Problem 1. Consider the function $f \colon [0, 2\pi] \to [-1, 1]$ given by $f(x) = \cos x$. Determine each of the following sets.

- (i) $f([0, \pi])$
- (ii) $f(\{\pi\})$
- (iii) $f\left(\left(0, \frac{\pi}{2}\right)\right)$
- (iv) $f((0, \pi))$
- (v) $f^{-1}(\{-1, 1\}) = \{0, \pi, 2\pi\}$
- (vi) $f^{-1}(\{0, 1\})$
- (vii) $f^{-1}((-1, 0))$
- (viii) $f^{-1}(\{0\})$

Solution.

- (i) $[-1, 1]$

- (ii) -1

- (iii) $(0, 1)$

- (iv) $(-1, 1)$

- (v) $\{0, \pi, 2\pi\}$

- (vi) $\left\{\frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi\right\}$

- (vii) $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$

- (viii) $\left\{\frac{\pi}{2}, \frac{3\pi}{2}\right\}$

$\square$

Problem 2. Consider $f \colon A \to B$.
   (i) Prove f is injective if and only if $X = f^{-1}(f(X))$ for all $X \subseteq A$.
   (ii) Prove f is surjective if and only if $Y = f(f^{-1}(Y))$ for all $Y \subseteq B$.

Solution.

   (i)

   Suppose some function f is injective.

   Then, for all $n, m \in A$ where $n \neq m$, $f(n) \neq f(m)$.

   Additionally, the inverse relation f^{-1}

   Consider each element $x \in X$. Because its unique, by definition of a set, $f(x)$ is

   also unique.

   □