

COMPUTATIONAL PRACTICUM

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1 Solving DE

$$y' = 3y^{\frac{2}{3}}$$

Divide by $y^{\frac{2}{3}}$, we can assume $y \neq 0$ because $y = 0$ is a trivial solution

$$\frac{y'}{y^{\frac{2}{3}}} = 3$$

Integrating both sides gives us:

$$3y^{\frac{1}{3}} = 3x + C$$

$$y = (x + C)^3$$

Express C for IVP:

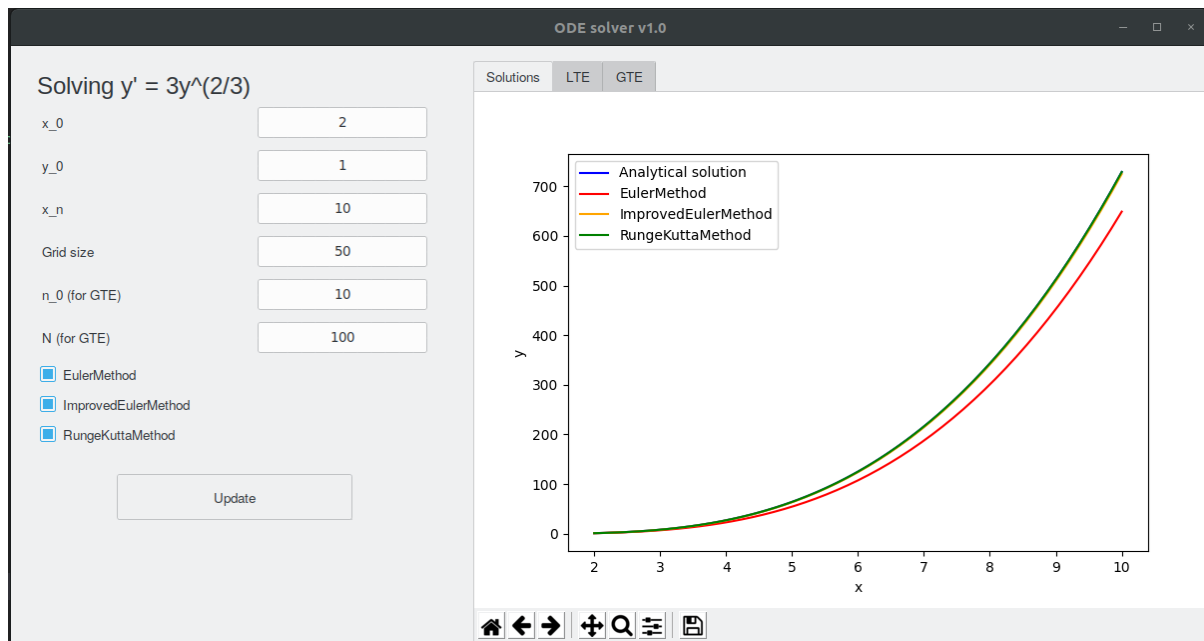
$$C = y^{\frac{1}{3}} - x, C = y_0^{\frac{1}{3}} - x_0$$

