For these problems, you should justify your answers. You do not need to provide a rigorous mathematical proof, but rather an informal argument.

**Problem 1.** Consider the sets

$$A = \{(x, y) \in \mathbb{R}^2 \,:\, x^2 + y^2 \leq 6\} \quad \text{and} \quad B = \{(x, y) \in \mathbb{R}^2 \,:\, y \geq x^2\}.$$

For each of the ten sets

$$A,\ B,\ A \cup B,\ A \cap B,\ A - B,\ B - A,\ \overline{A},\ \overline{B},\ \overline{A \cup B},\ \overline{A \cap B}$$

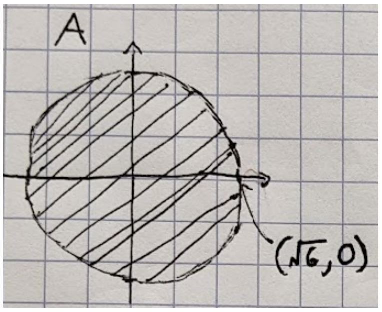
(i) Shade the relevant portion of the plane.
(ii) Describe the set in set-builder notation using inequalities.

(Note: Don't try to illustrate too many of the sets in the same picture; give multiple pictures, and consider using colors.)

*Solution.*

- $A$

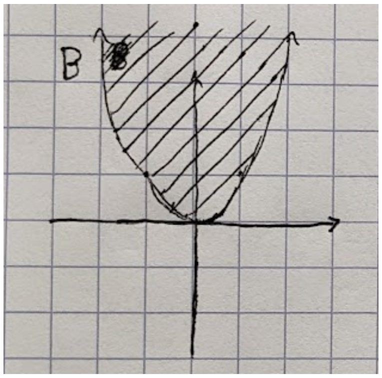
  (i)


  (ii) $\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 6\}$

- $B$

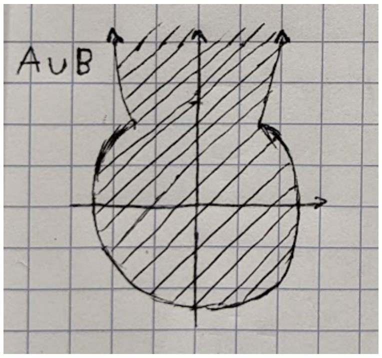
  (i)


  (ii) $\{(x, y) \in \mathbb{R}^2 : y \geq x^2\}$

- $A \cup B$

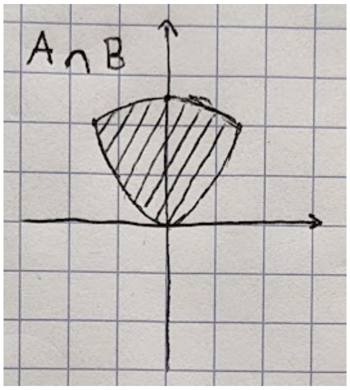
  (i)

  (ii) $\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 6 \text{ or } y \geq x^2\}$

- $A \cap B$

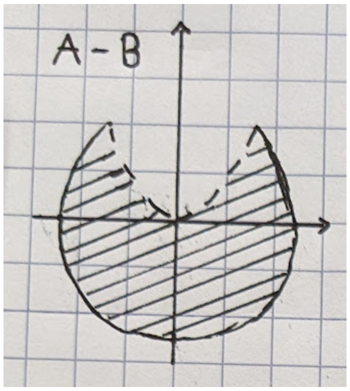
  (i)

  (ii) $\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 6 \text{ and } y \geq x^2\}$

- $A - B$

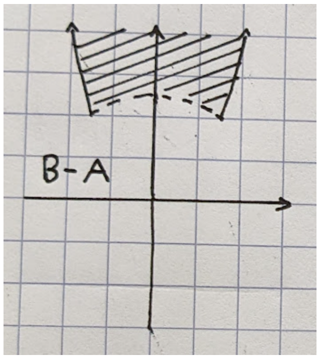
  (i)

