Prove the following propositions. Format your proof so each step of the proof is on its own line; each line should still be a complete sentence. Below, I have entered a nonsensical proof as a model.

Proposition 1. Suppose $a, b, c \in \mathbb{Z}$. If $a^2 + b^2 = c^2$, then a or b is even.

Proof.

Suppose $a, b, c \in \mathbb{Z}$ such that $a^2 + b^2 = c^2$ and it's not the case that a or b is even.

Therefore, a and b are both odd.

So $a = 2x + 1$ and $b = 2y + 1$ for some integers x and y.

Either c is even or odd.

Case 1: c is even.

Then $c = 2z$ for some integer x.

The expression $a^2 + b^2 = c^2$ becomes $(2x + 1)^2 + (2y + 1)^2 = 4z^2$.

Expanding the expression yields $4x^2 + 4x + 1 + 4y^2 + 4y + 1 = 4z^2$.

Factoring yields $4(x^2 + y^2 + x + y) + 2 = 4z^2$.

Simplifying the expression shows $4k + 2 = 4j$ where k and j are the integers $x^2 + y^2 + x + y$ and $4z^2$ respectfully.

Observe that $2 = 4(k - j)$, thus $4|2$.

We have arrived at contradiction.

Case 2: c is odd.

Then $c = 2z + 1$ for some integer x.

The expression $a^2 + b^2 = c^2$ becomes $(2x + 1)^2 + (2y + 1)^2 = (2z + 1)^2$.

Expanded and factored yields $2(2x^2 + 2x + 2y^2 + 2y + 1) = 2(2z^2 + 2z) + 1$

Simplifying the expression shows that $2k = 2j + 1$ for integers k and j.

Therefore, an even number equals an odd number. We have arrived at contradiction.

□

Proposition 2. Suppose $x, y \in \mathbb{Z}$. If $x + y$ is even, then x and y have the same parity.

Proof. $\square$

Proposition 3. If $a \equiv b \pmod{n}$, then $\gcd(a, n) = \gcd(b, n)$.

Proof. $\square$