For these problems, you should justify your answers. You do not need to provide a rigorous mathematical proof, but rather an informal argument.

Problem 1. The English alphabet has 26 letters and 5 vowels. How many lists of 8 English letters are there that...

(i) contain no vowels, if letters can be repeated?
(ii) contain no vowels, if letters cannot be repeated?
(iii) start with a vowel, if letters can be repeated?
(iv) start with a vowel, if letters cannot be repeated?
(v) contain at least one vowel, if letters can be repeated?
(vi) contain exactly one vowel, if letters cannot be repeated?
(vii) start with z and contain at least one vowel, if letters cannot be repeated?

Solution.

(i) $(26-5)^8$

(ii) $\frac{(26-5)!}{(25-5-8)!}$

(iii) $5 \cdot 26^7$

(iv) $5 \cdot \frac{(26-1)!}{(26-1-7)!}$

(v) $26^8 - (26-5)^8$

(vi) $5 \cdot 8 \cdot \frac{(26-5)!}{(26-5-7)!}$

(vii) $\frac{(26-1)!}{(26-1-8)!} - \frac{(26-1-5)!}{(26-1-5-8)!}$

$\square$

Problem 2. How many hands from a standard 52-card deck contain a full house? (A full house means three cards of one value and two cards of a second value.)

Solution.

$\frac{13}{(13-2)!} \cdot \binom{4}{3} \cdot \binom{4}{2} = 1872$ $\square$

Problem 3. A classroom has 2 rows of 8 desks. There are 14 students, 5 of whom always sit in the front row, and 4 of whom always sit in the back row. In how many ways can the students be seated?

Solution.

First intuition, there are two empty seats, which can be solved with stars in bars, but may make seating of the front and back row people difficult. Instead we might want to treat the two empty seats as two unique people, but simply divide by 2 at the end because all permutations where they are "swapped" should not be counted.

$\frac{1}{2} \cdot (16!)$ □