  (ii) $\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 6 \text{ and } y < x^2\}$

- $B - A$

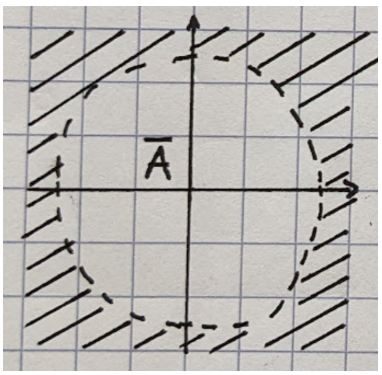
  (i)

  (ii) $\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 > 6 \text{ and } y \geq x^2\}$

- $\overline{A}$

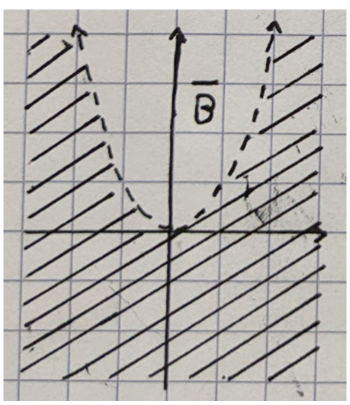
  (i)

  (ii) $\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 > 6\}$

- $\overline{B}$

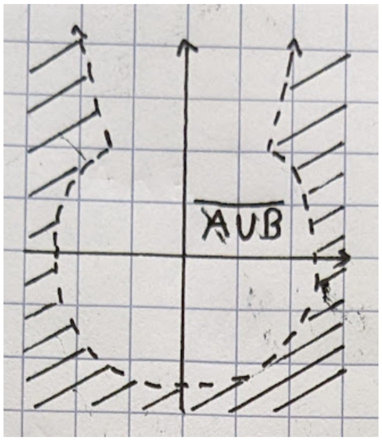
  (i)

  (ii) $\{(x, y) \in \mathbb{R}^2 : y < x^2\}$

- $\overline{A \cup B}$

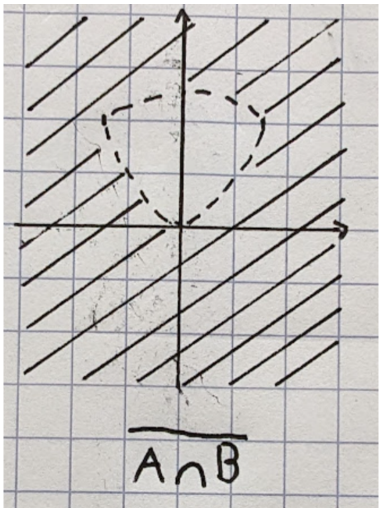
  (i)

  (ii) $\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 > 6 \text{ and } y < x^2\}$

- $\overline{A \cap B}$

  (i)

  (ii) $\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 > 6 \text{ or } y < x^2\}$

$\square$

**Problem 2.** For each $\alpha \in \mathbb{R}$, define $A_\alpha = \{(x, \alpha x) \in \mathbb{R}^2 : -1 \leq x \leq 1\}$.

(i) Describe in words the set $A_\pi$.

(ii) Describe $\bigcup_{\alpha \in \mathbb{R}} A_\alpha$ and $\bigcap_{\alpha \in \mathbb{R}} A_\alpha$ in set-builder notation.

*Solution.*

(i) This set can be described as a linear function with a slope of $\pi$ and a y-intercept of 0 on the closed interval $[-1, 1]$.

(ii) $\bigcup_{\alpha \in \mathbb{R}} A_\alpha = \{(x, y) \in \mathbb{R}^2 : x \in [-1, 1], \exists k(y = kx)\}$

$\bigcap_{\alpha \in \mathbb{R}} A_\alpha = \{(0, 0)\}$

$\square$