Prove the following propositions. Format your proof so each step of the proof is on its own line; each line should still be a complete sentence.

Proposition 1. If $n \in \mathbb{N}$, then

$$\frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \cdots + \frac{n}{(n+1)!} = 1 - \frac{1}{(n+1)!}.$$

Proof.

Suppose $n \in \mathbb{N}$.

Basis Step: $n = 1$

$$\begin{aligned} \frac{1}{2!} &= 1 - \frac{1}{(1)+1}. \\ &= \frac{1}{2!} \quad \checkmark\text{True} \end{aligned}$$

Inductive Step: $n = k + 1$

Assume $\frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \cdots + \frac{k}{(k+1)!} = 1 - \frac{1}{(k+1)!}$.

Show that:

$$\begin{aligned} 1 - \frac{1}{((k+1)+1)!} &= \frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \cdots + \frac{(k+1)}{((k+1)+1)!} \\ &= \frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \cdots + \frac{k}{(k+1)!} + \frac{(k+1)}{((k+1)+1)!} \\ &= (1 - \frac{1}{(k+1)!}) + \frac{(k+1)}{((k+1)+1)!} \\ &= (1 - \frac{1}{(k+1)!}) + \frac{(k+1)}{(k+2)!} \\ &= 1 + (-\frac{1}{(k+1)!} \cdot \frac{k+2}{k+2}) + \frac{(k+1)}{(k+2)!} \\ &= 1 + (-\frac{k+2}{(k+2)!}) + \frac{(k+1)}{(k+2)!} \\ &= 1 + \frac{(k+1) - (k+2)}{(k+2)!} \\ &= 1 - \frac{1}{(k+2)!} \\ &= 1 - \frac{1}{((k+1)+1)!} \quad \checkmark\text{True} \end{aligned}$$

□

Problem 2. A chocolate bar consists of unit squares arranged in an $n \times m$ rectangular grid. You may split the bar into individual unit squares by breaking along the lines. What is the number of breaks required? Prove your answer is correct.

Solution. □

Proposition 3. Let $n \in \mathbb{N}$ with $n \geq 2$, and let $A_1, A_2, \ldots, A_n$ be sets. Let B be a set. Then,

$$B \cap (A_1 \cup A_2 \cup \cdots \cup A_n) = (B \cap A_1) \cup (B \cap A_2) \cup \cdots \cup (B \cap A_n).$$

Proof. □