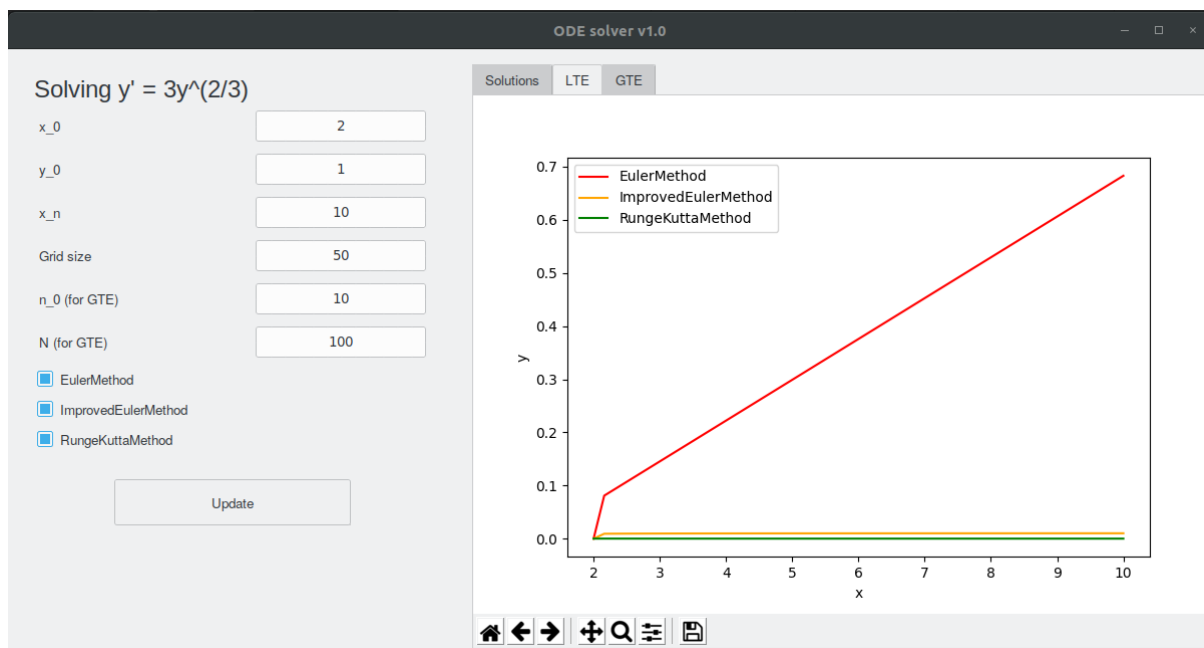
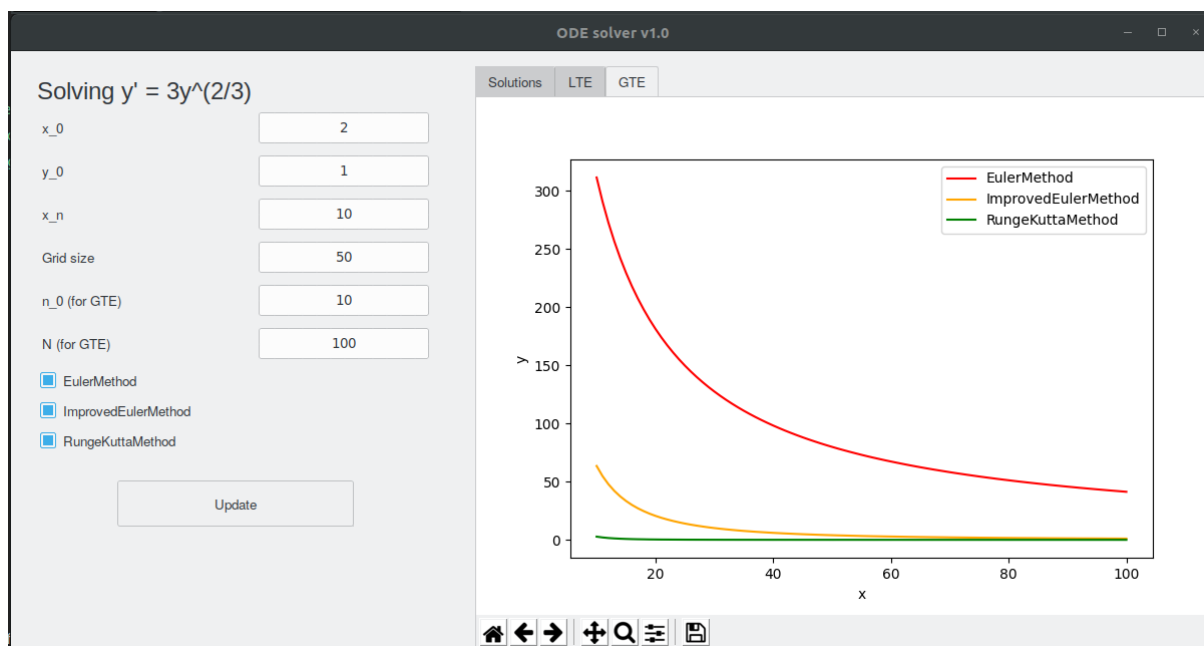
Eventually:

$$y = (x + (y_0^{\frac{1}{3}} - x_0))^3 \text{ where } x_0, y_0 \text{ are some constants from IVP}$$

