

Problem 1. Let \mathcal{C} be the unit circle, and let A be a collection of 20 evenly spaced points on \mathcal{C} . A straight segment connecting two points of A is called a *chord*. How many ways are there to draw 10 chords hitting all 20 points of A ?

Solution.

$$(i) \prod_{i=1}^{10} \binom{2 \cdot i}{2} = 3628800$$

(ii) all possibilities – (no cord diameters + exacty one cord diameters)

$$\prod_{i=1}^{10} \binom{2 \cdot i}{2} - \left(\binom{20}{1} \binom{18}{1} \right)$$

□

Problem 2. How many binary strings of length 10 contain somewhere within them the string 10001?

Solution.

There are 6 different placements of 10001 with two positions that risk double count. We can fill the rest. $6 \cdot 2^5$ is the result before accounting for doubly count strings.

It can be seen that 1000110001 is the only string that will be counted twice and it happens in position 1 and 6.

Thus, $6 \cdot 5^2 - 1$.

THIS IS WRONG, you can double count when 100010001?

□

Problem 3. Let $X = \{1, 2, 3, \dots, n\}$.

- (i) Determine the cardinality of the set

$$Z_k = \{A \in \mathcal{P}(X) : |A| = k\}$$

for $0 \leq k \leq n$.

- (ii) Find a combinatorial proof showing that $\sum_{k=0}^n k \binom{n}{k} = n2^{n-1}$.

Hint: The parts of this problem are related.

Solution.

