

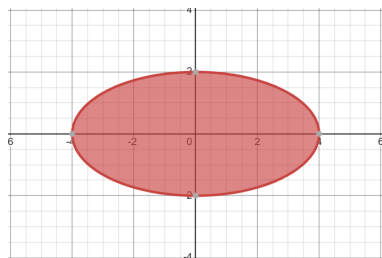
For these problems, you should justify your answers. You do not need to provide a rigorous mathematical proof, but rather an informal argument.

**Problem 1.** Let  $A = \mathbb{Z} \times \mathbb{R}$ , let  $B = \mathbb{R} \times \mathbb{Z}$ , and let  $C = \{(x, y) : x^2 + 4y^2 \leq 16\}$ . For each of the following sets, describe the set in set-builder notation and give a detailed sketch the set in  $\mathbb{R}^2$ .

- (i)  $C \cap (A \cup B)$
- (ii)  $C \cap A \cap B$
- (iii)  $C - (A \cup B)$
- (iv)  $C - (A \cap B)$

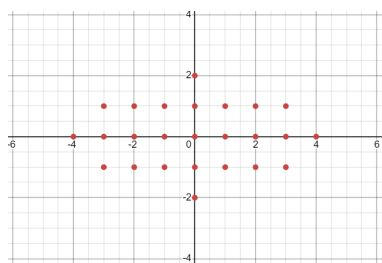
*Solution.*

(i)



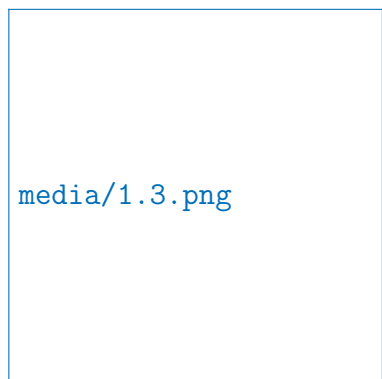
$$\{(x, y) \in \mathbb{R}^2 : x^2 + 4y^2 \leq 16\}$$

(ii)



$$\{(x, y) \in \mathbb{Z}^2 : x^2 + 4y^2 \leq 16\}$$

(iii)





**Problem 2.** Let  $\mathcal{F} = \{A \subseteq \mathbb{N} : |A| \text{ is finite}\}$ . Determine whether each of the following is true or false; justify your answer.

- (i)  $\mathbb{N} \in \mathcal{F}$ .
- (ii) If  $A \in \mathcal{F}$ , then  $\overline{A} \in \mathcal{F}$ .
- (iii)  $|\mathcal{F}|$  is finite.
- (iv) If  $A_1, A_2, \dots \in \mathcal{F}$ , then  $\bigcup_{n=1}^{\infty} A_n \in \mathcal{F}$ .
- (v) If  $A_1, A_2, \dots \in \mathcal{F}$ , then  $\bigcap_{n=1}^{\infty} A_n \in \mathcal{F}$ .

*Solution.*



**Problem 3.** For each  $\alpha \in \mathbb{R}$ , define  $A_\alpha = \{(x, \alpha \cos x) \in \mathbb{R}^2 : -2\pi \leq x \leq 2\pi\}$ . Describe the following sets in set-builder notation and draw them in the plane  $\mathbb{R}^2$ .

- (i)  $A_{\frac{1}{2}}$
- (ii)  $\bigcup_{\alpha \in \mathbb{Z}} A_\alpha$
- (iii)  $\bigcap_{\alpha \in \mathbb{Z}} A_\alpha$
- (iv)  $\bigcup_{\alpha \in \mathbb{R}} A_\alpha$
- (v)  $\bigcap_{\alpha \in \mathbb{R}} A_\alpha$

*Solution.*

