For these problems, you should justify your answers. You do not need to provide a rigorous mathematical proof, but rather an informal argument.

Problem 1. The English alphabet has 26 letters and 5 vowels. How many lists of 8 English letters are there that...

(i) contain no vowels, if letters can be repeated?
(ii) contain no vowels, if letters cannot be repeated?
(iii) start with a vowel, if letters can be repeated?
(iv) start with a vowel, if letters cannot be repeated?
(v) contain at least one vowel, if letters can be repeated?
(vi) contain exactly one vowel, if letters cannot be repeated?
(vii) start with z and contain at least one vowel, if letters cannot be repeated?

Solution.

(i) $(26-5)^8$

(ii) $\frac{(26-5)!}{(25-5-8)!}$

(iii) $5 \cdot 26^7$

(iv) $5 \cdot \frac{(26-1)!}{(26-1-7)!}$

(v) $26^8 - (26-5)^8$

(vi) $5 \cdot 8 \cdot \frac{(26-5)!}{(26-5-7)!}$

(vii) $\frac{(26-1)!}{(26-1-8)!} - \frac{(26-1-5)!}{(26-1-5-8)!}$

$\square$

Problem 2. How many hands from a standard 52-card deck contain a full house? (A full house means three cards of one value and two cards of a second value.)

Solution.

$\frac{13}{(13-2)!} \cdot \binom{4}{3} \cdot \binom{4}{2} = 1872$ $\square$

Problem 3. A classroom has 2 rows of 8 desks. There are 14 students, 5 of whom always sit in the front row, and 4 of whom always sit in the back row. In how many ways can the students be seated?

Solution.

$\binom{16}{2} \cdot 14! -$ whenever 1 of the 5 sitting in the back $-$ whenever one of the 4 siting in the front ∎