For these problems, you should justify your answers. You do not need to provide a rigorous mathematical proof, but rather an informal argument.

Problem 1. Symbolic logic can be used to find new expressions that are equivalent to old ones.

(i) Find an expression that is logically equivalent to the biconditional $P \Leftrightarrow Q$ that doesn't use $\Leftrightarrow$, $\Rightarrow$, or $\Leftarrow$.
(ii) Find an expression that is logically equivalent to the conditional $P \Rightarrow Q$ using only $\wedge$, $\vee$, and $\sim$.
(iii) Can you express $P \wedge Q$ using only $\vee$ and $\sim$? Justify your answer.
(iv) Can you express $P \vee Q$ using only $\wedge$ and $\sim$? Justify your answer.

This exercise shows that some of the symbols we use are redundant, but some are not. In any event, they are all useful.

Solution. □

Problem 2. Consider the following statement.

$$\forall N \in \mathbb{N}, \exists X \in \mathscr{P}(\mathbb{N}), |X| \geq N$$

(i) Write the statement as an English sentence.
(ii) Give the negation of the statement in symbolic logic. (You answer should have no $\sim$ symbols.)
(iii) Write the negation of the statement as an English sentence.
(iv) Is the original statement true or false? Justify your answer.

Solution. □

Problem 3. Consider the following English sentence.

If r is a rational number and $r \neq 0$, then $\frac{M}{r}$ is an integer for some natural number M.

(i) Write the statement using symbolic logic.
(ii) Give the negation of the statement in symbolic logic. (You answer should have no $\sim$ symbols.)
(iii) Write the negation of the statement as an English sentence.
(iv) Is the original statement true or false? Justify your answer.

Solution. □