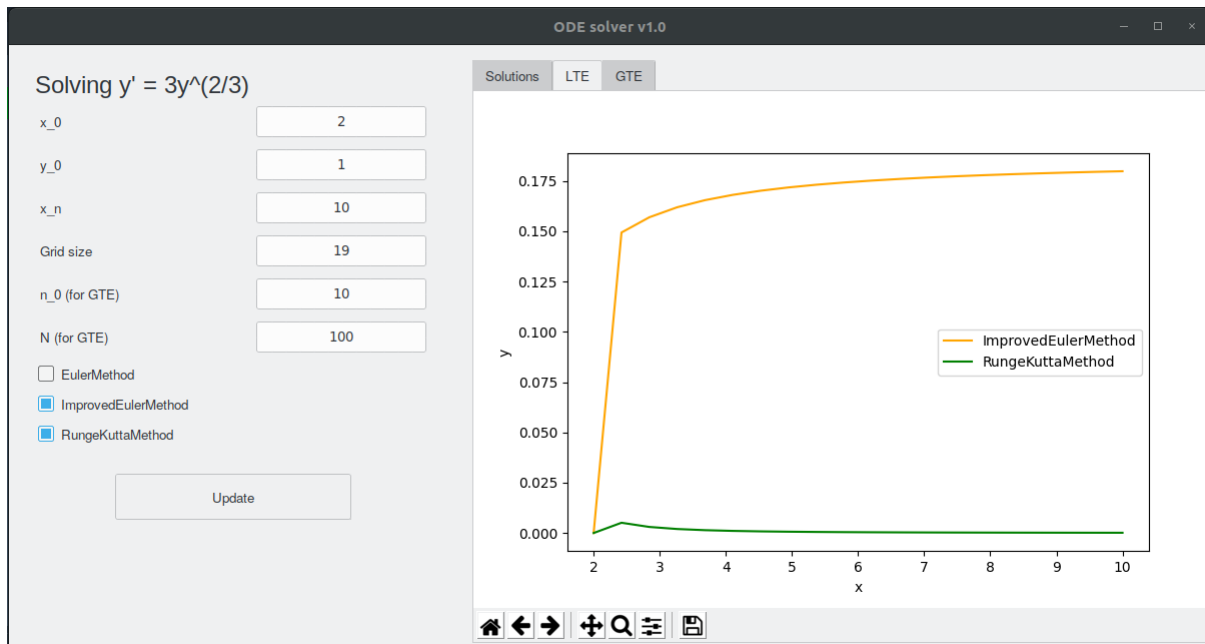
2 Screenshots of APP

Solution tab



LTE tab**GTE tab**

Example of chart analysis



3 Code https://github.com/sevenzing/de_solver

First, project has **constants.py** file, where you can easily change differential equation (line 20), solution (line 23), some initial values (lines 11-17) and other settings of the project

```

3 APP_TITLE = 'ODE solver v1.0'
4 APP_SIZE = '1200x600' # in pixels
5 LOG_FORMAT = '%(levelname) -10s %(asctime)s %(name) -15s %(funcName) -10s: %(message)s'
6 UPDATE_BUTTON_NAME = 'Update'
7 RESIZE = True
8 LOG_LEVEL = logging.INFO
9
10 INITIAL_VALUES = [
11     2,      # x_0
12     1,      # y_0
13     10,     # x_n
14     50,     # number of iterations
15     10,     # n_start
16     100,    # n_end
17     'y\' = 3y^(2/3)', # title of eq
18 ]
19
20 f = lambda x,y: 3 * y**(2/3)
21 def y_solution_getter(x_0, y_0):
22     def solution(x):
23         return (x + (y_0**(1/3) - x_0))**3
24     return solution

```

In the project you can find some supporting classes, like **Function**, **DifferentialEq** which makes working with data more convenient and **Manager** that help to connect front and back end of app

An interesting point that **DifferentialEq** class has fields **solution** and **h** that are actually properties (something like a function without arguments). It means that there is no need to update **solution** and **h**, you just need to change x_0 , x_n , y_0 or n

```
@property
def solution(self):
    return self.f_solution(self.x_0, self.y_0)

@property
def h(self):
    return (self.x_n - self.x_0) / self.n
```

Since we need to update values of differential equation, I suggest not to mix front and back end, but create a bridge between them, that will listen front-end, do some work and update shared values. That is why I need **Manager** (**manager.py**). It has all the necessary attributes for managing and updating and 2 public functions: **update()** and **hide_methods()**. These are the two functions that the front-end will call.

```
class Manager:
    """
    Class for managing changes of values of diff. eq
    """
    def __init__(self, diff: DifferentialEq, methods: List[Method], solution_color):
        self.diff = diff
        self.methods = methods
        self.functions: Dict[List[Function]] = {
            str(method): [Function(name=str(method), color=method.color) for _ in range(len(methods))]
            for method in methods
        }
        self.solution = Function(name='Analytical solution', color=solution_color)
        self.update()

    def update(self, x_0=None, y_0=None, x_n=None, n=None, n_start=None, n_end=None): ...

    def __update_solution(self): ...

    def hide_methods(self, methods: Dict[str, bool]): ...
```

