For these problems, you should justify your answers. You do not need to provide a rigorous mathematical proof, but rather an informal argument.

Problem 1. The English alphabet has 26 letters and 5 vowels. How many lists of 8 English letters are there that...

(i) contain no vowels, if letters can be repeated?
(ii) contain no vowels, if letters cannot be repeated?
(iii) start with a vowel, if letters can be repeated?
(iv) start with a vowel, if letters cannot be repeated?
(v) contain at least one vowel, if letters can be repeated?
(vi) contain exactly one vowel, if letters cannot be repeated?
(vii) start with Z and contain at least one vowel, if letters cannot be repeated?

Solution.

(i) $(26-5)^8 = 37,822,859,361$

(ii) $\dfrac{(26-5)!}{(26-5-8)!} = 8,204,716,800$

(iii) $5 \cdot 26^7 = 40,159,050,880$

(iv) $5 \cdot \dfrac{(26-1)!}{(26-1-7)!} = 12,113,640,000$

(v) $26^8 - (26-5)^8 = 171,004,205,215$

(vi) $5 \cdot 8 \cdot \dfrac{(26-5)!}{(26-5-7)!} = 23,442,048,000$

(vii) $\dfrac{(26-1)!}{(26-1-7)!} - \dfrac{(26-1-5)!}{(26-1-5-7)!} = 2,032,027,200$

□

Problem 2. How many hands from a standard 52-card deck contain a full house? (A full house means three cards of one value and two cards of a second value.)

Solution.

select the two ranks · pick 3 of the 4 available cards · pick 2 of the available 4 cards

*note that "order" matters, as we need to pick one rank for three, and the other for two

$$\frac{13!}{(13-2)!} \cdot \binom{4}{3} \cdot \binom{4}{2} = 1872$$

□

Problem 3. A classroom has 2 rows of 8 desks. There are 14 students, 5 of whom always sit in the front row, and 4 of whom always sit in the back row. In how many ways can the students be seated?

Solution.

pick 5 seats in the front · pick 4 seats in the back · place rest (including empty seats)

$$(\binom{8}{5} \cdot 5!) \cdot (\binom{8}{4} \cdot 4!) \cdot (\binom{7}{2} \cdot 5!) = 28,449,792,000$$

□