb{R}$, define $A_\alpha = \{(x, \alpha x) \in \mathbb{R}^2 : -1 \leq x \leq 1\}$.

- (i) Describe in words the set A_π .
- (ii) Describe $\bigcup_{\alpha \in \mathbb{R}} A_\alpha$ and $\bigcap_{\alpha \in \mathbb{R}} A_\alpha$ in set-builder notation.

Solution.

- (i) This set can be described as a linear function with a slope of π and a y-intercept of 0 on the closed interval $[-1, 1]$.
- (ii) $\bigcup_{\alpha \in \mathbb{R}} A_\alpha = \{(x, y) \in \mathbb{R}^2 : x \in [-1, 1], (x = 0) \implies (y = 0)\}$
 $\bigcap_{\alpha \in \mathbb{R}} A_\alpha = \{(x, y) \in \mathbb{R}^2 : (x, y) \in \{(0, 0)\}\}$

□