To implement 3 iteration methods, I decided to write an interface **Method** (**de_solver.py**). It has **get_next_value()** function, that we need to implement, and **solve()** function, that actually solves differential equation based on implemented function

```
class Method:
    def __init__(self, color=None): ...

    def get_next_value(self, f, x_i, y_i, h):
        raise NotImplementedError('you should implement it first')

    def solve(self, diff: DifferentialEq, precision=4) → Tuple[Function]: ...

    def __execute_method(self, f, y_solution_func, x_0, y_0, x_n, n, precision=4) → Tuple[Function]: ...
```

And then implement 3 derived classes

EulerMethod, **ImprovedEulerMethod** and **RungeKuttaMethod** (**de_solver.py**):

```

117 class EulerMethod(Method):
118     def get_next_value(self, f, x_i, y_i, h):
119         # https://en.wikipedia.org/wiki/Euler_method
120         return y_i + h * f(x_i, y_i)
121
122
123 class ImprovedEulerMethod(Method):
124     def get_next_value(self, f, x_i, y_i, h):
125         # https://en.wikipedia.org/wiki/Heun%27s_method
126         k_1i = f(x_i, y_i)
127         k_2i = f(x_i + h, y_i + h * f(x_i, y_i))
128         y_i = y_i + h/2 * (k_1i + k_2i)
129         return y_i
130
131 class RungeKuttaMethod(Method):
132     def get_next_value(self, f, x_i, y_i, h):
133         # https://en.wikipedia.org/wiki/Runge%E2%80%93Kutta_methods
134         k_1i = f(x_i, y_i)
135         k_2i = f(x_i + h/2, y_i + h/2*k_1i)
136         k_3i = f(x_i + h/2, y_i + h/2*k_2i)
137         k_4i = f(x_i + h, y_i + h*k_3i)
138
139         return y_i + h/6*(k_1i+2*k_2i+2*k_3i+k_4i)

```

Also I splitted front-end into 2 parts: **SettingsArea** and **GraphArea** (**frames.py**). SettingsArea should update the Manager, GraphArea should update the plotting

Another interesting thing about the project - you can easily add your own iteration method!

You need to add new class derived from **Method**, write function **get_next_value** and add your method to manager in **run.py** by specifying **color** and **title** if you need. That's it, method added!

de_solver.py

```

148 class SampleMethod(Method):
149     def get_next_value(self, f, x_i, y_i, h):
150         k_1i = f(x_i, y_i)
151         k_2i = f(x_i + h/2, y_i + h/2*k_1i)
152         k_3i = f(x_i + h/2, y_i + h/2*k_2i)
153         k_4i = 60
154         return y_i + h/6*(k_1i+2*k_2i+2*k_3i+k_4i)
155

```

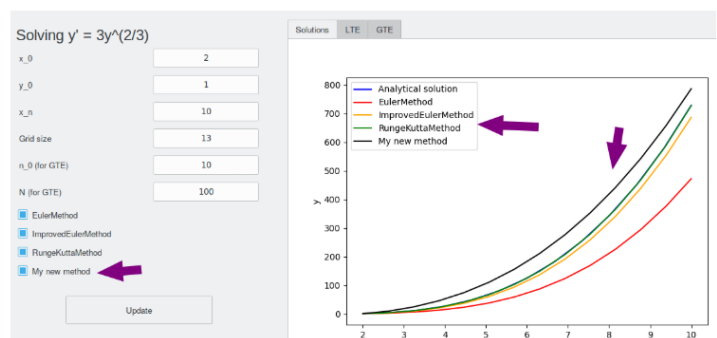
run.py

```

37 def get_manager(self) → Manager:
38     m = Manager(self.diff_eq, [
39         EulerMethod(color='red'),
40         ImprovedEulerMethod(color='orange'),
41         RungeKuttaMethod(color='green'),
42         SampleMethod(
43             color='black',
44             title='My new method'
45         ),
46     ],
47     solution_color='blue',
48     )
49     m.update()
50     return m

```

Method added to app!



4 UML diagram

Here you can see UML diagram of my project structure

